

A COMPARATIVE CONTENT ANALYSIS OF TWO CHILDREN'S
MATHEMATICS BOOKS: THE CAT IN NUMBERLAND AND JAYDEN'S
RESCUE

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A Comparative Content Analysis of Two Children's Mathematics Books: The Cat in
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August 2023

I certify that I have read this thesis and have found that it is fully adequate, in scope
and in quality, as a thesis for the degree of Master of Arts in Curriculum &
Instruction.

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ABSTRACT

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MA in Curriculum and Instruction

Advisor: Asst. Prof. Dr. Anita N. Alexander

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Children's mathematics books are receiving increasing attention by teachers and parents around the world. However, the choice of these books is highly depending on the popularity of the book or the publisher's suggestion lists. This study aims to shed light to how to measure the quality of children's mathematics books and observe the addressed mathematical skills and outcomes of it. By descriptive analysis of the content analysis and the content domain specific typology, this study shows two different methods to evaluate different aspects of any children's mathematics book. In this study two children's books were analyzed: *Jayden's Rescue* and *The Cat in Numberland*. The results of this study show that the two books have different mathematical skills and objectives, and different approaches to mathematical modelling. The authors' different motivations shaped these outcomes, skills, and levels for these books. *The Cat in Numberland* introduces sophisticated topics like countable infinities to its inquiring readers whereas *Jayden's Rescue* helps its readers to gain problem-solving skills for their mathematics lessons.

Keywords: Content Domain Specific Typology, Children's Mathematics Books

ÖZET

SAYILAR DİYARINDAKİ KEDİ VE KRALIÇEYİ KURTARMAK: İKİ ÇOCUK MATEMATİK KİTABININ İÇERİĞİNİN KARŞILAŞTIRMALI ANALİZİ

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Çocuklar için matematik okuma kitapları dünya genelinde öğretmenler ve aileler tarafından giderek daha fazla ilgi görmektedir. Ancak bu kitapların seçimi, çoğunlukla bu kitapların popülerliğine ya da yayın evlerinin öneri listelerine göre yapılır. Bu çalışma, çocuklar için yazılmış olan matematik kitaplarının kalitesini nasıl ölçüleceği ile kitaplarda ele alınan matematiksel becerilere ve kazanımlara ışık tutmayı amaçlamaktadır. Bu çalışma betimleyici analiz ile çocuklar için yazılmış bir matematik kitabının farklı yönlerini değerlendirmek için iki yöntem kullanır: içerik analizi ve içeriğe bağlı bir tipoloji. Çalışmada iki çocuk matematik kitabının analizi yapılmıştır; *Sayılar Diyarındaki Kedi* ve *Kraliçeyi Kurtarmak*. Bu çalışmanın sonucu kitapların farklı matematiksel beceri ve kazanımları hedeflediğini ve matematiksel modellemeye farklı alanlardan yaklaştıklarını göstermektedir. Yazarların farklı motivasyonları bu beceri ve kazanımların farklı şekillenmesine sebep olmuştur. *Sayılar Diyarındaki Kedi*, sayılabilir sonsuzluk gibi karmaşık konuları tanıtırken, *Kraliçeyi Kurtarmak* okuyucularının problem çözme becerisini kazanmalarına yardımcı olmaktadır.

Anahtar kelimeler: İçeriğe Bağlı Tipoloji, Matematik ile İlgili Çocuk Okuma Kitapları

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CHAPTER 1: INTRODUCTION

Introduction

Mathematics instructional materials often include a textbook, a notebook, and some mathematical instruments like graphing calculators, rulers, and protractors. Mathematics lessons are usually built around these objects and mathematics curriculum developed by a governing body or an organization such as Ministry of National Education (MoNE, 2018) in Türkiye or the International Baccalaureate (IB) program. The MoNE mathematics curriculum (MoNE, 2018) in Türkiye has been reformed seven times from 2005 to 2018 (İlhan & Aslaner, 2019). There have been changes in the objectives, mathematical skills, and competency expectations in the Ministry of National Education (MoNE) mathematics curriculum. Although the MoNE mathematics curriculum (MoNE, 2018) includes many different skills such as mathematical literacy, understanding mathematical concepts, problem solving skills, using the correct mathematical terminology, pattern recognition, and communicating mathematically; a comprehensive implementation of these skills by teachers in the classrooms may not be easy to achieve (Jakobsen et al., 2012). Also, mathematics is usually perceived by students as a set of rules that needs to be implemented rapidly and correctly (Guiett, 1999). This intimidates some of the students and this perception may hinder them from obtaining the desired mathematical skills (Hellwig et al., 2000).

Mathematics books for children can be used to create a natural connection between the abstract concept of mathematics and real-life cases for children to obtain mathematical skills (Whitin & Whitin, 1996). Using children's mathematics books as

an instructional tool may also allow students and teachers to expand their knowledge beyond grade level skills and objectives. Children's mathematics books can also provide a new learning opportunity for children who are not particularly competent in mathematics (Hunsader, 2004). These children's mathematics books have an advantage since they are different than textbooks or reference books. The books are written in such way that they communicate with the reader, the illustrations inside them are less formal than those in a textbook, and they are often more relevant to daily life. The choice of these children's mathematics books is also important since not all of these books are appropriate for instruction (Monroe et al., 2018).

This study aims to help parents and teachers to choose appropriate children's mathematics books by introducing different tools to evaluate different aspects of such books, by focusing on the different literary and mathematical aspects of the two children's mathematics books: *The Cat in Numberland* and *Jayden's Rescue*. The choice of these books is based on their different representation of mathematical skills and the different motivation of the authors who wrote them. *Jayden's Rescue* is written by a literary professor Vladimir Tumanov to help his mathematically struggling child whereas *The Cat in Numberland* is written by a mathematics professor Ivar Ekeland to introduce different infinities to children. The Turkish translated paperback versions of these books will be analyzed in a comparative content analysis and categorized by a content domain specific typology. The MoNE mathematics curriculum (MoNE, 2018) mathematics skills and objectives alignment of these books will also be explored.

Background

One of the many goals of mathematics education is to teach students how to communicate mathematically (MoNE, 2018; National Council of Teachers of

Mathematics [NCTM], 2000). Since the children's mathematics books facilitate this communication by familiarizing the mathematical concepts with real life (Moyer, 2000), many teachers and parents use children's books about mathematics. The four skills that are necessary for becoming competent in any topic are reading, writing, speaking, and listening (Hayes-Jacobs, 2014). By using children's books about mathematics, students ameliorate their reading and communication skills in mathematics. Gaston (2008) stated that students who read children's books about mathematics may become more motivated and interested towards mathematics. Additionally, Moyer (2000) stresses the importance of natural connection, where students obtain the information with familiarity and an integral part of their daily life rather than formal instruction and suggests this can be achieved by using children's books about mathematics in mathematics lessons.

Although there are many different children's mathematics books available that are mathematically inquiring and valuable, there are some children's mathematics books that provide little to no value mathematically (Hellwig et al., 2000; Nurnberger-Haag et al., 2021b). The use of such books will hinder the teachers' implementation and motivation towards using children's literature in the classroom and will not provide any thought-provoking ideas or familiarization of mathematical concepts for children (Hunsader, 2004). Teachers and parents have a plethora of choices for children's mathematical books by many different publishers with little to no explanation of the quality of those books. Some of the publishers create book lists for teachers and parents to use to find the contents and connection of these mathematics books for children but they do not state the reasoning behind the choice of these books or different qualities of those books (Hunsader, 2004).

The implementation of children's mathematics books is highly suggested by NCTM (2000) and when a mathematics book with a high quality is introduced to the children it is shown that it provokes the mathematical skills like communicating mathematically (Monroe et al., 2018), whereas the poor choice of these books directly affects these outcomes. There are some different standards related to the choice of children's mathematics books such as Schiro's (1997) standards. There are 11 literary and 11 mathematics standards that Schiro (1997) stated which allow for a detailed evaluation of a children's mathematics book. Hellwig et al. (2000) introduced an evaluation rubric that has five Likert charts which are similar to Schiro's (1997) standards, and also included the 'wow' factor. Whitin and Whitin (2004) talked about potential varied response in their rubric. Assessing the quality of children's mathematical books by using these rubrics is possible yet they are not easy nor fast for any teacher or parent to use. There are also typologies like Content Domain Specific Typology; Counting Book Typology by Nurnberger-Haag et al. (2021a) that allows a user to specify the counting type of the children's mathematics books with a rather straight forward classification with clearer separations. This Counting Book typology is "a conceptual lens through which to view children's books with arithmetic sequences for their learning affordances." (Nurnberger-Haag et al., 2021a, p. 150).

Problem

The use of children's literature as a tool for teaching has gained popularity in the past quarter century (Flevaris & Schiff, 2014). The benefits of using children's literature include making connections, different representations, communication, and reaching all level learners (Flevaris & Schiff, 2014). But one of the issues that is important to address is the appropriateness and the quality of these integrated books,

“because an unqualified book containing mistakes or misconceptions may harm students in numerous ways” (Durmaz, 2022, p. 606). This issue led to the focus problem of this study: How can parents or teachers choose a children’s book that aligns with the mathematical skills and objectives of a curriculum (MoNE, 2018), and that expands the students’ mathematical knowledge beyond their level’s objectives?

Purpose

The purpose of this study is to compare the contents of *The Cat in Numberland* and *Jayden’s Rescue*, two children’s mathematics books, according to their content-domain specific typology, and to explore their alignment to the skills and objectives of MoNE mathematics curriculum (MoNE, 2018). This research aims to shed light to provide a guide for teachers and parents to choose appropriate mathematical reading material for their students.

Research Question

This comparative content analysis study explores the following research question:

1. How do *The Cat in Numberland* and *Jayden’s Rescue*, the two children’s mathematics books relate to mathematical skills and objectives that are in MoNE mathematics curriculum (MoNE, 2018) based on their contents (e.g., pages, curricular outcomes represented in the books), presentation styles (narrative, question-answer based, real life examples, math history), numbers, images related to the written symbols, and number sequences?

Significance

Teachers often use children’s books to supplement mathematical instruction and skill building for students which establishes the need for a special tool to

evaluate and categorize children's mathematics books. Some of the rubrics created by researchers, e.g., Hellwig et al. (2000) and Hunsader (2004) that have been used to assess children's mathematics books had four to five criteria and mathematics and mathematical accuracy is one of these criteria. Mathematical aspects of children's mathematics books should be evaluated thoroughly, so the need for a content-specific tool led Nurnberger-Haag et al. (2021a) to analyze many children's mathematics books to create "a tool to identify potential problems and affordance for learning" (Nurnberger-Haag et al., 2021a, p. 146). The result of this work is the Counting Book Typology that will also be used as a method of content analysis in this study.

