REPUBLIC OF TURKEY

ALTINBAŞ UNIVERSITY

Institute of Graduate Studies

Civil Engineering

SIMULATION AND MODELING OF FLEXURAL BEHAVIOR IN CONCRETE BEAM UNDER DIFFERENT LOADING SHAPES

Ahmed AL-ALI

Master's Thesis

Supervisor

Prof. Dr. Tuncer CELIK

Istanbul, 2022

SIMULATION AND MODELING OF FLEXURAL BEHAVIOR IN CONCRETE BEAM UNDER DIFFERENT LOADING SHAPES

Ahmed AL-ALI

Civil Engineering

Master's Thesis

ALTINBAŞ UNIVERSITY

2022

The thesis/dissertation titled “SIMULATION AND MODELING OF FLEXURAL BEHAVIOR IN CONCRETE BEAM UNDER DIFFERENT LOADING SHAPES” prepared by AHMED AL-ALI and submitted on 23/02/2022 has been **accepted unanimously** for the degree of Master of Science in Civil Engineering.

____

Prof. Dr. Tuncer CELIK

Supervisor

Thesis Defence Committee Members:

Prof. Dr. Tuncer CELIK	Faculty of Engineering and Architecture, Altinbas University	__________________
Prof. Dr. Zeki HASGUR	Faculty of Engineering and Architecture, Altinbas University	__________________
Asst. Prof. Dr. Erdem DAMCI	Faculty of Engineering and Architecture, Istanbul University	__________________

I hereby declare that this thesis meets all format and submission requirements of a Master’s thesis in Civil Engineering.

Submission date of the thesis to the Graduate Education Institute: ___/___/___

I hereby declare that all information/data presented in this graduation project has been obtained in full accordance with academic rules and ethical conduct. I also declare all unoriginal materials and conclusions have been cited in the text and all references mentioned in the Reference List have been cited in the text, and vice versa as required by the abovementioned rules and conduct

Ahmed AL-ALI

Signature

DEDICATION

I dedicate this work to my dear father, my dear wife, and everyone who supported me to reach success.

ABSTRACT

SIMULATION AND MODELING OF FLEXURAL BEHAVIOR IN CONCRETE BEAM UNDER DIFFERENT LOADING SHAPES

Al-Ali, Ahmed

M.Sc. Civil Department, Altınbaş University, University,

Supervisor: Prof. Dr. Tuncer ÇELIK

Date: February /2022

Pages: 79

Using of beams in construction is common and very required as these elements can play important role for carrying the external loads as a part of the whole building. The problem of less ability for the concrete materials for handling the tensile stresses considered as obstacle for being freer in the design procedures. For improving the strength, the tensile zone in the beam is assisted by reinforcement, the cracking in concrete as well can still occurs due to reaching the limit of resistance so that understanding the maximum loading that can lead to this case is important key in the design steps. Four external forces were adopted in this study. The cases were single force, two concentrated force, rectangular shape load, and triangular shape load. These four states of loading were applied on simply supported beam so that flexural analyzing was carried out in the way of understand and calculation the maximum load in each case that can be lead to the initial cracking in the tensile fiber of concrete cross section. After the analysis was made for each case, different relationships were identified by graphs which involved the increasing or decreasing between the final cracking load and the changing of cross section width, cross section height, compressive strength, and also the point of action for the concentrated loading. The behavior of the relationships was derived toward regression correlation using SPSS software in which for predicting governing equation for the cracking moment that can be adopted to identify the maximum allowable load that can be act without cracking of concrete in tensile part. Three zones were assigned as the curve flow out to reduce the difference between the actual cracking moment and the predicting cracking moment for least values. Simulation for the all four cases was implemented using ANSYS workbench program and the results were explained in graphs that contained the behavior of stresses and strain and deformation along the span of the beam under each case study.

Keywords: Flexural Analysis, Concentrated Load, Uniform Load, Tensile Stress, Cracked Section.

TABLE OF CONTENTS

LIST OF TABLES

Pages

LIST OF FIGURES

LIST OF ABBREVIATIONS

FEM	:	Finite Element Method
Wu	:	Ultimate Weight
As	:	Area of Steel bars facing the tension stress
Fs	:	Compressive Strength of steel
fc	:	Compressive Strength of concrete
Mc	:	Moment in Cracked section
dc	:	Concrete Cover
db	:	Diameter of Bars
Ec	:	Modulus of Elasticity of Concrete
At	:	Equivalent Area
Es	:	Modulus of elasticity of Steel

LIST OF SYMBOLS

D	:	Dead Load
L	:	Live Load
T	:	Tension Force
I	:	Moment of Inertia
n	:	Modular Ratio
b	:	Width of Beam
L	:	Length of Beam
H	:	Depth of Beam

1. INTRODUCTION

Concrete elements are still a main material for construction because of its advantages like workability, fire resistance, low cost and its low maintenance requirements. It is formed from a hardened mixture of cement, fine aggregate, coarse aggregate, water and some admixture [1]. The concrete should be used as efficient as possible. The efforts are continuously looking for placing the concrete element in buildings in better state to resist the applying loadings, so that analysis the behavior of concrete parts for deformation and flexural is important step in design [2][3].Referring to the natural of the concrete, the compression capacity is so large than tension capacity. For the purpose of designing the reinforced concrete beams, tensile stress near to be neglected [4]. In reinforced concrete beam, the flexural capacity is supported by the part of compression stresses only of the concrete, so that the steel reinforcement plays important role in carrying out tensile stresses. Moments in relative high magnitudes can be reflected as the main issue for along the spans of simply supported beams. This large moment led to large deflection and wide cracking because there is no internal moment redistribution [5]. The matter of creation of cracking in reinforced concrete members is happened in the case of overloading on the element so that the element reaches maximum strength capacity. It contains several mechanisms and constraints associated with the physical shape of element, the interaction between reinforcement and concrete, loading state, and conditions of support. These factors can be considered in flexural analysis which is main scope in designing of concrete parts. These dominant factors are all covered the response of reinforced concrete beam and the compressive strength under the action of loading [6].

1.1 OVER VIEW OF FLEXURAL BEHAVIOR IN CONCRETE BEAM.

The mechanism of flexural state in concrete element works accumulatively starting from the initial resisting of the applying loading. The compression side of concrete works in parallel with the tension side. Equivalent rectangle that describes the forming of stresses can be adopted as better way illustrating purpose. A block of forces acting toward and outward the cross section of element can be used to evaluate the stress behavior. Compressive stress in the block forming the area of bending on the top part of simply supported beam while the other side is tension which considered very low and neglected in designing so that the tensile stresses is holding by reinforcing area [7]. Flexural in concrete beam that has simple support under concentrated and distributed loads will suffer shear stress and bending stress. Major bending stress happens at the top of cross section due the acting of bending moment which can be analysis to compressing forces and tensile force. The tensile forces create the tension stresses in the bottom of the section. The two parts of cross section must be designing well, in other word, increasing the loading or applying different loading shape can attack the ability of resistance especially in the tension part making cracks generation and leading to failure of element. For improving the resistance to the tension stress, the steel reinforcement can be used for carrying out this type of stress [8].

1.2 INTRODUCTION OF CURRENT STUDY

It so important in designing of construction parts to determine generation of stresses leading to deformations which can be occur in the internal area of a structure under applied load. For the case of reinforced beams this matter can be evaluate by assuming of elastic behavior occurred. The magnitude of ultimate strength for structural element is important scope to be evaluated for safe design. The lifetime of structure is corresponded by the state of resistance capacity which must be more than the influence of stresses caused by the actions of loads. So that factor of safety is adopted to prevent failure. Simulation and analysis of the member can give best view on the critical nodes and the chance of failure to be eliminated.

1.3 RESEARCH SCOPE

Concrete cracks occur at the stage of loading when maximum tension stress passes the modulus of rupture in concrete part. The response of beam for loading is varies because of different parameters including the loading status and the dimensions of the beam. The study aimed to simulate failure response of concrete beams that subjected to different loading shape including concentrated and distributed loads. The flexural behavior in beam cross section was simulated using ANSYS platform. The results were adopted under analysis for modeling the influence of the acting load along the beam for each loading case.

1.4 OBJECTIVE OF STUDY

i. Defining of the problems that involved in the study including the physical parameters of the beams and the loading state.
ii. Analysis the influence of each loading case on the resistance capacity of beam using working stress method.
iii. Building of simulation of each case using ANSYS workbench program including the input of necessary data.
iv. Explain the results of simulation of each case in graphs and tables.
v. Using the results for deriving applicable mathematical model the explain the variety of flexural response in each case.

2. LITERATURE REVIEW

2.1 OVEWRVIEW

Beams are one of main types of structural elements that are used for the purpose of extending distances as their use is very common. The loads are act on the longitudinal axis of the beams. The beams are subjected to the effect of bending and their external shape is curved, and the reason for this is due to the effect of the applied stresses. Three types of stresses affect the beams, where the effect of stress is on the upper part of the beam, the tensile effect is on the lower part of the beam, and the cross section is subjected to shear stress[1]. There is a decrease in the lengths of some of the upper parts of the beam and an increase in the length of the lower parts, and the reason for this is due to the effect of curvature. This causes the upper parts of the beam to be under the influence of compression and the lower parts to be under the influence of tension. In addition to the presence of internal bending within the beams, the shear effect is an important factor to be considered when determining the beam strength. Shear strength greatly influences the design of the beam strength. Number of materials that are weak in shear, such as wood, so the loads that are placed on the strength of the beam depend on the shear forces [2]. The principle of superposition, which is the principle that includes calculating each of the applied loads separately, then summing up the effects of all the loads to get the final value of the load, as it is assumed that all loads affect at the same time. The designers approved to calculate the effect of the internal forces required in the design (axial force, shear force, bending moment) based on this principle [3]. For any level of rigid structures or structural elements, they are often subjected to different loads. The equilibrium equations provide the mathematical basis for determining the amount of values that may reach unknown moments, such as the interactions that occur between loads and structures. Since this type of structure is statically determined because the amounts of reactions that are unknown are determined using the equations for static equilibrium [4]. There must be proportionality between the resistance to the loads and the structural elements, that is, the resistance to the design loads is greater for the purpose of avoiding failures of the structural elements and for the purpose of providing safety for the structure. The strength design where the member is designed to be able to support the loads that are converted into factors, as it is obtained by multiplying the load factors by the service loads, where different loads are used by different factors as well.

As the dead loads are calculated very accurately, and the load factors for dead loads are less than the factors for live loads whose amount is uncertain [5].

2.2 STRESS IN CONCRETE BEAMS

The stresses in beams relating to the internal forces that generated as reflecting to the action of external forces. Main stresses are that caused by bending moment. The internal moment's distribution along the span related to the direction and magnitude as well as the type of force [6]. For rectangular cross section beam with simple supports, the lower part of the section handles tensile stress while the upper part been under the action of compressive stress. The actual shape of compressive strength that distributed along the upper part of beam section is parabola; however, converting the shape to rectangular appearance can still give acceptable analysis data [7]. The behavior of compressive and tensile stresses on the rectangular cross section of a beam under loading was explained in figure (2.1) and figure (2.2). The distance from the top point of cross section to the center of reinforcement place is represented as d and width equal to b [8]:

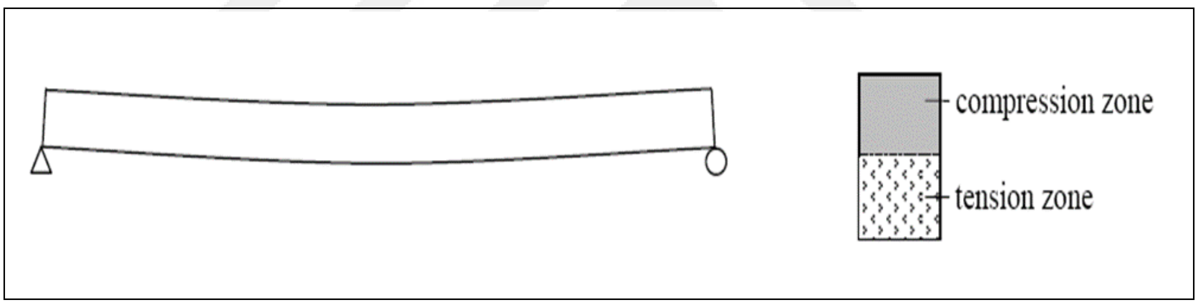

Figure 2.1: The Behavior of Beam Span Under Loading [9]

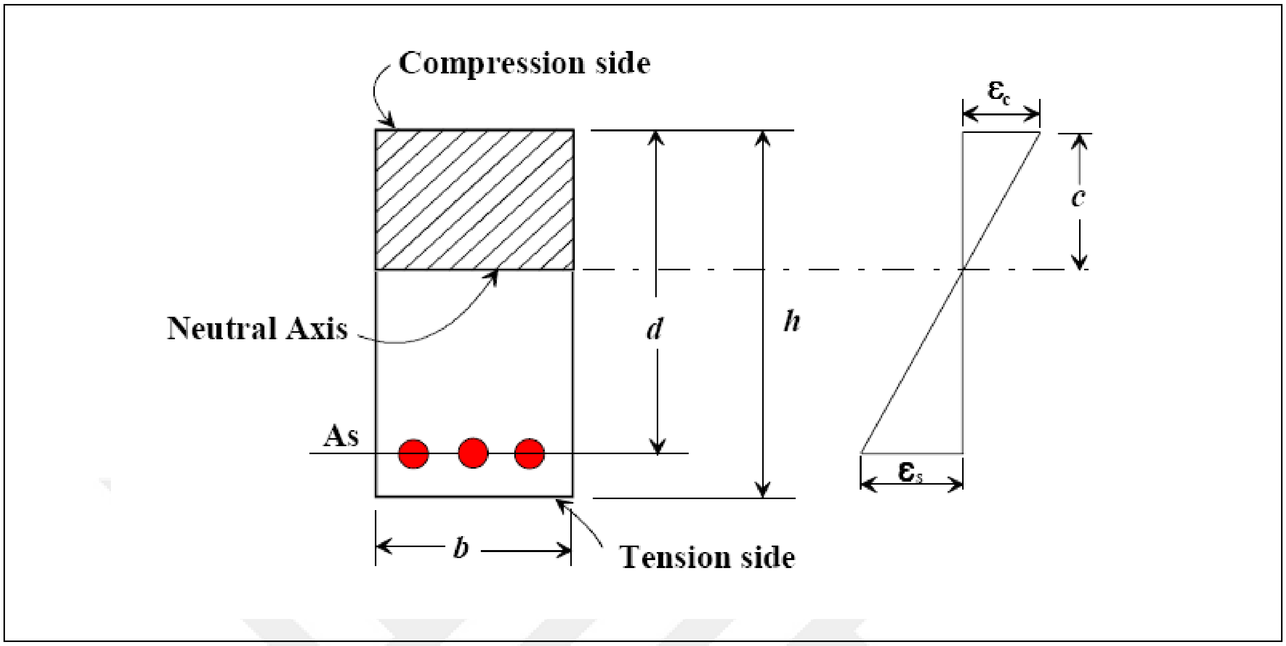

Figure 2.2: The Formation State of Strain and Stresses on Beam Cross Section [10].

The compression force C was illustrated in the figure as it acts as a result to the compressive stress on the area a*b which can be handled by the structure on concrete body. The tensile force T was shown in the section where the reinforcement been added to improve the tensile strength.

2.3 STRENGTH IN CONCRETE BEAMS

The analysis and design of the structural elements represents an appropriate process for selecting materials as well as for the purpose of determining the dimensions of these elements so that the strength required for the structure is less than the design strength that is calculated in advance when designing [11]. The product of multiplying all the applied loads on the structure leads to determining the required strength. These loads include live load, dead load, and other loads such as the effect of earthquake force, wind, ground pressure, rain loads and fluid pressure. The effect of these loads is determined by the load factors for each case. These loads represent external forces that may cause shear, bending, axial forces or torsion. The effect of these loads varies depending on how they are applied to the structures[12]. The proportionality between the structural elements of reinforced concrete must be achieved in the presence of three important elements [13] [14]:

i. Safety of the structures, where the internal design forces that are sufficient to maintain the structure must be provided.
ii. Reducing the deflection that occurs in the structural members, which is due to the service loads. Whereas, the maximum deflection value must be limited, and it is defined as an extension factor for the purpose of maintaining the appearance of the structure. [15].
iii. The ability to control crushing conditions under the influence of service loads. As moisture can enter the visible cracks in the concrete structure and this leads to corrosion of the rebar and thus weakens the resistance of the reinforced concrete.

Concrete is a brittle material and is subjected to a large compressive strength compared to the tensile strength, which is very little. There are many methods that are used to calculate the tensile strength, where the rupture coefficient is one of the important factors in the design [16]. One of the important indicators of the quality of reinforced concrete is the compressive strength and it is considered very important than the rest of the other properties. Compressive strength is defined as the ability of concrete to bear the loads imposed on it before reaching the stage of failure. The test that is used to determine the compressive strength of concrete is considered one of the most important tests because it gives an idea of the concrete properties [17]. The tensile strength zone in concrete is improved by using of reinforcement which involves the capability to resist the tension force and prevent cracks and failure until yielding [18].

2.4 STRESS- STRAIN IN CONCRETE BEAM

The stress curve of concrete represents the behavior of the load in the form of graphics, as it is obtained by drawing the stress affecting the concrete for different periods of the stress. Concrete is mostly used for compression, and this is due to the importance of determining its compression curve [19]. The machine is used to test the pressure of concrete, where a sample is taken from it at the age of 28 days, and the stress and strain are determined for it [20]. The curve used to represent the stresses of concrete allows engineers and designers to pre-determine the behavior of the concrete that is used in buildings, and by defining the relationship between the type of stress applied to the concrete and the stress curve, the performance of concrete can be controlled more accurately [21]. Figure (2.3) shows the strain and stress of light weight concrete and normal weight concrete. The curves on each shape represent the strength of the concrete, where the higher curves represent the higher concrete strength [22]. Depending on the loading speed, the shape of the concrete stress curve changes. There is a fact that the density of the concrete and the speed of the test affect the shape of the stress and strain curve. And almost all curves show the same character, that is, they are under loading subject to the same stages [23].

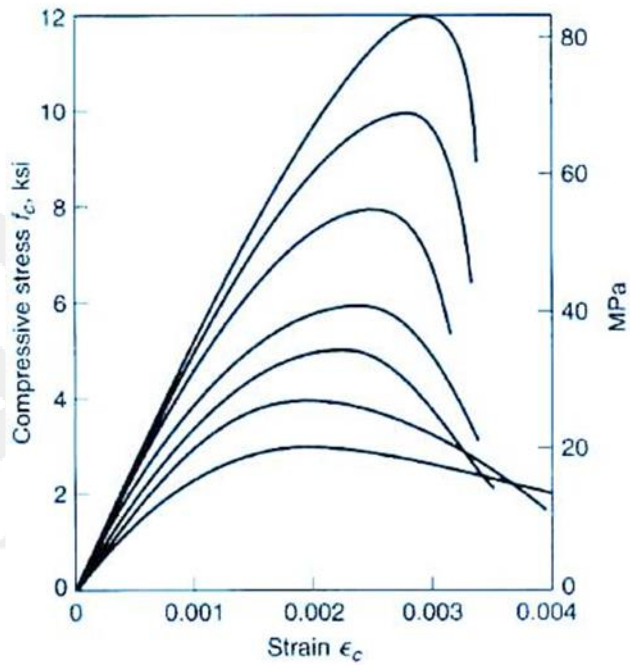

Figure 2.3: Compressive Stress with Strain for Normal Concrete [24]

Initially, all the stress and strain curves are straight, at this stage the pressure and stress are proportional, when the imposed loads are removed, the material must return to its original form [25]. The elastic range of the stress curve of concrete lasts up to the maximum strength of concrete then the relation curve falls down toward failure. The modulus of elasticity of concrete represents the slope of the elastic part of the stress-strain curve. The higher the strength of the concrete, the higher its elastic modulus. The ACI code provides several special equations for calculating elasticity coefficients [26]. Concrete begins to have a non-linear plastic behavior after it exceeds the elastic range and that is when the amount of applied load increases. After the elastic range phase ends, the curve goes horizontally until it reaches the maximum compressive strength[27]. When the weight of the concrete is normal, the maximum stress in the compressive state is between

0.002 to 0.003. The maximum compressive strength of lightweight concrete at stress ranges from 0.003 to 0.035[28].For the two curves, the stress results represent a greater strength. For concrete that is of normal weight, the maximum stress that the strain could reach was 0.003 and this value is used in the design of concrete elements. As for the European law, it is assumed that the stress of concrete reaches 0.0035, where this value is used to design the structural members [29].All curves have a descending direction when they reach the highest value of the compressibility. The properties of the stress-strain curve depend on the test method in the descending part. A long and constant descending fraction is obtained when a special test is used to ensure a constant rate of stress. The discharge is fast beyond the peak point if no special test procedure is followed as the descending portion of the curve will not be the same [30] .

2.5 FLEXURAL IN BEAMS

Reinforced concrete beams are made of two different materials, so they are not homogeneous. Where there are different methods used to analyze reinforced concrete beams and they differ from the methods that are used to examine or design beams that are entirely composed of one structural material such as wood, steel or others [31]. There are two types of problems facing the study of reinforced concrete:

a. Analyzing, given the strength of concrete, cross-section, location and size of reinforcement, and yield strength where the strength is calculated, the analysis must be in a unique way.
b. Design, based on the calculated design, a cross-section is determined that includes the calculation of concrete strength, dimensions and reinforcement [32].
c. There are constant equilibrium conditions for the method of designing force and stress where there are multiple solutions for the purpose of design.

The following are the assumptions can be adopted in design of reinforced concrete element:

a. The strains in concrete and reinforcement are directly proportional to the distance of the neutral axis [10]. That is, there is a variance of strains in the linear section, and through the linear relationships it is possible to calculate the known values of the strain from the unknown values.

b.The bending capability of the concrete sections reaches 0.003 as the maximum strain fibers develop.

c. The stress at beginning of loading increase with yield strength of reinforcement in linear state at range toward specific point. As the compressive strength remains constant after that point with continued strains.

d. The tensile strength of concrete is neglected.

e. The distribution of compressive stress can be represented for concrete by the relationship of strain and stress that is corresponding to concrete. Where the stress is distributed in a rectangular shape for the purpose of simplifying the distribution of stress [24].

f. The design of the structure is made by three important requirements, including stability (to prevent slipping or overturning that occur due to the influence of imposed loads), strength (for the purpose of providing appropriate resistance caused by the impact of the loads on the structures), and finally serviceability (to achieve adequate performance under load conditions). The designer must also provide the aesthetics and economy of the building [33].

