

ISTANBUL TECHNICAL UNIVERSITY ★ GRADUATE SCHOOL OF SCIENCE
ENGINEERING AND TECHNOLOGY

**SPARSE LINEAR PREDICTION MODELS FOR RADAR IMAGING AND
CLASSIFICATION**

M.Sc. THESIS

Bahar ÖZEN

**Department of Electronic and Communication Engineering
Telecommunication Engineering Programme**

MAY 2015

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Thesis Advisor: Assc. Doc. Dr. Işın YAZGAN ERER

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İSTANBUL TEKNİK ÜNİVERSİTESİ ★ FEN BİLİMLERİ ENSTİTÜSÜ

**RADAR HEDEF GÖRÜNTÜLEME VE SINIFLANDIRMA İÇİN SEYREK
DOĞRUSAL ÖNGÖRÜ MODELLERİ**

YÜKSEK LİSANS TEZİ

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To my parents,

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Bahar ÖZEN

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ABBREVIATIONS

SAR	: Synthetic Aperture Radar
ISAR	: Inverse Synthetic Aperture Radar
IFFT	: Inverse Fast Fourier Transform
FT	: Fourier Transform
SVD	: Singular Value Decomposition
BPDN	: Basis Pursuit Denoising
LASSO	: Least Absolute Shrinkage and Selection
OMP	: Orthogonal Matching Pursuit
MSTAR	: Moving Stationary Target Acquisition and Recognition
knn	: k nearest neighbours

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SPARSE LINEAR PREDICTION MODELS FOR RADAR IMAGING AND CLASSIFICATION

SUMMARY

RADAR, which is the abbreviation for Radio Detection and Ranging, is used for many applications including fire control, air traffic control, meteorology, target searching, detecting and tracking. Radar transmits pulses or burst to search for targets. Scattered signals from the target are processed by the radar receiver to extract information about the target such as range, velocity, image of target.

The most important radar application is the imaging. Imaging radars can provide range profile and radar image in addition to target's velocity. Resolution, which is the key point of imaging radar, is the problem for conventional radar because high resolution is succeeded with wide bandwidth and angular width. While wide bandwidth can be achieved by using stepped frequency or linear frequency modulated signal, wide angular width is obtained by increasing the dimension of antenna which is not desired or possible in real life application. Instead of huge antenna, synthetic aperture concept that is obtained by relative motion of target and radar is introduced. Synthetic Aperture Radar (SAR) and Inverse Synthetic Aperture Radar (ISAR) are based on the synthetic aperture concept.

Range profile and radar image are reconstructed by processing the backscattered signals which are collected in frequency and frequency-aspect domain, respectively. Fourier Transform based method is the most common method to reconstruct range profile and radar image. 1-D IFFT of frequency domain data gives the range profile and scattering centers are estimated from the peaks of range profile. Uniformity in spatial frequency domain is required to construct focused radar image but collected data are uniformly sampled in frequency aspect domain. Polar to Cartesian transform is used to convert frequency aspect data to spatial frequency data with uniform sampling. After employing polar reformatting, 2-D IFFT of data constructs focused radar image. Computational cost of this method is very low. In spite of this advantage, the achieved resolution is limited.

Spectral estimation methods are another method to generate high resolution range profile and radar image. Multiple Signal Classification (MUSIC) and Autoregressive (AR) model algorithms are the most popular spectral estimation methods.

The idea behind the Multiple Signal Classification algorithm is the decomposition of correlation matrix into signal and noise subspaces. Correlation matrix is computed from averaging over the number of snapshots. However, only one snapshot is available in radar application. Spatial smoothing method is used to compute correlation matrix. MUSIC algorithm with spatial smoothing process provides high resolution although it decreases the effective bandwidth. Success of this algorithm is not enough to find the location of scatterers if backscattered data are

limited or have low signal to noise ratio (SNR).

AR model which is based on the forward and backward linear prediction improves the resolution in range and cross range direction. Radar profile and radar image are obtained by AR model power spectrum. The simplest way to estimate AR model coefficients is to minimize the l_2 norm of the error between predicted and observed signal. Spurious peaks and sidelobes might appear in the range profile and radar image depending on the AR model order. Sidelobes can be suppressed by Singular Value Decomposition (SVD) truncation. The drawback of this method is that number of scatterers should be known or estimated.

Backscattered data can be represented as linear combination of small number of elements from overcomplete dictionary which consists of elementary signals. The idea behind the sparsity is to find the smallest subset of dictionary which represents the observed signal. Most of entries in signal representation vector/matrix should be zero to represent backscattered signal with less atoms. Radar image which is based on sparse signal representation provides high resolution in both range and cross range direction without predicting the number of scatterers. Sparse representations can be computed by three-different minimization problem which are Basis Pursuit Denoising (BPDN), Basis Pursuit Denoising with Penalty term and Least Absolute Shrinkage and Selection Parameter (LASSO). Basis Pursuit Denoising algorithm minimizes the sum of the absolute value of coefficients of the sparse representation vector/matrix subject to the residual sum of squares being less than the constant. BPDN with penalty terms is the unconstrained formulation of BPDN. Sparse coefficients are obtained by the l_2 norm minimization of residual by penalizing l_1 norm of the coefficients of the sparse representation vector/matrix. LASSO minimizes residual sum of squares while sum of the absolute value of coefficients of the sparse representation vector/matrix is smaller than constant threshold. Solution of BPDN with penalty term, BPDN and LASSO minimization problems take a lot of time. Orthogonal Matching Pursuit (OMP) algorithm is another method to generate sparse representations of the range profile and radar image. This method requires short time to find the sparse solution. High resolution radar image and range profile can be achieved by using these methods.

In this thesis, radar imaging based on sparse AR modeling is proposed. Tikhonov regularization is another way to solve the ill-posed equation system by introducing smoothness and sparsity. If l_2 norm of the AR model coefficients are penalized, smoother solution is produced. The inclusion of l_0 norm penalty function produces sparse solution. In this work, backscattered signals are modeled by AR model algorithm and sparse AR model coefficients are calculated. Sparse AR model coefficients are computed from BPDN, BPDN with penalty term and LASSO. Spurious peaks and side lobes are suppressed in the resulting radar image. Especially, proposed sparse AR model approaches yield better result than the other spectral estimation methods in case of the low SNR and narrowband data.

Although all scatterers and location of them are found correctly in radar image based sparse representation, classification success of these methods is the worst one. As a result, radar imaging based on sparse representation is a good candidate for imaging if data are collected at wide bandwidth and angular width.

Classification results of methods except sparse radar image representation are very close to each other under the wide bandwidth and aspect angle case. For the limited data case, classification of radar images based sparse AR model methods yields best

result among all the methods which are explained in the thesis. Classification results of MUSIC algorithm is in the second place. This result is expected since sparse AR model methods are more successful than MUSIC algorithm to find the scatterers and location of them.

As a conclusion, 1-D and 2-D sparse AR model are proposed and these models are applied on the radar data to reconstruct range profile and radar image. Sidelobes are suppressed in the generated range profile and radar image. In addition to that, this method is more successful than MUSIC algorithm to find the scatterers for the limited data case. Classification success will be enhanced if range profile and radar image are generated by using proposed method.

RADAR HEDEF GÖRÜNTÜLEME VE SINIFLANDIRMA İÇİN SEYREK DOĞRUSAL ÖNGÖRÜ MODELLERİ

ÖZET

RADAR (Radio Detecting and Ranging), uçak, gemi, doğa şekilleri gibi nesnelerin tespiti, yerlerinin belirlenmesi ve görüntülenmesi için kullanılan elektromanyetik tabanlı bir sistemdir. Radar görüntüsü, radarın anteninden gönderilen sinyallerin görüntülenmek istenen hedefe çarpıp yansmasıyla oluşan sinyaller kullanılarak elde edilir. Hedefin her noktasından yansıyan dalganın enerjisi aynı olmaz, yansıyan sinyalin bu özelliğinden yola çıkarak hedefin yeriyle birlikte hedefle ilgili diğer özellikler de tespit edilebilir.

Radarlarda görüntüleme son yıllarda ilgi çeken bir alandır. Görüntülemeye yüksek çözünürlüğü sağlamak önemli bir problemdir. Radar görüntülemeye menzil çözünürlüğü frekans bant genişliğine, çapraz menzil çözünürlüğü ise gözlem açısı aralığına bağlıdır. Darbe süresi kısaltılarak menzil çözünürlüğünü iyileştirmek mümkündür ancak bu durumda gönderilen darbenin enerjisi de süresiyle orantılı olarak azalacağı için yeterli SNR(işaret gürültü oranı) sağlanamayabilir. Module edilmiş sinyal kullanarak, hem yüksek band genişliği hem de gerekli olan SNR değeri sağlanır. Çapraz menzilde çözünürlüğü yükseltmek için pratikte gerçekleştirilemeyecek anten boyutları gerekmektedir. Bu gereksinimden ötürü, geleneksel radarlar ile yüksek çözünürlüklü görüntüleme yapmak mümkün değildir. Yapay açıklıklı radar, gerçek açıklıklı radar sisteminin kısıtlarını ortadan kaldıran bir yöntem olarak geliştirilmiştir. Yapay açıklıklı radar sistemlerinde, bir uydu veya bir uçak ile taşınan radar, mümkün olduğunca sabit bir hızla ileri yönde hareket ettirilmektedir. Radar, ileri yönlü hareketi esnasında belirli bir süre boyunca belirli sayıda darbe göndermektedir. Bu gönderilen işaretler hedef tarafından geri yansıtılır ve radar tarafından toplanır. Farklı frekanslarda ve açılarda toplanan yansıyan alanlar uygun bir biçimde kaydedilmekte ve birleştirilmektedir. Bu sayede yapay açıklık oluşturulmaktadır. Platformun hızı ve toplam darbe gönderme süresi yapay açıklığın boyunu belirlemektedir. Yapay açıklıklı radar yönteminde her nokta daha uzun süre gözlenmektedir. Bu sayede platformun ilerleme yönüne paralel olan azimut doğrultusunda aynı boydaki gerçek açıklıklı bir antene göre daha yüksek çözünürlük elde edilir. Ters yapay açıklıklı radarlardaki (ISAR) yapay açıklık da SAR sistemlerindeki gibi oluşturulur. Ancak, bu sistemde hedef hareketli, radar sabit olup; yapay açıklık gönderilen sinyalin hedefe farklı açılardan ulaşmasıyla elde edilir.

Yeterli band genişliğine sahip radardan gönderilen işaretin hedefe çarpıp geri dönmesiyle menzil profili elde edilir. Gönderilen sinyal zaman uzayında ise, yansıyan cevap radar alıcısı tarafından toplanarak RCS bilgisi oluşturulur. Eğer basamak frekanslı dalga ise, gelen cevabın ters Fourier dönüşümü alınarak menzil profili oluşturulur. Menzil profili, hedefin yalnızca konumunu değil biçimi hakkında da bilgi sahibi olunmasını sağlar. Darbe süresi uzun olduğunda birbirine yakın

saçılma merkezleri ayrı ayrı algılanamayabilir ve bu durum çözünürlüğü kötüleştirir. Menzil profiline benzer bir mantıkla, çapraz menzil profili de gelen cevapların açılı bilgileriyle elde edilir. Menzil profili için, tek bir açıdan toplanan farklı frekans cevapları değerlendirilirken; çapraz menzil profilinde tek bir frekans değerinde farklı açılardan gönderilen sinyallerin cevabı değerlendirilir. Radar görüntüsü ise farklı frekans ve açılarda gönderilen sinyallerin cevabının işlenmesiyle oluşturulur.

Sentetik açıklı radar görüntüleme için kullanılan geleneksel yöntem hızlı ve işlem maliyetinin düşük olması nedeniyle Fourier dönüşümüne dayalı yöntemlerdir. Menzil profili oluşturmak için, frekans uzayında toplanan verinin 1-B IFFT'si alınır. Frekans açılı uzayında toplanan verinin 2-B IFFT uygulanmasıyla radar görüntüsü oluşturulur. Odaklanma problemi olmayan bir radar görüntüsü oluşturmak için, öncelikle frekans açılı uzayında toplanan verinin kartezyen koordinatlarda sabit aralıklarla örneklenmelidir. Frekans-açılı uzayında sabit aralıklarla alınan verinin örnekleri, interpolasyonla kartezyen koordinatlarda eşit aralıklarla örneklenmesi sağlanır. Polar koordinat ile Kartezyen koordinat düzlemi arasında geçiş yapılarak elde edilen verinin ayrık Fourier dönüşümü (FT) alınarak radar görüntüsü elde edilir. Sıfır ekleme ve pencereleme fonksiyonları ile oluşturulan menzil profili veya radar görüntüsü iyileştirilir. Sıfır eklemek taraklama inceliğini artırarak yumuşak geçişler sağlarken, yan loblar pencereleme fonksiyonlarıyla bastırılır. Tüm bunlara rağmen Fourier dönüşümüne dayanan yöntemlerle yüksek çözünürlüklü görüntü elde edebilmek için geniş bant- gözlem açılı aralığında veri toplamak gerekmektedir. Ancak gerçek hayatta gerçekleştirilen uygulamalarda bu koşulları sağlamak oldukça zordur. Dar bant- açılı koşullarında Fourier dönüşümü istenen çözünürlüğü sağlayamamaktadır. Literatürde yüksek çözünürlüklü menzil profili ve radar görüntüsü elde etmek için spektral kestirim yöntemlerinin kullanılması önerilmiştir. Bu çalışmada spektral kestirim yöntemlerinden MUSIC (Multiple Signal Classification) ve özbağımlı (Auto-Regressive) modelleme metodlarına değinilecektir. MUSIC metodu verinin özilişki matrisinin sinyal ve gürültü alt uzayına ayrılmasına dayanmaktadır. Özbağımlı modelleme ileri ve geri lineer kestirime dayanmaktadır.

MUSIC algoritması için öncelikle birden fazla gözlem sonucunun ortalamasıyla hesaplanan özilişki matrisi bulunmalıdır. Ancak radar uygulamalarında yalnızca bir gözlem verisi bulunduğu için uzlamsal yumuşatma işlemi uygulanarak özilişki matrisi bulunur. Elde edilen özilişki matrisinin özvektörleri, özdeğerlerine bağlı olarak sinyal ve gürültü alt uzayına ayrılır. Yüksek özdeğerlere ait özvektörler sinyal alt uzayına dahil edilirken, geri kalan özvektörler gürültü alt uzayını oluşturur. MUSIC algoritmasında kullanılan uzlamsal yumuşatma işlemi bant genişliğini azaltarak çözünürlüğü düşürmesine rağmen Fourier yöntemine göre daha iyi çözünürlük değerleri sunmaktadır.

Özbağımlı modelleme ile toplanan radar verisi ileri ve geri lineer kestirimleriyle gösterilebilir. Özbağımlı modelleme katsayıları en küçük kareler çözümünden bulunabilir. Ancak en küçük kareler çözümüyle elde edilen menzil profilleri ve radar görüntülerinde, modelleme seviyesine bağlı olarak sahte saçıcılar ve yanloblar oluşabilir. Özbağımlı modellemede yan lobların bastırılması amacıyla tekil değer ayrışımı (SVD) kullanılır. Bu yöntem özbağımlı modelleme ile elde edilen radar hedef görüntüsündeki yan lobları bastırmayı başarsa da menzil profilinde ve radar görüntüsünde veri kaybını engelleyememektedir.

Tekil deęer ayrıştırımı ve MUSIC algoritmalarının performansı kestirilen saçıcı sayısına baęlı olarak deęişmektedir.

Son zamanlarda işaret işleme alanında yapılan çalışmalar ile sinyallerin sözlük kullanımıyla seyrek bir şekilde ifade edilmesi üzerine yoğunlaşmıştır. Burada radar işaretleri sözlük kullanılarak ifade edilmiş ve az sayıda sözlük elemanı kullanarak radar işaretleri seyrek bir şekilde gösterilmiştir. Radar işaretlerinin seyrek gösterimini elde etmek için BPDN, ceza terimi ile BPDN ve LASSO yöntemleri kullanılmıştır. BPDN yönteminde, hatanın 2- normunu sınırlandırarak seyrek gösterim vektörünün/matrisinin katsayılarının mutlak deęerleri minimize edilmeye çalışılmaktadır. İşaretlerin seyreklięi hatayı kısıtlayan deęere baęlıdır. Ceza terimli BPDN algoritmasında ise sıfırdan farklı olan seyrek gösterim vektörünün/matrisinin katsayılarını cezalandırarak hatanın 2 normunu minimize etmeye çalışır. Bu yöntemin dezavantajı ceza teriminin deęerinin bulunmasındaki işlem yüküdür. Farklı ceza terimleri için hatanın 2 normu ve katsayılarının bir normu hesaplanarak bir eğri oluşturulur ve bu eğrinin köşesi optimum ceza terim deęerini verir. LASSO probleminde seyrek gösterim vektörünün/matrisinin katsayılarının bir normu sınırlı tutularak hatanın iki normu minimize edilmeye çalışılmaktadır. Tüm bu yöntemler CVX optimizasyon programı ile çözülmüştür ancak işlem yükü çok fazladır. Radar sinyallerini seyrek olarak Dikgen Eşleştirme Takibi (OMP) ile daha kısa sürede elde oluşturulabilir. Bu yöntemlerle oluşturulan menzıl profilleri ve radar görüntüleri yüksek çözünürlüğe sahiptir.

Bu çalışmada, yüksek çözünürlüklü radar görüntüsü oluşturmak için seyrek yaklaşımlar yardımıyla AR modelleme katsayılarının bulunması önerilmiştir. Literatürde kötü konumlanmış (ill-posed) denklem sistemini çözmek için yumuşatma ve seyrekleştirme bilgilerini probleme katan Tikhonov regülarizasyon yöntemi önerilmiştir. AR model katsayılarının l_2 normu cezalandırılarak yumuşak geçiş sunan çözüm elde edilir. Eğer AR mode katsayılarının l_0 normu cezalandırılırsa, seyrek çözüme ulaşılır. Seyrek AR model katsayılarını bulmak için literatürde sıklıkla kullanılan BPDN, ceza terimli BPDN ve LASSO minimizasyon problemleri çözülür. Bu durumda oluşturulan AR model katsayılarının çok az bir kısmı sıfırdan farklı deęer alır. Seyrek AR modelleme katsayılarının kullanılmasıyla elde edilen görüntü ile en küçük kareler çözümü ile elde edilen görüntü karşılaştırıldığında yan lobların bastırıldığı ve görüntünün iyileştirildięi gözlemlenmiştir. Önerilen seyrek AR model yöntemlerinin performansının geri saçılan sinyallerin dar frekans bandı ve açılal sektörde toplanması durumunda bile yalancı saçıcıları başarılı bir şekilde bastırıldığı gözlemlenmiştir.

Bu çalışmada, yöntemlerin performanslarını karşılaştırmak amacıyla sınıflandırma yapılmıştır. Farklı yöntemlerle oluşturulan menzıl profilleri ve radar görüntüleri sınıflandırılmış ve önerilen seyrek AR model yaklaşımlarının sınıflandırma sonuçlarının daha yüksek olduęu görülürken, seyreklik yaklaşımıyla oluşturulan görüntülerin sınıflandırma sonuçlarının daha kötü olduęu gözlemlenmiştir.

Sonuç olarak önerilen seyrek AR modellerin, dięer yöntemlere göre daha iyi görüntü oluşturduęu ve sınıflandırma performansını arttırdığı gösterilmiştir.

1. INTRODUCTION

RADAR, the abbreviation for radio detection and ranging, gives information about the location of target via electromagnetic waves. The signal is transmitted from the radar antenna to detect the target. Transmitted signal propagates through the atmosphere and hits the target if exists. Small part of the signal, called echo, is backscattered from target to radar and radar listens echo to get information about target. Backscattered echo gives information about target such as location, size and speed. While the delay between the transmitted and received pulse gives the range, speed of the target is estimated from frequency shifts between transmitted and received pulses. Accuracy of information about target depends on the resolution in the range. Range resolution is inversely proportional to pulse width. On the other hand, higher average power is accomplished by long pulse width. To achieve high average power and range resolution at the same time, linearly frequency modulated or stepped frequency continuous wave signal is transmitted after the pulse compression [1].

Nowadays, imaging radars play an important role for military and civil applications. Resolution that is the minimum distance to resolve scatterers is an important parameter for imaging radar. Both range resolution and cross range resolution form image resolution. Range resolution and cross range resolution depend on the bandwidth of transmitted signal and antenna aperture, respectively. High cross range resolution is achieved by high antenna aperture. At this point, SAR/ISAR systems become important because implementation of huge antenna size is impractical in real life application. SAR systems are based on the synthetic aperture concept generated by relative movement of the radar and the target. SAR systems are used for imaging because of high cross range resolution.

Working principle of SAR and ISAR is the same. In ISAR system, the target is in motion while radar stays stationary. SAR system transmits linear frequency modulated pulse or stepped frequency signal into specific direction to detect the target. If the target exists on this direction, transmitted signal is reflected back and

radar collects reflected signal to reconstruct radar image. Range profile of the target is obtained by processing of the backscattered data collected at different frequency but constant look angle. On the other hand, cross range profile is obtained by the processing of data that are collected at different aspect angles but constant frequency. Range and cross range construct radar image.

Collected data are uniformly sampled in frequency aspect domain but backscattered data should be uniformly sampled in spatial-frequency domain to get focused image. Polar to Cartesian transform should be employed on the collected data to provide uniformly sampled data in spatial frequency domain [2].

In general, range profile and radar image of target are reconstructed by FT based method because of the low computational cost. If zero padding and windowing processes are applied on the collected data, FT based method provides better radar image. While zero padding provides smoother image, windowing process suppress the side lobes [3]. FT based method provides low-resolution radar image when bandwidth and angular width are limited. Spectral estimation methods such as MUSIC [4] and AR model [5] methods offer high resolution radar images .

In MUSIC method, construction of correlation matrix is the first step of this algorithm. Correlation matrix is computed by averaging over number of snapshots of 1-D and 2-D radar data but only one snapshot is available for radar applications. Correlation matrix is computed by employing the spatial smoothing preprocessing technique [6] . MUSIC algorithm analyses the eigenvalue of correlation matrix and constitutes signal subspace and noise subspace. As eigenvectors corresponding to largest eigenvalues belong to signal subspace, the rest of eigenvectors form the noise subspace [7] . Eigenvectors of data correlation matrix which are in the noise subspace are used to construct radar image.

Autoregressive (AR) model that finds location of scatterers is based on the forward and backward linear prediction. If RCS data are collected at narrow bandwidth and angular width, this algorithm will be an effective way to generate high-resolution radar image. The simplest way to calculate AR model coefficient is least square solution. It is possible that this solution will find more scatterers than actual one depending on the model order and cause high sidelobes. Singular Value Decomposition (SVD) truncation is used to suppress spurious scatterers [5]. At SVD

truncation, firstly eigenvalues of correlation matrix are computed and largest eigenvalues of the correlation matrix give the number of scatterers and radar image is reconstructed by using this information. In generated range profile and radar image, sidelobes vanish and actual scatterers might be lost depending on the estimated scatterers. Therefore, the success of SVD truncation depends on the correct choice of the number of scatterers.

Nowadays, sparsity and sparse representation are the most investigated topic in image processing. Signal can be represented as the superposition of elements from the dictionary which contain small amount of information about signal. $\mathbf{Ax} = \mathbf{b}$ is the matrix notation of signal representation where $\mathbf{A}$ is the dictionary, $\mathbf{b}$ is the observed signal and $\mathbf{x}$ is the signal representation. It is possible to represent original signal by using less information than whole information provided by signal. Finding the smallest subset which provides necessary information to approximate original signal refers the sparse signal representation. Sparse representation of signal, $\mathbf{x}$, which has fewer non-zero entries is found different minimization problem such as basis pursuit denoising (BPDN), basis pursuit denoising with penalty term and least absolute shrinkage and selection (LASSO). While BPDN optimization problem is the error constraint problem, LASSO optimization problem refers the sparsity constraints minimization problem [8]. BPDN finds the best representation of signal by minimizing the l_1 norm of the $\mathbf{x}$ while keeping l_2 norm of residue below the threshold which affects the sparsity level [9, 10]. In BPDN with penalty term, sparse solution is found by penalizing non-zero coefficients of the signal representation vector/matrix. BPDN with penalty term is the unconstrained formulation of BPDN [11]. LASSO minimization problem is based on minimizing l_2 norm of the residue when l_1 norm of $\mathbf{x}$ is limited.

In [12], the author claims that sparse signal representation is a good choice to get high resolution radar image. Backscattered signal can be represented by weighted superposition of Fourier dictionary atoms [13]. Sparse representation of signal is found by using basis pursuit minimization problem which seeks the sparsest exact representation of signal [9]. In this work, sparse representation of signal is found by solving BPDN, BPDN with penalty term and LASSO minimization problems. The resolution of reconstructed radar image based on sparse representation is very high.

In [10], AR model is used for linear prediction of speech signal. The authors use BPDN with penalty term to find the sparse AR model coefficients.

In this study, radar images are constructed by combining AR models and sparsity approach. AR model coefficients are calculated by solving the minimization problem of BPDN, LASSO and BPDN with penalty term. Most of the AR model coefficients will be zero. If range profile or radar image is reconstructed by using these coefficients, sidelobes will be suppressed. In addition to that, this method successfully finds the location of scatterers for the limited data case. As a result, this method yields better-resolved images than the conventional spectral estimation methods.

Target classification is also another important radar application. In military applications, target will be identified as hostile or friend as a result of a classification process. Target images or features of target can be used for the classification. Since image based classification is not practical, feature based classification is used widely.

In this thesis, range profiles and radar images are classified to determine which method yields better range profile/image and better classification result. In this thesis, two different feature extraction methods for range profile and radar image are investigated. For range profile, the feature extraction method in [14] is used in this thesis. Range profile gives information about the location of scatterers, size of target. Translational and level invariance features are desired for classification. While level invariance is achieved by normalizing the range profile, central moment of normalized range profile gives the translational invariance. The obtained feature that depends on the position and shape of target includes some redundant information. Principal component analysis (PCA) is used to eliminate redundant information and the dimension of feature vector is reduced [7]. For radar image classification, features are extracted from radar image by employing the algorithm in [15]. Polar mapping of radar image is used to get feature vector. Feature vector consists of the projection of polar image on r axis and θ axis and the compressed polar image [2]. The obtained features are classified by using k- nearest neighbors classifier.

As a conclusion, contributions of the thesis can be summarized as:

- Sparse AR models are proposed for 1-D and 2-D data to eliminate the shortcomings of SVD truncation.

- Sparse AR models are applied to radar imaging to generate high resolved range profile and radar image.
- The effect of obtained high resolution range profile and radar image on classification performance are demonstrated.

1.1 Scope of Thesis:

In chapter 1, general information about radar is given. Also, imaging techniques and classification method which is used in this thesis are explained briefly.

In chapter 2, fundamental information about ISAR systems are explained. Conventional methods to generate range profile and radar image are explained in detail.

In chapter 3, sparsity and sparse representation are introduced. Then sparse AR model based methods are proposed. Simulation results of each method are presented in this part.

In chapter 4, constructed range profile and radar images are classified. Feature extraction methods and classification results are presented in here.

Chapter 5 is the conclusion part which includes discussion about proposed methods.

2. SPECTRAL ESTIMATION BASED HIGH RESOLUTION RADAR IMAGING METHODS

2.1 Purpose

Resolution of image which is increased by wide bandwidth and angular width is important for target identification and classification applications. It is impossible to get high resolution image with conventional radar because of the physical restrictions. Synthetic aperture concept which is generated by relative movement of radar and target overcomes the physical restriction of conventional radars [3]. SAR systems provide high resolution image without huge antenna size. SAR and ISAR systems are similar but in ISAR systems target is moving while radar is stationary. ISAR is the most powerful candidate for imaging moving targets.

In this part, general information about ISAR and ISAR imaging techniques are presented.

2.2 General Information About Radar

▪ 1-D Radar Data

Range refers to the distance between the radar and the target. Range profile, also called radar signature, is reconstructed by using backscattered signal from the target. Radar transmits several pulses or burst and listens the returned signal from the target which gives desired information about target. The backscattered signal at range domain is denoted by:

$$E_s(f) = \sum_{i=1}^d A_i e^{-j2\pi \frac{2f}{c} x_i} \quad (2.1)$$

where , A_i is the amplitude of i^{th} scatterer, f is the frequency, x_i is the location of i^{th} scattering center and c is the speed of light and d is the number of scatterers.

Range resolution is the minimum distance between the scatterers along the line of sight (LOS). Since range resolution depends on the bandwidth of signal, high range

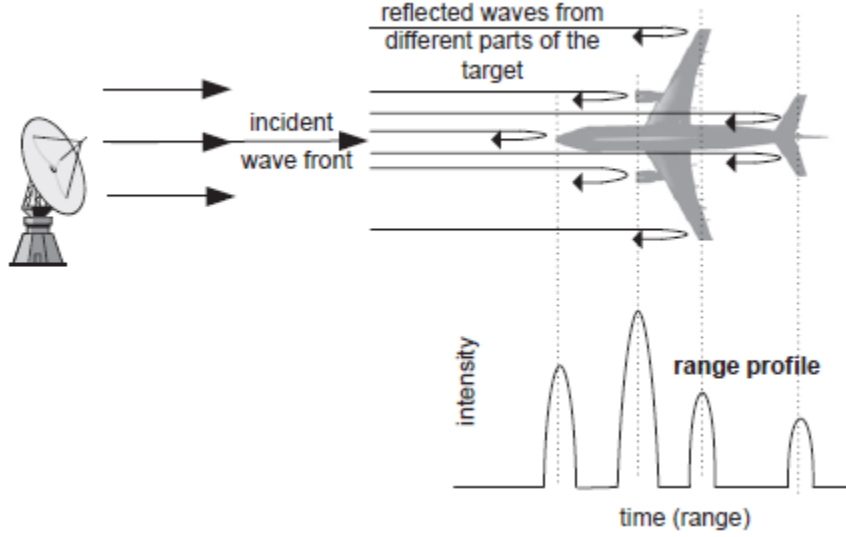


Figure 2.1: Range profile of target [5].

resolution is achieved by increasing the bandwidth of the signal. On the other hand, wide bandwidth causes low signal to noise ratio (SNR). Stepped frequency or linear frequency modulated signal is used to achieve high range resolution and desired SNR. Range resolution, Δr , is calculated from (2.2) where c is the speed of light and B is the bandwidth of signal.

$$\Delta r = \frac{c}{2B} \quad (2.2)$$

Backscattering signals from the target at different aspect angles form the cross-range profile. Backscattering signal for different aspect angle is approximated as:

$$E_s(\phi) = \sum_{i=1}^d A_i e^{-j2\pi \frac{2f}{c} \phi y_i} \quad (2.3)$$

where A_i is the amplitude of the i^{th} scatterer and y_i is the distance between the scattering center and origin.

$$\Delta y = \frac{R\lambda}{2D_{SA}} \quad (2.4)$$

Cross range resolution, which depends on the aperture of antenna, is given in Equation (2.4). High cross range resolution is achieved by the large aperture of the antenna. Since the implementation of huge aperture antenna is not practical in reality, synthetic aperture radar that is collected of small-size real aperture radars is used to

achieve high cross range resolution. Synthetic aperture is achieved by moving the radar antenna along the path.

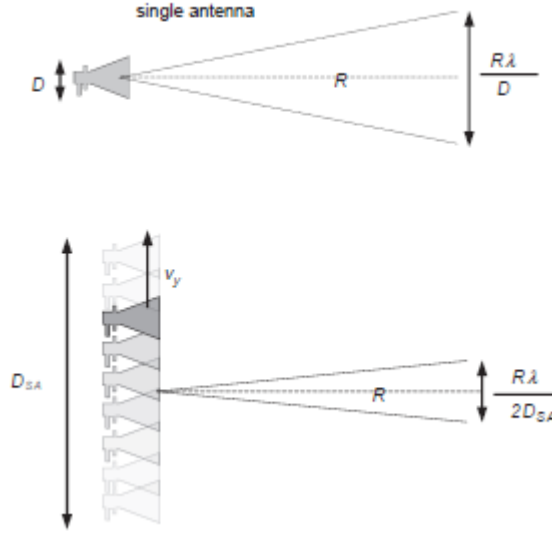


Figure 2.2: Real aperture of single antenna and synthetic aperture of N antenna [5].

In SAR, radar platform is moving while target stays stationary. On the other hand, in ISAR system radar is stationary while target is in motion [1].

Range profile is generated by processing of data collected in frequency domain. Range profile gives an idea about the length of target and location of scattering centers. Range profile is used for identification of the target.

Assume that the target consists of d different scatterers. 1-D frequency domain radar data are modeled by the sum of backscattered fields. Backscattered field of k^{th} scatterer can be represented as:

$$E(f) = A_k e^{-\frac{j4\pi f}{c}x_k} \quad (2.5)$$

$$f(l) = f_0 + l\Delta f; \quad l = 0, 1, 2, \dots, N \quad (2.6)$$

In (2.6), f_0 is the initial value of frequency and Δf is the frequency increment. Substitute (2.6) into (2.5) and obtain :

$$E(l) = A_k(l) e^{-\frac{j4\pi}{c}f_0 x_k} z_{x_k}^l \quad (2.7)$$

where $z_{x_k}^l = e^{-j\frac{4\pi}{c}l\Delta f x_k}$

From (2.7), total backscattered data in frequency domain can be written as:

$$E(l) = \sum_{k=1}^d A_k(l) e^{-\frac{j4\pi}{c} f_0 x_k z_{x_k}^l} + v(n) \quad (2.8)$$

▪ 2-D Radar Data

Radar image shows 2-D RCS distribution of the target in range and cross range direction. Range and cross range resolutions determine the quality of reconstructed image. High resolution is achieved by wide frequency bandwidth and angular width. Radar image should also provide information about the energy of each scattering center.

The radar image is obtained by processing the backscattered data that are collected at frequency-aspect domain. Backscattered field from the target at different frequencies and different angles can be approximated as:

$$E^s(f, \phi) = \sum_{i=1}^P A_i e^{-j4\pi \frac{f}{c} (x_i \cos \phi + y_i \sin \phi)} \quad (2.9)$$

The size of target and resolution are determined before the backscattered data are processed to get range profile and radar image.

Range profile is generated by using frequency domain data of target. Range profile provides necessary information such as location of scattering centers and length of the target. The range resolution is function of signal bandwidth. As indicated in (2-2), Δr is the range resolution or range bin. Range extension or length of the target in line of sight (LOS) direction is determined from:

$$X_{max} = M \Delta r \quad (2.10)$$

Cross range resolution of ISAR, which is formulated as (2.4), can be rewritten as:

$$\Delta y = \frac{R \cdot \lambda}{2D_{SA}} = \frac{R \cdot \lambda}{2R\Omega} = \frac{c}{2f_c \Omega} \quad (2.11)$$

Cross range, resolution depends on Ω , angular bandwidth. Cross range extension or length of target in cross range direction, Y_{max} is determined by:

$$Y_{max} = N \Delta y \quad (2.12)$$

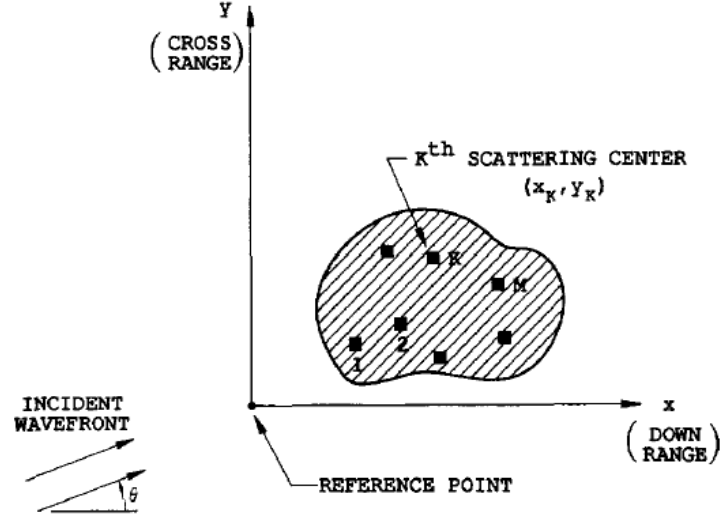


Figure 2.3: Coordinate system and target with M scattering center [5].

2-D radar data ,collected in frequency-aspect domain, contains two Fourier transform pairs not separable in terms of f and $\emptyset$. Two separable Fourier transform pairs are achieved by small angle approximation [16]. Unfocused radar image is constructed as a result of processing this approximation. Polar to Cartesian transform solves the focusing problem. Frequency aspect domain data are transformed into spatial frequency domain data. As a result of polar to Cartesian transform, data are uniformly spaced in spatial frequency domain.

Transformation of $E^s(f, \emptyset)$ to spatial frequency domain is shown below:

$$E^s(f_x, f_y) = \sum_{i=1}^P A_i e^{-j2\pi(f_x x_i + f_y y_i)} \quad (2.13)$$

where

$$f_x = \frac{2f \cos \emptyset}{c} \quad f_y = \frac{2f \sin \emptyset}{c} \quad (2.14)$$

2-D backscattered signal from d scatterers in spatial frequency domain is given by:

$$E(f_x(l), f_y(n)) = \sum_{i=1}^d A_i e^{-j2\pi(f_x(l)x_i + f_y(n)y_i)} \quad (2.15)$$

$$f_x(l) = f_x(0) + l\Delta f_x \quad l = 0, 1, \dots, N-1 \quad (2.16)$$

$$f_y(n) = f_y(0) + n\Delta f_y \quad n = 0, 1, 2, \dots, M-1$$

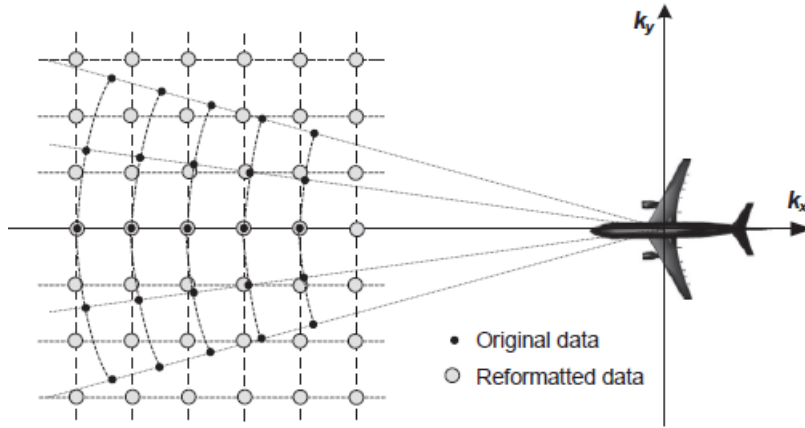


Figure 2.4: Rectangular reformatting of polar radar data [1].

If (2.16) is substituted in (2.15), total backscattered wave is represented by:

$$E_k(l, n) = A_k(l, n) e^{j4\pi/c (f_x(0) x_k + f_y(0) y_k)} z_{x_k}^l z_{y_k}^n \quad (2.17)$$

where

$$z_{x_k} = e^{j4\pi/c \Delta f_x x_k} \quad (2.18)$$

$$z_{y_k} = e^{j4\pi/c \Delta f_y y_k} \quad (2.19)$$

$A_k(f, \theta)$ is assumed to be constant. Then (2.17) can be written as:

$$E_k(l, n) = E_k(0, 0) z_{x_k}^l z_{y_k}^n \quad (2.20)$$

where

$$E_k(0, 0) = A_k(f_x(0), f_y(0)) e^{j4\pi/c (f_x(0) x_k + f_y(0) y_k)}$$

$E_k(0, 0)$ is the backscattered field of k^{th} scattering center. Total backscattered field is given by:

$$E(l, n) = \sum_{k=1}^M E_k(0, 0) z_{x_k}^l z_{y_k}^n + v(l, n) \quad (2.21)$$

2.3 Fourier Transform Based Radar Imaging

Generally, the range profile and radar image are reconstructed by Fourier Transform (FT) based methods. High resolution range profile and radar image is achieved by large amount of data which are collected at wide bandwidth and angular width. Collected data which is given in (2-8) are converted into space (time) domain by

applying 1-D IFFT to obtain range profile. Radar image is reconstructed by applying 2-D IFFT on the backscattered data given in (2-21) but to obtain focused image, polar to Cartesian transform is applied firstly. FT based methods gives the limited resolution radar image. Quality of radar image is improved by zero padding that increases the number of samples and windowing which suppress the sidelobes but image resolution is not improved sufficiently. Although the method is simple, it provides limited resolution and fails to find weak scatterers.

Spectral estimation methods such as MUSIC (Multiple Signal Classification) and AR (Auto Regressive) model provide high resolution radar images in the case of narrow bandwidth and angular width.

2.4 Autoregressive Model Algorithm

AR model algorithm provides better resolution in range and cross range at narrow bandwidth and angular width with respect to Fourier Transform. AR model algorithm is based on backward and forward linear prediction [5,17,18]. In AR model, data are represented as a linear combination of previous or future samples.

Forward prediction which is based on the linear combination of past samples is modeled as:

$$x(n) = \sum_{k=1}^K a_k x(n-k) + e(n) \quad (2.22)$$

where $n = 1, 2, \dots, K-1$

Backward prediction which is based on the linear combination of next samples is modeled as:

$$x(n) = \sum_{k=1}^K b_k x(n+k) + e(n) \quad (2.23)$$

where $n = K, K+1, \dots, N-1$

AR model coefficients of backward and forward prediction are complex conjugate, $b_i = a_i^*$ [19].

▪ **Range Profile Generation**

1-D radar data are used to generate the radar signature of target. Forward and backward prediction of 1-D radar data are modeled as:

$$\tilde{E}(l) = - \sum_{i=1}^P a_i E(l-i) \quad (2.24)$$

where $l = P+1, P+2; \dots N$

$$\tilde{E}(l) = - \sum_{i=1}^P b_i E(l+i) \quad (2.25)$$

where $l = 1, 2, \dots N-P$

Since forward and backward prediction coefficients are complex conjugate, (2-25) can be rewritten as:

$$\tilde{E}(l) = - \sum_{i=1}^P a_i^* E(l+i) \quad (2.26)$$

where $l = 1, 2, \dots N-P$

The unknown parameters, a_i , are calculated from the combination of (2.24) and (2.26). Matrix notation of this equation system is given by:

$$\mathbf{E}\mathbf{a} = -\mathbf{e} \quad (2.27)$$

$\mathbf{E}$ is the $2(N-p) \times p$ matrix, $\mathbf{a}$ is vector with p length, and $\mathbf{e}$ is the vector with $2(N-p)$ length.

