

Co-design Optimization for Lever-Type Variable Stiffness Actuation

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Keywords: Co-design, system-level optimization, multicriteria design optimization, variable stiffness actuation, prosthesis design

Abstract

Variable stiffness actuation (VSA) improves the dexterity of robots while ensuring the safety of physical human-robot interactions. VSAs can be implemented based on several principles. Among these, lever-type VSAs are advantageous, since no mechanical work against the spring is required to modulate the stiffness of the system under ideal conditions.

We study co-design optimization of lever-type VSAs where mechanical and control design variables are simultaneously optimized to achieve the best performance. We demonstrate that co-design optimization results in significant performance advantages in comparison to subsystem optimization.

Given that VSA optimization problems inherently consider multiple criteria, we model the co-optimization problem accordingly and compute all non-dominated system-level optima as a Pareto-front curve. We provide sample design selections from the Pareto-front curve and discuss the trade-offs involved in various solutions.

We also provide two single-actuator solutions: a series elastic actuation solution and a novel constant-torque solution that achieves the task through stiffness regulation. Through a modified optimization formulation, we further demonstrate that the constant torque requirement for the constant torque solution can be decreased at the expense of the lower stiffness of the actuator.

Finally, we compare Pareto solutions of lever-type VSA mechanisms with those of single spring preload adjustment based VSA, for the same task and objective functions. Through this comparison, we provide evidence that series type VSA or single spring preload based VSA can be favorable for different designer preferences.

Seri Tipteki Değişken Esnekliğe Sahip Eyleyicilerin
Eş-Tasarım Eniyileştirmesi

Zeynep Özge Orhan

ME, Yüksek Lisans Tezi, 2020

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Anahtar kelimeler: Eş-tasarım, sistem düzeyinde eniyileme, disiplinlerarası tasarım, değişken esnekliğe sahip eyleme, protez tasarımı

Özet

Değişken esnekliğe sahip eyleme (DESE), hem fiziksel insan-robot etkileşimlerinin güvenliğini sağlama, hem de robotların kabiliyetini artırma açılarından avantajlıdır. DESE'ler birkaç temel ilkeye göre uygulanabilir. Kaldıraç tipi DESE'ler, diğer yaklaşımlara kıyasla avantajlıdır, çünkü ideal koşullar altında sistemin esnekliğini değiştirmek için yaya karşı mekanik kuvvet uygulanması gerekli değildir.

Bu çalışmada, kaldıraç tipi DESE'lerde en iyi performansı elde etmek için, mekanik ve kontrol tasarım değişkenlerinin aynı anda ele alındığı, sistem seviyesinde eş-tasarım eniyilemesini ele alıyoruz. Eş-tasarım eniyilemesinin, alt sistem eniyilemesine kıyasla önemli performans avantajları sağladığını gösteriyoruz.

DESE eniyileme problemlerinin, doğası gereği birden fazla kriteri dikkate aldığını göz önünde bulundurarak, çok kriterli optimizasyon problemini modelliyoruz ve baskın olan sistem seviyesindeki tüm en iyi çözümleri bir Pareto eğrisi olarak hesaphıyoruz. Pareto eğrisinden örnek tasarım seçimleri sunuyor ve çeşitli çözümlerin avantajlarını ve dezavantajlarını tartışıyoruz.

Bunların yanı sıra, iki farklı tek eyleyicili çözüm sunuyoruz: seri elastik eyleyici çözümü ve sadece esneklik modülasyonu ile görevi yerine getiren özgün bir sabit tork çözümü. Güncellenmiş bir eniyileme formülasyonu sayesinde, sabit tork çözümü için gerekli tork seviyesinin, DESE'nin görev sırasındaki esnekliğini azaltmak kaydı ile, azaltılabileceğini gösteriyoruz.

Son olarak, aynı görev ve eniyileme ölçütleri için, kaldıraç tipi DESE mekanizmalarının Pareto çözümlerini tek yayın önerilmesi tabanlı DESE mekanizmalarının Pareto çözümleri ile karşılaştırıyoruz. Bu karşılaştırma sayesinde, kaldıraç temelli seri tip ve tek bir yayın önerilmesi tabanlı DESE'lerin birbirlerine üstünlük sağlayabildiği farklı tasarım seçenekleri olduğuna dair kanıtlar sunuyoruz.

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Nomenclature

Abbreviations

atan arc tangent function

cos cosine function

max maximum

min minimum

sin sine function

SIM simultaneous optimization method

Acronyms

NLP Nonlinear programming problem

OCP Optimal control problem

PEA Parallel elastic actuation

pHRI Physical human-robot interaction

SEA Series elastic actuation

SPEA Series-parallel elastic actuation

SQP Sequential quadratic programming

VSA Variable stiffness actuation

List of symbols

ϵ	Constraint bound on the objective function which is given as an inequality constraint
ϕ	Output link position
τ	Torque applied to the output link
τ_{load}	Torque required to move the human knee
$C(q, \dot{q})$	Coriolis matrix
$D(q)$	inertia matrix
dc	Control design variables
dm	Mechanical design variables
F	Force applied to the pivot
g_m	Inequality constraints
h_m	Equality constraints
IL_{33}	moment of inertia of lever body about the $\vec{n}_3$ direction
IO_{33}	moment of inertia of output body about the $\vec{n}_3$ direction
J	Design objective
J_1	Objective function corresponds to the minimization of torque range
J_2	Objective function corresponds to the minimization of force range
L	Lever body
l	Distance between M_2 and attachment point of the springs
l_l	Lever link length
l_o	Output link length
lb_c	Lower bounds on the control design variables
lb_m	Lower bounds on the mechanical design variables

M_1	Actuator controls the output angle
M_2	Actuator controls the stiffness of the mechanism
$N(q, \dot{q})$	Matrix which is including forces such as spring forces
O	Output body
P	Pivot point
s	Position of the pivot with respect to M_2
ub_c	Upper bounds on the control design variables
ub_m	Upper bounds on the mechanical design variables
x_s	Amount of spring deflection

Chapter 1

Introduction

The safety of human operators is the primary concern in physical human-robot interactions (pHRI). While industrial robots can perform a variety of tasks with high precision and accuracy in controlled environments without human interaction, due to their rigidity and large inertia, they are potentially unsafe for pHRI.

Series elastic actuation (SEA) has been proposed as a potential solution for this safety problem in pHRI. In SEA, by introducing a compliant element between the actuator and the link, the inertia of the actuator and the link are decoupled from each other. In this way, the reflected inertia at the link side is significantly decreased. Therefore during a contact, human users will be exposed to a lower reflected inertia. Moreover, during collisions, the compliant element behaves as a low-pass filter and allows safe interaction between the robot and the human user, and between the robot and the environment. SEA has been widely investigated and used in a variety of robotic designs in the literature [1–13].

In SEA, the bandwidth of the system is limited by the physical spring, and the physical stiffness cannot be changed during operation. However, humans can vary their joint stiffness depending on the performed task. This change in joint stiffness increases human’s adaptability and interaction performance. In the literature [14–17], it has been revealed that stiffness modulation is essential in the tasks that involve unstable dynamics. Moreover, it has also been stated that the stiffness

modulation is instrumental for stable robot-environment interactions, critical for the safety in pHRI, and beneficial to achieve better task performance through exploitation of the system dynamics [17–20].

Therefore stiffness adjustment mechanisms can bring a significant improvement in pHRI [17]. While there are substantial advantages that come with SEA, possible additional benefits, such as increased energy efficiency and adaptability to the environment, can be obtained using variable stiffness actuation (VSA). VSAs have a high potential to develop new generation robots, as well as biologically assistive devices, through their capability of controlling not only the output force but also the stiffness of the system [21].

1.1 Related Work

VSA

Over the last decade, various designs of VSAs have been developed based on different principles. In the literature, various categorizations have been proposed for VSAs [18, 19, 22, 23]. The categorization defined in [22] is followed in this study due to its comprehensiveness.

Figure 1.1 (reconstructed based on [22]) presents a graphic overview of variable impedance actuators. Variable impedance actuators are categorized depending on stiffness and damping adjustment methods.

Active impedance control is a software-based modulation approach where a control law is designed to follow a reference trajectory with the desired impedance level. Since the impedance modulation takes place in the software-level without requiring any change in the physical system, the active impedance control has great flexibility in the design and implementation. However, to perform the desired impedance properly, a significant amount of control effort is required since the continual operation of the controller is required to maintain the desired impedance level [24].

Furthermore, the impedance rendering is limited by the closed-loop bandwidth of the system, as the controller cannot effect frequencies above the closed-loop bandwidth. If the impedance is adjusted through software-based methods, no shock or impact can be absorbed. Moreover, the control performance can deteriorate due to many non-ideal factors, such as limited sensor precision, model accuracy, or low sampling frequency. These factors or selection of improper controller gains can lead to the instability of rendering.

In contrast to active impedance modulation, passive compliance methods contain an intrinsic compliant element. Therefore, the impedance implemented by the hardware-based techniques are always stable and valid over the whole frequency spectrum, since the impedance is physically realized through passive components, such as spring and dampers.

If fixed compliance is utilized in the hardware, the physical spring stiffness may be required to be altered for different tasks, since the bandwidth of the system is limited by the intrinsic spring stiffness.

As in the human muscles, different level of compliance can be utilized depending on the performed action. According to [22], the stiffness of an actuator can be adjusted based on three main principles: by changing spring preload, by changing the transmission between load and spring, and by changing the physical properties of spring. Within these three categories, spring preload methods are most commonly used in the design of VSAs. However, several arguments on the energetical expensiveness of the spring preload methods have been presented in the literature [14]. In particular, to adjust the stiffness of the mechanism, work is required to be done against the spring force in the preload methods.

VSAs based on changing transmission can also be implemented in three different means: by lever length adjustment, using a nonlinear mechanical link, and via a continuously variable transmission. Among these, there is a growing interest in the design and implementation of VSAs using the lever length principle [25–34].

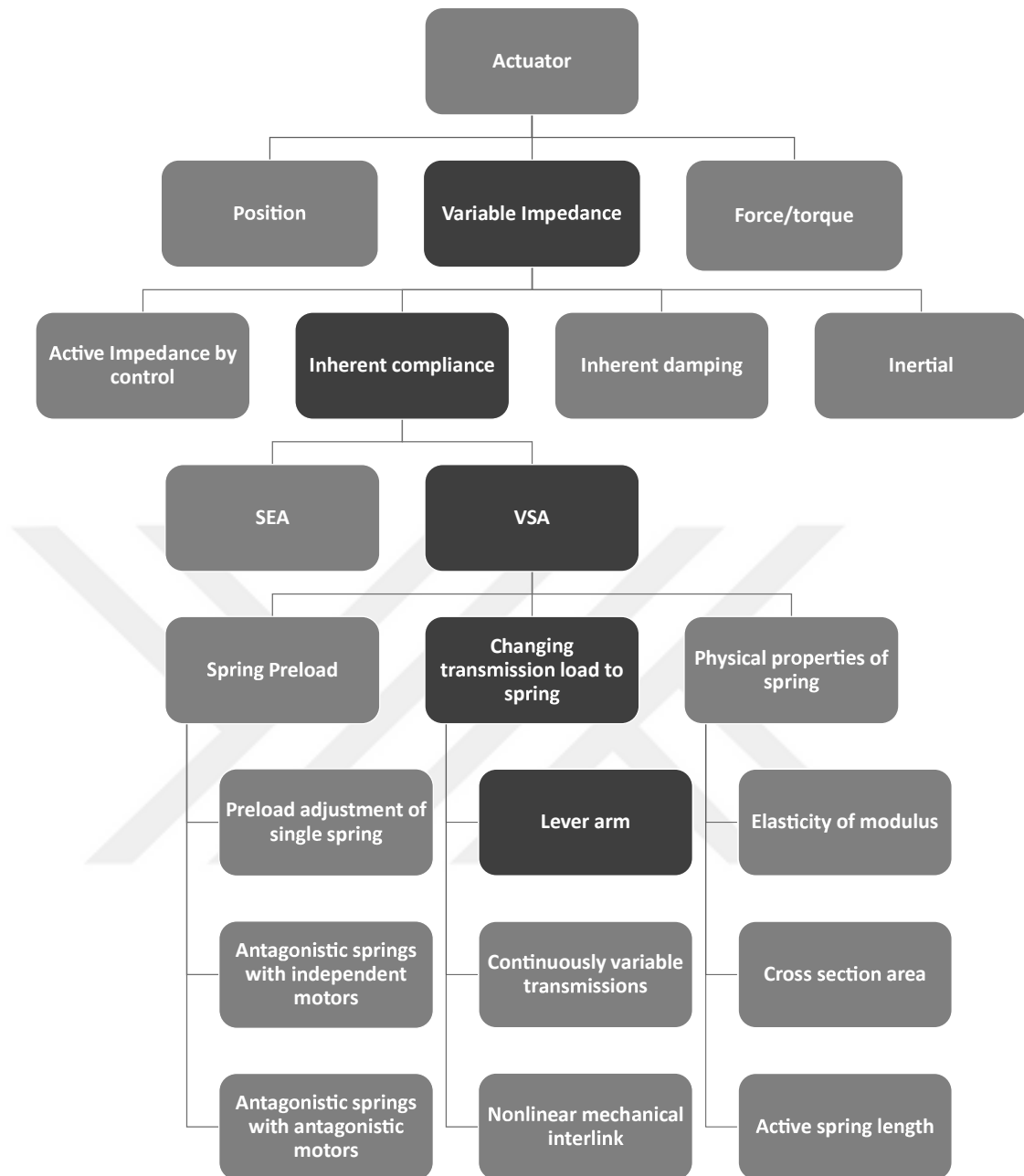


FIGURE 1.1: Overview of compliant actuators

Using the lever length principle, the apparent output stiffness can be adjusted by changing the transmission ratio between the internal springs and the actuator output. Mechanisms designed with the lever length principle can adjust their stiffness in an energetically efficient way, since the displacement needed to change stiffness is *almost perpendicular* to the force generated by the springs. In this way, low mechanical work against springs is required, and stiffness can be adjusted with a low energy cost.

In the lever arm principle, three possible configurations are possible, as presented in Figure 1.2 in [19]. To modulate the stiffness in lever arm based VSAs, the effective ratio of the lever mechanism is altered. The effective ratio of the lever mechanism can be changed in three different configurations: by moving the position of the pivot, by moving the location of springs, and by changing the point where the force applied to the system.

