Relational Contracting and On-The-Job Search

by

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To My Dear Grandfather

ABSTRACT

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In this study, I examine how firms can affect workers' job search efforts to be socially efficient with their counteroffer policies. I consider a repeated game where generations of workers take part in costly on-the-job searches. If an outside offer is realized for a worker, the employer firm can either make a counteroffer that matches the worker's outside offer or commit to not making a counteroffer based on the outside offer. I show that when firms choose to match workers' outside offers, workers spend too much effort on job search. Further, I derive an equilibrium where all firms are able to implement an efficient level of effort by not matching workers' outside offers and provide the conditions for the existence of this equilibrium.

ÖZETÇE

İlgisel Sözleşmeler ve İş Arama Süreci
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Bu makalede, firmaların karşı teklif politikaları ile çalışanların iş arama çabaları arasındaki ilişkiyi inceliyorum. Bu incelemeyi yapabilmek adına kurduğum modelde çalışanlar iş aramaya harcadıkları çabayı artırarak dışardan iş teklifi alma ihtimallerini arttırabilirler. Firmalar ise bu dışardan gelen teklife göre yeni eşdeğer bir karşı teklif sunabilirler veya çalışanlarla anlaşılmış olan teklife sadık kalabilirler. Sonuç olarak, firmaların eşdeğer karşı teklif sunma politikasının çalışanları iş aramaya daha çok çaba göstermeye teşvik ettiğini gösteriyor. Ayrıca, firmaların anlaşılmış tekliflere sadık kalarak, çalışanları iş aramaya daha az çaba göstermeye teşvik edebileceğini ve bu şekilde sosyal verimliliği arttırabileceğini ortaya koyuyorum.

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Chapter 1

INTRODUCTION

Workers' job search behavior and firm policies are important determinants of an efficient labor market (Karahan et al., 2017; Moscarini and Postel-Vinay, 2017; Haltiwanger et al., 2018). Economic literature has documented that a significant fraction of hiring is done through job-to-job transitions rather than hiring unemployed workers, (Fallick and Fleischman, 2004; Faberman et al., 2022). The fraction of job-to-job transitions being this high makes on-the-job search a significant topic in the literature. Even though various factors play a role in workers' on-the-job search behavior, one factor that directly influences it is counteroffers, as demonstrated by Barron et al. (2006). To explore this topic, I construct an infinite horizon model with costly on-the-job search where firms can choose between matching and no-matching counteroffer policies. The main finding is that depending on parameters such as firm productivity and cost of working, firms are able to maintain implement the no-matching counteroffer policy.

Faberman et al. (2022) report the results of a survey conducted through the Survey of Consumer Expectations, administered by the Federal Reserve Bank of New York and the results show that 22% of employed people look for a new job while still working. In the paper's sample, 72.4% of the offers were made to the already employed people. These numbers demonstrate the significance of job-to-job transitions among all job changes. Furthermore, Faberman et al. (2022) also show that job-search intensity declines as the current wages increase. This behavior can provide insight into the motivations of employed people actively searching for a job.

In principle, there are two potential paths for workers to increase their current wages. A worker can either switch to a higher-paying job or re-negotiate with the current employer for a salary increase. In order to retain their workers, firms may offer a new wage that will make workers stay. This strategy is common in the labor market and is termed *Matching Counteroffers*. This happens only if the worker receives an outside offer. This dynamic motivates workers to search for a job even more because even if they are happy with their current job, they can potentially increase their earnings by getting an offer from another firm.

In addition to the matching counteroffer policy, firms have an alternative strategy known as the *No-matching Counteroffer Policy*. Under this approach, rather than adjusting wages in response to outside offers received by their employees, firms choose to provide more information about future wages when the initial offer is made and commit to it when the time comes. Despite being non-contractible, trust can still be built through repeated interactions using a mechanism known as *relational contracting* (Macneil, 1985; Levin, 2003; Zábojník, 2012). When firms consistently demonstrate a commitment to the no-matching counteroffer policy, a history of trust is established, fostering confidence among workers that firms will uphold their initial offers. This highlights the role of relational contracting in mitigating uncertainty and building trust between employers and employees.

The model presented in this paper identifies an equilibrium that resolves potential problems arising from costly on-the-job search. In the model, firms choose between the mentioned counteroffer policies. For workers, both job search and work are costly. They choose how much effort they put into the job search. This decision is determined by the worker's skill and the firm's productivity. My initial findings show that the matching counteroffers policy influences workers to put too much effort into job search. The high effort levels of workers are driven by the incentive to increase their wages when they are old. Another outcome of this high job-search effort is the increased wages for young workers. Thus, matching counteroffers policy negatively impacts firms' profits while also being socially inefficient.

The main results show that an equilibrium where firms induce the socially efficient level of effort by committing to the no-matching counteroffers policy exists under certain conditions. Some of these conditions highlight the market characteristics that are required to sustain this no-matching counteroffers policy equilibrium.

One of these characteristics considered in this paper is the discount factor; the no-matching counteroffers equilibrium can be sustained in economies where the discount factor is high. Furthermore, the minimum required discount factor is influenced by the distribution of firm productivities, worker types, and cost of working. These results highlight the significance of the strategic use of counteroffer policies and their efficiency implications in labor markets.

Chapter 2

LITERATURE REVIEW

This paper combines several concepts used in theoretical work. The backbone of the model is the costly on-the-job search. Benhabib and Bull (1983) are among the first ones to conduct an analysis on job search. Their main contribution to the literature on search models was that they focus on search intensity by letting workers search for a job while working. By using the counteroffer policies, I delve deeper into the motivations of a worker's on-the-job search and find that the worker's effort in the job search is directly linked to the possibility of counteroffers.

Potter and Bernhardt (2022) use a similar approach. Their study focuses on how the employer's wage offers change depending on the employer's private information about the demand for their product and its implications on workers' job search decisions. In the model, firms' wage offers shape workers' beliefs regarding the possible shutdown of the firm, and indirectly, it influences their job search effort. Even though the usage of wage schemes to influence the worker's job search intensity exists in their model as well, their paper only considers the case where workers are influenced by wage offers. It does not recognize that firm policies such as counteroffer policies have a significant role in workers' job search efforts.

I analyze the wages through the effort decisions, outside offers of workers, and how they change depending on the counteroffer policies of firms. Yamaguchi (2010) analyzes wages through human capital, match-specific capital, and outside options. He shows that outside options are a significant factor in explaining the increasing wages. The model in this paper differentiates itself by giving firms in the model the choice between possible counteroffer policies. In this way, I am able to analyze the effects of counteroffer policies on wages, profits, and job search efforts of workers.

Several studies suggest that firms' monopsony power over their employees is limited because workers are able to search for a job while employed (Mortensen, 1986;

Mortensen and Pissarides, 1999). Despite their contributions to the literature, these papers lack the strategic use of the counteroffer policies. By integrating counteroffer policies, I investigate how workers can be prevented from putting inefficiently high effort into job search.

The papers mentioned lack a perspective where counteroffer policies can be used strategically. Barron et al. (2006) examine the selective use of counteroffer policies and its implications. Even though my use of counteroffer policies and costly job search are similar, I consider a different approach. While they consider a model where the beliefs of workers can change depending on the most recent counteroffer decisions of firms, this paper assumes that workers trust their employers will not match their counteroffers if and only if the employers have a clean history of committing to the no-matching counteroffers equilibrium. This makes the counteroffer policy decisions more crucial for firms because if they were to deviate, they would not be able to rebuild trust among workers.

Postel-Vinay and Robin (2004) explores labor market equilibrium involving two types of firms: those that match wage offers to retain workers and those that never make counteroffers. By including both policies, they analyze the strategic use of counteroffers and highlight the relationship between the counteroffer policies and workers' job search efforts. The main difference in my paper is that I put an emphasis on the social efficiency of the job-search effort and show that this social efficiency can be achieved by a no-matching counteroffers policy by all firms.