2.6 STAGES OF CONCRETE BEAM STRENGTH UNDER LOADING

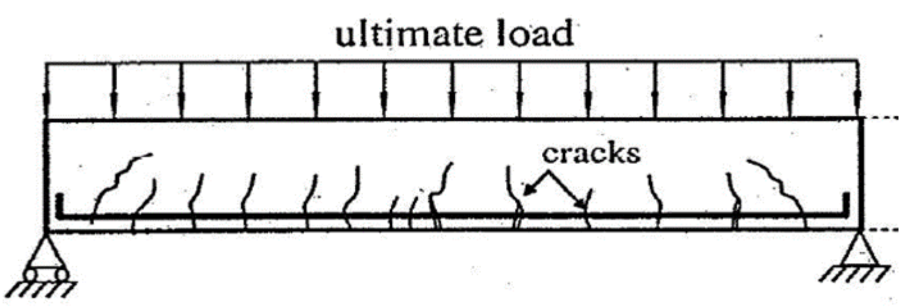

Figure 2.4: Appearance of Cracks in Tensile Zone [34].

Stage I: The distribution of stress is linear over the depth of the section, when the load applied to the member is low. The tensile stress in concrete is very low, which keeps the cross-section of the member from cracks. The stress distribution is linear when the concrete stresses are less than about 0.5 fc' where the concrete stresses are low[35].

Stage II: When the load is placed on the concrete member, the amount of tensile stress increases at the bottom and reaches the extent by which it exceeds the tensile force that causes the concrete to crack, in other word, it is subjected to cracking. Reinforcing steel works to resist the tensile strength. In this case, the stress distribution at the compressive zone is non-linear. The tensile stress in concrete contributes to a very small part of the bending strength [36].

Stage III: Upon reaching the final force acting to penetrate the bending cracks through the neutral axis where it is directed in the direction of compression. The distribution of stresses to which concrete is exposed in the compressive zone is highly non-linear and with the presence of reinforcing steel in the area. Cracks appear in many sections areas in the concrete below the neutral axis as shown in figure (2.4). Mainly there are two types of failure at the final stage; the first type is if the beam is reinforced with a small amount of steel, this type of failure causes cracks and large deflections. The second type of failure occurs when the amount of reinforcing steel is large in the concrete member, the failure in this case occurs suddenly as the concrete breaks and collapses in the area that is subjected to compression with the steel remaining and under relatively few cracks and deviations, this type of failure is considered dangerous because it cannot be done expect it and designer always be careful of it in advance [7] [37].

2.7 TYPES OF FAILURE IN BEAMS

2.7.1 Tension Failure

Failure in concrete in this case is due to the overstress happens in the tensile zone so that the crack appears in lower part of the beam. This type of failure can be take place when the concrete element not reinforced with sufficient steel amount so that yielding in steel bars occurs as shown in figure (2.5). This type of fail adopted in design because it gives signs before collapsing. The shape of member in this case be as curve toward down [38].

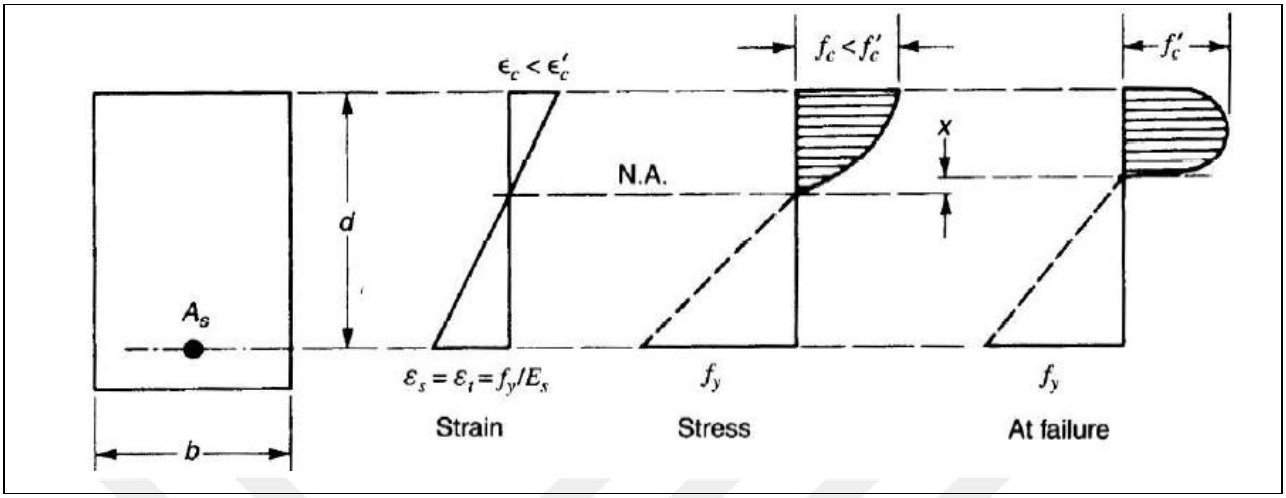

Figure 2.5: The State of Strains and Stresses Under Tension Failure [10]

2.7.2 Balanced Failure

As shown in figure (2.6) when the tension zone represented by the steel bars and the compressive zone represented by concrete with or without reinforcement reach the yielding state at same time. This case can assumed dangerous and could lead to rapid collapsing resulting from excessive amount of stresses when the beam subjected to loads in state more than what design for and more than the factor safety adopted in [3].

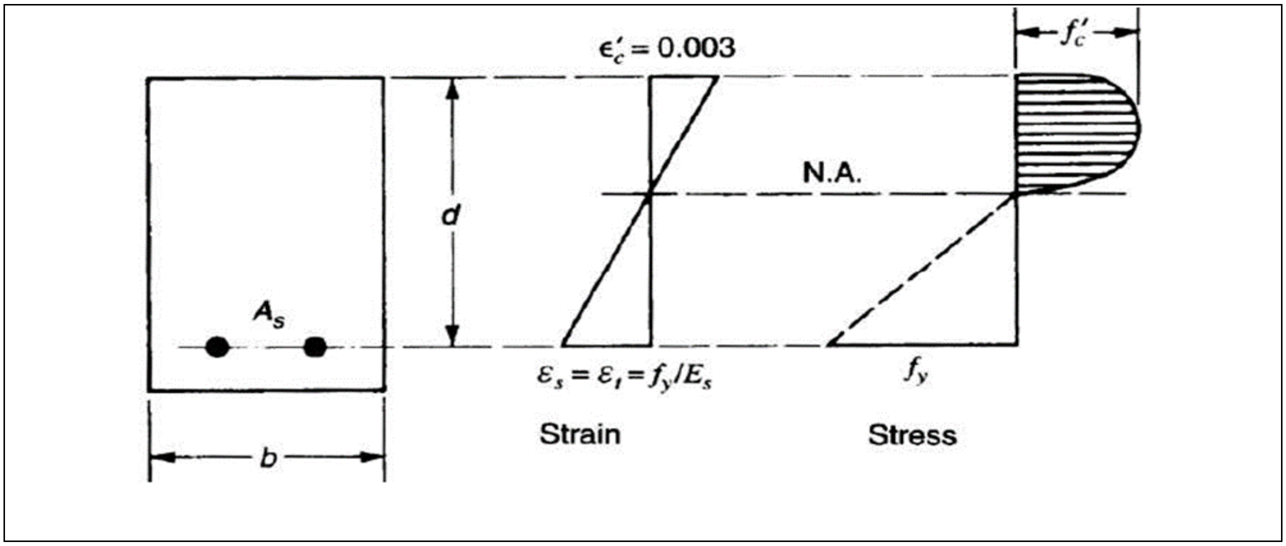

Figure 2.6: The State of Strains and Stresses Under Balanced Failure [3].

2.7.3 Compression Failure

Failure happens in the compressive zone before the tension zone reaches yielding state. The case also dangerous and called compressive controlled section. The failure occurs when the reinforcement amount so high that can carry the tension stresses under high loading as shown in figure (2.7). The failure of compression flexion begins with the removal of the reinforcing steel in the tension area of the beam, and on the side of compression, the concrete is subjected to crushing. This failure occurs when the rebar reinforcement ratio is in large quantities and exceeds the parallel reinforcement ratio. This sudden failure is considered a fragile failure and is undesirable during the design phase of concrete, to reduce this type of failure that occurs by reducing excessive reinforcement of concrete when designing or by increasing the compressive strength by increasing the geometry of the beams or strengthening the compressive side with steel [28].

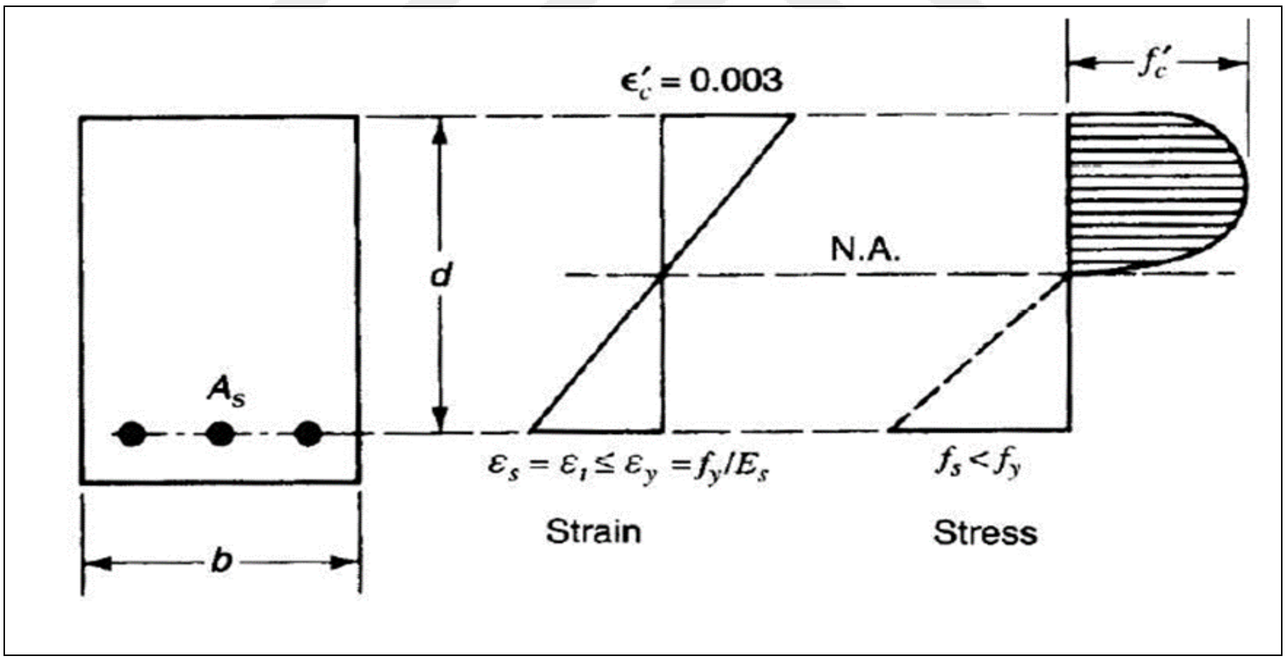

Figure 2.7: The State of Strains and Stresses Under Compressive Failure [3].

2.8 SAFETY FACTORS

Safety factors must be provided against the failure of the concrete structure, as the structural elements can withstand all the loads imposed on them, and their resistance is high to prevent failure. The load factors of dead loads are less than the load factors of live loads because the dead loads are estimated in relatively less impact as movable load. So that to allow a reserve that is additional to the capacity of the skeletal member, the ACI code specifies another factor [39] [40]. The structural analysis is acceptable, which depends on balance and statistics, and then the strength is calculated. In the method of designing the force, factors must be used to reduce the force, and the calculation of the nominal force should be with a large degree of accuracy and the presence of inverse changes that are in the dimensions and materials [41].

2.9 FINITE ELEMENT METHOD

Finite Element Method (FEM) is a widely used method for solving differential equations that are emerging in mathematical modeling and engineering. Typical areas of interest include conventional structural analysis, fluid flow, heat transfer, electromagnetic potential and mass transfer. FEM is a general numerical method for the purpose of solving all partial differential equations in different number of variables. [42]. For the purpose of solving a problem, FEM divides a large system into a number of smaller parts which are simpler where they are called finite elements. This is achieved by defining an area that exists in the dimensions of the space, as it is implemented by building a network of the object, which represents the numerical field of the solution, the field contains a limited number of points. When formulating the final value problem, this method finally uses a definite system of algebraic equations where the method approaches the unknown function to the domain[43]. These finite elements that we get from assembling simple equations into a system are greater than the equations that represent the whole problem. Where FEM is close to the solution, that is, when the error function decreases through differential calculus. This method is used to solve all mathematical, engineering and physical problems. This method is useful for some geometric shapes that are complex in addition to the properties of some materials for which analytical solutions cannot be found [44].

2.10 ANSYS MECHANICS

ANSYS Mechanical is a Workbench application that can perform a wide variety of system engineering simulations, including thermal, stress, electrical, vibration and magnetic simulations. The two typical simulation habits consist of preparing the model as well as preparing the set of loads applied to it, in addition to solving and responsiveness of that model to the applied loads, and then examining all the details of the response through the use of a variety of all tools [45].

The mechanical application contains a number of objects that are arranged in the form of a tree structure, which represents the steps of the different simulated steps. Through the process of extending these objects, all the details that are associated with the object can be clarified, and some specification tables can also be used for the corresponding tools for the purpose of executing this specific part of the simulation. These system contains the ability of using many purposes such as loading operations and determining some environmental conditions of contact surfaces, these functions are important in determining the results of previous reviews [46].

2.10.1 General Steps of ANSYS Program

Geometry: The modeling used in ANSYS Workbench provides users with a high percentage of flexibility in customization and depends on the type of application or analysis. Where many programs are used for this purpose, such as the use of the ANSYS Design Modeler program, which is characterized by many features for the purpose of organizing the CAD engineering to be present for analysis, which includes many options for repair, simplification and distortion, or for the purpose of creating parametric geometry from sketches [47].

Meshing It is a process for dividing the objects into number small parts like cubes forming a network. Where the network affects the solution, its size and convergence are so influence the rendering time, so that, creating a network model takes more time than it takes to solve single element and get the results, the solution is better when the networking tools are more and better [48].

2.10.2 Setup The Problem Conditions [49]:

A. Model Generation process:
   a. Job name specifying
   b. Entering of Preprocessor
   c. Define the types of element and options.
   d. Define the properties of the material
   e. Creating of the model
   f. Building the solid model
   g. Defining meshing controls.
   h. Creating the mesh.
   i. Finishing the Preprocessor.

B. Boundary and initial Conditions for running Solution
   a. Starting Solution Processor.
   b. Defining of analysis type
   c. Specifying of boundary and initial conditions:
   d. Specifying of any Degree of freedom and constraints.

C. Solving: Run the solver to get the result.

2.11 PRESENT WORK

The current study included the analysis strategy that carried out toward the flexural behavior of simply supported beam when subjected to different shapes of loading. Different parameters involving of the dimensions of the beam, strength of concrete, strength of steel, area of steel, effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel, modulus of elasticity of concrete, modulus of elasticity of steel, distance from the top to the neutral axis of a concrete beam, and moment of inertia where adopted in the analysis procedure for identifying the capacity of the beam under each case study. The input data where used in simulation of each case study under ANSYS workbench program for explaining in graphics the state of failure along the beam span. The data then tabled and used as input data in SPSS program for statistical analysis and deriving of applicable model that correlated the parameters for ease the processes of design for those types of members.

2.12 LITERATURE SURVEY

M. Rabi, et. al., (2019) had studied the behavior of beam that build by concrete and stainless steel under flexural case. The ductility and the strength of concrete beam were detected then recorded to analysis the improving of adding stainless steel on the strength capacity of member.

The displacement moment curve was sketched in the study and illustrated the increment in resistance zone before collapsing. The application of abaqus software was adopted as finite element analysis for explaining in 3d drawing the effect of stainless steel on the strength toward ultimate loading [50].

Zarko p., et. al., (2020) had analyzed the performance of concrete beam which its type was self-compacting type. The study covered the case of acting of two loadings on two beams in continuous member. Each force was acted in the middle of span. Different reinforcement area were placed in the casting for each study state. The performance of the beams was explained using strain stress curves. The study results were supported by madding finite element analysis using Abaqus software which demonstrated graphically the stress distribution along the beams [51].

Omer E., et., al., (2013) had studied the reliability of ultimate design method according to three construction codes: British Code, ACI code, Egyptian code, and Iraqi code. The study pass away through comparison between the case of ultimate load in beam with simply supporting and the theoretical conditions obtained from the three codes. The study achieved to the fact that ACI code provide maximum load followed by BS code. The relationship between the results and the adopted factor of safety was analyzed and explained in figures. The results illustrates that maximum factor of safety provide by Iraqi code [33].

Plasencia G., et. al., (2017), the study analyzed the flexural behavior of concrete beams with high depth under the two concentrated loads. The study obtained several results from previous researches and compared these results with a numerical analysis to the current case. The study found that the increment in vertical reinforcement by using small size diameters bars with close spacing can enhance the member strength, also adding of vertical web reinforcement can support noticeably the resistance of member toward shear stresses [52].

Salim T. and Majid A., (2009), had studied the influence of using carbon fiber on the flexural behavior on concrete beam using the test of two concentrated loads. The model of artificial neural network. The study based on pas experiments results to investigate the performance of concrete beam with the mentioned case. The study assign the model of ANN that used as effective and accurate procedure for prediction of ultimate load capacity. The effect of concrete strength and the carbon length and thickness were explained [53].

Jiarui L. and Xian C., (2021), the performance of concrete beam that constructed and reinforced by fiber plate with specific dimensions was investigated in the research. The tests of flexural made using three points' loads. The influence of different plate sized was analyzed in the way of the gaining better strength capacity for the member. The study found that using of plates can enhance the ductility of the beam as well as increasing the strength amount [54].

Wisam Kh., et. al., (2021), the study investigated the conditions of curved beams. failure state was investigated as the variation of strength capacity in the resistance to shear forces and moment and torsion. The study based on the past researches for illustrating the performance of beams with curvature in the projects. The influence of ring diameter, width of beam, the compressive strength of concrete where involved in the study. the study found that three dimensional finite element analysis can considered better procedure for analyzing that types of beams [55].

Rudy D., (2013) had studied the state of reducing the concrete body in the tension zone below the natural axis in simply supported beam on the performance of the beam for resisting applied loads. Two concentrated loads were subjected to concrete beam with filled concrete on the compressive zone and cut of volume in the tensile zone. The results of the study explained that cracks were lower in the beam with less fill in tensile zone compared to normal beam but the strength capacity in flexural reduced. The failure happened when the yielding in compression in the top of beam cross section occurred [56].

Payam s., et. al., (2011) had studied the performance of concrete beams that constructed by light weight concrete. Number of beams was tested by applied two concentrated loads. The flexural loads were recorded. The reinforcement area was varied according to each case. The reinforcement was placed in high, medium and low amount for analysis the strength until failure for each case. The fact that reached by the study pathway was that ACI 318 code of building was provide good prediction of the ultimate force magnitude [57].

Nazim N., et., al., (2021) had analyzed the condition of concrete beam when reinforced by additional steel plates and bracing as increment to the ordinary reinforcement. The study tested the flexural of two beams: first one was with ordinary reinforcement and the second one was updated reinforcement. The test was made by applying three concentrated loads. The results illustrated that the updated reinforcement in beam enhance the capability of loads resistance to about 15% more than ordinary reinforcement. The second benefit was the reducing of crack formation [58].

3. MATERIAL AND METHOD

3.1 FLEXURAL ANALYSIS STAGES :

3.1.1 Assigning the Dead Load and Live Load

The first steps in analysis the case study of simply supported state in concrete beams is by considering the external loads, the dead load represents mainly the self-weight of a member that depends on the density of materials and the size of it. The live load represents the mobilize loads that could be moved from place to place, the resultant of loads can be bonded with suitable factor of safety for the following formula [59] [60]:

$$Wu = 1.2\ D + 1.6\ L \tag{3.1}$$

Where:

Wu = ultimate weight kN

D = Dead load kN

L= Live load Kn

3.1.2 Driving of Stresses Equation

When simply supported beam be under the action of loading, internal stresses formulate depends on the position and magnitude of the load. The stresses are relating also to the height and width of the cross section. The stresses can be assigned for each distance (dx) as shown in figure (3.1) along the beam span from 0 to L where L is the total length of the span [1] [61] [62] .

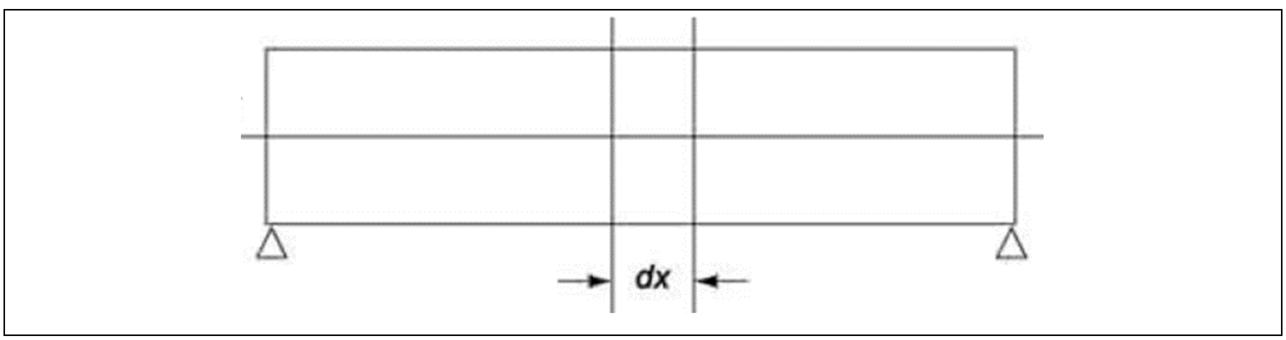

Figure 3.1: Strip of Beam Length [1].

