Modelling the integration of interaural time and level cues in sound lateralization based on a diversity combination perspective

by

Sina Mojtahedi

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Koç University

Graduate School of Sciences and Engineering

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examining committee have been made.

Committee Members:

Prof. Dr. Engin Erzin (Advisor)

Prof. Dr. Pekcan Ungan [Co-Advisor]

Prof. Dr. Burak Acar

Prof. Dr. Attila Gürsoy

Prof. Dr. Erdal Panayırcı

Prof. Dr. Kemal Türker

Date:

To my family

ABSTRACT

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A sound source with non-zero azimuth leads to interaural time and level differences. Studies on the hearing system imply that these cues are encoded in different parts of the brain, but combined to produce a single lateralization percept. According to the duplex theory of sound lateralization, ITD and ILD play a more significant role, respectively, in low- and high-frequency stimulations. The current study investigates the relative weights of these two cues based on the error rate data obtained from two major two-alternative forced-choice experiments. In the experiments, ITD and ILD values were extracted from a generic head-related transfer function (HRTF). In experiment I, three different sounds containing either only low frequencies, or only high frequencies, or both were lateralized by imposing these ITD and ILD values. The subjects' lateralization error rate data indicate that the performance of the binaural hearing system based on ITD and ILD cues undergo a linear combination based on the maximum ratio combining approach. In experiment II, a complex sound consisting of two low- and seven high-frequency harmonic tones was used as test sound. Each stimulus consisted of a reference sound having the ITD-ILD pair of a certain azimuth and a probe sound that was different from the reference sound in either its ITD or in its ILD or in both. Subjects indicated the direction of the shift they perceived in the lateralization of sound. The error rate data showed that the weights of the two cues change with the azimuth of the sound source. Based on the SNR values obtained from the error rate data, it was shown that the two pieces of spatial information supplied by ITD and ILD are combined in the brain, but not additively for shifts in lateral positions around small azimuths. The integration model of the ILD and ITD cues was defined by a diversity combination approach incorporating the observed directional bias with inward preference.

ÖZETÇE

Ses lateralizasyonunda kulaklar arası zaman ve seviye ipuçlarının entegrasyonunun çeşitlilik kombinasyonu perspektifine dayalı olarak modellenmesi

Sina Mojtahedi

Biyomedikal Bilimleri ve Mühendisliği, Doktora

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Sıfır azimut dışından gelen bir ses kaynağı, kulaklar arası zaman ve seviye farklılıklarına yol açar. İşitme sistemi üzerine yapılan çalışmalar, bu ipuçlarının beynin farklı bölgelerinde kodlandığını, ancak tek bir lateralizasyon algısı oluşturmak için birleştirildiğini ima ediyor. Dupleks teorisine göre, ITD ve ILD, sırasıyla, düşük ve yüksek frekanslı uyarılarda daha önemli bir rol oynamaktadır. Bu çalışma, iki alternatifli zorunlu seçim deneylerinden elde edilen hata oranı verilerine dayanarak, ses lateralizasyonunda bu iki faktörün kısmi ağırlıklarını araştırmaktadır. Deneylerde, ITD ve ILD değerleri genel bir transfer fonksiyonundan (HRTF) alındı. Deney I'de, ya sadece düşük frekansları, ya sadece yüksek frekansları, ya da her ikisini içeren üç farklı ses, bu ITD ve ILD değerlerine göre lateralize edildi. Bu sesleri dinleyen deneklerin hata oranı verileri, ITD ve ILD ipuçlarına dayalı işitme sistemi performansının, maksimum oran birleştirme yaklaşımına dayalı doğrusal bir kombinasyona tabi kaldığını gösterdi. Deney II'de, iki düşük ve yedi yüksek frekanslı harmonik tondan oluşan karmaşık bir ses, test sesi olarak kullanıldı. Her bir uyaran, belirli bir azimuta ait ITD-ILD çiftine sahip bir referans ses ile, referans sesten ya sadece ITD, ya sadece ILD, ya da her ikisi bakımından farklı olan bir prob sesten oluşuyordu. Deneklerden, sesin lateralizasyonunda algıladıkları kaymanın yönünü belirtmeleri istendi. Hata oranı verileri, iki işaretin ağırlıklarının ses kaynağının azimutu ile değiştiğini gösterdi. Hata oranı verilerinden elde edilen SNR değerlerine dayanarak, ITD ve ILD tarafından sağlanan iki uzaysal bilginin beyinde birleştirildiği, ancak küçük azimutlar etrafındaki yanal kaymalar için bu birleşimin toplamsal olmadığı gösterildi. ILD ve ITD ipuçlarının entegrasyon modeli, içe doğru kaymalar lehine gözlenen yanlılığı da içeren bir çeşitlilik birleşimi yaklaşımı ile tanımlandı.

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ABBREVIATIONS

ITD	Interaural Time Difference
ILD	Interaural Level Difference
HRTF	Head Related Transfer Function
2AFC	Two-Alternative-Forced-Choice
GUI	Graphical User Interface

Chapter 1

INTRODUCTION

The localization of sound sources plays a crucial role in animal behavior, and it is also a very important function for humans. Detecting sound source direction in horizontal plane is based on the disparities in time and the intensity of the sound reaching both ears, namely interaural time difference (ITD) and level differences (ILD) which are assumed to be responsible for sound lateralization in low and high frequencies, respectively (Middlebrooks and Green, 1991). This frequency-based behavioral dichotomy, which is known as the duplex theory of sound lateralization, has a physio-anatomical counterpart in the brain as is briefly explained below. Studies conducted on the hearing system suggest that ITD and ILD cues to sound lateralization are processed in the medial superior olive (MSO) and the lateral superior olive (LSO) sub-nulei of the superior olivary complex (SOC) located in the brainstem, respectively. However, there is some debate about the roles and neural mechanisms of these nuclei in mammalian sound lateralization (Rubel, 1990). The impulses from the cochlear nucleus ipsilateral to the ear are considered to go directly to the ipsilateral LSO. These impulses provide excitatory input to the neurons of ipsilateral LSO and send inhibitory impulses to the contralateral LSO via inhibitory interneurons in the medial nucleus of the trapezoid body (MNTB) (Rubel, 1990). This process provides the nucleus a neural mechanism capable of 22 encoding interaural intensity differences (Sanes, 1990). On the other hand, the MSO receives excitatory inputs directly from both ipsi- and contra-lateral cochlear nuclei. Meanwhile, a neural network forms axonal delay lines that travel in opposite directions over the impulses from the two ears (Joseph and Hyson, 1993). Such an arrangement makes the nucleus a neural mechanism capable of encoding interaural time differences as hypothesized by *Jeffress* in his classical model (Jeffress and Lioyd, 1948). In addition to this histo-anatomical arrangement, there exists a tonotopic organization in the auditory system of the brain so much so that the neurons display selectivity to the frequency of the stimulating sound. Interestingly, most LSO units that produce discharge rates related to interaural level difference are mostly responsive to sounds with high frequencies (Rubel, 1990). On the other hand, the majority of the units in MSO, which achieves the place-coding of interaural time differences, are most responsive to sounds with low frequencies (Rubel, 1990). This appears to be a physio-anatomical compliance of the auditory brain with the physics of the acoustical environment, which reveals itself as the frequency-dependent weights of ITD and ILD in sound lateralization as expressed by duplex theory (Strutt, 1907). This theory describes the complementary roles of ITD and ILD cues in the lateralization of pure tones at low and high frequencies, respectively. While the acoustic shadowing of the head is negligible for low-frequency sounds, it is quite large for the sounds at high frequencies. This results in the ILD assuming a dominant role in the lateralization of high-pitched sounds (Middlebrooks and Green, 1991; Macpherson and Middlebrooks, 2002). There is indeed a vast amount of information in the literature about the coding of the ITD and ILD cues in auditory nuclei of the brainstem and tegmentum in avaians and vertebrates. However, how these two pieces of information are combined in the higher centers of the brain and cortex still remains to be explained and modeled. The present modelling study using a communications approach based on the measured psychophysical error-rate data is an attempt in this direction.

Sound localization in humans has been studied intensively for several years. Some of the studies (Gaik, 1993; Wightman and Kistler, 1997; Cunningham, 2000; Lang and Buchner, 2008) focused on trading between ITD and ILD cues by adjusting these cues to produce a centered acoustic image that gives the point of equivalence between the cues, and the ratio of ITD to ILD in that situation gives the ITD/ILD trading ratio using single-frequency tones.

The frequency-based behavioral dichotomy which is known as the duplex theory of sound lateralization is a simplification. In the real world, the listener exposed mostly to the sounds, which are the combination of many frequencies rather than a single tone. Therefore, in later years researchers went beyond the classical duplex theory and studied the utility of ongoing ITD cues in the envelopes of low-frequency, high-frequency and complex sounds presented via headphones (Macpherson and Middlebrooks, 2002; Bernstein and Trahiotis, 1994).

Other groups of studies focused on processing information in different layers of the hearing system, from the external ears to the auditory cortex, by investigating the combination or integration of ITD and ILD cues by the means of electro-encephalography (EEG) and/or psychophysical experiments. Schroger (1996) used a single tone and a complex sound to study the availability of two separate regions in the brain to process ITD and ILD information. This is while, Salminen et al. (2015), used independently manipulated stimulations to produce sounds with either only one or both ITD and ILD cues by using head-related transfer function (HRTF) of the participants. The value of only one of the cues was applied by keeping the other cue at zero for ITD-only and ILD-only conditions. Their magneto-encephalographic (MEG) and psychoacoustical results, however, reveal the existence of neurons sensitive to both ITD and ILD cue information in the auditory cortex.

Physically in the real world, ITD and ILD change together with the azimuthal position of a sound source. However, in dichotic hearing conditions, where the sound to the ears is transmitted via stereophonic earphones, the ILD and ITD cues can be changed independently of each other. It is even possible to compensate an ITD-induced sound image shift towards one ear by changing the ILD in favor of the other ear (Blauert, 1997). There are two approaches available regarding the sound images based on the ITD and ILD cues. The first one suggests that the cues may not be completely combined and the subjects sometimes perceive two images when ITDs and ILDs conflict (Hafter and Carrier, 1972). The second scenario is that the two pieces of information carried by these two cues should either share a common pathway or undergo a combining process in the brain. The commonality in the coding of ITD and ILD cues is discussed by Hafter et al. (1990). The results show a combination of the cues by algebraically adding the d' values resulted from the judgments of the ITD-only and ILD-only stimulus types. They argue that the ITD and ILD cues are sharing a common neural representation.

1.1 Ears and Binaural Pathways

1.1.1 Cochlea

Cochlea with it‘s spiral-shape, as shown in Figure 1.1, is one of the inner ear parts. The core component of cochlea named ‘Organ of Corti‘ is the sensory organ of hearing and is distributed along the partition which separates the fluid chambers in the coiled organ of cochlea. The cochlea is filled with a liquid named the perilymph.

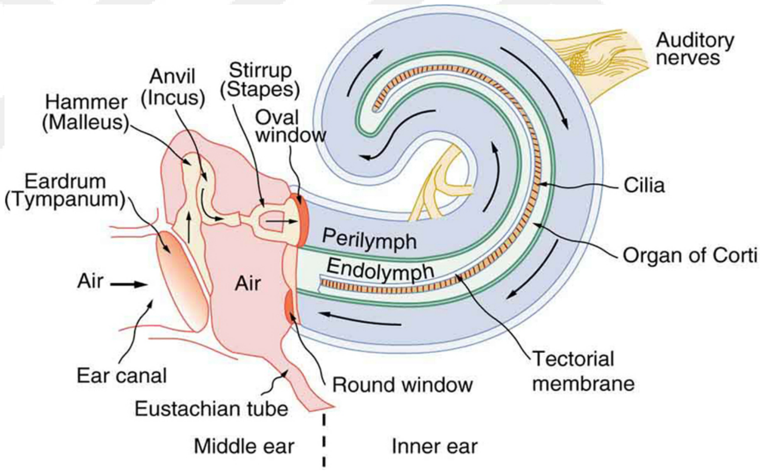

Figure 1.1: Schematic of inner ear and cochlea (socratic.org).

The vibrations from the middle ear results in the movement of the perilymph. This movement is sensed by thousands of hair cells in the cochlear portion. After amplifying, the motion is converted to the electrical signals. These signals are then transmitted to the nerve cells via neurotransmitters.

1.1.2 Medial and lateral nuclei of the superior olivary complex

The superior olivary complex (SOC) which is an important component of the auditory pathway of the human auditory system, is a collection of brainstem nuclei.

As it is shown in Figure 1.2, the SOC contains three main nuclei components: medial nucleus of trapezoid body (MNTB), medial superior olive (MSO) and lateral superior olive (LSO). Each MSO and LSO nuclei play distinct roles in horizontal sound source localization. As discussed in above lines, based on 'Duplex Theory' low and high frequencies are localized based on the differences in arrival time and differences in sound levels at two ears, respectively. The medial superior olive has a

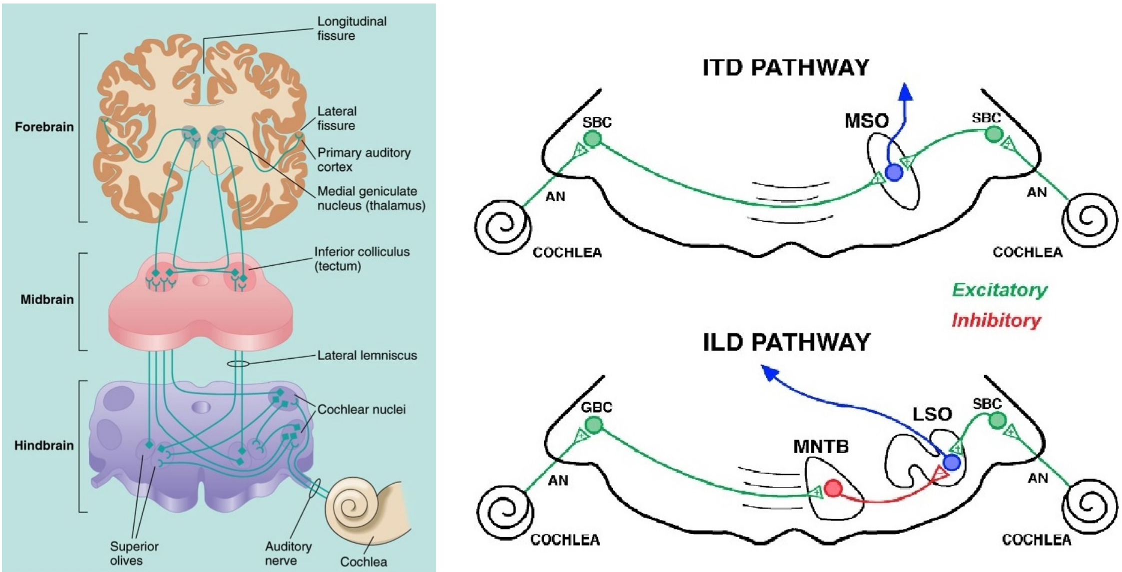

Figure 1.2: ITD and ILD pathways from Cochlea to the Cortex (Rumsey, 2001).

tonotopic organization and is a nucleus specialized to measure the time difference of sounds reaching ears (ITD). Neurons of MSO are bipolar and it's dendrites receive direct inputs from spherical bushy cells on ipsi- and contra-lateral sides of the brain. These cells which are sensitive to the relative timings of inputs reaching two ears, respond to action potentials that arrive simultaneously at the MSO. An inhibitory input from he MNTB shift the time difference of MSO neurons so that maximum spike rates be produced by binaural sounds at the contra-lateral ear.