Range profile of the target is generated by (2.28) and peaks of this profile give the scatterers in the range direction.

$$P(r) = \frac{1}{|1 + \sum_{i=1}^P a_i z^i|^2} \quad (2.28)$$

where $z = e^{-j\frac{4\pi}{c}\Delta f r}$

▪ 2-D Radar Image Generation

2-D radar image can be obtained by finding the location of scatterers from 2-D AR model. 2-D backward prediction in both directions is given in (2.29) and forward prediction in both directions is given in (2.30).

$$\hat{E}(l, n) = - \sum_{i=0}^p \sum_{j=0}^p a_{ij} E(l - i, n - j) \quad (2.29)$$

$$i = j \neq 0$$

$$l, n = L, L + 1, \dots, N - 1$$

$$\hat{E}(l, n) = - \sum_{i=0}^p a_{ij}^* E(l + i, n + j) \quad (2.30)$$

$$i = j \neq 0$$

$$l, n = 0, 1, \dots, N - L - 1$$

$(p + 1)^2 - 1$ unknown parameters and $2(N - p)^2$ linear equations are obtained by combining (2-29) and (2-30). Matrix notation of this equation system is:

$$\mathbf{Ea} = -\mathbf{e} \quad (2.31)$$

It is possible to apply backward prediction in cross range direction and forward prediction in range direction and vice versa. If forward prediction is applied on f_x direction and backward prediction is applied on f_y direction, predicted variables are written as:

$$\hat{E}(l, n) = - \sum_{i=0}^p \sum_{j=0}^p b_{ij} E(l - i, n + j) \quad (2.32)$$

$$i = j \neq 0$$

$$l = p, p + 1, \dots, N - 1$$

$$n = 0, 1, \dots, N - p - 1$$

Similarly, backward prediction may be applied on f_x direction and forward prediction may be applied on f_y direction . Then predicted values can be written as:

$$\hat{E}^*(l, n) - \sum_{i=0}^p \sum_{j=0}^p b_{ij} E(l + i, n - j) \quad (2.33)$$

$$i = j \neq 0$$

$$l = 0, 1, \dots, N - p - 1$$

$$n = p, p + 1, \dots, N - 1$$

As a result of (2-31) and (2-32), $(p + 1)^2 - 1$ unknown parameters and $2(N - P)^2$ linear equations are obtained.

$$\tilde{\mathbf{E}}\mathbf{b} = -\tilde{\mathbf{e}} \quad (2.34)$$

2-D radar image is generated by:

$$P(x, y) = \frac{1}{\left|1 + \sum_{i=0}^L \sum_{j=0}^L a_{ij} z_1^{-i} z_2^{-j}\right|^2 + \left|1 + \sum_{i=0}^L \sum_{j=0}^L b_{ij} z_1^{-i} z_2^{-j}\right|^2} \quad (2.35)$$

$$i = j \neq 0$$

where

$$z_1 = e^{j4\pi/c\Delta f_x x} \quad z_2 = e^{j4\pi/c\Delta f_y y} \quad (2.36)$$

The peaks of $P(x, y)$ correspond to the locations of the scatterers.

AR model coefficients can be calculated by l-2 norm minimization and SVD truncation [5].

2.4.1 L2 norm minimization

Unknown AR coefficients at (2.27), (2.31) and (2.34) are determined by l-2 norm minimization of residue which is the difference between predicted and observed data. AR coefficients are calculated from;

$$\mathbf{a} = -(\mathbf{E}^H \mathbf{E})^{-1} \mathbf{E}^H \mathbf{e} \quad (2.37)$$

Whenever unknown parameters are calculated, range profile and 2-D radar images can be computed by (2-28) and (2-35).

AR model order is an important parameter to find the location of scatterers in the range profile and radar image. If AR model order is too low, some scatterers are missing. If AR model order is increased, resolution of image is increased but spurious scatterers may arise. Separation of image from background will be difficult. In [20], effects of AR model order and simulation results are presented in detail.

2.4.2 Singular value decomposition truncation

Range profile and radar images which are generated by finding the unknown coefficients from least square error solution may have more peaks than real number of the scatterers. SVD truncation is employed to suppress these spurious peaks and estimates the actual scatterers [5].

Least square solution on (2.37) is rewritten as:

$$\mathbf{a} = -\mathbf{R}^{-1}\mathbf{r} \quad (2.38)$$

where $\mathbf{R} = \mathbf{E}^H \mathbf{E}$ and $\mathbf{r} = \mathbf{E}^H \mathbf{e}$.

$\mathbf{R}$ is the covariance matrix of backscattered signal. Eigenvalues and eigenvectors are found by computing SVD. While D largest eigenvalues represent the scatterers, the remaining $(p-D)$ eigenvalues represent noise in the backscattered field [5]. After eliminating eigenvalues which belong to noise, unknown coefficient vector is determined as:

$$\mathbf{a} = \sum_{i=1}^D \frac{(\mathbf{U}_i^H \mathbf{r})}{\lambda_i} \mathbf{U}_i \quad (2.39)$$

where D is the estimated number of signals, λ_i is eigenvalue belongs the scatterers, $\mathbf{U}_i$ is the eigenvector of corresponding eigenvalue.

After calculating unknown coefficients from (2.39), range profile and radar images are constructed.

The important point of SVD application is to predict the number of scatterers. If number predicted scatterers is larger than actual scatterers ($D \gg d$), spurious scatterers appear. If $D \ll d$, actual scattering centers are disappeared. In [20], the

estimation number of scatterers and simulation results of SVD truncation are presented in detail.

2.5 Multiple Signal Classification (MUSIC) Algorithm

MUSIC algorithm depends on the orthogonality of signal and noise eigenvectors of data correlation matrix. Firstly, correlation matrix is computed in this algorithm then correlation matrix is decomposed into signal and noise subspaces. By using the noise subspace, range profile and radar image are reconstructed. As a result, this algorithm yields better image than FT based methods when data are limited.

▪ 1-D MUSIC Algorithm:

Vector notation of the 1-D backscattered data which is given in (2.8) can be represented as:

$$\mathbf{E} = \mathbf{X}\mathbf{a} + \mathbf{n} \quad (2.40)$$

where

$$\begin{aligned} \mathbf{E} &= [E_1, E_2, \dots, E_N]^T \\ \mathbf{X} &= [\mathbf{x}(r_1), \mathbf{x}(r_2), \dots, \mathbf{x}(r_d)] \\ \mathbf{x}(r_k) &= \left[e^{-j\frac{4\pi}{c}f_1 r_k}, e^{-j\frac{4\pi}{c}f_2 r_k}, \dots, e^{-j\frac{4\pi}{c}f_N r_k} \right]^T \\ \mathbf{a} &= [a_1, a_2, \dots, a_d]^T \\ \mathbf{n} &= [n_1, n_2, \dots, n_d]^T \end{aligned} \quad (2.41)$$

The first step of MUSIC algorithm for 1-D data is the estimate the correlation matrix of the observation signal. Correlation matrix is defined as:

$$\mathbf{R}_{yy} = E(\mathbf{E}\mathbf{E}^H) = \mathbf{X}\mathbf{a}\mathbf{X}^H + \sigma^2 \mathbf{I} \quad (2.42)$$

Where H denotes the complex conjugate transpose, σ is the variance of uncorrelated noise. Exact correlation matrix is the average over the number of snapshots. Since one snapshot is available in radar application, correlation matrix can be estimated by using spatial smoothing preprocessing (SSP).

After employing SSP, correlation matrix is estimated as:

$$\tilde{\mathbf{R}}_{yy} = \frac{1}{2M} \sum_{k=1}^M \mathbf{R}_k + \mathbf{J}\mathbf{R}_k^* \mathbf{J} \quad (2.43)$$

where

$$\begin{aligned}
\mathbf{R}_k &= \mathbf{E}_k \mathbf{E}_k^H \\
\mathbf{E}_k &= [E_k, E_{k+1}, \dots, E_{k+m-1}]^T \\
\mathbf{J} &= \begin{bmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & 0 \\ \vdots & & \vdots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix}^T
\end{aligned} \tag{2.44}$$

In (2.43), M is the number of subarrays, m is the subarray dimension, $\mathbf{J}$ is the $m \times m$ exchange matrix.

Range profile of the target is computed by (2-45).

$$p_m(r) = \frac{1}{\sum_{i=d+1}^m |\mathbf{u}_i^H \mathbf{x}(r)|^2} \tag{2.45}$$

where $\mathbf{u}_i$ is the eigenvectors corresponding to minimum valued eigenvalues of the correlation matrix.

▪ 2-D MUSIC Algorithm:

Vector notation of the 2-D backscattered data which is given in (2.21) can be expressed as:

$$\mathbf{x} = \mathbf{A}\mathbf{s} + \mathbf{u} \tag{2.46}$$

where

$$\begin{aligned}
\mathbf{x} &= [x_{00} x_{10} \dots x_{M-10} x_{01} \dots x_{M-1N-1}]^T \\
\mathbf{s} &= [s_1 s_2 \dots s_d]^T \\
\mathbf{u} &= [u_{00} u_{10} \dots u_{M-10} u_{01} \dots u_{M-1N-1}]^T
\end{aligned} \tag{2.47}$$

$$\mathbf{A} = [a(x_1, y_1) a(x_2, y_2) \dots a(x_d, y_d)]$$

$$a(x_k, y_k) = \left[e^{j \frac{4\pi}{c} (f_0^x x_k - f_0^y y_k)} e^{j \frac{4\pi}{c} (f_1^x x_k - f_1^y y_k)} \dots e^{j \frac{4\pi}{c} (f_{M-1}^x x_k - f_0^y y_k)} e^{j \frac{4\pi}{c} (f_0^x x_k - f_1^y y_k)} \dots e^{j \frac{4\pi}{c} (f_{M-1}^x x_k - f_{N-1}^y y_k)} \right]^T$$

Autocorrelation matrix given below is estimated.

$$R_{xx} = E[xx^H]$$

Since one snapshot is available in radar application, correlation matrix can be estimated by spatial smoothing preprocessing.

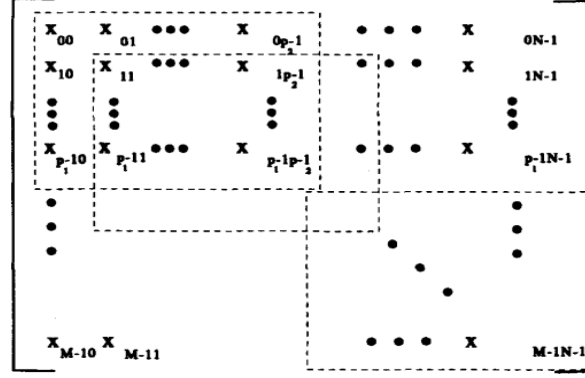


Figure 2.5 : 2-D Spatial Smoothing Preprocessing [20].

As shown from Figure 2.5, correlation matrix is divided submatrix whose dimension is $p_1 \times p_2$. Then correlation matrix can be estimated from (2.43).

2-D radar image is generated by:

$$P_{\text{MUSIC}}(x, y) = \frac{a(x, y)^H a(x, y)}{a(x, y)^H E E^H a(x, y)} \quad (2.48)$$

Although SSP reduces the resolution of image by decreasing effective bandwidth, performance of MUSIC algorithm combined with SSP is higher than IFFT.

Subarray dimension on 1-D MUSIC algorithm and submatrix dimension on 2-D MUSIC algorithm affect the image resolution. High sizes improve the resolution of image, but decorrelation performance between the signals from different scatterers is reduced.

The other important parameter on MUSIC algorithm is the number of predicted scatterers. If more scatterers than actual ones are estimated, spurious peaks might appear on the reconstructed image. Effect of smoothing parameter and number of scatterers are explained in [20].

For range profile, F117 and F14 simulated data are used. Properties of data are given below:

- F117 data: 15 GHz central frequency, 4GHz bandwidth and view angle is 30° . Additive white Gaussian noise (AWGN) is added to achieve 10 dB signal to noise ratio (SNR). Real image and radar image are given in Fig. 2.6.
- F14 noisy data: 15 GHz central frequency, 4GHz bandwidth and view angle is 30° . Additive white Gaussian noise (AWGN) is added to achieve 10 dB signal to noise ratio (SNR). Real image and radar image are given in Fig. 2.6.

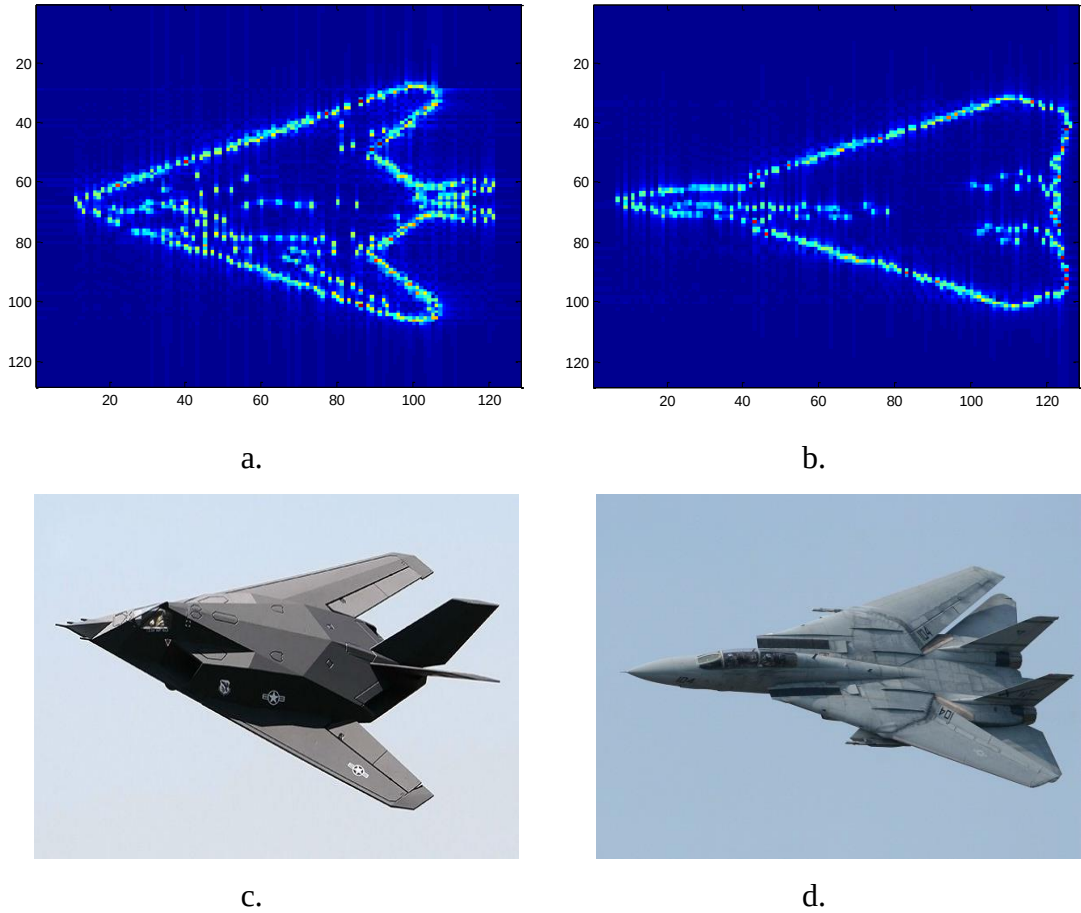


Figure 2.6: Radar image of a.) F117 simulated data b.)F14 simulated data , Real image of c.) F117 plane d.) F14 plane.

For radar image, Mig 25 simulated data which are generated by moments method and MSTAR real data whose properties are given below are used.

- Mig25 data: 9 GHz central frequency, 531 MHz bandwidth and view angle is 3.67° . Real image and radar image are given in Fig. 2.7.

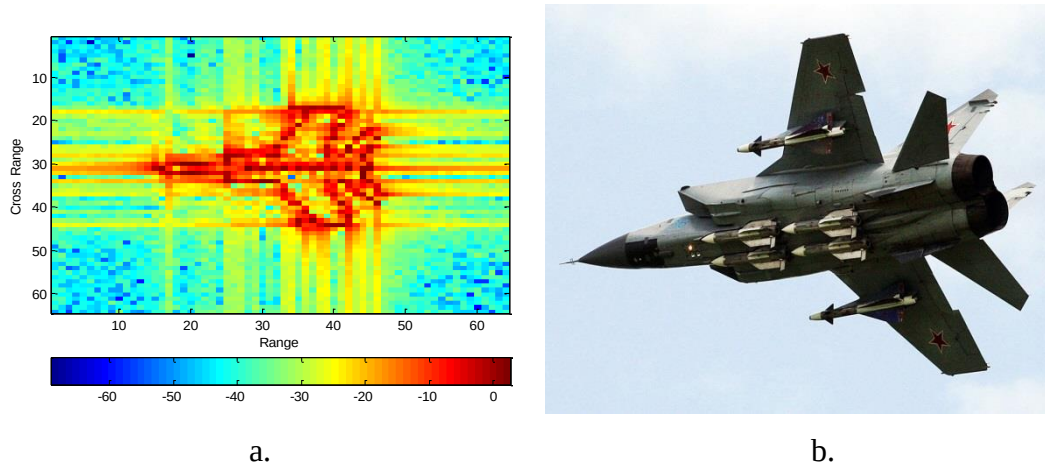


Figure 2.7: a.) Radar image of Mig-25 data b.) Real image of Mig-25 plane.

- T72, BMP2 and BTR70 backscattered data are collected by Sandia National Laboratory for MSTAR (Moving Stationary Target Acquisition and Recognition) program. The data are collected at 9.6 GHz center frequency and 591 MHz bandwidth. Real image and radar image of the MSTAR data are given in Fig. 2.8 and Fig. 2.9, respectively.



Figure 2.8: Real images of MSTAR data [21].

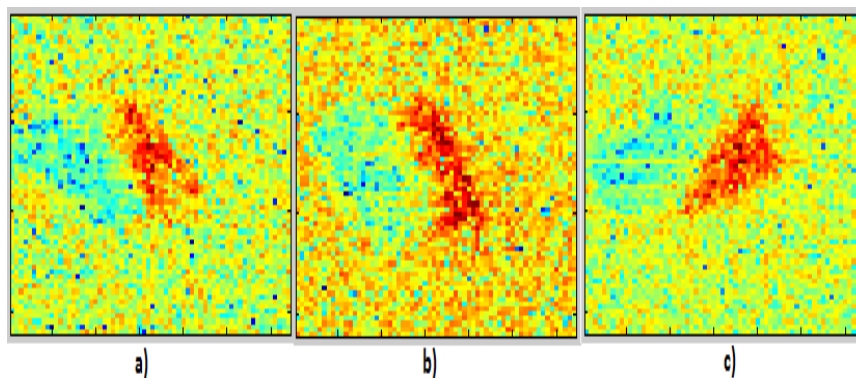


Figure 2.9: Radar images of MSTAR data.

For the generation of range profile, several AR model order and smoothing parameters for MUSIC algorithm are experimented but best results are presented in Fig. 2.10 and Fig. 2.11.

In Fig. 2.10, range profile of F117 plane based on FFT based method, AR model with least square solution, SVD truncation and MUSIC algorithm are presented, respectively. Fig. 2.10a shows the result of FT based method and it is seen that many spurious peaks exist in the range profile.

Range profile of F14 plane based on FFT based method, AR model with least square solution, SVD truncation and MUSIC algorithm are presented, respectively, in Fig. 2.11.

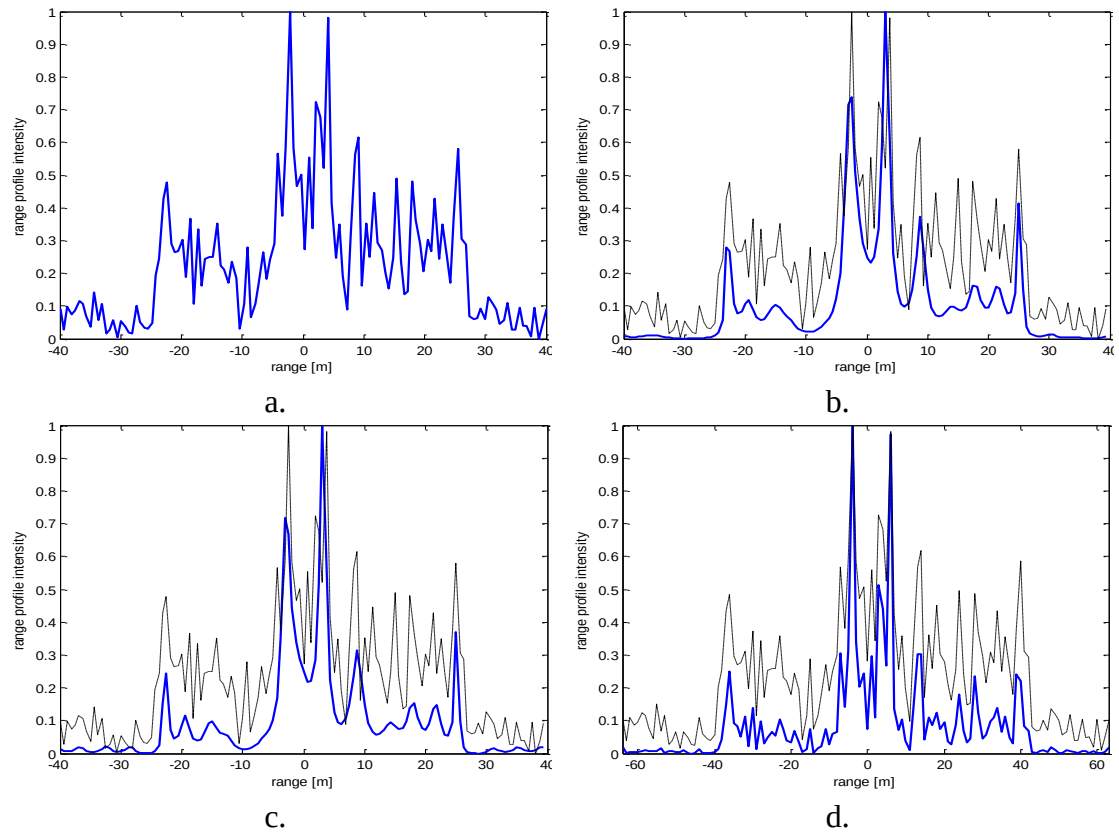


Figure 2.10:Range profile of F117 plane a.) FFT based method b.) least square solution when order 24, c.) SVD truncation d.) MUSIC.

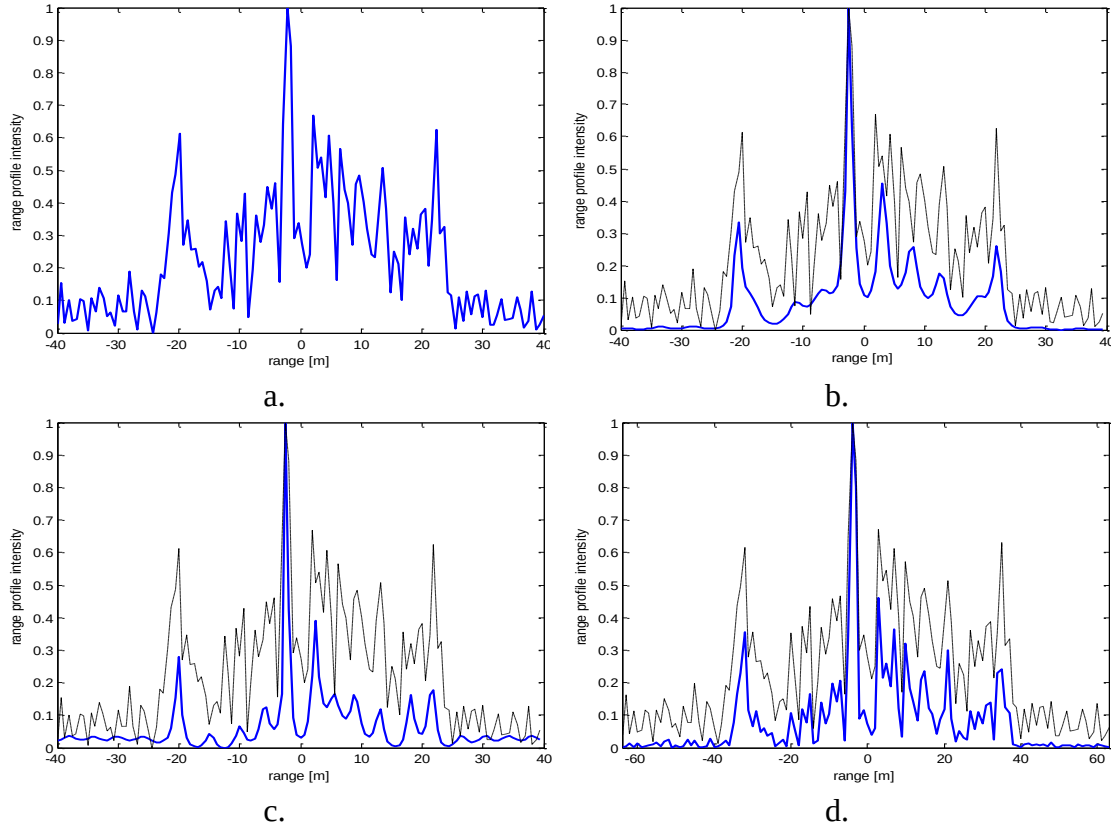


Figure 2.11:Range profile of F14 plane a.) FFT based method b.) least square solution when order 24, c.) SVD truncation d.) MUSIC.

In [20], the author analyzed the AR model order, SVD truncation and MUSIC algorithms for radar image. According to this work, optimum smoothing parameters and AR model orders are chosen for Mig-25 plane.

In Fig. 2.12, radar images of Mig-25 based on FFT based method, AR model with least square solution, SVD truncation and MUSIC algorithm are presented, respectively. Fig. 2.12a shows the result of FT based method. In this figure, there are many sidelobes and spurious scatterers. In Fig. 2.12b, radar image is generated by using AR model. In here, AR model order is given 24 and l-2 norm minimization is used to find the AR model coefficients. In Fig. 2.12c, SVD truncation result is given and some scatterers are lost in this subfigure. MUSIC algorithm result is given in Fig. 2.12d. MUSIC algorithm is the very successful for finding the scatterers and suppressing the side lobes.

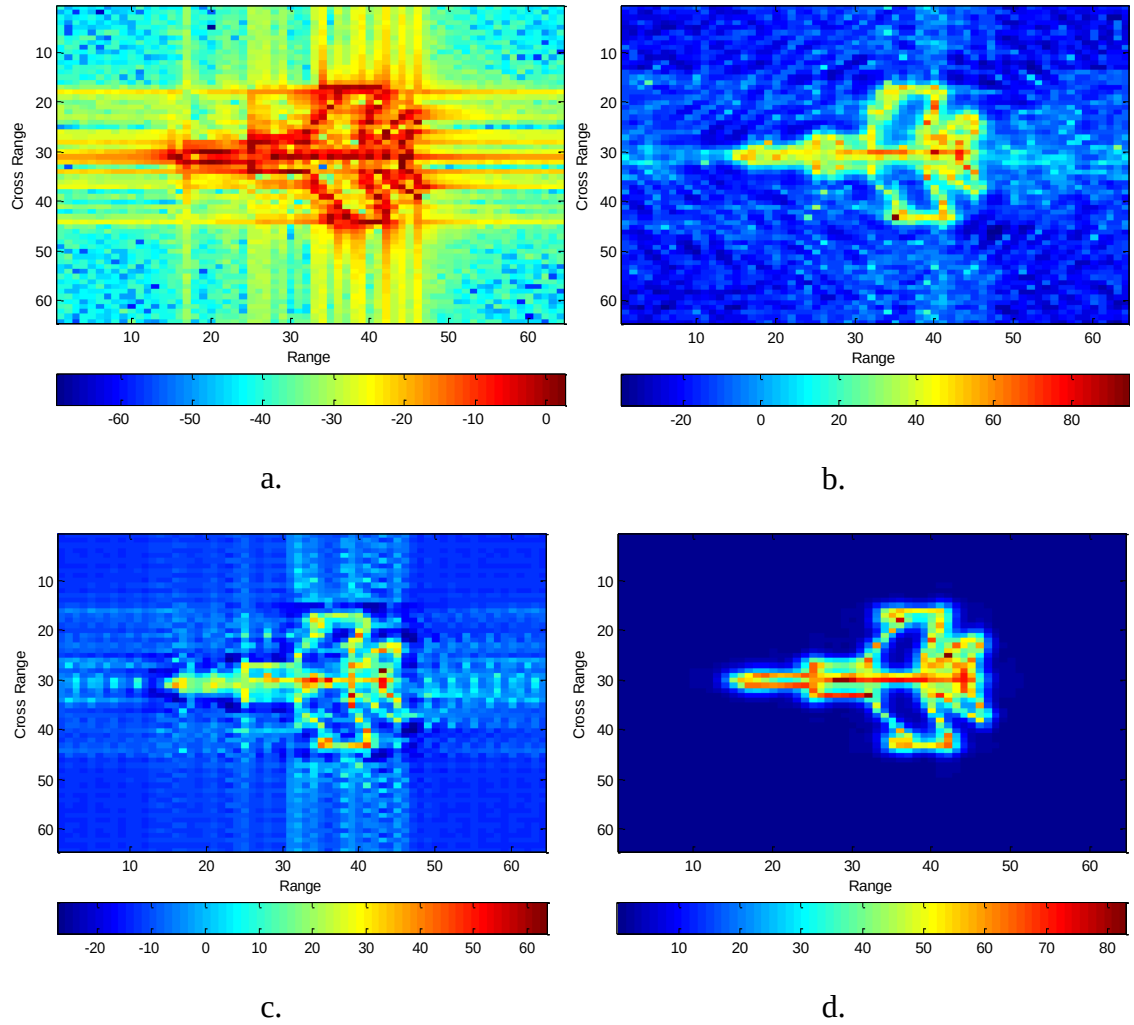


Figure 2.12: Radar image of Mig25 wideband data (SNR=30 dB) a.) FFT based method, b.) least square solution when order 24, c.) SVD truncation d.) MUSIC smoothing parameter 24.

In Fig. 2.13, radar images of Mig-25 based on FFT based method, AR model with least square solution, SVD truncation and MUSIC algorithm are presented, respectively, when SNR is 10 dB. AR model order and smoothing parameter are chosen 20. Spurious scatterers around the target are not completely eliminated but again MUSIC algorithm yields the best image.

In Fig. 2.14, radar images of Mig-25 based FFT based method, AR model with least square solution, SVD truncation and MUSIC algorithm are presented for the limited data case, respectively. AR model order and smoothing parameter is chosen 12. In this case, the performance of MUSIC algorithm decreases. In Fig. 2.14 c, scatterers at tail of the plane cannot be resolved. Fig. 2.14a shows the radar image which is generated by FT based method. As the expected performance of this method decreases for the narrowband data case.

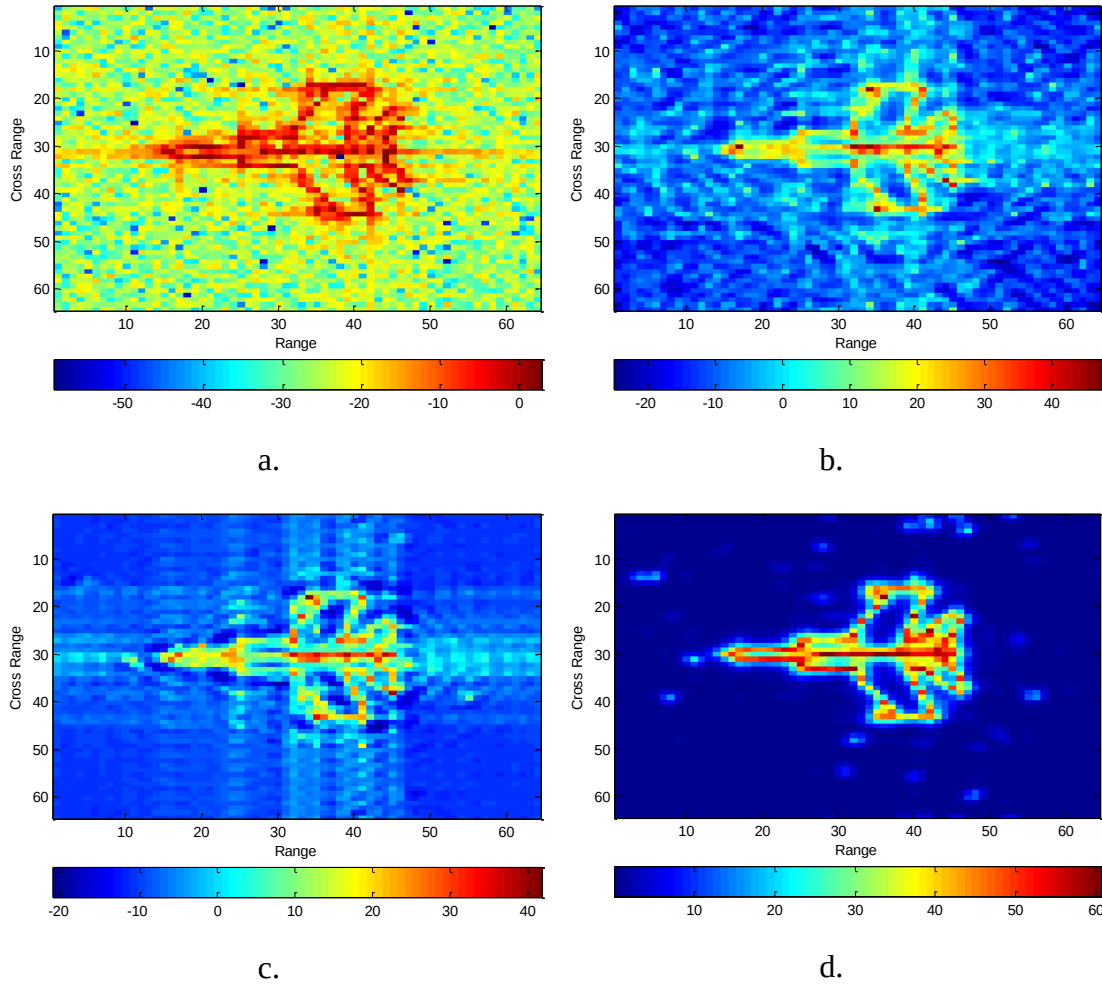


Figure 2.13: Radar image of Mig25 wideband data (SNR=10 dB) a.) FFT based method, b.) least square solution when order 20, c.) SVD truncation d.) MUSIC smoothing parameter 20.

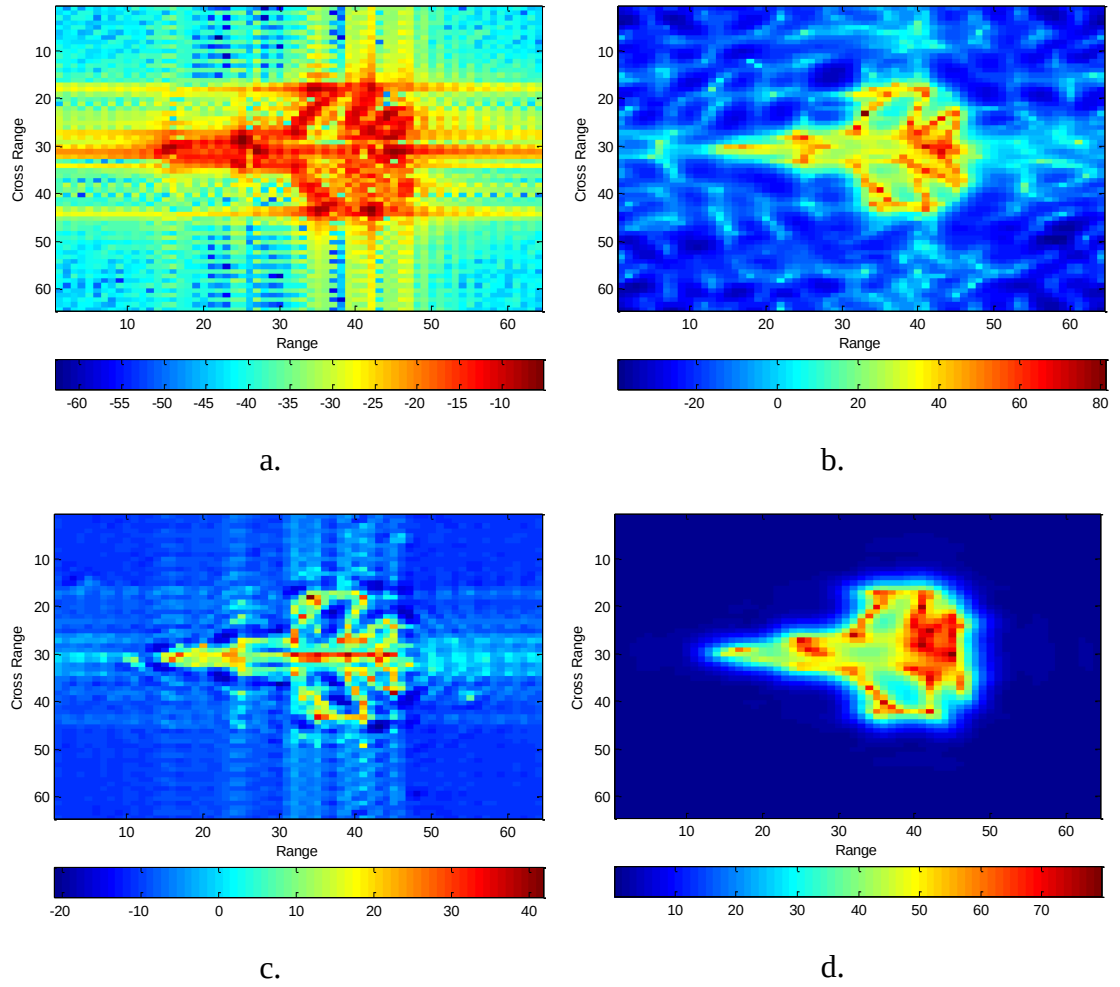


Figure 2.14: Radar image of Mig25 narrowband data (SNR=10 dB) a.) FFT based method, b.) least square solution when order 12, c.) SVD truncation d.) MUSIC smoothing parameter 12.

3. SPARSE AUTOREGRESSIVE MODELING FOR RADAR IMAGING

3.1 Review of Sparsity in Radar Imaging

3.1.1 Sparse representations

The idea behind the construction of sparse signal representation is to represent the observed signal by using the few elements from dictionary which consists of elementary signals. Sparsity problem is related with finding the smallest subset of dictionary which gives the best signal representation [12, 22]. Signal is represented as:

$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{n} \quad (3.1)$$

where $\mathbf{A}=m \times n$ dictionary matrix, $\mathbf{b}=m \times 1$ observed signal vector and $\mathbf{x}$ is the $n \times 1$ coefficient or signal representation vector.

Column of dictionary called atom contains small amount of information about signal. Dictionary is complete if m is equal to n . In this condition, system has unique solution and solution is found with zero error. If rank of $\mathbf{A}$ $m < n$, $\mathbf{A}$ is the under-determined dictionary. The solution of this system is based on the minimum norm problem. l_0 norm solution yields the sparse solution which has minimum number of non-zero coefficients. If dictionary is over complete, $m > n$, equation system is solved with least squares method.

Sparse approximation problem subjects to minimize cost function which refers the error. Since signal is represented with fewer atoms from dictionary, most elements in the coefficient vector will be zero. Minimization problem which gives the sparse solution can be described as:

$$\min(\|\mathbf{x}\|_0) \text{ subject to } \mathbf{e} = \mathbf{b} - \mathbf{A}\mathbf{x} \quad (3.2)$$

where $\|\cdot\|_0$ is the l_0 norm. Sparse solution which has minimum number of non-zero element is found l_0 norm which is the NP hard problem. That's why l_1 norm which refers linear optimization problem, is used to find sparse solution.

In this thesis, 3 minimization problems are applied to find sparse signal representation.

- **Basis Pursuit Denoising**

The most basic approach to get sparse solution is basis pursuit (BP) minimization problem is given in (3-3).

$$\min(\|x\|_1) \text{ subject to } b = Ax \quad (3.3)$$

$$\text{where } \|x\|_1 = \sum_{i=1}^n x_i.$$

Basis pursuit algorithm aims to minimize $l-1$ norm of coefficients vector to find best representation of the signal. If this optimization problem is solved, most of elements in coefficient vector will be zero. The necessary atoms of dictionary to represent signal are given by the indices of nonzero elements of the coefficient vector.

Basis Pursuit Denoising gives sparse solution by minimizing $l-1$ norm of coefficients while $l-2$ norm of residue (difference between the observed signal and predicted signal) is bounded with threshold [23,24]. Basis pursuit denoising is the general form of basis pursuit since error tolerance is 0 in basis pursuit.

BPDN optimization problem can be given as:

$$\min(\|x\|_1) \text{ subject to } \|b - Ax\|_2 \leq \epsilon \quad (3.4)$$

where ϵ is the threshold value which is related to noise bound.

- **Basis Pursuit Denoising with Penalty Term**

The BPDN with penalty term minimization problem is defined as

$$\min(\|b - Ax\|_2 + \lambda \|x\|_1) \quad (3.5)$$

where λ is the penalty parameter. This parameter controls the trade-off between $l-1$ norm of the coefficients and $l-2$ norm of residue. If λ is set to 0, the problem refers to minimum least square solution [25, 26].

The optimal penalty parameter value can be determined by L-curve method [27]. L-curve is generated by plotting $\log(\|b - Ax\|_2)$ versus $\log(\|x\|_1)$ for several values

of the penalty term. Corner of this curve gives the best penalty value. This penalty term affects the sparsity level.

- **Least Absolute Shrinkage and Selection Parameter**

The last method used to obtain sparse representation is the Least Absolute Shrinkage and Selection Parameter (LASSO). The minimization problem of LASSO is described as:

$$\min(\|b - Ax\|_2) \text{ subject to } (\|x\|_1) < \tau \quad (3.6)$$

In contrast to BPDN, this time $l - 1$ norm of the coefficients are bounded and $l - 2$ norm of residue is minimized [28, 29]. Threshold value which bounds the $l - 1$ norm of the coefficients can be estimated by cross-validation, generalized cross validation and analytical unbiased estimate of risk [8].

In the thesis, all of the optimization problems which are described above are solved with CVX optimization program [30].

3.1.2 Sparse range profile generation

Observed signal can be expressed as a linear combination of the atoms of the dictionary [31]. The signal representation is given by:

$$s = \sum_{i=1}^M \varphi_i \phi_i \quad (3.7)$$

where ϕ_i is the atom and $D = \{\phi_i, i=1,2..M\}$ is the dictionary. Matrix representation of (3.7) is given in (3-8).

$$s = D\varphi \quad (3.8)$$

In [12], backscattered signal is represented by linear combination of discrete Fourier dictionary. In this paper, high resolution radar image is reconstructed from sparse representation of backscattered data.

By ignoring the constant phase $e^{-\frac{j4\pi}{c}f_0 x_k}$, 1-D radar data are given in (2.8) can be rewritten as:

$$E(l) = \sum_{k=1}^d A_k(l) e^{-j\frac{4\pi}{c} l \Delta f x_k} \quad l = 1, 2 \dots N \quad (3.9)$$

According to this equation, dictionary is given by

$$D = [\phi_1 \ \phi_2 \ \phi_3 \ \dots \ \phi_M] \text{ where } \phi_m(l) = e^{-j\frac{4\pi}{c} l \Delta f x_k} \quad (3.10)$$

$$l = 1, 2 \dots N \quad m = 1, 2 \dots M$$

Matrix form of linear system can be given as:

$$\mathbf{E} = \mathbf{D}\boldsymbol{\varphi} \quad (3.11)$$

where $\mathbf{E}$ is $(N \times 1)$ observed vector, $\mathbf{D}$ is $(N \times M)$ Fourier dictionary matrix and $\boldsymbol{\varphi}$ is the $(M \times 1)$ unknown coefficient vector.