The shape of the member under bending can be assumed as in figure (3.2):

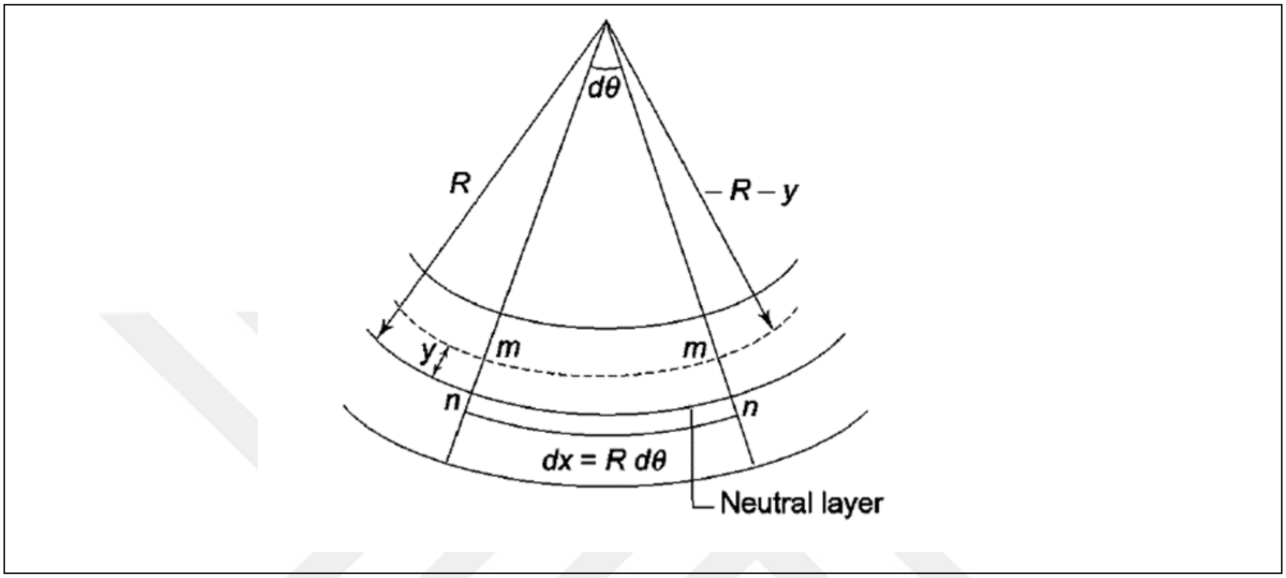

Figure 3.2: Shape of the Beam Element Under Bending Stresses [62].

The strain of the beam in tension and compression zone can be determined as following:

$$\varepsilon = \frac{\Delta L}{L} \tag{3.2}$$

From the figure (3.2), it can be noticed that:

$$dx = R\, d\theta \tag{3.3}$$

The line (m-m) lies at the center of curvature in the stress zone, the change in m-m length can be identified from the following formulas:

$$\Delta L \ for\ (m - m)\ line = R\, d\theta - (R - y)\, d\theta = y\, d\theta \tag{3.4}$$

Strain =

$$\varepsilon = \frac{\Delta L}{L} = \frac{y\, d\theta}{R\, d\theta} = \frac{y}{R} \tag{3.5}$$

The stress at m-m is gained by multiplying the Young modulus by the strain which explained in figure (3.3) :

$$\sigma = \frac{E\, y}{R} \tag{3.6}$$

$\frac{\sigma}{y} = \frac{E}{R}$ (3.7)

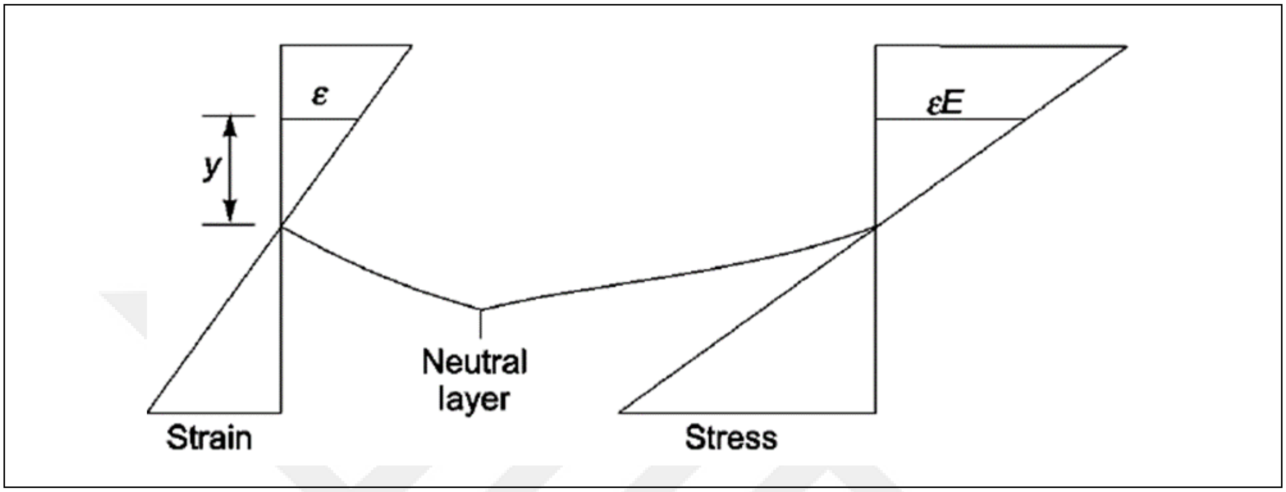

Figure 3.3: Stress –Strain Behavior in Cross Section of Beam [1]

As the stress applied on the area (b width and dy height), the force that given by dF is:

$dF = \frac{E\,y}{R}$ b dy (3.8)

The moment can be obtained by multiplying dF by the distance from the axis that discrete compression and tension which represented by y

$dM = \frac{E\,y^2}{R}$ b dy (3.9)

The term (b dy) y^2 is refers to the magnitude of area that under stress multiplying by the distance from the nature axis NA, so that for the whole cross section:

$\sum$ b dy y^2 = I, where I is the moment of Inertia

The result equation after substituting became as following:

$M = \frac{\sigma\, I}{y}$ (3.10)

3.1.3 Calculations of Internal Strength

The main stresses according to bending moments are compression and tension, for rectangular section of beam, tension force can be explained by the following formula [16]:

$$T = As\ Fy \tag{3.11}$$

Where:

T = tension force

As= area of steel bars facing the tension stress

Fs= tension strength of steel bars

For converting the capacity of steel resistance to what equivalent in concrete, modular ratio (n) can be used

$$n = \frac{Es}{Ec} \tag{3.12}$$

So that T = As n fc (3.13)

$$fc = \frac{M\ C}{I}$$

By adding the equivalent area of steel to the resultant of concrete area minus what area of steel occupied in the cross section, At was calculated as following [63] [64]:

$$At = (Ac - As) + nAs = Ac + (n-1)As \tag{3.14}$$

$$C = \frac{Ac * \frac{h}{2} + (n-1)As * d}{At} \tag{3.15}$$

$$I = \frac{b\ h^3}{12} + Ac\ (C - \frac{h}{2})^2 + (n-1)As\ (d - C)^2 \tag{3.16}$$

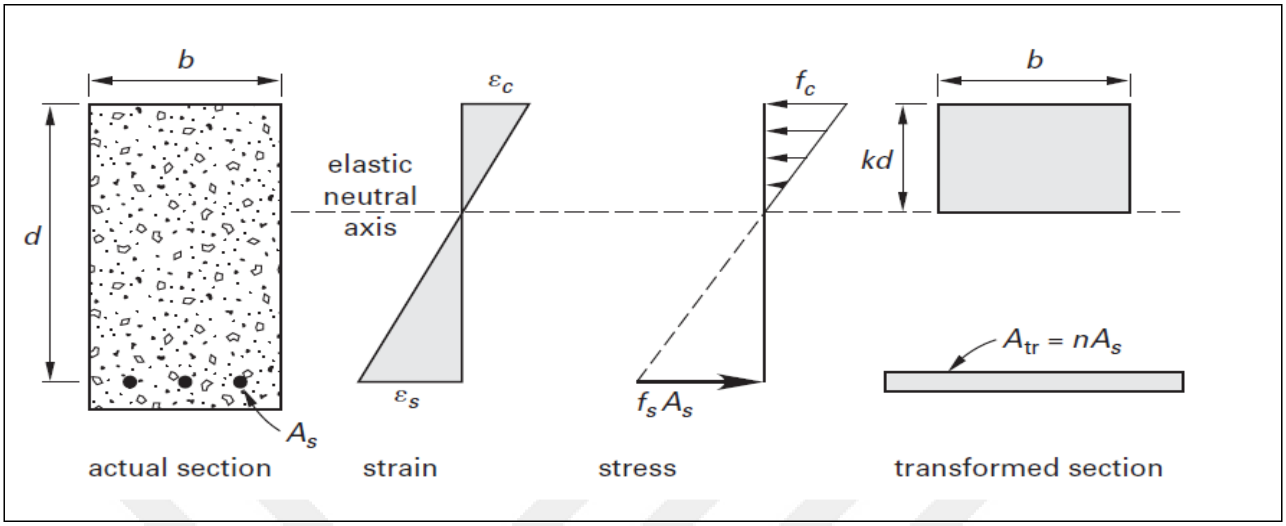

Figure 3.4: Tension and Compression Stresses in Beam Cross Section [16]

As the concrete in this study assumed to reach failure before steel, the magnitude of flexural moment in cracked section (Mc) was identified from equation (3.13):

3.2 LOADING CASES :

The study aimed to analysis the flexural behavior of concrete simply supported beam when the member is under the action of concentrated and distributed forces.The cases of study were explained by the following figure:

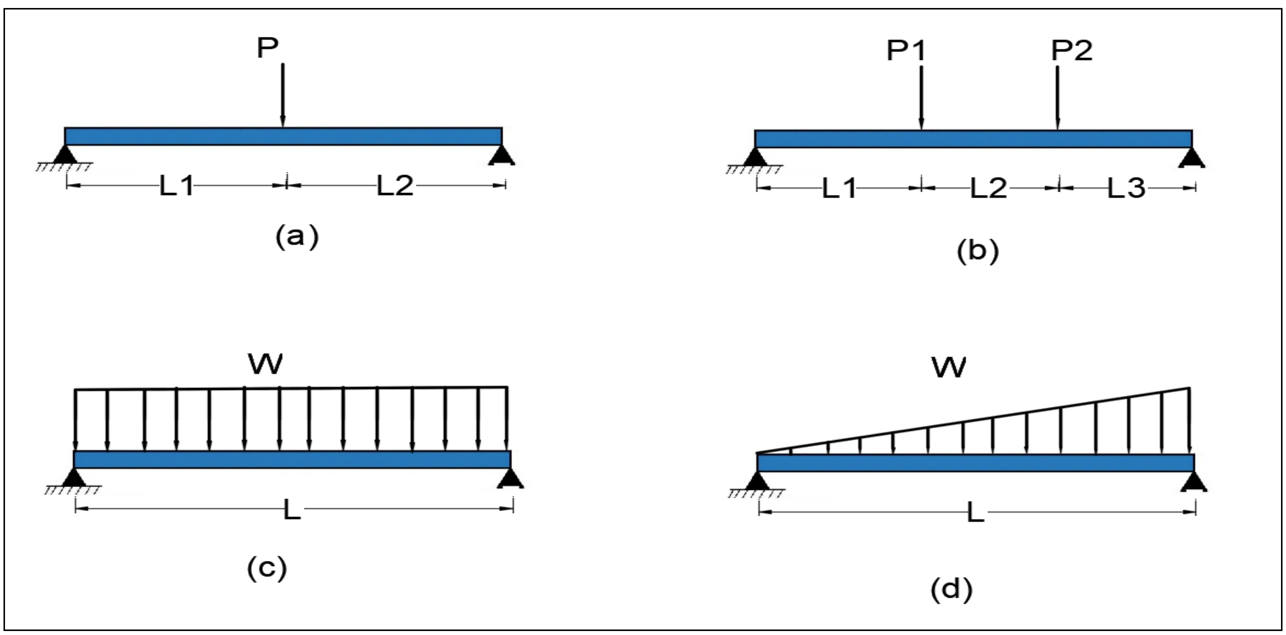

Figure 3.5: Cases of Simply Supported Beam Under Loading: (a) concentrated load, (b) Two concentrated loads; (c) Uniform distributed load, and (d) Triangular load.

3.2.1 Case One: Beam under Single Concentrated Load

The case represented the concrete member under concentrated pressure of weight like placing of column on specific place on the beam span. The distribution of the shear and moment according to the concentrated state of force on the beam was illustrated in the figure (3.6):

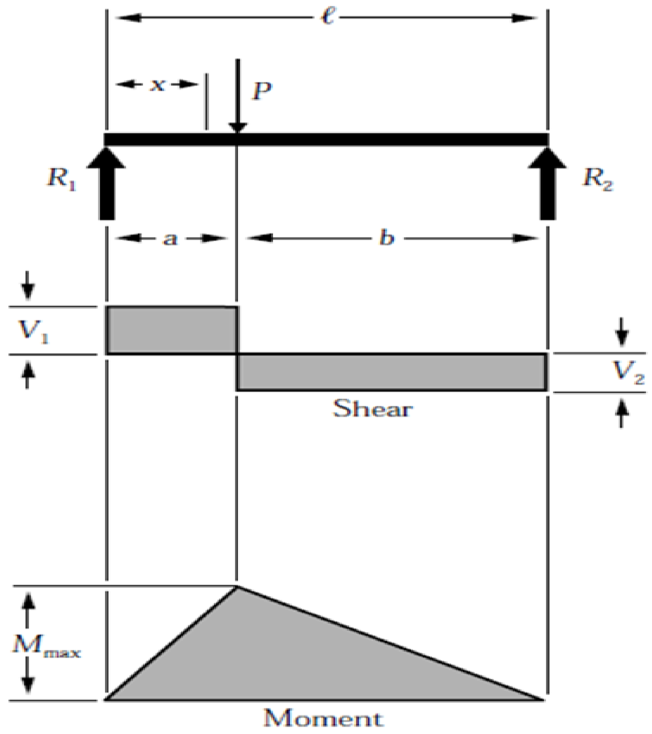

Figure 3.6: Shear Force and Bending Moment Distribution in Case One [65].

Max moment occurs at the point of loading and its magnitude is equal to:

$$\text{Mmax} = \frac{P\ a\ b}{L} = \frac{P\ a\ (L-a)}{L} \qquad (3.25)$$

For cracked section:

$$\frac{M\ C}{I} = \frac{P\ a\ (L-a)}{L} \qquad (3.26)$$

$$P = \frac{L\ M\ C}{I\ a\ (L-a)} \qquad (3.27)$$

3.2.2 Case Two: Beam under Two Concentrated Loads

The case represented the concrete member under couples of concentrated pressures of weight, the distribution of the shear and moment according to the concentrated state of force on the beam was illustrated in the following figure:

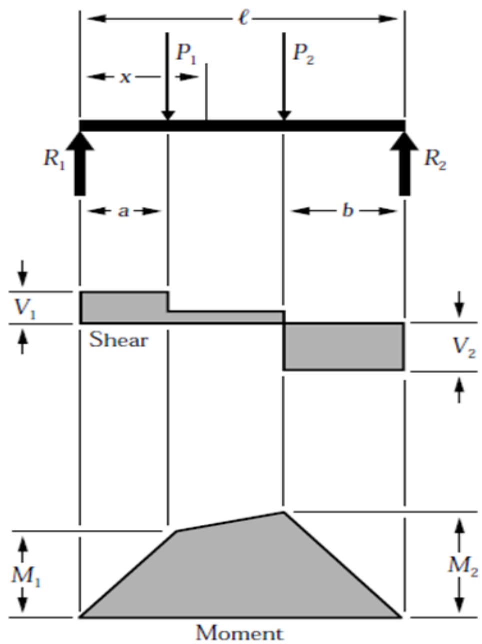

Figure 3.7: Shear Force and Bending Moment Distribution in Case Two [65].

Max moment occurs at the point of second load P2 when the reaction R2 < P2, the magnitude of maximum moment is equal to:

$$\text{Mmax} = \frac{(P1\ a+P2\ (L-b))b}{L} \tag{3.28}$$

For cracked section:

$$\frac{M\ C}{I} = \frac{(P1\ a+P2\ (L-b))b}{L} \tag{3.29}$$

When P1=P2 = P:

$$P = \frac{L\ M\ C}{I(a+(L-b))b} \tag{3.30}$$

3.2.3 Case (3): Beam under Rectangular Distributed Load

The case represented the concrete member under regular load in rectangular shape. The distribution of the shear and moment according to the concentrated state of force on the beam was illustrated in the following figure:

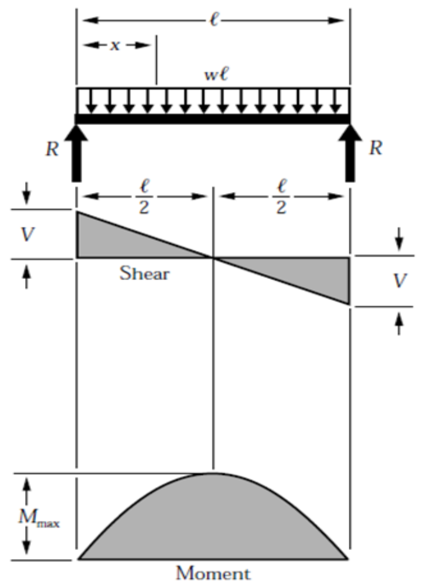

Figure 3.8: Shear Force and Bending Moment Distribution in Case Three [65].

Max moment occurs at the middle of span. Maximum moment is equal to:

Mmax = $\frac{w\,L^2}{8}$ (3.31)

For cracked section:

$\frac{M\,C}{I}$= =$\frac{w\,L^2}{8}$ (3.32)

W = $\frac{8\,M\,C}{I\,L^2}$ (3.33)

3.2.4 Case (4): Beam under Triangular Distributed Load

The case represented the concrete member under loading in triangular shape. The distribution of the shear and moment according to the loading state of force was illustrated in the following figure:

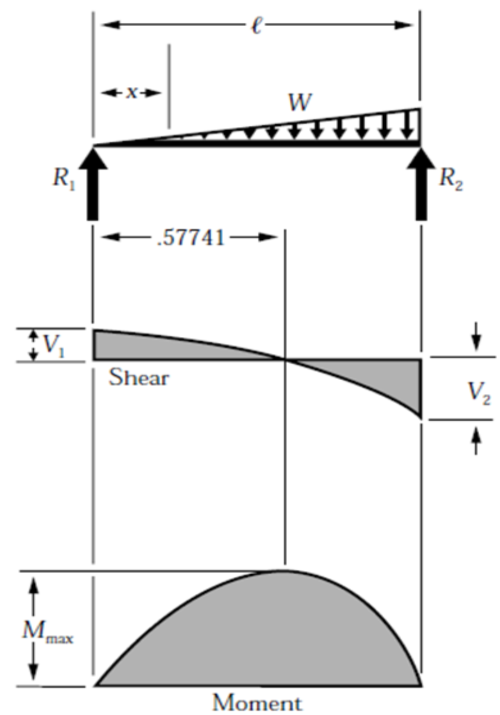

Figure 3.9: Shear Force and Bending Moment Distribution in Case Four [65]

Mmax = 0.1283 w L

For cracked section:

$$\frac{M\,C}{I} = = 0.1283 \text{ w L} \qquad (3.34)$$

$$\text{W} = \frac{M\,C}{0.1283\,L\,I} \qquad (3.35)$$

3.3 ANSYS WORK

For implementing preliminary steps for simulating the case studies of the concrete beam with reinforcement, the beam was drawing in the geometrical platform inside the ANSYS program as shown in figure (3.10). The shape that been built was made under the properties that mentioned in table (3.1).