With the use of the Counting Book typology this research aims to be one of the pioneers of the analysis and comparison of children's mathematics books in Türkiye. By comparing the two children's mathematics books in terms of a content analysis and a content-domain specific typology, this thesis aims to provide an easier tool for choosing children's mathematics book to help parents, mathematics teachers, and educators to have a tool besides opinion and Likert-scale surveys (Nurnberger-Haag et al., 2021a).

Definition of Key Terms

Content-domain specific typology: "Typologies are rigorous forms of classification." (Nurnberger-Haag et. al, 2021a, p. 150) The content-domain specific typology is used to classify different children's mathematics books according to their broadened understanding of their learning affordances. Specifically, this Counting Book typology is "a conceptual lens through which to view children's books with arithmetic sequences for their learning affordances." (Nurnberger-Haag et al, 2021a, p. 150).

Children's mathematics books: The books that have numbers, shapes, or patterns in them could be counted as mathematics books. "Mathematics literature references any piece that has the potential to engage children in mathematical conversations" (Nesmith & Cooper, 2010, p. 280) is defined as a children's mathematics book. The children's books about mathematics can be divided into two different categories, mathematics concept books, and mathematics related books (Gaston, 2008). The mathematics concept books are the books that are written for a specific mathematical idea, whereas the mathematics related books have multicultural and/or multidisciplinary themes which are integrated with other curricular ideas.

CHAPTER 2: REVIEW OF RELATED LITERATURE

Introduction

G.H. Hardy, who was an English Mathematician, discovered the talent of the Indian mathematician prodigy S. Ramanujan, and they had a numerical conversation when Ramanujan was on his deathbed from tuberculosis (Hardy, 1999). Hardy remarked that the license plate of the taxi he arrived at the sanitarium in was a dull number; 1729. But Ramanujan answered that it is not a dull number, 1729 is the smallest number that one can express as the sum of two cubes in two different ways. Which are $10^3 + 9^3$ and $12^3 + 1^3$.

The question is, how did Ramanujan calculate this seemingly complicated operation in his hospital bed so easily? These kinds of anecdotes about genius mathematicians performing computational skills were thought to only belong to those who had a special kind of brain development. Demystifying this anecdote, Ramanujan calculated the cubes and the squares of the numbers throughout his entire career, and it is very likely that he memorized the cubes of these small numbers because of repetition. Also, in his journal these numbers and their sum were observed. The results answer to time spent on them rather than having a brain for computation (Dehaene, 2001).

The number sense that Ramanujan demonstrates can also be acquired by formal education. In the hands of a seasoned mathematics teacher who possesses the mathematical knowledge for teaching (Pittalis et al., 2018), a child can obtain this much-needed number sense in their school years. To prepare such a seasoned mathematics teacher, Jakobsen et al. (2012) use a different tool in teacher education;

Fermat's Last Theorem, a documentary, and a book by Simon Singh (1997) that has been used to prepare some future mathematics teachers for their classrooms. This unorthodox approach of teacher education allows teachers to gain insight into what it is good for them to learn, rather than what they ought to know.

The documentary unfolds the difficult process of doing mathematics, which involves a high resiliency and a good peripheral vision of connections. Not only do the teachers benefit from reading different mathematics books, but also their students who read a good quality children's mathematics book will have different mathematical skills like problem solving and communicating mathematically. Communicating mathematically is a goal of many curricula (MoNE, 2018; NCTM, 2000). Moyer (2000) found that in the children's books about mathematics this communication occurs with natural connections. Children see the words in the books that communicate with them mathematically such as, shorter, equal, more, half, quarter, and counting. The children's books about mathematics make a natural connection with mathematics and a normal day of a student by demonstrating the use of mathematics in daily life through stories.

Moyer (2000) also suggests that mathematics naturally emerges from reading a story in a children's book about mathematics. These daily life stories in children's mathematics books may lead students to use mathematical modelling since the questions or discussions would be different than a textbook and require a complex thinking and understanding (English & Watters, 2004). Familiarization by the help of a narrative will also assist students with the abstract concepts of mathematics (Moyer, 2000). The children's books about mathematics will provide children with a broader perspective than with direct instruction only. Abstract concepts in literature have a way of being represented in a context that uses the story line and pictures

which ameliorates the understanding of students (Ward, 2005). High quality literature helps children to connect the patterns and increase their problem-solving skills by using mathematical modelling in this natural emergence context (English & Watters, 2004).

The studies around children's mathematics books usually focus on the implementation of these books into practice (Guiett, 1999, Monroe et al., 2018). There is also research about measuring the quality of such books (Hellwig et al., 2000; Schiro, 1997, Whitin & Whitin, 2004) with different standards. The content analysis with the Schiro's (1997) standards will give a perspective to comment on the different aspects for the two children's mathematics books. Although a general analysis will be done by Schiro's standards, the need for an easier and faster tool to evaluate mathematical aspects of the two children's mathematics books brings the Counting Book Typology by Nurnberger et al. (2021a). This research will provide readers with insight and a guide for choosing children's books about mathematics. These high-quality children's mathematics books will help children gain number sense by exposure to different representations of numbers (Griffin, 2004).

Subitizing and Number Sense

Number sense is a concept that is easier to observe but it is more difficult to define (Griffin, 2004). Numbers are the fundamental building blocks of how the world is perceived. All the spoken languages have words for numbers and almost all children calculate and count with their fingers. Every one of us has an arithmetic intuition, like $3+4$ cannot be equal to 68. These intuitions are recognizing the number of things, and relating the number word to the quantity, distinguishing the different sets of objects, counting one by one and knowing after a certain number there will be one more of that number, logic, and geometry of space. Just as we are exposed to the

color spectrum and we see the colors by processing light passing through our eyes to our brain cells, these intuitions about numbers are also inevitable. These intuitions are named as number sense (Dehaene, 2001).

Children form a number sense of different levels prior to school. This number sense aspect affects their future success in mathematics lessons since number sense could be defined as a prerequisite for early mathematics (Howell & Kemp, 2005). Children develop these number recognition skills as early as two years of age. Preverbal infants are implicitly aware of numbers up to four (Dehaene, 2001). At four years of age, children have the recognition of the concepts more or less than a number, and they can count how many objects there are in a set (Aunio et al., 2006).

The implicit awareness of the numbers up to four leads us to the term subitizing. Subitizing is defined as recognition of a quantity without counting (Bruce & Threlfall, 2004). Subitizing can be done by small children if all objects to be counted are present in sight (Dehaene, 2001). Subitizing can be categorized into two types: perceptual and conceptual. While perceptual subitizing means instantly giving the number of mostly smaller quantities like 1, 2, or 3 objects that are present without counting; conceptual subitizing means giving the number of a bigger quantity by grouping smaller quantities that make a whole, e.g., instead of 4, saying 2 and 2 (Conderman et al., 2014). Since grouping leads to calculating smaller quantities into a whole, children implicitly perform multiplication (Russo, 2018).

Children are expected to operate with quantities and understand the number word system in school. Children who are more than six years old develop an understanding of a number line in their minds which means they understand number to word relation, iteration, and before and after a number without starting to count

from 1 (Aunio et al., 2006). This concept of number sense is a powerful predictor of the mathematical success of a child (Pittalis et al., 2018).

Number sense can be divided into two dimensions. The first dimension of number sense is counting, number recognition and knowledge, number patterns, and nonverbal calculation. The second dimension of number sense is to understand story problems and different number combinations. It is remarked that the first dimension is intuitional while the second dimension is acquired by formal teaching. Repeated structure of number sense becomes stronger with time, and although the first dimension seems intuitive, both dimensions can be acquired by repetition. Arithmetic concepts learned at an early age will transform into algebraic concepts in following school years (Pittalis et al., 2018). Pittalis et al. (2018) conducted a longitudinal study with 204 grade 1 students to measure their early number sense and their algebraic understanding. They concluded that there is a relation between early number sense and students' future algebraic understanding. They also suggest that early algebraizing by using shapes further enhances students' arithmetic ability.

Measurement and geometry topics in mathematics are also supported by pre-K through grade 12 mathematics curricula. Children who can represent a number and understand their relations with these topics can be said to have a number sense (Conderman et al., 2014). Estimation skills are also highly dependent on number sense. Children who depend on paper-pencil procedures to calculate rather than estimating show a lower level of number sense. Rounding would be difficult for those students who mostly use the written computational strategy. When a problem is given, understanding the meaning of the problem should be viewed as more important than solving the question by routine exercise and finding a correct answer.

This paper-pencil routine helps students to memorize strategies rather than understanding and developing a number sense (Yang, 2005).

Number sense is often said to be easy to observe, but it is difficult to define and teach, yet may be taught using some principles. For example, new mathematical concepts should be built upon children's current or prior knowledge, and this will bridge the familiarity with the novelty, and help children to make connections. This is a natural development progression which must be followed when selecting and teaching new knowledge. Computational fluency is as important as conceptual understanding for number sense; providing hands on explorations will help children to count rationally and provide opportunities for problem solving and communication.

Lastly, the exposure of the different ways a number can be represented e.g., on a number line, on a scale, with dots, and more sophisticated concepts may positively impact the number sense of children (Griffin, 2014). The use of children's books will provide "layers of meaning" which will facilitate learning with familiar examples in students (Gaston, 2008, p. 8). These "layers of meaning" might also provide a window for broader mathematical learning for children. Besides the content knowledge of their curriculum, students might be introduced to new and extensive versions of their content (Zazkis & Mamolo, 2011).

Children's Books in Mathematics Education

Vaughan and Estes (1986) defined reading as "thinking cued by text" (p. 11). The act of reading is usually defined as an active process for thinking. Reading is for understanding, not only to hear the vocal sounds of the written words. A reader is receiving the information that has been given by the author by actively understanding the text. So reading is a way of thinking and learning (Draper, 2002). There are four

skills that are necessary for a learning environment, which are reading, writing, listening, and speaking (Hayes-Jacobs, 2014). In a mathematics lesson a student needs to be able to read and understand the text, and they can listen and actively communicate mathematically by speaking or writing, as communication is essential for mathematics education (MoNE, 2018, NCTM, 2000). Mathematics lessons are usually delivered by direct instruction, where the teacher transfers the knowledge to the student, and by checking their knowledge with exercises where they can apply the knowledge directly. Different reforms in educational ideology changed this instructional transfer method to a more constructivist method.

Constructivism (Dewey, 1938/1997) is a learning theory where the learners construct their knowledge with interaction and communication, rather than direct instruction. This change in pedagogy allowed learners to approach the information with their prior knowledge, understanding, and experiences. Learners are expected to be able to communicate with their mathematical knowledge, not just solve some exercises with the given direct information. Communication goals found their place in math education, then mathematical literacy and mathematical literacy instruction became necessary aspects of mathematical instruction (Draper, 2002).