Chapter 3

MODEL SETUP

I consider an infinitely repeated model with two types of players: firms and workers (Mukherjee, 2010). Firms have infinite lives, while workers live only for two periods. All players are risk-neutral and discount their future profits at a rate $\delta \in (0, 1)$. Workers are considered young in the first period of their lives, while in the second period, they are considered old. Firms' profits from a worker in period t are denoted by π_t. For the simplicity of notation, I assume that period t denotes the period when a generation is born. Therefore, π_t and π_{t+1} denote the firm's payoff when workers are young and old, respectively. Young and old workers' payoffs are denoted by u_t and u_{t+1}. Workers have two possible types: low productivity or high productivity, and the types are denoted by θ_L and θ_H, respectively, where $0 < \theta_L < \theta_H \leq 1$. These types become public information at the beginning of workers' lives.

The number of workers born in each generation in the model exceeds the number of firms. The productivity of firm j is denoted by h^j. Firms' productivities have the distribution $h \sim G(h)$ over $[\underline{h}, \overline{h}]$ where $0 \leq \underline{h} \leq \overline{h} \leq 1$. The firm where the young worker is employed is denoted by firm F, and h^F indicates its productivity, while the firm that makes an offer to the same worker at the end of the first period is denoted as firm R, and its productivity is indicated by h^R. The output produced by worker i at firm j in a given period is denoted by $y_i^j = \theta_i h^j$. If a firm is indifferent between hiring a worker or not producing at that period, the firm chooses to hire the worker. This can be considered as the firm's eagerness to produce.

After accepting the job offer, the young worker decides on the effort to put into the job search. Both working and job-search are costly to the worker. The cost of working is denoted by $\overline{c} \in [0, C]$, which is constant for all firms and workers. I set the upper bound of the cost of working to $C = \theta_L \underline{h}$. The reason for this upper bound is that if the cost of working exceeds the minimum possible output, some firms would not prefer to hire workers, and they would go out of business immediately. The job search effort is denoted by $e \in [\underline{e}, \overline{e}]$, where $0 < \underline{e} < \overline{e} < 1$ and the effort is costly. Cost function $c(e)$ is strictly increasing, twice differentiable, convex, satisfies the Inada conditions, and $c(\overline{e})$ is sufficiently large. The first four assumptions regarding the cost function are standard assumptions, while the last assumption becomes useful while dealing with the job-search efforts. The young worker who exerts effort e on the job search receives an outside offer with probability e, and the productivity of firm R is drawn from the distribution function $G(h)$. If the worker accepts the outside offer, she gets paid w^R. With probability $1 - e$, the worker does not receive an outside offer. If the worker receives no offers, her outside option is normalized to 0. If the worker is indifferent between working or going into unemployment, she chooses to work. This is because workers are eager to produce as firms are. If the worker is indifferent between accepting the outside offer and staying at the initial firm, she prefers to stay. This is because even though it is very small, changing jobs is costly for the worker.

The output y_i^j and job-search effort e are non-contractible. Wages are paid at the beginning of the corresponding period. Old workers' wages depend on the firm's two possible counteroffer policies: Matching counteroffers or no-matching counteroffers. If they choose to match, that period's wages are decided on the spot, and the worker's wage in the second period depends on the outside offer. If they choose the no-matching policy, the firm pays the amount selected at the beginning of the worker's career. Long-term contracts are not enforceable but can be sustained by repeated interaction. So, the only way to sustain the no-matching counteroffers policy is to commit to it in each generation.

Firms' payoffs and workers' outside offers are publicly observable. After the outside offer of the young worker is observed, firm F decides on committing to the no-matching counteroffers policy or switching to the matching counteroffers policy. After the commitment decision is made, firm F offers a wage to the worker depending on the commitment decision. After observing the wage offer, the worker decides where to work in the second period.

Chapter 4

STAGE GAME

In this chapter, I consider a single stage of the game described in the previous chapter. The timeline of the stage game is as follows.

Period 1.0: Firm F decides on a counteroffer policy
Period 1.1: Firm F observes the type of the worker and makes an offer
Period 1.2: Worker is hired by firm F and paid w_1
Period 1.3: Worker exerts effort e to seek outside offers
Period 1.4: Worker's output is realized
Period 1.5: Worker's outside offer from firm R is realized and observed by firm F
Period 2.0: Firm F decides on a counteroffer policy and makes an offer to the worker accordingly
Period 2.1: Worker decides where to work in period 2, and she is paid w_2
Period 2.2: Output is realized, and then the worker retires

The worker's expected lifetime payoff at the beginning of her career is given by the summation of the payoffs when she is young and old,

$$U_t = u_t + \delta u_{t+1}, \tag{4.1}$$

where the following equations give u_t and u_{t+1}. In these equations, w_t and w_{t+1} denote the young and old worker's wages, respectively;

$$u_t = w_t - c(e) - \overline{c} \tag{4.2}$$

$$u_{t+1} = \begin{cases} max\{w^F_{t+1}, w^R_{t+1}\} - \overline{c} & \text{if there is an outside offer} \\ w^F_{t+1} - \overline{c} & \text{otherwise.} \end{cases} \tag{4.3}$$

Firm F's expected profit from hiring a young worker in period t, which is denoted by Π_t, is given by

$$\Pi_t = \pi_t + \delta\pi_{t+1} \tag{4.4}$$

where π_t and π_{t+1} are given by

$$\pi_t = \theta_i h^F - w_t \tag{4.5}$$

$$\pi_{t+1} = \begin{cases} \theta_i h^F - w_{t+1} & \text{if worker i remains with firm F} \\ 0 & \text{otherwise.} \end{cases} \tag{4.6}$$

The main goal of this paper is to achieve an equilibrium where all firms commit to the no-matching counteroffers policy. I consider pure strategies and use Subgame Perfect Equilibrium as a solution concept. In this section, I derive the equilibrium for matching counteroffers policy and the efficient job search effort and discuss them together. The first step in deriving the matching counteroffers policy equilibrium is to find the old worker's wage offer correspondence for firm F, which chooses the matching counteroffers policy, as given in Lemma 1.

Lemma 1. *Let h^F and h^R be the productivity of firms F and R, respectively. If firm F chooses to match counteroffers, then its optimal second-period wage offer correspondence is given by*

$$w^F_{t+1} = \begin{cases} \theta_i h^R & \textit{if } h^R \leq h^F \\ [0, \theta_i h^R) & \textit{if } h^R > h^F \\ \bar{c} & \textit{if no outside offer is received} \end{cases}$$

and the optimal wage offer of firm R is $w^R_{t+1} = \theta_i h^R$.

Notice that to produce output, firm R offers all its output to the worker. This is because if an outside offer is realized, firm F and firm R would be competing to hire the worker. Since, in this case, two firms are competing for one worker, firm R needs to leave all the rent to the worker in order to hire her. As a result, if firm R is more productive than firm F, firm F cannot propose a better offer. Otherwise, firm F offers the lowest possible wage to keep the worker. Given this wage offer correspondence, the equilibrium behavior of the young worker and firm F is given in *Proposition 1.*

Proposition 1. *For a young worker employed in firm F, the equilibrium behavior for matching counteroffers policy is described as follows.*

1. *The optimal job-search effort, e_i^*, satisfies $\delta(\theta_i E_t[h] - \overline{c}) = c'(e_i^*)$.*

2. *Firm R's offer and firm F's counteroffer satisfy Lemma 1.*

3. *If the worker receives an outside offer from firm R with $h^R \leq h^F$ or she does not receive any offers, she stays with firm F and gets paid w_{t+1}^F, which is equal to the outside offer, w_{t+1}^R.*

4. *If the worker receives an outside offer from firm R with $h^R > h^F$, she moves to firm R and gets paid w_{t+1}^R.*

In *Proposition 1*, e_i^* is used instead of e^*. This is because the optimal effort level depends on the worker's type. I use e_i^* in the notation where $i \in \{L, H\}$ to differentiate them.

