

Procurement Strategies of a Commodity with a Spot Market

by

Mustafa Gökçen Göksel

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Procurement Strategies of a Commodity with a Spot Market

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Mustafa Gökçen Göksel

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and that any and all revisions required by the final
examining committee have been made.

Committee Members:

Prof. Fikri Karaesmen

Prof. Gürhan Kök

Asst. Prof. Bora Çekyay

Date: _____



Dedicated to my late grandfather.

ABSTRACT

We investigated the single period procurement policy of a commodity which is traded in a spot market. The presence of a spot market differentiates the problem definition from the classical single period inventory models, and requires different solution strategies. We have proposed two data-driven linear programs to find the optimal procurement strategies for two different objectives: (i) expected profit maximization (risk-neutral), (ii) minimization of CVaR (risk-averse). Our proposed solution approaches use historical demand and price data as well as the historical data of exogenous variables which are assumed to be correlated with demand and/or price. We have conducted a simulation to ensure the validity of the models. According to the simulation results, the risk-neutral case is able to yield near-optimal solutions for all price processes and all price-demand dependence cases. The risk-averse model is able to outperform the risk-neutral one in terms of CVaR risk measure for certain cases of price processes and price-demand dependence cases. The presence of the exogenous variables improved the performance of both programs.

ÖZETÇE

Spot piyasası bulunan emtiaların tek dönemlik satın alma politikalarını inceledik. Spot piyasasının varlığı, problem tanımını diğer tek dönemlik envanter modellerinden ayırmaktadır ve bu sebeple daha farklı çözüm stratejileri gerektirir. Beklenen kar maksimizasyonu (riske nötr) ve CVaR minimizasyonu (riskten kaçınan) olmak üzere iki farklı amaca yönelik optimal satın alma stratejilerini belirlemek adına iki veri güdümlü doğrusal program öneriyoruz. Önerdiğimiz çözüm yaklaşımı, geçmiş talep ve fiyat verilerini, ayrıca talep ve fiyat ile aralarında bir korelasyon olduğu varsayılan geçmiş dışsal değişken verilerini de kullanmaktadır. Modellerin geçerliliğinden emin olmak için bir simülasyon gerçekleştirdik. Simülasyon sonuçlarına göre, riske nötr durum, tüm fiyat süreçlerinde ve fiyat ve talebin birbirine bağlı olduğu durumlarda optimale yakın çözümler üretebilmektedir. Riskten kaçınan model ise belirli fiyat süreçlerinde ve fiyat ve talebin birbirine bağlı olduğu durumlarda CVaR risk ölçümlerinde riske nötr durumdan daha iyi performans göstermiştir. Ayrıca, dışsal değişkenlerin varlığı programların performansını yükseltmiştir.

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NOMENCLATURE

X_t	the random variable X at period t
F_X	the cumulative distribution function a random variable X
F_X^{-1}	inverse of the cumulative distribution function a random variable X
$E[X_t]$	expected value of the random variable X_t
$E_t[X_t]$	expected value of the random variable X_t , given that all the information until time t is known.
$Var(X_t)$	variance of the random variable X_t
$Var_t(X_t)$	variance of the random variable X_t , given that all the information until time t is known.
$Corr(X_t, Y_t)$	correlation function of the random variables X_t and Y_t
$Corr_t(X_t, Y_t)$	correlation function of the random variables X_t and Y_t , given that all the information until time t is known.
$\Pi(.)$	Profit function
$\Lambda(.)$	Loss function
$f_t^{(i)}$	a data point of feature $f^{(i)}$, at time t
h	the parameters array
$\theta^{(i)}$	i th element of the parameters array, the coefficient of feature $f^{(i)}$.
μ_P	expected value of the random variable $E[P]$.
ϵ_u	penalty cost per item if the demand is satisfied from the spot market
ϵ_o	penalty cost per item if the excess inventory is sold to the spot market



CVaR	Conditional Value-at-Risk
VaR	Value-at-Risk
ARMA	Auto-regressive Moving Average Model
EP-LP	the data-driven linear program with expected profit maximization in the simulation model
CVAR-LP	the data-driven linear program with CVaR minimization in the simulation model
TH-EP	the theoretical benchmark of the expected profit maximization in the simulation model

Chapter 1

INTRODUCTION

Procurement decisions of commodities are essential managerial decisions in most sectors, and due to different problem settings, the types of procurement decisions vary. This dissertation focuses on the procurement decisions of the single period inventory models which are widely studied in Operations Research literature. Different from the classical single period inventory model, we focus on the procurement policies of commodities which have a spot market. The presence of the spot market entails several modifications to the classical model. The selling price of the commodity is determined by the market price which is volatile. In addition, all the excess inventories can be liquidated in the spot market by paying a penalty costs per unit item. Similarly, any unsatisfied demand can be met by the spot market. Although this problem definition covers many commodities, it would be most suitable for electricity procurement, since the electricity can be traded in real-time via its spot market.

The presence of a spot market changes the dynamics of the procurement problem in several ways. While it provides an immediate trade opportunity which causes high service levels, the volatility of the price increases the complexity of the procurement problem. Therefore, using solution methods derived for the constant price settings may not address the characteristics of the problem and the solutions might be sensitive to the price fluctuations and may yield risky outcomes. Although there are certain solution methodologies that are specifically modeled for the volatile price environments, most of the solution methodologies in the existing literature require probability distribution functions of the demand, and/or the price, and even the underlying stochastic model of the price process if the problem is defined in a multi-period environment,

to be known. However, in practice these distributions are unknown and need to be estimated from previous data, or by any other technique. The methods which do not require the estimation of such distributions but use the historical data directly are called data-driven optimization.

Data-driven optimization is a well studied topic in Operations Research literature, yet it became more popular in recent years because of the availability of abundant data in almost every sector. [Ban and Rudin, 2014] proposed a data-driven linear programming approach to solve the classical Newsvendor Problem that uses the historical demand data as well as the data of exogenous variables (features) that are assumed to describe demand information. Their aim is to exploit the information provided by the exogenous variables to improve the maximum expected profit yielded. We have modified the method provided by [Ban and Rudin, 2014] to solve the single period procurement problem of commodity with a spot market with expected profit maximization objective (risk-neutral), instead of the classical newsvendor formulation. In addition to historical data of demand and the exogenous variables, our model uses historical price information to develop a procurement policy with respect to the underlying price process of the spot market. We showed that the solution approach is able to generate optimal strategies for single period procurement problems of commodities which have a spot market.

Another difficulty in finding optimal procurement policies for commodities which are traded in a spot market is the price-demand dependence. Since both demand and price are uncertain, they can possibly be correlated. The simulation results showed that our risk-neutral model is capable of not only exploiting the information provided by the exogenous variables, but also finding optimal procurement policies in the presence of the price-demand dependence.

In addition to the risk-neutral case, we have proposed a new solution approach for the risk-averse decision makers. Although finding theoretical results might be straightforward for the expected profit maximization case, this is not true for the risk-averse case. The existing literature does not have a closed form solution that

covers our problem definition, when the risk aversion is applied via minimization of Conditional Value-at-Risk measure (CVaR is explained in detail in Chapter 2). We have proposed a data-driven linear model which uses the Conditional Value-at-Risk measure to implement risk-averse objective. Our model uses historical demand and price data as well as the exogenous variables to produce a risk-averse procurement policy. We showed that the approach is successful for certain price processes and price-demand dependence levels.

In order to test the performance of the proposed solution approaches we have constructed a simulation model. The simulation model generates stationary price processes with different characteristics in order to test the performance of the approaches under different cases. We also tested the effect of price-demand dependence, which would be important in case of commodities with spot market, by generating several additive demand model to generate different correlation levels between the price and the demand. We have generated synthetic features (exogenous variables) that are correlated with demand and tested different correlation levels and different functional forms.

The simulation results showed that the expected profit maximization case is able to generate near-optimal policies for out-sample tests for all price processes and different price-demand dependence levels, for no feature cases. Although we do not have a theoretical benchmark for the cases when exogenous variables are available, the data-driven approach with features was able to outperform the no feature cases, and therefore we conclude that the data-driven approach is able to exploit the information provided by the features.

The simulation results of the risk-averse case showed that the performance of the data-driven approach depends on the variance of the price process and the correlation level between the price and the demand. Although the data-driven approach is able make notable improvements for some cases, it is not able make improvements in terms of risk-aversion with respect to the expected profit maximization objective. Therefore, while the data-driven approach for the risk aversion case works well for certain cases,

the approach may not be applicable for all cases.

The structure of this document is as follows: In Chapter 2, problem definition and the review of the related literature is presented. We also provided background information for the related concepts. Chapter 3 is on the data-driven linear programming approach for solving the single period problem under two different objectives: expected profit maximization, and the CVaR minimization (risk-averse). First the solution methods are introduced, then the simulation model is explained, and finally the simulation results are discussed. Detailed simulation results can be found in Appendix A, B and C. Finally, Chapter 4 is the conclusion part where the summary of the dissertation, as well as the key findings and future research directions are presented.

Chapter 2

LITERATURE REVIEW AND BACKGROUND

In this chapter the problem definition and solution strategies are introduced to position the contribution made to the existing literature. After a brief problem definition, we present the preliminary information which is needed to understand the model and solution approaches. Then, the literature that is comparable to this work is reviewed.

Problem Definition:

Our problem is to find the optimal procurement strategy of a commodity which is traded in a spot market. The retail price of the commodity changes over the discrete time intervals (periods) and it is determined by the spot market which evolves randomly. The decision maker is assumed not to have an impact on the market prices, and s/he needs to satisfy an uncertain demand in each period. The decision maker cannot carry inventory, i.e. s/he always starts all periods with zero inventory. If the realized demand for the commodity happens to be greater than the total procurement quantity for that period, the shortage amount is satisfied from the spot market immediately. However, a penalty cost ϵ_u TL incurs per item purchased from the market. The analogous case, excess inventory, has the same condition. If the realized demand happens to be smaller than the total procurement for that period, the excess amount is sold to the spot market with a penalty cost of ϵ_o TL per item. The mathematical relationship of the selling price and demand is modeled differently for each chapter.

In the following section (Section 2.1), we present a detailed definition of the Conditional Value-at-risk (CVaR), as well as its properties which make it more favorable than its alternatives. Section 2.2 will cover the existing literature on risk-sensitive decision making for the newsvendor problem, particularly via the CVaR risk measure,

and the newsvendor problem in Price-Demand dependent environment. Section 2.3 introduces the existing literature on data-driven optimization and its applications on the newsvendor problem, and discusses the extensions to the problem definitions such as the features affecting the demand, and the newsvendor problems with mean-CVaR objective function.

2.1 Conditional Value-at-Risk

Conditional Value-at-Risk (CVaR), also known as expected shortfall, is a widely accepted risk measure and studied in the Operations Research literature extensively because of its intuitive definition and convenient mathematical properties. β -CVaR of a random variable is the expectation of the values less than $(1 - \beta)$ percentile of the random variable, i.e. β -CVaR equals to average of the smallest $100\beta\%$ of the values of the corresponding random variable. [Rockafellar and Uryasev, 2000] first proposed the formal definition of CVaR and came up with the mathematical properties which made CVaR more preferable to alternative downside risk measures. Using the notation provided by [Rockafellar and Uryasev, 2000], $f(x, y)$ is the loss function where $x \in \mathbb{R}^n$ is the decision vector and $y \in \mathbb{R}^m$ is the random vector with probability density function $p(y)$. The probability of loss which does not exceed a given threshold α is defined as $\Psi(x, \alpha)$.

$$\Psi(x, \alpha) = \int_{f(x, y) \leq \alpha} p(y) dy$$

We assume that $\Psi(x, \alpha)$ is continuous. A formal CVaR definition and its mathematical properties for general loss distributions, without requiring a continuity assumption on the distribution function, can be found in [Rockafellar and Uryasev, 2002].

Let $\alpha_\beta(x)$ be Value-at-Risk with level β (β -VaR), and $\phi_\beta(x)$ be the Conditional Value-at-Risk with level β (β -CVaR).

$$\alpha_\beta(x) = \min\{\alpha \in \mathbb{R} : \Psi(x, \alpha) \geq \beta\}$$

$$\phi_\beta(x) = \frac{1}{1 - \beta} \int_{f(x, y) \leq \alpha_\beta(x)} f(x, y) p(y) dy$$

This representation requires calculation of $\alpha_\beta(x)$, to calculate the $\phi_\beta(x)$.

[Rockafellar and Uryasev, 2000] showed crucial mathematical properties can be found via the function $F_\beta(x, \alpha)$ which is defined as;

$$F_\beta(x, \alpha) = \alpha + \frac{1}{1-\beta} \int_{y \in \mathbb{R}^m} [f(x, y) - \alpha]^+ p(y) dy$$

[Rockafellar and Uryasev, 2000] proved that if $F_\beta(x, \alpha)$ is convex and continuously differentiable in α , the following expression holds.

$$\phi_\beta(x) = \min_{\alpha \in \mathbb{R}} F_\beta(x, \alpha)$$

This expression indicates that the CVaR of a loss function can be calculated without calculation of the VaR of the function. Since it only requires to calculate an integral, numerical methods can be used to compute the CVaR value.

[Rockafellar and Uryasev, 2000] also proved that minimization β -CVaR ($\phi_\beta(x)$) can be achieved by minimizing $F_\beta(x, \alpha)$ over x and α . This theorem provides a handy optimization technique since $F_\beta(x, \alpha)$ can be minimized, to find optimal β -CVaR, otherwise direct minimization β -CVaR requires finding β -VaR first which is not necessarily convex. Furthermore, since the integral can be approximated by a finite sum, in stochastic programming, CVaR can be modeled via linear equations which is not the case in VaR.

Another advantage of using CVaR over VaR is its being a "Coherent Risk Measure". A risk measure is called coherent, if it satisfies the following properties: monotonicity, subadditivity, homogeneity, and translational invariance. The definition is proposed in [Artzner et al., 1999], it defines the properties a risk measure should have. CVaR satisfies all the properties, while VaR fails to satisfy one of the properties (subadditivity). A detailed review on the coherence of CVaR, its advantages over VaR and its mathematical properties can be found in [Acerbi and Tasche, 2002a], [Acerbi and Tasche, 2002b], and [Tasche, 2002].

Since CVaR considers the expectation under a certain threshold, it is sensitive to the long tails. Therefore it is a more useful risk measure than its competing

risk measures such as Value at Risk (VaR) to capture worst case scenarios with relatively high losses with respect to the mean of the distribution. A more formal and general proposition stating the relation between VaR and CVaR can be found in [Bertsimas et al., 2004]. They examined the mathematical properties of CVaR and its relation with other risk measures such as VaR and standard deviation. [Bertsimas et al., 2004] also showed that mean-shortfall optimization can be modeled as a linear optimization model whereas the classical mean variance optimization method proposed by [Markowitz, 1952] for portfolio selection can only be modeled as quadratic problem.

For continuous distributions, [Krokhmal et al., 2002] showed minimizing negative expected return (i.e. maximizing expected return) with a CVaR constraint yields the same efficiency frontier with minimizing CVaR with a constraint on expected return. Therefore, a stochastic optimization problem having a risk constraint can be modeled as a linear model by using CVaR as a risk measure.

2.2 Newsvendor Problem

The Newsvendor Problem is a well-known and researched problem in Operations Research. The problem is to find the order quantity which maximizes expected profit when there is an uncertain demand. If the realized demand happens to be greater than the order quantity, there is shortage penalty (s TL) for per unit excess demand. If the realized demand happens to be smaller than the order quantity, the excess inventory can be sold to salvage value (r TL). The unit selling price of item is p TL, and purchase cost of per item is c TL. The solution of the problem is straightforward and balances costs occurring in excess and shortage cases. The profit function $\Pi(q, D)$ is,

$$\Pi(q, D) = (p - c)D - (p + s - c)(D - q)^+ - (c - r)(q - D)^+$$

where q is the order quantity, and D is the random variable that represents the demand. F_D denotes the cumulative distribution function of demand and optimal

order quantity (q^*) which maximizes the expected value of the profit function is,

$$q^* = F_D^{-1}\left(\frac{u}{u+o}\right)$$

where $u = p + s - c$ is the underage cost, and $o = c - r$ is the overage cost.

In our problem definition, when the decision maker is restricted to make a decision to satisfy the demand of a single period, the resulting problem has a newsvendor problem representation. For the single period case, the cost of a unit of product is c TL, the retail price of the item, equivalently the market price during the selling period is P TL, and random demand realized in the selling period is D units. F_D is the cumulative distribution function of the random demand D , and μ_P is the expected retail price during the selling period. The single period profit function of the decision maker is

$$\Pi(q, P, D) = (P - c)q - \epsilon_u(D - q)^+ - \epsilon_o(q - D)^+$$

Using the relation $q = D - (D - q)^+ - (q - D)^+$, the problem is equivalent to the classical newsvendor problem

$$\Pi(q, P, D) = (P - c)D - (P - c + \epsilon_u)(D - q)^+ - (-P + c + \epsilon_o)(q - D)^+$$

with the corresponding underage cost $u = P - c + \epsilon_u$, and overage cost $o = -P + c + \epsilon_o$. Therefore, our problem is a version of the newsvendor problem where price is uncertain and the underage and overage costs are functions of this random price.

2.2.1 Newsvendor Problem with Risk Consideration

Risk sensitive decision making in single period problems has been a popular topic. There is a number of different approaches to represent the risk in stochastic models. One of these approaches is to use utility theory which focuses on the optimization of the utility functions, since a utility function is the representation of one's risk profile by definition. Another method of modeling risk sensitivity is to use risk measures. Although there are plenty of different risk measures, I will focus on the literature on the CVaR, since it is the risk measure that I used in my model.

[Ahmed et al., 2007] analyzed a newsvendor problem with a coherent risk measure minimization objective. They have both investigated single and multi-period problems. In the single period case, they found that the structure of the optimal solution is similar to the original expected cost minimization problem. In addition to their solution framework for general coherent risk measures, for a special condition they provided a specific solution for CVaR risk measure.

[Gotoh and Takano, 2007] proposed a closed form solution to the single period problem with CVaR minimization objective. Using theorems proposed by [Rockafellar and Uryasev, 2000], $F_\beta(q, \alpha)$ function being the objective to be minimized,

$$F_\beta(q, \alpha) = \alpha + \frac{1}{1 - \beta} \int_{\xi \in \mathbb{R}^m} [f(q, \xi) - \alpha]^+ p(\xi) d\xi$$

where $f(q, \xi)$ is the loss function associated with the newsvendor problem.

[Gotoh and Takano, 2007] found optimal solutions for two different loss functions defined from the newsvendor problem. For $f(q, \xi) = -\Pi(q, \xi)$, i.e. negative profit or net loss, the optimal solution (q^*, α^*) which minimizes $F_\beta(q, \alpha)$ is,

$$q^* = \frac{o + v}{o + u} F_D^{-1} \left(\frac{u(1 - \beta)}{o + u} \right) + \frac{u - v}{o + u} F_D^{-1} \left(\frac{o\beta + u}{o + u} \right)$$

$$\alpha^* = \frac{o(u - v)}{o + u} F_D^{-1} \left(\frac{o\beta + u}{o + u} \right) + \frac{u - v}{o + u} F_D^{-1} \left(\frac{u(1 - \beta)}{o + u} \right)$$

where $v = u - s$. Note that when there is no penalty cost ($s = 0$), the optimal solution has a much simpler form which is,

$$q^* = F_D^{-1} \left(\frac{u}{o + u} (1 - \beta) \right)$$

$$\alpha^* = -uq^*$$

[Chen et al., 2014] modeled the single period newsvendor model with CVaR as a satisficing measure which is used to ensure that the expected profit reaches to a target with a given confidence level (β of CVaR). Since their model does not have shortage penalty, [Chen et al., 2014] have reached the same optimal solution with the

no shortage penalty cost case of [Gotoh and Takano, 2007] which is intuitive because minimizing negative profit and maximizing positive profit are identical. To find the optimal β they implemented a binary search on β . Finally, [Chen et al., 2014] have showed that mathematical characteristics of the solution are the same with the Certainty Equivalent formulation which is the utility function approach to model the risk.

[Jammernegg and Kischka, 2007] used a mean-CVaR approach to model risk - sensitive behavior of the decision maker. The objective function of their model is the convex combinations of CVaR and the expected profit. According to the weights of these components, the problem can be turned into the original newsvendor problem to CVaR minimization problem. Therefore their model can grasp a wider spectrum of risk preferences. [Xu and Li, 2010] also worked on the same problem setting and they reached the same result with [Gotoh and Takano, 2007] for the CVaR minimization case, and a similar result with [Jammernegg and Kischka, 2007] for mean-CVaR objective. However, the model of [Xu and Li, 2010] also includes the shortage penalty. [Choi and Ruszczyński, 2008] proved a broader result which establishes that any risk-averse newsvendor problem with general law invariant coherent measure of risk, has an equivalent mean-risk objective representation, i.e. mean-CVaR objective.

[Xinsheng et al., 2015] approach to the risk-averse newsvendor problem from a different perspective. [Xinsheng et al., 2015] use a function called legacy loss rather than the profit function, which only takes into account the cost parts of the original profit function. [Xinsheng et al., 2015] found closed form solutions for minimization of expected legacy loss function, minimization of CVaR of legacy loss function and, minimization of their convex combinations (similar to the mean-CVaR approach).

2.2.2 Newsvendor Problem with Price-Demand Dependence

Although there are several papers on the newsvendor problem having price-demand dependence, all of the papers cited above assume the price and demand are independent, moreover none of them assumes that the price is a random variable. Price-

demand dependence is usually used to model the newsvendor problems with pricing options. [Petruzzi and Dada, 1999] investigated a special version of the problem where the demand distribution is a function of the price and their model suggests that the decision maker can control not only the stock levels but also the selling price. [Petruzzi and Dada, 1999] have found the optimal strategy of choosing selling price and stock levels for maximizing expected profit under given conditions for both linear and multiplicative demand-price dependence.

[Agrawal and Seshadri, 2000] took a step further and considered the risk-averse decision makers, too. Similar to [Petruzzi and Dada, 1999], the decision maker should decide on the price and the order quantity, and the demand depends on the price. Nevertheless their objective is to maximize a utility function rather than the profit maximization. For the risk neutral utility functions, which is identical to profit maximization, [Agrawal and Seshadri, 2000] found slightly different results with respect to [Petruzzi and Dada, 1999], because of differences in problem definitions. For the risk averse case, [Agrawal and Seshadri, 2000] reported how optimal decisions change with respect to the risk neutral case. Although their work encapsulates both risk aversion and price-demand dependence for single period inventory models, there are two major factors which differentiate my work from theirs. First, they have used utility functions to measure risk rather than a risk measure, particularly CVaR. Second, they had control over the price, which my model does not cover.

