T.R.
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QCD SUM RULES FOR PION IN VACUUM AND DENSE MEDIUM

MASTER OF SCIENCE

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ABSTRACT

QCD SUM RULE FOR PION IN VACUUM AND DENSE MEDIUM
MSC THESIS
ROKAN SALIH MOHAMMED
BOLU ABANT İZZET BAYSAL UNIVERSITY
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Investigation of the light pseudoscalar mesons ($J^P = 0^-$) is one of the important research topics in order to better understand the perturbative and non-perturbative nature of Quantum Chromodynamics (QCD). We investigate the spectroscopic parameters of pion as a member of light meson family in a medium with finite density using QCD sum rules. The decay constant of pion linearly decreases with the increase in medium density and melts at higher values of the density $\rho \simeq 1.6\ \rho_{sat}$. At the saturation density, the in-medium mass of the pion is 198 MeV and the decay constant is 26 MeV. The results obtained in the current study may be also useful for future theoretical studies using different approaches and experimental studies, both in vacuum and in a dense medium.

KEYWORDS: QCD, Pion, Vacuum, Nuclear matter

ÖZET

PİON İÇİN VAKUM VE YOĞUN ORTAMDA KRD TOPLAM KURALLARI

YÜKSEK LİSANS TEZİ
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Hafif psödoskalar mezonların $(J^P = 0^-)$ incelenmesi, Kuantum Renkdinamiğinin (QCD) pertürbatif ve pertürbatif olmayan doğasını daha iyi anlamak için önemli araştırma konularından biridir. Hafif mezon ailesinin bir üyesi olarak pionun spektroskopik parametrelerini, QCD toplam kurallarını kullanarak sonlu yoğunluğa sahip bir ortamda araştırıyoruz. Pionun bozunma sabiti, orta yoğunluktaki artışla doğrusal olarak azalır ve ortam yoğunluğunun ρ $\simeq$ 1. 6 ρ_{sat} dan daha yüksek değerlerinde erir. Doyma yoğunluğunda, piyonun ortam içi kütlesi 198 MeV ve bozunma sabiti 26 MeV'dir. Mevcut çalışmada elde edilen sonuçlar, hem vakumda hem de yoğun bir ortamda farklı yaklaşımların kullanıldığı gelecekteki teorik çalışmalar ve deneysel çalışmalar için de yararlı olabilir.

ANAHTAR KELİMELER: KRD, Pion, Vakum, Nükleer madde

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LIST OF ABBREVIATIONS AND SYMBOLS

ψ_q	: Quark field spinor
q	: Light quark flavor
$\Pi(p^2)$	: Two-point correlation function
$\vert 0\rangle$	: Vacuum state
$\vert\psi_0\rangle$	: In-medium state
$\mathcal{T}$	: Time ordering operator
$J(x)$	: Interpolating current
Γ	: Dirac matrices
ϵ_{ijk}	:Levi-Civita tensor
C	: Charge conjugation operator
B	: Borel transformation
M^2	: Borel mass
d	: Operator dimension
Σ_s	: Scalar self-energy
Σ_v	: Vector self-energy
$S_{u(d)}^{ij}$	: Light-quark propagators
$Tr[\ldots]$	: Tracer
QCD	: Quantum Chromodynamics
EWT	: Electroweak theory
QED	: Quantum Electrodynamics
SM	: Standard Model
GR	: General Relativity
s_0	: Continuum threshold
ρ	: Medium density
ρ^{sat}	: Saturation nuclear matter density
m	: Vacuum mass
m^*	: In- medium mass
f	: Vacuum decay constant
f^*	: In-medium decay constant

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1. INTRODUCTION

Today, the standard model is the best theory that can explain elementary particles and their interactions. According to this theory, matter consists of quarks and leptons in different generations (I, II and III). The interactions between these particles occur via gauge bosons, which are the force carriers of the fundamental interactions in nature [1]. The standard model is self-consistent, and the empirical predictions have also shown great success. Despite all these achievements, it leaves phenomena such as baryon asymmetry and dark matter as unexplained.

In theoretical physics, the theory of strong interaction between quarks, the force carriers of which are gluons, is quantum chromodynamics (QCD). QCD is a type of quantum field theory like quantum electrodynamics (QED). The analogue of electrical charge in QED is color charges in QCD. One of the most important features of QCD is color confinement. According to this feature, color-charged particles such as quarks and gluons cannot be isolated, that is, free quarks and gluons cannot be mentioned [2,3]. Therefore, quarks and gluons must come together to form hadrons. Another important feature of QCD is asymptotic freedom. Accordingly, with increasing energy scale and corresponding decrease in length scale, the interactions between particles decrease asymptotically. Asymptotic freedom was discovered simultaneously and independently in 1973 by David Gross-Frank Wilczek [4] and David Politzer [5].

The two subgroups of the standard hadron family are baryons and mesons. Baryons are made up of three quarks or three anti-quarks, while mesons are made up of a quark and an anti-quark pair. A good understanding of the non-perturbative region of the QCD is required to improve our knowledge of the strong interactions of standard hadrons. For this reason, the QCD sum rules, first developed for mesons in 1979 [1] by M.A. Shifman, A.I. Vainshtein and V.I. Zakharov and then extended for baryons by Ioffe [2] in 1981, are an important tool.

First, the spectroscopic properties of hadrons were studied extensively in the literature in vacuum environment. However, in order to analyze the results of heavy ion collision experiments well, these studies should also be studied at dense medium and finite temperature. For example, PANDA collaboration plans to study the effects of medium density on the physical properties of charmed hadrons in the near future.

The simplest particles of the light meson family, consisting of up, down and strange quarks, are the pion triplet (π^-, π^+, π^0). Charged pions were discovered in 1947 and uncharged ones were discovered in 1950. Pions are not stable. The pion is the lightest mesons and have an important role in understanding the low-energy properties of the strong nuclear force. In recent studies, the static and dynamic properties of pions have been studied in a wide framework by using different methods [6-19].

The main parts of this thesis are: In chapter 1 it's the introduction and in the chapter 2 we will explain about the standard model and mesons as for the chapter 3 we will study QCD sum rules and related topics and in chapter 4 it's the numerical analysis and we will discuss the results obtained in chapter 5 and the last chapter (6) it will be for references.

