

DEVELOPMENT AND IMPLEMENTATION OF SBR TECHNIQUE FOR SCATTERING AND RCS PROBLEMS

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By
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FOR SCATTERING AND RCS PROBLEMS

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We certify that we have read this thesis and that in our opinion it is fully adequate,
in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Full wave solution techniques are preferred to calculate the electromagnetic characteristic of an object. These methods give very accurate results. However, when the object is electrically large regarding wavelength, computation time may take very long, or required computer resources may not be provided. In this case, high-frequency approximation techniques are employed to solve electromagnetic scattering problems involving electrically large objects. Shooting and Bouncing Ray (SBR) method is a high-frequency technique that combines Geometric Optics (GO) and Physical Optics (PO). It can be used for the solution of electrically large objects. Multiple reflection feature of SBR makes it possible to calculate current distribution accurately on the target for the complex objects.

In this study, an SBR code is developed in MATLAB[®] which solves different kinds of problems that involve electrically large objects. Required formulas are derived and implementation procedure of the code is discussed. Radar Cross Section (RCS) of some basic shapes is calculated using SBR. An RCS reduction technique is explained and implemented. As the antenna-platform interaction, different kind of antennas are placed on a tank-like and a ship-like object to observe the effect of a scatterer on antenna pattern. An antenna coupling formula is derived. This formula is used to calculate coupling between two antennas in different environments. Results show fairly good agreement with Method of Moment (MoM) solutions.

Keywords: Shooting and Bouncing Ray, Physical Optics, Radar Cross Section, High Frequency Techniques, Antenna Coupling.

ÖZET

SAÇILMA VE RKA PROBLEMLERİ İÇİN SIY TEKNİĞİNİN GELİŞTİRİLMESİ VE UYGULANMASI

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Tam dalga elektromanyetik çözüm teknikleri herhangi bir objenin elektromanyetik karakteristiğinin hesaplanmasında kullanılır. Bu teknikler oldukça doğru sonuçlar verir. Ancak, obje dalga boyu cinsinden elektriksel olarak büyük olduğunda hesap süresi uzun olabilir yada gerekli bilgisayar kaynakları sağlanamayabilir. Bu durumda, elektriksel olarak büyük problemlerin çözümünde yüksek frekans teknikleri kullanılır. Seken Işın Yöntemi (SIY), Geometrik Optik (GO) ve Fiziksel Optik (FO) yöntemlerini birleştiren bir yüksek frekans tekniğidir. Bu teknik, elektriksel olarak büyük problemlerin çözümünde kullanılabilir. SIY'nin çoklu yansıma özelliği, kompleks objeler üzerindeki akım dağılımının doğru bir şekilde hesaplanmasını mümkün kılmaktadır.

Bu çalışmada, elektriksel olarak büyük objeleri içeren farklı türde problemleri çözebilen bir SIY kodu MATLAB®'da geliştirilmiştir. Gerekli formüller çıkarılmış ve kodun uygulanma şekli anlatılmıştır. Bazı basit şekillerin Radar Kesit Alanı (RKA) SIY kullanılarak hesaplanmıştır. Bir RKA düşürme tekniği anlatılmış ve uygulanmıştır. Anten-platform etkileşimi olarak, bir saçıcının anten örüntüsü üzerine etkisini gözlemlemek amacıyla tanka benzer ve gemiye benzer objelerin üzerlerine farklı türden antenler yerleştirilmiştir. Bir anten etkileşim formülü çıkarılmıştır. Bu formül iki anten arasındaki etkileşimin farklı ortamlarda hesaplanması için kullanılmıştır. Sonuçlar, Momentler Metodu kullanılarak elde edilen sonuçlarla oldukça uyum içindedir.

Anahtar sözcükler: Seken Işın Yöntemi, Fiziksel Optik, Radar Kesit Alanı, Yüksek Frekans Teknikleri, Anten etkileşimi.

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Chapter 1

Introduction

This chapter provides the objective of this thesis and the general background knowledge about Shooting and Bouncing Ray (SBR) method. Current distribution on an electrically small object can be determined using full wave electromagnetic methods such as Method of Moments (MoM). However, once the object becomes electrically large, full wave solutions require an enormous amount of computation time and resources. Therefore, it becomes impractical to solve electrically large problems with full wave solution methods.

Approximate solution methods (High-Frequency Techniques) such as Physical Optics (PO), Geometric Optics (GO) and Uniform Theory of Diffraction (UTD) are employed to solve the electrically large problems. One of these methods is Shooting and Bouncing Ray (SBR) method [1]. GO propagation properties such as reflection, refraction, and divergence are utilized to find the path of rays in this method. PO surface currents are found through Equivalence Principle. These currents are used for near or far field calculations. SBR method combines GO and PO techniques. It has multiple reflection feature compared to traditional PO. Even if SBR method is an approximation, it gives reasonably accurate results at high frequencies. It is also faster than full wave solution methods. Therefore, it can be used for the solution of the electrically large electromagnetic problems, if some error level is acceptable.

In SBR method, a dense grid of GO rays is shot onto the object. Ray tubes are formed by these rays. These ray tubes carry field information. If the source is a plane wave, the magnitude of these fields is same for every ray tube. If the source is an antenna, field magnitude at hit point depends on the antenna pattern and the distance between the antenna and the target. Once rays hit the object, they bounce according to Snell's Law. Depending on the material type of the target, reflection coefficients are calculated to determine the reflected field. Rays that are bounced from the hit points generate surface currents on the target. These currents are used for field calculations. Once there is no hit point for rays or field values attached to each ray tube becomes very small, computation is stopped.

There are many applications of SBR method. This technique is first developed for the RCS calculation of cavities. Researchers [2, 3, 4] try to improve SBR method for RCS calculation. It is well suited for RCS calculation as it predicts the RCS of complex objects efficiently. RCS calculations are critical for military applications. It is desirable to have low RCS values for some military applications. Therefore, RCS predictions should be carried out for revisions before the mass production.

SBR method is not limited to RCS calculation. Indoor propagation of electromagnetic waves can also be investigated using SBR method [5, 6, 7]. For indoor propagation, it is desired to find locations for transmitting antennas so that larger areas are covered in an indoor environment. With some trials and errors, the best place for the transmitting antenna can be found for the best coverage. Similar to indoor propagation, outdoor signal propagation can be investigated with SBR method [8, 9, 10]. As an example, placement of base stations in a city with many skyscrapers can be analyzed. This problem is similar to indoor propagation problem. However, the area is very large and the required computation time is much longer.

It is hard to calculate the effect of small features on electrically large objects with high-frequency techniques. They may be ignored in high-frequency method solution processes. Some researchers [11, 12] have combined SBR method with full wave solutions. Electrically small objects are calculated with full-wave methods,

and larger objects are calculated with SBR. Final field values are calculated by adding contributions of full wave and SBR techniques. This way, the accuracy of the solution is improved.

Since propagation of electromagnetic field is satisfied with rays in SBR method, it is required to find the intersection location of rays on the target object. Central Processing Unit (CPU) based calculations to determine the intersection point of rays are time-consuming. Instead, Graphic Processing Units (GPU) can be used to find the intersection points [13, 14, 15]. As ray tracing is a graphical problem, it is faster to employ GPU for ray tracing.

In this thesis, an SBR code is developed to solve RCS and scattering problems. This code can calculate RCS of complex targets, investigate the interaction between platforms and antennas, and compute the coupling between two or more antennas. For calculations requiring an antenna, far field patterns can be imported so that any antenna can be studied. Note that diffraction effect is ignored for simplicity. The computer used in calculation phase had AMD FX-8350 8-core processor with 8 GB RAM.

This thesis is organized as follows: In Chapter 2, SBR method is described, implementation of SBR is explained, and the required formulas are derived. In Chapter 3, RCS of some canonical shapes is calculated. Also, coating is applied to these shapes to reduce RCS. In Chapter 4, antenna-platform interaction is investigated. SBR method is used to find the best place to locate an antenna on a tank so that the pattern of antenna does not deteriorate too much. Naval radar analysis is also conducted in Chapter 4. The interaction between a rotating antenna and a ship is studied. In Chapter 5, the coupling between antenna to antenna is analyzed. First, a coupling formula is derived. Then, coupling between antennas is calculated in some basic environment. Coupling between two antennas is also computed in a corridor. Summary of the thesis is given in Chapter 6.

Chapter 2

Shooting and Bouncing Ray Method

This chapter describes Shooting and Bouncing Ray (SBR) method and its implementation. Section 2.1 describes the original method [1], which is used to calculate RCS of arbitrarily shaped cavities. In Section 2.2 it is shown that how SBR is implemented in a numerical computing environment. Required equations including ray tracing algorithm and reflection coefficient calculations are also derived in this section.

Even the original paper has investigated SBR for cavities, it can also be implemented for another type of problems including calculations of Radar Cross Section (RCS) of electrically large objects, the coupling between a source and a platform, and the coupling between multiple sources in a multiple object environments. Excitation can be a plane wave, an antenna or any other type of electromagnetic source. Objects under investigation can be Perfect Electric Conductor (PEC), Perfect Magnetic Conductor (PMC), or a dielectric layer which may be ended with PEC or free space.

2.1 Formulation of Shooting and Bouncing Ray

Shooting and Bouncing Ray (SBR) combines Geometric Optics (GO) and Physical Optics (PO) rules. First, a grid of dense rays coming from a source is "shot" onto a target. These ray grids are then traced with GO rules. Finally, Kirchhoff's approximation (PO) is used for exiting ray grids on an aperture to calculate the far field. The bouncing part can be divided into three phases:

1. Depending on the geometry, ray paths are traced. Snell's law is used to determine the bounced ray direction.
2. Electric field amplitude of the ray tubes is also traced. It involves field of bounced rays, free space path loss and phase shift.
3. Existing ray fields are integrated using Kirchhoff's approximation over an aperture to calculate the far field.

Note that there are differences between the method that is described from the original work and the method used in this thesis. Differences are explained in Section 2.2. These differences are in the second and the third phases.

In the beginning, a dense grid of rays is launched on the target. These ray tubes have four corners. Corners determine the shape of the ray tubes. During the tracing part, directions of rays for an individual ray tube may change. Some rays in this ray tube can also get more distance. Therefore, the initial rectangular shape of the ray tube may change. This case is shown in Figure 2.1. Note that the form of the ray tubes determines the area, which is used in Kirchhoff's approximation. To obtain a more accurate result, initial grid size should be as small as possible. Otherwise, the final area of the ray tube at the aperture would be large, and Kirchhoff's approximation may diverge from the correct result.

Each ray has a globally referenced initial point $\bar{S} (S_x, S_y, S_z)$ and a direction vector $\bar{d} (d_x, d_y, d_z)$. Each ray hits the target at a point $\bar{H} (H_x, H_y, H_z)$, which is

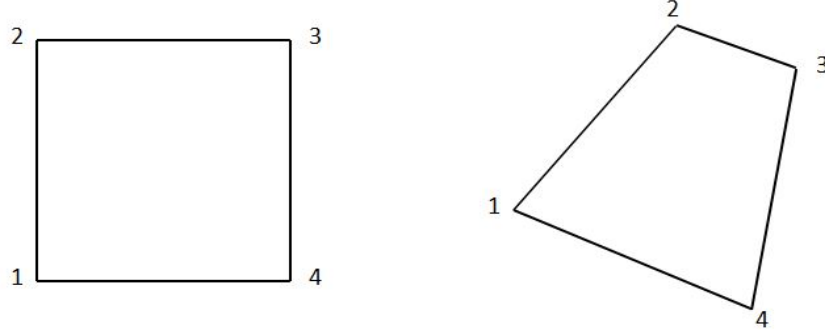


Figure 2.1: Change of the input and output shapes of a ray tube after reflections.

also referenced to global $(0, 0, 0)$. Travel of rays can be formulized as

$$(H_x, H_y, H_z) = (S_x, S_y, S_z) + (d_x, d_y, d_z)t, \quad (2.1)$$

where t is a parameter and should be positive.

After finding the path of each ray, the field amplitude should be determined. Amplitude tracking can be divided into two sections: Tracking at hitting point, and tracking at free space. At the hit point, the divergence factor (DF) and the reflection coefficients (Γ) should be calculated. Detailed derivation of divergence factor can be found in the literature [16]. In this thesis, planar surfaces are considered. For planar surfaces and plane wave sources, DF value is 1. Detailed calculation of Γ can be found in Section 2.2. In free space, there is also a phase term, which is $\theta = \beta d$. Here, β is the phase constant, and d is the distance between i th hit point and the next hit point.

Figure 2.2 shows a bouncing ray. Letting the incident E-Field $\bar{E}_i^i$, the reflected E-Field $\bar{E}_i^r$, the divergence factor DF_i , and the reflection coefficient matrix $\bar{\bar{\Gamma}}_i$ at i th hit point, following equations are used for amplitude tracking:

$$\bar{E}_i^r = DF_i \cdot \bar{\bar{\Gamma}}_i \cdot \bar{E}_i^i. \quad (2.2)$$

$$\bar{E}_{i+1}^i = \bar{E}_i^r \cdot e^{-j\theta}. \quad (2.3)$$

For the Equation 2.3, it is assumed that the source is a plane wave. If the excitation is a near field source, such as an antenna, then there will be an attenuation

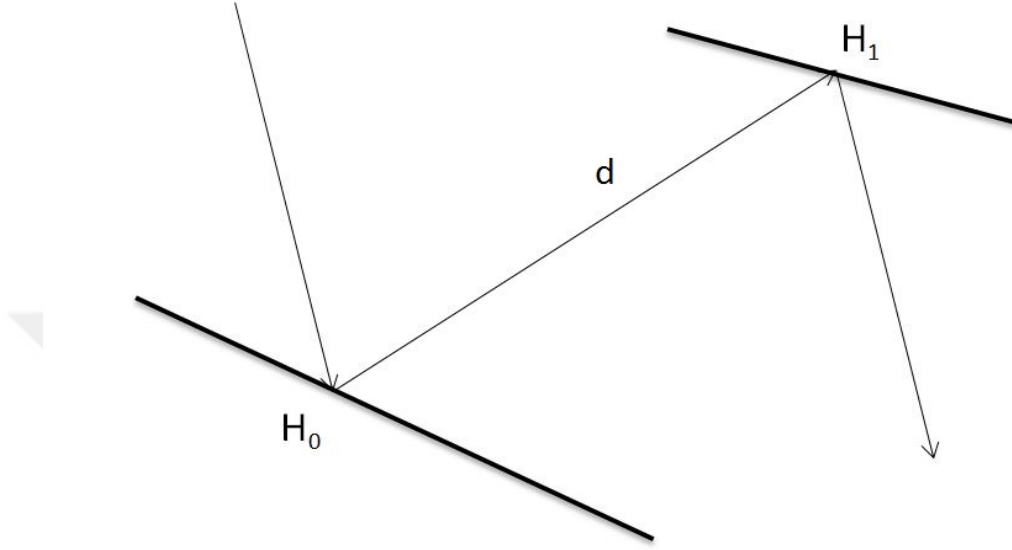


Figure 2.2: Bouncing of a ray in a multiple reflection environment.

term which is proportional to $\frac{r-d}{r}$, where r is the total traveled distance of a ray from the source point to the hit point H_{i+1} and d is the distance between H_i and H_{i+1} . In this case, Equation 2.3 should be modified as

$$\bar{E}_{i+1}^i = \bar{E}_i^r \cdot e^{-j\beta r} \cdot \frac{r-d}{r}, \quad (2.4)$$

where β is the phase constant and the last term is the divergence factor for planar surfaces if the excitation is a near field source. Finally, ray tubes hit the aperture. The field on the aperture is replaced by an equivalent magnetic current sheet $\bar{M}_s$:

$$\bar{M}_s = 2\bar{E} \times \hat{n}, \quad (2.5)$$

where $\hat{n}$ is the unit normal vector that points outward from the target region to free space, separated by the aperture. $\bar{M}_s$ currents are then used in far field calculations. Each ray tube contribution for the far field is then added up, and the final far field values are obtained. Notice that corners of ray tubes specify the area of the tube.

2.2 Implementation of Shooting and Bouncing Ray

This section describes the implementation of SBR method in a numerical computing environment. In this thesis, MATLAB[®] programming language is used. There are some implementation differences with the previous section. Those differences are explained when they are mentioned.

Implementation starts with meshing process. This is equivalent to generation of dense grid ray part of the previous section. Mesh type can be rectangular or triangular. FEKO[™] Software is used to create triangular meshes. Mesh size depends on the solution frequency and the type of the solution. Standard Physical Optics mesh size of $\frac{\lambda}{8}$ is used. Target is meshed using FEKO[™] software. The created meshes are used for the "shooting" part. Simply, created rays from the sources are forced to hit the corners of meshes. Also, another ray going towards the center of the mesh is generated for each mesh. This central ray is used to trace the field. Calculation of PO current is performed referencing the central ray location. Therefore, four rays are created and stored for each mesh: Three of them specify the area of the triangle, and the last one carries the field information. Ray tube generation is depicted in Figure 2.3.

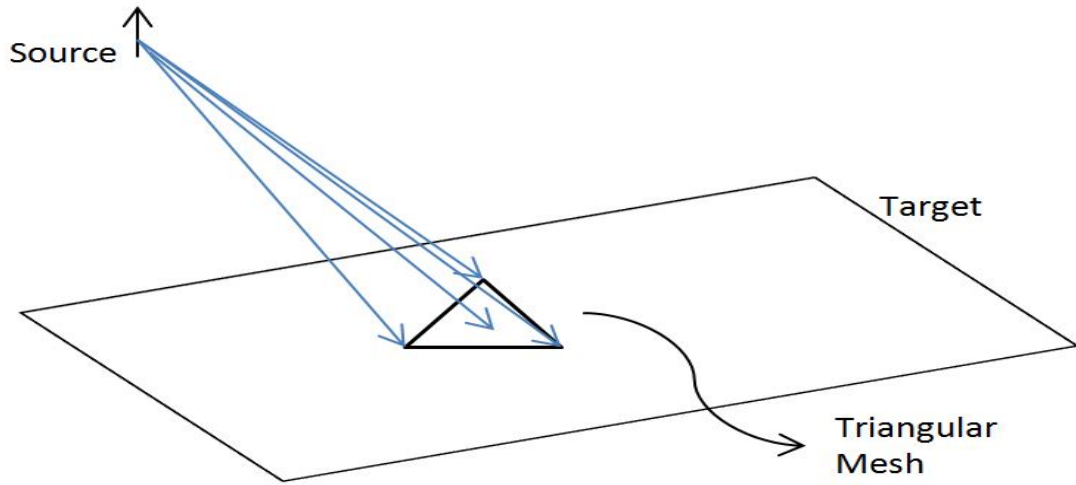


Figure 2.3: Generation of a ray tube using a triangular mesh.

Electromagnetic sources can be a plane wave, or an antenna. Field expression of simple radiation sources, such as a Hertzian dipole, can easily be calculated in MATLAB[®]. However, there are some antenna types, which can be hard to calculate their near or far field in MATLAB[®], for example horn antennas. As a solution, simulation of such sources are first conducted in FEKOTM and their far field data are exported from FEKOTM. Data involves the direction, directivity, E-Field magnitude, and phase. Data set includes θ and ϕ polarization information separately. Note that, this process assumes that the source antenna has a single phase center.