Teaching mathematical literacy and mathematical skills with children's literature also depends on the teachers' ideological orientation. Although the same teachers in a department could decide to use one children's book to teach mathematical literacy and mathematical skills, their classes' outcomes, skills, and level of their discussions will be different. Teachers who have a different approach to learning will also affect their lesson delivery according to their ideological orientation. Ideological orientation of the teacher refers to teachers' beliefs that are

affecting classroom actions such as purpose of schooling, nature of knowledge, and the knowledge most worth teaching (Cotti & Schiro, 2004).

There are four ideologies that Cotti and Schiro (2004) examined while they searched the use of children's literature to teach mathematics, which are Scholar Academic, Social Efficiency, Child Study (Constructivist), and Social Reconstruction. Scholar Academic ideology is where the information transformation occurs by direct instruction. Mathematical content understanding is a base for this ideology, and it tries to educate mini scholars; a smaller version of a researcher who absorbs and accumulates knowledge. Social Efficiency ideology is for creating a society where every student knows the skills that it needs to function productively. It uses automatic recall to assess knowledge.

Child Study (Constructivism) ideology focuses on the interaction and construction of the outcome by experience rather than the direct goal or outcome. It creates an environment for students to construct mathematical meanings themselves. This ideology focuses on the needs and the interests of the student and uses different learning tools like books and manipulatives. The last ideology examined was Social Reconstruction, focusing on bringing social justice by providing the maximum benefit from education to all its members (Cotti & Schiro, 2004). In this research study, the constructivism and social justice aspects of children's literature will be emphasized.

The use of children's books in mathematics is a way of introducing students to mathematical literacy and mathematical skills. Even basic picture books, which are a type of children's book that have a story that is depending on the relation between its text and pictures, will contribute to students' learning of mathematics (Heuvel-Panhuizen & Elia, 2012). The use of such books will help students with

concepts like classification, number concept, and shape tasks. Students not only will try to solve the problem, as they have the problems clearly stated in the textbooks, they need to understand the main goal of the problematic event that is presented in the children's mathematics book (English & Watters, 2004). Students who interacted with such books showed positive attitudes towards mathematics, and they are able to form a relationship between narratives and the mathematical concepts (Heuvel-Panhuizen & Elia, 2012). Students can familiarize themselves with the mathematical words such as shorter, equal, more, half, quarter, and counting, and they can form a natural connection to mathematics with this integral part of their everyday experiences (Moyer, 2000).

Children's Books as an Instructional Tool in Mathematics Lessons

Introducing children's literature into mathematics lessons as an instructional tool allows students to be interested and motivated towards learning mathematics. Students become better critical thinkers and problem solvers. Students also start to connect mathematics with their personal experiences and thus they appreciate real life situations that involve mathematics (Gaston, 2008).

Different types of children's literature about mathematics enriches their education as well. There are two ways of introducing mathematics in literature; Math concept books, which are specific books that have been written about a mathematical idea, and Math related books, which have different and sometimes multicultural themes than mathematics but integrate different concepts and curricular ideas into its narrative (Gaston, 2008). The math related books could reach students who may or may not have previously found success in mathematics. The quality of the literature promotes not only the mathematical literacy, but also the language literacy. If there

are more layers for students to understand, the literature will facilitate the mathematical connection and promote students' inquiry (Gaston, 2008).

Children's books about mathematics as a teaching tool in the classroom also help as a bridge between the abstract and symbolic world of mathematics with more world perspectives and examples. Children's books often include a problem, which allows the reader to observe it being solved by another child. This perspective of a solution helps students to familiarize themselves with the mathematical concept (Ward, 2005). For example, a suitcase supposed to have one billion dollars enters the class of pre-service mathematics teachers carried by Ward (2005). Ward asks the famous question "How long would it take to count the one billion dollars enclosed in this suitcase if I were to count a dollar a second?" (2005, p. 132). The answers varied but the case of number sense is the most important concept in this question. Many of the teachers responded that it would take one billion seconds but when it comes to calculating how many days, or weeks, or even years, one billion seconds is a mystery to many of the teachers. Clearly, they can calculate it, but estimating was a difficult task for those pre-service mathematics teachers. While the pre-service teachers were faced with this question, they also had to use their mathematical modelling skills. Ward (2005) gave away the answer; almost 32 years. Many of the pre-service mathematics teachers were surprised that even if they counted for all the time that they were alive, they would have only counted approximately two thirds of the one billion dollars.

The relation with this problem to children's literature is the book *How much is a million?* (Schwartz et al., 1985). By the illustrations in the book, young readers could conceptualize the larger numbers as million, billion, and even a trillion with ease. The book forms a bridge between quantities in mathematics with real life.

Even if the mathematical concepts are symbolic and abstract, children's literature has a way of picturing them in a context of the story using informal words as a medium (Ward, 2005). Different books like *If the world were a village* (Smith, 2011), and pre-service teacher authored book *The story of a young map colorer* were also given by Ward (2005) as good integrations of broader and more sophisticated mathematics. In Smith's (2011) book the world population is represented by 100 people. Their proportionate demographics, their access to facilities, and their socio-economic backgrounds were examined. Ward (2005) suggests that this would allow readers to have a broader understanding of the world. *The story of a map colorer* was introduced as good literature in which an introduction to the four-color map problem is provided, where it states only four colors are enough to color any map where all the neighbors have different colors. Ward (2005) also encourages teachers to not only use literature as reading but also to introduce poetry based on word count for their students to write their own poems. Haikus, limericks, cinquain, and diamantes are examples of those poetry types that are based on word count. Students could also "got a kick out of the rhymes" (Ward, 2011, p. 141).

Literature as a teaching tool in the mathematics classroom is not just a mathematics teaching tool. Literature not only helps students struggling in mathematics to be motivated, but it also helps students who like mathematics to increase their enthusiasm about literature (Ward, 2005). According to Schiro (1997), using children's literature about mathematics is a wonderful way of helping students learn mathematics while enjoying an enriching book, and understanding and learning about literature. To accomplish this goal, high quality literature about mathematics is needed (Schiro, 1997).

Qualities of Effective Children's Mathematics Books

In his book *Integrating Children's Literature and Mathematics in the Classroom* (1997), Schiro stresses the lack of the assessment of the quality of the current children's mathematics books. The first quality he suggests is the accuracy of mathematics, but the literary qualities of the books are also important. Schiro (1997) has mathematical concerns and literary issues about children's books, and he suggested two sets of standards for the evaluation of the children's books: mathematical and literary standards.

Schiro (1997) was mainly concerned about the mathematical accuracy of such literature. In his research he found a concerning number of books which had incorrect mathematics. To evaluate the correctness of mathematics in such books, not only the mathematical aspects given must be evaluated but also the "text, pictures, illustrations, diagrams, graphs, charts, numbers, equations, algorithms and scale must be accurate and correct" (Shiro, 1997, p. 77). This mathematical accuracy has also been mentioned in the Hellwig et al. (2000) study for the rubric of selecting children's books for mathematics instruction. If the literature has mathematics represented correctly, it should be presented effectively as well. The mathematics should be consistent with the storyline, graphs, and illustrations. The mathematics should also be worthy of learning, it should be age appropriate with the reader, and it should broaden the horizon of the reader about mathematics (Mosvold & Fauskanger, 2014). The mathematics in the books should also be visible to the reader. The numbers should not only be placeholders for quantity words like some or many; when the numbers appear in a sentence in the story, the reader should be able to interact with the numbers mathematically and operate with them.

Mathematics often may appear as a difficult topic for students to comprehend. One of the ways to reduce the anxiety of mathematics for students is to show them relatable examples. Mathematics literature for children is a nice way of presenting this relatable content. It is also one of Shiro's (1997) good quality properties in children's mathematics books. An appropriate view of mathematics is particularly important for the quality of the book since students often need a peer example. The examples in these quality books should not be taking away the joy of doing mathematics by asking irrelevant or excessively difficult questions, and/or people having a negative opinion about mathematics. The examples in these books may introduce some everyday use qualities of mathematics, sociable qualities of solving or doing mathematics, and its accessibility by everyone.

This accessibility is a good quality asset for a children's book about mathematics where it is also supported with the intellectually challenging aspects for the reader. A good quality children's mathematics book should also have some challenging aspects, and it should be enjoyable and have enough intellectual stimulation for all age groups (Hellwig et al., 2000). The contents should consider the prior knowledge of the children where it can build onto that knowledge with good challenging questions (Shiro, 1997). The questions or the storyline in the book should involve the reader as well.

There are three different levels of involvement according to Shiro (1997); observe, listen in, do. The observation level is where the reader is just being told about a mathematical concept. The listen in level is where the reader has a chance to see how the characters thought pattern of mathematics has occurred. The do level is where the reader is required to do mathematics to appreciate the story. The listen in level is a safe level for most of the children's mathematics books because if the

necessary mathematics have not been provided within the book, it would not be accessible for every reader. Providing the necessary information to read the mathematics book is also stated as a quality by Shiro (1997). It is important that if the book provides the answers to the questions that it has, or if there are some questions that require some materials to solve, whether they are given with the book, or if they are easily accessible.

Besides the literary qualities of the books, the mathematics and the story of the book should be intertwined. In good quality children's literature about mathematics, the storyline should have mathematics as a way of continuum of the story. The story should not be cut out to do mathematics. They should be complementary of each other (Schiro, 1997). The story and the characters should present the mathematics in a way such that the reader would like to join in. The pleasantness of the story and generalization of the facts should facilitate the understanding for the reader. Lastly, Schiro (1997) suggests that the book should be able to be read by itself. The research that is needed to understand the book must not be excessive. This property of a good quality book should not be crossed with the book's horizon broadening information, or some closure questions about extensions of the given mathematics.

The literary standards are also important for good quality children's literature about mathematics. The plot of the book should be logical and believable, and it should have a sequence. The characters should also be believable and relatable. The style of the book should be engaging for the reader, where its language is grammatically correct and age appropriate for its readers (Hellwig et al., 2000; Shiro, 1997). The book should be interesting and worth the read. The reader should receive some worthwhile information after reading the book. The illustrations, graphs and

pictures should be good, accurate, and relevant to the story. The mathematical representations, e.g., numbers, charts, shapes, etc. must be used appropriately and correctly. The characters should be represented diversely, and they should be role models to the readers. Inclusion of all cultures and appreciation of the cultures and historical events, if represented, should be correct and respectful. The book and its cover should also be appealing, and good quality materials should be used.

An effective children's book about mathematics should have the mathematical and literary standards which Shiro (1997) suggested. There are different tools to measure this effectiveness quality, like Shiro's literary and mathematics standards which will be used to examine the books' literary and mathematical quality in this study. Rubrics such as the rubric of Hellwig et al. (2000) has five Likert charts about accuracy, visual and verbal appeal, connections, audience, and 'wow' factor. The Whitin and Whitin (2004) rubric added some standards like mathematical integrity, potential for varied response, an aesthetic dimension, and ethnic, gender, and cultural inclusiveness. Typologies like the content domain specific typology; Counting Book typology by Nurnberger-Haag et al. (2021a) help teachers and researchers to classify good quality mathematics books easily and rapidly. In this research study, one of the aims will be analyzing the books *The Cat in Numberland* and *Jayden's Rescue* according to this typology and classifying them to help teachers and researchers with the books' mathematical content.