Table 3.1: Initial Properties of Beam Used in this Study

Property	Value
Dimensions of the beam: width (b), depth (H), and length (L)	300 mm, 300 mm, and 4000 mm
Compressive strength of concrete (fc)	25 MPa
Compressive strength of steel (fs)	400 MPa
Concrete cover (dc)	50 mm
Number of steel bars (N)	3
Diameter of bars (db)	12.7 mm
Modulus of elasticity of concrete (Ec)	$4700\sqrt{fc} = 4700\sqrt{25} * 10^{-3} = 23.5$ GPa
Modulus of elasticity of steel (Es)	200 GPa

The second step was the converting of the geometrical shape of the beam to be in mesh state, this process was so important for applying the finite element analysis for each small part and node that under the investigation. The mesh state of the bean was explained in figure (3.11)

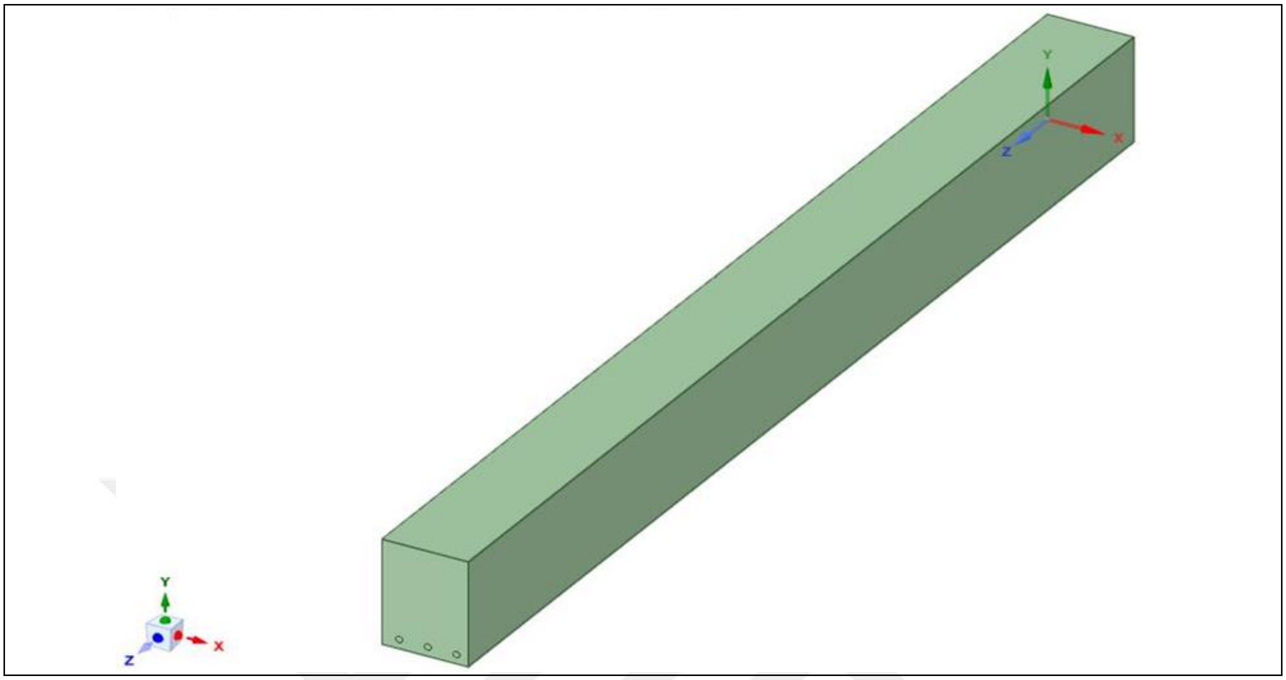

Figure 3.10: Geometric state of the beam (ANSYS)

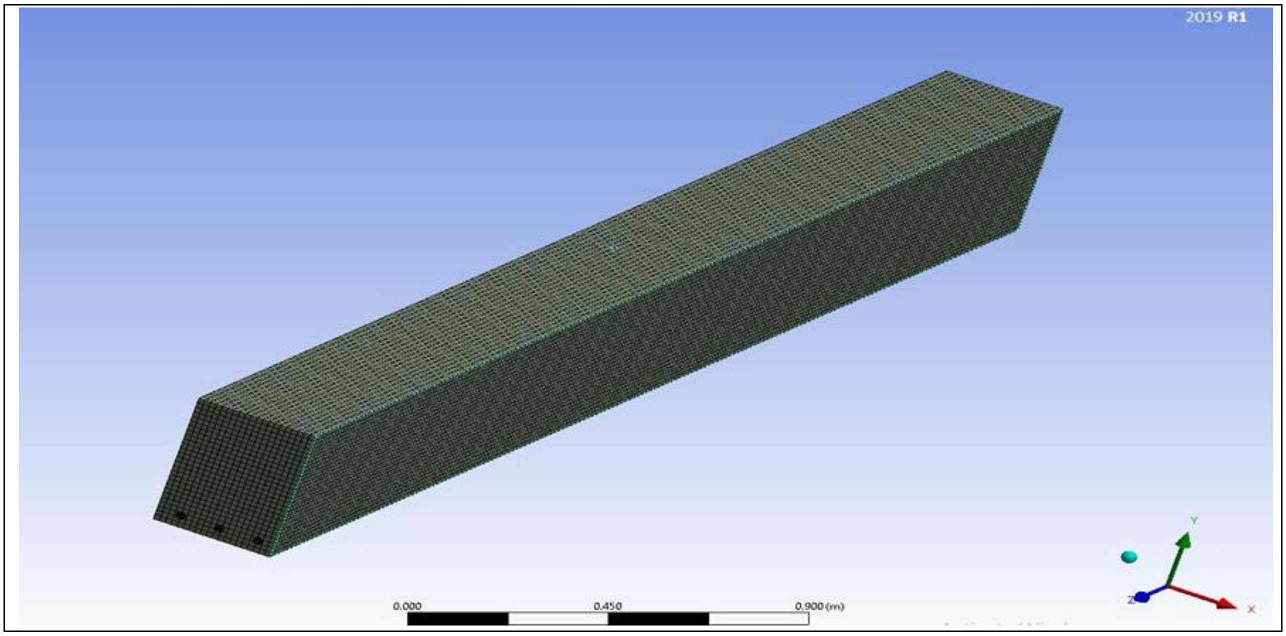

Figure 3.11: Mesh behavior of the beam (ANSYS)

4. RESULT AND DISCUSSION

4.1 RESULTS

The flexural analysis on the four cases that previously mentioned and identified in the study was implemented. The properties were arranged and used in the specific equations for explaining of the maximum forces that lead to cracked section of member to be occurred.

4.1.2 General Calculation Steps:

The analysis procedure start with the assumption of the concrete beam with specific properties to be the initial steps for identifying the influence of the parameters like dimensions and strength capacity on the allowable loading state that can be applied on the member without cracking occurs. The initial properties of beam used in this study were explained in table (3.1):

Area of steel = $3 * \frac{\pi\ db^2}{4} = 3 * \frac{\pi\ 12.7^2}{4}$ = 380 mm^2

n = $\frac{200}{23.5}$ = 8.51

Equivalent area:

$$At = 300 * 300 + (8.51 - 1)\ 380 = 92854\ mm^2$$

$$C = \frac{300*300*\frac{300}{2} + (8.51-1)*380*250}{92854} = 153.07 \text{ mm}$$

Moment of inertia of the member:

$$I = \frac{300\ 300^3}{12} + 300 * 300\ (153.07 - \frac{300}{2})^2 + (8.51 - 1) * 380 * (250 - 153.07)^2 = 702660905\ mm^4$$

$$fr = 0.62\sqrt{25} = 3.1\ MPa$$

$$M = \frac{3.1 * 702660905}{146.93} * 10^{-6} * = 14.23 \text{ kN.m}$$

4.1.3 Case One: Beam under Concentrated Load

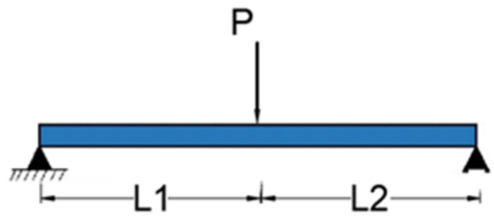

Figure 4.1: Beam under Concentrated Load

Maximum force that can be applied before cracking concrete section was as the following:

$P = \frac{M\ .L}{L1\ \ L2}$

For initial properties that assigned to the member, the force calculated as the following:

$P = \frac{14.23\ \ 4000}{2000\ \ \ 2000}$ *1000 = 14.23 kN

The relationship between the width of the beam and the maximum force that can be applied in which non-cracking section been conserved was illustrated in figure (4.2). The influence of the width of member examined from 300 mm to 500 mm, the magnitude of force increased from 15 kN to 25 kN. Increasing in the force as well was due to the enhancing of the concrete area leading to better resistance to the internal stresses. Linear behavior of the chart reflected uniform increment of the force with constant factor

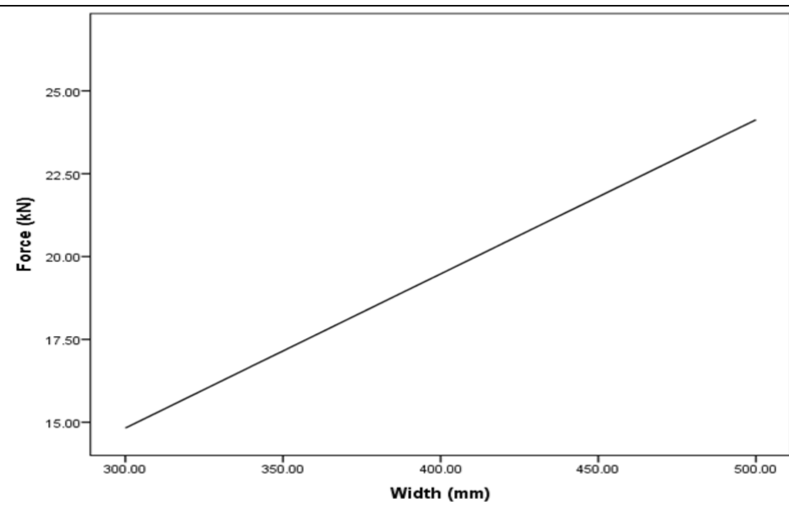

Figure 4.2: The Relationship Between Beam width and the Maximum cracking force for case one

In figure (4.3), the increment of force due to the enlarging of the height of the cross section was explained. The depth of beam when enlarged, the magnitude of the effective depth (d) become high leading to enhance the facing area of concrete to stresses. The strength capacity toward internal forces improved. The depth of cross section raised the allowable force to about 260 % when increased from 300 mm to 500 mm compared to the width of section which raised the force for about 160% when increased in same magnitude of depth.

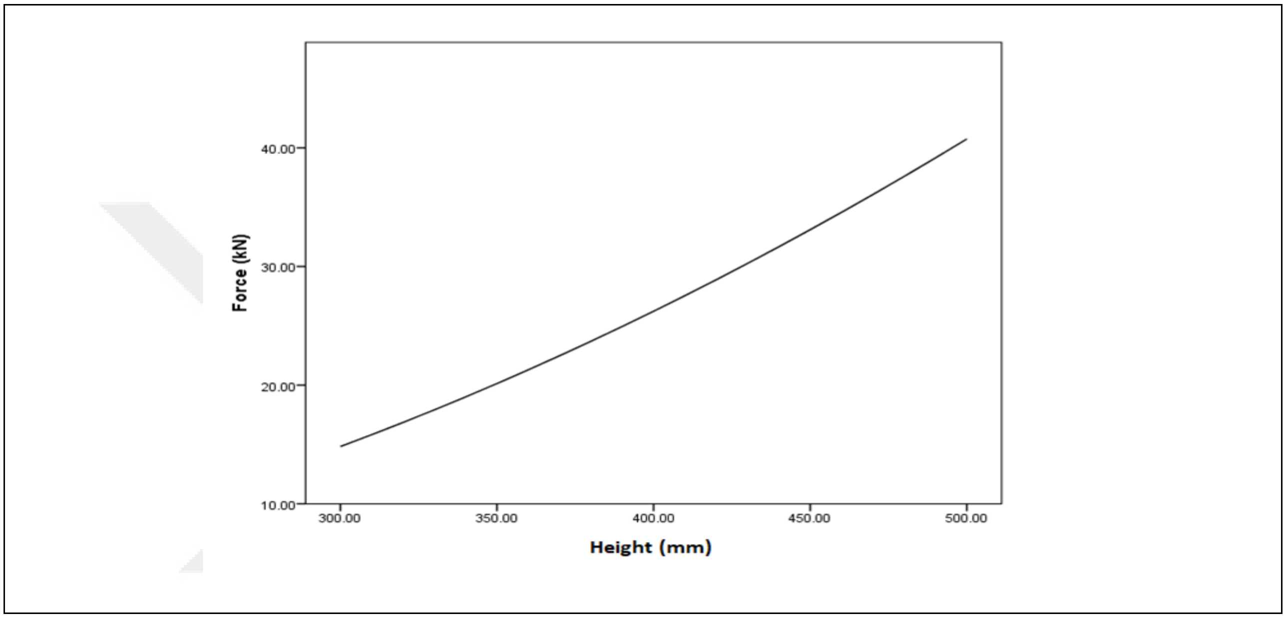

Figure 4.3: The Relationship Between Beam Height and the Maximum Cracking Force

Increasing in the length influenced negatively the possibility to carry loading for designers, the allowable force in figure(4.4) appeared in decreasing when the length changed from 2000 to 4000 mm, the length when become bigger, it effects the magnitude of external moment that can be formed by lower force value. The magnitude of that moment also influenced by the place of force action. Maximum moment occurred when the force acted in the middle of the span which leading to maximum deformation. The relationship between the point of action and the moment value illustrated in figure (4.5).

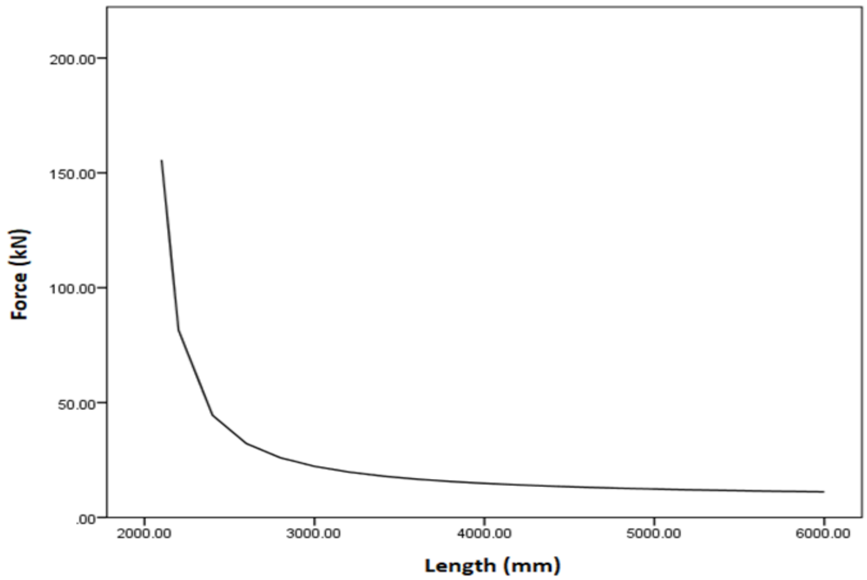

Figure 4.4: The Relationship Between Beam Length and the Maximum Cracking Force

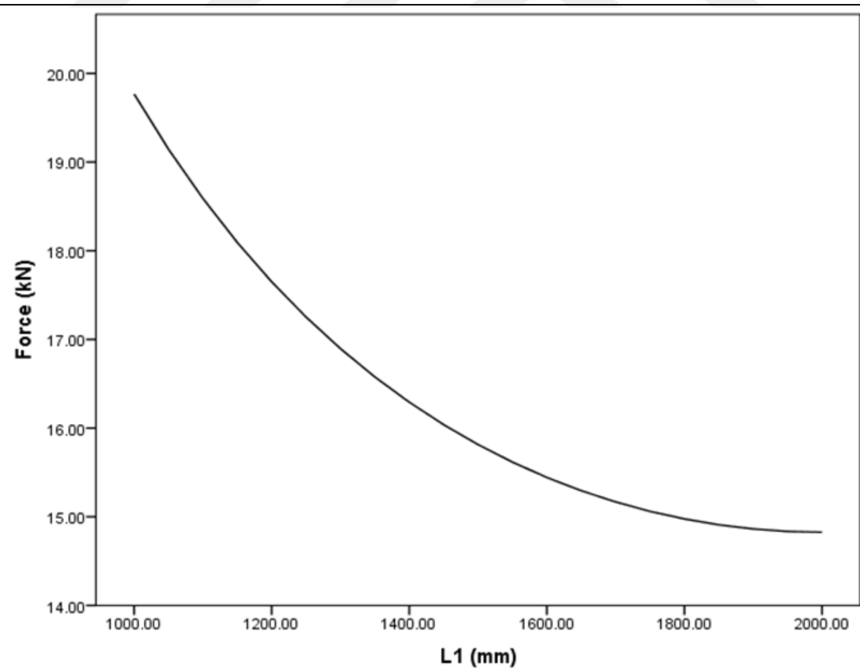

Figure 4.5: The Relationship Between Length Toward Force Action Point and the Maximum Cracking Force

Increasing in the strength of concrete leading to better resistance to applied force, figure (4.7) showed increasing in the magnitude of force when concrete with better compressive strength was used.

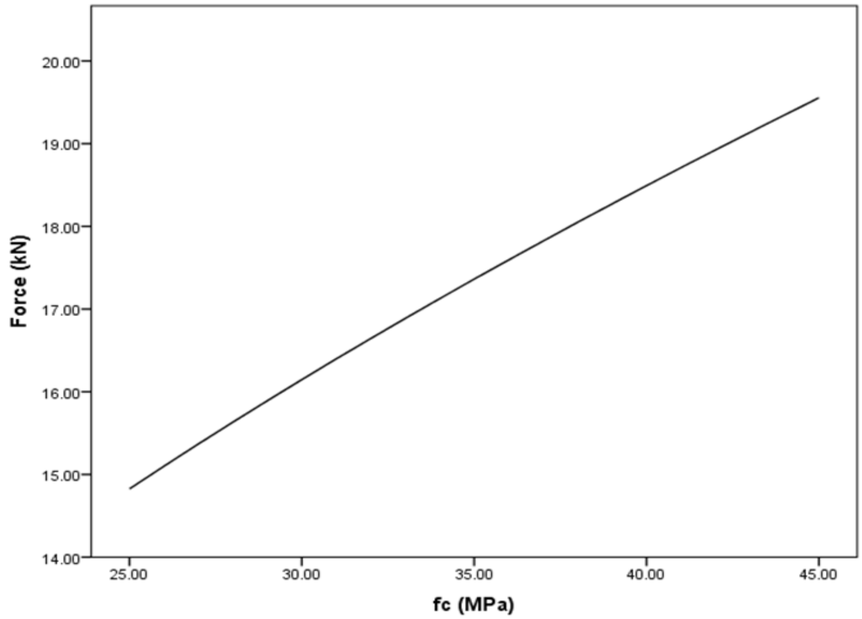

Figure 4.6: The Relationship Between Compressive Strength of Concrete and the Maximum Cracking Force.

4.1.4 Case Two: Beam under Two Concentrated Loads

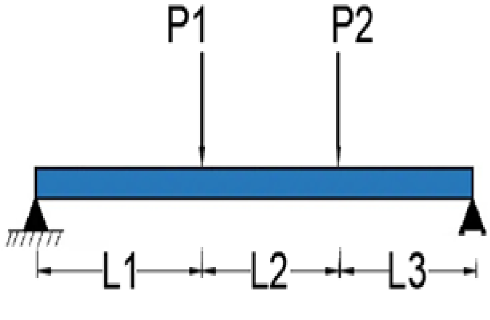

Figure 4.7: Beam under Two Concentrated Load

Figure (4.9) showed increment of force magnitude when the width of cross section increased as mentioned in the previous case. The force magnitude increased by 160 % when the cross section increased from 300mm to 500 mm. as compared with previous case with single load, decreasing of 36% of each allowable force can be noticed due to double action of forces occurred in this case. The magnitude of the two forces together were more than what reached in single force with about 13%.

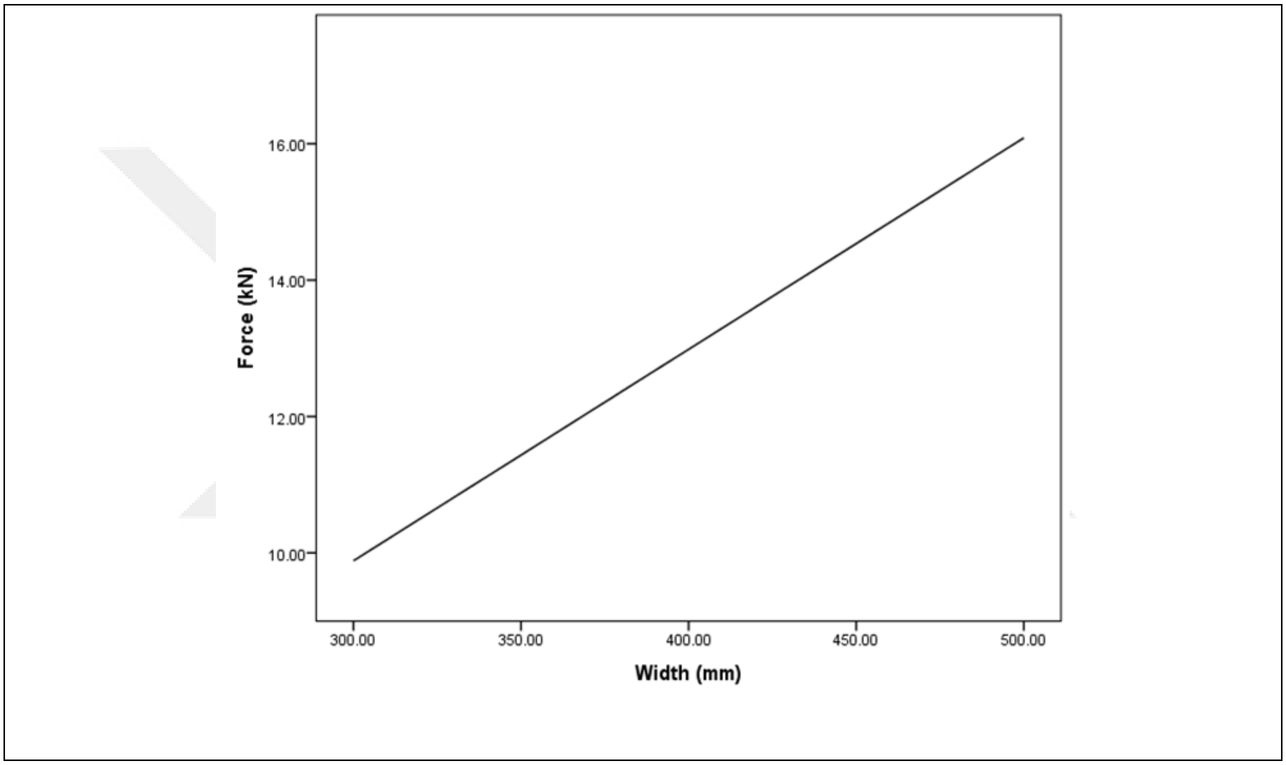

Figure 4.8: The Relationship Between Beam width and the Maximum Cracking Force for Case Two

Figure (4.9) showed the increment in the magintude of force with respect to the increasing in the member depth. the force magintude increased when the height changed to about 160 %.The magintude of the two forces was about 136% of the single force in the previous case.

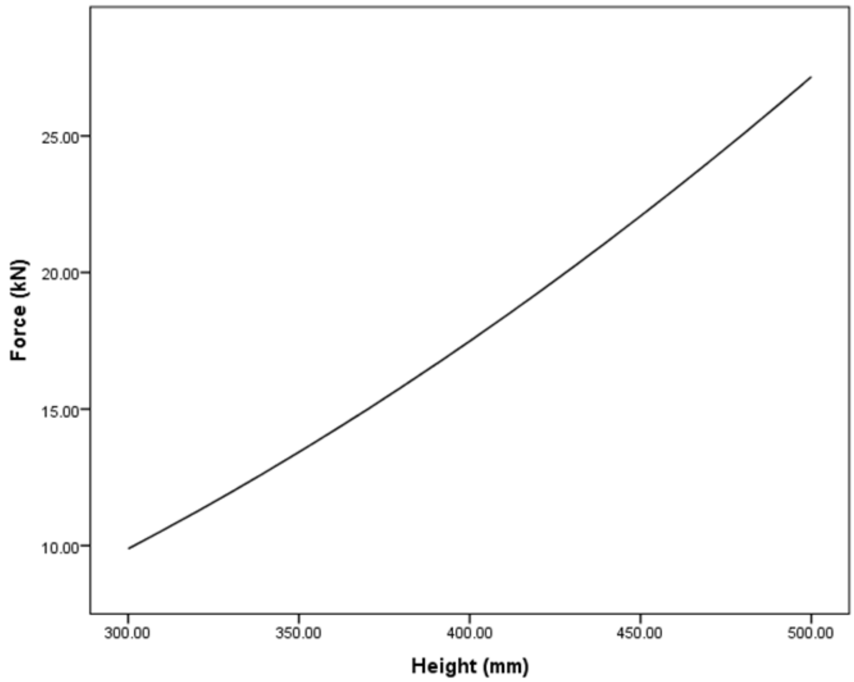

Figure 4.9: The Relationship Between Beam Height and the Maximum Cracking Force for Case Two.