The model proposed by [Chen et al., 2009] covers both the CVaR criterion and price-demand dependence. They modeled the demand as a function of price and a random variable ($D(p) = d(p, X)$). If the demand function satisfies certain mathematical properties whose details can be found in their paper, for a fixed price and general demand distributions, they found similar results to the independent price case. The optimal order quantity as a function of price is $q^*(p) = F^{-1}\left(p, \frac{u}{\sigma+u}(1-\beta)\right)$ where $F(p, x)$ is the c.d.f. of $d(p, X)$ (note that, there is no shortage penalty cost). They also investigated the cases which price is also a decision variable. [Chen et al., 2009] showed that, for both multiplicative and additive cases of demand a unique optimal

price exists; in addition to that, for the additive case it is less than the risk-neutral optimal price. [Xu, 2010] studied a very similar problem, the only difference is that, the excess demand can be satisfied via emergency procurement which inhibits the lost sale case. The results are significantly similar to [Chen et al., 2009], except for the small deviations due to the lack of lost sales.

While its problem definition is similar to the problem proposed, [Chen et al., 2009] fails to respond the modifications on newsvendor problem where price is a random variable and underage and overage costs are the functions of price. The other major difference of their model is the role of price in their problem. Since they have suggested price-demand dependence in order to observe the effect of the pricing decisions and price is a decision variable in their model. Therefore, the solution provided by [Chen et al., 2009] does not cover the problem proposed.

For a more broad and detailed review of literature and recent publications on newsvendor problem, please refer to [Qin et al., 2011].

2.3 Data-Driven Optimization

Different optimization approaches have been used in order to find the optimal order quantities in single period inventory problems (the Newsvendor Problem) under different objectives. Even though we assume that the demand distributions are known and apply the probability laws to find the optimal levels, in real life the demand distributions are not known and they should be estimated from historical demand data. The classical approach first estimates the demand probability distribution and its parameters based on the historical data, then performs optimization based on the estimated parameters.

There are several alternatives to this classical separated estimation and optimization approach. [Liyanage and Shanthikumar, 2005] proposed operational statistics which combines the estimation of distribution parameters and optimization to solve inventory models. [Akçay et al., 2011] showed the effect of length of historical demand data and several other factors on the accuracy of the optimization in the

separated parameter estimation and optimization case, and they showed optimal ordering levels considering the uncertainty caused by the estimation of the demand parameters. These approaches are called parametric approaches, since they still estimate the distribution parameters. However there are non-parametric approaches where the optimization model does not require the estimation of a parametric distribution function. More information can be found in [Godfrey and Powell, 2001], [Huh and Rusmevichientong, 2009], and [Huh et al., 2011] about different non-parametric approaches used to solve inventory problems.

Data-driven optimization is another non-parametric approach which does not require to estimate the distributions of the random variables but uses historical observations directly in the mathematical models. In this section, we will focus on the data-driven optimization methods where the demand distribution is unknown but the historical demand data is available. [Kleywegt et al., 2002] proposed a method called Sample Average Approximation (SAA) to solve stochastic optimization problems via Monte Carlo simulation approach. The method can be summarized as follows; suppose that the minimum value of the expectation of a function $g(q, D)$ is r^* , where q is a real valued decision vector and defined in finite set $\mathcal{Q}$, and D is a random variable. r^* can be found by the following expression:

$$r^* = \min_{q \in \mathcal{Q}} E[g(q, D)]$$

If a set containing n realizations of the random variable D (i.i.d. random samples, $d = [d_1, \dots, d_n]$) is available, the SAA of r^* is $\tilde{r}^*$ and it is defined as;

$$\tilde{r}^* = \min_{q \in \mathcal{Q}} \frac{1}{n} \sum_{i=1}^n g(q, d_i)$$

[Kleywegt et al., 2002] showed that $\tilde{r}^*$ converges to r^* as n increases.

[Kleywegt et al., 2002] were able to report the bounds on the number of samples required to achieve a specific performance level. [Shapiro, 2003] investigated the statistical properties of the results of the algorithm. [Levi et al., 2007] applied Sample Average Approximation to the newsvendor problem, and found the bounds of number

of samples required to have a close performance to the case where the demand distribution is known. Later, the bound was significantly improved by [Levi et al., 2015].

[Ban and Rudin, 2014] used data-driven optimization techniques to find optimal order quantity of a newsvendor problem, where the demand distribution is unknown but the historical demand data is known prior. Contrary to others, their work also considers the extra information available which might help to describe the demand distribution. [Ban and Rudin, 2014] assume that for each period there is a feature vector f which can describe the demand information with the following relation $D = g(f) + \epsilon$. The demand can be represented as Taylor Series Expansion of g . In other words, there exists a multivariate polynomials of feature vector which describes the demand. The feature vector f is known prior to the demand realization. Some of the possible examples of features are the dependent random variables, seasonality, location, weather, or any information which is related to demand and available prior to the realization of demand.

[Ban and Rudin, 2014] proposed a modified version of SAA which aims to exploit the information provided by the features. Suppose that the $C(q, D)$ is the cost function of the newsvendor model, where q is the order quantity, the minimization of expected cost of the newsvendor model can be expressed as,

$$\min_q E[C(q, D)]$$

In case of the presence of a feature vector f , the minimization of the expected cost of the newsvendor problem is expressed as,

$$\min_q E[C(q(f), D(f))|f]$$

The features are denoted as $f = [[f_1^{(1)}, \dots, f_n^{(1)}], [f_1^{(2)}, \dots, f_n^{(2)}], \dots, [f_1^{(m)}, \dots, f_n^{(m)}]]$, the demand is denoted as $d = [d_1, \dots, d_n]$, where there are n periods, and m features. The proposed formulation is as follows;

$$\min_{q: q(f) = \sum_{j=1}^m \theta^{(j)} f^{(j)}} \frac{1}{n} \sum_{i=1}^n C(q(f), d_i)$$

A more detailed LP formulation of this model is explained in Chapter 3.2.2. For two example cases, they showed that the suggested algorithm has a finite bias and yields asymptotically optimal solutions whereas the Sample Average Approximation method which does not include features in the decision criteria has finite bias and yields sub-optimal (inconsistent) solutions. In addition, they showed the bounds on the out-of-sample performances of the algorithm. [Ban and Rudin, 2014] suggested a quadratic programming version of the model which limits the number of features in the decision criteria by adding a regularization term to the objective function.

[Sachs and Minner, 2014] also worked on the data-driven newsvendor problem when the demand data is censored. Similar to [Ban and Rudin, 2014], thier model estimates the order quantity as a linear function of exogenous variables, by constructing a linear program which uses historical data. Although their approaches were similar, [Sachs and Minner, 2014] focused on the numerical comparison of the algorithm with benchmarks cases and measured the effect of problems settings such as the number of available data, price-demand dependence, etc. However their paper does not provide the mathematical model definitively for the exogenous variables case. Therefore we will refer to [Ban and Rudin, 2014] for introducing exogenous variables in data-driven optimization literature.

Risk sensitive data-driven newsvendor formulations also exist in the literature. [Zhang et al., 2009] used Sample Average Approximation method to solve newsvendor problem with a CVaR constraint and analyzed the convergence rates of solutions with respect to the number of samples. Similarly, [Choi and Ruszczyński, 2008] used a sample based linear problem (data-driven) to solve newsvendor problem with mean-CVaR objective. For portfolio optimization, [Karoui et al., 2011] proposed a different technique called Performance Based Regularization, and they showed that it performs better than Sample Average Approximation method for mean-CVaR objectives.

Among the literature cited throughout this section, [Ban and Rudin, 2014], in terms of solution methodology, and [Choi and Ruszczyński, 2008] and [Zhang et al., 2009], in terms of the risk minimization approaches, are the most related pieces of works

that are most related to Chapter 3 of my thesis. Nevertheless, [Ban and Rudin, 2014] does not address the risk averse behavior but focuses on the expected profit maximization. Another important difference between [Ban and Rudin, 2014] and Chapter 3 is the definition of the price. While [Ban and Rudin, 2014] uses a constant price, my model has a random process which might be correlated with the random demand. In addition the main focus of Chapter 3 is to show the impact of the price process on the decision of data-driven optimization model.

In contrast to [Ban and Rudin, 2014], works of [Choi and Ruszczyński, 2008] and [Zhang et al., 2009] both consider risk important and use CVaR as a risk measure. However their models do not address the effect of neither the exogenous features nor the price uncertainty, and possible correlation between price and demand. Therefore [Zhang et al., 2009] fail to solve the problem proposed.

Robust optimization is another method for solving stochastic problems in a risk averse manner. While the robust optimization is a way to solve the stochastic problems, its way of handling uncertainty is different than the other models. It aims to protect the system from the worst case scenarios caused by the uncertainty. In other words, it finds to best decision when all the decisions are countered by the worst possible realizations within the uncertainty set. The data-driven approach is widely used in robust optimization problems. Rather than being a way of avoiding risk in decision making, and its relevance to the data-driven optimization, robust optimization theory and its applications are less relevant to my work, because of the unique way of defining its objective. More information on data-driven robust optimization can be found in [Bertsimas and Thiele, 2006]. A data-driven robust optimization formulation of newsvendor problem can be found in [Bertsimas and Thiele, 2005].

Chapter 3

SINGLE-PERIOD PROCUREMENT POLICY VIA DATA-DRIVEN LP

As explained in detail in Chapter 2, data-driven optimization is a non-parametric optimization approach where the decision maker uses the historical data directly to make a decision without estimating the parameters of the underlying distributions of the data available.

In this chapter, a data-driven linear programming approach is proposed to determine the procurement policy of the problem defined in the following section. We will investigate the procurement strategies for the cases where the decision maker is risk neutral and risk sensitive. The risk neutral case aims to maximize its expected profit whereas the risk averse case aims to minimize the Conditional Value at Risk of the loss function (negative profit), as it is reviewed in Chapter 2.

The structure of this chapter is as follows; after the problem definition, the first section is on the solution methods of the risk neutral case (expected profit maximization). First, a closed form solution for the expected profit maximization objective is proposed when no features are available. Then, we introduce the data-driven techniques to solve cases which can exploit the information provided by the available features. After a detailed explanation of [Ban and Rudin, 2014], the data-driven LP formulation which provides a basis for our solution methodology, we proposed a data-driven linear program to generate a solution to our problem when the features are available. The next section, another data-driven LP formulation is proposed for the risk averse case (CVaR minimization) when features are available. Since the performance of the data-driven approaches are evaluated by simulation, the next section is the detailed explanation of simulation procedure, selection of parameters, and prepa-

ration of simulation environment. Finally, the last section is on the analysis of the simulation results.

3.1 Problem Definition

Similar to the classical newsvendor problem, the decision maker needs to satisfy an uncertain demand (D units) occurring in a specific time interval, and will make an order before the demand is observed. While the wholesale price per item (c TL) is known before the decision is made, the retail price of the commodity (P TL) is uncertain and will not be realized until the demand is observed. The retail price is determined by the spot market of the commodity. The decision maker is assumed to have no effect on the spot market price. The commodity is perishable, i.e. excess amount of items cannot be stored for the upcoming periods. However, the decision maker is able to sell excess inventory to the spot market with a penalty cost per item (ϵ_o TL) after the demand and the price is realized. Therefore, in case of excess inventory, the decision maker will sell any amount of goods to the spot market with price $(P - \epsilon_o)$ TL. The analogous case is also true. In case of shortage, the decision maker may satisfy the demand from the spot market with a penalty cost per item (ϵ_u TL). In other words, any amount of goods can be purchased from the spot market with price $(P + \epsilon_u)$ TL. Therefore, the profit function of the decision maker is $\Pi(q, P, D)$, where q is the order quantity.

$$\Pi(q, P, D) = (P - c)q - \epsilon_u(D - q)^+ - \epsilon_o(q - D)^+$$

The decision maker does not know the distribution function of the demand as well as underlying price process of the spot market prices. S/he needs to use the historical data of the demand and the spot market prices for n periods and historical data of exogenous variables (features), to find the optimal order quantity. The features are the variables which are possibly correlated with the demand and/or the price, and can be observed prior to demand, and price realization. Examples of the features could be the weather data (such as temperature, wind, humidity etc.), the data of

financial indicators (such as historical exchange rates, stock market prices, etc.), date information in order to capture the seasonality if it exists, or any possible available data which might be related to the demand and/or price.

Now, assume that there are m features each having the information of n periods. The vector $[f_1^{(i)}, f_2^{(i)}, \dots, f_n^{(i)}]$ contains the data of feature i for $i = 1, \dots, m$. The vector f contains all the features, $f = [[f_1^{(1)}, f_2^{(1)}, \dots, f_n^{(1)}], [f_1^{(2)}, f_2^{(2)}, \dots, f_n^{(2)}], \dots, [f_1^{(m)}, f_2^{(m)}, \dots, f_n^{(m)}]]$.

Using the historical data available for demand $d = [d_1, d_2, \dots, d_n]$, the historical data available for price $p = [p_1, p_2, \dots, p_n]$, and the feature vector f the decision maker aims to find the optimal order quantity which yields the best outcome for his/her objective function.

3.2 Maximization of Expected Profit

In this section, we will investigate the solution methods when the objective of the decision maker is to maximize his/her expected profit. We first propose a theoretical solution for when no exogenous variables (features) are available. In the following part, the data-driven linear programming approach proposed by [Ban and Rudin, 2014] is explained. Then a data-driven linear model is formulated by using their approach to solve our problem. Finally, a modified version of the linear model is proposed to improve the performance of the model in terms of the memory requirements.

3.2.1 Theoretical Solution for No Feature Case

Proposition 1. *The closed form solution for the optimal order quantity q^* which maximizes the expected profit at period t when all the information until time t is known,*

$$q^* = \arg \max_{q \geq 0} E_t[(P_t - c_t)q - \epsilon_u(D_t - q)^+ - \epsilon_o(q - D_t)^+]$$

is

$$q^* = \begin{cases} 0 & E_t[P_t] - c_t \leq -\epsilon_u \\ F_{D_t}^{-1} \left(\frac{E_t[P_t] - c_t + \epsilon_u}{\epsilon_u + \epsilon_o} \right) & -\epsilon_u < E_t[P_t] - c_t < \epsilon_o \\ +\infty & \epsilon_o \leq E_t[P_t] - c_t \end{cases}$$

where P_t is the price in period t , D_t is the demand in period t and c_t is the procurement price in period t , and $E_t[\cdot]$ is a conditional expectation where all the information until time t is known.

Proof. Let us note that:

$$E_t[\Pi(q, P_t, D_t)] = E_t[(P_t - c_t)q] - \epsilon_u E_t[(D_t - q)^+] - \epsilon_o E_t[(q - D_t)^+]$$

The derivative with respect to q is:

$$\frac{\partial E_t[\Pi(q, P_t, D_t)]}{\partial q} = (E_t[P_t] - c_t) + \epsilon_u(1 - F_{D_t}(q)) - \epsilon_o F_{D_t}(q)$$

and the second derivative is:

$$\frac{\partial^2 E_t[\Pi(q, P_t, D_t)]}{\partial q^2} = -(\epsilon_u + \epsilon_o)f_{D_t}(q)$$

Since the second derivative is always negative (ϵ_u , and ϵ_o are non-negative by definition), the function is concave, and q which satisfies $\frac{\partial E_t[\Pi(q, P_t, D_t)]}{\partial q} = 0$ will be the optimal order quantity.

$$(E_t[P_t] - c_t) + \epsilon_u(1 - F_{D_t}(q)) - \epsilon_o F_{D_t}(q) = 0$$

if $E_t[P_t] - c_t \leq -\epsilon_u$,

$$q^* = 0$$

if $-\epsilon_u < E_t[P_t] - c_t < \epsilon_o$,

$$q^* = F_{D_t}^{-1} \left(\frac{E_t[P_t] - c_t + \epsilon_u}{\epsilon_u + \epsilon_o} \right)$$

if $\epsilon_o \leq E_t[P_t] - c_t$,

$$q^* = +\infty$$

□

3.2.2 [Ban and Rudin, 2014], Data-Driven Linear Programming Approach for the Newsvendor Problem

As it is briefly explained in Chapter 2.3, [Ban and Rudin, 2014] proposed a data-driven linear program to generate an ordering policy which minimizes the expected cost of the classical Newsvendor Problem (Chapter 2.2).

Suppose that the decision maker does not know the true distribution of the demand (D), yet s/he has the historical data of the demand ($d = [d_1, \dots, d_n]$) and a set of features $f = [[f_1^{(1)}, \dots, f_n^{(1)}], [f_1^{(2)}, \dots, f_n^{(2)}], \dots, [f_1^{(m)}, \dots, f_n^{(m)}]]$ which is believed to explain the demand information in the form of linear combinations.

The proposed linear programming model which combines Sample Average Approximation approach with information of feature set is,

Sets

$\mathcal{I}$: set of periods, historical data of which are available, $\mathcal{I} = \{1, \dots, n\}$

$\mathcal{J}$: set of features, $\mathcal{J} = \{1, \dots, m\}$

Parameters

c : wholesale price of the item

u : underage cost per unsatisfied demand

o : overage cost per unsold item

d_i : demand in period i , $\forall i \in \mathcal{I}$

$f_i^{(j)}$: the value of feature j in period i , $\forall i \in \mathcal{I}$, and $\forall j \in \mathcal{J}$

Decision variables

x_i : order quantity in period i , $\forall i \in \mathcal{I}$

q_i^+ : $(x_i - d_i)^+$, i.e. amount of unsold products in period i , $\forall i \in \mathcal{I}$

q_i^- : $(d_i - x_i)^+$, i.e. amount of unsatisfied demand in period i , $\forall i \in \mathcal{I}$

$\theta^{(j)}$: weight of feature j on the order quantity, $\forall j \in \mathcal{J}$

Mathematical Model

$$\begin{aligned}
& \min_{\theta=[\theta^{(1)}, \dots, \theta^{(m)}]} \frac{1}{n} \sum_{i=1}^n (uq_i^- + oq_i^+) \\
& s.to \ \forall i = 1, \dots, n: \\
& q_i^- \geq d_i - \theta^{(1)} - \sum_{j=2}^m (\theta^{(j)} + f_i^{(j)}) \\
& q_i^+ \geq \theta^{(1)} + \sum_{j=2}^m (\theta^{(j)} + f_i^{(j)}) - d_i \\
& q_i^-, q_i^+ \geq 0
\end{aligned}$$

Note that this model provides a vector $\theta = [\theta^{(1)}, \dots, \theta^{(m)}]$ which minimizes expected cost of newsvendor problem over the sample data set. [Ban and Rudin, 2014] claim that the decision maker can use the resulting vector θ to find the order quantity ($q_t(f_t)$) for a period t which is not used to train the model (out-of-sample), using corresponding period's features f_t , i.e. $q_t(f_t) = \sum_{i=1}^m \theta^{(i)} f_t^{(i)}$.

3.2.3 Mathematical Model for Maximization of Expected Profit

Our claim is that the decision maker can modify the data-driven linear programming approach proposed by [Ban and Rudin, 2014], to solve the problem defined in Section 3.1.

Although the problem is similar to the classical newsvendor model, there are major differences such as the random price and underage and overage costs are being functions of that random demand. Our claim is that a similar approach to [Ban and Rudin, 2014] suffices to generate optimal solutions when the exogenous variables (features) are available.

Since the decision maker aims to maximize the expected profit function when the historical data is available, the objective function is,

$$\max_{x \geq 0} E[\Pi(x, P, D) | (d, p, f)]$$

We constructed h , the parameters array, by using the features and other available information such as historical data of price process. The parameters array is similar

to the feature vector f of [Ban and Rudin, 2014]. We chose to denote it differently to emphasize that it does not only consist of exogenous variables, but it may also include endogenous variables such as previous price realizations.

The claim of [Ban and Rudin, 2014] requires a linear dependence between exogenous features (or their functions) and the demand, in order to be able to use the solution methodology. We extend their claim, and suggest that if the selling price is volatile and generated by a random process, historical price information (or the functions of which) can be used alongside with the available exogenous variables in the parameters array h . *We claim that, there exists an array of parameters, h , a linear combination of which yields a near-optimal solution for the problem.*

$$\tilde{x}_i^* = \sum_{j=1}^{|h|} \theta_j h_{ij}$$

where $\tilde{x}_i^*$ is the near-optimal solution for the i^{th} period.

Since θ is the vector of coefficients, the aim of the mathematical model is to find the θ vector to evaluate order quantity $\tilde{x}_i^*$ given a parameters array h . The mathematical model chooses the optimal θ vector to maximize the sample mean profit of the previous periods. The resulting θ vector which generates optimal $\tilde{x}_i^*$ for each period to yield the maximum profit is used to find the optimal order quantity for the upcoming period.

The decision maker is faced with the question of how to construct a correct parameter array h which suffices to represent the characteristics of an ordering decision. While the scope of this thesis does not cover the optimal strategy of obtaining the optimal parameter array h , the performances of different parameter arrays under different scenarios will be tested by simulation in the following section. Even though this does not provide an optimal strategy, it will help the decision maker in the process of selecting the parameter array.

For instance, if the price is assumed to have an auto-regressive process where price of i^{th} period is heavily depends on the previous price. Then, the price of $(i - 1)^{th}$ period can possibly be selected as one of elements of the parameters array to reflect the effect of i^{th} price on the $(i + 1)^{th}$ price, consequently on the $\tilde{x}_i^*$. Furthermore,

let's assume that the demand is correlated with a feature observed in i^{th} period. Therefore, the resulting parameters array for the i^{th} period to be used to evaluate the order quantity can be selected as $h_i = [1, p_{i-1}, f_i^{(1)}]$, for all i . Note that, the first element of the parameters array is selected as 1. Even though it is not mandatory to allocate first term to 1, it is a useful practice since it provides a historical data free coefficient θ_1 which may serve as an intercept term.

Note that any function of the historical data can be used while constructing the parameters array. For instance, addition to the example case above, if the demand during the weekends happens to be greater than the weekdays, the decision maker may add an indicator function of date data, i.e. $I_{\{f_i^{(2)} \in [Saturday, Sunday]\}}$ where $f_i^{(2)}$ is the date information of i^{th} period and I is the indicator function, $I_{true} = 1$ and $I_{false} = 0$. Then the resulting parameters array for the i^{th} period can be $h_i = [1, p_{i-1}, f_i^{(1)}, I_{\{f_i^{(2)} \in [Saturday, Sunday]\}}]$.