From the otherside, LSO organization is responsible for high frequencies by receiving inputs directly from spherical bushy cells of ipsilateral AVCN. The contra-lateral pathway to the LSO is indirect which involves an excitatory projection from contra-lateral AVCN to ipsi-lateral MNTB and an inhibitory projection from ipsilateral MNTB to the LSO. In other words where, LSO neurons are excited by sounds at the ipsi-lateral ear, they are inhibited by sounds at the contra-lateral ear. The greatest change in LSO spike rates happens when there is not any level difference between the ears (near the mid-line of auditory space) (Middlebrooks, 2015).

1.2 Binaural Hearing and Sound Lateralization

1.2.1 *Interaural time and level differences (ITD - ILD) based on duplex theory of Rayleigh*

As mentioned, two most important binaural cues in lateralization are Interaural Time Difference (ITD) and Interaural Level Difference (ILD). We localize sound based on phase (period), intensity level, and spectral differences between the portions of the sound arriving at each ear. Interaural differences in arrival time (phase) and intensity constitute the most important sound localization cues. The theory outlining their contribution to sound localization judgments is referred to as the duplex theory of sound localization and was introduced by Lord Rayleigh. Rayleigh observed that sound arriving at a listener from sources located away from the median (mid-sagittal) plane would result in differences in the signals observed at a listener's ears. He noted that the sound received at the far (contralateral) ear would be effectively shadowed by the head, resulting in a difference in the level, or intensity, of the sound reaching the two ears.

Interaural time difference is the temporal difference between ipsi- and contra-lateral ears reaching sound waves from a sound source. Time difference happens because as shown in Figure 1.3, waves arrive one ear earlier than the other one and this path length difference between sound source and each of the ears results in time difference. Based on duplex theory, ITD is an important cue at frequencies lower than 1000 Hz because the wavelengths of frequencies are comparable with human head size at low frequencies (wavelength of the sound is similar to, or larger than, the distance between the two ears). Minimum and maximum ITD values are 0 μs for 0° (in front of subject) of horizontal azimuth and 700 μs for 90° (left or right side of the subject) of horizontal azimuth, respectively.

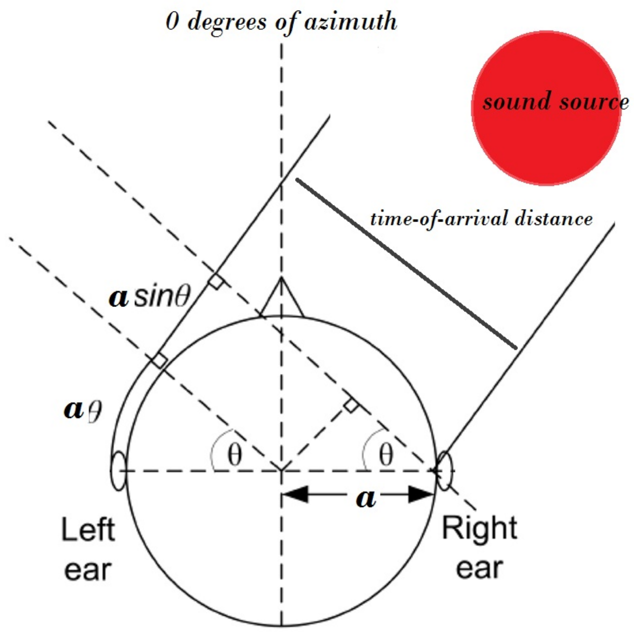

Figure 1.3: Sound source and time difference between ipsi- and contra-lateral ears (Rumsey, 2001).

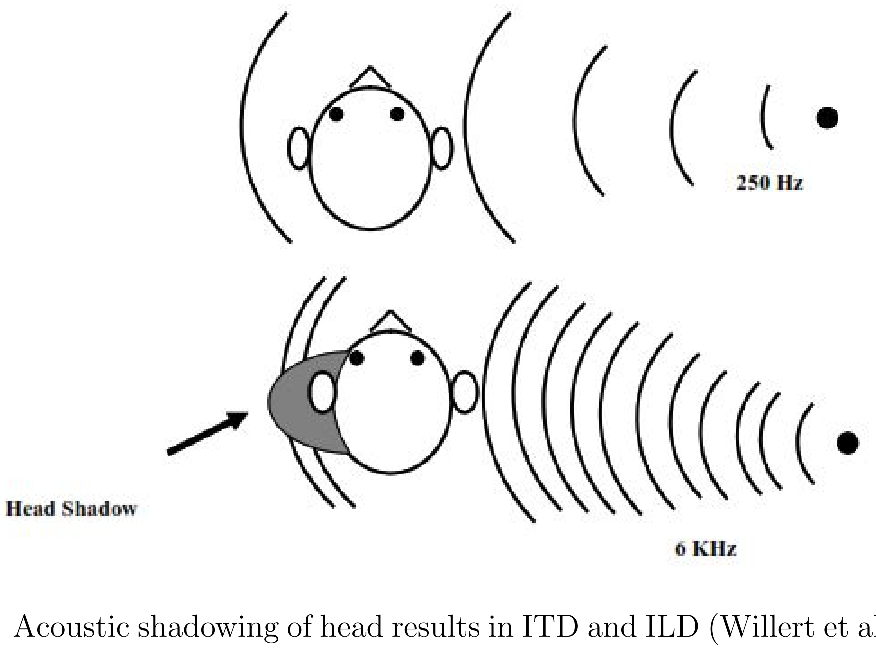

Figure 1.4: Acoustic shadowing of head results in ITD and ILD (Willert et al., 2006).

In Rayleigh spherical head model with a source at infinity, ITD could be calculated as below:

$$ITD = \frac{a}{c}(\theta + sin(\theta)) \qquad -\frac{\pi}{2} \leqslant \theta \leqslant +\frac{\pi}{2} \tag{1.1}$$

where, c is speed of sound (approximately 343 m/s) and θ is the angle between the median plane and sound wave emitted from source. Using the equation ITD value will be zero when the source is just in front of the subject.

Interaural level difference is the level or intensity difference between ipsi- and contra-lateral ears reaching sound waves from a sound source. Based on duplex theory, ILD generally affects contra-lateral signals of almost all frequencies but is dominant at frequencies above 1000 Hz. Therefore, ILD is a frequency dependent cue and because at higher frequencies the head diameter is larger than the wavelength, waves are scattered over audible spectrum. As it‘s shown in Figure 1.4, this results in ILD increment more rapidly by frequency increment. The smallest detectable ILD (regardless of frequency) is ~ 0.5 dB. Note that, where near-field ILD exceeds even 15 dB, far-field ILD maximally reaches 6 dB.

1.2.2 ITD-ILD Trade-Off

Based on the duplex theory, ITD and ILD are mostly used in localization of low and high frequency sounds, respectively. However, natural sounds have both low and high frequencies and these frequency ranges can overlap. Therefore, human auditory system in most cases will have to combine information from both ITD and ILD cues to be able to localize the sound source. In real world, a sound is originated from a single source and, therefore, there is a strict relationship between the ITD and ILD values for a certain acoustical setting.; i.e., as shown in Figure 1.5, they change together with the azimuthal position of the sound source. However, in dichotic hearing conditions, where the sound to the ears is transmitted via stereophonic earphones, the ILD and ITD cues can be changed independently of each other. It is even possible to compensate an ITD-induced sound image shift towards one ear by changing the ILD in favor of the other ear (Blauert, 1997), showing that there is a trade-off between ITD and ILD cues. The ratio of ITD and ILD cues in this situation gives the ITD-ILD trading ratio. The trading ratio depends on sound parameters including frequency, intensity and duration of the stimulus (Harris, 1960; David et al., 1959; Hafter and Jeffress, 1968). Trading curves become less and less steep with increasing loudness of the sound (David et al., 1959); and, they have only a moderate slope for sound frequencies below 1.6 kHz even at a low loudness level (Harris, 1960), as can be seen respectively in Figure 1.6 (a) and (b). The ITD-ILD trading briefly mentioned above clearly shows that the two pieces of spatial information carried by these interaural time and level cues (namely, ITD and ILD) must be undergoing a combining process in the brain.

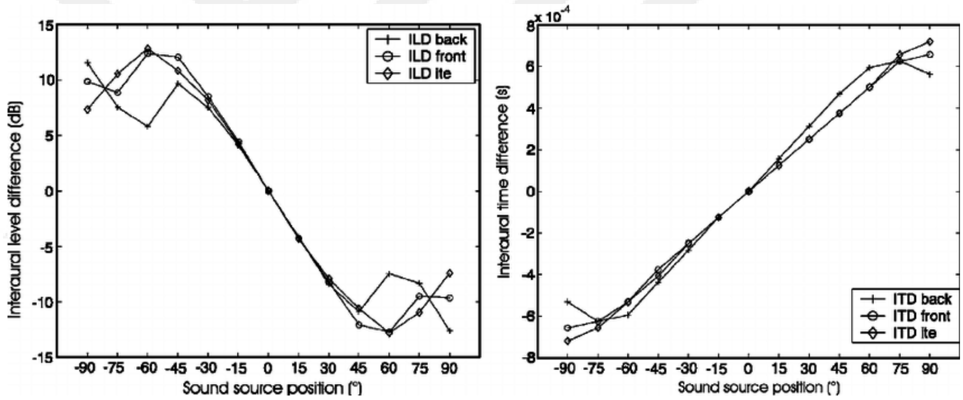

Figure 1.5: ITD and ILD values for different frequencies and azimuth of a Kemar manikin in horizontal plane (Bogaert et al., 2006)

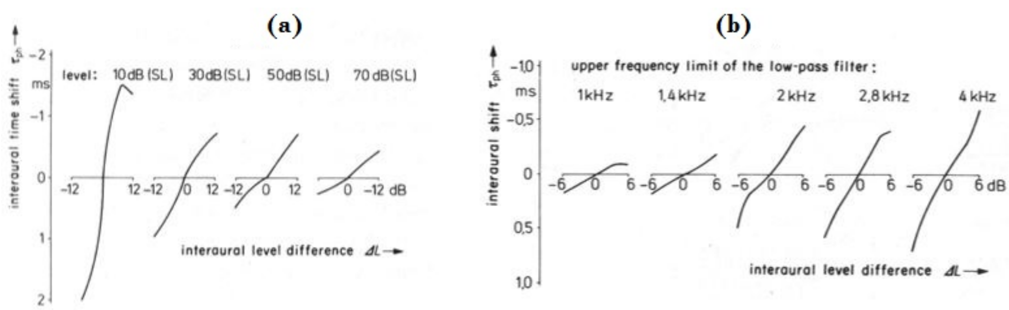

Figure 1.6: (a) Trading curves for broadband clicks at various sound intensity levels, (b) Trading curves for low-pass, band-limited clicks at a level of 20 dBSL (David et al., 1959; Harris, 1960) respectively).

1.2.3 Minimum Audible Angle

The minimum audible angle (MAA) is a standard measurement used for spatial auditory acuity and is the smallest angular distance at which two sounds are separable for hearing system. In 1958, Mills obtained the MAA (Minimum Audible angle) as a function of frequency and azimuth. As shown in Figure 1.7 the MAA, by using headphones, is about 1 degree when the sound source is in midline (front of head) at frequency range about 500 Hz - 750 Hz. A 1 degree MAA is nearly 10 μsec (the smallest detectable ITD). With increasing frequency the MMA increases. MAA is symmetric around 90^o in the spherical head model. For human hearing system, MAA ranges from 1^o at frequency of 1000 Hz in front to 7.5^o at the left or right side. This is however, for a direct sound with frequency of 2000 Hz, the MAA is almost 3^o in front.

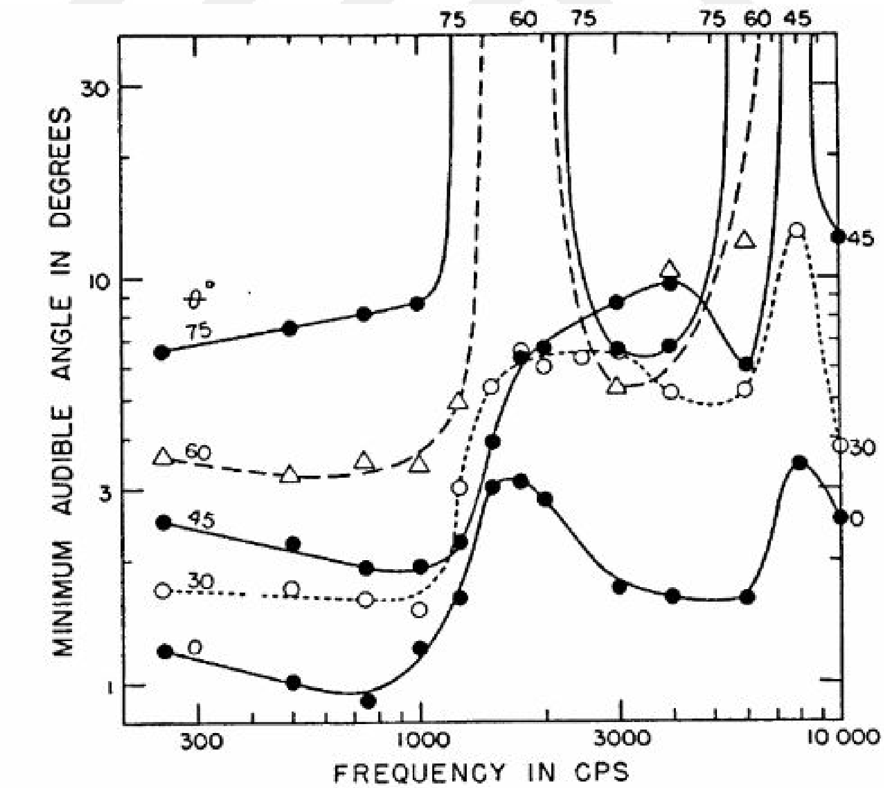

Figure 1.7: The minimum audible angle versus frequency (Mills, 1958)

1.2.4 Cone of Confusion

One of the coolest things about hearing is it‘s ability in locating the sounds. Interaural time and level differences are the main tools which help us in detecting the direction of the sound source. However, there are lots of uncertainty in sound lateralization. For example, the ITD and ILD for a sound from front-right and back-right are the same. This is also true for top and down sound sources. In fact, there is a cone of confusion which is not distinguishable by the hearing system. As it is shown in Figure 1.8, the cone of confusion consists of the points that have identical interaural time and level differences in 3-D hearing space. Using only one of the cues for synthesis binaural signals in virtual auditory models results in confusion. It doesn’t usually happen in the real world because there are many other parameters for localizing the sound source such as reflection of the sound waves from the environment and vision hearing. Furthermore, our ears and head (torso) attenuate certain frequencies.