Sparse range profiles can be generated either by using BPDN with penalty term, BPDN or LASSO minimization problem.

3.1.3 Sparse radar image generation

For 2-D radar data given (2.21) is used. $\mathbf{E}$ is the $N \times M$ signal matrix. If constant terms are ignored, (2.21) can be written as:

$$E(l, n) = \sum_{i=1}^d \varphi e^{-j2\pi(l\Delta f x_i + n\Delta f y_i)} \quad (3.12)$$

Then, the 2 dictionaries, D_y and D_x for cross range direction and range direction respectively, are determined from,

$$\mathbf{D}_y = \sum_{n=1}^N e^{-j2\pi n \Delta f y_i} \quad i = 1, 2 \dots Q \quad (3.13)$$

$$\mathbf{D}_x = \sum_{l=1}^M e^{-j2\pi l \Delta f x_i} \quad i = 1, 2, \dots P$$

Representation of backscattered data with dictionary is given in (3-14).

$$\mathbf{E} = \mathbf{D}_x \boldsymbol{\varphi} \mathbf{D}_y' \quad (3.14)$$

$\mathbf{E}$ is the $(M \times N)$ signal matrix, $\mathbf{D}_x$ is the $(M \times P)$ Fourier dictionary, $\mathbf{D}_y$ is the $(N \times Q)$ Fourier dictionary, and $\boldsymbol{\varphi}$ is the $(P \times Q)$ coefficient matrix. When $P = \xi M$ and $Q = \xi' N$, image resolution is improved ξ and ξ' times in range and cross range direction, respectively. In Fig. 3.1, if scatterer does not exist in the intersection of grid, amplitude of intersection is zero. If not, amplitude is nonzero.

In [12], sparse representation of observed signal is used to reconstruct high resolution range profile and radar image. In this paper, author uses BP optimization problem to find the sparse representation of observed signal.

$$\min |\boldsymbol{\varphi}| \quad \text{subject to } \mathbf{E} = \mathbf{D} \boldsymbol{\varphi} \quad (3.15)$$

$$\min |\boldsymbol{\varphi}| \quad \text{subject to } \mathbf{E} = \mathbf{D}_x \boldsymbol{\varphi} \mathbf{D}_y' \quad (3.16)$$

Sparse signal representation based methods increase the resolution of radar image without predicting the number of scatterers. Radar image based on sparse representation yields better image than spectral estimation method.

In this thesis, sparse representation of range profile and radar images are reconstructed by using basis pursuit denoising, basis pursuit denoising with penalty term and least absolute shrinkage and selection parameter minimization problem. Minimization problems are solved with CVX optimization packet. In addition to that orthogonal matching pursuit approach which gives faster solution than minimization problems which is explain in section 3.1.1. is used to obtain sparse representation.

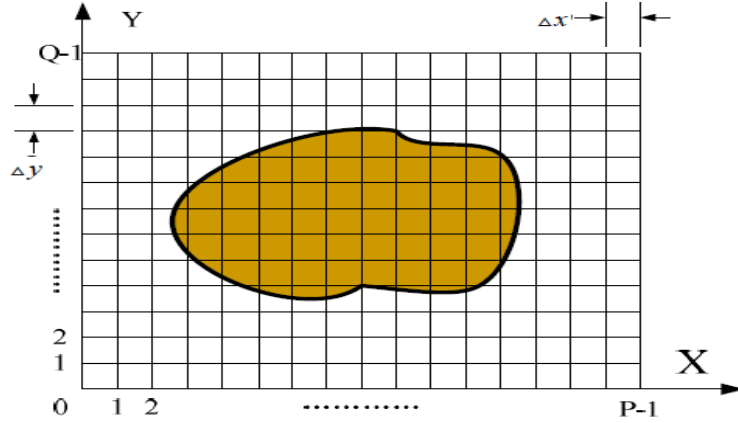


Figure 3.1: Map of target dissecting by two-dimensional grid [12].

- **Sparse radar Image based on BPDN**

Sparse radar image based on BPDN has very high resolution. As indicated in (2-42), l_1 norm coefficient vector or matrix is minimized subject to the residual sum of squares being less than the threshold. Threshold value is the important parameter for resolution. Threshold value is a compromise between sparse solution and restricting recovery error [32].

$$\text{Range Profile:} \quad \min(\|\boldsymbol{\varphi}\|_1) \text{ subject to } \|\mathbf{E} - \mathbf{D}_x \boldsymbol{\varphi}\|_2 \leq \epsilon \quad (3.17)$$

$$\text{Radar Image:} \quad \min(\|\boldsymbol{\varphi}\|_1) \text{ subject to } \|\mathbf{E} - \mathbf{D}_x \boldsymbol{\varphi} \mathbf{D}_y'\|_2 \leq \epsilon \quad (3.18)$$

Fig. 3.2 shows the range profiles of F117 plane based on FFT method and BPDN for different threshold values, respectively. In Fig. 3.2b, error is bounded with 10 and number of zero element is very high. To find the sparse representation and location of scatterers, threshold value is increased. In Fig. 3.2c and Fig. 3.2d, l_2 norm of error is constrained by 20 and 50, respectively but these thresholds are not enough to obtain sparse representation. When l_2 norm of error is limited with 100 in Fig. 3.2e, some elements will be 0. In Fig 3.72, threshold is increased to 200 and sparse representation is found while some dominant scatterers are eliminated.

Fig. 3.3 shows the range profiles of F14 plane based on FFT method and BPDN for different threshold values, respectively. As expected, when l_2 norm of error is limited with high threshold value, range profile is represented by fewer element.

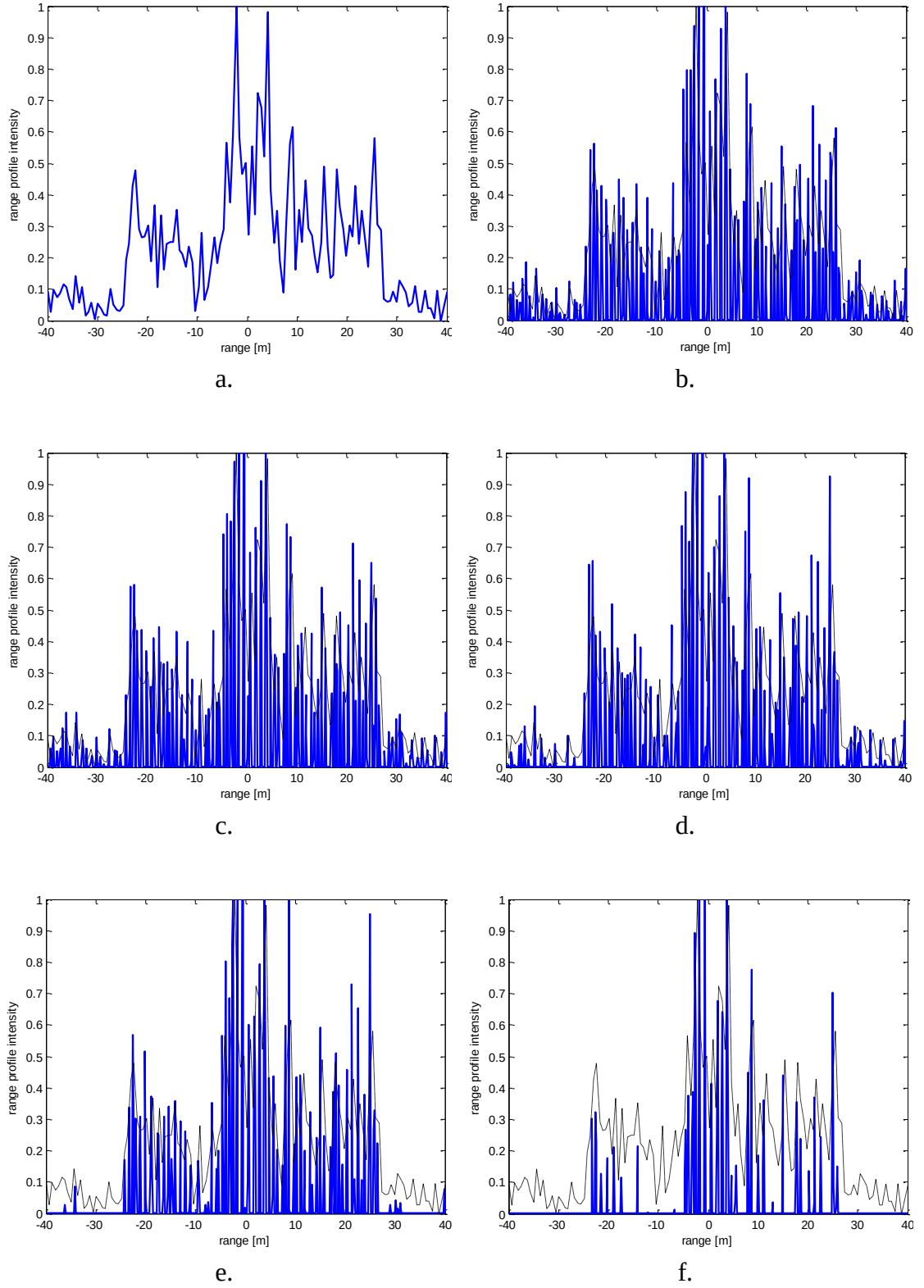


Figure 3.2: Radar image of F14 data (SNR=10dB) a.) FFT based method, Sparse signal representation based on BPDN when threshold b.)10, c.)20 d.) 50 e.)100 f.)200.

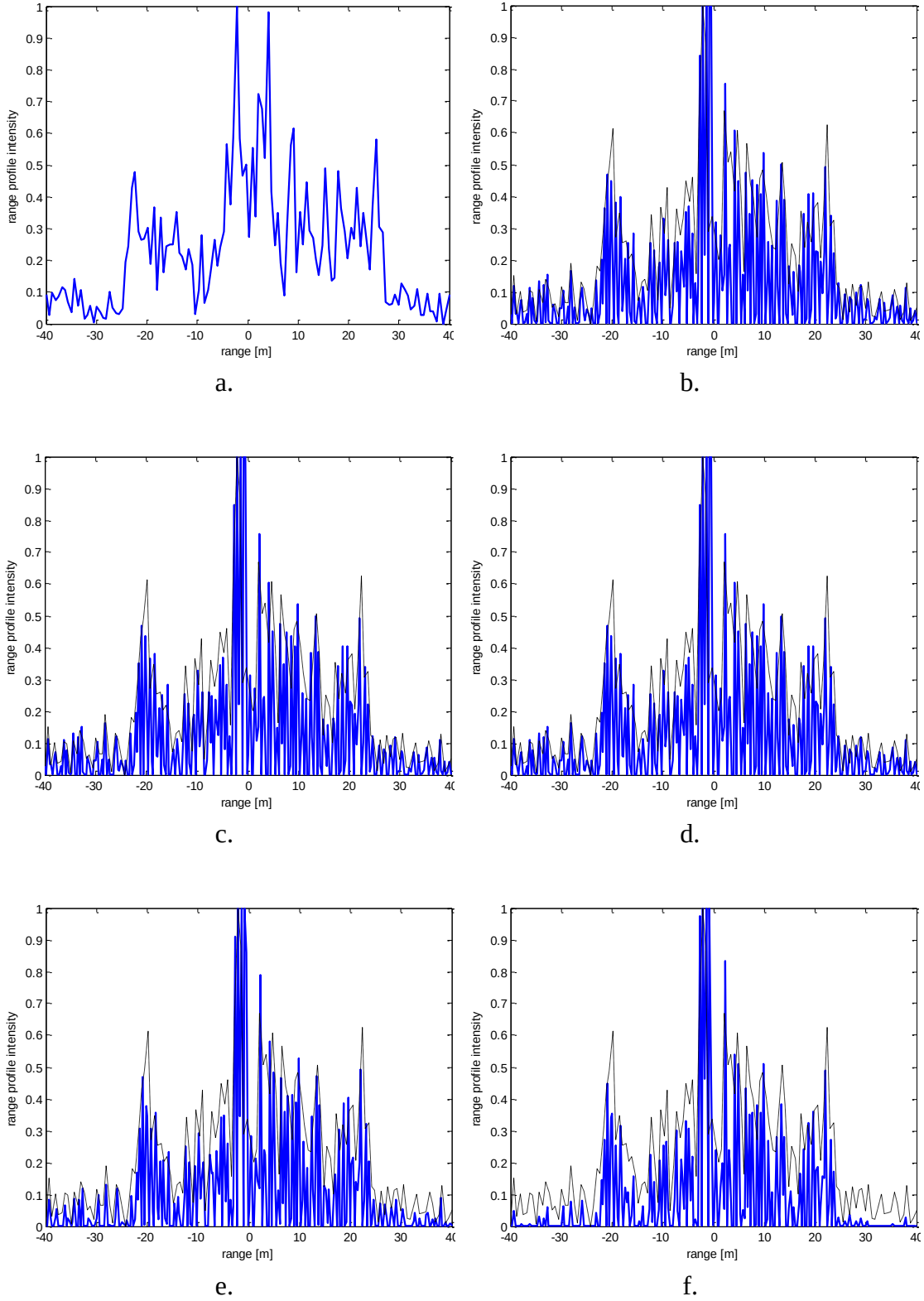


Figure 3.3: Radar image of F14 data (SNR=10dB) a.) a.) FFT based method, Sparse signal representation based on BPDN when threshold b.)0.1, c.)0.5 d.) 10 e.)50 f.)100.

Sparse representation of radar images based on FFT based method and BPDN are presented , respectively, in Fig. 3.4. Firstly, summation of the absolute value of backscattered data is calculated and threshold value is determined by deviation from this value. In Fig. 3.4 b,c and d , threshold values are determined as 0.001%, 0,05% and 0.02% deviation from summation of the absolute value of each sample, respectively. In these sub-figures, while location of scatterers are found properly, side lobes appear. In Fig. 3.4e, it is assumed that 0.1% deviation and in this sub-figure, while sidelobes are suppressed and scatterers are successfully found. In Fig 3.4f, some scatterers disappear because of the high deviation assumption.

In Fig. 3.5, sparse representation of radar images based on FFT based method and BPDN are presented, respectively when SNR is 10 dB. Scatterers of target are successfully found. If threshold value is chosen to high, it is not possible to find the all the scatterers. On the other hand, side lobes are observed with low threshold values. In Fig. 3.5e, while all scatterers display properly, sidelobes are suppressed. In Fig. 3.5f, scatterers disappear in spite of the small increment.

In Fig. 3.6, sparse representation of radar images based on FFT based method and BPDN are shown, respectively, in the case of limited data. Few scatterers and side lobes appear in the Fig. 3.6b. Side lobes effect can be mitigated when l_2 norm of residue is restricted by high threshold values. On the other hand, some of the existing scatterers are also eliminated. In Fig. 3.6e, some scatterers are eliminated because of the high valued threshold choice.

As a result of these figures, it is observed that this method yields high resolution image for wideband and low SNR value cases. However, this method couldn't provide proper image for the limited data case. In addition to that, computational cost of this method is very high.

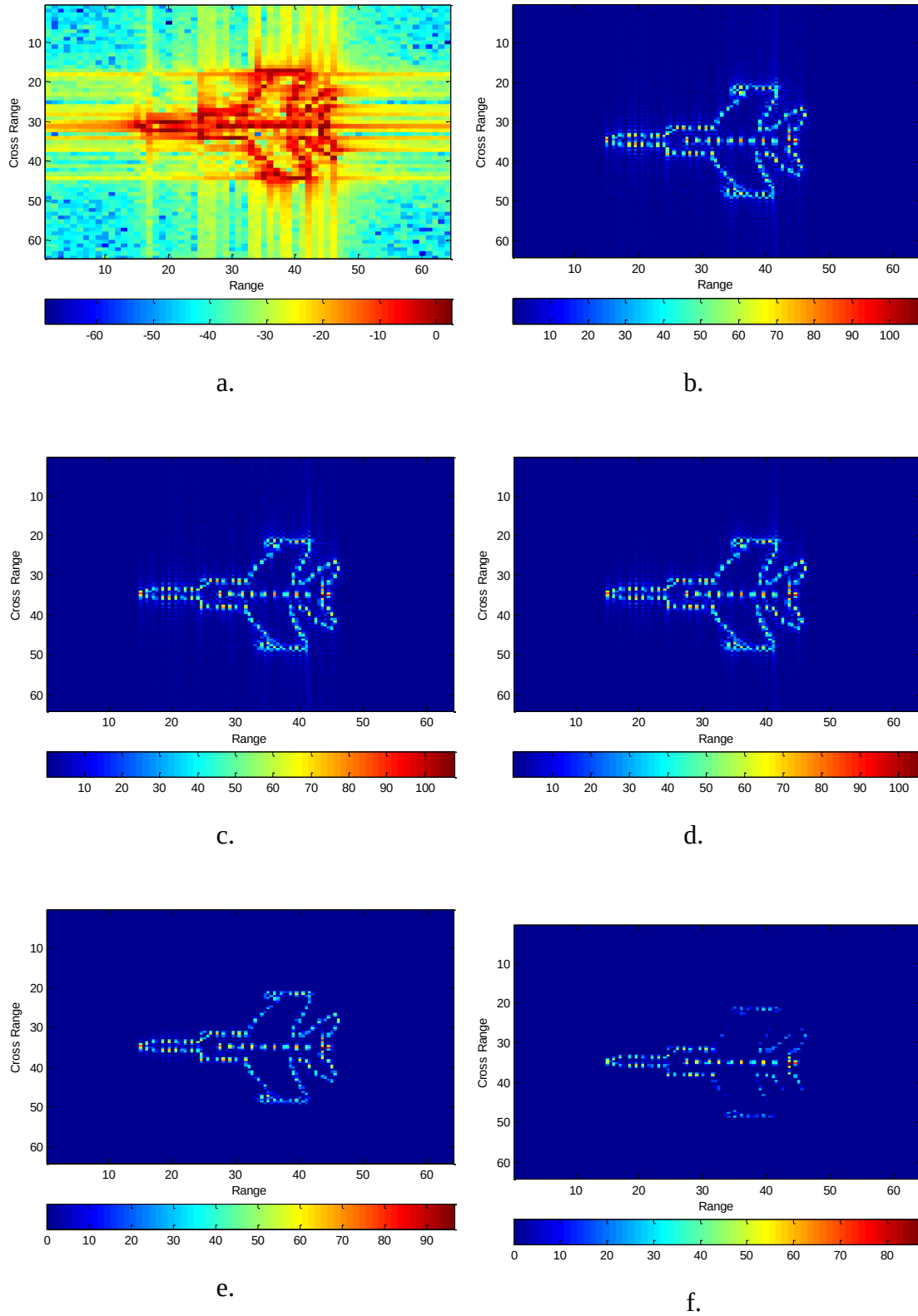


Figure 3.4: Radar image of wideband Mig25 data (SNR=30dB a.) FFT based method, Sparse radar image based on BPDN , b.)0.001%, c.)0.05%, d.)0.02% , e.)1%, f.)2% deviation from initial value.

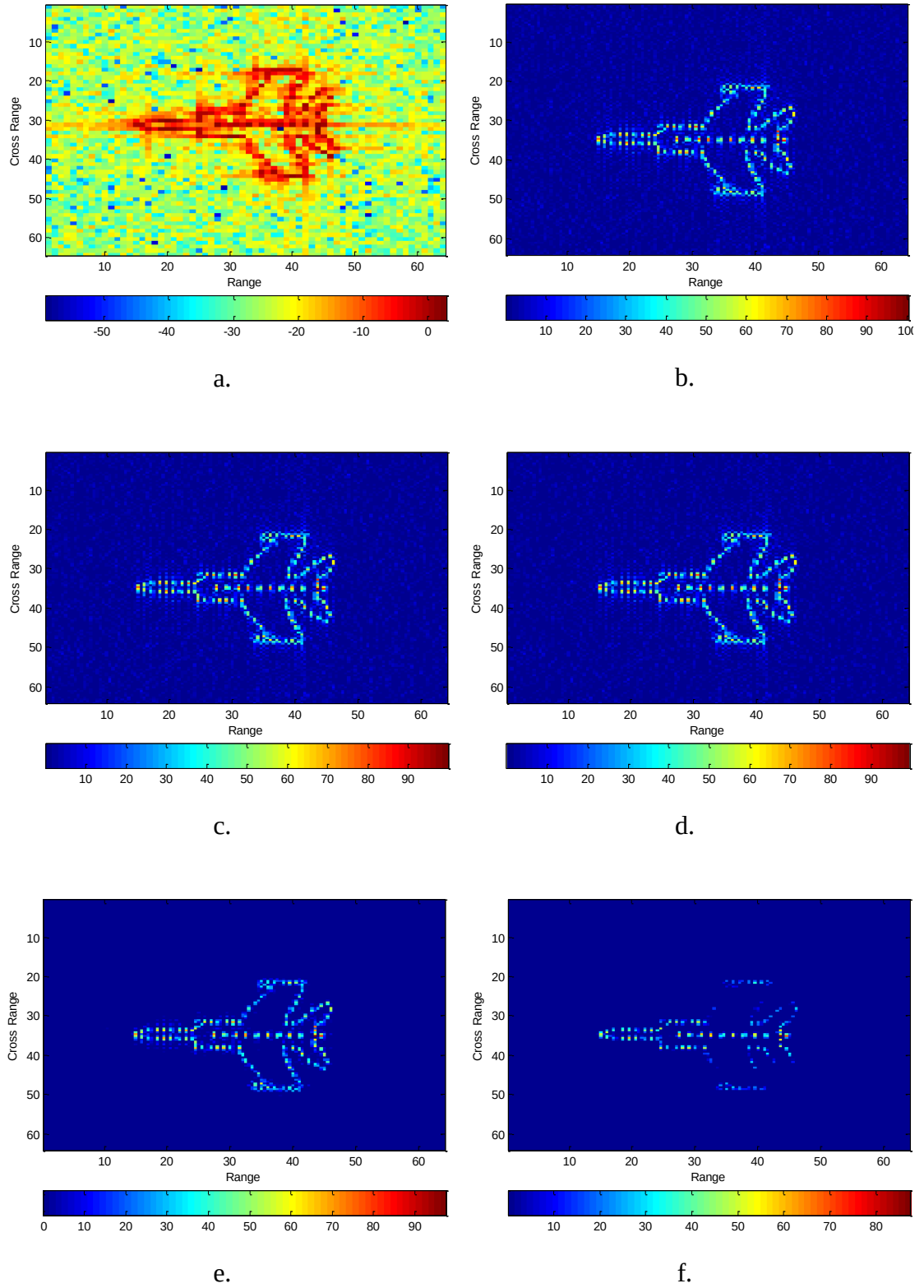


Figure 3.5: Radar image of wideband Mig25 data (SNR=10dB) a.) FFT based method, Sparse radar image based on BPDN , b.)0.001%, c.)0.05%, d.)0.02% , e.)1%, f.)2% deviation from initial value.

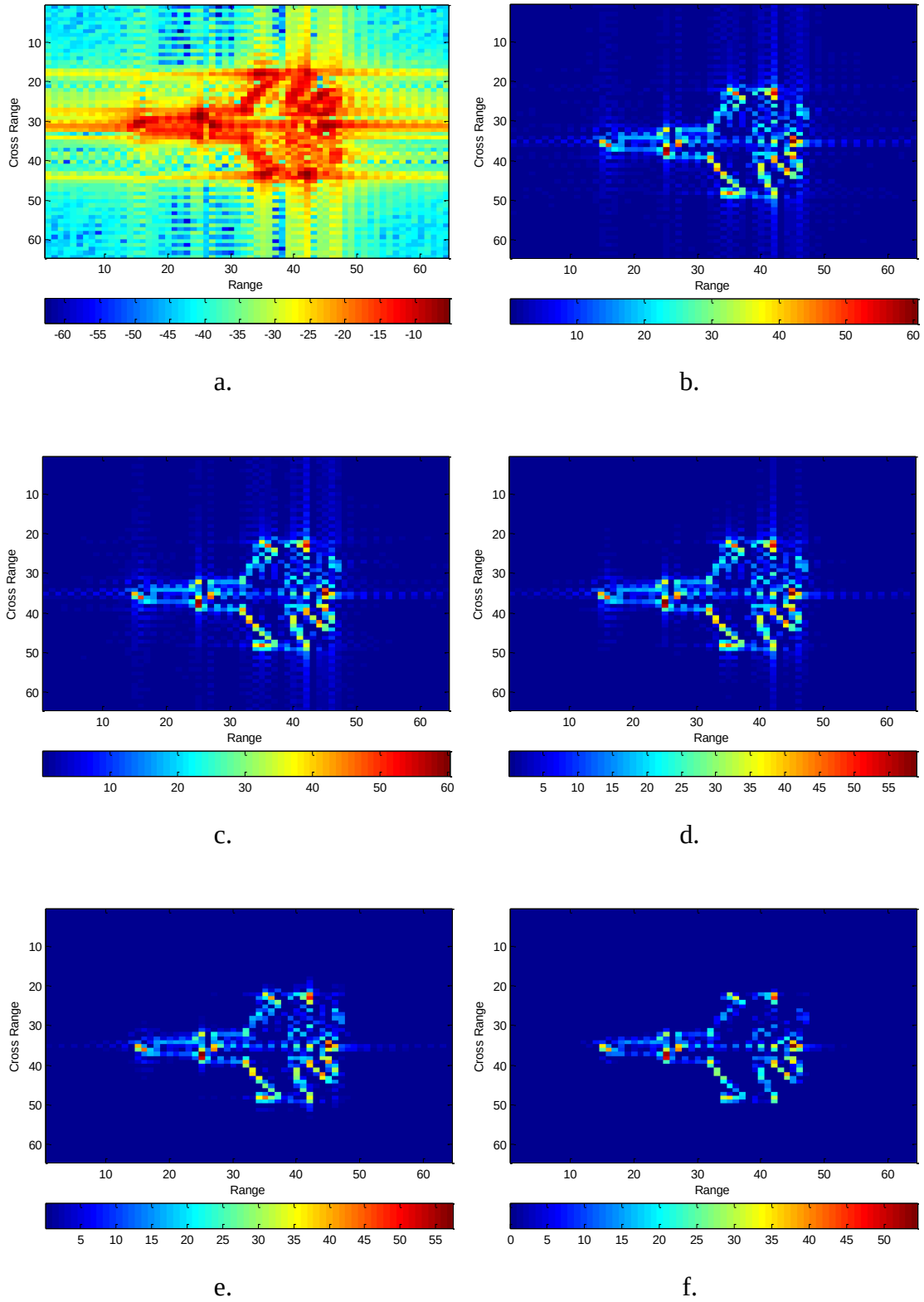


Figure 3.6: Radar image of narrowband Mig25 data (SNR=30dB) a.) FFT based method, Sparse radar image based on BPDN , b.)0.001%, c.)0.05%, d.)0.02% , e.)1%, f.)2% deviation from initial value.

▪ **Sparse radar image based on BPDN with penalty term**

As explained before, signal is represented with superposition of atoms over the Fourier dictionary. BPDN with penalty term optimization problem for 1-D and 2-D radar data are given by:

$$\text{Range Profile:} \quad \min(\|E - D_x \varphi\|_2 + \lambda \|\varphi\|_1) \quad (3.19)$$

$$\text{Radar Image:} \quad \min(\|E - D_x \varphi D_y'\|_2 + \lambda \|\varphi\|_1) \quad (3.20)$$

Sparse radar image based on BPDN with penalty term provides high resolution. If penalty term value is very small, minimization problem converges to the l_2 norm minimization. Penalty term that penalizes the non-zero coefficients to find the best sparse representation decreases the side lobes on the radar image. On the other hand, high penalty values may miss the actual scatterers.

In Fig. 3.7, range profiles of F117 plane based FFT method and sparse representation are presented when penalty term is 0.5, 1.5, 2, 2.5 and 5, respectively. When penalty term is chosen as 0.5, range profile is presented in Fig. 3.7b. As seen from the figure, range profile is not sparse since penalty term is too low. Penalty term is increased to 1.5 in Fig. 3.7c and range profile is represented by sparse way. The dominant scatterers are found in this figure. When penalty term is chosen as 2 in Fig. 3.7d, few dominant scatterers are found while some of them are missed. Penalty terms are chosen as 2.5 and 5 in Fig 3.7e and Fig. 3.2f, respectively and most of the dominant scatterers are missing.

In Fig. 3.8, range profiles of F14 plane based FFT method and sparse representation when penalty term is 0.5, 1.5, 2, 2.5 and 5 are given, respectively. It is observed that low penalty terms aren't successful to find sparse representation while high penalty term misses the scatterers, as expected.

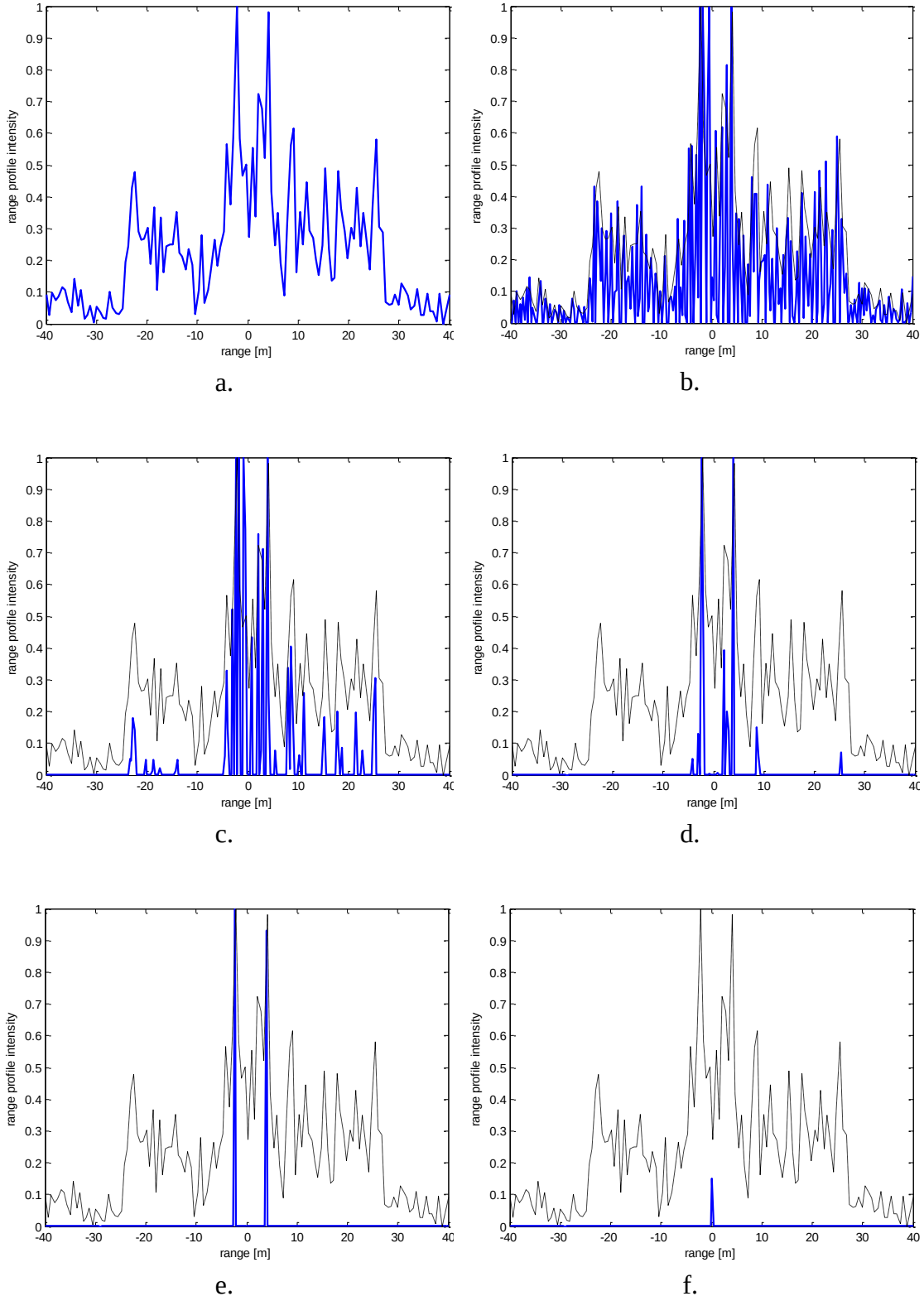


Figure 3.7: Range profile of F117 data (SNR=10dB) a.) FFT based method, Sparse range profile based on BPDN with penalty term is b.)0.5 , c.)1.5 , d.)2 , e.)2.5 , f.)5.

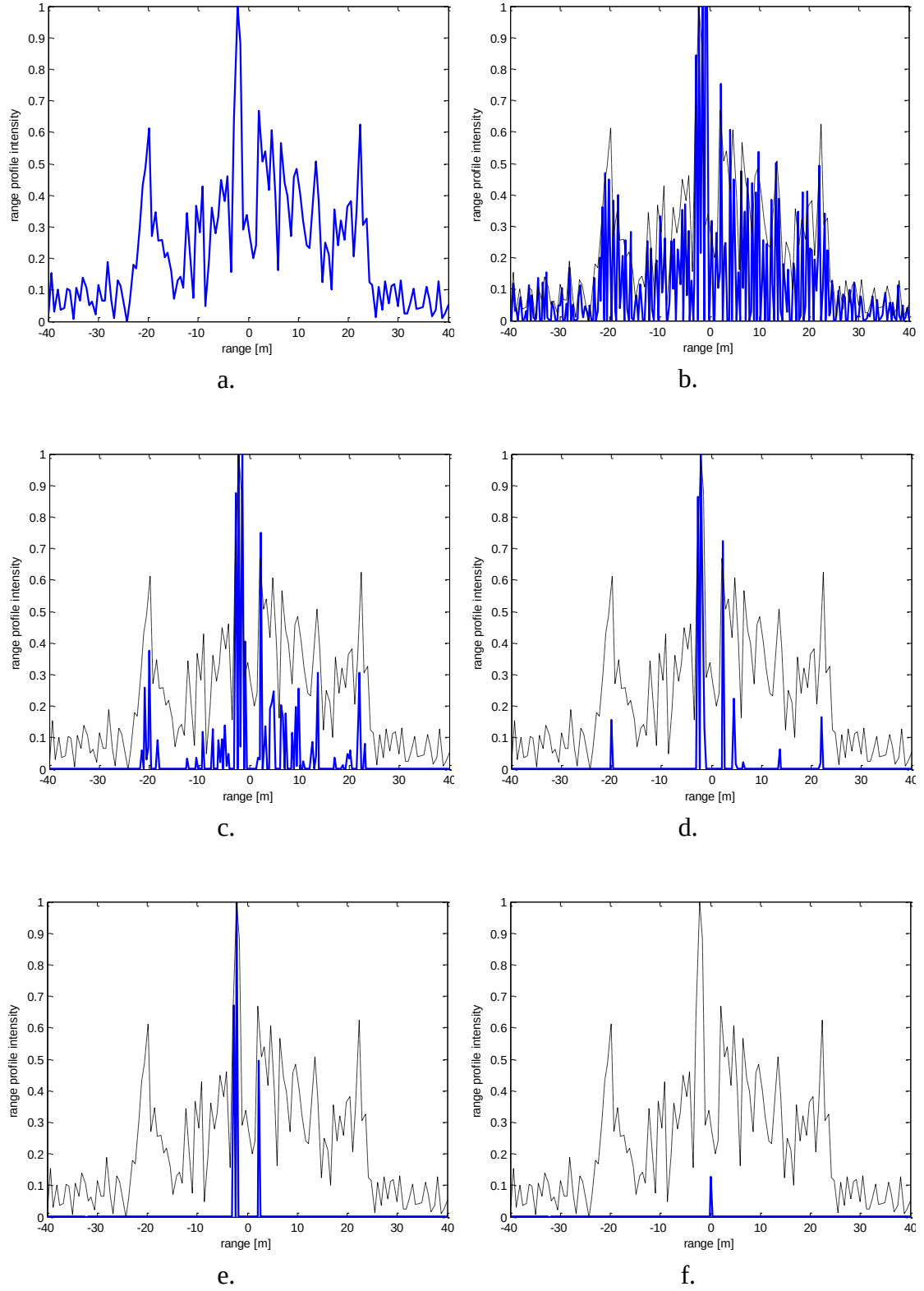


Figure 3.8: Range profile of F14 data (SNR=10dB) a.) FFT based method, Sparse range profile based on BPDN with penalty term is b.)0.5 , c.)1.5 , d.)2 , e.)2.5 , f.)5.

Sparse representation of radar images based on BPDN with penalty term when penalty term is 0, 0.5, 1.25, 2, 3 are represented, respectively, in Fig. 3.9. In Fig. 3.9b, low penalty term is chosen and more than real scatterers display on tail of the plane. To eliminate the spurious scatterers, penalty term is increased to 0.5 and 1.25 in Fig. 3.9c and Fig. 3.9d, respectively. In Fig. 3.9c, scatterers on the plane are found properly. Scatterers of plane are dispersed properly but some scatterers located in the middle of the target disappear in Fig. 3.9e. As penalty term is increased to 3, many scatterers are missing in Fig. 3.49. As seen from Fig. 3.9, side lobes around the target are suppressed while penalty term is increased.

In Fig. 3.10, Sparse representation of radar images for 10 dB SNR based on BPDN with penalty term when penalty term is 0, 0.2, 1.25, 2, 2.5 are represented, respectively. Noise effect is analyzed in this figure. Scatterers of the plane are obtained correctly but some spurious scatterers display around the wings of the plane in Fig. 3.10d when penalty term is 1.25. To eliminate these spurious scatterers, penalty value is increased to 2 in Fig. 3.10e. While these spurious scatterers are eliminated, few real scatterers are also eliminated. As a result, best sparse representation is obtained when penalty term is 1.25.

For limited data case, sparse representation of radar images for limited data based on BPDN with penalty term when penalty term is 0.1, 1, 1.2, 1.5, 2 are represented, respectively, in Fig. 3.11. It is observed that whole scatterers cannot be found in the sparse representation. In Fig. 3.11b, penalty term value is chosen as 0.1. Scatterers at the tail of plane cannot be found even for low valued penalty term. As expected, real scatterers are eliminated with high penalty term value. Since low valued penalty term fails to find scatterers, information loss increases as penalty term increases.

It can be said that, sparse representation of radar image based on BPDN with penalty term provides very clear image and this is robust to noise. On the other hand, this approach is not suitable for narrowband radar data. Moreover, computational cost is very high. Because of high computational cost, this approach is not suitable for target identification system.

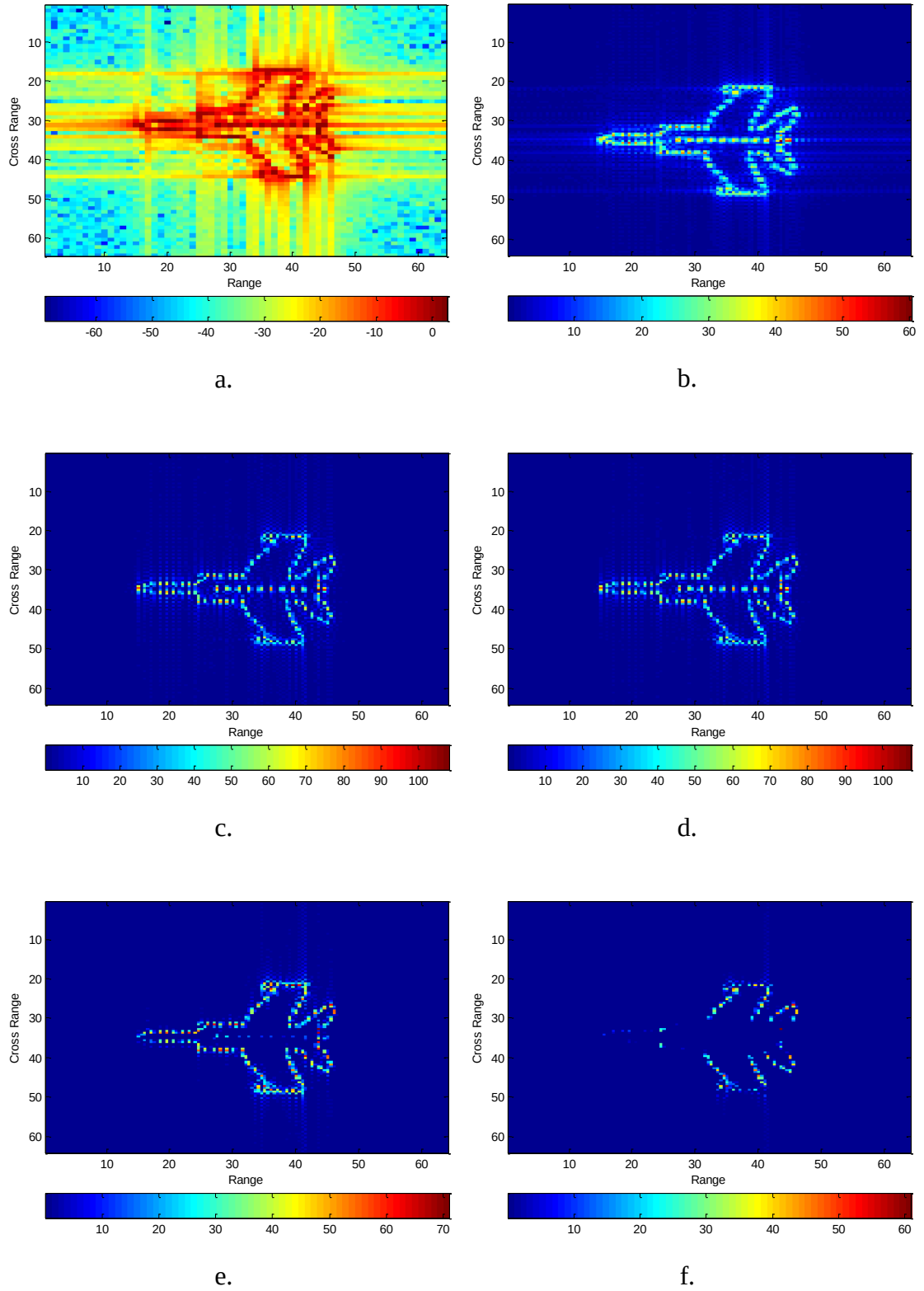


Figure 3.9: Radar image of wideband Mig25 data (SNR=30 dB). a.) FFT based method ,Sparse radar image based on BPDN with penalty term is b.) 0, c.) 0.5, d.)1.25 , e.)2 , f.)3.