Exported far field data contains E_θ and E_ϕ values for each (θ, ϕ) direction with respect to the source. Pointing vector from the source to the hit point gives the angular direction of (θ, ϕ) in the spherical coordinate system. If the direction (θ, ϕ) is not in the exported data, simple linear interpolation can be conducted to find the field values. Exported data does not include $\frac{e^{-j\beta r}}{r}$ dependency. Final field value at hit point can be calculated from

$$\bar{E}^i = (\hat{\theta}E_\theta + \hat{\phi}E_\phi) \cdot \frac{e^{-j\beta r}}{r}, \quad (2.6)$$

where E_θ and E_ϕ are far field list provided by FEKOTM. Components of $\bar{E}^i$ is in the spherical coordinate. They can be transformed into the cartesian coordinate system using simple transformation formula given by

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \sin\theta \cos\phi & \cos\theta \cos\phi & -\sin\phi \\ \sin\theta \sin\phi & \cos\theta \sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} E_r \\ E_\theta \\ E_\phi \end{bmatrix}. \quad (2.7)$$

Excitation type can also be a plane wave. Linearly polarized plane wave can be expressed by

$$\bar{E}^i = \hat{e}E_0e^{-j\hat{k}\vec{r}}, \quad (2.8)$$

or in cartesian coordinates

$$\bar{E}^i = \hat{e}E_0e^{-j(k_x x + k_y y + k_z z)}, \quad (2.9)$$

where $\hat{e}$ and E_0 are the directions and the magnitude of E-Field, $\hat{k}$ is the propagation vector of the plane wave, and $\vec{r}$ is the vector from global (0,0,0) to the hit point (x, y, z) .

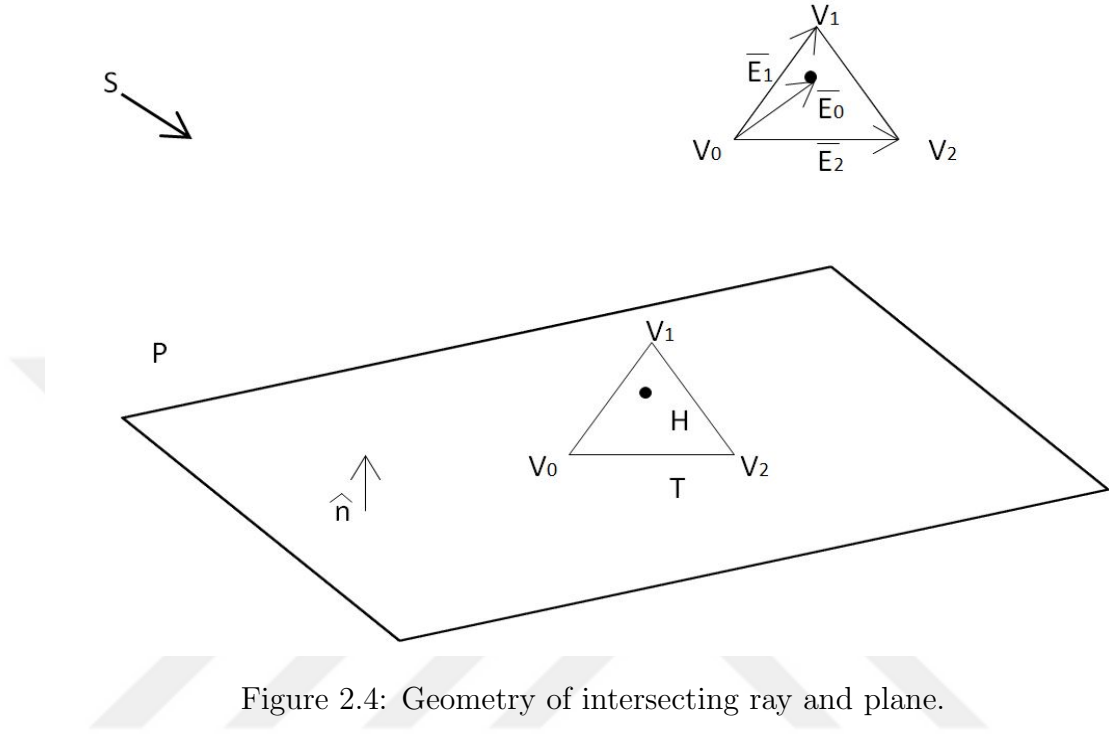


Figure 2.4: Geometry of intersecting ray and plane.

After shooting ray tubes from the source, some rays may not reach the intended destination. This is because of the other objects between the source and the intended location. To handle this shadowing problem, ray tubes that are shot into shadowing region should be removed from the list. For bouncing part, the next destination of the rays should also be determined. Therefore, a ray tracing algorithm is needed.

There are many ray tracing algorithms in the literature [17, 18, 19]. However, a simple mathematical ray tracing algorithm is used in this thesis as this algorithm saves more computation time. Ray tracing algorithm employed in this thesis requires less mathematical operations as more terms do not have to be recalculated compared to other algorithms. These terms are pre-calculated, stored and reused, which leads to improvement of computation time.

Referring to Figure 2.4 triangle T is on plane P with a normal vector $\hat{n}$. Vertices of the triangle are labeled as V_0 , V_1 , and V_2 . Two vectors are also defined for the triangle T . $\bar{E}_1$ is the vector from V_0 to V_1 , and $\bar{E}_2$ is the vector from V_0 to V_2 . A ray having initial point of S and direction of $\hat{d}$ hits the plane at

point H . Another vector is also defined for the hit point as $\bar{E}_0$, which is a vector from V_0 to T .

Ray tracing can be decomposed into two sections. First of all, the distance between the source and the plane of the mesh is calculated. Following equation shows the relation between the initial point and the hit point:

$$(V_{0x}, V_{0y}, V_{0z}) + \bar{E}_0 = (S_x, S_y, S_z) + \hat{d} t. \quad (2.10)$$

Taking dot product of all side of Equation 2.10 with the normal vector of plane P gives

$$\hat{n} \cdot (V_{0x}, V_{0y}, V_{0z}) + \hat{n} \cdot \bar{E}_0 = \hat{n} \cdot (S_x, S_y, S_z) + \hat{n} \cdot \hat{d} t. \quad (2.11)$$

Note that dot product of $\bar{E}_0$ and $\hat{n}$ is zero. Arranging terms to calculate the distance gives

$$t = \frac{\hat{n} \cdot ((V_{0x}, V_{0y}, V_{0z}) - (S_x, S_y, S_z))}{\hat{n} \cdot \hat{d}}. \quad (2.12)$$

Notice that the sign of $\hat{n}$ does not change the result. If the distance is negative or zero for a mesh, that mesh will be removed from the calculation, and the remaining meshes will be investigated further. Notice also that H can be inside or outside of the triangle. After finding the distance, vector $\bar{E}_0$ can be calculated using (2.10) as

$$\bar{E}_0 = (S_x, S_y, S_z) + \hat{d} t - (V_{0x}, V_{0y}, V_{0z}). \quad (2.13)$$

The second part of ray tracing is to determine whether the hit point H is inside of the triangle T or not. The barycentric coordinate system is adopted at this point which has V_0 as its origin. Any point on plane P can be represented by the summation of $\bar{E}_1$ and $\bar{E}_2$ vectors. Therefore, $\bar{E}_0$ can be expressed as

$$\bar{E}_0 = u\bar{E}_1 + v\bar{E}_2, \quad (2.14)$$

where u and v are scalar quantities. It can be shown that [20] vector $\bar{E}_0$ ends in triangle T if following conditions are satisfied:

$$u \geq 0 \quad (2.15a)$$

$$v \geq 0 \quad (2.15b)$$

$$u + v \leq 1. \quad (2.15c)$$

Only (2.14) is available to find u and v values. However, two equations are needed to solve the linear system of equation with two unknowns. To obtain two equations, both sides of (2.14) are dot producted by $\bar{E}_1$ and $\bar{E}_2$:

$$\bar{E}_1 \cdot \bar{E}_0 = u \bar{E}_1 \cdot \bar{E}_1 + v \bar{E}_1 \cdot \bar{E}_2 \quad (2.16a)$$

$$\bar{E}_2 \cdot \bar{E}_0 = u \bar{E}_2 \cdot \bar{E}_1 + v \bar{E}_2 \cdot \bar{E}_2. \quad (2.16b)$$

Now, two equations are obtained. Dot product terms can be rewritten as

$$E_{ij} = \bar{E}_i \cdot \bar{E}_j. \quad (2.17)$$

Equation (2.16) can be expressed in matrix form as

$$\begin{bmatrix} E_{10} \\ E_{20} \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}, \quad (2.18)$$

then, (2.18) can be rearranged as

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{E_{11}E_{22} - E_{12}E_{21}} \begin{bmatrix} E_{22} & -E_{12} \\ -E_{21} & E_{11} \end{bmatrix} \begin{bmatrix} E_{10} \\ E_{20} \end{bmatrix}, \quad (2.19)$$

or simply

$$u = \frac{1}{E_{11}E_{22} - E_{12}E_{21}} (E_{22}E_{10} - E_{12}E_{20}) \quad (2.20a)$$

$$v = \frac{1}{E_{11}E_{22} - E_{12}E_{21}} (E_{11}E_{20} - E_{21}E_{10}). \quad (2.20b)$$

At this point, one might think that the denominator term may vanish. However, it cannot be zero if the angle θ between $\bar{E}_1$ and $\bar{E}_2$ cannot be 0 or 180 degrees as it is impossible for a triangle T

$$\begin{aligned} E_{11}E_{22} - E_{12}E_{21} &= |E_1|^2|E_2|^2 - |E_1||E_2|\cos\theta|E_2||E_1|\cos\theta \\ &= |E_1|^2|E_2|^2(1 - \cos^2\theta) \\ &> 0. \end{aligned} \quad (2.21)$$

If a mesh satisfies both (2.15) and $t > 0$ conditions, then ray intersects that mesh. If more than one mesh satisfies conditions, closer mesh is hit by the ray. The hit point is expressed from (2.10) as

$$(H_x, H_y, H_z) = (S_x, S_y, S_z) + \hat{d} t. \quad (2.22)$$

The advantage of this method is the precalculations of E_{11} , E_{21} , E_{22} and the fractional term in (2.20). These terms are geometry dependent and do not change from ray tube to ray tube. Their precalculations can speed up the procedure to save a great amount of computation time. If there is not any mesh satisfying above conditions, then ray does not hit the target and goes to free space.

After finding hit points of rays, some ray tubes should be discarded as they are not suitable for current calculations. There are three tests to be conducted to decide if a ray tube is to be removed or not. First of all, all rays of a ray tube should hit the target at some point. If at least one ray of a ray tube fails to hit the target and goes to free space, that ray tube is discarded. Remaining ray tubes are tested further. Rays of a ray tube should hit meshes such that normal vector of those hit meshes should be the same or approximately same with some error margin:

$$\hat{n}_i \cdot \hat{n}_j > 0.999, \quad (2.23)$$

where i and j are one of four rays of ray tube. This test is conducted between each ray of a ray tube. Rays may hit meshes having the same normal vector, but they may not be on the same plane. In that case, ray tube cannot form a filled triangle, which violates Kirchhoff's approximation. Therefore, another test should be carried out to check if rays of ray tubes are on the same plane. If a ray tube fails in any of these tests, it is failed and discarded from further calculations.

Direction of reflected rays from the hit points need to be calculated to search for the next hit points. Following equation gives the direction of the reflected ray:

$$\hat{d}^{ref} = \hat{d}^{inc} - 2\hat{n}(\hat{d}^{inc} \cdot \hat{n}). \quad (2.24)$$

Once a ray hits the target, its electrical reflection characteristics should also be investigated. Depending on the material that is hit, part of the field is reflected to the region where ray comes from, and some are transmitted into the target. To determine the incident field on the next hit point and the current created by field information carrying ray tube, reflection coefficients have to be calculated.

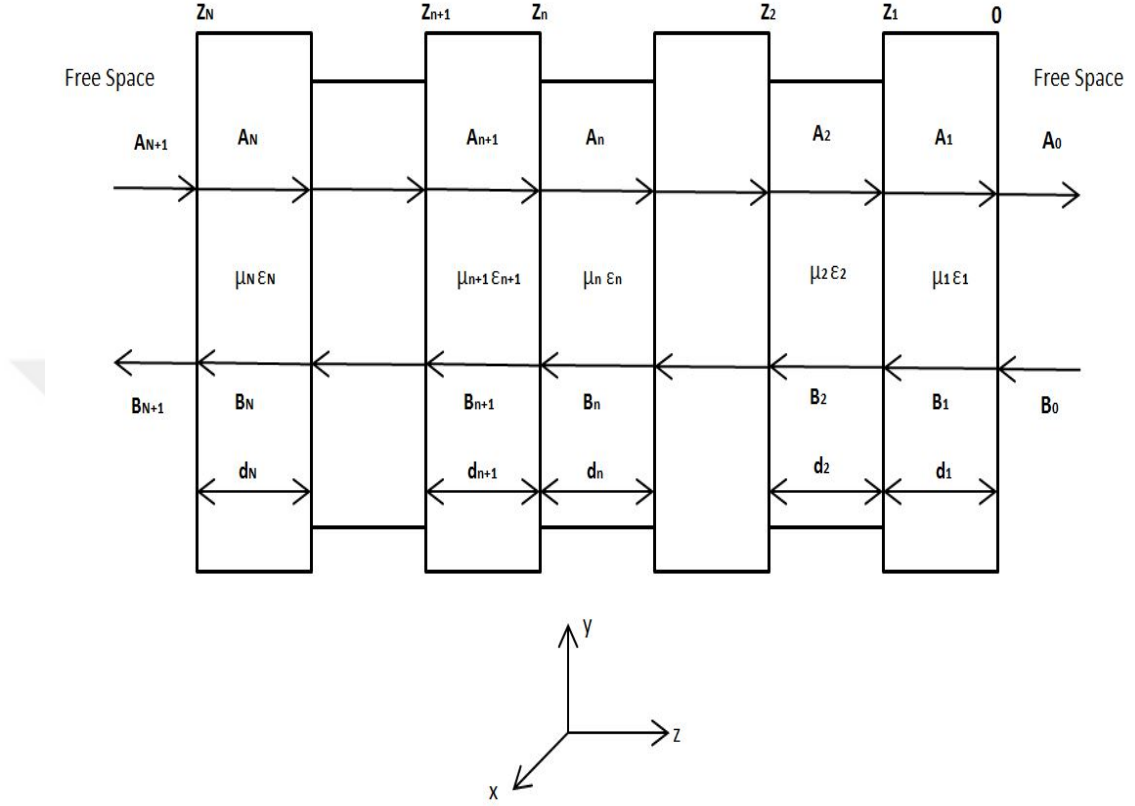


Figure 2.5: Ingoing and outgoing waves in a plane multilayer.

In order to calculate reflection and transmission coefficients, field expression must first be decomposed into transverse electric (TE) polarized and transverse magnetic (TM) polarization components using following unit vectors:

$$\hat{e}_{TE} = \frac{\hat{k}^{inc} \times \hat{n}}{|\hat{k}^{inc} \times \hat{n}|} \quad (2.25a)$$

$$\hat{e}_{TM} = \frac{\hat{e}_{TE} \times \hat{k}^{inc}}{|\hat{e}_{TE} \times \hat{k}^{inc}|}. \quad (2.25b)$$

Once unit vectors in (2.25) are dot producted with E-Field, components of each polarization can be found.

There are many reflection coefficient calculation formulas in the literature [1, 19, 21, 22]. Richmond has proposed a recursive method to calculate the reflection and transmission coefficient calculation for multilayered dielectrics [23]. This approach is adopted in this thesis.

Figure 2.5 represents a multilayer structure. It consists of N layers and ended with a free space. Each layer has its own permittivity ϵ_i , and permeability μ_i . The thickness of the layers is d_i . Assume that a TE polarized plane wave is coming from the left-hand surface. Incident field can be expressed as

$$\bar{E}^i = \hat{x} E_0 e^{jk_0 y \sin \theta} e^{jk_0 z \cos \theta}, \quad (2.26)$$

where k_0 is the free space propagation constant, θ is the angle of incidence. The reflected field is given by

$$\bar{E}^r = \hat{x} \Gamma E_0 e^{jk_0 y \sin \theta} e^{-jk_0 z \cos \theta}, \quad (2.27)$$

where Γ is the reflection coefficient. The transmitted field on the right hand side is given by

$$\bar{E}^t = \hat{x} \mathcal{T} E_0 e^{jk_0 y \sin \theta} e^{jk_0 z \cos \theta}, \quad (2.28)$$

where $\mathcal{T}$ is the transmission coefficient. In layer n , the field is represented by

$$E_n = (A_n e^{\gamma z} + B_n e^{-\gamma z}) e^{jk_0 y \sin \theta}. \quad (2.29)$$

Notice that A_n and B_n are magnitude of outgoing and ingoing waves in layer n . After enforcing boundary conditions between layers (n) and $(n+1)$ at $z = z_n$, one can obtain

$$A_{n+1} = P_n A_n + Q_n B_n \quad (2.30a)$$

$$B_{n+1} = R_n A_n + N_n B_n, \quad (2.30b)$$

where

$$P_n = 0.5 \left(1 + \frac{\mu_{n+1} \gamma_n}{\mu_n \gamma_{n+1}} \right) e^{(\gamma_n - \gamma_{n+1}) z_n} \quad (2.31a)$$

$$Q_n = 0.5 \left(1 - \frac{\mu_{n+1} \gamma_n}{\mu_n \gamma_{n+1}} \right) e^{(\gamma_{n+1} + \gamma_n) z_n} \quad (2.31b)$$

$$R_n = 0.5 \left(1 - \frac{\mu_{n+1} \gamma_n}{\mu_n \gamma_{n+1}} \right) e^{(\gamma_n + \gamma_{n+1}) z_n} \quad (2.31c)$$

$$S_n = 0.5 \left(1 + \frac{\mu_{n+1} \gamma_n}{\mu_n \gamma_{n+1}} \right) e^{(\gamma_{n+1} - \gamma_n) z_n}, \quad (2.31d)$$

where γ_n is the propagation constant in medium n , given by

$$\gamma = j \sqrt{\omega^2 \mu_n \epsilon_n - k_0^2 \sin^2 \theta}. \quad (2.32)$$

Recursive calculation of reflection coefficient starts by forcing the bottom layer field magnitude coefficients as

$$A_0 = 1 \quad (2.33a)$$

$$B_0 = 0, \quad (2.33b)$$

which simply states that there is not any impinging field from the right hand surface of Figure 2.5. However, there exists a wave from the bottom side with a magnitude of 1. After implying (2.33) and solving (2.30) give A_{N+1} and B_{N+1} . Finally, E_0 , Γ and $\mathcal{T}$ can be expressed as

$$E_0 = A_{N+1} \quad (2.34)$$

$$\Gamma = \frac{B_{N+1}}{A_{N+1}} e^{-2jk_0 d \cos \theta} \quad (2.35)$$

$$\mathcal{T} = \frac{1}{A_{N+1}}. \quad (2.36)$$

There is an exponential factor for Γ term. This factor should be added, because all calculations are carried out with respect to $z = 0$, but we need the reflection coefficient at $z = d$, where d is the total thickness of the multilayer structure.