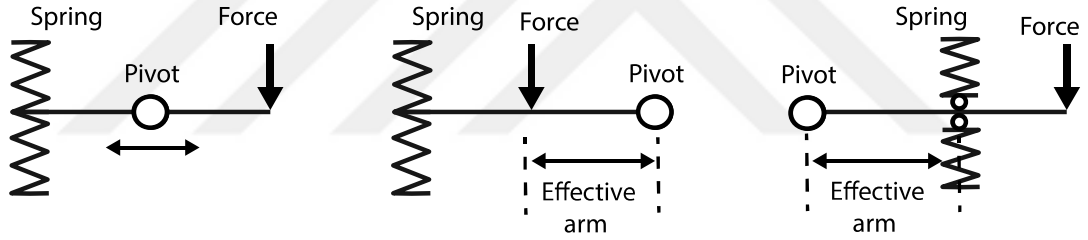


FIGURE 1.2: Concept of lever-type VSAs

Among these configurations, the most energy-efficient one is based on moving the pivot, as analyzed in [35]. Although the lever arm methods that change the pivot location look most promising in terms of reducing required mechanical work to change stiffness, the effect of friction has not been investigated in detail for this case in lever arm based VSAs. Friction effects are considered as a potential drawback of these types of mechanisms, especially with high stiffness inherent springs and under high load torque conditions.

Design Optimization

The design of efficient assistive robotic devices promises a significant impact on pHRI and human augmentation. In order to place a VSA in a daily use device or a wearable device, the energy consumption, the weight, and the volume of the mechanism should be rigorously designed. Multidisciplinary analysis and design techniques that address the specific needs of the system are required for a successful design.

In literature, significant performance improvements for SEA and VSA have been shown through optimization studies [36–42]. The nonlinear spring of an SEA is optimized to obtain a more compact design for the desired torque profile in [36]. In [37, 38], SEA design is optimized to decrease energy consumption by proper selection of the joint stiffness, preload value, and stiffness of the elastic element. Moreover, [39] presents the importance of actuator unit selection in the early stages of the design by considering a possible increase in peak power due to the drivetrain inertia, and the limitations come from the drivetrain and the controller.

Recently, researchers presented optimization frameworks to choose the best motors and gearboxes to minimize the overall effective inertia and electrical losses for a SEA design [40]. In [41], a multi-objective convex optimization framework is proposed to study the trade-offs of simultaneous minimization of energy consumption and peak power for a design of SEA for ankle joint. In [42], it has been illustrated that the SEA with an optimal spring stiffness and geometry can decrease the peak power of the actuator. Moreover, it has been explored that VSA has the potential benefits to reduce motor power more than SEA.

The performance of the existing designs has been increased through the implementation of optimal control theory in addition to design optimization, with a focus on energy consumption of the system. These works mostly focused on task performance and safety of human-robot interaction [43–51]. Study in [47] presents a general optimization framework for torque and impedance profiles of robots

driven by compliant actuators to enhance task performance. In [47], the generation of motor trajectories is formulated as an optimal control problem (OCP). To reliably track the generated trajectories from the solution of OCP, nonlinear model predictive control is utilized for the closed loop control of VSA robots in [49]. To minimize actuator torques, a simultaneous optimization method of joint trajectory and force profile of nonlinear spring is proposed in [52]. For periodic and non-periodic motions, optimal control approaches to stiffness and equilibrium position adjustment are presented in [53, 54]. A substantial reduction in control input magnitudes is demonstrated through the utilization of optimized trajectories by taking advantage of the passive dynamics of the plant.

To reach a given goal within the minimum time while guaranteeing safety constraints, time-optimal control for VSA robots is implemented in [48]. It has been shown that VSAs perform better with respect to rigid actuators and SEAs. A closed form solution is provided in [46] for the optimal control problem of reaching the maximum link side velocity with an elastic joint to execute explosive motion with a suitable stiffness and deflection. In [46, 48], the safety concern is introduced as a constraint in the problem. In contrast to the safety concern, the velocity of the link at a given goal position is maximized in [44] to increase the impact. It has been demonstrated that VSA can increase the maximum speed at the desired position up to 30% with respect to SEA. Similar to [44], in [45], the impact is maximized at the goal position with a fixed terminal time to provide a kick effect. In [45], it has been stated that after a particular terminal time, VSA can achieve better performance over SEA in terms of speed.

Although a substantial effort has been spent in the literature to get the maximally beneficial performance from compliant actuators, most of these studies consider either the mechanical system or the design of the controller. However, the mechanical system and the controller are the subsystems of the whole controlled mechanical systems. Therefore, considering independent optimization of these subsystems cannot yield a system-level optimal solution that performs at the full potential of the system [55]. There exist significant couplings and trade-offs between optimal mechanical design and optimal control design of the system. By

considering the interactions between the mechanical subsystem and the control subsystem, the system-level optimal solution can be obtained, which guarantees the best performance of a controlled mechanical system [56].

In [57], to improve the dynamic response of the system, the optimum structure and the optimal control system have been designed through a sequential optimization method. However, due to the coupling between the mechanical plant and the controller, a sequential approach fails to guarantee system-level optimal solutions [58]. In [59], the structural design and the control design problems are treated as a unified single optimization problem. By improving these subsystems simultaneously, an integrated design of the structure and the vibration control system is achieved.

Integrated optimization of the design and the control of engineering systems has been utilized to improve the performance of numerous engineering systems, such as flexible structures [60–62], automotive suspensions [63, 64] and powertrains [65], electrical motors [66], thermodynamic systems [67–70], renewable energy systems [71, 72].

In [72], co-design optimization has been implemented for horizontal axis wind turbines, and a significant increase in peak power output has been demonstrated. The analysis in [72] interprets that the synergistic interactions between the rotor speed and the structural changes can only be exploited if a co-design approach is considered. Through the utilization of system-level optimization strategies where the optimality of integrated plant and control design is guaranteed, similar performance improvements have been shown in [63, 73].

As well as the other fields, improvements through co-design techniques have been applied to various mechatronic systems [74–81].

In [79], two alternative co-design optimization methods that guarantee system-level optimal designs are presented. The implementation and comparison of these methods are demonstrated over a single spring preload type VSA. As a result of optimization, more compact and energy-efficient design is obtained. The study in

[80] shows the performance improvement through the implementation of the co-design method over a domain-specific optimal system for a single spring preload type VSA. By integrating plant structure with the controller design, 16% reductions are observed in the dynamic torque range requirements. In [79], a similar but more dramatic performance improvement, up to 70%, has been shown.

By utilizing Pareto-optimization, [81] presents a systematic design comparison methodology for compliantly actuated robots. Moreover, [81] develops a homotopic process to make a fair comparison between the control architectures of SEA and single spring preload type VSA.



1.2 Contributions

- We demonstrate that co-design optimization of the mechanical and the control design variables for a lever-type VSA can result in significant performance advantages (i.e., up to 75% decrease in torque ranges) in comparison to subsystem optimization via optimal control or plant optimization.
- Since two actuators are utilized, VSA co-optimization problems inherently consider multiple optimization criteria, based on prioritization of one actuator over the other. We present all non-dominated system-level optima for a lever-type VSA as a Pareto-front curve, to enable the designer to study fundamental trade-offs involved in these designs and to make informed decisions among all possible solutions. In addition to the Pareto-front curve, we provide sample design selections from the Pareto-front curve and discuss the trade-offs involved in various solutions.
- At the boundaries of the Pareto-front curve, we obtain two single-actuator solutions: an SEA solution and a constant-torque solution that achieves the task through stiffness regulation. To the best of authors' knowledge, the constant-torque solution is novel and has not been presented in the literature. Through a modified optimization formulation, we further demonstrate that the constant-torque requirement for this novel solution can be decreased, at the expense of higher compliance of the actuator while performing the task.
- Since Pareto-front curves provide all optimal solutions for a given VSA, they can be used to compare different VSA designs with each other. Along these lines, we compare Pareto solutions of lever-type VSA mechanisms with those of single spring preload adjustment based VSA [81], for the same task and objective functions. This comparison provides evidence that none of the mechanisms dominate the other for all design preferences. On the contrary, series type VSAs or single spring preload based VSAs can be favorable for different preferences of the designer.

1.3 Outline

The rest of the thesis is organized as follows:

Chapter 2 reviews the optimization approach utilized in this study, together with its implementation details. This section presents the model of the mechanism, the desired task trajectory, and the mathematical formulation of the multicriteria optimization problem.

Chapter 3 presents and discusses system-level optimization results of the mechanism. Section 3.1 demonstrates a sample system-level optimal design, while Section 3.2 presents all non-dominated solutions for this optimization problem. The novel constant torque design is studied in Section 3.3. Finally, the single-actuator designs are compared with each other in Section 3.4.

In Chapter 4, optimization results for possible configurations are discussed. Section 4.1 presents the advantages of implementation of co-design techniques for compliant actuators over sample design selections. Over the Pareto-front curve, design trade-offs are thoroughly discussed in Section 4.2. Section 4.3 shows that, depending on the required stiffness ranges from the mechanism for the desired task, there exist different constant torque designs. A comparison between lever-type VSA and single spring preload based VSA that are optimized for the same task with the same objectives is presented in Section 4.4.

Chapter 5 concludes the thesis and elaborates on future research directions.

Chapter 2

Methodology and Problem Formulation

This chapter presents the architecture and implementation details of the system-level optimization of a lever-type VSA with an application to knee prosthesis design. The chapter starts with an explanation of the system-level optimization procedure. Particularly, the simultaneous optimization method and the implementation details are introduced. The chapter continues with the description of the multicriteria optimization formulation. Later, the mechanical model of a generic lever-type VSAs is presented together with simplifying assumptions. Finally, the optimization problem for a lever-type VSA is constructed for a performance index that is formulated to improve the energy efficiency of the system by decreasing its weight through the minimization of actuators size.

2.1 Simultaneous Co-Design Approach

To ensure the maximum performance of a mechatronic system, a system-level optimization needs to be ensured such that both the mechanical design and the controller are optimized simultaneously. Hence, the utilization of co-design methodologies is a necessity.

There are different co-design approaches in the literature [79]. In this study, the simultaneous optimization (SIM) method is preferred due to its implementation simplicity. In the SIM approach, a single optimization problem simultaneously handles the mechanical variables and the control variables, and the corresponding static/dynamic constraints.

The SIM method can be formulated as a dynamic optimization problem as follows:

$$\begin{aligned}
 & \underset{d_m, d_c}{\text{minimize}} && J(d_m, d_c) \\
 & \text{subject to} && g_m(dm, dc) \leq 0, \quad g_c(dm, dc) \leq 0, \\
 & && h_m(dm, dc) = 0, \quad h_c(dm, dc) = 0, \\
 & && lb_m \leq d_m \leq ub_m, \quad lb_c \leq d_c \leq ub_c.
 \end{aligned}$$

where a system-level design objective J needs to be minimized by changing the mechanical and the control design variables, d_m and d_c , respectively. The mechanical design variables consist of time-independent variables such as the link lengths, the pivot mass, and the spring coefficient. The control design variables are time-varying trajectories, such as the torque trajectories supplied by the actuators. The symbols h_m and h_c respectively define the equality constraints on the mechanical and the control variables. The symbols g_m , and g_c present the inequality constraints with the lower and the upper bounds, the symbols lb_m , ub_m , lb_c , ub_c , on the mechanical and the control design variables. The system dynamics, modeled as ordinary differential equations, are enforced by the equality constraints.

By definition, SIM strategy results in system-level optimal designs. However, it only guarantees optimality within a local neighborhood, due to the non-convex nature of general dynamic optimization problems. Therefore, it is essential to initialize the optimization with a favorable initial design. To achieve a proper initialization, we solve for the optimal control problem for a feasible predetermined mechanical design, before searching for system-level optimized design. This co-design methodology, illustrated in Figure 2.1, can be summarized as follows:

1. Select a feasible mechanical design.
2. Find the optimal control solution for the initial design.
3. Use the obtained optimal control solution as an initial guess to system-level co-optimization problem.

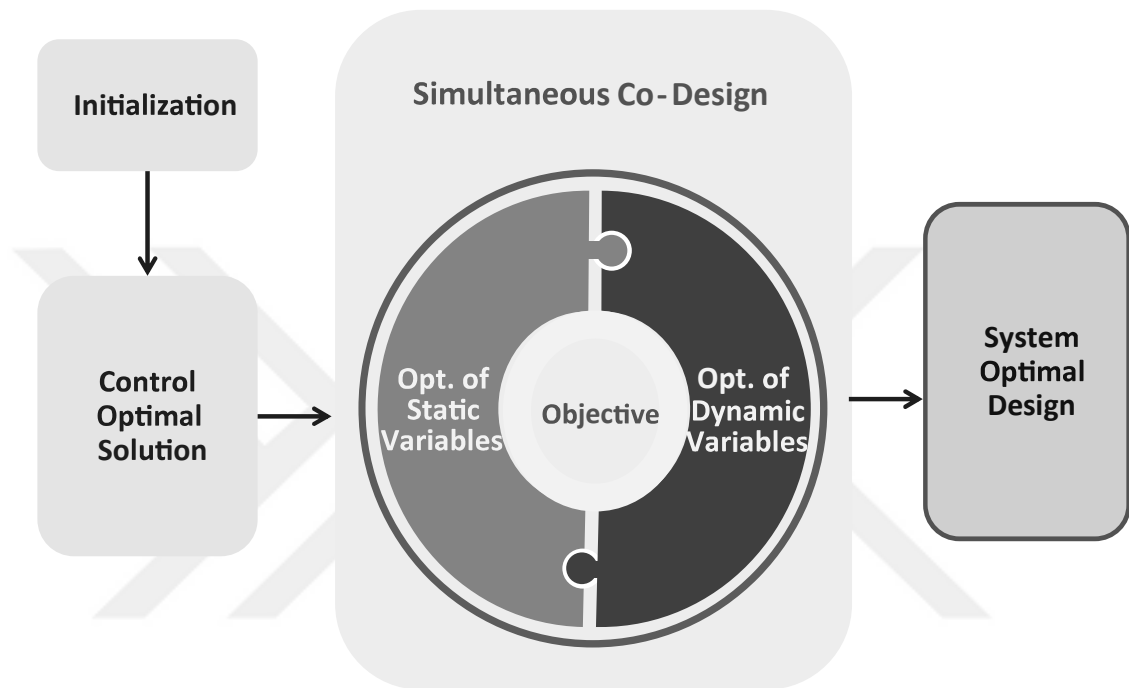


FIGURE 2.1: System-level optimization procedure

2.2 Implementation Details

The simultaneous optimization method is implemented to solve the co-design optimization problem for a lever-type series VSA. This co-design optimization problem is defined as a dynamic optimization problem and solved numerically employing the PROPT package of the TOMLAB Optimization Environment. TOMLAB is an optimization platform and modeling language to solve optimization problems in MATLAB.