The decision maker is free to make any linear or non-linear transformation to the historical data to come up with a parameters array h . A sample parameters array for the i^{th} period h_i could be

$$h_i = [1, (p_{i-1})^2, d_{i-7}, (d_{i-1}p_{i-1}), I_{\{f_i^{(1)} \leq 0\}}, \sqrt{f_{i-1}^{(2)}}, \log(\frac{f_i^{(3)}}{1 - f_i^{(3)}})]$$

After the parameters array h is constructed, the mathematical model is solved to find θ vector which maximizes the optimal expected profit of the sample data available.

Sets

$\mathcal{I}$: set of periods, historical data of which are available, $\mathcal{I} = \{1, \dots, n\}$

$\mathcal{H}$: set of parameters, $\mathcal{H} = \{1, \dots, m\}$

Parameters

p_i : retail price of the item in period i , $\forall i \in \mathcal{I}$

c_i : wholesale price of the item in period i , $\forall i \in \mathcal{I}$

d_i : demand in period i , $\forall i \in \mathcal{I}$

ϵ_u : penalty cost per item if the demand is satisfied from the spot market

ϵ_o : penalty cost per item if the excess inventory is sold to the spot market

h_{ij} : value of parameter j in period i , $\forall i \in \mathcal{I}$, and $\forall j \in \mathcal{H}$

Decision variables

x_i : order quantity in period i

q_i^+ : $(x_i - d_i)^+$, i.e. amount of product sold to spot market in period i

q_i^- : $(d_i - x_i)^+$, i.e. amount of product purchased from the spot market in period i

θ_j : weight of parameter j on the order quantity, $\forall j \in \mathcal{H}$

Mathematical Model

$$\begin{aligned}
 \max \quad & \frac{1}{n} \sum_{i=1}^n (p_i - c_i)x_i - \epsilon_o q_i^+ - \epsilon_u q_i^- \\
 \text{s.t.} \quad & \forall i \in \mathcal{I} \\
 & x_i = \sum_{j=1}^m \theta_j h_{ij} \\
 & q_i^+ \geq x_i - d_i \\
 & q_i^- \geq d_i - x_i \\
 & x_i, q_i^+, q_i^- \geq 0
 \end{aligned}$$

The linear model has the following memory requirements; for $|\mathcal{I}| = n$, and $|\mathcal{H}| = m$, the number of decision variables is $3n + m$, the number of constraints is $3n$, and the number of non-negativity constraints is $3n$.

Although the model above is straightforward and easy to comprehend, there is a better way of implementing the problem. By not defining the variable x , the user may decrease the memory need of the model.

Modified Version of the Model

$$\begin{aligned}
 \max \quad & \frac{1}{n} \sum_{i=1}^n (p_i - c_i) \sum_{j=1}^m \theta_j h_{ij} - \epsilon_o q_i^+ - \epsilon_u q_i^- \\
 \text{s.t.} \quad & \forall i \in \mathcal{I} \\
 & q_i^+ \geq \sum_{j=1}^m \theta_j h_{ij} - d_i
 \end{aligned}$$

$$q_i^- \geq d_i - \sum_{j=1}^m \theta_j h_{ij}$$

$$\sum_{j=1}^m \theta_j h_{ij} \geq 0$$

$$q_i^+, q_i^- \geq 0$$

The modified version has the following memory requirements; for $|\mathcal{I}| = n$, and $|\mathcal{H}| = m$, the number of decision variables is $2n + m$, the number of constraints is $3n$, and the number of non-negativity constraints is $2n$.

Although, we assure that this model maximizes in-sample expected profit, its out-sample performance should be tested. In Section 3.4, a simulation model is proposed to test both in-sample and out-sample performance of the model for different price processes, different demand models to simulate different price-demand dependence, different features, and different parameters array selections.

3.3 Minimization of CVaR

Consider the case where the decision maker is risk-averse and willing to manage its risk by having a different objective function. Rather than maximizing the expectation of the profit function, s/he will minimize the CVaR of the distribution of a loss function $\Lambda(x, y)$ where x is the decision variable and y is the uncertain vector. First, the mathematical model for the data-driven CVaR minimization of the problem is presented in this section. Then, a modified version is proposed to improve the performance of the model in terms of the memory requirements.

In addition to the variables and parameters defined for the expected profit maximization model (Section 3.2.3), Proposition 2 requires the introduction of the following parameters and variables: β is parameter denoting the CVaR level, α variable is the resulting VaR level of the β -CVaR, π_i is the profit acquired in period i , and z_i is an auxiliary variable.

Proposition 2. *The following linear program minimizes the in-sample β -CVaR of the loss function $-\Pi(q, P, D)$, where $\Pi(q, P, D) = (P - c)q - \epsilon_u(D - q)^+ - \epsilon_o(q - D)^+$, when features are available.*

Mathematical Model

$$\begin{aligned}
\min \quad & \alpha + \frac{1}{(1 - \beta)n} \sum_{i=1}^n z_i \\
\text{s.to} \quad & \forall i \in \mathcal{I} \\
& x_i = \sum_{j=1}^m \theta_j h_{ij} \\
& \pi_i = (p_i - c_i)x_i - \epsilon_o q_i^+ - \epsilon_u q_i^- \\
& z_i \geq -\alpha - \pi_i \\
& q_i^+ \geq x_i - d_i \\
& q_i^- \geq d_i - x_i \\
& x_i, z_i, q_i^+, q_i^- \geq 0
\end{aligned}$$

Proof. Note that, CVaR is the expectation of the losses exceeding a threshold α , which is β percentile of the loss distribution. Suppose that p.d.f. of a loss function $\Lambda(x, y)$ is $f_\Lambda(y)$ (where x is the decision vector, and y is the random vector. See Section 2.1 for a detailed definition of CVaR risk measure).

If $\Lambda(x, y)$ is convex for a fixed y , [Rockafellar and Uryasev, 2000] showed that minimization of CVaR of loss function $\Lambda(x, y)$, can be achieved by minimization of $F_\beta(x, \alpha)$ which is defined as,

$$F_\beta(x, \alpha) = \alpha + \frac{1}{1 - \beta} \int_{y \in \mathbb{R}^m} [\Lambda(x, y) - \alpha]^+ f_\Lambda(y) dy$$

Furthermore, [Rockafellar and Uryasev, 2000] also proposed that $F_\beta(x, \alpha)$ can be approximated by the sampling of the random vector y .

$$\hat{F}_\beta(x, \alpha) = \alpha + \frac{1}{1 - \beta} \sum_{i=1}^n [\Lambda(x, y_i) - \alpha]^+$$

where $[y_1, y_2, \dots, y_n]$ vector consists of n realizations of the random vector y .

In our problem the loss function is the negative of the profit function ($\Lambda(x, [p, d]) = -\Pi(q = x, p, d)$) which is,

$$\Lambda(x, [p, d]) = -(p - c)x + \epsilon_u(d - x)^+ + \epsilon_o(x - d)^+$$

Note that in order to use $\hat{F}_\beta(x, \alpha)$ to minimize CVaR of the loss function, $\Lambda(x, y)$ should be convex in x . $\Lambda(x, [p, d])$ can be represented as piece-wise linear functions.

$$\Lambda(x, [p, d]) = \begin{cases} (-p + c - \epsilon_u)x + \epsilon_u d & x < d \\ (-p + c + \epsilon_o)x - \epsilon_o d & d \leq x \end{cases}$$

$\Lambda(x, y)$ is continuous, i.e. $\Lambda(d, [p, d]) = (-p + c)d$, and it is convex in x for a fixed $[p, d]$, when $\epsilon_o > -\epsilon_u$ which is always true since ϵ_o and ϵ_u are non-negative by definition.

Therefore our problem is to minimize $\hat{F}_\beta(x, \alpha)$,

$$\min_{x, \alpha} \alpha + \frac{1}{1 - \beta} \sum_{i=1}^n [-\Pi(x, p_i, d_i) - \alpha]^+$$

The expression above can be turned into a linear form by removing $[\cdot]^+$ function. We introduce the auxiliary variables z_i , as [Rockafellar and Uryasev, 2000] suggested.

$$\begin{aligned} \min_{x, \alpha} \quad & \alpha + \frac{1}{1 - \beta} \sum_{i=1}^n z_i \\ \text{s.to} \quad & \forall i \in \mathcal{I} \\ & z_i \geq -\Pi(x, p_i, d_i) - \alpha \\ & z_i \geq 0 \end{aligned}$$

A data-driven approach similar to [Ban and Rudin, 2014], can be implemented to this problem as follows;

$$\begin{aligned} \min_{x, \alpha} \quad & \alpha + \frac{1}{1 - \beta} \sum_{i=1}^n z_i \\ \text{s.to} \quad & \forall i \in \mathcal{I} \\ & x_i = \sum_{j=1}^m \theta_j h_{ij} \\ & \pi_i = (p_i - c_i)x_i - \epsilon_o q_i^+ - \epsilon_u q_i^- \end{aligned}$$

$$\begin{aligned}
z_i &\geq -\pi_i - \alpha \\
q_i^+ &\geq x_i - d_i \\
q_i^- &\geq d_i - x_i \\
x_i, z_i, q_i^+, q_i^- &\geq 0
\end{aligned}$$

□

The CVaR minimization model has the following memory requirements; for $|\mathcal{I}| = n$, and $|\mathcal{H}| = m$, the number of decision variables is $5n + m + 1$, and the number of constraints is $5n$, and the number of non-negativity constraints is $4n$.

The memory need of this model can be decreased by eliminating the variables x and π , leading to the following modified model.

Modified Version

$$\begin{aligned}
\min \quad & \alpha + \frac{1}{(1-\beta)n} \sum_{i=1}^n z_i \\
s.to \quad & \forall i \in \mathcal{I} \\
z_i &\geq -\alpha - (p_i - c_i) \sum_{j=1}^m \theta_j h_{ij} + \epsilon_o q_i^+ + \epsilon_u q_i^- \\
q_i^+ &\geq \sum_{j=1}^m \theta_j h_{ij} - d_i \\
q_i^- &\geq d_i - \sum_{j=1}^m \theta_j h_{ij} \\
\sum_{j=1}^m \theta_j h_{ij} &\geq 0 \\
z_i, q_i^+, q_i^- &\geq 0
\end{aligned}$$

The modified version of the CVaR minimization has the following memory requirements; for $|\mathcal{I}| = n$, and $|\mathcal{H}| = m$, the number of decision variables is $3n + m + 1$, the number of constraints is $3n$, and the number of non-negativity constraints is $3n$.

Although this model generates minimum β -CVaR for the periods that is trained (in-sample), its performance should be tested for out-sample data. The simulation model is proposed to test both in-sample and out-sample performance of the model for different settings in the following section.

3.4 Simulation Model

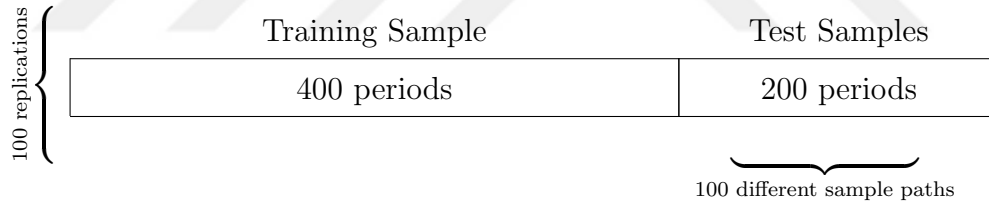
The performance of the data-driven linear programs that are proposed to solve the procurement problem is measured by a simulation model. The simulation model is constructed to test as many scenarios as possible in order to show the solution methodology is capable of providing solution to a wide range of different scenarios.

The simulation model tests a combination of 11 different price processes, 5 different price-demand relations and 2 exogenous variables (one continuous, one binary) each having 2 different correlation levels with demand, which results $11(\text{price process}) \times 5(\text{demand model}) \times 3(\text{continuous feature}) \times 3(\text{binary feature}) = 495$ different scenarios. For each problem scenario, the parameters are selected in such a way that the comparison between the scenarios is possible, i.e. the same and stationary mean for all the price processes, the same variance or the conditional variance (variance at time t when all the information until time $t - 1$ is known), same mean for each demand scenario, and the same variance or the conditional variance for demand. In addition, the random seed, which is used to generate realizations of random variables, is different for all iterations, but the same within an iteration for different problem scenarios in order to eliminate the possible anomalies caused by chance.

Within an iteration, 495 different but equivalent scenario data is generated for 400 consecutive periods each. For each scenario two data-driven approaches are applied to the problem, data-driven linear programs having expected profit maximization(risk neutral) and CVaR minimization(risk averse). Furthermore, for each linear program, 12 different parameter arrays is used to solve each 495 unique problem. Therefore, the simulation model provides the most suitable parameters arrays for the specific scenario.

Each different scenario with 400 periods is solved with 2 different objective and each objective have 12 different solution setup (parameters array) which yields 24 different θ vectors which are the outcomes of the linear programs. Each θ vector is the vector yields the optimal objective value for its objective function given the scenario data. The data that are used to evaluate θ vector is called training data. The accuracy of the model can only be measured via out of sample tests. Therefore, for each iteration and for each scenario, 100 different sample paths of 200 periods are simulated which are called test data. The θ vector evaluated using training data is tested for 100 different test data, for each theta, for each scenario and for each iteration.

Figure 3.1: Simulation setup



The procedure is repeated for 100 iterations and the resulting statistics are analyzed at the last section of this chapter. The following part of this chapter presents the detailed explanations for model and parameter selections to construct different scenarios, selection of parameters array, and selecting benchmarks.

3.4.1 Price Process Settings

While constructing different scenarios to be tested, the selection of the price process model is the primary decision. Since the price is determined via a spot market, the model to be selected should show the characteristics of a spot market model while having the flexibility to reflect different patterns. Furthermore, despite reflecting different patterns, the process should be comparable among different cases. Therefore, I chose the price process to be an ARMA (Auto-regressive - Moving Average Model)

model with different parameters which is able to reflect a wide range of different patterns.

ARMA is a time series model, and it has the following mathematical representation,

$$P_t = k + \varepsilon_t + \sum_{i=1}^{\bar{p}} \phi_i P_{t-i} + \sum_{j=1}^{\bar{q}} \rho_j \varepsilon_{t-j}$$

This process is called $ARMA(\bar{p}, \bar{q})$. P_t is the price at period t (auto-regressive terms), and ε_t is the moving average term at period t , and k is a constant. ε_t are i.i.d. random variables with variance σ^2 .

Note that the price process is stationary, i.e. $\mu_p = E[P_t] = E[P_{t+1}], \forall t \geq 0$, if all zeros of ϕ lie outside of the unit circle. If the process is stationary, then the mean of the process can be found by using this formula:

$$\mu_p = \frac{k}{1 - \phi_1 - \phi_2 - \dots - \phi_{\bar{p}}}$$

Variance of the process $Var(P_t)$ depends on the coefficients of the auto-regressive and moving average terms. I will provide the variances of $ARMA(0, 0)$, $ARMA(1, 0)$, $ARMA(2, 0)$, and $ARMA(1, 1)$, since these are the ones that are used in the simulation.

$$Var(P_t) = \begin{cases} \sigma^2 & ARMA(0, 0) \\ \frac{\sigma^2}{1 - \phi_1^2} & ARMA(1, 0) \\ \frac{1 - \phi_2}{1 + \phi_2} \frac{\sigma^2}{(1 - \phi_2)^2 - \phi_1^2} & ARMA(2, 0) \\ \frac{\sigma^2(1 + \rho_1^2 + 2\phi_1\rho_1)}{1 - \phi_1^2} & ARMA(1, 1) \end{cases}$$

Note that, $Var_t(P_t)$ and $Var(P_t)$ are different measures. $Var_t(P_t)$ is a conditional variance of P_t where all the information until time t is known, while $Var(P_t)$ is the variance of P_t at time 0.

$$Var_t(P_t) = \sigma^2, \forall t \geq 0$$

4 different ARMA process will be simulated; $ARMA(0, 0)$, $ARMA(1, 0)$, $ARMA(2, 0)$, and $ARMA(1, 1)$. All of these processes have the following mathematical representation.

$$P_t = k + \varepsilon_t + \phi_1 P_{t-1} + \phi_2 P_{t-2} + \rho_1 \varepsilon_{t-1}$$

Each different set of parameters k , ϕ_1 , ϕ_2 , ρ_1 and σ^2 variance of ε_t , will generate a different price process. Note that all moving average terms ε are normally distributed with mean 0, and variance σ^2 .

Table 3.1: Coefficient sets of $P_t = k + \varepsilon_t + \phi_1 P_{t-1} + \phi_2 P_{t-2} + \rho_1 \varepsilon_{t-1}$ with resulting $Var_t(P_t)$, and $Var(P_t)$. Note that, all of them are stationary processes with $\mu_p = 100$. The first row is the i.i.d. case where $Var_t(P_t) = Var(P_t) = 25$. $Var_t(P_t)$ of P1-P10 are the same with the i.i.d. case.

Tag ($P_{t-1}, P_{t-2}, \varepsilon_{t-1}$)	k	ϕ_1	ϕ_2	ρ_1	$Var(\varepsilon_t)$ σ^2	$Var_t(P_t)$	$Var(P_t)$
IID : ($\cdot, \cdot, \cdot$)	100	0	0	0	25.00	25.00	25.00
P1 : ($+, \cdot, \cdot$)	30	0.7	0	0	25.00	25.00	49.02
P2 : ($-, \cdot, \cdot$)	170	-0.7	0	0	25.00	25.00	49.02
P3 : ($+, \cdot, +$)	30	0.7	0	0.5	25.00	25.00	95.59
P4 : ($-, \cdot, +$)	170	-0.7	0	0.5	25.00	25.00	26.96
P5 : ($+, \cdot, -$)	30	0.7	0	-0.5	25.00	25.00	26.96
P6 : ($-, \cdot, -$)	170	-0.7	0	-0.5	25.00	25.00	95.59
P7 : ($+, +, \cdot$)	10	0.7	0.2	0	25.00	25.00	111.11
P8 : ($-, +, \cdot$)	150	-0.7	0.2	0	25.00	25.00	111.11
P9 : ($+, -, \cdot$)	50	0.7	-0.2	0	25.00	25.00	39.47
P10 : ($-, +, \cdot$)	190	-0.7	-0.2	0	25.00	25.00	39.47

The simulation model generates 11 different price process for each iteration. The coefficients of each price process are chosen arbitrarily yet they are adjusted to cover a large set of possible price scenarios, while having comparable statistical properties. All of these are stationary processes with mean $\mu_p = 100$. The first process is $ARMA(0,0)$, with mean 100 and variance 25, i.e. prices are identically and independently distributed. Note that variance and conditional variance are equal for the i.d.d case. The next 10 processes are different variations of $ARMA(1,0)$, $ARMA(2,0)$, and $ARMA(1,1)$ which cover all positive and negative parameter combinations, and

the parameters are adjusted to make the conditional variance of the process equals to 25. However their variances do not equal 25. All the simulated processes, their respective parameter, and resulting variance and conditional variance values are listed in Table 3.1.

Within the same iteration, all the processes are generated by using the same random seed, therefore the same random variables. A 100-period sample path of i.i.d. price process is shown in Figure 3.2. The sample paths of other 10 price processes is shown in Figure 3.3 (blue or light processes). Note that all the different price processes are generated via the same random seed (including Figure 3.2). Figure 3.3 is provided to depict the behavior of different ARMA configurations. The graphs on the left consist of the processes with positive ϕ_1 , while the price processes shown in the right graphs have a negative ϕ_1 coefficient which increases the spikiness of the process. We use the term "spikiness" to assess the magnitude of the oscillation around their mean. In addition, in order to emphasize the effect of the variance on the shape of process for different price models, Figure 3.3 provides reference price processes (red or dark) having variance of 25. Note that the price processes, P1 to P10 (blue or light) have a conditional variance of 25, however their variances differ.

Figure 3.2: A 100-period sample path of normally distributed (mean: 100, variance: 25) i.i.d. price (case 1). x axis is the price and y axis is the periods.

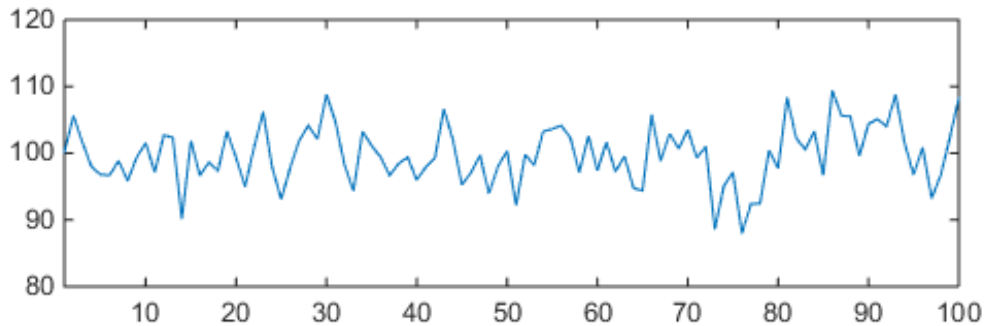
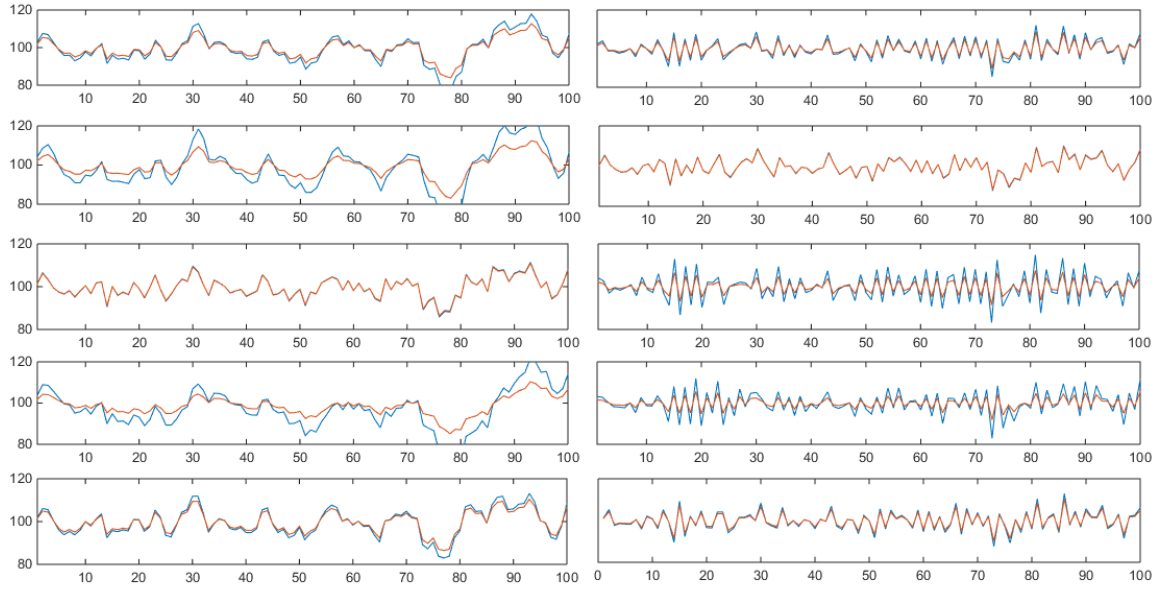


Figure 3.3: 100-period sample paths of prices processes case P1 to case 10 (upper left is case P1, bottom right corner is case P10). x axis is the price and y axis is the periods.