The magnitude of the allowable force decreased noticeably when the length in figure (4.10) and the place of P1 in figure (4.11) increased.

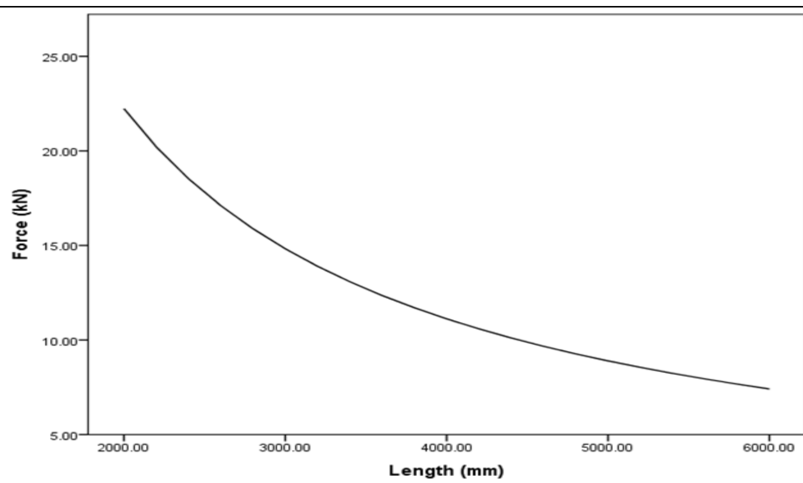

Figure 4.10: The Relationship Between Beam Length and the Maximum Cracking Force for Case Two

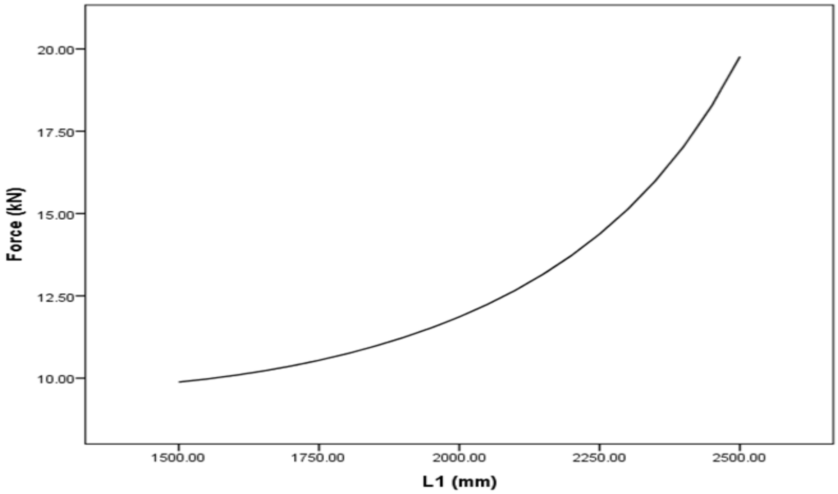

Figure 4.11: The Relationship Between Length Toward Force Action Point and the Maximum Cracking Force

In figure (4.12) the allowable force increased with upgrading of compressive strength of concrete. The magnitude of the two forces was 130 % of that in case of single force.

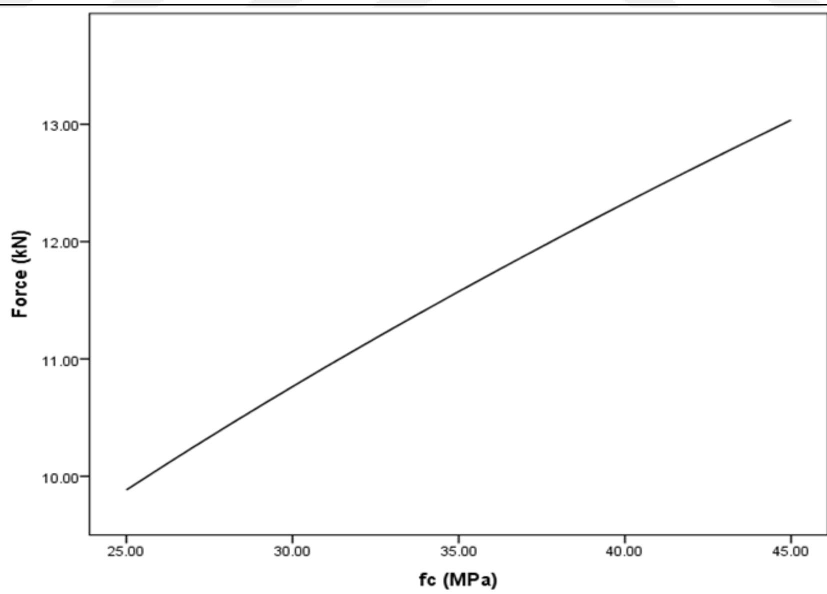

Figure 4.12: The Relationship Between Compressive Strength of Concrete and the Maximum Cracking Force

4.1.5 Case Three: Beam under Rectangular Loading

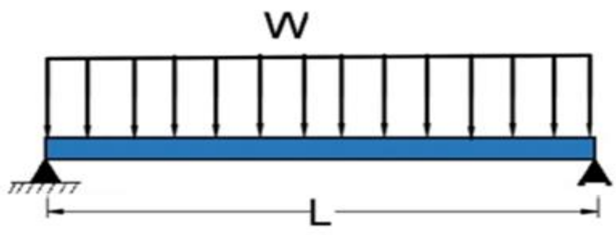

Figure 4.13: Beam Under Uniform Rectangular Load

In rectangular shape loading, the maximum allowable loading per meter was calculated. The magnitude of that loading with respect to the changing in width of beam, height of beam, length of beam, and compressive strength of concrete were showed in figures (4.14), (4.15), (4.16), and (4.17) respectively.

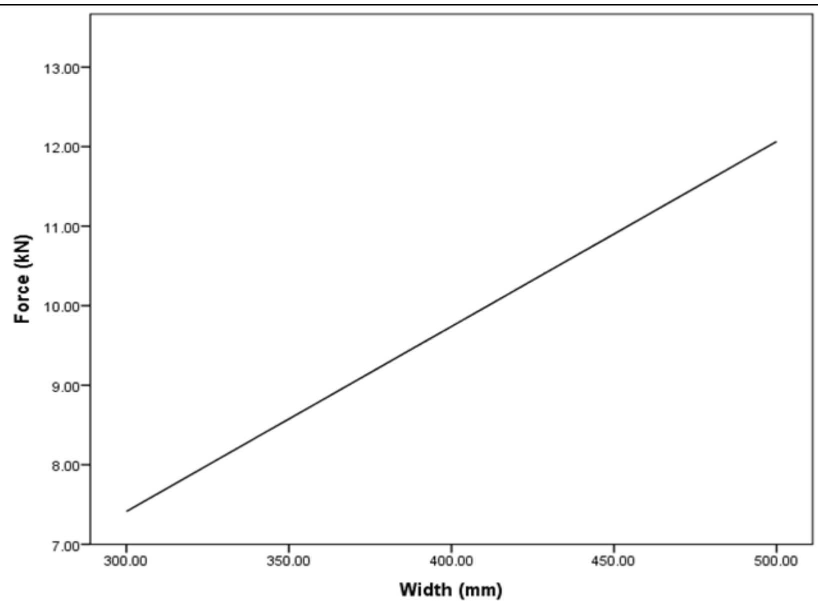

Figure 4.14: The Relationship Between Beam width and the Maximum Cracking Force for Case Three

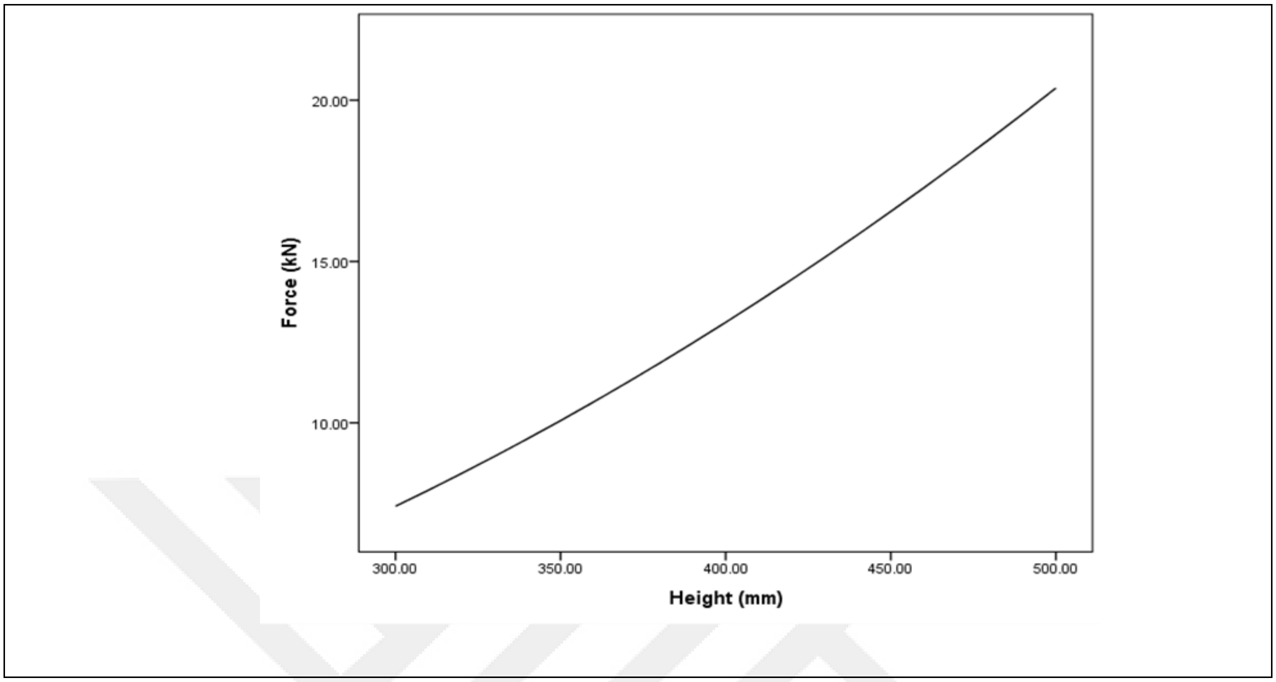

Figure 4.15: The relationship between beam height and the maximum cracking force for case three.

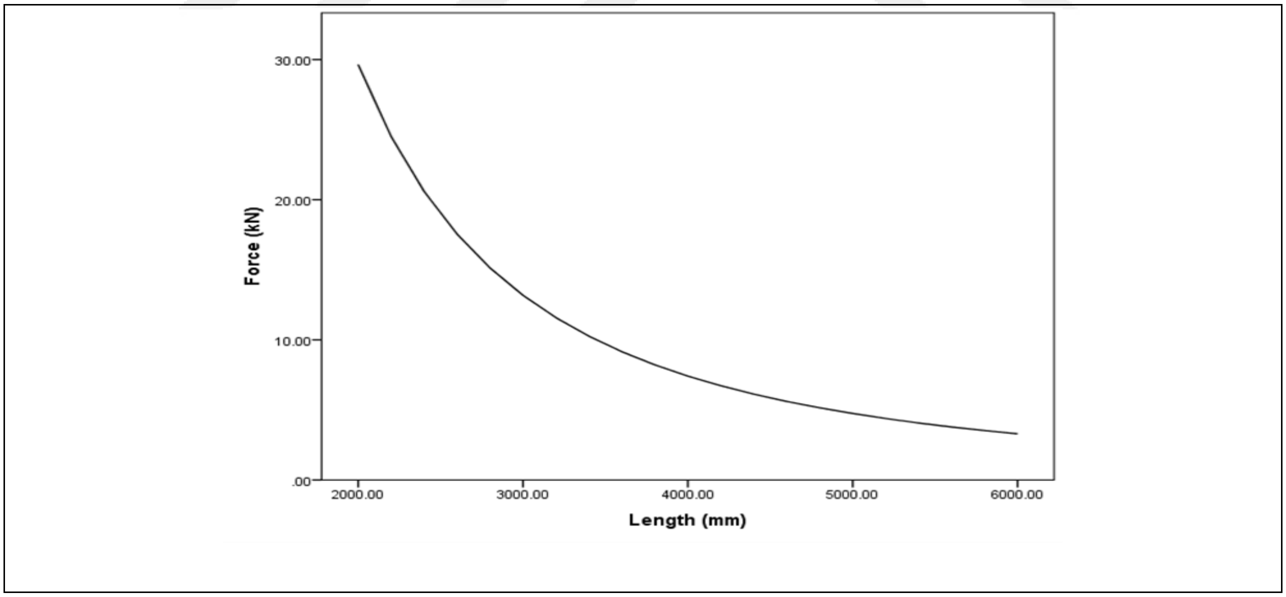

Figure 4.16: The Relationship Between Beam Height and the Maximum Cracking Force for Case 3

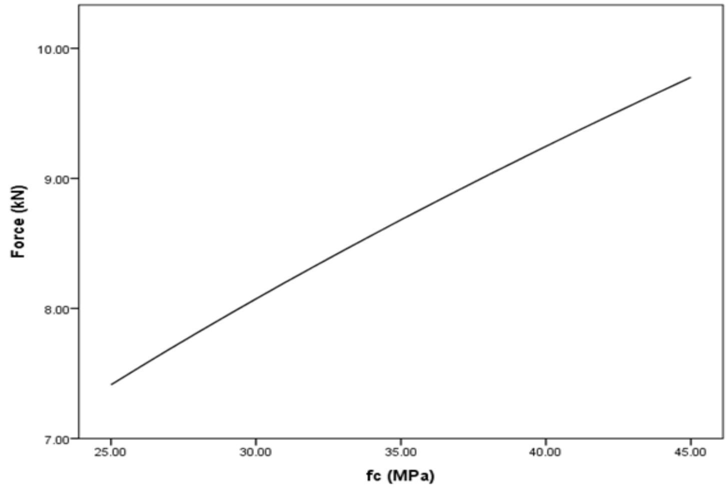

Figure 4.17: The Relationship Between Compressive Strength and Concrete and the Maximum Cracking Force for Case Three

4.1.6 Case Four: Beam under Triangular Loading

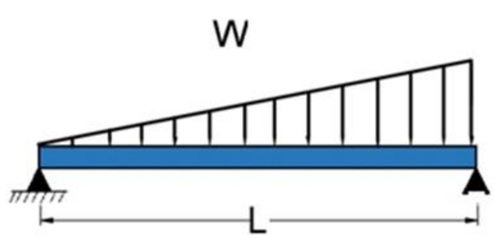

Figure 4.18: Beam Under Triangular Load

The influence of changing width of beam of the allowable load was explained in figure (4.19).Changing of height in the figure (4.21) showed the improved of assigning the allowable loading in design. More than 400 % of the magnitude of loading per meter for rectangular case can be carried in this case as illustrated in related figures.

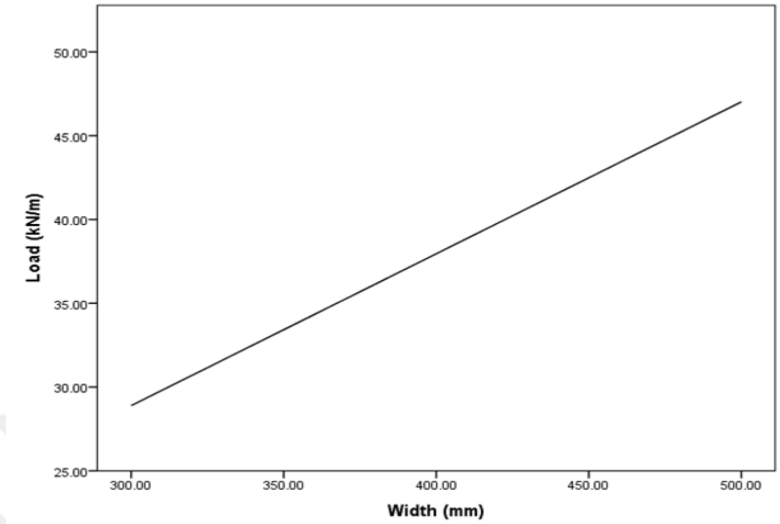

Figure 4.19: The Relationship Between Beam width and the Maximum Cracking Force for Case Four.

In figure (4.20), the decreasing in allowable loading explained as enlarging in the member length. About 300% of the loading in the rectangular shape can be adopted in the design in the current case.

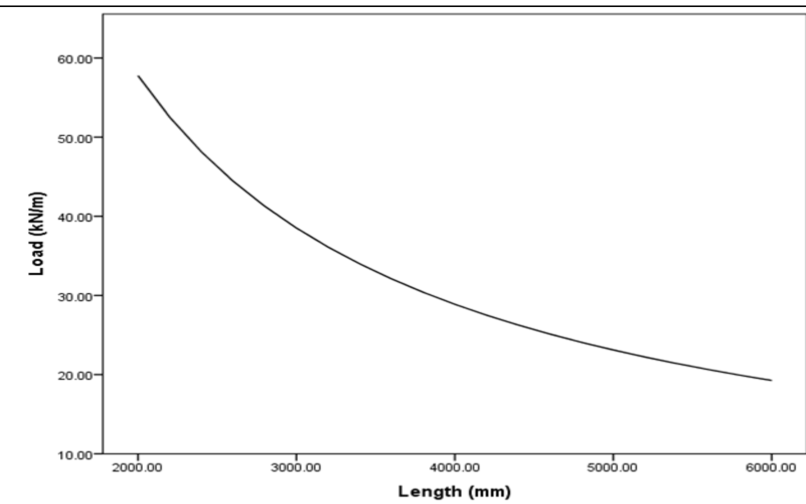

Figure 4.20: The Relationship Between Beam Length and the Maximum cracking force for case four.

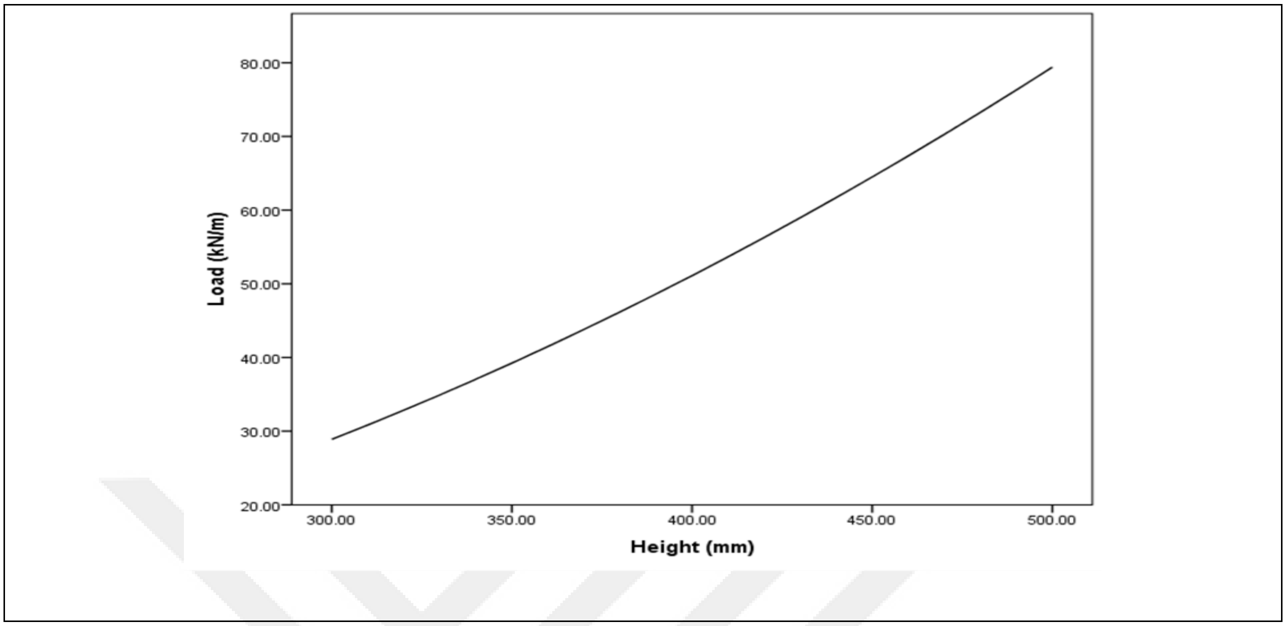

Figure 4.21: The Relationship Between Beam Height and the Maximum cracking force for case four.

In the state on improving compressive strength for concrete beam that carried out triangular shape of loading, fig(4.22) showed the increment of loading when this property improved

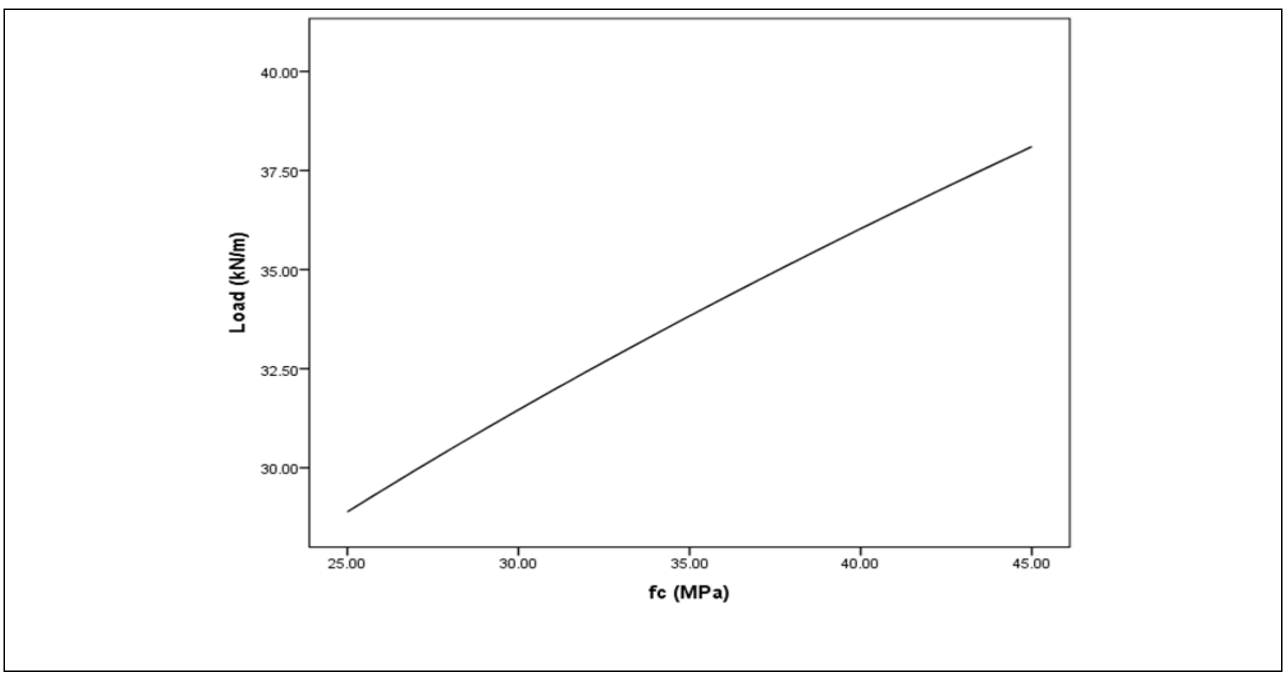

Figure 4.22: The Relationship Between Compressive Strength of Concrete the Maximum cracking force for case four.