To solve the dynamic optimization problem, the PROPT package uses a pseudospectral collocation method to transcribe the continuous optimization problem into a nonlinear programming (NLP) problem. In the pseudospectral collocation method, the state is approximated using an appropriate set of basis functions, and the dynamics are orthogonally collocated with this approximated state [82]. This transcription creates a discrete, finite-dimensional problem. Once the problem is efficiently discretized, the problem can be solved using a nonlinear programming solver. In this study, sequential quadratic programming (SQP) based large-scale nonlinear programming solver SNOPT is utilized [83].

The periodicity of solutions is enforced by the choice of “cyclic” basis function in TOMLAB due to the nature of task trajectories in this design. All other settings of the solver are left at their default values except the user-specified number of collocation points. Twenty collocation points are employed over a single period of design trajectories. The number of collocation points is determined empirically by considering the representation performance of a single period of design trajectories.

2.3 Problem Formulation

In this section, the optimization problem is formulated. Firstly, the mechanism model is introduced with its simplifying assumptions. Secondly, desired task requirements are provided. Later, after providing the definition of multi-objective

optimization, two frequently used methods are explained. Finally, the optimization formulation is constructed by using the mechanism model, requirements of the task, and selected multi-objective optimization approach.

2.3.1 Modeling of the Mechanism

While there exist various implementations of lever arm based series VSAs, a simplified yet comprehensive model is studied without compromising the working principles of the mechanism. The kinematic model of this mechanism is proposed in [25], as illustrated in Figure 2.2. Descriptions of the variables and parameters are presented in Table 2.1.

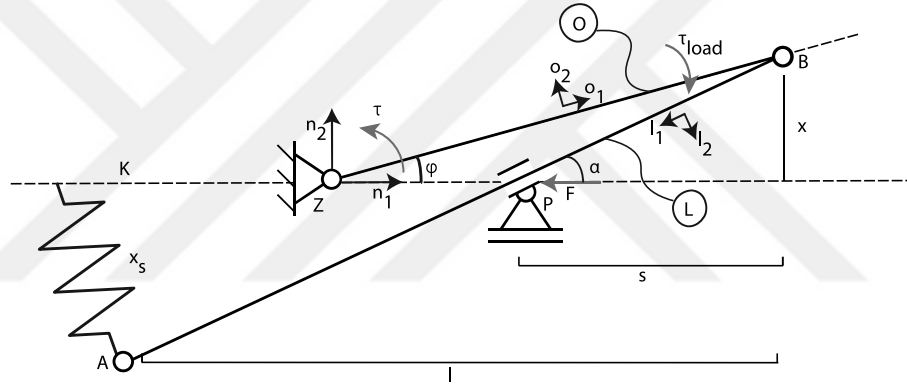


FIGURE 2.2: Simplified model of a lever-type series VSA

The lever arm based series VSA is driven by two actuators where the first actuator (M_1) controls the output angle of the system, and the second actuator (M_2) controls the stiffness of the mechanism under quasi-static conditions. However, there exists some coupling due to the dynamics of the system. Typically a motor M_1 applies τ torque to the output body to adjust the output angle. A linear actuator or a combination of a motor with a ball-screw type mechanism can be used as the second actuator. In this configuration, M_2 transmits force (F) to the pivot Point P . Therefore with the help of F , Point P can be linearly translated along the $\vec{n}_1$ axis.

TABLE 2.1: Parameters used in the numerical analysis

Variables	Description	Unit
τ	Torque applied to the output link	Nm
F	Force applied to the pivot	N
ϕ	Output link position	rad
s	Position of the pivot with respect to M_2	m
l_o	Output link length	m
l_l	Lever link length	m
l	Dist. btw. M_2 and attachment point of the springs	m
m_P	Mass of pivot	kg
D	Inertia matrix	-
C	Coriolis matrix	-
N	The matrix that includes load torque and other forces such as spring forces	-

The location of the pivot changes the stiffness of the system. The closer the pivot is to Point K , the stiffer the mechanism and vice versa for the opposite end of Link AB . A representative example is presented in Figure 2.3, where the stiffness is adjusted using Link AB as a lever mechanism while a desired fixed output angle is satisfied. As the pivot moves toward Point A , the extension of the spring decreases while creating a smaller moment on Link AB . Therefore, the required effort to stay in the same output position decreases under the effect of disturbances.

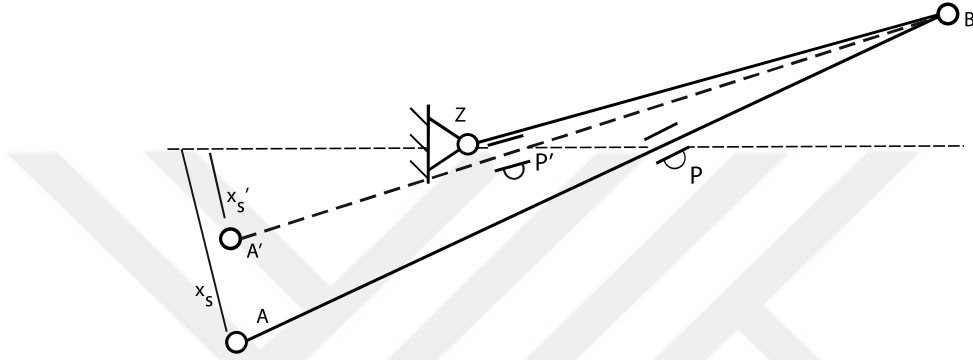


FIGURE 2.3: Stiffness change depends on the pivot position

2.3.1.1 Simplifying Assumptions

- Gravitational forces are not taken into consideration, since the mechanism is considered to work in a horizontal plane.
- Friction effects are considered only at the pivot. All other dissipative effects, such as actuator and link dampings, are neglected as their effect is typically negligible compared to other forces.

Under these simplifying assumptions, the dynamics of this system are obtained in the form of Eqn. (2.1), where $D(q)$, and $C(q, \dot{q})$ represent inertia, and Coriolis matrix, respectively. $N(q, \dot{q})$ matrix composed of load torque and other forces, such as spring forces.

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q, \dot{q}) = u$$

$$\text{where } q = \begin{bmatrix} \phi \\ s \end{bmatrix} \quad u = \begin{bmatrix} \tau \\ F \end{bmatrix} \quad (2.1)$$

2.3.2 Desired Output Trajectory

The series lever arm based VSA is optimized by considering the nominal trajectories for a human knee prosthesis. The desired output trajectory of the knee joint and the load torque required to be maintained at a human knee joint are presented in Figure 2.4. These task trajectories are obtained using the human walking data in [84] for a healthy person with a bodyweight of 80 kg.

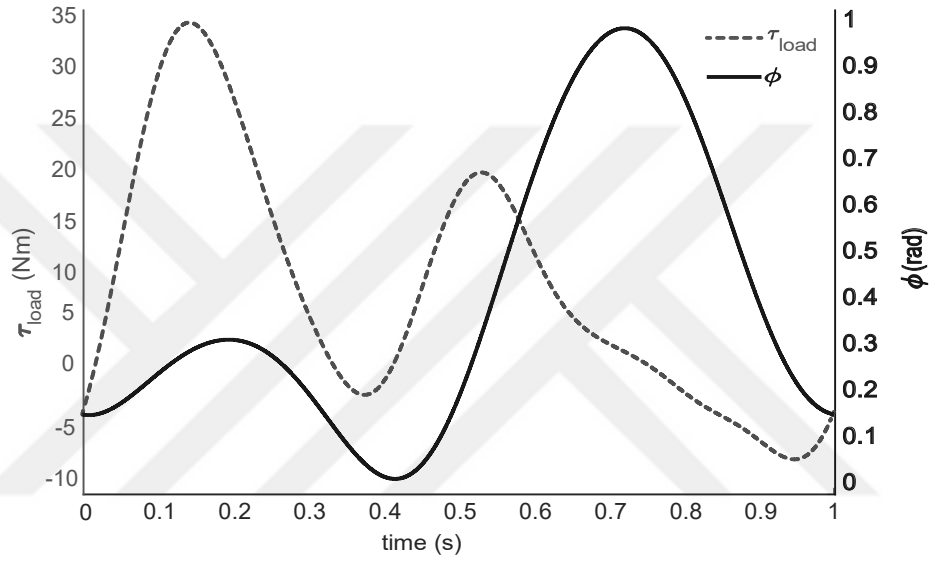


FIGURE 2.4: Nominal output angle and load torque trajectories for a knee prosthesis

2.3.3 Optimization Model for a Lever Arm Based Series VSA

To make the system more energy-efficient for human use, a user-friendly and lightweight design is pursued. In the mathematical formulation of the system-level design optimization problem, the objectives are chosen by considering the weight of the system. Since the motor weights bring a significant contribution to the overall system weight, a design objective with the purpose of minimization of motor weights can result in a lightweight design. Considering the fact that the

weight of the actuators is directly proportional to their torque outputs, the difference between the maximum and minimum of the control inputs are minimized, such that the torque requirement is reduced.

As presented in Section 2.3.1, a lever-type VSA requires two actuation units where M_1 applies τ torque to the output Link ZB , and M_2 transmits F force to the pivot P . Since the dynamic ranges of the control inputs are aimed to be minimized, these two objectives are formulated as Eqn. 2.2.

$$\begin{aligned} J_1 &= [\max(\tau(t)) - \min(\tau(t))]^2 \\ J_2 &= [\max(F(t)) - \min(F(t))]^2 \end{aligned} \tag{2.2}$$

Since the weight of the motors decreases significantly as the required dynamic torque range reduces, such a design objective serves the purpose of weight minimization in an overall robotic system. Therefore, the minimization of dynamic torque ranges in a robotic system may bring substantial weight savings.

Note that other design objectives can be utilized for different aspects, such as kinematics/dynamics, motion ranges, and power consumption. However, in this study, the objective has been selected to minimize the required dynamic torque range by aiming to reduce the physical system weight. As a consequence of a design with two actuators, the design optimization problem turns into a multi-objective optimization problem.

2.3.3.1 Multi-objective Optimization

Multi-objective optimization (multicriteria optimization, Pareto optimization) is concerned with the minimization of a vector of objectives $F(x)$ that can be the subject of several constraints or bounds [85].

$$\begin{aligned}
& \underset{x}{\text{minimize}} && F(x) = [F_1(x), F_2(x), \dots, F_k(x)]^T \\
& \text{subject to} && g_j(x) \leq 0, \quad j = 1, 2, \dots, m, \\
& && h_l(x) = 0, \quad l = 1, 2, \dots, e,
\end{aligned}$$

where k is the number of objective functions, m is the number of inequality constraints, and e is the number of equality constraints.

The feasible design space (feasible region, decision space, constraint set) X in the parameter space x is defined as $\{x \mid g_j(x) \leq 0, j = 1, 2, \dots, m; \text{ and } h_i(x) = 0, i = 1, 2, \dots, e\}$ [85, 86]. The feasible criterion space (feasible cost space, attainable set) Z is defined as the set $\{F(x) \mid x \in X\}$. The vector of objectives, $F(x)$, maps the parameter space into objective function space, as presented in Figure 2.5 for a two dimensional case.

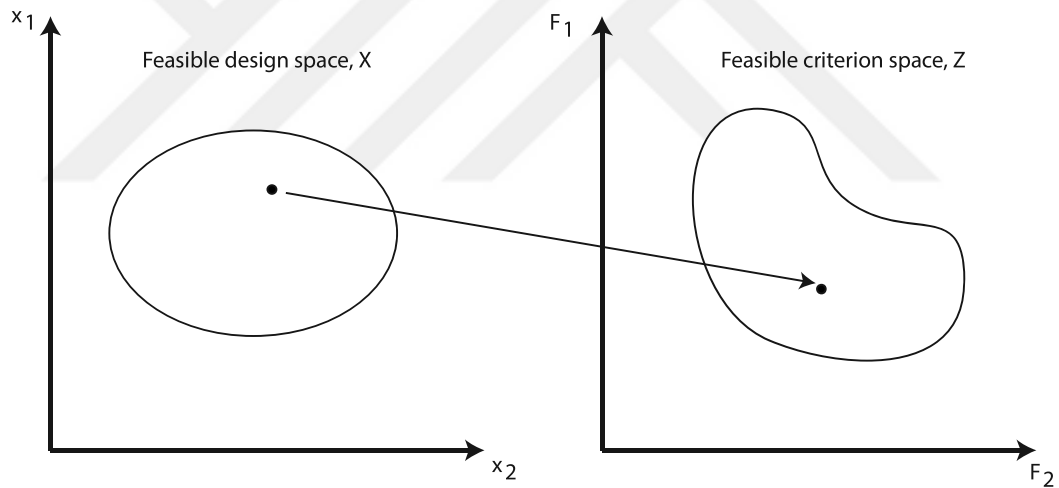


FIGURE 2.5: Mapping from parameter space into objective function space

Note that, since $F(x)$ is composed of more than one objective, if any of these objectives are conflicting, there exists no unique solution that simultaneously minimizes both objectives. Since the trade-offs between the objectives are not entirely apparent in most of the cases, the relative importance of these objectives is not known.

Instead of a unique solution that minimizes all objectives simultaneously, a set of mathematically equally good solutions can be computed. These solutions are

presented in [87–90] as non-dominated, efficient, Pareto optimal, and non-inferior solutions. As stated in [85], a non-dominated solution is a solution that is one, in which an improvement in one objective requires the degradation of another objective.

A point $x^* \in X$ is Pareto-optimal iff there does not exist another point $x \in X$ such that $F(x) \leq F(x^*)$, and $F_i(x) \leq F_i(x^*)$ for at least one objective function. As presented in Figure 2.6 all Pareto optimal solutions lie on the boundary of the feasible criterion space Z [86] and all these non-dominated solutions constitute the Pareto-front [91, 92].

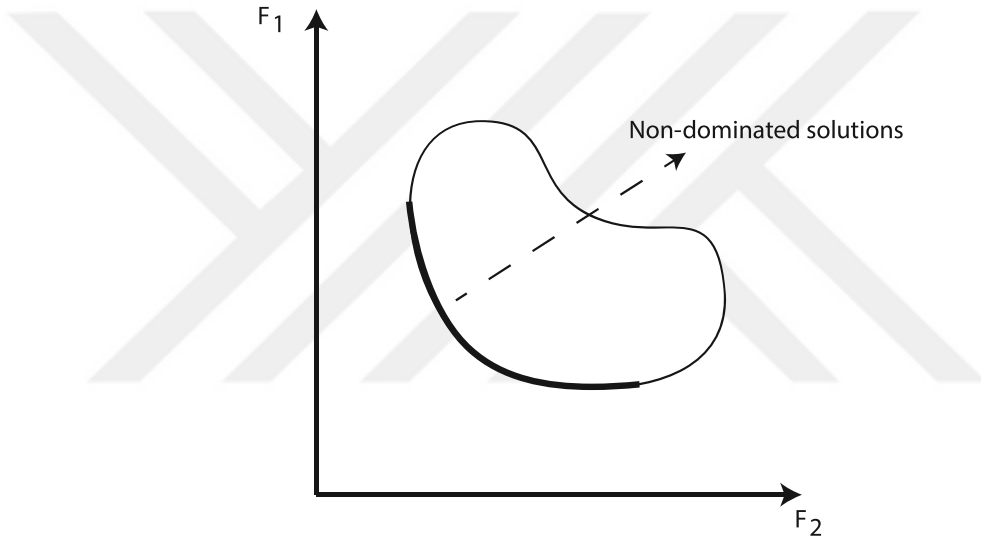


FIGURE 2.6: Pareto solutions lies on the boundary of feasible criterion space

Other than the non-dominated solutions, any point in the feasible design space X cannot improve all the objectives simultaneously. Therefore, the concern of multi-objective optimization is the computation and selection of non-dominated solutions forming the Pareto-front. Although there exists a wide range of techniques for multi-objective optimization [86, 93, 94], frequently used weighted sum and ϵ -constrained methods are described in the subsequent sections.