Right hand surface could be another material, for example PEC. In that case, boundary conditions should be enforced accordingly. In the case of TE field

$$A_1 = 1 \quad (2.37a)$$

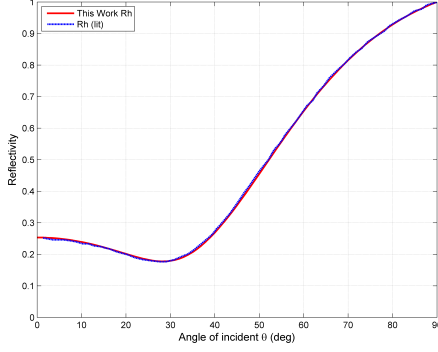
$$B_1 = -1, \quad (2.37b)$$

and in the case of TM field

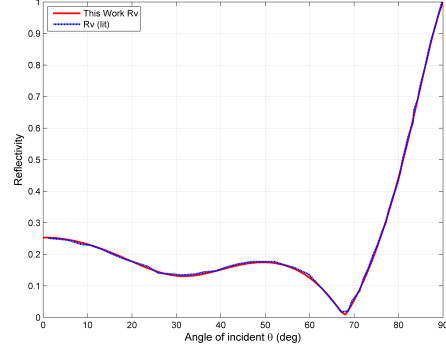
$$A_1 = 1 \quad (2.38a)$$

$$B_1 = 1. \quad (2.38b)$$

In the case of TM polarization, the procedure is same with some differences. For TM case, all μ expressions must be replaced with ϵ in (2.31). In the final expression of reflected and transmitted fields, TE and TM contributions are added vectorially using their vector definitions, which are given in (2.25).

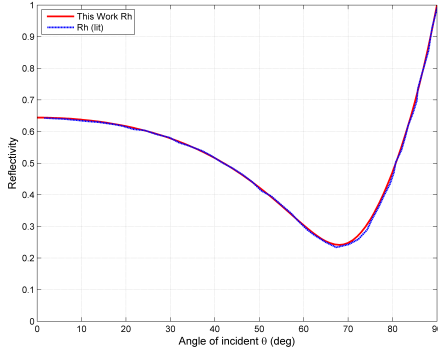


(a) Horizontal (TE) Polarization.

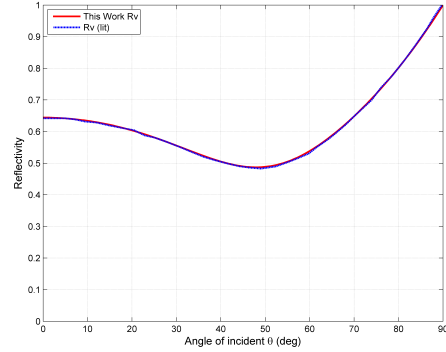


(b) Vertical (TM) Polarization.

Figure 2.6: Reflection coefficients of a wall with $\epsilon = 6 - j0.36$ and $t = 25$ cm.



(a) Horizontal (TE) Polarization.



(b) Vertical (TM) Polarization.

Figure 2.7: Reflection coefficients of a coated object with $\epsilon = 4 - j0.18$ and $t = 13$ cm.

Calculation of reflection coefficients is implemented in MATLAB[®] and compared with literature results [24]. Figure 2.6 shows the reflection coefficients of a wall with a thickness of 25 cm and $\epsilon = 6 - j0.36$. Figure 2.7 shows the reflection coefficients of a PEC coated with a material having thickness of 13 cm and $\epsilon = 4 - j0.18$. Both figures show that reflection coefficient calculations agree with literature results. Note that the solution frequency is 1 GHz.

Calculations are ended when there is no ray left to hit the target, or the number of hitting for each ray reached a specified number. This number is given as input to prevent infinite calculations. Finally, the contribution of each ray is summed up to calculate field.

Chapter 3

RCS Calculation and Reduction

In this chapter, monostatic Radar Cross Section (RCS) of some basic shapes are calculated and their RCS are reduced using radar absorber materials (RAM). In Section 3.1, monostatic RCS of a simple plate, a corner dihedral, and a corner trihedral are calculated and compared with commercially available software results, namely FEKOTM. In Section 3.2, RCS of calculated basic shapes are reduced applying dielectric coating. In this section, it is also shown that if main reflection locations are identified, less dielectric material can be applied to target to reduce RCS so that it is not necessary to be fully coated.

3.1 RCS Calculation

RCS is a figure of merit that denotes how much energy is scattered from the object to the direction from which direction the object is illuminated. It simply states how an object is observable. It is formulated by

$$\sigma = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|\bar{E}^s|^2}{|\bar{E}^i|^2}. \quad (3.1)$$

RCS equation of (3.1) can also be expressed in its components as

$$\sigma_{\theta\theta} = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|E_{\theta}^s|^2}{|E_{\theta}^i|^2} \quad (3.2a)$$

$$\sigma_{\theta\phi} = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|E_{\theta}^s|^2}{|E_{\phi}^i|^2} \quad (3.2b)$$

$$\sigma_{\phi\theta} = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|E_{\phi}^s|^2}{|E_{\theta}^i|^2} \quad (3.2c)$$

$$\sigma_{\phi\phi} = \lim_{r \rightarrow \infty} 4\pi r^2 \frac{|E_{\phi}^s|^2}{|E_{\phi}^i|^2}. \quad (3.2d)$$

where the first subscript states the scattering polarization and the second one states the incident polarization.

In this section; a plate of 0.3m x 0.3m lying in the xy plane, a corner dihedral composed of 0.25m x 0.5m plates [25] and a corner trihedral formed by right triangles with 1m legs [26] are investigated. Monostatic RCS of these structures are calculated for two different frequencies. SBR solutions are compared with FEKOTM-MoM/MLFMM (Method of Moment/Multilevel Fast Multipole Method) and FEKOTM-PO solution, which does not take into account multiple reflection effect.

Figures 3.3-3.8 show the calculated RCS results. Notice that multiple reflection feature of SBR is necessary to accurately calculate RCS of dihedral and trihedral. As it can be seen from plots, traditional PO (FEKOTM-PO) fails as it does not support multiple reflection feature. SBR and FEKOTM-PO results also diverge from FEKOTM-MoM results due to ignored electromagnetic phenomenas, such as diffractions. This is more obvious from lower frequency results. Physical Optics based results also fails at low frequencies as dimensions in terms of wavelength decrease. Shapes of dihedral and trihedral are shown in Figure 3.1 and 3.2, respectively.

Frequency dependency of RCS in the maximum RCS direction for the plate and the trihedral is shown in Figure 3.9, and 3.10. For the plate case; SBR, FEKOTM-PO, and FEKOTM-MoM results agree with each other. For the trihedral case, the maximum error is around 1.5 dB. Notice the ripple on the plot. This is caused

by the multiple reflection effect as currents generated by reflected waves have constructive and destructive effects in far field with changing frequency.

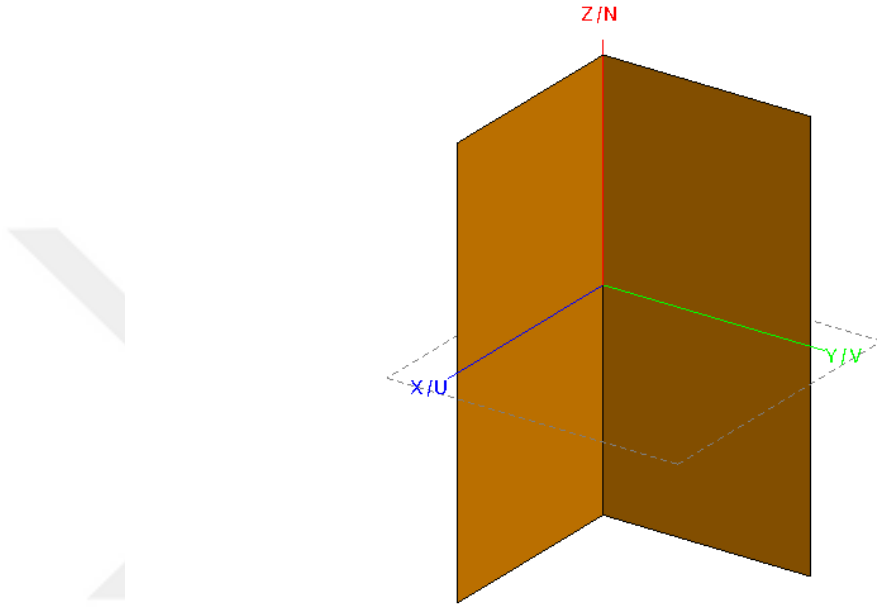


Figure 3.1: Shape of the dihedral for RCS calculations in FEKO™.

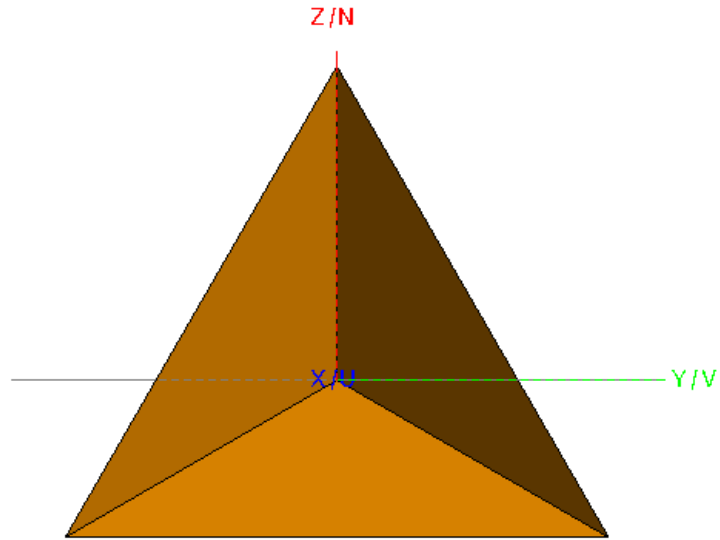


Figure 3.2: Shape of the trihedral for RCS calculations in FEKO™.

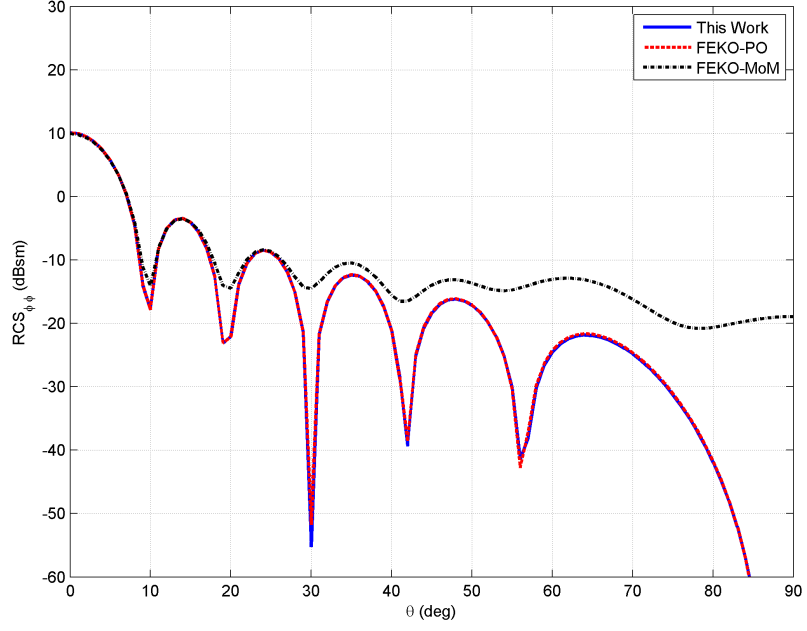


Figure 3.3: Comparison of SBR, PO, and MoM results for HH-polarized monostatic RCS of PEC plate in $\phi = 90^\circ$ plane cut at 3 GHz.

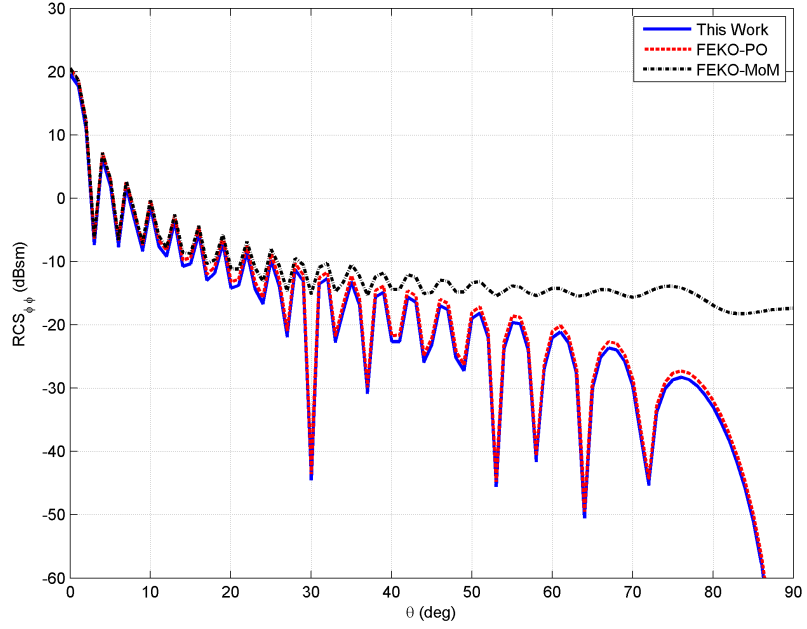


Figure 3.4: Comparison of SBR, PO, and MoM results for HH-polarized monostatic RCS of PEC plate in $\phi = 90^\circ$ plane cut at 10 GHz.

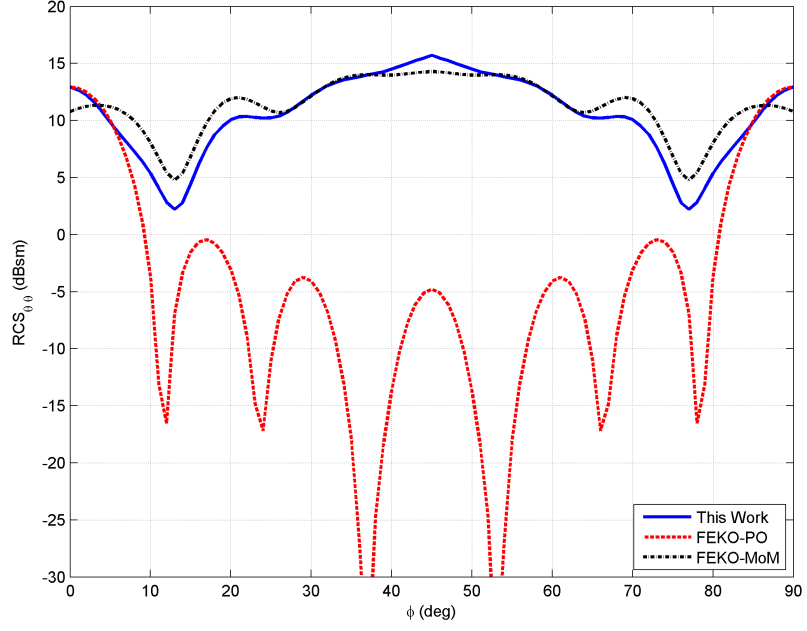


Figure 3.5: Comparison of SBR, PO, and MoM results for VV-polarized monostatic RCS of PEC dihedral in $\theta = 90^\circ$ plane cut at 3 GHz.

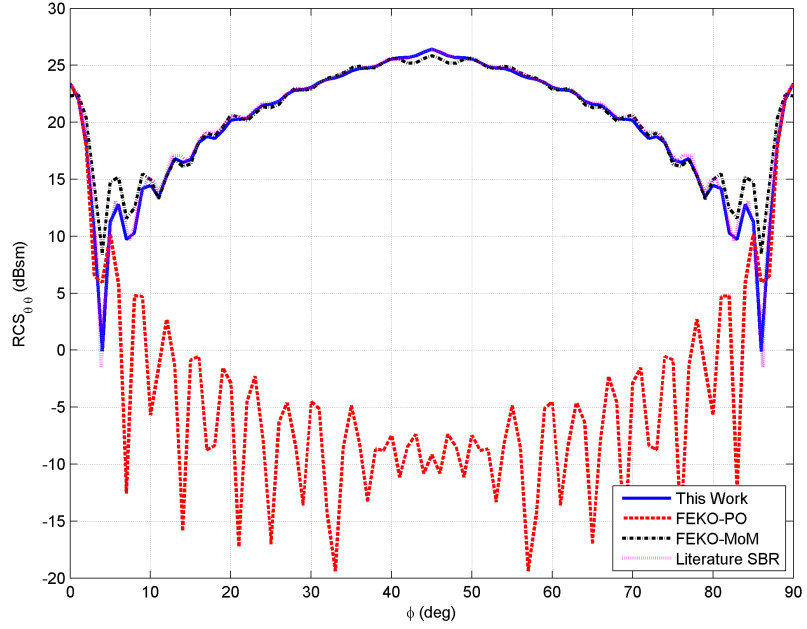


Figure 3.6: Comparison of SBR, PO, and MoM results for VV-polarized monostatic RCS of PEC dihedral in $\theta = 90^\circ$ plane cut at 10 GHz.

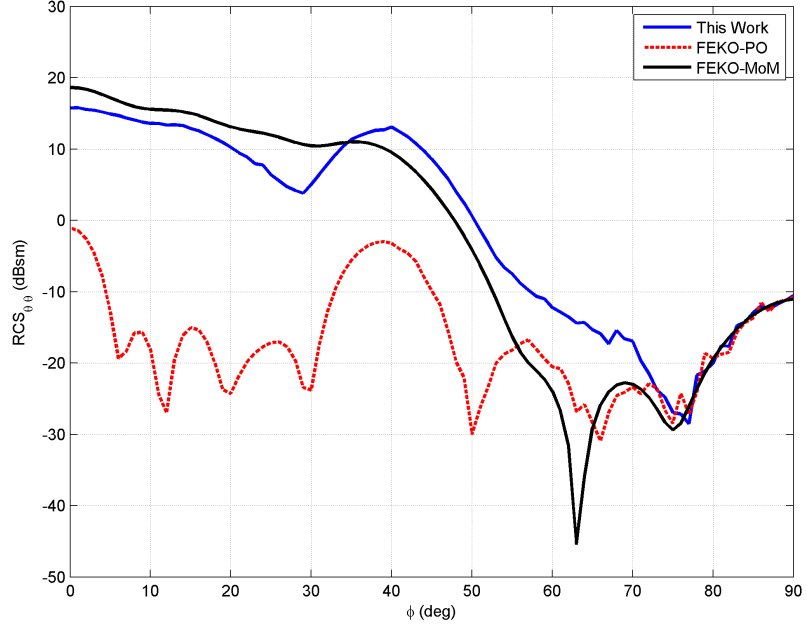


Figure 3.7: Comparison of SBR, PO, and MoM results for VV-polarized monostatic RCS of PEC trihedral in $\theta = 90^\circ$ plane cut at 1 GHz.

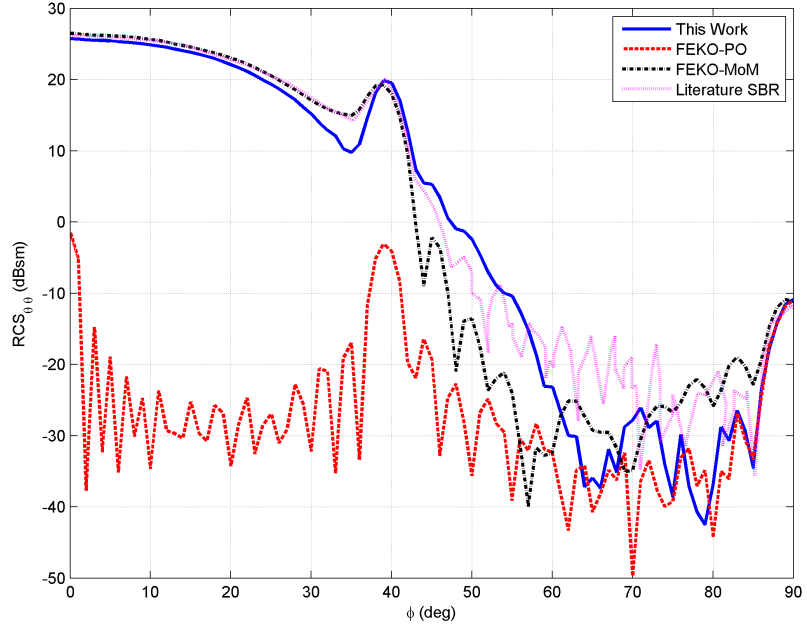


Figure 3.8: Comparison of SBR, PO, and MoM results for VV-polarized monostatic RCS of PEC trihedral in $\theta = 90^\circ$ plane cut at 3 GHz.