4.2 PREDICTION OF CORRELATION FORMULA

The bonding strategy for the parameters the played as independent variables was identifying using SPSS linear regression, the influence of each parameter with magnitude of moment that led to crack-section state in concrete was analyzed for predicting the governing formula. Arranging the statistical steps in regression procedure was so important outline to control the result for less error formation with the calculation values. The following table represented the magnitude of cracking moment in single concentrated case with the increment of member width, member height, and the compressive strength of concrete

Table 4.2: Cracking moment with the increment of member width, member height, and the compressive strength of concrete.

Width (mm)	Height (mm)	fc (MPa)	Moment KN.M
300	300	25	14.83
310	310	26	16.62
320	320	27	18.57
330	330	28	20.68
340	340	29	22.96
350	350	30	25.40
360	360	31	28.03
370	370	32	30.85
380	380	33	33.87
390	390	34	37.09
400	400	35	40.52
410	410	36	44.18
420	420	37	48.07
430	430	38	52.20
440	440	39	56.58
450	450	40	61.21
460	460	41	66.11
470	470	42	71.29
480	480	43	76.75
490	490	44	82.50
500	500	45	88.56

In the previous table, the linear regression between the changing of width or height and the compressive strength indicates the following formula

fc = 0.1 * H- 5 (3.36)

by assuming F = 10*(fc+5) (3.37)

F became a variable changed in same rate with the height and width variables

The linear regression for the width of the member and the cracking moment when the height and the compressive strength remained constant was:

M= 0.878 + 0.047 *W (3.38)

The procedure re-calculated for the other parameters and the results were summarized in the following table.

Table 4.3: Linear Regression Results for Changing width, Height and Compressive Strength

W	H	F	Linear regression equation	slope	R^2
300 to 500	300	300	M= 0.878 + 0.047 *W	0.047	0.999
300	300 to 500	300	M= -25.046 + 0.13 *H	0.13	0.996
300	300	300 to 500	M= 7.881 + 0.024 *F	0.024	0.998

The results in table (4.3) indicated maximum influence on the allowable moment before cracking occurred in the beam was by the value of cross section height. This fact is acceptable as the height reflected the effective depth which considered so important parameters and found in different calculation steps in design.

The linear regression for the height and cracking moment when all parameters change from 300 to 500 was as following:

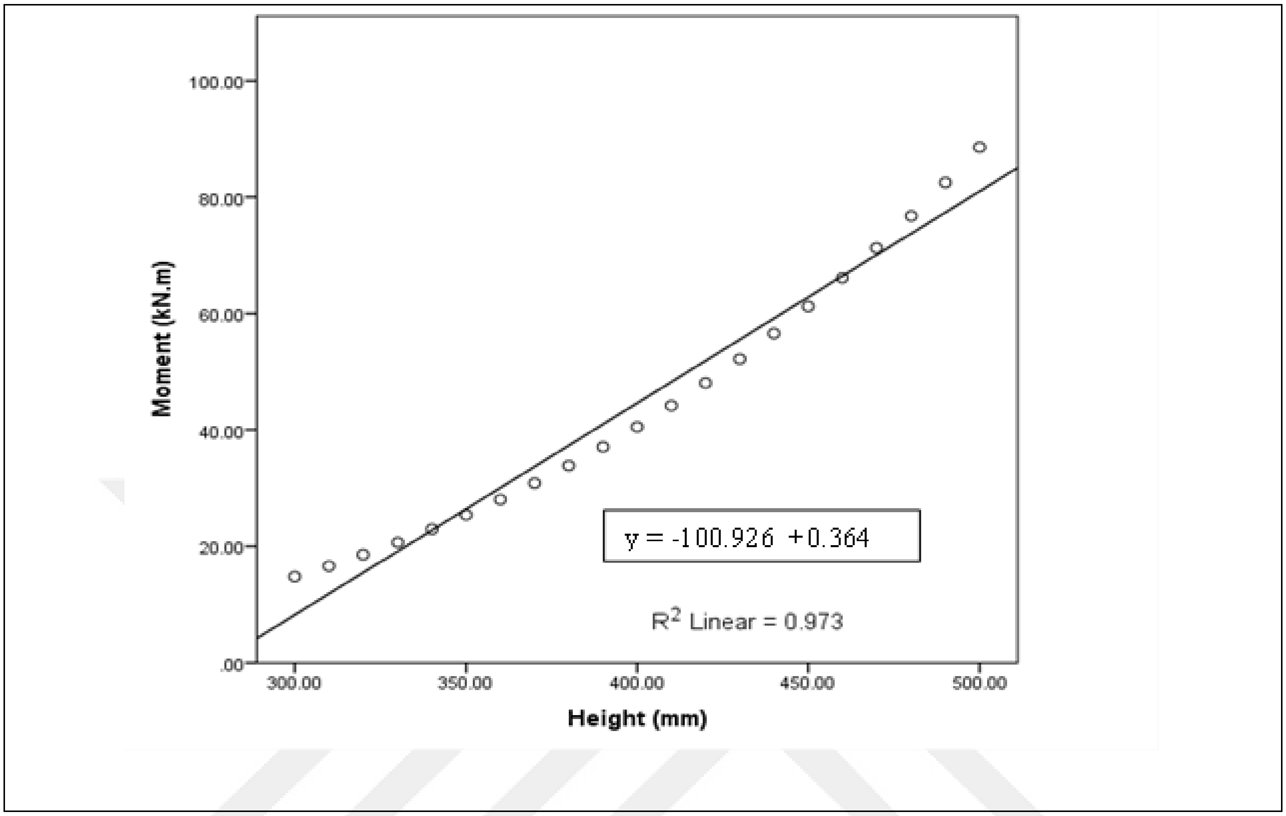

Figure 4.23: The Linear Regression for the Height and Cracking Moment

The governing formula that included the influence of width, height, and compressive strength was derived as following:

M = a1+ a2 W + a3 H + a4 F

a1 = 100.926

$$a2 = \frac{0.047}{(0.047 + 0.13 + 0.024)} * (\,0.364) = \;0.0851$$

$$a3 = \frac{0.13}{(0.047 + 0.13 + 0.024)} * (\,0.364) = \;0.2354$$

$$a4 = \frac{0.024}{(0.047 + 0.13 + 0.024)} * (\,0.364) = \;0.0434$$

M = -100.926 +0.0851 W + 0.2354 H + 0.0434 F

M = -100.926 + 0.0851 W + 0.2354 H + 0.0434 * 10*(fc+5)

M = -100.926 + 0.0851 W + 0.2354 H + 0.0434 * 10*(fc+5)

M= - 98.756 +0.0851 W + 0.2354 H + 0.434 fc (3.39)

Table 4.3: calculation of the difference between calculated and predicted cracking moment

Calculated Moment (kN.m)	Predicated Moment (kN.m)	Difference (kN.m)
14.83	8.244	6.58
16.62	11.883	4.74
18.57	15.522	3.05
20.68	19.161	1.52
22.96	22.8	0.16
25.40	26.439	-1.04
28.03	30.078	-2.05
30.85	33.717	-2.87
33.87	37.356	-3.49
37.09	40.995	-3.91
40.52	44.634	-4.11
44.18	48.273	-4.09
48.07	51.912	-3.84
52.20	55.551	-3.35
56.58	59.19	-2.61
61.21	62.829	-1.62
66.11	66.468	-0.36
71.29	70.107	1.18
76.75	73.746	3.00
82.50	77.385	5.12
88.56	81.024	7.53

The differences between the calculated and predicted moment as explained in table were noticeable and can lead to converging in design state. The reason related to the compatibility between the regression line and the Height-Moment curve in figure (4.23). For moving more toward precision, three zones in figure (4.24) can be used as shown in the following figure.

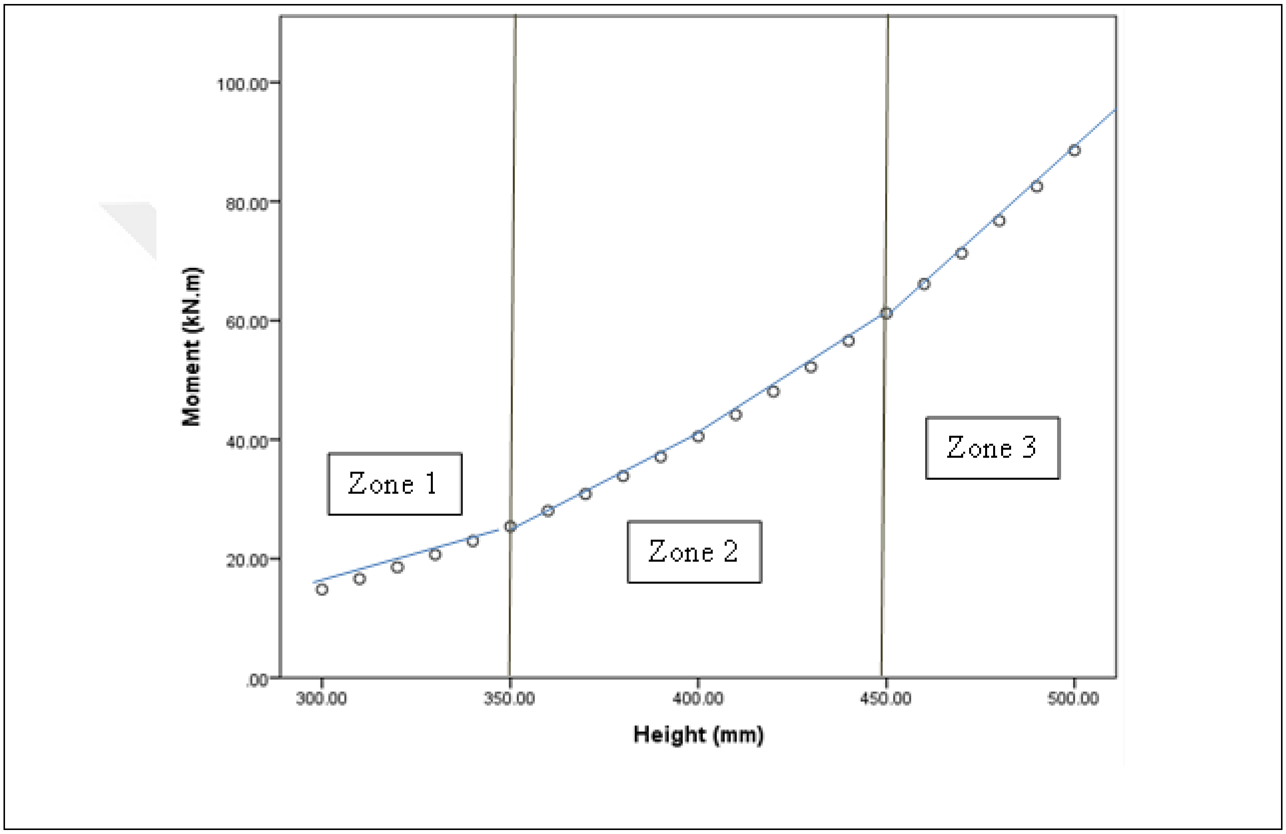

Figure 4.24: The Linear Regressions for the Zones of Beam Height and Cracking Moment Relationship.

Table 4.4: Linear Regression Results for Zone1,Zone2 and Zone3

Region	Linear regression equation	slope	R^2
Zone 1	M= -48.8 + 0.21 *H	0. 21	0.998
Zone 2	M= -101 + 0.36 *H	0.36	0.994
Zone 3	M= -185 + 0.55 *H	0.55	0.999

M = a1+ a2 W + a3 H + a4 F (H < 350 mm)

a1 = -48.8

$$a2 = \frac{0.047}{(0.047 + 0.13 + 0.024)} * (0.21) = 0.0491$$

$$a3 = \frac{0.13}{(0.047 + 0.13 + 0.024)} * (0.21) = 0.1358$$

$$a4 = \frac{0.024}{(0.047 + 0.13 + 0.024)} * (0.21) = 0.0250$$

M = -48.8+ 0.0491 W + 0.1358 H + 0.0250 F (H < 350 mm)

M = a1+ a2 W + a3 H + a4 F (350 ≤ H < 450 mm)

a1 = -101

$$a2 = \frac{0.047}{(0.047 + 0.13 + 0.024)} * (0.36) = 0.0842$$

$$a3 = \frac{0.13}{(0.047 + 0.13 + 0.024)} * (0.36) = 0.232$$

$$a4 = \frac{0.024}{(0.047 + 0.13 + 0.024)} * (0.36) = 0.043$$

M = -101+ 0.0842 W + 0.2328 H + 0.043 F (350 ≤ H < 450 mm)

M = a1+ a2 W + a3 H + a4 F (450 mm ≤ H)

a1 = -185

$$a2 = \frac{0.047}{(0.047 + 0.13 + 0.024)} * (0.55) = 0.1286$$

$$a3 = \frac{0.13}{(0.047 + 0.13 + 0.024)} * (0.55) = 0.3557$$

$$a4 = \frac{0.024}{(0.047 + 0.13 + 0.024)} * (0.55) = 0.0656$$

M = -185+ 0.1286 W + 0.3557 H + 0.0656 F (450 mm $\leq$ H $\leq$ 500 mm)

Table 4.5: Calculation of the Difference Between Calculated and Predicted Cracking Moment after Using Three Zones Procedure

Calculated Moment (kN.m)	Predicated Moment (kN.m)	Difference (kN.m)
14.83	14.17	0.66
16.62	16.27	0.36
18.57	18.37	0.21
20.68	20.47	0.22
22.96	22.57	0.39
25.40	24.37	1.03
28.03	27.95	0.08
30.85	31.53	-0.68
33.87	35.12	-1.25
37.09	38.70	-1.61
40.52	42.28	-1.76
44.18	45.86	-1.68
48.07	49.44	-1.37
52.20	53.03	-0.83
56.58	56.61	-0.03
61.21	61.42	-0.21
66.11	66.90	-0.79
71.29	72.37	-1.09
76.75	77.85	-1.10
82.50	83.32	-0.82
88.56	88.80	-0.24

The influence of the low difference between calculated and predicted moment can be extinguished by using the factor of safeties that adopted by ACI code for dead and live loads.

By applying the relating between the compressive strength fc and the factor F as explained in equation (3.39), the following table included the results of the three equations that governed the value of cracking-moment:

Table 4.6: Final Derived Equations

H (mm)	Linear regression equation
H < 350 mm	M = -47.55+ 0.0491 W + 0.1358 H + 0.250 fc
350 ≤ H < 450 mm	M = -98.85+ 0.0842 W + 0.2328 H + 0.43 fc
450 mm ≤ H ≤ 500 mm	M = -181.72+ 0.1286 W + 0.3557 H + 0.656 fc

For section with initial properties as mentioned in table(3 .1):

Case one: single concentrated load if L1= L2= 2000 mm :

$$P = \frac{-47.55 + 0.0491\,(300) + 0.1358\,(300) + 0.250 * 25 \quad *(4000)}{2000 \quad 2000} * 1000 \ = 14.17 \text{ kN}$$

Case Two: two equal concentrated loads if L1= 1500 and L2= 1000 mm :

$$P1 = \frac{-47.55 + 0.0491\,(300) + 0.1358\,(300) + 0.250 * 25 \quad *(4000)}{(2 * 1500 + 1000) \quad 1500} * 10^3 \ = 9.45 \text{ kN}$$

Case Three: rectangular shape loading:

$$w = \frac{-47.55 + 0.0491\,(300) + 0.1358\,(300) + 0.250 * 25 \quad *8}{4000^2} * \ 10^6 \ = 7.08 \ \text{ kN/m}$$

Case four: triangular shape loading:

$$w = \frac{-47.55 + 0.0491\,(300) + 0.1358\,(300) + 0.250 * 25}{0.1283 * 4000} * \ 10^3 \ = 27.61 \ \text{ kN/m}$$

4.3 ANSYS SIMULATING RESULTS

The previous data were arranged as input data for ANSYS work to simulate each case and explained the results in diagram and graphs:

4.3.1 Case One: Single Concentrated Load

The appearing of internal stresses in figure (4.25) along the beam by the output of ANSYS platform explained the high relative values that found in the part of force action which handled maximum bending moments. The magnitude of the stress passed the cracking threshold which indicated the state of cracked section at the middle of the beam span in the tensile zone of cross section.

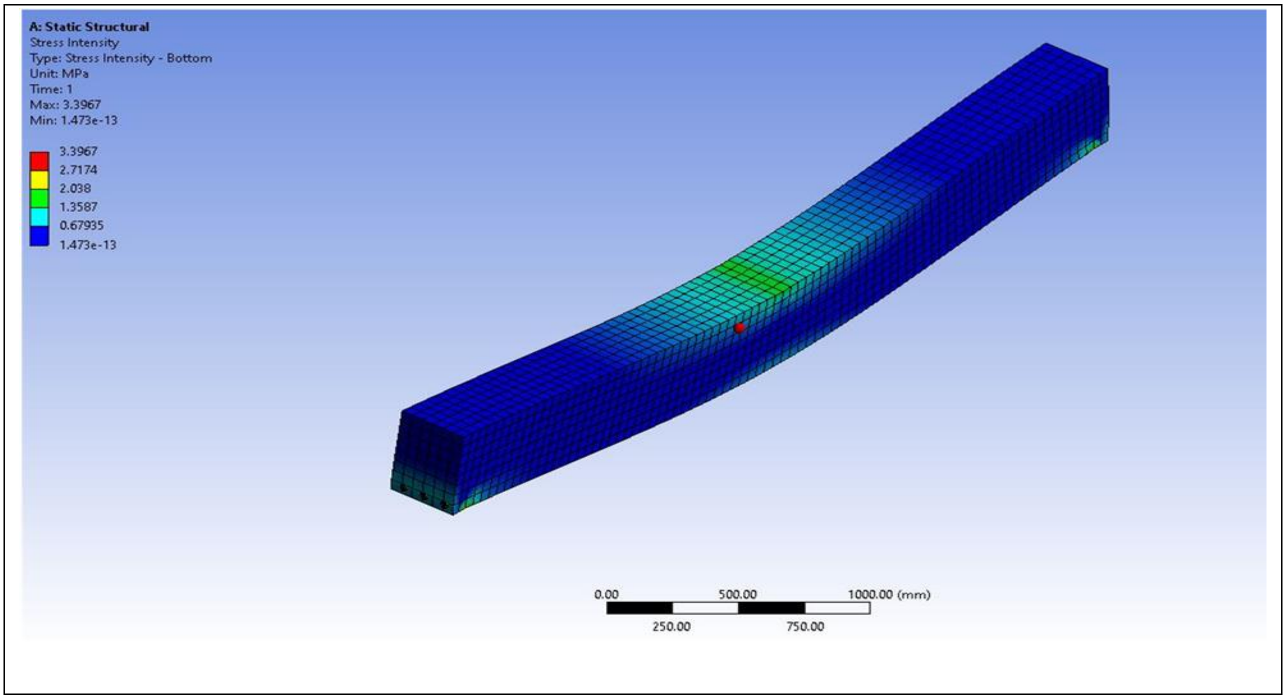

Figure 4.25: Stress Intensity Along Beam span in Case One (ANSYS)

In the figure (4.26) illustrate the deformation in the cracked beam which the pink color indicated high displacement as this zone carried the maximum magnitude of bending moment.

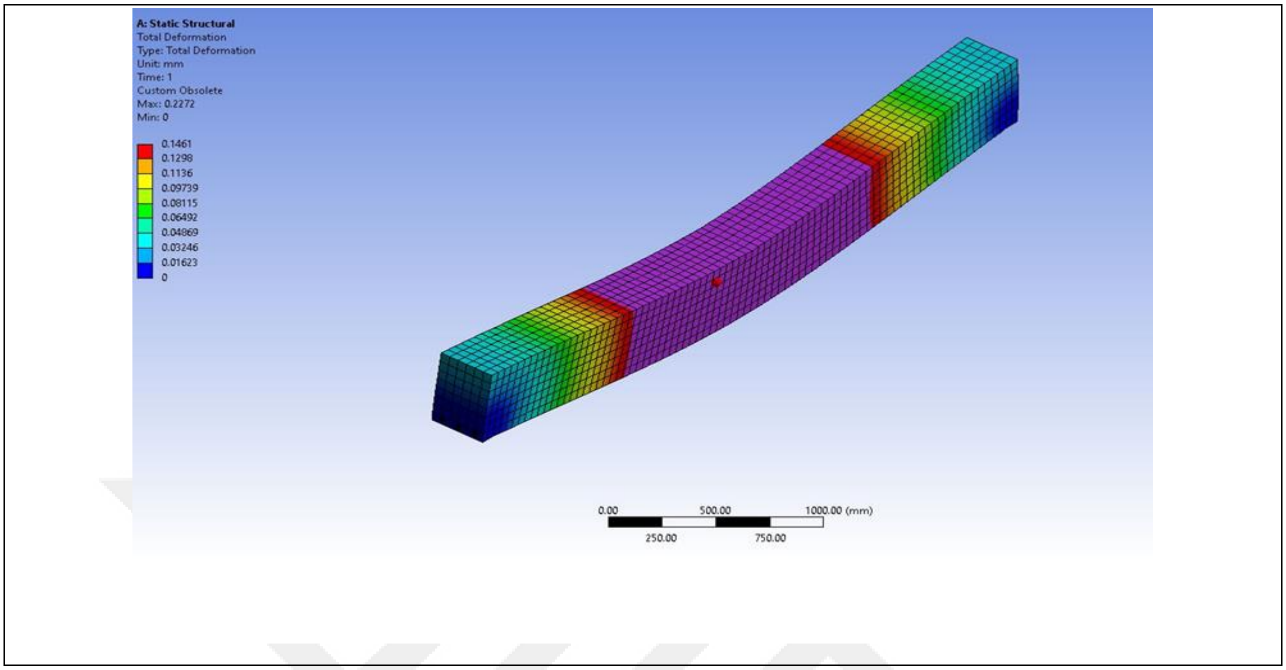

Figure 4.26: Deformation Along Beam Span in Case One (ANSYS)

Figure (4.27) showed the magnitudes of strain along the span beam which accompanied the cracking section state.