The optimal effort level defined in *Lemma 4.1* does not depend on h^F. This is a direct result of the second-period wage offer policy of firm F. The second-period wage offer policy of firm F depends on the outside offer, and the outside offer does not depend on the productivity of firm F. This leads to the optimal effort to be independent of the productivity of firm F.

The efficiency of the optimal effort level remains to be seen. For *Proposition 1*, the objective function for the worker was her expected lifetime payoff before making the effort decision. While deriving the expected lifetime payoff, the wage offer correspondence defined in *Lemma 1* is used. To evaluate the efficiency, I need to define the efficient level of effort, and to define it, a new objective function is required. The efficient level of job-search effort needs to provide the effort level that maximizes the expected total payoff in the market. So, the new objective function becomes the sum of the expected output produced by the worker in the second period of her life and the cost of her job-search effort in the first period of her life. The cost of job-search effort is simply given by $c(e)$. For the subsequent period's expected output, three cases need to be considered. The first case is where the worker does not receive an outside offer with probability $1-e$ and produces $\theta_i h^F$. In the second case, the worker receives an outside offer but stays with firm F with probability $eG(h^F)$ and produces $\theta_i h^F$. Lastly, the worker receives an outside offer and switches to firm R with probability $e(1-G(h^F))$ and produces $\theta_i h^R$. Since the productivity of firm R is not known when the worker is deciding on the job-search effort, the expected output in the last case would be $\theta_i E_t[h|h > h^F]$. Combining all these and discounting them at the correct rate, the new objective function is provided in equation (4.7), and its solution is given in *Lemma 2*.

$$\delta\Big((1-e)\theta_i h^F + eG(h^F)\theta_i h^F + e(1-G(h^F))\theta_i E_t[h|h > h^F]\Big) - c(e) \tag{4.7}$$

Lemma 2. *The efficient level of effort, e_i^{fb}, is characterized as follows:*

$$\delta\theta_i Prob(h > h^F)[E_t[h|h > h^F] - h^F] = c'(e_i^{fb}).$$

Further, the efficient effort level decreases with h^F and satisfies $e_H^{fb}(h^F) > e_L^{fb}(h^F)$ for all $h^F \in [\underline{h}, \overline{h}]$.

Recall that the output is not driven by the worker's effort. The only relationship between the output produced by the old worker and the job-search effort of the young worker is through the probability of getting an outside offer from a firm with higher productivity.

Lemma 2 provides the relationship between firms' productivities and the marginal benefit of increased job-search effort. There are two terms in the equation that are dependent on h^F. The first one is $1-G(h^F)$. Observe that as h^F increases, the likelihood of getting an offer from a firm with higher productivity, $1-G(h^F)$, decreases. Thus, as the probability of getting an offer with a higher wage decreases, less effort is put into the job search. The other term in the equation, which is dependent on h^F, is $(E_t[h|h > h^F] - h^F)$. This term is decreasing in h^F as well. As h^F increases, the difference between the productivity of firm F and the average productivity of more productive firms decreases as well. As a result, the marginal benefit of increased effort level decreases with h^F. Hence, the expected gain is less for workers who work for more productive firms. Another factor that changes the efficient effort level is workers' types. The efficient effort level for the high-type worker is higher than that for the low-type worker. This is because the marginal benefit of increased effort level is higher for high-type workers. For both types of workers, the change in the probability of receiving a better offer and the cost of job search effort remains the same for an increase in the effort level. The only factor that is different is the types of workers. So if a better offer is received, the increase in the total output when the worker is high type exceeds the increase in the total output when the worker is low type. Taking all these factors into account, the main result of this section is presented in *Proposition 2*.

Proposition 2. *The effort spent on job search is inefficiently high, $e_i^* > e_i^{fb}$ for $i \in \{L, H\}$.*

As can be seen, matching counteroffers policy is not efficient. The main reason is that the worker's effort determines her wage when she is old. If the worker does not receive any outside offers, her total payoff is 0 when she is old. The only action for the worker to increase the probability of getting an outside offer is to exert more effort in the job search. Recall that for all firms, e_i^* is constant, but e_i^{fb} decreases as firms get more productive. Consequently, the difference $(e_i^* - e_i^{fb})$ increases as firms get more productive. Thus, more productive firms are more incentivized to implement the efficient effort level.

Chapter 5

REPEATED GAME

This section considers a game where the stage game is repeated indefinitely. The history of the game is public information. As stated in the previous chapters, the main goal of this paper is to find an equilibrium where all firms implement the efficient effort level by committing to the no-matching counteroffers policy. To sustain this equilibrium, firms use a grim trigger strategy, meaning that firms do not defect from the no-matching counteroffers policy in any generation. If a firm reneges on its no-matching counteroffers policy, workers hired in the subsequent periods behave as if the firm implements the matching counteroffers policy. In other words, the firm's commitment to the no-matching counteroffers policy makes workers trust the firm will continue to do so. Thus workers' wage expectations for period $t+1$ rely on the firm's history of commitment decisions.

In the previous section, I derived the matching counteroffers equilibrium and showed that the job search effort for the matching counteroffers policy is inefficiently high. Since the main goal of this paper is to find an equilibrium where the efficient level of effort is implemented, the matching counteroffers equilibrium will be the off-path equilibrium in the repeated game, meaning that if firms deviate from the no-matching counteroffers policy, the new equilibrium would be the one defined in *Proposition 1*.

Another key result from the previous section is the efficient level of job search effort described in *Lemma 2*. This efficient effort level is the effort level that needs to be sustained in the on-path equilibrium, which is the no-matching counteroffer policy equilibrium. To show that the efficient effort level can be implemented, I need to find the old worker's wage function that influences workers to exert the efficient level of effort. This wage function for the old worker is given in *Lemma 3*.

Lemma 3. *The efficient effort level, e_i^{fb}, is achieved by the following wage function for the old worker $w_{t+1}^F = \theta_i h^F$.*

Another interpretation of *Lemma 3* is that, to induce the efficient effort level, firm F must allocate all its output to the experienced worker. Firms increase the total welfare by giving up on their profits in period $t+1$. To see whether increasing the total welfare is more profitable for firms, I derive the young worker wages and expected profits of firms for both matching counteroffers and no-matching counteroffers policies.

5.1 *Young Worker Wages*

To define the no-matching counteroffers policy equilibrium, the wages and profits need to be derived explicitly. The old worker wages for matching and no-matching counteroffer policies are defined in *Lemma 1* and *Lemma 3*, respectively. This section uses the findings in these Lemmas and derives the wages for the young workers. The derivations start with the expected lifetime payoff of a worker who is born at period t, which is given in equation (5.1).

$$U_t = w_t - c(e) - \bar{c} + \delta E_t[u_{t+1}] \geq 0 \tag{5.1}$$

Notice that the lifetime payoff of the worker is restricted to be non-negative. The outside option of the worker is 0 for both periods. Hence, her individual rationality constraint requires the lifetime payoff of the worker to be non-negative. Since workers compete to get a job, the inequality in (5.1) binds. This equality is used to compute the young worker's wage for both counteroffer policies. Let w_t^s, π_t^s denote the wage and profit in period t for the matching counteroffers policy, while w_t^r and π_t^r denote the wage and profit in period t for the no-matching counteroffers policy. Keep in mind that optimal and efficient effort levels change with the type of worker. Having established a clear method for deriving young workers' wages, the derivations start with the young worker's wages for the matching counteroffers policy.