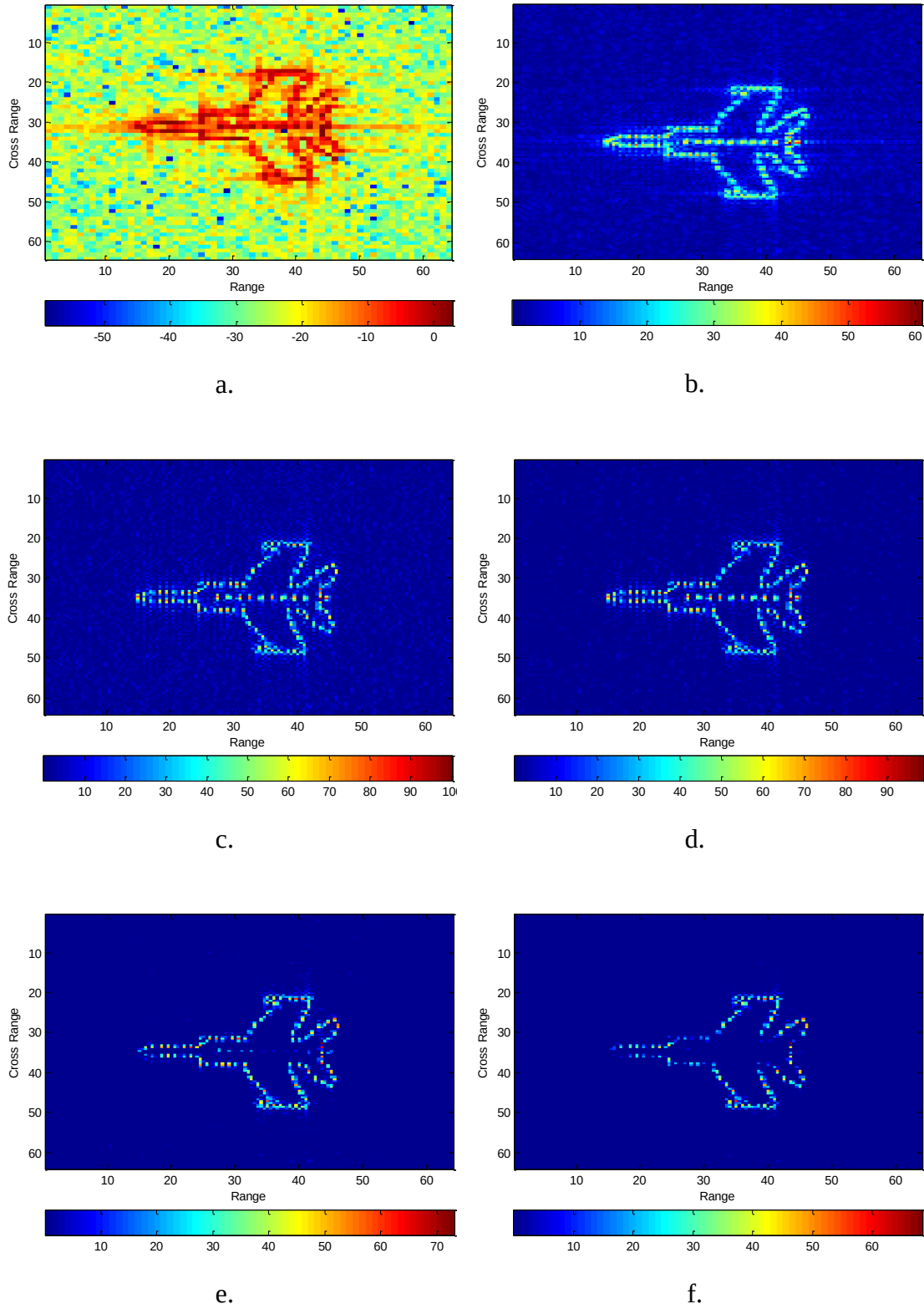


Figure 3.10: Radar image of wideband Mig25 data (SNR=10 dB). a.) FFT based method ,Sparse radar image based on BPDN with penalty term where b.) 0, c.) 0.2, d.)1.25 , e.)2, f.)2.5.

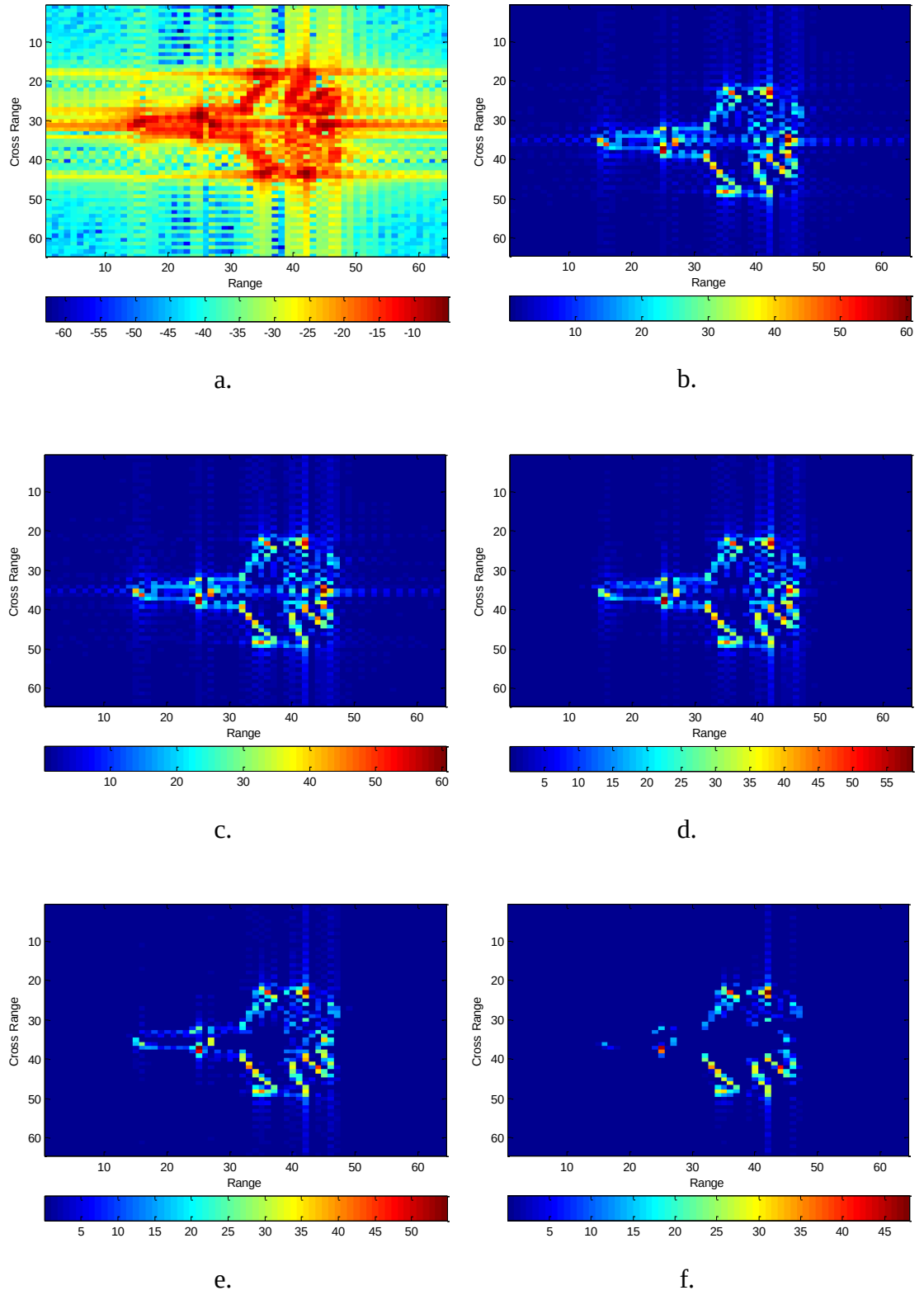


Figure 3.11: Radar image of narrowband Mig25 data (SNR=30dB). a.) FFT based method, Sparse radar image based on BPDN with penalty term is b.) 0.1, c.) 1, d.) 1.2, e.) 1.5, f.) 2.

▪ **Sparse radar image based on LASSO**

LASSO minimizes residual sum of squares (l_2 norm) by keeping sum of the absolute value (l_1 norm) of coefficient matrix/ vector element lower than constant threshold.

$$\text{Range Profile:} \quad \min(\|E - D_x \varphi\|_2) \text{ subject to } \|\varphi\|_1 \leq \tau \quad (3.21)$$

$$\text{Radar Image:} \quad \min(\|E - D_x \varphi D_y'\|_2) \text{ subject to } \|\varphi\|_1 \leq \tau \quad (3.22)$$

To determine τ , firstly range profile is found by FT based method. Initial τ is determined by l_1 norm of the range profile sample is calculated. The Fig. 3.12 shows the range profiles of F117 plane based on FFT method and LASSO for different τ values ,respectively. Initial threshold value is determined from FT transform based method. Then by making small changes on this initial threshold value, range profiles are obtained. In Fig. 3.12b, only 2 dominant scatterers are found when l_1 norm of φ is limited by 3. To find more scatterers, threshold value is increased.

In Fig. 3.13, the range profiles of F14 plane based on FFT method and LASSO for different τ values are presented ,respectively. As seen from the range profile figures, finding the scatterers is impossible with low threshold values. As threshold increases, scatterers are found properly.

Sparse representation of radar images based on FFT method and LASSO for different τ values are represented, respectively, in Fig. 3.14. Firstly, the mean of the l_1 norm of the pixels of image which is generated from FT based method is calculated and this value is multiplied with 0.05, 0.1, 0.2, 1, 2 to determine the τ value. In Fig. 3.14b, low τ value is chosen and most of the scatterers aren't found. To find the actual scatterers, τ value should be increased. τ value is determined from the multiplication of 0.1 and 0.2 with the mean of the l_1 norm of the pixels in Fig. 3.14c and Fig. 3.14d, respectively, and image can be represented properly. In Fig. 3.14e. τ value is increased and more scatterers are found. On the other hand, side lobes appear in the Fig.3.14e. Spurious scatterers and side lobe displays in the Fig. 3.14f.

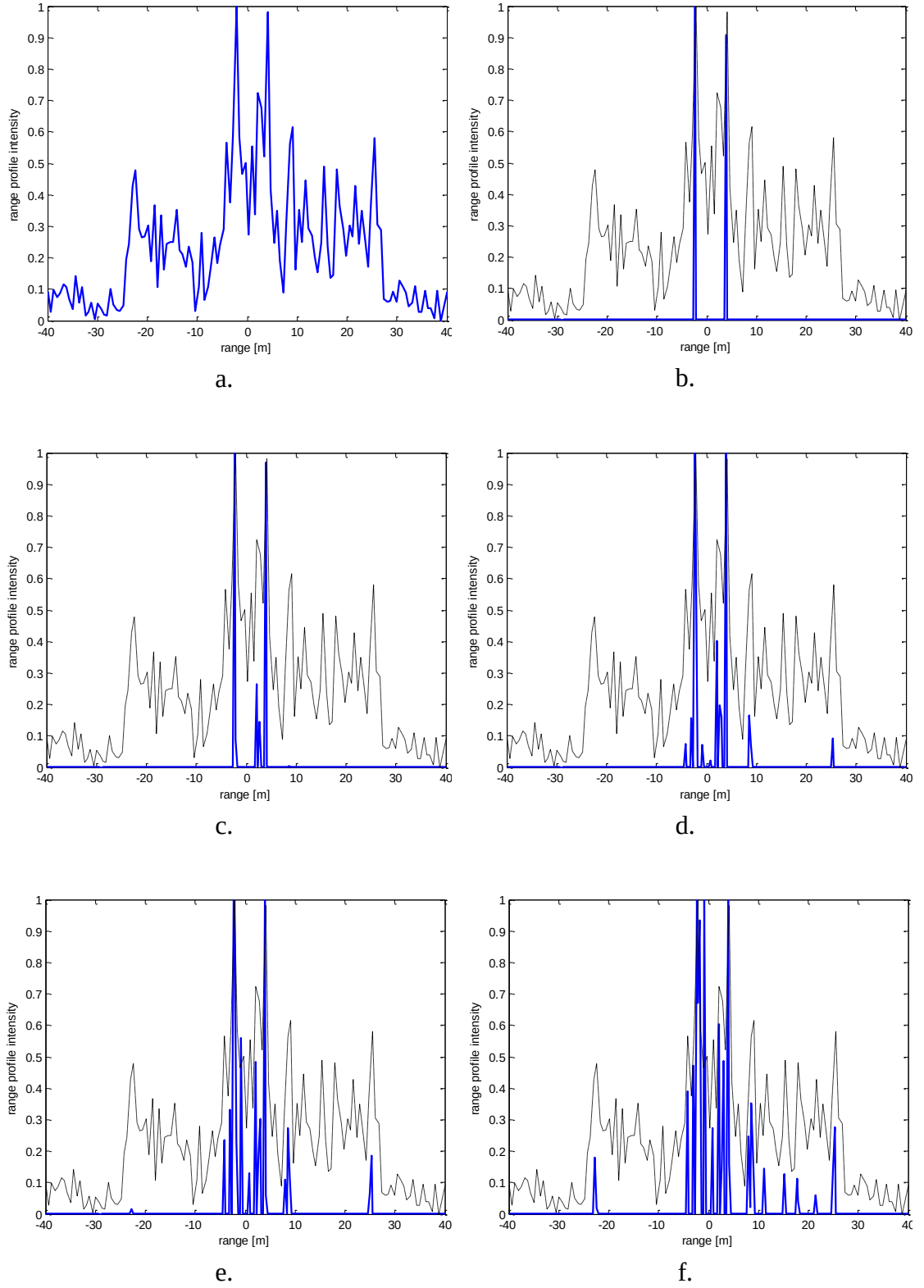


Figure 3.12:Range profile of F117 data (SNR=10 dB) a.) FFT based method , Sparse range profile, based on LASSO. Threshold: b.)3 , c.)7 , d.)12, e.)18 , f.)30.

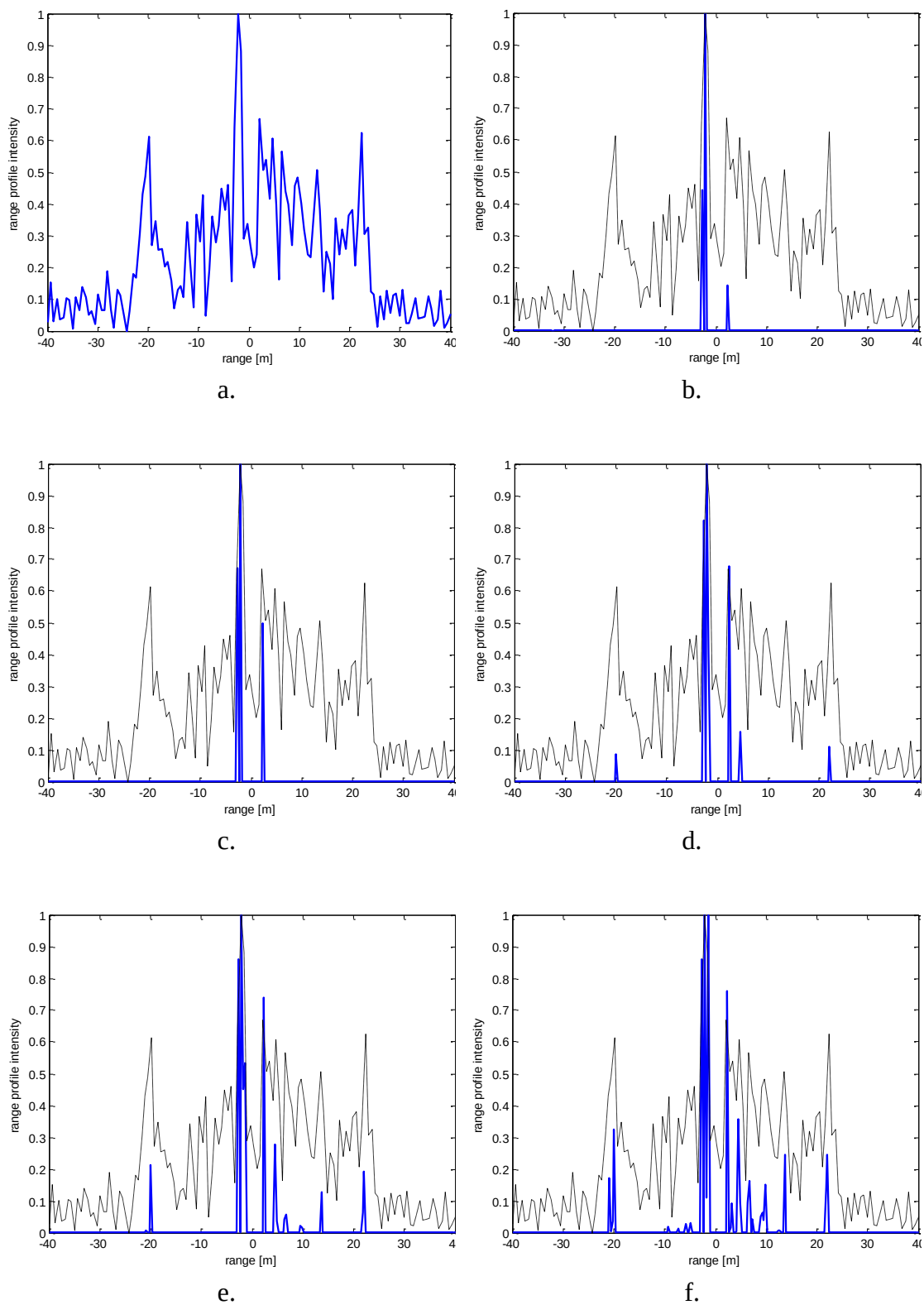


Figure 3.13:Range profile of F14 data (SNR=10 dB) a.) FFT based method , Sparse range profile based on LASSO threshold: b.)3 , c.)7 , d.)12, e.)18 f.)30.

For low SNR, sparse representation of radar images based on FFT method and LASSO for different τ values are presented, respectively, in Fig. 3.15. In Fig. 3.15b, low τ value is chosen and most of the scatterers can not be found. Side lobes cannot be prevented even low τ value. The actual scatterers can be found by increasing the τ value. In Fig. 3.15e, many spurious scatterers are found because of the high τ value.

For the limited data case, sparse representation of radar images based on FFT method and LASSO for different τ values are presented, respectively, in Fig. 3.16. As expected, fewer scatterers are found with low τ value. As seen in the Fig. 3.16f, actual scatterers cannot be found even if high τ value is chosen.

It can be said that, sparse representation of radar image based on LASSO provides very clear image and this is robust to noise. On the other hand, this approach is not suitable for narrowband radar data. In addition to that, computational cost is very high. Because of high computational cost, this approach is not suitable for target identification algorithms.

▪ **Orthogonal Matching Pursuit (OMP)**

Generation sparse range profile and radar image based on BPDN, BPDN with penalty term and LASSO require high computational cost such as time, RAM, etc. Because of this drawback, these optimization problems are not suitable for radar imaging and classification. Orthogonal Matching Pursuit (OMP), which requires less computational cost, is employed to generate sparse range profile and radar image in a short time.

OMP constructs sparse signal representation by calculating locally optimum solution iteratively [33]. At each iteration, atom of the dictionary which is highly correlated with the remaining part of observed signal is found. Then, contribution of this atom is subtracted from the observed signal and residue is calculated. After several iteration, right set of atoms is obtained [34]. In this algorithm, sparsity level can be determined by the user.

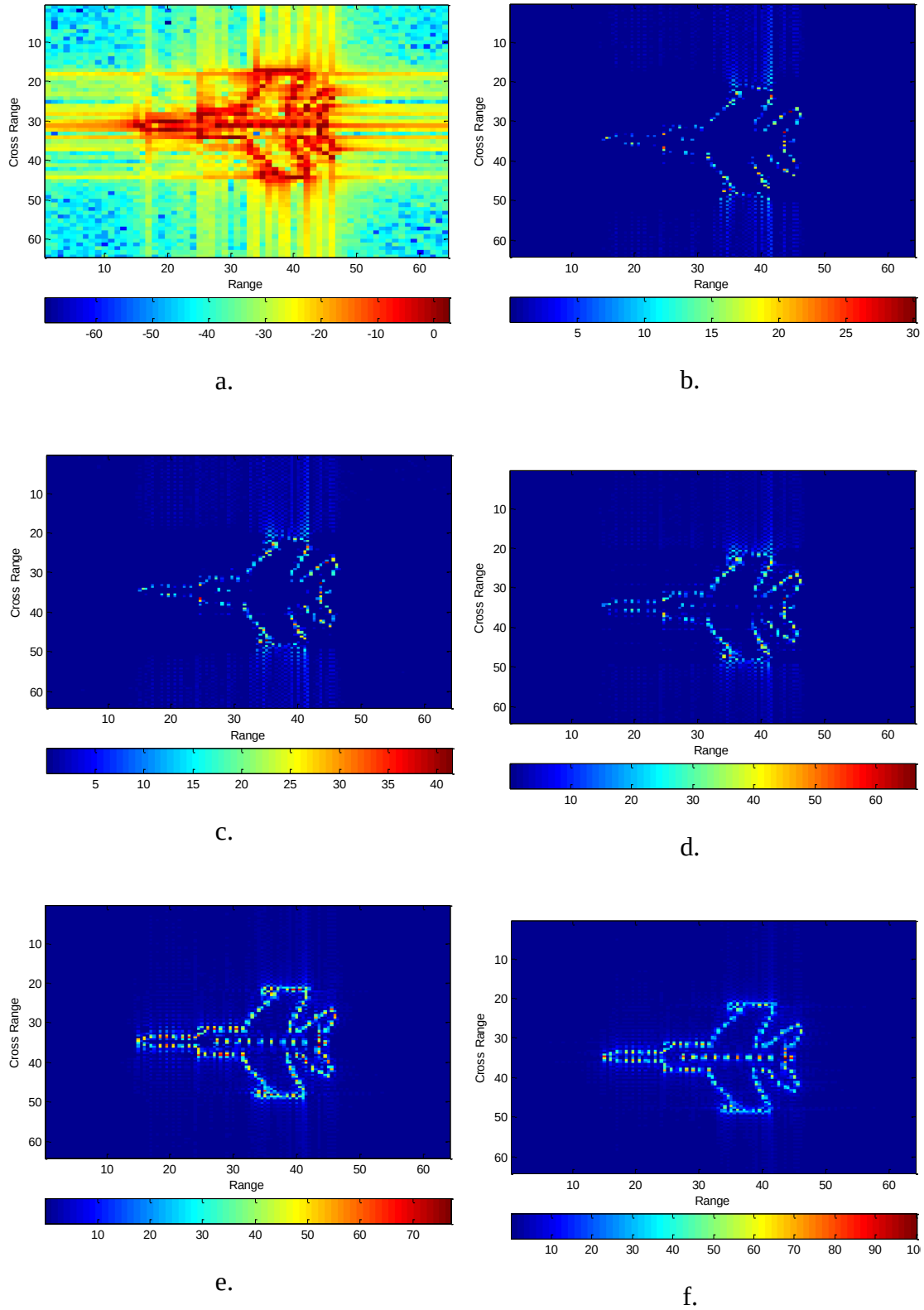


Figure 3.14: Radar image of wideband Mig25 data (SNR=30dB) a.) FT based method, Sparse radar image based on LASSO , initial threshold is multiplied with b.)0.05, c.)0.1 , d.)0.2 , e.)1 , f.) 2.

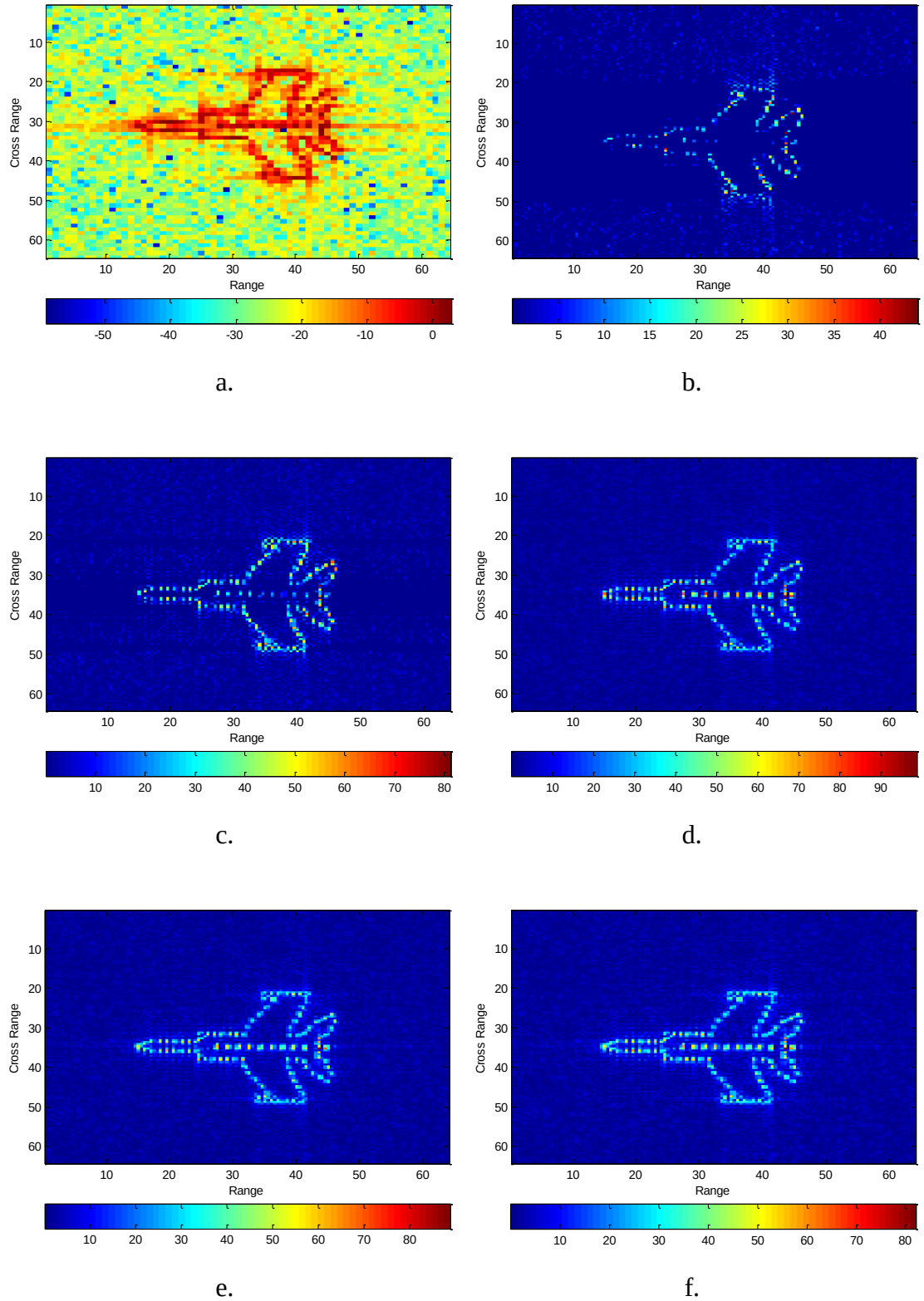


Figure 3.15 :Radar image of wideband Mig25 data (SNR=10dB) a.) FT based method, Sparse radar image based on LASSO , initial threshold is multiplied with b.)0.05, c.)0.1 , d.)0.2 , e.)1 , f.) 2.

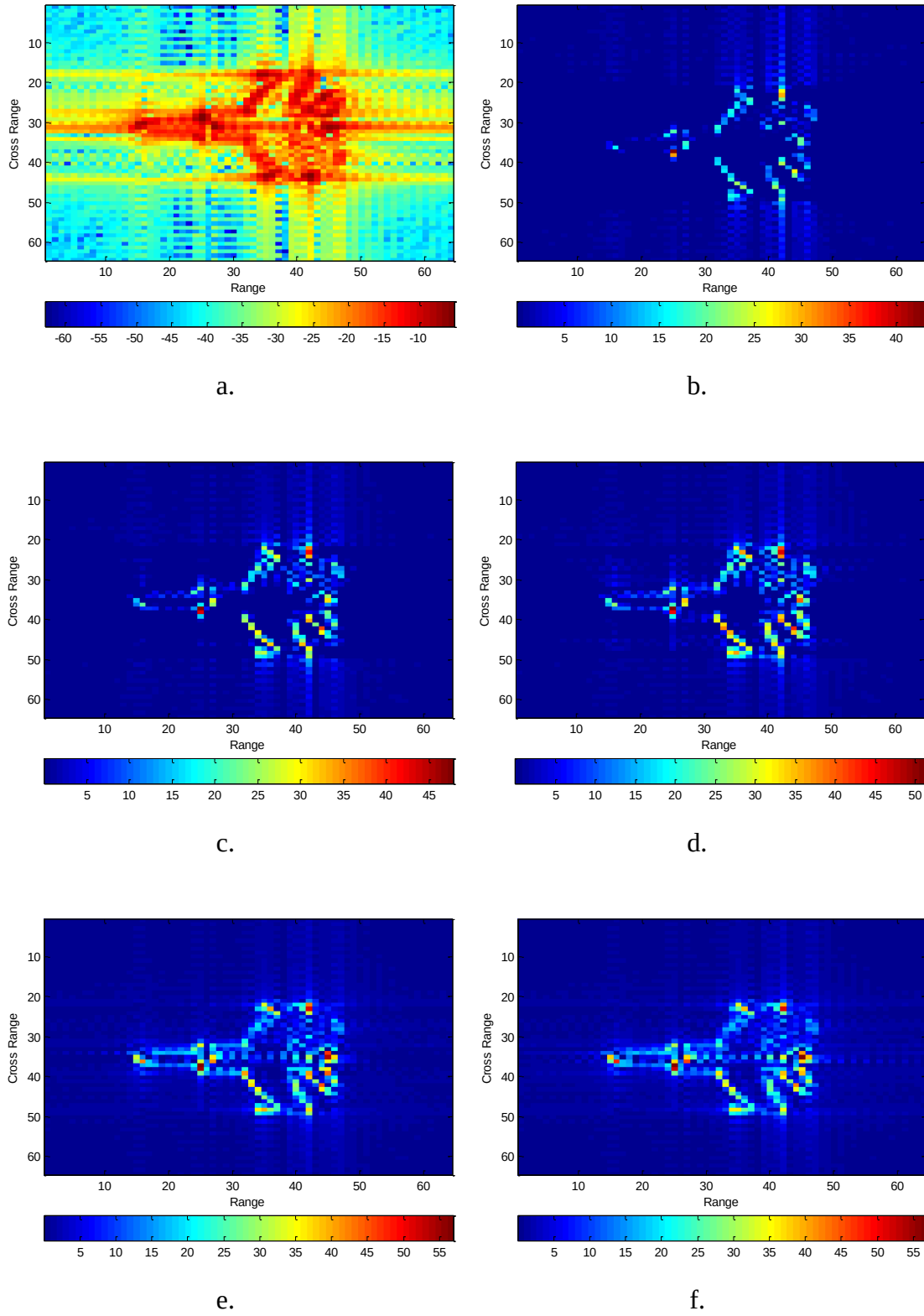


Figure 3.16: Radar image of narrowband Mig25 data a.) FT based method, Sparse radar image based on LASSO , initial threshold is multiplied with b.)0.05, c.)0.1 , d.)0.2 , e.)1 , f.) 2.

1-D radar data is given in (3.11). $\mathbf{E}$ is observed vector, $\mathbf{D}$ is Fourier dictionary matrix and $\boldsymbol{\varphi}$ is the unknown coefficient vector. According to these notation, steps of OMP algorithm are given as:

Step 1: Initialize the residual . Since iteration does not start, index is equated to 0 and the index of dictionary which is highly correlated with the residue is defined as empty set.

$$\begin{aligned}\mathbf{r}_t &= \mathbf{E} \\ t &= 0 \\ \Lambda_t &= \emptyset\end{aligned}\tag{3.23}$$

Where $\mathbf{r}_0$ is the initial residue, t is the iteration index and Λ is the vector where indexes of the dictionary which are highly correlated with the residue.

Step 2: Dictionary matrix is multiplied by last obtained residue and maximum value of this multiplication result gives the atom of dictionary which is highly correlated with residue. Index of atom can be found from (3-24).

$$\Lambda_t = \arg \max(\mathbf{D} * \mathbf{r}_t)\tag{3.24}$$

After finding the index i , Λ will be updated as:

$$\begin{aligned}\Lambda_t &= i \\ \Lambda_t &= \Lambda_{t-1} \cup \{\Lambda_t\}\end{aligned}\tag{3.25}$$

Step 3: Least square optimization problem is solved to obtain $\boldsymbol{\varphi}$. This minimization problem is given by:

$$\min(\mathbf{E} - \mathbf{D}_t \boldsymbol{\varphi})\tag{3.26}$$

Step 4: Residue vector and iteration index are updated after each iteration. How residue vector is updated is given in (3.25).

$$\begin{aligned}\mathbf{r}_t &= \mathbf{r}_{t-1} - \mathbf{D}_t \boldsymbol{\varphi} \\ t &= t + 1\end{aligned}\tag{3.27}$$

Last 4 steps are repeated until reaching the break criteria which is the sparsity level.

As a result of several iteration, sparse range profile and radar image is generated.

In Fig. 3.17, range profiles of F117 plane which are generated by FFT based method and OMP solution for different sparsity level are presented. In Fig. 3.17b, sparse range profile is generated when it is assumed that 5 % of the unknown coefficients vector are sparse. In this case, dominant scatterers are found. It is assumed that 8% and 15% of unknown coefficients vector are sparse in Fig. 3.17c and Fig. 3.17d, respectively. In Fig.3.17d, most of the scatterers are found. In Fig. 3.17e and Fig. 3.17f, sparse range profile is generated when it is assumed that 20 % and 25 % of unknown coefficients vector are sparse.

Range profiles of F14 plane which are generated by FFT based method and OMP solution for different sparsity level are presented in Fig. 3.18. As seen in the figure, more scatterers are found by increasing the sparsity level.

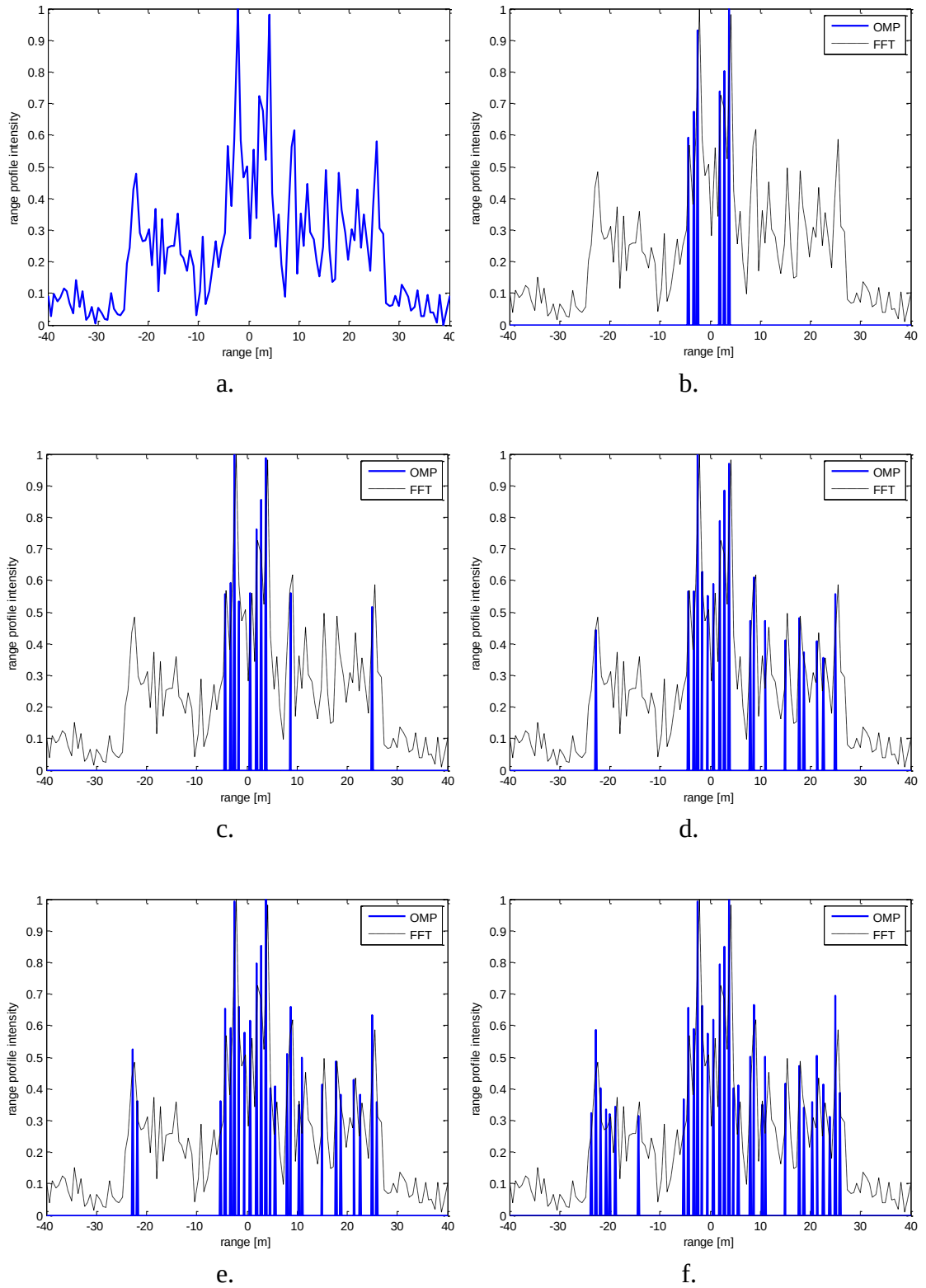


Figure 3.17:Range Profile of F117 plane a.) FT based method , OMP solution when b.) 5% ,c.)8 % , d.)15% , e.) 20% , f.) 25% of unknown coefficients vector are sparse.

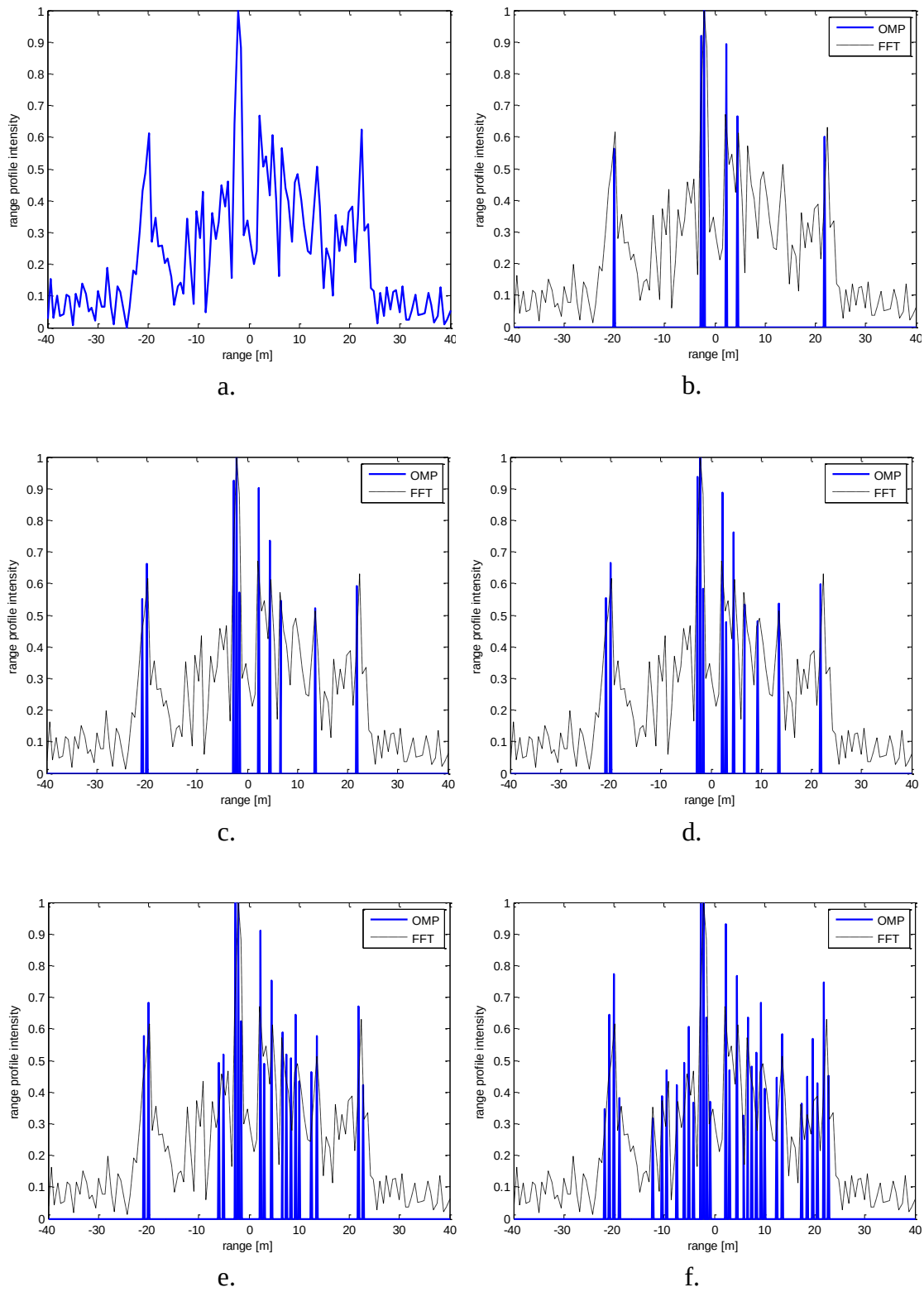


Figure 3.18:Range profile of F14 plane a.) FT based method , OMP solution when b.) 5% ,c.)8 % , d.)10% , e.) 15% , f.) 25% of unknown coefficients vector are sparse.

In Fig. 3.19, sparse radar images are generated for different sparsity levels by employing OMP. As seen in Fig.3.19b, if sparsity level is chosen as 1% of unknown coefficients vector, few scatterers are found but these scatterers are not enough to represent the target. When 3 % of unknown coefficients vector are sparse, radar image of target is successfully generated in Fig. 3.19c. Spurious scatterers appears in the image as sparsity level is increased. In Fig. 3.19d, Fig. 3.19e and Fig. 3.19f, 5%, 8% and 15% of unknown coefficients vector are sparse. As seen in the Fig. 3.19, spurious scatterers are increased when it is assumed that high percentage of data are sparse.

For low SNR, sparse radar images are generated for different sparsity levels by using OMP and these radar images are given in Fig. 3.20. As seen in the figure, spurious scatterers are increased when sparsity level is increased. Also, this solution is robust to noise.

In Fig. 3.21, sparse radar images are generated for different sparsity levels by employing OMP for limited data case. In Fig. 3.21b and Fig. 3.21c, 2 % and 5% of unknown coefficients vector are nonzero, respectively, but scatterers cannot be found with these sparsity levels and image cannot be represented properly. In Fig. 3.21e, 40% of unknown coefficients vector are sparse but still target cannot be represented properly.

As a conclusion, radar images which have high resolution are generated by OMP. But this method is not suitable for limited data case.

Scatterers of target can be successfully found by using sparse representation approach in case of the wideband data and low SNR value data. On the other hand, this approach is not a good candidate to find the scatterers of target when data are limited.