Weighted Sum Method: The most common technique used in multi-objective optimization is the weighted sum method. By constructing a weighted sum of objectives in the vector $F(x)$, the weighted sum method converts the multi-objective problem into a scalar problem in the form of:

$$U = \sum_{i=1}^k w_i F_i(x)$$

where w is a vector of weights generally set by the designer. However, assigning weights to each objective is not a trivial task since the trade-offs between different objectives may not fully be known a priori optimization. It should also be noted that the assigned weights do not necessarily correlate directly with the relative importance of objectives [85]. Moreover, even if all possible weight combinations are implemented, specific solutions cannot be reachable if the boundary of non-dominated solutions is concave. In particular, it is not possible to obtain points on non-convex regions of the Pareto-front in the criterion space, as explained in [95–97]. This drawback of the weighted sum approach is geometrically illustrated in [85, 96].

By considering two dimensional criterion space, the two objectives in the problem creates a line, L , $w^T F(x) = c$. The multi-objective optimization can be interpreted as finding the point where L just touches the boundary of Z while moving outwards from the origin. The slope of L is determined by the weights w_1 and w_2 . Through systematic variation of weights set of solutions on the boundary of Z can be obtained. However, when the feasible criterion space is non-convex, the aforementioned convexity problem occurs. Because it is not possible to obtain the solutions between A and B points with any combination of weights.

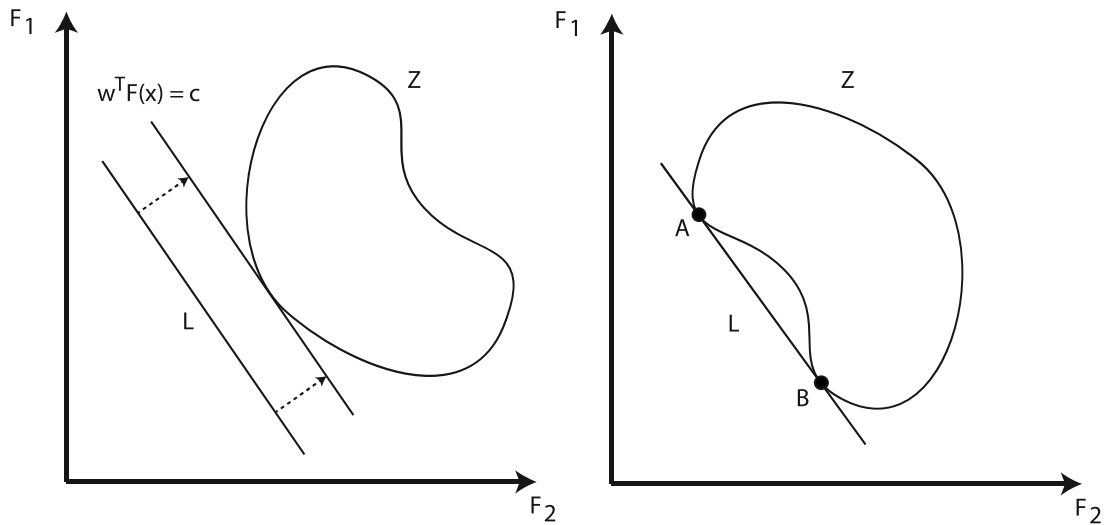


FIGURE 2.7: Geometrical representation of the weighted sum method

ϵ -Constraint Method: The ϵ -constraint method proposes minimizing one of the objectives and expressing the other objectives in the form of inequality constraints, as given in Eqn. 2.3 [93]. In this way, the ϵ -constraint method also handles a multi-objective problem as a series of single-objective problems [98–100]. Through the systematic variations of the constraint bounds, ϵ_j , set of Pareto-optimal solutions can be obtained.

$$\begin{aligned} & \underset{x \in X}{\text{minimize}} && J_1(x) && i = 1, 2, \dots, k \\ & \text{subject to} && J_j(x) \leq \epsilon_j, && j = 1, 2, \dots, k, i \neq j \end{aligned} \quad (2.3)$$

As presented in Figure 2.8, on the objective function constraints by varying the upper bounds, which correspond to the ϵ values, a new single objective optimization problem is created along the Pareto-front curve. Unlike the weighted sum approach, the ϵ -constraint method can solve for non-convex regions of the Pareto-front, as illustrated in Figure 2.8.

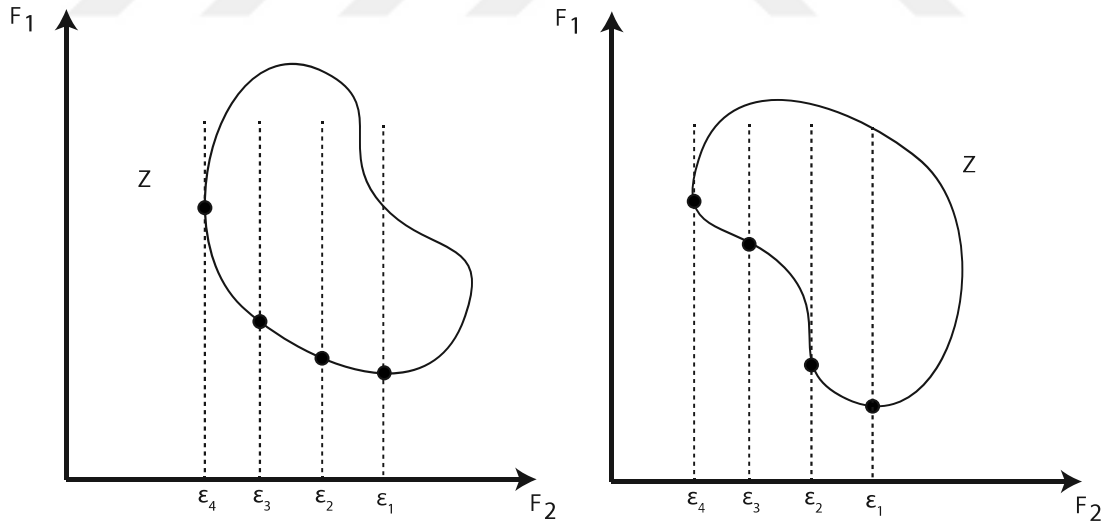


FIGURE 2.8: Geometrical representation of the ϵ constraint method and generation of solutions on non-convex regions

In the co-design optimization of VSA, not only each dynamic optimization problem is inherently non-convex, but also the resulting Pareto-front for multi-objective

optimization may be non-convex. Therefore to handle possible non-convex Pareto-sets, the ϵ -constraint method is implemented for multi-objective optimization since the ϵ -constraint method works for both convex and non-convex problems.

The design objective is formulated as the minimization of required dynamic torque range where the minimization of required dynamic force range is enforced as an inequality constraint, as shown in Eqn. (2.4) using the ϵ -constraint method.

Depending on the application, different torque and force ranges may be favorable. Along these lines, ϵ value can be used to restrict individual requirements from the actuators. All possible solutions can be obtained with a systematic variation of ϵ if the interval between consecutive ϵ values is fine enough.

The mathematical formulation of the simultaneous co-design problem is constrained by the physical requirements and the dynamical behavior of the system, as presented in Eqn. (2.4).

$$\begin{aligned}
 & \underset{(k_s, l_l, l_o, l, m_P, \tau(t), F(t))}{\text{minimize}} && J_1 \\
 & \text{subject to} && J_2 \leq \epsilon, \\
 & && lb_{k_s} \leq k_s \leq ub_{k_s}, \\
 & && lb_l \leq l_l \leq ub_l, \\
 & && lb_l \leq l_o \leq ub_l, \\
 & && lb_l \leq l \leq ub_l, \\
 & && lb_{m_P} \leq m_p \leq ub_{m_P}, \\
 & && D(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q, \dot{q}) = u
 \end{aligned} \tag{2.4}$$

where

$$q = \begin{bmatrix} \phi \\ s \end{bmatrix} \quad u = \begin{bmatrix} \tau \\ F \end{bmatrix}$$

$$J_1 = [\max(\tau(t)) - \min(\tau(t))]^2$$

$$J_2 = [\max(F(t)) - \min(F(t))]^2$$

Chapter 3

System-level Optimal VSAs

In this chapter, the effectiveness of the co-design technique in the lever-type VSAs is presented through the comparison with a conventional domain-specific approach, such as optimal control. Since this optimization is an inherently multicriteria optimization problem, all non-dominated system-level optimal designs are computed as a Pareto-front curve. Two single-actuator solutions consisting of a series elastic actuation solution, and a novel constant torque solution are presented. Moreover, by modifying the optimization formulation, a set of constant torque designs at various stiffness levels are computed. Two single actuator designs with no additional force/torque input requirements are presented.

3.1 A Sample System-Level Optimal Solution

To demonstrate the difference between a system-level optimal solution with an optimal control solution, a sample result is provided in Figure 3.1. This solution is obtained as a result of the optimization problem stated in Eqn. (2.4) using the upper and lower bounds presented in Table 3.1. In this specific problem, the force range of M_2 is penalized far more than the torque range of M_1 to allow a large torque range for M_1 .

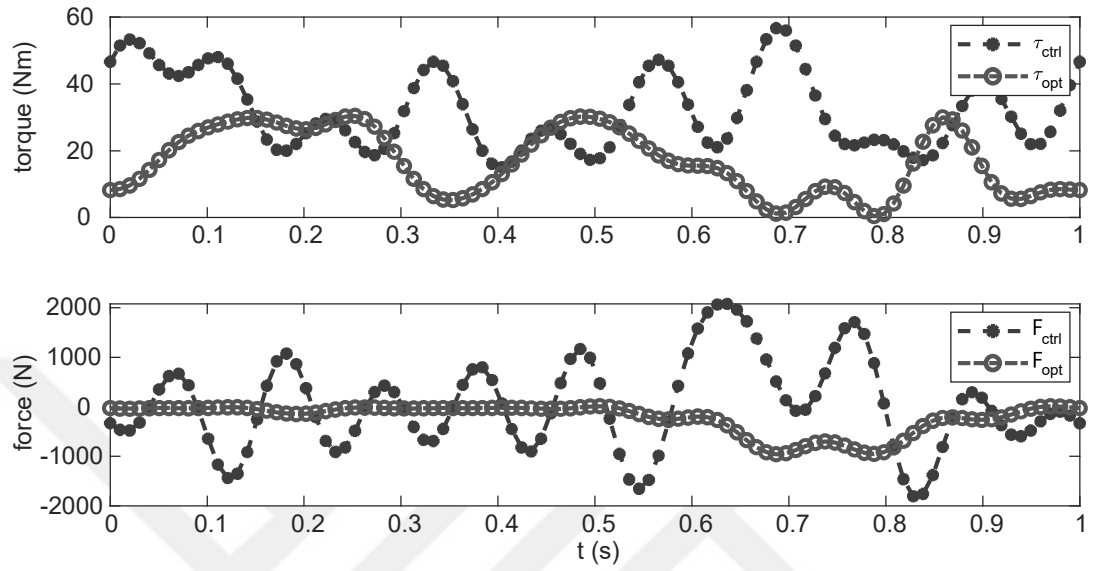


FIGURE 3.1: Comparison of trajectories of control optimal and system-level optimal solutions

TABLE 3.1: Bounds on mechanical design variables

Parameter	Numerical Value	Description	Unit
$[lb_{k_s} \ ub_{k_s}]$	$[1000 \ 100000]$	Bounds on spring constant	N/m
$[lb_l \ ub_l]$	$[0.26 \ 0.30]$	Bounds on link lengths	m
$[lb_{m_P} \ ub_{m_P}]$	$[0.1 \ 0.5]$	Bounds on pivot mass	kg

As a result, the force range of M_2 is decreased to 970 N from 3883 N as shown in Table 3.2. Since the penalization on the force range of M_2 is higher, 75% change has occurred on force range of M_2 while the torque range of M_1 is decreased by about 28%.

The torque and force trajectories of M_1 and M_2 are presented in Figure 3.1. The trajectories with the subscripts “ctrl” and “opt” depict the optimal control and the system-level optimal trajectories, respectively.

Initial mechanical variables of the system and the optimized mechanical variables after SIM optimization are presented in Table 3.3. After optimization using the SIM approach, the mass of the device is higher with longer link lengths, also the system utilizes a higher spring constant. Although the weight of the system increases by choosing longer link lengths, a smaller control input range allows the designer to choose a smaller actuator. Therefore the final version of the design can be lighter, and more compact with lower inertia depending on the actuator selection.

TABLE 3.2: Change in the torque and force ranges

	$\Delta\tau(\text{Nm})$	$\Delta F(\text{N})$
optimal control	41.62	3882.5
system optimal	30.01	969.79
change in %	27.90	75.02

TABLE 3.3: Initial system and system-level optimized mechanism mechanical variable values

	$k_s(\text{N/m})$	$m_P(\text{kg})$	$l_1(\text{m})$	$l_o(\text{m})$	$l(\text{m})$
initial design	5000	0.10	0.20	0.20	0.20
system optimal	12600	0.10	0.26	0.26	0.26

The use of co-design techniques can achieve substantial performance improvements (e.g., a 75% decrease in the dynamical torque range requirement), as seen from the comparison of optimal control solutions of a predesigned mechanism and system-level optimal solution.

3.2 Pareto Optimization

In the previous section, the effectiveness of a system-level optimization is presented on a sample design of the lever-type variable stiffness mechanism. However, the original problem is a multicriteria optimization problem, where the objective is composed of minimization of the torque and force ranges of two different actuators. There exists a trade-off between these actuators, decreasing the torque/force range of one actuator causes an increase in the torque/force range of the other actuator. Therefore, while optimal, a solution for a particular choice of ϵ values in the constraints of the optimization problem might not end up with an insightful solution for the designer. Different design options for the designer need to be computed in the presence of trade-offs between conflicting objectives.