3.4.2 Demand Settings

The simulation model generates different demand streams in order to test a wide set of scenarios with different correlations between the price and the demand. An additive model is used to generate demand streams correlated with the price process. The mathematical formulation of the additive model is as follows,

$$D_t = a - bP_t + \delta_t$$

Note that D_t is the amount of demand occurred in period t , $\forall t \geq 0$ where P_t is the spot price, a , and b are constant, and δ_t is i.i.d. random variable with Normal distribution, with mean 0, and standard deviation σ_δ . The values of b , and σ_δ determine the correlation between the price and demand in a given period.

Similar to the selection of parameters of price process, different parameters are selected to test a wide range of scenarios for demand. 5 different demand models are generated which are: independent from price, positively or negatively correlated

(correlation coefficient is either 0.7 or -0.7, respectively), and weakly positively or negatively correlated (correlation coefficient is either 0.3 or -0.3, respectively). To make comparison between scenarios possible, parameters selected to satisfy the following conditions; the expected profit is equal to $\mu_D = 1000$, and $Var_t(D_t) = 100^2$. All the respective parameters and their resulting variance and can be found in Table 3.2.

Table 3.2: Coefficient sets of $D_t = a - bP_t + \delta_t$ with resulting $Var_t(D_t)$ and $Corr_t(D_t, P_t)$. Note that, there are 5 different demand scenario and all have the same expected value which is $\mu_d = 1000$.

Tag	a	b	$Var(\delta_t)$ $\sigma_\delta^2 =$	$Var_t(D_t)$ $t \in [2, 11]$	$Corr_t(D_t, P_t)$ $t \in [2, 11]$
<i>iid</i>	1000	0	10000	100^2	0
<i>l+</i>	400	-6	9100	100^2	0.3
<i>l-</i>	1600	6	9100	100^2	-0.3
<i>h+</i>	-400	-14	5100	100^2	0.7
<i>h-</i>	2400	14	5100	100^2	-0.7

The mathematical relations between variance, correlation, conditional variance, conditional correlation, and parameters are provided in below formulations.

$$\begin{aligned}
 Var(D_t) &= Var(a - bP_t + \delta_t) \\
 &= Var(a) + Var(bP_t) + Var(\delta_t) \\
 &= b^2Var(P_t) + \sigma_\delta^2
 \end{aligned}$$

$$Var_t(D_t) = b^2Var_t(P_t) + \sigma_\delta^2$$

$$\begin{aligned}
Corr(D_t, P_t) &= \frac{Cov(D_t, P_t)}{\sqrt{Var(D_t)Var(P_t)}} \\
&= \frac{Cov(a - bP_t + \delta_t, P_t)}{\sqrt{(b^2Var(P_t) + \sigma_\delta^2)Var(P_t)}} \\
&= \frac{Cov(a, P_t) - bCov(P_t, P_t) + Cov(\delta_t, P_t)}{\sqrt{(b^2Var(P_t)^2 + \sigma_\delta^2Var(P_t))}} \\
&= \frac{-bVar(P_t)}{\sqrt{(b^2Var(P_t)^2 + \sigma_\delta^2Var(P_t))}} \\
Corr_t(D_t, P_t) &= \frac{-bVar_t(P_t)}{\sqrt{(b^2Var_t(P_t)^2 + \sigma_\delta^2Var_t(P_t))}}
\end{aligned}$$

3.4.3 Feature Set Settings

The presence of exogenous variable (feature) might have an important impact on the decision criteria. In order to measure the effect of a feature that is correlated to the demand (and/or price) stream, two different feature models are proposed. For each model, two correlation levels are tested (high/low).

Scalar Feature

Suppose that there is a feature that is correlated with the demand and/or the price. The feature is observed before the realization of demand. The first model for the feature is a noise added over the demand. The correlation between feature and demand streams are controlled by the standard deviation of the noise. The mathematical formulation of the model is,

$$f_t^{(1)} = D_t + w_t$$

where, w_t is the random noise added to demand to adjust the rate of correlation. w_t is normally distributed with mean 0 and variance σ_w^2 . Values of σ_w^2 selected to simulate high (0.9) and low (0.6) correlation values between $f_t^{(1)}$ and D_t . Corresponding coefficients and their respective correlation values are listed in Table 3.3.

The correlation values shown in Table 3.3 are computed via following expressions.

Table 3.3: Coefficient Set of $f_t^{(1)}$.

Tag	$Var(w_t)$ $\sigma_w^2 =$	$Corr_t(D_t, f_t^{(1)})$ $t \in [2, 11]$
high	2500	≈ 0.9
low	17500	≈ 0.6

$$Corr(D_t, f_t^{(1)}) = \frac{Var(D_t)}{\sqrt{(Var(D_t))^2 + \sigma_w^2 Var(D_t)}}$$

$$Corr_t(D_t, f_t^{(1)}) = \frac{Var_t(D_t)}{\sqrt{(Var_t(D_t))^2 + \sigma_w^2 Var_t(D_t)}}$$

Binary Feature

Suppose that there is an exogenous variable (feature) which signals whether the demand is going to be higher than its mean with a level of certainty, before the demand is observed. This feature can be modeled via the following mathematical expression,

$$f_t^{(2)} = \begin{cases} 1 & D_t + w_t > \mu_D \\ 0 & D_t + w_t \leq \mu_D \end{cases}$$

where, w_t is the random noise added to demand to adjust the level of accuracy of the indicator function. w_t is normally distributed with mean 0 and variance σ_w^2 .

3.4.4 Parameters Array Selection

The solution methodology requires the selection of the parameters array h_i . The parameters array is important, because the accuracy of the model depends on this selection. According to the selection of h_i the data-driven linear program finds θ

Table 3.4: Coefficient Set of binary feature $f_t^{(2)}$. Since the explicit solution of $P\{D_t > \mu_D | f_t^{(2)} = 1\}$ does not exist, the corresponding probabilities are calculated via simulation.

Tag	$Var(w_t)$ $\sigma_w^2 =$	$P\{D_t > \mu_D f_t^{(2)} = 1\}$ $t \in [2, 11]$
high	1000	≈ 0.9
low	10000	≈ 0.75

vector which optimizes the objective function of the problem.

$$\tilde{x}_i^* = \sum_{j=1}^{|h|} \theta_j h_{ij}$$

The simulation model solves data-driven linear programs for every different scenario, with each of parameters array configurations to test the performance of the respective configuration. All the parameters array configurations are listen in Table 3.5.

3.4.5 Run Configuration

Parameters selected for simulation model are as follows; mean demand is 1000, standard deviation of demand is 100 (Note that standard deviation is measured with two different methods, one step ahead, i.e. standard deviation of next period when all the information is known until the current period, and the standard deviation of a period without any information). Price is stationary and the mean price is 100, and standard deviation of each price is 5 (two different standard deviation calculation applies for the price process, too). The underage penalty $\epsilon_u = 40$, and overage penalty $\epsilon_o = 60$. The procurement cost is constant for all periods, $c_i = 80, \forall i > 0$.

Note on procurement cost, c , being a constant:

The procurement cost is modeled as a constant and it is the same for all periods. Therefore it is independent from the spot market price. One might argue that the assumption on the procurement cost is misleading, and not relevant. Since the price

Table 3.5: Parameters array configuration

#	P_{t-1}	P_{t-2}	$f_t^{(1)}$	$f_t^{(2)}$	h_t
1	0	0	0	0	$[1]$
2	0	0	1	0	$[1, f_t^{(1)}]$
3	0	0	0	1	$[1, f_t^{(2)}]$
4	0	0	1	1	$[1, f_t^{(1)}, f_t^{(2)}]$
5	1	0	0	0	$[1, P_{t-1}]$
6	1	0	1	0	$[1, P_{t-1}, f_t^{(1)}]$
7	1	0	0	1	$[1, P_{t-1}, f_t^{(2)}]$
8	1	0	1	1	$[1, P_{t-1}, f_t^{(1)}, f_t^{(2)}]$
9	1	1	0	0	$[1, P_{t-1}, P_{t-2}]$
10	1	1	1	0	$[1, P_{t-1}, P_{t-2}, f_t^{(1)}]$
11	1	1	0	1	$[1, P_{t-1}, P_{t-2}, f_t^{(2)}]$
12	1	1	1	1	$[1, P_{t-1}, P_{t-2}, f_t^{(1)}, f_t^{(2)}]$

is modeled as an ARMA process, and the profit function requires only the margin of the price and the procurement cost ($P - c$) and the margin ($P - c$) turns out to be another ARMA process. Therefore even if the procurement cost is modeled as a function of the previous price, or even if it has a noise added, i.e. $c_i = P_{i-1} + \epsilon$, the margin ($P_t - c_t$) is going to be another ARMA process. Hence, a procurement cost modeled as an ARMA process can be modeled as a constant procurement cost and modified ARMA coefficients. Therefore the current simulation model is sufficient to test the cases where the procurement costs depends on the previous periods, since it test a wide range of ARMA processes.

3.4.6 Benchmarks

A theoretical benchmark can be found for no feature cases by Proposition 1. The simulation model uses ARMA price processes, and an additive demand-price model,

where all the error terms are normally distributed.

Note that the price and demand models used in simulation are,

$$P_t = k + \varepsilon_t + \phi_1 P_{t-1} + \phi_2 P_{t-2} + \rho_1 \varepsilon_{t-1}$$

and

$$D_t = a - bP_t + \delta_t$$

Corollary 1. *Theoretical benchmark of the simulation model, q^* , i.e. optimal order quantity of the single period problem where P_t is an ARMA process, D_t an additive price-demand model and all the information until time t is known, can be computed by the following expression:*

$$q^* = \begin{cases} 0 & \mu_p - c_t \leq -\epsilon_u \\ \mu_d + \sigma_d \Phi^{-1} \left(\frac{\mu_p - c_t + \epsilon_u}{\epsilon_u + \epsilon_o} \right) & -\epsilon_u < \mu_p - c_t < \epsilon_o \\ +\infty & \epsilon_o \leq \mu_p - c_t \end{cases}$$

where $E_t[P_t] = \mu_p = \left(k + \phi_1 \hat{P}_{t-1} + \phi_2 \hat{P}_{t-2} + \rho_1 \hat{\varepsilon}_{t-1} \right)$, and $E_t[D_t] = \mu_d = a - b \left(k + \phi_1 \hat{P}_{t-1} + \phi_2 \hat{P}_{t-2} + \rho_1 \hat{\varepsilon}_{t-1} \right)$ and $\sqrt{\text{Var}_t(D_t)} = \sigma_d = \sqrt{b^2 \sigma^2 + \sigma_\delta^2}$

Proof. The benchmark should be calculated using the same information which is available for the simulation. In other words, the theoretical optimal decision for time period t , should be calculated given all the information until time t is known. Since all the information until time t is known, values of P_{t-1} , P_{t-2} , and ε_{t-1} are already observed (all realized random variables are signed by a hat $\hat{\cdot}$). The uncertainty of P_t is caused only by ε_t (which has a 0 mean and σ standard deviation), and δ_t (which has a 0 mean and σ_δ standard deviation). Since both ε_t , and δ_t are normally distributed variables, the profit function can be rewritten using the information available.

$$P_t \sim \text{Normal} \left(\mu_p = \left(k + \phi_1 \hat{P}_{t-1} + \phi_2 \hat{P}_{t-2} + \rho_1 \hat{\varepsilon}_{t-1} \right), \sigma \right)$$

$$D_t \sim \text{Normal} \left(\mu_d = \left(a - b \left(k + \phi_1 \hat{P}_{t-1} + \phi_2 \hat{P}_{t-2} + \rho_1 \hat{\varepsilon}_{t-1} \right) \right), \sigma_d = \sqrt{b^2 \sigma^2 + \sigma_\delta^2} \right)$$

Since $E_t[P_t] = \mu_p = \left(k + \phi_1 \hat{P}_{t-1} + \phi_2 \hat{P}_{t-2} + \rho_1 \hat{\epsilon}_{t-1}\right)$, and D_t is normal mean $\mu_d = a - b \left(k + \phi_1 \hat{P}_{t-1} + \phi_2 \hat{P}_{t-2} + \rho_1 \hat{\epsilon}_{t-1}\right)$ and standard deviation $\sigma_d = \sqrt{b^2 \sigma^2 + \sigma_\delta^2}$, by using Proposition 1 the optimal order quantity can be calculated as,

$$q^* = \mu_d + \sigma_d \Phi^{-1} \left(\frac{\mu_p - c_t + \epsilon_u}{\epsilon_u + \epsilon_o} \right)$$

The above equation is valid only when $-\epsilon_u < \mu_p - c_t < \epsilon_o$. Therefore the resulting optimal order quantity with respect to the different parameter values is,

$$q^* = \begin{cases} 0 & \mu_p - c_t \leq -\epsilon_u \\ \mu_d + \sigma_d \Phi^{-1} \left(\frac{\mu_p - c_t + \epsilon_u}{\epsilon_u + \epsilon_o} \right) & -\epsilon_u < \mu_p - c_t < \epsilon_o \\ +\infty & \epsilon_o \leq \mu_p - c_t \end{cases}$$

□

3.5 Simulation Results of Expected Profit Maximization

This section covers the analysis of data-driven linear program with expected profit maximization objective.

Before we begin the analysis of the results, we will define some terms which will be used through this section. "EP-LP" stands for the data-driven linear program with expected profit maximization which is proposed in Section 3.2.3. "In-sample" refers any direct result of the linear program. For instance, expected profit result of EP-LP is called "in-sample" expected profit. When we apply the policy generated by EP-LP to another data set rather than the data used in the EP-LP, resulting statistics are called as "out-sample". "Theoretical" term is used to indicate the resulting statistics when the theoretical optimal policy is used.

The aim of this simulation is *to show the power of the data-driven LP formulation in generating optimal decisions to maximize out-sample expected profit, for all possible scenarios, i.e. for all price processes, for all price-demand dependency, and for all feature availability cases.*

We first analyze the in-sample performance of the algorithm, then, before moving into the analysis of out-sample performance which is more important, we provide an analysis for the theoretical results. Out-sample performance is analyzed in two parts. We first looked at the when the features are not available. The effect of price process and demand model and the impact of parameters array selection is investigated. Then the effect of the exogenous feature availability and the dispersion of the results are analyzed.

The tables presented in this section are constructed to provide insights to the simulation results. More detailed simulation results are listed in Appendix A (expected profits) and Appendix B (dispersion of the out-sample profits).

3.5.1 In-sample Performance

In-sample performance of the data-driven LP formulation (EP-LP):

- EP-LP is able to maximize in-sample mean profit for **all scenarios**.
- In-sample performance of EP-LP increases as the number of elements in the parameters array increases for all scenarios.

The fact that the performance of in-sample results increases as the size of the parameter array increases is not surprising because a larger parameters array causes an increment in the dimension of the LP which enables to find θ values improves the optimal policy. In other words, the feasible region of a data-driven LP with a given parameters array is always a subset of feasible region of a data-driven LP parameters array of which is a super set of the initial parameters array. Therefore, the improvement in the objective function is expected as the size of parameters array increases. However it may cause over fitting, a poorer performance in the out-sample results. The comparison of the out-sample results are more meaningful to evaluate the performance of the solution method.

These observations are consistent for all possible scenarios, i.e. different price processes, demand models and availability of features. As an example case, in-sample mean profit values of a scenario are presented in Table A.1 (Appendix A). The table

shows the in-sample and out-sample mean profits for all price processes when the demand is i.i.d. and no feature is available.

In-sample results may perform better than the theoretical benchmark which is not surprising since the LP uses the information of all periods in the computation, while the theoretical benchmark only uses the information of the previous periods.

Note that in Table A.1, there is a difference between in-sample and out-sample results of theoretical benchmarks. This difference is not expected and it is caused by the variance of the simulation results. Number of iterations are increased, we expect that both values converge to one another. However this variance does not cause a problem for measuring the outcome of EP-LP since, in-sample and out-sample values should be compared with their respective benchmarks.

Although in-sample results are valuable to depict the capability of EP-LP, the true indicator of the performance is the out-sample results. Before the analysis of the out-sample performance of the data-driven LP for no feature case, results of the theoretical benchmark should be investigated.

3.5.2 Analysis of Theoretical Expected Profit

Our findings on the theoretical expected profit are as follows,

- For all price processes, as the correlation level of price and demand increases, the theoretical expected profit increases. (As the correlation level of price and demand decreases, the theoretical expected profit decreases.)
- The effect of correlation on the theoretical expected profit increases with respect to the variance of the price process.
- The variance of the theoretical expected profit increases as the correlation increases (both positive and negative correlation increases the variance of the results.), as variance of the price process increases.
- The variance of the theoretical expected profit decreases as the spikiness of the price process increases. (Smoother price processes result bigger variances on their theoretical expected profits with respect to the spikier processes with the same

variance.)

Table 3.6: Expected Value of the theoretical benchmark. (15000-15500: Very Low, 15500-16000: Low, 16000-16500: Moderate, 16500-17000: High, 17000-: Very High)

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID-P4-P5	Moderate	Moderate	Moderate	Moderate	Moderate
Low	P9-P10	Low	Moderate	Moderate	Moderate	Moderate
Moderate	P1-P2	Low	Moderate	Moderate	Moderate	High
High	P3-P6	Very Low	Low	Moderate	High	Very High
Very High	P7-P8	Very Low	Moderate	Moderate	High	Very High

Table 3.6 shows the effect of variance of the price process and the correlation between price and demand.

The variance of the theoretical results does not only depend on the variance of the price and the correlation level between the price and the demand, but also they depend on the spikiness of the price process (Table 3.7). The sign of ϕ_1 which is the coefficient of P_{t-1} of the ARMA process determines the spikiness of the process; therefore, the variance of the theoretical results depends heavily on the sign of the parameter ϕ_1 . Negative and zero ϕ_1 cases result small variances for demand models whereas positive ϕ_1 cases, the variance of the theoretical outcome increases as the correlation level, and the variance of the price process.

3.5.3 Out-sample Performance for No Feature Cases

The true performance of EP-LP can only be understood by the analysis of the out-sample results. The analysis of the out-sample results for the cases where the features are not available is straightforward, since we have theoretical benchmarks for these cases. The out-sample results are compared with the respective theoretical results.

The simulation results show that,

Table 3.7: Variance of the theoretical benchmark. (0-500: Very Low, 500-1000: Low, 1000-1500: Moderate, 1500-2000: High, 2500-: Very High)

ϕ_1	Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
			Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
-	Very Low	P5	Very Low	Very Low	Very Low	Very Low	Very Low
-	Low	P9	Very Low	Very Low	Very Low	Very Low	Very Low
-	Moderate	P1	Very Low	Very Low	Very Low	Very Low	Very Low
-	High	P3	Very Low	Very Low	Very Low	Very Low	Very Low
-	Very High	P7	Very Low	Very Low	Very Low	Very Low	Very Low
0	Very Low	IID	Very Low	Very Low	Very Low	Very Low	Very Low
+	Very Low	P4	Very Low	Very Low	Very Low	Low	Low
+	Low	P10	Very Low	Very Low	Low	Low	Low
+	Moderate	P2	Low	Low	Low	Low	Moderate
+	High	P6	Low	Moderate	Moderate	Moderate	High
+	Very High	P8	High	Very High	Very High	Very High	Very High

- EP-LP is able to find a procurement rule which yields an expected profit which is as good as the theoretical optimal policy for **all no feature cases**.

- The simulation results show that the performance of the data-driven LP depends on the correlation of demand and price. As the correlation between demand and price decreases (positively or negatively) the performance of the solution methodology increases. (Table 3.8)

- The performance of the data-driven LP is affected by the number of parameters exist in the price process. For simpler price processes the solution method performs better than for more complex price processes.(Table 3.8)

The simulation results of out-sample mean profit show that even the scenario with the worst expected profit is as good as the theoretical upper bound, the data-driven LP is only -0.44% off the theoretical result, whereas the scenario with the

Table 3.8: Effect of the price process and the demand model on the performance of EP-LP with respect to the theoretical benchmark. (Percentage deviation from the theoretical benchmark, a more detailed table can be found in Appendix A A.3)

# of Parameters of Price Process	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
0	IID	0.06	0.04	0.04	0.05	0.06
1	P1-P2	0.08 - 0.09	0.07 - 0.09	0.08 - 0.09	0.09	0.10 - 0.12
2	P3-...-P10	0.13 - 0.32	0.07 - 0.13	0.06 - 0.13	0.14 - 0.22	0.15 - 0.44

best performance is as good as -0.04% for all price process - demand model pairs. (See Appendix A Table A.1 and, Table A.2 for resulting out-sample mean profits, and see Table A.3 for the percentage deviation of the out-sample mean profits to the theoretical benchmark)

Despite the performance of the EP-LP depends on the correlation level between the price and the demand, interestingly, it does not directly depend on neither the variance of the price process, nor the spikiness of but it depends on the number of parameters exist in the price process. For simpler price processes, i.e. i.i.d. or ARMA(1,0), the solution method performs better than more complex price processes such as ARMA(1,1) or ARMA(2,0). Nevertheless, the difference in terms of performance negligible. (Table 3.8)

The Effect of the Parameters Array Selection:

Selection of the parameters array plays an important role on the performance of the data-driven approach. Note that all in-sample and out-sample performance statistics presented in the previous sections are the results of the best performing parameters array selection.

Our aim is to investigate the best parameters array selection for different price processes and demand models. Without the presence of the features, the parameters array is constructed by using only the previous price information. We have tested

three different parameters array configurations, $h_1 = [1]$, $h_2 = [1, P_{t-1}]$, and $h_3 = [1, P_{t-1}, P_{t-2}]$.