2. THE STANDARD MODEL AND MESONS

2.1 Standard Model :

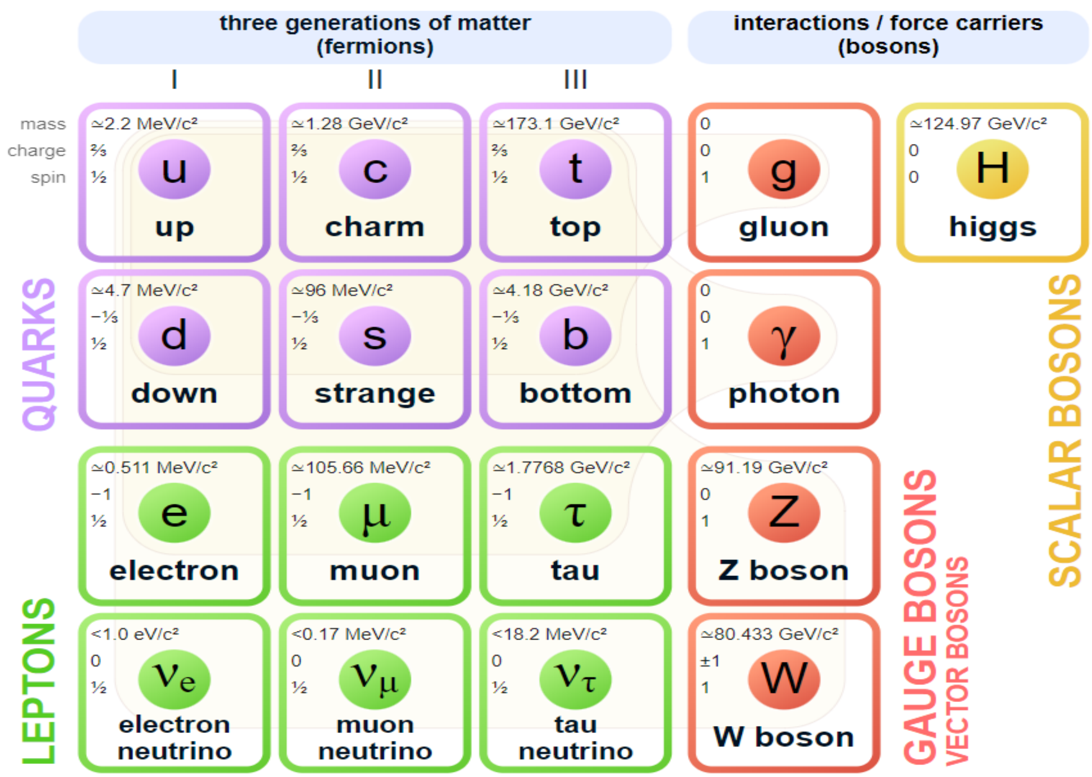
Since the early 1900s, there have been remarkable developments both theoretically and experimentally regarding the basic structure of matter. Everything in the universe is governed by four fundamental forces: gravitational force, electromagnetic force, strong nuclear force and weak nuclear force. The properties of these four fundamental forces (current theory, force carrier, relative strength, long-distance behavior, range) are given in Table 2.1.

Table 2.1. The properties of four fundamental forces.

Interaction	Current theory	force carrier	Relative strength	Long-distance behavior	Range (m)[20]
Weak	Electroweak theory (EWT)	W and Z bosons	10^{25}	$\frac{1}{r}\, e^{-m_{w,z}r}$	10^{-18}
Strong	Quantum chromodynamics (QCD)	Gluons	10^{38}	$\sim r$ (Color confinement)	10^{-15}
Electromagnetic	Quantum electrodynamics (QED)	Photons	10^{36}	$\frac{1}{r^2}$	∞
Gravitation	General relativity (GR)	Gravitons (hypothetical)	1	$\frac{1}{r^2}$	∞

It was also discovered that everything in the universe consists of fundamental building blocks called elementary particles. The standard model, a theory of fundamental particles and how they interact with three of these four fundamental forces (excluding gravity), was developed in the early 1970s. Historically, experiments have shown that the standard model has been established as a well-tested physics theory. It included everything known about subatomic particles as of its foundation, and also predicted the existence of additional particles. The existence of exotic hadrons, which have a different quark content than standard hadrons and have been observed in various experiments, is in agreement with Standard Model. The elementary particles of the standard model are given in Table 2.1 [21]. The elementary particles in this table can be collected into two groups: fermions that interact and have half integer spin, or bosons that mediate the interaction and have integer spin. As seen in the table, quarks and leptons have spin-1/2 fermions and gauge bosons have spin-1. The higss boson, known as the origin of mass in matter, is also a spin-0 particle. Each fermion column in the table are named the first, second, and third generation, in order from left to right. Leptons, in turn, are divided into six particles: the electron, the muon, the tau, the electron neutrino, the muon neutrino, and the tau neutrino.

Table 2.2. Elementary particle of SM and their properties.



2.2 Mesons:

Mesons are not fundamental particles. A pair of a quark (q) and its antiquark $(\bar{q})$ bounded by strong interaction is known as meson. For light quarks (u, d, s) nine possible combinatios are:

Table 2.3. Different combinations of u,d and s quarks

$u\bar{u}$	$d\bar{d}$	$s\bar{s}$
$u\bar{d}$	$d\bar{u}$	$s\bar{d}$
$u\bar{s}$	$d\bar{s}$	$s\bar{u}$

In table 2.3, each combination correspond to a diffrent particle. For example $u\bar{s}$ and $d\bar{s}$ are named as K^+ and K^0 mesons, respectively. They are bosons because they all have integer spin. As with the π^0, different spin combinations are possible

$$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \qquad (2.1)$$

known as the singlet. Considering the heavy quarks (c, b), it is possible to reach a longer meson family list.

Mesons have different quantum numbers such as isospin (I), total angular momentum (J), parity (P), G-parity (G) or C-parity (C). Considering these quantum numbers and quark contents, although classifying mesons has complex rules as defined in the Particle Data Group (PDG), it is also possible to simplify it as in the table 2.4 [22].

Table 2.4. Types of mesons with respect to their quantum numbers

Type	Spin	Angular momentum operator	Parity transformation	Total angular momentum	J^P
Pseudoscalar meson	**0**	**0**	**-**	**0**	$\mathbf{0^-}$
Pseudovector meson	**0,1**	**1**	**+**	**1**	$\mathbf{1^+}$
Vector meson	**1**	**0,2**	**-**	**1**	$\mathbf{1^-}$
Scalar meson	**1**	**1**	**+**	**0**	$\mathbf{0^+}$
Tensor meson	**1**	**1,3**	**+**	**2**	$\mathbf{2^+}$

Since pions are the lightest of the meson family, this makes them more stable than other mesons. Depending on their quark content, pions can be $\pi^+(u\bar{d})$, π^- $(d\bar{u})$ and π^0 $(u\bar{u}$ or $d\bar{d})$. Positive pions and negative pions are antiparticles of each other, while the antiparticles of neutral pions are themselves. Originally, all types of pions are discovered in the interaction of hadronic cosmic components with matter in the earth's atmosphere and observed in high energy collisions between pions are the exchange particles of the strong interaction that holds the proton and neutron together in the nucleus. In this case, pions can also be called force carrier particles of strong interaction within the nucleus. The pion created for this purpose is a temporary violation of the energy and mass interaction, but is not directly observed as it is a virtual particle.

3. QCD SUM RULES FOR PION

Quantum Chromodynamics is the theory of strong interactions in the Standard model. The interaction between the quarks that make up the hadrons occurs through gluons, gluons are massless particles with a spin-1, they can also interact with each other due to the color charge (red, green, blue). Although the interaction between quarks is very strong at long distances, the absence of isolated quarks has led to the idea that quarks are trapped inside hadrons. To study a system with quarks and gluons, perturbative and non-perturbative QCD must be considered, because the strong interaction constant α depends on the conditions of the interaction. At short interaction distances, the interaction constant is also small and is called asymptotic freedom. Because of this feature, perturbative expansion can be made according to the interaction constant α at short distances or at large momenta. However, at large distances or small momenta, non-perturbative effects will dominate due to strong quark-gluon interactions.