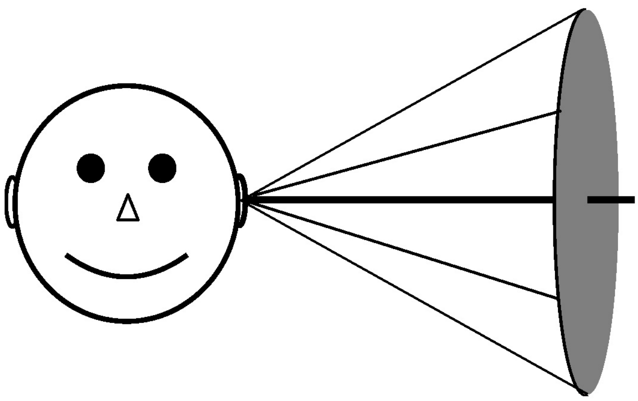

Figure 1.8: Cone of confusion. For the given cone, the sound source located at any point on the surface, will produce the same ITD and ILD differences.

1.2.5 Head Related Transfer Function (HRTF)

The location-dependent pattern is called Head related transfer function. The HRTF is the transformation of a sound wave from a source to the subject's ears which as a function could characterize the binaural cues for source localization during the cone of confusion. As HRTF depends on frequency and azimuth of sound source, it considered as an individual function for any subject and any sound source. In other words, HRTF can be described as the filtering of a sound source before reaching the ears (Figure 1.9). Where, near-field HRTF suits the ILD changes, far-field HRTF inversely attenuated by the range. HRTF is the Fourier transform of head related impulse response (HRIR). Therefore, it is a complex function having both level and phase shift information for each of the ears. So if impulse responses for left and right ears be M-point signals and are respectively shown as $h_L[n]$ and $h_R[n]$, the frequency domain's transfer functions will be denoted as $H_L(w)$ and $H_R(w)$. If the N-point signal x[n] be the sound signal at the source, $x_L[n]$ and $x_R[n]$ with $N+M-1$ points will be the signals at left and right ears, respectively. So for each ear in time domain the corresponding signal can be written as:

$$x_{L,R}[n] = x[n] * h_{L,R}[n] = \sum_{k=0}^{N+M-2} x[k]h_{L,R}[n-k] \tag{1.2}$$

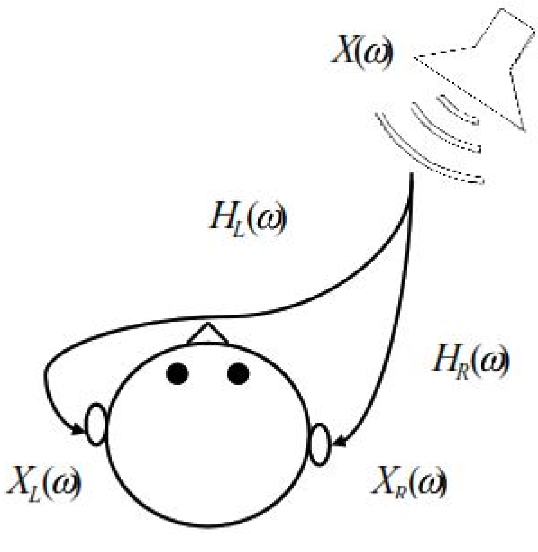

Figure 1.9: Filtering of a signal x(t) by separate transfer functions of $h_L(t)$ and $h_R(t)$.

1.3 Visions and Missions

In order to understand the significance of cues in different frequencies, (Macpherson and Middlebrooks, 2002) used a virtual auditory space approach to quantify the relative strength (weight) of ITD and ILD cues in low-pass, high-pass, and wideband noise bursts. In their stimulation sounds, the interaural time and level differences were manipulated by delaying or attenuating the stimuli sound to one ear. Their results revealed that, while the ITD is weighted weakly for high-pass stimuli the ILD is weighted strongly for high-pass stimuli even when substantial biases were introduced. In some studies, the performance of the participants in judgments differs depending on the direction and the cue used in the stimulation sounds. To test the existence of bias in lateralization, Wood and Bizley (2015) studied the relative sound localization (the target sound originated from the left or right of a reference sound) with rather greater values of the time and level differences when a multi-source noise background is present besides stimulation sounds. Their results showed that the subjects were more biased in the band-pass noise condition. They claimed this type of bias which shifts the decision criteria in the direction of the hemifield in which the stimulation is presented as the ”response bias” (Wood and Bizley, 2015; Hartmann and Rakerd, 1989). To test the effect of direction in localization, however, Magezi and Krumbholz (2010) studied EEG responses for the low-pass filtered stimulations (1500 ms - 250 ms pair of noise sounds with a frequency less than 1kHz) in which the ITD change was either away from (outward) or toward the midline (inward). Their findings suggest that ITDs are coded non-topographically and the majority of the neurons in each hemisphere are sensitive to ITD. Results of a similar study, (Ihlefeld et al., 2018) confirm the effect of biasing perceived laterality toward midline with decreasing intensity of stimulation sound. Regardless of whether its origin is a real source or is an intracranial image (created by dichotically presented acoustic signals), a sound is always associated with a pair of directional ITD and ILD cues. To have a proper assessment of the weights of the ITD and ILD cues in the localization of a sound source each of the cues must be presented separately for the listener and their lateralization performance must be measured. ILD and ITD cues can be changed independently from each other under dichotic hearing conditions. However, this does not provide the opportunity to present one of these cues to the listener on its own and discarding the other cue completely. That is, only a single cue cannot be isolated and studied by ignoring the other. For instance, when a sound is presented with a cue combination of ITD = 0 sec and ILD = 6 dB, it does not mean that the sound will carry no ITD information but that the ITD cue will still be pointing the median plane, and it will have a certain effect on the perceived lateralization of the source.

The experiments of the current study are in the 2 alternative-forced-choice format. Using paired-comparison in psychological experiments has several advantages in the cross-moderation context. Firstly, the securities of the judges are not based on the individuals, rather the relative merit turns out to be important. Secondly, the analysis model handles missing data as the estimate of the scale separation between any two series does not depend on which other scripts they are compared with. Thirdly, fitting an explicit model to the results allows investigating the residuals to detect misfitting series and bias in judgments (Bramley and Oates, 2011). Choosing such a method makes it possible to impose either of the ITD and ILD cues independently from each other as is also the case when conventional methods are used to assess the relative roles of these cues in sound lateralization and the trading between them.

A study by Buus et al. (1986) showed that the psychometric functions, which were used for the auditory discrimination and detection tasks were parallel for single and 18-tone complex stimulation. The weights of the ITD and ILD cues were found to depend on the sound frequency (Middlebrooks and Green, 1991; Macpherson and Middlebrooks, 2002). To have both the ITD and ILD cues and a complex stimulus rather than a single low or high-frequency pure tone in the present study, the test stimulus sounds are chosen from a combination of low- and high-frequency harmonics of a 261 Hz tone. In a study by Rakerd and Hartmann (2010), they used complex signals of mid-band (noise with a center frequency of 750 Hz) and high-band (noise with a center frequency of 2850 Hz) to find the effectiveness of ITD and ILD cues by measuring the ITD when there was an opposing ILD of 0, 1 or 2 dB in different reverberation conditions. In the present study, however, the first two harmonics of a 261-Hz tone are chosen as low-frequency components and its 7th through 19th odd harmonics are chosen as high frequency components of the test stimulus. As the effect of the ITD and ILD cues are of equal importance for mid-frequencies (783 Hz $\sim$ 1566 Hz) this range are discarded from the stimulus and instead, the ranges in which either of the ITD and ILD cues is mostly effective in lateralization are selected. Another reason why the second harmonic of the 261 Hz-tone, which would have been at 783 Hz, is not included in the low-frequency band of the sound stimulus is connected to the findings suggesting that humans may use different ITD-based sound lateralization strategies for different sound frequencies. For instance, based on data from the barn owl, gerbil, cat, and human, Harper and McAlpine (2004) proposed a model in which two types of optimum coding strategy were apparent: In humans, for the lowest frequencies, the optimum strategy involves two distinct sub-populations, as for the classical case of the gerbil, whereas for the highest frequencies above 700 Hz a homogeneous distribution is optimal. Because 500 Hz appears to be the frequency where the transition from the classical strategy to a different strategy takes place, we prefer the former to avoid possible complications which might be caused by the involvement of two different ITD-coding strategies respectively for frequencies lower than 700 Hz and between 700 Hz and 1500 Hz. To impose location information to sound bursts, various studies conducted previously used arbitrary ITD and/or ILD values from available trading ratio tables (Ungan et al., 2001) or the cue values extracted from the head-related transfer function (HRTF) (Salminen, 2015; Asahi and Matsuoka, 1977; Morimoto, 2003). The results of a study by Begault et al. (2001), in which they compared the localization performances of the subjects when they were presented with virtual sounds simulated by using individualized and generic HRTFs, showed that there was no clear advantage in using individualized HRTFs in this respect (under both echoic and anechoic conditions). Therefore, in the current study, we use the ITD and ILD values that are extracted from a generic HRTF dataset. The main aim of the present study is to investigate the relative contribution of ITD and ILD cues in sound lateralization and their weight values by the means of stimuli consisting of low and high frequencies. For this reason, two scenarios are considered. First scenario which is presented in the form of experiment I series, performed to examine the ability of detecting azimuthal changes in lateralization with smaller azimuth. The stimulation sounds for these experiment series are presented as, Low-Frequency (LF), High-Frequency (HF) and combination of Low and High Frequencies (LHF). In LF stimuli, sounds contain only low frequencies and only interaural time difference (ITD) is applied to the stimuli. In HF stimuli, sounds contain only high frequencies and only interaural level difference (ILD) is applied to the stimuli. In LHF stimuli, sounds contain both low and high frequencies and both interaural time and level difference (ITD and ILD) are applied to the stimuli. This is while, for experiment II series, the changes in the azimuth of a sound source are detected by applying only one or both of the cues to the complex stimuli sound in which all the frequencies are available.

1.4 Contributions

In the current study, we present two main contributions in modeling roles of ITD and ILD in sound lateralization in small (near midline) and large (far from midline) azimuth as in experiment series I and II, respectively. First, we model the direction of arrival of a single tone separately for ITD and ILD cues in the form of phase and magnitude differences, which are extracted from the frequency domain characterization of the right and left HRTF responses. Then the error rates of absolute and relative lateralizations are evaluated in experiments I and II, respectively. The approach of experiment II, which provides a reference to the subject for lateralization assessment within each stimulus, provides more sensible and reproducable results, though. This avails us to monitor contribution of each cue in perceived lateralization change precisely, whereas in the literature common practice sets one of the ITD or ILD cue as the control signal and sets the other one to zero that results one cue to point the center. Second, we estimate the relative contribution of the ILD and

ITD cues in sound lateralization using the diversity combination assumption for the integration of these cues based on the binary additive white-Gaussian noise channel model.

Chapter 2

GENERAL EXPERIMENTAL METHODOLOGY

2.1 Participants

Three male and two female paid subjects, all student at *KOÇ University* and aged between 25 and 35, participated in the test. None of these subjects reported to have any audiological or neuorological disease. The experiment procedure approved by the *Ethics Committee of the School of Medicine at the KOÇ University*.

2.2 Test Equipment

Stimulation sounds are presented by using a pair of Etymotic Research 61-Martin Lane (Etymotic R-2A, shown in Figure 2.1) audiological earphones with flat frequency response that are driven by an external USB sound card (MUSE Mini DAC).

Figure 2.1: Etymotic R-2A earphone

2.3 Hearing Level Threshold Test

Prior to the main experiments, subjects need to pass the threshold level test. All the frequencies used in producing sounds are used in the test. Subjects listen to the single-sine-wave tones one-by-one using a Matlab Graphical User Interface (shown in Figure 2.2). At the beginning of each section, the sounds are presented to the subjects with zero volume. Subjects use an slider to increase the volume until they start hearing the sound. Then they press **OK** key and press **PROCEED** key to go to next step. Next step starts the same sound with maximum volume level. Again, subjects use the slider to decrease the volume of the sound until they are not able to hear any sound. In this moment they need to press **OK** and proceed to the next step. In the next step of the experiment, the intensity levels for each frequency which are collected from the subjects are compared by the sounds which are 60 dB louder than minimum hearing level threshold. Maximum of 5 dB difference between real value and the subjects results are accepted and averaged over five subjects took part in the experiment.