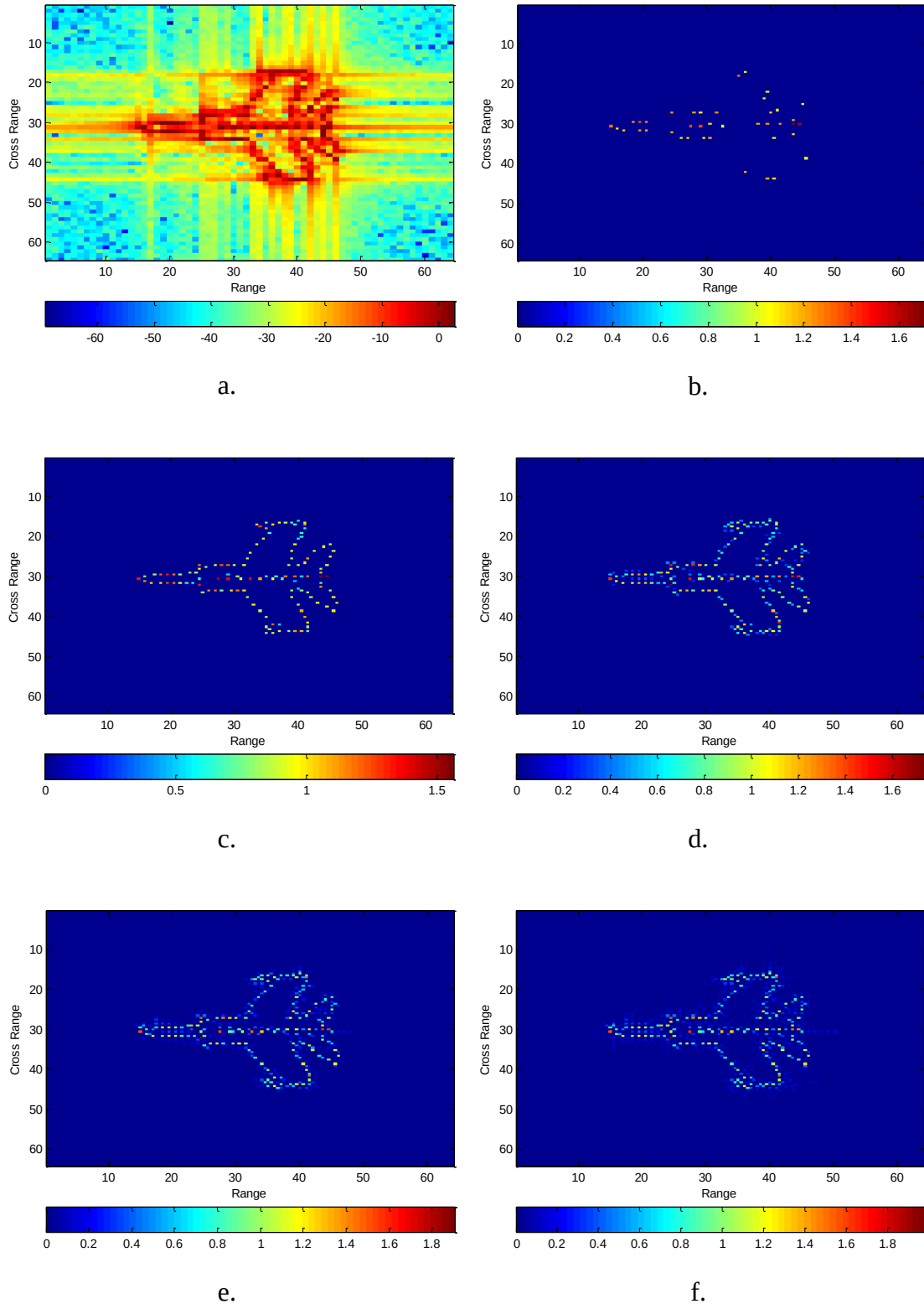


Figure 3.19 : Radar image of wideband Mig25 data (SNR 30 dB) a.) FT based method, Sparse radar image based on OMP, sparsity level b.)1%, c.)3% , d.)5 % , e.)8 % , f.) 15%.

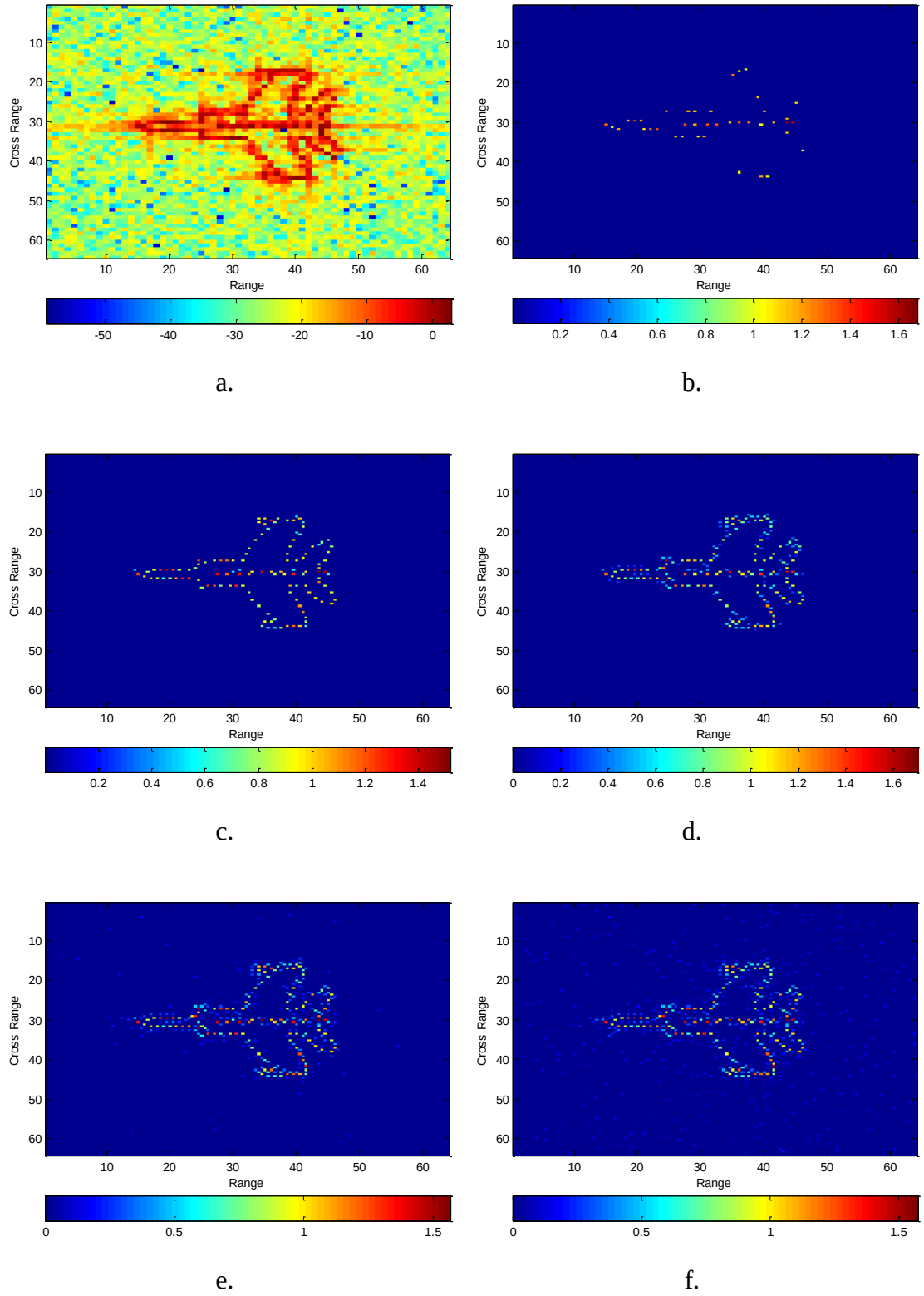


Figure 3.20 : Radar image of wideband Mig25 data (SNR 10 dB) a.) FT based method, Sparse radar image based on OMP, sparsity level b.)1%, c.)3% , d.)5 % , e.)8 % , f.) 15%.

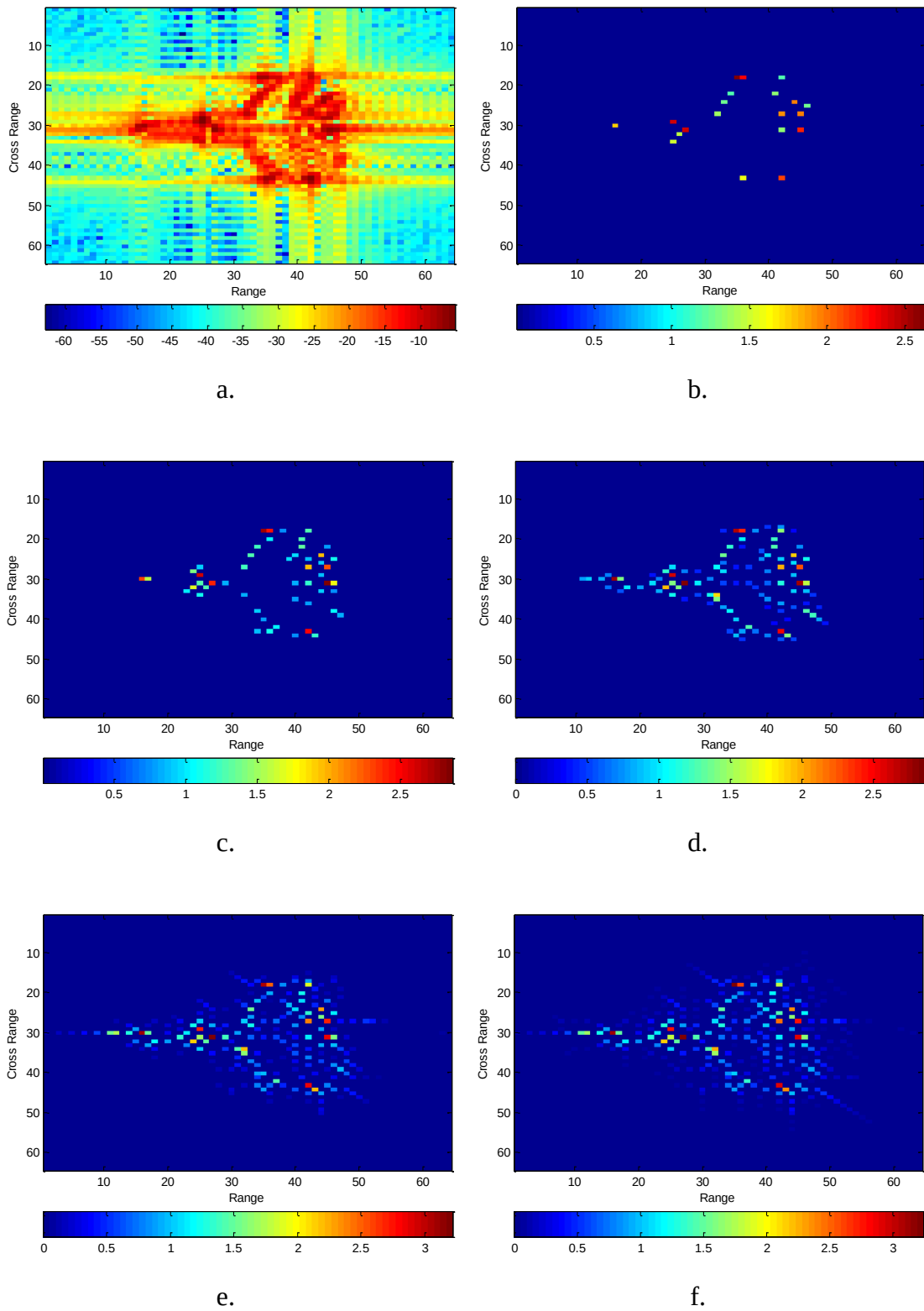


Figure 3.21 :Radar image of narrowband Mig25 data a.) FT based method, Sparse radar image based on OMP, sparsity level b.)2%, c.)5% , d.)10 % , e.)20 % , f.) 40%.

3.2 Sparse Autoregressive Modeling for Radar Imaging

3.2.1 Sparse autoregressive modeling

Signal can be modeled by linear combination of past and future values of the available data and range profile and radar images can be derived by using AR model. Least square solution is the most common way to determine the AR coefficients but this solution causes some spurious picks. SVD truncation overcomes this problem by estimating the number of scatterers. The performance of SVD truncation is highly related with the predicted scatterers. Unfortunately, it is not possible to know or predict exact number of scatterers.

Tickhonov regularization is another way to solve the ill-posed equation system by introducing some prior such as smoothness, sparsity. Formulation of Tickhonov regularization is given in (3-28). In these equation system, if l_2 norm of AR model coefficients are penalized, smoother solution is obtained. Sparsity is introduced by penalizing the l_1 norm of the AR model coefficients.

$$\min(\|\mathbf{x} - \mathbf{X}\mathbf{a}\|_2 + \lambda\|\mathbf{a}\|_p) \quad (3.28)$$

In this thesis, sparsity approach is proposed to determine the AR model coefficients. As a result of proposed method, few AR model coefficients are non-zero. l_0 norm minimization yields the sparse coefficients but application of l_0 norm minimization is not feasible because of the computational cost. Because of that, sparse coefficients are obtained by l_1 norm minimization which is approximated the l_0 norm minimization .

In [25], the authors proposed sparse linear prediction for speech signal. As a result of sparse linear prediction, sparse prediction coefficients are obtained. Their proposed method minimizes the l_2 norm of residue by penalizing the l_1 norm of linear prediction coefficients. 1-D speech signal is modeled as:

$$x(n) = \sum_{k=1}^K a_k x(n-k) + e(n) \quad (3.29)$$

In (3.29), a_k is the prediction coefficients and $e(n)$ is the error term. The authors predict the coefficients by minimizing the prediction error. Prediction error is defined

as $\hat{e} = \mathbf{x} - \mathbf{X}\mathbf{a}$. Prediction coefficients are determined by solving this minimization problem:

$$\min(\|\mathbf{x} - \mathbf{X}\mathbf{a}\|_2 + \lambda\|\mathbf{a}\|_1) \quad (3.30)$$

where

$$\mathbf{x} = \begin{bmatrix} x(N_1) \\ \vdots \\ x(N_q) \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} x(N_1 - 1) & \cdots & x(N_1 - k) \\ \vdots & & \vdots \\ x(N_q - 1) & \cdots & x(N_q - k) \end{bmatrix} \quad (3.31)$$

In this thesis, the authors' approach is applied to 1-D AR model and 2-D AR model for radar data and high-resolution range profile and radar images are generated. Matrix notation of 1-D AR model and 2-D AR model are given in (2.27), (2.31) and (2.34).

3.2.2 Range profile and radar image generation

From (2.27), sparse AR coefficients for range profile generation are found by solving this minimization problem:

$$\min(\|\mathbf{a}\|_0) \quad \text{when } \mathbf{e} = \mathbf{E}\mathbf{a} \quad (3.32)$$

For radar imaging, basis pursuit (BP) minimization problem can be formulated by using (2.31) and (2.34),

$$\begin{aligned} \min(\|\mathbf{a}\|_0) \quad \text{when } \mathbf{e} = \mathbf{E}\mathbf{a} \\ \min(\|\mathbf{b}\|_0) \quad \text{when } \tilde{\mathbf{e}} = \tilde{\mathbf{E}}\mathbf{b} \end{aligned} \quad (3.33)$$

Since l_0 norm is NP hard problem, l_1 norm is used. This minimization problem is called Basis Pursuit. In this minimization problem $\mathbf{e} = \mathbf{E}\mathbf{a}$ condition should be satisfied.

BPDN is another alternative way to find sparse coefficients. The objective of this problem is to minimize the l_1 norm of the AR model coefficients while keeping l_2 norm of prediction error under the determined threshold. It can be said that this problem is the error constrain problem. The minimization problem is:

$$\min(\|\mathbf{a}\|_1) \quad \text{subject to } \|\mathbf{E}\mathbf{a} - \mathbf{e}\|_2 < \epsilon \quad (3.34)$$

where $\mathbf{a}$ is the AR model coefficients and ϵ is the error threshold. Sparsity level is affected by the choice of the error threshold.

For 2-D radar imaging, 2 system of linear equations which are given in (2.31) and (2.34) exist. Minimization problem is given in (3-35).

$$\begin{aligned} \min(\|\mathbf{a}\|_1) \quad \text{subject to } \|\mathbf{E}\mathbf{a} - \mathbf{e}\|_2 < \epsilon_1 \\ \min(\|\mathbf{b}\|_1) \quad \text{subject to } \|\tilde{\mathbf{E}}\mathbf{b} - \tilde{\mathbf{e}}\|_2 < \epsilon_2 \end{aligned} \quad (3.35)$$

BPDN with penalty term is the unconstraint formulation of BPDN. The idea behind of this method is to minimize l_2 norm of residual while penalizing the l_2 norm of the AR model coefficients. Minimization problem for equation system to generate range profile:

$$\min(\|\mathbf{E}\mathbf{a} - \mathbf{e}\|_2 + \lambda\|\mathbf{a}\|_1) \quad (3.36)$$

If penalty term is chosen as 0, formulation is reduced to least square minimization solution. In this minimization problem, determination of penalty term, λ , is a difficult problem. In the literature, optimum penalty value is determined with L-curve [27,35]. The corner of L-curve gives the optimum penalty value but in this thesis, penalty value is determined empirically. Minimization problem for 2-D AR model is given:

$$\begin{aligned} \min(\|\mathbf{E}\mathbf{a} - \mathbf{e}\|_2 + \lambda_1\|\mathbf{a}\|_1) \\ \min(\|\tilde{\mathbf{E}}\mathbf{b} - \tilde{\mathbf{e}}\|_2 + \lambda_2\|\mathbf{b}\|_1) \end{aligned} \quad (3.37)$$

LASSO is the another minimization problem to obtain sparse AR model coefficients. LASSO minimizes residual sum of squares (l_2 norm) while sum of the absolute value (l_1 norm) of coefficients is smaller than constant threshold [36]. LASSO minimization problem is given in (3.38).

$$\min(\|\mathbf{E}\mathbf{a} - \mathbf{e}\|_2) \quad \text{subject to } \|\mathbf{a}\|_1 < \tau \quad (3.38)$$

For the radar image, minimization problem is given in (3.39).

$$\begin{aligned}
& \min(\|\mathbf{E}\mathbf{a} - \mathbf{e}\|_2) \quad \text{subject to } \|\mathbf{a}\|_1 < \tau_1 \\
& \min(\|\mathbf{E}\mathbf{b} - \mathbf{e}\|_2) \quad \text{subject to } \|\mathbf{b}\|_1 < \tau_2
\end{aligned} \tag{3.39}$$

All minimization problem is solved with CVX optimization package. As a result of this minimization problems, sparse AR model coefficients are calculated and range profile of target is determined from (2.28). Radar image is constructed from (2.35) after sparse AR model coefficients are calculated from the minimization problems.

Sparse linear prediction based on BPDN with penalty and penalty selection

Optimum penalty term value is determined by the L-curve. L-curve is generated by plotting $\log(\|\mathbf{E}\mathbf{a} - \mathbf{e}\|_2)$ versus $\log(\|\mathbf{a}\|_1)$ for several penalty term values. The corner of L-curve gives the optimum penalty value. In this thesis, penalty term is determined empirically.

Fig. 3.22 shows the range profile of F117 plane when $l-1$ norm of the AR model coefficients are penalized by different penalty term values. $l-1$ norm of AR model coefficients are penalized with 0.1 in Fig. 3.22b and dominant scatterers are found successfully with this penalty value. As seen in Fig. 3.22c, scatterer at -20 m range is suppressed when penalty term is 8. Scatterers cannot be dissolved as value of penalty term increases. Fig. 3.22d, Fig. 3.22e and Fig. 3.22f show effect of the the penalty term on the location and number of scatterers.

In Fig. 3.23, range profile of F14 plane are presented when $l-1$ norm of the AR model coefficients are penalized by different penalty term values. In Fig. 3.23b,c,d, it is observed that scatterer around the -10 m range disappears while value of penalty term is increased.

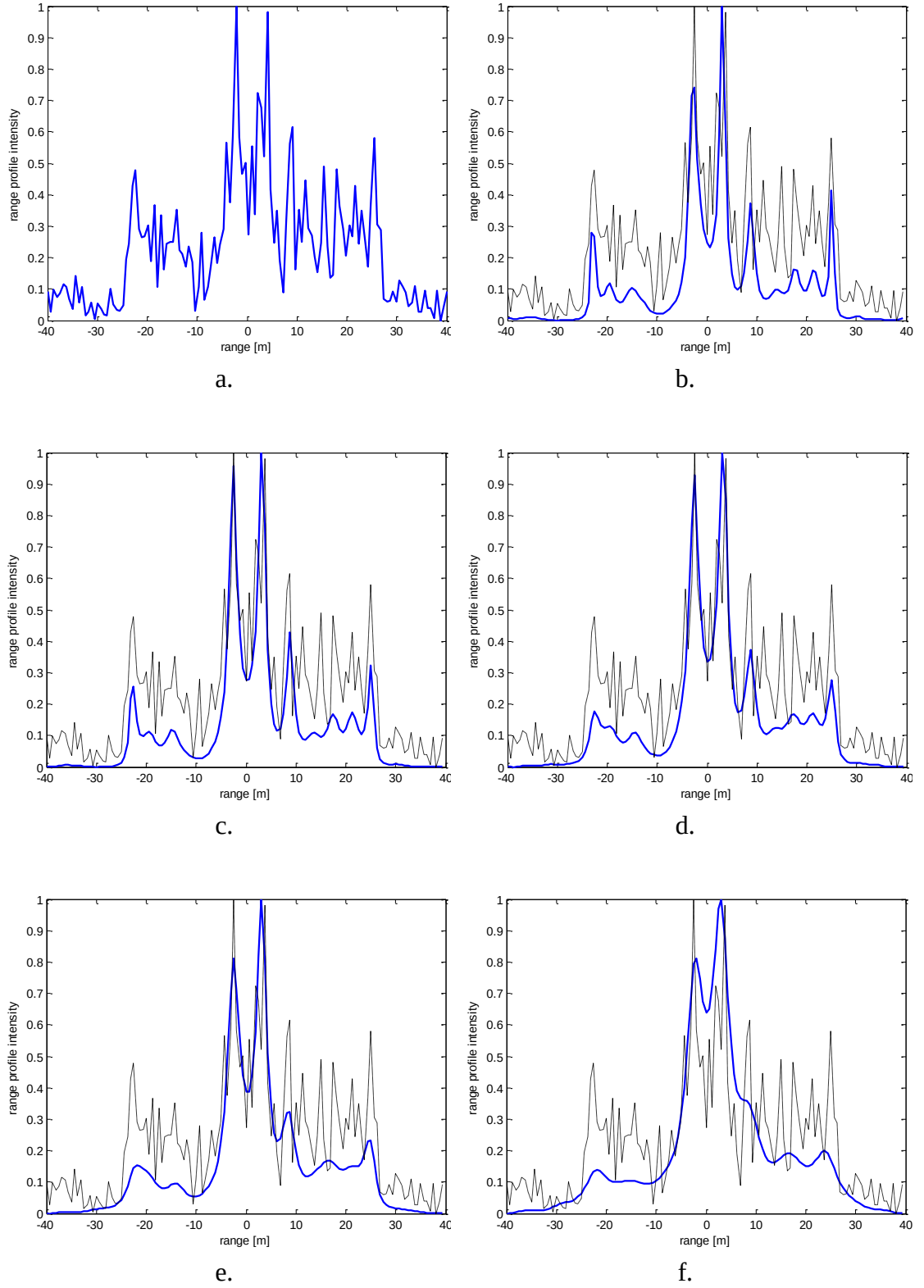


Figure 3.22:Range profile of F117 data (SNR=10dB) a.)FFT based method , Sparse AR model based on BPDN with penalty . AR model order=20, penalty term value b.)0.1, c.)8, d.)20, e.)30, f.)50.

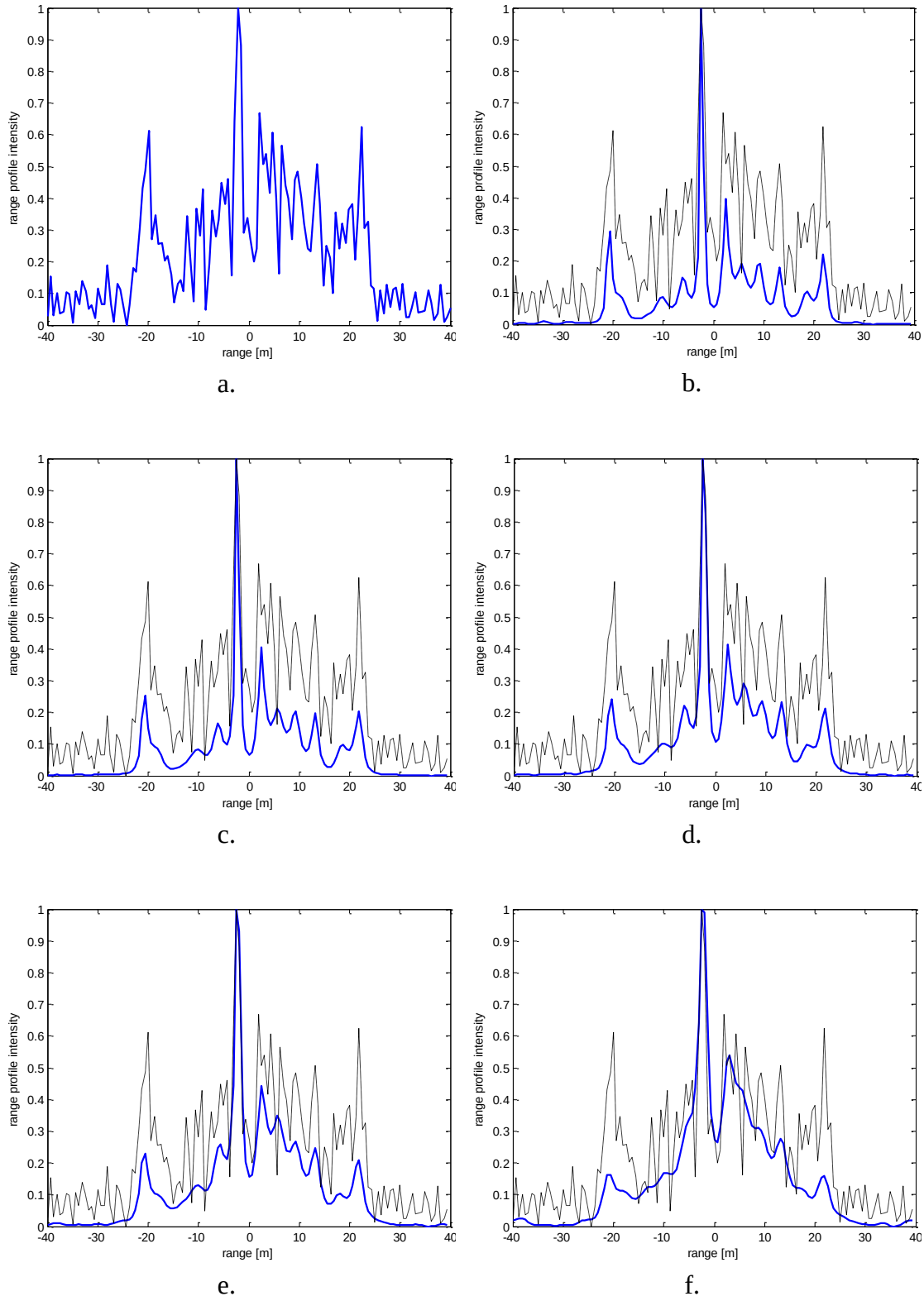


Figure 3.23:Range profile of F14 data (SNR=10dB) a.) FFT based method , Sparse AR model based on BPDN with penalty . AR model order=20, penalty b.)0.1, c.)8, d.)20, e.)30, f.)50.

In Fig. 3.24, radar images are presented for different penalty term. Radar image in Fig. 3.24b converges to least square error solution since value of penalty term is very low. As seen in this figure, side lobes exist and scatterers cannot be resolved. When penalty term is increased to 0.1 in Fig. 3.24c, sidelobe effect decreases and scatterers are dispersed. As seen in Fig. 3.24d, many of the scatterers are dispersed while side lobes are decreased. Also few spurious scatterers appear around the tail of plane. In Fig. 3.24e and Fig.3.24f, many spurious scatterers are observed around the tail of plane.

In Fig. 3.25, performance of sparse AR model coefficients is investigated in case of the low SNR. Many spurious scatterers appear around the target in Fig. 3.25b when penalty term is closed to zero. In Fig 3.25c, while few scatterers disappear, sidelobes are reduced. Penalty term is increased to 0.5 in Fig 3.25d and spurious scatterers and few real scatterers are lost but this penalty terms is the most suitable one to obtain better radar image. In Fig 3.25e and Fig. 3.25f, scatterers are getting disappear and shape of target is not clear since value of penalty term is too high.

In Fig. 3.26, radar images are constructed with narrow band radar data when AR model coefficients are calculated by basis pursuit denoising with penalty term. In Fig. 3.26b, radar image has sidelobes when value of penalty term is 0.001 and this image converges to least square error solution. These side lobes are suppressed by increasing the threshold value. In Fig. 3.26c and Fig. 3.26d, side lobes are suppressed but Fig. 3.26d has more spurious scatterers around the target than Fig. 3.26c. In Fig. 3.26e, scatterers at the body of the plane disappear as value of penalty term is 1. When penalty term is increased to 1.5, many scatterers at the body of the plane disappear and some properties of target are missed.

As a conclusion, less dominant scatterers are lost with high value penalty term while sidelobes are suppressed. On the other hand, low value penalty term cannot eliminate the spurious scatterers and high sidelobes are observed.

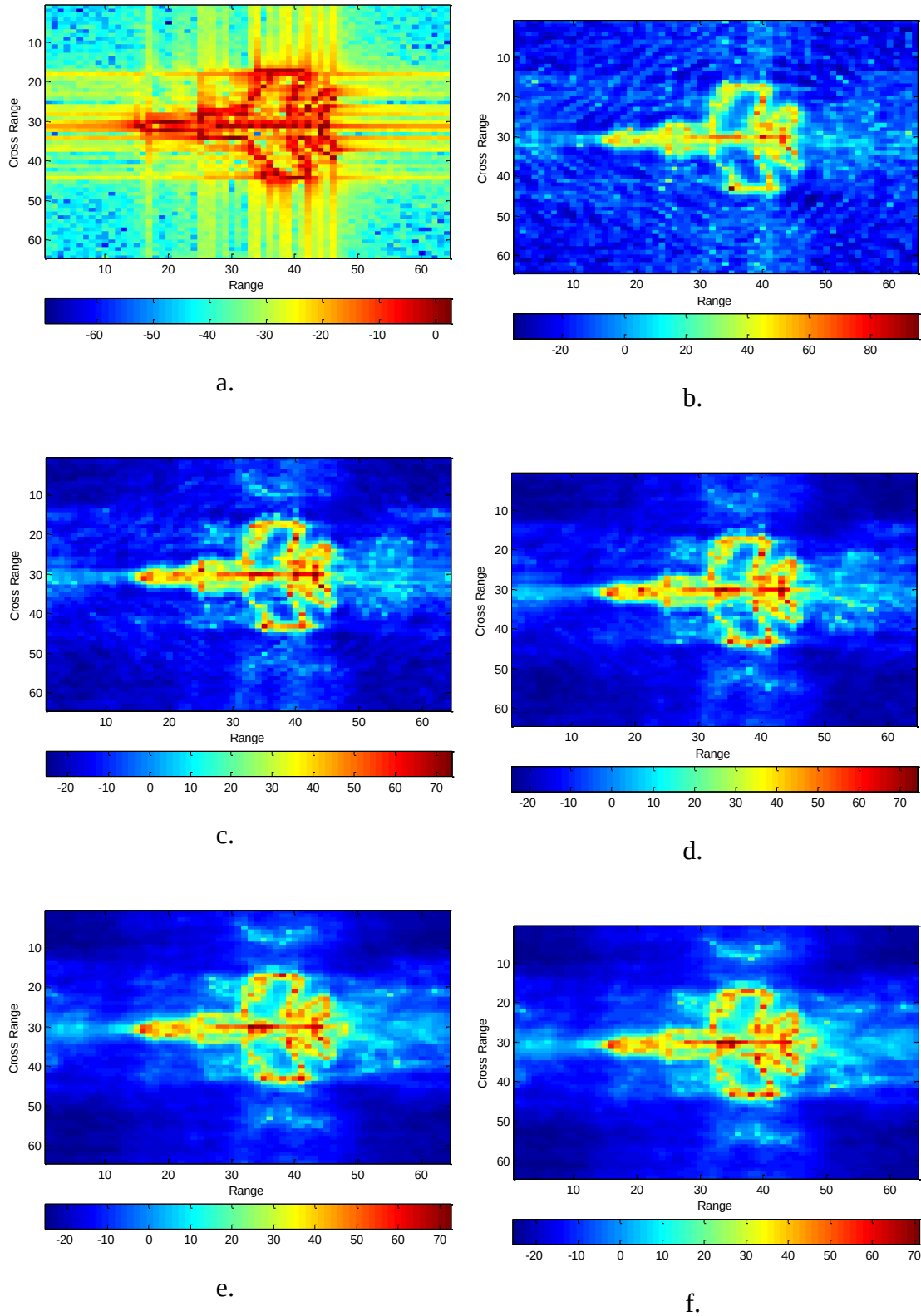


Figure 3.24: Radar image of wideband Mig25 data (SNR=30dB). a.) FFT based method, Sparse AR model based on BPDN with penalty. AR model order=24, penalty b.)0.001 , c.) 0.1, d.) 0.5, e.)1, f.)4.

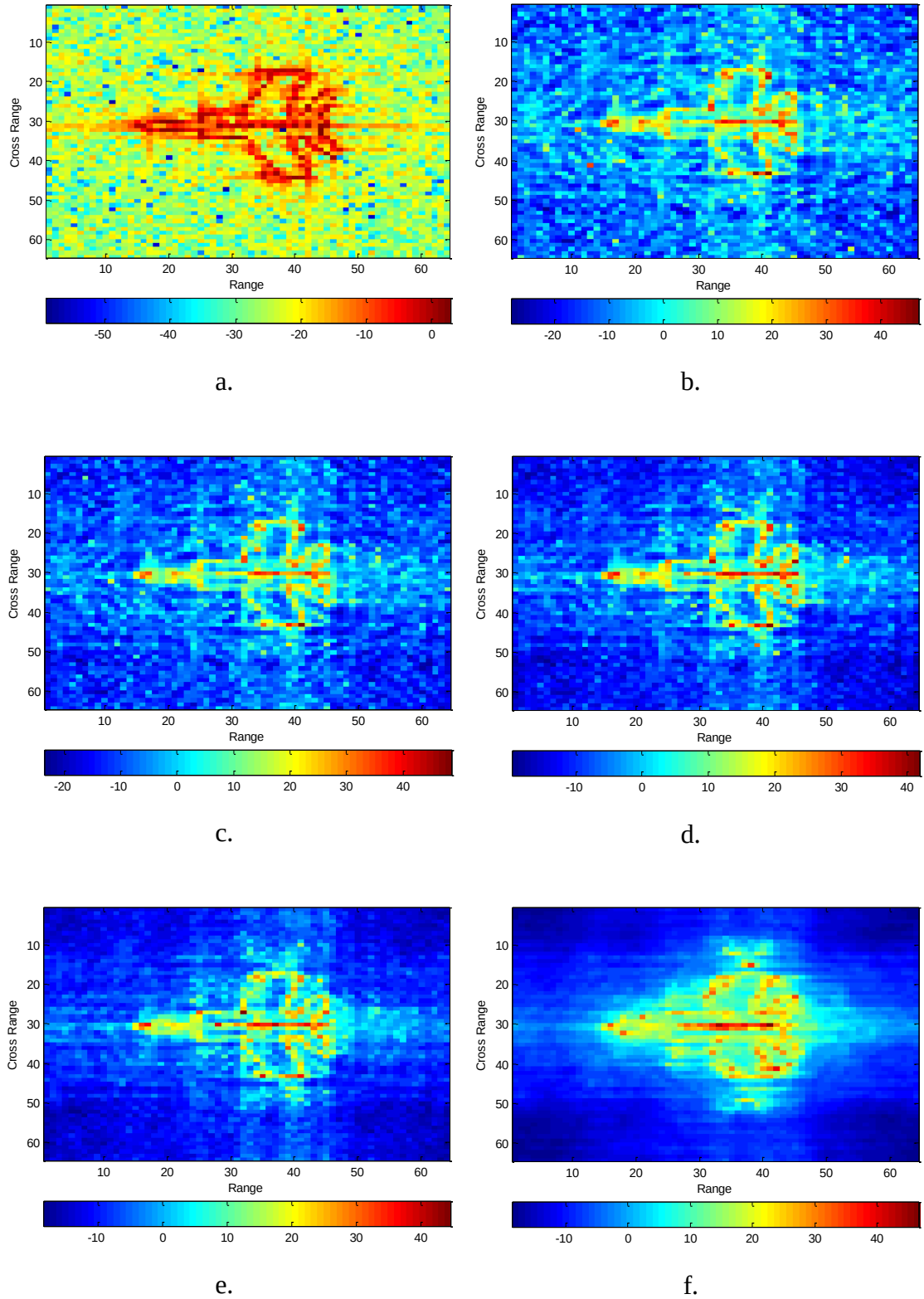


Figure 3.25: Radar image of wideband Mig25 data (SNR=10dB). a.) FFT based method, Sparse AR model based on BPDN with penalty. AR model order=20, penalty b.)0.001 , c.) 0.1, d.) 0.5, e.)1 f.)4.

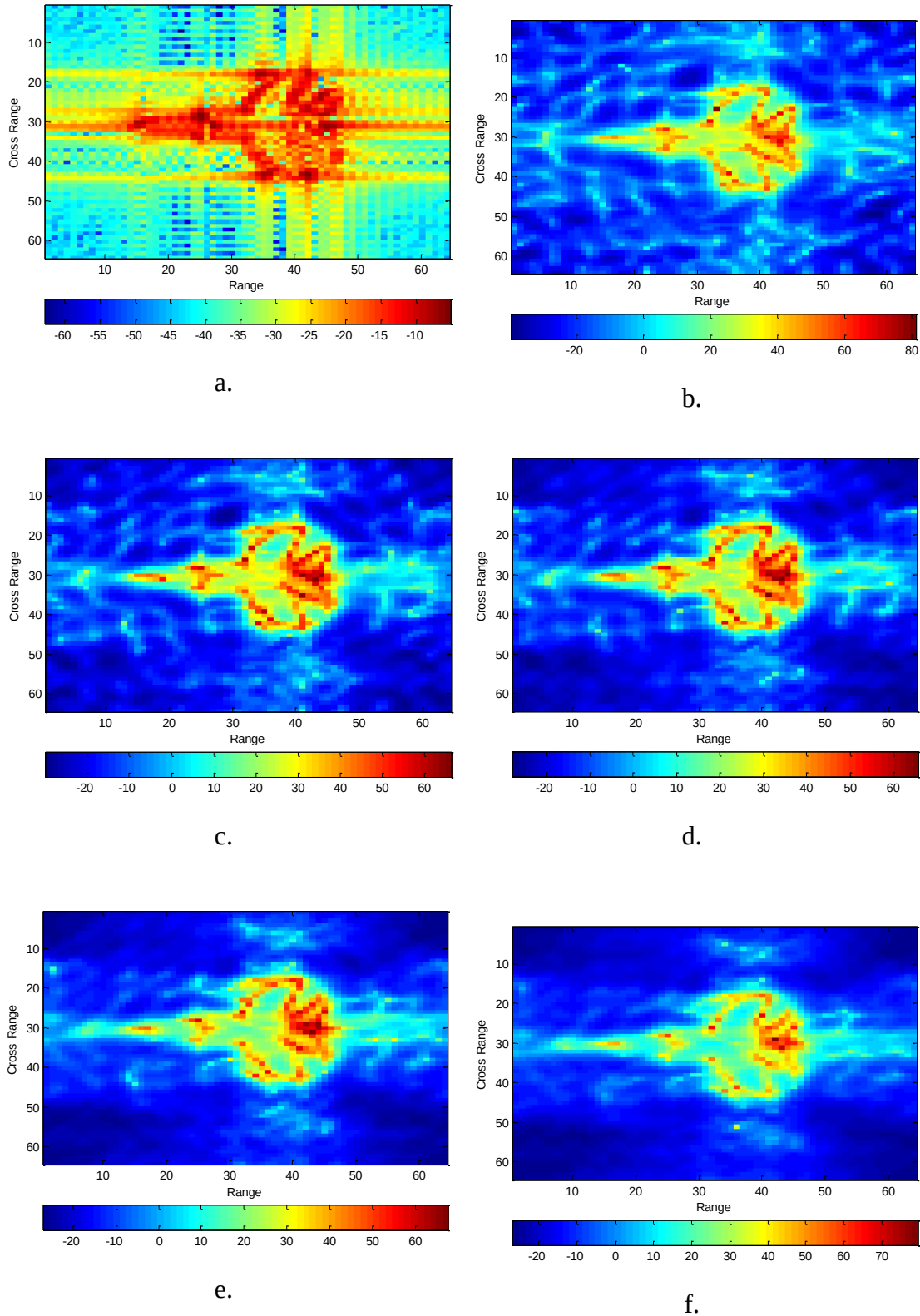


Figure 3.26: Radar image of narrowband Mig25 data (SNR=30dB). a.) FFT based method, Sparse AR model based on BPDN with penalty. AR model order=12, penalty b.)0.001, c.) 0.25, d.) 0.5, e.)1, f.)1.5

▪ Sparse linear prediction based on BPDN and threshold value selection

Unknown coefficients are calculated while cost function which is the l_2 norm of error is smaller than threshold. Initial value of threshold is determined by cost function of least square error solution. It means that firstly, AR model coefficients are calculated from least square solution, then error ϵ_0 , is calculated by using these AR coefficients. This error is the our initial point. Different threshold values are derived from multiplication of ϵ_0 and one constant varies between 1 and 2.

In Fig. 3.27 and Fig. 3.28, range profiles are obtained from AR model coefficients which are calculated by basis pursuit denoising sparsity problem. In Fig. 3.27 b, range profile is obtained when cost function is limited by $\epsilon_0 * 1.01$. Scatterers around the -20 m are merged when threshold value is $\epsilon_0 * 1.05$ in Fig. 3.27c. Location of scatterers cannot be found in Fig. 3.27e and Fig. 3.27f.

In Fig. 3.28, range profiles of F14 plane are presented for different threshold value. As the expected, scatterers will disappear for high threshold value.

For radar image, ϵ_0^1 and ϵ_0^2 initial threshold value are derived from least square error solution.

Fig. 3.29 shows the radar images for different threshold values. In Fig. 3.29b, sidelobes appear when threshold value is determined by multiplying ϵ_1^0 and ϵ_2^0 by 1.01. Sidelobes are suppressed in Fig. 3.29c and Fig. 3.29d when 1.05 and 1.10, respectively multiply initial threshold value. As threshold value is increased, scatterers are merged in Fig. 3.29e and Fig. 3.29f.

In Fig. 3.30, the performance of the methods in low SNR value is investigated. Initial threshold values are multiplied with 1.01 in Fig. 3.30b and sidelobes display in the figure. Sidelobes are suppressed when initial threshold value is multiplied by 1.02 and 1.05 instead of 1.01 and shape of target isn't affected in Fig. 3.30c and Fig. 3.30d. In Fig. 3.30e, distortion is observed at the tail part of plane. As seen in Fig. 3.30e, shape of image is deteriorated because of the high threshold value.