In 1979 QCD sum rules were first developed for mesons[23], then in 1981 this method was extended to baryons by Ioffe [24]. Depending on its energy, the QCD has two limit regions. These are: quark gluon region at high energies and hadron region at low energies. QCDSR is an attempt to interpolate between these two regions. The main purpose here is to obtain physical quantities such as mass and decay constant from microscopic parameters of QCD such as condensates or quark masses. At each step, the QCD sum rules method starts from the asymptotic freedom that is valid at short distances and approaches the long distances in which the bound states in the QCD occur. In this process, as the asymptotic freedom property begins to destroy, resonances occur corresponding to the bound quark states trapped in hadrons. With the deterioration of asymptotic freedom, non-perturbative effects occur in the QCD vacuum. As a result, they appear as nonzero values of the quark and gluon operators in vacuum.

QCD sum rules are a powerful theoretical tool for studying the ground states of hadrons, so both the finite temperature and finite density properties of various hadrons are expressed in terms of QCD parameters. For this purpose, the correlation function expressed by the time-ordering product of the interpolating currents written in terms of quark fields is calculated. The investigated hadrons and interpolating currents must have the same quantum numbers. Interpolating current can be in the form of scalar, pseudoscalar, vector, axial-vector or tensor depending on the quantum numbers of hadrons.

3.1 Correlation Function

QCD sum rules of hadrons including baryon and meson families; it is a phenomenological calculation method that expresses various phenomenological parameters such as mass, decay constant, coupling constant in terms of expected values of local operators that make up quark and gluon fields, and QCD parameters. The starting point for the physical parameters to be calculated in the vacuum or nuclear medium within the framework of QCD sum rules is the two-point correlation function for vacuum and nuclear medium, respectively

$$\Pi\,(p^2) = i\int d^4x\, e^{ip.x}\langle 0|\,\mathcal{T}[\,J(x)\bar{J}(0)]|0\rangle, \tag{3.1}$$

$$\Pi\,(p^2) = i\int d^4x\, e^{ip.x}\langle \psi_0|\,\mathcal{T}[\,J(x)\bar{J}(0)]|\psi_0\rangle, \tag{3.2}$$

where p represents the four momentum of the hadron and, $|0\rangle$ is the ground state of vacuum, $|\psi_0\rangle$ is the ground state of the nuclear medium. $\mathcal{T}$ is the ordering operator when the multipliers are arranged so that the time arguments increase from right to left. $J(x)$ is the interpolating current generated by Dirac-gamma matrices and quark fields.

In order to construct the interpolating current, quark fields are created by considering the quantum numbers of the hadron whose properties are examined. For mesons the general structure is

$$J(x) = \bar{q}_i(x)\Gamma q_i(x), \tag{3.3}$$

and for baryons it is

$$J(x) = \epsilon_{ijk}\left[q_i^a(x)\Gamma_1 q_j^b(x)\,\Gamma_2 q_k^C(x)\right], \tag{3.4}$$

where the letters i, j, k represent the type of quark flavor, ϵ_{ijk} is Levi-Civita tensor, the letters a, b, c, are used for the color of the quark, and $\Gamma = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$ are the Dirac matrices, for example, the interpolating current of nucleons as a member of the baryon family, is expressed as

$$J(x) = 2\epsilon_{abc}\{[q^{T,a}(x)Cq^b(x)]\gamma_5 q^c(x) + [q^{T,a}(x)C\gamma_5 q^b(x)]\beta q^c(x)\} \qquad (3.5)$$

where C represents the charge conjugate operator and q represents light quarks. On the other hand, the interpolating current of kaon, which we can give as an example of the interpolating current of mesons, can be written as

$$J_\mu(x) = \bar{s}^a(x)\gamma_\mu\gamma_5 u^a(x) \qquad (3.6)$$

In accordance with the standard philosophy of the method used, the correlation function given in Eq. (3.1) can be calculated using two different approaches: hadronic and operator product expansion. In the first, the correlation function is calculated in terms of hadronic parameters. For this, a full set of vacuum or in-medium hadronic states with the same quantum numbers as the interpolating quark currents is inserted into the correlation function to obtain the following expression:

$$\begin{aligned} 2\mathrm{Im}\,\Pi(q^2) &= \sum_h \int \langle 0|J|h\rangle\langle h|\bar{J}|0\rangle \; d\tau_h (2\pi)^4 \delta^4(q - p_h) \\ &= 2\pi f_h^2 \delta(q^2 - m_h^2) + 2\pi\rho^h(q^2) \qquad (3.7) \end{aligned}$$

where the summation is taken over all possible hadronic states that the quark current can create. f_h represents the leptonic decay constant for mesons or residue baryons and $\rho^h(q^2)$ includes the higher states and continuum.

As a second method, in operator product expansion, the time sequence product of two nonlocal operators is serially expanded in terms of various local operators and Wilson coefficients $C_m(x^2)$ involving perturbatively computed short-distance physics and local operators O_m containing non-perturbative long-distance effects:

$$\mathcal{T}[J(x)\bar{J}(0)] = \sum_m C_m(x^2) O_m \;. \qquad (3.8)$$

The nuclear matter expected values of QCD operators, namely their condensates, are expressed in terms of $\langle\psi_0|0_m|\psi_0\rangle$, as follows:

$$\Pi(p^2) = \sum_m C_m \langle \psi_0 | O_m | \psi_0 \rangle \ . \tag{3.9}$$

Using wick's theorem and interpolating currents, the correlation function is expressed in terms of light quark or heavy quark propagators, depending on the quark content of the particle concerned.

The results of the correlation function calculated using both ways are equated with a dispersion relation to obtain the QCD sum rules for the interested physical quantity.

3.2 Borel Transformation

In applications of the QCD sum rules method, the contributions to the higher states and continuum need to be suppressed after equating the hadronic and QCD sides of the correlation function. Therefore, besides the analytical continuation, the Borel transformation is often used to improve the reliability of the sum rule and the overlap of OPE and phenomenological description.