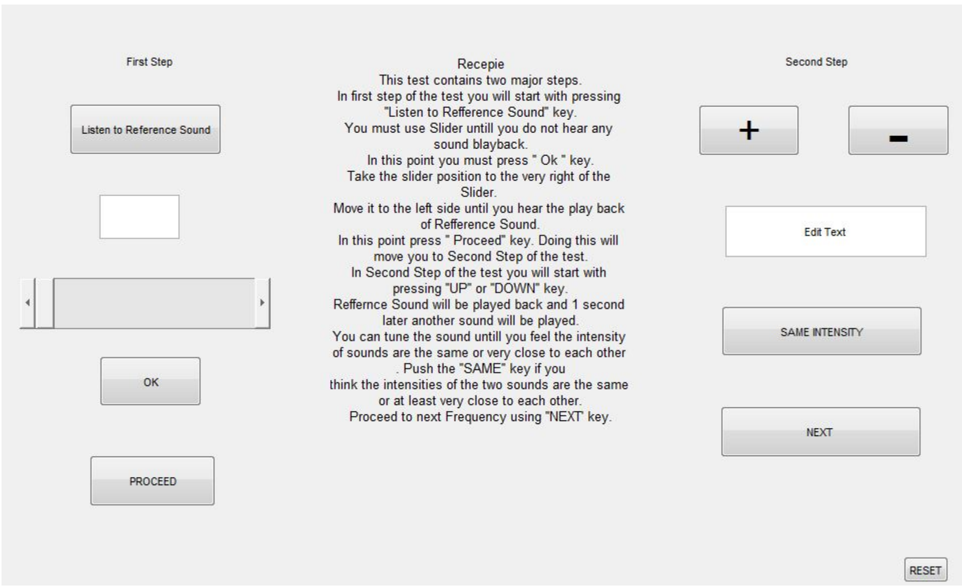

Figure 2.2: Graphical User Interface (GUI) for hearing level threshold test.

Chapter 3

EXPERIMENT I

3.1 Experimental Methodology

3.1.1 Stimulation

For test setup, we considered three sources: a single low-frequency tone (LF), a collection of high-frequency tones (HF) and combination of low- and high-frequency tones (LHF). We set the low-frequency source at $f_0 = 261$ Hz as, $s_L(t) = a_0 \sin(2\pi f_0 t)$, where a_0 is the amplitude of the LF source. We target to construct high-frequency tones from 7th to 19th odd harmonics of the LF tone. But we also try to avoid the missing fundamental, which is suggested by the overtones even though the sound lacks the fundamental frequency itself. Hence the HF source is set as,

$$s_H(t) = \sum_{i=1}^{7} a_i \sin(2\pi f_i t), \tag{3.1}$$

where the high-frequency tones are selected from 7th to 19th odd harmonics of the LF tone f_0 with random perturbations as $f_i = (5 + 2i)f_0 + \alpha_i f_0$ for $i = 1, 2, ..., 7$. Here the α_i values are randomly selected from a uniform distribution in $[-0.3\ 0.7]$ interval. The α_i values are fixed to a set of location for the experimental studies. The amplitudes, a_i, of the LF and HF sources are set 60 dB over the respective hearing threshold levels for the individual tones.

Onsets and offsets of these three sources are tapered by a Blackman window. Then each source is convolved with head-related impulse response (HRIR) to impose 3°, 5°, and 10° azimuth to left and right. We use the multichannel in-ear and behind-the-ear HRIR database (Kayser et al., 2009), in which HRIR sets are available in increments of 5°s. In order to introduce a challenging condition to the test, we interpolate 3° HRIR from 0° and 5° HRIRs. Three sources and three azimuth angles set nine conditions in the experiment. We also include 60° azimuth with the LHF source just to verify subject's attention during the test.

3.1.2 Procedure

As a preprocessing, an audiometry test was performed with six subjects, and their average hearing thresholds at all 8 frequencies $\{s_i\}_{i=0}^{7}$ were extracted. Then the amplitude of each tone was set +60dB above the threshold of hearing at the corresponding frequencies. The test was performed using 122 sound stimuli. Distribution of the sound stimuli to the conditions are presented in Table 3.1. During the test the sound stimuli were randomly presented so that the subjects were not able to predict order of the sources. Subjects were asked to report lateralization as left or right for each stimulus. Each sound stimuli was followed by a reset tone, which was constructed as a combination of all tones in the experiment with a sweeping azimuth. The test was executed in blocks of twenty source tones with 30-second resting periods between blocks. Since the test was in two-alternative forced choice (2AFC) format, each response to the stimuli was taken as a HIT or a FALSE. If the subject was unable to answer the test in 3 seconds, a MISS is reported for that stimuli.

Table 3.1: Number of conditions for different azimuths and sources in the experiment.

	Number of Conditions			
Source	±3°	±5°	±10°	±60°
LF	10	20	10	0
HF	10	20	10	0
LHF	10	20	10	2

3.1.3 Data Collection

In the listening tests we collect each response to the stimuli as a HIT, FALSE or MISS. Then over the total number of HIT, FALSE and MISS responses, we define the error rate E_R as,

$$E_r = \frac{N_F}{N_H + N_F + N_M}, \tag{3.2}$$

where N_H, N_F and N_M are respectively total number of HIT, FALSE and MISS responses. Then the SNR of the lateralization decisions in the experimental tests that corresponds to the error rate E_R can be computed as,

$$SNR_{\bar{\Theta}} = (\text{erfc}^{-1}(2E_r))^2 \tag{3.3}$$

where $\bar{\Theta}$ represents lateralization in the experimental tests and $\text{erfc}^{-1}()$ is the inverse of complementary error function (Shankar, 2011).

3.1.4 Analytical Methodology

In general diversity combining is a technique used to combine the received signals with diversity in their reception into a single improved signal by increasing the signal-to-noise ratio (SNR) and thereby reducing bit-error-rate in the following three ways:
* Equal-gain combining: the received signals are summed equally.
* Maximum-ratio combining: the signals are summed based on their SNR weight.
* Selection combining: the most powerful signal is selected.

Sound lateralization takes place in the MSO and LSO respectively using the ILD and ITD cues. In this study, we hypothesize a diversity combining perspective for the fusion of ILD and ITD cues, as if unbiased estimates of lateralization combines in the brain. In this data model, let $\hat{\Theta}_L$ and $\hat{\Theta}_T$ represent the unbiased estimates of lateralization for each diversity branch, namely ILD and ITD. We can write these estimates as,

$$\hat{\Theta}_L = \Theta_L + n_L \quad \text{and} \quad \hat{\Theta}_T = \Theta_T + n_T \tag{3.4}$$

where n_L and n_T represent estimation errors with variances σ_L^2 and σ_T^2, respectively. Then in this framework a diversity combination model can define the fusion as,

$$\hat{\Theta} = \lambda\hat{\Theta}_L + (1-\lambda)\hat{\Theta}_T = \Theta + \lambda n_L + (1-\lambda)n_T \tag{3.5}$$

where λ is the weight of the ILD mechanism in $(0, 1)$ interval. If we further assume that the estimation errors of different mechanisms are mutually uncorrelated, SNR of the fusion can be written as,

$$SNR_{\hat{\Theta}} = \frac{\Theta^2}{\lambda^2\sigma_L^2 + (1-\lambda)^2\sigma_T^2}. \tag{3.6}$$

In this framework, when the SNR values of the ILD, ITD and the fusion mechanisms are known, then the linear weight λ of the ILD mechanism can be extracted from the following quadratic equation:

$$\frac{\lambda^2}{SNR_{\hat{\Theta}_L}} + \frac{(1-\lambda)^2}{SNR_{\hat{\Theta}_T}} = \frac{1}{SNR_{\hat{\Theta}}}. \tag{3.7}$$

We can consider several well-known diversity combination approaches, such as equal gain combining, selection combining and maximum ratio combining. Equal gain combining (EGC) sets $\lambda = \frac{1}{2}$, where the corresponding SNR can be written as,

$$SNR_{\hat{\Theta}} = \frac{4\Theta^2}{(\sigma_L^2 + \sigma_T^2)}. \tag{3.8}$$

In selection combining (SC), the branch with the best SNR is selected, which leads to the selection of maximum SNR as,

$$SNR_{\hat{\Theta}} = \max\{SNR_{\hat{\Theta}_L}, SNR_{\hat{\Theta}_T}\}. \tag{3.9}$$

Finally in maximal ratio combining (MRC), the SNR of the fusion is maximized over λ ($\lambda = \frac{\sigma_T^2}{\sigma_L^2+\sigma_T^2}$) and the maximum SNR is achieved as,

$$SNR_{\hat{\Theta}} = SNR_{\hat{\Theta}_L} + SNR_{\hat{\Theta}_T}. \tag{3.10}$$

3.2 Results and Data

Table 3.2 presents experimental measurements and results. Experimentally measured error rate (E_R) values for the LF, HF and LHF sources are given in the first three columns. Corresponding SNR values for the experimentally measured error rates and SNR values of the LHF source under EGC, SC and MRC as defined in section 3.1.4 are given in the middle three columns. Furthermore the last three

Table 3.2: Experimentally measured error rate (E_R) and corresponding computed SNR and ILD weight (λ) values.

	E_R			SNR			λ		
	3°	5°	10°	3°	5°	10°	3°	5°	10°
LF	0.381	0.263	0.098	0.046	0.201	0.833	-	-	-
HF	0.097	0.054	0.015	0.840	1.290	2.355	-	-	-
LHF	0.111	0.064	0.025	0.745	1.157	1.921	0.86	0.68	0.51
MRC	-	-	-	0.885	1.490	3.186	0.89	0.78	0.62
SC	-	-	-	0.840	1.290	2.355	1.00	1.00	1.00
EGC	-	-	-	0.135	0.315	0.616	0.50	0.50	0.50

columns of Table 3.2 present the calculated ILD weight λ for the LHF source and the MRC, SC and EGC combining schemes. Based on MRC results, ILD weight (λ) is decreasing by increasing the azimuth. This shows that, for azimuth near midline, hearing system trust mostly on level information rather than time. By increasing the azimuth of stimuli, the importance of the time and level informations turn to be equal for human hearing system.

3.3 Discussion

In order to assess properly the weights of the ITD and ILD cues in the function of sound lateralization, one has to provide one at a time for the listener and measure his/her lateralization performance. It is indeed possible under dichotic hearing conditions that the ILD and ITD cues can be changed independently of each other. However, it does not provide the possibility to present one of these cues alone for the listener. No matter if its origin is a real source or it is an intracranial image created dichotically presented acoustic signals, a sound is always associated with a pair of directional cues, namely ITD and ILD. One of them cannot be isolated and studied by zeroing the other. For instance, when a sound is presented with a cue combination of $ITD = 0$ and $ILD = 6$ dB it will not mean that the sound carries no ITD information; the ITD cue will still tell the listener that the source is on the median plane, and it will have a certain effect on the perceived lateral position of the source. To delineate the weights of the two cues, we therefore chose another approach based on different operating frequency ranges of the neural mechanisms encoding them, and tried to disentangle their individual roles in sound lateralization by presenting a complex sound and its low- and high-frequency components in separate sessions of lateralization tests. We should admit that associating the ITD cue only with the lateralization performance at low-frequencies and the ILD cue only with the lateralization performance at high-frequencies may not be an accurate assumption. This is because for amplitude-modulated (AM) high-frequency complex sounds, LSO may also encode ITDs through interplay between the arrival times of the ipsilateral excitatory and contralateral inhibitory inputs (Goksoy et al., 2005; Joris and Yin, 1995; Ungan and Yagcioglu, 2002). However, due to the slight jittering of frequency positions of high-frequency harmonic components we made in our stimulating sounds prevented forming AM patterns which otherwise would have been processed by such an ITD coding mechanism in LSO. This measure also prevented the high-frequency harmonics from forming low-frequency periodicity pitch which would have been processed by the ITD coding mechanism in MSO.

There are still some concerns remain in the integration of the time and level information, though. Here, we considered ITD in LF and ILD in HF. Based on previous studies, LSO neurons are sensitive to both cues and both of the cues play roles in either of the low and high frequency ranges. In addition, in the stimulation sounds we used for the experiment series I, even applying a jitter to the high-frequency harmonic components did not prevent forming AM patterns and the stimuli still have envelope fluctuations which could be used for lateralization on the basis of envelope ITD at higher frequencies.

3.4 Concerns and Solutions

Considering the mentioned concerns, experiment series II (Chapter 4) with the aim of investigating the relative contribution of ITD and ILD cues in sound lateralization and their weight values by the means of complex stimuli consisting of low and high frequencies are designed. For this reason, instead of absolute azimuthal positions, the changes in the azimuth of a sound source are used in the experiments. The stimuli are designed as pairs of sounds to create the necessary azimuthal changes. The first sound (reference) of the stimulus pair is made to have the ITD and ILD of a reference azimuth (θ). The azimuth of the second sound (probe) differed from the first one, assuming an azimuthal change of $10°$ for both or either of the ITD and ILD cues. This change is made in the following three ways to produce azimuthal shifts toward the midline or the periphery:

(i) TO: Only ITD cue moves from θ to either $\theta - 10°$ or $\theta + 10°$.

(ii) LO: Only ILD cue moves from θ to either $\theta - 10°$ or $\theta + 10°$.

(iii) TL: Both ITD and ILD cues move from θ to either $\theta - 10°$ or $\theta + 10°$.

Chapter 4

EXPERIMENT II

4.1 Experimental Methodology

4.1.1 Stimulation

A study by Buus et al. (1986) showed that the psychometric functions, which are used for discrimination and detection hearing tasks, are parallel for single and 18-tone complex stimulation. The weights of ITD and ILD cues depend on the sound frequency (Middlebrooks and Green, 1991; Macpherson and Middlebrooks, 2002). In order to have both the ITD and ILD cues and to have a complex stimulus rather than a single low or high frequency pure tone, test stimulus is chosen as a combination of low- and high-frequency harmonics of a 261 Hz tone. The harmonics included are at frequencies of 261, 512, 1827, 2349, 2871, 3393, 3915, 4437 and 4959 Hz. In the test stimulus, the first two harmonic components are chosen for low and seven odd harmonic components starting from the seventh harmonic are chosen for high frequencies. As effect of the ITD and ILD cues are of equal importance for mid-frequencies this range is discarded from the stimulus and instead, the ranges in which the ITD and ILD cues are mostly effective in source detection tasks are selected. In order to impose location information to sound bursts some previous studies used arbitrary ITD and/or ILD values from available trading ratio tables (Ungan et al., 2001) or the values extracted from the HRTF (Salminen, 2015). We used discrete Fourier transform (DFT) to extract time and level differences between left and right channels of the impulse responses, from a previously recorded *head related transfer function* (HRTF) database (Kayser et al., 2009), for each frequency component. The DFT of a single channel HRTF impulse response $h_\theta^c(n)$ at frequency index k_i is given as,

$$H^c_\theta(k_i) = \frac{1}{N}\sum_{n=0}^{N-1} h^c_\theta(n)e^{\frac{-j2\pi k_i n}{N}}, \tag{4.1}$$

where the HRTF $h^c_\theta(n)$ is defined for channel c, left or right, at azimuth angle θ, and k_i is the frequency index that corresponds to $k_i = \left\lfloor \frac{f_i}{f_s} + 0.5 \right\rfloor$ at frequency f_i with sampling frequency f_s.