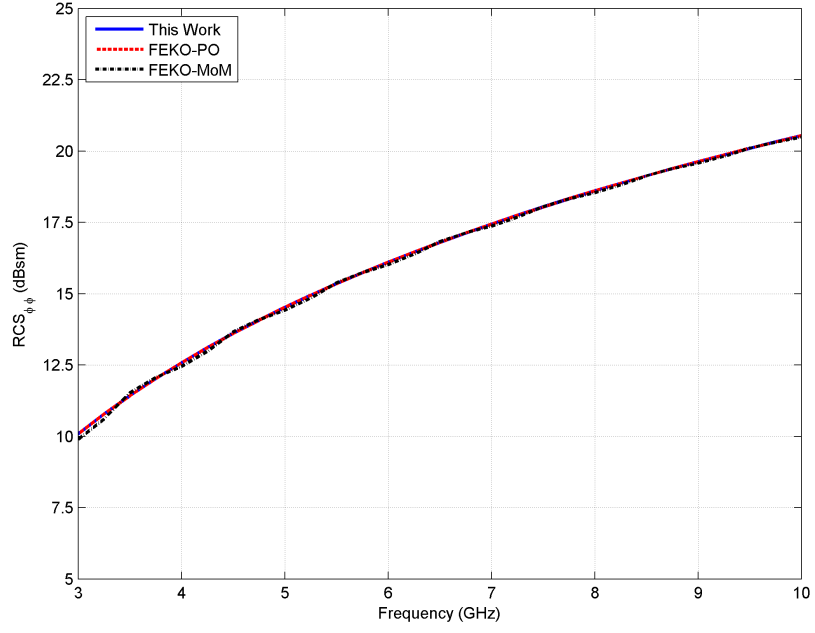


Figure 3.9: Comparison of SBR, and MoM results for HH-polarized monostatic RCS of PEC plate in $\theta = 0^\circ$, $\phi = 90^\circ$ from 3 GHz to 10 GHz.

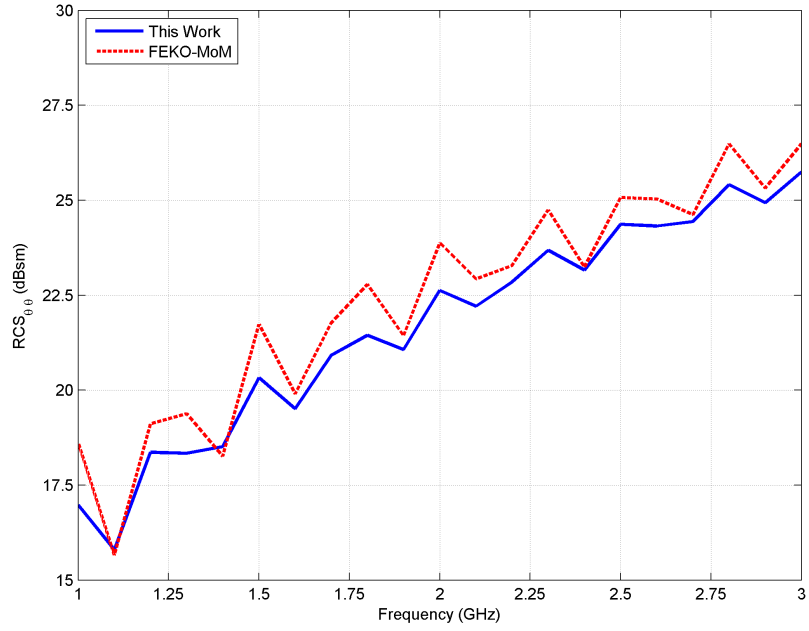


Figure 3.10: Comparison of SBR, and MoM results for VV-polarized monostatic RCS of PEC trihedral in $\theta = 90^\circ$, $\phi = 0^\circ$ from 1 GHz to 3 GHz.

3.2 RCS Reduction

As RCS is the electromagnetic detectability of an object, it is wanted to be reduced in some cases, especially in military applications. There are many options to reduce RCS. One of them is coating the object. The object can completely be coated by a lossy dielectric to reduce RCS. However, this approach may be costly and disturb communication of the target object as antennas mounted on the object may be affected negatively.

High frequency phenomena state that only some locations of the object have a higher impact on the scattered field. Therefore, it would be wisely to coat only some areas of the object, where the effect on RCS is high. Significant contributors to the scattered field include edges, corners, and specular reflections points. Other than these locations, phases of surface currents change rapidly. Therefore, adjacent currents have cancellation effect which leads to less contribution from these locations.

Main contributors of scattered field points should be identified so that they can be coated to reduce RCS. One way to extract main contributors is to filter currents in 3-D [27]. As shown in Figure 3.11, an eye is looking at the object and filter the current at the point where it is looking at using the formula given in (3.3). For the traced point and its adjacent points, eye function is implemented to calculate the impact of the traced point on the scattered field. This way, fewer contribution points are filtered out, and main contributors are extracted. Eye function is given by

$$eye(a) = \begin{cases} \cos\left(\frac{a}{a_0}\pi\right) + 1 & (a \leq a_0), \\ 0 & (a > a_0). \end{cases} \quad (3.3)$$

Here, the radius of the area that the eye is tracking is a_0 . Note that this is done in 2-dimension. It can be extended to 3-dimension by changing the area to a volume and the circle radius to a sphere radius.

For each small volume segment, weighted currents inside the volume are used to calculate far field. Adding these far field values gives a hotspot value for that

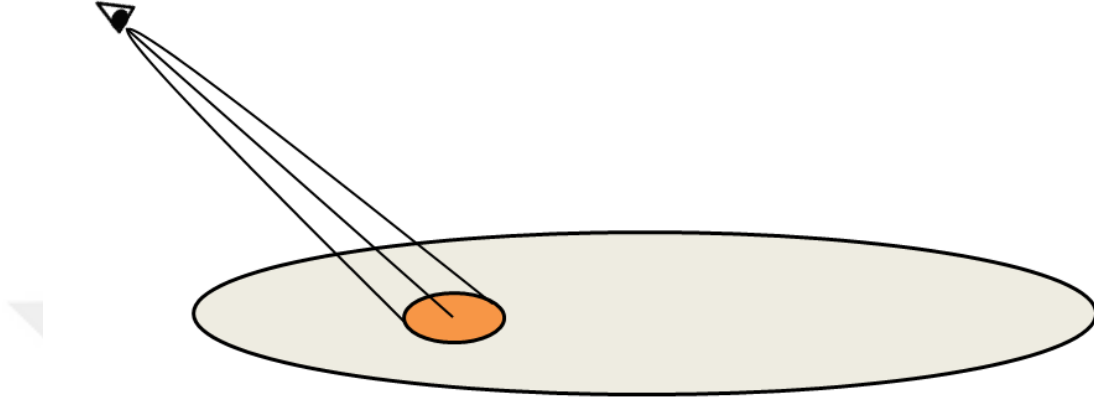


Figure 3.11: Sweeping of the eye on the object.

point, to where the eye is looking on. This procedure is repeated for each small volume segment, and the target object is colored depending on the calculated far field values. Finally, main contributors will have larger hotspot values, while others will have fewer hotspot values.

After identifying main contributors, these locations are coated with a lossy dielectric (RAM) so that most of the incident field is absorbed and less amount of field is scattered. This way, RCS of the object is reduced without coating the whole object. Note that the eye should look from the location where the RCS value to be reduced in that direction.

Figures from 3.12 to 3.17 show the normalized hotspot plots for the previously solved targets. Because of frequency, resolution differs. Notice that for plate or dihedral; there is not any significant contributor location for scattered field. However, the main contributors of the scattered field for trihedral are around the intersection location of three right angle plates.

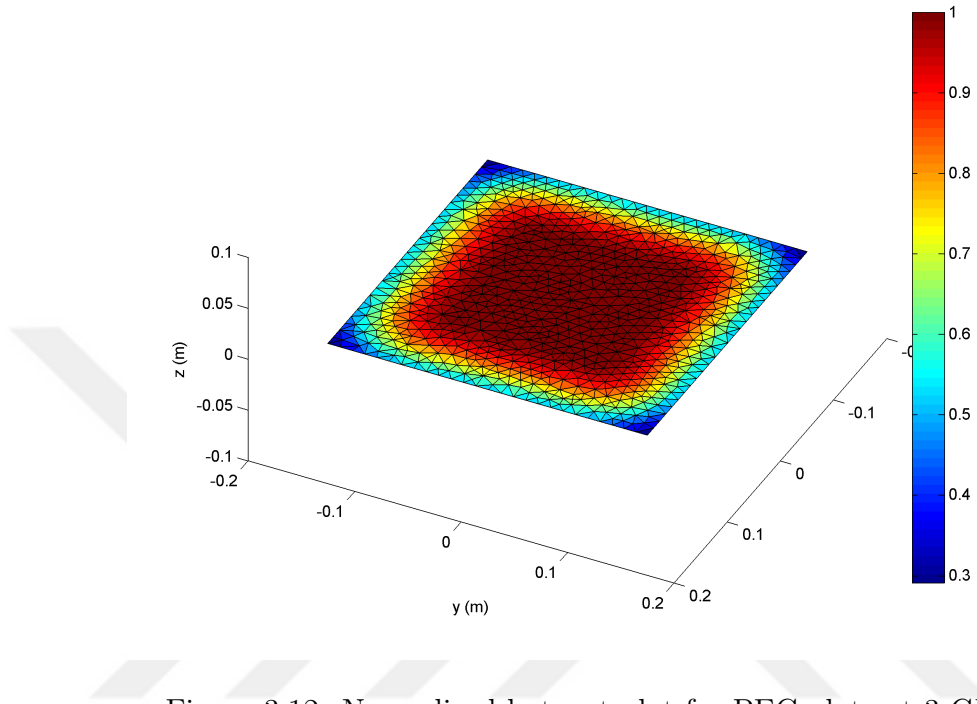


Figure 3.12: Normalized hotspot plot for PEC plate at 3 GHz.

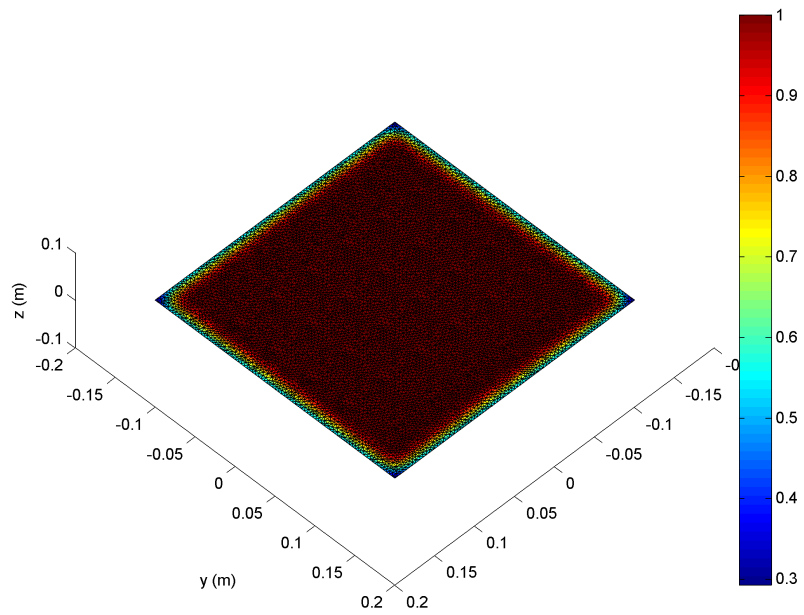


Figure 3.13: Normalized hotspot plot for PEC plate at 10 GHz.

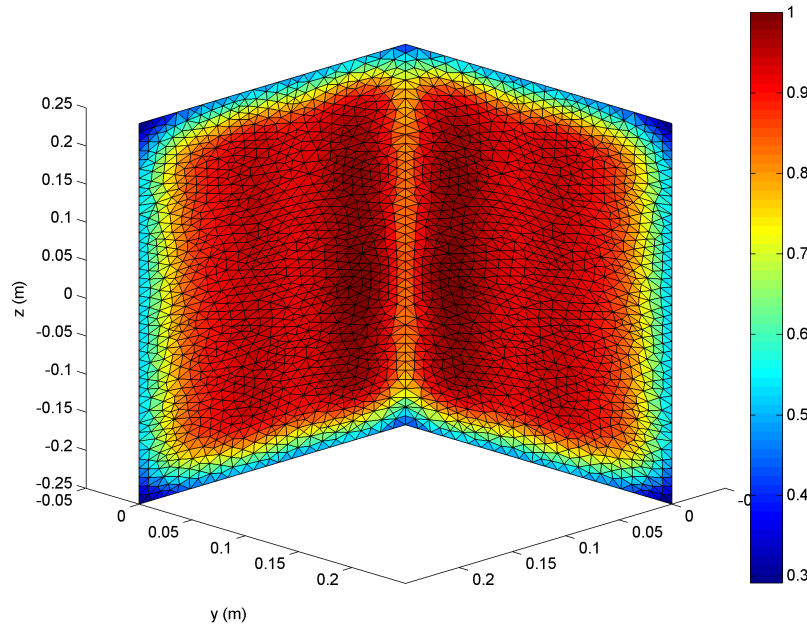


Figure 3.14: Normalized hotspot plot for PEC dihedral at 3 GHz.

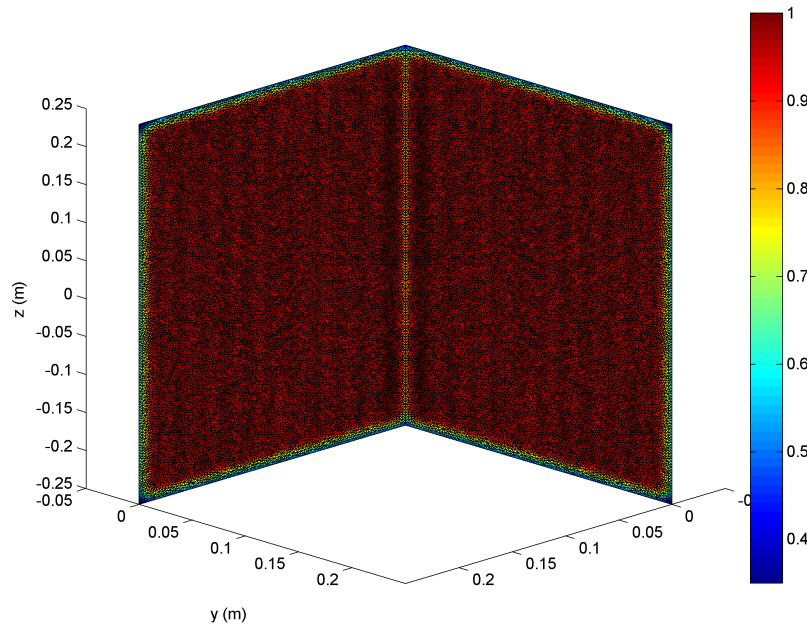


Figure 3.15: Normalized hotspot plot for PEC dihedral at 10 GHz.

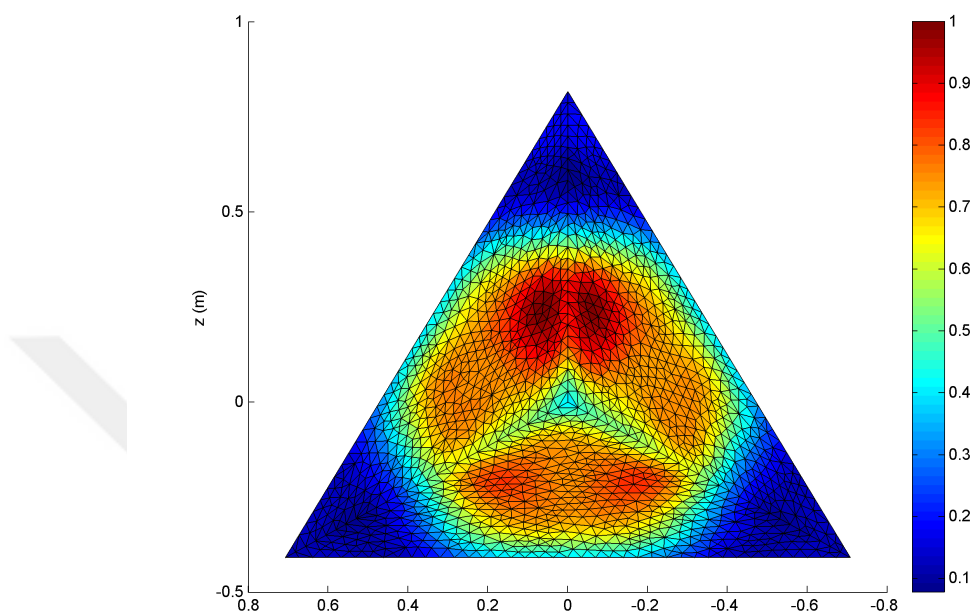


Figure 3.16: Normalized hotspot plot for PEC trihedral at 1 GHz.

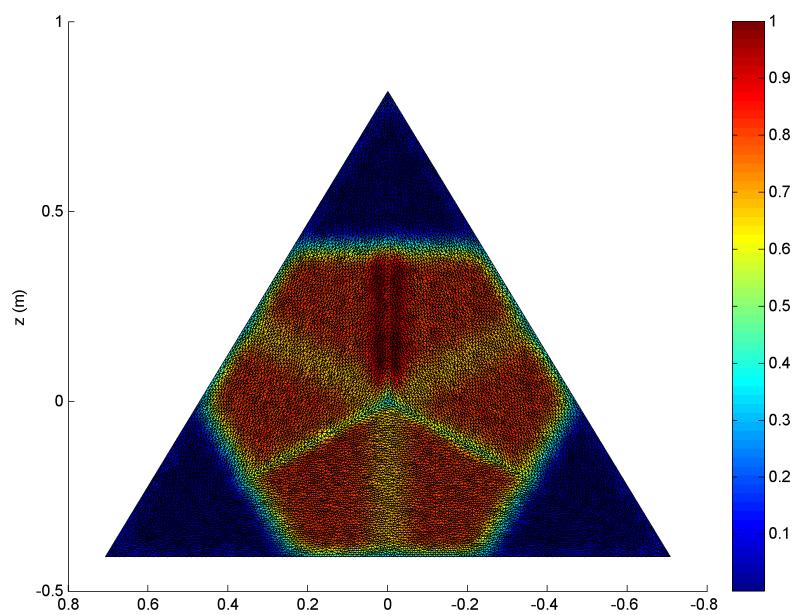


Figure 3.17: Normalized hotspot plot for PEC trihedral at 3 GHz.