Simulation results show that the selection of the best performed parameters array depends on the following conditions:

- The performance of the parameters array depends on the structure of the underlying ARMA process of the price most. Parameters array should include the variables of the ARMA process, i.e. $h_1 = [1]$ is the best option for i.i.d. price (ARMA(0,0)), $h_2 = [1, P_{t-1}]$ is the best option for P1 and P2 which are ARMA(1,0).
- In case of more complex ARMA processes (ARMA(1,1) and ARMA(2,0)) both variance of the price process, and the correlation level between the price and the demand affect the best performing parameters array selection, and h_1 and h_2 might perform better than $h_3 = [1, P_{t-1}, P_{t-2}]$ which we would expect that it would be the best option according to the first observation. However always performs almost as good as the best performing parameters array selection. Therefore the first observation still holds.
- Spikiness of the price process does not affect the performance of the parameters array selection, if the variances of the processes are the same.

Note that EP-LP with the parameters array $h_1 = [1]$ is the same with Sample Average Approximation (SAA) method. The results of Appendix A Table A.2 shows that SAA is not able adapt price-demand correlation especially with processes with bigger variance. It lacks responding to different price levels, since it does not possess the price information of the previous periods. Nevertheless, if the price is i.i.d., then since the previous realization of the price does not affect the future, then the information of the previous price does not contribute to the solution, and in that case SAA results are immune to the price-demand correlation. Therefore, for i.i.d. price processes, it is preferable to use SAA (or data-driven LP formulation with parameters array $[1]$) for all correlation levels of the price and the demand.

The SAA case (h_1) yields higher profits if price process is i.i.d., but for the rest of the price processes, other two parameters array configurations perform better.

Table 3.9: Percentage difference of out-sample mean profit and theoretical benchmark when parameters array is $h_1 = [1]$ (SAA). (0-0.10: Very Good, 0.10-0.25: Good, 0.25-1.00: Moderate, 1.00-5.00: Bad, 5.00-10.00: Very Bad, 10.00-: Extremely Bad) (See Appendix A A.3 for the values)

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	Very Good	Very Good	Very Good	Very Good	Very Good
Very Low	P4-P5	Moderate	Very Good	Very Good	Good	Moderate
Low	P9-P10	Bad	Good	Good	Bad	Bad
Moderate	P1-P2	Bad	Moderate	Good	Bad	Very Bad
High	P3-P6	Very Bad	Moderate	Moderate	Very Bad	Extremely Bad
Very High	P7-P8	Very Bad	Bad	Moderate	Very Bad	Extremely Bad

Table 3.10: Percentage difference of out-sample mean profit and theoretical benchmark when parameters array is $h_2 = [1, P_{t-1}]$. (0-0.10: Very Good, 0.10-0.25: Good, 0.25-1.00: Moderate, 1.00-5.00: Bad, 5.00-10.00: Very Bad, 10.00-: Extremely Bad) (See Appendix A A.3 for the values)

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	Very Good	Very Good	Very Good	Very Good	Good
Very Low	P4-P5	Good	Very Good	Very Good	Good	Moderate
Low	P9-P10	Good	Very Good	Very Good	Good	Moderate
Moderate	P1-P2	Very Good	Very Good	Very Good	Good	Good
High	P3-P6	Moderate	Good	Good	Moderate	Bad
Very High	P7-P8	Moderate	Very Good	Very Good	Good	Moderate

Simulations results show that the parameters array should be constructed to provide information to represent underlying ARMA process. Although, the variance of the price process and the correlation of the demand and the price affect the parameters

Table 3.11: Percentage difference of out-sample mean profit and theoretical benchmark when parameters array is $h_3 = [1, P_{t-1}, P_{t-2}]$. (0-0.10: Very Good, 0.10-0.25: Good, 0.25-1.00: Moderate, 1.00-5.00: Bad, 5.00-10.00: Very Bad, 10.00-: Extremely Bad) (See Appendix A A.3 for the values)

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	Good	Good	Good	Good	Good
Very Low	P4-P5	Good	Good	Good	Good	Good
Low	P9-P10	Good	Good	Good	Good	Good
Moderate	P1-P2	Good	Good	Good	Good	Good
High	P3-P6	Moderate	Good	Good	Good	Moderate
Very High	P7-P8	Good	Good	Good	Good	Good

Table 3.12: Best Performing parameters arrays for different price processes and different demand models (1: $h_1 = [1]$, 2: $h_2 = [1, P_{t-1}]$, 3: $h_3 = [1, P_{t-1}, P_{t-2}]$)

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	1	1	1	1	1
Very Low	P4-P5	3	1	1	2	3
Low	P9-P10	3	2	2	3	3
Moderate	P1-P2	2	2	2	2	2
High	P3-P6	3	3	2	3	3
Very High	P7-P8	3	2	2	3	3

array performance especially for complex price processes, for complex price processes, it is useful to construct larger parameters arrays.

3.5.4 Out-sample Performance when the Features are Available

In this section, the effect of features (exogenous variables) on the out-sample performance of the EP-LP is analyzed.

First of all, it is important to remind that a theoretical closed form solution of the problem does not exist when the features are available. Therefore, it is not possible to calculate the deviation of the EP-LP from the optimal value directly, and only the relative performance of the method can be measured, i.e. comparison of the out-sample mean profits of the theoretical benchmark of the featureless case and results the EP-LP. Although this comparison does not prove the optimality of the method, it reveals the ability of EP-LP to capture and exploit the extra information provided by the features on the demand and/or the price.

The simulation results show that

- The decision maker would better off if s/he uses the EP-LP with features rather than using theoretical optimal values which does not use the feature information, for all price-demand scenarios and all parameters array configurations.
- As the information provided by the feature become more accurate, i.e. the correlation between the demand and feature increases, the gain increases with respect to the theoretical benchmark of no feature case. This condition holds for all price processes and all demand models. (Table 3.13)
- Although both scalar and binary features yield greater profits than the theoretical results, scalar features performs better than binary feature cases. (Table 3.13)
- Presence of multiple features increases the gain acquired. (Table 3.13)
- The effects of variance of the price process and the correlation level between price and demand on the performance of EP-LP are consistent with no feature cases.

Table 3.13 presents the percentage gains acquired when EP-LP is preferred instead of the theoretical benchmark of no feature case. It lists the minimum and the maximum gains acquired among all scenarios. Therefore, we can conclude that having extra information improves the performance of the solution method regardless of the

Table 3.13: Minimum and maximum percentage gain acquired in out-sample mean profit with respect to the theoretical benchmark of no feature cases, for different feature availability cases. ($f^{(1)}$: Scalar Feature, $f^{(2)}$: Binary Feature)

Scalar Feature	Binary Feature		
	No	Low	High
No	(0.44) - (0.04)	2.69 - 4.82	4.77 - 9.72
Low	3.94 - 5.59	5.65 - 8.21	6.94 - 11.43
High	11.49 - 14.96	11.90 - 15.44	12.44 - 16.26

scenario, and even when the amount of information provided by the features is small, EP-LP still performs better than the theoretical benchmark of the featureless case in terms of the out-sample mean profit.

More detailed tables presenting the out-sample mean profits for all scenarios and feature availability cases can be found in Appendix A. Table A.4 and Table A.5 presents the out-sample mean profits of EP-LP with scalar feature cases, and Table A.6 and Table A.7 presents the out-sample mean profits of EP-LP with binary feature cases. Simulation results of multiple feature cases can be found in Appendix A Tables A.8, A.9, A.10, and A.11.

The simulation results coincide with the findings of no feature cases in terms of the effect of variance of the price process, the correlation level between price and demand on the out-sample mean profit, and the effect of parameters array selection. For instance, when the prices are i.i.d. the presence of previous prices does not improve the performance. However, for price processes which evolves conditionally on the previous realizations, the parameters array with the price information (h_2, h_3) yields better outcomes. Furthermore, the gain acquired by selection of the parameters array with price information accumulates as the variance of the price process and as the price and demand become more correlated. Nevertheless, this gain is proportionally smaller than the no feature case. This is reasonable since the feature already induced the model to result a better solution, and a smaller room for improvement is left with

respect to the no feature case.

In conclusion, having a feature which is correlated with the demand improves the performance of the data-driven LP approach and a proper price information should be included to parameters array to yield a better mean profit.

3.5.5 Dispersion of the Simulation Results

Although the analysis on sample means of the simulation results provides insights on the data-driven LP, the dispersion of the results should be investigated in order to ensure the validity of the strategy. An out-sample test with a smaller variance is more preferable, because smaller variance indicates that solution methodology is capable of generating outcomes near the out-sample mean. However, being able to assess the magnitude of a statistical property requires a reference point. In this case, a reference statistic is required to measure the dispersion generated by data-driven LP.

Simulation results of a theoretical benchmark can be used as a reference to assess the performance of the proposed solution methodology. Table 3.14, representing coefficient of variation of simulation results of theoretical benchmark, provides these reference points for each scenario (price-demand pair) to assess the performance of data-driven LP. The table presents the coefficient of variations of the sample mean profit within an iteration accumulated when the theoretical optimal procurement strategy is used. In other words, for no feature cases, the variation of the data-driven LP results can only be compared with these theoretical values.

Table 3.14 shows the out-sample coefficient of variations among separate out-sample simulation iterations, for each scenario (Price-Demand pair). Note that coefficient of variation is $CV = \frac{\sigma}{\mu}$ which scales the standard deviation of the collection with its mean to measure the dispersion, and enables to compare the dispersion of different data sets with different mean values. The coefficient of variation values presented in Table 3.14 show that they vary among different scenarios. While its value is around 0.02 to 0.05 for most of the scenarios (8 out of 11 scenarios), the value can be as high as 0.26 (P7- $h+$ pair). This dispersion prevents direct comparison of the

Table 3.14: Coefficient of Variation ($CV = \frac{\sigma}{\mu}$) of out-sample theoretical mean profits for all price-demand scenarios

	Demand				
	IID	$l+$	$l-$	$h+$	$h-$
IID	0.03	0.03	0.03	0.03	0.02
P1	0.08	0.08	0.07	0.09	0.06
P2	0.02	0.02	0.02	0.02	0.02
P3	0.11	0.12	0.10	0.13	0.09
P4	0.03	0.03	0.02	0.03	0.02
P5	0.04	0.04	0.04	0.05	0.03
P6	0.02	0.02	0.02	0.02	0.02
P7	0.22	0.23	0.20	0.26	0.17
P8	0.02	0.03	0.02	0.04	0.04
P9	0.05	0.05	0.04	0.06	0.04
P10	0.02	0.02	0.02	0.02	0.02

out-sample variances of the solution methodology.

Therefore, we measure the dispersion of the EP-LP by investigating the statistical properties of the difference between results of theoretical optimal strategy and the solution approach for the same realizations of the random environment. For instance, for a given scenario, and a given realization of demand-price pair, theoretical optimal strategy and a EP-LP is used concurrently to compute out-sample mean profit. Difference between the resulting out-sample mean profits is collected for all out-sample realizations. Analysis of the statistical properties of these differences reveal the true dispersion of the solution strategy which is mostly purified from the volatility already existing in theoretical results.

Table 3.15 shows the percentage of the mean values of the difference between out-sample results of theoretical benchmark and EP-LP in each simulation run to the theoretical benchmark for all scenarios when no features available. Table 3.16

shows the percentages of the standard deviations. The simulation results on the tables show that the solution methodology is successfully able to imitate the behavior of the theoretical benchmark. None of the results conflicts this claim. First of all, the distribution of the differences between theoretical and data-driven LP are notably small when they are scaled with the mean of the theoretical benchmark (the percentage values). The mean of the differences between the data-driven LP and the theoretical benchmark are very small with respect to the mean of the benchmark. The scenario with the worst performance, performed only 0.42% less than the theoretical benchmark for correct parameters array selection. Not only the mean difference but also the standard deviation is very small with respect to the benchmark mean value. The deviation of result generated by the data-driven LP is as small as 0.42% of the theoretical benchmark mean for the worst performing scenario. More detailed statistics can be found in Appendix B Tables B.1, B.2, B.3, B.4, and B.5. Among all scenarios, the worst performing one realized within a range of -3.99% , $+0.98\%$ of the theoretical benchmark mean. Therefore, we can conclude the following,

- Since the dispersion of the simulation results are very small with respect to the mean values of the theoretical benchmark, the analysis performed using the out-sample mean profits are accurate.

Although the performance of EP-LP is satisfactory for all scenarios, the variance of the price process and the correlation level between the price and the demand, still affects the performance of EP-LP. The correlation level decreases the performance of the solution methodology for both positive and negative cases. Furthermore, the effect of the price variance can be observed in Table 3.15, and Table 3.16.

Dispersion of data-driven LP is investigated not only for no-feature cases, but also for the cases when the features are available. They show the dispersion on the results of data-driven LP when the features are available. Since there are so many scenarios and parameters array configurations, only three scenarios are selected to be able to represent the characteristics of a wide range of possible scenarios (among 55 possible scenarios), and result of the rest can be found in Appendix Table B.6, Table B.7, and

Table 3.15: The percentage of the mean values of the difference between out-sample results of theoretical benchmark and EP-LP in each simulation run to the theoretical benchmark for all scenarios when no features available.

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	0.01	0.00	0.00	0.01	0.02
Very Low	P4-P5	0.04 - 0.07	0.01 - 0.01	0.00 - 0.00	0.00 - 0.00	0.09 - 0.11
Low	P9-P10	0.09 - 0.07	0.02 - 0.00	0.00 - 0.01	0.05 - 0.05	0.08 - 0.09
Moderate	P1-P2	0.08 - 0.06	0.02 - 0.01	0.01 - 0.01	0.06 - 0.06	0.08 - 0.08
High	P3-P6	0.29 - 0.25	0.07 - 0.05	0.04 - 0.05	0.16 - 0.16	0.42 - 0.41
Very High	P7-P8	0.11 - 0.08	0.06 - 0.04	0.04 - 0.04	0.10 - 0.17	0.13 - 0.10

Table 3.16: The percentage of the standard deviations of the difference between out-sample results of theoretical benchmark and EP-LP in each simulation run to the theoretical benchmark for all scenarios when no features available.

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	0.16	0.15	0.15	0.18	0.19
Very Low	P4-P5	0.22 - 0.26	0.17 - 0.17	0.16 - 0.16	0.24 - 0.25	0.30 - 0.32
Low	P9-P10	0.55 - 0.22	0.20 - 0.19	0.19 - 0.19	0.24 - 0.23	0.26 - 0.26
Moderate	P1-P2	0.23 - 0.20	0.20 - 0.19	0.20 - 0.20	0.22 - 0.21	0.24 - 0.23
High	P3-P6	0.35 - 0.32	0.24 - 0.22	0.23 - 0.22	0.30 - 0.31	0.42 - 0.41
Very High	P7-P8	0.30 - 0.24	0.23 - 0.21	0.21 - 0.22	0.27 - 0.25	0.28 - 0.26

Table B.8 . The first scenario is i.i.d. demand i.i.d. price case. The second scenario is where price process is P1 (ARMA(1,0) with positive coefficient) and demand model is $l-$ (when there is a -0.6 correlation between price and demand). The third scenario is where price process is P7 (ARMA(2,0) with positive coefficients) and demand is

$h+$ (when there is 0.9 correlation between price and demand). Note that their price processes have different underlying ARMA processes, they all have different demand-price relations, and therefore different coefficient of variation values for the theoretical case. The dispersion is measured according to the methods aforementioned.

The results show that the presence of features does not increase the variance of the data-driven LP out-sample mean profits. For all selected scenarios, the relative percentage of the standard deviation of the difference between theoretical benchmark for no-feature case, and the data-driven LP results is smaller than 1.4% for the best parameters array configuration in each feature configuration. That means the data-driven LP generate results, such that the deviation of the results are not more than 1.4% when the volatility of the theoretical results is eliminated.

The results also support the previous results suggesting the mean improvement of EP-LP when the features are available. Tables show that the presence of features improves the performance of the data-driven LP from 3.1% ($f^{(1)}$: o , $f^{(2)}$: low), ($f^{(1)}$: o , $f^{(2)}$: low) for the worst case, and up to 13.9% for the least performed scenario. 'min. diff.' column lists the simulation run with the worst the performance of data-driven LP with respect to the theoretical benchmark. The results shows that, if the decision maker uses the decision maker uses the data-driven LP when at least a feature is available, s/he will almost always better off than using the theoretical benchmark for all simulation runs, for each scenario with a correct parameters array selection.

3.6 Simulation Results of CVaR Minimization

In the case of risk-averse decision maker, data-driven LP formulation with expected profit maximization objective (EP-LP) may produce unsatisfying results. Minimization of Conditional Value-at-Risk of the profit formulation is proposed for the risk-averse decision makers. The LP formulation can be found in Section 3.2.2. In this section, "CVAR-LP" is used to denote the data-driven LP with CVaR minimization objective.

The performance of the solution approach is measured via simulation. However,

since a theoretical benchmark for CVaR minimization case does not exist, assessment of the simulation results requires a different approach. The simulation results are assessed with respect to the improvement on the sample CVaR values of theoretical values with expected profit maximization objective, and sample CVaR values of data-driven LP with expected profit maximization objective. Although this comparison does not prove neither the validity of the solution approach nor the accuracy of the results, it still provides a worst case benchmark for all scenarios. The decision maker can measure the improvement on the CVaR of the profit that s/he may achieve if the data-driven LP with CVaR Minimization objective (CVAR-LP) is used instead of the data-driven LP with Expected Profit Maximization objective (EP-LP).

Sample CVaR values are calculated as follows; for in-sample CVaR calculation, since there are 400 periods, and β value is 0.95, the average of 20 periods with least profit among that 400 periods, gives the in-sample CVaR value of that specific iteration, for out-sample CVaR calculation, since there are 200 periods for out-sample tests, and β value is 0.95, the average of 10 periods with least profit among that 200 out-sample periods, results the out-sample CVaR value of that specific out-sample test.

3.6.1 In-sample Performance

In-sample performance of the data-driven LP formulation (CVAR-LP):

- CVAR-LP generates better in-sample CVaR values than out-sample tests, and in-sample and out-sample CVaR values of EP-LP, for all scenarios.
- In-sample performance of CVAR-LP increases as the number of elements in the parameters array increases for all scenarios.

However in-sample results do not guarantee the capability of the CVAR-LP on minimizing out-sample CVaR value of the cost function.

3.6.2 Out-sample Performance

Out-sample performance of the data-driven LP formulation (CVAR-LP):

The Effect of the Variance of the Price Process:

- The variance of the price process has a negative impact of the performance of CVAR-LP. For the processes with a greater variances, the CVAR-LP yields poorer performance. (Table 3.17)

The Effect of the Correlation Between the Price and the Demand:

- CVAR-LP is incapable of improving CVaR values of EP-LP when the demand model is $l-$ (negatively correlated with a small correlation coefficient). This applies for all price processes and features configuration.

- CVAR-LP outperforms EP-LP in terms of CVaR value, in case of positively correlated price and demand, for almost all price processes and feature configurations.

- The performance of CVAR-LP also depends on the price process and configuration of features. In addition to $l-$ demand model, for the demand models $i.i.d.$ and $h-$, the CVAR-LP may not be able to outperform EP-LP in terms of CVaR depending on the price process and configuration of features.

- The gains acquired in terms of CVaR by using CVAR-LP instead of EP-LP with respect to the demand model can be sorted as $h+ > l+ > i.i.d. > l-$, and $h- > l-$ for all price processes, and feature configurations. (Table 3.17)

Table 3.17: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means (no feature cases).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	8.48	(0.25)	5.45	16.83	35.92
Very Low	P4-P5	8.05 - 8.24	(0.27) - (0.32)	5.27 - 5.22	16.39 - 16.26	34.93 - 34.81
Low	P9-P10	7.00 - 6.82	(0.60) - (0.65)	3.83 - 3.97	12.60 - 12.71	28.30 - 28.82
Moderate	P1-P2	5.79 - 6.10	(0.99) - (0.78)	3.18 - 3.17	10.57 - 10.68	24.40 - 25.33
High	P3-P6	3.48 - 4.17	(1.08) - (0.96)	1.48 - 1.55	5.47 - 5.59	14.25 - 15.55
Very High	P7-P8	1.41 - 3.01	(1.64) - (1.10)	0.99 - 1.26	3.72 - 4.76	10.35 - 12.93

The Effect of Parameters Array Selection:

- CVAR-LP requires a fewer number of elements in parameters array than EP-LP, for most of the price process-demand model-feature configuration selections. (Table 3.18) Rather than this difference, the parameters array selection criteria is parallel to the parameters array selection criteria of EP-LP.

Table 3.18: The best performing parameters arrays for different price processes and different demand models for CVaR Minimization (1: $h_1 = [1]$, 2: $h_2 = [1, P_{t-1}]$, 3: $h_3 = [1, P_{t-1}, P_{t-2}]$)

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	1	1	1	1	1
Very Low	P4-P5	1	1	1	1	1
Low	P9-P10	2	1 - 2	1	1	2
Moderate	P1-P2	2	2	1	2	2
High	P3-P6	2	2	1	2	2 - 3
Very High	P7-P8	2	2	1	2	2 - 3

The Effect of Feature Availability:

- The gains acquired in terms of CVaR by using CVAR-LP instead of EP-LP, decreases when the features are available. (Table 3.19)
- The presence of a scalar feature has a bigger impact on the difference than the presence of a binary feature.

Simulation results for out-sample mean CVaR values of all price process-demand model pairs with all possible feature configurations are listed in Appendix C, Table C.2 to Table C.19. The results show that CVAR-LP does not improve CVaR values when price and demand has a negative correlation with a small correlation coefficient ($l-$). While the information provided by the table is not sufficient to explain the

Table 3.19: Percentage gain acquired by the CVAR-LP with respect to EP-LP in terms of out-sample CVaR for different feature availability cases (IID Price, $h+$ Demand) ($f^{(1)}$: Scalar Feature, $f^{(2)}$: Binary Feature)

Scalar Feature	Binary Feature		
	No	Low	High
No	35.92	21.78	15.19
Low	20.54	14.62	10.60
High	4.04	3.32	3.25

reason behind this situation, we can guess that it may be caused by the two reasons. Either the CVAR-LP is incapable of generating a meaningful policy for a CVaR objective or the expected profit maximization policies (EP-LP and TH-EP: Theoretical benchmark of expected profit maximization case) already yield near optimal results for the CVaR value. In any case, to be able to conclude that the CVAR-LP provides near-optimal solutions for results for all demand models, the other demand and price process relationships should be investigated.

On the other hand, the performance of CVAR-LP for the other demand models varies. For the cases when the price process is positively correlated with the demand model ($h+$ and $l+$), CVAR-LP is able to outperform EP-LP with respect to CVaR values almost all of the cases. In addition, the gain in CVaR increases as the positive correlation coefficient is increased. The variance of the price process has a negative effect on the performance of CVAR-LP. Moreover, the presence of a scalar feature highly correlated with demand decreases the gain generated by CVAR-LP. While the presence of binary feature also decreases the performance of CVAR-LP, interestingly its effect on the gain is negligible when it is compared with a scalar feature. The detailed results on the effect of the different features on the CVAR-LP performance can be found in Appendix C Table C.20 to Table C.27.