Borel transform is a standard mathematical method used to improve the radius of convergence by taking infinite derivatives of any function. It is expressed by the following formula for any function $f(Q^2)$

$$\mathcal{B}_{M^2}[f(Q^2) = \lim_{\substack{Q^2, n \to \infty \\ \frac{Q^2}{n} = M^2}} \frac{(Q^2)^{n+1}}{n!} \left(\frac{-d}{dQ^2}\right)^n f(Q^2), \tag{3.10}$$

Borel transformations of some functions used in this thesis and similar QCD sum rules applications are as follows:

$$\mathcal{B}_{M^2}[(Q^2)^k] = 0 \; for \; k \geq 0,$$

$$\mathcal{B}_{M^2}\left[\frac{1}{(Q^2)^k}\right] = \frac{1}{(k-1)!}\left(\frac{1}{M^2}\right)^{k-1},$$

$$\mathcal{B}_{M^2}\left[\frac{1}{s+Q^2}\right] = \exp^{-s/M^2},$$

$$\mathcal{B}_{M^2}[(Q^2)^k \log(Q^2/\Lambda^2] = k!\,(-M^2)^{k+1}. \tag{3.11}$$

3.3 QCD Sum rules for pion

In this study, we calculate the mass and decay constant of pion in nuclear medium and their in-medium results shifts from vacuum values. The calculation of the mass and decay constant in the nuclear medium begins with the time ordered two-point correlation amplitude $\Pi_{\mu\nu}$ and it is given as:

$$\Pi_{\mu\nu} = i\int d^4x e^{ip.x} \langle \psi_0 | \mathcal{T}[J_\mu(x) J_\nu^\dagger(0) | \psi_0 \rangle, \tag{3.12}$$

where $|\psi_0\rangle$ the ground state of nuclear medium and it is replaced by the vacuum ground state for vacuum calculations. The interpolating current (J) in Eq. (3.12) is given as follows for the pion with the quark content ($u\bar{d}$),

$$J_\mu(x) = \bar{d}^a(x)\gamma_\mu\gamma_5 u^a(x). \tag{3.13}$$

According to the generally accepted philosophy of the QCD sum rule method, the correlation function in Eq. (3.12) can be expressed in terms of hadronic parameters known as the physical or hadronic side, and alternatively in terms of QCD degrees of freedom named with the QCD or phenomenological side. Then, by equating the results of both sides, the QCD sum rules for the physical parameters are obtained. The next section details how the hadronic and QCD sides are obtained, respectively.

3.3.1 Hadronic Side

The hadronic side of the correlation function is obtained in terms of the in-medium mass and the in-medium decay constant of the pion. The axial current of the pion is coupled to both pseudo-scalar (PS) and axial vector (AV) pion simultaneously.

The correlation function has the same quantum numbers as the interpolating current in Eq. (3.13). After taking the four space -time integral and isolating PS and AV states, we get the necessary expression for the hadronic side of $\Pi_{\mu\nu}$ as:

$$\Pi_{\mu\nu}^{Phys}(p) = \frac{\langle \psi_0 | J_\mu | n_{AV} \rangle \langle n_{AV} | J_\nu^\dagger | \psi_0 \rangle}{m_n^{*AV^2} - p^{*2}} + \frac{\langle \psi_0 | J_\mu | n_{PS} \rangle \langle n_{PS} | J_\nu^\dagger | \psi_0 \rangle}{m_n^{*PS^2} - p^{*2}} + \cdots, \tag{3.14}$$

where p^* is the in-medium momentum and it is represented in terms of the vector self energy ($\sum_\nu$) and the four-velocity of nuclear matter u_μ as: $p^*_\mu = p_\mu - \Sigma_\nu u_\mu$. The calculations are done in the rest frame of the nuclear medium ($p_\mu = p_0$). In the above equation, the dots represent the contributions of higher resonances and continuum states in both channels (PS and AV). Matrix elements for PS and AV states in Eq. (3.14) are defined in terms of corresponding mass, decay constant, momentum and polarization vector of AV state as follows:

$$\langle \psi_0 | J_\mu | n_{AV}(p^*) \rangle = f^*_{AV} m^*_{AV} \epsilon^*_\mu, \tag{3.15}$$

and

$$\langle \psi_0 | J_\mu | n_{PS}(p^*) \rangle = f^*_{PS}\, m^*_{PS} p^*_\mu. \tag{3.16}$$

Inserting Eq. (3.15) and (3.16) into Eq. (3.14) yields the hadronic part of the correlation function for nuclear matter:

$$\begin{aligned}\Pi^{Phys}_{\mu\nu} &= \frac{f^{*2}_{AV}\, m^{*2}_{AV}\, \Sigma_u \Sigma_\nu}{m^{*2}_{AV} - p^{*2}} + \frac{f^{*2}_{PS} m^{*2}_{PS} p^*_\mu p^*_\nu}{m^{*2}_{PS} - p^{*2}} \\ &= \frac{f^{*2}_{AV} m^2_{AV}}{m^{*2}_{AV} - p^{*2}} \left[-g_{\mu\nu} + \frac{p^*_\mu p^*_\nu}{p^{*2}} \right] + \frac{f^{*2}_{PS}\, m^{*2}_{PS}\, p^*_\mu p^*_\nu}{m^{*2}_{PS} - p^{*2}} + \cdots \ .\end{aligned} \tag{3.17}$$

Since only the contributions from the PS state are needed, to get rid of the contributions from the AV state, multiply the correlation function obtained for the nuclear matter medium by p^*_μ. Multiply both side of the correlation function by (p^*_μ) we get

$$\begin{aligned} p^*_\mu \ \ \Pi^{Phys}_{\mu\nu} &= \frac{f^{*2}_{AV}\, m^2_{AV}}{m^{*2}_{AV} - p^{*2}} (-p^*_\nu + p^*_\nu) + \frac{f^{*2}_{PS}\, m^{*2}_{PS}\, p^{*2} p^*_\nu}{m^{*2}_{PS} - p^{*2}} \\ &= -f^{*2}_{PS}\, m^{*2}_{PS} \left(\frac{m^{*2}_{PS}}{p^{*2} - m^{*2}_{PS}} + \frac{p^{*2} - m^{*2}_{PS}}{p^{*2} - m^{*2}_{PS}} \right) p_\nu \\ p^*_\mu \ \ \Pi^{Phys}_{\mu\nu} &= -f^{*2}_{PS}\, m^{*2}_{PS} \left(1 + \frac{m^{*2}_{PS}}{p^{*2} - m^{*2}_{PS}} \right) p^*_\nu \end{aligned} \tag{3.18}$$

where $\qquad p^*_\nu = p_\nu - \Sigma_u u_\nu \Rightarrow p^{*2} = p^2 - 2\Sigma_u p_0 + \Sigma^2_u$

$$p^{*2} - m^{*PS2} = p^2 - 2\Sigma_\nu p_0 + \Sigma_u^2 - m^{*PS2}$$
$$\mu^2 = m^{*PS2} - \Sigma_u^2 + 2\Sigma_\nu p_0\,, \qquad (3.19)$$

and p_0 is the energy of the quasi-particle in nuclear medium, then the physical part of the correlation function is

$$p_\mu^* \quad \Pi_{\mu\nu}^{Phys} = -f_{PS}^{*2} m_{PS}^{*2} \left(1 + \frac{m_{PS}^{*2}}{p^2 - \mu^2}\right)(p_\nu - \Sigma_u u_\nu). \qquad (3.20)$$

Then we take the Borel transformation according to the momentum squared to suppress the contributions of the higher states and continuum. As a result, the following equation is obtained for the hadronic part the QCD sum rules to be used for the mass and decay constant of the pion in the nuclear medium:

$$\widehat{\mathrm{B}}\left[p_\mu^* \Pi_{\mu\nu}^{Phys}\right] = -f_{PS}^{*2} m_{PS}^{*2}\, e^{-\frac{\mu^2}{M^2}} (p_\nu - \Sigma_u u_\nu), \qquad (3.21)$$

where M^2 is the Borel mass parameter.