Next, target objects are coated with a RAM having $\epsilon = 12.3 - j0.4$ and $\mu = 2.4 - j1.1$ and then their RCS values are calculated again. Figure 3.18 to 3.20 show the coated and the non-coated RCS results. As a partially coating example, roughly one-fourth of the trihedral area is coated. Locations, where hotspot values are high, are selected. These locations are shown in Figure 3.21 with green color. It can be observed that coating leads to 10 dBsm RCS reduction for plate and dihedral. RCS reduction for trihedral is about 20 dBsm for full coating, and 15 dBsm for partial coating. It can be concluded here that if primary RCS contributor locations are identified and these areas are coated, RCS reduction can be similar to the fully coated case.

One may question about RCS calculation time of SBR. For the plate case at 10 GHz, FEKOTM-PO is the fastest one with 7.3 sec. SBR solves this problem in 36.2 seconds, and FEKOTM-MoM calculated it in 454 seconds. For the case of dihedral at 10 GHz, it takes 358 seconds for SBR and 2851 seconds for FEKOTM-MoM. Note that FEKOTM-PO is excluded as it does not support multiple reflection. Finally, trihedral at 10 GHz is solved in 266 seconds by SBR and 1424 seconds by FEKOTM-MoM.

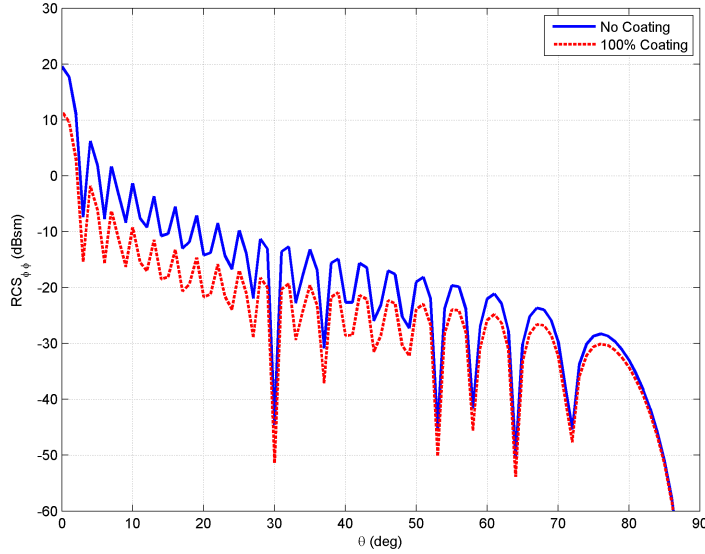


Figure 3.18: HH-polarized monostatic RCS of coated PEC plate in $\phi = 90^\circ$ plane cut at 3 GHz.

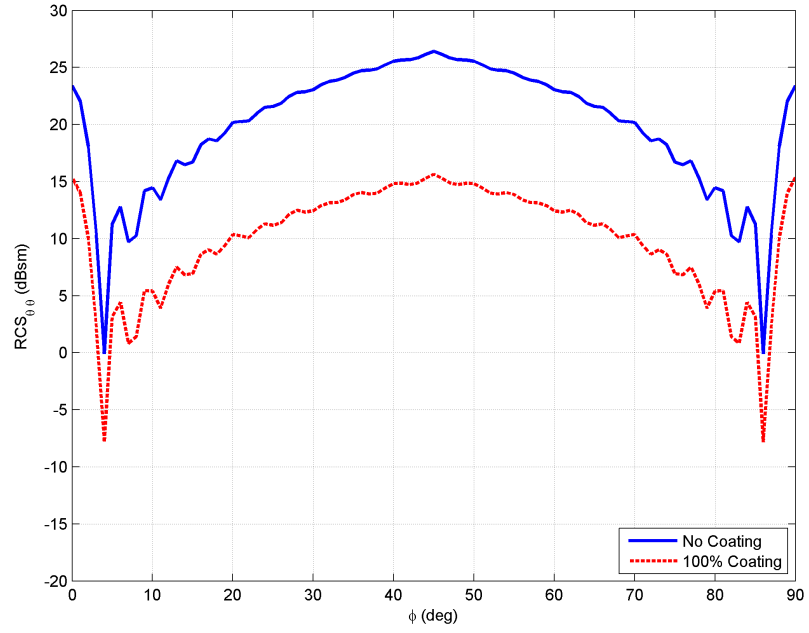


Figure 3.19: VV-polarized monostatic RCS of coated PEC dihedral in $\theta = 90^\circ$ plane cut at 10 GHz.

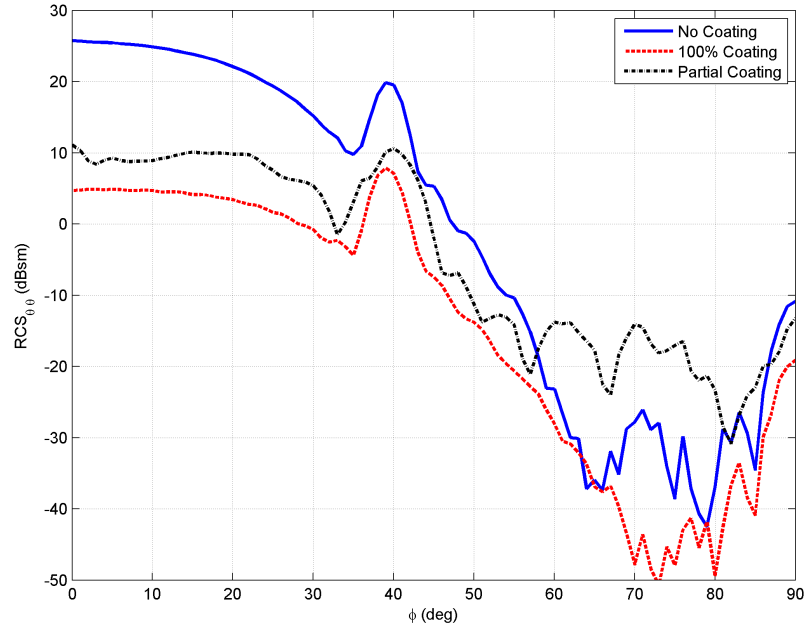


Figure 3.20: VV-polarized monostatic RCS of coated PEC trihedral in $\theta = 90^\circ$ plane cut at 3 GHz.

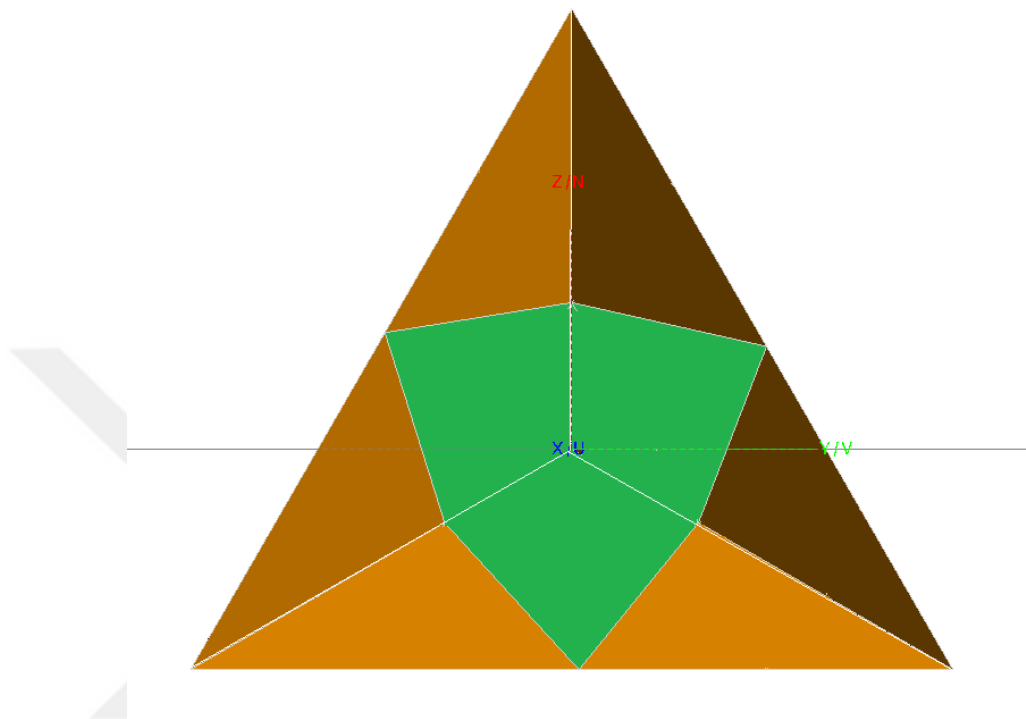


Figure 3.21: Partially coated locations of trihedral in FEKOTM.

Chapter 4

Antenna-Platform Interaction

In this chapter, antenna-platform interaction is investigated using SBR Code. In general, antennas are designed without considering platform effects. However, their electrical characteristic such as pattern may change depending on the platform on which they are mounted. Therefore, antenna-platform interaction should be investigated before antennas are mounted on the platform.

First, the effect of the distance between antenna and platform is analyzed. The distance between a Hertzian dipole antenna and a plate is changed, and variation of the pattern is observed. Next, a half-wave dipole antenna is placed on a tank, and its interaction with the tank is studied. SBR results are compared with FEKOTM-MoM and FEKOTM-PO results as a secondary check. Lastly, a horn antenna is located on a ship-like object and a naval radar analysis is conducted. SBR results are compared with FEKOTM-UTD (Uniform Theory of Diffraction) as other methods in FEKOTM, i.e. MoM or PO, require a huge amount of memory to solve naval radar analysis problem.

4.1 Antenna-Plate Interaction

Before studying antenna placement on a tank and naval radar analysis, antenna plate interaction is studied. As stated in (2.6), antenna patterns can be imported and used to find the hit point E-Field. In this chapter, this technique is used as it is a general case and applicable to any type of antenna.

A 300 MHz Hertzian dipole is used in this section. A $5\text{m} \times 5\text{m}$ plate (5λ by 5λ) centered at the origin is the platform. Hertzian dipole is above the center point of the plate and its distance is set to be 0.1m , 0.5m , 1m and 2m in vertical direction. E-Field patterns of SBR and FEKOTM-MoM are compared in elevation cut. As it can be seen from plots 4.1 to 4.4, as the distance increases, error between SBR and FEKOTM-MoM decreases. This is expected as SBR Code uses far field pattern, but FEKOTM-MoM makes calculations in near-field. Distance between platform and antenna around 0.4λ is selected for tank calculations and 3.33λ is selected for the ship calculations.

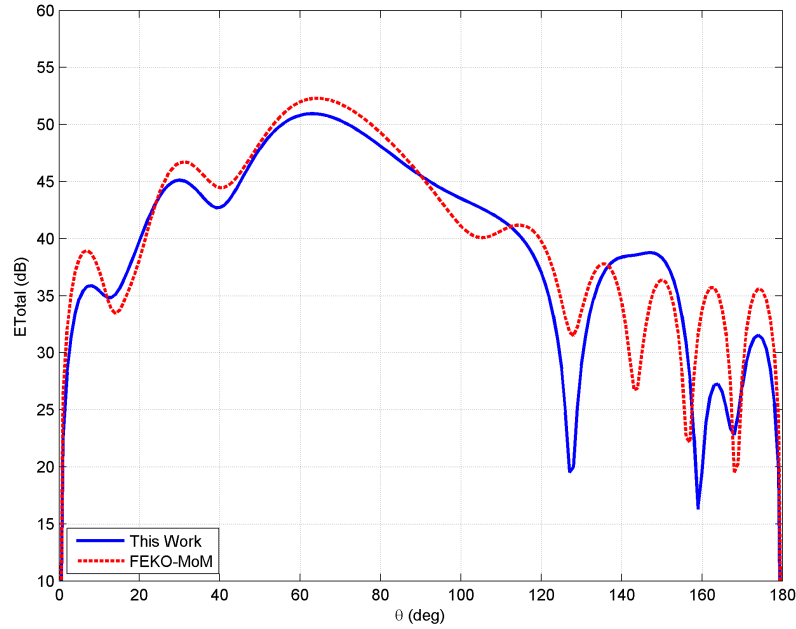


Figure 4.1: Elevation pattern of Hertzian dipole at $\phi = 0^\circ$ on a 5λ by 5λ plate with a distance of 0.1λ .

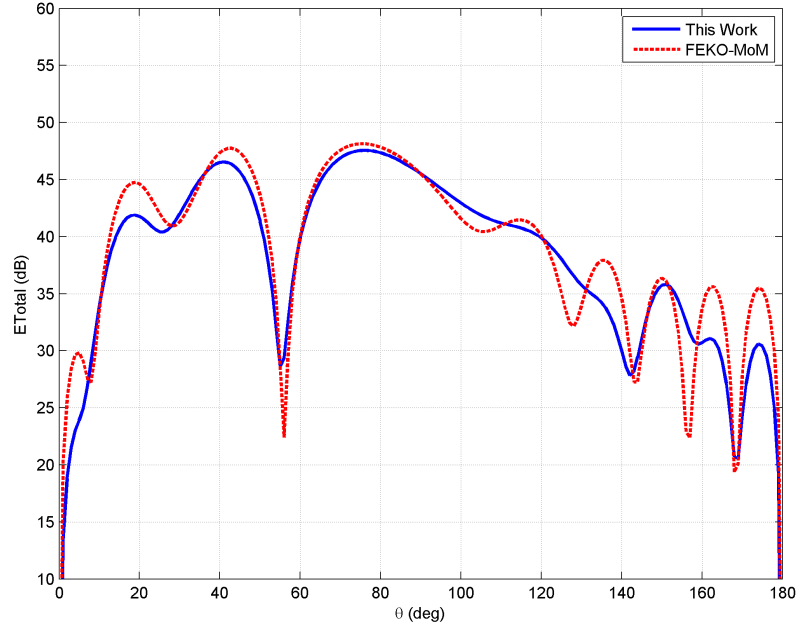


Figure 4.2: Elevation pattern of Hertzian dipole at $\phi = 0^\circ$ on a 5λ by 5λ plate with a distance of 0.5λ .

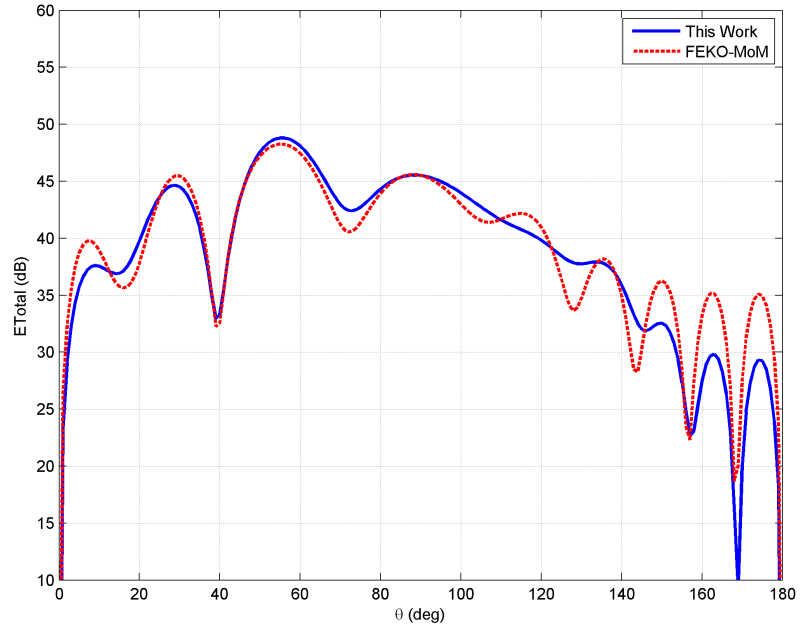


Figure 4.3: Elevation pattern of Hertzian dipole at $\phi = 0^\circ$ on a 5λ by 5λ plate with a distance of 1λ .

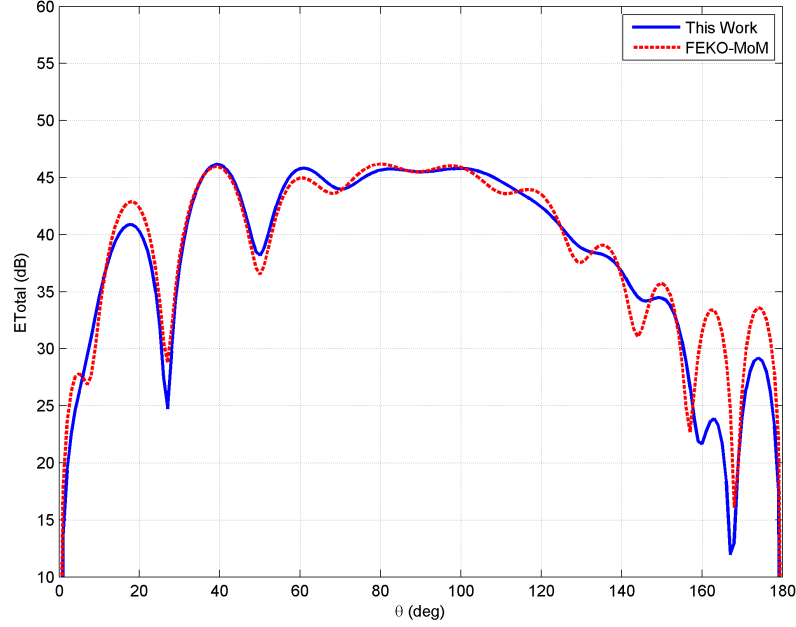


Figure 4.4: Elevation pattern of Hertzian dipole at $\phi = 0^\circ$ on a 5λ by 5λ plate with a distance of 2λ .

4.2 Antenna-Tank Interaction

The purpose of this section is to investigate an antenna and a tank-like object interaction. An half-wave dipole antenna at 300 MHz is placed at different locations on a tank. The tank has maximum dimensions of 3.5m x 7m x 3m. It is tried to find the best location for the antenna so that its azimuth pattern does not deteriorated so much. FEKOTM-MoM and FEKOTM-PO results are also added as supplementary results.

Five different locations are tested. First one is at the front corner, the second is the front middle, third one is the upper front plate, the fourth one is the upper turret, and the last one is the commander hatch. Antenna is placed 0.4λ above the surface. These locations are shown using FEKOTM in Figure 4.5. An ideal half-wave dipole has a constant magnitude pattern in the azimuth plane. Its

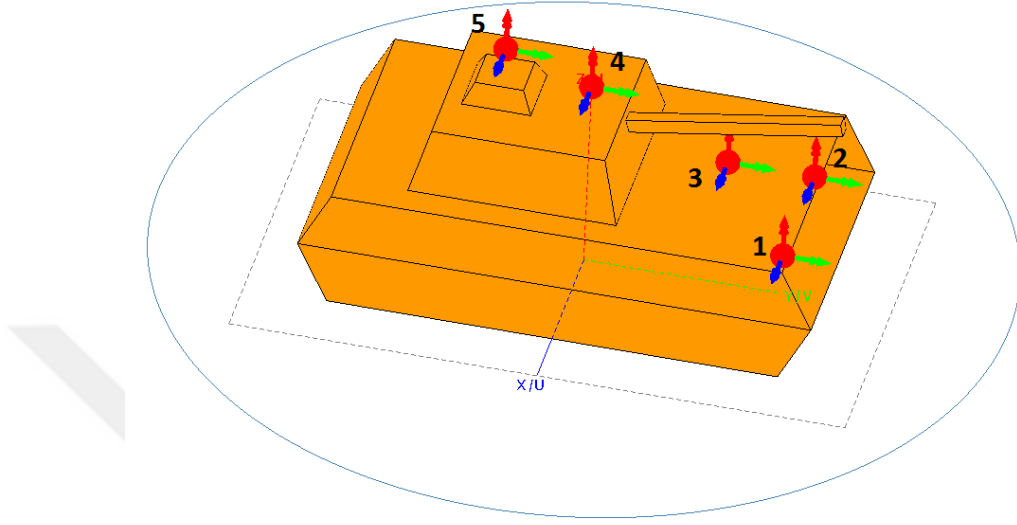


Figure 4.5: Location of test antennas on the tank-like object.