Corresponding to altering design preferences, by systematic variation of ϵ in the optimization problem, all possible system-level optimal designs can be obtained as a Pareto-front curve. From all available system-level optimal solutions, the solutions that are non-dominated by any other solutions form the set of Pareto optimal solutions. All solutions in the set of Pareto optimal solutions are considered equally good in terms of the objectives considered and guarantee to perform at their full potential. By using the set of Pareto optimal solutions, designers can study trade-offs and make insightful design preferences, by introducing additional design criteria, not considered as part of original problem.

The multicriteria optimization methodology to obtain all system-level optimal designs is previously presented in Section 2.1. Within this procedure, the change of parameters results in different system-level optimized designs. Moreover, the dynamic optimization problem has a non-convex design space, and criterion space hence has a non-convex Pareto solution. As a result of the nature of the problem, there exist multiple local optima, and each individual optimization guarantees only local optimality. Therefore, the solutions should be obtained in a homotopic manner within the same locality to enable the study of design trade-offs. For this reason, all the solutions are initialized by using previous system-level optimal design with small changes in the resulted design variables. The lower and

upper bounds used in this optimization problem for the mechanical variables are presented in Table 3.4, and the resulting Pareto-front curve is presented in Figure 3.2.

In Figure 3.2 the system-level optimal solutions are initialized with the case $\epsilon = 0$, where $\Delta\tau = 50.36 \text{ Nm}$. In this initial configuration, the torque range of M_1 is not penalized, while the force range of M_2 is minimized. Later, all other system-level optimal solutions are computed until the torque range of M_1 is minimized, while the force range of M_2 is not penalized. Two solutions in the corners indicated with rectangles in Figure 3.2 correspond to these two single optimization cases, while all other solutions are multicriteria optimization.

TABLE 3.4: Bounds on mechanical design variables

Parameter	Numerical Value	Description	Unit
$[lb_{k_s} \ ub_{k_s}]$	$[1000 \ 100000]$	Bounds on spring constant	N/m
$[lb_l \ ub_l]$	$[0.26 \ 0.30]$	Bounds on link lengths	m
$[lb_{m_P} \ ub_{m_P}]$	$[0.1 \ 0.5]$	Bounds on pivot mass	kg

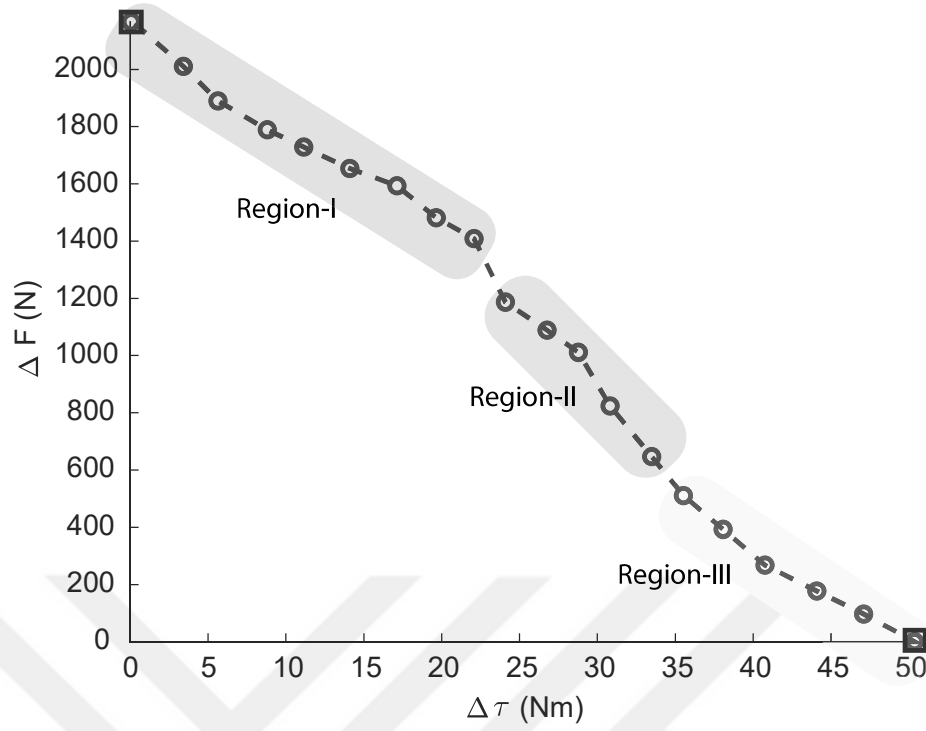


FIGURE 3.2: Obtained Pareto-front curve using the set of Pareto optimal solutions

When ϵ is small, the force range of M_2 is minimized, and the solution on the right lower corner is obtained. In this solution, the torque range of actuator is about 31 Nm , where the system needs no force input. Since there is no stiffness variation in this solution, this configuration corresponds to a series elastic actuation.

The other extreme on the left upper corner is obtained for large values of ϵ . Since the torque range of M_1 is minimized in this design, an almost constant torque present throughout the trajectory.

The solutions on the Pareto-front curve are grouped into three according to the dominance of particular actuators. In Region-I, more significant force ranges are required from M_2 . The solutions in Region-II require moderate efforts from both actuators. In Region-III, almost all work needs to be supplied by M_1 . Sample force and torque trajectories from these three ranges are presented in Figure 3.3.

As depicted in Figure 3.3, for the SEA design, a 50 Nm torque range is required, when there is no force input. While the ϵ values increase, a broader range of force input can be provided to the system when the torque range demand decreases.

However, the mean torque values rise for the solutions with larger ϵ values. When the torque range demand is minimized, a force range of about 2167 N within the presence of 30.7 Nm torque is required.

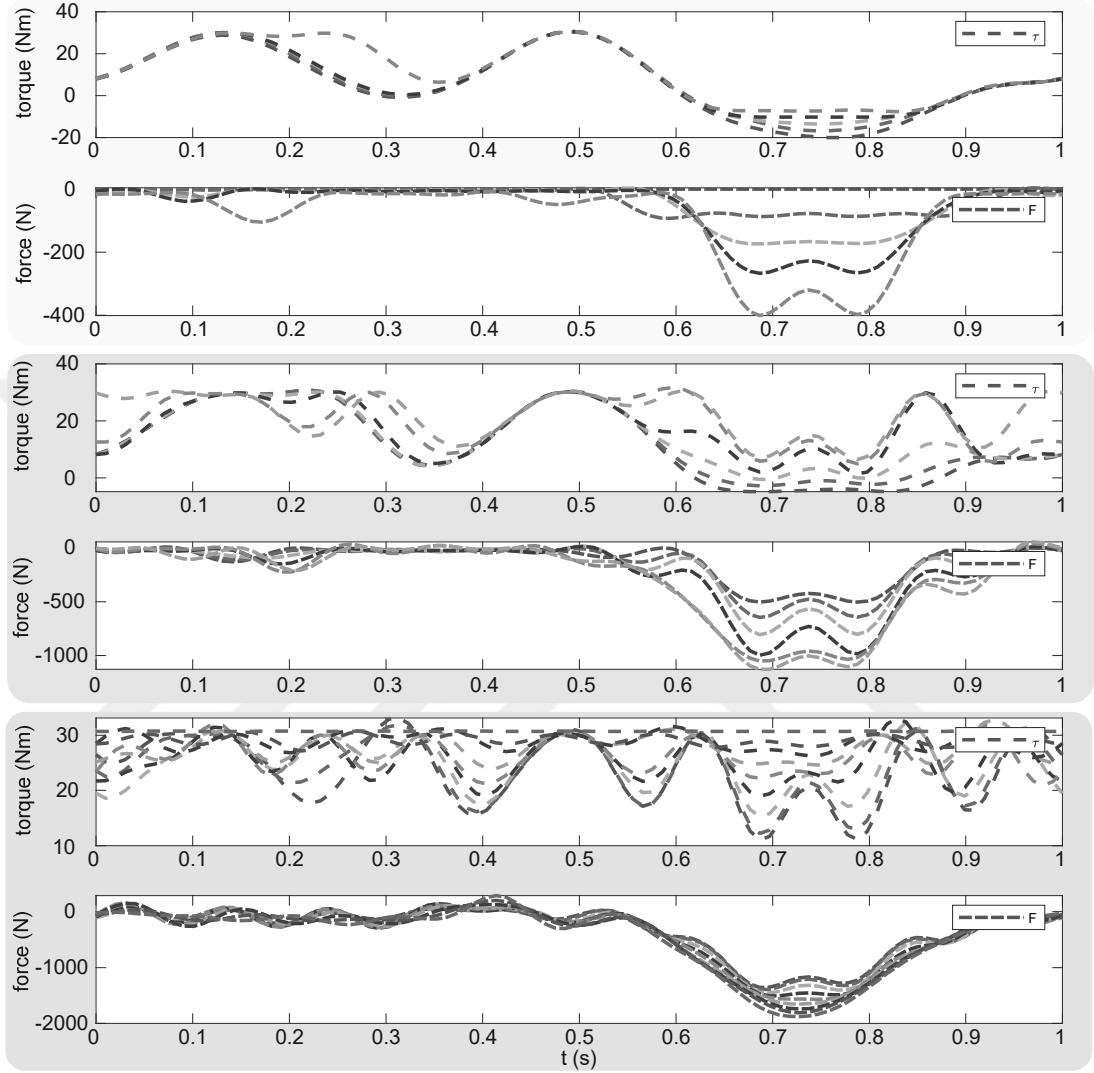


FIGURE 3.3: Solutions from different regions on Pareto-front curve

3.3 Constant Torque Solutions

As stated in [101], the multidisciplinary analysis not only allows us to analyze existing systems, but also provides insight into the design of new system configurations. The system-level optimal designs on the Pareto-front curve have two limit cases. The limit on the right lower corner corresponds to an SEA design, while

the other limit on the left upper corner indicates an interesting new design. In this design, an almost constant 32 Nm torque is supplied to the system. In the presence of this torque, the desired human knee trajectory is followed by the stiffness changes of the mechanism. This solution may present an interesting design that can be implemented in a more compact manner with only stiffness changing mechanism under the presence of continuous torque. A conceptual model of this mechanism is presented in Figure 3.4. Since this system-level optimal design solution is unique in terms of the work distribution of the actuators, similar designs are further investigated.

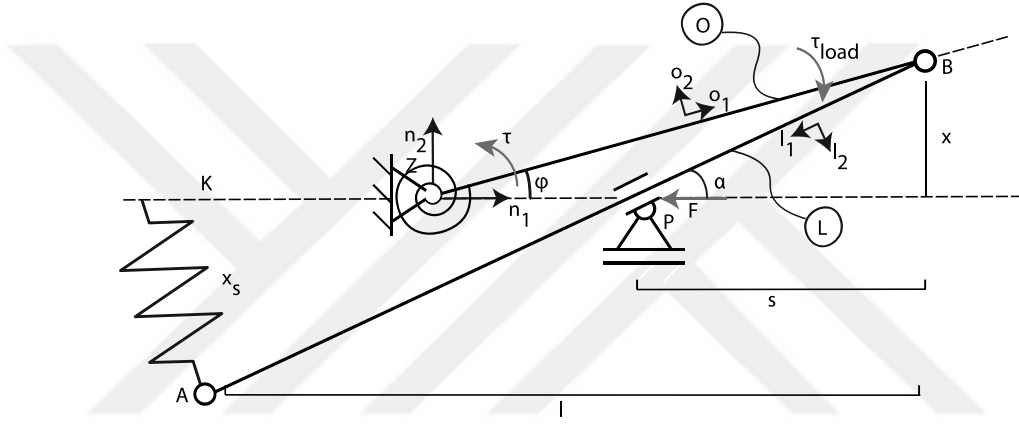


FIGURE 3.4: VSA with pretensioned parallel elastic element

While the work distribution of this case is interesting, it requires a significant amount of constant torque to be applied to the system. Since the supply and maintenance of considerable amount torque might also result in an oversized design, systems that require lower constant torque values are explored.

The optimization problem for the designs with the presence of constant torques is formulated as follows, where the bounds on the mechanical variables are presented in Table 3.5. This design consists of only one actuator M_2 ; hence, the problem turns into a single objective optimization problem in which the amount of constant torque also needs to be optimized.

While formulating the new problem, the inertial effects due to the second actuator are extracted, as this actuator is replaced by a spring. Furthermore, the stiffness range of the spring coefficient in the system is limited to $[20000, 30000]$.

$$\begin{aligned}
& \underset{(k_s, l_l, l_o, l, m_P, F(t))}{\text{minimize}} && J = [\max(F(t)) - \min(F(t))]^2 \\
& \text{subject to} && \tau = \epsilon, \\
& && lb_{k_s} \leq k_s \leq ub_{k_s}, \\
& && lb_l \leq l_l \leq ub_l, \\
& && lb_l \leq l_o \leq ub_l, \\
& && lb_l \leq l \leq ub_l, \\
& && lb_{m_P} \leq m_P \leq ub_{m_P}, \\
& && D(q)\ddot{q} + C(q, \dot{q})\dot{q} + N(q, \dot{q}) = u
\end{aligned} \tag{3.1}$$

where

$$q = \begin{bmatrix} \phi \\ s \end{bmatrix} \quad u = \begin{bmatrix} \tau \\ F \end{bmatrix}$$

TABLE 3.5: Bounds on mechanical design variables

Parameter	Numerical Value	Description	Unit
$[lb_{k_s} \ ub_{k_s}]$	[20000 30000]	Bounds on spring constant	N/m
$[lb_l \ ub_l]$	[0.22 0.30]	Bounds on link lengths	m
$[lb_{m_P} \ ub_{m_P}]$	[0.1 0.5]	Bounds on pivot mass	kg

Supplied constant torque values are changed linearly within the $0 - 45 \text{ Nm}$ range. The force range of the actuator is minimized while the constant torque amount is changed. Constant torque amount is decreased by certain amounts until all the work is done by M_2 , which is the only actuator in this design. As a result of this optimization, plentiful non-identical designs are developed and these design are presented in Figure 3.5.

As shown in Figure 3.5, in these constant torque designs, the provided torque values can range from 0 Nm to 44.75 Nm for this optimization problem.

Table 3.6 presents optimal design variables for system-level optimized mechanical design for constant torque VSA mechanism. In particular five sample results are represented in Table 3.6.

Figure 3.5 demonstrates the supplied torque and force trajectories during the execution of motion. The optimal force trajectories can be grouped into three according to required force ranges. As seen from Figure 3.5, these groups are related to the provided continuous torque values, and there are similar optimal force trajectories for all of the obtained solutions.

