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DEVELOPING OF THE STRUCTURAL ANALYSIS FOR BEAMS BASED ON STIFFNESS MATRIX METHOD

Basma Faeq JUMAAH

Master's Thesis

Supervisor

Prof. Dr. Tuncer ÇELİK

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The thesis titled "DEVELOPING OF THE STRUCTURAL ANALYSIS FOR BEAMS BASED ON STIFFNESS MATRIX METHOD" prepared by BASMA FAEQ JUMAAH and submitted on 05/12/2022 has been **accepted unanimously** for the degree of Master of Science in Civil Engineering.

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Basma Faeq JUMAAH

Signature

DEDICATION

I want to thank GOD for his uncountable gifts

((And my success is not but through Allah. upon him I have relied and to him I return))

I dedicate this

To my supervisor Prof. Dr. TUNCER ÇELİK, thank you very much for the knowledge you gave it to us.

To my mom and my mother in low you have been always there for me.

To my father’s soul, I wish you were here with us.

To my beloved husband, you have been very supportive and I love you very much.

To my children, I am very proud of you.

ABSTRACT

DEVELOPING OF THE STRUCTURAL ANALYSIS FOR BEAMS BASED ON STIFFNESS MATRIX METHOD

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The using of beams in the form of steel or concrete materials is widely adopted in the construction projects. The conditions that coming with the design of the beams and the internal forces and moments depend on the cases of loading, the geometrical and physical properties of the beams, and the supporting states. The stiffness matrix is a general tool that take into account these parameters for evaluating the shear force and the bending moment diagram along the spans of the beams in part or all the construction body. The stiffness matrix can become more bigger when increasing the number of beam spans or that influence the overall internal stresses inside the member. In the present study, a technique based on Gaussian elimination theory was applied on the body of the stiffness matrix toward finding the required variables in less steps with same degree of accuracy. The study converted the matrix that used for two spans beam from six rows and six columns to only two rows and two columns that the magnitudes of the slope of deformation can be determined then the reactions and moments can be identified easily. The study move to more analytical steps by analyzing the relationship between the uniform loading, the length of span, and the beam stiffness condition EI with each of the four parts of the derived matrix: A1, A2, B1, and B2. The resultant of that analysis was building of new simple equations that can be used to obtained the four main parts of the matrix without the previous steps of the conventional stiffness matrix and the Gaussian elimination steps. The results of the new equations were checked by evaluation the values of the main parts according to the original steps and the derived equation. The differences were so small indicating the feasibility of the derived method to contribute in positive role for the design purpose. The study explained the variation of the shear forces and the bending moments along the beam spans by building simulation of specific case of beams under uniform loading by using of ANSYS workbench program. A full mathematical model for calculate the displacements and reactions has been made in MATLAB/Simulink from the main four parts of the stiffness matrix. In aim of making this model being used in analysis of continuous beam easily. Besides make use of it in field of understanding the analysis method.

Keywords: Structural Analysis, Reinforcement, Stuffiness Matrix

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ABBREVIATIONS

SM	:	: Stiffness Matrix
LTB	:	Lateral-Torsional Buckling
PSI	:	Pounds per Square Inch

LIST OF SYMBOLS

Es : Modulus of elasticity of Steel

L : Length of beam

M : Moment on beam

W : Wight of beam

Ө : Rotation of the member

1. INTRODUCTION

1.1 INTRODUCTION

Beam structures are a kind of structural component that construction companies and certain types of engineers must be familiar with. These structures play an important role in transmitting weight besides carrying and with stand vertical loads and ensuring that the foundation of a building is firmly rooted in the ground. The most common types of beam structures are overhanging, fixed, trussed, continuous, and easily supported beams. In the this position, The beam structure generally can be made from concrete in square and rectangular shape with adding the metal bars to resist the tension force or can be made from aluminum, steel or other metal with popular (I) section to resist the shear force and bending moment in efficient way. an over view has been reviewed on what an installation of various is, why understanding them is important, and the most common types of beams used by building project engineers When a structure is subjected to dynamic or static loads, cracks may develop and the modal frequencies of the cracked structure may change. The most popular approach which is particularly well suited for modeling structures is the finite element method. However, the traditional FEM cannot obtain satisfactory results for eigen-value problems. therefore stiffness matrix method used for analysis the structure [1].A beam structure, also known as both a beam, is a structure used in engineering and construction to create a safe and effective load path that distributes weight evenly across the base of a constructing. These spent as much time the load by resisting bending under load pressure. However, in each new theory are doing hypotheses about how the system behaves or element. Therefore, we must always be aware of these hypotheses when evaluating solutions, the result of applying or developing theories .As the force is applied to the line, the beams resist the load laterally. In the structural systems analysis has been studied by diverse researchers in the past, making a brief historical review of progress in this subject The load-bearing pattern in most cases consists of a slab, beam, section, and foundation. This means that the beam is installed underneath the base and section to provide additional collective support throughout the structure [2]. All structural steel design codes allow for the design of beams that are susceptible to buckling. A possible mode of buckling for slender beams is lateral-torsional buckling (LTB). An elastic critical moment (Mcr) is determined, where Mcr dictates the resistance of slender beams. The plastic moment of resistance (Mp) limits the capacity of stocky beams. A beam structure uses a variety of reinforcement material depending on the type of building as well as the purpose of the beam. following are the most commonly used reinforcements kind of beams. [3].

a) Main bars are the type of reassurance used to carry loads.

b) Support bars: A additionally completed is a reassurance that is connected to the upper level of the beam and serves to hold the stirrups in place.

c) Stirrups: This type of reassurance is used to compensate for shear force or shear applied stress and strain in the structure.

1.2 SIGNIFICANCE OF BEAM STRUCTURE

Recognizing element structure is critical in construction as well as structural engineering because these beams support the majority of the building's weight. Beams ensure that the weight of the building's roofs, ceilings, and floors is properly staffed by providing a stable loading channel at the building's base. Design and construction professionals must understand which type of beam is best suited for a project and how they can influence beam installation to ensure that the structure being built can withstand its own weight[4].

If a construction is taller, larger and heavier beams will almost certainly be needed in the future to sustain the load efficiently. Smaller pieces are required for smaller constructions because they carry a lighter load and do not require as much assistance. Choosing the wrong type of element can result in the framework not being strong enough to support its own weight, which can be detrimental to the building's durability and safety. Understanding beams necessitates a thorough understanding of statics, design, as well as fundamental physics. This knowledge helps construction and design professionals to determine the loads that will affect this same beam and choose the right size, shape, and component of the required beam in a timely manner[5].

1.3 BEAM STRUCTURE TYPES

Beams of various types are used in engineering and construction. These beams are in a standard condition and are classified according to their duration, balance, pass, and supporting style and they include:

1.3.1 Continuous Beams

When the beam has more than two supports, the beam can be classified according to that to continuous beam. This kind of beams is consisting of numbers of beam which gathered to be one structure. An energy is possible is an execution that is reinforced by one or more aids. These aids are usually vertical and are used inside and between the beams. Continuous beams are thought to be less expensive than other types of beams[6].

1.3.2 Simply Supported Beams

Simply measured beams have help and support at both ends. This is widely used in general construction and are quite able to adapt in regards to the types of constructions that can be used with them. A simple support beam has no tensile stress at the help area and is designed to allow the ends on columns or walls to freely rotate[7].

1.3.3 Fixed Beams

This kind of beams is known also as built-in beam because of the beam both two ends are built-in or fixed to a wall. A repaired beam is a single element with no supports on both ends. The type of element does not support trying to bend action of present time creation and will not support future vertical movement or rotation. Fixed beams are typically used in trusses and other similar structures[8].

1.3.4 Overhanging Beams

A cantilevered component is one that is backed for two separate linked locations, usually at one finish and in the center of the beam, but lacks support for the other end, leaving it dangling. This beam type extends beyond the wall or sections and has an unsupported overhanging section. Combining a braced piece and a deflection beam results in a cantilevered beam[9]. The overhang beam can represent to be continues beam with free or cantilever end span only, the warping resistance is the major difference between the overhang beam and cantilever beam. The warping resistance in cantilever beam is banding, while the warping resistance can be allowed, besides it rely on the neighboring span relative stiffness in the overhang beam[10].

1.3.5 Cantilever Beam

A cantilever beam is a single element that is free at one end as well as fixed at the other. This type of element can carry loads preceded by both bending activity of moment as well as sheer force and therefore is commonly used in the construction of bridge trusses or similar structures. In its normal state, the fixed end is attached to a column or a wall. A cantilever beam's tension zone is located above the element, with compaction at the bottom of the beam[11].

1.4 INTERNAL FORCE IN BEAMS

Beams are structural components that can be used in many different design applications, including roofs, roads, specialized equipment, and so on. A beam is a narrow, straight, rigid structure made of isotropic materials that is subjected to lots and lots orthogonally to its latitudinal line. When the same structural part is subjected to longitudinal forces rather than perpendicular loads, it is made reference to as a column or post. If the same member can be torqued, it is referred to and handled as a shaft. As a result, when defining mechanical or shaping, it is critical to consider how forces act[12]. Shearing powers were internal forces created in the beam material to counteract externally accepted forces in rank in order to maintain the balance of all beam elements. Bending moments are internal moments formed in a beam's material to counterbalance the tendency of external forces to cause rotation of any beam element. By adding the vertical forces to the left and right of the section under consideration, the shear load at any point on an element can be calculated. Similarly, the having joined moment at each segment of a beam can be computed by summing the moments from the left and right sides of the section under consideration[13]. The location under scrutiny is the pivot point of the moment. Internal shearing forces having to act downwards are traditionally regarded as positive. They are used to offset positive external factors. As a direct consequence, you can draw shear forces in the path of the external factors when expressing them. It is more visually pleasing than the sign convention[14].

Deflection in structural engineering describes the movement of a component or node away from its original position. It occurs with intention of imparting forces to the body. Deflection and displacement are both ended up causing by applied external loads or by the weight of the body structure itself. It can occur in beams, girders, frames, and almost any other type of body building projects[15].

1.5 BEAM DEFLECTION FORMULA

Deflection is the amount that a specific structural element can be shifted with the help of a significant amount of load. It is also known as an angle or a distance. The distance a member deflects when subjected to a force is proportional to the slope of the body's deflected shape under that load. It can be calculated by integrating the state of function used to characterize the slope of the member under that load [16].

A few beam deflection formulae can be used to calculate a fundamental value for diversion in various types of beams. In general, the deflection calculation has been done by dividing the product of E and I (Young's Modulus and Moment of Inertia) by the double integral of the Bending Moment Equation. The length unit of deflection, or displacement, is commonly measured in millimeters. This rank indicates how far the beam could be diverted from its original location[17].

The stiffness matrix is being used for structure analysis specially the displacement method which can be used for indeterminate and determinate static structures besides by applying the stiffness matrix, the forces and displacement values can be gained immediately. The beam stiffness matrix analysis method working on dividing the flexural beam to small pieces (finite element) known as members, and for each support is found, location of external force or connected point of two members beside the point where the beam cross section is being changed, the location of nodes are there. Development of the stiffness method for beams follows a similar procedure as that used for trusses. First we must establish the stiffness matrix for each element, and then these matrices are combined to form the beam or structure stiffness matrix. This analysis method is dealing with the loads where located on the joints of the beam only. For each node the effect of bending moment and shear force are taking in concern, then these effects will cause a rotation and vertical displacement (two degree of freedom) in each node. In order to apply the stiffness method to beams, we must first determine how to subdivide the beam into its component finite elements. In general, each element must be free from load and have a prismatic cross section. For this reason the nodes of each element are located at a support or at points where members are connected together, where an external force is applied, where the cross-sectional area suddenly changes, or where the vertical or rotational displacement at a point is to be determined. Finally the overall stiffness for whole beam is being calculated after applying the stiffness matrix for each member to determine the displacements, and from known displacements all the reactions and internal forces can be also known [18].

1.6 PROBLEM STATEMENT

The formation of stresses in different beams joints is non-easy to calculate. The weak detection of the available stresses that founded by the external dead loads and live loads can lead to severe problems like unwanted degree of deflection and cracking. In other hand, the excessive supporting to the construction can affect the cost without play significance role in enhancing the member strength.

1.7 CONTRIBUTION

The design of beams lead to put precise view toward the internal forces that take place in the member body. The study aimed to take the advantages of using stiffness matrix analysis for creation analytical system for the flexural analysis beams under different loading. The analysis dealt with the structural properties of the reinforced beams toward more effective design results.

Besides using the stiffness matrix analysis method which depends on divided the beam structure in finite element, considering to be very important method in the field of engineering both analysis and design. Because this method can be used for determinate and indeterminate structure furthermore the displacements can be directly obtained by fixed steps and procedures. This analysis method also is a base for many analysis software programs for engineering properties. Furthermore Made a mathematical model to calculate the reactions and displacements directly and easily is a step towards understanding the analysis method in simplest way specially four students.

1.8 OBJECTIVE

a) Dealing with various cases of reinforced concrete beams in the way of different geometrical characteristics and loading shapes.

b) Applying the stiffness matrix procedure for each case and forwarding the data into statistical analysis under ANSYS program for gaining flexible steps toward better solutions

c) Simulating the cases under ANSYS workbench program for explaining in more supporting figures the behavior of stresses formation.

d) Explain the Link between the stuffiness matrix techniques and the statistical analysis in effective graphs and charts by using excel sheets and charts.

e) Built full mathematical model in MATLAB Simulink to estimate the reactions and displacements directly from the main parts of the beam structure stiffness matrix after elimination. This model can be used to solve any continuous beam subjected to uniform load with update the variables very easily. That's will help in analysis field as well as the students to understand the stiffness matrix method in simple way.

2. LITERATURE REVIEW

2.1 OVERVIEW

In most cases, a statics analysis is the first step in determining how an engineering system deforms under load and how the scheme supports the load inside. For a system to be in equilibrium, the vector sum of all the forces Fi acting on it must equal zero. Similarly, the velocity sum of all the moments Mi acting on a scheme must equal zero. Regardless of how complex the scheme, it must keep the applied loads from accelerating. Free body diagrams of the entire system, separate components, and parts of personal mechanisms are critical to the resolution of minor issues. The form of interest is isolated from its surroundings in an FBD. Power and moment vectors are then used to represent all of the forces and moments acting on the form from its surroundings. The rank of FBD is also indicated in the coordinate system[19].

To ensure the serviceability of tall reinforced concrete structures, it is essential to accurately assess the tension and deflection beneath lateral and gravity loads. In recent times, high-rise and slim structures have been built with high-strength steel and real. As a result, the serviceability limit condition for lateral drift becomes an increasing importance design standard that must be satisfied in order to prevent large additional data in a structured P-delta effects. In addition, control of deformation in bolstered true beams is critical for ensuring serviceability requirements. Because of the low tensile strength of concrete, cracking can occur at service loads, reducing the flexural and shear stiffness of armor-plated concrete memberships. Cracked members in reinforced concrete buildings must be identified and their efficient flexural and shear stiffness determined for accurate deflection determination. An exact stiffness matrix of a beam element on elastic foundation is formulated. A single element is required to exactly represent a continuous part of a beam on a Winkler foundation. Thus only a few elements are sufficient for a typical problem solution[20].

2.2 DIFFICULTY IN MATERIALS

Complexity is defined as the rigidity of any object prior to material. When subjected to physical force, objects with a high stiffness determination maintain balance in form. Loose, wet earthen, for example, has low stiffness and can change shape with only a few pounds of strain. Aluminum's difficulty is markedly stiffer than wet unglazed clay. Steel alloys, on the other hand, have a high stiffness, allowing them to withstand thousands of pounds per square inch (PSI) of stress without deforming the metal's surface[21].

Structural beam stiffness or rigidity is a scale for how much the stiffness of a structure is to overcome the bending moment. Rigidity is the capacity of a structure scheme to withstand external loads without undergoing significant changes in its geometrical (distortions). It is one of the most important design parameters systems and components. While strength is the most important project criterion, there are many instances where stress and strain in elements and their connections are significantly below allowable levels and aspects as healthy as performance characteristics of systems and their components are determined by rigidity requirements[22] . Recently, significant progress has been made in refining the strength of schemes and components. These advancements are based on the creation of great strength metals, composites, and other resources, as well as a better understanding of fracture failure marvels and the development of better techniques for stress analysis and computation, which has resulted in tensile stress and safety variables plummeting. Nonetheless, these advancements frequently result in the reduction of irritated -sections of physical components. Because payload-induced loads in structures typically do not change, systemic deformations in systems made of high-strength materials and/or designed with reduced safety factors are more noticeable[23].

The modulus of elasticity, also known as Young's Modulus, determines stiffness. While strength varies from grade to grade, Young's Modulus remains constant for a given steel and is unaffected by external stressors such as heat action, handling, and cold work. When it comes to strength vs. stiffness in modern simple footings, the power of metal can change all through the item, but its rigidity determination remains constant[24].

2.2.1 Benefits of Stiffness

The impact of deformations on static then tiredness strength, materials resistance, efficiency, dynamic/shaking, stability, and manufacturability causes difficulty effects on structure. To sum up, the stiffness standard's rank is currently rising as a result of:

a) Cumulative precision requirements requiring deformation lowering

b) Increasing the use of high-strength resources, which results in fewer irritated sections of the components and, as a result, increased structural deformations;

c) Improved analytical methods lead to smaller care factors, which lead to shortened cross sections and accumulated deformations[25].

2.3 STRENGTH IN RESOURCES

Physical strength is the amount of stress that can be applied to a material before it permanently deforms or disrupts. This can be perplexing because opposition to deformation caused is an important component of both strength and difficulty. marlin-strengthening -custom-bags One of the most noticeable differences is the use of the word "perpetual." A production instance can curve from a coercion that is less than its maximum yield strength, but it will return to its original shape once the force is removed. This means that a strong physical can curve without resistance and return to his original condition. Another significant difference is that the strength of a material is determined by its chemical composition and the heat treatments it undergoes when being converted in size or shape. Simply put, forte in a metal can vary depending on the processes that metal has been subjected to [26].

2.4 YOUNG MODULUS

It is the degree of difficulty or resistance of a solid to flexible deformation. It describes the relationship between stress (force per unit part) and strain (proportional deformation) along an axis or streak. The basic code states that when a physical is compressed before being extended, it undergoes elastic deformation and returns to its original shape once the load is removed. The Young's modulus is frequently affected by the orientation of a physical. Isotropic materials have mechanical characteristics that are identical in all directions. Pure metals and porcelains are two examples. Working a physical or going to add impurities to it can result in ounce structures that direct motorized properties. These anisotropic components may have very distinct Young's modulus value systems depending as to whether force is applied perpendicular to the grain or along it. Anisotropic components involve wood, concrete structures, and carbon fiber[27].

The elastic modulus is essentially an unit of measure of a material's stiffness. The modulus of elasticity of concrete is an important factor in estimating the deformation of buildings and representatives, as well as a key factor in determining the decisive modular ratio, m, which is rummage-sale for the design of flexure-prone sections of members. The precise self-control of the elasticity modulus of concrete is critical for constructions which require strict deformability switching. The modulus of bounciness is frequently used in the sizing of reinforced and non-reinforced structural members, determining the amount of reinforcement, calculating stress for observed straining, and is especially important in the project of pre- harassed structural structures. As a result, the young's modulus of concrete is a critical parameter indicating the ability of real to deform elastically[28] . Furthermore, in order to make full utilization of the compressive strength possibility, structures made of high strength concrete often seem to be slimmer and necessitate a higher elastic modulus to maintain their stiffness. As a result, understanding the elasticity modulus of high strength concrete is essential for preventing extreme deformation, providing acceptable serviceability, and avoiding the most expense designs. Un - reinforced concrete is a brittle material with low ductile strength, ductility, and little crack resistance. Although conventional bolstered strengthen bars can increase the tensile strength of the concrete, they need not increase the inherent concrete's Tensile Strength. Cracks in bolstered real members spread freely until they come into contact with a reinforcement saloon. There is a need for omnidirectional and widely dispersed reinforcement for real. This emphasizes the use of relatively simple methods for improving various static and dynamic properties of concrete. One such method is the arbitrary addition of reinforcement fibers to concrete, which is referred to as Steel Grit Reinforced Concrete [29].

2.4.1 Stress in Beams

When a beam is subjected to external force or load a resistance force in the structure will generated, this resistance force on unite area is called stress. There are two kinds of stresses, the first is shear stress which is generated in a structure due to the effect of two opposite forces in direction and equal in magnitude. The second one is normal stress which is generated in the structure due to the two opposite pull forces in direction and equal in magnitude, normal stress is being divided in compressive stress and tensile stress. In the same time due to the same external force the dimensions of the structure will be effected and change, the ratio between the changing in dimensions of the structure above the original dimensions of the structure before the load applied is called the strain[30]. A beam must be constrained in order to maintain static symmetry when external loads are applied to it. Constraints are established at single points along the beam, as well as the essence of the constraint is controlled by the boundary disorder at that point. The border condition implies whether the beam is fixed (not moving) or free to move in all directions. The x-direction (axial orientation), y-direction (circumferential direction), and rotation are the directions of interest for a two-dimensional beam. To have a constraint on a point, the parameter must specify that at least one unique direction is fixed on that juncture. If there is any wonder that the condition designates that the beam is fixed in a direction, then an outer reaction within this direction may exist on the location of a boundary. If a beam is secured in the y-direction at a specific point, a transverse (y) outer force acting may develop at that point. Similarly, if the beam is not secure against spinning at a specific point, an external response instant could develop at a certain point [31].

The instant diagram can be used to calculate the winding moment, M, along the length of the beam. The twisty moment anywhere at point along the beam could then be used to determine the bending stress over the irritated section of the beam at that point. The bending stress is zero at the beam's pivot point, which coincides with the beam's cross section centroid. Bending stress increases linearly aside from of the neutral axis until it reaches maximum values at the top and bottom of the beam's extreme fibers[32].

A linear elastic examination is predicated on the assumption that all elements' difficulty is independent of weight. This assumption only applies to reinforced concrete rays in the untraceable stage. As a result, even in elastically designed beams, a redistribution of interior forces occurs before the ultimate load is reached. As a purpose of the load, the moment at the fixed finish and the moment in the span are shown, followed by the curvature. The stiffness in the region of the fixed end decreases after reaching the cracking weight . As a result, as weight increases, the fixed end instant grows at a slower rate than the range moment. Following that, blows might very well develop within the span, resulting in a reduction in stiffness in the ipsilateral side. This, in turn, will result in an increased growth of the secure end moment. Because the yield strength of the reassurance at the secure end is greater than within the span, this same yield strength of a reassurance at the secure end will be touched first. The consistent pile is the harvest load Qy, which is less than the final load[33].

The stress-stain behaviors for steel and concrete were explained in figure 2.1 and 2.2 respectively

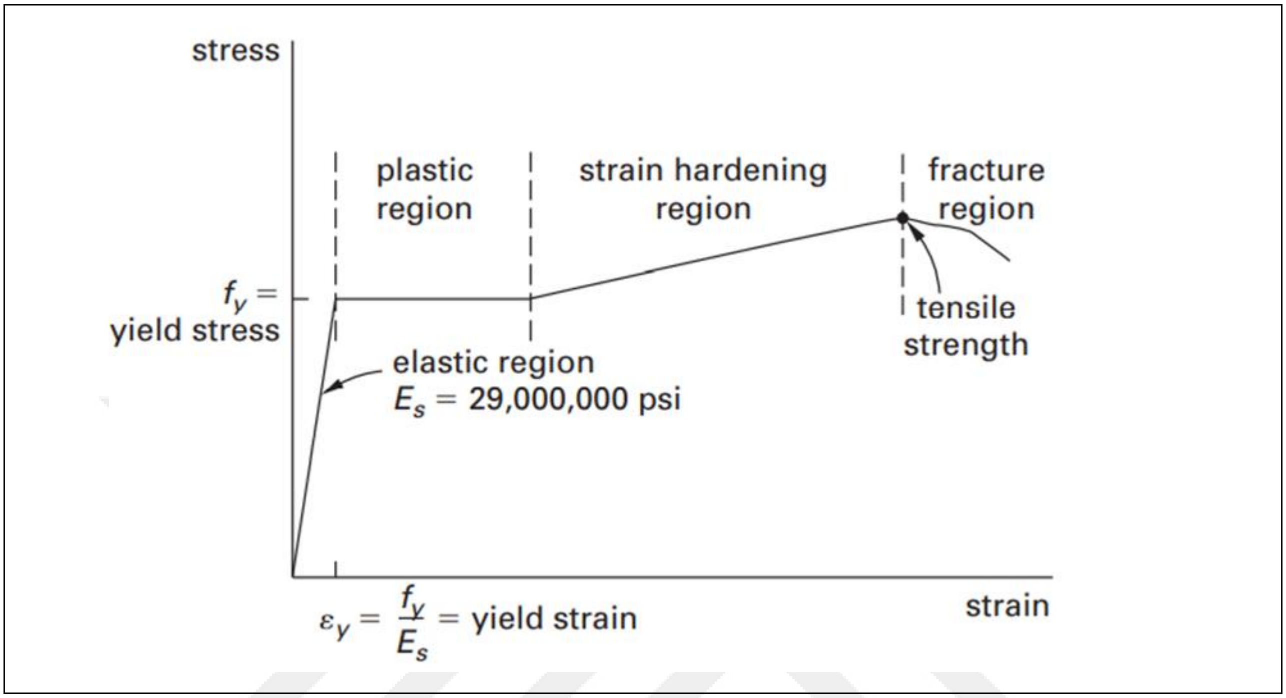

Figure 2.1: Stress- Strain Behavior in Steel [51]

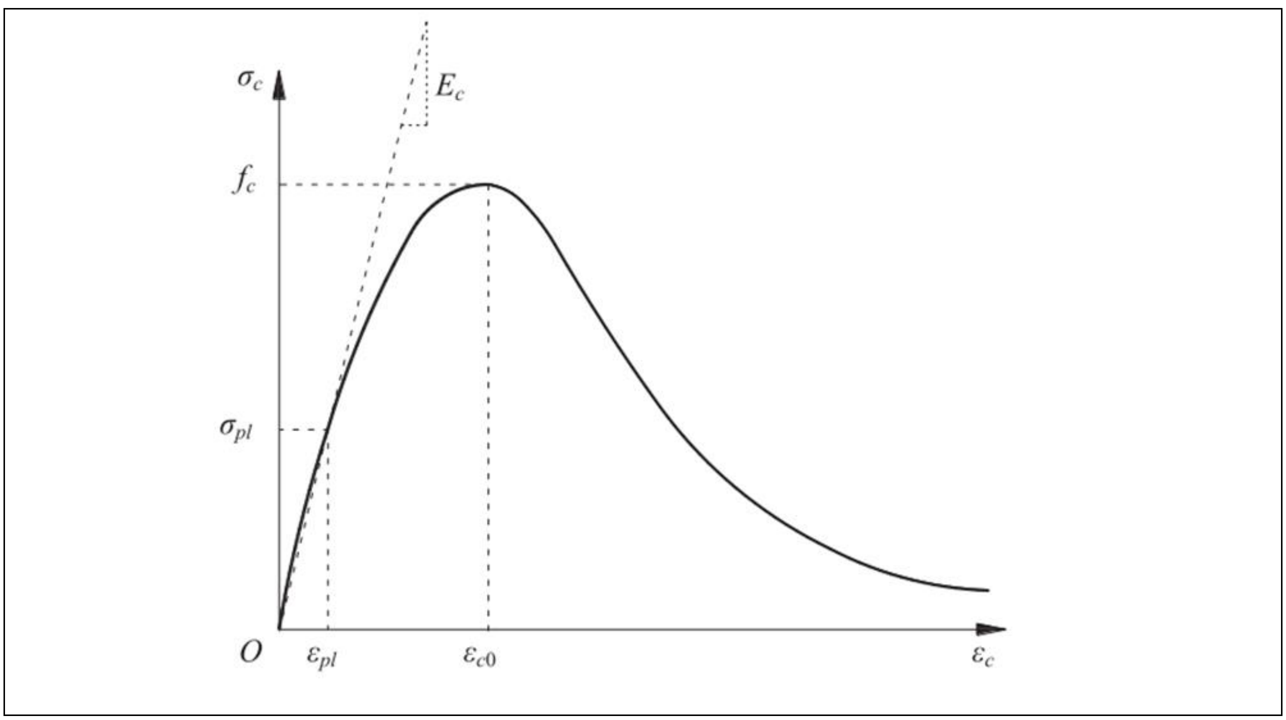

Figure 2.2: Stress -Strain Behavior in Concrete [52]

2.4.2 Bending Stress

When a beam is loaded to external loads, all artillery pieces of the beam dedication experience bending and shear militaries. Some practical bending stress requests will also be handled by. These are the [34]:

a) Instant carrying capacity of a section

b) Assessment of extreme usual stresses due to winding

c) Design of ray for bending

d) Assessment of load manner capacity of the ray

The primary stresses induced by winding seem to be tension and compression stresses. However, the stress state within the ray would include shear stresses due to shear power as well as major regular stresses due to bending, though the former are usually of lower order when compared to the latte [35] . The state of stress on cross section can be recognized by identifying the bending moment sign as shown in figure 2.3.

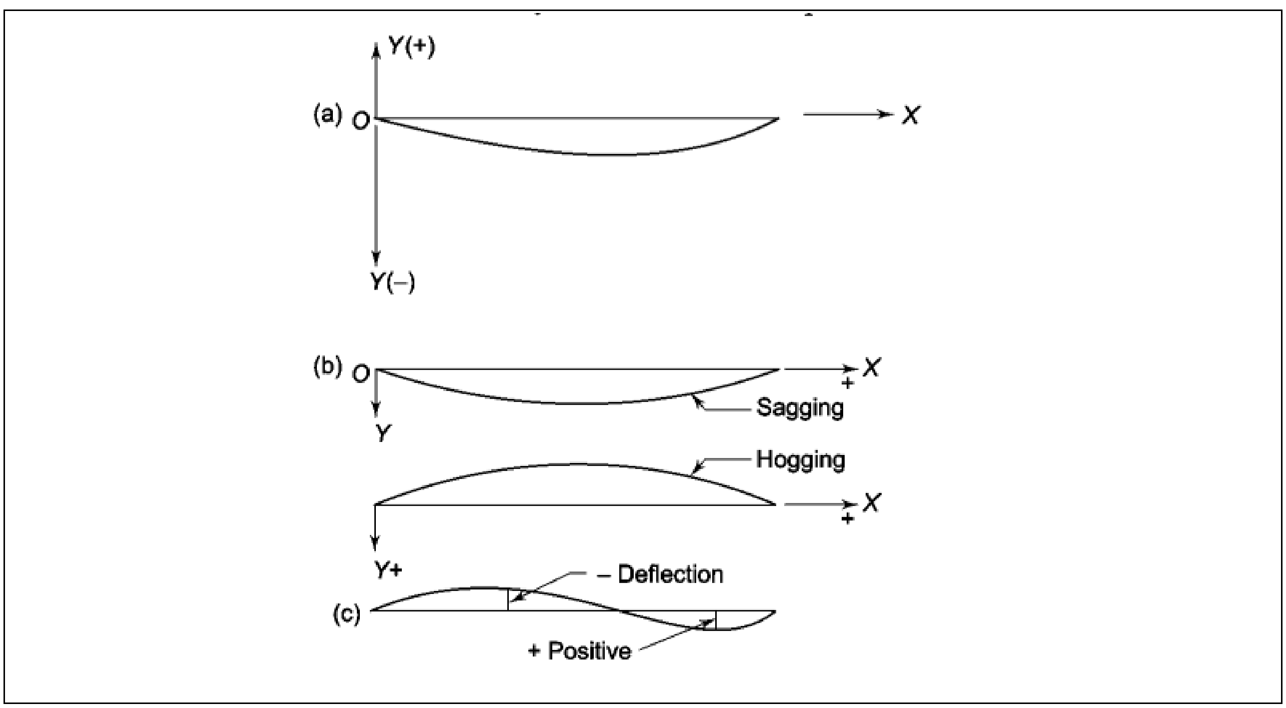

Figure 2.3: Winding Moment in negative than positive formation [30]

2.4.3 Molds for Theory of Unadulterated Bending

The assumptions complete in the theory of simple bending are by way of follows:

a) The beam's material is flawlessly racially homogenous (of the same kind throughout) and orthotropic (same elastic properties in all directions).

b) The content is hounded first within about an elastic boundary and then according to Hooke's rule.

c) The material's modulus of bounciness is the same under tension and compression.

d) The beam undergoes pure bending and bends in the manner of an arc of a circle.

e) After having to bend, the transverse sections that were plane and ordinary to the long direction prior to actually bending remain plane and ordinary to the longitudinal alliance of a beam.

f) The curvature radius of a bent axis of the beam is large in comparison to the sizes of the beam sections.

g) Each layer of the beam is allowed to expand and contract independently.

h) The cross-sectional area seems to be symmetric around a perpendicular to the neutral axis axis[36] [37].

2.5 CURRENT WORK

The study aimed to build active analysis of reinforced concrete beam based on the analytical technique of stuffiness matrix under various loading conditions. The study arranged the steps from assigning the effective parameters that influence the design progress of such members including the geometrical and physical properties toward achieving the results for the output member state. Moreover, the study simulated the assigned cases of beams under the finite element technique that built-in of ANSYS workbench program. Furthermore The study develop a full mathematical model by MATLAB/Simulink in aim of using the beam stiffness matrix analysis method in easiest way, besides the model is updated easily so it can be used for different cases.

2.6 LITERATURE SURVEY

The results indicate that numerical approaches are commonly used in static structural troubles. As analytical approaches prove ineffective, they are commonly the only way to obtain responses to the most complex then practical problems. The formation of this newspaper was motivated by a desire to broaden the scope of numerical techniques to cover the problems of creating constitutive equations of raw objects. The goal of this research was to develop a medium or numerical methods discrete constitutive model of materials. It outlines the underlying assumptions of a proposed method for designing material possessions. Only when combined with an appropriate numerical algorithm is the matrix model useful. This paper designed and described one such algorithm. Its findings led to the development of processer software for numerical simulations. The calculation examples presented have long demonstrated the effectiveness of the advanced method for creating constitutive composite models of various fairly standard materials, such as steel, but also, for example, overenthusiastic -elastic substances. It also demonstrates how fundamental medium designs can be used to simulate simple stress states before analyzing structural rudiments such as reinforced real estate. Each of the examples listed involved the material requirements' physiological non - linearity. The advanced composite modeling of materials is shown to be effective as well as versatile. In complex exams of structures made of nonlinear materials, he can be used as a true alternative to traditional foundational or rational models based on basic arithmetic operations[38].

The research indicate that compression examination can be used to determine the low dielectric tangent and mechanical characteristics secant of concrete, but for simple structure, connections with fairly standard power are used. Friendships are far too commonly used it to calculate transversal elastic properties, resulting in a safe value of 0.20 for the Elastic modulus (v). The purpose of this research was to assess the use of ultrasonic transmitting waves to regulate these characteristics. Samples with f ck tend to range from ten to 35 MPa were employed in the tests. Ultrasonic tests were performed on conical and cubic specimens. The yield point of deflection obtained via ultrasound has been found to be very comparable to that acquired via compression tests. The results of units obtained using family members with f ck differed greatly from those obtained prior to compression tests using ultrasound. The ultrasound Poisson's ratio was greater than the fixed amount. Authors can show that the ultrasound concrete summary is reliable, and the cylindrical example, which would be normally used to decide f ck, could be applied to this portrayal[39].

The indefinite vibrations of the functionally erected supports were investigated using the dynamic rigidity method. It was found that the dynamic properties of the material are supposed to evolve in the direction of thickness. After that, the differential equations of motion are solved, and the force, shear force, and everything related to the maximum bending expressions are derived. Then the stiffness matrix is calculated by relating the relative intensity to the force and the displacement. For some illustrative examples. The results were discussed and several conclusions were drawn. Where a development took place in the dynamic shear modulus of a reinforced composite beam structure with a special reference towards an algorithm for the purpose of investigating the properties of free vibration, starting from deriving the variation equations that direct the movement in the free vibration[40].