E-Field value in azimuth cut is around -3.23 dBi in every direction.

Figures from 4.6 to 4.10 show the azimuth pattern of the half-wave dipole antenna on the tank. Best location looks like the location-5 as the antenna has field intensity value around -3 dBi in every direction, which is very close to the ideal, free space case for the selected antenna. This result is not surprising as half-wave dipole pattern is like a donut shape and highest field intensity is in the azimuth plane. As there is no obstacle in the azimuth plane, there is little interaction between the antenna and the tank. Therefore, location-5 is the best choice to locate the antenna. Notice that MoM result is slightly higher for location-5. This is expected as SBR and FEKOTM-PO do not take into account diffraction and other types of secondary effects. These effects lead to higher field intensity values for MoM case.

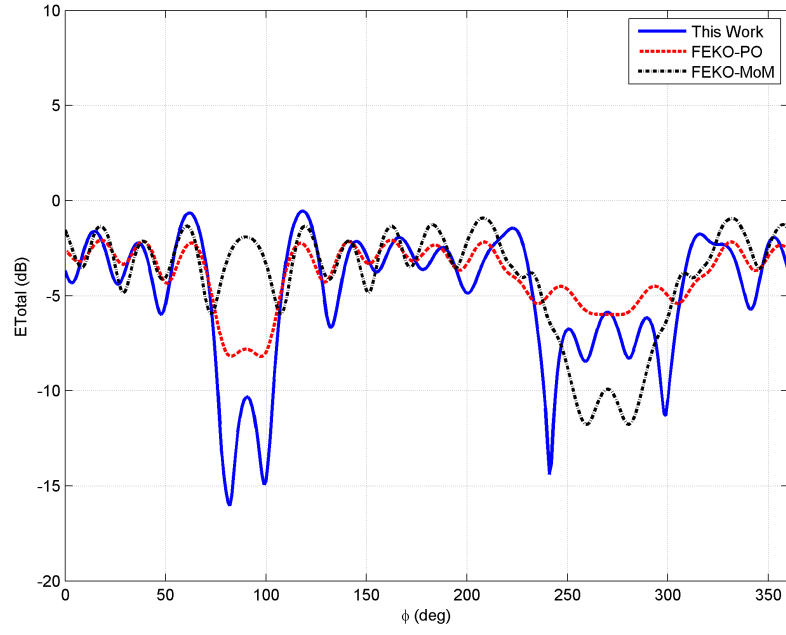


Figure 4.6: Azimuth pattern of half-wave dipole on test location-1 at 300 Mhz.

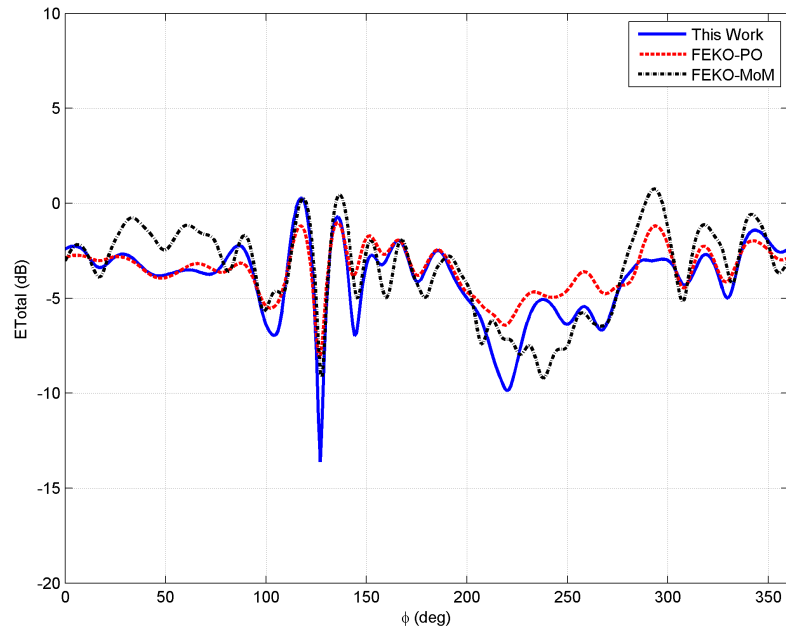


Figure 4.7: Azimuth pattern of half-wave dipole on test location-2 at 300 Mhz.

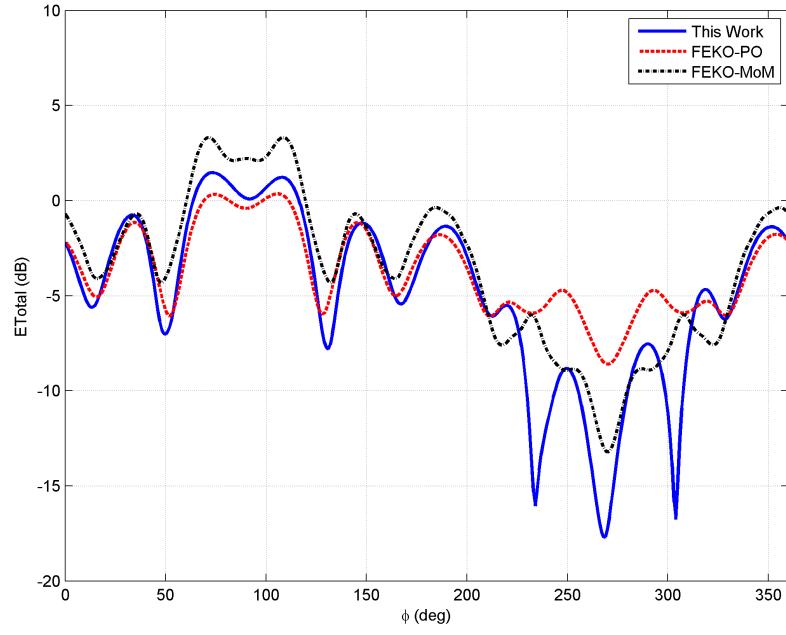


Figure 4.8: Azimuth pattern of half-wave dipole on test location-3 at 300 Mhz.

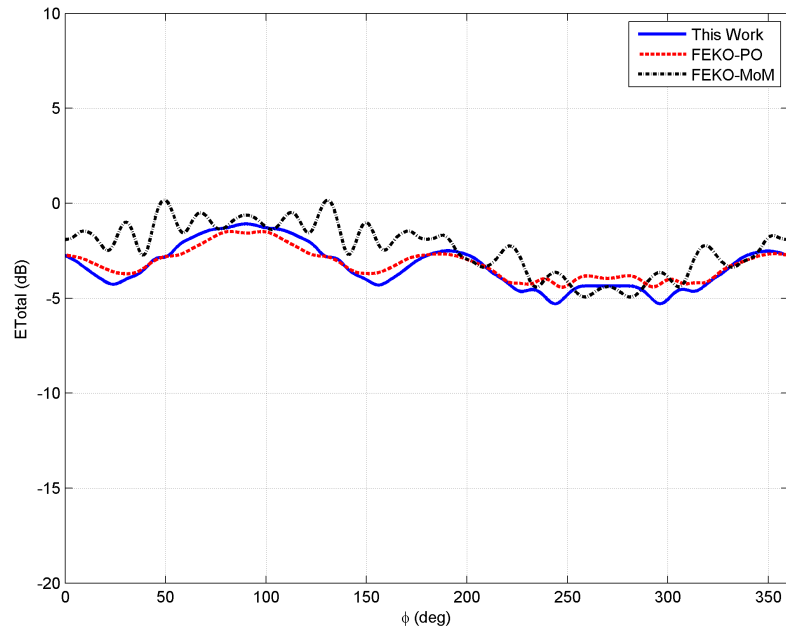


Figure 4.9: Azimuth pattern of half-wave dipole on test location-4 at 300 Mhz.

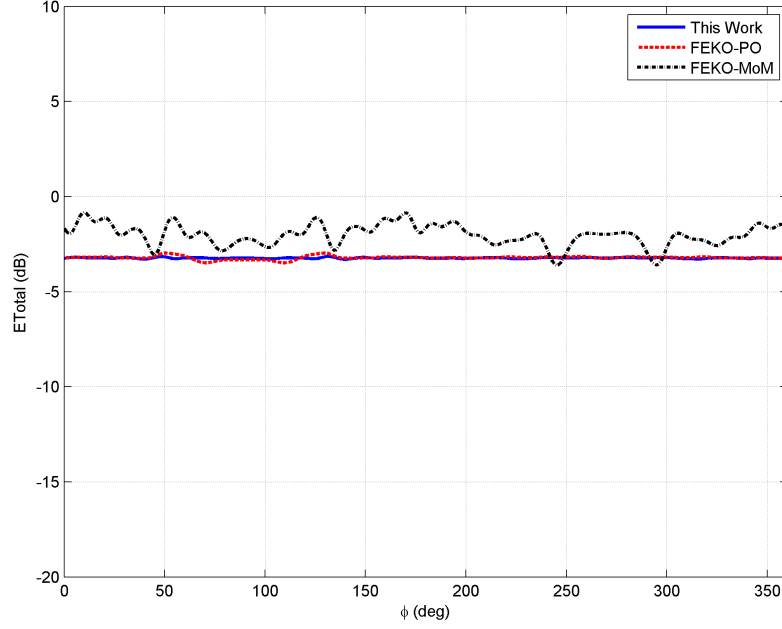


Figure 4.10: Azimuth pattern of half-wave dipole on test location-5 at 300 Mhz.

4.3 Antenna-Ship Interaction

In this section, a naval radar analysis is conducted. Radars are used to find the location and velocity of a target. A directive antenna can be used to scan a region that is under interest. The directive antenna rotates around its own axis for scanning purpose.

Different from the tank case, the antenna location is fixed. A directive antenna is located on a ship-like object and it is rotated in the azimuth plane. Frequency is 1 GHz, and the directive antenna is selected to be an L-Band horn antenna. The pattern of this horn antenna is given in Figure 4.11. This antenna is rotated from 0° to 180° by 30° in the azimuth plane. As a supplementary result, FEKOTM-UTD solution is provided. The ship-like object and the antenna location are shown using FEKOTM in Figure 4.12. The ship has maximum dimensions of 5m x 35m x 6.5m (width, length, height).

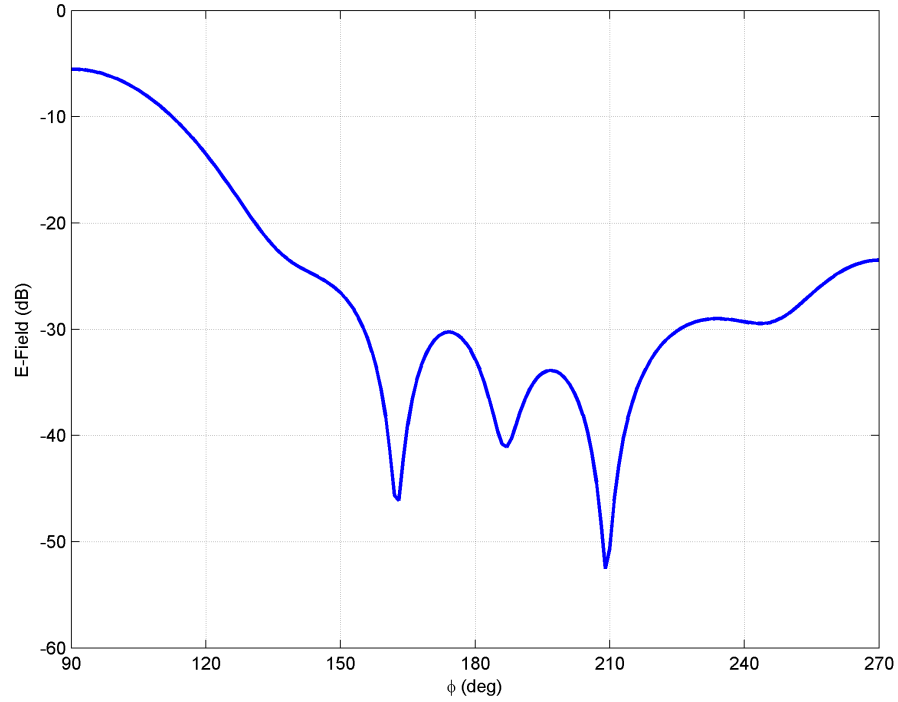


Figure 4.11: Pattern of horn antenna looking toward $\phi = 90^\circ$ direction at 1 GHz.

From Figure 4.13 to 4.19 analysis results are shown. It can be observed that pattern deteriorates when the horn is turned to the back part of the ship. This is because of the main mast. Reflections from the main mast change the pattern. It can also be observed that the worst case looks like when the horn is pointed toward $\phi = 240^\circ$, where two main lobes exist, which worsens the antenna's ability to distinguish target.

For the case of $\phi = 270^\circ$, main mast acts like a reflector so that it illuminates the prow side of the ship. Therefore, this antenna cannot be used to find the location of a target if the target is in stern direction of the ship. One solution may be to use another antenna in backside of the ship. Another solution is to relocate the horn antenna to another location on the ship, and then conduct the analysis again.

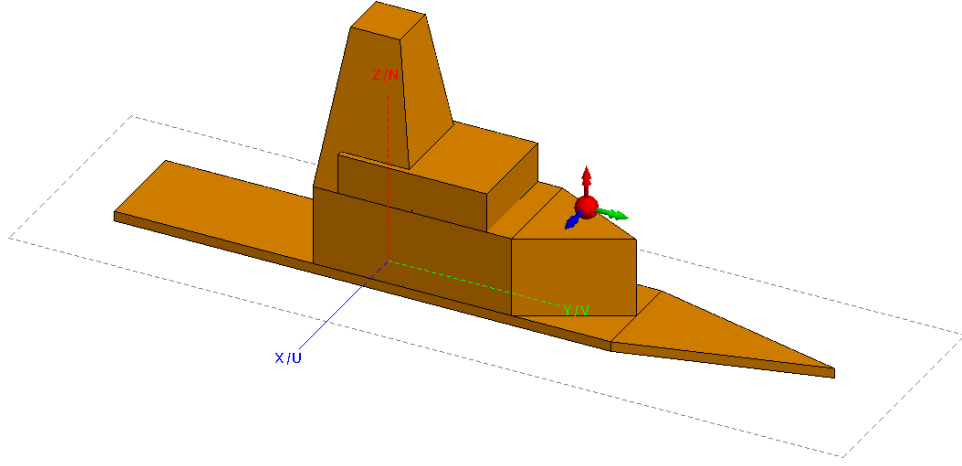


Figure 4.12: Location of the horn antenna on the ship-like object.

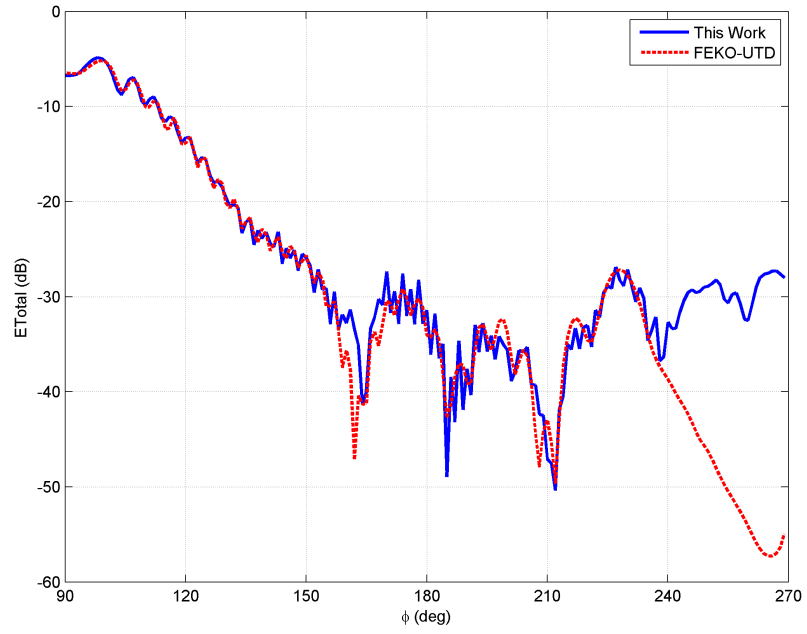


Figure 4.13: Azimuth pattern of horn antenna pointing towards $\phi = 90^\circ$ at 1 Ghz.

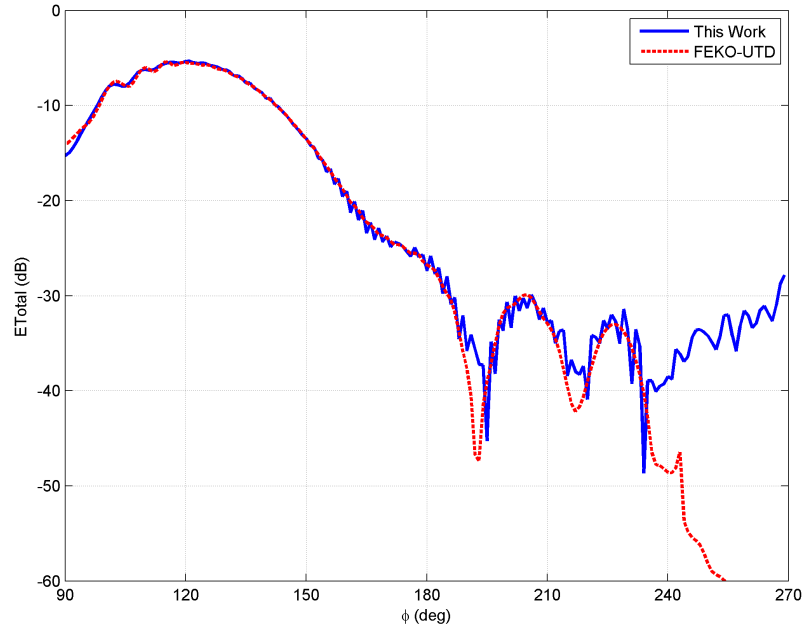


Figure 4.14: Azimuth pattern of horn antenna pointing towards $\phi = 120^\circ$ at 1 Ghz.

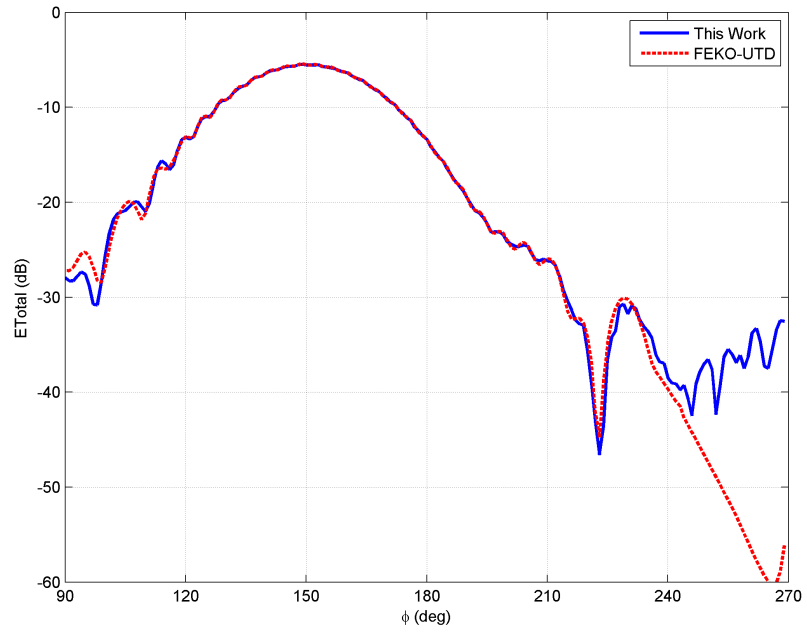


Figure 4.15: Azimuth pattern of horn antenna pointing towards $\phi = 150^\circ$ at 1 Ghz.