5.1.1 Matching Counteroffers

To compute the young worker's wage for the matching counteroffers policy, I explicitly write the term $E_t[u_{t+1}]$ in equation (5.2).

$$\begin{aligned} E_t[u_{t+1}] &= e_i^* \int_{\underline{h}}^{\overline{h}} (\theta_i h^R - \overline{c})\, dG(h^R) \\ &= e_i^* \theta_i \int_{\underline{h}}^{\overline{h}} h^R\, dG(h^R) - e_i^* \overline{c} \int_{\underline{h}}^{\overline{h}} dG(h^R) \\ &= e_i^* \theta_i E_t[h] - e_i^* \overline{c} \end{aligned} \tag{5.2}$$

The first detail to consider in equation (5.2) is that the case where the worker does not receive any counteroffers does not show itself in the equation. This is because, in that case, the worker gets paid $\overline{c}$ and is subject to the cost of working, making the total payoff of the old worker equal to 0. Another detail to consider is the wage term in the integral. Observe that if firm F can match the counteroffer, she offers $\theta_i h^R$ to the worker, and if it cannot, then the worker gets paid $\theta_i h^R$ at firm R. As a result, in both cases, the worker gets paid an amount equal to the outside offer. In the third line, $E_t[h]$ denotes the average productivity of raiding firms. Combining equations (5.1) and (5.2), I obtain the young worker's wage given in equation (5.3).

$$\begin{aligned} w_t^s &= c(e_i^*) + \overline{c} - \delta e_i^* (\theta_i E_t[h] - \overline{c}) \\ &= c(e_i^*) + \overline{c} - e_i^* c'(e_i^*) \end{aligned} \tag{5.3}$$

Recall the equation for the optimal effort level satisfies $c'(e_i^*) = \delta(\theta_i E_t[h] - \overline{c})$. This provides the second line in equation (5.3). Notice that the term above is not necessarily positive. Since the first derivative of the cost function is increasing and $c(0) = 0$, it can be shown that the term $c(e_i^*) - e_i^* c'(e_i^*)$ is strictly negative. That is why, depending on $\overline{c}$, wages may be negative. However, the young workers in the model are subject to a liquidity constraint, so they cannot make payments to firms to receive an offer. To ensure that the liquidity constraint does not cause any problems in the derivation, I need to put a restriction on $\overline{c}$. This restriction will be defined and discussed in more detail in section 5.2, after the derivations of the young workers' wages. Until that discussion, I assume that $\overline{c}$ is sufficiently large to make both w_t^r and w_t^s positive.

5.1.2 No Matching Counteroffers

The wage of the old worker for the no-matching counteroffers policy is given in *Lemma 3*, and the effort level for this policy is given in *Lemma 2*. Using these results, I derive the expected payoff of the old worker in period $t+1$ where $i \in \{L, H\}$.

$$\begin{aligned} E_t[w_{t+1}^r] &= (1 - e_i^{fb})\theta_i h^F + G(h^F) e_i^{fb} \theta_i h^F + e_i^{fb}(1 - G(h^F))\theta_i E_t[h|h > h^F] \\ &= \theta_i h^F + e_i^{fb}(1 - G(h^F))\theta_i(E_t[h|h > h^F] - h^F) \\ &= \theta_i h^F + e_i^{fb} c'(e_i^{fb})/\delta \end{aligned} \tag{5.4}$$

The first two terms in the first line are the probability of not receiving a better offer multiplied by the wage firm F promises to pay. The third term is the expected wage if the worker receives a better offer. The second term in the second line is the $c'(e_i^{fb})$ without the discount factor, and the cost of working does not appear in the equation. In the case of no-matching counteroffers, firms promise to pay an amount already larger than the cost of working. This means even in the worst case, the worker receives a wage larger than the cost of working, implying that in all cases, the worker will work, and she will be subject to the cost of working. The resulting expected payoff of the old worker is given in equation (5.5).

$$\begin{aligned} E_t[u_{t+1}^r] &= E_t[w_{t+1}^r] - \overline{c} \\ &= \theta_i h^F + e_i^{fb} c'(e_i^{fb})/\delta - \overline{c} \end{aligned} \tag{5.5}$$

Similar to the matching counteroffers case, I use equation (5.5) and the individual rationality constraint from inequality (5.1), and I obtain the young worker's wage given in equation (5.6).

$$w_t^r = c(e_i^{fb}) + \overline{c} - \delta\theta_i h^F - e_i^{fb} c'(e_i^{fb}) + \delta\overline{c} \tag{5.6}$$

Given that wage terms for both policies are derived explicitly, their differences can be discussed in more detail. Similar to the matching counteroffers case, the no-matching counteroffers policy wage differs for different types of workers. This result is again expected because the worker's type directly affects the output. One significant difference between the wage equations is that the wage function for no-matching counteroffers depends on h^F. The exact relationship between the parameters and the wage is given in *Lemma 4*.

Lemma 4. *The young worker's wage for the no-matching counteroffers case, w_t^r, is decreasing in both h^F and θ_i. Thus, it achieves its minimum value at $(\overline{h}, \theta_H)$, which is $w_t^r(\overline{h}) = (1+\delta)\overline{c} - \delta\theta_i\overline{h}$.*

The interpretation of *Lemma 4* is that a high-type worker employed at a more productive firm gets paid less than a low-type worker working for a less productive firm. The main aspect of the model that leads to this result is the competition among workers to get a job. The competition among workers makes it possible for firms to offer young workers a wage that makes the workers' expected lifetime payoffs equal to 0. Since the expected wages for the old workers are increasing in both h^F and θ_i, it makes sense that the young workers' wages are decreasing in these parameters. This is because output is the product of worker types and firm productivity and in the no-matching counteroffers case, they get paid that period's output. This implies that higher wages when they are old incentivize these workers to work for lower wages when they are young.

5.2 Cost of Working

Lemma 4 states that the minimum value of the expected wage of the young worker is given by $w_t^r(\overline{h}, \theta_H) = (1+\delta)\overline{c} - \delta\theta_H\overline{h}$. To analyze the firm decision without considering the liquidity constraint, the parameters are required to provide a non-negative $w_t^r(\overline{h}, \theta_H)$. The non-negativity of the wage term can be achieved by imposing restrictions on one of the parameters, and in this case, the parameter of interest is $\overline{c}$. Recall that the upper bound for $\overline{c}$ is $\theta_L\underline{h}$. So, by restricting $\overline{c}$ to be in the interval given in inequality (5.7), I ensure that all young workers receive positive amounts.

$$\theta_L\underline{h} \geq \overline{c} \geq \frac{\delta\theta_H\overline{h}}{1+\delta} \tag{5.7}$$

One problem with inequality (5.7) is that the defined interval may not exist. I impose an additional assumption on the model to ensure that such an interval exists for all $\delta \in (0, 1)$.

Assumption 1. *The lowest possible to highest possible output ratio satisfies the following inequality.*

$$2 \geq \frac{\theta_H \overline{h}}{\theta_L \underline{h}}$$

Assumption 1 restricts the ratio of lowest to highest possible output. Putting this restriction on the model makes it possible to analyze the case where the liquidity constraint is irrelevant. From this point on, I assume that *Assumption 1* and inequality (5.7) hold to ensure that all wages remain positive.

5.3 Profits

In this section, I use the wages defined previously to derive the expected profits for firms. The expected profits derived in this section play a crucial role in the no-matching counteroffers equilibrium because the sustainability of the equilibrium is determined by the profitability of the no-matching counteroffers policy. Similar to the previous sections, the derivations start with the matching counteroffers policy.