The study revealed a novel concrete analyze method called continuous, organic growth, tangential analysis . To calculate stiffness counting parts with negative standards, uses a part-wise linear stress-strain curve followed by a tangent elasticity modulus. Indefinite construction stiffness matrices have traditionally been avoided because they generally denote instability. However, because analysis involves introducing concussion in stages, the full range of real behavior can be addressed, including the unstiffening portion under tensile fast. CITA is validated here against numerical and new results for concrete rays, demonstrating earlier remedies for quasi problems than successively -linear analysis whilst also reducing identity stiffness constraint. In this effort, a second model, which is essentially a macro-modeling technique that uses the stiffness matrix technique to analyze an equal one-strut perfect used to replace the infilled panel, is also used, and the results are validated against those of the micro-demonstrating procedure. It was discovered that the difficulty matrix technique for infilled frame instruction-modeling can quickly and effectively model the shave power response of reinforced concrete members with openings waking to a failure weight [41].

The research presented a technique for determining the pervasive stiffness of reinforced real structural processes of massive buildings using the decay method. The secondary calculated arrangement for deciding dynamic extra loading in the periphery of a monolithic structure is implemented in the form of infrastructure secluded from the periphery of a building inside the zone of likely local failure underneath a special action. Aiming for such a scheme, generalized company offers a range are defined, with the stiffness of the overlying floors determined by the angular rigidity of the support restraints (head-to- skull structural elements). Logical requirements are proposed, as well as a technique for determining the overall stiffening of reinforced true structural components with stipulated border conditions. The impact of border conditions on infrastructure internal power factors is investigated. References are provided for improving the survival rate of the regarded engineering structures of monolithic building structures, as well as suggestions for protecting them from liberal collapse[42].

On the grounds of the systemic stiffness matrix, which would be described as the variant of and out vector field with respect to a datum coordinate vector, a high efficiency tensegrity element method is presented. This same stiffness matrix as well as potential maximum power of the framework are used to guide the structural configuration's rapid convergence to a soul and stable position. The probability selecting algorithm is used in the coded procedure to exclude rigid-body movements, the restricted walk algorithm to ensure the positive specificity of the systemic stiffness matrix, as well as the line query optimizer to reduce potential maximum energy. From such an arbitrary initial state, we can quickly predict the self-equilibrated as well as stable setup of a tensegrity using this form-finding method. A number of typical examples are provided to show its accuracy as well as efficacy both for regular as well as irregular large-scale tensegrity structures. A novel SMFF technique for tensegrity framework form-finding has indeed been established. The structural stiffness matrix is used to guide this same computation to rapidly cohere to an ego and stable arrangement, even when the blocks assumed at the start of the computation differs greatly from the final procured state. A variety of examples show that this method is extremely efficient for different kinds of tensegrities. This research could aid in the design and building of tensegrity constructions for practical uses[43].

The study demonstrated that the advancement of structural project methods nowadays allows for the inclusion of the effects of spatial and mechanical nonlinearity of components in the analysis itself. The resolution via media methods usually entails an iterative treatment for weight application and a line stiffness matrix that takes non - linearity into account. The current paper demonstrates a method for evaluating bar difficulty while taking mechanical nonlinearity of materials into account. For reinforced concrete structures formed by two materials in which each of them does have a resistant behavior that differs from the other, an accurate assessment is both necessary and particularly complex. The iteration described here provides the symmetry stance of the unit to the general situation of axial force then biaxial bending while taking nonlinear constituent friendships of resources into account. Once the symmetry is achieved, the connection between the quasi - isotropic moment and the curving allows the section increasingly difficult module to be calculated, from which the bar stiffness is calculated. The iterative process of symmetry research in a reinforced concrete unit is demonstrated as an implementation of the bare procedure. On the moment-curvature drawing, the stiffness gradually decreases as the biaxial winding moments increase, due to material nonlinearity stresses and strains, and the section inertia decreases due to concrete fast [44].

The research revealed that the necessity of developing a unified design procedure for non-linear analysis of reinforced real. A highly efficient form-finding method of temerity is presented on the basis of the structural stiffness matrix, which is defined as the derivative of the out-of-balance force vector with respect to the nodal coordinate vector. The stiffness matrix and the total potential energy of the structure are utilized to direct the rapid convergence of the structural configuration to the self-equilibrated and stable state. Besides gain ground on the subject of engineering's future prospects. The ease of use of fast computing machines, as well as the variety of processor programming environments and arithmetic methods, provides excellent potential for solving manufacturing tasks that were previously impossible to solve. The free vibration of functionally graded Timoshenko beams is investigated by developing the dynamic stiffness method. Material properties of the beam are assumed to vary continuously in the thickness direction. The governing differential equations of motion are solved and expressions for axial force, shear force and bending moment are derived. The dynamic stiffness matrix is then formulated by relating the amplitudes of forces and displacements at the ends of the beam However, in addition to understanding the engineering problem, there is an urgent need for programming skills. As a result, actions should be taken to encourage such cooperation at all levels of engineering/technical work. use the derived arithmetic model The process of redistributing wealth of internal military powers in steel and concrete rudiments could then be evaluated and defined in terms of its actual magnitude and nature. The numerical analysis results could be compared to the currently used simplified approaches. Most likely, a more efficient and all-around algorithm for tangible member design and reinforcement will emerge[45] .

According to the findings, the article sums up and analyzes the basic requirements affiliated with initial conceptual design then hypothesis of reinforced real structural frames easily understandable synopsis accompanied by one numerical model then pictures. One of the main hypotheses is the stiffness solid block and the product EI, which have an impact on the perfect analysis and construction behavior. Following the model analysis, it is discovered that the value of the winding moment on the column is increasing while the elasticity modulus is decreasing. On struts, the alteration of the creation EI causes the value of the bending instant to decrease. This information is crucial because, in reality, the construction behavior is directly impacted by the concrete condition, which must be perfectly provided in simulation. Because the lump and girder pull are combined, the slab must have the same troubles as the beam[46] .

The research revealed that the having to cut this daily paper a structural collimated (3 span constant ray type) that can resist tangent loading solely by transferring only winding moment and shear power at its start and end nodes is investigated. Sadly, the effect of axial filling along the beam's span is not taken into account in this newspaper. For the etymology of the native rigid body of ray elements, the stiffness medium method employing the Castilian strategy is rummage-sale. The first code mathematical formalism is presented. An investigation example clarifies the methodology for using the stiffness moderate approach, which would be similar to the limited element approach. Due to the ease of achieving the global major difficulties matrix of the ray structure and carrying out other matrix manipulation as needed, MATLAB software is employed in the data processing approach. The analysis example available in the paper considers various types of crosswise loading and assistance movement conditions. These included transverse loading such as partial evenly dispersed loading on a beam, juncture loading for forces and moments both along the time frame and at support areas, vertical support settlement, and the application of rotation at the support node. Finally, the Stationary distribution structural software is used to verify the findings of a study data in this paper[47].

3. METHODOLOGY

3.1 BEAM STIFFNESS MATRIX

A prismatic beam of a constant cross section that is fully restrained at ends in local orthogonal co-ordinate system. The beam ends are denoted by nodes zyx and j. The axis coincides with the centroid axis of the member with the positive sense being defined from k x' j to k. Let L be the length of the member, A an area of cross section of the member, I is the moment of inertia about axis, and E the young modules of elasticity for the beam. At each end of the member, two degrees of freedom are considered. As a result, this member has four degrees of freedom, and the resulting stiffness matrix is of the order. Counterclockwise moments and rotations are considered positive in this method. Positive displacements in the direction of the coordinate axis are considered. When unit displacements are imposed at each end of the member, the stiffness matrix elements indicate the forces exerted on the member by the restraints.

The forces which developed in the above beam member have been computed when a unit displacement is imposed along each degree of freedom while all other displacements are held constant. Now, at the j end of the member, impose a unit displacement along the y' axis while holding all other displacements to zero. In the beam, this displacement causes shear and moment. They are, by definition, elements of the member stiffness matrix .They form the first column of the element stiffness matrix, and the unit rotation in the positive sense is imposed at the j end of the beam while all other displacements are held to zero.

The elimination method applied for the stiffness matrix is Gaussian elimination method. In this method a series of equations are being applied to the rows of the matrix, which aimed for making the lower matrix triangle values equal to zero[48]. Gaussian elimination or row reduction algorithm is usually used to solve systems of linear equations. The algorithm consists of a sequence of operations which are applied to the matrix representing the set of equations. Linear equations are used vastly in all the fields of Science and Engineering. Researchers have come up with many ways to solve any set of linear equations The sequence of equations of This elimination method depending on multiplying a non-zero number by any matrix row, swap two row and place one instead other, and multiplying or adding one row to another [49].This elimination method is well known and widely used for solving the linear equations.

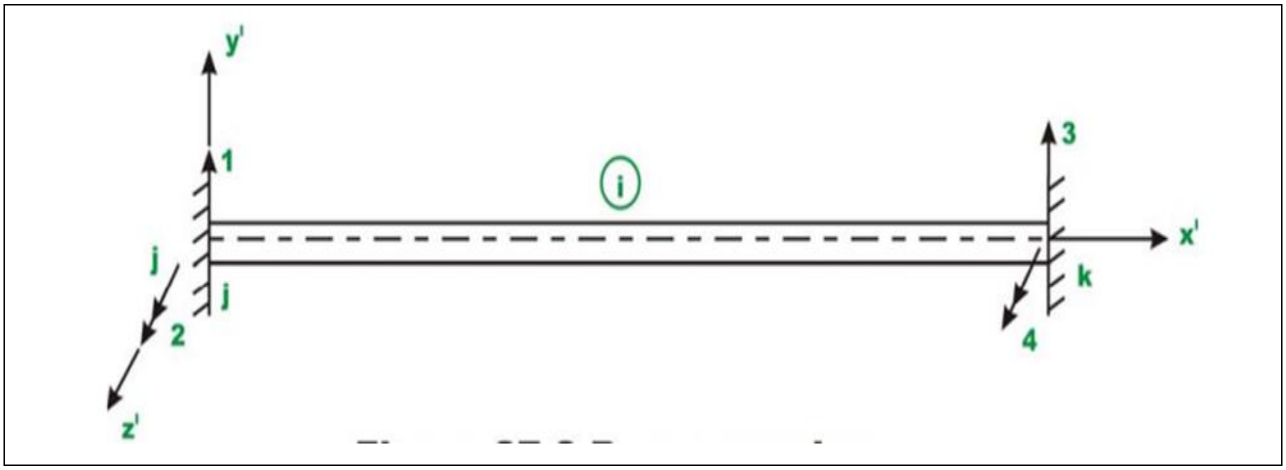

Figure 3.1: Assigning Nodes for stiffness matrix analysis[50]

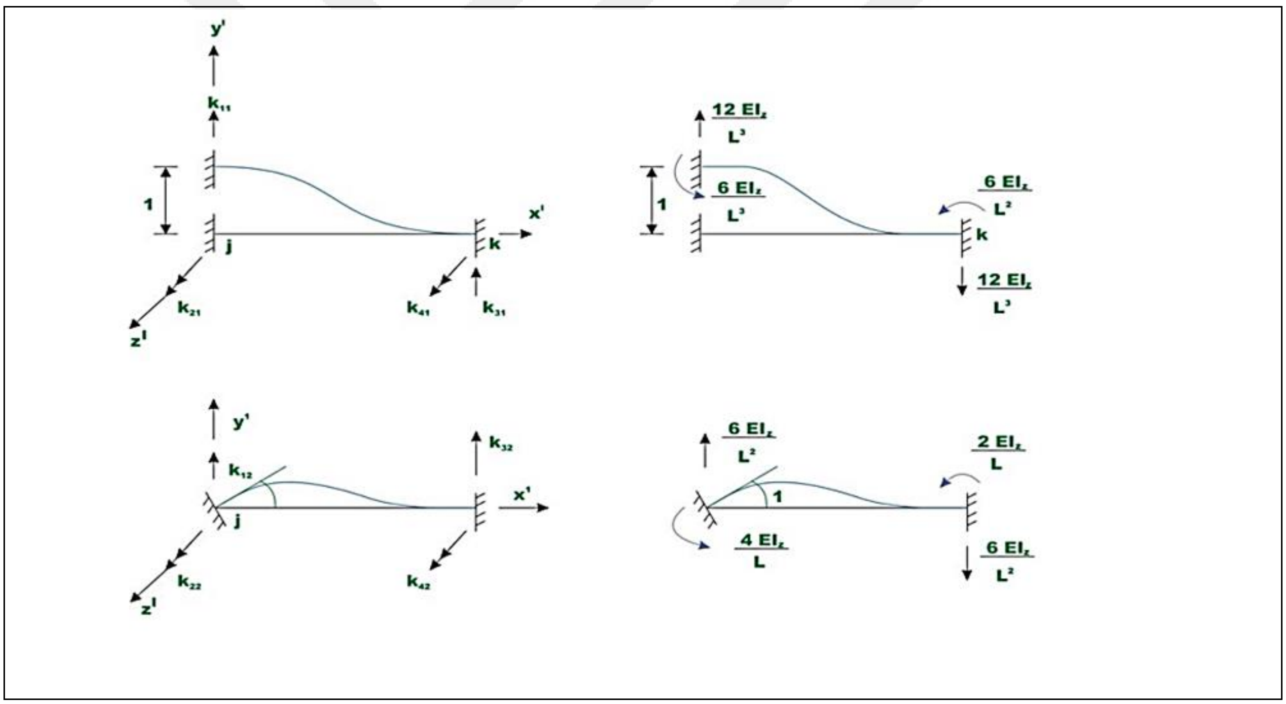

Figure 3.2: Deformation and displacement behavior for stiffness analysis[50]

The general form of the stiffness matrix for beam is:

$$SM = \frac{EI}{L^3} \begin{vmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{vmatrix}$$

Ac=Aj+AE

By assuming no loading on joints

Aj=0

3.2 STIFFNESS REACTIONS AND MOMENTS

Table 3.1: Stiffness Reactions and Moments F.E.M

$RAB = \frac{wL1}{2}$	$MAB = \frac{wL1^2}{12}$
$RBA = \frac{wL1}{2}$	$MBA = -\frac{wL1^2}{12}$
$RBC = \frac{wL2}{2}$	$MBC = \frac{wL2^2}{12}$
$RCB = \frac{w\,L2}{2}$	$MCB = -\frac{wL2^2}{12}$

C

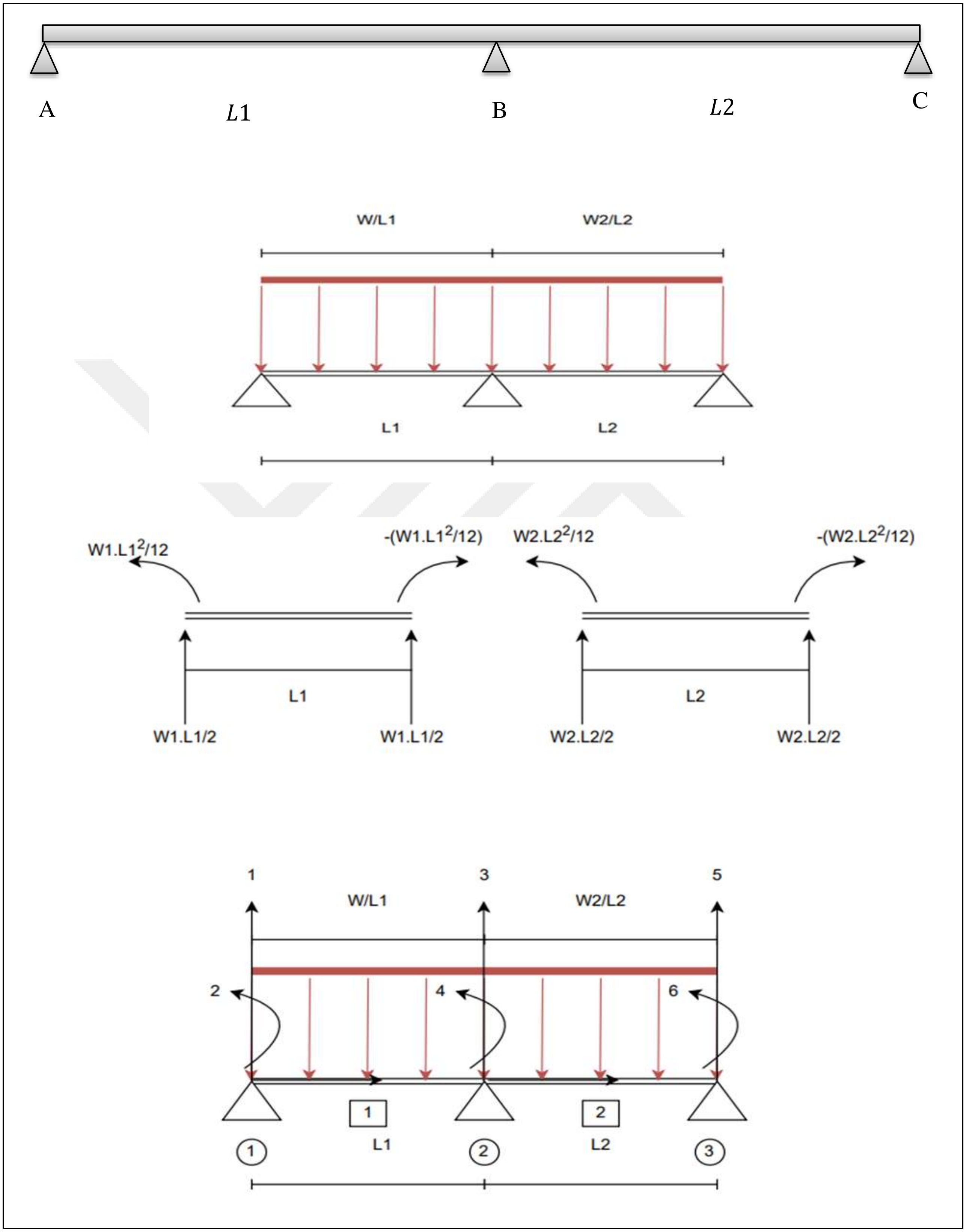

Figure 3.3: The Distribution load, F.E.M, vertical displacements and rotation on the beam

The stiffness matrix for the first member

		1	2	3	4
	1	12	6L1	-12	6L1
	2	6L1	$4\ L1^2$	-6L1	$2L1^2$
EI/L^3	3	-12	-6L1	12	-6L1
	4	6L1	$2L1^2$	$-6L1^2$	$4L1^2$

The stiffness matrix for the second member

		3	4	5	6
	3	12	6L2	-12	6L2
	4	6L2	$4\ L2^2$	-6L2	$2L2^2$
EI/L^3	5	-12	-6L2	12	-6L2
	5	6L2	$2L2^2$	$-6L1^2$	$4L2^2$

From the collection of the two matrices for the two beam members The overall stiffness matrix for the two spans became:

		1	2	3	4	5	6
	1	12	6L1	-12	6L1		
	2	6L1	$4\ L1^2$	-6L1	$2L1^2$		
EI/L^3	3	-12	-6L1	12+12	-6L1+6L2	-12	6L2
	4	6L1	$2L1^2$	-6L1+6L2	$4L1^2$+ $4\ L2^2$	-6L2	$2L2^2$
	5			-12	-6L2	12	-6L2
	6			6L2	$2L2^2$	-6L2	$4L2^2$

From the collection of the stiffness matrix, summing the effect of the spans occurred at the cells as explained below:

		3	4
	3	12+12	-6L1+6L2
EI/L^3	4	-6L1+6L2	$4L1^2$+ 4 $L2^2$

The variation of the core of the matrix as explained above can examined for each cell to identified its effect on the change of the overall reaction magnitude.

From the boundary conditions of the joints and the non-displacement at the middle joints, the matrix can be reduced to:

		2	4	6
	2	4 $L1^2$	$2L1^2$	
EI/L^3	4	$2L1^2$	$4L1^2$+ 4 $L2^2$	$2L2^2$
	6		$2L2^2$	$4L2^2$

The equivalent matrix for forces and moment AE can be obtained as following

$$\left|\begin{matrix} (\frac{w1\ L1}{2}) \\ -(\frac{w1\ L1^2}{12}) \\ (\frac{w1\ L1}{2}+\frac{w2\ L2}{2}) \\ -(-\frac{w1\ L1^2}{12}+\frac{w2\ L2^2}{12}) \\ (\frac{w2\ \ L2}{2}) \\ -(-\frac{w2\ L2^2}{12}) \end{matrix}\right|$$

By eliminating the matrix to satisfy the results in simple steps:

	C1	C2	C3	Q
R1	$4k1/L1$	$2k1/L1$	0	$-\frac{w1\ L1^2}{12}$
R2	2k1/L1	4k1/L1+ 4 k2/L2	2k2/L2	$-(-\frac{w1\ L1^2}{12}+\frac{w2\ L2^2}{12})$
R3	0	2k2/L2	4k2/L2	$\frac{w2\ L2^2}{12}$

By dividing R1 by 4k1/L1:

	C1	C2	C3	Q
R1	1	0.5	0	$-\frac{w1\ L1^3}{48\ k1}$
R2	2k1/L1	4k1/L1+ 4 k2/L2	2k2/L2	$-(-\frac{w1\ L1^2}{12}+\frac{w2\ L2^2}{12})$
R3	0	2k2/L2	4k2/L2	$\frac{w2\ L2^2}{12}$

By Adding $-2\mathrm{k1/L1}\ \ R1\ to\ R2$:

	C1	C2	C3	Q
R1	1	0.5	0	$-\frac{w1\ L1^3}{48\ k1}$
R2	0	3k1/L1+ 4 k2/L2	2k2/L2	$-\left(-\frac{w1\ L1^2}{12}+\frac{w2\ L2^2}{12}\right)+\frac{w1\ L1^2}{24}$
R3	0	2k2/L2	4k2/L2	$\frac{w2\ L2^2}{12}$

By dividing R2 by 3k1/L1+ 4 k2/L2:

	C1	C2	C3	Q
R1	1	0.5	0	$-\frac{w1\,L1^3}{48\,k1}$
R2	0	1	$\frac{\frac{2k2}{L2}}{(\frac{3k1}{L1}+\ 4\frac{k2}{L2})}$	$\frac{-\left(-\frac{w1\,L1^2}{12}+\frac{w2\,L2^2}{12}\right)+\frac{w1\ \ L1^2}{24}}{\frac{3k1}{L1}+\ 4\frac{k2}{L2}}$
R3	0	2k2/L2	4k2/L2	$\frac{w2\,L2^2}{12}$

By adding -2k2/L2 R2 to R3

	C1	C2	C3	Q
R1	1	0.5	0	$-\frac{w1\,L1^3}{48\,k1}$
R2	0	1	$\frac{\frac{2k2}{L2}}{(\frac{3k1}{L1}+\ 4\frac{k2}{L2})}$	$\frac{-\left(-\frac{w1\,L1^2}{12}+\frac{w2\,L2^2}{12}\right)+\frac{w1\,L1^2}{24}}{\frac{3k1}{L1}+\ 4\frac{k2}{L2}}$
R3	0	0	$\frac{-4\ (\frac{k2}{L2})^2}{(\frac{3k1}{L1}+\ 4\frac{k2}{L2})}+\frac{4k2}{L2}$	$\frac{\left(-\frac{-\left(-\frac{w1\,L1^2}{12}+\frac{w2\,L2^2}{12}\right)+\frac{w1\,L1^2}{24}}{\frac{3k1}{L1}+\ 4\frac{k2}{L2}}\right)(-2k2)}{L2}+\frac{w2\,L2^2}{12}$

From the last form of the matrix conversion steps, the magnitude of four parts of the matrix can be obtained as following:

$$k = EI \tag{3.1}$$

$$A1 = \frac{\frac{2k2}{L2}}{(\frac{3\mathrm{k1}}{\mathrm{L1}} + 4\frac{\mathrm{k2}}{\mathrm{L2}})} \tag{3.2}$$

$$A2 = \frac{-4\,(\frac{k2}{L2})^2}{(\frac{3\mathrm{k1}}{\mathrm{L1}} + 4\frac{\mathrm{k2}}{\mathrm{L2}})} + \frac{4k2}{L2} \tag{3.3}$$

$$B1 = \frac{-\left(-\frac{w1\,L1^2}{12} + \frac{w2\,L2^2}{12}\right) + \frac{w1\,L1^2}{24}}{\frac{3\mathrm{k1}}{\mathrm{L1}} + 4\frac{\mathrm{k2}}{\mathrm{L2}}} \tag{3.4}$$

$$B2 = \frac{\left(\frac{-\left(-\frac{w1\,L1^2}{12} + \frac{w2\,L2^2}{12}\right) + \frac{w1\,L1^2}{24}}{\frac{3\mathrm{k1}}{\mathrm{L1}} + 4\frac{\mathrm{k2}}{\mathrm{L2}}}\right)(-2k2)}{L2} + \frac{w2\,L2^2}{12} \tag{3.5}$$

$$\theta 3 = \frac{B2}{A2} \tag{3.6}$$

$$\theta 2 = B1 - A1\frac{B2}{A2} \tag{3.7}$$

$$\theta 1 = -\frac{w1\,L1^3}{48\,k1} - 0.5\,(B1 - A1\frac{B2}{A2}) \tag{3.8}$$

For the stiffness matrix that derived first:

		2	4	6
	1	6L1	6L1	
	3	-6L1	-6L1+6L2	6L2
EI/L^3	5		-6L2	-6L2

The magnitude of angles of deformations ($\theta 1, \theta 2, and\ \theta 3$) can be used in the stiffness matrix to find the reaction and moments.

$$REACTION\ 1 = 6\ L1\ \theta 1 + 6\ L1\ \theta 2 + F.E.M \tag{3.9}$$

$$R1 = 6\ L1\ \left(\left(\frac{-W1\ L1^3}{48\ K1}\right) - 0.5\ \left(B1 - A1\frac{B2}{A2}\right)\right) + 6\ L1\ \left(B1 - A1\frac{B2}{A2}\right) + F.E.M \tag{3.10}$$

$$REACTION\ 2 = \ -6\ L1\ \Theta 1 + (-6\ L1 + 6\ L2\ )\ \Theta 2 + 6\ L2\ \Theta 3 + F.E.M$$

$$R2 = -6\ L1\ (\frac{-W1\ L1^3}{48\ K1} - 0.5\ \left(B1 - A1\frac{B2}{A2}\right) + (-6\ L1 + 6\ L2)\left(B1 - A1\frac{B2}{A2}\right) + 6L2\frac{B2}{A2} + F.E.M \tag{3.11}$$

$$REACTION\ 3 = \ -6\ L2\ \Theta 2 - 6\ L2\ \Theta 3 + F.E.M \tag{3.12}$$

$$R3 = -6\ L2\ \left(B1 - A1\ \frac{B2}{A2}\right) - 6\ L2\ \frac{B2}{A2} + F.E.$$

3.3 ANSYS GEOMERTYY AND MODELING

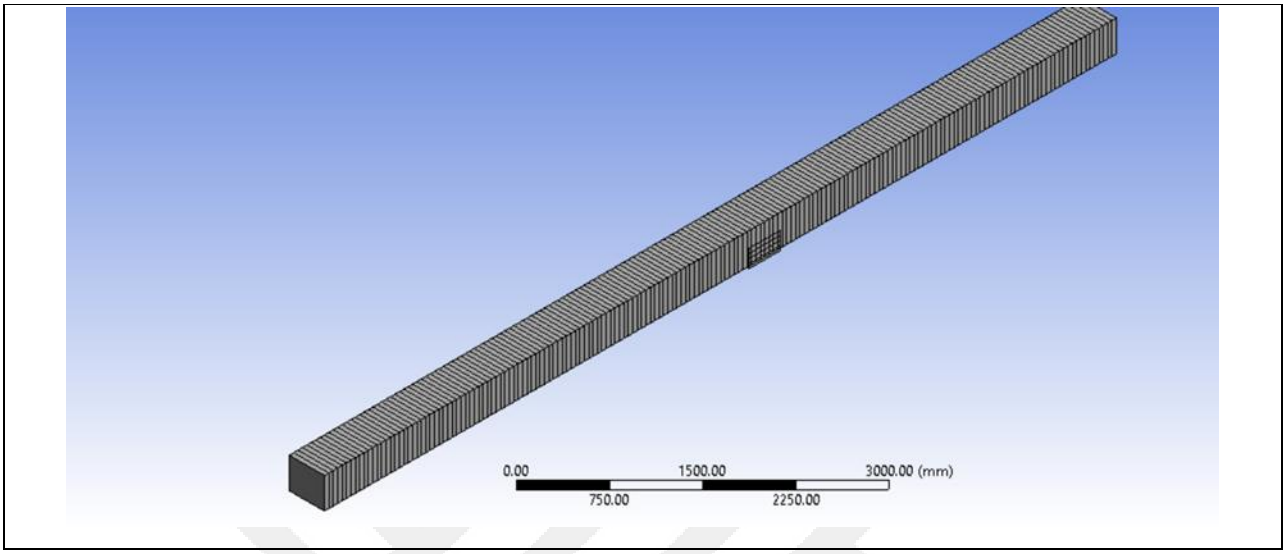

Figure 3.4: Mesh Assigning for simulation case (ANSYS)

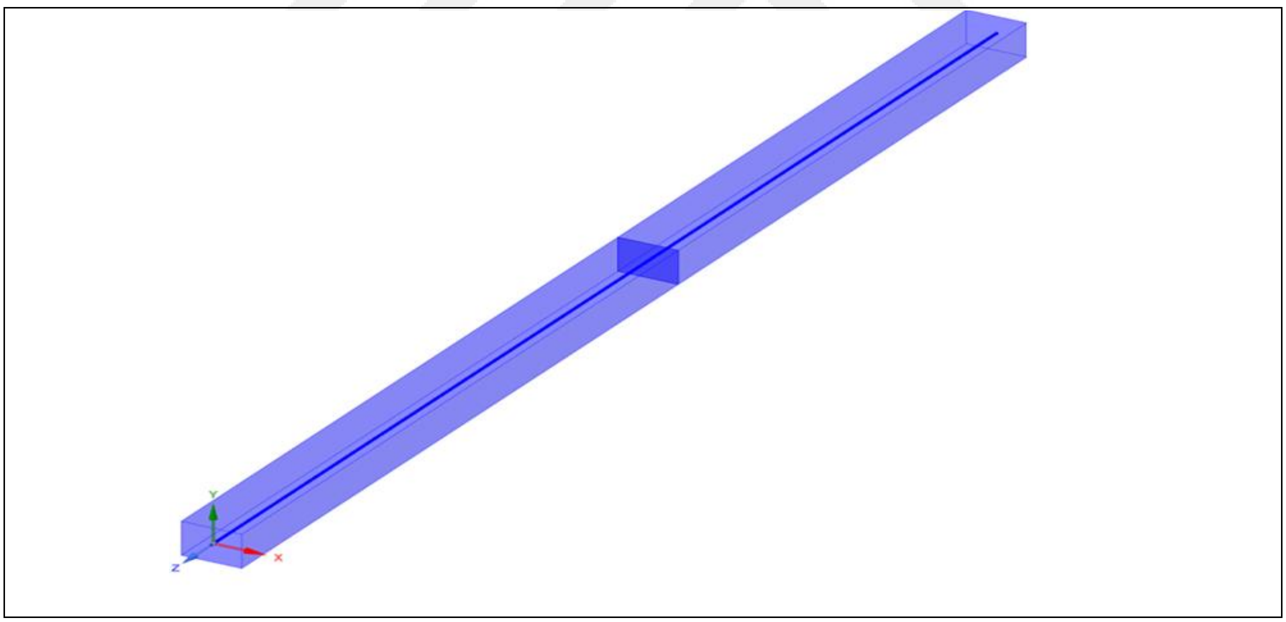

Figure 3.5: Geometrical shape for simulation case (ANSYS)

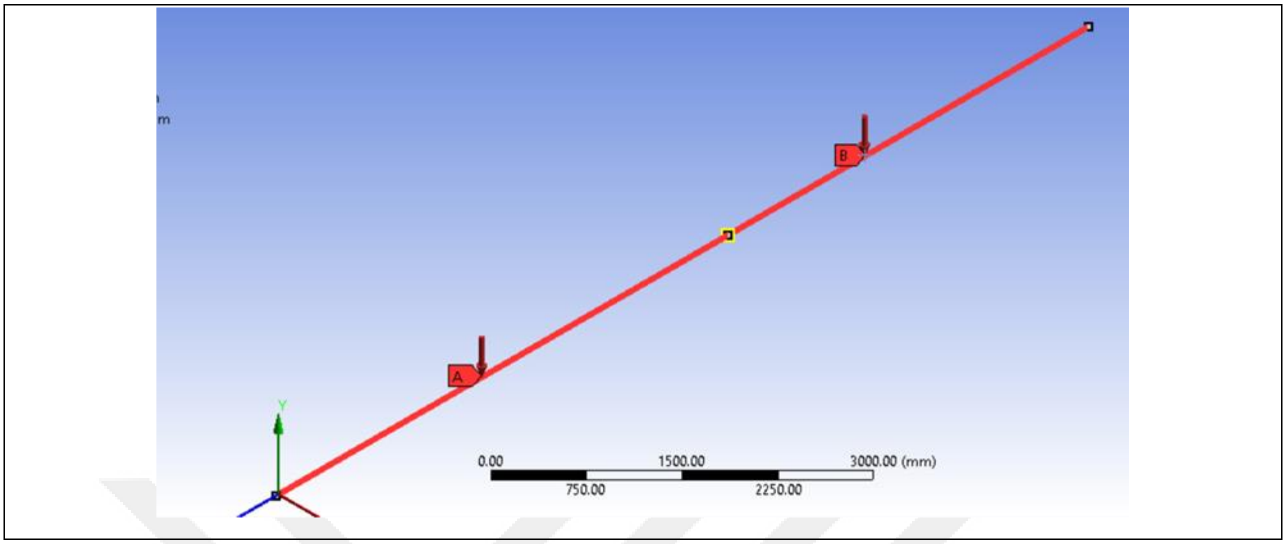

Figure 3.6: The Uniform loading as line path (ANSYS)

4. RESULTS

4.1 CALCULATION AND RESULTS

The achieved linear equations depending on the length of the two spans, the magnitude of the uniform loading, and value of stiffness coefficient EI (young modules of elasticity and moment of inertia). The relationship between the parameters inside the stiffness matrix that derived after applying Gaussian elimination method which are (A1, A2, B1, and B2) and the variation values of other influence items (L1, L2, w1, k1,k2 and w2) were explained in tables below (w2 =20kN/m , L1=L2=2m , and k1=k2=16000 N.mm^2 :

Table 4.1: Variation of B1 and B2 with uniform loading increasing W1

W1	W2	L1	L2	K1	K2	B1	B2
20	20	2	2	16000	16000	5.95238E-05	5.714286
25	20	2	2	16000	16000	0.000104167	5
30	20	2	2	16000	16000	0.00014881	4.285714
35	20	2	2	16000	16000	0.000193452	3.571429
40	20	2	2	16000	16000	0.000238095	2.857143
45	20	2	2	16000	16000	0.000282738	2.142857
50	20	2	2	16000	16000	0.000327381	1.428571
55	20	2	2	16000	16000	0.000372024	0.714286
60	20	2	2	16000	16000	0.000416667	0
65	20	2	2	16000	1600	0.00046131	-0.71429
70	20	2	2	16000	16000	0.000505952	-1.42857
75	20	2	2	16000	16000	0.000550595	-2.14286
80	20	2	2	16000	16000	0.000595238	-2.85714
85	20	2	2	16000	16000	0.000639881	-3.57143
90	20	2	2	16000	16000	0.000684524	-4.28571