3.3.2 QCD side

The QCD part of the correlation function in Eq. (3.12) can be obtained in terms of quarks and gluon degrees of freedom in the deep Euclidean region. The current in Eq. (3.13) and its dagger are

$$J_\mu(x) = \bar{d}(x)\, \gamma_\mu\, \gamma_5\, u(x), \qquad (3.22)$$
$$J_\nu^\dagger(0) = u^\dagger(0) \gamma_5^\dagger \gamma^\dagger \mathrm{d}(0). \qquad (3.23)$$

After these currents are placed in Eq. (3.12) and contracting out all pairs via Wick's theorem, the QCD part of the correlation function in terms of the full color carried light quark propagators (S_q) in the dense medium carried is obtained as

$$\Pi_{\mu\nu}^{QCD} = i \int d^4x e^{ip.x}\, Tr[\gamma_5 S_u^{ab}(x) \gamma_5 \gamma_\nu S_d^{ba}(-x) \gamma_\mu] \qquad (3.24)$$

The expression of the light quark propagator carried in full color in dense medium is

$$S_q^{nl}(x) = \left(\frac{i}{2\pi^2}\frac{\not{x}}{x^4} - \frac{m_q}{4\pi^2 x^2} + i\frac{m_q^2}{8\pi^2}\frac{\not{x}}{x^2}\right)\delta^{nl} + x_q^n(x)\bar{x}_q^l(0) \qquad (3.25)$$
$$- i\frac{g_s}{32\pi^2}\frac{\not{x}\sigma_{\mu\nu} + \sigma_{\mu\nu}\not{x}}{x^2}\, F_A^{\mu\nu}(0)t_A^{nl} + \cdots,$$

where x_q^n and $\bar{x}_q^l$ are Grassmann background quark fields. $F_A^{\mu\nu}$ is a classical background gluon field. And $t_A^{nl} = \frac{\lambda_A^{nl}}{2}$ with λ_A^{nl} named as Gell-Mann matrices. The dense medium matrix elements of the two-quark, two-gluon, mixed quark-gluon and four quarks are respectively,

$$x_{a\alpha}^q(x)\bar{x}_{b\beta}^q(0) = \langle q_{a\alpha}(x)\bar{q}_{b\beta}(0)\rangle_{\rho,} \qquad (3.26)$$
$$F_{n\lambda}^A F_{\mu\nu}^B = \langle G_{n\lambda}^A G_{\mu\nu}^B\rangle_{\rho}, \qquad (3.27)$$
$$x_{a\alpha}^q \bar{x}_{b\beta}^q F_{\mu\nu}^A = \langle q_{a\alpha}\bar{q}_{b\beta} G_{\mu\nu}^A\rangle_{\rho} \qquad (3.28)$$
$$x_{a\alpha}^q \bar{x}_{b\beta}^q x_{c\gamma}^q \bar{x}_{d\delta}^q = \langle q_{a\alpha}\bar{q}_{b\beta} q_{c\gamma}\bar{q}_{d\delta}\rangle_{\rho}. \qquad (3.29)$$

In order to continue with the QCD calculations, we substitute the light quark propagator in Eq. (3.25) containing quarks, gluons and mixed condensates depending on the medium density in the equation Eq. (3.25) as a result, we have obtained the perturbative and non-perturbative parts of the correlation function. Thus, in order to calculate the correlation function with density dependent medium operators, the calculations need to be moved to the momentum space. The following parameterization in D dimension is used for this:

$$\frac{1}{(x^2)^n} = \int \frac{d^D t}{(2\pi)^D} e^{-it.x} i(-1)^{n+1} 2^{D-2n} \pi^{D/2} \qquad (3.30)$$
$$\times \frac{\Gamma[D/2 - n]}{\Gamma[n]}\left[-\frac{1}{t^2}\right]^{D/2-n}.$$

For the continuum subtraction, the following replacement is used

$$(M^2)^N e^{-m^2/M^2} \rightarrow \frac{1}{\Gamma(N)}\int_{m^2}^{s_0} dq e^{-\frac{s}{M^2}}(s - m^2)^{N-1} \qquad (3.31)$$

For $N > 0$, where s_0 in the continuum threshold.

Then, when the four integral over the x space is performed, a Dirac Delta function arises from the above equation, which includes the momentum of particle p and t. We use the Dirac Delta obtained in the previous step to take the D-dimensional integral over the momentum t. For the dimensional regularization, we replace D with (4-2ϵ). As applied the hadronic side, the QCD side obtained after multiplying by p_μ^* to get rid of the axial contributions is as follows,

$$\begin{aligned}
p_\mu^* \Pi_{\mu\nu}^{QCD} = & \frac{\left(3\, m_d^2\, p_\nu\right)}{4\pi^2} - \frac{\left(3\, EulerGamma\, m_d^2\, p_\nu\right)}{8\,\pi^2} \\
& - \frac{\left(3\, EulerGamma\, m_d\, m_u\, p_\nu\right)}{4\,\pi^2} + \frac{\left(3\, m_u^2\, p_\nu\right)}{4\,\pi^2} \\
& - \frac{\left(3\, EulerGamma\, m_u^2\, p_\nu\right)}{8\,\pi^2} + \frac{\left(3\, m_d^2\, m_u^2\, p_\nu\right)}{2\, p^2 \pi^2} - \frac{\left(p^2 p_\nu\right)}{4\,\pi^2} \\
& + \frac{\left(3\, m_d^2 \log[2] p_\nu\right)}{4\,\pi^2} + \frac{(3\, m_d\, m_u \log[2] p_\nu)}{2\,\pi^2} + \frac{3\, m_u^2 \log[2] p_\nu}{4\pi^2} \\
& - \frac{3\, m_d^2 \log[-p^2] p_\nu}{8\,\pi^2} - \frac{3\, m_d\, m_u \log[-p^2] p_\nu}{4\,\pi^2} - \frac{3\, m_u^2 \log[-p^2] p_\nu}{8\pi^2} \\
& + \frac{3\, m_d^2 \log[\pi] p_\nu}{8\,\pi^2} + \frac{3\, m_d\, m_u \log[\pi] p_\nu}{4\,\pi^2} + \frac{3\, m_u^2 \log[\pi] p_\nu}{8\,\pi^2} \\
& + \frac{-2\left\langle \bar{u} g_s \sigma G\, u \right\rangle_\rho m_d\, p_\nu}{3\,\pi^4} + \frac{16\,\langle \bar{d} i D_0 i D_0 d\,\rangle_\rho m_u\, p_\nu}{3\,\pi^4} \\
& - \frac{2\left\langle \bar{d} g_s \sigma G\, d\,\right\rangle_\rho m_u\, p_\nu}{3\,\pi^4} + \frac{4\langle \bar{d}\, d\rangle_\rho m_u\, p_\nu}{p^2} - \frac{8\langle\, d^\dagger d\rangle_\rho m_q\, m_u\, p_0 p_\nu}{p^4} \\
& + \frac{8\left\langle \bar{u} g_s \sigma G\, u\,\right\rangle_\rho m_d\, p_0^2 p_\nu}{3\, p^6} - \frac{64\langle \bar{d} i D_0 i D_0 d\,\rangle_\rho m_u\, p_0^2 p_\nu}{3\, p^6} \\
& + \frac{8\left\langle \bar{d} g_s \sigma G\, d\,\right\rangle_\rho m_u\, p_0^2 p_\nu}{3\, p^6} + \frac{16\, m_d\, \langle \bar{u} i D_0 i D_0 u\,\rangle_\rho p_\nu}{3\, p^4} \\
& - \frac{64\, m_d\, p_0^2 \langle \bar{u} i D_0 i D_0 u\,\rangle_\rho p_\nu}{3\, p^6} + \frac{4\, m_d\, \langle \bar{d}\, d\rangle_\rho p_\nu}{p^2} \\
& + \frac{8\, m_q\, m_d\, p_0 \langle\, u^\dagger u\rangle_\rho p_\nu}{p^4} + \frac{\left\langle\, d^\dagger g_s \sigma G\, d\right\rangle_\rho\, p_0 p_\nu}{3\, p^4} \\
& - \frac{\left\langle\, u^\dagger g_s \sigma G\, u\right\rangle_\rho p_0\, p_\nu}{3\, p^4} + \frac{\left\langle\, d^\dagger g_s \sigma G\, d\right\rangle_\rho\, u_\nu}{6\, p^2} - \frac{\left\langle\, u^\dagger g_s \sigma G\, u\right\rangle_\rho\, u_\nu}{6\, p^2},
\end{aligned} \tag{3.32}$$