5.3.1 Matching Counteroffers

The expected profit from a generation of a firm that chooses to match counteroffers is given by

$$\Pi_t^s = \theta_i h^F - w_t^s + \delta E_t[\pi_{t+1}^s]. \tag{5.8}$$

The first two terms in equation (5.8) give me the firm's profit in period t. The last term is the expected profit in period $t+1$. The expected profit in period $t+1$ is given explicitly in equation (5.9).

$$E_t[\pi_{t+1}^s] = (1 - e_i^*)(\theta_i h^F - \overline{c}) + e_i^* G(h^F)\theta_i(h^F - E_t[h|h \leq h^F]) \tag{5.9}$$

In period $t+1$, there are three cases. With probability $(1-e_i^*)$, the worker does not receive any outside offers; in that case, firm F pays $\overline{c}$ to the worker. With probability $e_i^* G(h^F)$, the worker receives an outside offer from firm R with productivity $h^R \leq h^F$. In this case, the worker stays with firm F and gets paid $\theta_i h^R$. Then, firm F's profit for that period becomes $\theta_i(h^F - h^R)$. This possibility is included in the second term in the equation. I use the expectation operator since h^R is randomly drawn from a distribution. Lastly, the worker receives an outside offer from firm R with productivity $h^R > h^F$. In this case, the worker moves to firm R, and firm F makes zero profit. Combining equations (5.8) and (5.9) and substituting w_t^s, I obtain the expected profit from the generation born in period t.

$$\begin{aligned}\Pi_t^s &= \theta_i h^F - (c(e_i^*) + \overline{c} - e_i^* c'(e_i^*)) \\ &\quad + \delta[(1-e_i^*)(\theta_i h^F - \overline{c}) + e_i^* G(h^F)\theta_i(h^F - E_t[h|h \leq h^F])]\end{aligned} \tag{5.10}$$

In equation (5.10), the first term is the firm's output that comes from the young worker, the negative term is the young worker's wage, and the remaining part is the expected second-period profit of the firm.

5.3.2 No Matching Counteroffers

To conclude this section, I consider the profits for firms who choose the no-matching counteroffers policy. To do that, I need to compute the expected profit from generation for the no-matching counteroffers policy. In this case, the expected profit for firm F from an old worker is zero. This is because the firm offers all its output to implement the efficient level of job-search effort. Thus, the expected profit for a firm only consists of the profit from the young workers. Equation (5.11) provides the expected profit from the generation that is born in period t.

$$\Pi_t^r = \theta_i h^F - \Big(c(e_i^{fb}) + (1+\delta)\overline{c} - \delta\theta_i h^F - e_i^{fb} c'(e_i^{fb})\Big). \tag{5.11}$$

As stated, the profit when the worker is old is 0 in this case. So, the expected profit from a generation simplifies to the expected profit from the young worker.

5.4 *Expected Gain from No-Matching*

Every term necessary to define an equilibrium for both matching and no-matching counteroffers policies is derived. The findings until this point show that the efficient level of job-search effort cannot be reached by matching counteroffers. In this section, I conclude the analysis by showing that the efficient job-search effort can be achieved by the no-matching counteroffers equilibrium and derive the conditions required to sustain this equilibrium. Recall that the equilibrium considered in this paper is a grim trigger strategy. By definition, the desired no-matching counteroffer policy equilibrium can only be sustained if firms commit to it in each period. If firms switch to the matching counteroffers policy, the relational contract is broken and cannot be repaired. So, in equilibrium, deviating from the no-matching counteroffers policy must not be more profitable for firms. To see whether deviating is more profitable for firms, I introduce a new term called the expected gain from no-matching counteroffers and denote it by $\Delta\Pi_t$. Observe that the subscript t denotes the period when the policy decision is being made. In the analysis, I evaluate this term for periods t and $t+1$ assuming a new generation is born at period t. The equation for the expected gain, $\Delta\Pi_t$, is given in equation (5.12).

$$\begin{aligned}\Delta\Pi_t =& E_t\Big[\sum_{j=0}^{\infty}\delta^{2j}\Pi^r_{t+2j}\Big] - E_t\Big[\sum_{j=0}^{\infty}\delta^{2j}\Pi^s_{t+2j}\Big] \\ &= \sum_{j=0}^{\infty}\delta^{2j}E_t[\Pi^r_{t+2j}-\Pi^s_{t+2j}] \\ &= \Pi^r_t - \Pi^s_t + \frac{\delta^2 E_t[\Pi^r_{t+2j}-\Pi^s_{t+2j}]}{(1-\delta)^2}\end{aligned} \tag{5.12}$$

The main takeaway from equation (5.12) is seen in the last line. The term in the expectation operator on the last line is the expected gain per generation. If the expected gain per generation is positive, then the lifetime expected gain is positive as well. So, I need to derive the equation for expected gain per generation as well, and it is given in equation (5.13).

$$E_t[\Pi^r_{t+2j}-\Pi^s_{t+2j}] = E_t[c(e^*_i)] - E_t[c(e^{fb}_i)] - E_t[c'(e^{fb}_i)e^*_i] + E_t[c'(e^{fb}_i)e^{fb}_i] \tag{5.13}$$

Considering that the equations are written explicitly, I can conclude this part of the analysis in *Proposition 3.*

Proposition 3. *The no-matching counteroffers policy is more profitable for all firms, and the expected gain is increasing in* $h^F \in [\underline{h}, \overline{h}]$.

The first interpretation of this result is that as firms get more productive, the expected gain increases. This is because $(e_i^* - e_i^{fb})$ is increasing in h^F. Recall that the efficient effort, e_i^{fb}, is decreasing in h^F, and e_i^* is independent of h^F. Under the right circumstances, workers would not want to leave more productive firms because the wages they receive when they are old are higher than most other firms. However, with a matching counteroffers policy, workers exert more effort to increase their payoff when they are old. Because of that, the difference between e_i^{fb} and e_i^* is larger for more productive firms. Therefore, implementing the efficient effort level brings more payoff to more productive firms.

The only detail that remains to be shown is whether firm F would stick to the no-matching counteroffers policy at the beginning of period $t+1$. Recall that a new generation is born at period t. So, period $t+1$ corresponds to the middle of that generation's life. Considering this and using logic similar to that of equation (5.12) and the findings from equation (5.13), I derive the expected lifetime gain starting from period $t+1$, which is given in equation (5.14).

$$\begin{aligned}\Delta\Pi_{t+1} =& (\pi^r_{t+1} - \pi^s_{t+1}) + \sum_{j=1}^{\infty} \delta^{2j-1} E_t[\Pi^r_{t+2j} - \Pi^s_{t+2j}] \\ =& -\pi^s_{t+1} + \frac{\delta^2 E_t[\Pi^r_{t+2j} - \Pi^s_{t+2j}]}{(1-\delta)^2}\end{aligned} \tag{5.14}$$

The term $(\pi^r_{t+1} - \pi^s_{t+1})$ is the expected gain from period $t+1$ for no-matching counteroffers policy. Recall that $\pi^r_{t+1} = 0$. Therefore, this term becomes $-\pi^s_{t+1}$. The expected gain per generation is given in equation (5.13). Every term in the above equation is written explicitly, except π^s_{t+1}. Recall the formula for π_{t+1} in equation (4.6). It is implied in equation (4.6) that the minimum possible value for π_{t+1} is 0, and the maximum value of π_{t+1} is achieved by the minimum value of w_{t+1}. The interval of possible values of w_{t+1} is included in *Lemma 1*, and it can be seen that the minimum possible value of w_{t+1} is $\bar{c}$. Considering that, I conclude $\pi^s_{t+1} \in [0, \theta_H h^F - \overline{c}]$. If equation (5.14) ends up positive for the maximum value of π^s_{t+1}, firm F commits to the no-matching counteroffers policy in all possible scenarios. Since I search for an equilibrium where all firms commit to the no-matching counteroffers policy for all possible outside offers, I use the maximum value of π^s_{t+1}, which is $\theta_H h^F - \overline{c}$. With all terms in equation (5.14) derived explicitly, I can conclude the analysis with the main result of this paper.