Table 4.2: Variation of B1and B2 with uniform loading increasing W2

W1	W2	L1	L2	K1	K2	B1	B2
20	20	2	2	16000	16000	5.95238E-05	5.714286
20	25	2	2	16000	16000	2.97619E-05	7.857143
20	30	2	2	16000	16000	7.93016E-21	10
20	35	2	2	16000	16000	-2.9762E-05	12.14286
20	40	2	2	16000	16000	-5.9524E-05	14.28571
20	45	2	2	16000	16000	-8.9286E-05	16.42857
20	50	2	2	16000	16000	-0.00011905	18.57143
20	55	2	2	16000	16000	-0.00014881	20.71429
20	60	2	2	16000	16000	-0.00017857	22.85714
20	65	2	2	16000	16000	-0.00020833	25
20	70	2	2	16000	16000	-0.0002381	27.14286
20	75	2	2	16000	16000	-0.00026786	29.28571
20	80	2	2	16000	16000	-0.00029762	31.42857
20	85	2	2	16000	16000	-0.00032738	33.57143
20	90	2	2	16000	16000	-0.00035714	35.71429

Table 4.3: Variation of B1, B2, A1, and A2 with uniform span length increasing L1

W1	W2	L1	L2	K1	K2	A1	A2	B1	B2
20	20	2	2	16000	16000	0.285714	27428.57	5.95238E-05	5.714286
20	20	2.5	2	16000	16000	0.3125	27000	0.000174967	3.867188
20	20	3	2	16000	16000	0.333333	26666.67	0.000329861	1.388889
20	20	3.5	2	16000	16000	0.35	26400	0.000524089	-1.71875
20	20	4	2	16000	16000	0.363636	26181.82	0.000757576	-5.45455
20	20	4.5	2	16000	16000	0.375	26000	0.001030273	-9.81771
20	20	5	2	16000	16000	0.384615	25846.15	0.001342147	-14.8077
20	20	5.5	2	16000	16000	0.392857	25714.29	0.001693173	-20.4241
20	20	6	2	16000	16000	0.4	25600	0.002083333	-26.6667
20	20	6.5	2	16000	16000	0.40625	25500	0.002512614	-33.5352
20	20	7	2	16000	16000	0.411765	25411.76	0.002981005	-41.0294
20	20	7.5	2	16000	16000	0.416667	25333.33	0.003488498	-49.1493
20	20	8	2	16000	16000	0.421053	25263.16	0.004035088	-57.8947
20	20	8.5	2	16000	16000	0.425	25200	0.004620768	-67.2656
20	20	9	2	16000	16000	0.428571	25142.86	0.005245536	-77.2619

Table 4.4: Variation of B1, B2, A1, and A2 with uniform span length increasing L2

W1	W2	L1	L2	K1	K2	A1	A2	B1	B2
20	20	2	2	16000	16000	0.285714	27428.57	5.95E-05	5.714286
20	20	2	2.5	16000	16000	0.258065	22296.77	-8.4E-06	10.52419
20	20	2	3	16000	16000	0.235294	18823.53	-0.00011	16.17647
20	20	2	3.5	16000	16000	0.216216	16308.88	-0.00025	22.66892
20	20	2	4	16000	16000	0.2	14400	-0.00042	30
20	20	2	4.5	16000	16000	0.186047	12899.22	-0.00062	38.1686
20	20	2	5	16000	16000	0.173913	11686.96	-0.00086	47.17391
20	20	2	5.5	16000	16000	0.163265	10686.46	-0.00113	57.01531
20	20	2	6	16000	16000	0.153846	9846.154	-0.00144	67.69231
20	20	2	6.5	16000	16000	0.145455	9130.07	-0.00179	79.20455
20	20	2	7	16000	16000	0.285714	8512.315	-0.00216	91.55172
20	20	2	7.5	16000	16000	0.285714	7973.77	-0.00257	104.7336
20	20	2	8	16000	16000	0.285714	7500	-0.00302	118.75
20	20	2	8.5	16000	16000	0.285714	7079.895	-0.0035	133.6007
20	20	2	9	16000	16000	0.285714	6704.762	-0.00402	149.2857

Table 4.5: Variation of B1, B2, A1, and A2 with uniform EI increasing K1

W1	W2	L1	L2	K1	K2	A1	A2	B1	B2
20	20	2	2	10000	16000	0.340426	26553.19	7.0922E-05	5.531915
20	20	2	2	15000	16000	0.293578	27302.75	6.1162E-05	5.688073
20	20	2	2	20000	16000	0.258065	27870.97	5.3763E-05	5.806452
20	20	2	2	25000	16000	0.230216	28316.55	4.7962E-05	5.899281
20	20	2	2	30000	16000	0.207792	28675.32	4.329E-05	5.974026
20	20	2	2	35000	16000	0.189349	28970.41	3.9448E-05	6.035503
20	20	2	2	40000	16000	0.173913	29217.39	3.6232E-05	6.086957
20	20	2	2	45000	16000	0.160804	29427.14	3.3501E-05	6.130653
20	20	2	2	50000	16000	0.149533	29607.48	3.1153E-05	6.168224
20	20	2	2	55000	16000	0.139738	29764.19	2.9112E-05	6.200873
20	20	2	2	60000	16000	0.131148	29901.64	2.7322E-05	6.229508
20	20	2	2	65000	16000	0.123552	30023.17	2.574E-05	6.254826
20	20	2	2	70000	16000	0.116788	30131.39	2.4331E-05	6.277372
20	20	2	2	75000	16000	0.110727	30228.37	2.3068E-05	6.297578
20	20	2	2	80000	16000	0.105263	30315.79	2.193E-05	6.315789

Table 4.6: Variation of B1, B2, A1, and A2 with uniform EI increasing K2

W1	W2	L1	L2	K1	K2	A1	A2	B1	B2
20	20	2	2	16000	10000	0.227273	17727.27	7.57576E-05	5.909091
20	20	2	2	16000	15000	0.277778	25833.33	6.17284E-05	5.740741
20	20	2	2	16000	20000	0.3125	33750	5.20833E-05	5.625
20	20	2	2	16000	25000	0.337838	41554.05	4.5045E-05	5.540541
20	20	2	2	16000	30000	0.357143	49285.71	3.96825E-05	5.47619
20	20	2	2	16000	35000	0.37234	56968.09	3.5461E-05	5.425532
20	20	2	2	16000	40000	0.384615	64615.38	3.20513E-05	5.384615
20	20	2	2	16000	45000	0.394737	72236.84	2.92398E-05	5.350877
20	20	2	2	16000	50000	0.403226	79838.71	2.68817E-05	5.322581
20	20	2	2	16000	55000	0.410448	87425.37	2.48756E-05	5.298507
20	20	2	2	16000	60000	0.416667	95000	2.31481E-05	5.277778
20	20	2	2	16000	65000	0.422078	102564.9	2.1645E-05	5.25974
20	20	2	2	16000	70000	0.426829	110122	2.03252E-05	5.243902
20	20	2	2	16000	75000	0.431034	117672.4	1.91571E-05	5.229885
20	20	2	2	16000	80000	0.434783	125217.4	1.81159E-05	5.217391