Magnitude ratio and phase difference of the right, H^r_θ, and the left, H^l_θ, channels at frequency f_i are respectively computed as,

$$\alpha_\theta(k_i) = \frac{|H^r_\theta(k_i)|}{|H^l_\theta(k_i)|}, \tag{4.2}$$

$$\phi_\theta(k_i) = arg(H^r_\theta(k_i)) - arg(H^r_\theta(k_i)). \tag{4.3}$$

Then, the power ratio of $\alpha^2_{\theta_1}$ at azimuth θ_1 and the phase difference of ϕ_{θ_2} at azimuth θ_2 are used to define a sound stimulus with the right and left channels as,

$$s^r(t;\alpha_{\theta_1},\phi_{\theta_2}) = \frac{1}{9}\sum_{i=1}^{9} \alpha_{\theta_1}(k_i) sin(2\pi f_i t + \frac{1}{2}\phi_{\theta_2}(k_i)), \tag{4.4}$$

$$s^l(t;\alpha_{\theta_1},\phi_{\theta_2}) = \frac{1}{9}\sum_{i=1}^{9} \frac{1}{\alpha_{\theta_1}(k_i)} sin(2\pi f_i t - \frac{1}{2}\phi_{\theta_2}(k_i)) \tag{4.5}$$

where the azimuth angles θ_1 and θ_2 are respectively refers to the ILD and ITD cues and the used setup is flexible to set the ILD and ITD cues from different azimuth angles. Duration of the sound signals is set to 200 msec and a cosine window of 100 msec is used to eliminate the onset/offset effects of the rise and fall of the stimuli. In the experiments, stimuli are presented to the subjects in pairs of sound bursts consisting of a reference sound and a probe sound. Reference sound is the first sound of the stimulus pair. The reference sound contains interaural time and level cue information of azimuth θ. That is, both α and ϕ values are set to azimuth θ and the reference sound is defined as, $s^c(t;\alpha_\theta,\phi_\theta)$, at channel c. Probe sound is the second sound of the stimulus pair. Probe sounds are defined under three conditions based on time and level cues they use. These conditions are time-only (TO), level-only (LO) and time-level (TL). The TO probe sound uses the ILD information from azimuth

θ and the ITD information from azimuth $(\theta \pm \Delta)$, where we set $\Delta = 10°$ as the azimuthal difference of the probe sound with respect to the reference sound. Then, the resulting TO stimulus can be given as: $s^c(t; \alpha_\theta, \phi_{\theta \pm 10°})$. The LO probe sound uses the ITD information from azimuth θ and the ILD information from azimuth $(\theta \pm 10°)$. Then, the resulting LO stimulus can be given as: $s^c(t; \alpha_{\theta \pm 10°}, \phi_\theta)$. The

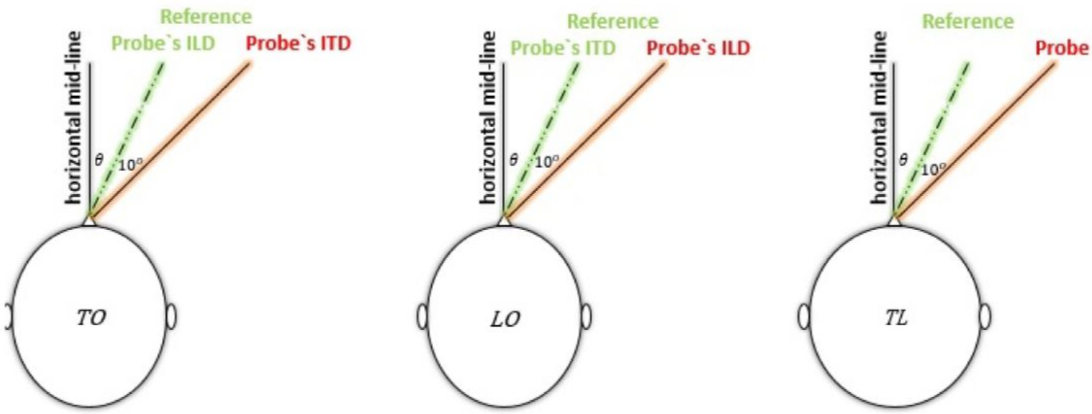

Figure 4.1: A sample visualization of probe sounds from azimuth $(\theta + 10°)$ for TO, LO and TL conditions where the reference sound is from azimuth (θ).

TL probe sound uses both of the ITD and ILD information from azimuth $(\theta \pm 10°)$. Hence, the resulting TL stimulus can be given as: $s^c(t; \alpha_{\theta \pm 10°}, \phi_{\theta \pm 10°})$. A sample visualization of the reference and probe sounds are given in Figure 4.1. Thus, for the TO and LO probe sounds, either the time or the level cue of azimuth $(\theta \pm 10°)$ is imposed, leaving the other cue as it is in the reference sound (θ). In other words, azimuthal change in the probe sound is made by changing only ITD, or only ILD, or both of the cues. A 3-second interval separated the offset of a probe and the onset of the next pair‘s reference. To adjust the intensity of the sound stimuli to be used in experiments, the reference sound for zero azimuth was presented to fifteen normal hearing subjects with no reported audiological problem, and their absolute hearing thresholds for this sound were determined. Taking the average of these individual thresholds as 0 dB nHL, the intensity of this reference sound was set to 67 dB above that level; i.e., to 67 dB nHL. Figure 4.2(a) shows one pair of the sample TL stimulus with reference sound of $s^c(t; \alpha_{20}, \phi_{20})$ and probe sound of $s^c(t; \alpha_{30}, \phi_{30})$.

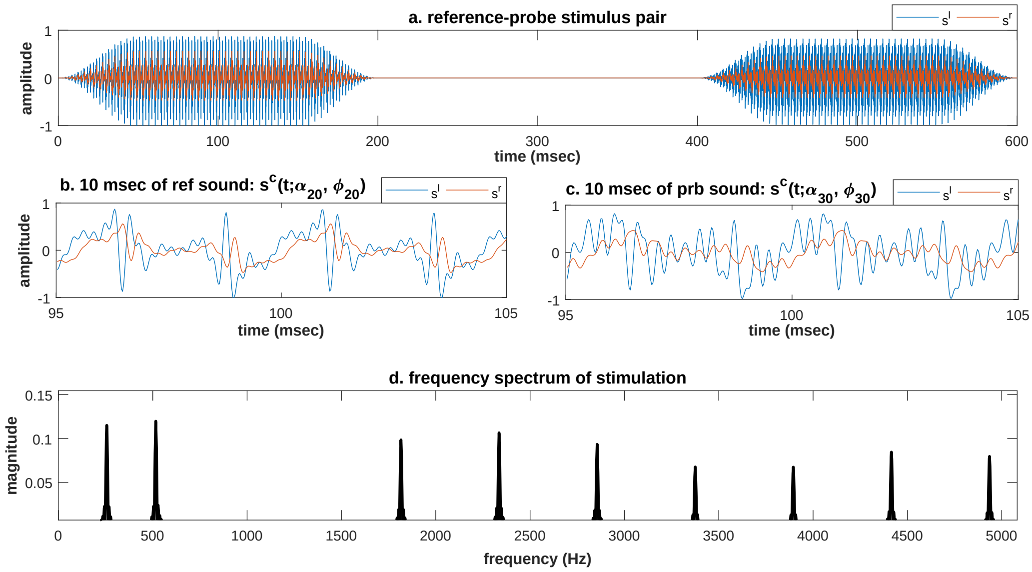

Figure 4.2: Reference-Probe stimulus sound samples with s^l (blue) and s^r (red) channels: (a) 200 msec reference $s^c(t;\alpha_{20},\phi_{20})$ - 200 msec gap - 200 msec probe $s^c(t;\alpha_{30},\phi_{30})$, (b) 10 msec zoomed sample of the reference sound, (c) 10 msec zoomed sample of the probe sound, (d) frequency spectrum of the stimulus sound

4.1.2 Procedure

Stimuli are presented to the subjects using a MATLAB Graphical User Interface (Figure 4.3 shows the main test GUI used by the subjects). The psychophysical lateralization experiment is conducted inside a chamber with 40-dB sound attenuation (schematic of the experiment room is shown as Figure 4.4). There are always 10 degrees of azimuth difference between the reference and probe sounds and the subject's task is to report whether the probe sound is heard from the left or right side of the reference sound by pressing one of the two respective keys on a keyboard.

During the test, the sound pairs with different reference azimuthal angles and shift directions are randomly presented so that the subjects are not able to learn and predict type of the next pair of sounds in stimulation sequence. An experiment with one subject is performed in three separate sessions for each of the TO, LO and TL conditions.

Table 4.1 shows all the available stimulation pairs presented in the experiments.

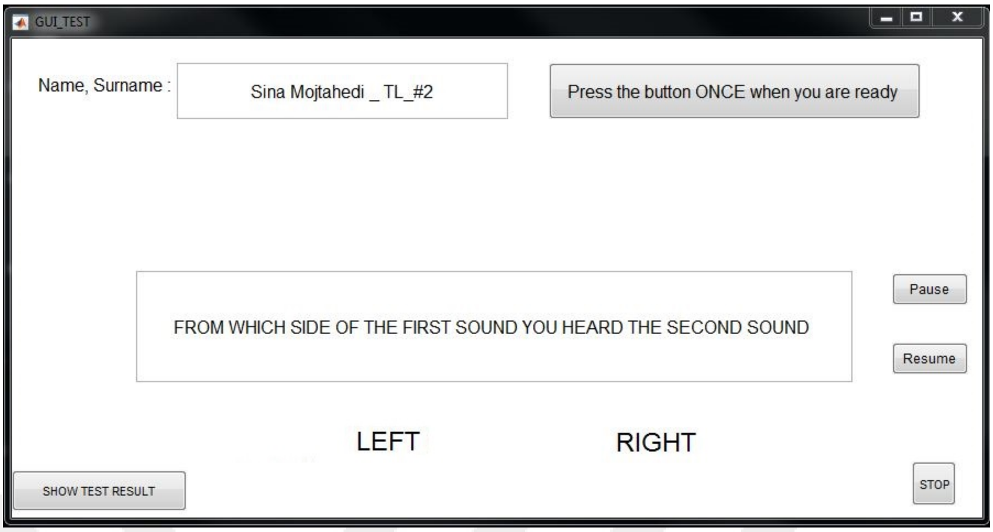

Figure 4.3: Graphical User Interface (GUI) of the main experiments. Participants used the GUI to detect the changes in stimuli sounds.

Note that only the ones, which are symmetric around the reference are used in the error rate calculations. Each session includes 10 experimental blocks. Each block includes 16 combination of reference-probe sounds. Each combination is repeated 10 times in a block. Hence, the number of stimuli evaluated by each subject is, 10 blocks x 16 combinations/block x 10 repeats/combination, which adds up to 1600 stimulations per condition. In the whole test, the total number of stimulations is 1600 stimuli/condition x 3 condition/subject x 5 subjects, which is in total 24000.

4.1.3 Data Collection

Two-alternative forced choice (2AFC) is a measuring method in pyschophysical experiments in which the subject is forced to have one choice between two possible answers. The options are presented to the subject and the subject is asked for the correct one. Since the test is in 2AFC format, each response to the stimulus pair is taken as a HIT or a FALSE. If the subject is unable to answer the test in 2 seconds, a MISS is reported for the stimuli. In our experimental data, total MISS rate appears to be 3 out of 24000 trials (0.0125%), so we simply discarded the MISS responses. Then over the total number of HIT and FALSE responses, the error rate is defined

Figure 4.4: Sound proof room used for performing experiments

as,

$$E_r = \frac{N_F}{N_H + N_F}, \tag{4.6}$$

where N_H and N_F are respectively total number of hit and false responses. The experimental SNR values are then extracted from the error rate, E_r, under the binary additive white-Gaussian noise channel model as,

$$SNR = (\text{erfc}^{-1}(2E_r))^2, \tag{4.7}$$

where $\text{erfc}^{-1}()$ is the inverse of complementary error function (Shankar, 2011).

The error rate at each azimuth θ is also defined based on the probe sound's azimuth being inward (towards center) and outward (away from center) with respect to the reference sound's azimuth. The inward and outward azimuth angles are defined as $\theta_{in} = sign(\theta)(|\theta| - 10°)$ and $\theta_{out} = sign(\theta)(|\theta| + 10°)$, where $sign(\theta)$ is positive if the azimuth θ is at the right side and negative if the azimuth is at the left side of the horizontal mid-line.

Let's define $N_F(\theta_r, \theta_p)$ as the number of false responses for the azimuth of the reference and probe stimuli for θ_r and θ_p, respectively. In this case, the number of

Table 4.1: The reference (columns) and probe (rows) sounds from azimuth angles that are used in the tests: (*) conditions are used in the error rate calculations, (+) conditions are presented and responded to in the experiments but not included in the error rate calculations, right and left azimuth angles are respectively denoted on positive and negative axis.

PR	-40	-30	-20	-10	0	+10	+20	+30	+40
-40	-	*	-	-	-	-	-	-	-
-30	+	-	*	-	-	-	-	-	-
-20	-	*	-	*	-	-	-	-	-
-10	-	-	*	-	*	-	-	-	-
0	-	-	-	*	-	*	-	-	-
+10	-	-	-	-	*	-	*	-	-
+20	-	-	-	-	-	*	-	*	-
+30	-	-	-	-	-	-	*	-	+
+40	-	-	-	-	-	-	-	*	-

false responses for inward $N_F^{in}(\theta)$, and outward $N_F^{out}(\theta)$, cases are defined as,

$$N_F^{in}(\theta) = N_F(\theta_{out}, \theta) + N_F(\theta, \theta_{in}), \tag{4.8}$$

$$N_F^{out}(\theta) = N_F(\theta, \theta_{out}) + N_F(\theta_{in}, \theta), \tag{4.9}$$

for all non-zero θ values, i.e. $\theta = \pm 10, \pm 20, \pm 30$. The number of hit responses for inward and outward cases can be computed similarly and represented as $N_H^{in}(\theta)$ and$N_H^{out}(\theta)$.