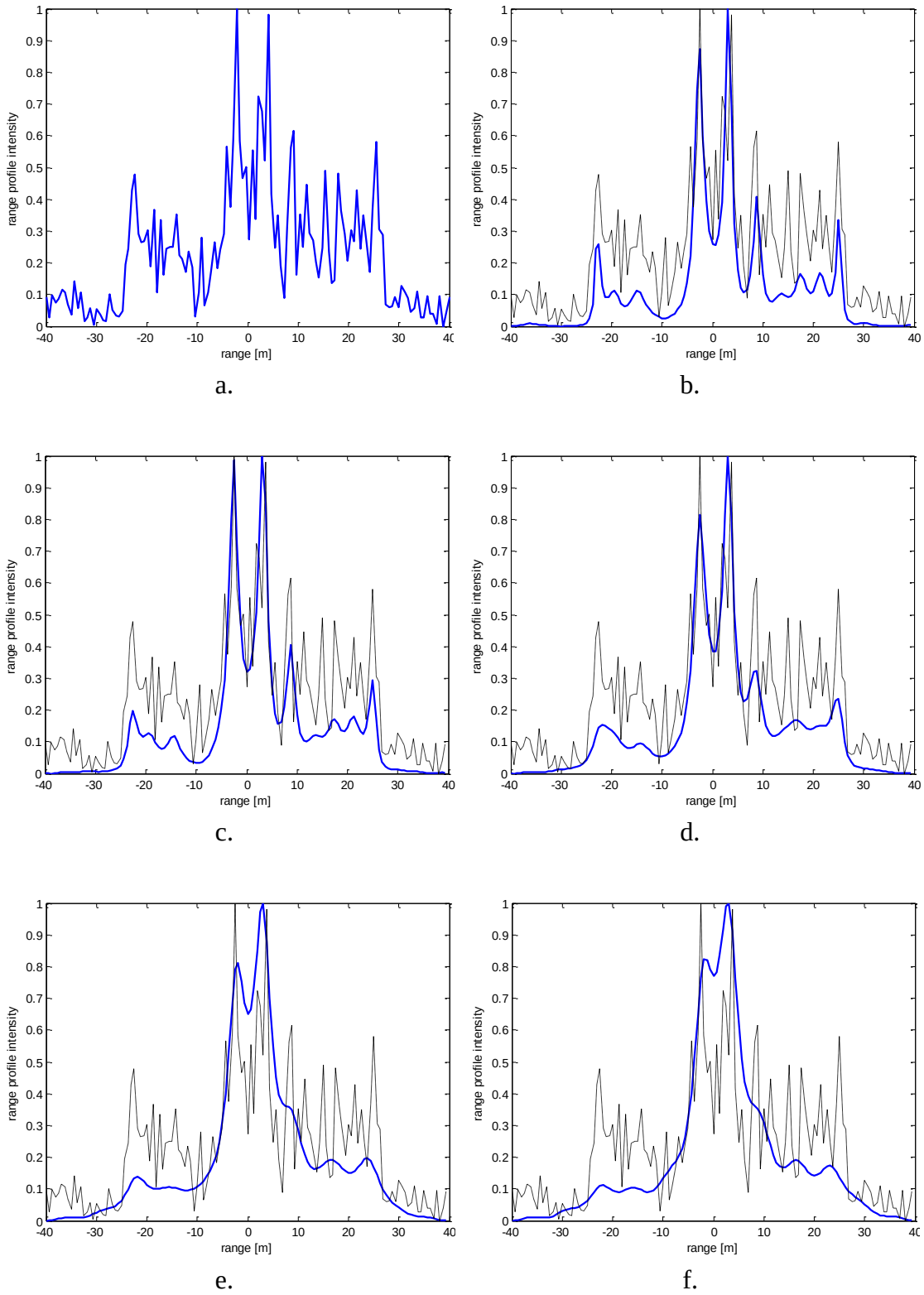


Figure 3.27:Range profile of F117 data (SNR=10dB) a.) FFT based method, Sparse AR model based on BPDN. AR model order=20. Initial threshold value ϵ_0 is multiplied by : b.)1.01 , c.) 1.05 , d.)1.10 , e.)1.20 , f.)1.25.

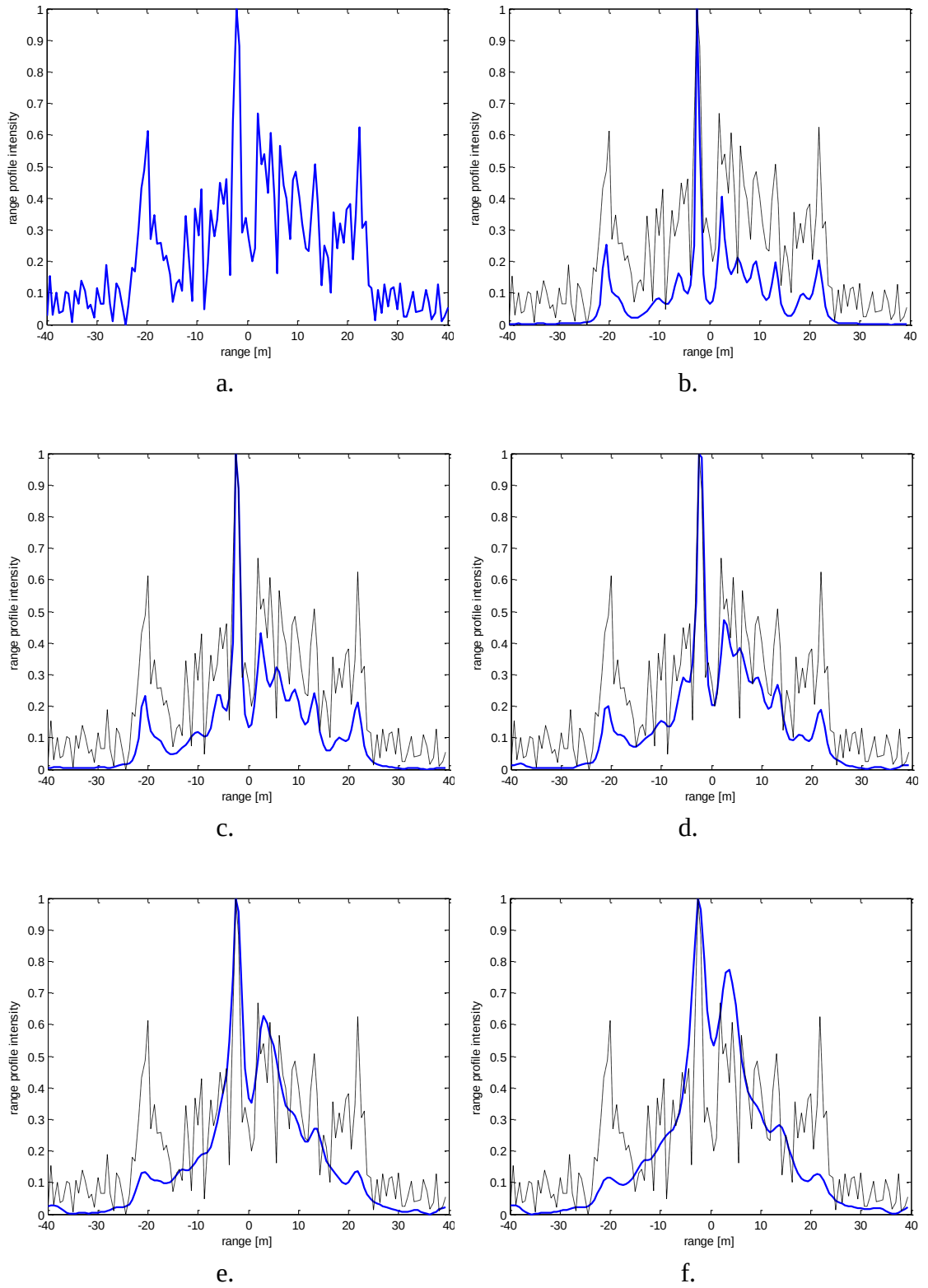


Figure 3.28:Range profile of F14 data (SNR=10dB) a.) FFT based method , Sparse AR based on BPDN. AR model order=20. Initial threshold value ϵ_0 is multiplied by: b.)1.01 , c.) 1.05 , d.)1.10 , e.)1.20 , f.)1.25.

In Fig. 3.31, radar image is reconstructed from collection of narrowband data. First step is to find the error of least square error solution. After finding this, AR model coefficients are determined by solving BPDN optimization problem. In Fig. 3.31b, threshold value is multiplied by 1.01 and radar image is reconstructed. This image is similar to least square solution but sidelobes are suppressed little. Sidelobes are eliminated in Fig. 3.31c and Fig. 3.31d by increasing the threshold value. Although high threshold value suppresses the sidelobes, real scatterers also disappear. In Fig. 3.31e, scatterers between the nose of plane and engine ducts disappear since threshold value is the multiplication of initial threshold value and 1.10. Many scatterers are eliminated in Fig. 3.31f and less information about plane is obtained.

As a conclusion, AR model coefficients are calculated by solving the (2-65) and (2-66). By increasing the threshold, sparse AR coefficients are obtained and better radar images are observed even in narrowband or low SNR cases.

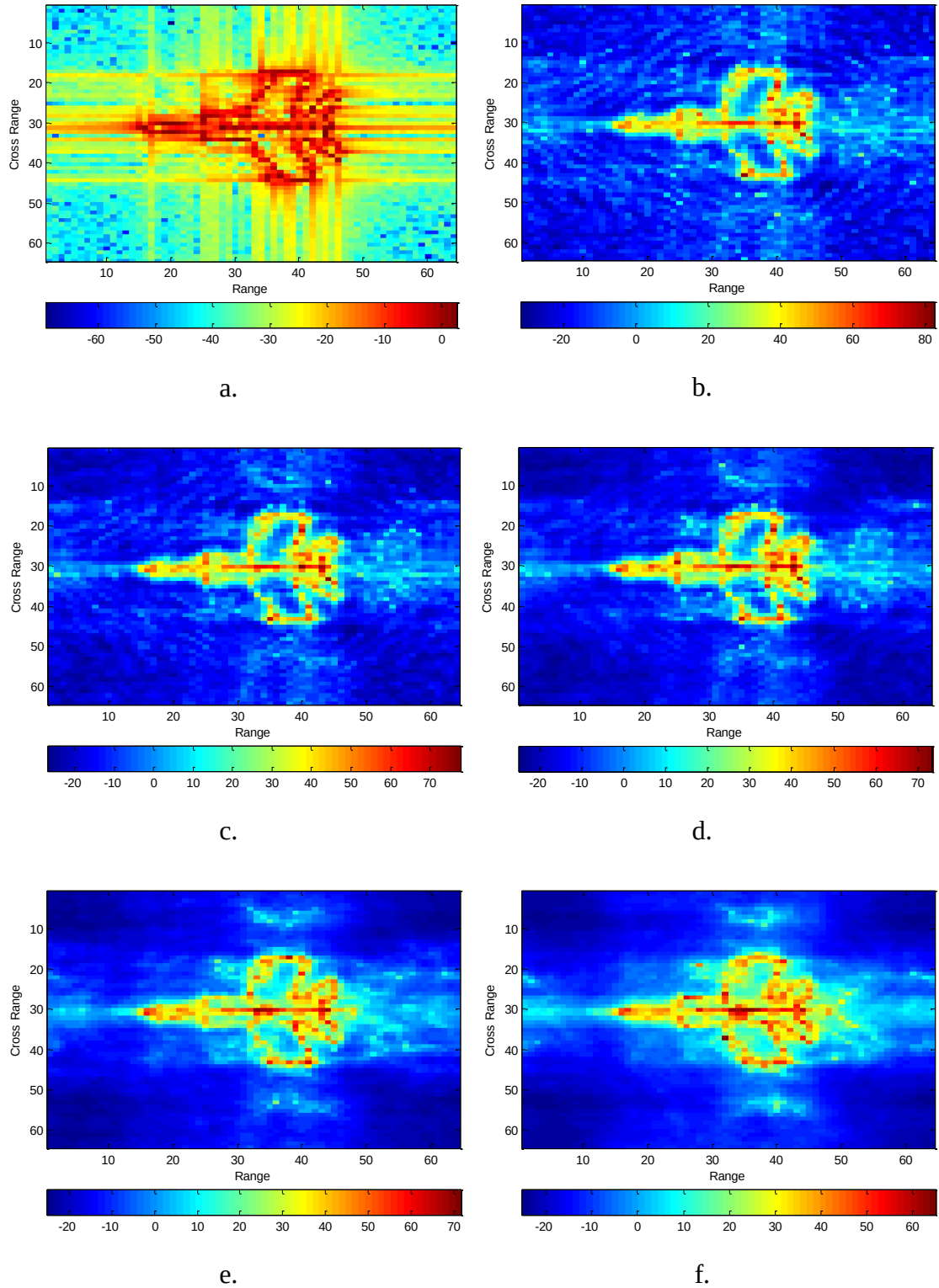


Figure 3.29: Radar image of wideband Mig25 (SNR=30 dB). a) FFT based method, Sparse AR based on BPDN. AR model order=24. Error coefficient: b)1.01, c)1.05 ,d) 1.10 , e) 1.25 , f) 1.5.

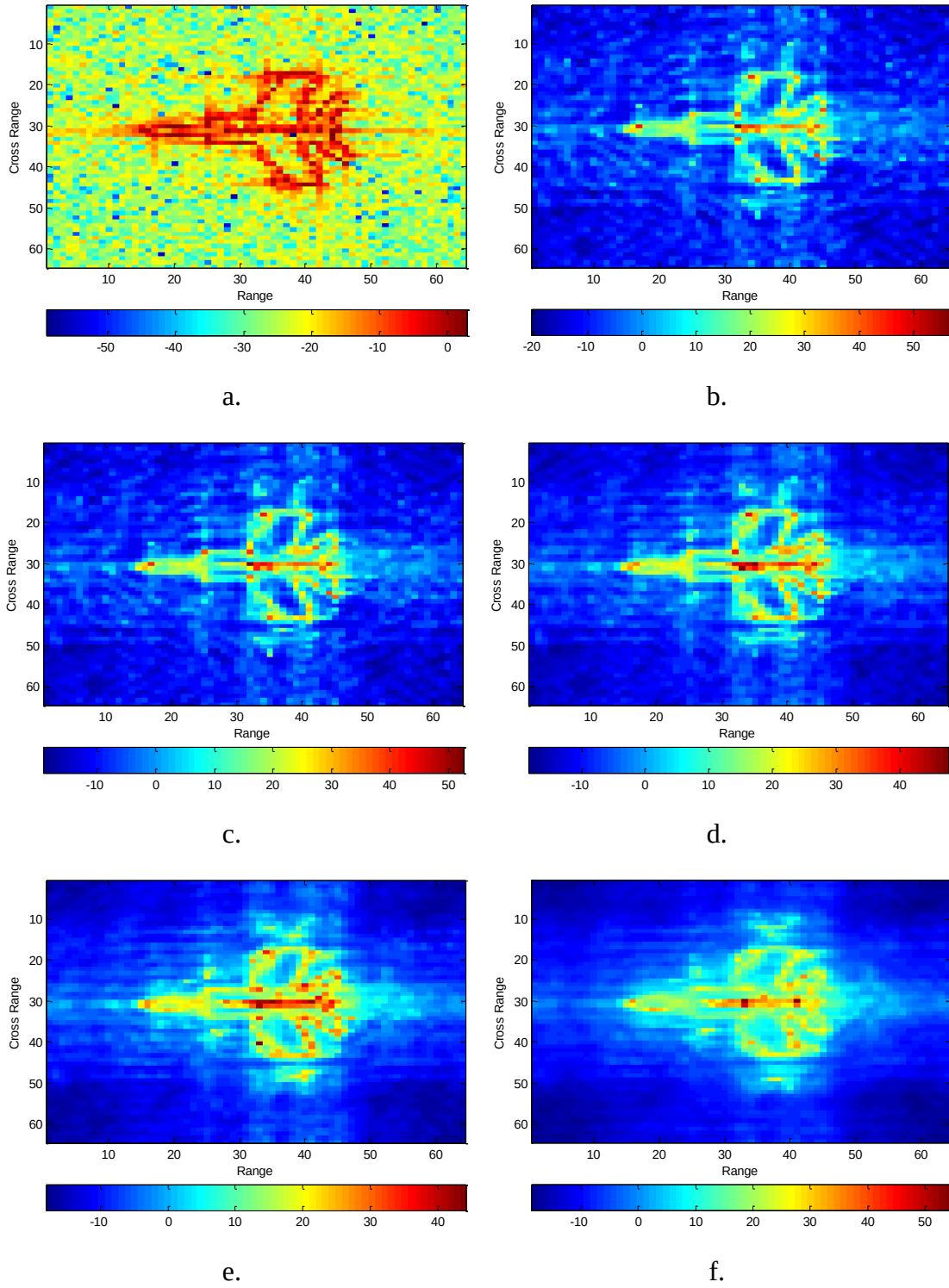


Figure 3.30: Radar image of wideband Mig25 data (SNR=10 dB). a) FFT based method, Sparse AR based on BPDN. AR model order=20. Error coefficient: b)1.01, c)1.02 , d) 1.05) , e) 1.10 , f) 1.15 .

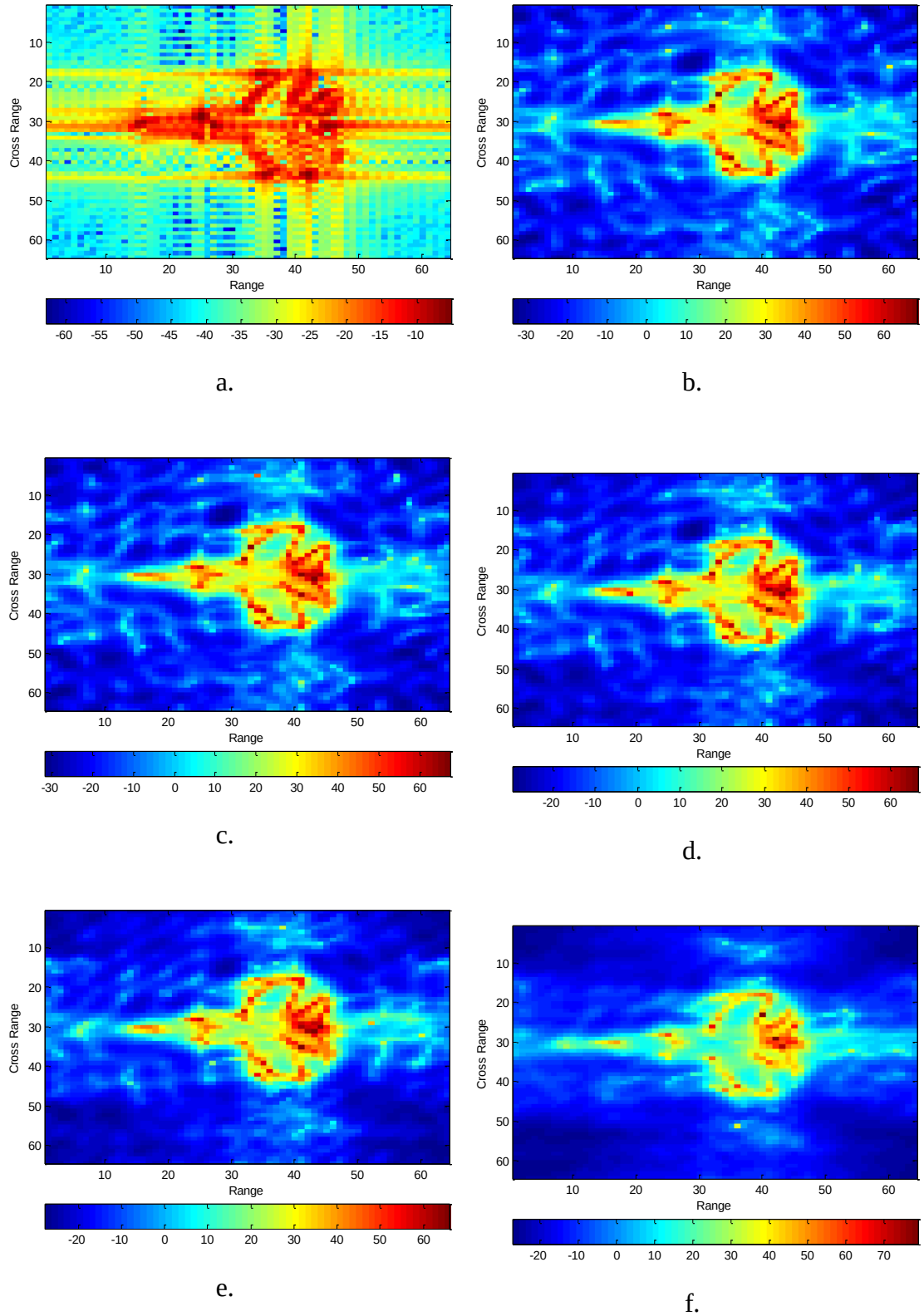


Figure 3.31: Radar image of narrowband Mig25 data (SNR=30 dB). a) FFT based method, Sparse AR based on BPDN. AR model order=12. Error coefficient: b)1.01 , c)1.02 , d) 1.05) , e) 1.10 , f)1.25.

▪ **Sparse linear prediction based on LASSO and threshold value selection**

In this thesis, threshold value in (3-38) and (3-39) is derived from the l_2 norm minimization. Firstly, AR model coefficients are calculated from l_2 norm minimization. Then, it is assumed that l_1 norm of the coefficients is the initial threshold value, $\tau_{1,0}$. By modifying the initial threshold value, problem is solved.

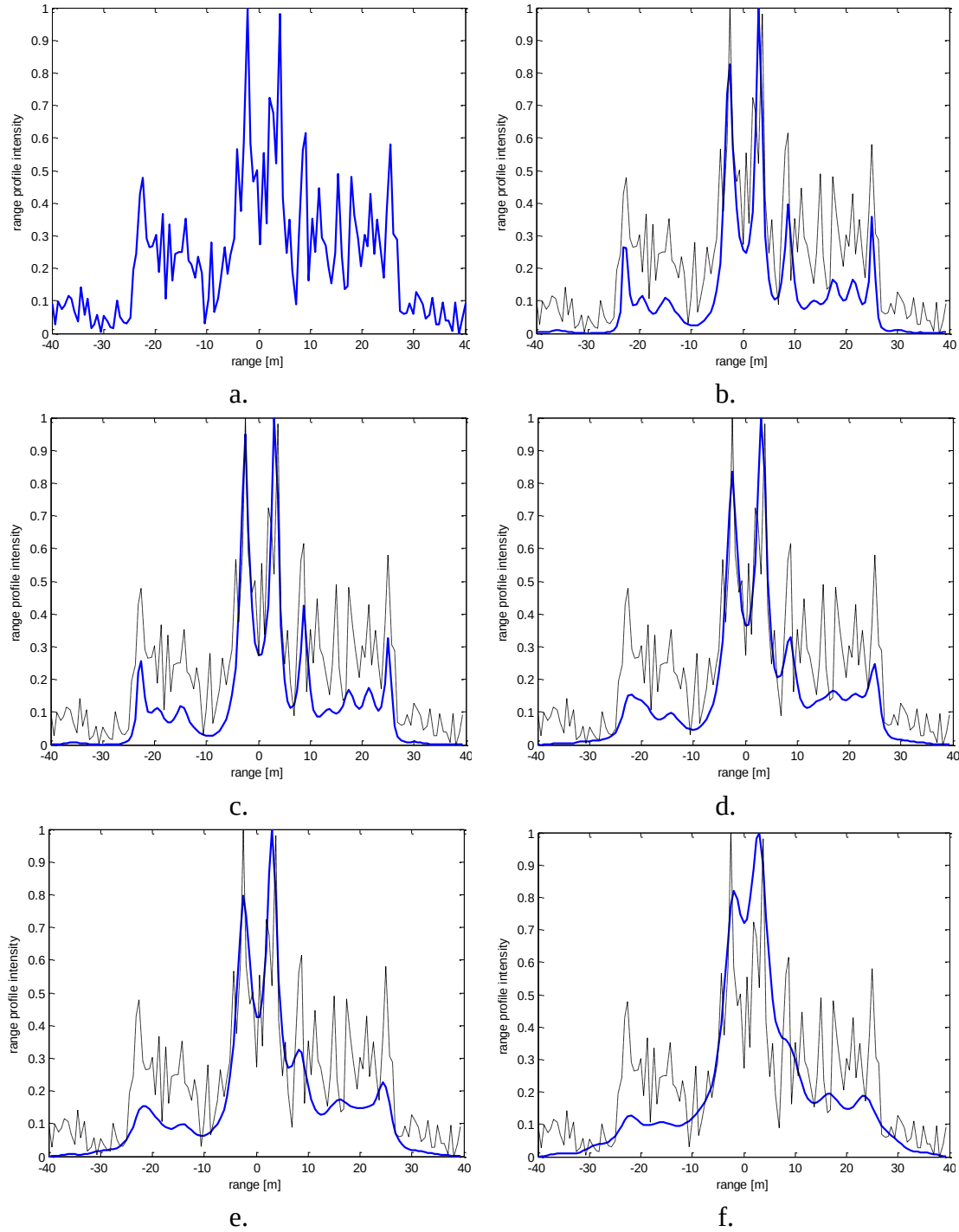


Figure 3.32:Range profile of F117 data (SNR=10 dB) a.) FFT based method , Sparse AR model based on LASSO. Sparse AR order=20 , τ_0 is multiplied by b.)1/1.25 , c.) 1/1.50 , d.)1/2.50 , e.) 1/3 , f.)1/5.

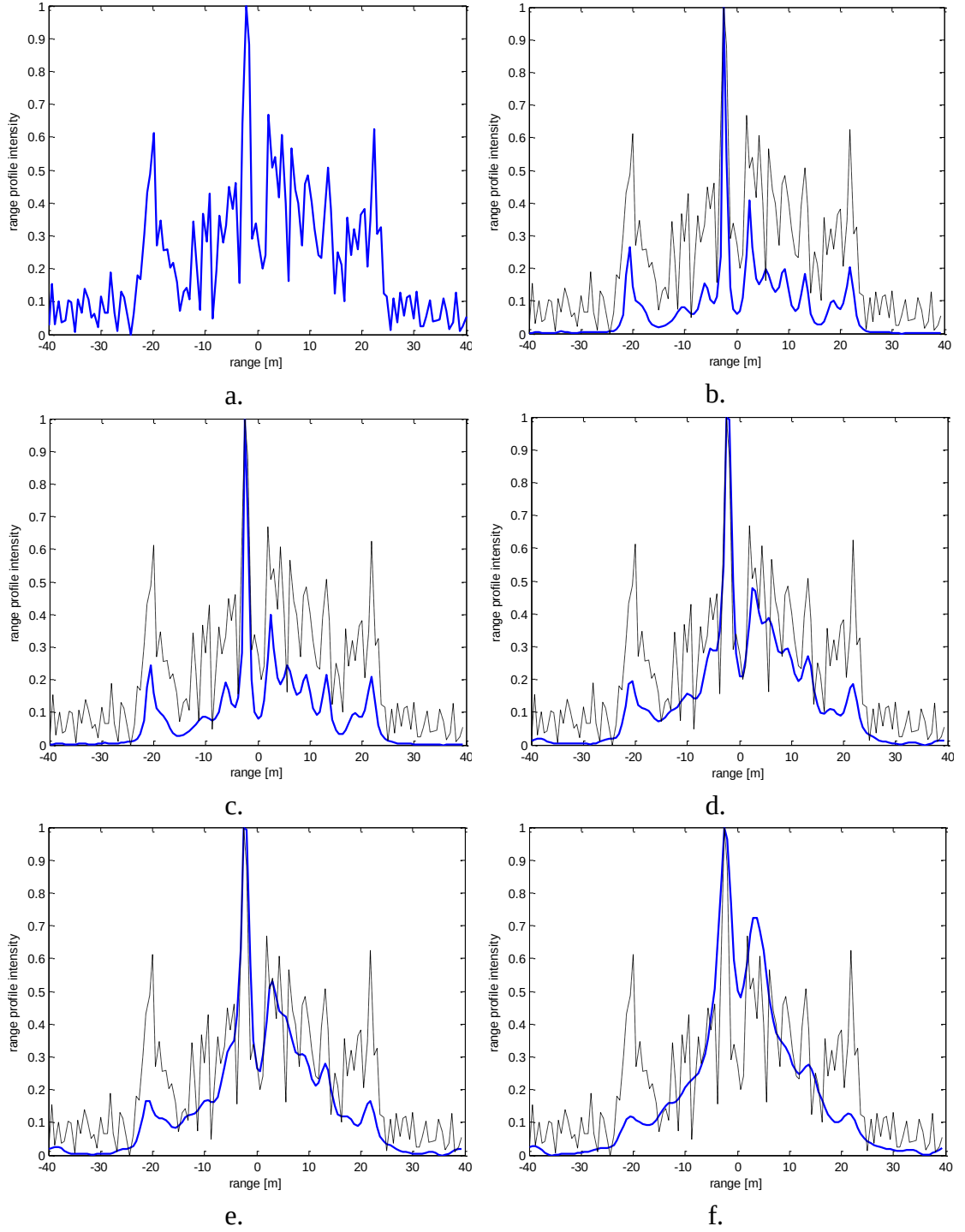


Figure 3.33:Range profile of F14 data (SNR=10 dB) a.) FFT based method, Sparse AR model based on LASSO. Sparse AR order=20, τ_0 is multiplied by b.) 1/1.25 , c.) 1/1.50 , d.) 1/2.50 , e.) 1/3 , f.) 1/5.

In Fig. 3.32 and Fig. 3.33, range profiles are obtained for different threshold values. It is observed that less dominant scatterers are merged as threshold value increases. In Fig. 3.32b,c,d and Fig. 3.33b,c,d , less dominant scatterers disappear by reducing

the threshold value. In Fig. 3.32 e,f and Fig 3.33e,f , dominant scatterers are merged since threshold value is chosen high.

In Fig. 3.34, initial threshold values are found, firstly . Effect of these threshold values on radar image is analyzed. As seen in Fig 3.34b, Fig. 3.34c and Fig. 3.34d, side lobes are suppressed as threshold value reduces. In Fig. 3.34e, scatterers around the wings of the plane disappears when initial threshold value is multiplied by $1/3$. Few dominant scatterers are suppressed in Fig. 3.34f and necessary information about the target may not be obtained.

In Fig. 3.35, radar imaging performance of LASSO is investigated in case of the low SNR. As seen in Fig. 3.35, side lobes are suppressed as threshold value reduces. On the other hand, spurious scatterers cannot be eliminated even if high threshold value is chosen.

In the Fig. 3.36, radar images are constructed with narrow band radar data when AR model coefficients are calculated by using LASSO. As seen in the figure, radar images can be obtained properly when optimum threshold value is chosen. In the Fig. 3.36b,c,d , side lobes are suppressed by reducing the threshold. The last sub-figure shows that scatterers are eliminated if small threshold is chosen.

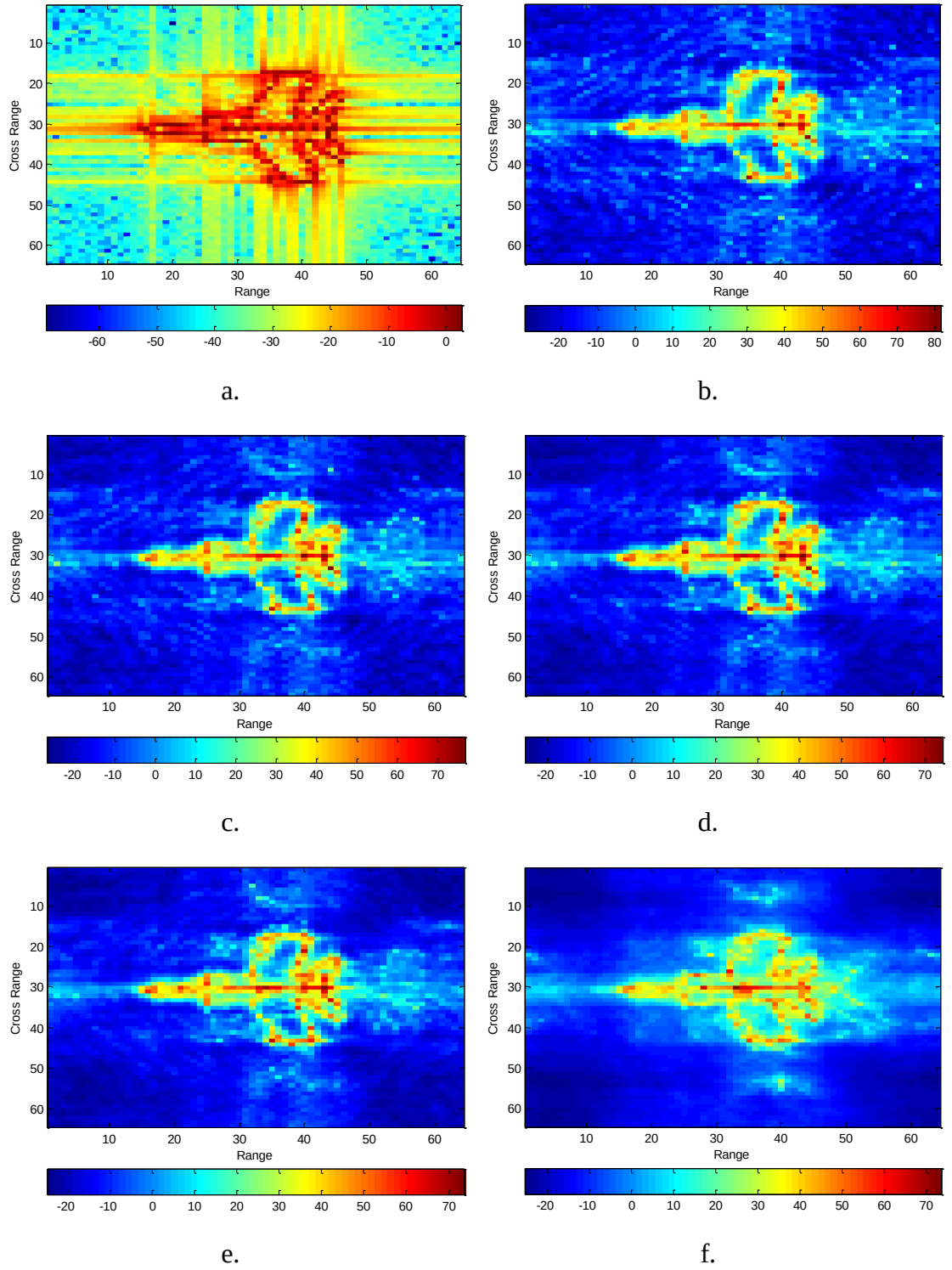


Figure 3.34: Radar image of wideband Mig25 data (SNR=30dB) . a.) FFT based method , Sparse AR model based on LASSO. Sparse AR order=24 , $\tau_{1,0}$ and $\tau_{2,0}$ are multiplied by : b.)1/1.5 , c.)1/2, d.)1/2.5, e.)1/3, f.) 1/6.

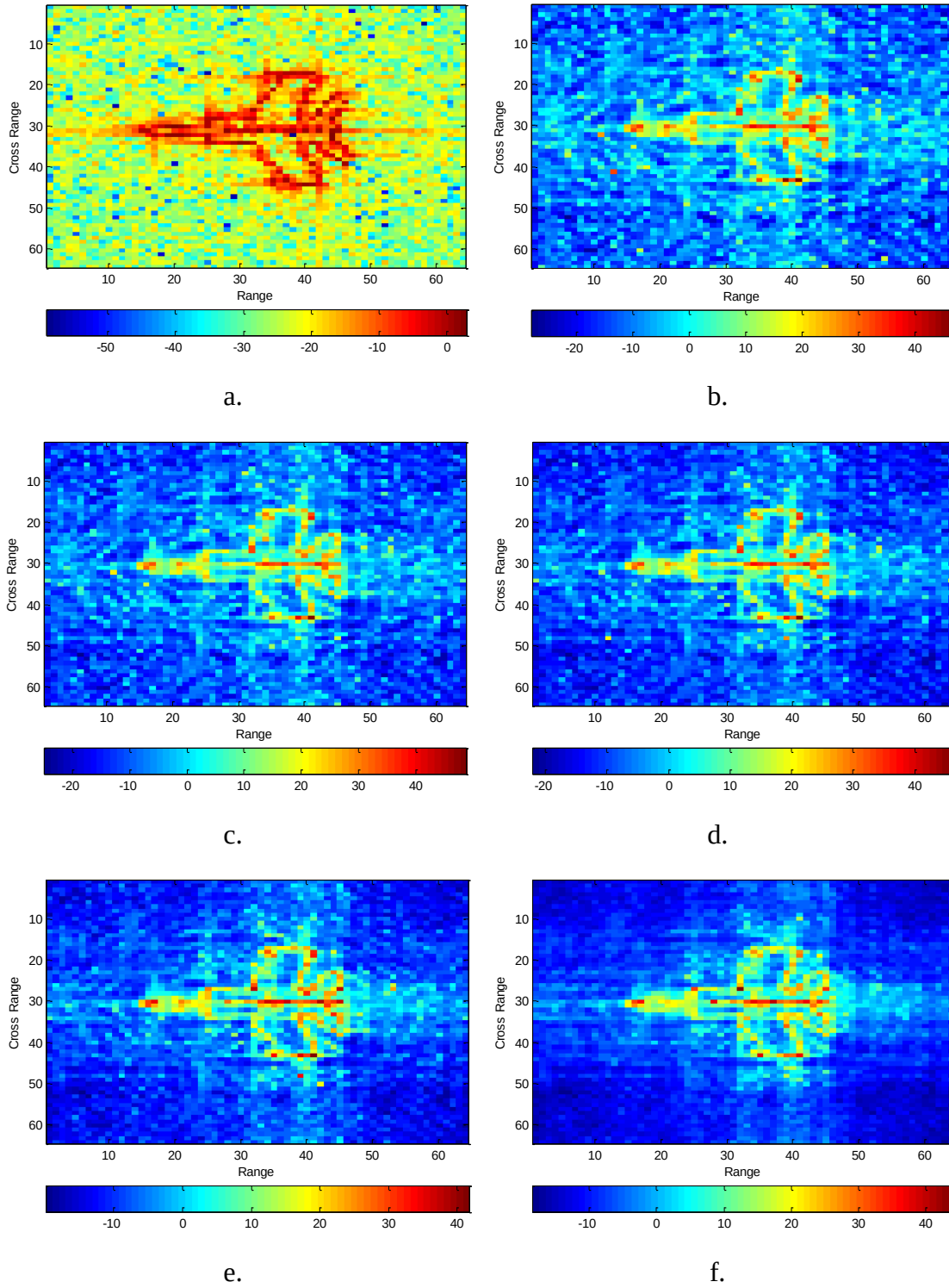


Figure 3.35: Radar image of wideband Mig25 data (SNR=10dB) . a.) FFT based method, Sparse AR model based on LASSO. Sparse AR order=20 , $\tau_{1,0}$ and $\tau_{2,0}$ are multiplied by : b.)1/1.1 , c.)1/1.2, d.)1/5, e.)1/2, f.) 1/3.

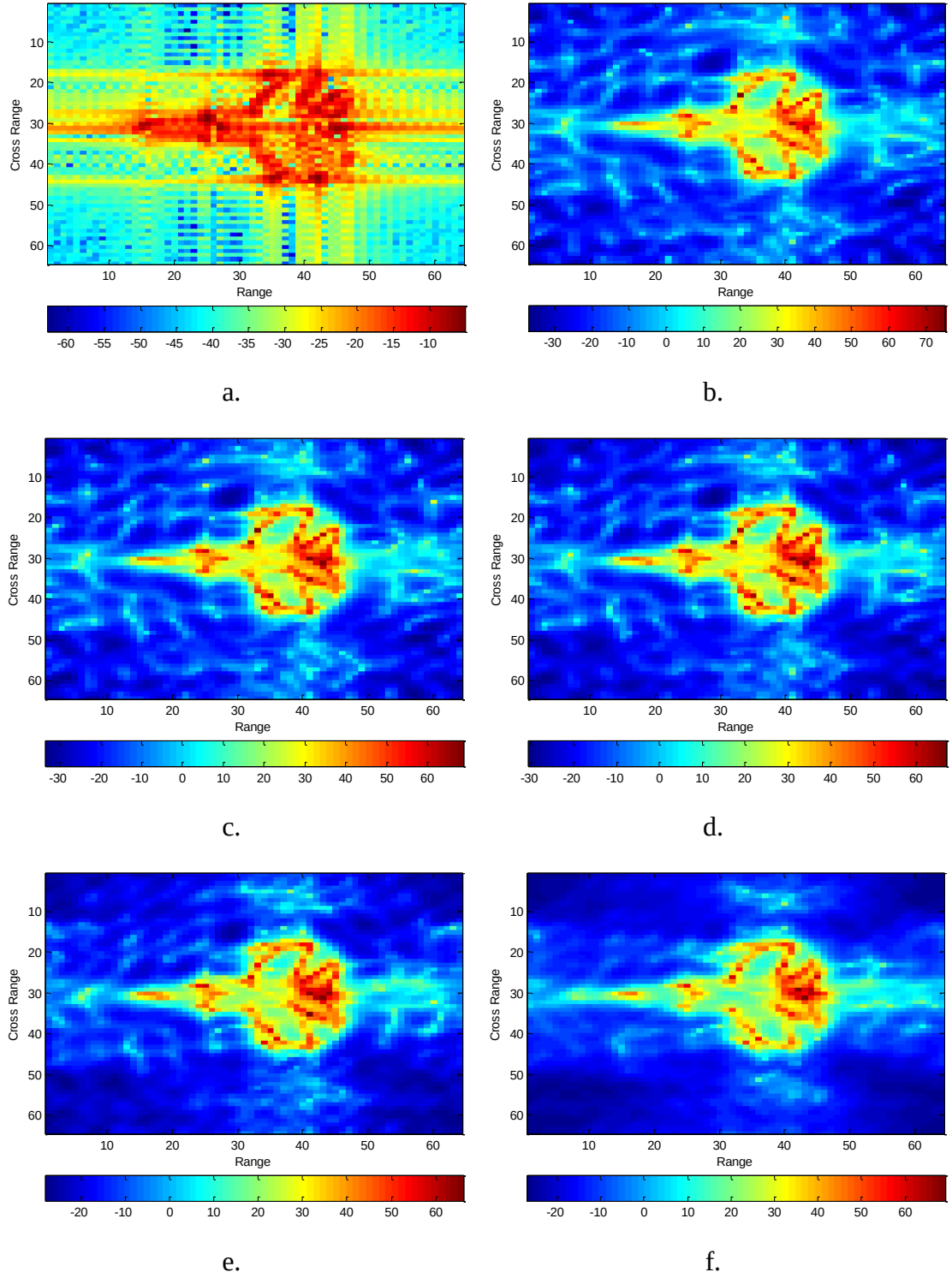


Figure 3.36: Radar image of narrowband Mig25 data (SNR=30dB) . a.) FFT based method , Sparse AR model based on LASSO. Sparse AR order=20 , $\tau_{1,0}$ and $\tau_{2,0}$ are multiplied by : b.) 1.10 , c.) 1/1.25 , d.) 1/1.5 , e.) 1/2 , f.) 1/3.