TABLE 3.6: Mechanical variable results of co-design optimization of constant torque VSA mechanism

		$\mathbf{k_s}$	$\mathbf{m_P}$	$\mathbf{l_l}$	$\mathbf{l_o}$	$\mathbf{l}$
@ $\tau = 43$	Nm	25500	0.10	0.30	0.30	0.30
@ $\tau = 36.5$	Nm	24270	0.10	0.28	0.28	0.28
@ $\tau = 25.75$	Nm	20000	0.10	0.26	0.26	0.26
@ $\tau = 11$	Nm	25207	0.10	0.30	0.30	0.30
@ $\tau = 0$	Nm	20000	0.22	0.30	0.30	0.30

As demonstrated in Figure 3.5, when the supplied torque values are decreased, the required force range is also reduced. Although the exact opposite of this trend can be expected, this reduced force range can be explained with the compliance level of the mechanism.

As stated in Section 2.3.1, when the pivot moves towards to the springs, the stiffness of the mechanism decreases. As presented in Figure 3.6, when the supplied torque value decreases, optimal trajectories result in a system that operates at a lower stiffness region.

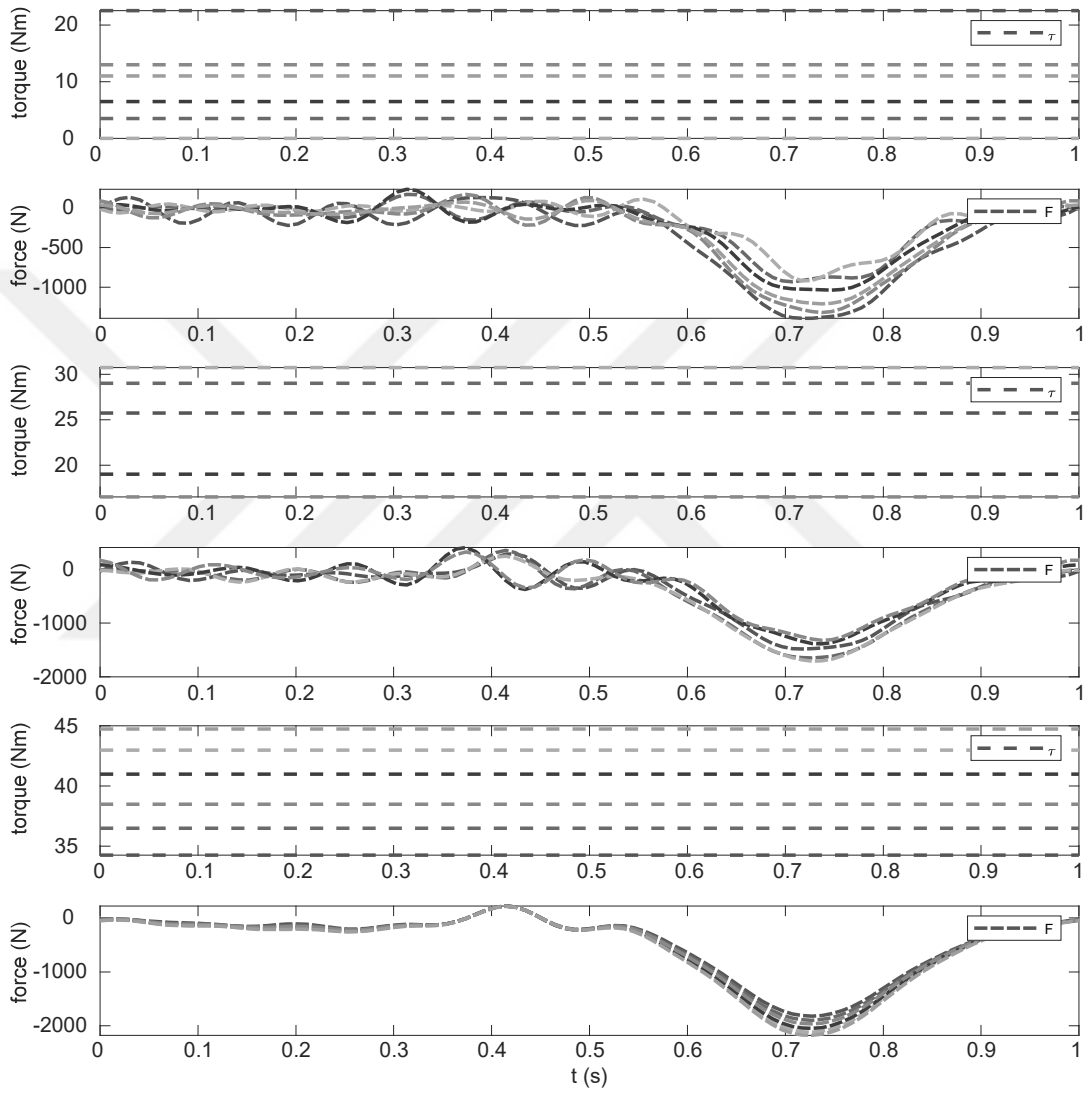


FIGURE 3.5: System-level optimized trajectories of VSAs with continuous constant torque

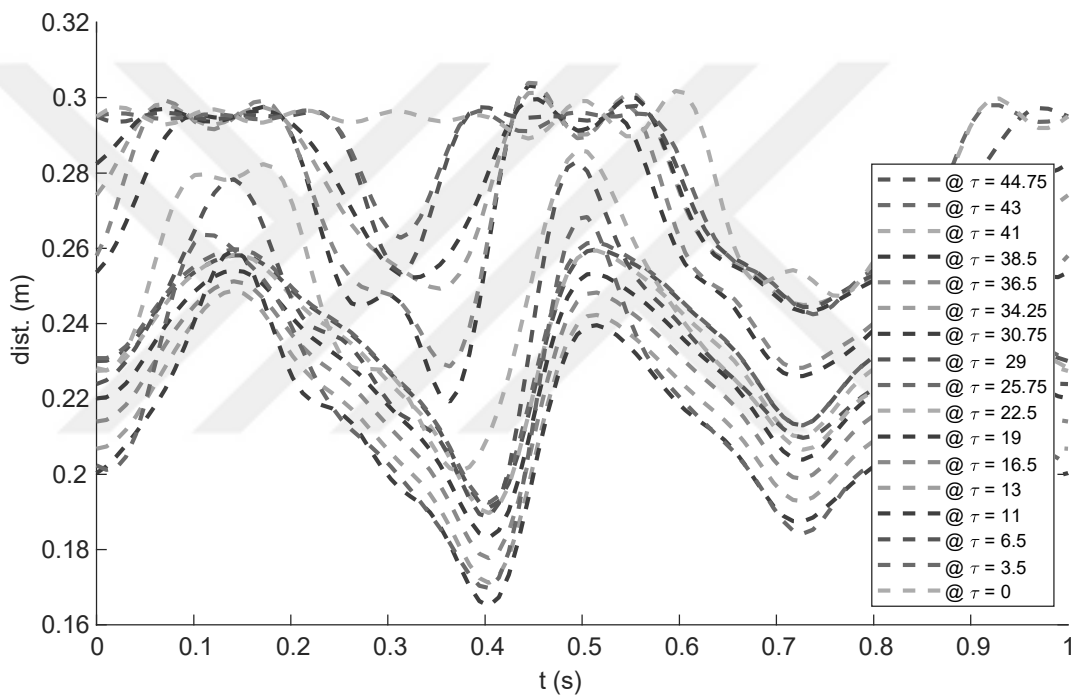


FIGURE 3.6: System-level optimized trajectories of VSAs with continuous constant torque

3.4 Single Actuator Designs without Additional Torque/Force Inputs

Up to this point, two conceptual models for system-level optimized lever-type VSA are investigated. At the limit cases of both these concepts, the required dynamical task can be achieved using a single actuator.

In Figure 3.5, constant torque designs with the torque ranges between $0 - 44.75 \text{ Nm}$ are presented. In the 0 Nm solution, the stiffness change motor M_2 is the only active source in the system.

In the SEA solution of the system, stiffness of the mechanism is kept constant when no force input is required and all the work is performed by a M_1 responsible for output position change as presented in Figure 3.3.

Table 3.7 presents co-design optimization result of mechanical variables for single actuator solutions. Optimal trajectories for the systems that employ only a single actuator are presented in Figure 3.7. Results indicate that when only M_1 is present in the system, about 50 Nm torque range, when only M_2 is employed, about 1000 N force is required to fulfill the dynamical task requirements.

TABLE 3.7: Mechanical variable results of co-design optimization for single motor designs

	$\mathbf{k_s}$	$\mathbf{m_P}$	$\mathbf{l_l}$	$\mathbf{l_o}$	$\mathbf{l}$
@ $F = 0 \text{ N}$	20000	0.22	0.30	0.30	0.30
@ $\tau = 0 \text{ Nm}$	1037	0.10	0.26	0.26	0.26

Consequently, it has been presented that the desired task can be successfully performed through both an SEA mechanism and a stiffness modulation mechanism where there is no additional torque/force input is required. Depending on the designer's preferences, both single actuator designs are available options to perform for the same task.

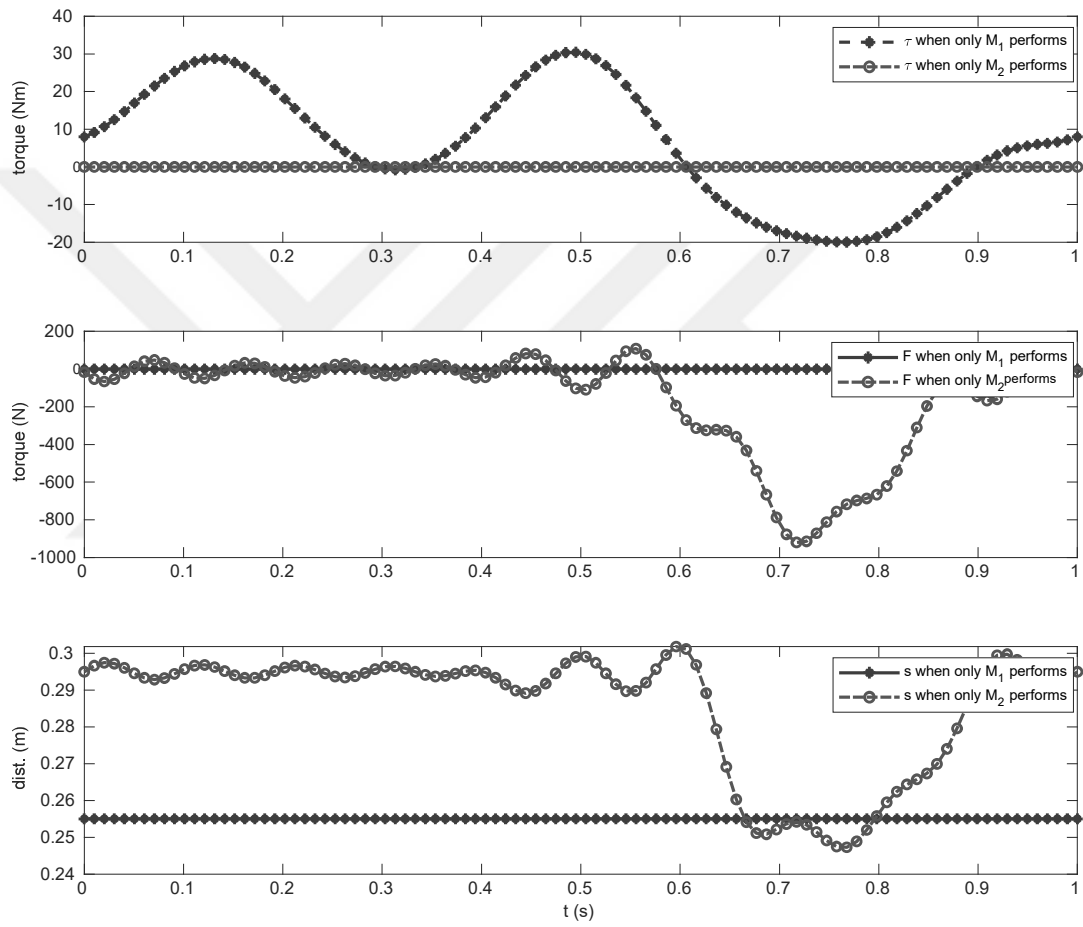


FIGURE 3.7: Comparison of trajectories of single actuator designs

Chapter 4

Design Selection and Discussion

In this chapter, we provide sample design realizations for various solutions. Firstly, we demonstrate the effectiveness of co-design techniques in lever-type VSAs for the objective of minimization of range of required actuator inputs, which enables the use of smaller actuators, resulting in significant weight reductions and increased energy efficiency. Since the problem is a multicriteria optimization problem, all the non-dominated system-level optimal solutions are computed as a Pareto-front curve in Section 3. In this chapter, we demonstrate sample design selections from the Pareto-front curve and discuss the advantages and the disadvantages of various solutions. Later, we state that numerous constant torque design selections are available depending on the desired stiffness ranges from the mechanism for the desired task. Lastly, we provide a comparison between two VSAs such as lever-type VSA mechanisms and single spring preload adjustment based VSA for the same task and objectives.

4.1 Advantages of Co-Design Optimization

Significant performance improvements with respect to isolated design approaches have been achieved by utilizing co-design optimization. Performing system-level optimization rather than the more commonly utilized domain-specific approach,

such as optimal control, results in a more compact and lightweight design. The effectiveness of simultaneous co-design optimization is demonstrated in Tables 3.2 and 3.3 through the change in the required torque/force input ranges, and changes in the mechanical variables.

The effect of these reductions can be better appreciated by sample motor selections together with the direct changes in the mechanical variables.

The peak-to-peak amplitude of the motor M_2 is decreased from 3882.5 N to 969.79 N . For system-level optimized design, a Maxon RE-25 motor, in combination with a 1:14 gearhead in combination with a screw drive, can provide the required force range within the desired speeds. This actuation unit comes at a weight of only 0.27 kg .

On the other hand, the initial system design with an optimal control subsystem required 3882.5 N . This force can be generated with a Maxon RE-35 motor with a 1:14 gearhead in combination with a screw drive, and the total actuation unit comes at a weight of 0.51 kg [102]. Moreover, this design requires a constant 15 Nm torque throughout the trajectory, as demonstrated in Figure 3.1. Therefore, a constant torque providing element such as a pretensioned torsional spring is required.

Through these design selections, the mass of the actuation unit of M_2 decreases by 47% by performing the system-level optimization. Together with the reduction of actuation unit size, the total weight of the system can also decrease in system-level optimal design since 15 Nm constant torque is no longer required.

Although the reduction ratio in the actuation unit of M_2 is significant, through the utilization of the co-design method, the weight of the mechanical system is increased by the selection of longer links, as shown in Table 3.3.