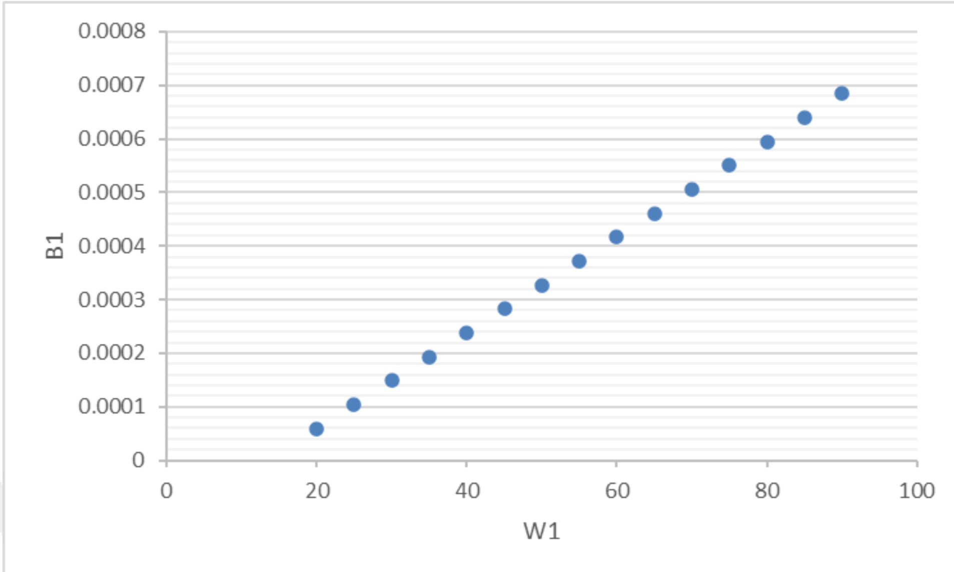

Figure 4.1: Variation of B1 with uniform loading W1

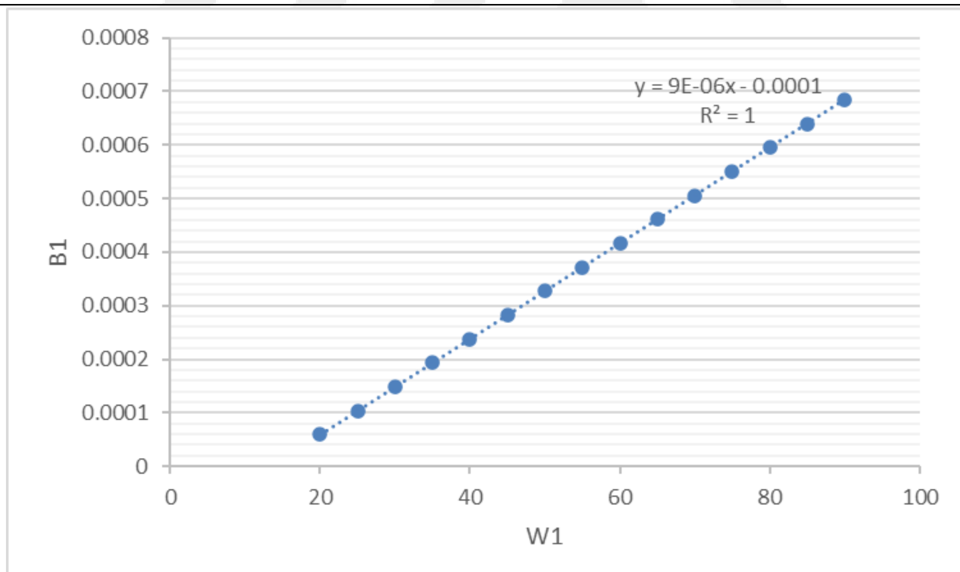

Figure 4.2: Curve Fitting for variation of B1 with uniform loading W1

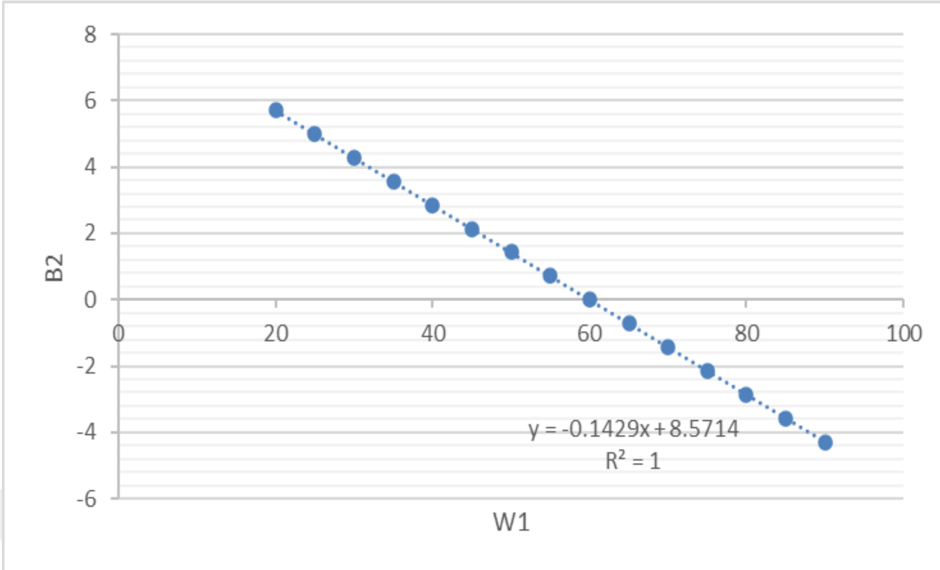

Figure 4.3: Curve Fitting for variation of B2 with uniform loading W1

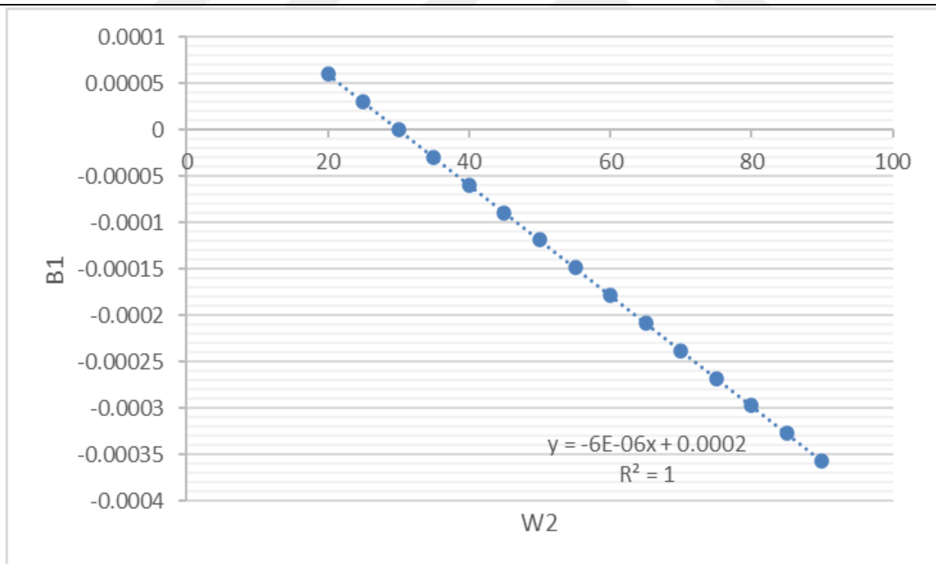

Figure 4.4: Curve Fitting for variation of B1 with W2

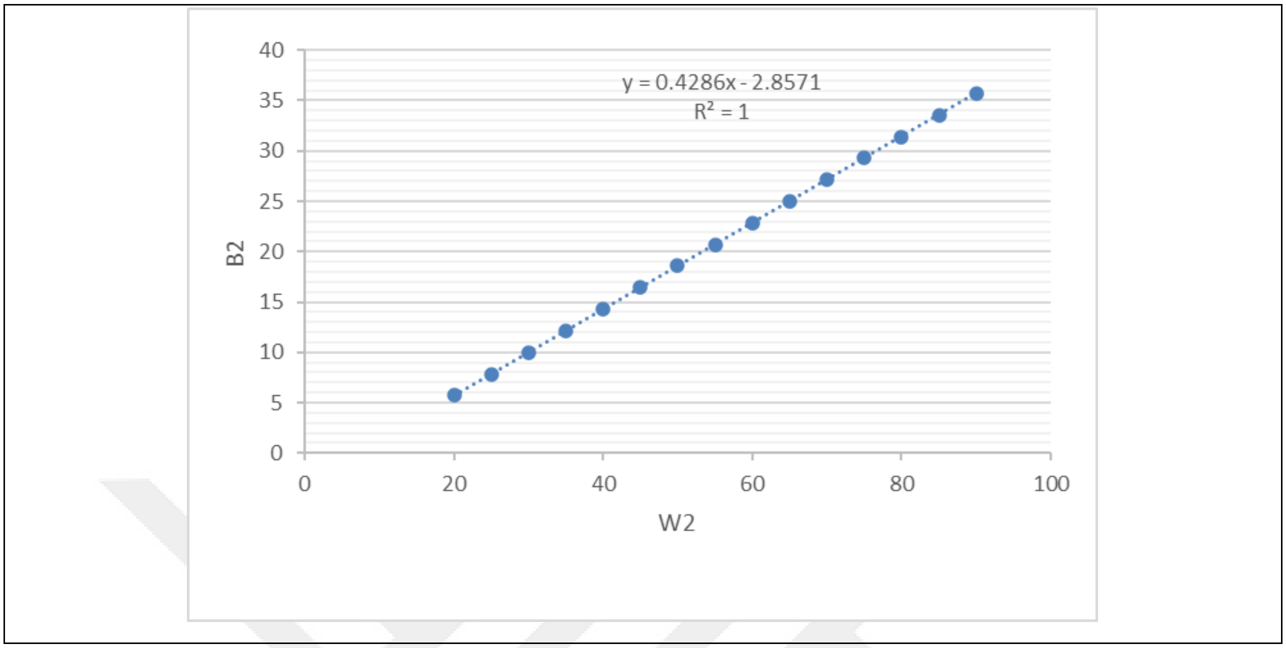

Figure 4.5: Curve Fitting for variation of B2 with W2

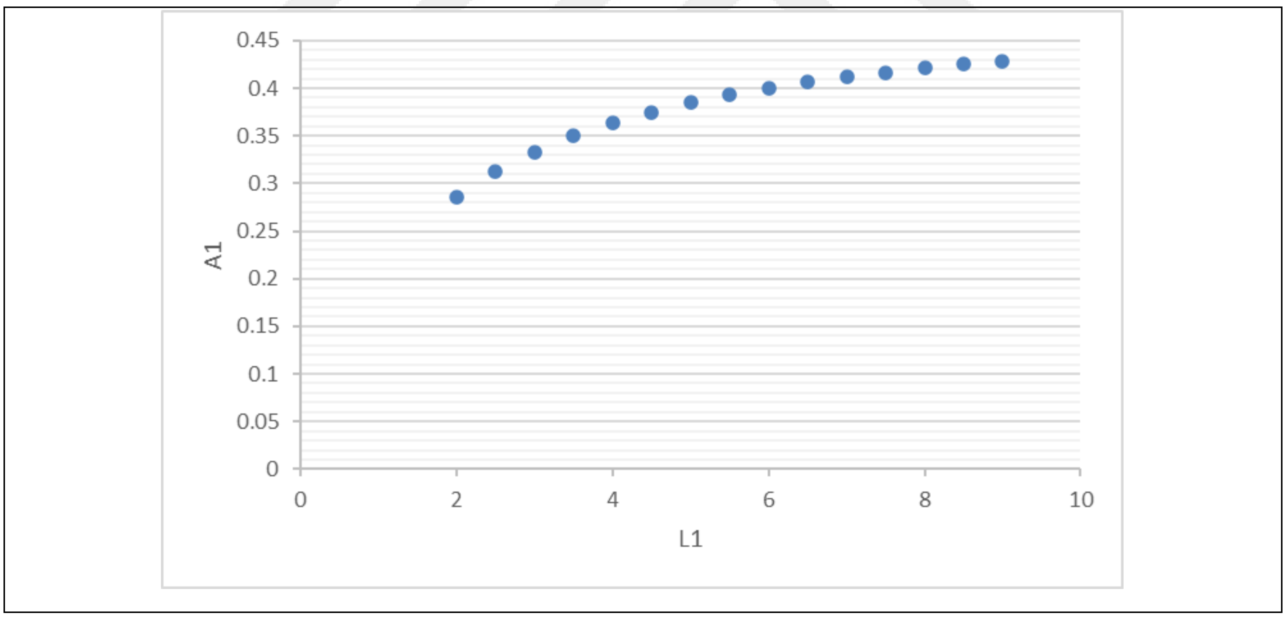

Figure 4.6: Variation of A1 with L1

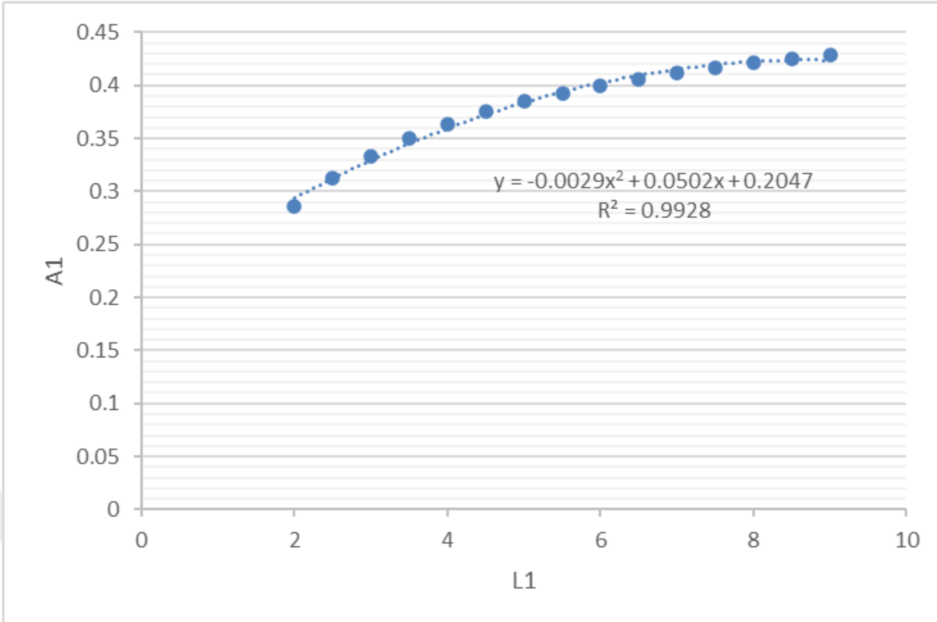

Figure 4.7: Curve Fitting for variation of A1 with L1

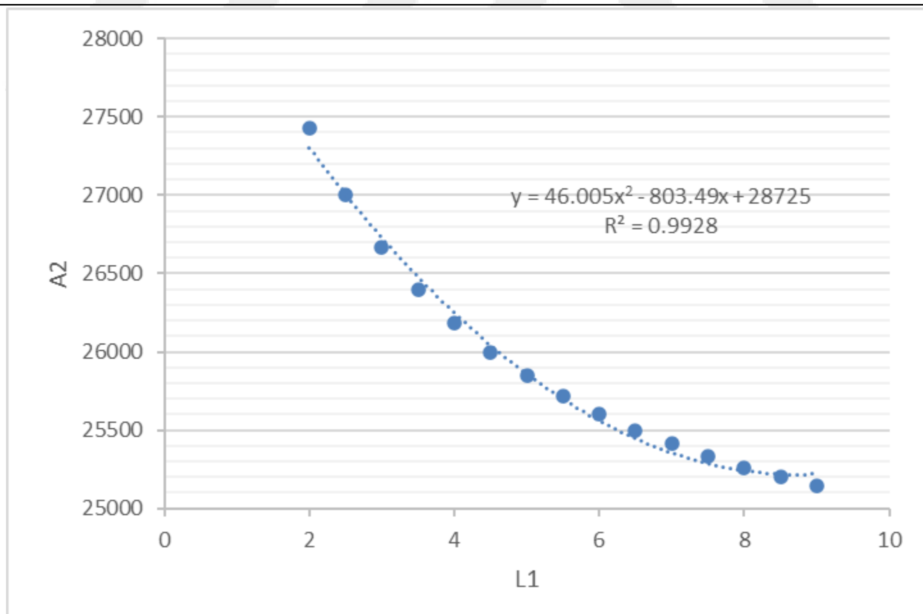

Figure 4.8: Curve Fitting for variation of A2 with L1

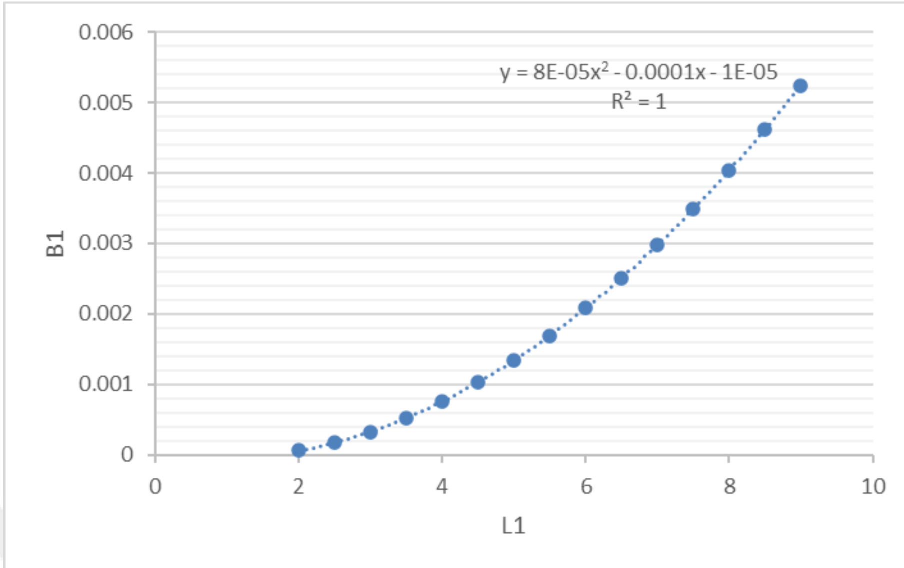

Figure 4.9: Curve Fitting for the variation of B1 with L1

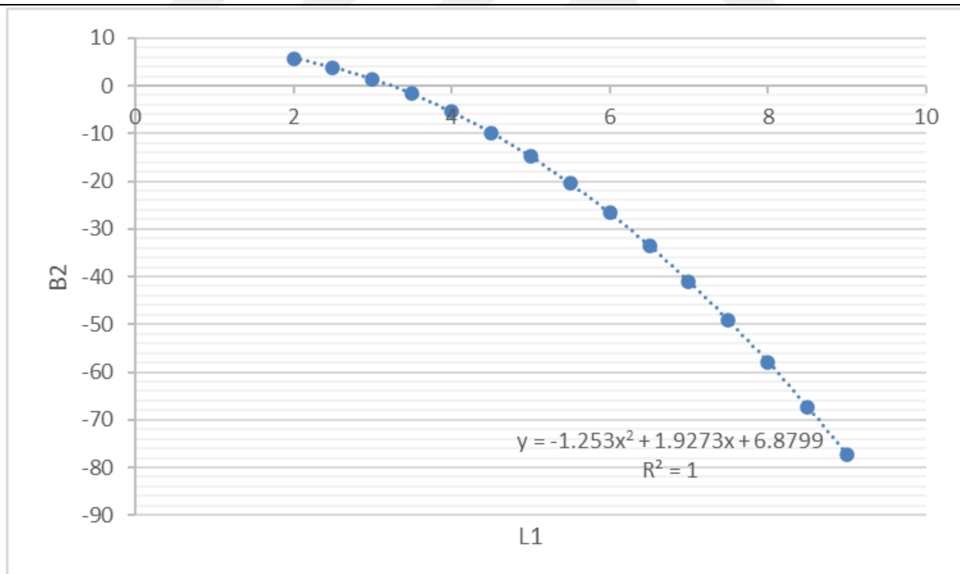

Figure 4.10: Curve Fitting for the variation of B2 with L1

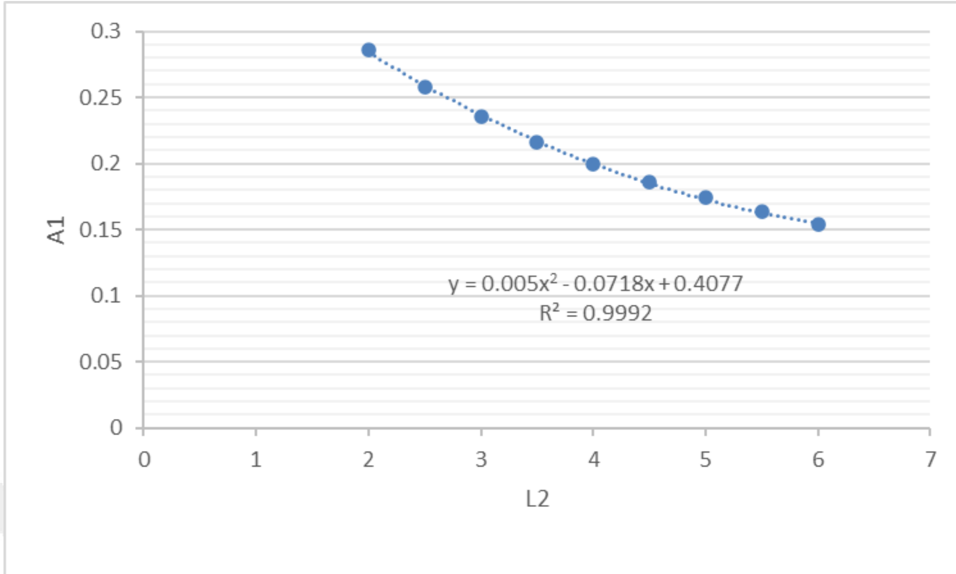

Figure 4.11: Curve Fitting for the variation of A1 with L2

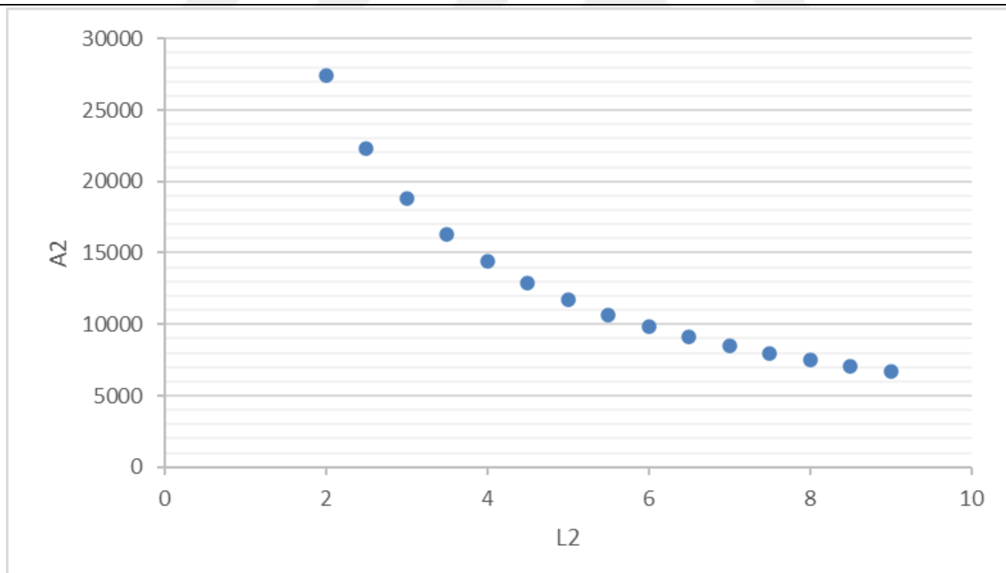

Figure 4.12: The Variation of A2 with L2

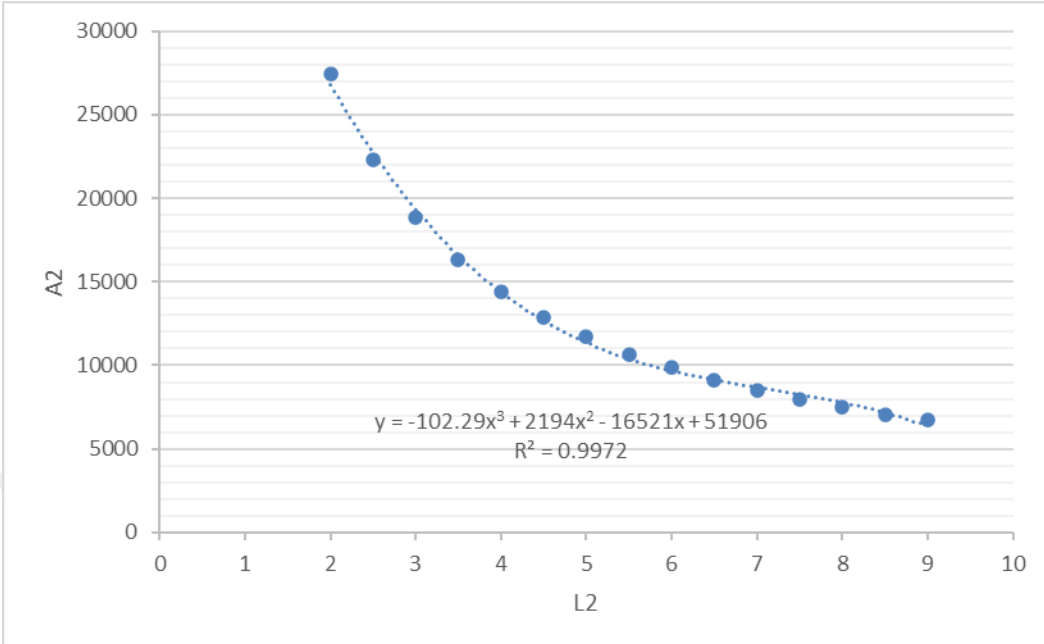

Figure 4.13: Curve Fitting for the variation of A2 with L2

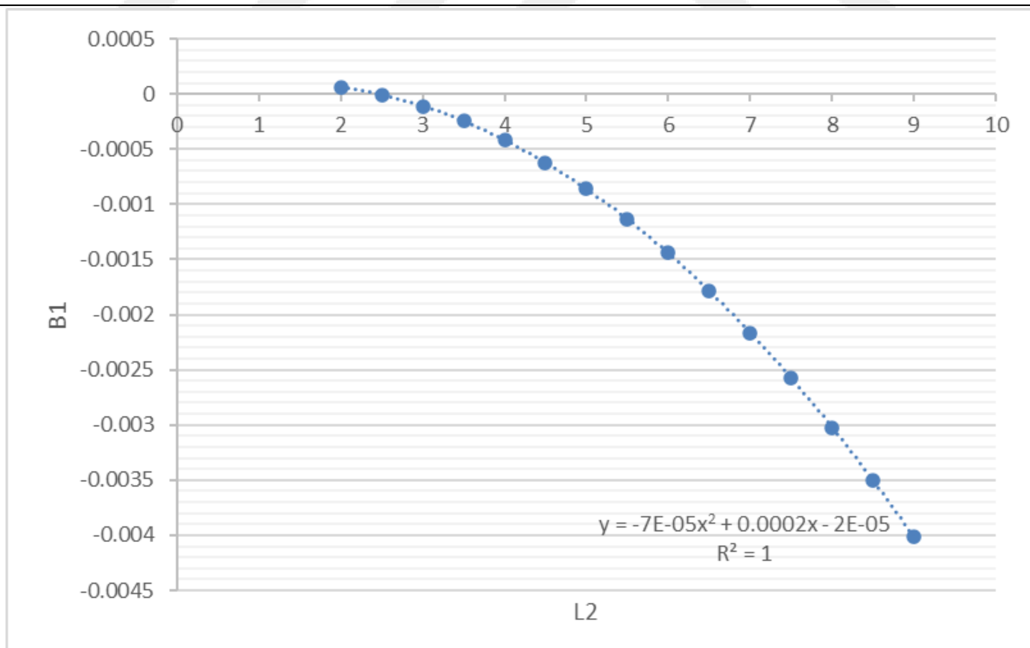

Figure 4.14: Curve Fitting for the variation of B1 with L2

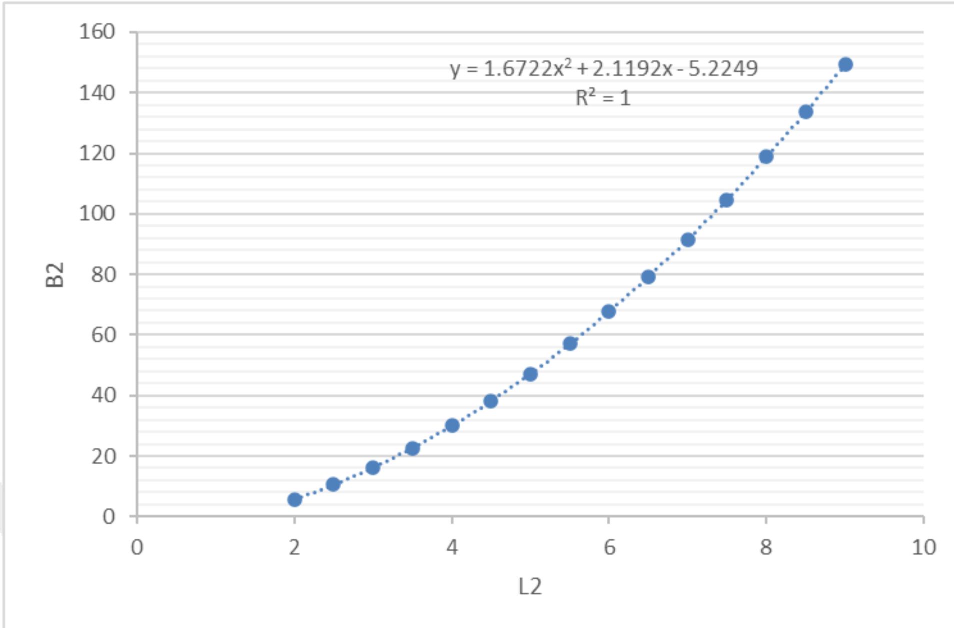

Figure 4.15: Curve Fitting for the variation of B2 with L2

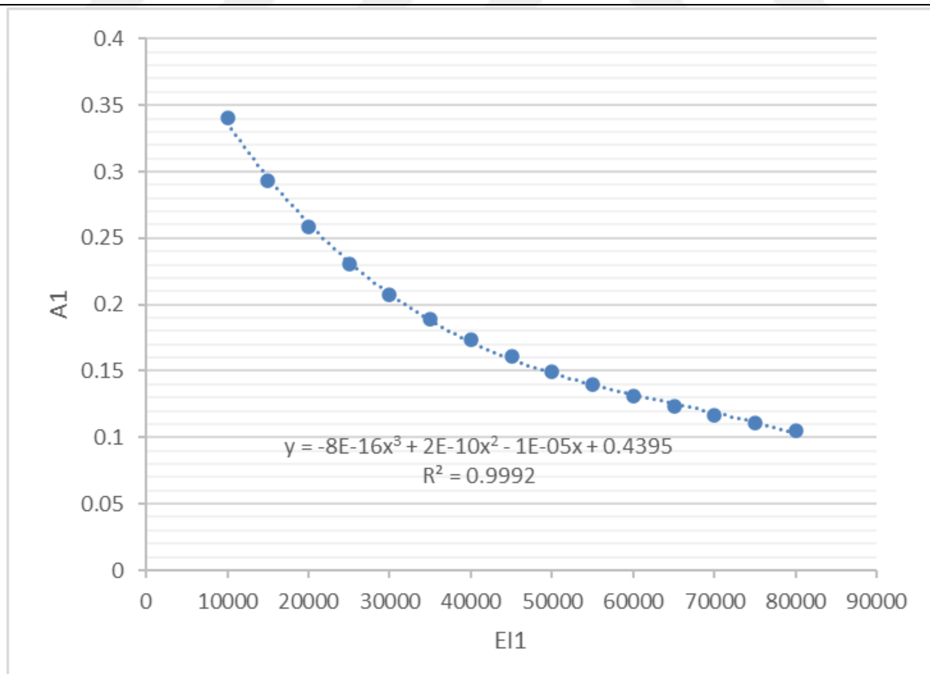

Figure 4.16: Curve Fitting for the variation of A1 with K

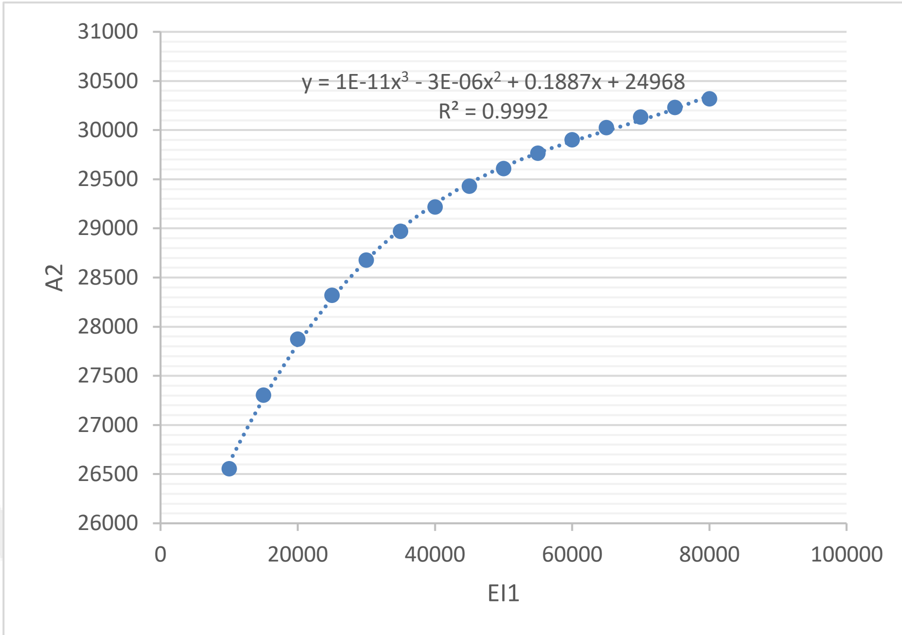

Figure 4.17: Curve Fitting for the variation of A2 with K1

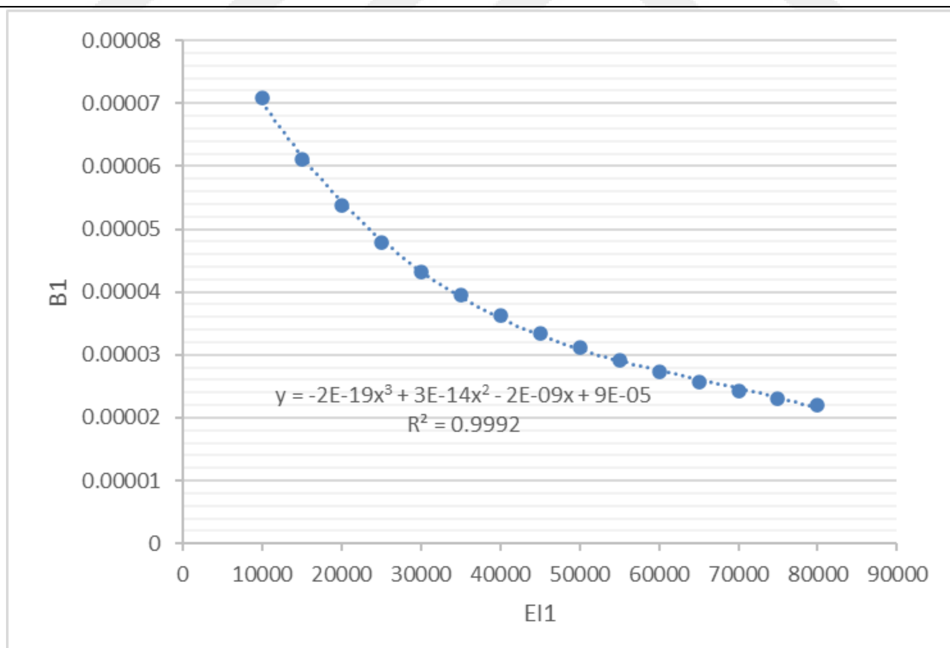

Figure 4.18: Curve Fitting for the variation of B1 with K1

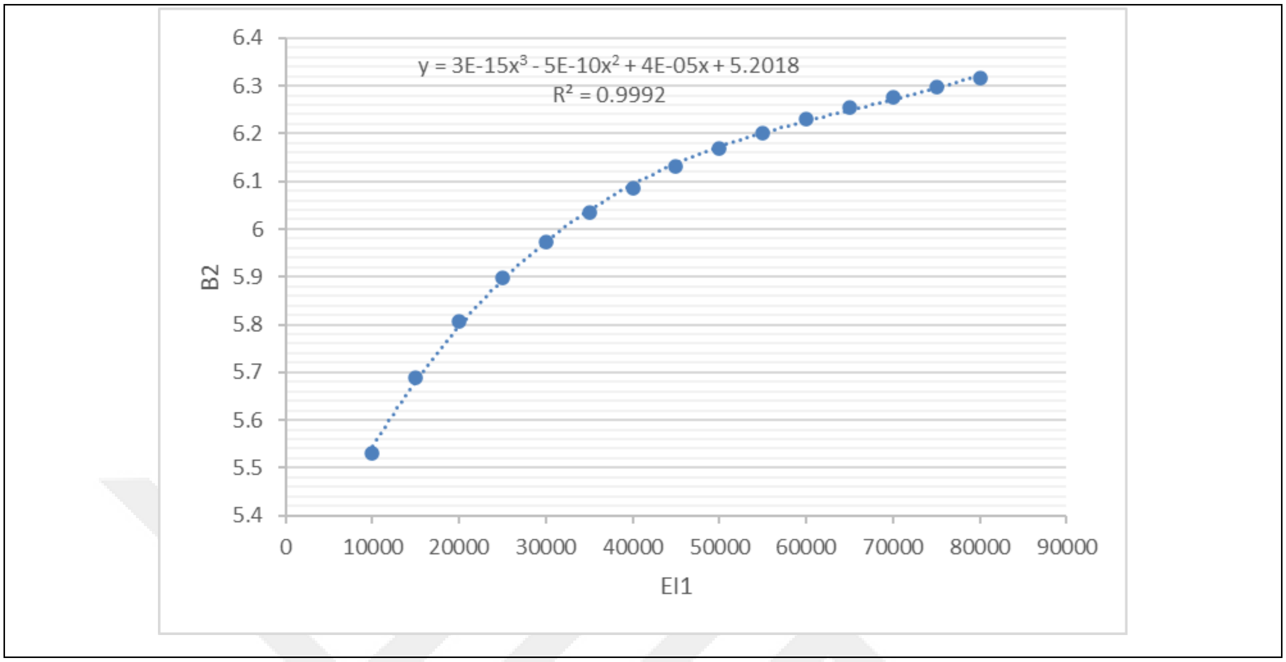

Figure 4.19: Curve Fitting for the variation of B2 with K1

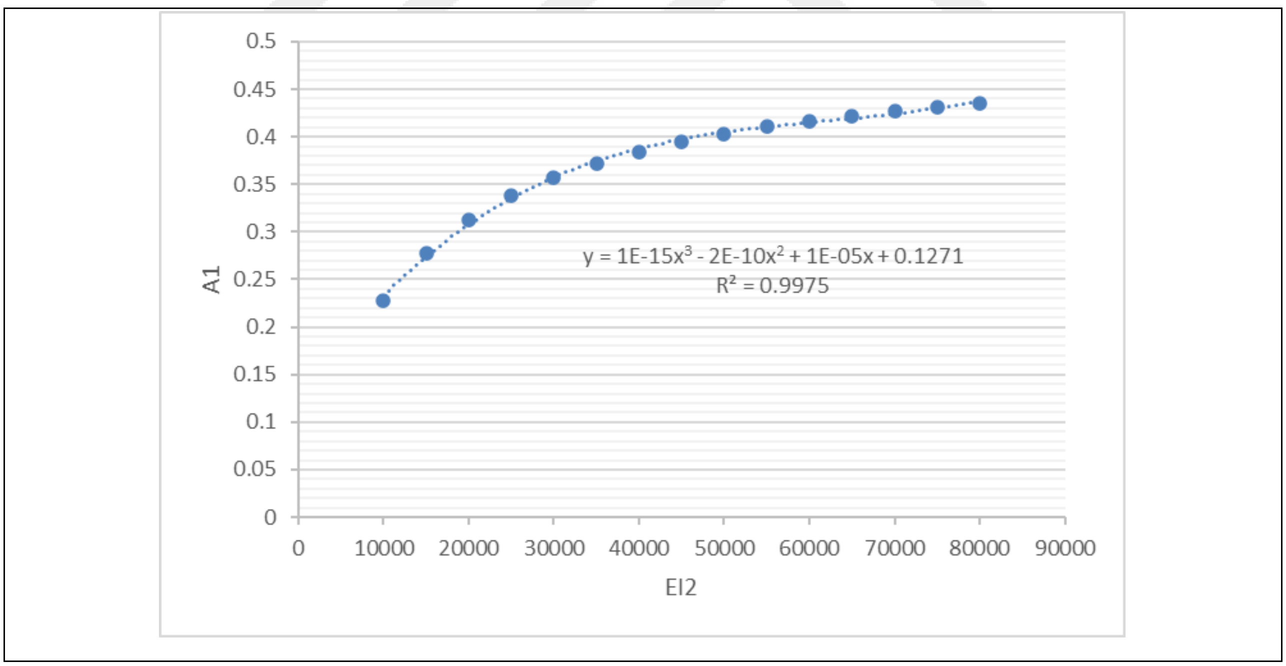

Figure 4.20: Curve Fitting for the variation of A1 with K2

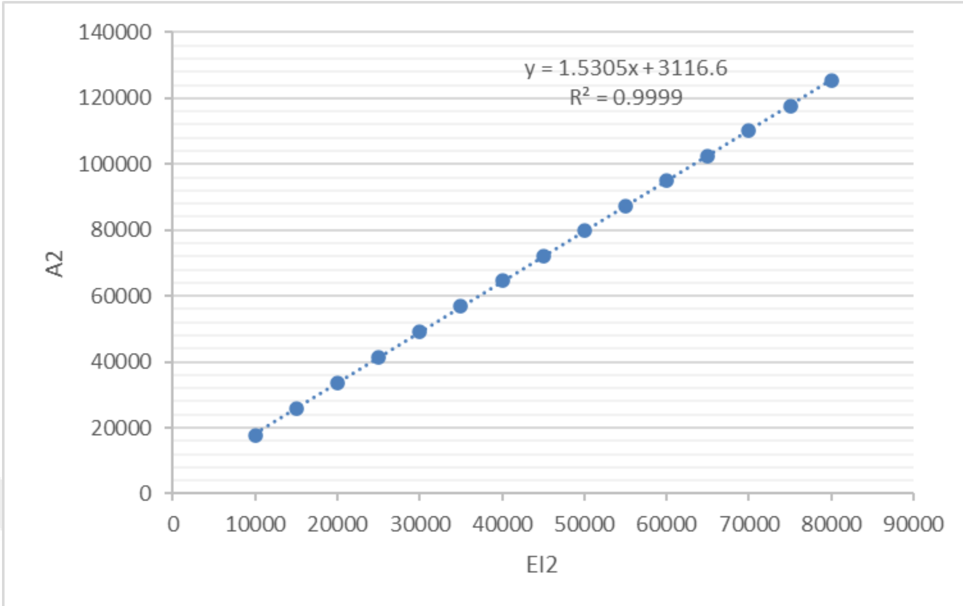

Figure 4.21: Curve Fitting for the variation of A2 with K2

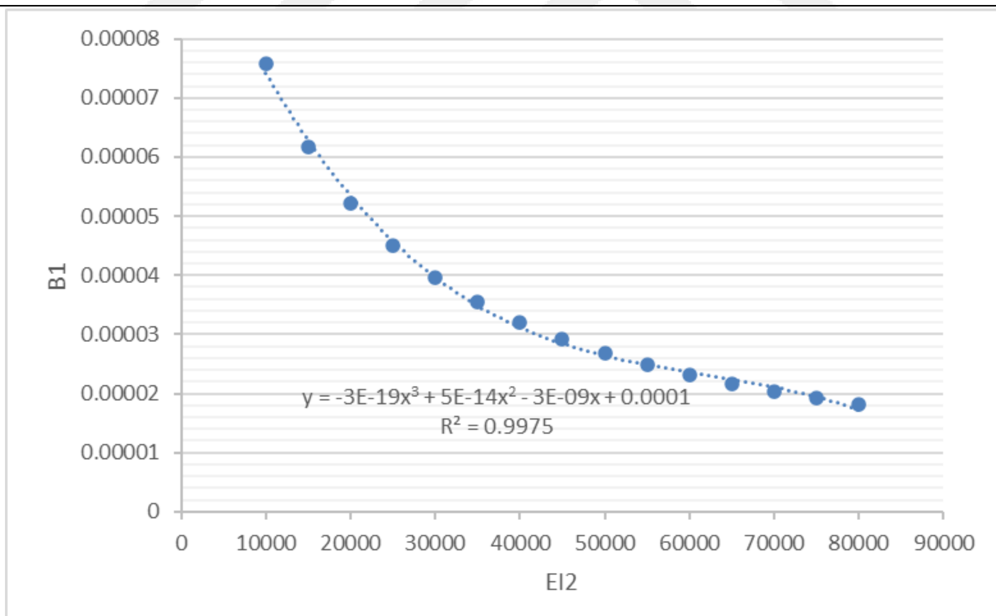

Figure 4.22: Curve Fitting for the variation of B1 with K

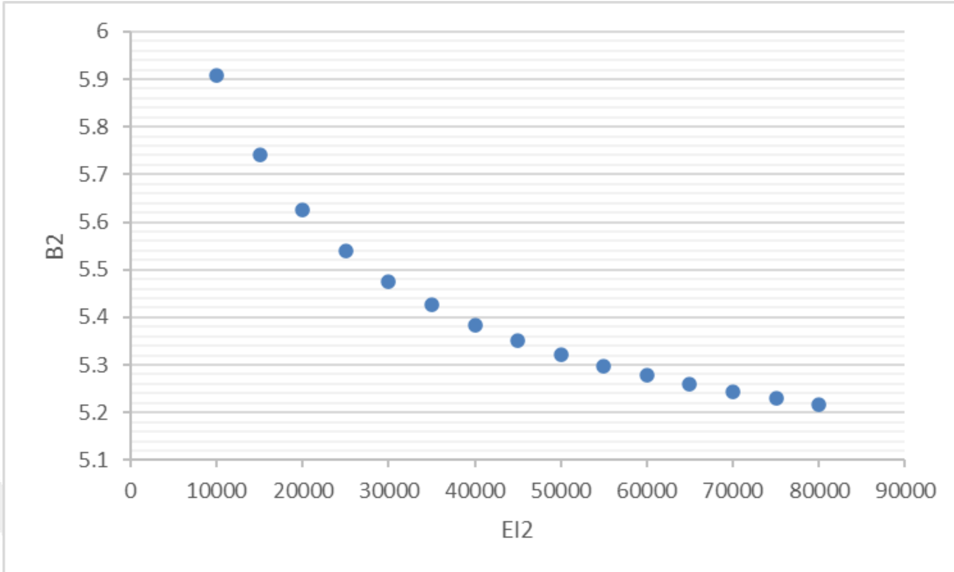

Figure 4.23: The Variation of B1 with K2

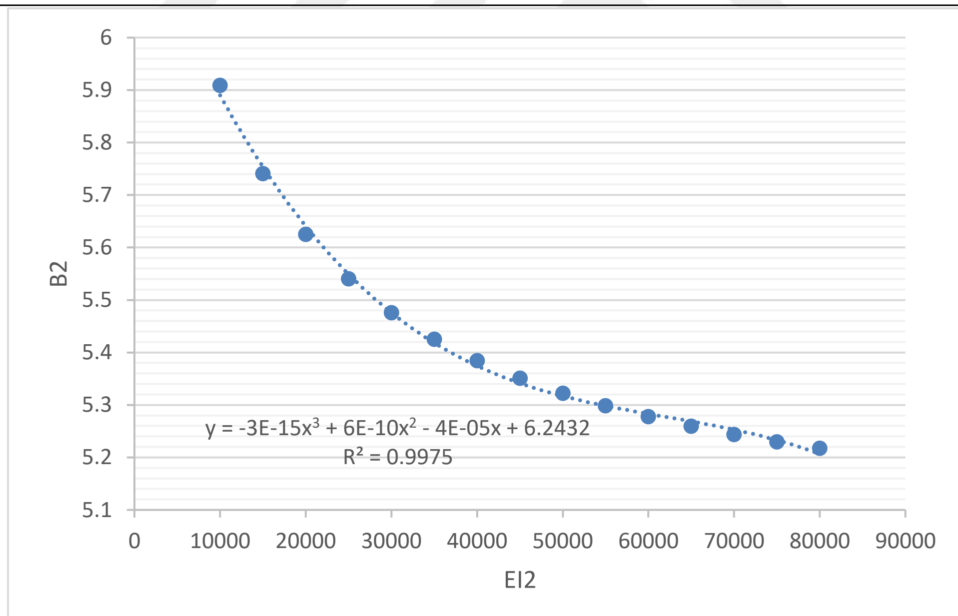

Figure 4.24: Curve Fitting for the variation of B2 with K2

Table 4.7: The Derived equations for A1

Item	A1
W1	-
W2	-
L1	$y = 0.0029x^2 +0.0502x + 0.2047$
L2	$y = 0.005x^2 -0.0718x+ 0.4077$
(EI)1	$y = -8E-16x^3 +2E-10x^2 - 1E-05x + 0.4395$
(EI)2	$y = 1E-15x^3-2E-10x^2 + 1E-05x + 0.1271$

Table 4.8: The Derived equations for A2

Item	A2
W1	-
W2	-
L1	$y = 46.005x^2 + 803.49x + 28725$
L2	$y = -102.29x^3+2194x^2 -16521x +51906$
(EI)1	$y =1E-11x^3 -3E-06x^2 + 0.1887x + 24968$
(EI)2	$y = 1.5305x + 3116.6$

Table 4.9: The Derived equations for B1

Item	B1
W1	$y = 9E\text{-}06x\text{-}0.0001$
W2	$y = -6E\text{-}06x + 0.0002$
L1	$y = 8E\text{-}05x^2 - 0.0001x – 1E\text{-}05$
L2	$y = -7E\text{-}05x^2 + 0.0002x – 2E\text{-}05$
(EI)1	$y = -2E\text{-}19x^3+3E\text{-}14x^2\text{-}2E\text{-}09x+9E\text{-}05$
(EI)2	$y = -3E\text{-}19x^3+5E\text{-}14x^2 -3E\text{-}09x+0.0001$

Table 4.10: The Derived equations for B2

Item	B2
W1	$y = -0.1429x + 8.5714$
W2	$y = 0.4286x - 2.8571$
L1	$y = -1.253x^2 + 1.9273x+ 6.8799$
L2	$y = 1.6722x^2 +2.1192x – 5.2249$
(EI)1	$y = -3E\text{-}15x^3 -5E\text{-}10x^2 +4E\text{-}05x+5.2018$
(EI)2	$y = -3E\text{-}15x^3 +6E\text{-}10x^2 -4E\text{-}05x+6.2432$

For identifying the impact of parameters magnitude (W1, W2, L1, L2, K1, and K2) on the four matrix parts (A1, A2, B1, and B2) when changing the parameters in various values and apply it in the derivative linear equations, then checked the deviation between the two results. The achieved derivative equations for each parameter with high deviation value must be multiplied by a factor that can correct the overall results to the actual magnitudes.

Table 4.11: Variation of B1’ and B2’ with uniform loading increasing W1

W1	W2	L1	L2	K1	K2	B1’	B2’
20	20	2	2	16000	16000	0.00008	5.7134
25	20	2	2	16000	16000	0.000125	4.9989
30	20	2	2	16000	16000	0.00017	4.2844
35	20	2	2	16000	16000	0.000215	3.5699
40	20	2	2	16000	16000	0.00026	2.8554
45	20	2	2	16000	16000	0.000305	2.1409
50	20	2	2	16000	16000	0.00035	1.4264
55	20	2	2	16000	16000	0.000395	0.7119
60	20	2	2	16000	16000	0.00044	-0.0026
65	20	2	2	16000	16000	0.000485	-0.7171
70	20	2	2	16000	16000	0.00053	-1.4316
75	20	2	2	16000	16000	0.000575	-2.1461
80	20	2	2	16000	16000	0.00062	-2.8606
85	20	2	2	16000	16000	0.000665	-3.5751
90	20	2	2	16000	16000	0.00071	-4.2896

Table 4.12: Variation of B1' and B2' with uniform loading increasing W2

W1	W2	L1	L2	K1	K2	B1'	B2'
20	20	2	2	16000	16000	0.00008	5.7149
20	25	2	2	16000	16000	0.00005	7.8579
20	30	2	2	16000	16000	0.00002	10.0009
20	35	2	2	16000	16000	-0.00001	12.1439
20	40	2	2	16000	16000	-0.00004	14.2869
20	45	2	2	16000	16000	-0.00007	16.4299
20	50	2	2	16000	16000	-0.0001	18.5729
20	55	2	2	16000	16000	-0.00013	20.7159
20	60	2	2	16000	16000	-0.00016	22.8589
20	65	2	2	16000	16000	-0.00019	25.0019
20	70	2	2	16000	16000	-0.00022	27.1449
20	75	2	2	16000	16000	-0.00025	29.2879
20	80	2	2	16000	16000	-0.00028	31.4309
0	85	2	2	16000	16000	-0.00031	33.5739
20	90	2	2	16000	16000	-0.00034	35.7169

Table 4.13: Variation of A1’, A2’, B1’ and B2’ with span length increasing L1

W1	W2	L1	L2	K1	K2	A1’	A2’	B1’	B2’
20	20	2	2	16000	16000	0.2935	27302.04	0.00011	5.7225
20	20	2.5	2	16000	16000	0.312075	27003.81	0.00024	3.8669
20	20	3	2	16000	16000	0.3292	26728.58	0.00041	1.3848
20	20	3.5	2	16000	16000	0.344875	26476.35	0.00062	-1.7238
20	20	4	2	16000	16000	0.3591	26247.12	0.00087	-5.4589
20	20	4.5	2	16000	16000	0.371875	26040.9	0.00116	-9.8205
20	20	5	2	16000	16000	0.3832	25857.68	0.00149	-14.8086
20	20	5.5	2	16000	16000	0.393075	25697.46	0.00186	-20.4232
20	20	6	2	16000	16000	0.4015	25560.24	0.00227	-26.6643
20	20	6.5	2	16000	16000	0.408475	25446.03	0.00272	-33.5319
20	20	7	2	16000	16000	0.414	25354.82	0.00321	-41.026
20	20	7.5	2	16000	16000	0.418075	25286.61	0.00374	-49.1466
20	20	8	2	16000	16000	0.4207	25241.4	0.00431	-57.8937
20	20	8.5	2	16000	16000	0.421875	25219.2	0.00492	-67.2673
20	20	9	2	16000	16000	0.4216	25220	0.00557	-77.2674

Table 4.14: Variation of A1', A2', B1' and B2' with span length increasing L2

W1	W2	L1	L2	K1	K2	A1'	A2'	B1'	B2'
20	20	2	2	16000	16000	0.2841	26821.68	0.0001	5.7023
20	20	2	2.5	16000	16000	0.25945	22717.72	4.25E-05	10.52435
20	20	2	3	16000	16000	0.2373	19327.17	-5E-05	16.1825
20	20	2	3.5	16000	16000	0.21765	16573.32	-0.0001775	22.67675
20	20	2	4	16000	16000	0.2005	14379.44	-0.00034	30.0071
20	20	2	4.5	16000	16000	0.18585	12668.82	-0.0005375	38.17355
20	20	2	5	16000	16000	0.1737	11364.75	-0.00077	47.1761
20	20	2	5.5	16000	16000	0.16405	10390.5	-0.0010375	57.01475
20	20	2	6	16000	16000	0.1569	9669.36	-0.00134	67.6895
20	20	2	6.5	16000	16000	0.15225	9124.609	-0.0016775	79.20035
20	20	2	7	16000	16000	0.1501	8679.53	-0.00205	91.5473
20	20	2	7.5	16000	16000	0.15045	8257.406	-0.0024575	104.7304
20	20	2	8	16000	16000	0.1533	7781.52	-0.0029	118.7495
20	20	2	8.5	16000	16000	0.15865	7175.154	-0.0033775	133.6048
20	20	2	9	16000	16000	0.1665	6361.59	-0.00389	149.2961

Table 4.15: Variation of A1', A2', B1' and B2' with EI increasing K1

W1	W2	L1	L2	K1	K2	A1'	A2'	B1'	B2'
20	20	2	2	10000	16000	0.3587	26565	0.0000728	5.5548
20	20	2	2	15000	16000	0.3318	27157.25	0.000066075	5.699425
20	20	2	2	20000	16000	0.3131	27622	0.0000604	5.8258
20	20	2	2	25000	16000	0.302	27966.75	0.000055625	5.936175
20	20	2	2	30000	16000	0.2979	28199	0.0000516	6.0328
20	20	2	2	35000	16000	0.3002	28326.25	0.000048175	6.117925
20	20	2	2	40000	16000	0.3083	28356	0.0000452	6.1938
20	20	2	2	45000	16000	0.3216	28295.75	0.000042525	6.262675
20	20	2	2	50000	16000	0.3395	28153	0.00004	6.3268
20	20	2	2	55000	16000	0.3614	27935.25	0.000037475	6.388425
20	20	2	2	60000	16000	0.3867	27650	0.0000348	6.4498
20	20	2	2	65000	16000	0.4148	27304.75	0.000031825	6.513175
20	20	2	2	70000	16000	0.4451	26907	0.0000284	6.5808
20	20	2	2	75000	16000	0.477	26464.25	0.000024375	6.654925
20	20	2	2	80000	16000	0.5099	25984	0.0000196	6.7378

Table 4.16: Variation of A1', A2', B1' and B2' with EI increasing K2

W1	W2	L1	L2	K1	K2	A1'	A2'	B1'	B2'
20	20	2	2	16000	10000	0.2081	18421.6	0.0000747	5.9002
20	20	2	2	16000	15000	0.235475	26074.1	6.52375E-05	5.768075
20	20	2	2	16000	20000	0.2551	33726.6	0.0000576	5.6592
20	20	2	2	16000	25000	0.267725	41379.1	5.15625E-05	5.571325
20	20	2	2	16000	30000	0.2741	49031.6	0.0000469	5.5022
20	20	2	2	16000	35000	0.274975	56684.1	4.33875E-05	5.449575
20	20	2	2	16000	40000	0.2711	64336.6	0.0000408	5.4112
20	20	2	2	16000	45000	0.263225	71989.1	3.89125E-05	5.384825
20	20	2	2	16000	50000	0.2521	79641.6	0.0000375	5.3682
20	20	2	2	16000	55000	0.238475	87294.1	3.63375E-05	5.359075
20	20	2	2	16000	60000	0.2231	94946.6	0.0000352	5.3552
20	20	2	2	16000	65000	0.206725	102599.1	3.38625E-05	5.354325
20	20	2	2	16000	70000	0.1901	110251.6	0.0000321	5.3542
20	20	2	2	16000	75000	0.173975	117904.1	2.96875E-05	5.352575
20	20	2	2	16000	80000	0.1591	125556.6	0.0000264	5.3472

Table 4.17: Estimate the average value for A1 from the stiffness matrix values

Sum of A1	A1-k1	A1-k2	A1-L1	A1-L2	A1*A1	A1*A1	A1*A1	A1*A1	Final A1
1.139127	0.199515	0.298848	0.250819	0.250819	0.045344	0.101735	0.071662	0.071662	0.290405
1.14192	0.243255	0.257091	0.273662	0.225992	0.067571	0.075476	0.085519	0.05832	0.286887
1.139192	0.274317	0.226533	0.292605	0.206545	0.085724	0.05846	0.097535	0.048599	0.290318
1.13427	0.297846	0.202964	0.308569	0.190621	0.100624	0.046725	0.107999	0.041215	0.296564
1.128571	0.316456	0.18412	0.322209	0.177215	0.11302	0.038259	0.117167	0.035443	0.303889
1.122736	0.331637	0.16865	0.334005	0.165708	0.123482	0.031934	0.125252	0.030829	0.311497
1.117057	0.344311	0.155689	0.344311	0.155689	0.132427	0.027076	0.132427	0.027076	0.319007
1.111663	0.355087	0.144652	0.353396	0.146866	0.140166	0.023261	0.138834	0.023978	0.326239
1.106605	0.364381	0.135127	0.361466	0.139025	0.146928	0.020206	0.144586	0.021389	0.333109
1.10189	0.372494	0.126817	0.368685	0.132005	0.152889	0.017721	0.149778	0.019201	0.339589
1.09751	0.379647	0.119496	0.375181	0.125676	0.158186	0.015672	0.154486	0.017335	0.345679
1.093444	0.386008	0.112994	0.381059	0.11994	0.162925	0.013961	0.158775	0.01573	0.35139
1.08967	0.391705	0.107178	0.386404	0.114714	0.167191	0.012517	0.162696	0.014339	0.356744
1.086164	0.396841	0.101943	0.391285	0.109931	0.171052	0.011288	0.166296	0.013126	0.361762
1.082903	0.401497	0.097205	0.395762	0.105536	0.174564	0.010232	0.169612	0.012061	0.36647

Table 4.18: Estimate the average value for A2 from the stiffness matrix values

Sum of A2	A1-K2	A2-K1	A2-L1	A2-L2	A2*A2	A2*A2	A2*A2	A2*A2	Final A2
99137.61	0.178815	0.267842	0.276672	0.276672	3169.899	7112.054	7588.71	7588.71	25459.37
102432.9	0.252198	0.266543	0.263587	0.217672	6515.108	7277.355	7116.857	4853.385	25762.7
107111.2	0.315093	0.260206	0.248963	0.175738	10634.4	7252.193	6639.001	3308.014	27833.6
112579.5	0.369109	0.251525	0.234501	0.144865	15337.96	7122.318	6190.826	2362.594	31013.7
118542.9	0.415763	0.241898	0.220864	0.121475	20491.17	6936.514	5782.614	1749.241	34959.54
124837.7	0.456337	0.232065	0.20827	0.103328	25996.65	6723.007	5415.03	1332.85	39467.54
131365.9	0.491873	0.222412	0.196749	0.088965	31782.59	6498.308	5085.214	1039.729	44405.84
138064.7	0.52321	0.21314	0.186248	0.077402	37795.04	6272.104	4789.236	827.1508	49683.53
144892.3	0.551021	0.204341	0.176683	0.067955	43992.8	6050.028	4523.082	669.095	55235
151819.6	0.57585	0.19605	0.167962	0.060138	50343.92	5835.261	4283.043	549.0606	61011.29
158825.7	0.59814	0.188267	0.159998	0.053595	56823.29	5629.491	4065.826	456.2203	66974.83
165895.2	0.618251	0.180977	0.152707	0.048065	63410.91	5433.493	3868.573	383.2601	73096.24
173016.5	0.636482	0.174153	0.146016	0.043348	70090.68	5247.479	3688.823	325.1135	79352.1
180180.7	0.65308	0.167767	0.13986	0.039293	76849.51	5071.324	3524.462	278.1925	85723.49
187380.8	0.668251	0.161787	0.134181	0.035781	83676.64	4904.703	3373.682	239.9063	92194.93

Table 4.19: Estimate the average value for B1 from the stiffness matrix values

Sum of B1	B1-K2	B1-K1	B1-W1	B1-W2	B1-L1	B1-L2
0.000385	0.196888	0.184321	0.154698	0.154698	0.154698	0.154698
0.000423	0.145797	0.144459	0.246032	0.070295	0.413258	-0.01984
0.000474	0.109829	0.113372	0.313796	1.67E-17	0.695582	-0.23258
0.000534	0.084284	0.089741	0.361968	-0.05569	0.980621	-0.46093
0.000602	0.065868	0.071856	0.39521	-0.0988	1.257485	-0.69162
0.000677	0.052359	0.058245	0.417468	-0.13183	1.521219	-0.91746
0.000758	0.04227	0.047783	0.431755	-0.157	1.770044	-1.13485
0.000845	0.034604	0.039647	0.440272	-0.17611	2.003786	-1.3422
0.000937	0.028684	0.033242	0.444608	-0.19055	2.22304	-1.53903
0.001035	0.024045	0.02814	0.445908	-0.20138	2.428726	-1.72544
0.001137	0.020359	0.024031	0.444998	-0.20941	2.621871	-1.90185
0.001244	0.017395	0.020686	0.44248	-0.21526	2.803496	-2.0688
0.001357	0.014983	0.017936	0.438795	-0.2194	2.974567	-2.22688
0.001473	0.013001	0.015656	0.434267	-0.22218	3.135972	-2.37671
0.001595	0.011357	0.013748	0.42914	-0.2239	3.28852	-2.51887

Table 4.19: Estimate the average value for B1 from the stiffness matrix values "Table Continued"