Considering the symmetry of the left and right sides of the horizontal mid-line, the number of false and hit responses can be calculated for absolute azimuth angles over both the left and right sides. The number of false responses around an absolute azimuth θ in inward and outward directions are defined as,

$$N_F^{in}(|\theta|) = N_F^{in}(\theta) + N_F^{in}(-\theta), \tag{4.10}$$

$$N_F^{out}(|\theta|) = N_F^{out}(\theta) + N_F^{out}(-\theta), \tag{4.11}$$

at absolute azimuth values, $\theta = 10°, 20°, 30°$. Similarly, $N_H^{in}(|\theta|)$ and $N_H^{out}(|\theta|)$ can be defined for hit responses. At $\theta = 0°$, the inward and outward false responses can be defined as,

$$N_F^{in}(\theta) = N_F(10°, \theta) + N_F(-10°, \theta), \tag{4.12}$$

$$N_F^{out}(\theta) = N_F(\theta, 10°) + N_F(\theta, -10°). \tag{4.13}$$

Finally, the error rates for inward (in) and outward (out) cases are defined as,

$$E_r^{in}(|\theta|) = \frac{N_F^{in}(|\theta|)}{N_F^{in}(|\theta|) + N_H^{in}(|\theta|)}, \tag{4.14}$$

$$E_r^{out}(|\theta|) = \frac{N_F^{out}(|\theta|)}{N_F^{out}(|\theta|) + N_H^{out}(|\theta|)}, \tag{4.15}$$

at absolute azimuth values, $\theta = 0°, 10°, 20°, 30°$.

4.1.4 Analytical Methodology

We propose a diversity combination approach for the fusion of the ITD and ILD cues for the lateralization decision in the brain. We can set lateralization decision due to each cue as an estimation problem under additive white Gaussian noise as,

$$\hat{\Delta}_L = \Delta_L + n_L \pm \beta_L \quad \text{and} \quad \hat{\Delta}_T = \Delta_T + n_T \pm \beta_T, \tag{4.16}$$

where $\hat{\Delta}_L$ and $\hat{\Delta}_T$ respectively represent observed lateralization changes due to the ILD and ITD cues while n_L and n_T are zero-mean Gaussian random noise sources with σ_L^2 and σ_T^2 variances, and β_L and β_T are positive bias factors for LO and TO conditions representing the inward and outward decisions respectively with additive (plus) and subtractive (minus) contributions.

Then the fusion of the ITD and ILD cues for the lateralization decision is defined in the diversity combination framework as,

$$\begin{aligned} \hat{\Delta}_{TL} &= \lambda\hat{\Delta}_L + (1-\lambda)\hat{\Delta}_T \\ &= \lambda(\Delta_L + n_L \pm \beta_L) + (1-\lambda)(\Delta_T + n_T \pm \beta_T) \\ &= \Delta_{TL} + \lambda n_L + (1-\lambda)n_T \pm \lambda\beta_L \pm (1-\lambda)\beta_T \end{aligned} \tag{4.17}$$

where $\Delta_{TL} = \Delta_L = \Delta_T$ is the noise-free lateralization change, $\hat{\Delta}_{TL}$ is the observed lateralization change due to weighted fusion of ILD and ITD cues with weights λ and $(1-\lambda)$, respectively. The weight parameter λ should be in $(0,1)$ interval. Assuming noise sources are uncorrelated, the estimated SNR of the observed lateralization change of the combined cues can be written as,

$$SNR^{\pm}_{TL} = \frac{(\Delta_{TL} \pm \lambda\beta_L \pm (1-\lambda)\beta_T)^2}{\lambda^2\sigma_L^2 + (1-\lambda)^2\sigma_T^2}, \tag{4.18}$$

where plus representing inward and minus representing outward decisions.

Under the biased diversity combination assumption, the task is to find the optimal model parameters, which are noise variances, bias factors and the fusion weight, such that the difference between the estimated and experimental SNR values is minimized. This results into a multivariable optimization problem, where we can define the following six SNR difference functions as,

$$E_1 = SNR^{+}_{TO} - SNR^{in}_{TO} = \frac{((1-\lambda)(\Delta + \beta_T))^2}{\lambda^2\sigma_L^2 + (1-\lambda)^2\sigma_T^2} - SNR^{in}_{TO} \tag{4.19}$$

$$E_2 = SNR^{+}_{LO} - SNR^{in}_{LO} = \frac{(\lambda(\Delta + \beta_L))^2}{\lambda^2\sigma_L^2 + (1-\lambda)^2\sigma_T^2} - SNR^{in}_{LO} \tag{4.20}$$

$$E_3 = SNR^{-}_{TO} - SNR^{out}_{TO} = \frac{((1-\lambda)(\Delta - \beta_T))^2}{\lambda^2\sigma_L^2 + (1-\lambda)^2\sigma_T^2} - SNR^{out}_{TO} \tag{4.21}$$

$$E_4 = SNR^{-}_{LO} - SNR^{out}_{LO} = \frac{(\lambda(\Delta - \beta_L))^2}{\lambda^2\sigma_L^2 + (1-\lambda)^2\sigma_T^2} - SNR^{out}_{LO} \tag{4.22}$$

$$E_5 = SNR^{+}_{TL} - SNR^{in}_{TL} = \frac{(\Delta + \lambda\beta_L + (1-\lambda)\beta_T)^2}{\lambda^2\sigma_L^2 + (1-\lambda)^2\sigma_T^2} - SNR^{in}_{TL} \tag{4.23}$$

$$E_6 = SNR^{-}_{TL} - SNR^{out}_{TL} = \frac{(\Delta - \lambda\beta_L - (1-\lambda)\beta_T)^2}{\lambda^2\sigma_L^2 + (1-\lambda)^2\sigma_T^2} - SNR^{out}_{TL} \tag{4.24}$$

which are jointly minimized for λ, β_T, β_L, σ_T and σ_L when the experimental lateralization change is set as $\Delta = 10$. Note that a set solution for the parameters can be extracted for each absolute azimuth values, $\theta = 0°, 10°, 20°, 30°$.

Optimization

Let‘s define the parameter vector for the optimization problem as $\mathbf{x} = [\lambda, \beta_T, \beta_L, \sigma_T, \sigma_L]$ and the mean square SNR error over the six SNR differences as $F(\mathbf{x}) = \frac{1}{6}\sum_{k=1}^{6} E_k^2$.

Then the optimization problem is defined to minimize the mean square error,

$$x^* = \underset{\text{s.t. } \mathbf{x_0}=0,\ \mathbf{l_b} \leq \mathbf{x} \leq \mathbf{u_b}}{\arg\min} F(x), \tag{4.25}$$

where $\mathbf{x_0}$ is the initial parameter vector and $\mathbf{l_b}$ and $\mathbf{u_b}$ are set to [0, 0, 0, 0, 0] and [1, 10, 10, 100, 100] as the lower and upper bounds for the $\mathbf{x}$ vector, respectively. The above optimization problem was solved by the nonlinear solver (fmincon) based on the Rosenbrock function (Rosenbrock, 1960) using the MATLAB Optimization Toolbox (MATLAB Optimization Toolbox, ver).

4.2 Results and Data

Experimental results of the five participants are pooled together and presented as SNR values of TO, LO and TL cases for inward and outward stimuli in Table 4.2. Note that, SNR^{in} and SNR^{out} are calculated using E_r^{in} and E_r^{out} values.

Table 4.2: SNR values calculated from the experimental error rates

	Experimental SNR values					
	SNR_{TO}^{in}	SNR_{TO}^{out}	SNR_{LO}^{in}	SNR_{LO}^{out}	SNR_{TL}^{in}	SNR_{TL}^{out}
0°	0.154	0.073	0.490	0.497	0.941	0.866
10°	0.249	0.074	0.474	0.293	0.915	0.634
20°	0.496	0.023	0.528	0.064	0.907	0.298
30°	0.618	0.0004	0.494	0.054	0.924	0.055

The results in Table 4.2 show the differences between the SNR values of inward (SNR^{in}) and outward (SNR^{out}) cases, where the SNR values of the inward cases are always greater than the outward ones except for the SNR_{LO}^{out} at 0° azimuth. For the TO, LO and TL conditions, the SNR difference, (SNR^{in} - SNR^{out}), increases by increasing azimuth. This can be considered as a bias between inward and outward decisions, such that the subjects mostly prefer to make a "toward center" decision for sound's azimuth changes under difficult to decide conditions. Note that, this bias occurs regardless of the type and azimuth of the imposed cue, and it gets smaller as the azimuth moves toward the horizontal mid-line.

4.2.1 Analytical data and fitting the model to experimental data

As a result of the optimization in (4.25) optimal values of λ, β_T, β_L, σ_T and σ_L were extracted and presented in Table 4.3. The estimated SNR values were calculated using the optimal parameter values in (4.18) at each azimuth and presented in Table 4.4. The differences between the experimental and estimated SNR values were then calculated for TO, LO and TL (inward and outward) cases as TO^+, TO^-, LO^+, LO^-, TL^+ and TL^-. The experimental and estimated SNR differences are given in Figure 4.5. All of the SNR values, apart from TO^+ (all azimuths) and LO^+ (20^o and 30^o), were close to the experimental data.

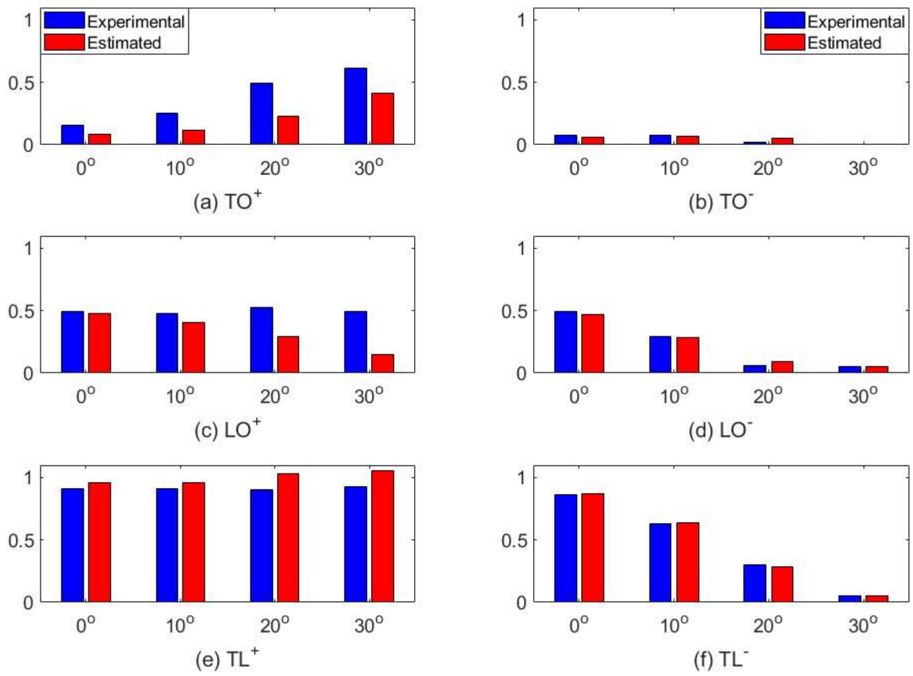

Figure 4.5: Experimental and estimated SNR values for inward (+) and outward (-) cases of the azimuths 0^o, 10^o, 20^o and 30^o: (a) TO^+, (b) TO^-, (c) LO^+, (d) LO^-, (e) TL^+ and (f) TL^-.

Table 4.3 shows that the ILD cue weight, λ, is decreasing with the increasing azimuth and the bias values, β_T and β_L are increasing as azimuth increases. The increase in azimuth has more influence on ITD than ILD as the β_T increased drastically from 0.900 for 0^o to 9.868 for 30^o. The increase in azimuth makes the differences in β_T more apparent, which shows the inability of the ITD cue to compensate for the ILD cue in larger azimuths.

Table 4.3: Estimated λ, β_T, β_L, σ_T and σ_L values

	λ	β_T	β_L	σ_T	σ_L
0°	0.721	0.900	0.000	25.316	10.697
10°	0.660	1.418	0.825	22.777	12.364
20°	0.540	3.142	2.872	18.600	17.900
30°	0.489	9.868	2.508	23.365	22.453

4.2.2 Heat-map Presentation

In order to double-check the precision in calculations, heat-map plots of the mean square SNR error $F(\mathbf{x})$ as a function of λ vs β_T and β_L are presented in Figure 4.6 and Figure 4.7 for azimuth 0^o, 10^o, 20^o and 30^o values. Figure 4.6 plots the mean square SNR error $F(\mathbf{x})$ as a function of λ vs β_T after fixing the β_L, σ_T and σ_L at the extracted optimal values as given in Table 4.3. Similarly, Figure 4.7 plots the mean square SNR error $F(\mathbf{x})$ as a function of λ vs β_L after fixing the β_T, σ_T and σ_L at the extracted optimal values. The optimal λ and β values for azimuth 0^o, 10^o, 20^o and 30^o extracted from the proposed model are shown with white dots in Figure 4.6 and Figure 4.7.

Similar to the results given in Table 4.4, the SNR values of the inward cases are found to be greater than the outward ones in all azimuth values. For the TO, LO, and TL conditions, the difference between the inward and outward SNR values are decreasing with the increasing azimuth.

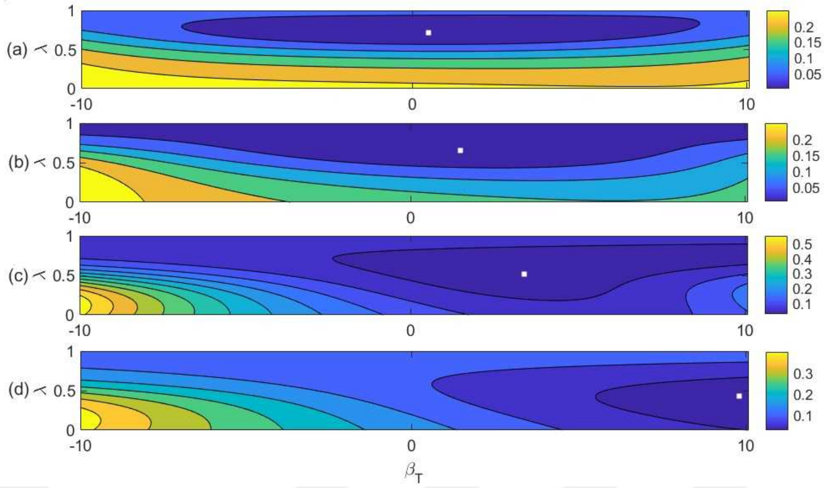

Figure 4.6: Heat-map plots of the mean square SNR error $F(\mathbf{x})$ as a function of λ vs β_T after fixing the β_L, σ_T and σ_L at the extracted optimal values for azimuth (a) 0^o, (b) 10^o, (c) 20^o and (d) 30^o. White dots are the optimal λ and β_T values extracted from the model.