where in the limit $\rho \to 0$, we obtain the vacuum results.

Then, as in the hadronic part, we apply the Borel transform according to the square of the momentum to suppress the contributions of the higher states and the continuum

After all, in order to apply the continuum subtraction procedure to the result obtained, we apply the replacement (3.31).

After all, the fallowing expansion is obtained for the in-medium QCD side of the correlation function,

$$\widehat{\mathrm{B}}\left[p_\mu^* \Pi_{\mu\nu}^{QCD}\right] = \Omega_1^*(M^2, s_0, \rho) p_\nu + \Omega_2^*(M^2, s_0, \rho) u_\nu. \tag{3.33}$$

Here s_0 is the continuity threshold, which will later be fixed according to the standard procedure of the QCD sum rules method. As a result of all these calculations, the following relation is obtained for the QCD part of the correlation function in the nuclear medium:

$$\begin{gathered} f_{PS}^{*2} m_{PS}^{*4} e^{-\mu^2/M^2} = \Omega_1^*(M^2, s_0, \rho), \\ -f_{PS}^{*2} m_{PS}^{*4} \Sigma_u e^{-\frac{\mu}{M^2}} = \Omega_2^*(M^2, s_0, \rho), \\ \Sigma_u = -\frac{\Omega_2^*(M^2, s_0, \rho)}{\Omega_1^*(M^2, s_0, \rho)} \end{gathered} \tag{3.34}$$

where Ω_1^* for the p_ν structure and Ω_2^* for the u_ν structure are the Borel transformed amplitudes of the QCD side. The sum rules for the mass and decay constant of pion are

$$\left(p_\mu^* \Pi_{\mu\nu}^{had}\right)_{p_\nu} = f_{PS}^{*2}\, m_{PS}^{*4}\, e^{-\frac{\mu^2}{M^2}}, \tag{3.35}$$

$$\left(p_\mu^* \Pi_{\mu\nu}^{had}\right)_{u_\nu} = -f_{PS}^{*2}\, m_{PS}^{*4}\, e^{-\frac{\mu^2}{M^2}} \Sigma_u, \tag{3.36}$$

where

$$\mu^2 = \frac{\frac{d}{d\,(-\frac{1}{M^2})}\left(\left(p_\mu^* \Pi_{\mu\nu}^{had}\right)_{p_\nu}\right.}{\left(p_\mu^* \Pi_{\mu\nu}^{had}\right)_{p_\nu}}. \tag{3.37}$$

Therefore the express for the mass and decay constant of the pion are

$$\Sigma_u = \frac{-\left(p_\mu^* \Pi_{\mu\nu}^{had}\right)_{u_\nu}}{\left(p_\mu^* \Pi_{\mu\nu}^{had}\right)_{p_\nu}} \quad and \quad m_{PS}^{*2} = \mu^2 + \Sigma_u^2 - 2\Sigma_u p_0,$$

$$f_{PS}^{*2} = -\frac{\left(p_{\mu}^{*}\Pi_{\mu\nu}^{had}\right)_{p_{\nu}}}{m_{PS}^{*4}e^{-\frac{\mu^2}{M^2}}}. \tag{3.38}$$

Thus, for the numerical calculations we have obtained QCD sum rules to be used for the mass and decay constant of the pion in the nuclear medium.

4. NUMERICAL ANALYSIS

In this section, we obtain numerical results for the physical quantities of the pion which obtained by equalizing the hadronic and QCD results of the correlation function and given in the Eq. (3.12). In order to obtain these numerical results, light quark masses and numerical values of different dimension of quark, gluon and mixed condensates with different dimensions for the in-medium are needed as seen in the Eq. (3.12). The numerical values of these parameters are given in the Table 4.1.

In addition, as can be seen in the Eq. (3.33), there is a need for two auxiliary parameters in the in-medium sum rules of the mass and decay: the Borel mass parameter M^2 and the in-medium continuum threshold s_0. For the working regions of these auxiliary parameters to be fixed, the conditions for the convergence of pole dominance and operator product expansion must be satisfied.

For this purpose, Borel and continuity threshold parameters can be fixed according to the standard prescriptions of the QCD summation rules method. It is also required that; the dependence of the physical quantities should be relatively weak on these auxiliary parameters. So, the fixed Borel mass and continuity threshold parameters within the limits are as follow:

$M^2(MeV^2)$	$s_0(GeV^2)$
$0.1 \leq M^2 \leq 0.3$	$0.19 \leq S_0 \leq 0.4$

Table 4.1. Numerical parameters [25-28]