B1*B1	B1*B1	B1*B1	B1*B1	B1*B1	B1*B1	Final B1
1.49E-05	1.31E-05	9.21E-06	9.21E-06	9.21E-06	9.21E-06	6.48E-05
9E-06	8.84E-06	2.56E-05	2.09E-06	7.23E-05	1.67E-07	0.000118
5.72E-06	6.1E-06	4.67E-05	1.33E-37	0.000229	2.57E-05	0.000314
3.8E-06	4.3E-06	7E-05	1.66E-06	0.000514	0.000114	0.000707
2.61E-06	3.11E-06	9.41E-05	5.88E-06	0.000953	0.000288	0.001347
1.86E-06	2.3E-06	0.000118	1.18E-05	0.001567	0.00057	0.002271
1.35E-06	1.73E-06	0.000141	1.87E-05	0.002376	0.000977	0.003515
1.01E-06	1.33E-06	0.000164	2.62E-05	0.003393	0.001522	0.005107
7.71E-07	1.04E-06	0.000185	3.4E-05	0.004631	0.00222	0.007072
5.98E-07	8.19E-07	0.000206	4.2E-05	0.006102	0.00308	0.009432
4.71E-07	6.57E-07	0.000225	4.99E-05	0.007816	0.004112	0.012204
3.77E-07	5.32E-07	0.000244	5.77E-05	0.00978	0.005326	0.015408
3.05E-07	4.36E-07	0.000261	6.53E-05	0.012003	0.006727	0.019057
2.49E-07	3.61E-07	0.00028	7.27E-05	0.014491	0.008323	0.023165
2.06E-07	3.01E-07	0.00094	8E-05	0.01725	0.01012	0.027745

Table 4.20: Estimate the average value for B2 from the stiffness matrix values

Sum of B2	B2-K2	B2-K1	B2-W1	B2-W2	B2-L1	B2-L2
34.29815	0.172286	0.161289	0.166606	0.166606	0.166606	0.166606
38.67734	0.148426	0.147065	0.129275	0.203146	0.099986	0.272102
43.28253	0.12996	0.134152	0.099017	0.23104	0.032089	0.373741
48.10428	0.115178	0.122635	0.074243	0.252428	-0.03573	0.471245
53.13853	0.103055	0.112424	0.053768	0.268839	-0.10265	0.564562
58.38336	0.092929	0.103377	0.036703	0.281391	-0.16816	0.653758
63.83779	0.084348	0.09535	0.022378	0.290916	-0.23196	0.738965
69.5013	0.07699	0.088209	0.010277	0.298042	-0.29387	0.820349
75.37359	0.070616	0.081835	0	0.303251	-0.35379	0.898091
81.45448	0.065049	0.076127	-0.00877	0.30692	-0.4117	0.972378
87.74388	0.06015	0.070996	-0.01628	0.309342	-0.4676	1.043397
94.24172	0.055811	0.06637	-0.02274	0.310751	-0.52152	1.111329
100.948	0.051947	0.062184	-0.0283	0.311334	-0.57351	1.176349
107.8626	0.048487	0.058385	-0.03311	0.311243	-0.62362	1.23862
114.9856	0.045374	0.054927	-0.03727	0.310598	-0.67193	1.2983

Table 4.20: Estimate the average value for B2 from the stiffness matrix values "Table Continued"

B2*B2	B2*B2	B2*B2	B2*B2	B2*B2	B2*B2	Final B2
1.018054	0.892237	0.952036	0.952036	0.952036	0.952036	5.718434
0.852078	0.836515	0.646373	1.596146	0.386664	2.863658	7.181434
0.731025	0.778949	0.424359	2.310401	0.044568	6.045816	10.33512
0.638147	0.72346	0.265155	3.065195	0.06141	10.68262	15.43599
0.564349	0.671622	0.153622	3.840559	0.559896	16.93686	22.72691
0.504192	0.623933	0.07865	4.622858	1.65094	24.95304	32.43361
0.454184	0.580393	0.031969	5.402724	3.434764	34.85988	44.76392
0.411962	0.54078	0.007341	6.173721	6.001962	46.77244	59.9082
0.375859	0.504779	0	6.93146	9.434487	60.79382	78.04041
0.344661	0.472053	0.006264	7.672997	13.80657	77.01675	99.31929
0.317457	0.442273	0.023259	8.396422	19.18553	95.52481	123.8898
0.293552	0.415133	0.048724	9.100566	25.63253	116.3935	151.8841
0.272403	0.390354	0.080866	9.784794	33.20325	139.6914	183.4231
0.253579	0.367685	0.118253	10.44886	41.94841	165.4805	218.6173
0.236736	0.346906	0.159736	11.09279	51.91436	193.8176	257.5681

Table 4.21: Estimate the average value for A1’ from the derivative linear equations values

Sum of A1’	A1’-K2	A1’-K1	A1’-L1	A1’-L2	*A1’	*A1’	*A1’	*A1’	Final A1’
1.1444	0.181842	0.313439	0.256466	0.248252	0.037841	0.112431	0.075273	0.070528	0.296073
1.1388	0.206775	0.291359	0.274038	0.227828	0.04869	0.096673	0.085521	0.05911	0.289994
1.1347	0.224817	0.275932	0.290121	0.20913	0.057351	0.086394	0.095508	0.049627	0.288879
1.13225	0.236454	0.266726	0.304593	0.192228	0.063305	0.080551	0.105046	0.041838	0.290741
1.1316	0.242223	0.263256	0.317338	0.177183	0.066393	0.078424	0.113956	0.035525	0.294299
1.1329	0.242718	0.264984	0.328251	0.164048	0.066741	0.079548	0.122068	0.030488	0.298846
1.1363	0.238581	0.271319	0.337235	0.152865	0.064679	0.083648	0.129228	0.026553	0.304108
1.14195	0.230505	0.281624	0.344214	0.143658	0.060675	0.09057	0.135302	0.023567	0.310114
1.15	0.219217	0.295217	0.34913	0.136435	0.055265	0.100226	0.140176	0.021407	0.317073
1.1606	0.205476	0.311391	0.351952	0.131182	0.049001	0.112537	0.143763	0.019972	0.325273
1.1739	0.19005	0.329415	0.352671	0.127864	0.0424	0.127385	0.146006	0.019192	0.334983
1.1905	0.173711	0.348557	0.351309	0.126423	0.03591	0.144581	0.146873	0.01902	0.346386
1.2092	0.157211	0.368095	0.347916	0.126778	0.029886	0.163839	0.146368	0.019435	0.359528
1.2315	0.141271	0.387333	0.34257	0.128827	0.024578	0.184758	0.144522	0.020438	0.374295
1.2571	0.126561	0.405616	0.335375	0.132448	0.020136	0.206824	0.141394	0.022053	0.390406

Table 4.22: Estimate the average value for A2' from the derivative linear equations values

Sum of A2'	A2'-K2	A2'-K1	A2'-L1	A2'-L2	*A2'	*A2'	*A2'	*A2'	Final A2'
99110.32	0.18587	0.268035	0.275471	0.270624	3424.016	7120.34	7520.926	7258.604	25323.89
102952.9	0.253262	0.263783	0.262293	0.220661	6603.591	7163.629	7082.906	5012.922	25863.05
107404.3	0.314015	0.257178	0.248859	0.179948	10590.67	7103.762	6651.656	3477.881	27823.96
112395.5	0.368156	0.248824	0.235564	0.147455	15233.97	6958.811	6236.876	2443.824	30873.48
117857.2	0.416026	0.239264	0.222703	0.122007	20398.4	6747.011	5845.307	1754.397	34745.12
123720.1	0.458164	0.228954	0.210482	0.102399	25970.62	6485.419	5481.15	1297.276	39234.47
129915	0.495221	0.218266	0.199035	0.087478	31860.81	6189.144	5146.59	994.1694	44190.71
136372.8	0.527885	0.207488	0.188435	0.076192	38001.93	5871.035	4842.309	791.6719	49506.95
143024.2	0.55684	0.196841	0.178713	0.067606	44347.63	5541.659	4567.939	653.7112	55110.94
149800	0.582738	0.186484	0.169867	0.060912	50869.56	5209.468	4322.432	555.7977	60957.26
156630.9	0.60618	0.17653	0.161876	0.055414	57554.76	4881.044	4104.34	480.9665	67021.11
163447.9	0.627718	0.167055	0.154707	0.05052	64403.26	4561.39	3912.027	417.1652	73293.84
170181.5	0.647847	0.158108	0.14832	0.045725	71426.18	4254.203	3743.816	355.8086	79780
176762.7	0.667019	0.149716	0.142673	0.040592	78644.29	3962.128	3598.089	291.2539	86495.76
183122.2	0.685644	0.141894	0.137722	0.03474	86087.11	3686.982	3473.354	220.999	93468.45

Table 4.23: Estimate the average value for B1' from the derivative linear equations values

Sum of B1'	B1'-K2	B1'-K1	B1'-W1	B1'-W2	B1'-L1	B1'-L2
0.000518	0.144348	0.140676	0.154589	0.154589	0.21256	0.193237
0.000589	0.110795	0.112217	0.212292	0.084917	0.4076	0.072179
0.000668	0.086228	0.090419	0.254491	0.02994	0.613772	-0.07485
0.000755	0.068323	0.073706	0.284886	-0.01325	0.821532	-0.2352
0.000849	0.055274	0.060813	0.306423	-0.04714	1.025339	-0.40071
0.000949	0.045716	0.050761	0.32137	-0.07376	1.222259	-0.56635
0.001056	0.038636	0.042803	0.331439	-0.0947	1.410985	-0.72917
0.001169	0.033289	0.036379	0.337914	-0.11121	1.591189	-0.88756
0.001288	0.029126	0.031068	0.341748	-0.12427	1.763107	-1.04078
0.001411	0.025747	0.026553	0.343652	-0.13463	1.927284	-1.18861
0.00154	0.022857	0.022597	0.344156	-0.14286	2.084416	-1.33117
0.001673	0.020238	0.019021	0.343655	-0.14942	2.235255	-1.46875
0.001811	0.01773	0.015686	0.342447	-0.15465	2.380558	-1.60177
0.001952	0.015212	0.01249	0.340753	-0.15885	2.521057	-1.73066
0.002096	0.012595	0.009351	0.33874	-0.16221	2.657443	-1.85592

Table 4.23: Estimate the average value for B1’ from the derivative linear equations values “Table Continued”

* B1’	* B1’	* B1’	* B1’	* B1’	* B1’	Final B1’
1.08E-05	1.02E-05	1.24E-05	1.24E-05	2.34E-05	1.93E-05	8.85E-05
7.23E-06	7.41E-06	2.65E-05	4.25E-06	9.78E-05	3.07E-06	0.000146
4.97E-06	5.46E-06	4.33E-05	5.99E-07	0.000252	3.74E-06	0.00031
3.52E-06	4.1E-06	6.13E-05	1.33E-07	0.000509	4.17E-05	0.00062
2.59E-06	3.14E-06	7.97E-05	1.89E-06	0.000892	0.000136	0.001116
1.98E-06	2.45E-06	9.8E-05	5.16E-06	0.001418	0.000304	0.00183
1.58E-06	1.93E-06	0.000116	9.47E-06	0.002102	0.000561	0.002793
1.3E-06	1.55E-06	0.000133	1.45E-05	0.00296	0.000921	0.004031
1.09E-06	1.24E-06	0.00015	1.99E-05	0.004002	0.001395	0.005569
9.36E-07	9.95E-07	0.000167	2.56E-05	0.005242	0.001994	0.00743
8.05E-07	7.86E-07	0.000182	3.14E-05	0.006691	0.002729	0.009635
6.85E-07	6.05E-07	0.000198	3.74E-05	0.00836	0.003609	0.012206
5.69E-07	4.45E-07	0.000212	4.33E-05	0.01026	0.004645	0.015162
4.52E-07	3.04E-07	0.000227	4.92E-05	0.012404	0.005845	0.018526
3.33E-07	1.83E-07	0.000241	5.52E-05	0.014802	0.00722	0.022318

Table 4.24: Estimate the average value for B2’ from the derivative linear equations values

Sum of B2’	B2’-K2	B2’-K1	B2’-W1	B2’-W2	B2’-L1	B2’-L2
34.3081	0.171977	0.161909	0.166532	0.166576	0.166797	0.166209
38.71555	0.148986	0.147213	0.129119	0.202965	0.09988	0.271838
43.3376	0.130584	0.134428	0.098861	0.230767	0.031954	0.373406
48.17425	0.115649	0.123223	0.074104	0.252083	-0.03578	0.470723
53.2255	0.103375	0.113344	0.053647	0.268422	-0.10256	0.563773
58.49135	0.093169	0.104595	0.036602	0.280895	-0.1679	0.652636
63.9718	0.084587	0.096821	0.022297	0.290329	-0.23149	0.737452
69.66685	0.077294	0.089895	0.010219	0.297357	-0.29316	0.818391
75.5765	0.07103	0.083714	-3.4E-05	0.30246	-0.35281	0.895642
81.70075	0.065594	0.078193	-0.00878	0.306018	-0.41042	0.969396
88.0396	0.060827	0.07326	-0.01626	0.308326	-0.46599	1.039842
94.59305	0.056604	0.068855	-0.02269	0.30962	-0.51956	1.107167
101.3611	0.052823	0.064924	-0.02822	0.310088	-0.57116	1.171549
108.3438	0.049404	0.061424	-0.033	0.309883	-0.62087	1.233156
115.541	0.04628	0.058315	-0.03713	0.309127	-0.66874	1.292148

Table 4.24: Estimate the average value for B2’ from the derivative linear equations values “Table Continued”

*B2’	*B2’	*B2’	*B2’	*B2’	*B2’	Final B2’
1.014698	0.899374	0.95146	0.951964	0.954498	0.947771	5.719769
0.859362	0.839028	0.645451	1.594878	0.386225	2.860916	7.185862
0.739001	0.783152	0.4235	2.30788	0.04425	6.042635	10.34048
0.644321	0.731473	0.264544	3.061268	0.061682	10.67448	15.43777
0.568791	0.683783	0.153184	3.83492	0.559874	16.91719	22.71775
0.507731	0.639907	0.078361	4.615069	1.648829	24.91343	32.40332
0.457719	0.599689	0.031805	5.392261	3.427989	34.79009	44.69955
0.416214	0.562981	0.007275	6.16001	5.987167	46.66038	59.79403
0.381303	0.529641	8.94E-08	6.913913	9.407486	60.62557	77.85791
0.351523	0.49953	0.006294	7.651031	13.76228	76.77647	99.04713
0.325742	0.472514	0.023279	8.369479	19.1179	95.19475	123.5037
0.303075	0.448463	0.04869	9.06812	25.53452	115.954	151.3569
0.282825	0.427254	0.080731	9.746357	33.06673	139.1209	182.7248
0.264437	0.408773	0.11797	10.40399	41.7642	164.7555	217.7149
0.247467	0.392916	0.159257	11.04108	51.67214	192.9127	256.4256

Table 4.25: Checking for A1, A2, B1, and B2 with A1’, A2’, B1’, and B2’ equations

A1/A1’	A2/A2’	B1/B1’	B2/B2’
1.01952	0.994678	1.364738	1.000234
1.010829	1.003895	1.239667	1.000616
0.995045	0.999654	0.987471	1.000519
0.980365	0.995479	0.87677	1.000115
0.968442	0.993867	0.828487	0.999597
0.959387	0.994095	0.805633	0.999066
0.953295	0.995155	0.794466	0.998562
0.950573	0.996446	0.789301	0.998094
0.951862	0.997754	0.787521	0.997662
0.957843	0.999115	0.787816	0.99726
0.969058	1.000691	0.789492	0.996884
0.985757	1.002703	0.792165	0.996529
1.007805	1.005393	0.795615	0.996193
1.034644	1.009009	0.799716	0.995872
1.065317	1.013813	0.804392	0.995564

After estimate the final value for each item (A1, A2, B1, B2, A1’, A2’, B1’, and B2’) by applying the weighted arithmetic mean. And checking the deviation. All the values is ok, except there is little deviation in values of B1/B1’.the correction factor procedure for the B1/B1’has been applied as fallowing

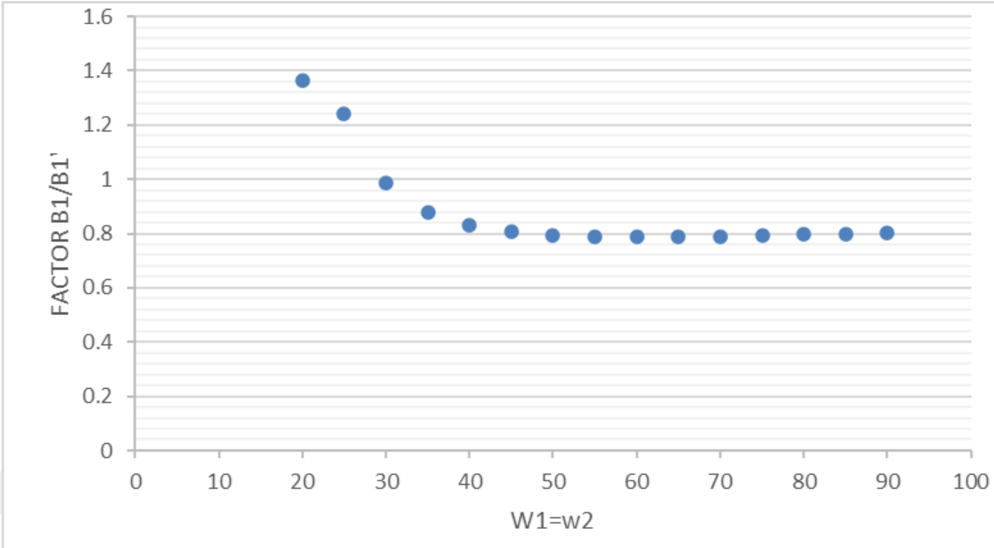

Figure 4.25: Correlation variation for W and B1 / B1’ factor

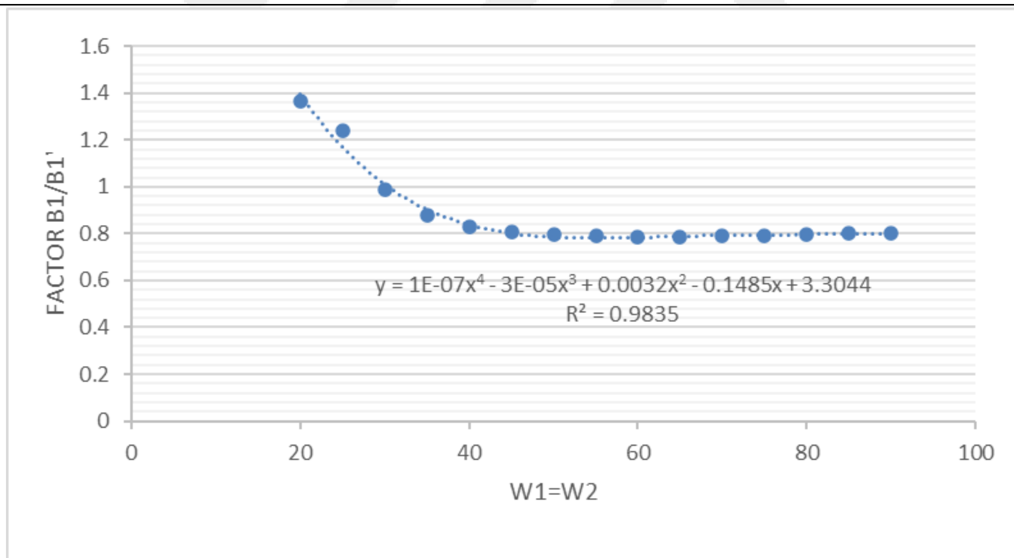

Figure 4.26: Correlation equation for W and B1 / B1’ factor

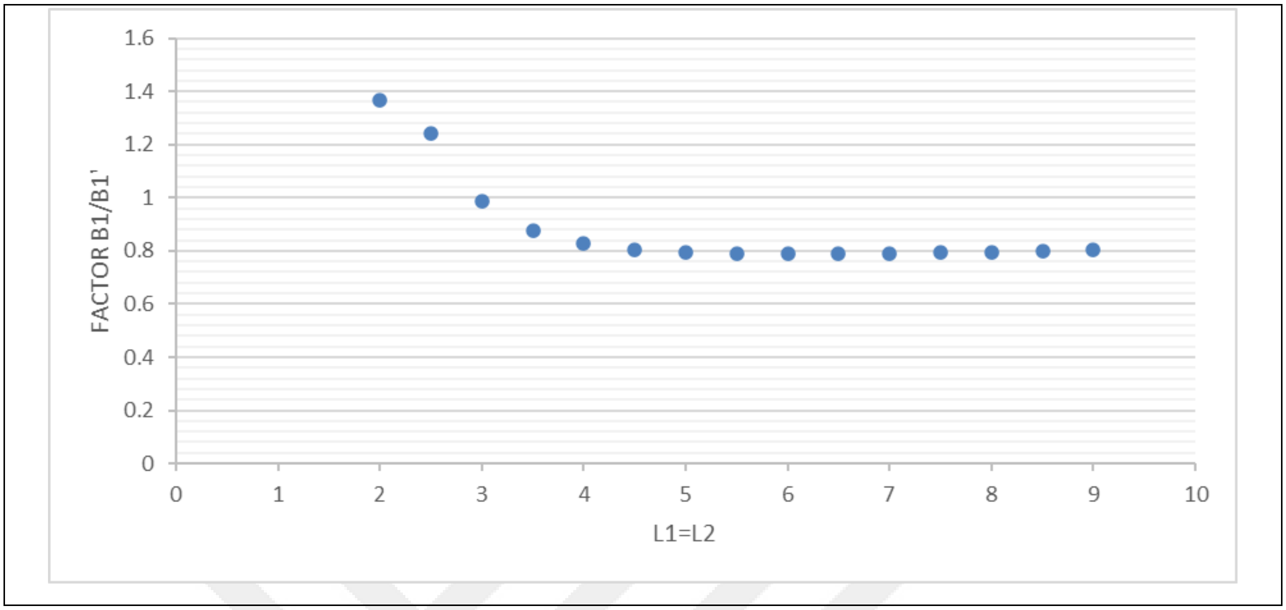

Figure 4.27: Correlation variation for L and B1 / B1' factor

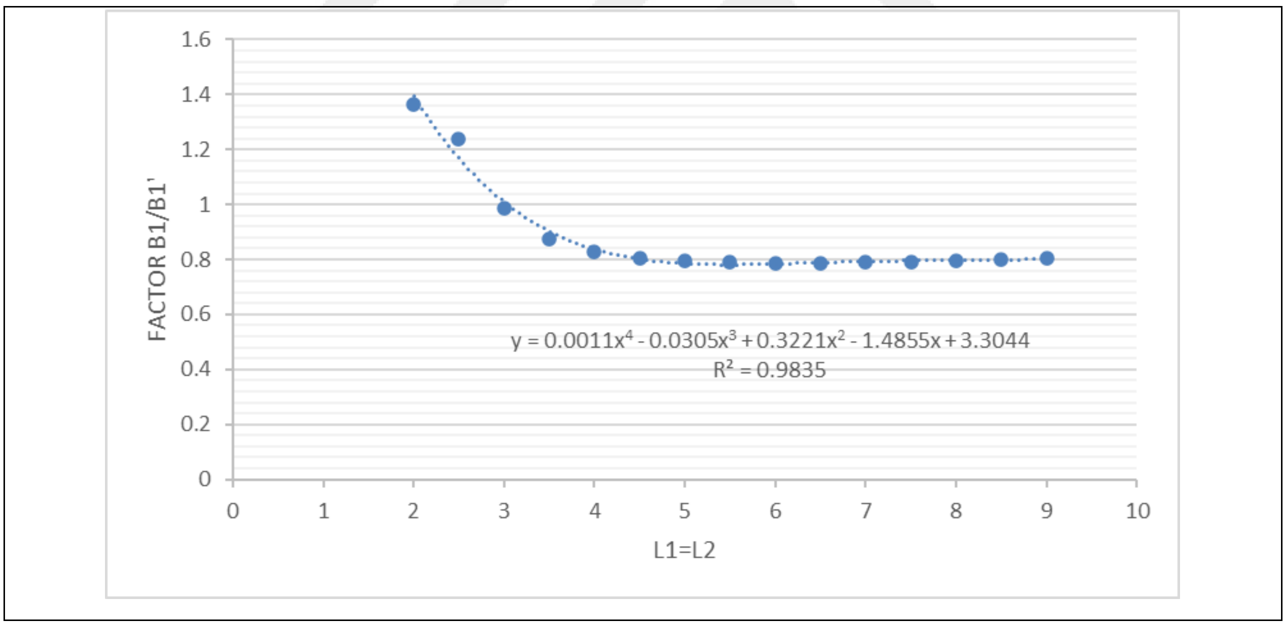

Figure 4.28: Correlation equation for L and B1 / B1' factor

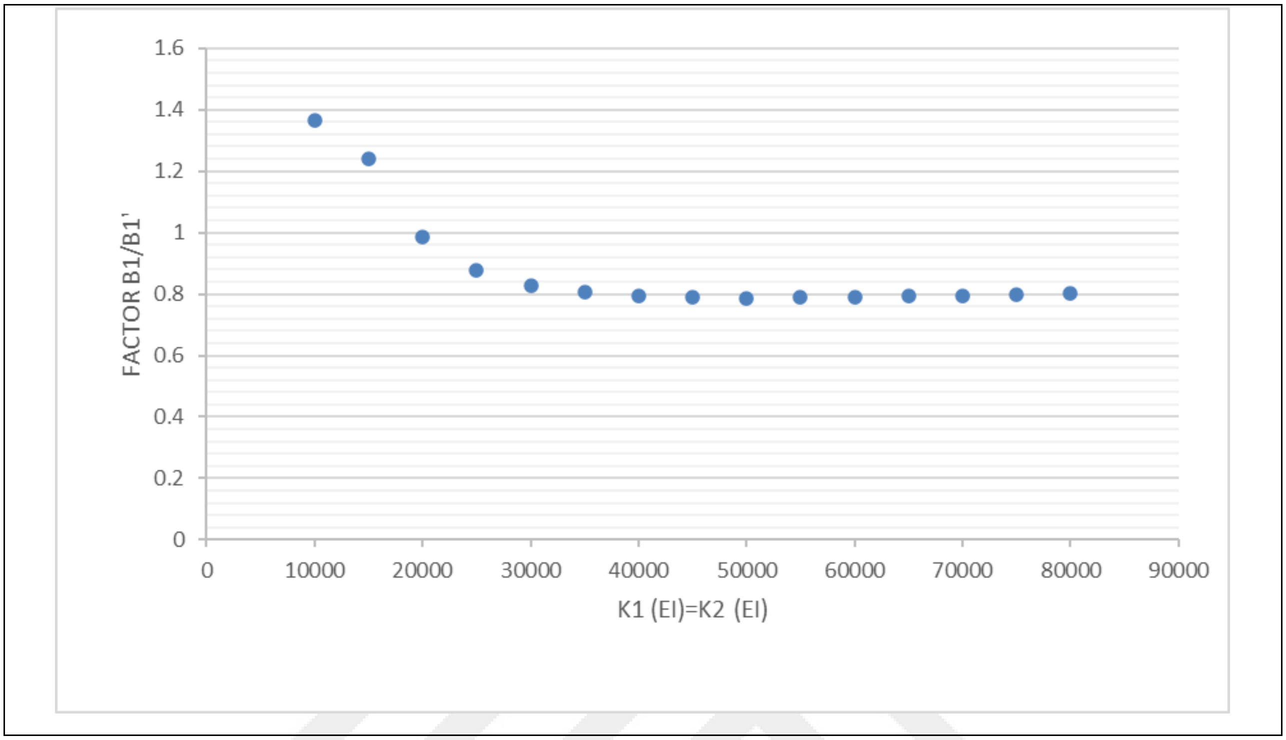

Figure 4.29: Correlation variation for K and B1 / B1’ factor

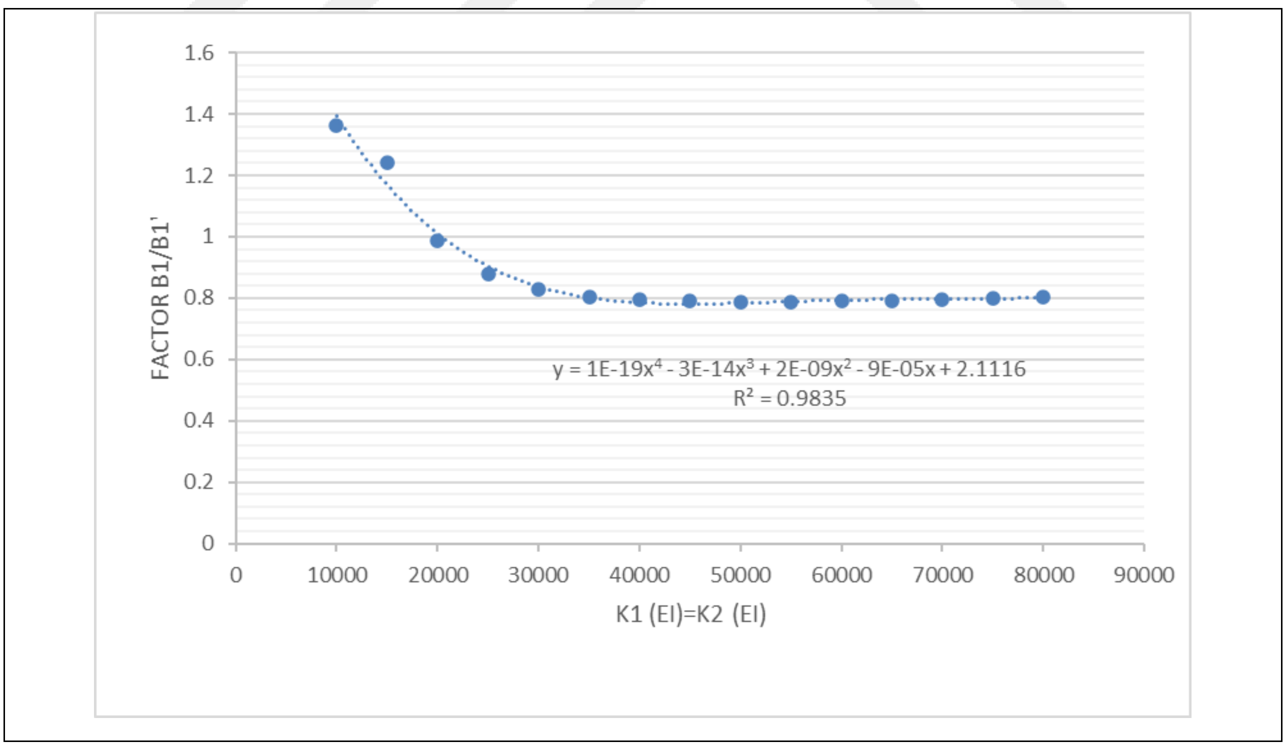

Figure 4.30: Correlation equation for (EI) and B1/ B1’ factor

Table 4.26: The Derived linear equations for B1 and B1/B1'

M1		M2
W1	y = 9E-06x-0.0001	$y = 1E\text{-}07x^4\text{-}3E\text{-}05x^3\text{+}0.0032x^2\text{-}0.1485x\text{+}3.3044$
W2	y = -6E-06x + 0.0002	
L1	$y = 8E\text{-}05x^2 - 0.0001x - 1E\text{-}05$	$y = 0.0011x^4\text{-}0.0305x^3\text{+}0.3221x^2\text{-}1.4855x\text{+}3.3044$
L2	$y = -7E\text{-}05x^2 + 0.0002x - 2E\text{-}05$	
(EI)1	$y = -2E\text{-}19x^3\text{+}3E\text{-}14x^2\text{-}2E\text{-}09x\text{+}9E\text{-}05$	$y = 1E\text{-}19x^4\text{-}3E\text{-}14x^3\text{+}2E\text{-}09x^2\text{-}9E\text{-}05x\text{+}2.1116$
(EI)2	$y = -3E\text{-}19x^3\text{+}5E\text{-}14x^2 \text{-}3E\text{-}09x\text{+}0.0001$	

$$B1'' = M1 \times M2$$

Table 4.27: The Magnitude of M2 with the variation of values of L, K, and W

L1=L2	K1=K2	W1=W2	M2 for L	M2 for K	M2 for W
2	10000	20	1.3954	1.3826	1.3904
2.5	15000	25	1.170181	1.115413	1.162213
3	20000	30	1.0124	0.8876	1.0004
3.5	25000	35	0.908256	0.681913	0.890713
4	30000	40	0.8456	0.4826	0.8204
4.5	35000	45	0.813931	0.275413	0.778213
5	40000	50	0.8044	0.0476	0.7544
5.5	45000	55	0.809806	-0.21209	0.740712
6	50000	60	0.8246	-0.5134	0.7304
6.5	55000	65	0.844881	-0.86459	0.718213

Table 4.27: The Magnitude of M2 with the variation of values of L, K, and W "Table Continued"

L1=L2	K1=K2	W1=W2	M2 for L	L1=L2	K1=K2
7	60000	70	0.8684	-1.2724	0.7004
7.5	65000	75	0.894556	-1.74209	0.674713
8	70000	80	0.9244	-2.2774	0.6404
8.5	75000	85	0.960631	-2.88059	0.598213
9	80000	90	1.0076	-3.5524	0.5504

Table 4.28: The Magnitude of M1*M2 (B1'') for W1, W2, L1, L2, K1, and K2

M1*M2 for L1	M1*M2 for L2	M1*M2 for k1	M1*M2 for k2	M1*M2 for W1	M1*M2 for W2
0.000153494	0.00013954	0.000100653	0.00010328	0.000111232	0.000111
0.000280844	4.97327E-05	7.37009E-05	7.27667E-05	0.000145277	5.81E-05
0.000415084	-5.062E-05	5.3611E-05	5.11258E-05	0.000170068	2E-05
0.000563119	-0.000161215	3.79314E-05	3.51611E-05	0.000191503	-8.9E-06
0.000735672	-0.000287504	2.49022E-05	2.26339E-05	0.000213304	-3.3E-05
0.00094416	-0.000437488	1.3268E-05	1.19495E-05	0.000237355	-5.4E-05
0.001198556	-0.000619388	2.15152E-06	1.94208E-06	0.00026404	-7.5E-05
0.00150624	-0.000840174	-9.01902E-06	-8.25285E-06	0.000292581	-9.6E-05
0.001871842	-0.001104964	-0.000020536	-1.92525E-05	0.000321376	-0.00012
0.002298077	-0.001417288	-3.24004E-05	-3.14169E-05	0.000348333	-0.00014
0.002787564	-0.00178022	-4.42795E-05	-4.47885E-05	0.000371212	-0.00015
0.00334564	-0.002198372	-5.54419E-05	-5.89914E-05	0.00038796	-0.00017
0.003984164	-0.00268076	-6.46782E-05	-7.31045E-05	0.000397048	-0.00018
0.004726306	-0.003244532	-7.02143E-05	-8.55174E-05	0.000397811	-0.00019
0.005612332	-0.003919564	-6.9627E-05	-9.37834E-05	0.000390784	-0.00019

Table 4.29: Estimate The Average value for B1'' for the correlation equations values

Sum B1''	B1''-L1	B1''-L2	B1''-K1	B1''-K2	B1''-W1	B1''-W2
0.000719	0.213355	0.193959	0.139907	0.143558	0.154611	0.154611
0.00068	0.412744	0.07309	0.108315	0.106942	0.213507	0.085403
0.000659	0.629605	-0.07678	0.081318	0.077548	0.257961	0.030348
0.000658	0.856335	-0.24516	0.057682	0.05347	0.291219	-0.01355
0.000676	1.087963	-0.42518	0.036827	0.033473	0.315449	-0.04853
0.000715	1.32093	-0.61207	0.018563	0.016718	0.332072	-0.07621
0.000772	1.552812	-0.80246	0.002787	0.002516	0.342082	-0.09774
0.000845	1.782358	-0.99419	-0.01067	-0.00977	0.346216	-0.11394
0.000932	2.009273	-1.18609	-0.02204	-0.02067	0.344972	-0.12544
0.001029	2.23365	-1.37755	-0.03149	-0.03054	0.338567	-0.13263
0.001135	2.455138	-1.56792	-0.039	-0.03945	0.326944	-0.13571
0.001252	2.671988	-1.75572	-0.04428	-0.04711	0.309843	-0.13471
0.001383	2.880069	-1.93787	-0.04675	-0.05285	0.287018	-0.12962
0.001538	3.072207	-2.10902	-0.04564	-0.05559	0.258586	-0.12054
0.001733	3.238496	-2.26171	-0.04018	-0.05412	0.225495	-0.10798