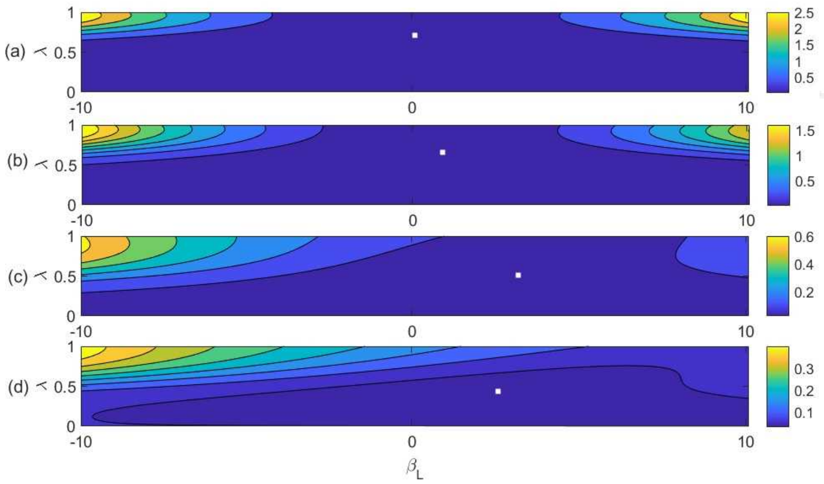

Figure 4.7: Heat-map plots of the mean square SNR error $F(\mathbf{x})$ as a function of λ vs β_L after fixing the β_T, σ_T and σ_L at the extracted optimal values for azimuth (a) 0^o, (b) 10^o, (c) 20^o and (d) 30^o. White dots are the optimal λ and β_L values extracted from the model.

4.2.3 3-way Analyses of Variance Test (3-way ANOVA)

Analysis of variance (ANOVA) is used to calculate the significance of the difference between three or more independent means in a normally distributed series. ANOVA

Table 4.4: SNR values calculated from the experimental error rates

	SNR^{+}_{TO}	SNR^{-}_{TO}	SNR^{+}_{LO}	SNR^{-}_{LO}	SNR^{+}_{TL}	SNR^{-}_{TL}
0°	0.084	0.058	0.476	0.473	0.961	0.869
10°	0.119	0.067	0.403	0.289	0.960	0.636
20°	0.227	0.054	0.295	0.089	1.032	0.283
30°	0.410	0.000	0.149	0.053	1.053	0.055

alone compares the arithmetic means of three or more groups cumulatively; ANOVA result is found to be significant when at least one of these comparisons is significant. In this case, the hypotheses is as follows:

H0: There is no difference between the averages.

H1: There is a significant difference between at least two means.

The term "3-way" refers to the three factors (independent variables) and one dependent variable in tests. A 3-way ANOVA over the SNR values with the independent factors being Azimuth (0°, 10°, 20°, 30°), Cue (TO, LO, TL) and shift Direction (inward, outward) confirmed the statistical significance of Azimuth [F(3,57) = 254.52, p<0.000, $\eta^2_{partial} = 0.931$], Cue [F(2,38) = 2282.04, p<0.000, $\eta^2_{partial} = 0.992$] and Direction [F(1,19) = 1945.69, p<0.000, $\eta^2_{partial} = 0.990$]. The values confirmed that each of the factors individually affect the performance of detecting shift direction. Interactions among all pairs of factors were also significant as follows. Azimuth x Cue: [F(6,114) = 365.42, p<0.000, $\eta^2_{partial} = 0.951$], Cue x Direction: [F(2,38) = 13.48, p<0.001, $\eta^2_{partial} = 0.415$] and Azimuth x Direction: [F(3,57) = 1011.58, p<0.000, $\eta^2_{partial} = 0.982$]. The differences in all of the tests are confirmed to be statistically significant. The Azimuth x Cue x Direction interaction is also found to be significant [F(6,114) = 5.66, p = 0.002, $\eta^2_{partial} = 0.230$]. This shows that each of the pairwise interactions may change significantly depending on the levels of the remaining factors.

The stimuli used in the study have three types: TO, LO and TL. The sum of the SNR values of the TO and LO conditions is denoted by SNR_{T+L} and compared

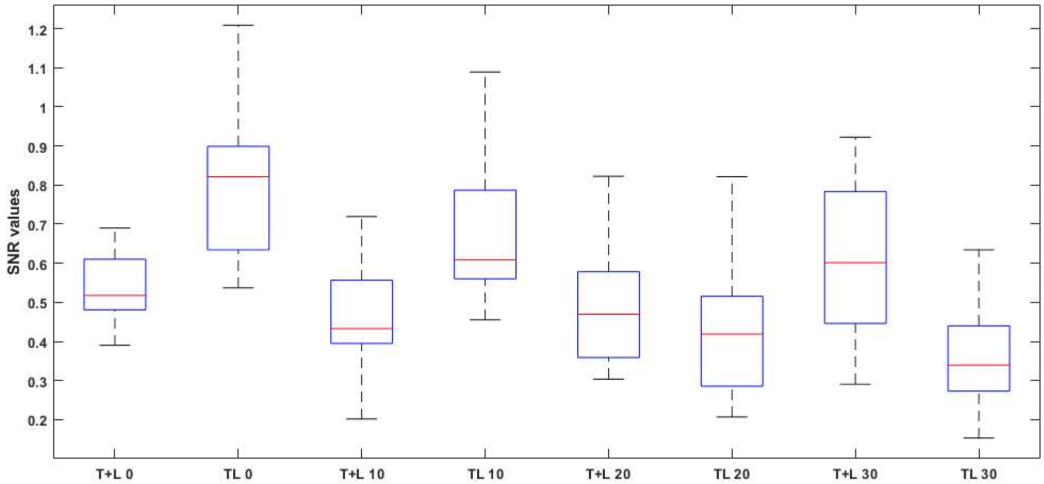

Figure 4.8: Box-plots of the SNR_{T+L} and SNR_{TL} values computed from the re-sampled error rates of T+L and TL cases, respectively, for the four azimuthal angles studied. Note that the SNR values of time and level cues are combined into SNR_{TL} super-additively for 0° and 10°, whereas their combination turns to be additive and sub-additive for 20° and 30°, respectively.

to the SNR_{TL} to see if the cue combination is additive ($SNR_{TL} = SNR_{T+L}$), super-additive ($SNR_{TL} > SNR_{T+L}$) or sub-additive ($SNR_{TL} < SNR_{T+L}$).

4.2.4 2-way Aalyses of Variance (2-way ANOVA)

In order to test if SNR_{TL} and SNR_{T+L} are significantly different from each other, the error rate data of the five participants are pooled and 20 re-sampled (bootstrapped) data of size 100 samples were drawn. The median, quartile, minimum, and maximum SNR values computed from these error rates for the mentioned combinations are given as boxplots in Figure 4.8. The SNR values of T+L are calculated ($SNR_{T+L} = SNR_{TL} + SNR_L$). Then the SNR data used for testing the additivity, super- and sub-additivity of the cues in different azimuths. Based on the data a super-additivity is observed for 0° and 10°. Increasing the azimuth leads to additivity and sub-additivity in rather peripheral azimuths of 20^o and 30^o. To test the statistical significance of the differences between combinations, a 2-way ANOVA with the factors Combination (T+L and TL) and Azimuth (0, 10, 20, and 30) is applied. Significant effects are noted for Combination [F(1,19) = 5.99, p = 0.024, $\eta^2_{partial} = 0.240$] and for Azimuth [F(3,57) = 9.41, p<0.000, $\eta^2_{partial} = 0.331$]. Interaction of Azimuth with Combination also proves to be significant [F(3,57) = 30.56, p<0.000, $\eta^2_{partial} = 0.617$), which indicates that the effect of Combination strongly depends on Azimuth.

4.2.5 post-hoc t-test

The LSD (least square difference) is a post-hoc test and applies standard t-tests to all possible pairs of group means. Mathematically, it is highly vulnerable to type I error. Because, although Type I error level (α) is chosen as 5%, as the number of groups increases, the amount of error per group also increases. To study this dependency $post - hoc$ t-tests are also applied for testing the significance of the differences between SNR_{TL} and SNR_{T+L} in the four azimuthal conditions. The differences are found significant, except for 20° where T+L and TL profile lines intersect each other, indicating a transition in the type of cue-combination from super-additivity to sub-additivity.

4.3 Discussion

Main aim of the current study was to find the weight values for each of the ITD and ILD cues by the means of behavioral experiments. The first sound of the stimulus pair is made to have the ITD and ILD of a reference azimuth (θ). The azimuth of the second sound differed from the first one, assuming an azimuthal change of 10°. This change is made in the following three ways to produce azimuthal shifts toward center or toward periphery:

a) Only ITD is changed and made equal to its value for either $(\theta - 10°)$ or $(\theta + 10°)$, respectively.

b) Only ILD is changed and made equal to its value for either $(\theta - 10°)$ or $(\theta + 10°)$, respectively.

c) ITD and ILD are changed together and both made equal to their values for either $(\theta - 10°)$ or $(\theta + 10°)$, respectively.

To delineate the weights of the two cues, we chose an approach based on different operating frequency ranges of the neural mechanisms encoding them by presenting a complex sound with imposed either of the TO, LO or TL cues in the lateralization tests. We should admit that associating the ITD cue only with the lateralization performance at low-frequencies and the ILD cue only with the lateralization performance at high-frequencies may not be an accurate assumption. This is because for amplitude-modulated (AM) high-frequency complex sounds, LSO may also encode ITDs through interplay between the arrival times of the ipsilateral excitatory and contralateral inhibitory inputs (Goksoy et al., 2005; Joris and Yin, 1995; Ungan and Yagcioglu, 2002). Therefore, in our stimulation sounds, while the stimulation's first sound (reference) contains both ITD and ILD cue information from the same azimuth (θ), the second sound (probe) contains either one of the ITD or ILD or both of the ITD and ILD cues. Choosing such a method, enabled us to impose either of the ITD or ILD cues independently from each other where both of the cues, each with different values, are available at a time. From the other side, this approach helped us to study the distorting effect of the fixed cue (from azimuth θ) while the changing cue (from azimuth $\theta \pm 10°$) shows the proper direction of the sound.

The results of current study are in conformity with the results of a previous study by Schroger (1996) - in which the mismatch negativity results are compared for complex and ITD-ILD deviant stimulations - the most correct answers were recorded for the TL case in comparison to the worst responses to TO case. Simultaneous presentation of ITD and ILD cues to the subjects, results in increased SNR values in all of the four azimuthal cases we tried. Although the hearing system utilizes both ITD and ILD information equivalently (similar SNR values and response times for TO and LO conditions), combining these two pieces of information when they are simultaneously available leads to more confident lateralization of acoustical sources.

An important observation of the current study is the decrease in SNR values and increase in the bias values with increasing azimuth. Despite the fact that Yost et al. (2013) do not impress the bias in their finding, this observation is in harmony with their results of a previous study in which the subjects responded to the stimulation by determining the location of the loud speakers that presents the stimulation (Yost et al., 2013). According to their results of low-pass, high-pass and broadband stimulations, the most correct answers were recorded for the azimuth near horizontal mid-line. Increasing the azimuth of the stimulation resulted in decrease of the SNR values.

Diversity combination model considers the ITD and ILD cues as independent and uncorrelated cues. Therefore, the experimental SNR values will closely fit or not to the SNR values estimated by the model depending upon the level of independence of these cues from each other. While the results of a previous study by Kuwada and Yin (1983) provides evidences with regards to the availability of some colliculus neurons which are responsible for both ITD and ILD cue coding, similar study (Joris and Yin, 1995) proved the availability of some LSO cells which are influenced by both ITD and ILD cues. Similarly, there are some MSO cells which are also sensitive to ILD cues (Yin and Sujaka, 1984). The results of other studies (Calvert and Thesen, 2004; Edmonds and Krumbholz, 2013) related with the significance of ITD and ILD cues, show a subadditivity of ITD and ILD cues, which indicated a coupling between the neural mechanisms processing the cues, rather than their independency in cue coding.

For inward stimulation sounds, while our experimental results show a super-additivity of the cues ($SNR_{TL} > SNR_{TO} + SNR_{LO}$) for 0°, 10° and 20° azimuths which suggests a nonlinear combination of modality-specific influences as the multi-cue whole is greater than the sum of its uni-cue parts. This is while the results for outward stimulation sounds show super-additivity in all azimuthal conditions except for 30 degree, for which a slight subadditivity is observed like in the case of inward shifts. However, the above hypothesis is valuable in the case when the differences between sum of the uni-cue parts and multi-cue part are significantly different.

The present study investigated the effects of azimuth, cue-combination and shift-direction on lateralization for paired-stimuli presented to the subjects with earphones. The significant effect of all three factors indicates that azimuth of stimulation, direction of lateralization and combination of cues are all effecting cues of lateralization. The non-significant interaction between azimuth and shift-direction shows that the pattern of performance for the four groups of azimuth was not statistically different, suggesting similar effects of azimuths despite the different overall performance of the subjects as it revealed by the pairwise comparisons between azimuth in which the differences between the $20°$ and $30°$ azimuth are statistically non-significant. The repeated-measurement ANOVA did show an azimuth, cue-combination and shift-direction main effect and interaction of factors except for interaction between azimuth and shift direction, suggesting that while azimuth increased (indicating better performance), shift direction was largely affected. The only issue of the azimuth effect on subject‘s performance was the insignificant azimuth change from $20°$ to $30°$ and vice-versa.

However, the results of the azimuthal cases tried show that, the TL stimulation results in a higher SNR value than the cases in which TO or LO is imposed. This means that, the hearing system uses each of the cues individually, but in order to have a better azimuthal detection, it uses both of the information provided for the ears by simultaneous availability of ITD and ILD.