Proposition 4. *There exists a $\overline{\delta}$ such that for all $\delta > \overline{\delta}$, firm F implements the efficient effort level e_i^{fb} by using a relational contract with the following features:*

1. *Firm F commits to pay $h^F\theta_i$ regardless of the outside offers.*
2. *The worker exerts e_i^{fb} on the job search.*
3. *If $h^R \leq h^F$ or the worker does not receive outside offers, she stays with firm F and gets paid $h^F\theta_i$.*
4. *If $h^R > h^F$, the worker moves to firm R and gets paid $h^R\theta_i$.*

In other words, for a sufficiently high δ, firms commit to the no-matching counteroffers policy. By implementing the efficient effort level, firms see a significant decrease in young workers' wages. Since the no-matching counteroffer policy is more profitable for each generation, it is reasonable for firms in markets with higher δ to commit to the no-matching counteroffer policy.

The two equilibria provided in *Propositions 1 and 4* also convey information regarding the turnovers. In the matching counteroffers equilibrium, the probability of a turnover happening is $e_i^*(1 - G(h^F))$, while for the no-matching counteroffers equilibrium, it is $e_i^{fb}(1 - G(h^F))$. In both equilibria, firms do not lose their workers if the outside offer comes from a firm with lower productivity or does not come at all. Consequently, the main factor that differentiates the probabilities of turnovers is the job-search effort. Since the job-search effort for the matching counteroffers policy equilibrium is higher, in that case, the probability of a turnover is higher. This outcome further highlights the advantages of a no-matching counteroffers policy, as it also reduces the likelihood of unwanted turnover.

The last factor that requires attention is $\overline{\delta}$. The $\overline{\delta}$ defined in *Proposition 4* is the minimum discount factor that is required for all firms to commit to the no-matching counteroffers policy. The significance of $\overline{\delta}$ is that it provides us with a signal regarding the likelihood of all firms sustaining a no-matching counteroffers equilibrium. A higher $\overline{\delta}$ implies that all firms are less likely to sustain such an equilibrium. The minimum discount factor required, $\overline{\delta}$, is defined by equation (5.15).

$$\frac{\overline{\delta}^2}{(1-\overline{\delta})^2} = \frac{\theta_H \overline{h} - \overline{c}}{E_t[\Pi^r_{t+2j} - \Pi^s_{t+2j} | \underline{h}]} \tag{5.15}$$

Equation (5.15) holds significant information on how $\overline{\delta}$ changes with market characteristics. The main market characteristic that can be seen is firm productivity. Observe that the term in the denumerator is the expected gain per generation for a firm with productivity $\underline{h}$. From *Proposition 3*, I know that the expected gain is increasing in h^F. So, as $\underline{h}$ increases, $\overline{\delta}$ decreases. The relationship between $\overline{\delta}$ and $\overline{h}$ is in the opposite direction. So, in markets where the difference between the highest firm productivity, $\overline{h}$, and the lowest firm productivity, $\underline{h}$, is relatively higher, the minimum discount factor needed for the no-matching counteroffers policy equilibrium $\overline{\delta}$ is higher. So, in markets where the markup between firm productivities is higher, it is less likely for all firms to commit to the no-matching counteroffers policy

The influence of the two other factors, θ_H, and $\overline{c}$, can be clearly observed in equation (5.15). $\overline{\delta}$ increases as θ_H increases while $\overline{\delta}$ decreases as $\overline{c}$, increases. So, for the likelihood of all firms committing to the no-matching counteroffers equilibrium to be relatively high, θ_H needs to be closer to θ_L while $\overline{c}$ needs to be closer to its upper bound, $\theta_L \underline{h}$.

Chapter 6

CONCLUSION

The model presented in this paper illustrates that a no-matching counteroffers policy is more effective for achieving efficient labor market outcomes. In the model, firms choose to match counteroffers, which increases the job search effort of workers rather significantly, leading to inefficient outcomes and more turnovers. This makes the market inefficient. However, this paper shows that this can be resolved by firms committing to no-matching counteroffers policy.

The findings also suggest that, under certain conditions, firms have more incentive to commit to the no-matching counteroffers policy as it is more profitable. Particularly, those with higher productivity stand to gain more from adopting a no-matching counteroffers policy. The main condition that is required for firms to be better off in a no-matching counteroffer policy regards the discount rate. If the discounting in the economy happens at a sufficiently high rate, all firms choose the no-matching counteroffers policy.

The analysis also provides more insight into market characteristics that are influential on firms' counteroffer policy decisions. These characteristics include the distribution of firm productivity, worker productivity, and the cost of working. The results show that as the difference between the highest and lowest firm productivity increases, it becomes less likely for firms to sustain the no-matching counteroffers policy equilibrium. The same result also holds for worker types. As the difference increases, no matching counteroffers policy equilibrium becomes less likely. Lastly, the cost of working influences the likelihood of the no-matching counteroffers policy equilibrium in a different way. as the cost of working increases, the likelihood of the no-matching counteroffers policy increases.

Although the results are compelling, there is still an opportunity for further improvement. One addition to the model would be that workers have private information regarding their types. This addition would shed more light on how firms' counteroffer decisions change depending on the type of their workers. when handling highly talented workers. Another addition to the model would be effort-driven output. In the model, the cost of working is constant among all workers, and the output is not driven by worker effort. Adding effort-driven output to the model would help examine the problem from a broader perspective and provide more insight into the inefficiency of the matching counteroffers policy. Although these do not diminish the significance of the findings, they offer avenues for future research to explore.

BIBLIOGRAPHY

Barron, J. M., Berger, M. C., and Black, D. A. (2006). Selective counteroffers. *Journal of Labor Economics*, 24(3):385–409.

Benhabib, J. and Bull, C. (1983). Job search: The choice of intensity. *Journal of Political Economy*, 91(5):747–764.

Faberman, R. J., Mueller, A. I., Şahin, A., and Topa, G. (2022). Job search behavior among the employed and non-employed. *Econometrica*, 90(4):1743–1779.

Fallick, B. and Fleischman, C. A. (2004). Employer-to-employer flows in the us labor market: The complete picture of gross worker flows. *Available at SSRN 594824*.

Haltiwanger, J. C., Hyatt, H. R., Kahn, L. B., and McEntarfer, E. (2018). Cyclical job ladders by firm size and firm wage. *American Economic Journal: Macroeconomics*, 10(2):52–85.

Karahan, F., Michaels, R., Pugsley, B., Şahin, A., and Schuh, R. (2017). Do job-to-job transitions drive wage fluctuations over the business cycle? *American Economic Review*, 107(5):353–357.

Levin, J. (2003). Relational incentive contracts. *American Economic Review*, 93(3):835–857.

Macneil, I. R. (1985). Relational contract: What we do and do not know. *Wis. L. Rev.*, page 483.

Mortensen, D. T. (1986). Job search and labor market analysis. *Handbook of labor economics*, 2:849–919.

Mortensen, D. T. and Pissarides, C. A. (1999). New developments in models of search in the labor market. *Handbook of labor economics*, 3:2567–2627.

Moscarini, G. and Postel-Vinay, F. (2017). The relative power of employment-to-employment reallocation and unemployment exits in predicting wage growth. *American Economic Review*, 107(5):364–368.

Mukherjee, A. (2010). The optimal disclosure policy when firms offer implicit contracts. *The RAND Journal of Economics*, 41(3):549–573.

Postel-Vinay, F. and Robin, J.-M. (2004). To match or not to match?: Optimal wage policy with endogenous worker search intensity. *Review of Economic Dynamics*, 7(2):297–330.

Potter, T. and Bernhardt, D. (2022). Wage offers and on-the-job search. *Canadian Journal of Economics/Revue canadienne d'économique*, 55(1):74–105.

Yamaguchi, S. (2010). Job search, bargaining, and wage dynamics. *Journal of Labor Economics*, 28(3):595–631.

Zábojník, J. (2012). Promotion tournaments in market equilibrium. *Economic Theory*, 51:213–240.