Connections Between the Books *The Cat in Numberland* and *Jayden's Rescue* and Fifth Grade Mathematics MoNE Curriculum (MoNE, 2018) in Türkiye

Middle school mathematics objectives of MoNE mathematics curriculum (MoNE, 2018) included in the children's books can be examined under five

categories; Numbers and Operations, Algebra, Geometry, Measurement, and Data analysis and Probability (Genc et al., 2017). The numbers and operations have different subcategories such as counting, recognizing, and expressing numbers, ordinal numbers, and four operations. Algebra contains the subcategories pattern recognition, substitution, and sorting. Geometry focuses on geometric shapes and their properties, symmetry and translations, and geometrical modelling. The measurements category has physical measurements like length, weight, area and volume, and non-physical measurements like time, measuring tools, comparison, and estimation. The data analysis and probability category are all about data, collecting, sorting, and interpreting. The fifth-grade mathematics objectives of MoNE mathematics curriculum (MoNE, 2018) will be a focus of this study because of the age correspondence with the books' content and accordance of the publishers' age group for these books. The six units of middle school mathematics are represented in the fifth-grade mathematics Ministry of National Education (MoNE) curriculum in Türkiye in Table 1.

Table 1

Mathematics Units in MoNE Curriculum (MoNE, 2018)

Unit	Topics
Unit 1	M.5.1.1 Natural Numbers M.5.1.2 Operations with Natural Numbers
Unit 2	M.5.1.3 Fractions M.5.1.4 Operation with Fractions
Unit 3	M.5.1.5 Decimals M.5.1.6 Percentages
Unit 4	M.5.2.1 Point-Line-Plane Postulate

Unit	Topics
	M.5.2.2 Triangles and Quadrilaterals
Unit 5	M.5.3.1 Collecting and Interpreting Data
	M.5.2.3 Length and Time Measurements
Unit 6	M.5.2.4 Area of Rectangles
	M.5.2.5 Solids

The Unit 1 on the natural numbers set is introduced to fifth grade students where they learn how to read and write the numbers up to 9 digits. Students should be able to write the 9-digit numbers in their expanded form, and be able to identify the group, and the place values of the digits of the given number. Operational goals of the curriculum are that students should be able to add and subtract at most five-digit numbers. Students should be able to estimate the sum or difference of two-digit numbers, multiply at most three-digit numbers, and divide a four-digit number into two-digit numbers.

Students should be able to estimate the results of multiplication and division by using various strategies. In division problems, students should be able to interpret the remainder. Students should be able to find a relation between division and multiplication, find the missing element of one operation by using the other one, represent a number multiplied by itself multiple times as an exponent, and solve operations with parenthesis and with two operations.

In the Unit 2, Fractions and operations with fractions, students are expected to achieve the following goals: students should be able to order unit fractions on a number line, define fraction types and understand the relationship between mixed fractions and improper fractions, and convert them to each other. Students should

understand the equivalent fractions by simplification and expansion. Students should be able to order fractions where they have equal numerators or denominators, find the fraction of an amount or vice versa. Students should also be able to add and subtract the fractions that have the same denominators or the denominators that are multiples of each other.

In the Unit 3, students understand the relationship between the fractions that have a denominator of 10, 100, and 1000, and the decimals. Students should be able to tell the place value of a decimal given up to thousandths, to convert a fraction with a denominator that is a factor of 10, 100, or 1000 to a decimal, and add or subtract decimals. Students should also be able to compare decimals, fractions, and percentages. Students are expected to find the percentage of an amount or find the total if a percentage of it is given.

Middle school mathematical skills that are in the MoNE mathematics curriculum (MoNE, 2018) are improving the literacy in mathematics, understanding and implementing the mathematical concepts in their daily life, problem solving, communicating mathematically, estimating and pattern recognition, different representation of the concepts, positive attitude towards mathematics, understanding the mathematics relation with art and beauty, and understanding and appreciating that mathematics is the global language of humans.

Although there are many different objectives under these six units, the objectives that are directly addressed within the books analyzed are highlighted. In this study, *The Cat in Numberland* and *Jayden's Rescue* will be analyzed according to their alignment with the outcomes and skills of fifth grade MoNE mathematics curriculum (MoNE, 2018) in addition to their comparative content analysis.

Mathematical Modelling and Mathematics Skills

Mathematical modelling is a model that can describe, explain, or estimate a system by using systems of relations and mathematical connections between the case and its result (Doerr & English, 2003). Instead of focusing on the solution of a well-defined problem, mathematical modelling focuses on creating new systems and generalization of those systems to many similar not so well-defined problems (Blum, 1993). In mathematics lessons usually there are some routine situations, where students given a problem for one solicited case of a general problem, and they are expected to use some mathematical operations and/or formulas to solve that question (English, 1998). Although it is a system that may improve routine situation question solving ability, students need to familiarize themselves with nonroutine situation questions. Nonroutine situation questions are defined as questions where students need to understand the system behind it by trying different approaches and the solution is not one formula and/or one step away from them (English, 1998). These type of nonroutine questions where students can create a relationship between mathematics objectives and skills that they learn in the classroom and the question, help them to create generalizable and reusable systems of solutions (Doerr & English, 2003).

English (1998) examined the problem posing skills and its effect on fifth grade students. Students were chosen from five different fifth grade classrooms according to their number sense and novel problem-solving skills. This study is important since in the beginning weeks of the study, a similar problem was introduced in different familiarization levels, like the routine question students faced which have the computational solutions to the nonroutine, new type of problems which require students to use their reasoning skills or to think beyond the objectives

of fifth grade to solve these questions. In the appendix of this study the number sense and novel problem-solving questions are given. One of the number sense examples was as follows: “Which is greater: 1235 or 999? How do you know?” (English, 1998, p. 212) and one of the novel problem-solving examples: “Crusty’s Bakery bakes whole meal bread, rye bread, and white bread. These come in round loaves and square loaves, and have either poppy seeds or sesame seeds. How many different kinds of loaves are there to choose from?” (English, 1998, p. 212). In the examined books *The Cat in Numberland* and *Jayden’s Rescue*, mathematical questions in those books will also be analyzed according to their different problem posing skills and whether they are capable of inducing mathematical modelling skills of their readers.

There are different arguments around mathematical modelling where Blum (1993) argues that some of the questions that are asked in the mathematics lessons are “dress ups” (Blum, 1993, p. 4) of a purely mathematical problem. In order for a question to be solved by mathematical modelling there are different steps that a question needs to address. According to Blum (1993) the starting point should be a real-world case which is outside of the mathematics, then this real model will be mathematised around its components where one or more different models may be required. The results of these models will be applied to the real-world, where sometimes they might need to be changed since the models might not be applicable. This cycle of modelling and returning to the original case with different points of view is considered mathematical modelling of the real-world situation. Students not only need to know the mathematical objectives, but they also need to apply their mathematical skills such as problem solving, pattern recognition, different representation of mathematical concepts, and mathematical communication.

CHAPTER 3: METHOD

Introduction

This study used comparative content analysis and content-domain specific typology to address the following research question:

1. How do *The Cat in Numberland* and *Jayden's Rescue*, the two children's mathematics books relate to mathematical skills and objectives that are in MoNE mathematics curriculum (MoNE, 2018) based on their contents (e.g., pages, curricular outcomes represented in the books), presentation styles (narrative, question-answer based, real life examples, math history), numbers, images related to the written symbols, and number sequences?

Research Design

In this study a comparative content analysis (Karasar, 1986) and a content domain specific typology (Nurnberger-Haag et al., 2021a) are used to classify the two children's mathematics books: *The Cat in Numberland* and *Jayden's Rescue*. A plan for analysis of these two children's books based on the content analysis and the content-domain specific typology (presentation styles, numbers, images related to the written symbols, and number sequences) is described here. Although the methods of this study are largely qualitative, there are quantitative aspects from the analysis of the books, such as descriptive statistics, page numbers, and number of pictures.

Context

The contents of the two books; *The Cat in Numberland* and *Jayden's Rescue* were analyzed. These books are widely used in fifth grade mathematics classrooms in Türkiye. Turkish translated paperback versions were analyzed in this study.

The Cat in Numberland (*Sayılar Diyarındaki Kedi*, 2006/2018) was written by Ivar Ekeland. The book has been written for children to understand difficult concepts of mathematics like infinity. Ivar Ekeland is a French mathematician of a Norwegian descent. He has written many books about university and higher-level mathematics but also popular science and children's books. His books have been published in French, English, and many other languages in addition to Turkish.

Jayden's Rescue (*Kraliçeyi Kurtarmak*, 2002/2004) was written by Vladimir Tumanov. This book was written to motivate Vladimir Tumanov's mathematically struggling son Alex. Vladimir Tumanov is an Associate Professor of Modern Languages at a university in Canada and is of Russian descent. His work focuses on acquiring second language and literature comparisons. He wrote his first children's books in 2002. Not only is he interested in children's mathematics books, but he also writes geography books for children.

In this study, these two children's books, *The Cat in Numberland* and *Jayden's Rescue*, were chosen because they have been written for similar age groups, mathematical objectives, and skills. The choice of the books is based on their different representation of mathematical skills and the different motivation of the authors in writing them. These books were analyzed according to their presentation styles and how they introduce mathematics, and their relation to the mathematical objectives and mathematical skills in MoNE fifth grade mathematics curriculum (MoNE, 2018).

Instrumentation

The two children's mathematics books were analyzed using the evaluation standards for children's mathematics books in Schiro's (1997) *Integrating Children's Literature and Mathematics*. There are two sets of standards defined for evaluating the children's mathematics books which are literary standards and mathematics standards. The literary standards include the literary qualities of the books such as plot, characters, and audience appropriate use of language. The mathematics standards include not only the correctness of the mathematics presented but also its use of horizon concepts and its interactivity.

Furthermore, the books were analyzed for counting and arithmetic by using the Counting Book Typology (Nurnberger-Haag et al., 2021a) which is a content domain specific typology. This typology facilitates readers in the process of specifying the types and qualities of children's mathematics books by three categories: sequences, explicitness, and rationality.

Literary and Mathematics Standards for Children's Books about Mathematics

Schiro (1997), in his book *Integrating Children's Literature and Mathematics* in Chapter 4 Evaluation Standards for Children's Mathematics Trade Books, defines two sets of standards for evaluating children's literature about mathematics: Mathematics Standards and Literary Standards. The Mathematics Standards of Schiro (1997) are:

A book's mathematics should be correct and accurate.

A book's mathematics should be effectively presented.

A book's mathematics should be worthy of being learned.

A book's mathematics should be visible to the reader.

A book should present an appropriate view of mathematics.

A book's mathematics should be intellectually and developmentally appropriate for its audience.

A book should involve the reader in its mathematics.