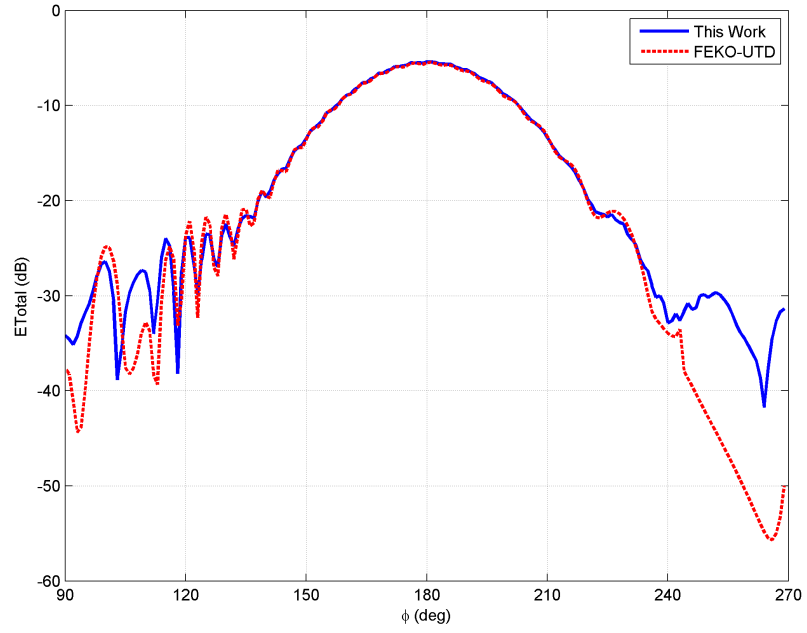


Figure 4.16: Azimuth pattern of horn antenna pointing towards $\phi = 180^\circ$ at 1 Ghz.

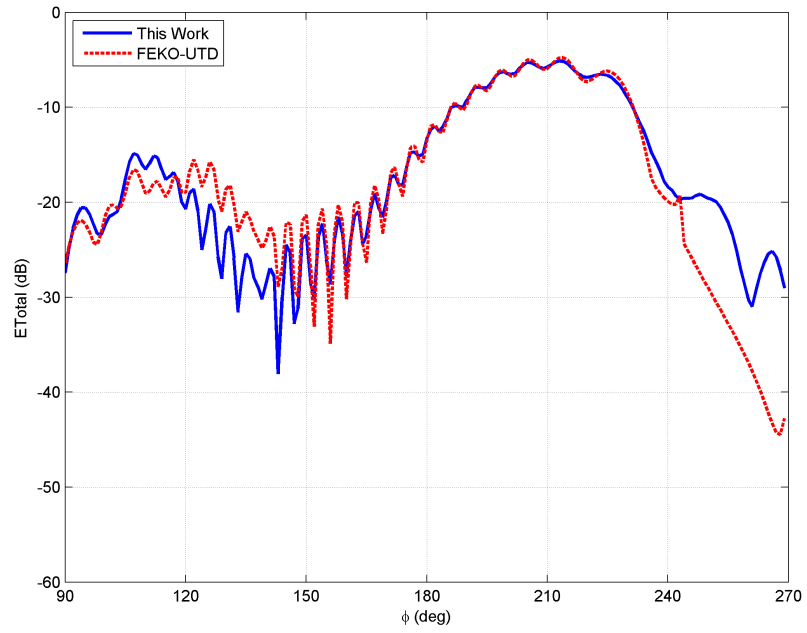


Figure 4.17: Azimuth pattern of horn antenna pointing towards $\phi = 210^\circ$ at 1 Ghz.

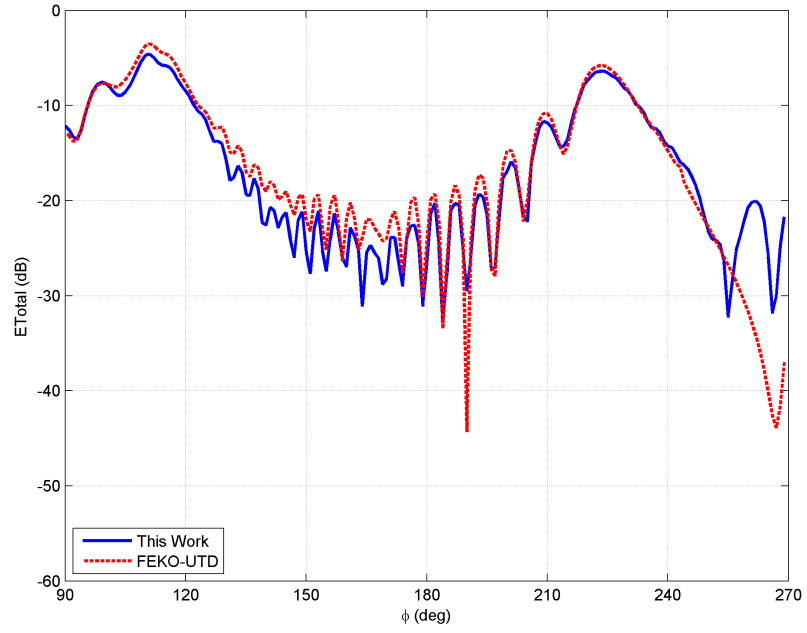


Figure 4.18: Azimuth pattern of horn antenna pointing towards $\phi = 240^\circ$ at 1 Ghz.

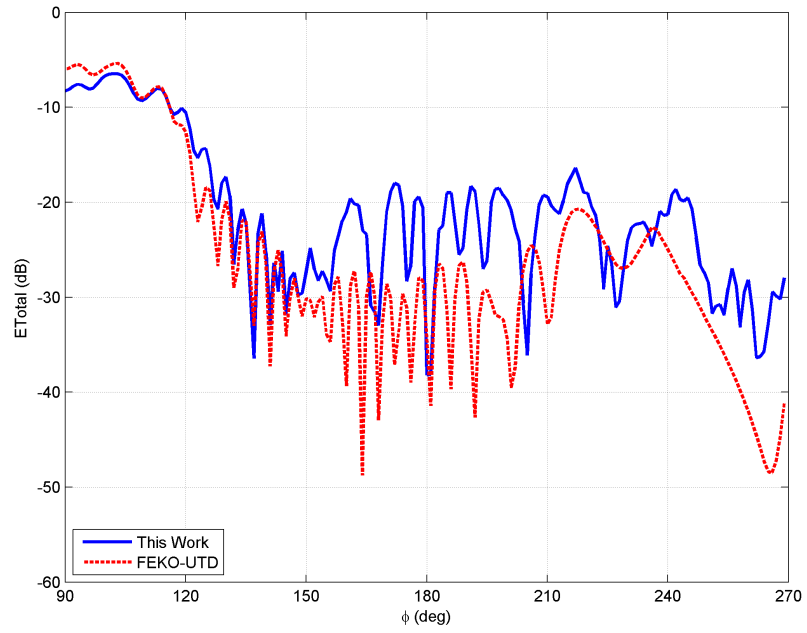


Figure 4.19: Azimuth pattern of horn antenna pointing towards $\phi = 270^\circ$ at 1 Ghz.

Chapter 5

Antenna-Antenna Coupling

In this chapter, the coupling between two different antennas is investigated. Because of the tremendous growth in wireless communications, it is in-demand that signal coverage should be as large as possible. Transmitter antenna should provide enough signal level so that receiver can successfully process it. For example, the received signal level should be higher than the noise level. Therefore, it is required to locate transmitter antennas carefully so that it covers the maximum area to provide telecommunication channels to the highest amount of receiver.

As frequency bands allocated for wireless communication increase, it is not possible to predict the signal coverage of transmitter antenna with full-wave solutions. SBR method can be used to predict the received signal level. First, a coupling formula is derived for SBR using Friis Transmission Formula. Then, this formula is tested for different environments such as in free-space or in the presence of a scatterer. Lastly, two antennas are placed in a corridor. The transmitter antenna is set to be stationary, and the receiver antenna is moved. Coupling between these two antennas are calculated, and results are compared with FEKOTM-MoM/MLFMM.

5.1 Antenna-Antenna Coupling Formula

Power is not a vectorial unit. Therefore, total power due to different sources cannot be summed directly. Instead, voltage values can be used to calculate the total power. In the case of multiple transmitting antennas, terminal voltage values due to these transmitting antennas should be added to find the total terminal voltage value at the receiving antenna. Note that if a scatterer exists, this is also a multiple antenna problem as the surface current due to incident field caused by a transmitter, acts like an electric/magnetic dipole.

Derivation of Antenna-Antenna coupling formula starts from the Friis Transmission Formula which is given by [28]

$$\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2}, \quad (5.1)$$

where D_t and D_r are directivity, e_t and e_r are radiation efficiency of transmitter and receiver antennas, respectively. P_r is the received power and P_t is the total power radiated from transmitting antenna. Also, R is the distance between two antennas. Received power can be represented by

$$P_r = e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2} e_r \frac{D_r(\theta_r, \phi_r) \lambda^2}{4\pi}. \quad (5.2)$$

Equation (5.2) can be divided into two different formula. First part gives the power density W_t of the transmitting antenna in the direction of (θ_t, ϕ_t) , and the second one gives the effective area A_r of the receiving antenna. They can be represented as follows

$$W_t = e_t \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R^2} \quad (5.3)$$

$$A_r = e_r \frac{D_r(\theta_r, \phi_r) \lambda^2}{4\pi}. \quad (5.4)$$

Here, $D_t(\theta_t, \phi_t) P_t$ product can also be represented by

$$D_t(\theta_t, \phi_t) P_t = 4\pi R^2 \frac{(E_\theta^{inc})^2 + (E_\phi^{inc})^2}{2\eta_0}, \quad (5.5)$$

then, plugging (5.5) into (5.2) gives

$$P_r = e_t \frac{(E_\theta^{inc})^2 + (E_\phi^{inc})^2}{2\eta_0} e_r \frac{D_r(\theta_r, \phi_r) \lambda^2}{4\pi}. \quad (5.6)$$

Note that the polarization of incident power is with respect to receiver location. The received power should be separated into different polarization parts. Assuming that each antenna is ideally matched and defining $\hat{p}_\theta$ and $\hat{p}_\phi$ as unit vectors of directivity in their respective directions, (5.6) can be written as

$$P_{r\theta} = (\bar{E}^{inc} \cdot \hat{p}_\theta)^2 \frac{D_{r\theta} \lambda^2}{8\pi\eta_0} \quad (5.7a)$$

$$P_{r\phi} = (\bar{E}^{inc} \cdot \hat{p}_\phi)^2 \frac{D_{r\phi} \lambda^2}{8\pi\eta_0}, \quad (5.7b)$$

then, terminal voltage of receiving antenna can be found by

$$V_{r\theta} = (\bar{E}^{inc} \cdot \hat{p}_\theta) \sqrt{\frac{D_{r\theta} \lambda^2}{8\pi}} \quad (5.8a)$$

$$V_{r\phi} = (\bar{E}^{inc} \cdot \hat{p}_\phi) \sqrt{\frac{D_{r\phi} \lambda^2}{8\pi}}. \quad (5.8b)$$

Equation (5.8) is correct if the receiving antenna is located on the reference point of coordinate system. Therefore, its phase value should be corrected.

$$V_{r\theta} = (\bar{E}^{inc} \cdot \hat{p}_\theta) \sqrt{\frac{D_{r\theta} \lambda^2}{8\pi}} e^{j\angle \bar{E}_\theta^{rec}} \quad (5.9a)$$

$$V_{r\phi} = (\bar{E}^{inc} \cdot \hat{p}_\phi) \sqrt{\frac{D_{r\phi} \lambda^2}{8\pi}} e^{j\angle \bar{E}_\phi^{rec}}, \quad (5.9b)$$

where the phase is the far field pattern phase of receiving antenna in the direction pointing toward the transmitting antenna. After calculating terminal voltages due to each transmitting antenna, they are summed up

$$V_{total\theta} = \sum_i V_{r\theta i} \quad (5.10a)$$

$$V_{total\phi} = \sum_i V_{r\phi i}, \quad (5.10b)$$

and finally, the total received power can be calculated as

$$P_r = \frac{(V_{total\theta} + V_{total\phi})^2}{\eta_0}. \quad (5.11)$$

5.2 Antenna-Antenna Coupling Results

In this section, the formula derived in Section 5.1 is verified. First, the coupling between two antennas is calculated in free-space. Then, basic shapes such as plate

and dihedral are added to observe the effect of the scatterer. Last of all, a corridor is used as indoor scatterer environment and coupling is calculated. Power gains in dB scale are plotted. P_r values are computed using (5.11). P_t values are taken from FEKOTM Software. Results are compared with FEKOTM-MoM solutions.

For the free space example, a z-directed Hertzian dipole is used as transmitting antenna. It is located at (0,0,0). Receiving antenna is also a z-directed Hertzian dipole. It is located at (X_r ,0,0) and it is moved in x-direction. Frequency is selected as 300 MHz and $P_t = 395.057$ W.

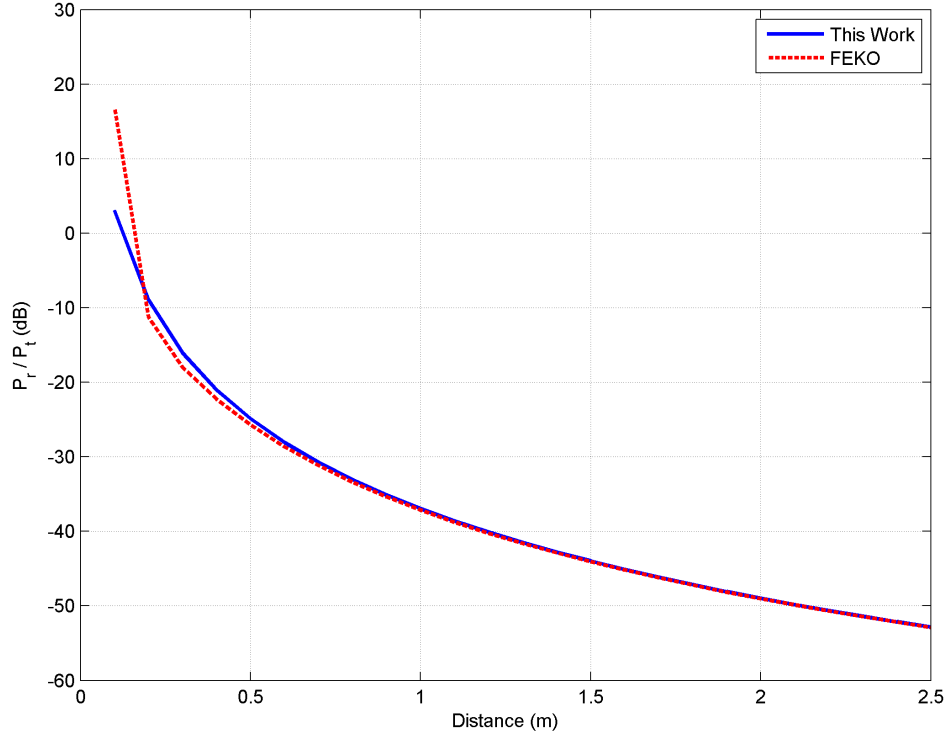


Figure 5.1: Free space antenna coupling between two Hertzian dipoles at 300 MHz.

As it can be seen from Figure 5.1, error between SBR and FEKOTM results are acceptable starting from 0.2m (or 0.2λ). Results agree with each other after 1λ . Therefore, it can be concluded that two antennas should be separated in

free space by at least 0.2λ for SBR.

Next, the impact of the scatterer is investigated. A 5m by 5m plate centered at $(0,0,-1)$ is placed at $z = -1\text{m}$ plane. The receiving antenna is moved in the z -direction. Transmitting antenna is at $(0,0,0)$, and the receiving antenna is at $(1,0,Z_r)$, where Z_r changes from -0.9m to 1.5m . Figure 5.4 denotes the location of transmitter with a red arrow and receivers with blue spheres. All antennas are z -directed Hertzian dipoles. Figure 5.2 shows the result. Different error levels are obtained for different distance. 1 dB error is observed if the receiver is located 1λ above the scatterer. It can also be deduced that the maximum coupling occurs when the receiver 1m above the scatterer as directivity of these two antennas are maximum in the horizontal direction, and they are on the same plane in z -coordinate.

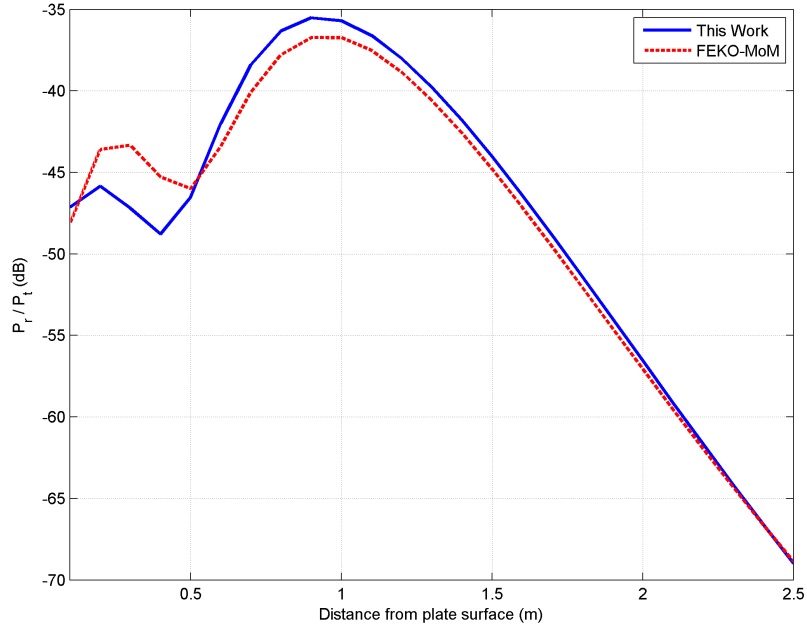


Figure 5.2: Antenna coupling between two Hertzian dipoles at 300 MHz in the presence of a plate as receiver moves vertically.

Effect of scatterer should also be investigated in horizontal direction. In this case, receiver antenna is moved horizontally. Transmitting and receiving antennas

are again z-directed Hertzian dipoles. Receiving antenna is located at $(X_r, 0, 0)$ and it is moved in the x-direction. Transmitting antenna is at $(0, 0, 0)$. A 5m by 5m plate is placed at $z = -1$ m plane so that two antennas are 1λ above the platform. Figure 5.5 denotes the location of the source and receivers. Figure 5.3 shows the results of calculations. It can be stated that error is reasonably small after 0.2λ distance.