Appendix A

PROOFS

A.1 Proof of Lemma 1

Assume that the worker did not receive any outside offers. Then, she is willing to stay at the firm for any $w_{t+1} \geq 0$. The firm's objective function is given by $\pi_{t+1} - w_{t+1}$, and π_{t+1} does not depend on w_{t+1}. So, the firm's optimal wage offer is the minimum wage possible, which is $\overline{c}$.

To continue, assume that the worker received an outside offer from firm R with productivity $h^R \leq h^F$. In this case, the worker stays for any $w_{t+1} \geq \theta_i h^R$. By the same logic I used previously, it is optimal for the firm to offer $w_{t+1} = \theta_i h^R$.

Lastly, consider the case where the worker received an outside offer from firm R with productivity $h^R > h^F$. If the firm is to offer $w_{t+1} \geq \theta_i h^R$, the firm's profit becomes $\theta_i(h^F - h^R) < 0$. If the firm is to offer $w_{t+1} < \theta_i h^R$, the firm's profit is 0. □

A.2 Proof of Proposition 1

The objective function of the worker who maximizes future profits is as follows.

$$\max_{e \in [\underline{e}, \overline{e}]} \delta E_t[u_{t+1}] - c(e) \tag{A.1}$$

where

$$\begin{aligned} E_t[u_{t+1}] &= e \int_{\underline{h}}^{\overline{h}} (\theta_i h^R - \overline{c}) \, dG(h^R) \\ &= e\theta_i \int_{\underline{h}}^{\overline{h}} h^R \, dG(h^R) - e\overline{c} \int_{\underline{h}}^{\overline{h}} dG(h^R) \\ &= e(\theta_i E_t[h] - \overline{c}) \end{aligned} \tag{A.2}$$

accordingly, the objective function becomes

$$\max_{e\in[\underline{e},\overline{e}]} \delta e(\theta_i E_t[h] - \overline{c}) - c(e). \tag{A.3}$$

First order conditions, then provide

$$\delta(\theta_i E_t[h] - \overline{c}) - c'(e_i^*) = 0. \tag{A.4}$$

To conclude that the above equality provides us with the optimal effort, two more conditions need to be checked. The first condition is the non-negativity of the first derivative. In the model, I already assumed that $\overline{c} < \theta_L \underline{h}$. This restriction implies $\theta_i E_t[h] - \overline{c} > 0$. Thus, non-negativity is not violated. To conclude the proof, I need to check the second order condition, which is $-c''(e)$. By the characteristics of the cost function, I know that $-c''(e) < 0$ for all possible e. Hence, the value that satisfies the first-order condition provides the value that maximizes the objective function. Another detail in the proof that requires attention is the corner solutions. Since $\delta(\theta_i E_t[h] - \overline{c})$ is positive, I assume that $\underline{e} < e_i^*$. For the upper bound, it is assumed in the model setup that $c(\overline{e})$ is sufficiently large. These assumptions ensure that there are no corner solutions. In conclusion, the optimal effort level is given by the equation:

$$c'(e_i^*) = \delta(\theta_i E_t[h] - \overline{c}) \tag{A.5}$$

The other parts of *Proposition 1* are direct results of *Lemma 1*.

□

A.3 Proof of Lemma 2

The objective function to find the first best effort is given in equation (4.7). So, the problem becomes the following.

$$\max_{e\in[\underline{e},\overline{e}]} \delta\Big((1-e)\theta_i h^F + eG(h^F)\theta_i h^F + e(1-G(h^F))\theta_i E_t[h|h > h^F]\Big) - c(e). \tag{A.6}$$

The first-order conditions give the following.

$$\delta\Big(-\theta_i h^F + G(h^F)\theta_i h^F + (1-G(h^F))\theta_i E_t[h|h > h^F]\Big) - c'(e) = 0. \tag{A.7}$$

Rearranging the terms, the following equation appears.

$$\delta\theta_i(1 - G(h^F))[E_t[h|h > h^F] - h^F] = c'(e_i) \tag{A.8}$$

where $1 - G(h^F) = Prob(h > h^F)$.

The efficient effort level function $e^{fb}(\theta_i, h^F)$ is given by

$$e^{fb}(\theta_i, h^F) = c'^{-1}(\delta\theta_i(1 - G(h^F))[E_t[h|h > h^F] - h^F]) \tag{A.9}$$

In order to compute the derivative of the efficient effort level, I first need to derive the derivative of $E_t[h|h > h^F]$. Before that, consider its functional form.

$$E_t[h|h > h^F] = \int_{h^F}^{\overline{h}} \frac{g(h)h}{1 - G(h^F)} dh = \frac{\int_{h^F}^{\overline{h}} g(h)hdh}{1 - G(h^F)} \tag{A.10}$$

To find the derivative of the term above, I use the derivative rule for division. To do that, I require the derivatives of the numerator and denumerator. The derivative of the denumerator is simply $-g(h^F)$. By using the fundamental theorem of calculus, I compute the derivative of the numerator, and it is given by $-g(h^F)h^F$. Combining these results with the derivative rule for division, I reach the following equation.

$$\begin{aligned} \frac{\partial E_t[h|h > h^F]}{\partial h^F} &= \frac{-g(h^F)h^F(1 - G(h^F)) - (-g(h^F)\int_{h^F}^{\overline{h}} g(h)hdh)}{(1 - G(h^F))^2} \\ &= \frac{g(h^F)(-h^F + E_t[h|h > h^F])}{1 - G(h^F)} \end{aligned} \tag{A.11}$$

To complete the proof, I combine all the results with the inverse function theorem in order to compute the derivative of the efficient effort level with respect to h^F. For simplicity of the notation, I denote the above term as $E'_t[h|h > h^F]$

$$\begin{aligned} \frac{\partial e^{fb}(\theta_i, h^F)}{\partial h^F} &= \delta\theta_i \frac{(1 - G(h^F))(E'_t[h|h > h^F] - 1)}{c'(e_i^{fb})} \\ &- \delta\theta_i \frac{g(h^F)(E_t[h|h > h^F] - h^F)}{c'(e_i^{fb})} \\ &= \delta\theta_i \frac{(g(h^F)(E_t[h|h > h^F] - h^F) - (1 - G(h^F))}{c'(e_i^{fb})} \\ &- \delta\theta_i \frac{g(h^F)(E_t[h|h > h^F] - h^F)}{c'(e_i^{fb})} \\ &= -\frac{\delta\theta_i(1 - G(h^F))}{c'(e_i^{fb})} \leq 0 \end{aligned} \tag{A.12}$$

Notice that the above inequality is strict for the interior values of h^F. I conclude that the first best effort level is decreasing in h^F for $h^F \in [\underline{h}, \overline{h})$. To conclude the proof, I need to derive the derivative with respect to θ_i, and it is given below.

$$\frac{\partial e^{fb}(\theta_i, h^F)}{\partial \theta_i} = \delta(1 - G(h^F))[E_t[h|h > h^F] - h^F] \geq 0 \tag{A.13}$$

Thus, I conclude that the efficient effort level for high-type workers is higher. $\square$

A.4 Proof of Proposition 2

The difference between the first derivatives of the cost of optimal and first best effort levels are

$$c'(e_i^*) - c'(e_i^{fb}) = \delta\theta_i E_t[h] - \delta\theta_i(1 - G(h^F))[E_t[h|h > h^F] - h^F]. \tag{A.14}$$

Note that,

$$E_t[h] = G(h^F)E_t[h|h < h^F] + (1 - G(h^F))E_t[h|h > h^F] \tag{A.15}$$

Putting it into the equation and rearranging the terms, I get the following.

$$c'(e_i^*) - c'(e_i^{fb}) = \delta\theta_i\big(G(h^F)E_t[h|h < h^F] + (1 - G(h^F))h^F\big) > 0 \tag{A.16}$$