A book should provide readers the information needed to do its mathematics.

A book's story and mathematics should complement each other.

A book should facilitate readers' use, application, transfer, and generalization of its mathematics.

The resources needed to help readers benefit from a book's mathematics should not be too great. (p. 77)

The mathematics standards present the qualities expected from a good children's book about mathematics. Although they appear like yes/no type of standards, different levels of these standards could be evaluated. After the mathematics standards, Schiro (1997) explains that there are some literary standards that should be present in the book as well. The Literary Standards of Schiro (1997) are listed as follows:

Plot: A book's plot or story should be well developed and imaginative, flowing logically, believably, and sensibly from one idea to the next.

Characterization: A book's characters should be well portrayed and believable.

Style A book should contain a vivid and interesting writing style that actively involves the child.

Language: A book should use correct grammar and punctuation and age-appropriate language and style.

Readability: A book's readability and knowledge content should be ap-

appropriate to the reader's age.

Interest: A book's interest level should be relevant and developmentally age-appropriate to the reader.

Enrichment: A book should enrich the child.

Graphics: A book's illustrations, pictures, or graphics should be well chosen, appealing, relevant to the text, and representative of a child's view of the world.

Unity: A book's plot, content, and graphics should convey the same stylistic message.

Respect: A book's tone should respect the reader, its characters should provide positive role models for readers, and its characters should be culturally diverse and free from stereotype.

Physical traits: A book should be visually appealing, well organized, durable, and laid out to produce easy comprehension. (p. 78)

Although the literary standards also appear like yes/no type of standards, the different levels of them could be evaluated. These standards that Schiro developed could also be enhanced by different standards that are used by different researchers, like Whitin and Whitin (2004) suggested that varied responses to the same question in the book could improve the books' readability by different levels. Hellwig et al. (2000) include that although the understandability by the target audience is important, it is also a good asset of a quality book to be targeting a broader range of audience. Hellwig et al. (2000) also include the 'wow' Factor in their rubric, meaning the richness of the book with its unpredictability. The content analysis of the books *The Cat in Numberland* and *Jayden's Rescue* was subjected to these literary and mathematics standards in this study.

Content-domain Specific Typology

The Counting Book typology by Nurnberger-Haag et al. (2021a) was one of the appropriate methods to analyze the two children's books *The Cat in Numberland* and *Jayden's Rescue* because it allows for a categorization of the books according to their mathematical features so that it identifies the possible problems and opportunities for learning. This Counting Book typology analyzes a number book under three categories: counting sequences, explicitness, and rationality. This typology will help teachers and parents to understand the books' alignment with their curriculum by these different categorizations.

Nurnberger-Haag et al. defined counting sequences in a book as a single or a multiple sequence (2021a). The single sequence means there is exactly one sequence in the book. There are different examples of this type of sequence which are Whole Number Counting (WC), Skip Counts (SC), and Ordinal Sequence (OS). Examples of these different types of single sequences are given respectively. In a book if there is a sequence of a number that is counting one by one, increasing or decreasing, it is called whole number sequencing. If in a book there is any different scaled counting, such as counting by fives, it is identified as skip counts. If in a book there are at least three ordinal numbers following each other, for example second, third, and fourth, it is an ordinal sequence.

The multiple sequences are examined under two titles; starts new and counts on. Starts new is when in the book there are multiple scenarios when the book uses different sequences as examples. In the book, the character(s) can be shown counting things and then stop and start counting different things. Starts new has three subtitles which are same scalar to different values (SDV), different scalars to same value (DSV), and multiple different scalars (MDS). Examples of the given titles are for

SDV; the book can start counting by twos and finish that sequence it has started and start a new sequence from a different number, and then start with a different number but again count by twos. An example for a DSV is different in that although it has different scalars like counting by fives or by fours, the ending number of both sequences is the same, for example twenty. An MDS is when there are at least two different scaled sequences ending with different numbers present in the book.

Counts on is when in the book there is such context that counting occurs with continuity. For example, counting money or time. In the book there can be different measurements or scales that are adding onto each other, and there can be different scalars to count them on. Counts on has two subtitles: Multiple Scalars with a Scenario (MWS) and Multiple Scalars with No Scenario (MNS). In MWS type of books there is a context for the reader to understand the scale change, for example if they are counting money, there can be different types like quarters, nickels, and dimes in USD. In MNS there is no context, for example after counting one by one up to ten, the book may continue counting by tens without giving a reason or context to it. Examples related with counting sequence types could be seen in Table 2.

Table 2

Examples of Counting Sequence Types (Nurnberger-Haal et al., 2021a, p. 153)

Category Terms	Code	Data Examples
Single Sequence		
Whole Number Counting	WC	0, 1, 2, 3, 4, 5, ... , 25, 26
Skip Counts	SC	5, 10, 15, ... , 40, 45
Ordinal Sequence	OS	First, second, third, ...
Multiple Sequences		
Starts New		

Category Terms	Code	Data Examples
Same scalar to different values	SDV	0, 2, 4, 6, ... then starts 1, 3, 5, ...
Different scalars to same value	DSV	4, 8, 12, ..., 20 then 5, 10, ..., 20
Multiple different scalars	MDS	2, 4, 6, ..., 20 and 3, 6, 9, ..., 30
Counts on		
Multiple scalars within a scenario	MWS	25, 30, 40, 50, 51 scenario where they counted a quarter, then a nickel, then two dimes and one penny.
Multiple scalars no scenario	MNS	1, 2, 3, ..., 10, 20, 30 ...

Nurnberger-Haag et al. define explicitness in a counting book as how openly the reader can understand if the book makes them count (2021a). The explicitness was expressed in three subtitles. These are explicit scalar (ES), explicit counting (EC), and implicit counting (IC). The examples of these subtitles are for ES, text explicitly announces that it is counting and by which scalar. EC, however, explicitly announces counting but does not state a scalar. In IC the book does not use the word counting or state a scalar, it only uses the numbers in context.

Nurnberger-Haag et al. define rationality as how well the pictures and illustrations in the book draw the attention of its readers to the changes in a sequence (2021a). Rationality also was expressed under three subtitles. These are rational counting sequence (RS), rational counting terms (RT), and rote counting (RC). The difference between the RS and the RT is that although the books have relevant pictures to accompany the given numbers or numerals, in RS the sequences are shown by adding or taking away from the previously presented illustration, where in

RT the pictures are presented separately. In RC the numerals or numbers are shown without images. Data obtained from counting sequence types will be quantitative and data collected from explicitness and rationality will be qualitative.

Method of Data Collection

In this study the content analysis method is used to collect data. According to Karasar (1986), the analysis of the related documents to the research questions are identified as a content analysis. Content analysis could be a sole data collection tool in a research study, though in this study the books chosen for analysis were also selected according to the researcher's professional experience of teaching mathematics in fifth grade level.

The books that are studied in this research are acquired in their Turkish translated paperback versions. Related literature review for this study was conducted by reading various articles and books about the use of children's mathematics books, and the broader effects of reading and using such books both at home and in the classroom. *The Cat in Numberland* by Ivar Ekeland, and *Jayden's Rescue* by Vladimir Tumanov were chosen for this content analysis because of their familiarity in Türkiye and their use in mathematics classrooms.

The data was collected according to the presentation style of these books; mathematics and literary standards of Schiro (1997). Literary standards and the presentation style include the collection of the cover pages, count of pages, plots of these books, explaining the main and supporting characters in these books, books' interaction style with its audience, whether the books are grammatically correct or not, and the illustrations in the book and their relevance to the narrative. Mathematics standards data collection involved examining every question and mathematical concept for accuracy, the presentation of mathematical concepts or

problems, affordances of learning, the ways that books represent mathematics (illustrations, graphs etc.), and facilitation of understanding of mathematics (whether the book gives out the answers to the questions in the book or not).

The Counting Book typology requires three types of data to be collected; to analyze the sequence aspect, all different number sequences presented in the books must be collected. For the explicitness types, whether the word “count” is explicitly stated or not, and whether book states it is counting by a scalar or not must be collected. For the rationality aspect, whether the relationship between images and the illustrations with the sequences in the book are present.

Methods of Data Analysis

Koh et al. (2000) define descriptive analysis as a “study of status” (p. 219), where the researcher acquires information for the research question by observation, and the problems could be solved, or the practices could be improved by this observation, analysis, and description. The two children’s mathematics books, *The Cat in Numberland*, and *Jayden’s Rescue* were analyzed by using Schiro’s (2007) literary and mathematical standards, according to their similarities and the differences. The mathematical presentations of the problems in the books are also analyzed by the different mathematical skills and objectives that they refer to. Additionally, the books were classified according to the Counting Book typology by Nurnberger-Haag et al. (2021a) by the number sequences they have and the explicitness and the rationality aspects. Results of this content analysis are presented in the next chapter, followed by recommendations for teachers and parents for choosing appropriate children’s literature.

CHAPTER 4: RESULTS

Introduction

In this chapter, results from the analysis of the children's books about mathematics; *Jayden's Rescue* and *The Cat in Numberland* were presented to address following research question:

1. How do *The Cat in Numberland* and *Jayden's Rescue*, the two children's mathematics books relate to mathematical skills and objectives that are in MoNE mathematics curriculum (MoNE, 2018) based on their contents (e.g., pages, curricular outcomes represented in the books), presentation styles (narrative, question-answer based, real life examples, math history), numbers, images related to the written symbols, and number sequences?

Presentation Styles

The contents of the two children's books about mathematics; *Jayden's Rescue* (*Kraliçeyi Kurtarmak*) and *The Cat in Numberland* (*Sayılar Diyarındaki Kedi*) were analyzed in two perspectives; Literary Standards and Mathematical Standards (Schiro, 1997). The literary standards for evaluating the children's books include the plot, characterization, style and language, readability, interest, enrichment, graphics, unity, respect, and physical traits. The mathematical standards for evaluating the children's books include accuracy, presentation, worthy of learning, visibility, appropriate for its audience, involvement of the reader, information needed to do the mathematics, story and mathematics completion, and facilitation.

Literary Standards of the Children's Books about Mathematics

In this section, physical traits, plot, characterization, style, and language in both children's books are represented. The paperback versions of these books used the same quality cover and end pages. *The Cat in Numberland* is printed on white paper while *Jayden's Rescue* is printed on a much softer, off-white paper.

Figure 1

Cover Pages of the Children's Books about Mathematics



Note. From Cover illustration by Sadi Güran; *Jayden's Rescue* (*Kraliçeyi Kurtarmak*), by Vladimir Tumanov, 2004. Cover illustration by J. O'Brien; *The Cat in Numberland* (*Sayılar Diyarındaki Kedi*), by Ivar Ekeland, 2018.