Apart from the weight alternations through system-level optimization, the co-design method also provides a considerable decrease in apparent inertia of the actuation units. High apparent inertia values cause shrinking of the operational bandwidth of the mechanism, due to the inverse relation between the apparent

inertia and the natural frequency. The initial design, which utilizes Maxon RE-35 motor, has apparent inertia of an 0.0015 kgm^2 , whereas the system-level optimal design with the RE-25 motor has apparent inertia of an 0.0003 kgm^2 . In this way, the bandwidth of the mechanism may also be improved through the utilization of the co-design technique.

4.2 Studying Design Trade-offs Through Pareto Optimization

Based on designers' preferences for specific application scenarios, there exist multiple system-level optimal designs whose performances are guaranteed to be at their best. These system-level optimal designs are presented in Figure 3.2 as a Pareto-front curve. Then Pareto-optimal solutions enable studying the trade-offs among the different designs. To demonstrate how the design of a VSA changes for different design preferences, three designs from Pareto-set, as shown in Figure 4.1 are investigated.

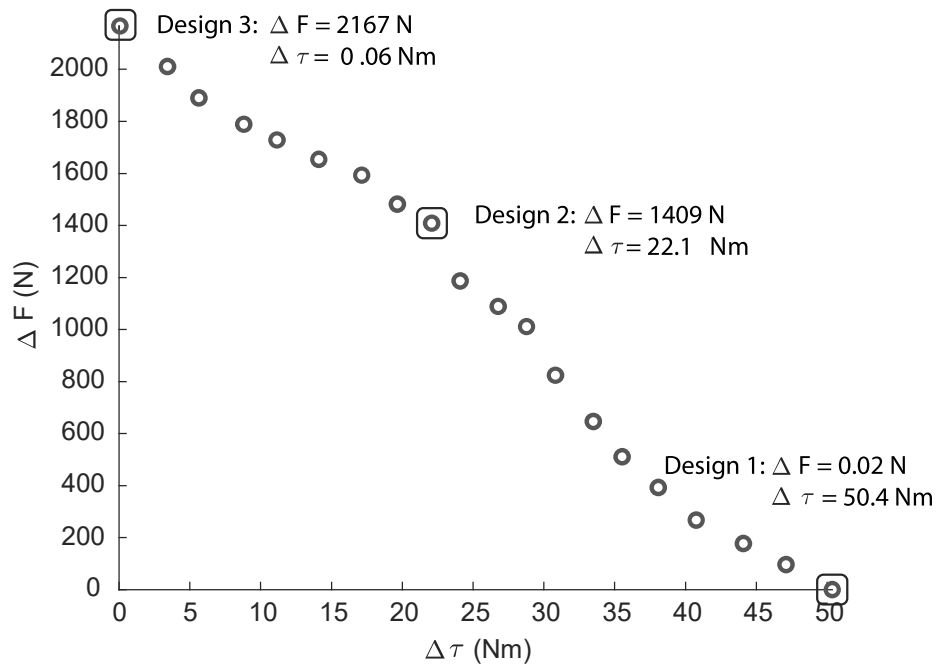


FIGURE 4.1: Three different design from Pareto-set to study design trade-offs

Table 4.1 presents the summary of selected designs. We already know that Design 1 and Design 3 on the Pareto-front correspond to a traditional SEA design and a constant torque design, respectively.

TABLE 4.1: Summary of three different designs on Pareto-set

Design#	$k_s(\text{N/m})$	$m_P(\text{kg})$	$l_1(\text{m})$	$l_o(\text{m})$	$l(\text{m})$	M_1	GH_1	M_2	GH_2
Design 1	1037	0.10	0.26	0.26	0.26	RE-40	1:81	-	-
Design 2	18595	0.10	0.26	0.26	0.26	RE-35	1:43	RE-25	1:33
Design 3	26351	0.10	0.26	0.26	0.26	-	-	RE-40	1:3.5

Design 1 keeps the stiffness of mechanism at a fixed level and through the utilization of M_1 , it meets the dynamical task requirements. An RE-40 motor, in combination with a 1:81 gearhead, can deliver the demanded peak-to-peak torque range for this SEA design. The total weight of the RE-40 motor and the chosen gearhead is 1.25 kg . Different from Design 2 and Design 3, Design 1 does not necessitate constant torque contribution in the system. However, by keeping the stiffness at a fixed level, human-like joint stiffness change modulation cannot be performed by this mechanism.

Unlike the limit solutions, Design 2 is composed of two actuators, M_1 and M_2 , which are responsible for output position and stiffness change. In Design 2, an RE-35 motor in combination with a 1:43 gearhead can be selected for M_1 , while an RE-25 motor with a 1:33 gearhead in combination with a screw drive with a transmission ratio of 1:314 is utilized in M_2 . Although peak-to-peak torque value is 22.1 Nm in Design 2, 11 Nm torque should always be maintained in the system. Neglecting the complexity of this VSA design compared to SEA, Design 2 has 0.18 kg less weight than the Design 1 when only the weight of the motors are taken into consideration. If the mass of torque providing element is higher than 0.18 kg , Design 2 will become heavier and more complicated at the same time. On the other hand, depending on the application, a potential increase in the total weight of the system may be compensated by the advantages provided by the ability to modulate stiffness.

Design 3 is composed of a rotary motor M_2 , in combination with a screw drive in the presence of 30 Nm constant torque. An RE-40 motor, in combination with a screw drive with a 1:3.5 gearhead, is chosen to satisfy both force range and the velocity requirements. Although a smaller motor can also supply the same amount of force a range; however, since the velocity of the pivot is high for this solution, a larger motor is necessitated to avoid velocity saturation. Even with this motor selection, Design 3 is 0.51 kg lighter than a traditional SEA design. However, once again, it should be noted that the calculated weight does not include the mass of the element that provides constant torque to the system.

4.3 Design Development as a Result of Pareto Optimization

Through the Pareto optimization approach, an interesting new design is revealed. It has been shown that the dynamic task can be performed at various constant torque levels.

Although the constant torque results are distinguished from the rest of the designs presented in Pareto-optimal solutions, it basically implies another energy storage element in parallel to the series compliance in the system. This conceptual design has already reported in the literature as series-parallel elastic actuation (SPEA) [103–106]. The same conceptual outcome from a system-level optimization that has been performed for a VSA with preload adjustment of a single spring is presented in [79].

It has already been shown in the literature that parallel elasticity can decrease the peak torque requirements [107, 108]. For instance, the study in [109] reports that through the implementation of parallel elastic actuation (PEA), theoretical peak torque requirements of actuators are reduced by 48% and 66% for ankle and hip actuation of an exoskeleton. In [110], researchers show that through the use of clutched parallel elastic actuators (CPEA), the energy consumption of the

actuator is reduced by about 80% and the peak torque demand by about 66%. The study in [107] concludes that SPEA can be useful to reduce peak power, but SEA is preferable for energy reduction. Presented Pareto-optimal solutions enable designers to study such trade-offs and make well-informed decisions to choose appropriate for actuation.

4.4 Comparison between Series and Single Spring Preload Type VSAs

A comparison of two non-identical designs is not a trivial task since their performance depends on both static and dynamic variables of each design. For this reason, all the non-dominated system-level optimized designs should be presented in the comparison. To make fair comparisons between two mechanisms for any set of design preferences, a systematic optimization methodology should be followed. In this study, the following steps are pursued to ensure a fair comparison.

- Compared designs should be at their maximum performance for all the possible preferences of the designer.
- Both mechanisms should be optimized under the same dynamical task requirements using the same objective functions.
- For each mechanism, optimization results should be in the same neighborhood, if a global solution cannot be ensured.

Since the set of non-dominated system-level optimal solutions for the chosen mechanism constitutes a Pareto-front, the study of Pareto-front curves of different VSA mechanisms enables a comparison between non-identical designs. Given that the Pareto solutions for each mechanism perform at their maximum performance for any potential design preferences of that mechanism drawing inferences through such a comparison is fair [111]. Therefore, a comparison of system-level optimized

designs on the Pareto-front of each mechanism can provide more reliable conclusions.

In this section, two VSA mechanisms based on different working principles are compared through their Pareto-front curves. The work in [81] presents co-design optimization of mechanically adjustable compliance and controllable equilibrium position actuator (MACCEPA). Co-design optimizations computed in this study and the study in [81] are both conducted for a knee prosthesis purpose under the requirements of the same dynamical task with the same objectives.

Both designs employ two actuators, where one of them supplies force and the other torque. In [81], force solutions are converted into torque values by using the characteristic length of the mechanism. For comparison, the torque values given in [81] are converted back to force values since torque and force values are used in this analysis. By using converted force values, Pareto-optimal sets for these two designs are presented in Figure 4.2.

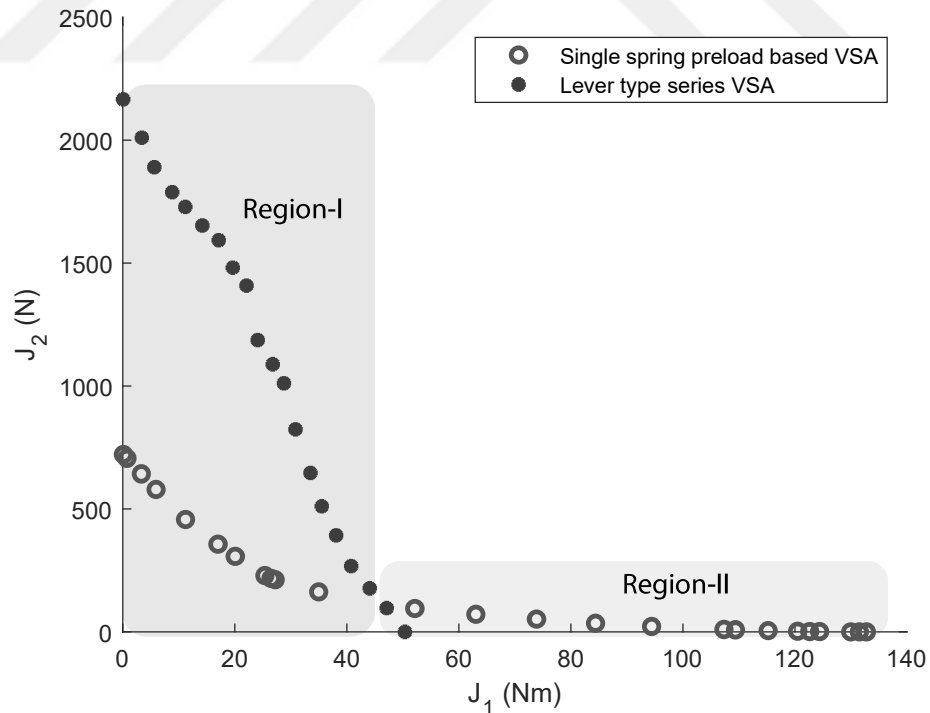


FIGURE 4.2: Pareto-optimal designs of VSA by preload adjustment of a single spring, and lever-type series VSA

As demonstrated in Figure 4.2, either of the VSA mechanisms may be favorable depending on the design considerations. For example, in Region-I single spring preload based VSA designs can be more advantageous in terms of the required torque and force ranges. In contrast to Region-I, the system-level optimal solutions of lever-type VSA can be a more appropriate selection in the Region-II by considering the required torque ranges.

In terms of the constant torque and the constant force optimal control inputs, similar solutions from both Pareto-optimal sets are presented in Figure 4.3 and Figure 4.4, respectively.

Figure 4.3 presents that, by using a VSA by preload adjustment of a single spring, about 700 N force range is required where there is no torque provided to the system. At the same time, this solution requires -600 N continuous force throughout the trajectory. On the other hand, through the utilization of a lever-type VSA, about 2167 N force range is applied to the system when there is a requirement of almost constant 30 Nm continuous torque.

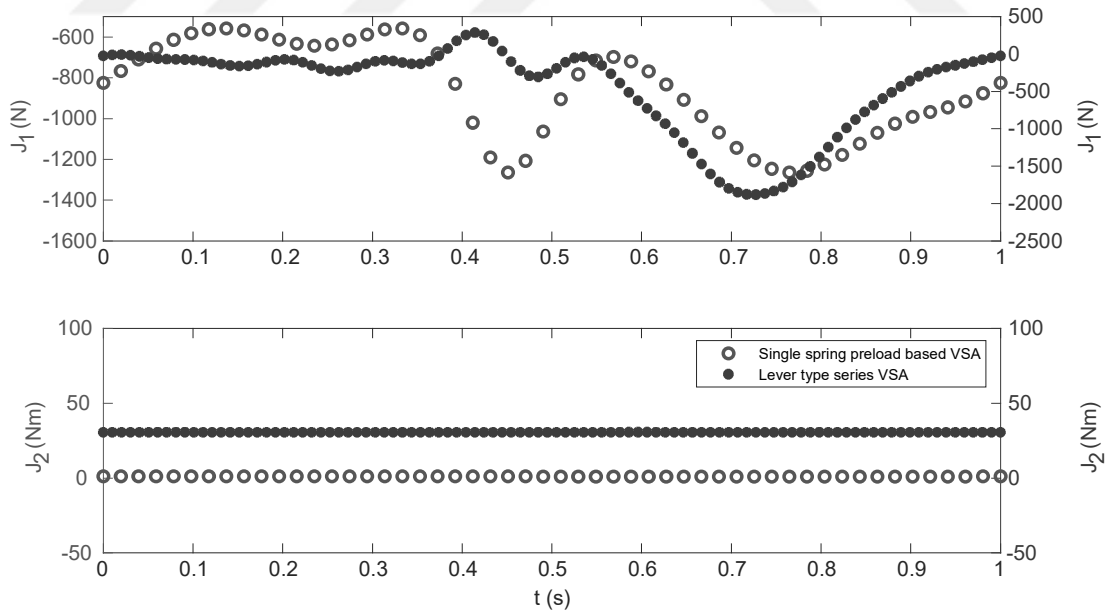


FIGURE 4.3: Constant torque solutions from both Pareto-optimal sets

In Figure 4.4, the corresponding SEA design of lever-type VSA and the constant force solution for a VSA by preload adjustment of a single spring are presented. In SEA solution of a lever-type VSA, 50 Nm torque range is required, whereas about

130 Nm torque range is required for a VSA by preload adjustment of a single spring. Also, VSA with preload adjustment of a single spring requires about -1300 N force, continuously.