Table 4.29: Estimate The Average value for B1'' for the correlation equations values "Table Continued"

*B1''	*B1''	*B1''	*B1''	*B1''	*B1''	Final B1''
3.27486E-05	2.7065E-05	1.40821E-05	1.48267E-05	1.71977E-05	1.71977E-05	0.000123
0.000115916	3.63496E-06	7.98291E-06	7.78183E-06	3.10175E-05	4.9628E-06	0.000171
0.000261339	3.88666E-06	4.35954E-06	3.96471E-06	4.3871E-05	6.07211E-07	0.000318
0.000482218	3.95236E-05	2.18797E-06	1.88005E-06	5.57693E-05	1.20648E-07	0.000582
0.000800384	0.000122241	9.17073E-07	7.57618E-07	6.72865E-05	1.59258E-06	0.000993
0.001247169	0.000267773	2.46289E-07	1.9977E-07	7.88188E-05	4.1517E-06	0.001598
0.001861132	0.000497034	5.99724E-09	4.88647E-09	9.03233E-05	7.37333E-06	0.002456
0.002684658	0.000835294	9.62542E-08	8.05952E-08	0.000101296	1.0972E-05	0.003632
0.003761042	0.001310588	4.52691E-07	3.97873E-07	0.000110866	1.46599E-05	0.005198
0.005133099	0.001952391	1.02036E-06	9.59353E-07	0.000117934	1.80994E-05	0.007224
0.006843855	0.002791248	1.72686E-06	1.76679E-06	0.000121365	2.09117E-05	0.009781
0.008939511	0.003859736	2.45489E-06	2.77929E-06	0.000120207	2.27234E-05	0.012947
0.011474666	0.005194952	3.02399E-06	3.86326E-06	0.00011396	2.32426E-05	0.016814
0.014520189	0.006842783	3.20465E-06	4.75377E-06	0.000102869	2.23544E-05	0.021496
0.018175516	0.008864935	2.79741E-06	5.07518E-06	8.81198E-05	2.02076E-05	0.027157

Table 4.30: Final Values of deviation after correlation factor

A1/A1'	A2/A2'	B1'/B1''	B2/B2'
1.01952	0.994678	0.718528	1.000234
1.010829	1.003895	0.854173	1.000616
0.995045	0.999654	0.973749	1.000519
0.980365	0.995479	1.066019	1.000115
0.968442	0.993867	1.123233	0.999597
0.959387	0.994095	1.144826	0.999066
0.953295	0.995155	1.137196	0.998562
0.950573	0.996446	1.109798	0.998094
0.951862	0.997754	1.071465	0.997662
0.957843	0.999115	1.028626	0.99726
0.969058	1.000691	0.985116	0.996884
0.985757	1.002703	0.942703	0.996529
1.007805	1.005393	0.901762	0.996193
1.034644	1.009009	0.861806	0.995872
1.065317	1.013813	0.821811	0.995564

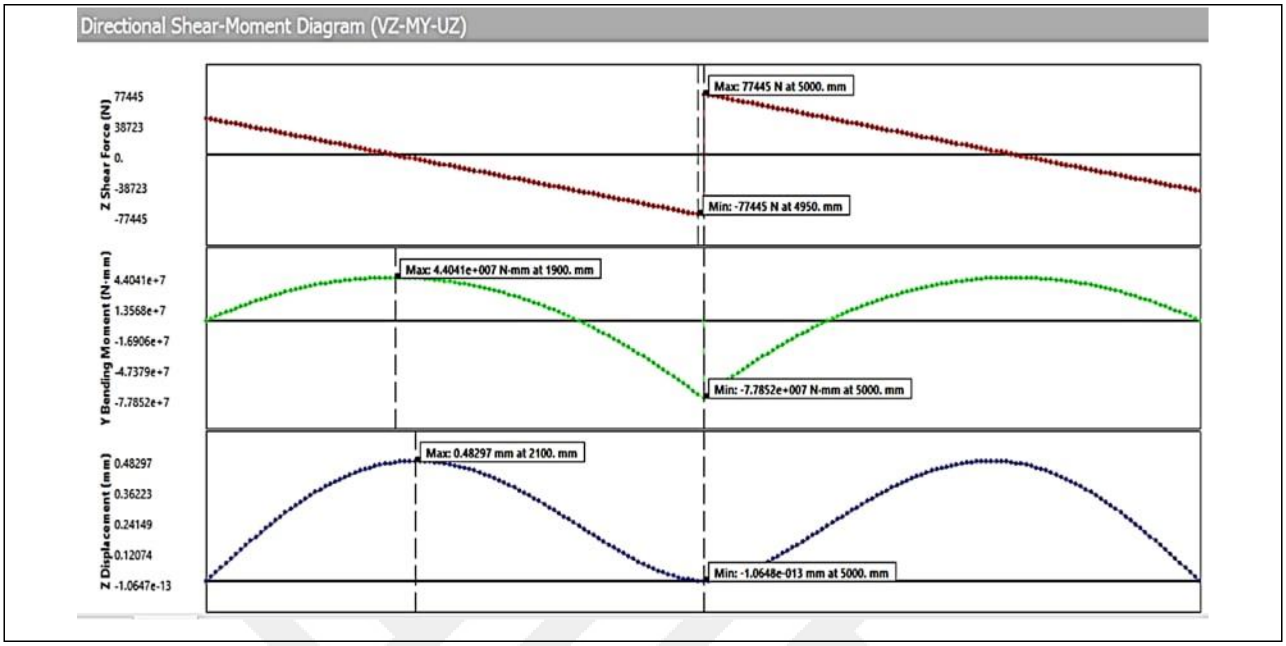

Figure 4.31: Shear Force, bending moment and displacement diagram (ANSYS: W1=W2= 25) kN/m)

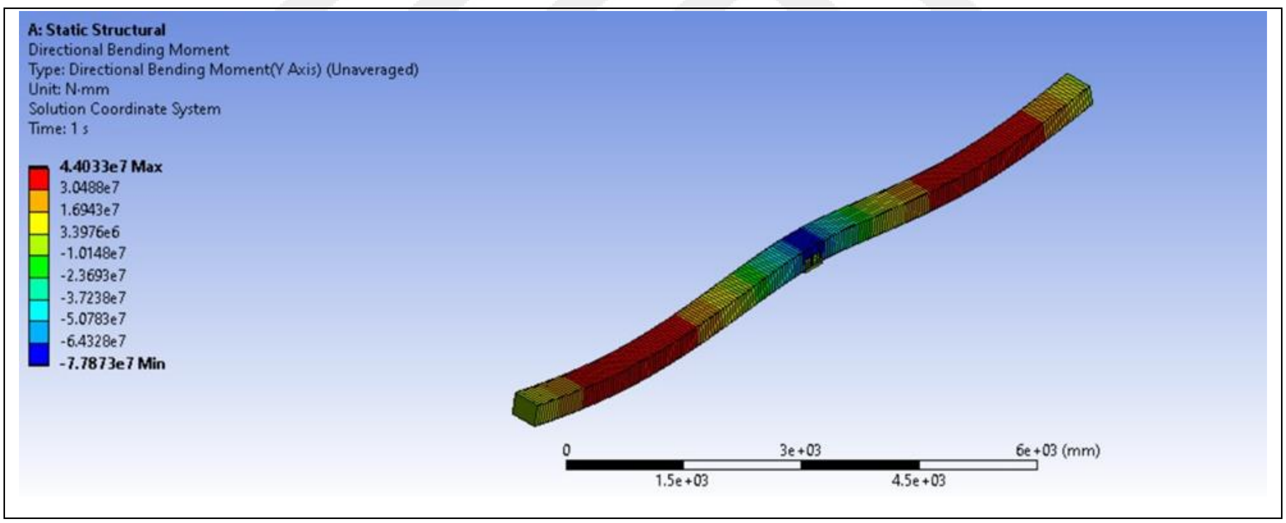

Figure 4.32: Bending Moment variation (ANSYS: W1=W2= 25 kN/m)

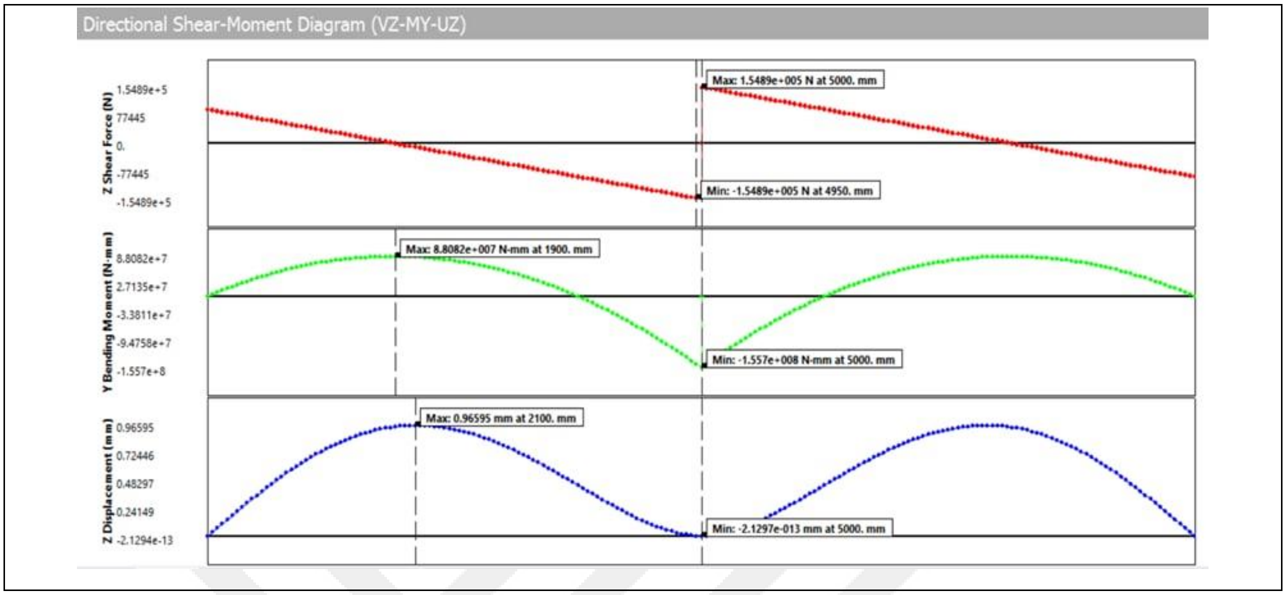

Figure 4.33: Shear Force, bending moment and displacement diagram (ANSYS: W1=W2= 50 kN/m)

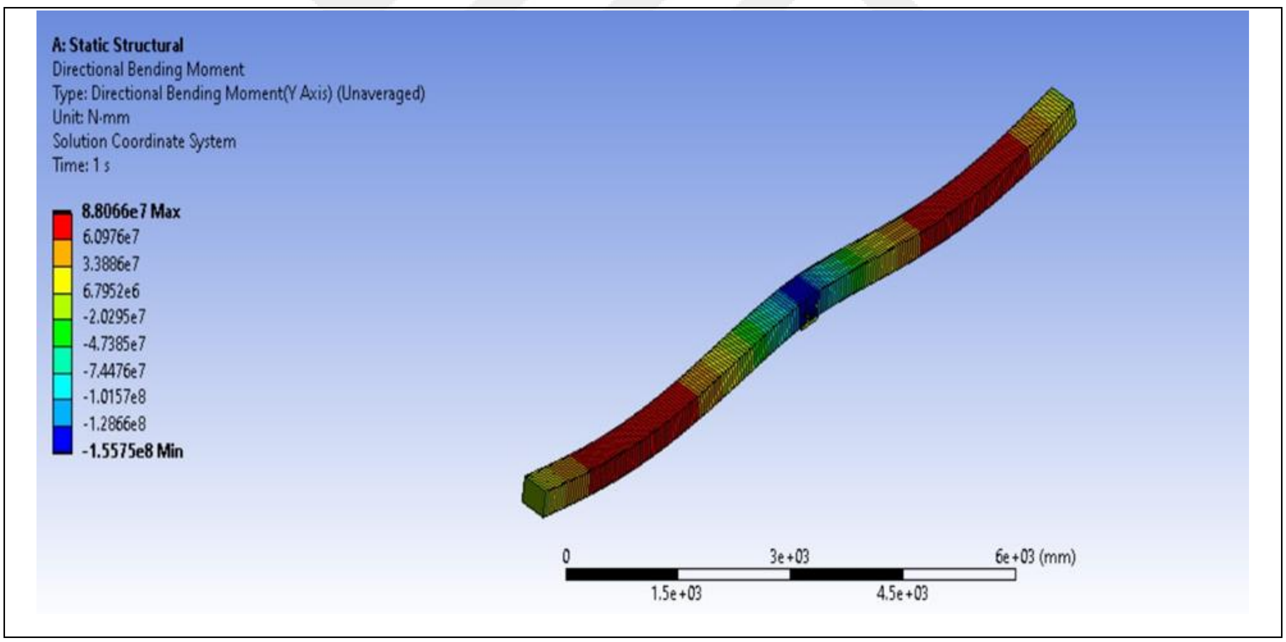

Figure 4.34: Bending Moment variation (ANSYS: W1=W2= 50 kN/m)

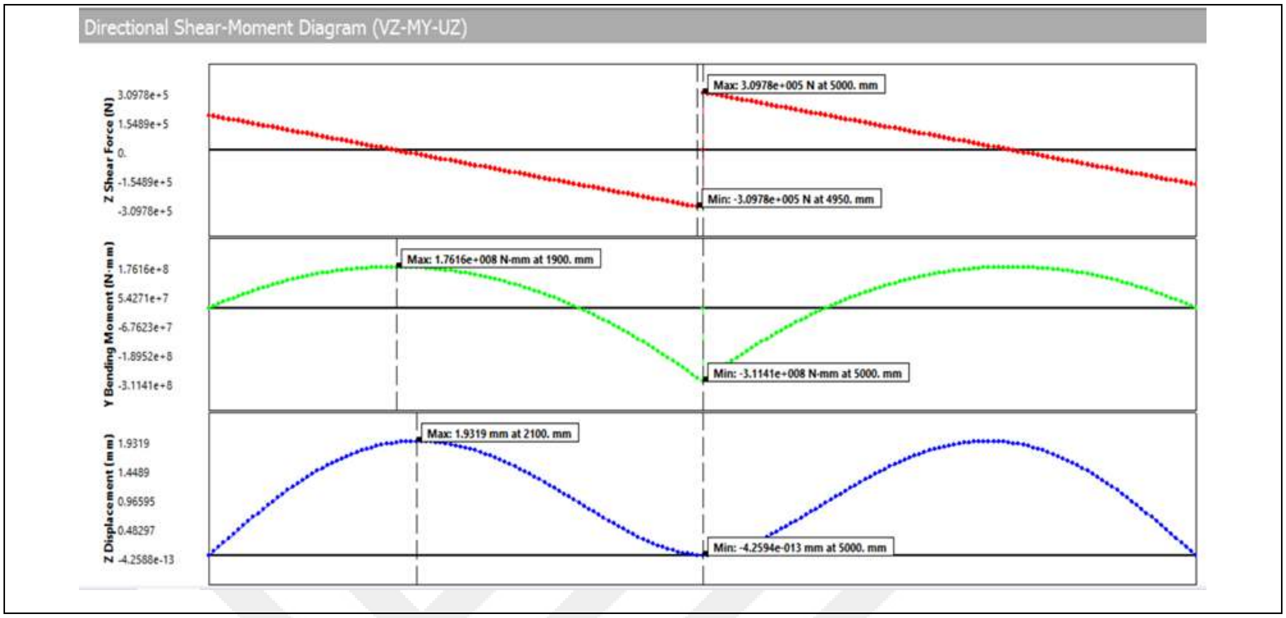

Figure 4.35: Shear Force, bending moment and displacement diagram (ANSYS: W1=W2= 100 kN/m)

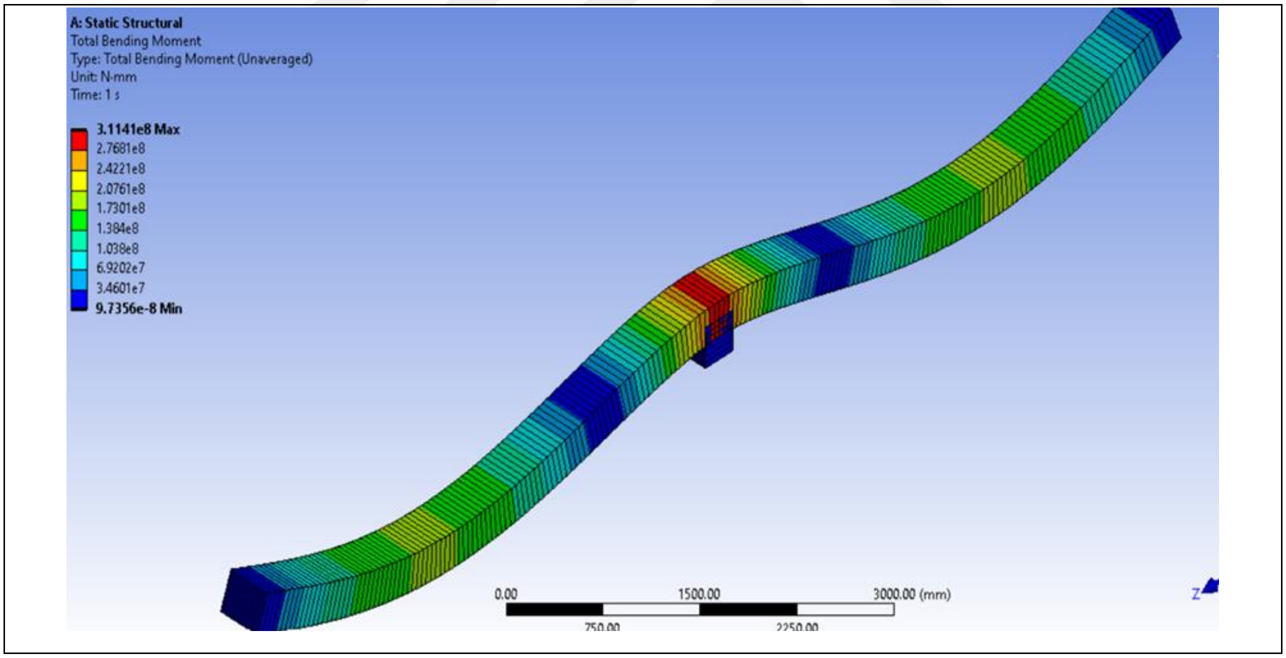

Figure 4.36: Bending Moment variation (ANSYS: W1=W2= 100 kN/m)

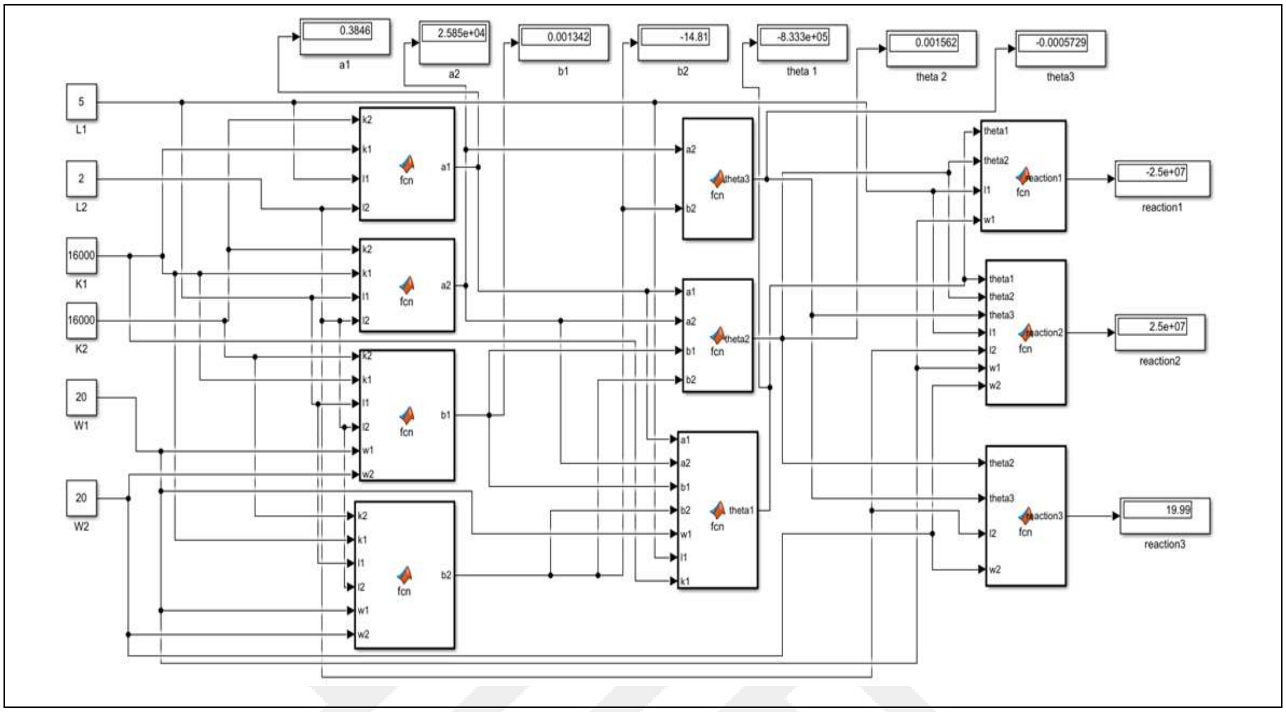

Figure 4.37: The Mathematical model in MTLAB/Simulink with changing L1

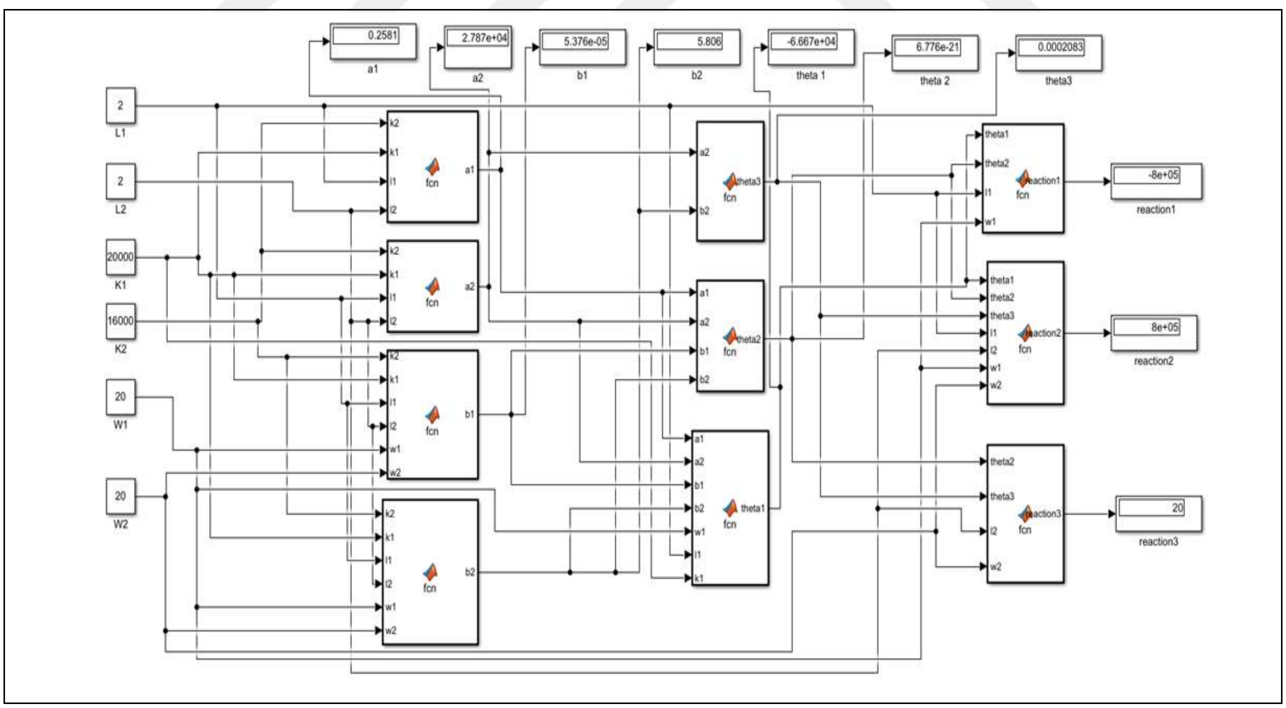

Figure 4.38: The Mathematical model in MTLAB/Simulink with changing K1

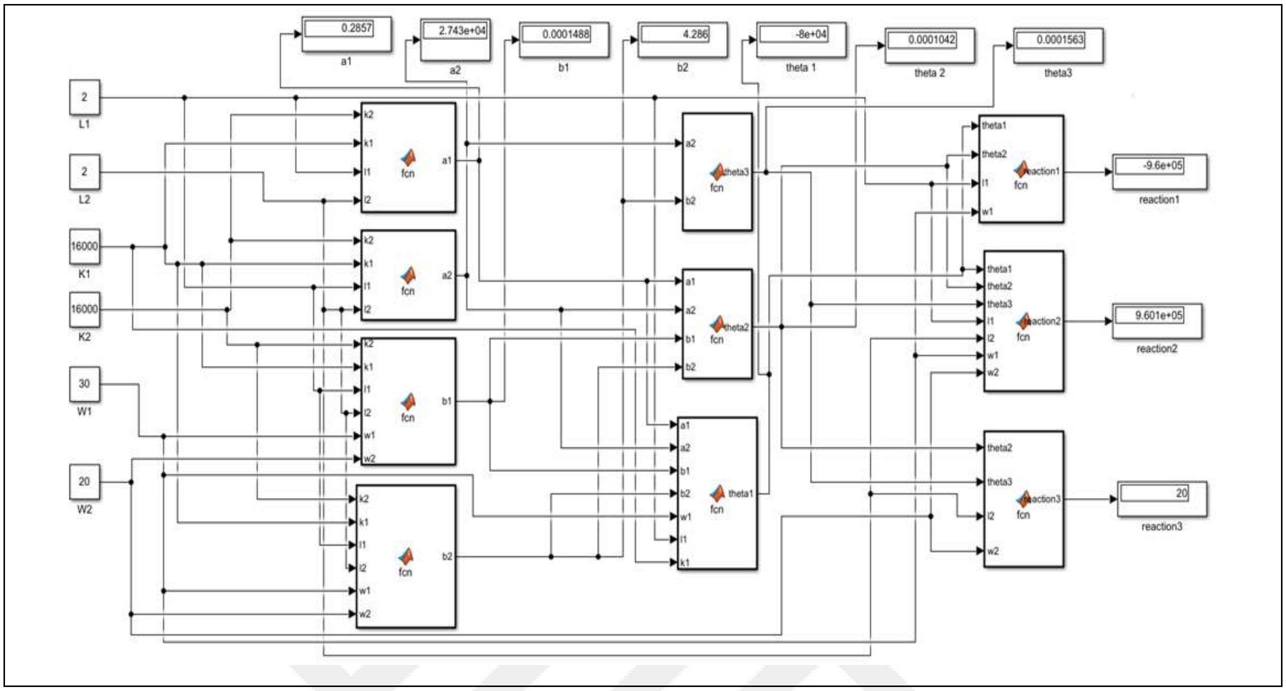

Figure 4.39: The Mathematical model in MTLAB/Simulink with changing W2

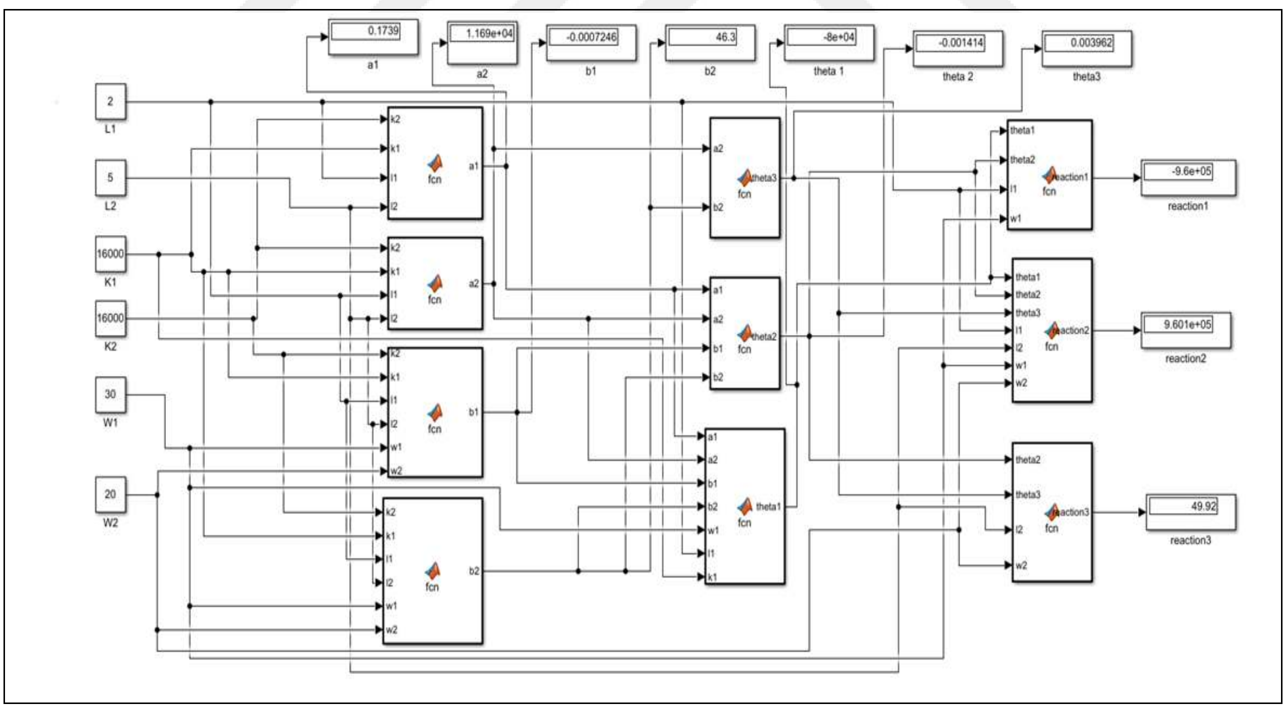

Figure 4.40: The Mathematical model in MTLAB/Simulink with changing L1 and W1

5. DISCUSSION AND CONCLUSIONS

5.1 CONCLUSIONS

The study builds its analytical pathway by three main steps:

a) The first step: analyzing the stiffness matrix body that used to identify the reactions and the moments for two spans of beams for various conditions including the magnitude of loading, the length of the spans, and the stiffness of the beams EI. The step included the converting of the stiffness matrix from its original steps to simpler one with only four parts by using Gaussian elimination theory. The original body of the stiffness matrix in which can contain difficulty and more number of stages to reach the magnitude of variables while the study reach only four parts named as A1, A2, B1, and B2 in which these four parts can used easily to identify the magnitude of the angles of deformation for determined the internal forces and moments.

b) The second step: drawing and analyzing the relationships for the parameters that influenced the results of the stiffness matrix and each derived part (A1, A2, B1, and B2) then curve fitting analysis used for these drawings toward assigning number of simple equations that can be used easily to find the value of each derived part of the matrix in which the steps of identifying the angles of deformation were reduced noticeably.

c) The third step: building effective simulation of the two span cases using of uniform loading of (25, 50, and 100 kN/m). The simulation results were showed in the study by shear force and bending moment diagrams as well as 3d images of the bending moment distribution along the spans.

d) The study checked the feasibility of using the derived equation in the design processes by comparing the results of the original and the derived equations in which small differences were get and explained in related tables.

e) The study achieved full mathematical model in MATLAB/Simulink to estimate the displacements and reactions. This model can be updated easily with different variables besides it can be used to solve any continuous beam subjected to uniform load as well as the one used in this study. This mathematical model represent a calculator can be used in the field of analysis and by the any student who wants to understand the beam-structure stiffness matrix in simplest way.

5.2 FUTURE WORK

The analysis of more cases of beams and columns can be obtained in coming studies to find out the capability of Gaussian elimination and the deriving of new equation to reduce the steps for identifying the required reactions and moments. The future studies can make link between the stiffness matrix parts and the flexural design steps to give more specific figure about the design output as cost effective or risk assessment analysis line. The future studies can analysis the possibility to deal with other analysis techniques like unit load method or slope deflection method to be converted into simpler form based on the original ways of the methods. The future studies can study and analysis the impact of lateral loads besides the vertical loads. Furthermore the future studies can built a mathematical model in MATLAB/ Simulink for another analysis method.