Figure 1 shows the cover pages of these two books: *Jayden's Rescue* and *The Cat in Numberland*. On the cover page of *Jayden's Rescue*, the picture of the Queen Jayden whom the reader should save by solving mathematics questions was illustrated. This cover page does not have any mathematical symbols on it. On the cover page of *The Cat in Numberland*, the cat and the many numerical characters are illustrated. Not only does the title imply it is a mathematical book, but the illustrations also show that there will be some mathematics in the book.

The illustrations continue inside these books as well. In *Jayden's Rescue*, there are 20 pages of illustrations in the 176-page book. The illustrations are mostly character illustrations. Illustrations are not directly related with any of the given mathematics problems in the book. In *the Cat in Numberland*, there are 61 pages and there is not even one page without an illustration. Illustrations of the places, story characters, and all the numbers are all illustrated even if it is only a small design next to the text. Most of the illustrations are related to the mathematics that are written in the story.

The plots of these two books are well developed, they occur sequentially, and they have believability in their own context. In *Jayden's Rescue*, the plot starts when Aleks, the main child character in the book, finds a magical pencil. He and his friends figure out the properties of this magical pencil, followed by them finding a book that has many mathematical questions that they need to solve to save Queen Jayden from an unwanted marriage with the evil king Rechner. The narrative is in third person, so the reader observes those children having an adventure.

The Cat In Numberland states that it starts on a different planet, where there is an infinity hotel. The story starts when an extra guest comes into their hotel, and it confuses the hotel's cat. The story is established with natural or counting numbers as the only guests of the hotel, and it continues to introduce new number sets as guests of the hotel, and this confuses the cat more and more. The narrative of this book is also in third person.

The characters of these two books are well portrayed and believable in their context. In *Jayden's rescue*, the main characters are three children that are friends: Aleks, Sam, and Vanessa. Aleks is the main character who has his difficulties with mathematics lessons which he overcomes at the end of the book. Two important side

characters are the Queen Jayden and the evil king Rechner, where the children try to save Queen Jayden from the evil king Rechner. *The Cat in Numberland* has three main characters that are the cat of the hotel, and Mrs. and Mr. Hilbert who own the Hilbert Hotel in the story. All the side characters are numbers and fractions from number sets and letters of the alphabet.

The style of these two books differs in drawing the attention of the reader. *Jayden's Rescue* actively involves the readers by introducing mathematical problems. Although the storyline seems to cut to do mathematics, these questions include the reader. The answers are also presented after the problems in the narrative as main characters solving them for the pursuit of saving Queen Jayden. *The Cat in Numberland* has a style of confusing the reader by creating an event seemingly impossible to solve. In this book the infinity hotel is full of natural, or counting numbers that are 1, 2, 3, ... and it goes to infinity. Every number has its own room which corresponds to its number, such as 1 to the first room, 2 to the second room, and so on. One night the door of the hotel was knocked on by the number and character Zero and this complicated things. If the hotel is full, how would they fit this new number? The narrative then explains the concept of infinity by moving every number one room over, e.g., 1 to the second room, 2 to the third room, and so on, to find a room for their new guest.

In addition to how the books involve the reader, the styles of these books also differ in mathematical sense. *Jayden's Rescue* chooses to explicitly ask its readers to solve mathematics questions and learn about them by solving, whereas *The Cat in Numberland* implicitly gives away the mathematical concepts like number sets and infinity. The story tries to create an inquiry around these topics.

The language of these books is age appropriate to nine and ten years old children which are the children's ages in fifth grade. Both books are grammatically correct. The words chosen in the text are simple and familiar to the age group's everyday language. The fonts are with serif and their sizes are easy to read in both books.

Illustrations in both books are explanatory of the texts. In *Jayden's Rescue*, on page 55 (Tumanov, 2002/2004) there is a problem about a child who has five heads that needs to comb the hair on their heads. The problem is written in a lyrical way, as there are rhyming syllables at the end of each verse. All the problems that children solve to save Queen Jayden are written in a lyrical way with rhymes. The combing problem in *Jayden's Rescue* (Tumanov, 2002/2004) is as follows (translated to English by the researcher):

Yığınla saç var başlarımda, (There are many hairs on every one of my heads)

taramaya yetmez zamanım. (There is not enough time to comb all of them)

Her baştaki saç taradığımda, (When I comb one of the head's hairs)

anam babamdan bir onluk kazanırım. (I'll get a dime from my parents)

Ancak, her baş taranmalı sırasıyla, (All heads must be combed in order)

bir, iki, üç, dört, beş diye mutlaka. (Must be one, two, three, four and five)

Anam der ki düzen hiç şaşmamalı, (My mom says the order shall never change)

her iyi canavar bu hedefe erişmeye çabalamalı. (all good monsters must try to achieve this goal)

Gururla söylemeliyim ki şimdi kazandığımı (I am proud to say that I earned)

tam tamına hakkıyla on iki dolar. (Exactly twelve dollars by my own efforts)

Söyle bakalım, kaç kez taramışım, (say to me how many times have I combed)

her kafamdaki saç yumağını? (The hairs on every head of mine) (p. 55)

The illustration in Figure 2 shows a five-headed child with combs around them.

Although in *Jayden's Rescue*, the main children's characters are never illustrated, in *the Cat in Numberland* all the main characters illustrated as can be seen in Figure 3, Mrs. and Mr. Hilbert, the cat, and the numbers.

Figure 2

The Five Headed Children in Kraliçeyi Kurtarmak



Note. From *Jayden's Rescue* (p. 56), V. A. Tumanov, 2004. Günışığı Kitaplığı.

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Figure 3

Mrs. & Mr. Hilbert, the Cat, and the Numbers in Sayılar Diyarındaki Kedi



Note. From *The Cat in Numberland* (pp. 14-15), I. Ekeland, 2006. Doğan Egmont.

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Mathematics Standards of the Children's Books about Mathematics

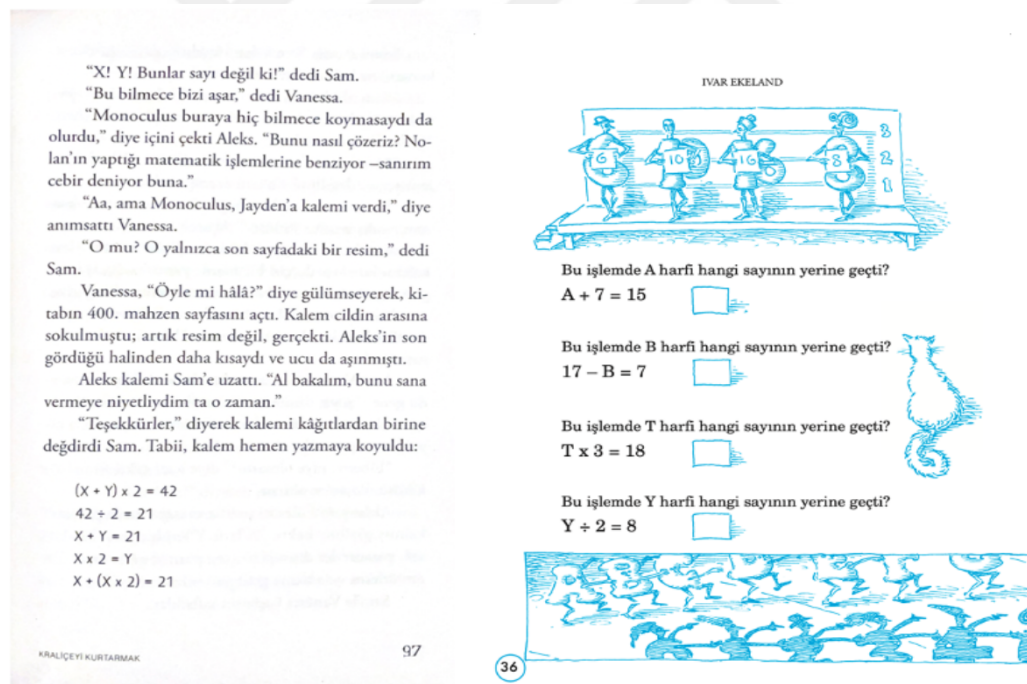
In *Jayden's Rescue* and *The Cat in Numberland*, the mathematics presented is correct and it aligns with the objectives and mathematical skills of the fifth-grade curriculum of the Ministry of National Education (2018). *Jayden's Rescue* has four operation problems, exponents, and some algebra. Operations and exponents are in the objectives of the first unit of the fifth grade MoNE (MoNE, 2018) curriculum. Although the algebraic expressions are not introduced in the objectives in fifth grade in a way that letters are used for the variables, students are familiar with the problems where they need to use a placeholder for their unknown value.

In *Jayden's Rescue*, the problems Aleks and his friends were solving immediately introduce letter substitutes for variables, whereas in *the Cat in Numberland*, the author used letters and familiar boxes as shown in Figure 4. The

questions in *the Cat in Numberland* are also probing the reader that the letters are indeed placeholders for a number, “Bu işlemde A harfi hangi sayının yerine geçti? (In this operation which number is the letter A substituting?)” (translated to English by the researcher) (Ekeland, 2006/2018, p. 36) In *Jayden’s Rescue*, the children are confused with the question that included algebra and the only explanation of it is familiar to Aleks since his big brother Nolan is solving some questions about algebra beforehand. The magic pen that solves mathematics problems appears to save the children from asking Aleks’s big brother and exploiting their secret mission to save Queen Jayden.

Figure 4

Introduction to Algebra from Kraliçeyi Kurtarmak & Sayılar Diyarındaki Kedi



Note. From *Jayden’s Rescue* (p. 97), V. A. Tumanov, 2004. Günışığı Kitaplığı.

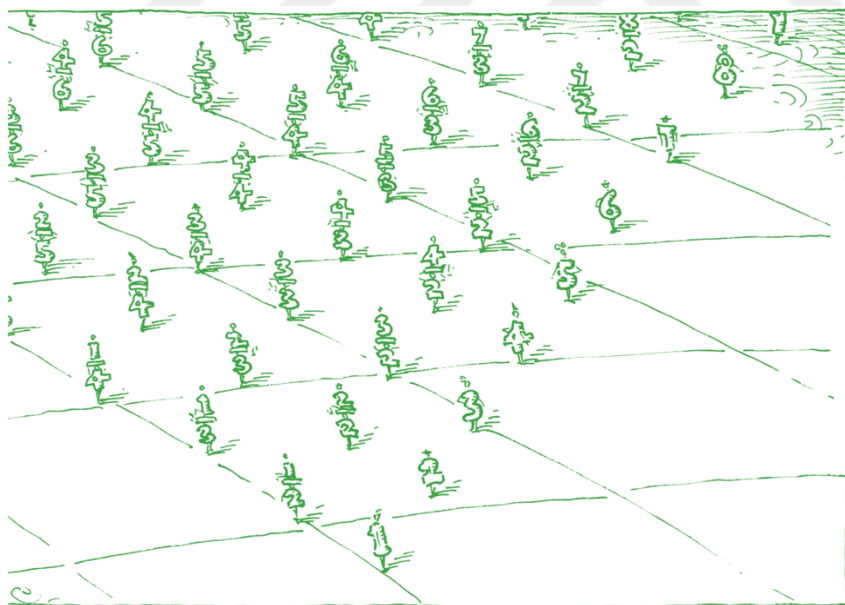
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From *The Cat in Numberland* (p. 36), I. Ekeland, 2006. Doğan Egmont. Copyright year 2018 by Doğan Egmont. Reprinted with permission.