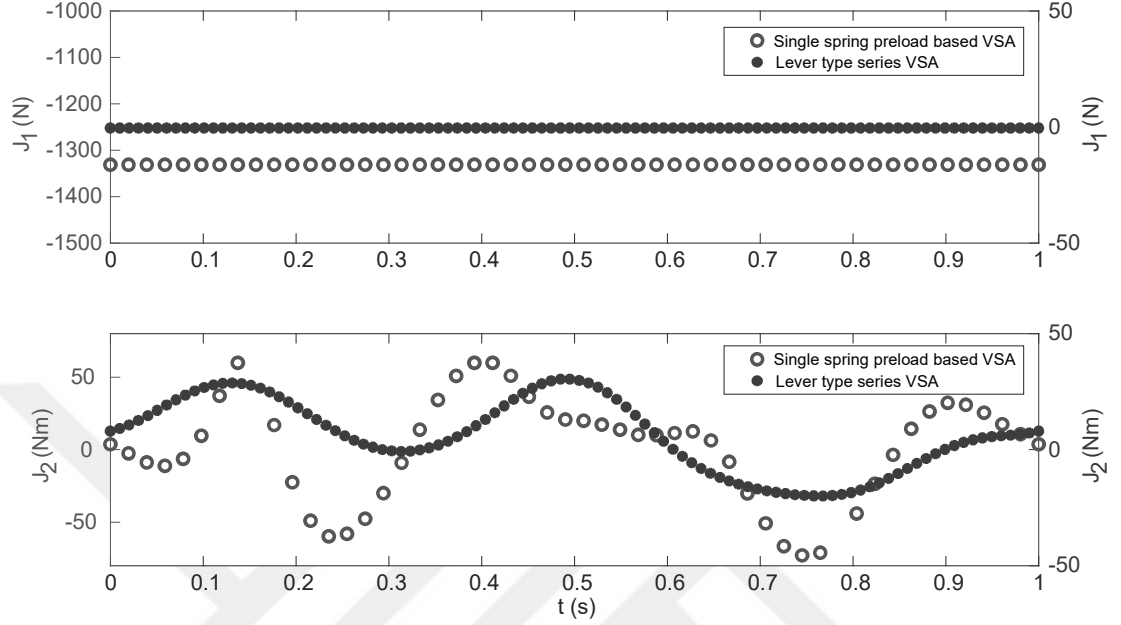


FIGURE 4.4: Constant force solutions from both Pareto-optimal sets

It can be emphasized that the comparison of Pareto-front curves of non-identical designs provide a fair comparison method since they allow the designer to study and compare all system-level optimized designs and trade-offs simultaneously. As a consequence, this comparison provides evidence that depending on the designer's preferences, each of the mechanisms can be favorable depending on the required force and torque ranges. However, the results need to be considered with caution, as both Pareto-front curves may represent local optima due to the non-convex nature of the problem. A conclusive comparison cannot be claimed without ensuring global optimality for both mechanisms.

Chapter 5

Conclusion and Future Work

In this study, we have shown that system-level optimization of a lever-type VSA can provide significant performance improvements compared to isolated subsystem optimization. As a result of system-level optimization, about a 75% decrease has been observed in the torque range demand. These results indicate more compact and lightweight designs.

Since the design of lever-type VSA has multi-DoF, the system-level optimization of the system turns into multi-objective optimization. Through the implementation of a co-design methodology for this multi-objective optimization problem, a set of non-dominated solutions that fully utilize their performance potential are presented. All non-dominated system-level solutions are presented as a Pareto-front curve, to enable the designer to study fundamental trade-offs involved in these designs and make informed decisions among all possible solutions. Various design possibilities from the Pareto-front curve are discussed.

At the limit designs of the Pareto-front curve, two single-actuator solutions are computed. While one of the limit solutions corresponds to a well-known SEA, the other limit solution suggests a VSA in the presence of constant torque that achieves the desired task through stiffness modulation as an alternative design. The design with the constant torque is further investigated, and the system-level optimized

results demonstrate that the constant torque requirement can be decreased at the expense of operational stiffness.

Different VSA designs can be compared with each other through Pareto-front curves since all system-level optimal solutions are provided for selected mechanisms. Since the Pareto-front curve of a VSA with a preload adjustment of a single spring is already studied in the literature [81], a fair comparison between two VSA types is presented for the same task and objective functions. This comparison provides evidence that none of the mechanisms dominates the other for all designer preferences. On the contrary, both series type VSAs or single spring preload based VSAs can be favorable depending on the designer's preferences.

Future works include the comparison of lever-type VSA with antagonistic spring preload based VSAs along with preload adjustment based VSA. Moreover, the advantages of lever-type VSAs can be explored for a larger range of applications with different performance objectives and various task requirements.

Appendix

In Section 2.3.1, system dynamics are presented in a compact manner. In this appendix, an explicit form of the system dynamics are presented.

The kinematic model of a lever-type VSA mechanism which is presented in [25], as illustrated in Figure A.1. Descriptions of the variables and parameters are presented in Table A.1.

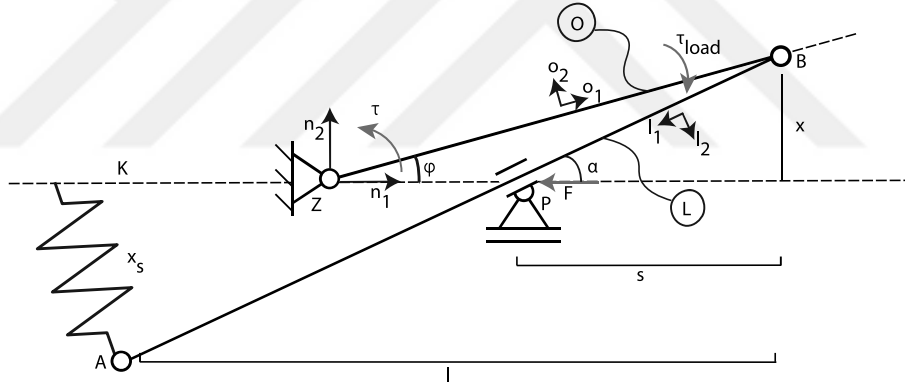


FIGURE A.1: Simplified model of a lever-type series VSA

The lever-type VSA is driven by two actuators where the first actuator (M_1) controls the output angle of the system, and the second actuator (M_2) controls the stiffness of the mechanism under quasi-static conditions. Due to the system dynamics, there exists some coupling between output position control and the stiffness adjustment in the system. Typically the motor M_1 applies τ torque to the output body to adjust the output angle. A linear actuator or a combination of a motor with a ball-screw type mechanism can be used as the second actuator. In this configuration, M_2 transmits force (F) to the pivot Point P . Therefore with the help of F , Point P can be linearly translated along the $\vec{n}_1$ axis.

Simplifying Assumptions

- Since the mechanism is considered to work in an horizontal plane, gravitational forces are not taken into consideration,
- Friction effects is considered only at the pivot. Like actuator and link dampings, all dissipative effects are neglected since their effect is typically negligible compared to other forces.

Under these simplifying assumptions, the generic formulation for the system dynamics has already presented in Eqn. 2.1. The explicit form of the system dynamics are obtained in the form of Eqn. (1).

TABLE A.1: Parameters used in the numerical analysis

Parameters	Description	Unit
τ	Torque applied to the output link	Nm
τ_{load}	Torque required to move the human knee	Nm
F	Force applied to the pivot	N
ϕ	Output link position	rad
s	Position of the pivot with respect to M_2	m
x_s	Amount of spring deflection	m
l_o	Output link length	m
l_l	Lever link length	m
l	Dist. btw. M_2 and attachment point of the springs	m
M_1	The actuator controls the output angle of the mechanism	-
M_2	The actuator controls the stiffness of the mechanism	-
O	Output body	-
L	Lever body	-
P	Pivot point	-
m_O	Mass of output body	kg
m_L	Mass of lever body	kg
m_{eff}	Effective mass in the system	kg
IO_{33}	Moment of inertia of output body about the $\vec{n}_3$ direction	kgm^2
IL_{33}	Moment of inertia of lever body about the $\vec{n}_3$ direction	kgm^2

$$\begin{aligned}
\tau = & \left(IO_{33} + \frac{l_o^2 m_O}{4} - l_o^2 m_L \left(\frac{l_l \cos(\sigma_6) \sigma_7}{\sigma_8 \sigma_9^2} - 1 \right) + \frac{l_o^2 \sigma_7^2 \sigma_1}{\sigma_8^2 \sigma_9^4} \right) \ddot{\phi} \\
& + \left(- \frac{l_o^2 \sin(\phi) \left(\frac{\sigma_7 \sigma_1}{\sigma_8} - l_l m_L \cos(\sigma_6) \sigma_9^2 \right)}{\sigma_8 \sigma_9^4} \right) \ddot{s} \\
& + \frac{l_o^2 \left(\sigma_7 \left(l_l m_L \left(\sin(\sigma_6) 2 + \frac{l_l \sigma_2}{\sigma_8 \sigma_9^5} \right) + \frac{IL_{33} \sigma_2 4}{\sigma_8 \sigma_9^5} \right) \sigma_9^3 - l_l m_L \left(\cos(\sigma_6) \sigma_2 + \frac{l_o \sin(\sigma_6) \sigma_7^2 \sigma_9}{\sigma_8} \right) 2 \right)}{\sigma_5} \dot{\phi}^2 \\
& + \left(- \frac{l_o^2 \left(\left(\frac{IL_{33} \sigma_3 4}{\sigma_8 \sigma_9^5} + \frac{l_l^2 m_L \sigma_3}{\sigma_8 \sigma_9^5} \right) \sigma_7 \sigma_9^3 - l_l m_L \left(\cos(\sigma_6) \sigma_3 + \frac{l_o \sin(\sigma_6) \sin(\phi) \sigma_7 \sigma_9^2}{\sigma_8} \right) 2 \right)}{\sigma_5} \right) \dot{\phi} \dot{s} \\
& + \left(- \frac{l_o^2 \left(l_l m_L \left(\cos(\sigma_6) \sigma_4 + \frac{l_o \sin(\sigma_6) \sin(\phi)^2 \sigma_9}{\sigma_8} \right) 2 - \left(\frac{IL_{33} \sigma_4 4}{\sigma_8 \sigma_9^5} + \frac{l_l^2 m_L \sigma_4}{\sigma_8 \sigma_9^5} \right) \sigma_7 \sigma_9^3 \right)}{\sigma_5} \right) \dot{s}^2 \\
& + \tau_{load} \\
& - k_s l_o \left(-l_l \sin(\sigma_6) + \sin(\phi) (l - l_o) + \frac{l_l \left(l_o \sin(\sigma_6) - \frac{l_o \sin(\phi) (l - l_o)}{\sqrt{\sigma_8 \sigma_9}} \right) \sigma_7}{\sigma_8 \sigma_9^2} \right)
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
\sigma_1 &= m_L l_l^2 + 4IL_{33} \\
\sigma_2 &= \left(\sin(\phi) \left(2l_o^2 \sin(\phi)^2 - \sigma_9^2 \right) + l_o \cos(\phi) \sin(\phi) \sigma_9^3 \right) \sigma_9^2 - \frac{l_o^2 \sin(\phi) \sigma_7^2 2}{\sigma_8} \\
\sigma_3 &= \left(4l_o \sin(\phi)^2 + \cos(\phi) \sigma_9^2 \right) \sigma_9^2 - \frac{l_o^2 \sin(\phi)^2 \sigma_7 4}{\sigma_8} \\
\sigma_4 &= \sin(\phi) \sigma_9^2 2 - \frac{l_o^2 \sin(\phi)^3 2}{\sigma_8} \\
\sigma_5 &= \sigma_8 \sigma_9^5 4 \\
\sigma_6 &= \phi - \text{atan} \left(\frac{l_o \sin(\phi)}{\sigma_9} \right) \\
\sigma_7 &= l_o - \cos(\phi) (l_o - s) \\
\sigma_8 &= \frac{l_o^2 \sin(\phi)^2}{\sigma_9^2} + 1 \\
\sigma_9 &= s - l_o + l_o \cos(\phi)
\end{aligned}$$

$$\begin{aligned}
F = & \frac{l_o^2 \sin(\phi) \left(l_l m_L \left(\cos(\sigma_5) 2 - \frac{l_l \sigma_6}{\sigma_7 \sigma_8^2} \right) - \frac{I L_{33} \sigma_6^4}{\sigma_7 \sigma_8^2} \right)}{\sigma_4} \ddot{\phi} \\
& + \left(m_{\text{eff}} + \frac{l_o^2 \sin(\phi)^2 (m_L l_l^2 + 4 I L_{33})}{\sigma_7^2 \sigma_8^4} \right) \ddot{s} \\
& + \left(- \frac{l_o \sin(\phi) \left(l_l l_o m_L \left(\sin(\sigma_5) 2 + \frac{l_l \sigma_1}{\sigma_7 \sigma_8^5} \right) + \frac{I L_{33} l_o \sigma_1^4}{\sigma_7 \sigma_8^5} \right)}{\sigma_4} \right) \dot{\phi}^2 \\
& + \frac{l_o \sin(\phi) \left(\frac{I L_{33} l_o \sigma_2^4}{\sigma_7 \sigma_8^5} + \frac{l_l^2 l_o m_L \sigma_2}{\sigma_7 \sigma_8^5} \right)}{\sigma_4} \dot{\phi} \dot{s} + \left(- \frac{l_o \sin(\phi) \left(\frac{I L_{33} l_o \sigma_3^4}{\sigma_7 \sigma_8^5} + \frac{l_l^2 l_o m_L \sigma_3}{\sigma_7 \sigma_8^5} \right)}{\sigma_4} \right) \dot{s}^2 \\
& + \frac{k_s l_l l_o \sin(\phi) \left(l_o \sin(\sigma_5) - \frac{l_o \sin(\phi) (l - l_o)}{\sqrt{\sigma_7 \sigma_8}} \right)}{\sigma_7 \sigma_8^2}
\end{aligned}$$

where

$$\begin{aligned}
\sigma_1 &= \left(\sin(\phi) \left(2 l_o^2 \sin(\phi)^2 - \sigma_8^2 \right) + l_o \cos(\phi) \sin(\phi) \sigma_8 \right) \sigma_8^2 - \frac{l_o^2 \sin(\phi) \sigma_6^2 2}{\sigma_7} \\
\sigma_2 &= \left(4 l_o \sin(\phi)^2 + \cos(\phi) \sigma_8 \right) \sigma_8^2 - \frac{l_o^2 \sin(\phi)^2 \sigma_6^4}{\sigma_7} \\
\sigma_3 &= \sin(\phi) \sigma_8^2 2 - \frac{l_o^2 \sin(\phi)^3 2}{\sigma_7} \\
\sigma_4 &= \sigma_7 \sigma_8^2 4 \\
\sigma_5 &= \phi - \text{atan} \left(\frac{l_o \sin(\phi)}{\sigma_8} \right) \\
\sigma_6 &= l_o - \cos(\phi) (l_o - s) \\
\sigma_7 &= \frac{l_o^2 \sin(\phi)^2}{\sigma_8^2} + 1 \\
\sigma_8 &= s - l_o + l_o \cos(\phi)
\end{aligned}$$

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