3.6.3 Dispersion of the Simulation Results

Although mean CVaR values of different simulation runs are beneficial to understand the performance of the solution methodology, the distribution of CVaR values of different simulation runs are also important, especially for a risk sensitive decision-maker. The dispersion of the simulation results show the following:

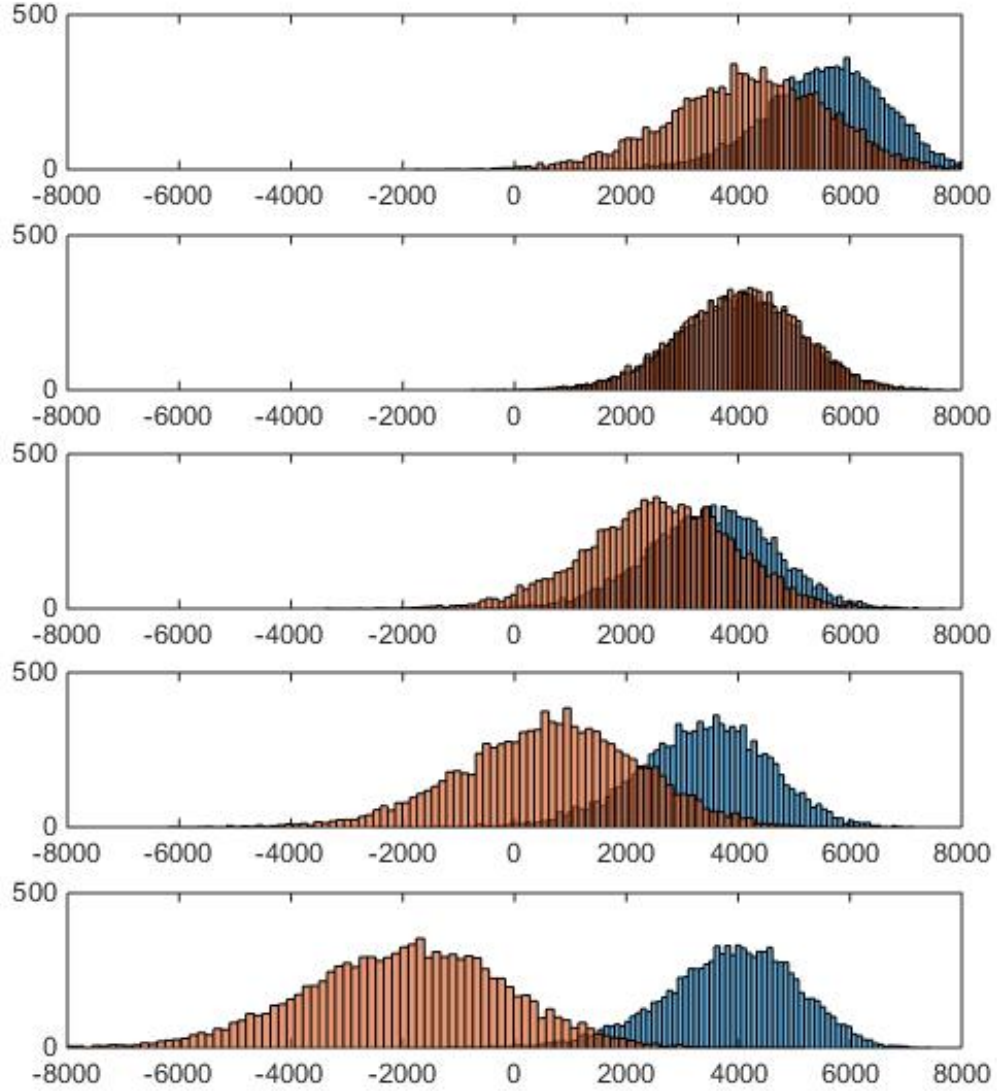
Table 3.20: The percentage of the difference between standard deviations of out-sample CVaR values of CVAR-LP and TH-EP for different scenarios to the theoretical out-sample means (no feature cases).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	(1.84)	0.04	(1.03)	(2.36)	(3.51)
Very Low	P4-P5	(1.77) - (1.87)	0.03 - 0.04	(0.95) - (0.91)	(2.29) - (2.25)	(3.34) - (3.40)
Low	P9-P10	(1.61) - (1.52)	0.07 - 0.01	(0.46) - (0.64)	(1.75) - (1.90)	(2.81) - (3.00)
Moderate	P1-P2	(1.32) - (1.21)	0.06 - 0.03	(0.13) - (0.33)	(1.13) - (1.32)	(1.98) - (2.36)
High	P3-P6	(0.94) - (0.71)	(0.05) - (0.04)	0.16 - 0.01	(0.21) - (0.32)	(0.52) - (1.13)
Very High	P7-P8	(1.25) - (0.03)	(0.16) - (0.05)	0.49 - 0.31	0.96 - 0.53	1.64 - 0.82

- CVAR-LP approach is able to generate a consistent policy for CVaR minimization of the loss function regardless of the demand model.
- However, since we measure the performance of CVAR-LP with respect to TH-EP, for demand model $l-$, both strategies turn out to be resulted a similar outcomes. Therefore, despite CVAR-LP is not able to out-perform TH-EP for demand model $l-$, it yields consistent results with respect to its results for the other demand models.
- The standard deviation of the results of CVAR-LP depends on both the correlation level between the price and the demand and the variance of the price process. However, the results on Table 3.20 show that we are not able to explain the effects of both variables separately, since they affect one another.

The comparison in terms of the mean values is already discussed in the previous

Figure 3.4: Distributions of CVaR values of CVAR-LP (Blue or Light) and TH-EP (Red or Dark) for IID Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)



chapter. Table 3.20 compares the standard deviations of the CVaR values of CVAR-LP and TH-LP.

Although the comparison with respect to the mean and the standard deviation

between the CVaR values of the two approaches is useful, the comparison of the histograms of the simulation results enables us to depict the gain generated by CVAR-LP for different scenarios. Figure 3.4 depicts the distributions of CVaR values of the simulation results of CVAR-LP and TH-EP for IID price process and all demand models. The corresponding figures for the rest of the price processes are listed in Appendix C (Figure C.1 to Figure C.10).

Detailed statistics of the CVaR values for all price process and demand model of the CVAR-LP which are compared with the CVaR values in the simulation results of EP-LP can be found in Appendix C Table C.28 to Table C.32.

Chapter 4

CONCLUSION

This dissertation proposes data-driven solution strategies for the single period procurement problems of commodities which are traded on a spot market. Since the presence of a spot market differentiates the problem definition from the classical single period inventory models, the existing data-driven optimization literature does not address the our problem definition. We have proposed two different data-driven solution approaches, expected profit maximization (risk-neutral case) and the CVaR minimization of the loss function (risk-averse case). We also investigated the effect of the presence of exogenous variables in both cases. The approaches appear to be promising for real life applications involving large feature data.

As for future research directions, we plan to implement our data-driven linear formulation for the CVaR minimization case to the original newsvendor problem. Although there are several papers on the CVaR minimization with Sample Average Approximation approach, a data-driven formulation with the CVaR minimization objective that uses the information of exogenous variables (features) has not been researched. In addition, a real-life application of the data-driven linear programming approach that uses real-data would be useful for both expected profit maximization case and the CVaR minimization case.

Appendix A

APPENDIX A

Description of Table A.1 :

Table A.1 depicts the performance of the data-driven approach when no feature is available. The i.i.d. demand case is shown in the table (other cases of demand are analyzed in the following part). The first column is the underlying price process whose tag is available in Table 3.1. For each price process three different data-driven LPs are solved (Column 4-5-6), both have identical formulations and inputs except their parameters arrays. The optimal ordering strategy which explained in Section 3.3.6 (Theoretical - Column 3) is used as a benchmark. For each price process - solution method pair, in-sample and out-sample expectations are calculated. In-sample expectations are the resulting expected profit of the LPs, i.e. the value which θ vector aims to maximize using the training data. While out-sample expectations are the expected profits when θ vector is used to find an order quantity on a different sample which is not used in the LP (test data).

Table A.1: Sample Mean Profit (Demand is i.i.d, no features)

			Parameters Array		
Price		Theoretical	$\{1\}$	$\{1, P_{t-1}\}$	$\{1, P_{t-1}, P_{t-2}\}$
IID	in-sample	16221.79	16228.42	16234.68	16241.70
	out-sample	16130.38	16123.63	16118.25	16110.30
P1	in-sample	16453.41	16428.09	16468.63	16476.71
	out-sample	16153.89	16114.82	16138.66	16130.71
P2	in-sample	16215.99	16192.04	16229.34	16235.38
	out-sample	16163.76	16125.90	16150.94	16144.48
P3	in-sample	16659.63	16571.49	16666.16	16679.22
	out-sample	16209.98	16108.21	16189.07	16185.61
P4	in-sample	16213.72	16218.05	16225.87	16232.41
	out-sample	16133.53	16124.24	16119.52	16114.54
P5	in-sample	16281.64	16285.19	16294.59	16302.88
	out-sample	16130.68	16121.17	16116.86	16109.78
P6	in-sample	16250.62	16166.03	16258.61	16269.19
	out-sample	16226.97	16127.44	16207.24	16204.09
P7	in-sample	17075.92	16968.12	17090.39	17096.85
	out-sample	16276.41	16145.86	16260.42	16255.97
P8	in-sample	16310.22	16199.89	16321.71	16330.10
	out-sample	16244.72	16123.96	16228.72	16223.85
P9	in-sample	16331.14	16317.27	16343.08	16354.70
	out-sample	16141.41	16115.59	16126.58	16118.53
P10	in-sample	16195.60	16186.32	16207.99	16214.10
	out-sample	16151.83	16126.38	16137.73	16134.16

Description of Table A.2 :

Table A.2 is an extension of Table A.1. It shows the simulation results when demand is correlated with the current price (See Section 3.3.2 for details). Each row represents a price process. For each process, 4 different demand model is generated. $l+$ and $h+$ represent positively correlated demand with a small and large correlation coefficient, respectively. $l-$ and $h-$ represent negatively correlated demand with a small and large correlation coefficient respectively. The coefficients of demand models are shown in the Table 3.2. For each price-demand pair, sample mean profits are calculated via theoretical benchmark and data-driven LP with 3 different parameters array configurations are presented. Note that, only the out-sample results are listed in the table, since the in-sample results are parallel with the i.i.d. demand case (Most of the in-sample results of data-driven LP are better than the theoretical for each case, and their performance increases as the number of elements in the parameters array).

Table A.2: Out-sample Mean Profit (no features, all correlated demand cases)

P	Demand	Theoretical		Data Driven LP					
				$\{1\}$		$\{1, P_{t-1}\}$		$\{1, P_{t-1}, P_{t-2}\}$	
IID	$l+ l-$	16130	16131	16122	16124	16114	16117	16108	16111
	$h+ h-$	16132	16132	16122	16123	16115	16118	16107	16112
P1	$l+ l-$	16298	16010	15941	15951	16283	15995	16275	15988
	$h+ h-$	16491	15819	15254	15278	16474	15802	16467	15794
P2	$l+ l-$	16308	16020	15960	15962	16293	16008	16288	16001
	$h+ h-$	16503	15828	15303	15303	16484	15814	16479	15807
P3	$l+ l-$	16634	15786	15622	15644	16553	15763	16597	15762
	$h+ h-$	17201	15222	13924	13984	16949	15099	17126	15173
P4	$l+ l-$	16145	16122	16109	16110	16123	16108	16121	16102
	$h+ h-$	16163	16107	16050	16050	16120	16080	16133	16084
P5	$l+ l-$	16142	16120	16105	16108	16119	16104	16117	16098
	$h+ h-$	16159	16105	16039	16044	16114	16076	16127	16078
P6	$l+ l-$	16651	15803	15661	15664	16572	15780	16615	15781
	$h+ h-$	17219	15238	14025	14023	16968	15118	17145	15195
P7	$l+ l-$	16804	15748	15501	15575	16769	15730	16778	15725
	$h+ h-$	17511	15045	13405	13572	17429	14999	17481	15019
P8	$l+ l-$	16763	15727	15561	15564	16731	15712	16740	15708
	$h+ h-$	17455	15038	13644	13648	17378	14998	17429	15018
P9	$l+ l-$	16227	16056	16013	16016	16199	16040	16205	16034
	$h+ h-$	16344	15942	15589	15599	16276	15903	16319	15917
P10	$l+ l-$	16239	16065	16026	16027	16210	16051	16219	16046
	$h+ h-$	16357	15950	15614	15613	16287	15911	16333	15929

Description of Table A.3 :

Table A.3 shows percentage difference of out-sample means of theoretical benchmark and EP-LP. The percentage difference is computed by the following equation;

$$\text{Percentage Difference} = -100 \frac{\text{Theoretical} - \text{LP Result}}{\text{Theoretical}}$$

The table covers all demand cases for 3 parameters array configuration $h_1 = [1]$, $h_2 = [1, P_{t-1}]$, $h_3 = [1, P_{t-1}, P_{t-2}]$ when no features are available. Smaller values mean more accurate results whereas the larger values mean the deviation from the theoretical benchmark.

Table A.3: Percentage difference of out-sample means of Theoretical benchmark and data-driven LPs. $\text{Percentage Difference} = -100 \frac{\text{Theoretical} - \text{LP Result}}{\text{Theoretical}}$. (no features, all demand cases, for 3 parameters array configuration $h_1 = [1]$, $h_2 = [1, P_{t-1}]$, $h_3 = [1, P_{t-1}, P_{t-2}]$). Bold numbers indicates the parameters array with the best performance

	i.i.d.			$l+$			$l-$			$h+$			$h-$		
P	h_1	h_2	h_3	h_1	h_2	h_3	h_1	h_2	h_3	h_1	h_2	h_3	h_1	h_2	h_3
IID	0.04	0.07	0.12	0.05	0.10	0.14	0.04	0.09	0.12	0.06	0.11	0.15	0.06	0.09	0.12
P1	0.24	0.09	0.14	2.19	0.09	0.14	0.37	0.09	0.14	7.50	0.10	0.15	3.42	0.11	0.16
P2	0.24	0.08	0.12	2.13	0.09	0.12	0.36	0.07	0.12	7.27	0.12	0.15	3.32	0.09	0.13
P3	0.63	0.13	0.15	6.08	0.49	0.22	0.90	0.15	0.15	19.05	1.47	0.44	8.13	0.81	0.32
P4	0.06	0.09	0.12	0.22	0.14	0.15	0.07	0.09	0.12	0.70	0.27	0.19	0.35	0.17	0.14
P5	0.06	0.09	0.13	0.23	0.14	0.15	0.07	0.10	0.14	0.74	0.28	0.20	0.38	0.18	0.17
P6	0.62	0.12	0.14	5.95	0.47	0.22	0.88	0.15	0.14	18.55	1.46	0.43	7.97	0.79	0.28
P7	0.80	0.10	0.12	7.75	0.21	0.15	1.10	0.11	0.15	23.45	0.47	0.17	9.79	0.31	0.17
P8	0.74	0.10	0.13	7.17	0.19	0.14	1.04	0.10	0.12	21.83	0.44	0.15	9.24	0.27	0.13
P9	0.15	0.09	0.14	1.32	0.17	0.14	0.25	0.10	0.14	4.62	0.42	0.15	2.15	0.24	0.16
P10	0.16	0.09	0.11	1.31	0.18	0.12	0.24	0.09	0.12	4.54	0.43	0.15	2.11	0.24	0.13

Description of Table A.4 and Table A.5 :

Table A.4 and Table A.5 show the out-sample mean profit values of data driven linear program simulation results when a scalar feature (see Section 3.3.3) is available. Two different configuration of the scalar feature are tested. The first configuration named "low" is the case where the correlation coefficient of the feature and the demand is 0.6. The second configuration named "high" is the case where the correlation coefficient of the feature and the demand is 0.9. Each different scenario is tested for both "low" and "high" cases, and results are presented as pairs to see the effect of correlation coefficient on the performance. Each scenario consists of a price-demand pair and a parameter array. The model is simulated for all price process and the first column lists tags of price processes to indicate corresponding rows. Similarly, the simulation tests all demand models and the first row of the table lists the demand models to indicate the particular columns. 3 different parameter array configuration is used to see the effect of price information while features are available. The tested parameters arrays are $h_1 = [1, f^{(1)}]$, $h_2 = [1, f^{(1)}, P_{t-1}]$, and $h_3 = [1, f^{(1)}, P_{t-1}, P_{t-2}]$. "Th" stands for the theoretical benchmark of the featureless case where the information of the feature is not taken into account. Since theoretical value does not depend on the feature being "high" or "low" correlated, a single value is written instead of a pair for each scenario.

Table A.4: Out-sample mean profit when a scalar feature ($f^{(1)}$) is available. (low/high correlated scalar feature, all demand cases, price processes IID, P1, P2, P3, P4 and P5, and 3 parameters array configurations, $h_1 = [1, f^{(1)}]$, $h_2 = [1, f^{(1)}, P_{t-1}]$, and $h_3 = [1, f^{(1)}, P_{t-1}, P_{t-2}]$)

		IID		l+		l-		h+		h-	
		low	high	low	high	low	high	low	high	low	high
IID	Th	16130		16130		16131		16132		16132	
	h_1	16899	18258	16953	18379	16846	18141	17033	18545	16777	17987
	h_2	16892	18255	16946	18376	16840	18137	17027	18542	16772	17984
	h_3	16887	18252	16942	18373	16836	18133	17021	18539	16768	17980
P1	Th	16154		16298		16010		16491		15819	
	h_1	16890	18250	16919	18478	16700	18003	16832	18793	16218	17626
	h_2	16910	18261	17108	18526	16714	18000	17381	18884	16455	17654
	h_3	16903	18257	17101	18522	16708	17996	17373	18881	16449	17651
P2	Th	16164		16308		16020		16503		15828	
	h_1	16901	18260	16935	18490	16712	18013	16858	18807	16231	17634
	h_2	16921	18272	17120	18537	16723	18010	17391	18896	16462	17663
	h_3	16914	18269	17114	18535	16718	18006	17385	18893	16457	17659
P3	Th	16210		16634		15786		17201		15222	
	h_1	16883	18244	16901	18697	16439	17745	16887	19387	15363	16969
	h_2	16949	18281	17396	18818	16474	17742	17959	19536	15794	17016
	h_3	16946	18279	17420	18822	16472	17738	18044	19550	15827	17019
P4	Th	16134		16145		16122		16163		16107	
	h_1	16899	18258	16951	18389	16835	18131	17011	18564	16728	17958
	h_2	16894	18257	16957	18389	16831	18128	17043	18569	16741	17957
	h_3	16889	18254	16956	18387	16827	18124	17048	18568	16742	17954
P5	Th	16131		16142		16120		16159		16105	
	h_1	16896	18255	16947	18385	16833	18128	17004	18560	16724	17956
	h_2	16892	18254	16954	18386	16828	18124	17041	18566	16738	17955
	h_3	16886	18251	16952	18384	16823	18121	17045	18564	16739	17952

Table A.5: (Cont. of Table A.4) Out-sample mean profit when a scalar feature ($f^{(1)}$) is available. (low/high correlated scalar feature, all demand cases, price processes P6, P7, P8, P9, P10, and 3 parameters array configurations, $h_1 = [1, f^{(1)}]$, $h_2 = [1, f^{(1)}, P_{t-1}]$, and $h_3 = [1, f^{(1)}, P_{t-1}, P_{t-2}]$)

		IID		l+		l-		h+		h-	
		low	high	low	high	low	high	low	high	low	high
P6	Th	16227		16651		15803		17219		15238	
	h_1	16902	18262	16931	18718	16456	17762	16926	19407	15383	16985
	h_2	16966	18298	17414	18837	16491	17759	17979	19555	15806	17031
	h_3	16964	18298	17435	18840	16488	17756	18064	19569	15842	17034
P7	Th	16276		16804		15748		17511		15045	
	h_1	16923	18285	16923	18823	16385	17695	16980	19660	15109	16772
	h_2	17012	18336	17589	18984	16432	17691	18351	19850	15648	16832
	h_3	17008	18334	17592	18983	16428	17688	18374	19854	15655	16830
P8	Th	16245		16763		15727		17455		15038	
	h_1	16899	18258	16933	18792	16367	17674	16996	19612	15123	16767
	h_2	16981	18307	17545	18943	16411	17672	18295	19796	15641	16826
	h_3	16977	18304	17549	18943	16406	17668	18320	19799	15649	16824
P9	Th	16141		16227		16056		16344		15942	
	h_1	16891	18250	16928	18434	16756	18055	16887	18683	16429	17766
	h_2	16899	18255	17033	18460	16761	18052	17208	18737	16568	17782
	h_3	16893	18252	17033	18459	16757	18048	17228	18740	16574	17780
P10	Th	16152		16239		16065		16357		15950	
	h_1	16901	18261	16942	18446	16766	18064	16907	18698	16436	17773
	h_2	16910	18267	17044	18473	16770	18062	17220	18751	16574	17789
	h_3	16905	18264	17047	18472	16766	18059	17241	18755	16582	17788

Description of Table A.6 and Table A.7 :

Table A.6 and Table A.7 show the out-sample mean profit values of data driven linear program simulation results when a binary feature (see Section 3.3.4) is available. Two different configuration of the binary feature are tested. The first configuration named "low" is the case where the probability of demand being higher than its mean is 0.75 given the feature is equal to 1. The second configuration named "high" is the case where the probability of demand being higher than its mean is 0.9 given the feature is equal to 1. Besides the model uses a binary feature rather than a scalar one, the layout of the tables A.6 and Table A.7 are exactly the same with the layouts of A.4 and Table A.5. The detailed explanation of the layout can be found in Description of Table A.4 and Table A.5 part.

Table A.6: Out-sample mean profit when a binary feature ($f^{(2)}$) is available. (low/high correlated scalar feature, all demand cases, price processes IID, P1, P2, P3, P4 and P5), and parameters array configurations $h_1 = [1, f^{(2)}]$, $h_2 = [1, f^{(2)}, P_{t-1}]$, and $h_3 = [1, f^{(2)}, P_{t-1}, P_{t-2}]$.)