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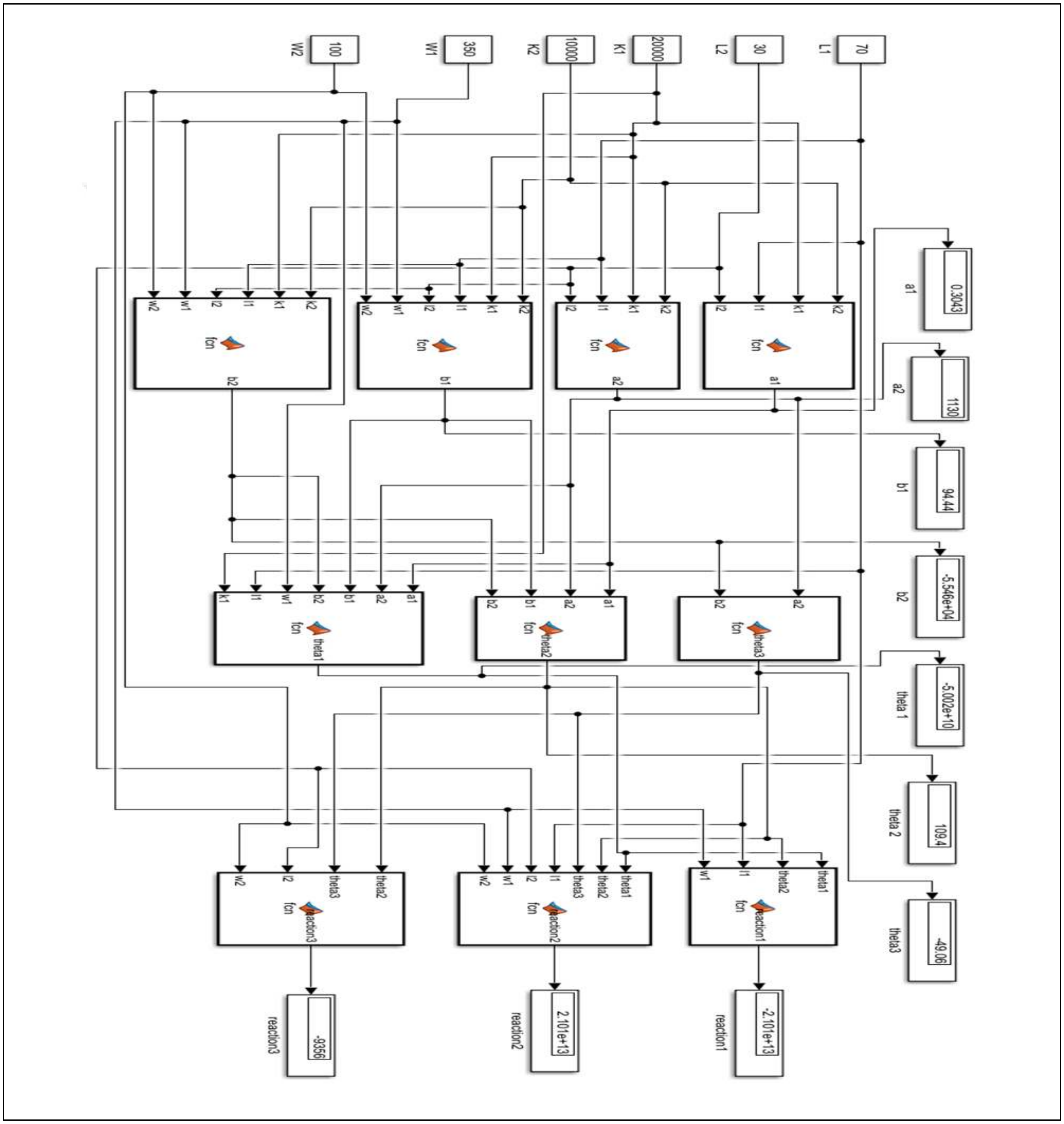
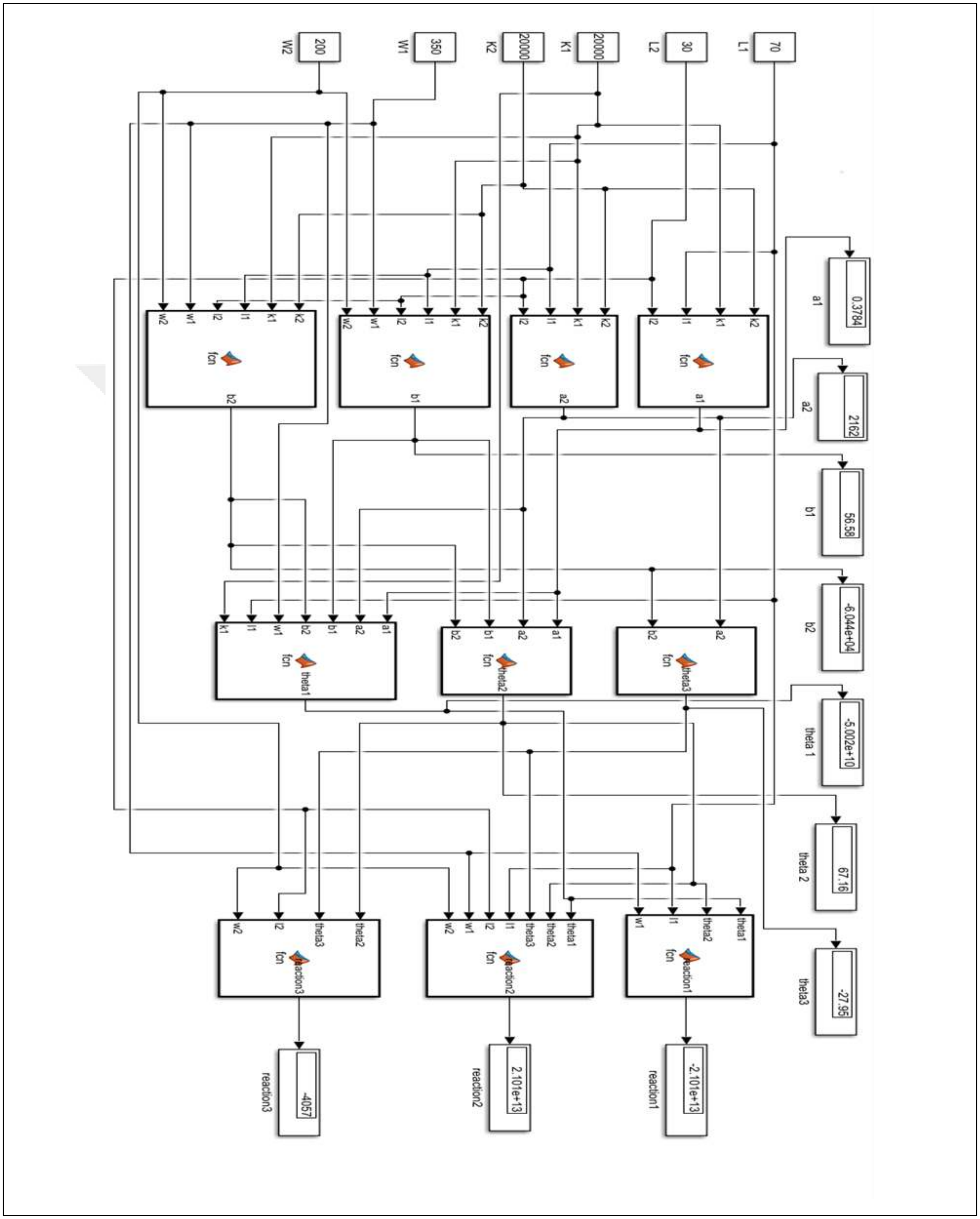
APPENDIX A

Testing for the mathematical model with different values to calculate the rotations and the reactions at the supports from the main parts of developed stiffness matrix





70
L1
30
L2
20000
K1
20000
K2
350
W1
200
W2
a1
a2
b1
b2
theta1
theta2
theta3
reaction1
reaction2
reaction3
0.3784
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67.16
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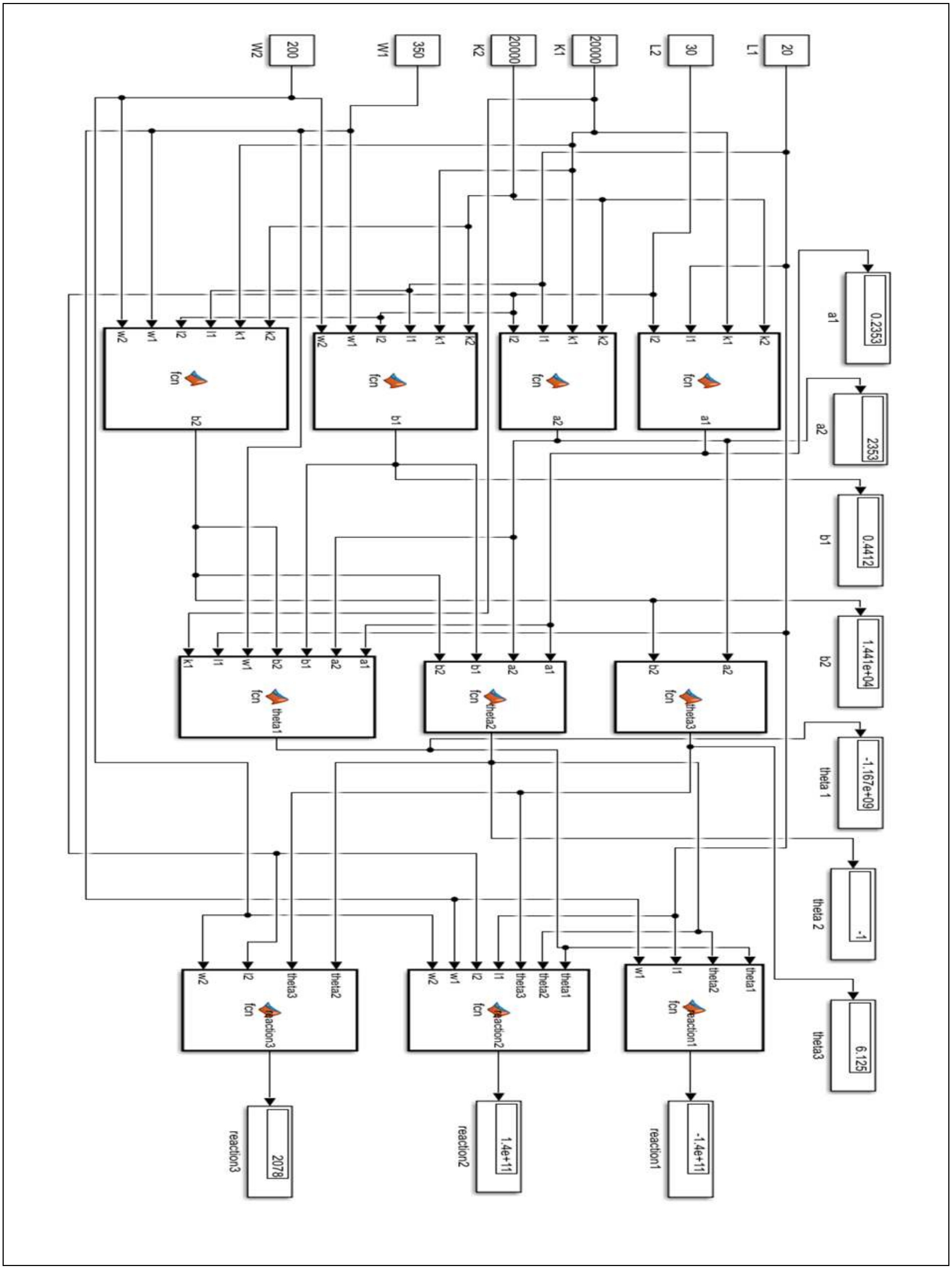
2.101e+13
-4057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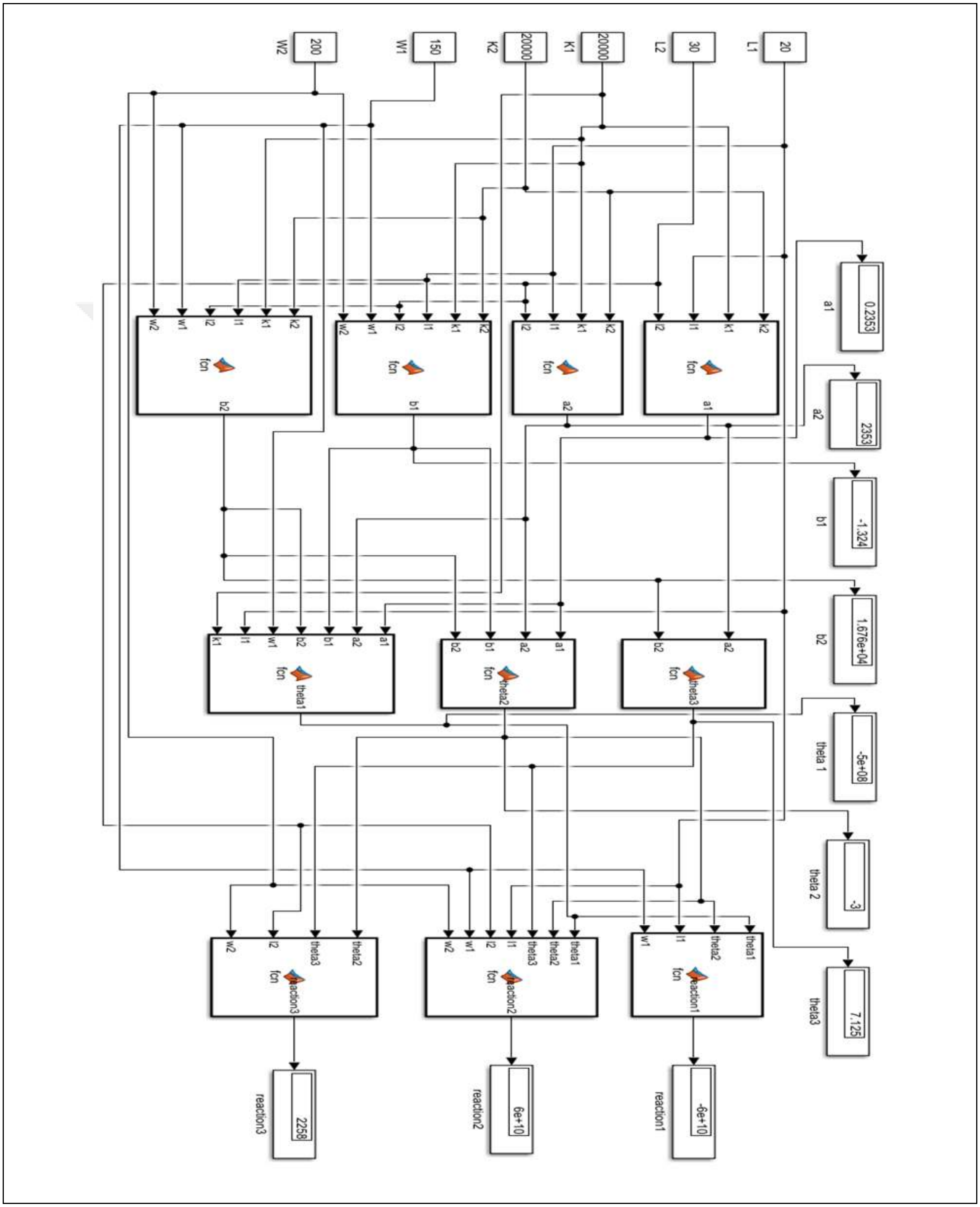
200
W1
350
K2
20000
K1
20000
L2
30
L1
20
fcn
a1
a2
b1
b2
theta1
theta2
theta3
reaction1
reaction2
reaction3
0.2353
2353
0.4412
1.441e+04
theta 1
-1.167e+09
theta 2
-1
theta3
6.125
-1.4e+11
1.4e+11
2078



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20
L2
30
K1
20000
K2
20000
W1
150
W2
200
a1
0.2353
a2
2353
b1
-1.324
b2
1.676e+04
theta 1
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theta 2
-3
theta3
7.125
reaction1
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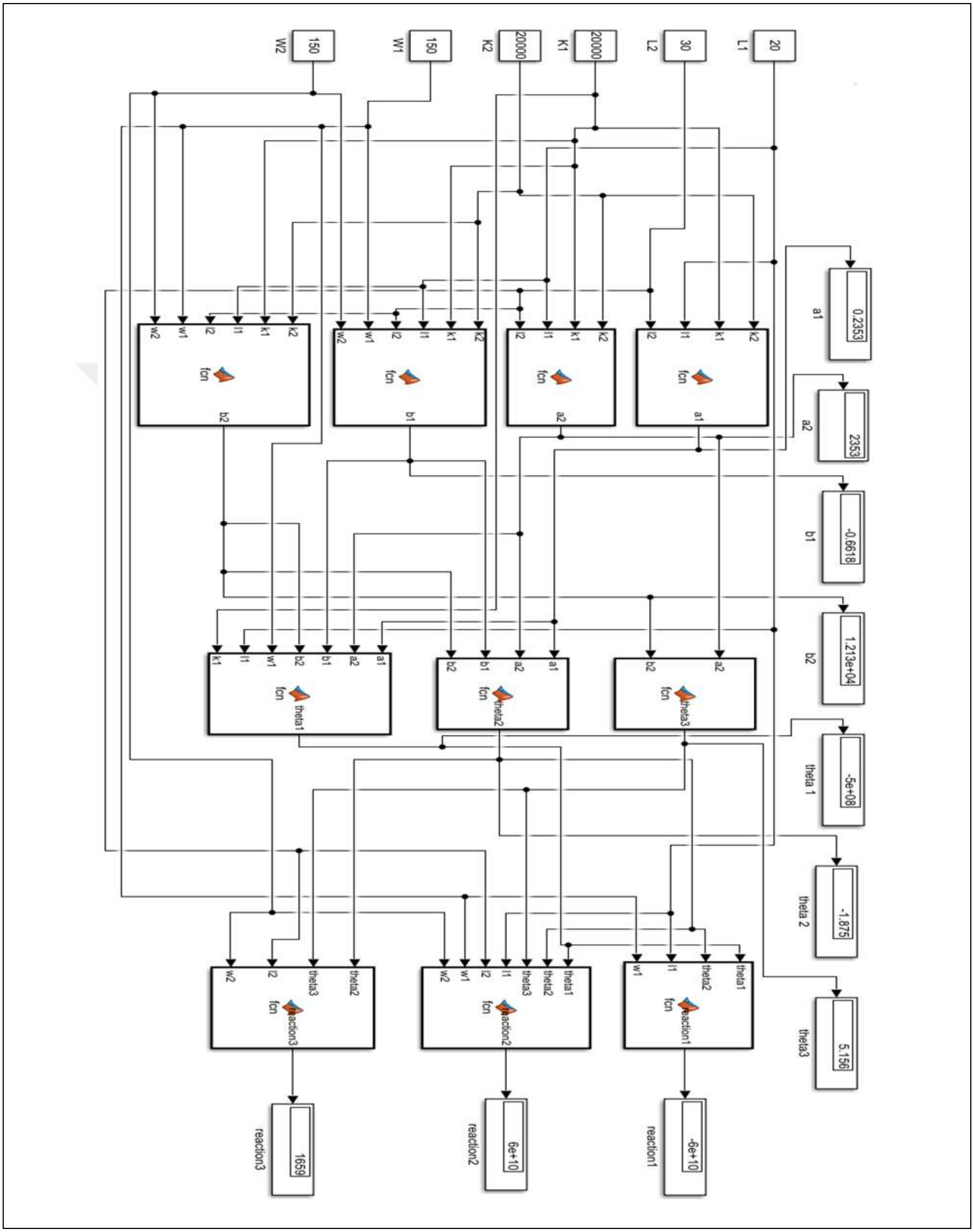
reaction2
6e+10
reaction3
2258



W2
150
W1
150
K2
20000
K1
20000
L2
30
L1
20
a1
0.2353
a2
2353
b1
-0.6618
b2
1.213e+04
theta 1
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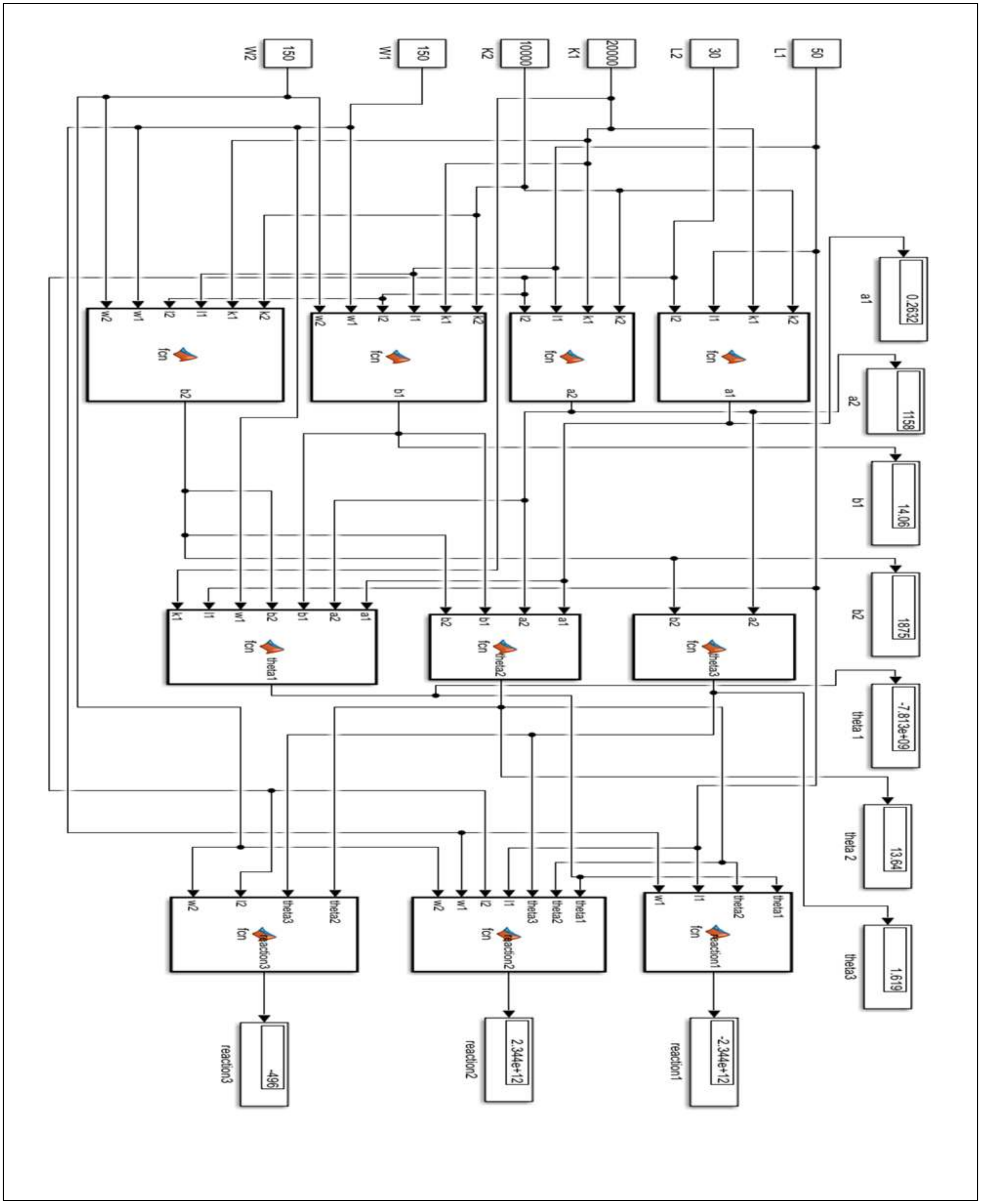
theta 2
-1.875
theta3
5.156
reaction3
1659
reaction2
6e+10
reaction1
-6e+10



L1
50
L2
30
K1
20000
K2
10000
W1
150
W2
150
a1
0.2632
a2
1158
b1
14.06
b2
1875
theta 1
-7.813e+09
theta 2
13.64
theta3
1.619
fcn
reaction1
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reaction2
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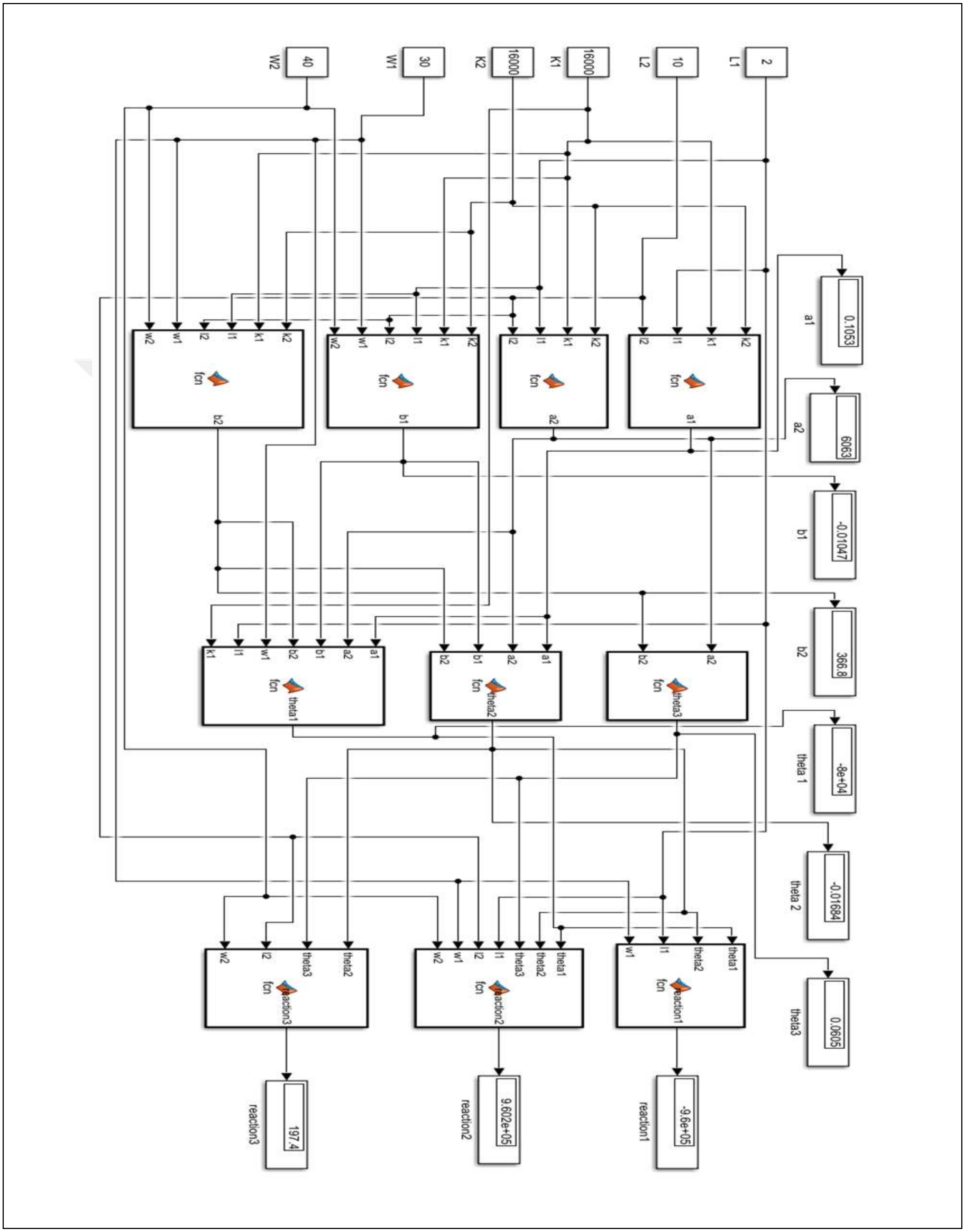
reaction3
-496



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W1 30
K2 16000
K1 16000
L2 10
L1 2
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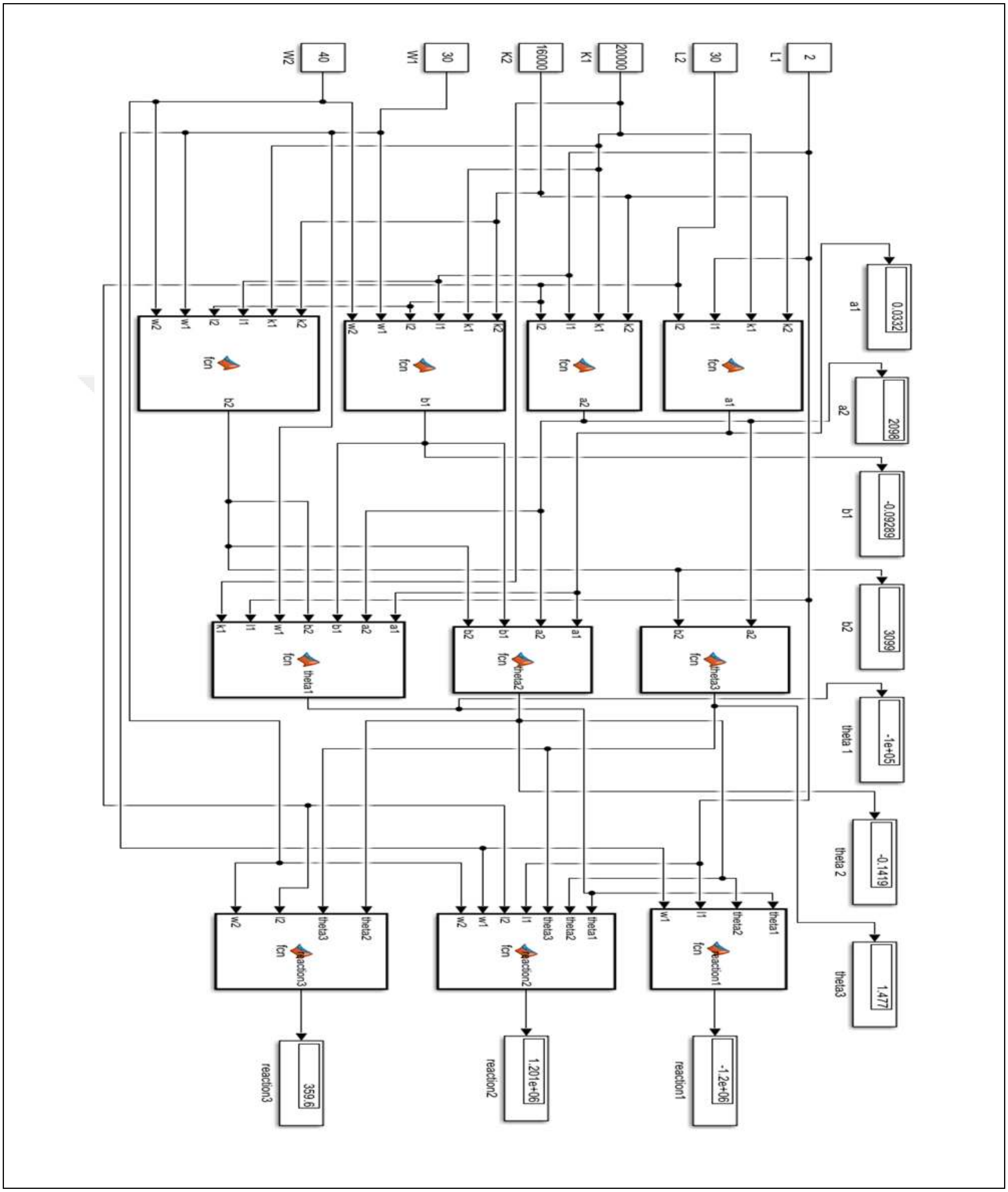
a2 6063
b1 -0.01047
b2 366.8
theta 1 -8e+04
theta 2 -0.01684
theta3 0.0605
reaction1 -9.6e+05
reaction2 9.602e+05
reaction3 197.4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2
L2
30
K1
20000
K2
16000
W1
30
W2
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a1
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a2
2098
b1
-0.09289
b2
3099
theta 1
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theta 2
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theta3
1.477
reaction1
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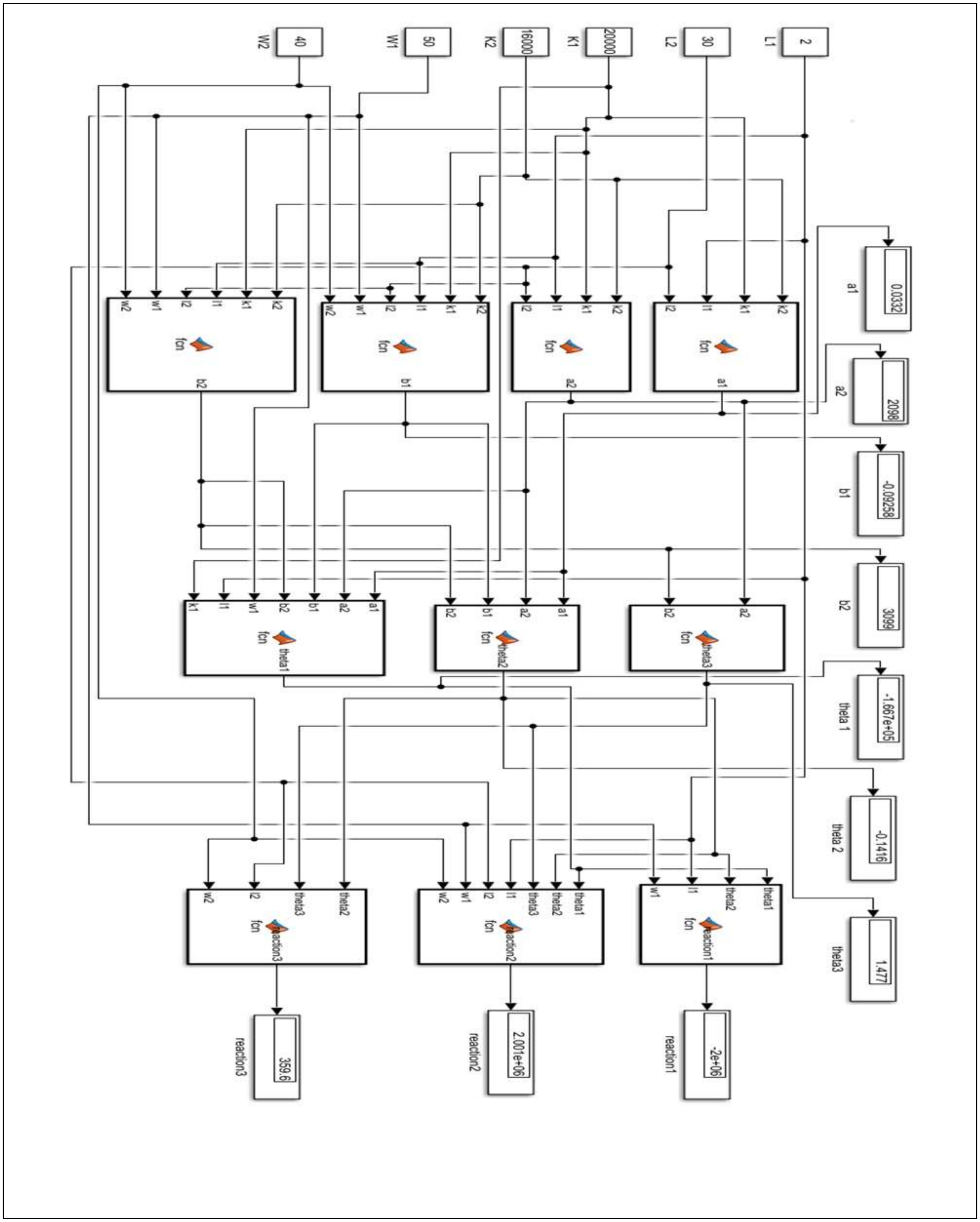
reaction2
1.201e+06
reaction3
359.6



40
W2
50
W1
16000
K2
20000
K1
30
L2
2
L1
fcn
b2
b1
a2
a1
theta1
theta2
theta3
reaction1
reaction2
reaction3
0.0332
2098
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3099
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theta 1
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theta 2
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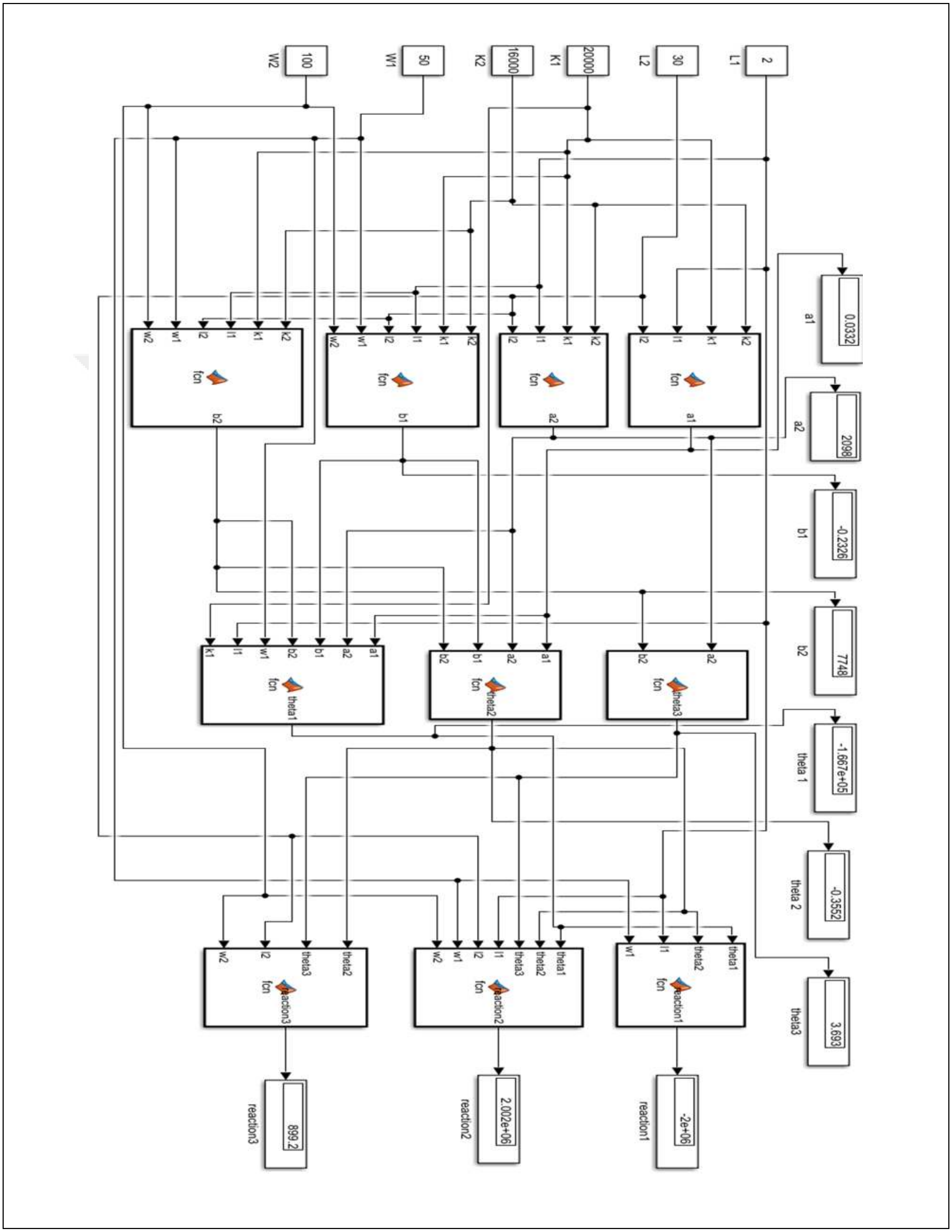
-2e+06
359.6