Parameter	Numeric Values
$\boldsymbol{m_u}$	$2.3\ [\boldsymbol{MeV}]$
$\boldsymbol{m_d}$	$4.8\ [\boldsymbol{MeV}]$
$\boldsymbol{p_0}$	$139\ [\boldsymbol{MeV}]$
$\boldsymbol{\rho^{sat}}$	$(0.11)^3\ [\boldsymbol{GeV^3}]$
$\boldsymbol{m_q}$	$0.5\,(m_u + m_d)\ [\boldsymbol{GeV}]$
$\boldsymbol{\sigma_N}$	$0.045\ [\boldsymbol{GeV}]$
$\boldsymbol{m_0^2}$	$0.8\ [\boldsymbol{GeV^2}]$
$\langle \boldsymbol{u^\dagger u} \rangle_{\boldsymbol{\rho}} = \langle \boldsymbol{d^\dagger d} \rangle_{\boldsymbol{\rho}}$	$\frac{3}{2}\rho\ [\boldsymbol{GeV}]$
$\langle \boldsymbol{\bar{u}u} \rangle_{\boldsymbol{0}} = \langle \boldsymbol{\bar{d}d} \rangle_{\boldsymbol{0}}$	$(-0.241)^3\ [\boldsymbol{GeV^3}]$
$\langle \boldsymbol{\bar{u}u} \rangle_{\boldsymbol{\rho}} = \langle \boldsymbol{\bar{d}d} \rangle_{\boldsymbol{\rho}}$	$\langle \bar{u}u \rangle_{\boldsymbol{0}} = \langle \bar{d}d \rangle_{\boldsymbol{0}} + \frac{\sigma N}{2m_a}\rho\ [\boldsymbol{GeV^3}]$
$\langle \boldsymbol{q^\dagger g_s \sigma G q} \rangle_{\boldsymbol{\rho}}$	$-0.33\,\rho\ [\boldsymbol{GeV^2}]$
$\langle \boldsymbol{q^\dagger i D_0 q} \rangle_{\boldsymbol{\rho}}$	$0.18\,\rho\ [\boldsymbol{GeV}]$
$\langle \boldsymbol{\bar{q} i D_0 q} \rangle_{\boldsymbol{\rho}}$	$\frac{3}{2} m_q \rho \simeq 0\ [\boldsymbol{GeV^4}]$
$\langle \boldsymbol{\bar{q} g_s \sigma G q} \rangle_{\boldsymbol{0}}$	$m_0^2 \langle \bar{u}u \rangle_{\boldsymbol{0}} = \langle \bar{d}d \rangle_{\boldsymbol{0}}\ [\boldsymbol{GeV^5}]$
$\langle \boldsymbol{\bar{q} g_s \sigma G q} \rangle_{\boldsymbol{\rho}}$	$\langle \boldsymbol{\bar{q} g_s \sigma G q} \rangle_{\boldsymbol{0}} + 3\,\rho\ [\boldsymbol{GeV^2}]$
$\langle \boldsymbol{\bar{q} i D_0 i D_0 q} \rangle_{\boldsymbol{\rho}}$	$0.3\,\rho - \frac{1}{8} \langle \boldsymbol{\bar{q} g_s \sigma G q} \rangle_{\boldsymbol{\rho}}\ [\boldsymbol{GeV^2}]$
$\langle \boldsymbol{q^\dagger i D_0 i D_0 q} \rangle_{\boldsymbol{\rho}}$	$0.031\,\rho - \frac{1}{12} \langle \boldsymbol{q^\dagger i D_0 i D_0 q} \rangle_{\boldsymbol{\rho}}\ [\boldsymbol{GeV^2}]$
$\left\langle \frac{\boldsymbol{\alpha_s}}{\boldsymbol{\pi}} \boldsymbol{G^2} \right\rangle_0$	$(0.33 \mp 0.04)^4\ [\boldsymbol{GeV^4}]$
$\left\langle \frac{\boldsymbol{\alpha_s}}{\boldsymbol{\pi}} \boldsymbol{G^2} \right\rangle_{\rho}$	$\left\langle \frac{\boldsymbol{\alpha_s}}{\boldsymbol{\pi}} \boldsymbol{G^2} \right\rangle_0 - 0.65\ \rho\ [\boldsymbol{GeV}]$

Considering the Borel mass parameter and the continuum threshold for the working regions, in Figure 4.1, we plot the ratio of the decay constant of the pion in-medium at the saturation density ($\rho^{sat} = 0.11\ GeV^3$) to the decay constant of the pion in vacuum (f^*/f), according to the Borel mass parameter at different values of continuum threshold values.

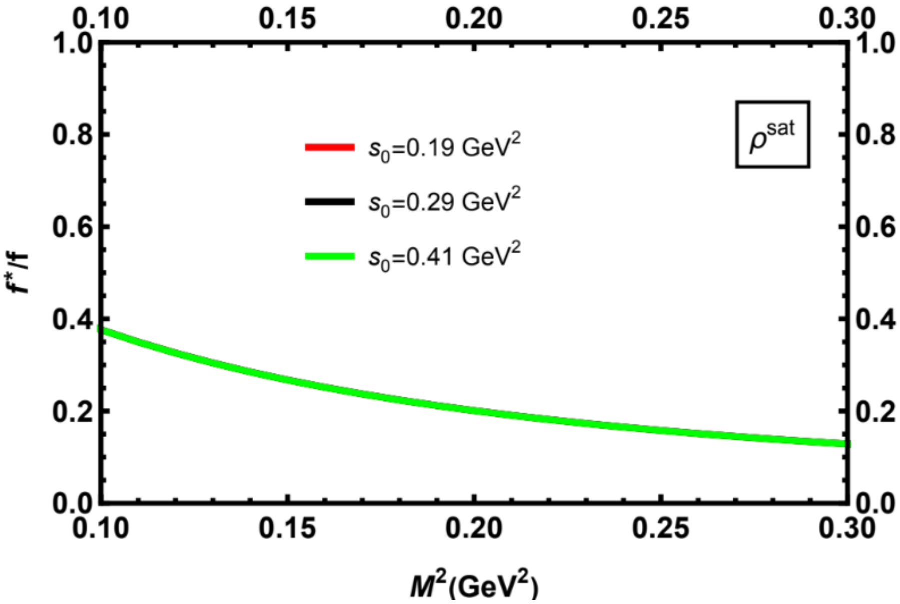


Figure 4.1. For different continuum threshold and at saturation density, the variation of f^*/f according to Borel mass.

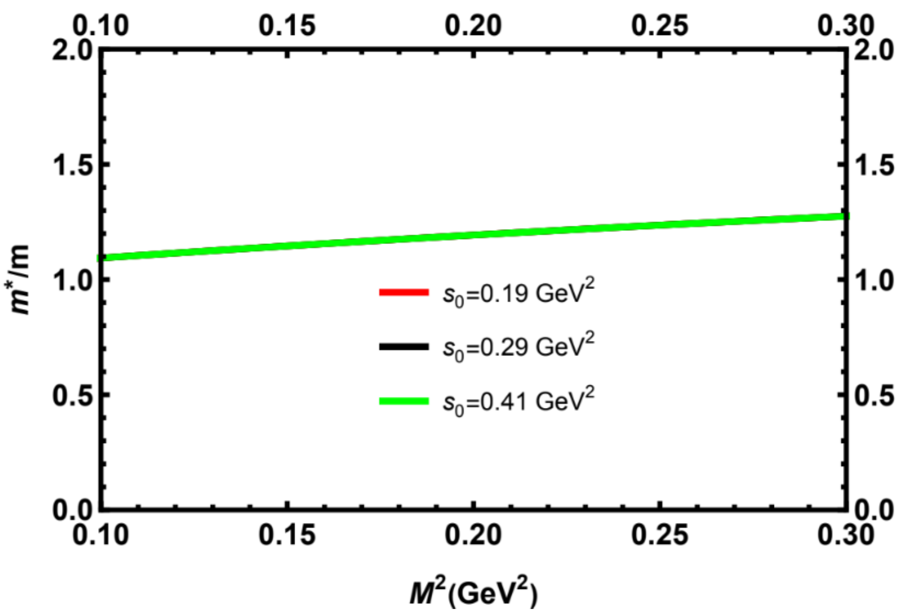


Figure 4.2. Same as Fig 4.1 for m^*/m.