The Cat in Numberland has mostly number sets, four operations, some zero properties, and some algebra in it. Compared with *Jayden's Rescue* which has limited its mathematics to the mathematical objectives from fifth grade MoNE mathematics curriculum (MoNE, 2018) and some algebra, *The Cat in Numberland* introduces a broader mathematics like isomorphism and Cantor's set theory without explicitly stating them. *The Cat in Numberland* does not explicitly cover the objectives that are in the fifth Grade MoNE curriculum (MoNE, 2018) other than Natural Numbers and Number Sets, rather it introduces different horizon broadening concepts to children. While using the fraction bars, students may remember the Cantor's number line from *the Cat in Numberland*, seen in Figure 5.

Figure 5

Fraction Guests of the Hilbert Hotel in The Cat in Numberland



Note. From *The Cat in Numberland* (p. 57), I. Ekeland, 2006. Doğan Egmont.

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In both books, the mathematics is visible, and it is appropriate for their readers' level. In *Jayden's rescue*, the mathematical problems presented were solved

by the main character children. In *The Cat in Numberland*, the problems characters face are mathematically sophisticated like comparing different infinities and observing what that means in a simple way. Both books involve the reader in the mathematics, in *Jayden's Rescue* by explicit mathematics problems, and in *The Cat in Numberland* by direct questions like “*Bu işlemde A harfi hangi sayının yerine geçti? $A+7=15$* (In this operation which number is the letter A substituting? translated by the researcher)” (Ekeland, 2006/2018, p. 36).

In the translated version of *The Cat in Numberland*, there are some probing questions at the end of the book added by the translator Feyza Demir, who is also a mathematics teacher. These probing questions like “Is the universe infinite?” (translated to English by the researcher) or “Can you give an example of an infinite thing?” (translated to English by the researcher) (Ekeland, 2006/2018, p. 64) are also suitable for Whitin and Whitin's (2004) varied response property, where the reader can answer this question correctly in many ways. The translator, Feyza Demir, also added some mathematical concepts like Roman numerals, even and odd numbers, prime numbers, number sets, four operations, and fractions. With those additions to *The cat in Numberland*, the readers of both books can benefit from the mathematics in the books without much further research.

Content – Domain Specific Typology

Researchers and teachers, while evaluating children's mathematics books, used different types of rubrics (Hellwig et al., 2000; Hunsader, 2004). The content – domain specific typology (Nurnberger-Haag et al., 2021a) allows teachers and researchers to filter the books on the mathematics they have inside, rather than the literary aspects or the correctness of the books. The two children's books about

mathematics, *Jayden's Rescue* and *The Cat in Numberland* were examined by their counting sequences, rationality, and explicitness.

Counting Sequences

Jayden's Rescue is categorized as multiple different scalars (MDS). On page 55, there is a problem which has the sequence “*bir, iki, üç, dört, beş*” and on page 77, there is a sequence where the children add $10 + 20 + 30 + 40 + 50 + 60 + 70 + \dots$. The problem has several brothers that have 10 years as age difference and their ages add up to 1200. On page 131, there are two sequences; where the number of genies that are having a dance night in the problem increases with a sequence at every five minutes. The leaving genies have the sequence pattern of leaving by 2, 4, 6, 8, 10, 12, 14, 16, ..., 24. The second sequence is the coming genies, where every five minutes a number of genies come to the dance. The increasing genies sequence is 4, 8, 16, 32, 64, 128, 256, 512, ..., 8192. This sequence is not an arithmetic sequence, which is not stated in the content-domain specific typology since counting & counting by different scalar are arithmetic sequences. There is a geometric increase which has a common ratio of two, which means each term can be found by multiplying the preceding term by the same value, in this case by two. While counting there is a common difference between the terms, that is why the geometric sequences were not included in the Counting Book typology.

The Cat In Numberland is also categorized as multiple different scalars (MDS). On page 7, there are two sequences with the same scalar. The even and odd numbers described, 1, 2, 3, and 7 are counted odd, 2, 4, 6, and 8 are counted even. On page 53, where the fractions are introduced, Mr. and Mrs. Hilbert order the fractions. There are multiple different scalars present. The sequences are $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \dots$ and $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \dots$ and $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots$ where the scalars are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. These

examples are the only instances where books have these different sequences. The books are in the category of multiple different scalars (MDS).

Rationality and Explicitness

Jayden's Rescue is categorized as rote counting (RC). The illustrations in *Jayden's Rescue* are representing the characters in the book. There are no mathematical illustrations. *Jayden's rescue* is also categorized as explicit counting (EC). On page 166, Aleks, one of the main characters, says he must count the chairs. He explicitly used the word counting.

The Cat in Numberland is categorized as rational counting terms (RT). There are images that are representing the number symbols, and on page 42 you can observe that the even and odd numbers are holding hands. On page 53, there are fractions that show another sequence, and the number symbols are also used to describe the fractions.

The Cat in Numberland is also categorized as explicit counting (EC). On page 21, Mr. Hilbert, one of the main characters, said the numbers are for counting. He explicitly used the word counting. The categorization shows that *the Cat in Numberland* is an MDS, RT, and EC, whereas *Jayden's Rescue* is MDS, RC, and EC. The difference between the MDS aspect of these books is in *The Cat in Numberland*, the author uses fractions as a common difference, where readers have more variety than *Jayden's Rescue* which only uses natural number scalars. *Jayden's rescue* does not have any mathematical illustrations, while *The Cat in Numberland's* illustrated characters besides the cat and the Hilberts are numbers and letters. Both books are explicitly stating that they are counting.

Connections to the Curriculum and Beyond

The Cat in Numberland and *Jayden's Rescue* are both aligned with the objectives and mathematical skills of the fifth grade MoNE mathematics curriculum (MoNE, 2018). *Jayden's Rescue* addresses many different mathematics objectives of the MoNE mathematics curriculum (MoNE, 2018) with the questions it poses. The questions require students to use natural numbers and operations, fractions and operations with fractions, time, length, and area measurements and their conversions. *Jayden's Rescue* introduces algebra and some of its operations in the book, but the first algebraic question is solved by the magical pencil and the second one is solved by trial and error by substituting different numbers to the place of the placeholder letter. Algebra objectives are not in the objectives of fifth grade mathematics curriculum of MoNE (MoNE, 2018).

The Cat in Numberland also addresses different mathematical objectives of fifth grade MoNE mathematics curriculum (MoNE, 2018). Again, the natural numbers and operations, fractions and operations with fractions were addressed. *The Cat in Numberland* introduced some algebra to its readers with a different approach than *Jayden's Rescue*, where it represents the placeholder with a box before the variable introduction. The placeholders for the elements of an operation that are not given is familiar for students and the finding these not given elements in an operation is covered in the objectives of fifth grade MoNE mathematics curriculum (MoNE, 2018) as: “Çarpma ve bölme işlemleri arasındaki ilişkiyi anlayarak işlemlerde verilmeyen öğeleri (çarpan, bölüm veya bölünen) bulur (Students understands the relation between the multiplication and division operation and can find the not given element in these operations, factor, quotient, dividend. Translated by researcher.)” (MoNE, 2018, p. 52). Finding the not given elements of addition and subtraction is a

mathematics objective of the 4th grade MoNE mathematics curriculum (MoNE, 2018).

The mathematical skills that are covered in the books varies. *Jayden's Rescue* mainly focuses on mathematical problems that address problem solving skills and mathematical literacy. The problems are not different than any problem you can find in a mathematics textbook besides its verbalization. The problems are presented in a beautiful way, regarding literary quality, by using rhymes and a backstory. Although the operations required to solve these questions are mainly calculations, understanding these problems may require higher literary understanding skills and following up with a text where the objective is given in parts.

Jayden's Rescue asks its readers to solve the mathematics questions to save the Queen, so it encourages its readers to be patient and responsible which are also skills that are covered in MoNE mathematics curriculum (MoNE, 2018). The Queen that needs to be saved is from a magical land where mathematics is also used may address the skill for universality of mathematics, and the main character Aleks's plot may encourage students to have a positive attitude towards mathematics, which is also a skill that is in the MoNE mathematics curriculum (MoNE, 2018).

The Cat in Numberland approaches mathematical skills in a different way. Although it has some direct questions as *Jayden's Rescue*, mainly it does not ask simple mathematics problems to involve its reader. It asks different discussion questions about infinity, like in the first chapter, when zero arrived in the hotel, all guests – all counting numbers in this case-- move one room next to theirs and make room for zero. The cat gets very confused with this movement since the hotel was full before zero and still full after zero but however, they find a way to fit them in. This problem-solving skill when compared to solving a problem related to a routine

type of a problem requires students to have a higher level of understanding of the abstract concepts of mathematics, yet it is explained very eloquently and simply in the book. The skill that these discussions address could be the recognition and improvement of their metacognition in the MoNE mathematics curriculum (MoNE, 2018). The discussions continue with the even and odd numbers isomorphism and infinity, and when fractions are included to the equation it is still a countable infinity, where Cantor's Theorem is addressed. The readers of this book could understand different countable infinities and their sizes.

The motivations of authors were different in writing these books which explains their different approach to mathematical skills. *Jayden's Rescue* is written by Tumanov to encourage his mathematically struggling child Alex. *The Cat in Numberland* is written by Ekeland to introduce more sophisticated topics to the children in an understandable manner. So, the modelling behind these books follow this motivation. *Jayden's Rescue* has well defined at most three step problems which the reader could also solve by trial and error. The reader may choose to solve it by generalizing a formula or modelling the problem accordingly but the problems themselves does not require a higher level of metacognition to be solved.

The Cat in Numberland poses many difficult discussions that may require higher level thinking and modelling, and although the answers are given in the book, the reader must think thoroughly about the question to even understand the given answer. This requires the reader to understand the problem thoroughly. For example, the problem of even and odd numbers and their equal infinity with the natural numbers. The numbers wanted to visit the letter world, but the hotel would be empty if they all would leave at the same time. So, they decide to go in halves, and here the first problem arises. The odds and evens are separated into two lines by a method

Mrs. Hilbert suggests, and there are an equal number of evens and odds. Then the odds leave with Mrs. Hilbert to visit the letters. Now another problem arises, all of even numbers stayed in their assigned rooms, such as two in second room and four in fourth. They said the two rooms next to them are empty and they feel worried. Mr. Hilbert tried to find a solution to this problem and zero came up with a solution. Zero suggested every number should assigned to the hotel room number half of them. Such as two in first room, four in second and so on. This discussion is posed by the cat, in the previous night the half of the hotel was full, but tonight all of the hotel is full. The comparison of two infinities and the modelling of this case required higher level thinking by readers.