2
L1
30
L2
20000
K1
16000
K2
50
W1
100
W2
fcn
0.0332
a1
2098
a2
-0.2326
b1
7748
b2
-1.667e+05
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theta 2
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theta3
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reaction1
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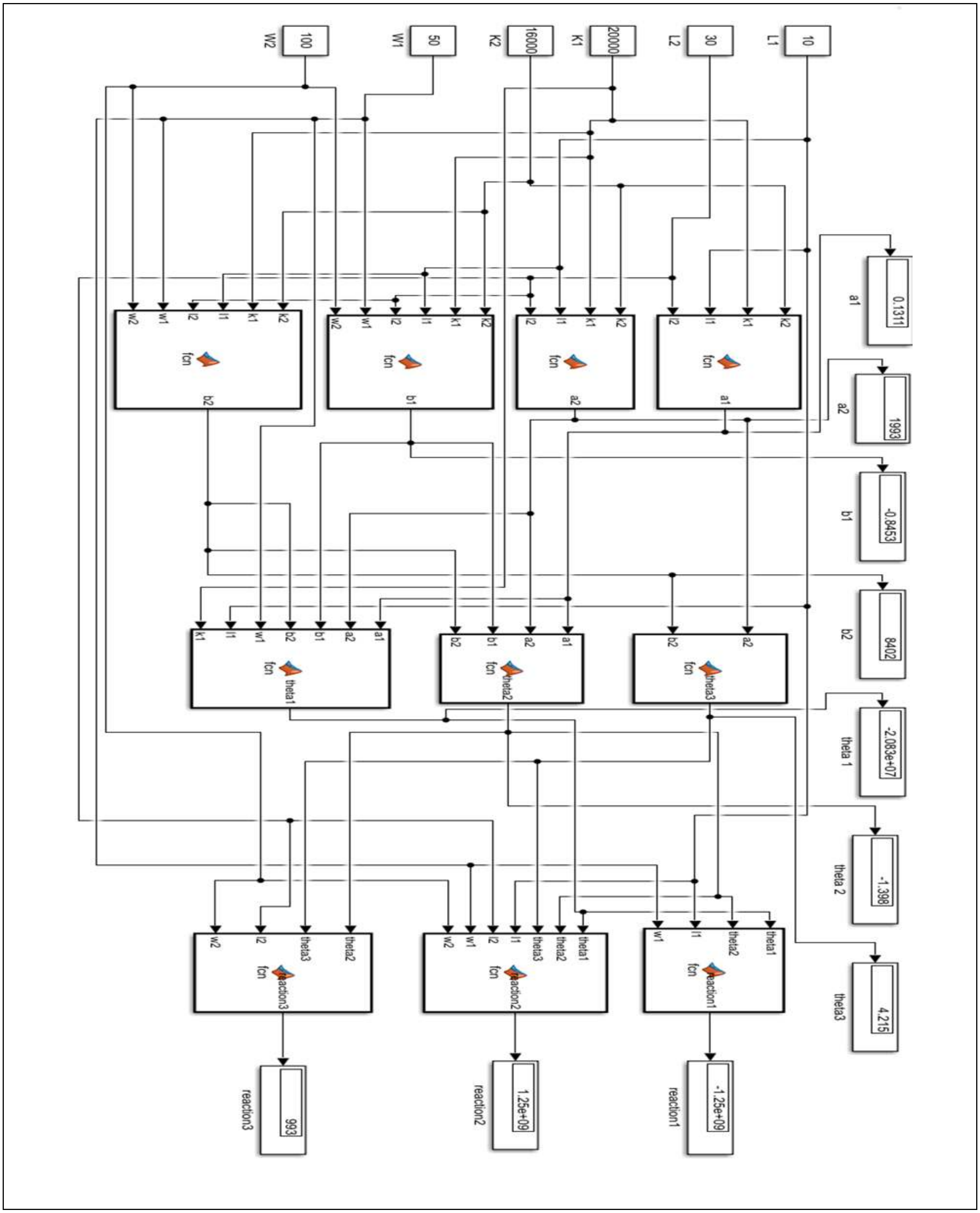
reaction2
899.2
reaction3



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W1
50
K2
16000
K1
20000
L2
30
L1
10
a1
0.1311
a2
1993
b1
-0.8453
b2
8402
theta 1
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theta 2
-1.398
theta3
4.215
reaction3
993
reaction2
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reaction1
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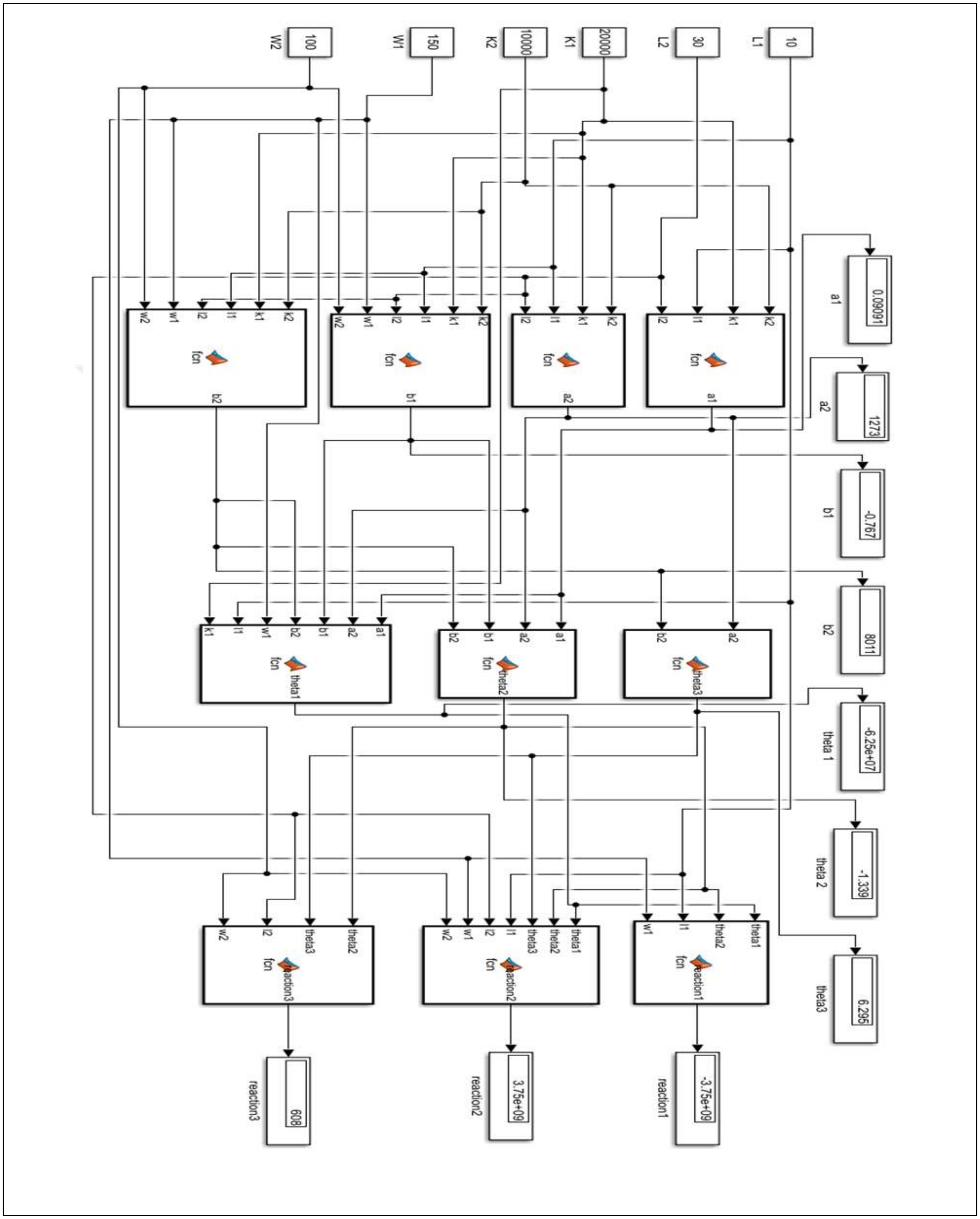

L1
10
L2
30
K1
20000
K2
10000
W1
150
W2
100
fcn
a1
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a2
1273
b1
-0.767
b2
8011
theta 1
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theta 2
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theta3
6.295
reaction1
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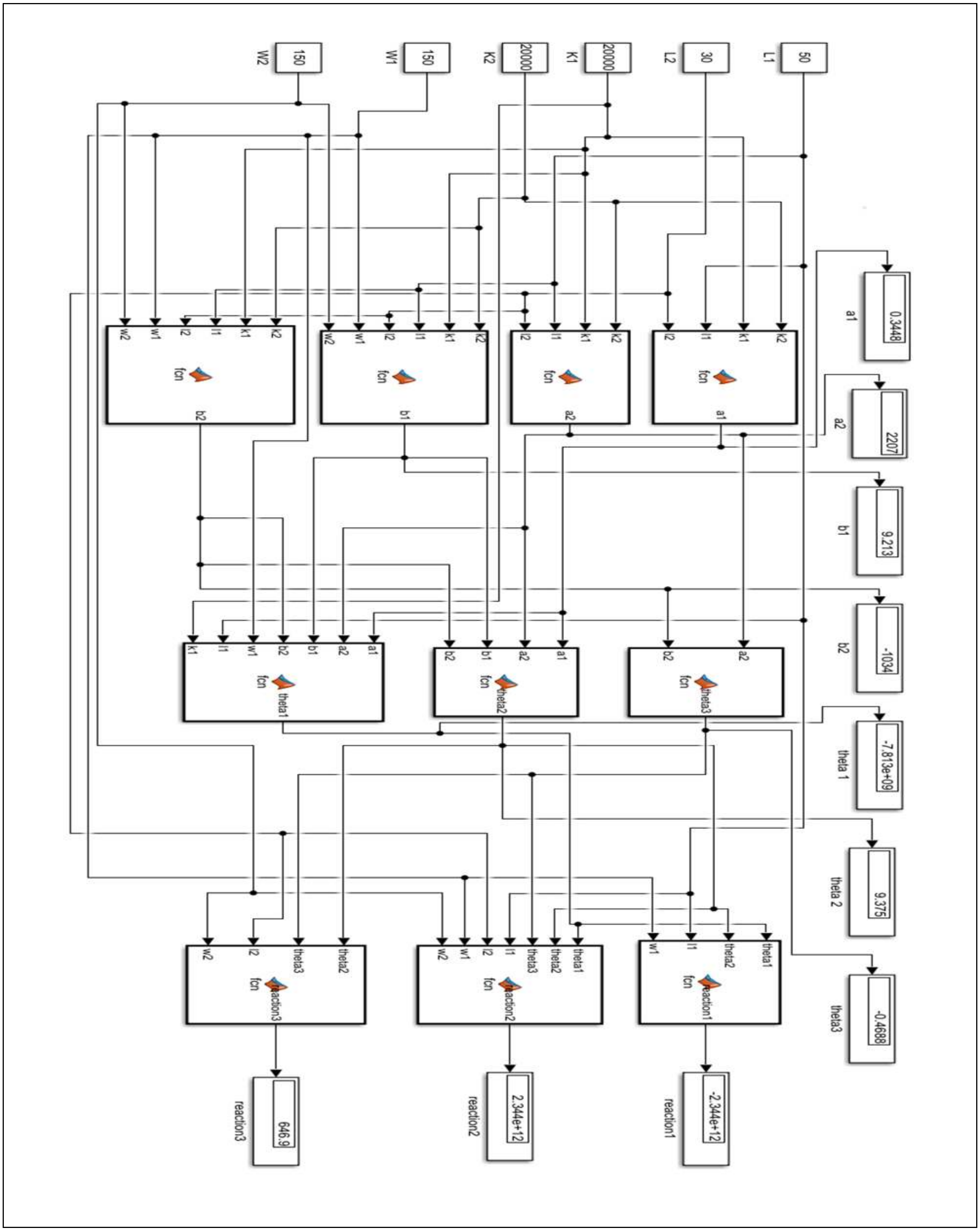
reaction2
3.75e+09
reaction3
608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L2
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K1
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K2
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W1
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W2
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a2
9.213
b1
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b2
-7.813e+09
theta 1
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theta 2
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theta3
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reaction1
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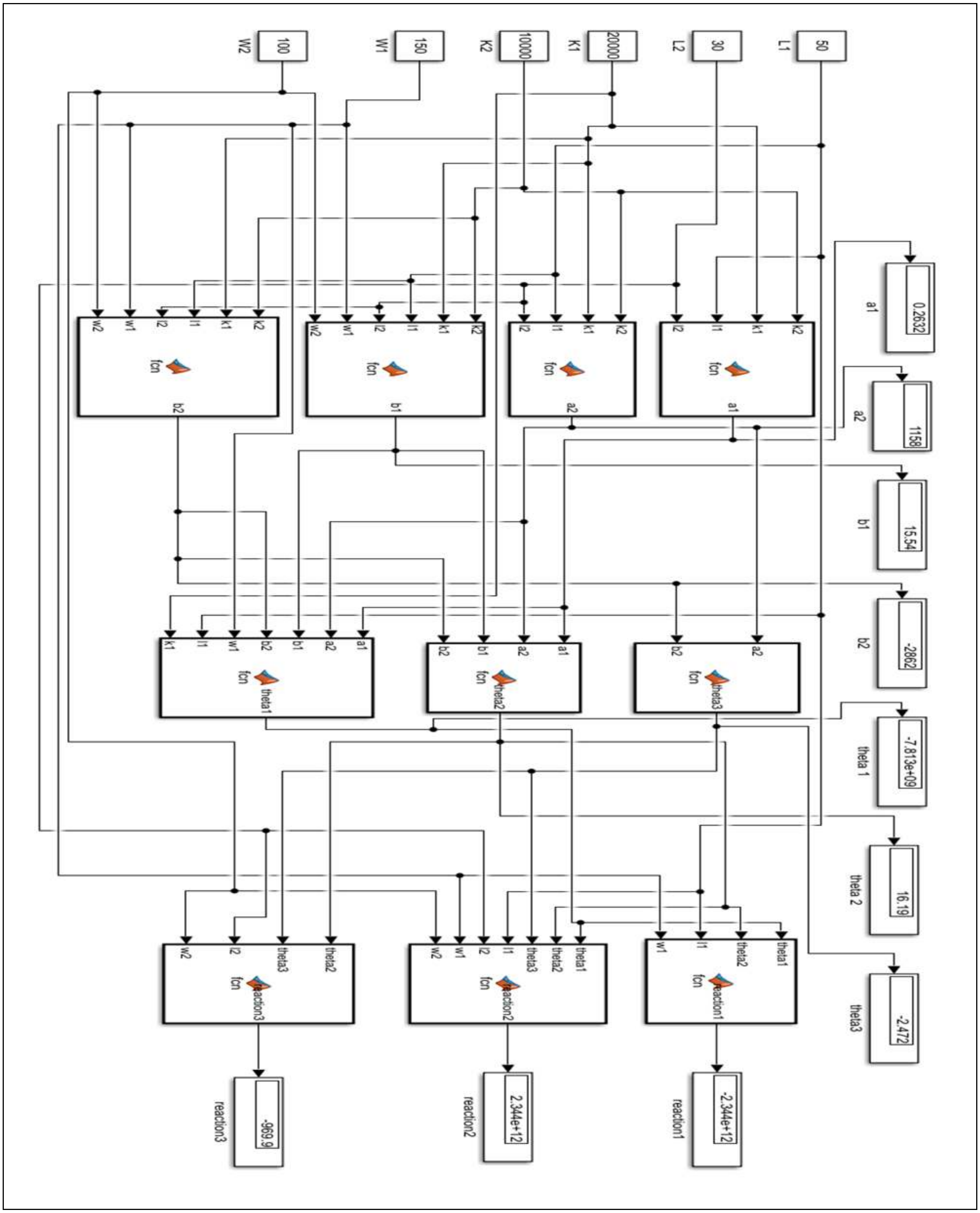
reaction2
646.9
reaction3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L1
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10000
K2
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W2
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a1
1158
a2
15.54
b1
-2862
b2
-7.813e+09
theta 1
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theta 2
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theta3
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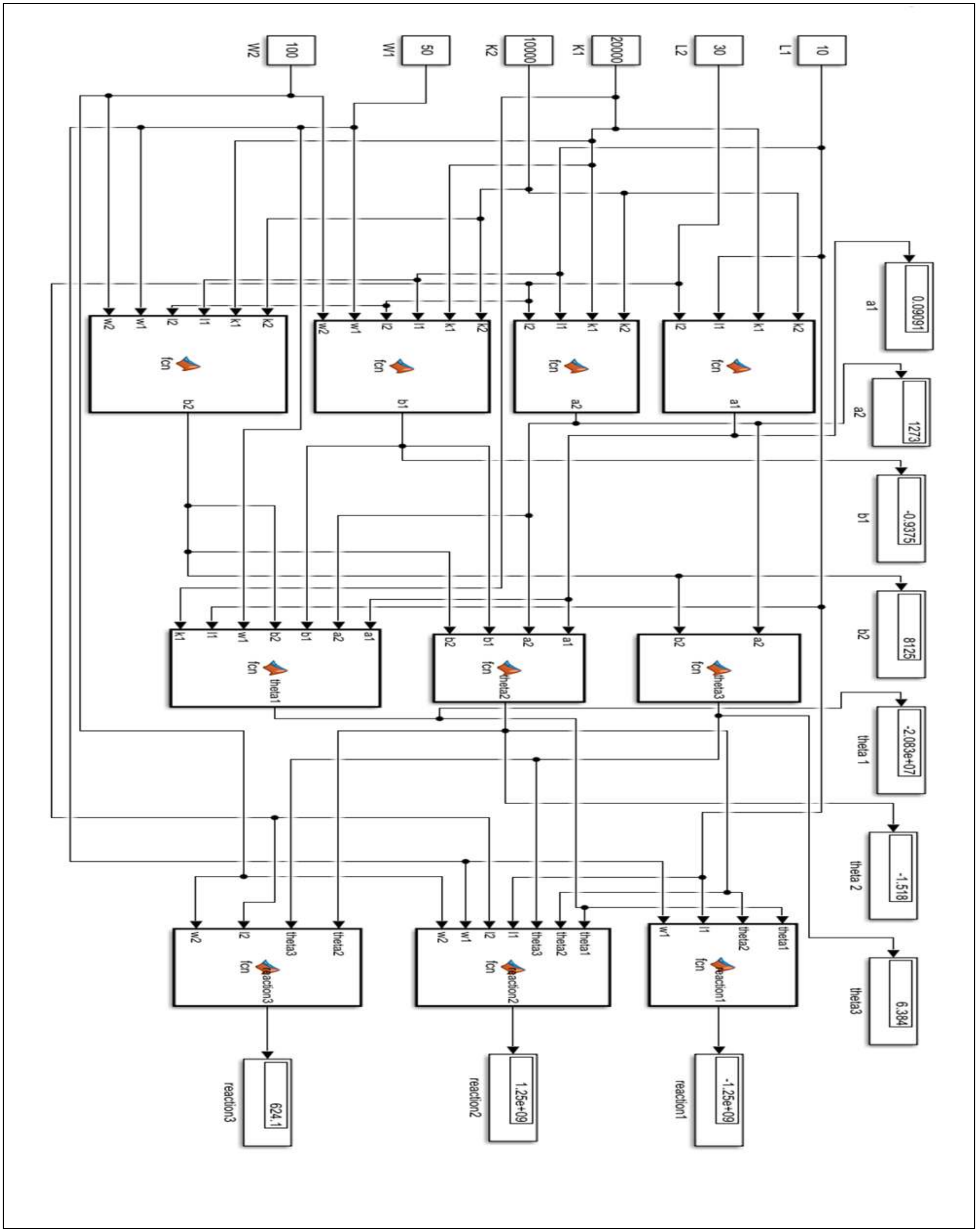
reaction3



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L2
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K2
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W2
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fcn
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a2
1273
b1
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b2
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theta 1
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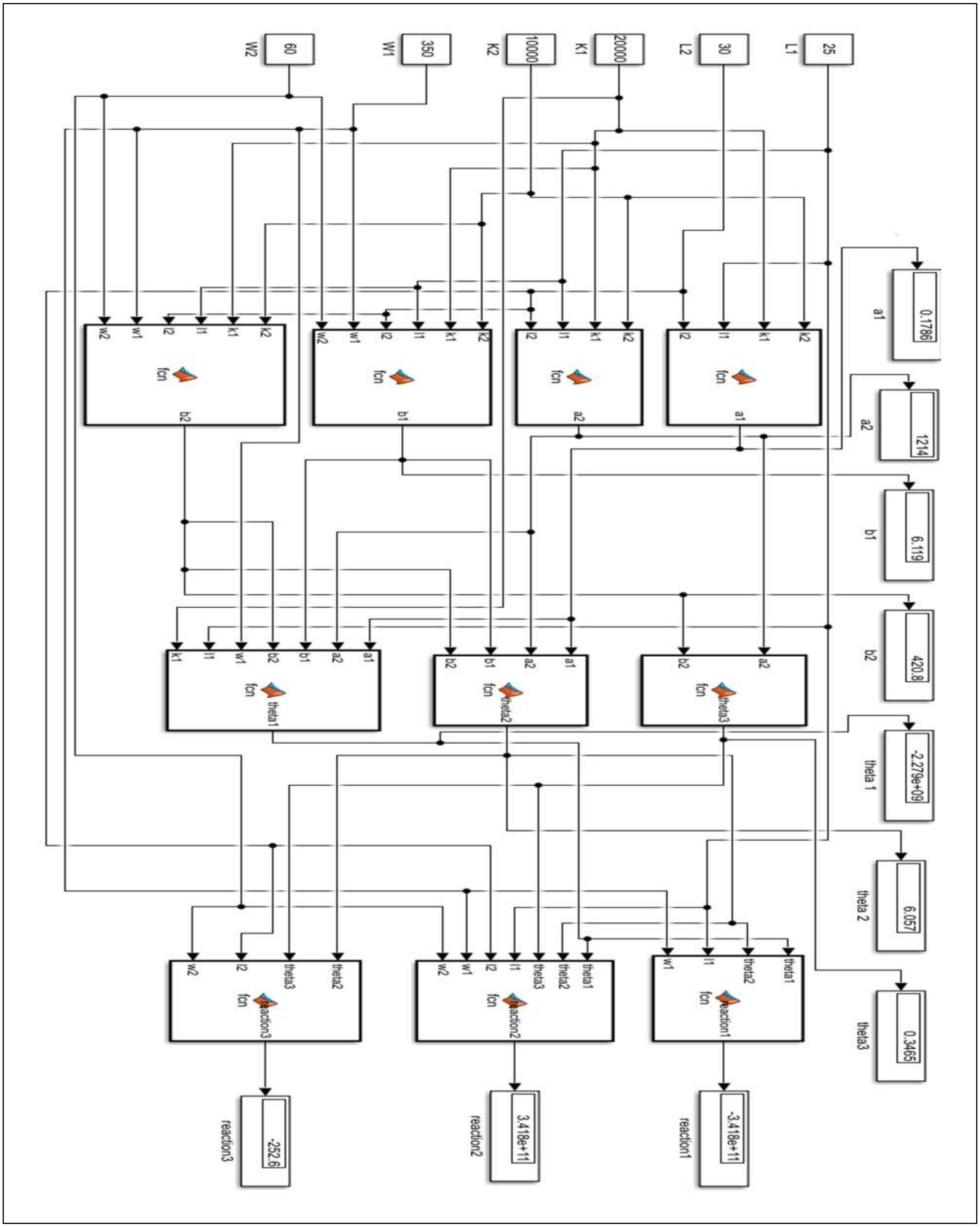
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b2
theta1
theta2
theta3
reaction1
reaction2
reaction3
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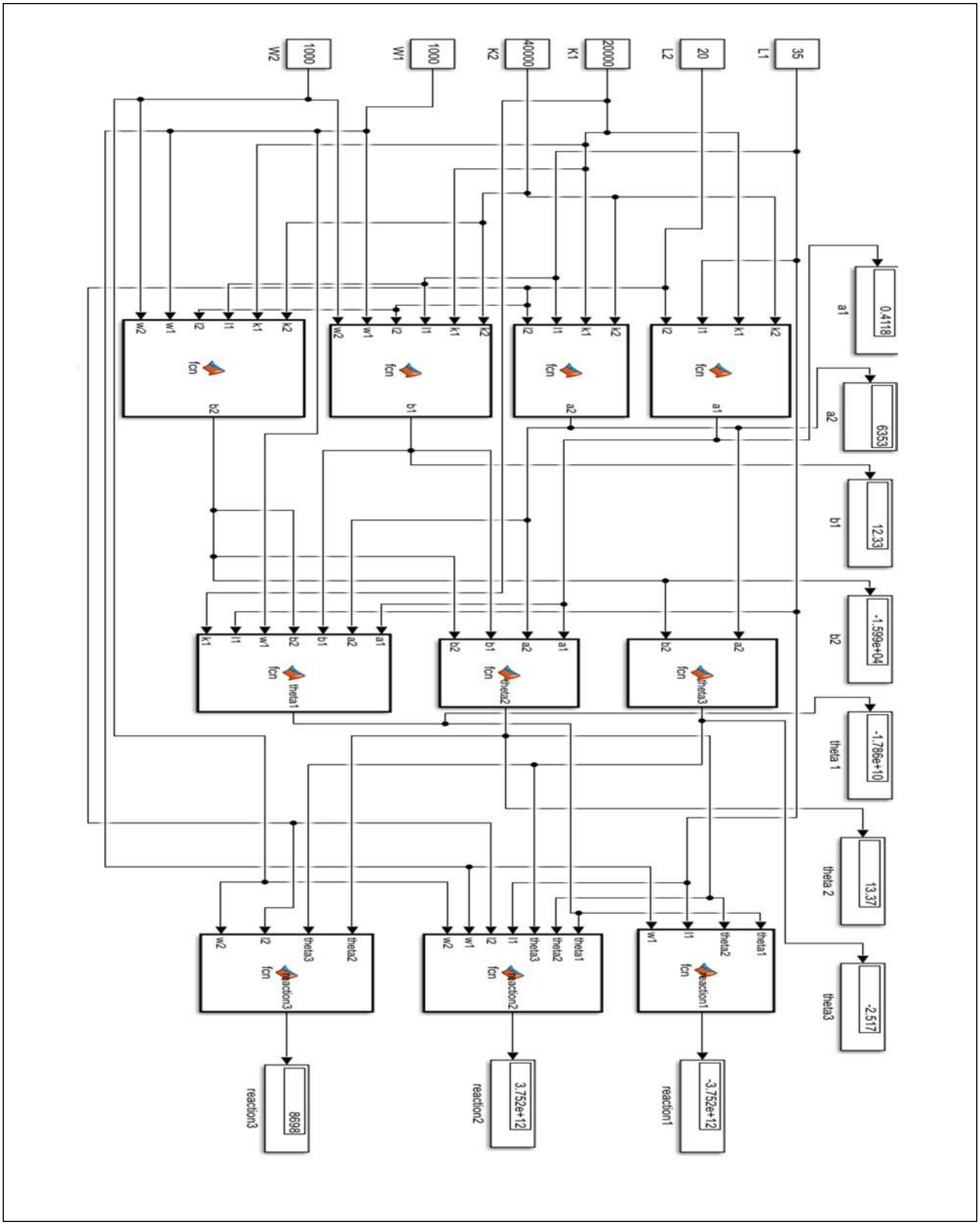
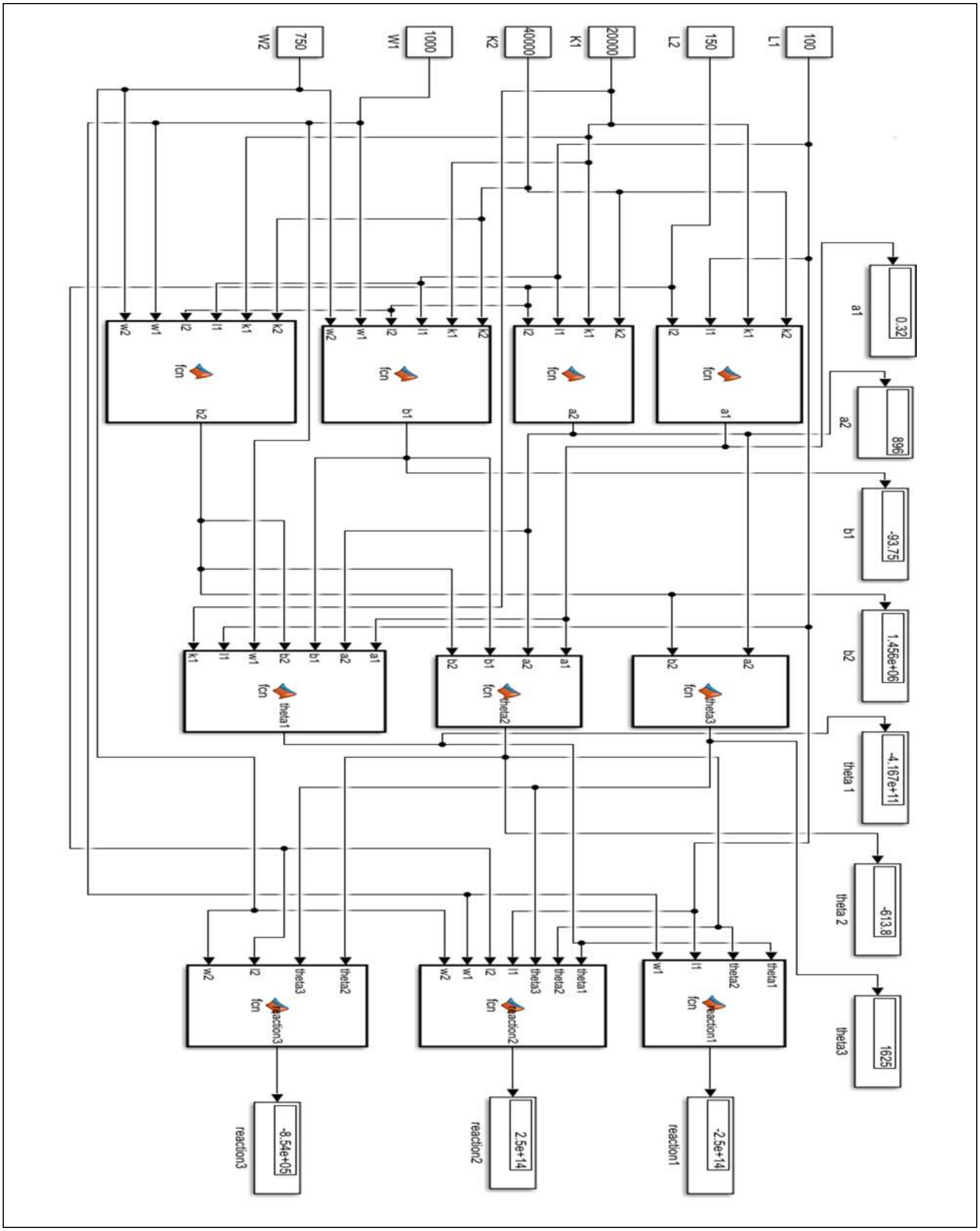
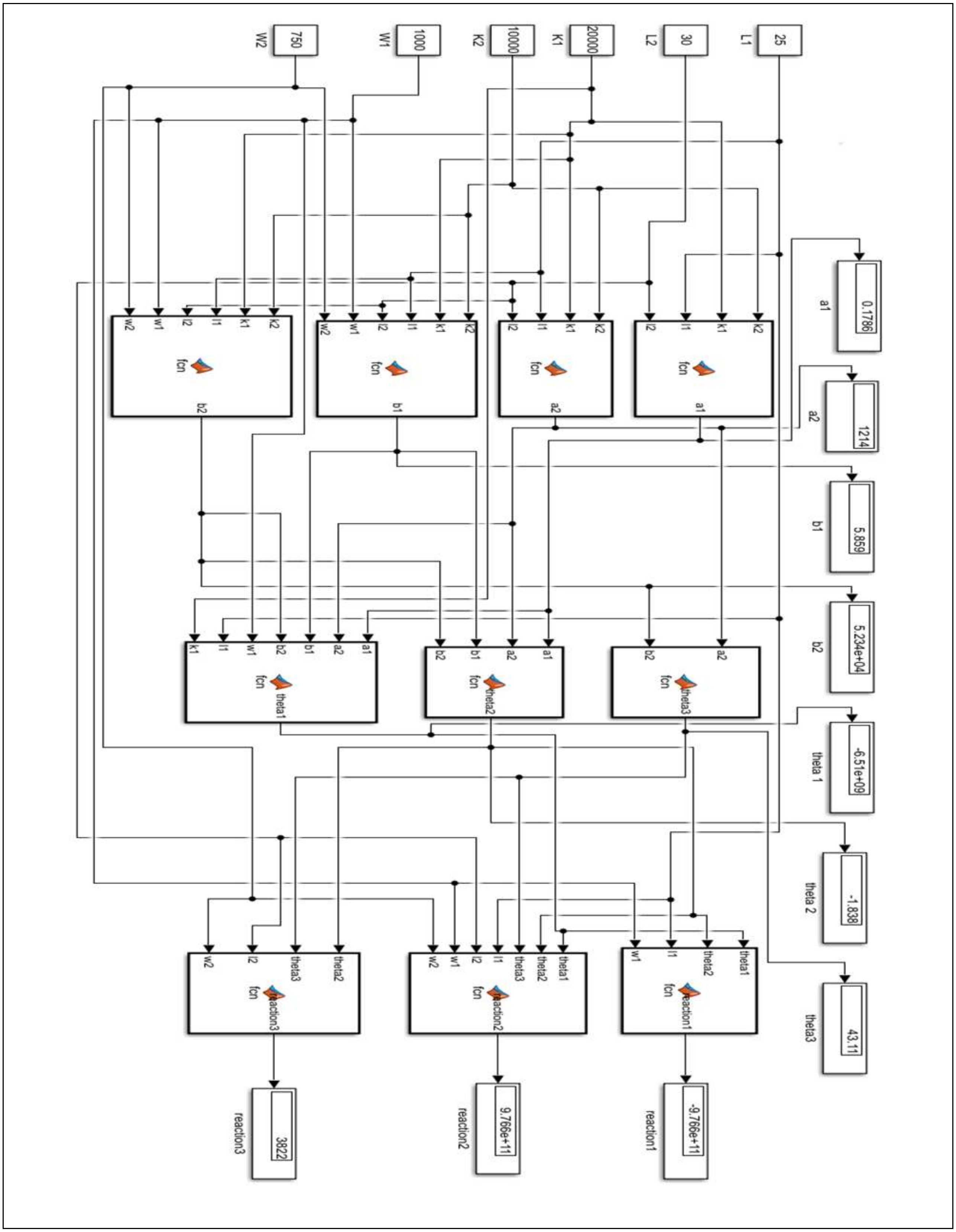
a2
b1
b2
theta 1
theta 2
theta3
reaction1
reaction2
reaction3



W2
1000
W1
1000
K2
40000
K1
20000
L2
20
L1
35
a1
0.4118
a2
6353
b1
12.33
b2
-1.599e+04
theta 1
-1.786e+10
theta 2
13.37
theta3
-2.517
reaction3
8698
reaction2
3.752e+12
reaction1
-3.752e+12


100
L1
150
L2
20000
K1
40000
K2
1000
W1
750
W2
fcn
0.32
a1
896
a2
-93.75
b1
1.456e+06
b2
-4.167e+11
theta 1
-613.8
theta 2
1625
theta3
-2.5e+14
reaction1
2.5e+14
reaction2
-8.54e+05
reaction3



25
L1
30
L2
20000
K1
10000
K2
1000
W1
750
W2
fcn
a1
a2
b1
b2
theta1
theta2
theta3
reaction1
reaction2
reaction3
0.1786
1214
5.859
5.234e+04
-6.51e+09
-1.838
43.11
-9.766e+11
9.766e+11
3822