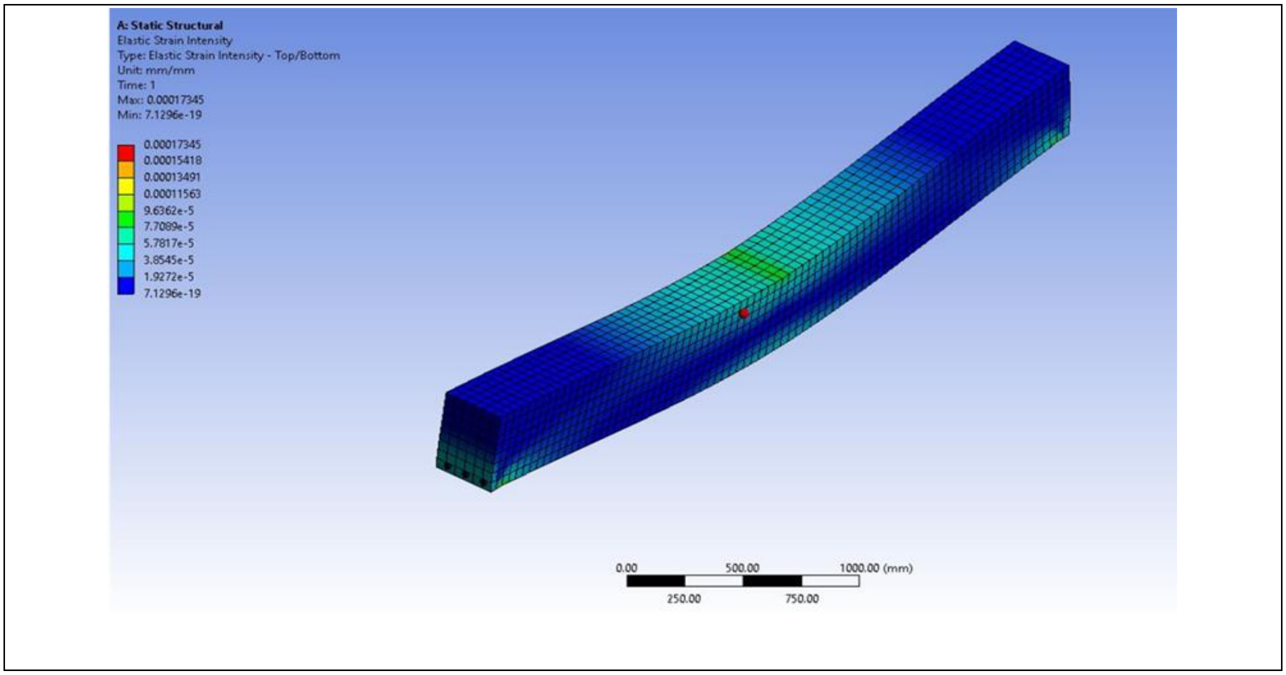

Figure 4.27: Strain Along Beam Span in Case One (ANSYS)

4.3.2 Case Two: Two Equal Concentrated Loads

The figures (4.28), (4.29), and (4.30) explained the internal stresses, the deformation, and the strain amounts respectively. Cracked section can be noticed by the distribution of stresses inside the beam as the middle span showed high values of stresses indicated by the colors reference.

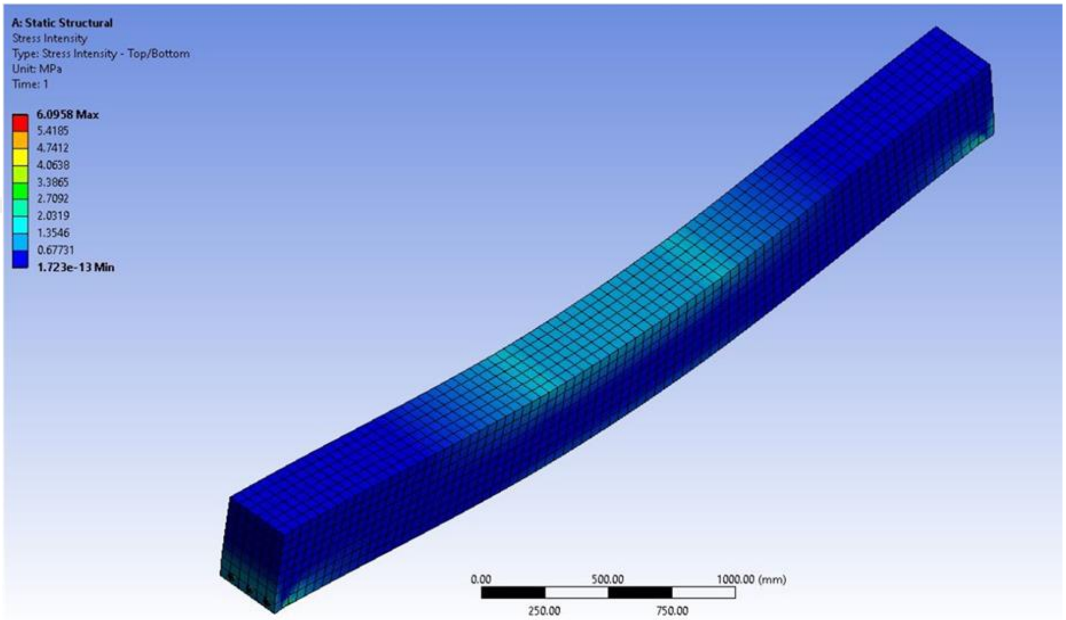

Fig 4.28: Stress Intensity Along Beam Span in Case Two (ANSYS).

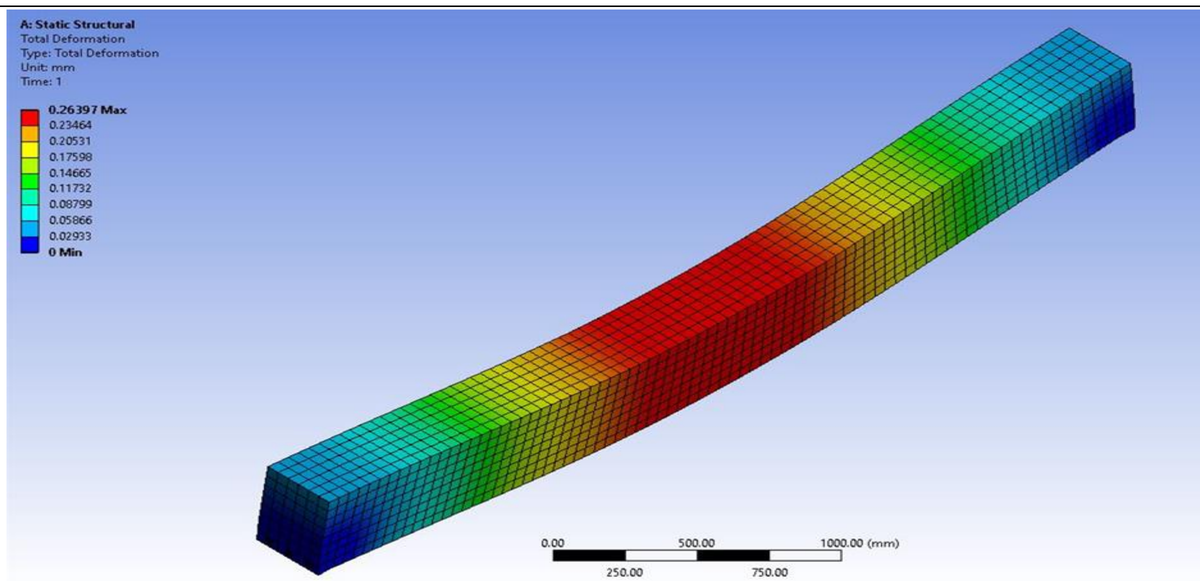

Figure 4.29: Deformation Along Beam Span in Case Two (ANSYS).

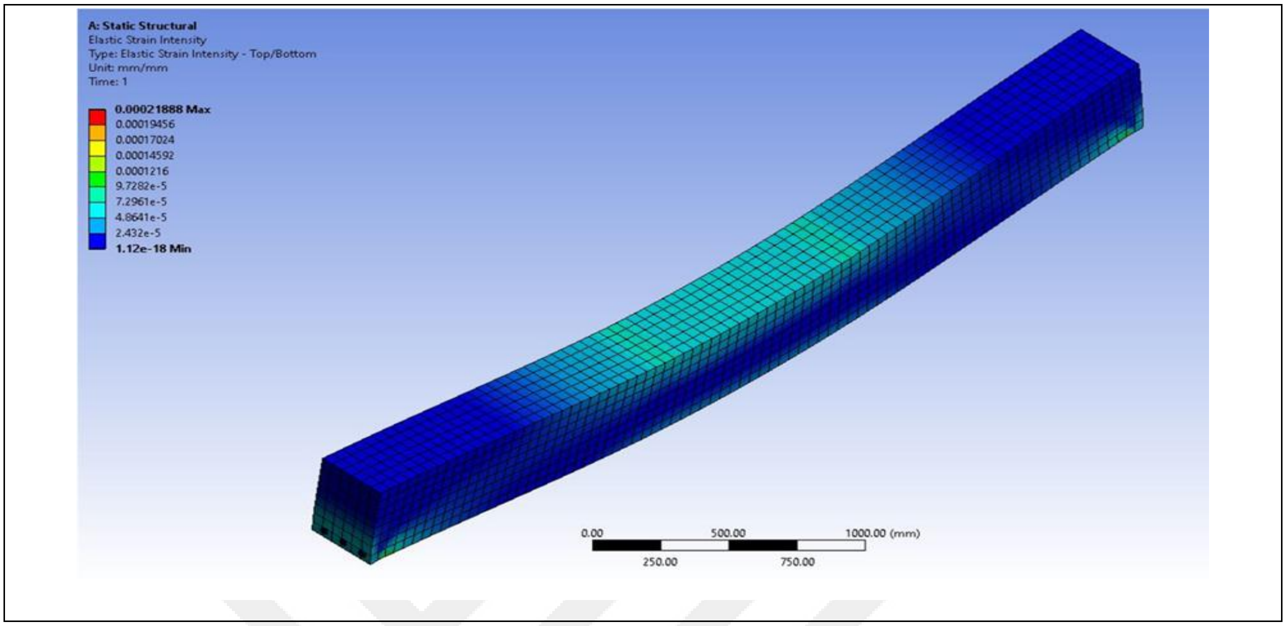

Figure 4.30: Strain Along Beam Span in Case Two (ANSYS).

4.3.3 Case Three: Rectangular Shape Loading

By applying uniform loading with rectangular behavior on the beam, the middle of the span faced the higher magnitude of the bending moment so it suffered of maximum stresses, this state was shown clearly in figure (4.29). The deformation occurred in main at the middle point as shown in figure (4.30). The strain that accompanied to this case was illustrated in figure(4.31)

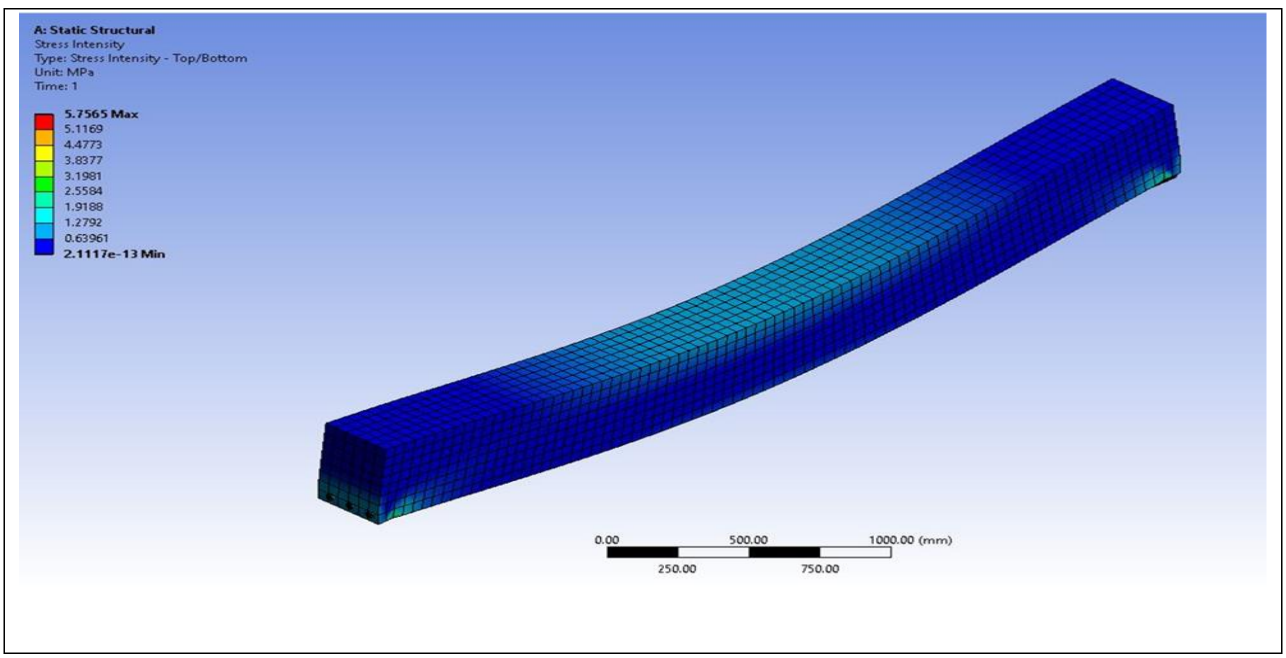

Figure 4.31: Stress Intensity Along Beam Span in Case Three (ANSYS).

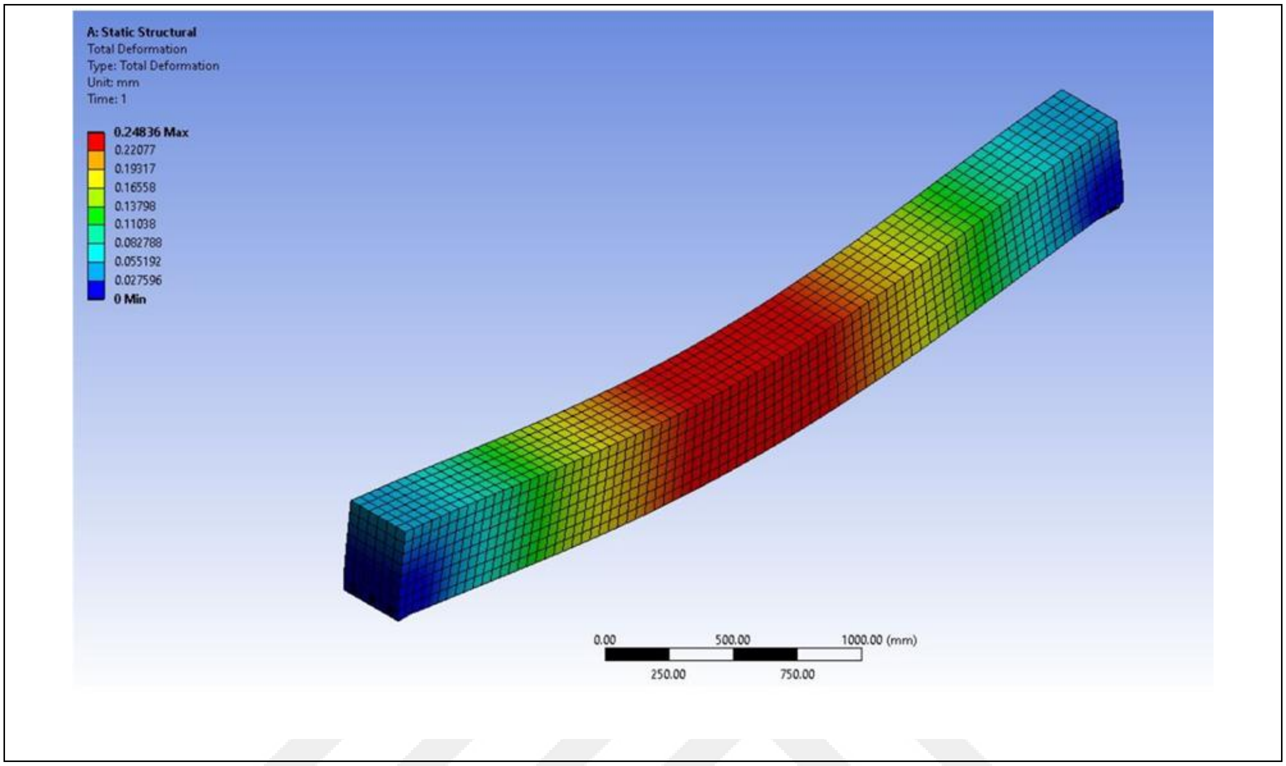

Figure 4.32: Deformation Along Beam Span in Case Three (ANSYS)

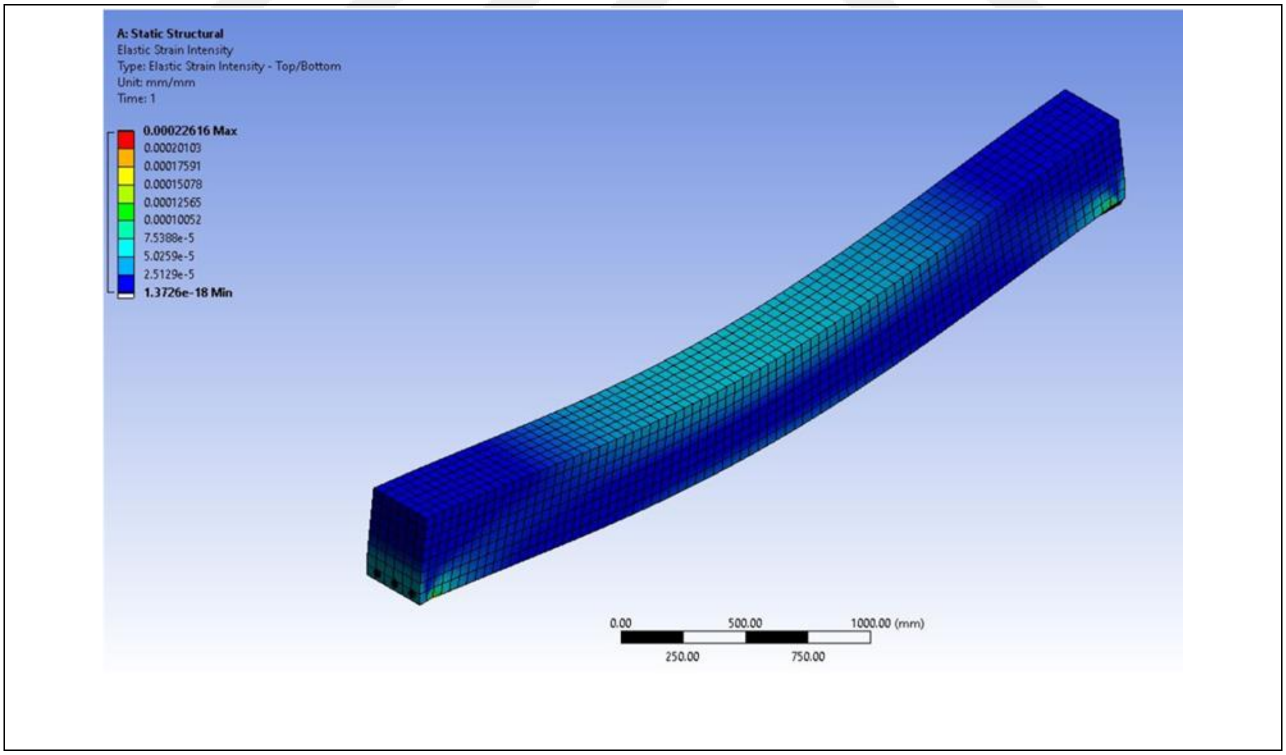

Figure 4.33: Strain Along Beam Span in Case Three (ANSYS)

4.3.4 Case Four: Triangular Shape Loading

For triangular shape intensity of loading, the maximum stress occurred at 0.577 L which equal to 2.3 m from left side of the beam. This distance explained by figure (4.34), and also can be noticed in figure (4.35) which related to deformation and figure (4.36) which explained the amount of strain.