In general ITD and ILD cues are jointly processed in the neurons of inferior colliculus which are sensitive to both cues. Based on the SNR values of the experiments, it seems that human subjects mostly trusted the ILD cue for lateralizing the sound sources near zero azimuth, whereas, they mostly rely upon the ITD cue for relatively larger azimuths. This is similar to the results of the study by Morikawa (2014). Their findings also suggest an increase in error rates for smaller azimuth in ITD and larger azimuth in ILD stimulation sounds. This could be related to the azimuth-dependent consistency of the cues; i.e. the auditory system relies more on the most consistent cue in lateralizing sound sources (Edmonds and Krumbholz, 2013). In connection to the directional bias in the SNR data, we should mention here the study of Ozmeral et al. (2016) where the P1-N1-P2 components of the human scalp-recorded event-related potentials (ERP) were recorded in response to auditory lateralization shifts produced by reference (adapter, as the call it) and probe sounds having different ITD values. They studied the inward and outward lateralization shifts produced by ITD-changes in young and old listeners. The latency and amplitude of both N1 and P2 components showed the main effect of inward vs outward where they record shorter latencies and larger amplitudes for outward shifts. They conclude this phenomenon happens as a result of facilitation of shifts within a presently dominant hemifield channel because, according to the opponent-channel model, the response to a shift is due to the activity in the channel that is rising in activity at the onset of the probe. The opponent-channel model predicts that the brain‘s electrical responses induced by inward shifts are smaller than those induced by outward shifts. These arguments seems to be in contradiction with the bias that we observed. However, the following argumentation may provide an explanation for this apparent contradiction. Even though the exact reason is unknown but it can be considered as the result of an attentional priority formed throughout life. Because inward shifts tend to elicit a weak neural activation, according to the opponent-channel model, the cortical mechanism that gives the final decision for the shift direction is expected to learn this correspondence. Therefore, in the most of the cases where activation is very weak (which would also include rather small outward shifts, besides moderate inward shifts), the brain will decide as inward. In other words, in every case where the shift is vague (which is difficult to make decisions), the activation will be weak and, regardless of the actual (physical) aspects, these shifts will be considered as inward, thus resulting in an inward bias. The resulting bias, which reveals in experimental results and fits the model appropriately and indicated that, accuracy of human subjects in detecting a change in the azimuth of an auditory source is lower for a change toward periphery than for a change toward center. Behavioral or response bias is discussed in many articles which used 2AFC discrimination tasks in sound lateralization. Experimental trials in these tests are often presenting stimuli that are very similar and observers find it difficult to make a judgment. The strategies that subjects use when they are unable to make a decision are known as bias 2AFC tasks (Morgan et al., 1990; Jakel and Wichmann, 2006). A possible resean for the directional bias in our results could be connected to an attentional priority developed through life for the sounds moving toward center relative to those moving toward sides. This priority may be in the focused attention and cognition mechanisms which are active during the tests or in an automatic (unintentional) and thus pre-attentive change detection mechanism similar to the results described by Näätänen (1992) and von Kriegstein et al. (2008).

Chapter 5

CONCLUSION AND FUTURE WORK

In the experiment I series, we investigate the error rate with the joint LHF source, and try to model it using a diversity combining perspective in terms of LF and HF sources. We observe the highest SNR values with the HF source, and SNR of the LHF source is extracted slightly lower than the SNR of HF source. This observation tells us that the LF and HF sources are not combined with the MRC model, but a linear combination model can be defined for the LF and HF sources and the corresponding contributions, λ for HF source, can be computed. Results also imply that the human hearing system mostly relies on high frequencies than low frequencies for sound lateralization. Moreover, considering just two challenging cases of $3°$ and $10°$ azimuths, SNR of the $10°$ LF is also lower than the SNR of the $3°$ HF. This fact also implies that even in very difficult situations, the ILD cues have more importance than the ITD cues.

In the experiment II series, results of the behavioral experiments show that, ILD information is more important than ITD in lateralizing sound sources with smaller azimuths. As the azimuth of the sound source increases, the roles of ITD and ILD cues turn to be reversed. A behavioral bias, which is independent of type of the cue, misleads the subjects in determining the direction of change in the lateral position of a sound source. This centripetal bias increases with increasing azimuth and is minimum for an azimuth of $\pm 10°$ around median plane. The SNR data show that the two pieces of spatial information supplied by ILD and ITD are combined in the brain, but not additively for shifts in lateral position around small azimuths. Nonetheless, integration of the ILD and ITD cues can be modeled using a diversity combination approach incorporating the directional bias mentioned.

Considering the possibility that centripetal and centrifugal movements of sound sources may have different importance in everyday life, it should be interesting to search for the electrophysiological correlate of this bias in the brain. This may be done by using P300 and mismatch negativity paradigms with deviant sounds from azimuthal angles smaller and larger than the standard sound, and comparing the electrical brain responses to these two deviant sounds.

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Appendix A

In this chapter, the important codes of the study are presented. Note that, only some basic parts of the codes are included.

A.1 Matlab Codes

A.1.1 ITD and ILD Extraction from HRTF

```
% Generate a stereo tone at frequency FREQ at level LEV from
% angle ANG with duration DUR seconds
% Returns stereo tone and sampling freq fs

function [tone, fs] = gtone(FREQ, LEV, ANG, DUR)

[h,fs] = audioread('HRTF/0.wav');
hl = h(:,1)'; hr = h(:,2)';

% DFT to find amp and phas in any frequency (for example 260 Hz)
[m N] = size(hl); t = 0:1/fs:(N-1)/fs;

HL = sum(hl .* exp(-i*2*pi*FREQ*t))*2/N;
amp_hl = abs(HL); phas_hl = angle(HL);
HR = sum(hr .* exp(-i*2*pi*FREQ*t))*2/N;
amp_hr = abs(HR); phas_hr = angle(HR);

% Amp ratio and phas differences
AR5 = amp_hr/amp_hl; AR0 = 1;
```

```
PD5 = phas_hr - phas_hl; PD0 = 0;

% Interpolation to find amp and phas for any degree
dir = sign(ANG); % Direction 1: right, -1: left
aang = abs(ANG);  % Absolute angle

ARd = ((30-aang)*AR0 + aang*AR5)/30;
PDd = ((30-aang)*PD0 + aang*PD5)/30/2;
AMP = sqrt(ARd);
if dir<1
    AMP = 1/AMP;
    PDd = -PDd;
end

% Producing Sound using sine_wave
t = (1:DUR*fs)/fs;
Nt = max(size(t));
% AMP = 1; %Just ITD
% PDd = 0; % Just ILD
CHr = LEV * AMP * sin(2*pi*FREQ*t + PDd);
% CHr = CHr/max(CHr);
CHl = LEV * (1/AMP) * sin(2*pi*FREQ*t - PDd);
% CHl = CHl/max(CHl);
tone = [CHl;CHr]';
```

A.1.2 Main Test

```
function varargout = GUI_TEST(varargin)
gui_Singleton = 1;
gui_State = struct('gui_Name',       mfilename, ...
                   'gui_Singleton',  gui_Singleton, ...
```

```
                    'gui_OpeningFcn', @GUI_TEST_OpeningFcn, ...
                    'gui_OutputFcn',  @GUI_TEST_OutputFcn, ...
                    'gui_LayoutFcn',  [] , ...
                    'gui_Callback',   []);
if nargin && ischar(varargin{1})
    gui_State.gui_Callback = str2func(varargin{1});
end

if nargout
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
    gui_mainfcn(gui_State, varargin{:});
end
% End initialization code - DO NOT EDIT


% --- Executes just before GUI_TEST is made visible.
function GUI_TEST_OpeningFcn(hObject, eventdata, handles, varargin)
% This function has no output args, see OutputFcn.
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)
% varargin   command line arguments to GUI_TEST (see VARARGIN)

% Choose default command line output for GUI_TEST
handles.output = hObject;

% Update handles structure
guidata(hObject, handles);
```

```
global jj

% Position = [200 300 450 30];
% jj = prog_bar (Position);

% UIWAIT makes GUI_TEST wait for user response (see UIRESUME)
% uiwait(handles.figure1);


% --- Outputs from this function are returned to the command line.
function varargout = GUI_TEST_OutputFcn(hObject, eventdata, handles)
% varargout  cell array for returning output args (see VARARGOUT);
% hObject    handle to figure
% eventdata  reserved - to be defined in a future version of MATLAB
% handles    structure with handles and user data (see GUIDATA)

% Get default command line output from handles structure
varargout{1} = handles.output;

global esm
esm = get(hObject,'String');

global j esm LR CC CL Fs jj
j = 1; Fs = 48000;

resfolder = ('Results\Final');
TxtName = strcat(esm,'.txt');
fullTxtName = fullfile(resfolder, TxtName);

fid = fopen(fullTxtName,'a');
```

```
fprintf(fid,['Name:' '\t' ,get(handles.edit1,'String') '\r\n'])
fprintf(fid,['Sound Test GUI' '\r\n'])
fclose(fid);
LR = 'SECOND STIMULI comes from: LEFT —— RIGHT  of the FIRST STIMULI';
fid = fopen(fullTxtName,'a');
set (handles.edit2,'String',LR);
pause (2)

% Angle Calculation
Theta = 0:10:40
% ITD_Only
Cond_T = ['LFT_T(Theta)L(Theta)_T(Theta+10)L(Theta)'];
% ILD_Only
Cond_L = ['LFT_T(Theta)L(Theta)_T(Theta)L(Theta+10)'];
% ITD_ILD
Cond_TL = ['LFT_T(Theta)L(Theta)_T(Theta+10)L(Theta+10)'];

Conds = Cond_TL;

[NC CL] = size(Conds);
A = cell(NC,4);
wav_ind = [];

for f = 1:NC
    wav_ind = [wav_ind ones(1,10)*f];
end

wav_shuf_ind = wav_ind(randperm(length(wav_ind)));
soundFolder = ('STIMS');
```

```
for n = 1:length(wav_ind)

    baseWavFileName = strcat(Conds(wav_shuf_ind(n),:),'.wav');
    fullWavFileName = fullfile(soundFolder, baseWavFileName);

    A(n,1) = {Conds(wav_shuf_ind(n),:)};

    try
        if exist(fullWavFileName, 'file')
            [waveFileData, Fs] = audioread(fullWavFileName);
            i = 2;
            ppp  = audioplayer(waveFileData, Fs);
            play(ppp)
            tic
            CC = getkeywait(2.5)
            t = toc
            statusbar('Processing %d of %d (%.1f%%)...
            Response Time = %f',n,length
            (wav_ind),100/(length(wav_ind))*n,toc);
%             statusbar
('Processing (%.1f%%)...',100/
(length(wav_ind))*n);
           pause(0.5)



        else
            warningMessage = sprintf

            ('Warning: sound file not found:\n%s', fullWavFileName);
            WarnUser(warningMessage);
```

```
        end
        % It worked.  It played because the audio format was OK.
        itWorked = true;
    catch

        tryCount =  1;
        if tryCount >= NC

        end
    end

  switch CC

        case 28
            direction = 'Left';
            A{n,2} = A{n,2} + 1;
        case 29
            direction = 'Right';
            A{n,3} = A{n,3} + 1;
%           case 31
%               direction = 'Same';
%               A{n,4} = A{n,4} + 1;

      otherwise
            direction = 'NR';
            A{n,4} = A{n,4} + 1;
           t = toc;
  end

    % Response Time
```

```
    tt = num2str(t);
    fprintf(fid,[Conds(wav_shuf_ind(n),:)
    '\t' direction '\t' tt '\r\n']);
    pause (1);
    set (handles.edit2,'String',LR);
end
set (handles.edit2,'String','END OF THE TEST');
```

A.1.3 Optimization

```
% Values to be find with optimization:
Lambda = x(1); Sigma_T = x(2); Sigma_L = x(3); BetaL = x(4);
BetaT = x(5);

% Experimental SNR values are available in Table 4.2 at the text;
% Rows are 0, 10, 20, 30 degrees
% Columns are SNR in/out for TO, LO, TL
SNR = [0.154 0.073 0.49 0.497 0.941 0.866
 0.249 0.074 0.474 0.293 0.915 0.634
 0.496 0.023 0.528 0.064 0.907 0.298
 0.618 0.0004 0.494 0.054 0.924 0.055];

 Estimated Lambda, BetaT, BetaL, SigmaT and SigmaL values for 0,
 10, 20 and 30 degrees.
 Estimated_L_bT_bL_sT_sL = [];

% Each loop computes 0, 10, 20 and 30 degree estimates
for i = 1:4
    SNR_TO_IN = SNR(i,1); SNR_TO_OUT = SNR(i,2);
    SNR_LO_IN = SNR(i,3); SNR_LO_OUT = SNR(i,4);
    SNR_TL_IN = SNR(i,5); SNR_TL_OUT = SNR(i,6);
```

```
fun = @(x)(1/6)*((((1-x(1))*(10+x(5)))^2/(((x(1)^2)*x(3)^2)+
((1-x(1))^2*x(2)^2)) - SNR_TO_IN)^2 ...
    + (((1-x(1))*(10-x(5)))^2/(((x(1)^2)*x(3)^2)+
((1-x(1))^2*x(2)^2)) - SNR_TO_OUT)^2 ...
    + ((x(1)*(10+x(4)))^2/(((x(1)^2)*x(3)^2)+
((1-x(1))^2*x(2)^2)) - SNR_LO_IN)^2 ...
    + ((x(1)*(10-x(4)))^2/(((x(1)^2)*x(3)^2)+
((1-x(1))^2*x(2)^2)) - SNR_LO_OUT)^2 ...
    + ((10+(x(1)*x(4))+(1-x(1))*x(5))^2/(((x(1)^2)*x(3)^2)+
((1-x(1))^2*x(2)^2)) - SNR_TL_IN)^2 ...
    + ((10-(x(1)*x(4))-(1-x(1))*x(5))^2/(((x(1)^2)*x(3)^2)+
((1-x(1))^2*x(2)^2)) - SNR_TL_OUT)^2);

    x0 = [0,0,0,0,0]; % starting point
    A = []; b = []; Aeq = []; beq = [];
    lb = [0 0 0 0 0]; % lower bound
    ub = [1 100 100 10 10]; % upper bound
    [x,fval] = fmincon(fun,x0,A,b,Aeq,beq,lb,ub);

    Estimated_L_bT_bL_sT_sL=
[Estimated_L_bT_bL_sT_sL; x(1) x(5) x(4) x(2) x(3)];
end
Estimated_L_bT_bL_sT_sL
```