		IID		l+		l-		h+		h-	
		low	high	low	high	low	high	low	high	low	high
IID	Th	16130		16130		16131		16132		16132	
	h_1	16796	17511	16845	17591	16750	17435	16910	17700	16695	17341
	h_2	16791	17507	16840	17587	16745	17431	16904	17694	16689	17336
	h_3	16783	17502	16832	17581	16739	17426	16896	17688	16684	17332
P1	Th	16154		16298		16010		16491		15819	
	h_1	16788	17503	16776	17565	16593	17265	16533	17411	16052	16700
	h_2	16808	17519	16984	17682	16609	17265	17187	17778	16325	16833
	h_3	16800	17513	16975	17676	16603	17261	17179	17773	16318	16828
P2	Th	16164		16308		16020		16503		15828	
	h_1	16798	17513	16797	17582	16605	17275	16564	17433	16067	16712
	h_2	16820	17529	16997	17695	16620	17276	17201	17792	16337	16844
	h_3	16814	17526	16993	17690	16615	17271	17195	17787	16332	16839
P3	Th	16210		16634		15786		17201		15222	
	h_1	16781	17497	16688	17544	16311	16955	16130	17052	15033	15644
	h_2	16848	17546	17241	17871	16355	16963	17626	18043	15593	15967
	h_3	16845	17545	17265	17885	16352	16960	17740	18121	15635	15994
P4	Th	16134		16145		16122		16163		16107	
	h_1	16796	17512	16842	17592	16739	17422	16876	17675	16639	17287
	h_2	16794	17509	16849	17594	16735	17418	16914	17692	16652	17291
	h_3	16786	17504	16845	17590	16729	17413	16919	17692	16652	17288
P5	Th	16131		16142		16120		16159		16105	
	h_1	16793	17509	16835	17587	16735	17420	16869	17670	16634	17282
	h_2	16790	17506	16844	17590	16731	17416	16907	17689	16649	17286
	h_3	16781	17501	16841	17585	16725	17411	16912	17688	16649	17285

Table A.7: (Cont. of Table A.6) Out-sample mean profit when a binary feature ($f^{(2)}$) is available. (low/high correlated scalar feature, all demand cases, price processes P6, P7, P8, P9, P10), and parameters array configurations $h_1 = [1, f^{(2)}]$, $h_2 = [1, f^{(2)}, P_{t-1}]$, and $h_3 = [1, f^{(2)}, P_{t-1}, P_{t-2}]$.)

		IID		l+		l-		h+		h-	
		low	high	low	high	low	high	low	high	low	high
P6	Th	16227		16651		15803		17219		15238	
	h_1	16800	17515	16723	17570	16332	16973	16181	17089	15059	15670
	h_2	16866	17566	17264	17893	16376	16983	17650	18063	15609	15983
	h_3	16865	17565	17291	17908	16374	16980	17762	18144	15655	16011
P7	Th	16276		16804		15748		17511		15045	
	h_1	16819	17536	16674	17564	16254	16887	16000	16941	14717	15322
	h_2	16911	17606	17430	18019	16312	16901	18016	18342	15436	15754
	h_3	16905	17603	17434	18021	16307	16897	18048	18366	15450	15762
P8	Th	16245		16763		15727		17455		15038	
	h_1	16796	17512	16696	17565	16238	16869	16076	16984	14751	15346
	h_2	16881	17579	17394	17983	16294	16881	17962	18294	15438	15757
	h_3	16877	17575	17400	17985	16288	16878	17997	18319	15448	15762
P9	Th	16141		16227		16056		16344		15942	
	h_1	16788	17503	16799	17572	16654	17331	16666	17516	16298	16946
	h_2	16797	17510	16914	17634	16660	17329	17037	17713	16456	17017
	h_3	16790	17505	16913	17632	16654	17325	17061	17724	16464	17018
P10	Th	16152		16239		16065		16357		15950	
	h_1	16799	17514	16818	17588	16664	17338	16687	17531	16306	16955
	h_2	16807	17521	16928	17648	16670	17336	17050	17727	16463	17026
	h_3	16803	17517	16932	17648	16666	17333	17075	17738	16469	17027

Table A.8: Out-sample mean profit when multiple features ($f^{(1)}$ and $f^{(2)} = high$) are available. (low/high correlated scalar feature, all demand cases, price processes IID, P1, P2, P3, P4, P5), and parameters array configurations $h_1 = [1, f^{(1)}, f^{(2)}]$, $h_2 = [1, f^{(1)}, f^{(2)}, P_{t-1}]$, and $h_3 = [1, f^{(1)}, f^{(2)}, P_{t-1}, P_{t-2}]$.)

		IID		l+		l-		h+		h-	
		low	high	low	high	low	high	low	high	low	high
IID	Th	16130		16130		16131		16132		16132	
	h_1	17753	18457	17848	18583	17661	18334	17976	18756	17546	18175
	h_2	17748	18454	17844	18581	17657	18332	17972	18752	17542	18171
	h_3	17744	18451	17840	18579	17653	18329	17968	18750	17538	18168
P1	Th	16154		16298		16010		16491		15819	
	h_1	17744	18448	17863	18677	17499	18187	17888	18970	16996	17780
	h_2	17757	18459	17955	18716	17498	18184	18132	19040	17084	17800
	h_3	17753	18456	17950	18713	17495	18181	18128	19037	17079	17797
P2	Th	16164		16308		16020		16503		15828	
	h_1	17755	18459	17878	18692	17509	18195	17904	18983	17007	17788
	h_2	17768	18469	17967	18728	17507	18193	18146	19052	17093	17807
	h_3	17765	18466	17963	18725	17502	18190	18141	19049	17089	17804
P3	Th	16210		16634		15786		17201		15222	
	h_1	17738	18442	17932	18891	17205	17913	17978	19516	16097	17082
	h_2	17784	18475	18176	18987	17206	17911	18543	19644	16291	17118
	h_3	17783	18474	18186	18990	17204	17908	18592	19655	16307	17119
P4	Th	16134		16145		16122		16163		16107	
	h_1	17753	18457	17851	18592	17648	18324	17966	18771	17498	18142
	h_2	17750	18455	17852	18593	17645	18321	17979	18774	17501	18140
	h_3	17747	18453	17849	18590	17640	18318	17978	18772	17498	18138
P5	Th	16131		16142		16120		16159		16105	
	h_1	17751	18454	17847	18589	17646	18320	17961	18767	17496	18140
	h_2	17747	18453	17849	18590	17642	18318	17973	18770	17497	18139
	h_3	17743	18450	17846	18588	17639	18316	17972	18769	17495	18136

Table A.9: (Cont. of Table A.8) Out-sample mean profit when multiple features ($f^{(1)}$ and $f^{(2)} = high$) are available. (low/high correlated scalar feature, all demand cases, price processes P6, P7, P8, P9, P10), and parameters array configurations $h_1 = [1, f^{(1)}, f^{(2)}]$, $h_2 = [1, f^{(1)}, f^{(2)}, P_{t-1}]$, and $h_3 = [1, f^{(1)}, f^{(2)}, P_{t-1}, P_{t-2}]$.)

		IID		l+		l-		h+		h-	
		low	high	low	high	low	high	low	high	low	high
P6	Th	16227		16651		15803		17219		15238	
	h_1	17756	18460	17955	18910	17221	17927	18007	19537	16118	17097
	h_2	17801	18493	18194	19006	17223	17925	18564	19662	16307	17133
	h_3	17799	18492	18206	19009	17220	17923	18613	19673	16321	17134
P7	Th	16276		16804		15748		17511		15045	
	h_1	17778	18483	17995	19016	17143	17855	18077	19776	15826	16872
	h_2	17840	18528	18329	19145	17146	17851	18845	19941	16086	16921
	h_3	17837	18526	18330	19144	17141	17848	18857	19944	16089	16919
P8	Th	16245		16763		15727		17455		15038	
	h_1	17753	18457	17978	18982	17123	17836	18064	19728	15837	16871
	h_2	17812	18499	18290	19104	17127	17833	18791	19887	16085	16916
	h_3	17809	18497	18291	19104	17124	17831	18806	19889	16086	16914
P9	Th	16141		16227		16056		16344		15942	
	h_1	17745	18449	17854	18636	17563	18244	17906	18872	17205	17931
	h_2	17750	18454	17903	18656	17560	18241	18045	18914	17254	17943
	h_3	17747	18452	17901	18655	17558	18239	18052	18915	17254	17941
P10	Th	16152		16239		16065		16357		15950	
	h_1	17755	18459	17867	18649	17570	18252	17923	18886	17215	17939
	h_2	17761	18464	17915	18669	17567	18250	18055	18927	17262	17950
	h_3	17758	18462	17914	18667	17563	18246	18063	18928	17263	17948

Table A.10: Out-sample mean profit when multiple features ($f^{(1)}$ and $f^{(2)} = \text{low}$) are available. (low/high correlated scalar feature, all demand cases, price processes IID, P1, P2, P3, P4, P5), and parameters array configurations $h_1 = [1, f^{(1)}, f^{(2)}]$, $h_2 = [1, f^{(1)}, f^{(2)}, P_{t-1}]$, and $h_3 = [1, f^{(1)}, f^{(2)}, P_{t-1}, P_{t-2}]$.)

		IID		l+		l-		h+		h-	
		low	high	low	high	low	high	low	high	low	high
IID	Th	16130		16130		16131		16132		16132	
	h_1	17274	18331	17351	18583	17200	18210	17457	18623	17107	18054
	h_2	17269	18328	17347	18581	17196	18207	17452	18620	17102	18051
	h_3	17265	18324	17341	18579	17192	18204	17447	18617	17098	18048
P1	Th	16154		16298		16010		16491		15819	
	h_1	17266	18322	17355	18677	17058	18071	17386	18871	16598	17690
	h_2	17283	18334	17496	18716	17061	18067	17760	18952	16751	17714
	h_3	17278	18330	17492	18713	17056	18064	17754	18948	16745	17710
P2	Th	16164		16308		16020		16503		15828	
	h_1	17276	18333	17374	18692	17064	18080	17409	18885	16609	17697
	h_2	17295	18344	17508	18728	17069	18078	17772	18965	16760	17723
	h_3	17290	18341	17504	18725	17065	18075	17766	18962	16756	17719
P3	Th	16210		16634		15786		17201		15222	
	h_1	17260	18317	17410	18891	16793	17810	17560	19456	15777	17025
	h_2	17316	18351	17771	18987	16808	17807	18300	19593	16060	17066
	h_3	17314	18349	17786	18990	16804	17804	18361	19605	16082	17068
P4	Th	16134		16145		16122		16163		16107	
	h_1	17274	18331	17353	18592	17190	18201	17447	18643	17063	18024
	h_2	17272	18330	17357	18593	17186	18198	17467	18646	17070	18023
	h_3	17268	18326	17354	18590	17183	18194	17470	18644	17069	18021
P5	Th	16131		16142		16120		16159		16105	
	h_1	17272	18328	17349	18589	17186	18197	17438	18638	17059	18022
	h_2	17268	18326	17354	18590	17183	18195	17462	18642	17066	18022
	h_3	17264	18323	17351	18588	17179	18191	17465	18641	17065	18019

Table A.11: (Cont. of Table A.10) Out-sample mean profit when multiple features ($f^{(1)}$ and $f^{(2)} = low$) are available. (low/high correlated scalar feature, all demand cases, price processes P6, P7, P8, P9, P10), and parameters array configurations $h_1 = [1, f^{(1)}, f^{(2)}]$, $h_2 = [1, f^{(1)}, f^{(2)}, P_{t-1}]$, and $h_3 = [1, f^{(1)}, f^{(2)}, P_{t-1}, P_{t-2}]$.)

		IID		l+		l-		h+		h-	
		low	high	low	high	low	high	low	high	low	high
P6	Th	16227		16651		15803		17219		15238	
	h_1	17277	18334	17438	18910	16808	17825	17591	19476	15797	17040
	h_2	17336	18370	17791	19006	16823	17823	18324	19612	16075	17081
	h_3	17332	18369	17808	19009	16821	17821	18389	19625	16101	17083
P7	Th	16276		16804		15748		17511		15045	
	h_1	17300	18357	17467	19016	16738	17758	17688	19725	15528	16824
	h_2	17376	18406	17950	19145	16759	17755	18652	19902	15890	16877
	h_3	17374	18403	17951	19144	16754	17751	18670	19905	15896	16875
P8	Th	16245		16763		15727		17455		15038	
	h_1	17274	18331	17461	18982	16719	17737	17681	19679	15537	16820
	h_2	17348	18377	17911	19104	16740	17734	18595	19846	15888	16872
	h_3	17344	18374	17912	19104	16735	17731	18614	19848	15892	16871
P9	Th	16141		16227		16056		16344		15942	
	h_1	17266	18323	17350	18636	17112	18124	17395	18760	16791	17831
	h_2	17274	18328	17428	18656	17113	18121	17613	18809	16882	17845
	h_3	17270	18324	17428	18655	17107	18117	17626	18810	16886	17843
P10	Th	16152		16239		16065		16357		15950	
	h_1	17276	18333	17367	18649	17119	18134	17414	18776	16796	17837
	h_2	17286	18339	17440	18669	17121	18131	17625	18823	16887	17851
	h_3	17283	18337	17442	18667	17118	18128	17638	18826	16889	17851

Appendix B

APPENDIX B

Table B.1: Dispersion of simulation results for i.i.d. Demand (diff. : difference between theoretical and data-driven LP out-sample mean profits in each out-sample simulation runs for the same demand-price realization), (values in parentheses: relative percentages of the values to the theoretical means, i.e. $\frac{100x}{\text{Theoretical Mean}}$)

Price	mean diff.		st.dev. diff.		max. diff.		min. diff.	
IID	-0.35	0.00 %	24.04	0.15 %	141.35	0.88 %	-153.73	-0.95 %
P1	1.49	0.01 %	32.48	0.20 %	285.80	1.77 %	-163.76	-1.01 %
P2	1.55	0.01 %	31.88	0.20 %	213.50	1.32 %	-149.03	-0.92 %
P3	6.84	0.04 %	36.72	0.23 %	260.73	1.61 %	-207.15	-1.28 %
P4	0.52	0.00 %	25.96	0.16 %	155.41	0.96 %	-135.91	-0.84 %
P5	-0.22	0.00 %	25.86	0.16 %	165.94	1.03 %	-147.00	-0.91 %
P6	7.46	0.05 %	36.19	0.22 %	211.75	1.30 %	-166.95	-1.03 %
P7	6.03	0.04 %	34.92	0.21 %	424.00	2.60 %	-179.66	-1.10 %
P8	5.74	0.04 %	35.10	0.22 %	266.40	1.64 %	-179.74	-1.11 %
P9	0.33	0.00 %	30.78	0.19 %	216.93	1.34 %	-151.82	-0.94 %
P10	2.09	0.01 %	31.05	0.19 %	207.51	1.28 %	-118.23	-0.73 %

Table B.2: Dispersion of simulation results for $l+$ Demand (diff. : difference between theoretical and data-driven LP out-sample mean profits in each out-sample simulation runs for the same demand-price realization), (values in parentheses: relative percentages of the values to the theoretical means, i.e. $\frac{100x}{\text{Theoretical Mean}}$)

Price	mean diff.		st.dev. diff.		max. diff.		min. diff.	
IID	0.87	0.01 %	29.12	0.18 %	284.95	1.77 %	-135.67	-0.84 %
P1	9.52	0.06 %	35.33	0.22 %	338.91	2.08 %	-218.66	-1.34 %
P2	9.43	0.06 %	34.72	0.21 %	205.85	1.26 %	-175.55	-1.08 %
P3	26.46	0.16 %	49.95	0.30 %	286.87	1.72 %	-204.78	-1.23 %
P4	5.78	0.04 %	38.95	0.24 %	306.41	1.90 %	-129.73	-0.80 %
P5	5.53	0.03 %	40.21	0.25 %	340.39	2.11 %	-134.77	-0.83 %
P6	29.14	0.17 %	51.27	0.31 %	281.22	1.69 %	-165.56	-0.99 %
P7	17.28	0.10 %	45.43	0.27 %	482.63	2.87 %	-132.74	-0.79 %
P8	12.38	0.07 %	41.27	0.25 %	279.95	1.67 %	-145.48	-0.87 %
P9	8.30	0.05 %	38.85	0.24 %	306.78	1.89 %	-215.53	-1.33 %
P10	8.71	0.05 %	37.64	0.23 %	209.83	1.29 %	-180.34	-1.11 %

Table B.3: Dispersion of simulation results for l – Demand (diff. : difference between theoretical and data-driven LP out-sample mean profits in each out-sample simulation runs for the same demand-price realization), (values in parentheses: relative percentages of the values to the theoretical means, i.e. $\frac{100x}{\text{Theoretical Mean}}$)

Price	mean diff.		st.dev. diff.		max. diff.		min. diff.	
IID	0.62	0.00 %	24.36	0.15 %	182.84	1.13 %	-137.47	-0.85 %
P1	3.88	0.02 %	32.65	0.20 %	207.40	1.30 %	-184.32	-1.15 %
P2	1.53	0.01 %	30.48	0.19 %	210.39	1.31 %	-137.40	-0.86 %
P3	11.15	0.07 %	37.14	0.24 %	190.56	1.21 %	-172.16	-1.09 %
P4	1.02	0.01 %	26.79	0.17 %	187.22	1.16 %	-128.65	-0.80 %
P5	1.76	0.01 %	27.18	0.17 %	207.85	1.29 %	-183.49	-1.14 %
P6	7.84	0.05 %	34.88	0.22 %	208.98	1.32 %	-179.36	-1.13 %
P7	9.59	0.06 %	35.92	0.23 %	294.40	1.87 %	-188.01	-1.19 %
P8	7.07	0.04 %	32.57	0.21 %	208.32	1.32 %	-152.20	-0.97 %
P9	3.24	0.02 %	32.11	0.20 %	184.73	1.15 %	-171.11	-1.07 %
P10	0.30	0.00 %	30.06	0.19 %	219.65	1.37 %	-129.45	-0.81 %

Table B.4: Dispersion of simulation results for $h+$ Demand (diff. : difference between theoretical and data-driven LP out-sample mean profits in each out-sample simulation runs for the same demand-price realization), (values in parentheses: relative percentages of the values to the theoretical means, i.e. $\frac{100x}{\text{Theoretical Mean}}$)

Price	mean diff.		st.dev. diff.		max. diff.		min. diff.	
IID	2.43	0.02 %	30.54	0.19 %	209.05	1.30 %	-127.14	-0.79 %
P1	12.57	0.08 %	38.85	0.24 %	365.38	2.22 %	-144.69	-0.88 %
P2	13.24	0.08 %	38.16	0.23 %	253.07	1.53 %	-126.89	-0.77 %
P3	72.17	0.42 %	72.06	0.42 %	434.19	2.52 %	-193.94	-1.13 %
P4	14.84	0.09 %	49.09	0.30 %	347.19	2.15 %	-198.39	-1.23 %
P5	17.13	0.11 %	51.71	0.32 %	308.48	1.91 %	-202.60	-1.25 %
P6	70.96	0.41 %	70.34	0.41 %	377.73	2.19 %	-220.73	-1.28 %
P7	22.34	0.13 %	58.78	0.34 %	698.88	3.99 %	-172.01	-0.98 %
P8	17.89	0.10 %	48.30	0.28 %	411.86	2.36 %	-158.06	-0.91 %
P9	13.45	0.08 %	43.23	0.26 %	231.91	1.42 %	-196.38	-1.20 %
P10	14.53	0.09 %	43.12	0.26 %	342.03	2.09 %	-188.07	-1.15 %

Table B.5: Dispersion of simulation results for h – Demand (diff. : difference between theoretical and data-driven LP out-sample mean profits in each out-sample simulation runs for the same demand-price realization), (values in parentheses: relative percentages of the values to the theoretical means, i.e. $\frac{100x}{\text{Theoretical Mean}}$)

Price	mean diff.		st.dev. diff.		max. diff.		min. diff.	
IID	2.31	0.01 %	25.17	0.16 %	270.36	1.68 %	-163.79	-1.02 %
P1	13.25	0.08 %	36.40	0.23 %	337.58	2.13 %	-167.93	-1.06 %
P2	9.63	0.06 %	31.23	0.20 %	263.48	1.66 %	-134.95	-0.85 %
P3	44.17	0.29 %	53.52	0.35 %	311.96	2.05 %	-159.77	-1.05 %
P4	6.66	0.04 %	35.55	0.22 %	252.97	1.57 %	-150.09	-0.93 %
P5	11.96	0.07 %	41.34	0.26 %	360.24	2.24 %	-181.74	-1.13 %
P6	37.93	0.25 %	48.30	0.32 %	305.80	2.01 %	-165.55	-1.09 %
P7	17.16	0.11 %	44.43	0.30 %	434.06	2.89 %	-142.70	-0.95 %
P8	12.58	0.08 %	36.71	0.24 %	244.89	1.63 %	-159.56	-1.06 %
P9	14.51	0.09 %	39.07	0.25 %	262.84	1.65 %	-180.58	-1.13 %
P10	10.57	0.07 %	35.15	0.22 %	267.65	1.68 %	-134.64	-0.84 %

Description of Table B.6, Table B.7, and Table B.8 :

First four columns of tables indicate the parameters arrays configurations of the data-driven LPs. While '1' implies the presence of the corresponding information in the parameters array, 'o' implies the absence of the corresponding information. The fifth column labeled as 'st.dev.' lists the standard deviations of out-sample mean profits in each out-sample simulation run. The column labeled as 'mean diff.' lists the mean values of difference between theoretical and data-driven LP out-sample mean profits in each out-sample simulation runs for the same demand-price realizations. The column labeled as 'st.dev. diff.' lists the standard deviations of these differences. In addition, minimum and maximum difference values are listed to observe the extreme values and understand the shape of the distributions better. These values are represented in columns labeled as 'min. diff.' and 'max. diff.'. Furthermore, the values in parentheses are the relative percentages of the values to the theoretical means of the corresponding cases, i.e. for any value x ,

$$100 \frac{x}{\text{Theoretical Mean}}$$

will be the value in parenthesis. This percentage provides a tool to assess the magnitude of the value with respect to the theoretical benchmark for all cases presented.

Table B.6: Dispersion of simulation results with features for scenario: IID Demand-IID Price case.