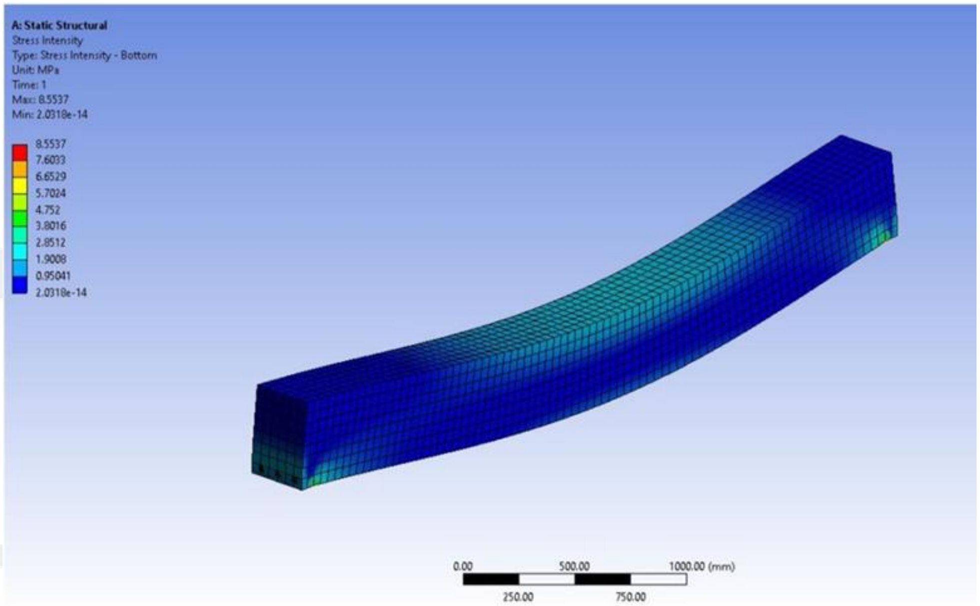

Figure 4.34: Stress Intensity Along Beam Span in Case Four (ANSYS)

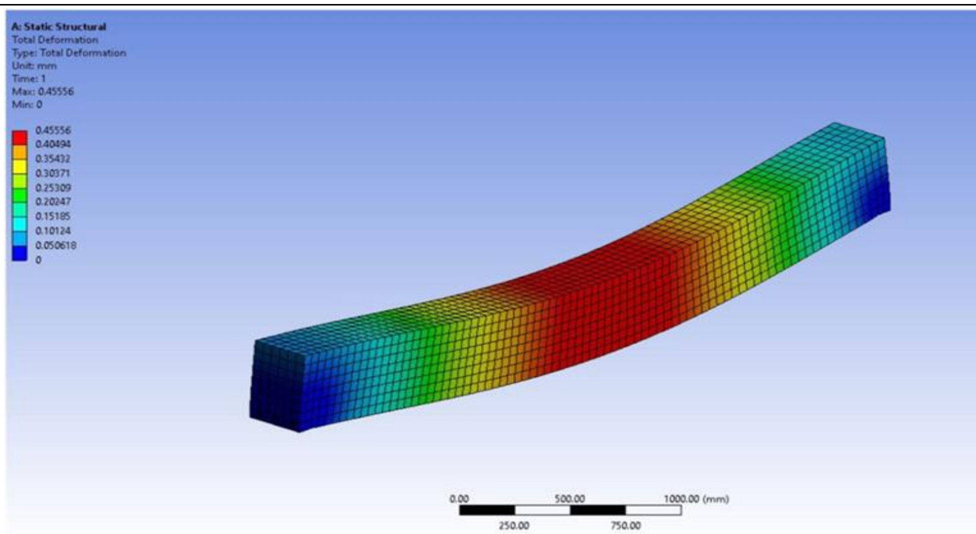

Figure 4.35: Deformation Along Beam Span in Case Four (ANSYS)

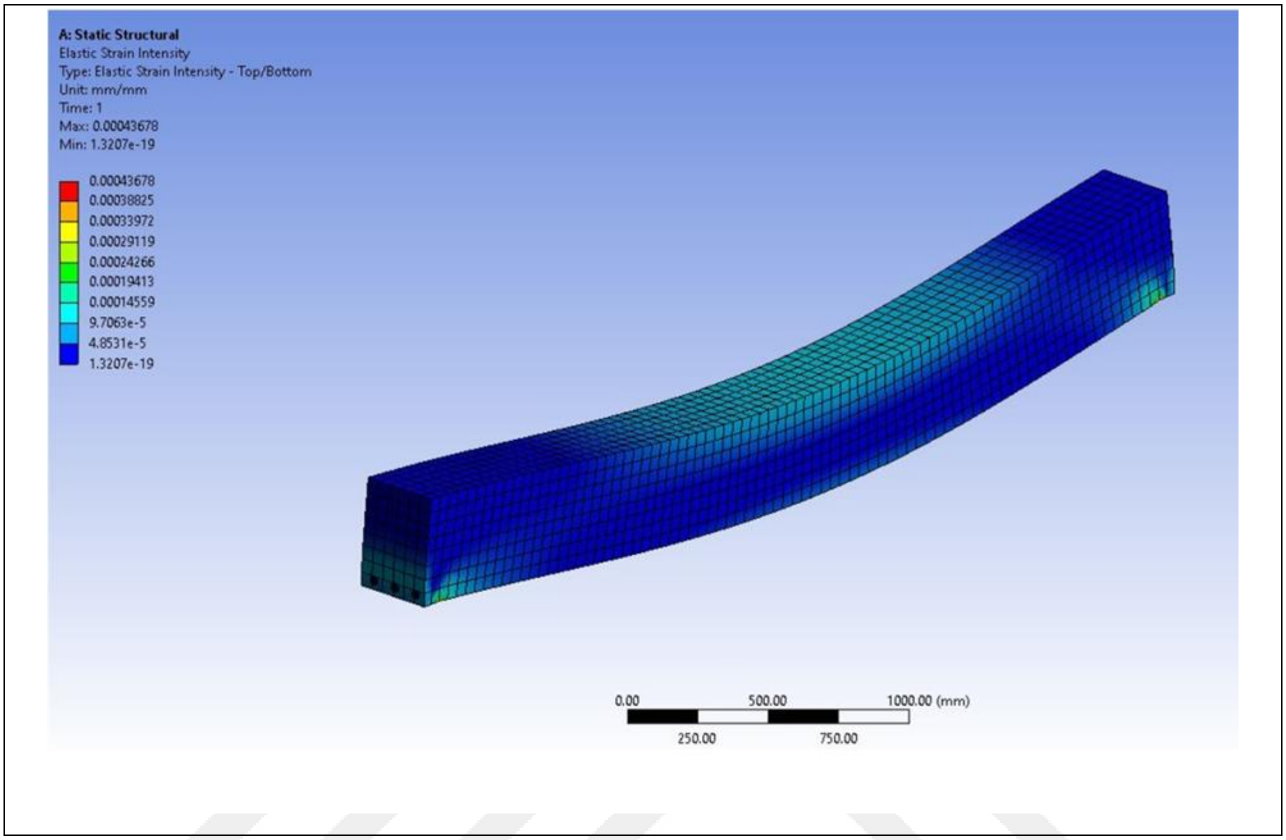

Figure 4.36: Strain Along Beam Span in Case Four (ANSYS

5. CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

The study aimed to assist the pathway of concrete beams design by applying flexural analysis on four cases of external loading that carried out onto simply supported beam. The cases were single concentrated force, two concentrated force, rectangular shape load, and triangular shape load. The maximum bending moment was determined for each case by the help of bending moment diagram. The parameters that influenced the magnitude of the maximum moment for each case and impacted as well the magnitude of the applying load before cracking in the tensile fiber of the beam cross section were assigned. These parameters were the width of the cross section, the height of the cross section, the length of the beam span and the compressive strength of the concrete. The magnitude of cross section height identified as the so effective parameter which increased the allowable loading. This fact was due to the increment of effective depth that enhanced the distance from natural axis to top or down level in compression and tension region. The mentioned parameters were changed from lower to upper limit in same rate (300 to 500) so that the curve between the cracking moment and the height of the member cross section was drawing. The height was chosen as it more effective one. The resultant curve then examined under regression correlation process in SPSS software for achieving the best fitting line that can be adopted for evaluating predicted cracking moment. The difference between the predicting moment and calculated one was large relatively, it reaches in some input data more than 7 KN.m.

For reducing the diverging in the resultant moment, three zones were assigned in the curve so that regression correlation increased noticeably to be more than 0.99 for all zones. The difference had been examined at this state which be reduced markedly (less than 2KN.m at maximum difference). Three linear equations were derived using the statistical result that can be used for easily supporting the design procedure by identify the cracking external loads in simple few steps. The behavior of each case was explained by simulating the state of beam using statistic structural option in ANSYS platform. The result supported the calculation by showing the deformation, the stresses, and also the strain that presented along the span of the beam under each case of loading.

5.2 FUTURE WORK

Analysis of more cases of beams can be marked as coming work that base in some points on the present study, statistical and flexural analysis can be implemented on the case of two fixed end beam or beam with holes or beam under other loading cases like inclined forces or non-uniform shape of loading intensity. Moreover, the analysis can be applied on the plastic relationship between concrete stresses and strain in which cracked section occurred. The statistic action can cover in future the behavior of reinforcement including the state toward ultimate strength in the steel bars under tension and compression for different loading case. The joining between beams and columns can be also assigned as destination for researchers as future scope in statistical analysis, these parts play important role in multi-story buildings and structures.

REFERENCES

[1] R. Subramanian, *Strength of Materials: Second Edition*. 2010.

[2] B. Onouye and Kevin Kane, *Statics and Strength of Materials for Architecture and Building Construction*, vol. 148. 2012.

[3] D. R. Salah and R. Al-Zaidee, "Design of Reinforced Concrete Structures," 2018.

[4] J. Ochshorn, *Structural Elements for Architects and Builders*, vol. 148. 2010.

[5] M. W. Nadir, "Design of Reinforced Concrete Structures I Flexural Analysis & Design of Beam Part6 Flexural Analysis & Design of T-Section," *2020*.

[6] F. A. N. Silva and B. Horowitz, "Bending moments in beams of reinforced concrete buildings Momentos fletores em vigas de edifícios de concreto armado Bending moments in beams of reinforced concrete buildings," *IBRACON Struct. Mater. J. •*, vol. 1, no. 2, pp. 193–211, 2008.

[7] Y.-C. Loo and S. H. Chowdhury, *Reinforced and Prestressed Concrete : Analysis and Design with Emphasis on Application of AS3600-2009*. 2013.

[8] M. V. Rama Rao, A. Pownuk, and I. Skalna, "Stress analysis of a singly reinforced concrete beam with uncertain structural parameters," *Int. J. Reliab. Saf.*, vol. 3, no. 1–3, pp. 307–329, 2009, doi: 10.1504/IJRS.2009.026848.

[9] R. Djamaluddin, P. L. Frans, and R. Irmawati, "Flexural Capacity of the Concrete Beams Reinforced by Steel Truss System," *MATEC Web Conf.*, vol. 138, 2017, doi: 10.1051/matecconf/201713802003.

[10] P. E. Jose-Miguel Albaine, M.S., "Design of Reinforced Concrete Beams per ACI 318-02," *Reinf. Concr. Des.*, vol. 152, pp. 154–191, 2012, doi: 10.1007/978-1-349-20929-3_7.

[11] M. H. Wan Ibrahim *et al.*, "Compressive and flexural strength of concrete containing palm oil biomass clinker and polypropylene fibres," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 271, no. 1, 2017, doi: 10.1088/1757-899X/271/1/012011.

[12] B. Dupen, "Applied Strength of Materials for Engineering Technology," *Appl. Strength Mater. Eng. Technol.*, vol. 6, p. 152, 2014.

[13] S. J. Narayanan, "Introduction to reinforced concrete 1," *Des. RC Struct.*, no. November, pp. 1–45, 2013.

[14] J. Hua, F. Zhou, L. Huang, Z. Chen, Y. Xu, and Z. Xie, "Influence of reinforcement bars on concrete pore structure and compressive strength," *Materials (Basel).*, vol. 13, no. 3,

2020, doi: 10.3390/ma13030658.

[15] A. A. Mansor, A. S. Mohammed, and W. D. Salman, “Effect of longitudinal steel reinforcement ratio on deflection and ductility in reinforced concrete beams,” *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 888, no. 1, 2020, doi: 10.1088/1757-899X/888/1/012008.

[16] C. Buckner, *Concrete Design, Second Edition*. 2014.

[17] Ş. Erdoğdu, Ş. Kurbetci, U. Kandil, M. Nas, and S. Nayır, “Evaluation of the Compressive Strength of Concrete by means of Cores Taken from Different Casting Direction,” no. September, pp. 12–14, 2018.

[18] Jean Pierre Jacobs, “Sustainable Benefits of Concrete Structures,” *Eur. Concr. Platf. ASBL*, p. 40, 2009.

[19] W. Tameemi, “By Waleed Tameemi Rémy D. Lequesne,” no. 114, 2015.

[20] Cttp, “Concrete Strength Testing Technician Course Topics • Failure of Either Exam,” vol. 39, no. T 22, pp. 1–87, 2020.

[21] I. Židonis, “Strength calculation method for cross-section of reinforced concrete flexural member using curvilinear concrete stress diagram of EN-2,” *Procedia Eng.*, vol. 57, pp. 1309–1318, 2013, doi: 10.1016/j.proeng.2013.04.165.

[22] F. Shanahan, A. Niederlehner, R. P. MacDermott, W. F. Stenson, M. G. Kane, and S. Targan, “Inhibition of cytotoxicity by sulfasalazine. II. Sulfasalazine and sulfapyridine inhibit different stages of the NK and NKCF lytic processes,” *Immunopharmacology*, vol. 11, no. 2, pp. 111–118, 1986, doi: 10.1016/0162-3109(86)90031-7.

[23] P. Jiratatprasot, “Mechanical properties and stress-strain behavior of high performance concrete under uniaxial compression,” *Public Health*, p. 125, 2007, [Online]. Available: http://scholar.google.com/scholar?hl=en&btnG=Search&q=intitle:Some+Contributions+on+MIMO+Radar#0.

[24] C. W. David Darwin and A. H. Nilson, *Design of concrete structures*. 2016.

[25] G. Campione and L. La Mendola, “Stress-strain behavior in compression of lightweight fiber reinforced concrete under monotonic and cyclic loads,” *Adv. Earthq. Eng.*, vol. 9, pp. 387–396, 2001.

[26] R. Su, “Mechanical Properties of Local Concrete,” *Annu. Concr. Semin. 2015*, no. April, p. 47, 2015, [Online]. Available: https://www.devb.gov.hk/filemanager/en/content_946/5_Mechanical_Properties_of_Local

_Concrete_By_Dr_Ray_Su.pdf.

[27] T. Liu, S. Wang, J. Gao, and X. Tian, "The energy analysis of concrete failure in structural engineering," *Syst. Eng. Procedia*, vol. 1, pp. 106–111, 2011, doi: 10.1016/j.sepro.2011.08.018.

[28] R. Mark, A. S. Cakmak, K. Hill, and R. Davidson, "Analysis of compression failure of reinforced concrete by the modified distinct element method," *Trans. Built Environ.*, vol. 3, 1993.

[29] A. Lect, A. Abdul, and H. Jawad, "Toward an Appeal for Revision on ACI Stress Block Parameters for High Strength Concrete," *J. Eng. Dev.*, vol. 11, no. 2, pp. 87–103, 2007.

[30] J. G. Kaufman and E. L. Rooy, "Stress-Strain Curves," *Alum. Alloy Cast.*, pp. 193–209, 2001, doi: 10.31399/asm.tb.aacppa.t51140193.

[31] A. Chhachhia, "Concrete Mix Design by IS, ACI and BS Methods: A Comparative Analysis," *J. Build. Mater. Sci.*, vol. 2, no. 1, pp. 30–33, 2021, doi: 10.30564/jbms.v2i1.2636.

[32] T. Izhar, S. Bano, and N. Mumtaz, "Comparative Study on Analysis and Design of Reinforced Concrete Building under Seismic Forces for Different Codal Guidelines," *Int. J. Trend Sci. Res. Dev.*, vol. Volume-3, no. Issue-4, pp. 536–551, 2019, doi: 10.31142/ijtsrd23819.

[33] O. Elzaroug, R. M. Abd Alkarim, A. F. Gabar, H. A. Alzalaka, and E. M. Ahshwsan, "Design of Reinforced Concrete Beams by Ultimate Strength According to Different Codes," *12th Arab Struct. Eng. Conf.*, no. June, 2017.

[34] J. W. Wallace and A. Safdari, "Design of reinforced concrete structural walls," *11th National Conference on Earthquake Engineering 2018, NCEE 2018: Integrating Science, Engineering, and Policy*, vol. 3. pp. 1324–1328, 2018.

[35] S. A. Daud, J. P. Forth, and N. Nikitas, "Time-dependent behaviour of cracked, partially bonded reinforced concrete beams under repeated and sustained loads," *Eng. Struct.*, vol. 163, pp. 267–280, 2018, doi: 10.1016/j.engstruct.2018.02.054.

[36] M. Li and H. Li, "Effect of loading rate on reinforced concrete member," *aEngineering Plast. Its Appl. - Proc. 10th Asia-Pacific Conf. AEPA 2010*, no. 1987, pp. 30–34, 2011, doi: 10.1142/9789814324052_0006.

[37] X. Chen, L. Sun, W. Zhao, and Y. Zheng, "Effect of loading rate on tensile and failure

behavior of concrete," *Sensors (Switzerland)*, vol. 20, no. 21, pp. 1–17, 2020, doi: 10.3390/s20215994.

[38] M. Słowik, "The analysis of failure in concrete and reinforced concrete beams with different reinforcement ratio," *Arch. Appl. Mech.*, vol. 89, no. 5, pp. 885–895, 2019, doi: 10.1007/s00419-018-1476-5.

[39] A. M. Arafah, "Factors affecting the reliability of reinforced concrete beams," *Manag. Inf. Syst.*, no. 1, pp. 379–387, 2000.

[40] S. W. Tabsh, "Safety of reinforced concrete members designed following ACI 318 building code," *Eng. Struct.*, vol. 19, no. 10, pp. 843–850, 1997, doi: 10.1016/S0141-0296(97)00169-7.

[41] C. Trentin and J. R. Casas, "Safety factors for CFRP strengthening in bending of reinforced concrete bridges," *Compos. Struct.*, vol. 128, pp. 188–198, 2015, doi: 10.1016/j.compstruct.2015.03.048.

[42] C. Clason, "Finite Element Methods," *Hist. Mech. Mach. Sci.*, vol. 20, pp. 141–183, 2017, doi: 10.1007/978-94-007-1165-5_13.

[43] O. C. Zienkiewicz, "The Finite Element Method: Its Basis and Fundamentals," *Finite Elem. Method its Basis Fundam.*, p. iii, 2013, doi: 10.1016/b978-1-85617-633-0.00020-4.

[44] V. Jagota, A. P. S. Sethi, and K. Kumar, "Finite element method: An overview," *Walailak J. Sci. Technol.*, vol. 10, no. 1, pp. 1–8, 2013, doi: 10.2004/wjst.v10i1.499.

[45] H. M. H. Al-khafaji, "Introduction to Simulation with ANSYS Mechanical APDL," no. 10, p. 252, 2016.

[46] S. Moaveni, "Finite element analysis—theory and application with ANSYS," *Minerals Engineering*, vol. 12, no. 8. pp. 992–993, 1999, doi: 10.1016/s0892-6875(99)90030-4.

[47] M. R. Bahrami, "ANSYS Tutorial : An Introduction to Elasticity and Contact Problems in," no. June, pp. 0–69, 2018.

[48] A. Javidinejad, "FEA Practical Illustration of Mesh-Quality-Results Differences between Structured Mesh and Unstructured Mesh," *ISRN Mech. Eng.*, vol. 2012, pp. 1–7, 2012, doi: 10.5402/2012/168941.

[49] KEGAN LECKNESS, "NOVEL FINITE ELEMENT METHOD TO PREDICT BLAST WAVE TRANSMISSION THROUGH HUMAN EAR," *Univ. OKLAHOMA*, vol. 9, no. August, p. 10, 2016, [Online]. Available:

https://www.infodesign.org.br/infodesign/article/view/355%0Ahttp://www.abergo.org.br/revista/index.php/ae/article/view/731%0Ahttp://www.abergo.org.br/revista/index.php/ae/article/view/269%0Ahttp://www.abergo.org.br/revista/index.php/ae/article/view/106.

[50] M. Rabi, K. A. Cashell, and R. Shamass, "Flexural analysis and design of stainless steel reinforced concrete beams," *Eng. Struct.*, vol. 198, no. August, p. 109432, 2019, doi: 10.1016/j.engstruct.2019.109432.

[51] B. M. Žarko Petrovi´, A. Zori´, S. Rankovi, and B. M. and D. Zlatkov, "applied sciences Flexural Behavior of Continuous Beams Made of Self-Compacting Concrete (SCC)—Experimental and Numerical Analysis."

[52] G. R. Plasencia, J. D. B. Rocha, J. J. H. Santana, and L. Pudipedi, "Study of the behavior of reinforced concrete deep beams. Estimate of the ultimate shear capacity," *Rev. la Constr.*, vol. 16, no. 1, pp. 43–56, 2017, doi: 10.7764/RDLC.16.1.43.

[53] D. Salim T.Yousif, M. A. AL- Jurmaa, and M. A. AL- Jurmaa, "Modeling of ultimate load for R.C. beams strengthened with Carbon FRP using artificial neural networks," *AL-Rafdain Eng. J.*, vol. 18, no. 6, pp. 28–41, 2010, doi: 10.33899/rengj.2010.34813.

[54] X. C. Jiarui Liu, "Research on flexural performance of reinforced of Damaged RC Beams Streangthen by FPR Plates," *E3S Web Conf.*, vol. 03007, 2021.

[55] W. H. Khaleel, A. A. Dawood, K. S. Abdul-Razzaq, A. A. Talal, and S. N. Al-Karawi, "Reinforced Concrete Curved Beams in Literature," *IOP Conf. Ser. Mater. Sci. Eng.*, vol. 1105, no. 1, p. 012098, 2021, doi: 10.1088/1757-899x/1105/1/012098.

[56] R. Djamaluddin, "Flexural behaviour of external reinforced concrete beams," *Procedia Eng.*, vol. 54, pp. 252–260, 2013, doi: 10.1016/j.proeng.2013.03.023.

[57] P. Shafigh, M. H. Hassanpour, S. V. Razavi, and M. Kobraei, "An investigation of the flexural behaviour of reinforced lightweight concrete beams," *Int. J. Phys. Sci.*, vol. 6, no. 10, pp. 2414–2421, 2011, doi: 10.5897/IJPS10.550.

[58] N. A. Nariman, M. Husek, I. I. Mohammad, K. O. Ahmed, D. Dilshad, and I. Khidr, "Analysis of stiffness and flexural strength of a reinforced concrete beam using an invented reinforcement system," *Front. Struct. Civ. Eng.*, vol. 15, no. 2, pp. 378–389, 2021, doi: 10.1007/s11709-021-0706-z.

[59] H. Baji and H. R. Ronagh, "Investigating the reliability of RC beams of tall buildings designed based on the new ACI 318-05/ASCE 7-05," *Struct. Des. Tall Spec. Build.*, vol.

21, no. 8, pp. 592–604, 2012, doi: 10.1002/tal.638.

[60] S. Nisreen Ali, “DESIGN OF REINFORCED CONCRETE STRUCTURES,” no. Part I, p. 6, 2021.

[61] C. T. F. R. JOHN CASE, LORD CHILVER, “Strength of materials and structures,” *Strength Mater.*, vol. 38, no. 4, pp. 330–347, 1999, doi: 10.1007/s11223-006-0048-z.

[62] P. J. Uno and A. S. Beletich, “Design Handbook for Reinforced Concrete Elements,” 2003.

[63] R. F. Mast, “Analysis of cracked prestressed concrete sections: A practical approach,” *PCI J.*, vol. 43, no. 4, pp. 80–90, 1998, doi: 10.15554/pcij.07011998.80.91.

[64] J. G. M. JAMES K. WIGHT, *REINFORCED CONCRETE Mechanics and Design*. 2012.

[65] American Wood Council, “Beam Design Formulas With Shear and Moment Diagrams,” *Des. AID No. 6*, p. 20, 2005, [Online]. Available: http://www.awc.org/pdf/codes-standards/publications/design-aids/AWC-DA6-BeamFormulas-0710.pdf.