CHAPTER 5: DISCUSSION

Introduction

The purpose of this study was to analyze and compare two children's mathematics books, *The Cat in Numberland* and *Jayden's Rescue* by using content analysis and content-domain specific typology. The comparative content analysis supported by the content-domain specific typology increased the strength of the research by supporting the content analysis with a tool to evaluate the mathematics in children's books effectively and efficiently. Although most people who read children's books about mathematics do not use any rubrics explicitly, their underlying reasoning about the books might be aligned with some of Schiro's (1997) standards, such as the plot and characters representation, language and the illustrations of the books, and the mathematical accuracy and worthiness of the content. This could be an important outcome for this study since parents and teachers are usually hesitant to use these kinds of rubrics or typologies, as the existing rubrics in educational research might appear intimidating or time consuming. The underlying reasoning might show that the use of such rubrics is already implicitly in the parents and teachers, and that their explicit use is also applicable and realistic.

Reflections on Different Presentation Styles

The Cat in Numberland and *Jayden's Rescue* have some similar aspects in literary qualities, as they are both written in simple everyday language and are grammatically correct. *Jayden's Rescue* differs in its mathematical questions in the book. These questions in *Jayden's Rescue* do not contribute to the storyline. The reader stops to do some mathematics, which devalues the literary aspect of the book.

The answers to the questions are immediately given by the children characters after the question, which leaves no room for thinking for the reader.

Although Tumanov (2002/2004) presented all the literary standards stated by Schiro (1997) in his book, there are some aspects in the characterization that are not well portrayed. The main child character Aleks was not very competent in mathematics, and he had no interest in mathematics until he finds a magical pencil that can solve mathematical questions. Aleks used the pencil in his mathematics exam without knowing it is magical and the pencil solved all the questions correctly. After this incident Aleks shows no regret or similar emotion. He was excited to use the magic pencil. The character Aleks also lies about losing the pencil to his friends. The character development shows Aleks evolving into a more mathematically capable person, but never returns to, or shows remorse about him cheating in an exam and lying to his friends about the magic pencil. Aleks does show other moral qualities like sharing and helping his friends in the pages following his discovery of the magical pencil.

The Cat in Numberland by its literary standards is simple in execution but the book is mathematically very sophisticated. There are some concepts that although they align with the Natural Numbers and Operations unit in the MoNE mathematics curriculum (MoNE, 2018), it goes beyond the objectives. Like the concept of infinity, which is difficult to understand even by adult readers, but Ekeland (2006/2018) simplified it so well that students understand it easily. The numbers are introduced like friends, and operations introduced as games the numbers play. This familiarization helps students to enjoy and personalize mathematics and make abstract things like numbers and operations more likable personas.

The owners of the infinity hotel, Mr. and Mrs. Hilbert also represent a balance. Mr. Hilbert represents the side of the reader that does not want to learn anything new or complicated, whereas Mrs. Hilbert enjoys learning new things. This duality is held very well in the book, so it balances the competent and not so competent reader by relatability, without devaluing mathematics or learning. *The Cat in Numberland* also states some of the properties of zero, and how natural numbers are closed under four operations. The book does not explicitly name these concepts like isomorphism and being closed under addition or multiplication, or density of rational numbers, but it glimpses into the ideas in an accessible manner. For example, it shows an eloquent proof of isomorphism of natural numbers to fractions by using the Cantor's number line method. When the numbers are aligned as lines, there are infinitely many lines and infinitely many rows, but when they aligned as a triangular form, all the numbers fit into the infinity hotel, as seen in Figure 5.

Often, the authors of these children's books are not in the field of mathematics, or even education. Most of the books do not have a research background of misconceptions or children's metacognition. Authors of children's books generally observe a behavior that they want to change in their children, then they write a children's book about it. Even B.F Skinner who is a pioneer in educational psychology research, started his experiments with his children (Slater, 2005). Similarly, the names of the main characters in *Jayden's Rescue*, Aleks and Vanessa, are the names of the author Vladimir Tumanov's children. There is an assumption that children's literature might be a medium for representing the children's point of view, which could hinder parents and teachers in evaluating the books for their use in connection to their curriculum.

An Effective Tool for Evaluating the Mathematics in Children's Books

The Counting Book typology is a fast and effective way of evaluating the mathematical aspects of children's mathematics literature. The typology is easy to use to evaluate any mathematics book, and useful to show the differences and similarities between the chosen books. In this study, both books are MDS and EC. The difference between *Jayden's Rescue* and *The Cat in Numberland* is that the *Jayden's rescue* is rote counting (RC), where *The Cat in Numberland* is rational counting terms (RT).

In *Jayden's Rescue* only the illustrations of a few characters are given, however there are no illustrations of the main characters, the children who the reader follows throughout the story. In *The Cat in Numberland*, the numbers are the characters, and they are on every page. The story aligns with the illustrations, which helps reader to follow along. *The Cat in Numberland* has different mathematical theorems such as Cantor's set theory and number line method. The illustrations in this book (see Figure 5) facilitate the reader to see that all the fractions could be aligned as a triangle rather than a rectangle, where the reader realizes that the countable infinities of the fractions and the numbers have the same greatness. These types of illustrations can be helpful in identifying appropriate children's books, in addition to the typology and the literary and mathematical standards.

Implications for Practice

The results of this study are important since it unifies the tools that parents, teachers, and researchers use to evaluate children's books about mathematics by creating a bridge between them. The content analysis made by the standards of Schiro (1997), and the mathematics aspects of these books evaluated by the Counting

Book typology (Nurnberger-Haag et al., 2021a) can both be used easily by parents and teachers.

The standards of Schiro (1997) require both a literary and a mathematical analysis of a book, so it can require more time to analyse the book thoroughly. But with the mathematical classification tool that has been used in this research; The Counting Book typology, the classification could be done rather efficiently. Skimming the number sequences that are in the book, and analysing the drawings and its relatability with the text would be enough to classify the book mathematically. Depending on the intended objectives of choosing a children's book about mathematics, either or both methods can be implemented by parents and teachers.

Implications for Further Research

The findings of this study also lead to some new research questions. The question this research leads to is how to measure *The Cat in Numberland's* mathematical sophistication broadening abilities in parents, teachers, and students. Though the book has a broad concept of mathematics and advanced concepts, children's feedback about the books did not provide a comprehensive perspective of the knowledge that they obtained (Ekeland, 2021). A tool to understand this aspect of the book could be useful in further research.

Another research question to explore is about the popularity of *Jayden's Rescue* among the private schools in Türkiye. Teachers and parents could be interviewed to observe their understanding of the book and the reasoning of this book choice. Students could also be interviewed or observed to identify what draws them to want to read this story, encouraging children to read for fun in addition to choosing books that are well aligned with the curriculum.

Limitations

In this research, only two children's books were analyzed because of their authors' different backgrounds. Although many children's books were analyzed beforehand by the researcher, these books were found appropriate because of their level and because they have been used for many years in classrooms in Türkiye. *The Cat in Numberland* was printed in 2006 and *Jayden's Rescue* was printed in 2004. The researcher is familiar with the use of these books as a teaching tool because of their prior experience in teaching middle school mathematics. The method of children's mathematics books selection was partly based on the professional experience of the researcher which might be a bias for the study. This could limit the studies' generalization ability, yet it can still provide a useful guideline for parents and teachers to choose appropriate children's books about mathematics.

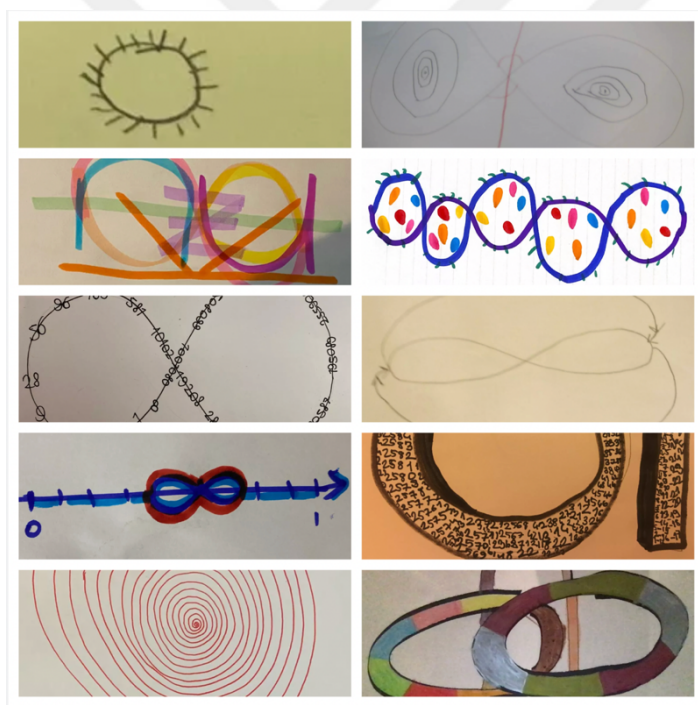
There are a few limitations in the study that will help the reader to interpret the data and the findings of this study in its scope. The first limitation is the children's books in this content analysis are limited to the two books that the researcher has consistently implemented in their middle school teaching experience. The second limitation of this research is the children's books that are content analyzed are translations of the original books, so some of the literary aspects might have been lost. For example, in *Jayden's Rescue* the problems are written with some rhyming syllables, while translating preserving this aspect of the book might result in different meanings because of different word choices. Also, in *Jayden's Rescue* the measurement units are American, like money, and even in the translated version, it is still in dimes and dollars. In *The Cat in Numberland*, the main character cat is a female, which is highly important to the author since it is representing the curiosity

and presence of female scientists that are questioning the truth, however it is not explicitly stated.

Although children could not evaluate a book by its mathematical and literary standards, they might be an area for researchers to observe. Student work of the question from *the Cat in Numberland* “Can you design your own infinity symbol?” (Ekeland, 2006/2018, p. 65) was asked to some fifth-grade students after they read the book (Ekeland, 2021). In Figure 6 there are some examples of the students’ works.

Figure 6

Students’ Work of Infinity (Ekeland, 2021)



The interview of Mr. Ekeland included these students’ infinity works (see Figure 6). Mr. Ekeland, while examining these infinity symbols, commented that the creativity and the broadened horizons of the children amazed him. He examined all the drawings, but the focus of the interview was on only a few of these drawings. He really liked the spiral and the idea behind it, as the spiral also goes on and on forever. The drawing that has all the numbers and letters onto each other was also a focus of

discussion during the interview, where the meaning behind this drawing was like mixing all the colours for the children.

With all the components of infinity, they tried to create a complete infinity. One of the students used the splitting cells where he defined a sort of organic infinity. Many of the students' drawings included the original infinity symbol with many numbers on to it, which also shows how the students connected the infinity idea with the number sets and counting theories (Ekeland, 2021).



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