As can be seen in the results obtained, we observe that the rate of the decay constant shows a good stability according to the variation of the auxiliary parameters within the working regions. In Figure 4.1 we see some residual dependency of (f^*/f), but within the limits allowed by the QCD sum rule method, this result is a reasonable one. With the same parameters, the dependence of the results obtained for the ratio of the in-medium mass of the pion to the mass of the vacuum to the auxiliary parameters is weak, as is the rate of the decay constant in figure 4.2 and it same a relatively could dependency on the auxiliary parameters, this weak dependence on auxiliary parameters is important so that the decay constant and mass of the pion can be studied with respect to the medium density.

The next step of the study was to investigate how these physical quantities of the pion, i.e.decay constant and mass, dependency on the medium density. For this, we plot how the ratio (f^*/f) in Figure 4.1 and (m^*/m) in Figure 4.2. change according to the medium density $M^2 = 0.2\ GeV^2$ (ρ/ρ^{sat}) at Borel mass and different continuum thresholds.

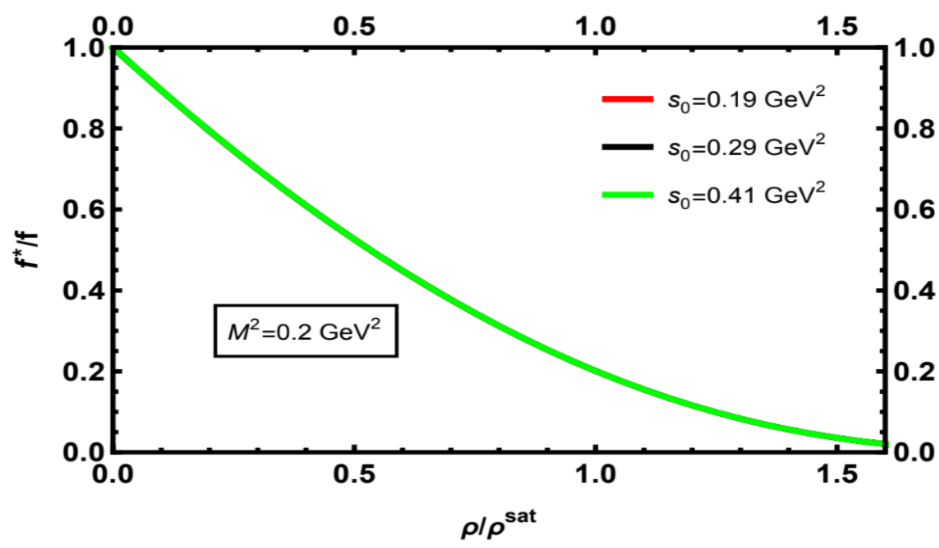


Figure 4.3. The density dependency of f^*/f for $s_0 = 0.29\ GeV^2$ and $M^2 = 0.2\ GeV^2$.

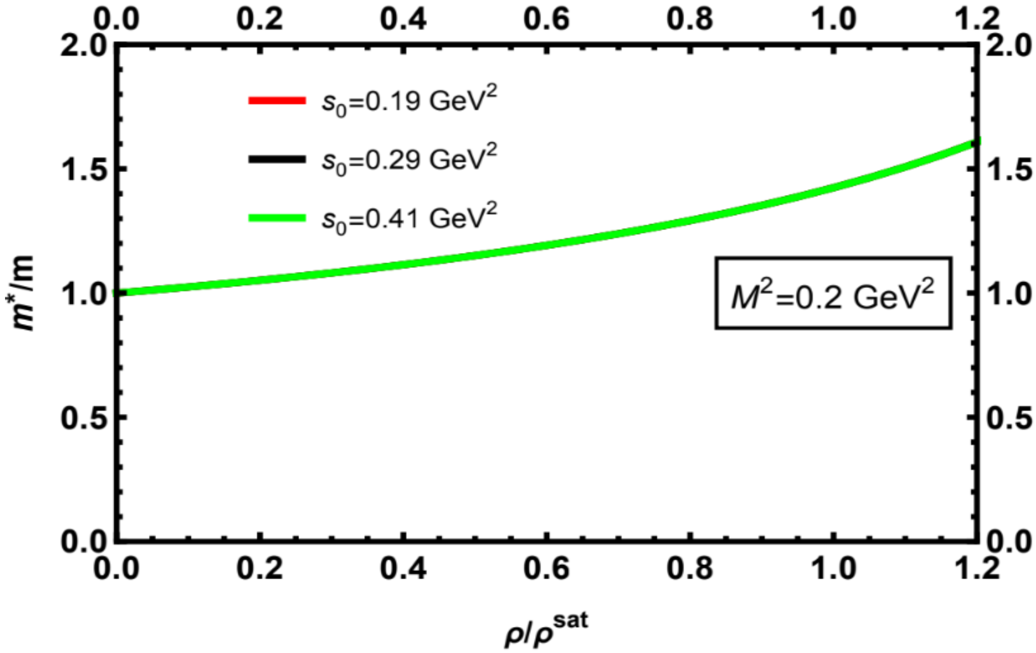


Figure 4.4. Same as Fig 4.3 for m^*/m.

As seen in Figure 4.3, the in-medium decay constant of the pion linearly decreaes with increasing medium density. The (f^*/f) ratio reaches zero at the 1.6 value of (ρ^*/ρ^{sat}). This result indicates that the meson melts at higher medium densities.

In Fig 4.4, the in-medium mass of the pion increases with increasing medium density. This rate of increase in medium mass appears to be small at low densities and increases at higher densities.

At the saturation density of the medium, the in-medium decay constant of the pion is obtained 26 MeV and in-medium mass is obtained 198 MeV.

5. CONCLUSION

The spectroscopic parameters of the π meson like mass and decay constant were investigated in the frame of QCD sum rules in a finite density of the medium. We discussed how the mass and decay constant of the pion change with the variation of the density of the nuclear medium. Previously, we showed the weak dependence of these physical quantities on the auxiliary parameters as the justification for the applicability of the QCD sum rules.

Next, we plotted the variation of (f^*/f) and (m^*/m) ratios with respect to ρ/ρ^{sat} to show the variation of mass and decay constant of pion with respect to the medium density. We concluded that although the increase in the mass of the pion was small where the medium density was low, its mass also increased with the increase in the medium density.

The decay constant of the pion, another physical quantity we examined, decreases linearly with the increase in the density of the nuclear medium and approaches zero at a value of $\rho \simeq 1.6\ \rho_{sat}$ and melts at higher densities.

As a result, the results obtained in the current study at finite density can be checked by different phenomenological models and approaches. In terms of experimental studies in the medium, they can be useful in the analysis of future data. In addition, the results obtained can provide useful information about the nature and internal structure of the light π meson, as well as the non-perturbative nature of QCD as the strong interaction theory.

6. REFERENCES

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