This implies that $c'(e_i^*) > c'(e_i^{fb})$. Since the cost function is convex,

$$c'(e_i^*) > c'(e_i^{fb}) \iff e_i^* > e_i^{fb}. \tag{A.17}$$

Thus, the optimal effort level is strictly higher than the first best effort. $\square$

A.5 Proof of Lemma 3

To encourage the efficient effort level, the firm must provide a wage scheme such that the optimal effort level for the worker is equal to the efficient effort level. Let the wage be denoted by w. In order to make calculations easier, I express the wage as the product of a new variable $h^* =$ and θ_i. This expression makes it possible for us to compare the firm's output and workers' wages in more detail. So I set $h^* = w/\theta_i$, and the resulting objective function for the worker becomes the following:

$$\max_{e \in [\underline{e}, \overline{e}]} (1 - e)w + eG(h^*)w + e(1 - G(h^*))\theta_i E_t[h|h > h^*] - c(e). \tag{A.18}$$

The first order condition provides:

$$\begin{aligned}c'(e) &= -w + G(h^*)w + (1-G(h^*))\theta_i E_t[h|h>h^*] \\ &= (1-G(h^*))\theta_i E_t[h|h>h^*] - (1-G(h^*))w \\ &= \theta_i(1-G(h^*))(E_t[h|h>h^*] - w/\theta_i) \\ &= \theta_i(1-G(h^*))(E_t[h|h>h^*] - h^*).\end{aligned} \tag{A.19}$$

The wage level that makes the effort level given by the above equation equal to the efficient effort level is $w = \theta_i h^F$. □

A.6 Proof of Lemma 4

The partial derivatives of w_t^r with respect to the parameters are given below.

$$\frac{\partial w_t^r}{\partial h^F} = \frac{\partial c(e_i^{fb})}{\partial e_i^{fb}}\frac{\partial e_i^{fb}}{\partial h^F} - \delta\theta_i - \frac{\partial c'(e_i^{fb})}{\partial h^F}e_i^{fb} - \frac{\partial c(e_i^{fb})}{\partial e_i^{fb}}\frac{\partial e_i^{fb}}{\partial h^F} = -\Big(\delta\theta_i + \frac{\partial c'(e_i^{fb})}{\partial h^F}e_i^{fb}\Big) \tag{A.20}$$

$$\frac{\partial w_t^r}{\partial \theta_i} = \frac{\partial c(e_i^{fb})}{\partial e_i^{fb}}\frac{\partial e_i^{fb}}{\partial \theta_i} - \delta h^F - \frac{\partial c'(e_i^{fb})}{\partial \theta_i}e_i^{fb} - \frac{\partial c(e_i^{fb})}{\partial e_i^{fb}}\frac{\partial e_i^{fb}}{\partial \theta_i} = -\Big(\delta\theta_i + \frac{\partial c'(e_i^{fb})}{\partial \theta_i}e_i^{fb}\Big) \tag{A.21}$$

Observe that both partial derivatives are negative for all values of θ_i and h^F. Since the w_t^r is decreasing in both parameters, its minimum value is achieved by the maximum values of θ_i and h^F. □

A.7 Proof of Proposition 3

To see whether $\Delta\Pi_t$ is increasing in h^F, I check the first derivative of $E_t[\Pi_t^r - \Pi_t^s|h^F]$ with respect to h^F. The reason behind this can be seen in equation (5.1). $\Delta\Pi_t$ is increasing in h^F if and only if $E_t[\Pi_t^r - \Pi_t^s|h^F]$ is increasing in h^F. The derivative of $E_t[\Pi_t^r - \Pi_t^s|h^F]$ is given below.

$$\begin{aligned}\frac{\partial E_t[\Pi_t^r - \Pi_t^s|h^F]}{\partial h^F} &= -\frac{\partial c(e_i^{fb})}{\partial e_i^{fb}}\frac{\partial e_i^{fb}}{\partial h^F} - \frac{\partial c'(e_i^{fb})}{\partial h^F}e_i^* + \frac{\partial c'(e_i^{fb})}{\partial e_i^{fb}}e_i^{fb} + \frac{\partial c(e_i^{fb})}{\partial e_i^{fb}}\frac{\partial e_i^{fb}}{\partial h^F} \\ &= \frac{\partial c'(e_i^{fb})}{\partial h^F}(e_i^{fb} - e_i^*) \geq 0\end{aligned} \tag{A.22}$$

To resume, I prove the first claim. I show that for all $h^F \in [\underline{h}, \overline{h}]$, $\Delta\Pi_t$ is strictly positive. Once again, I consider $\Delta\Pi_t$. The Mean Value Theorem states that for e_i^*

and e_i^{fb}, there exists an $x \in (e_i^*, e_i^{fb})$ that satisfies the following.

$$c(e_i^*) - c(e_i^{fb}) = c'(x)(e_i^* - e_i^{fb}) \tag{A.23}$$

By combining equations (A.23) and (5.13), I achieve the following.

$$\begin{aligned} E_t[\Pi_t^r - \Pi_t^s | h^F] &= c'(x)(e_i^* - e_i^{fb}) - c'(e_i^{fb})e_i^* + c'(e_i^{fb})e_i^{fb} \\ &= (e_i^* - e_i^{fb})(c'(x) - c'(e_i^{fb})) > 0 \end{aligned} \tag{A.24}$$

$E_t[\Pi_t^r - \Pi_t^s | h^F]$ being strictly positive means $E_t[\Pi_{t+2j}^r - \Pi_{t+2j}^s | h^F]$ is strictly positive as well. Thus, the following result is achieved.

$$\Delta\Pi_t = E_t[\Pi_t^r - \Pi_t^s | h^F] + \frac{\delta^2 E_t[\Pi_{t+2j}^r - \Pi_{t+2j}^s | h^F]}{(1-\delta)^2} > 0 \tag{A.25}$$

$\square$

A.8 Proof of Proposition 4

The proof of *Proposition 3* states that the expected gain per generation is increasing in h^F. Consequently, its minimum value is achieved by $\overline{h}$, and it is positive. Also, observe that the maximum value of $\theta_i h^F$ is achieved by the maximum value of h^F and θ_H, and it is also achieved by a positive number. To continue, I need to determine a value for $\overline{\delta}$. Let $\overline{\delta}$ satisfy the following equation.

$$\frac{\overline{\delta}^2}{(1-\overline{\delta})^2} = \frac{\theta_H \overline{h} - \overline{c}}{E_t[\Pi_{t+2j}^r - \Pi_{t+2j}^s | \underline{h}]} \tag{A.26}$$

Since both $\theta_H \overline{h}$, $min\{\Delta\Pi_{t+2}\}$ are positive numbers, I know that there exists a $\overline{\delta}$ that satisfies the equality above. Combining all these, I reach the following inequality.

$$\begin{aligned} &\frac{\delta^2}{(1-\delta)^2}\Big(E_t[c(e_i^*)] - E_t[c(e_i^{fb})] - E_t[c'(e_i^{fb})e_i^*] + E_t[c'(e_i^{fb})e_i^{fb}]\Big) - (\theta_i h^F - \overline{c}) \\ &= \frac{\theta_H \overline{h} - \overline{c}}{E_t[\Pi_{t+2j}^r - \Pi_{t+2j}^s | \underline{h}]}\Big(E_t[c(e_i^*)] - E_t[c(e_i^{fb})] - E_t[c'(e_i^{fb})e_i^*] + E_t[c'(e_i^{fb})e_i^{fb}]\Big) \\ &- (\theta_i h^F - \overline{c}) \geq \theta_H \overline{h} - \theta_i h^F \geq 0 \end{aligned} \tag{A.27}$$

Considering the inequality above, I conclude that there exists a $\overline{\delta}$ that for all $\delta > \overline{\delta}$, the above inequality holds. Hence, all firms commit to relational contracting for all $\delta > \overline{\delta}$. The other parts of *Proposition 4* are direct results of previous lemmas. $\square$