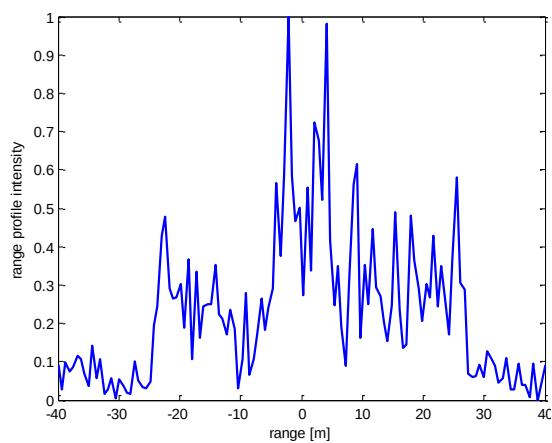
3.3 Comparison of Methods

In this section, range profiles and radar images based on different methods are presented.

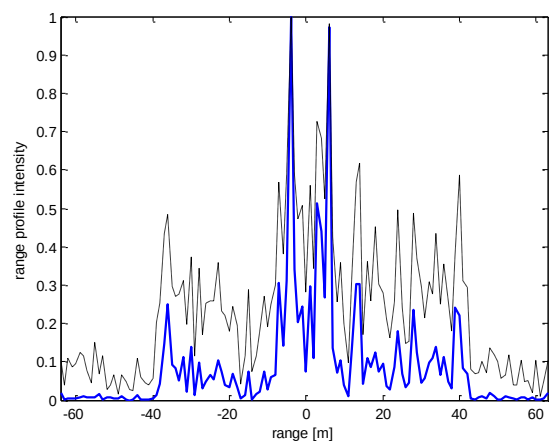
The best methods which provides the clear image and finds the location of scatterers are the sparse representation methods when wide bandwidth and angular width data are available. Then MUSIC algorithm is the second best method which suppresses the side lobes and detects the location of scatterers. If SNR ratio is decreased, sparse representation and MUSIC based radar image give the best result, again. Whenever narrower bandwidth and angular width are used, performance of sparse representation and MUSIC decrease. All scatterers aren't displayed anymore on the radar image based on sparse representation. The detailed information about the target couldn't be obtained from this radar image. In the radar image based on MUSIC, the locations of the most scatterers can be found. Least square solution based AR model causes spurious peak in the image. SVD truncation suppresses the side lobes but distinction of target from background will be harder.

Sparse AR models suppress the side lobes while locations of scatterers are detected properly. Radar images based on sparse AR models are closed to each other so it can be said that imaging performance of BPDN, BPDN with penalty term and LASSO are the same. Sparse AR model methods achieve to resolve the scatterers when bandwidth and angular width are narrow.

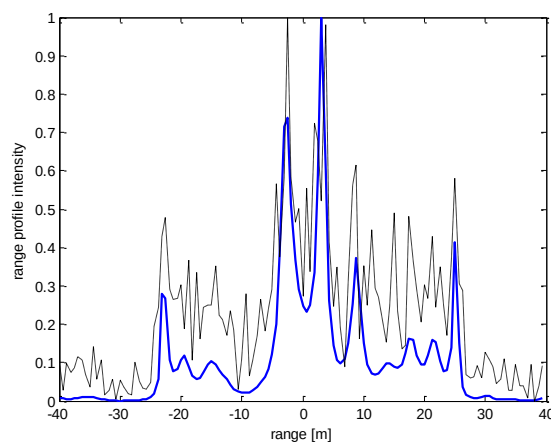
In Fig. 3.37, range profiles of F117 plane based on FFT based method, MUSIC, AR model least square error solution, SVD truncation, sparse linear prediction models, sparse representation models are presented, respectively. As seen from the figure, range profile based on FT based method has many peaks but MUSIC algorithm suppresses these spurious peaks. In Fig. 3.37 h,i,j, sparse representation of range profile are shown. As seen in these figure, choice of threshold value is critic to represent the all scatterers.



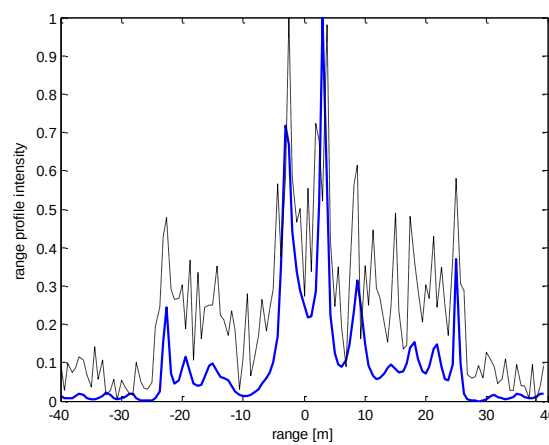
a.



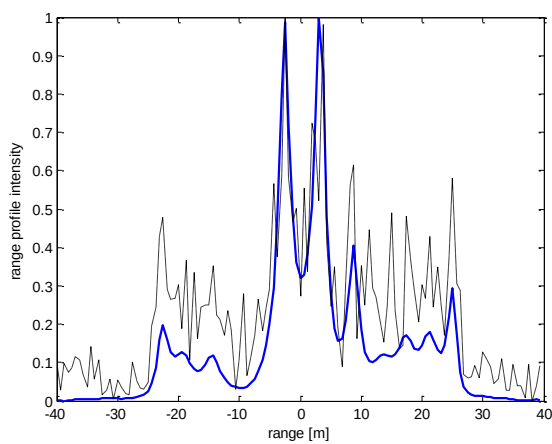
b.



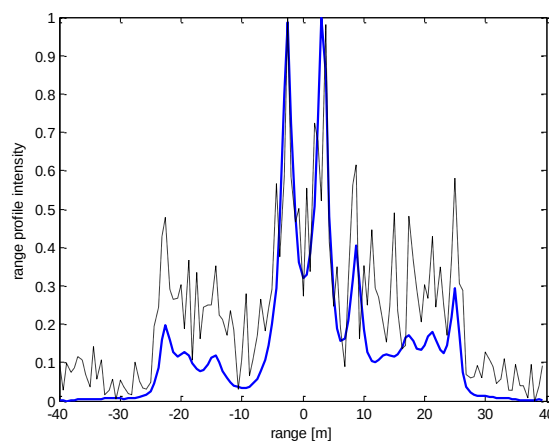
c.



d.



e.



f.

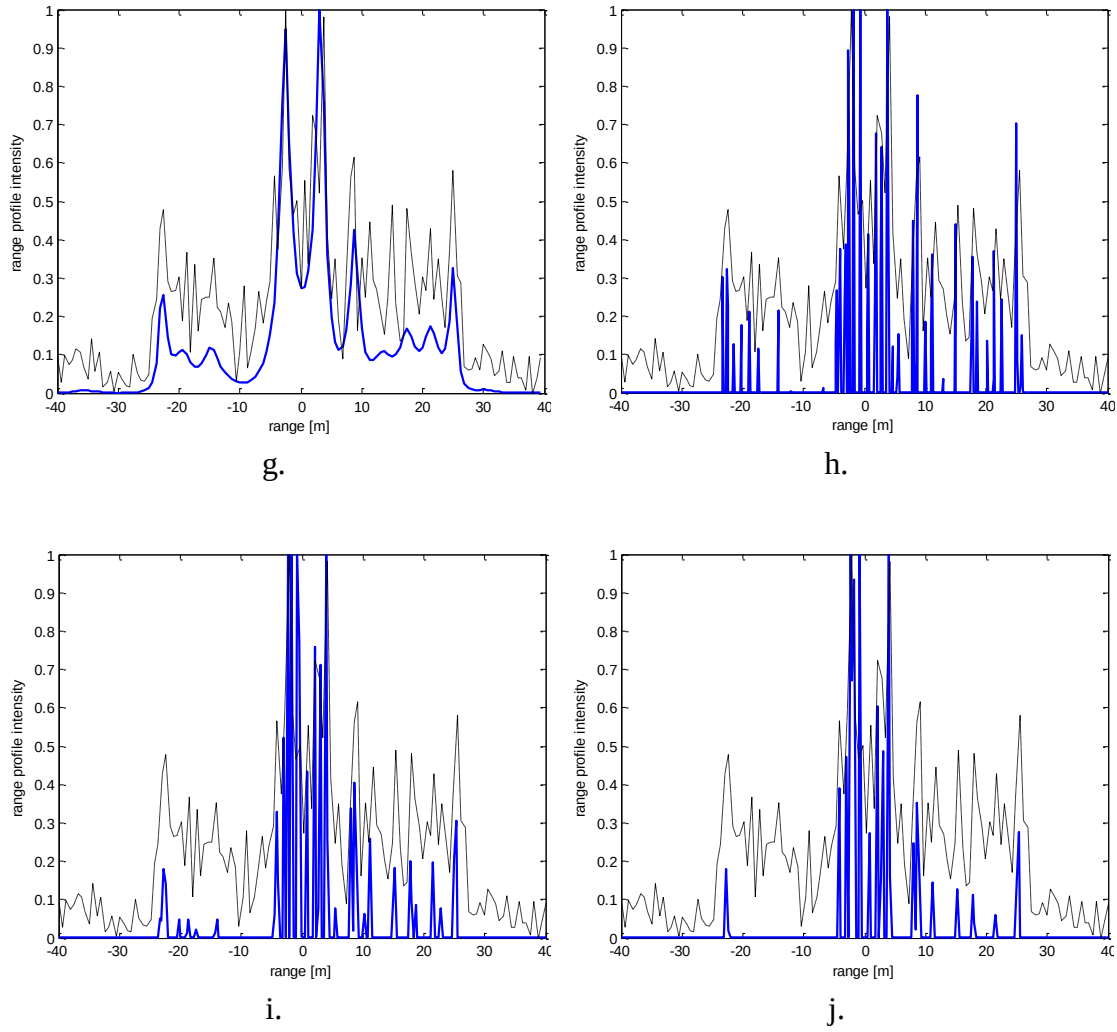
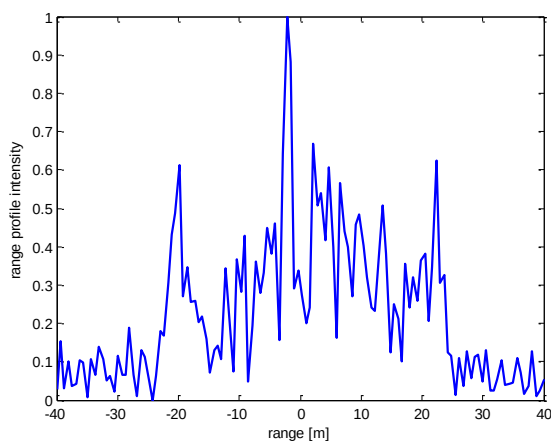
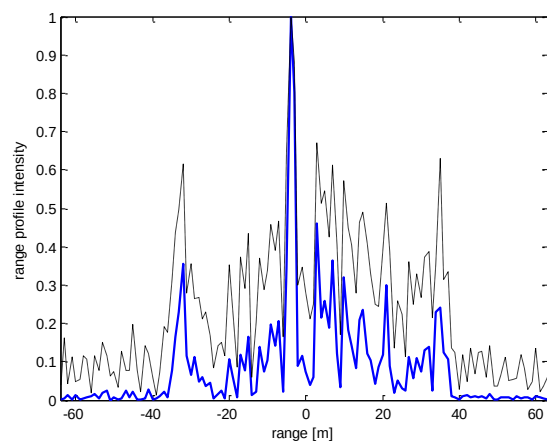


Figure 3.37:Range profile of F117 data (SNR=10 dB) a.)FFT based method , b.) MUSIC, AR model order :24, c.) Least Square Error solution , d.) SVD , Sparse linear prediction based one.)BPDN (1,05) , f.) BPDN with penalty(0.5) , g.)LASSO(0.4) , Sparse range profile representation based on h.) BPDN (200), i.) BPDN with penalty term 1.5, j.) Sparse LASSO when threshold is mean of absolute pixel value.

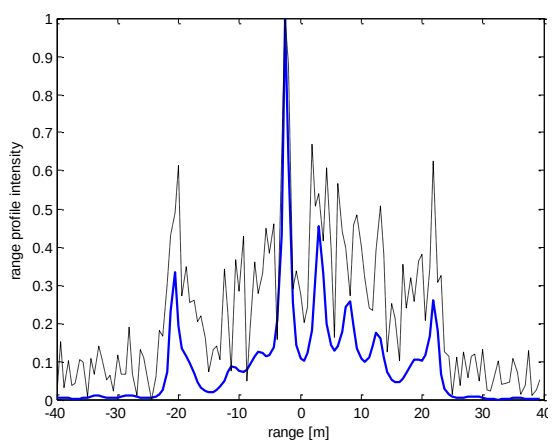
In Fig. 3.38, range profiles of F14 plane based on FFT based method, MUSIC, AR model least square error solution ,SVD truncation , sparse linear prediction models , sparse representation models are presented, respectively. As seen from the figure, range profile based on FT based method has many peaks but MUSIC algorithm suppresses these spurious peaks. Proposed AR models successfully find the location of scatterers



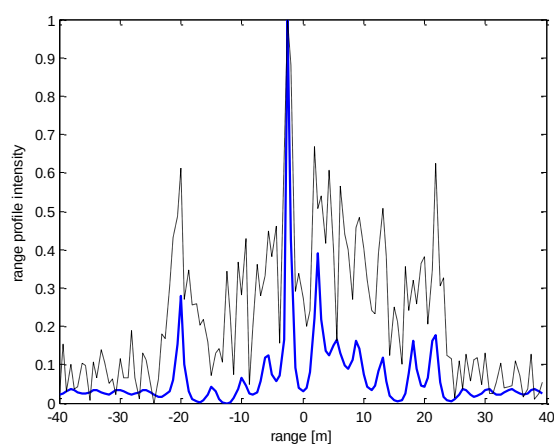
a.



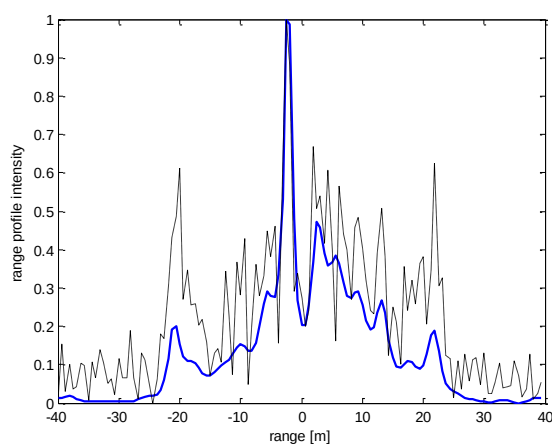
b.



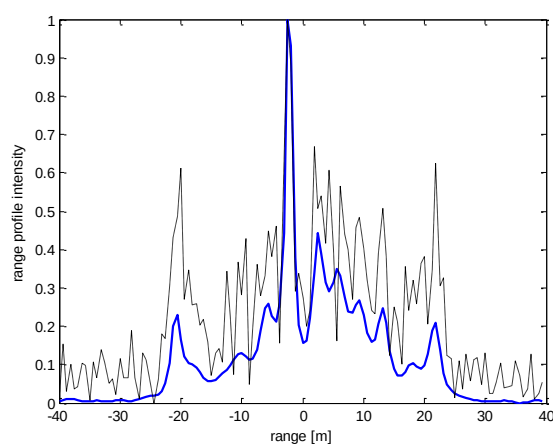
c.



d.



e.



f.

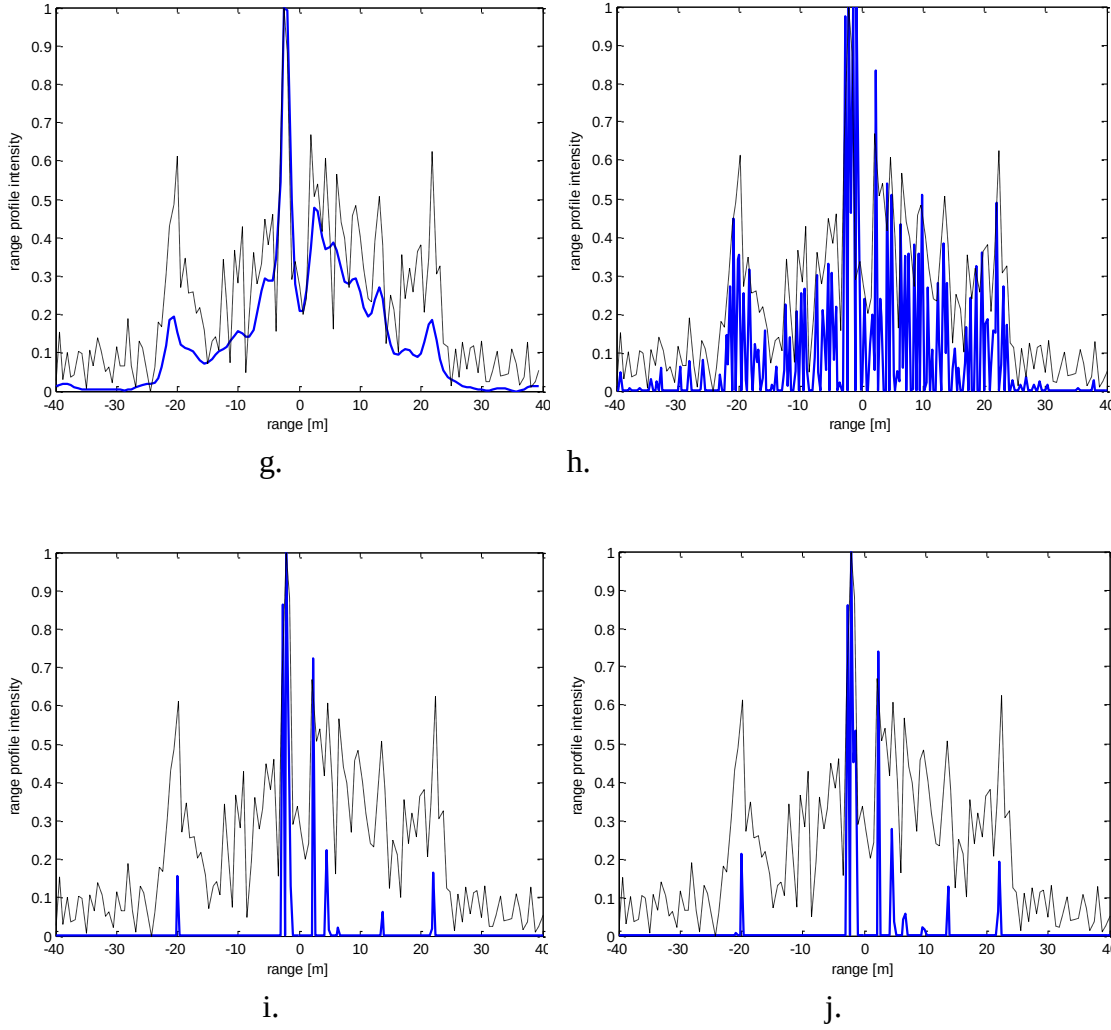
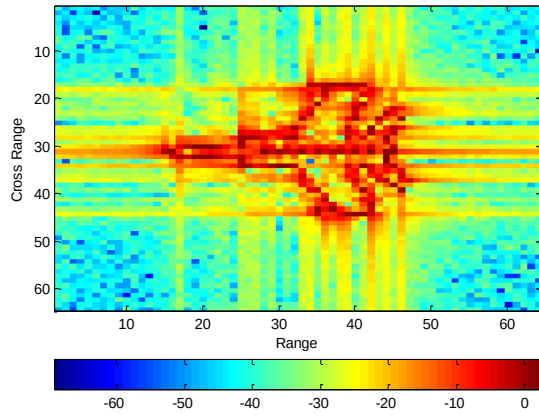
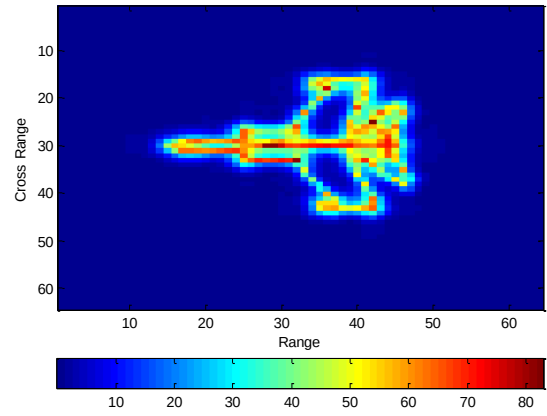


Figure 3.38:Range profile of F14 data (SNR=10 dB) a.)FFT based method , b.) MUSIC, AR model order :24, c.) Least Square Error solution , d.) SVD , Sparse linear prediction based on e.)BPDN (1,05) , f.) BPDN with penalty(0.5) , g.)LASSO(0.4) , Sparse range profile representation based on h.) BPDN (200), i.) BPDN with penalty term 1.5, j.) LASSO when threshold is mean of absolute pixel value.

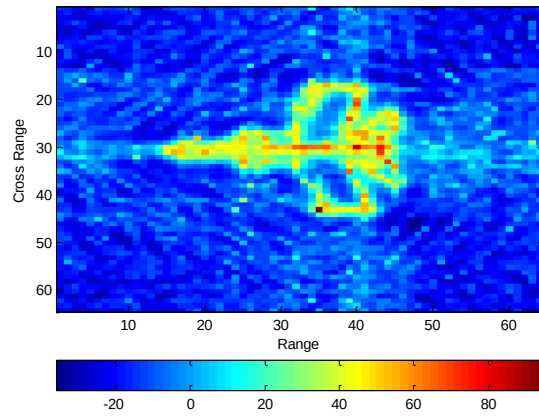
In Fig. 3.39, radar images of Mig25 whose data are collected at wide bandwidth and angular width are presented for different radar imaging techniques. The last 3 subfigures belong to the radar image based on sparse representation. This method provides better images. Almost all side lobes and spurious scatterers are suppressed. MUSIC algorithm yields better results than other spectral estimation methods such as least square error and SVD truncation. The proposed sparse AR model algorithm provides better result with respect to least square error solution and SVD truncation.



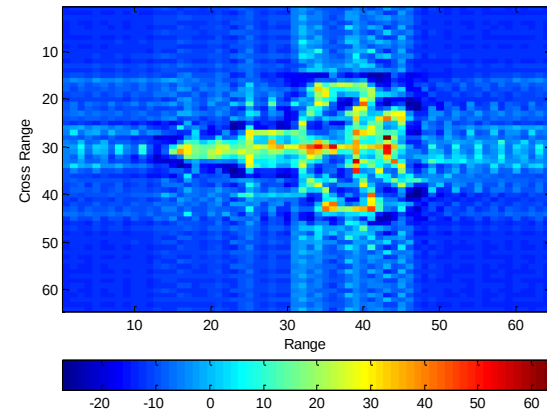
a.



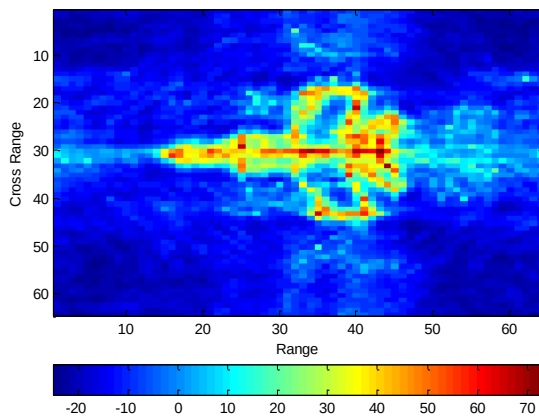
b.



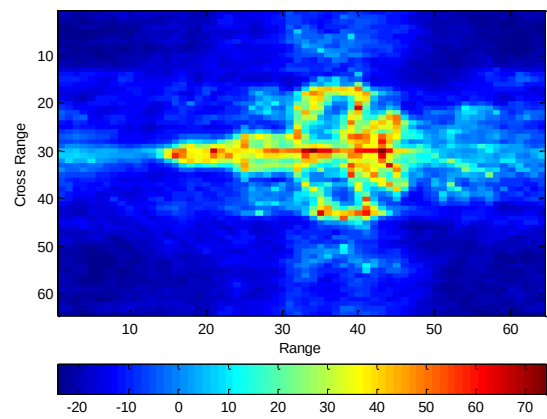
c.



d.



e.



f.

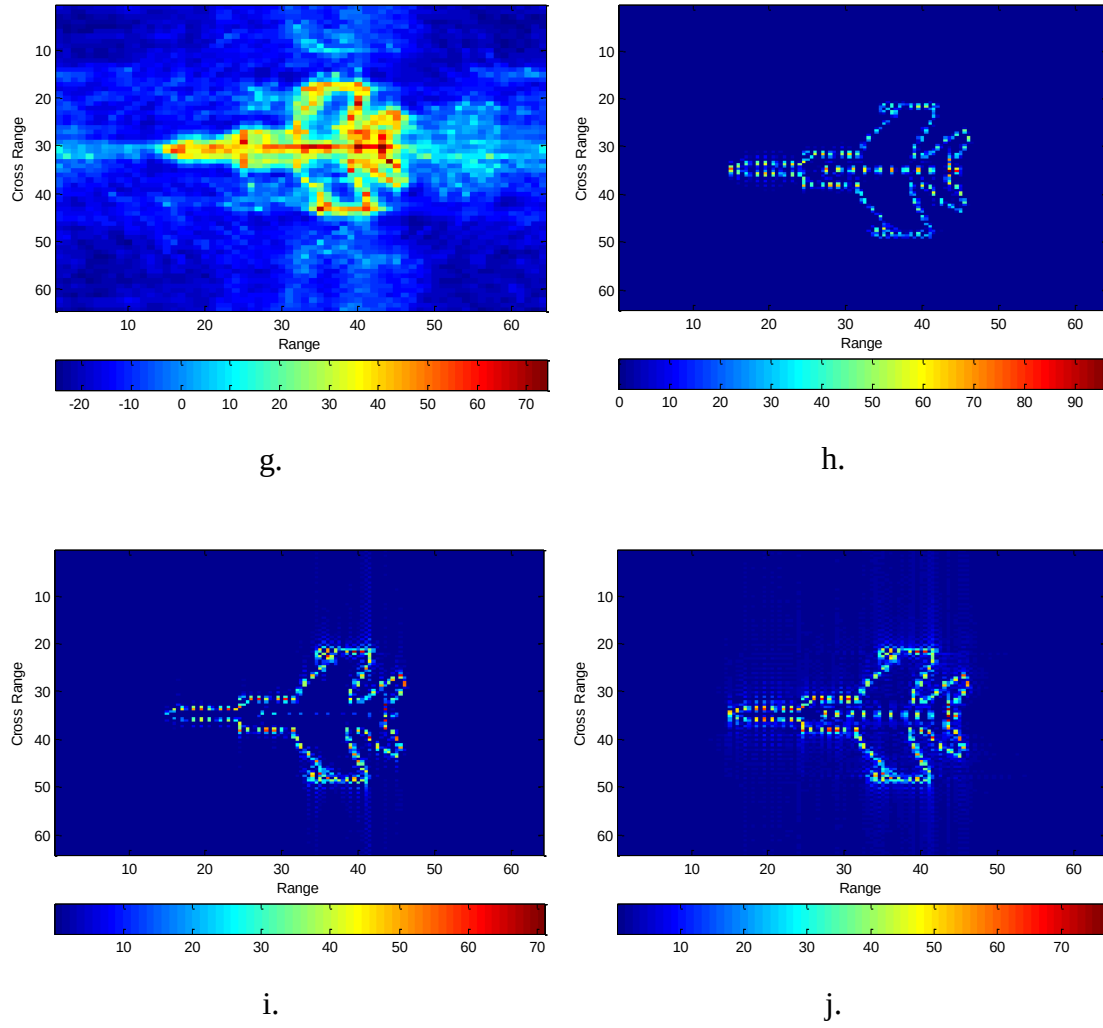
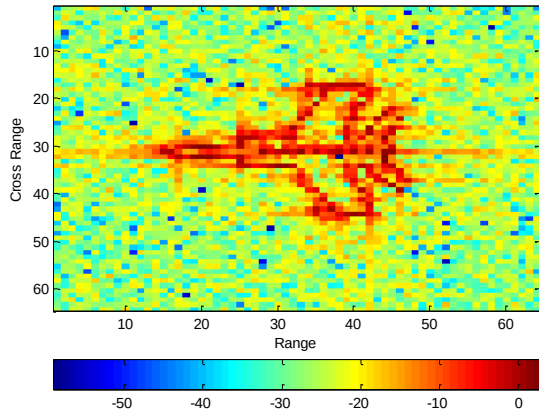
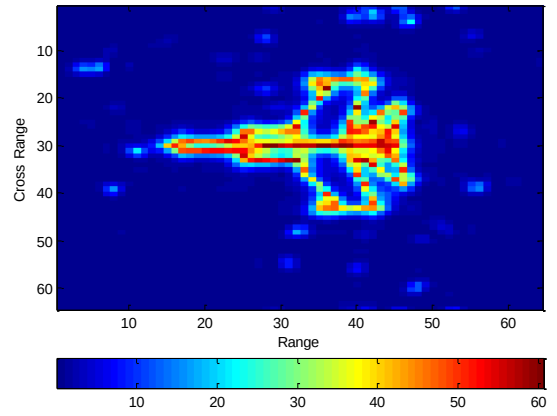


Figure 3.39: Radar image of Mig25 wideband data (SNR=30 dB) a.)FFt based method , b.) MUSIC (smoothing parameter=24), AR model order :24, c.) Least Square Error solution , d.) SVD , Sparse linear prediction based on e.)BPDN (1,05) , f.) BPDN with penalty(0.5) , g.)LASSO(0.4) , Sparse radar image representation based on h.) BPDN (2% deviation), i.) BPDN with penalty term(2) , j.)LASSO when threshold is mean of absolute pixel value.

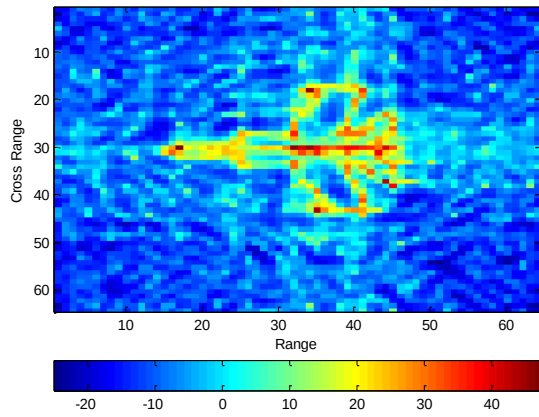
In Fig. 3.40, noise effect on the methods is presented. As seen in the last three sub-figures, sparse representations yield better result with respect to spectral estimation and proposed sparse AR model. In Fig. 3.40b, radar image based MUSIC algorithm is given. MUSIC algorithm fails to disperse the scatterers around the tail of plane. Proposed sparse AR models successfully eliminate the spurious scatterers around the tail of plane. Because of that imaging performance of sparse AR models yields better result than MUSIC algorithm.



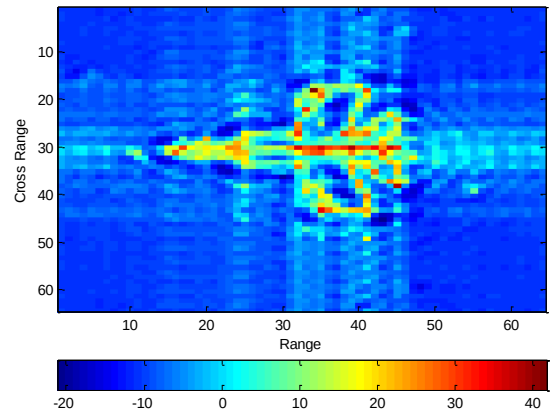
a.



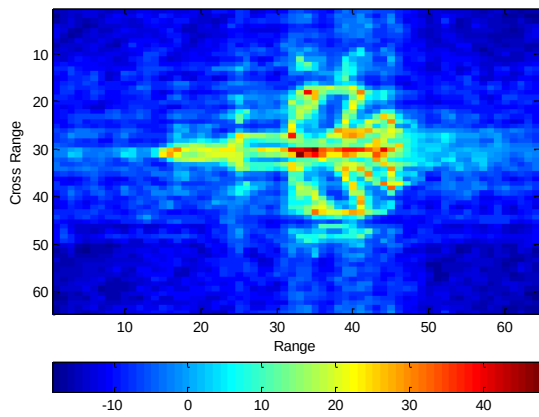
b.



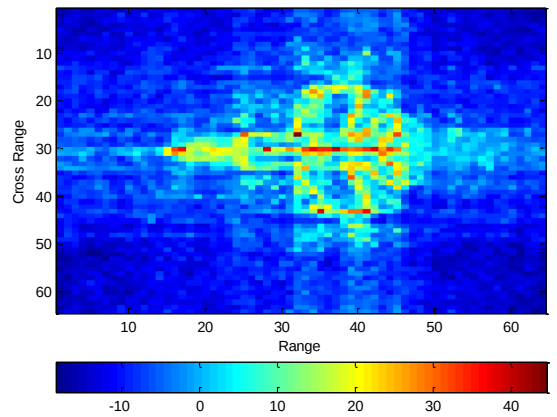
c.



d.



e.



f.

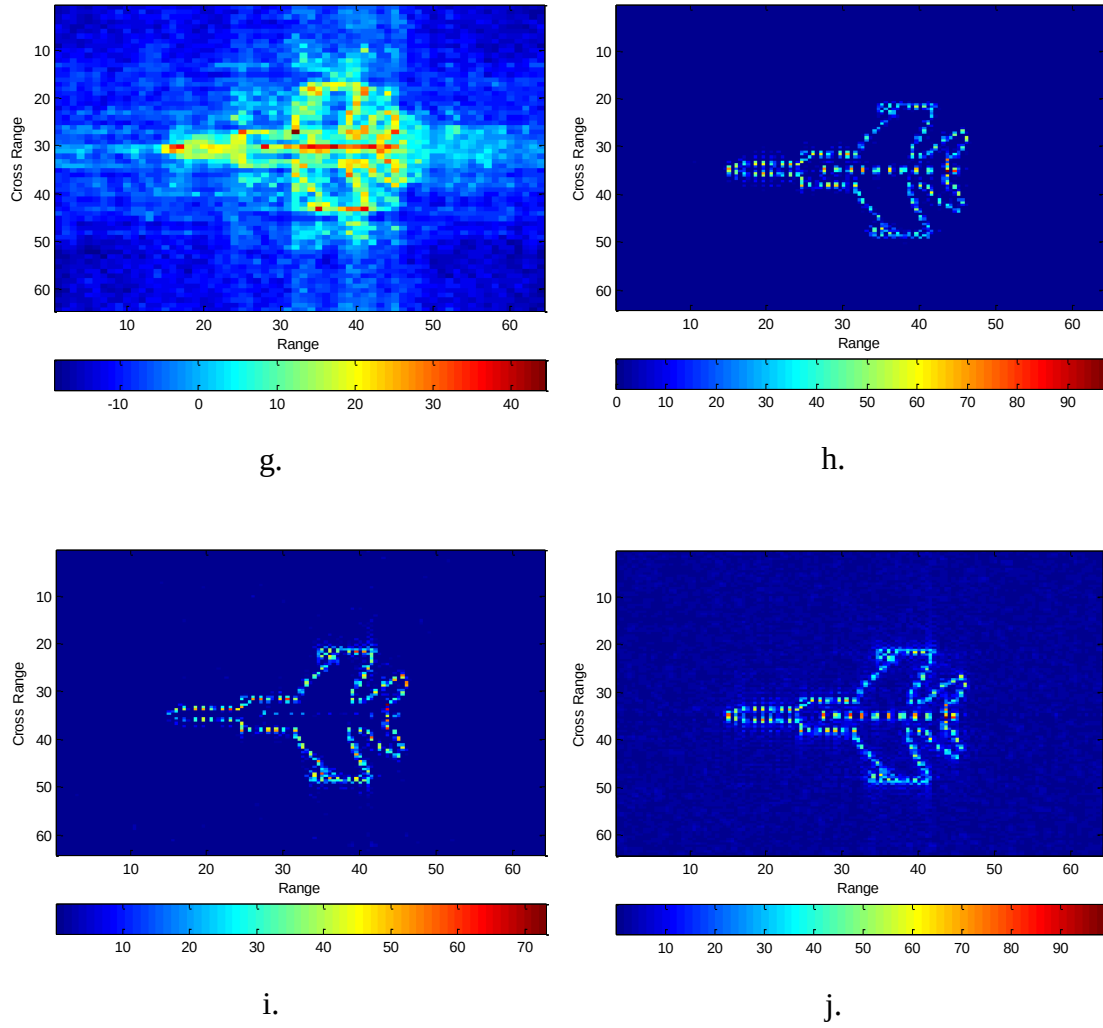
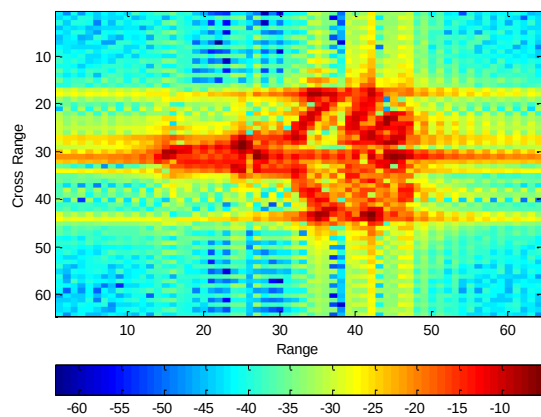


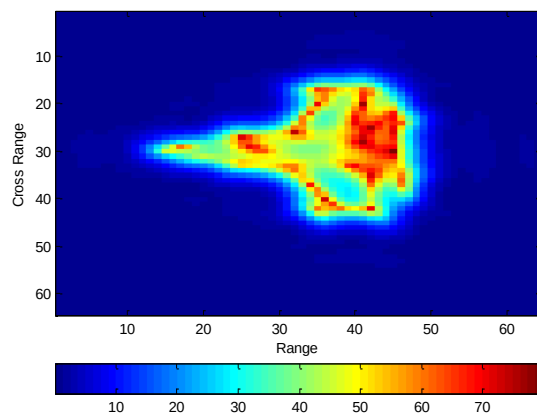
Figure 3.40: Radar image of Mig25 wideband data (SNR=10 dB) a.)FFT based method , b.) MUSIC (smoothing parameter:20), AR model order :20, c.) Least Square Error solution , d.) SVD , Sparse linear prediction based on e.)BPDN (1,05) , f.) BPDN with penalty(0.5) , g.)LASSO(0.4) , Sparse radar image representation based on h.) BPDN (1% deviation), i.) BPDN with penalty term 2, j.)LASSO when threshold is $0.2 \times \text{mean of absolute pixel value}$.

The success of the methods to reconstruct radar image is compared in Fig. 3.41 when narrow band data are available. In Fig. 3.41b, scatterers around the tail cannot be found properly by applying MUSIC algorithm. In Fig. 3.41d, SVD truncation eliminates the real scatterers. Proposed sparse AR models yield better result than the other methods. When least square error solution and sparse AR models are compared, it is observed that side lobes around the target are suppressed by applying sparse AR models. Performance of sparse AR model is better than least square error solution. Sparse AR models achieve to find the scatterers around the tail of the plane while MUSIC algorithm fails. It can be said that sparse AR model yields better result

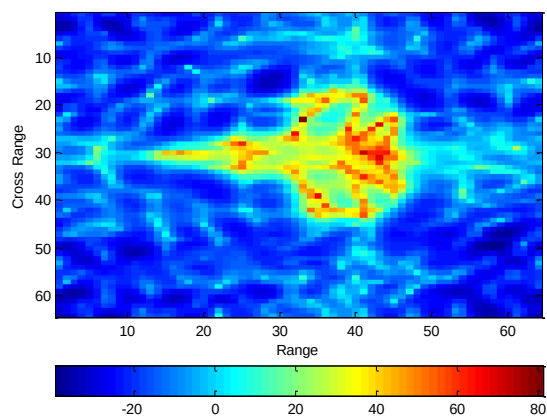
than MUSIC. Sparse image representations fail to find the scatterers when narrow band data are used.



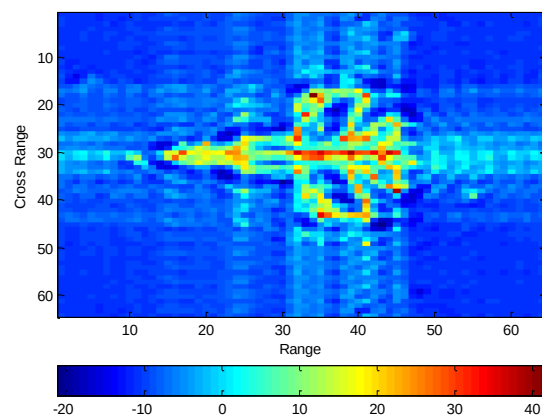
a.



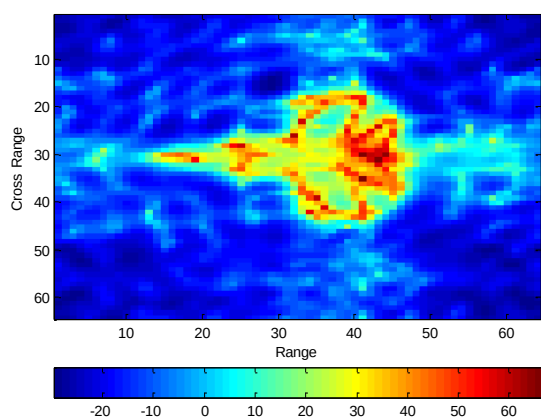
b.



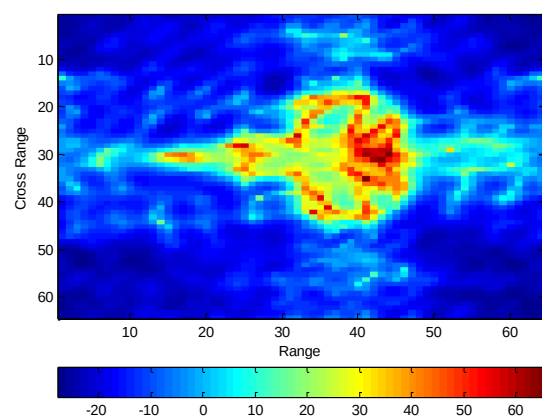
c.



d.



e.



f.