Antenna coupling in a multiple reflection environment can also be investigated with SBR. A dihedral is used as the scatterer. Here, a z-directed 500 MHz half-wave dipole antenna is used as both transmitter and receiver. The transmitter is at $(0, 0, 3)$, and the receiver is at $(X_r, 4, 5)$. X_r changes from -2m to 2m. Plates of dihedral are 2.828m x 2.828m squares. Figure 5.6 shows a FEKOTM picture of the location of antennas and the scatterer. Figure 5.7 denotes the result of coupling. It can be stated that even the power gain level is very low, the maximum error is around 0.7 in dB scale. Note that $P_t = 4.8$ mW in this example.

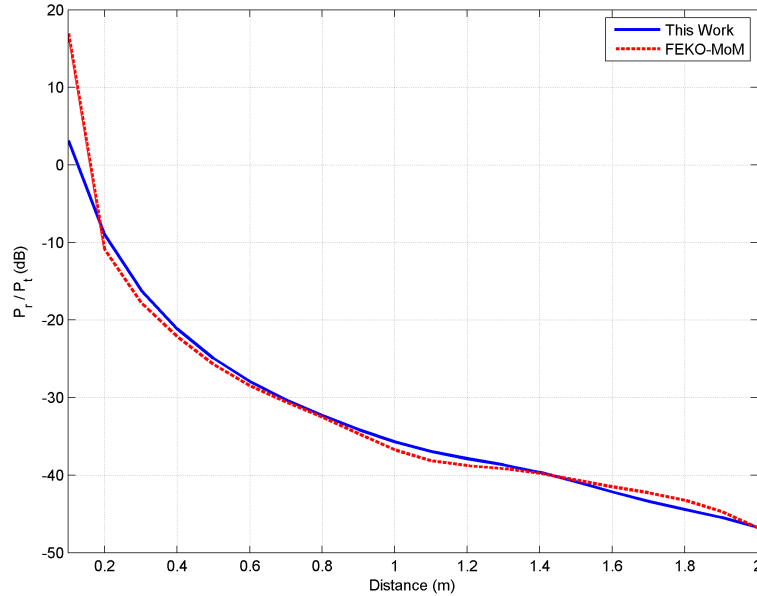


Figure 5.3: Antenna coupling between two Hertzian dipoles at 300 MHz in the presence of a plate as receiver moves horizontally.

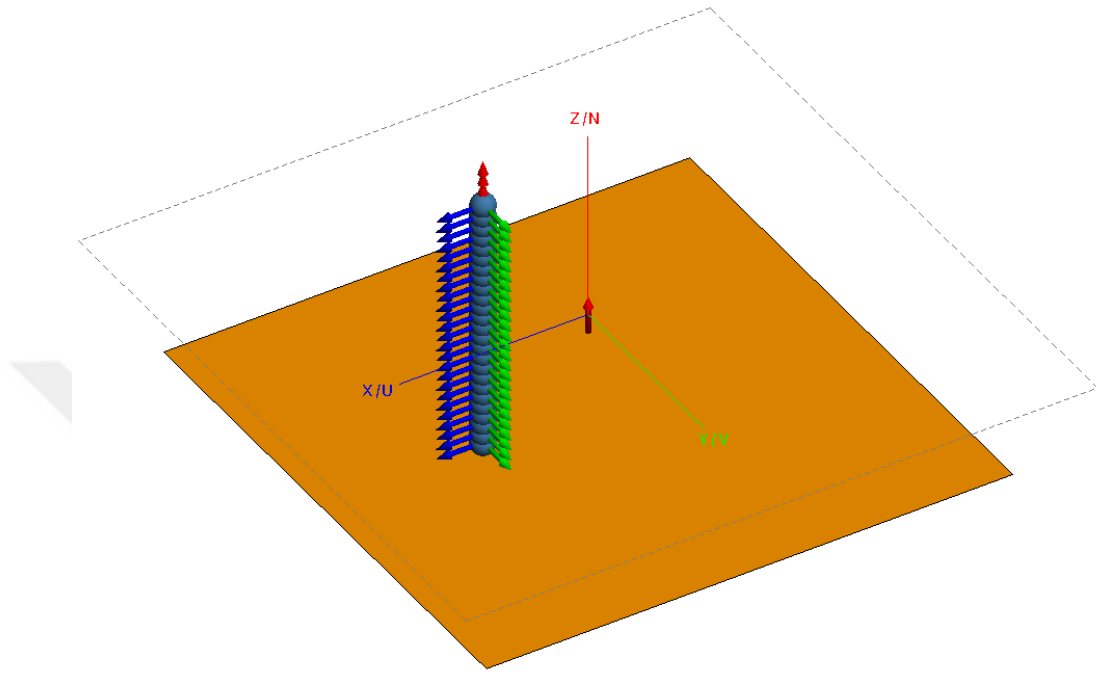


Figure 5.4: Locations of source Hertzian dipole and vertically moving receivers in FEKOTM.

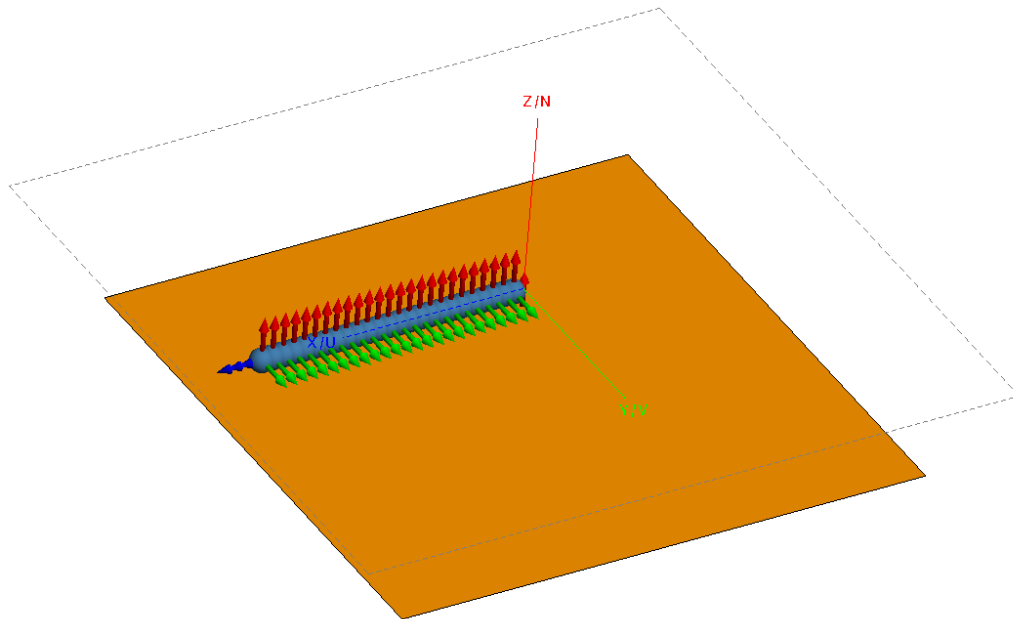


Figure 5.5: Locations of source Hertzian dipole and horizontally moving receivers in FEKOTM.

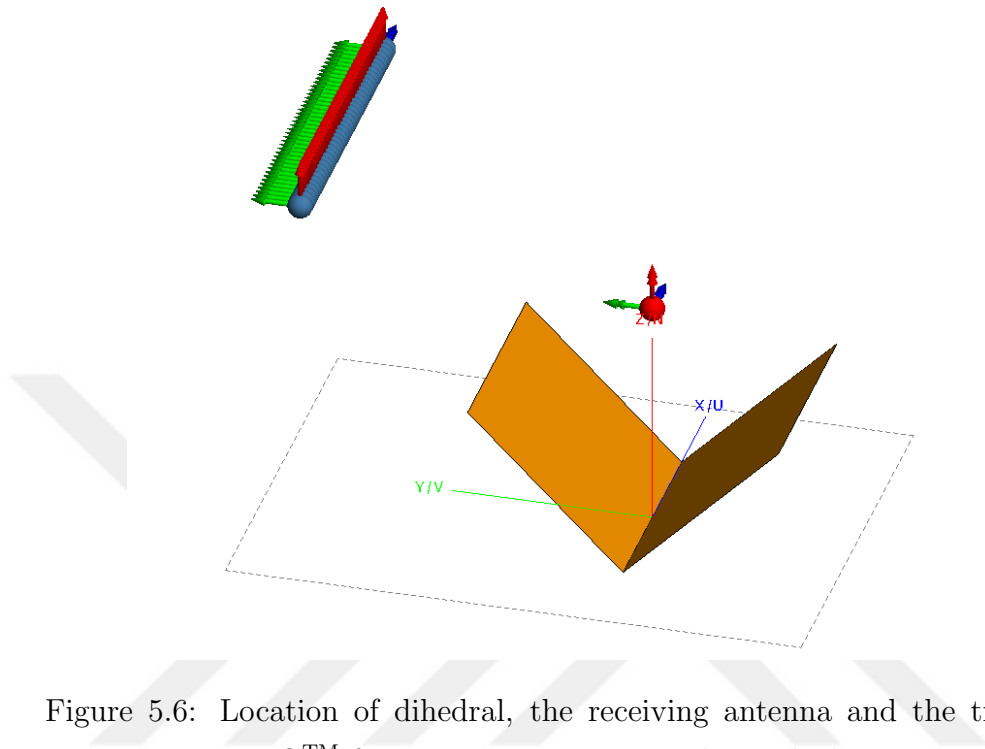


Figure 5.6: Location of dihedral, the receiving antenna and the transmitting antennas in FEKOTM for antenna-antenna coupling calculation.

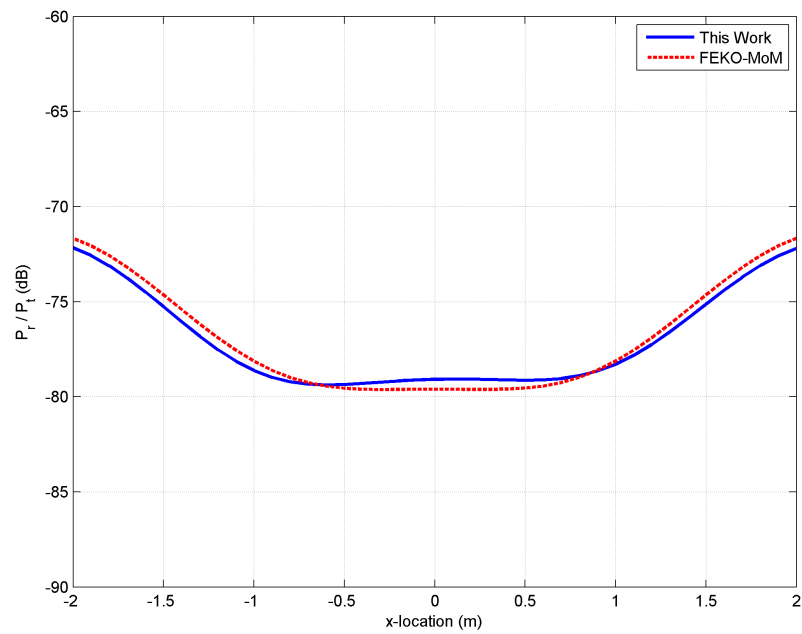


Figure 5.7: Antenna coupling between half-wave dipoles at 500 MHz with dihedral scatterer.

The last example is the antenna coupling in an indoor environment. A straight corridor with 2m width, 3m height, and 10m length is used as the scatterer. The transmitter is located at $(1, 1.5, 0)$ and the receiver is at $(1.4, 2.4, Z_r)$. Receiver moves in the z-direction. An x-directed 500 MHz half-wave dipole antenna is used as transmitting and receiving antenna. For SBR, the exit of all ray tubes from the corridor is waited. Figure 5.8 denotes the location of source with a red sphere and receivers with blue spheres.

Figure 5.9 shows the SBR and FEKOTM-MoM results. Even if the error is small at some locations, it may exceed 10 dB. It may be concluded that the developed SBR code may not be suitable for indoor environments where there are too much multiple reflections occur. However, it gives a general idea about the indoor propagation. If an approximation is required with some error, a quick check can be conducted with SBR code.

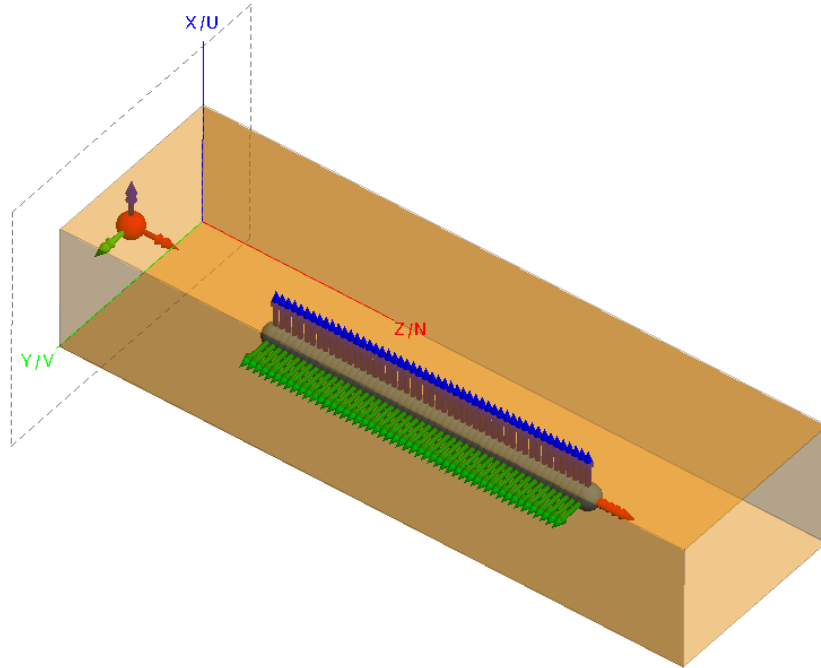


Figure 5.8: Transmitter and receiver locations in the example corridor in FEKOTM.

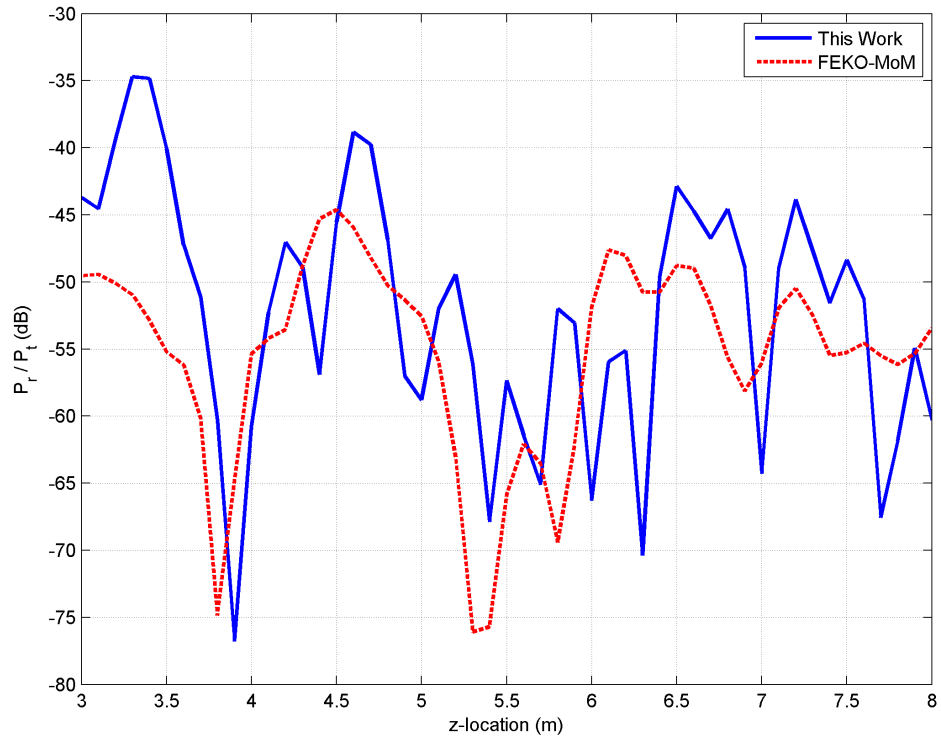


Figure 5.9: Antenna coupling in a corridor at 500 MHz.

Chapter 6

Conclusion

In this thesis, a high frequency electromagnetic solver code using SBR method is developed. The developed code can be used for RCS calculations of complex targets, RCS reduction of a target using coating, antenna-platform interaction calculations, and antenna-antenna coupling calculations. PO and GO techniques are combined with multiple reflection feature. Implementation of the code is done in MATLAB[®]. FEKO[™] software is used for mesh generation and comparison purposes.

First, formulas for SBR are derived. Ray tracing algorithm is explained. Reflection coefficient calculations for multilayer structures are shown. Ray-Triangle intersection algorithm is formulated. The method used for the intersection algorithm includes precalculation of repeated terms, which are same for each ray tube. Precalculated terms are reused instead of repeating the calculation. Hence, computation time is reduced.

SBR method is first implemented for RCS calculation of canonical shapes, including plate, dihedral and trihedral. It is shown in RCS calculation section of dihedral and trihedral that multiple reflection effects should be taken into account if the correct result is desired. Basic PO calculation fails as it does not support multiple reflection.

RCS reduction using radar absorber material (RAM) treatment is also conducted for the same canonical shapes. First, primary scatterer locations are identified by filtering the current on the target. Locations having larger hotspot values are coated with a RAM. It is shown that it is not required to coat the whole object. Coating main scatterer locations may reduce RCS to acceptable levels without wasting RAM.

Antenna-platform interaction is investigated. The impact of distance between antenna and platform is studied. SBR Code is compared with FEKOTM-MoM. As it is expected, larger distance leads to more accurate results.

A dipole antenna is placed on different locations of a tank so that antenna-platform interaction is studied. Results are compared with FEKOTM-MoM. It is shown that if the antenna is put in a position where the main lobe direction does not intersect the platform, antenna pattern does not deteriorate so much. Therefore, these locations are well suited if antenna pattern shape is to be preserved.

A naval radar analysis is conducted. A directive antenna is used to scan the horizontal plane. The interaction between the antenna and the ship is examined. The comparison is done with FEKOTM-UTD. It is shown that platform may act as a reflector if the directive antenna's main beam is faced on a flat scatterer. Also, if the main lobe does not look onto the platform, pattern shape does not change too much, which is a similar conclusion with tank case.

Antenna-antenna interaction is studied. First, received power formula for the receiving antenna is derived using Friis Transmission Formula. Free space coupling between antennas is investigated. A plate and a dihedral scatterer are introduced, and the limits of SBR is tested. Finally, antenna-antenna interaction in a corridor is examined. Power gain between transmitting and receiving antennas is calculated for different locations in the corridor. It is shown that SBR code can be utilized for quick analysis of indoor environments. For the case of indoor propagation, SBR code can be used if there is some error margin for the application. In this case, SBR can be used when full-wave solution methods are not available or obtainable due to computational cost.

The code can calculate the transmission coefficient of the incident field on a multilayer structure. As a future work, indoor propagation in a multiple room environment can be investigated as transmission coefficient of a wall can be calculated. Another future work can be the inclusion of diffraction effect to the code. The inclusion of diffraction to the code may lead to more accurate results. Current SBR code uses Central Processing Unit (CPU) for computation. SBR can also be implemented using Graphics Processing Unit (GPU). Calculation time can be reduced if a GPU-based code is used since the most time consuming part of the code is to find the ray-triangle intersection locations and it is faster to find the intersection points using GPU. As a future work, SBR can be implemented using GPU.

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