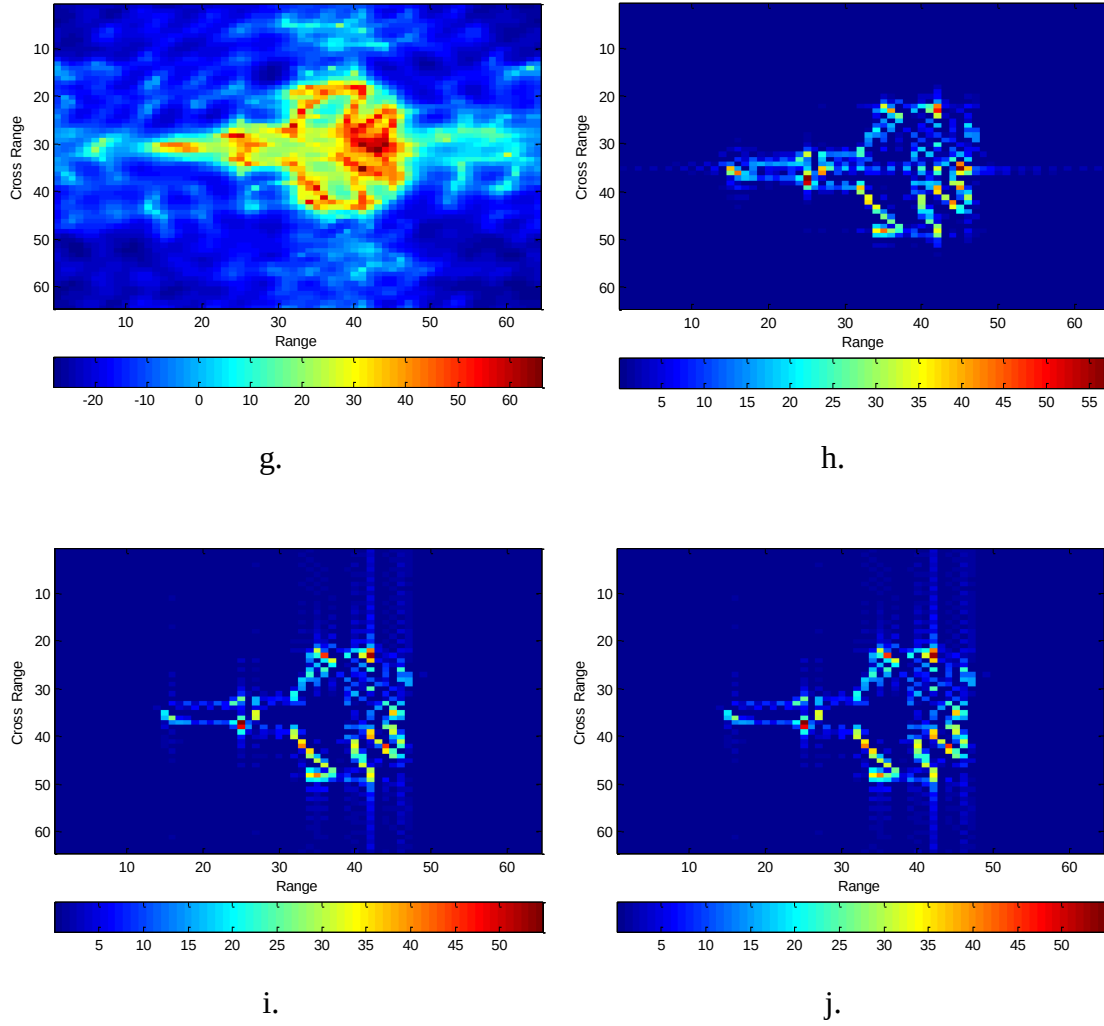


Figure 3.41: Radar image of Mig25 narrowband data (SNR=30 dB) a.)FFt based method , b.) MUSIC (smoothing parameter:12), AR model order :12, c.) Least Square Error solution , d.) SVD ,Sparse linear prediction based on e.)BPDN (1,05) , f.) BPDN with penalty(0.5) , g.)LASSO(0.5) , Sparse radar image representation based on h.) BPDN, i.) BPDN with penalty term 1.5, j.) LASSO when threshold is mean of absolute pixel value.

All methods are applied to different data and radar images of real MSTAR data are given in Appendix A.

As a result of these images, sparse representations have the best performance followed by MUSIC and sparse AR modeling for wideband data. For narrowband data, sparse AR modeling has best performance, and then MUSIC is in the second place.

4. RANGE PROFILE and RADAR IMAGE CLASSIFICATION

In this section, range profiles and radar images generated by Fourier Transform based method, AR model with l-2 norm minimization, SVD truncation, sparse representation and sparse AR models are classified.

4.1 Classification Method for Range Profile

Range profiles are classified by using the classification method in [14]. Detailed information about range classification method is explained in [14] and given in Appendix B.

The steps of classification method can be summarized as:

- Step1:** Normalize range profiles to obtain distance invariant features .
- Step2:** Calculate central moment
- Step3:** Construct training database.
- Step4:** Map elements of database into values between 0 and 1.
- Step5:** Apply PCA to eliminate redundant information and dimension reduction. Finally, transformation matrix is found.
- Step6:** Multiply training database by transformation matrix.
- Step7:** Repeat the first 4 steps to construct test database
- Step8:** Then, classify each test data by using knn classifier.

4.1.1 Classification result

F-16, F-117, F-14 and C-5 simulated data are used for range profile classification. These data are collected from 13 GHz to 17 GHz for 128 discrete frequencies and azimuth aspect angle varies from 0° and 60° with 0.4 °increments. Collected data set is divided into training database and test database. 63 aspects for each target are used for training database. So training database consists of 252 dataset (63(aspect angle) x4(target number)).The remaining of database is used for testing.

Correct recognition rate of FFT, l-2 minimization solution, SVD, Sparse AR Model with BP, sparse AR model with BPDN, sparse AR model with LASSO, sparse image with BPDN, sparse image with BPDN and sparse image with LASSO are investigated. Classification performance of all methods for broadband data is

calculated and results are presented in Table 4.1. As seen from the table, FFT based range profile classification result 91.26. Classification performance of l_2 minimization solution is very low with respect to MUSIC algorithm. SVD truncation fails to find the scatterers ,that's why classification performance of it is lower than l_2 minimization solution. Classification result of sparse representation of range profile is the lowest because it is impossible to obtain accurate aspect-invariant feature vector. Classification result of sparse AR modeling for range profile is better than l_2 minimization but classification performance of sparse AR methods cannot pass the classification performance of MUSIC.

Table 4.1 : Classification results of range profile reconstructed by using different method in the case of the wideband data with 30 dB SNR.

Method	Parameters	Result (%)
FFT		91.26 %
MUSIC		92.06 %
Least Square Error	AR model order: 24	83.3 %
SVD	AR model order: 24	76.9 %
Sparse radar imaging based on OMP	Sparsity level 5%	78.6 %
Sparse Linear Prediction based on BPDN with penalty	AR model order: 24 $\lambda=6$	86.9 %
Sparse Linear Prediction based on BPDN	AR model order: 24 1.15	90.4 %
Sparse Linear Prediction based on LASSO	AR model order: 24	89.6 %

Confusion matrix of one simulation is presented in Table 4.2. As seen from table, C5 plane is correctly classified.

Dimension of backscattered data is reduced to 64 to obtain narrowband data. By using these data, range profiles are generated from Fourier Transform based method, AR model with l_2 norm minimization, SVD truncation, sparse representation and sparse AR modeling. Classification result of these range profile are given in Table 4.3. As the expected, classification performance of Fourier Transform based method is reduced. Classification performance of MUSIC and AR model with l_2 norm minimization are very close to each other. Sparse AR model enhances the classification result. Confusion matrix of one simulation result is given in Table 4.4.

Table 4.2: Confusion matrix of methods in the case of the wideband data with 30 dB SNR. 1: FFT 2:MUSIC 3:Least Square Error 4: SVD 5: Sparse Image Representation based on OMP 6:Sparse Linear Prediction based on BPDN 7: Sparse Linear Prediction based on BPDN with penalty 8: Sparse Linear Prediction based on LASSO.

		F117	F16	F14	C5			C5	F14	F16	F117		
1	F117	61	0	11	0	2		0	1	0	62	F117	2
	F16	0	54	0	0			0	0	61	0	F16	
	F14	2	9	52	0			10	56	1	1	F14	
	C5	0	0	1	63			53	6	1	0	C5	
3	F117	50	5	4	0	4		6	4	4	48	F117	4
	F16	4	55	13	0			0	16	54	4	F16	
	F14	9	3	44	2			1	36	4	11	F14	
	C5	0	0	2	61			56	7	1	0	C5	
5	F117	37	0	3	131	6		0	6	0	47	F117	6
	F16	0	63	0	0			0	1	60	1	F16	
	F14	19	0	59	25			0	53	0	15	F14	
	C5	7	0	1	37			63	3	3	0	C5	
7	F117	57	1	5	0	8		0	4	2	49	F117	8
	F16	2	55	7	0			3	2	60	1	F16	
	F14	4	5	49	0			0	57	0	13	F14	
	C5	0	1	2	63			60	1	3	0	C5	

Table 4.3: Classification results of range profile reconstructed by using different method in the case of the narrowband data with 30 dB SNR.

Method	Parameters	Result (%)
FFT		63.57 %
MUSIC		81.3%
Least Square Error	AR model order: 24	81.63 %
SVD	AR model order: 24	72.10 %
Sparse radar imaging based on OMP	Sparsity level 10 %	56 %
Sparse Linear Prediction based on BPDN	AR model order: 24 1.25	82.3 %
Sparse Linear Prediction based on BPDN with penalty	AR model order: 24 $\lambda=6$	84.4 %
Sparse Linear Prediction based on LASSO	AR model order: 24 $\tau = 1/3$	84.1 %

Table 4.4: Confusion matrix of methods in the case of the narrowband data with 30 dB SNR. 1: FFT 2:MUSIC 3:Least Square Error 4: SVD 5: Sparse Image Representation based on OMP 6:Sparse Linear Prediction based on BPDN 7: Sparse Linear Prediction based on BPDN with penalty 8: Sparse Linear Prediction based on LASSO.

		F117	F16	F14	C5		C5	F14	F16	F117		
1	F117	58	0	4	1		0	9	1	56	F117	2
	F16	1	51	4	14		0	4	42	2	F16	
	F14	4	6	47	11		4	48	10	1	F14	
	C5	0	6	8	37		59	2	10	4	C5	
3	F117	46	2	7	0		1	0	0	50	F117	4
	F16	3	48	3	1		2	0	63	0	F16	
	F14	7	4	45	7		9	39	0	12	F14	
	C5	7	9	8	55		51	24	0	1	C5	
5	F117	21	0	11	0		1	3	0	43	F117	6
	F16	12	9	0	16		1	4	60	5	F16	
	F14	14	43	10	26		8	53	0	7	F14	
	C5	13	11	12	21		53	3	3	8	C5	
7	F117	44	4	0	3		1	6	2	47	F117	8
	F16	6	56	3	1		3	1	54	1	F16	
	F14	4	0	54	0		0	52	0	8	F14	
	C5	9	3	6	59		59	4	7	7	C5	

4.2 Classification Method for Radar Image

Radar images are classified by using the classification method which is explained in [15]. The detailed information can be got from [15] and Appendix C. Figure 4.1 shows the classification steps.

4.2.1 Classification result

For classification of radar image, the data which are collected by Sandia National Laboratory for MSTAR (Moving Stationary Target Acquisition and Recognition) program. The data are collected at 9.6 GHz center frequency and 591 MHz bandwidth. The backscattered data at different aspect angles belongs to T-72 , BMP2 and BTR70. 232 different image for training and 196 different image for test are available for each target. So, training data set contains 696 images and 588 target image classified. Firstly, classification success of 10 different methods are investigated. For the second step, by reducing the frequency and angular width,

resolution of data is decreased and classification results of 10 different methods are analyzed.

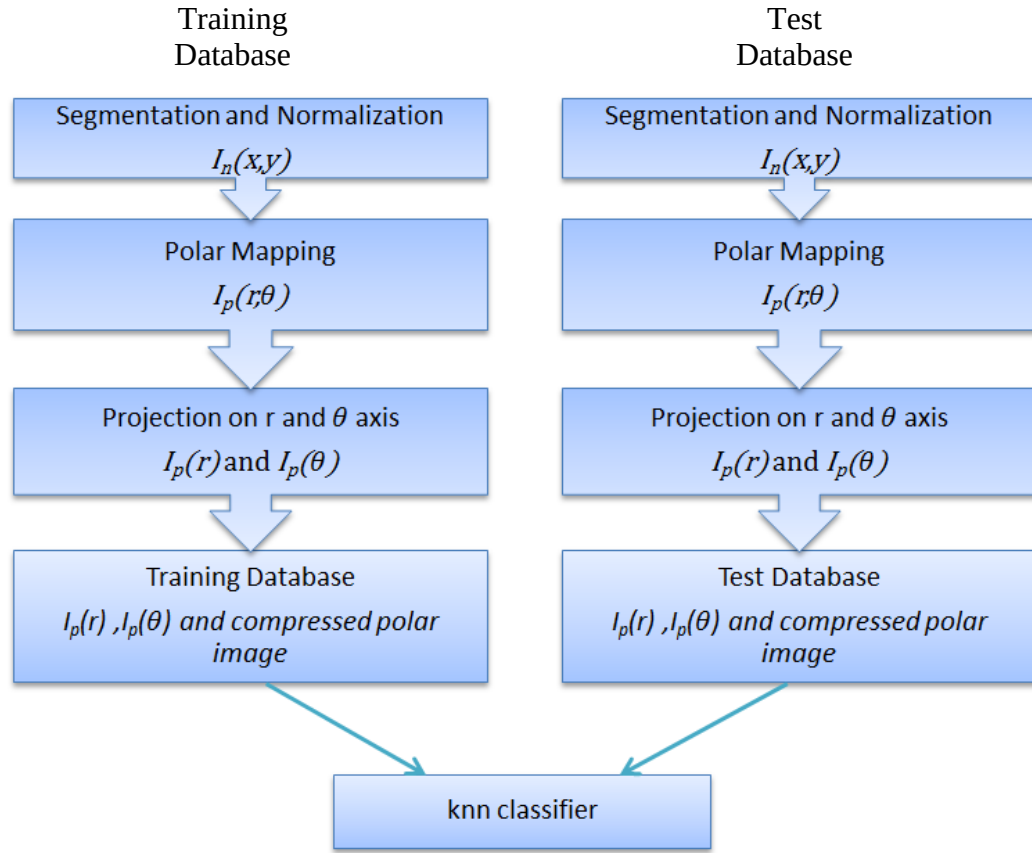


Figure 4.1:Feature Extraction from radar image.

4.2.1.1 Wide Band Data Classification

In this section , classification results of different imaging techniques are obtained. Classification success of radar imaging based on 2-D FFT based method is 86.7 %. At Table 4.5, classification results of MUSIC and AR model least square solution are given.

Table 4.5 : Effects of AR model order and smoothing parameter on classification performance in the case of broadband data.

Model order/ Smoothing parameter	MUSIC	AR Model with least square error
8	87.4	88.1
10	86.9	87.6
12	86.75	86.2

While AR model order is increased, more scatterers are detected but it is hard to distinct scatterers from background. Because of that, classification perforce decreases as order increases. SVD truncation suppresses the spurious peaks but classification performance is not increased since some scatterers are sensed as a noise.

As seen from the table 4.6, classification performance of sparse representations is lower than the other methods. Although sparse representation methods provide perfect image, classification result is not sufficient because invariant-rotation feature vectors couldn't be generated.

Classification performance of radar image reconstructed by sparse AR model methods are the higher than the others when sparsity parameters are given optimally.

Table 4.6: Classification results of radar image reconstructed by using different method in the case of the broadband data.

Method	Parameters	Result (%)
FFT based method	-	86.7 %
MUSIC	Smoothing parameter: 8	87.4 %
Least Square Error	AR model order: 8	88.1 %
SVD	AR model order: 8	88.2 %
Sparse radar imaging based on OMP	-	70.41 %
Sparse Linear Prediction based on BPDN	AR model order:8; $\epsilon=1.02$	88.2 %
Sparse Linear Prediction based on BPDN with penalty	AR model order: 8; $\lambda=0.5$	88.6 %
Sparse Linear Prediction based on LASSO	AR model order: 8; $\epsilon=0.5$	88.3 %

At table 4.7, confusion matrix are given. According to this table, most of T72 target are correctly classified.

4.2.1.2 Narrow Band Data Classification

Classification of radar image when data are collected at narrow bandwidth and angle width is investigated. Narrowband data are reconstructed by reducing the dimension of 64x64 MSTAR data to 32x32. The effect of AR model order and smoothing parameters on the classification performance which is given table 4.8 is investigated .

Table 4.7: Confusion matrix of methods in the case of broadband data. 1: FFT 2:MUSIC 3:Least Square Error 4: SVD 5: Sparse Image Representation based on OMP 6:Sparse Linear Prediction based on BPDN 7: Sparse Linear Prediction based on BPDN with penalty 8:Sparse Linear Prediction based on LASSO.

		BMP2	BTR70	T72			T72	BTR70	BMP2		
1	BMP2	158	20	14	2		17	8	153	BMP2	
	BTR70	5	171	2			4	182	10	BTR70	
	T72	33	5	180			175	6	33	T72	
3	BMP2	160	12	11	4		20	15	168	BMP2	
	BTR70	6	175	2			0	175	6	BTR70	
	T72	30	10	183			176	6	22	T72	
5	BMP2	116	29	24	6		11	11	155	BMP2	
	BTR70	21	140	14			0	176	7	BTR70	
	T72	59	27	158			185	9	34	T72	
7	BMP2	156	8	10	8		5	16	158	BMP2	
	BTR70	4	180	1			3	173	17	BTR70	
	T72	36	8	185			188	7	31	T72	

Table 4.8: Effects of AR model order and smoothing parameter on classification performance in the case of the narrowband data.

Model order/ Smoothing parameter	MUSIC	AR Model with least square error
8	74.8	79.2
10	72.3	71.4
12	70.4	70.6

At table 4.9, classification performance of radar image which is reconstructed by using spectral estimation methods, sparse representations and proposed sparse methods is given. Classification success of FFT based method is 72.1%. Classification success of MUSIC algorithm is better than least square error and SVD truncation. Classification success of sparse representation is the worst since translational invariance is not satisfied for feature vector.

The classification performance of sparse AR models are the best. It is noted that performance of MUSIC and SVD truncation is the related with the prediction of scatterers. While MUSIC and SVD truncation suppress the side lobes, information loss which is reason of lost scatterers occurs. Quality of radar image is improved by

the sparsity at sparse AR models without estimation of scatterer numbers. At table 4.10, confusion matrix are given. It is observed that classification of T72 data is more accurate.

Table 4.9: Classification results of radar image reconstructed by using different method in the case of narrowband data.

Method	Parameters	Result (%)
FFT based method		72.1 %
MUSIC	Smoothing parameter: 8	74.8 %
Least Square Error	AR model order: 10	71.4 %
SVD	AR model order: 8	71.2 %
Sparse radar imaging based on OMP		55.3 %
Sparse Linear Prediction based on BPDN	AR model order: 8; $\epsilon=1.05$	75.68 %
Sparse Linear Prediction based on BPDN with penalty	AR model order: 8; $\lambda=4$	75.7 %
Sparse Linear Prediction based on LASSO	AR model order: 8; $\epsilon=0.5$	75.8 %

Table 4.10: Confusion matrix of methods in the case of the narrowband data. 1: FFT 2: MUSIC 3: Least Square Error 4: SVD 5: Sparse Image Representation based on OMP 6: Sparse Linear Prediction based on BPDN 7: Sparse Linear Prediction based on BPDN with penalty 8: Sparse Linear Prediction based on LASSO.

		BMP2	BTR70	T72			T72	BTR70	BMP2		
1	BMP2	117	29	31		2	19	34	121	BMP2	
	BTR70	25	149	7			8	150	39	BTR70	
	T72	54	18	158			169	12	36	T72	
3	BMP2	108	42	22		4	16	51	109	BMP2	
	BTR70	38	140	2			3	133	37	BTR70	
	T72	50	14	172			177	12	50	T72	
5	BMP2	76	62	19		6	22	43	131	BMP2	
	BTR70	55	103	18			2	142	30	BTR70	
	T72	65	31	159			172	11	45	T72	
7	BMP2	133	36	22		8	28	30	137	BMP2	
	BTR70	24	142	14			5	146	21	BTR70	
	T72	41	28	170			163	20	38	T72	

CONCLUSIONS AND RECOMMENDATIONS

In radar imaging, Fourier transform based method will be a good candidate if backscattered data are collected within wide bandwidth and angular width. Unfortunately, application of this condition is not possible and practical in real life. Since FFT based method provides poor resolution under the narrow bandwidth and aspect angle condition, alternative ways such as spectral estimation methods, sparse representation approach are proposed to provide high resolution for the limited data case.

MUSIC algorithm based on eigenvalue analyses provides high resolution radar image. On the other hand, deterioration in the image will be observed when bandwidth and angular width is getting narrower. AR model based on least square solution detects the location of scatterers but spurious peaks may appear depending on the AR model order. Although SVD truncation suppresses spurious peaks successfully, it requires the knowledge of the number of the scatterer. Success of MUSIC and SVD truncation are directly related with the right choice of the number of scatterer.

Radar imaging based on sparse representation approach is very successful at detecting the location of scatterers without knowing scattered numbers. But location of scatterers aren't detected properly as bandwidth and angular width is getting narrower.

In this thesis, sparse approach is applied to find AR model coefficients instead of representing sparse radar image. The proposed sparse AR model detects the scatterers while suppressing the spurious peaks. Three different optimization problems are used to get sparse AR model coefficients. Although optimization parameters affect the quality of radar image obtained by the proposed sparse AR models, high resolution images with decreased sidelobes can be obtained with optimum valued parameters.

Classification is employed to decide which method yields better image result and classification result. The effects of FT based method, MUSIC, AR model with least square error, SVD truncation, sparse signal representation and sparse AR model method on the classification performance are investigated. When backscattered data are collected at wide bandwidth and angular width, effect of methods except sparse representation on classification performance are very close to each other. Although radar image based on sparse representation yields better result than the others, classification results are not good because feature vector which has translational invariance cannot be extracted. To compare the performance of methods under the narrow bandwidth and angular width, dimension of data are reduced. At that point, it is observed that classification performance of radar image which is constructed from the proposed sparse AR models are better than other methods.

As a conclusion, sparse AR model improves the quality of radar image and classification performance especially for the limited data case. The disadvantage of this method is the time consuming.

This proposed method can be enhanced by employing the OMP and compressive sensing which need less time to find the sparse AR coefficients. In addition to that, structured sparsity which uses groups of atoms in the dictionary instead of using atoms individually may be applied to sparse AR modeling problem.

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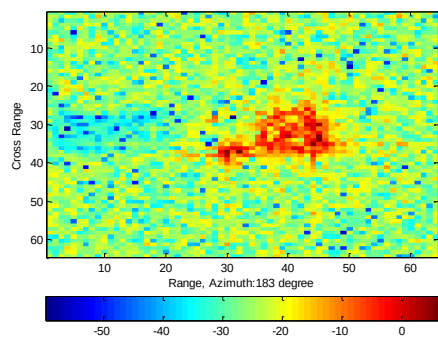
APPENDICES

Appendix A: Radar images of MSTAR data

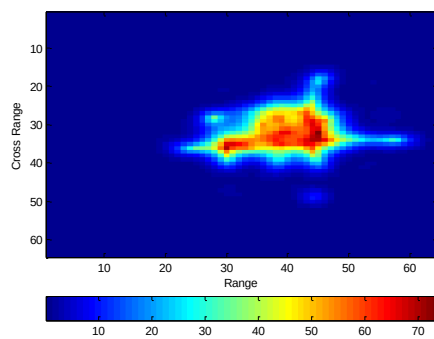
Appendix B: Range profile classification algorithm

Appendix C: Radar image classification algorithm

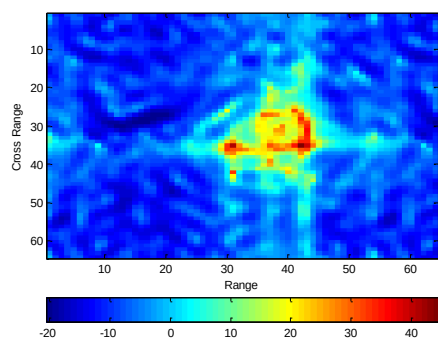
Appendix A.



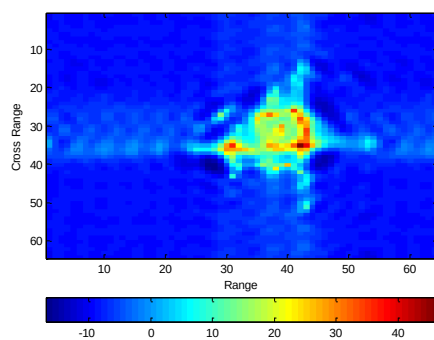
a.



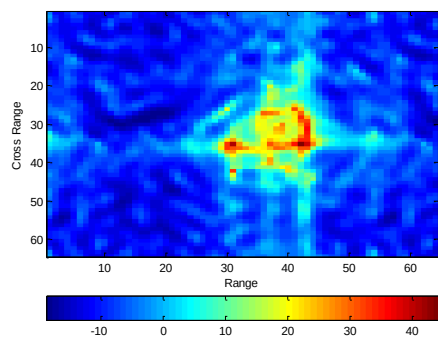
b.



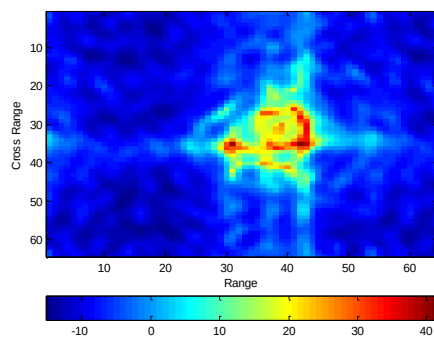
c.



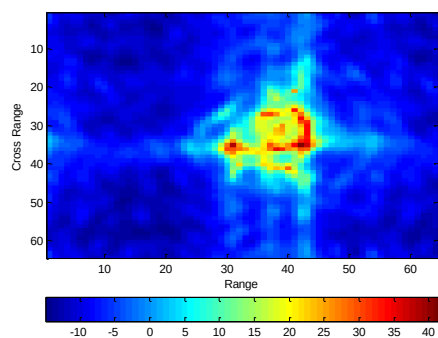
d.



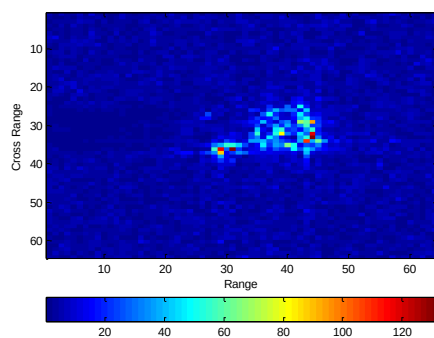
e.



f.



g.



h.

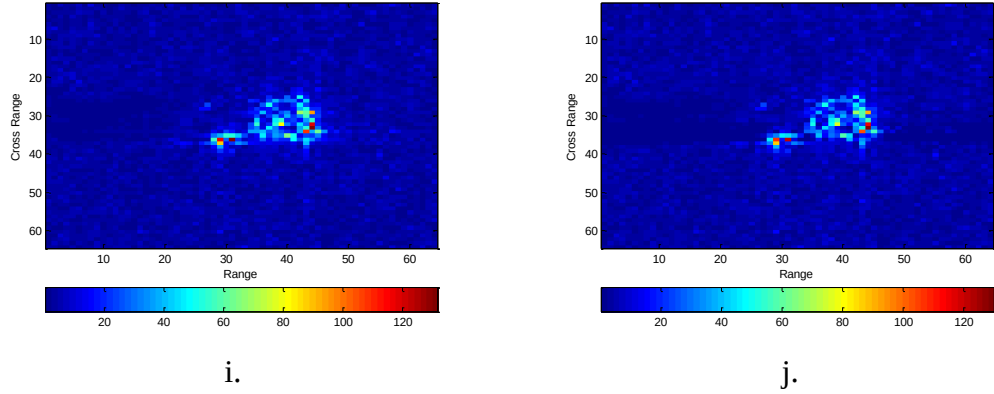
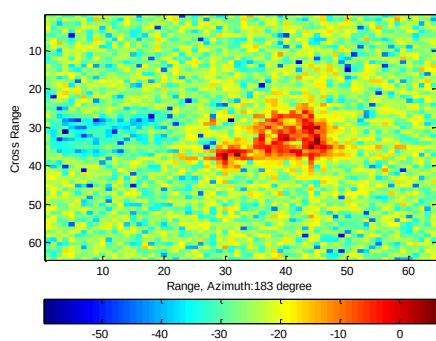
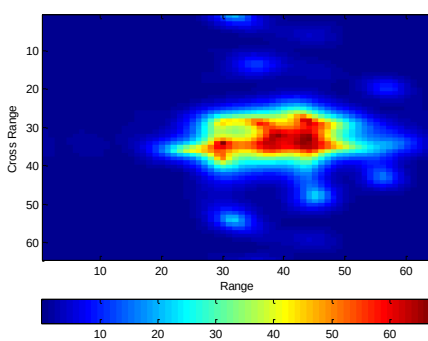


Figure A. 1: Radar image of MSTAR wideband data a.)2-D IFFT , b.) MUSIC smoothing parameter 8, AR model order=8 , c.) Least Square Error solution , d.) SVD , e.)BPDN (1.02) , f.) BPDN with penalty(4) , g.)LASSO(0.5) , h.)Sparse radar representation based on BPDN i.) Sparse radar representation based on BPDN with penalty term 1, j.) Sparse radar representation based on LASSO when threshold is mean of absolute pixel value.

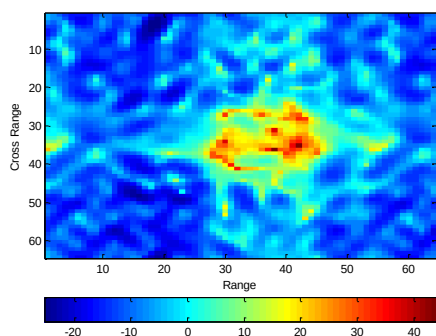
In Fig. A.1, radar image of one of the wideband MSTAR data based on FFT based method, MUSIC algorithm, l-2 minimization, SVD truncation, sparse AR modeling methods and sparse representations are presented, respectively. As seen from the sub-figures, Sparse representations have the best performance followed by MUSIC and sparse AR modeling for wideband data.



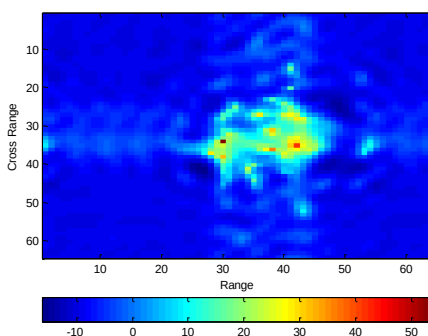
a.



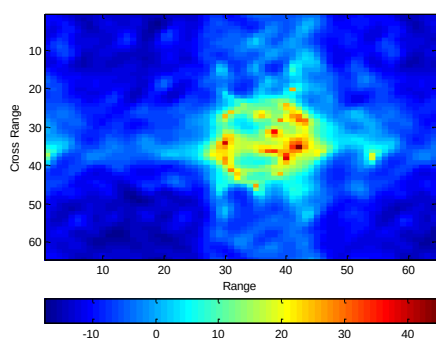
b.



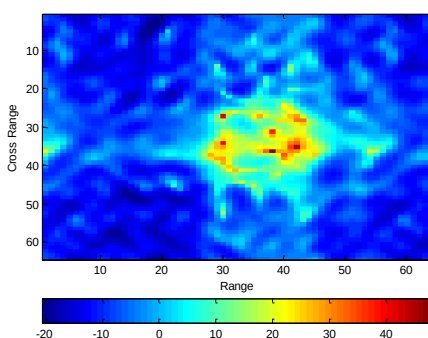
c.



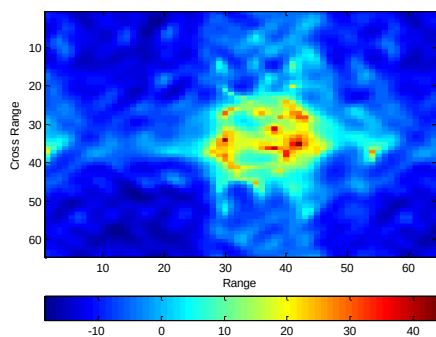
d.



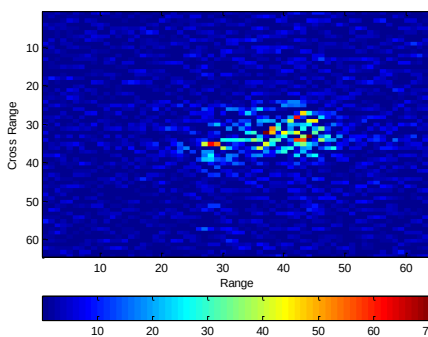
e.



f.



g.



h.

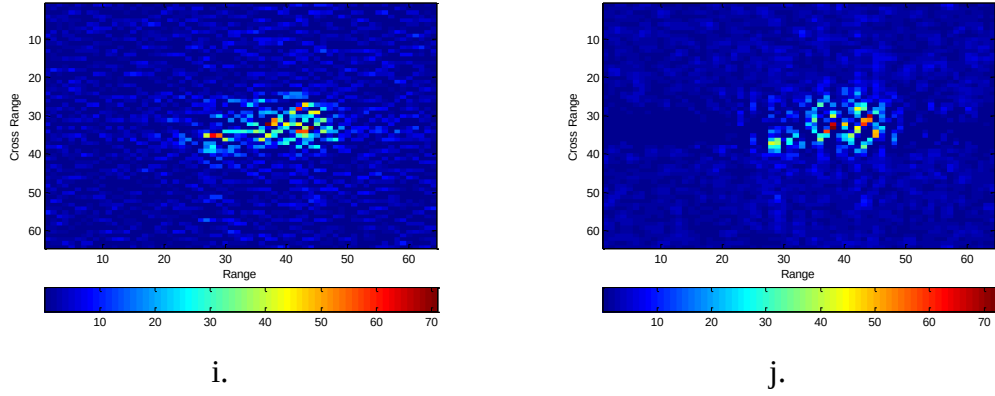


Figure A.2: Radar image of MSTAR narrowband data a.)2-D IFFT , b.) MUSIC smoothing parameter 12 , AR model order=8 , c.) Least Square Error solution , d.) SVD , e.)BPDN (1,02) , f.) BPDN with penalty(0.5) , g.)LASSO(0.33) , h.)Sparse radar representation based on BPDN, i.) Sparse radar representation based on BPDN with penalty term 1, j.) Sparse radar representation based on LASSO when threshold is mean of absolute pixel value.

Appendix B.

Classification Method for Range Profile

Classification method of range profile is explained in this part.

Feature Generation with Central Moment

Feature vectors are determined from range profiles for target recognition. Range profile of the target depends on the aspect angle and distance between the radar and target. To provide the distance invariance, each range profile is normalized to its maximum value. After this normalization, range profile depends only aspect angle [14].

Use of central moment provides range profiles independent of aspect angle. Central moment is computed by:

$$f_p = \sum_{i=1}^N (r_i - \delta)^p \left[\frac{\bar{p}(r_i)}{\sum_{l=1}^N \bar{p}(r_l)} \right] \quad (\text{B-1})$$

where $\delta = \sum_{i=1}^N r_i \left[\frac{\bar{p}(r_i)}{\sum_{l=1}^N \bar{p}(r_l)} \right]$

where p is the moment order, N is the range bin sampling, $\bar{p}(r_i)$ is the normalized range profile at range r_i . Central moments, f_p , are projection onto $(r_i - \delta)^p$, so they are independent of aspect angle which is caused by translation [7].

Feature vector of range profile is represented as:

$$\mathbf{f} = [f_1, f_2, f_3 \dots, f_p]^T \quad (\text{B-2})$$

For 2-D data, moment order should be high to determine the edges and corners. For range profile, working with high order moment is not necessary. Also, redundant information in the feature vector increases if central moment order is high.

Training and test database should be constructed for classification. Training and test database contain feature vectors of range profiles at different aspect angle which belong to different targets. Assume that N_c targets exist and N_a aspect angles each target are used. Then training database F_{train} is obtained as:

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1N_a} & \cdots & f_{1Q} \\ f_{21} & f_{22} & \cdots & f_{2N_a} & \cdots & f_{2Q} \\ \vdots & \vdots & & \vdots & & \vdots \\ f_{p1} & f_{p2} & \cdots & f_{pN_a} & \cdots & f_{pQ} \end{bmatrix} \quad (\text{B-3})$$

where $Q=N_c \times N_a$ and $\mathbf{F}$ is $(p \times Q)$ feature matrix of training database.

As seen from $\mathbf{f}$, while moment order is increased, element of $\mathbf{f}$ decreases. Elements of $\mathbf{F}$ is normalized between 0 and 1 to accurately identify the target. Normalized $\mathbf{F}$, $\bar{\mathbf{F}}$ is determined from:

$$\bar{f}_{ij} = \frac{f_{ij} - \min(f_{ij} \text{ given } i)}{\max(f_{ij} \text{ given } i) - \min(f_{ij} \text{ given } i)} \quad (\text{B-4})$$

where $i = 1, 2, \dots, p$ $j = 1, 2, \dots, Q$

Test database is also obtained by following (4.1), (4.2), (4.3) and (4.4)..

Application of Principal Component Analysis for Reducing Redundancy

Principle Component Analysis (PCA) is used to eliminate redundant information contained in the feature vector as moment order increases. While redundancy is reduced with PCA, dimension of feature vector decreases with little information loss.

Eigenvalues of covariance matrix play an important role to reduce redundant information and dimension. Covariance matrix is obtained by:

$$\bar{\mathbf{F}} = [\bar{\mathbf{f}}_1, \bar{\mathbf{f}}_2, \dots, \bar{\mathbf{f}}_{N_a}, \dots, \bar{\mathbf{f}}_Q] \quad (\text{B-5})$$

$$\mathbf{R}_{\bar{\mathbf{f}}\bar{\mathbf{f}}} = \frac{1}{Q-1} \sum_{k=1}^Q (\bar{\mathbf{f}}_k - \mathbf{m}_{\bar{\mathbf{f}}})(\bar{\mathbf{f}}_k - \mathbf{m}_{\bar{\mathbf{f}}})^T$$

where

$$\mathbf{m}_{\bar{\mathbf{f}}} = \frac{1}{Q} \sum_{k=1}^Q \bar{\mathbf{f}}_k$$

$\mathbf{m}_{\bar{\mathbf{f}}}$ is the $p \times 1$ mean vector, $\mathbf{R}_{\bar{\mathbf{f}}\bar{\mathbf{f}}}$ is the $p \times p$ covariance matrix.

Eigenvalues $\lambda_1, \dots, \lambda_p$ and eigenvectors $\mathbf{v}_1, \dots, \mathbf{v}_p$ are obtained from decomposition of covariance matrix $\mathbf{R}_{\bar{\mathbf{f}}\bar{\mathbf{f}}}$.

Transformation matrix $\mathbf{T}$ is constructed from eigenvectors and $\mathbf{T}$ is given by

$$\mathbf{P} = [\mathbf{v}_1, \dots, \mathbf{v}_l] \quad \text{where } l < p \quad (\text{B- 6})$$

$\mathbf{v}_1, \dots, \mathbf{v}_l$ are the eigenvectors which are associated with largest l eigenvalues.

Final training feature matrix is found from:

$$\mathbf{F}_{train} = \mathbf{P}^T \bar{\mathbf{F}} \quad \text{where } l < p \quad (\text{B- 7})$$

Normalized test database is obtained from ... equation. Final test feature matrix is also found from $\mathbf{F}_{test} = \mathbf{P}^T \bar{\mathbf{F}}_T$ where $\bar{\mathbf{F}}_T$ is normalized test database.

K Nearest Neighbours (knn) Classifier

Knn classifier is used to predict the class of new sample by using training database in which the data are separated into several separate classes. K entries in the training database which are closest to test data are selected and most common class of among these entries are found. This is the class of the test data.

Appendix C.

Classification Method for Radar Image

Classification method of radar image is explained in this part. Feature vector is obtained from polar mapping satisfies the requirements such as useful information with small dimension, invariant-rotation.

Segmentation and Normalization

Radar image is denoted by $I_0(x_i, y_j)$ where $x=1,2,\dots,M$ and $j=1,2,\dots,N$. M ve N are the pixel or sampling numbers at range and cross-range. In the segmentation process, target response is extracted from the rest of image including noise. It is assumed that Signal to noise ratio (SNR) is very high so separation of image from the background is hard. For this purpose, mean value is given at (C-1) is used as threshold. The values below threshold are set to 0.

$$\bar{I}_0 = \frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N I_0(x, y) \quad (\text{C-1})$$

Signal level depend on the distance between the scattering center and radar. To reduce the signal level variation, radar image is normalized:

$$I_n(x_i, y_j) = \frac{I_{seg}(x_i, y_j)}{\sum_{i=1}^M \sum_{j=1}^N I_{seg}(x_i, y_j)} \quad (\text{C-2})$$

Where $I_{seg}(x_i, y_j)$ is the radar data after segmentation.

Polar Mapping

Resampling of radar image is achieved by polar mapping procedure. Resampled polar image is denoted ,

$$I_p(r_m, \theta_n) \quad m = 1, 2, \dots, N_r \quad n = 1, 2, \dots, N_\theta \quad (\text{C-3})$$

In the above, N_r and N_θ are the number of samples in r direction and θ direction respectively. Relation between (r_m, θ_n) and (x_i, y_j) given by

$$(x_k, y_k) = (x_c, y_c) + (r_m \cos \theta_n, r_m \sin \theta_n) \quad (\text{C-4})$$

$$m = 1, 2, \dots, N_r \quad n = 1, 2, \dots, N_\theta$$

$$k = 1, 2, \dots, N_r N_\theta$$

Where

$$r_m = R_{min} + (m - 1)\Delta r$$

$$\theta_n = -\pi + (n - 1)\Delta \theta$$

$$\Delta r = \frac{R_{max} - R_{min}}{N_r - 1}$$

$$\Delta \theta = \frac{2\pi}{N_\theta - 1}$$

In the (4.12), (x_c, y_c) is the center position of polar mapping. Δr and $\Delta \theta$ are the sampling interval toward r and θ direction respectively.

Feature Database Construction

Feature vector of data for classification consist of 3 parts: r projected image, θ projected image and compressed polar image which is constructed with PCA.

Projection of image onto r axis:

$$I_r(r) = \int_{-\pi}^{\pi} I_p(r, \theta) d\theta = \sum_{n=1}^{N_\theta} I_p(r_m, \theta_n) \quad (\text{C-5})$$

The projection of image onto θ axis:

$$I_\theta(\theta) = \int_{R_{min}}^{R_{max}} I_p(r, \theta) dr = \sum_{m=1}^{N_r} I_p(r_m, \theta_n) \quad (\text{C-6})$$

Radar data contains useful information as well as redundant information . To reduce redundant information and dimension of the image, PCA is used on polar image.

Training database is shown at Figure 3.1. As seen from the figure, Feature vector of each target consists of $f_1(p, q), f_2(p, q), f_3(p, q)$ where p is the target, q is the aspect angle, $f_1(p, q), f_2(p, q), f_3(p, q)$ refer to $I_r(r), I_\theta(\theta)$ and compressed polar image, respectively. By using these feature vector, test and train database are constructed and test database are classified by knn classifier.

	p th target
1 st aspect	f ₁ (p,1) f ₂ (p,1) f ₃ (p,1)
2 nd aspect	f ₁ (p,2) f ₂ (p,2) f ₃ (p,2)
	⋮
q th aspet	f ₁ (p,q) f ₂ (p,q) f ₃ (p,q)

Figure C.3: Feature Vector.

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