P_{t-1}	P_{t-2}	$f^{(1)}$	$f^{(2)}$	σ	mean diff.	st.dev. diff.	min. diff.	max. diff.
o	o	o	o	441 (2.7)	-7 (0.0)	22 (0.1)	-157 (-1.0)	104 (0.6)
1	o	o	o	442 (2.7)	-12 (-0.1)	29 (0.2)	-199 (-1.2)	111 (0.7)
1	1	o	o	442 (2.7)	-20 (-0.1)	38 (0.2)	-242 (-1.5)	154 (1.0)
o	o	o	l	427 (2.6)	666 (4.1)	171 (1.1)	23 (0.1)	1363 (8.4)
1	o	o	l	427 (2.6)	660 (4.1)	172 (1.1)	19 (0.1)	1363 (8.4)
1	1	o	l	427 (2.6)	652 (4.0)	173 (1.1)	-16 (-0.1)	1368 (8.5)
o	o	o	h	413 (2.6)	1381 (8.6)	172 (1.1)	709 (4.4)	1962 (12.2)
1	o	o	h	414 (2.6)	1377 (8.5)	173 (1.1)	710 (4.4)	1961 (12.2)
1	1	o	h	413 (2.6)	1371 (8.5)	174 (1.1)	677 (4.2)	1965 (12.2)
o	o	l	o	423 (2.6)	768 (4.8)	175 (1.1)	17 (0.1)	1585 (9.8)
1	o	l	o	423 (2.6)	762 (4.7)	176 (1.1)	-14 (-0.1)	1503 (9.3)
1	1	l	o	423 (2.6)	757 (4.7)	177 (1.1)	-3 (0.0)	1526 (9.5)
o	o	l	l	416 (2.6)	1144 (7.1)	196 (1.2)	396 (2.5)	1973 (12.2)
1	o	l	l	416 (2.6)	1139 (7.1)	197 (1.2)	381 (2.4)	1976 (12.3)
1	1	l	l	416 (2.6)	1134 (7.0)	197 (1.2)	380 (2.4)	1974 (12.2)
o	o	l	h	407 (2.5)	1622 (10.1)	187 (1.2)	875 (5.4)	2358 (14.6)
1	o	l	h	407 (2.5)	1617 (10.0)	188 (1.2)	871 (5.4)	2389 (14.8)
1	1	l	h	407 (2.5)	1614 (10.0)	188 (1.2)	884 (5.5)	2385 (14.8)
o	o	h	o	398 (2.5)	2127 (13.2)	215 (1.3)	1329 (8.2)	3108 (19.3)
1	o	h	o	398 (2.5)	2125 (13.2)	215 (1.3)	1333 (8.3)	3105 (19.2)
1	1	h	o	398 (2.5)	2121 (13.2)	216 (1.3)	1335 (8.3)	3097 (19.2)
o	o	h	l	397 (2.5)	2200 (13.6)	216 (1.3)	1395 (8.6)	3151 (19.5)
1	o	h	l	397 (2.5)	2198 (13.6)	216 (1.3)	1378 (8.5)	3151 (19.5)
1	1	h	l	398 (2.5)	2194 (13.6)	216 (1.3)	1366 (8.5)	3152 (19.5)
o	o	h	h	395 (2.5)	2326 (14.4)	210 (1.3)	1542 (9.6)	3217 (19.9)
1	o	h	h	395 (2.5)	2323 (14.4)	210 (1.3)	1540 (9.5)	3212 (19.9)
1	1	h	h	396 (2.5)	2321 (14.4)	211 (1.3)	1531 (9.5)	3212 (19.9)

Table B.7: Dispersion of simulation results with features for scenario: l – Demand - P1 Price case.

P_{t-1}	P_{t-2}	$f^{(1)}$	$f^{(2)}$	σ	mean diff.	st.dev. diff.	min. diff.	max. diff.
o	o	o	o	1090 (6.8)	-60 (-0.4)	60 (0.4)	-335 (-2.1)	165 (1.0)
1	o	o	o	1086 (6.8)	-15 (-0.1)	33 (0.2)	-271 (-1.7)	155 (1.0)
1	1	o	o	1086 (6.8)	-22 (-0.1)	40 (0.3)	-269 (-1.7)	184 (1.2)
o	o	o	l	1080 (6.7)	583 (3.6)	164 (1.0)	-137 (-0.9)	1224 (7.6)
1	o	o	l	1079 (6.7)	599 (3.7)	160 (1.0)	-61 (-0.4)	1241 (7.7)
1	1	o	l	1079 (6.7)	593 (3.7)	160 (1.0)	-70 (-0.4)	1247 (7.8)
o	o	o	h	1073 (6.7)	1254 (7.8)	164 (1.0)	527 (3.3)	1828 (11.4)
1	o	o	h	1073 (6.7)	1255 (7.8)	164 (1.0)	558 (3.5)	1865 (11.6)
1	1	o	h	1073 (6.7)	1251 (7.8)	164 (1.0)	555 (3.5)	1862 (11.6)
o	o	l	o	1076 (6.7)	690 (4.3)	170 (1.1)	-48 (-0.3)	1533 (9.6)
1	o	l	o	1075 (6.7)	704 (4.4)	166 (1.0)	31 (0.2)	1536 (9.6)
1	1	l	o	1075 (6.7)	698 (4.4)	167 (1.0)	13 (0.1)	1489 (9.3)
o	o	l	l	1072 (6.7)	1048 (6.5)	187 (1.2)	277 (1.7)	1768 (11.0)
1	o	l	l	1071 (6.7)	1051 (6.6)	185 (1.2)	259 (1.6)	1779 (11.1)
1	1	l	l	1071 (6.7)	1046 (6.5)	184 (1.2)	217 (1.4)	1771 (11.1)
o	o	l	h	1067 (6.7)	1489 (9.3)	179 (1.1)	739 (4.6)	2159 (13.5)
1	o	l	h	1067 (6.7)	1488 (9.3)	179 (1.1)	760 (4.7)	2160 (13.5)
1	1	l	h	1067 (6.7)	1484 (9.3)	179 (1.1)	758 (4.7)	2159 (13.5)
o	o	h	o	1055 (6.6)	1993 (12.4)	203 (1.3)	1202 (7.5)	2939 (18.4)
1	o	h	o	1056 (6.6)	1990 (12.4)	203 (1.3)	1204 (7.5)	2929 (18.3)
1	1	h	o	1056 (6.6)	1986 (12.4)	203 (1.3)	1195 (7.5)	2915 (18.2)
o	o	h	l	1056 (6.6)	2061 (12.9)	203 (1.3)	1265 (7.9)	2932 (18.3)
1	o	h	l	1056 (6.6)	2057 (12.8)	204 (1.3)	1264 (7.9)	2935 (18.3)
1	1	h	l	1056 (6.6)	2054 (12.8)	204 (1.3)	1237 (7.7)	2935 (18.3)
o	o	h	h	1054 (6.6)	2177 (13.6)	199 (1.2)	1442 (9.0)	3063 (19.1)
1	o	h	h	1054 (6.6)	2174 (13.6)	199 (1.2)	1441 (9.0)	3059 (19.1)
1	1	h	h	1054 (6.6)	2171 (13.6)	199 (1.2)	1439 (9.0)	3055 (19.1)

Table B.8: Dispersion of simulation results with features for scenario: $h+$ Demand - P7 Price case.

P_{t-1}	P_{t-2}	$f^{(1)}$	$f^{(2)}$	σ	mean diff.	st.dev. diff.	min. diff.	max. diff.
o	o	o	o	4848 (27.7)	-4105 (-23.4)	1565 (8.9)	-13817 (-78.9)	-463 (-2.6)
1	o	o	o	4514 (25.8)	-81 (-0.5)	86 (0.5)	-699 (-4.0)	172 (1.0)
1	1	o	o	4512 (25.8)	-29 (-0.2)	57 (0.3)	-753 (-4.3)	157 (0.9)
o	o	o	l	4563 (26.1)	-1510 (-8.6)	825 (4.7)	-6787 (-38.8)	602 (3.4)
1	o	o	l	4492 (25.7)	506 (2.9)	189 (1.1)	-422 (-2.4)	1220 (7.0)
1	1	o	l	4493 (25.7)	537 (3.1)	181 (1.0)	-293 (-1.7)	1250 (7.1)
o	o	o	h	4509 (25.7)	-569 (-3.3)	787 (4.5)	-5807 (-33.2)	1227 (7.0)
1	o	o	h	4469 (25.5)	831 (4.7)	256 (1.5)	-931 (-5.3)	1836 (10.5)
1	1	o	h	4471 (25.5)	856 (4.9)	246 (1.4)	-603 (-3.4)	1866 (10.7)
o	o	l	o	4543 (25.9)	-531 (-3.0)	497 (2.8)	-3007 (-17.2)	1130 (6.5)
1	o	l	o	4497 (25.7)	841 (4.8)	202 (1.2)	24 (0.1)	1578 (9.0)
1	1	l	o	4496 (25.7)	863 (4.9)	198 (1.1)	60 (0.3)	1597 (9.1)
o	o	l	l	4508 (25.7)	177 (1.0)	433 (2.5)	-2242 (-12.8)	1631 (9.3)
1	o	l	l	4485 (25.6)	1142 (6.5)	220 (1.3)	213 (1.2)	1977 (11.3)
1	1	l	l	4484 (25.6)	1159 (6.6)	216 (1.2)	244 (1.4)	1996 (11.4)
o	o	l	h	4480 (25.6)	566 (3.2)	457 (2.6)	-2203 (-12.6)	1787 (10.2)
1	o	l	h	4470 (25.5)	1334 (7.6)	239 (1.4)	287 (1.6)	2291 (13.1)
1	1	l	h	4471 (25.5)	1347 (7.7)	236 (1.3)	341 (1.9)	2314 (13.2)
o	o	h	o	4465 (25.5)	2149 (12.3)	266 (1.5)	1139 (6.5)	3230 (18.4)
1	o	h	o	4461 (25.5)	2340 (13.4)	243 (1.4)	1352 (7.7)	3287 (18.8)
1	1	h	o	4461 (25.5)	2343 (13.4)	243 (1.4)	1360 (7.8)	3293 (18.8)
o	o	h	l	4463 (25.5)	2215 (12.6)	265 (1.5)	1227 (7.0)	3250 (18.6)
1	o	h	l	4459 (25.5)	2392 (13.7)	244 (1.4)	1462 (8.4)	3321 (19.0)
1	1	h	l	4460 (25.5)	2394 (13.7)	243 (1.4)	1482 (8.5)	3322 (19.0)
o	o	h	h	4458 (25.5)	2266 (12.9)	266 (1.5)	1255 (7.2)	3268 (18.7)
1	o	h	h	4456 (25.4)	2431 (13.9)	243 (1.4)	1441 (8.2)	3363 (19.2)
1	1	h	h	4457 (25.5)	2433 (13.9)	243 (1.4)	1474 (8.4)	3365 (19.2)

Appendix C

APPENDIX C



Table C.1: i.i.d. Demand - i.i.d. Price in-sample/out-sample CVaR results of CVaR Minimization LP and Expected Profit Maximization LP

		CVAR LP		EP LP	
		In-sample	Out-sample	In-sample	Out-sample
IID	Th	-	-	2645	2508
	h_1	3674	3327	2643	2501
	h_2	3753	3253	2635	2488
	h_3	3818	3190	2652	2466
l+	Th	-	-	723	663
	h_1	3521	3254	676	610
	h_2	3597	3179	680	630
	h_3	3664	3125	700	618
l-	Th	-	-	4110	3860
	h_1	4214	3829	4112	3857
	h_2	4293	3768	4105	3837
	h_3	4357	3678	4111	3832
h+	Th	-	-	-1832	-1918
	h_1	4026	3731	-1910	-1999
	h_2	4081	3671	-1912	-2010
	h_3	4142	3626	-1906	-2016
h-	Th	-	-	4282	4135
	h_1	5789	5385	4243	4098
	h_2	5854	5337	4240	4084
	h_3	5913	5244	4260	4083

Table C.2: Mean CVaR no feature IID-P5

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2557		628		3988		-1952		4184	
	h_1	3427	2545	3297	575	3935	3975	3784	-2031	5510	4140
	h_2	3348	2538	3230	568	3858	3962	3737	-2049	5439	4129
	h_3	3272	2528	3148	572	3783	3949	3663	-2053	5378	4122
P1	Th	-198		-1650		866		-3837		1178	
	h_1	285	-563	6	-3919	669	682	-176	-9167	1636	-1375
	h_2	174	-234	-15	-1734	669	829	149	-3923	2012	1090
	h_3	90	-236	-120	-1748	578	812	77	-3948	1944	1075
P2	Th	-353		-1807		708		-3996		1007	
	h_1	129	-698	-115	-4042	536	518	-172	-9338	1624	-1601
	h_2	44	-383	-167	-1857	558	682	70	-4114	1945	978
	h_3	-41	-399	-248	-1862	472	670	6	-4116	1879	961
P3	Th	-4742		-5420		-4345		-6628		-4444	
	h_1	-4553	-5485	-4715	-10860	-4627	-4979	-5364	-19642	-4622	-9514
	h_2	-4672	-4812	-4626	-5785	-4583	-4413	-4400	-7552	-4086	-4866
	h_3	-4782	-4796	-4697	-5550	-4657	-4410	-4408	-6894	-4117	-4622
P4	Th	2303		414		3702		-2149		3927	
	h_1	3124	2254	2976	146	3640	3684	3380	-2727	5170	3620
	h_2	3068	2272	2914	293	3581	3677	3356	-2368	5158	3821
	h_3	2987	2266	2836	325	3516	3664	3316	-2282	5096	3873
P5	Th	2332		447		3729		-2109		3950	
	h_1	3153	2280	3007	182	3660	3711	3416	-2714	5176	3643
	h_2	3083	2308	2924	339	3585	3701	3374	-2328	5134	3811
	h_3	2983	2298	2819	370	3517	3688	3324	-2243	5066	3847

Table C.3: Mean CVaR no feature P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th	-5059		-5723		-4689		-6910		-4846	
	h_1	-4870	-5770	-4925	-11120	-4913	-5318	-5277	-19960	-4738	-9915
	h_2	-4974	-5131	-4898	-6029	-4888	-4744	-4520	-7865	-4329	-5249
	h_3	-5059	-5122	-4963	-5828	-4969	-4737	-4503	-7175	-4382	-4966
P7	Th	-4980		-5549		-4682		-6611		-4871	
	h_1	-4848	-5803	-5284	-11505	-5058	-5365	-6631	-20671	-5779	-9935
	h_2	-5084	-5014	-4994	-5680	-5010	-4745	-4882	-6944	-4773	-5051
	h_3	-5184	-5019	-5076	-5638	-5120	-4750	-4891	-6756	-4805	-4989
P8	Th	-6037		-6499		-5849		-7443		-6180	
	h_1	-5876	-6826	-5943	-12582	-6102	-6545	-6575	-22094	-6408	-11493
	h_2	-6022	-6085	-5776	-6617	-6059	-5887	-5325	-7777	-5786	-6288
	h_3	-6115	-6081	-5853	-6575	-6166	-5893	-5319	-7578	-5843	-6239
P9	Th	798		-830		1995		-3198		2300	
	h_1	1391	551	1149	-2294	1864	1894	1220	-6662	3112	539
	h_2	1297	755	1079	-1004	1828	1960	1362	-3538	3321	2115
	h_3	1200	769	1006	-908	1753	1946	1309	-3297	3278	2203
P10	Th	743		-887		1942		-3264		2248	
	h_1	1340	506	1103	-2357	1809	1841	1210	-6721	3090	488
	h_2	1281	698	1054	-1053	1810	1915	1326	-3611	3304	2136
	h_3	1188	699	996	-962	1737	1902	1279	-3394	3239	2215

Table C.4: Mean CVaR f1 high IID-P1

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2508		663		3860		-1918		4135	
	h_1	7214	7094	6725	6359	7800	7851	6224	5475	8769	8731
	h_2	7168	7089	6678	6356	7751	7845	6173	5468	8719	8725
	h_3	7123	7081	6627	6354	7709	7840	6123	5466	8678	8723
P1	Th	-256		-1661		806		-3805		1212	
	h_1	3677	3525	3254	2749	4135	4226	2742	2013	4854	4778
	h_2	3630	3643	3208	3071	4087	4215	2731	2343	4826	4888
	h_3	3569	3634	3157	3062	4035	4209	2685	2341	4761	4884
P2	Th	-145		-1581		884		-3776		1202	
	h_1	3498	3347	3085	2576	3961	4041	2592	1827	4682	4577
	h_2	3449	3462	3029	2889	3916	4033	2558	2175	4641	4692
	h_3	3401	3457	2976	2882	3866	4027	2508	2175	4601	4689
P3	Th	-4747		-5435		-4292		-6578		-4312	
	h_1	-1520	-1772	-1517	-2225	-1531	-1460	-1538	-2141	-1554	-1627
	h_2	-1594	-1519	-1569	-1596	-1614	-1473	-1520	-1693	-1590	-1498
	h_3	-1661	-1516	-1620	-1577	-1678	-1479	-1554	-1656	-1645	-1488
P4	Th	2334		490		3654		-2068		3933	
	h_1	6868	6743	6373	5995	7450	7503	5855	5099	8402	8355
	h_2	6824	6748	6333	6018	7401	7498	5824	5137	8362	8360
	h_3	6773	6749	6278	6023	7354	7491	5769	5140	8323	8360
P5	Th	2266		458		3611		-2074		3914	
	h_1	6904	6775	6414	6025	7482	7537	5903	5138	8434	8392
	h_2	6863	6783	6364	6055	7446	7531	5860	5175	8394	8398
	h_3	6821	6778	6323	6058	7411	7525	5811	5172	8366	8399

Table C.5: Mean CVaR f1 high P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th		-4748		-5413		-4358		-6592		-4432
	h_1	-1906	-2122	-1846	-2545	-1956	-1843	-1775	-2423	-1973	-2060
	h_2	-1963	-1875	-1906	-1930	-1992	-1850	-1788	-1985	-1979	-1919
	h_3	-2026	-1870	-1959	-1908	-2060	-1856	-1843	-1944	-2052	-1910
P7	Th		-4786		-5385		-4481		-6479		-4671
	h_1	-1769	-2021	-1692	-2393	-1887	-1782	-1734	-2138	-2114	-2077
	h_2	-1865	-1730	-1766	-1686	-1978	-1792	-1634	-1631	-2126	-1931
	h_3	-1949	-1733	-1841	-1682	-2056	-1796	-1710	-1614	-2195	-1931
P8	Th		-5886		-6325		-5727		-7280		-6017
	h_1	-2923	-3174	-2737	-3453	-3123	-3032	-2527	-3079	-3425	-3456
	h_2	-3015	-2883	-2790	-2750	-3216	-3039	-2483	-2576	-3455	-3313
	h_3	-3060	-2888	-2844	-2751	-3261	-3045	-2542	-2565	-3499	-3309
P9	Th		697		-882		1843		-3211		2292
	h_1	4890	4753	4427	3963	5421	5497	3889	3109	6266	6179
	h_2	4849	4829	4388	4172	5383	5488	3866	3354	6224	6249
	h_3	4814	4827	4342	4178	5333	5482	3829	3365	6175	6255
P10	Th		874		-753		2054		-3091		2388
	h_1	4820	4692	4352	3899	5346	5432	3833	3053	6191	6109
	h_2	4762	4763	4298	4106	5298	5426	3780	3296	6152	6184
	h_3	4711	4764	4241	4113	5245	5420	3739	3319	6100	6187

Table C.6: Mean CVaR f1 low IID-P5

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2508		663		3860		-1918		4135	
	h_1	4895	4387	4588	2990	5439	5519	4639	1136	6719	6026
	h_2	4812	4372	4507	2985	5369	5508	4575	1140	6640	6007
	h_3	4758	4368	4446	2975	5324	5502	4517	1131	6599	5995
P1	Th	-256		-1661		806		-3805		1212	
	h_1	1590	1071	1246	-1008	2023	2137	900	-3593	2924	1600
	h_2	1511	1311	1188	273	2002	2162	1044	-1210	3043	2664
	h_3	1436	1299	1116	272	1923	2150	987	-1212	2962	2647
P2	Th	-145		-1581		884		-3776		1202	
	h_1	1404	925	1089	-1121	1860	1972	851	-3705	2823	1470
	h_2	1334	1155	1018	123	1809	2004	920	-1364	2900	2502
	h_3	1280	1150	954	107	1749	1996	859	-1380	2831	2496
P3	Th	-4747		-5435		-4292		-6578		-4312	
	h_1	-3426	-4026	-3504	-6770	-3418	-3415	-3782	-9002	-3388	-4995
	h_2	-3526	-3516	-3475	-4008	-3471	-3265	-3369	-4894	-3199	-3461
	h_3	-3597	-3504	-3567	-3898	-3538	-3267	-3414	-4606	-3254	-3359
P4	Th	2334		490		3654		-2068		3933	
	h_1	4568	4071	4254	2602	5114	5208	4244	654	6384	5614
	h_2	4491	4075	4181	2696	5062	5200	4206	818	6343	5690
	h_3	4426	4078	4116	2706	4996	5189	4147	868	6296	5703
P5	Th	2266		458		3611		-2074		3914	
	h_1	4592	4098	4270	2632	5141	5238	4270	663	6404	5613
	h_2	4533	4111	4215	2737	5080	5224	4226	873	6367	5716
	h_3	4459	4102	4158	2743	4998	5214	4174	909	6312	5722

Table C.7: Mean CVaR f1 low P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th	-4748		-5413		-4358		-6592		-4432	
	h_1	-3763	-4336	-3738	-6988	-3780	-3767	-3814	-9189	-3655	-5370
	h_2	-3843	-3845	-3786	-4327	-3823	-3618	-3571	-5177	-3504	-3828
	h_3	-3963	-3832	-3882	-4227	-3904	-3625	-3619	-4876	-3587	-3718
P7	Th	-4786		-5385		-4481		-6479		-4671	
	h_1	-3677	-4316	-3847	-7148	-3796	-3763	-4570	-9223	-4235	-5431
	h_2	-3800	-3726	-3776	-4014	-3835	-3597	-3739	-4585	-3859	-3779
	h_3	-3929	-3729	-3871	-3999	-3950	-3602	-3763	-4509	-3938	-3761
P8	Th	-5886		-6325		-5727		-7280		-6017	
	h_1	-4764	-5383	-4677	-8103	-4953	-4962	-4836	-10037	-5222	-6770
	h_2	-4890	-4825	-4681	-5022	-5024	-4782	-4312	-5462	-4973	-5087
	h_3	-4974	-4825	-4745	-5003	-5104	-4789	-4373	-5392	-5022	-5057
P9	Th	697		-882		1843		-3211		2292	
	h_1	2737	2241	2389	389	3228	3359	2189	-2073	4328	3128
	h_2	2677	2391	2342	1176	3181	3360	2217	-528	4378	3816
	h_3	2598	2397	2270	1226	3119	3353	2158	-403	4325	3878
P10	Th	874		-753		2054		-3091		2388	
	h_1	2672	2193	2339	341	3177	3303	2160	-2042	4294	3122
	h_2	2604	2331	2279	1116	3138	3306	2155	-586	4313	3792
	h_3	2525	2335	2187	1155	3079	3298	2106	-481	4262	3836

Table C.8: Mean CVaR f2 high IID-P5

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2508		663		3860		-1918		4135	
	h_1	5784	5317	5206	3981	6395	6357	4681	1984	7237	6596
	h_2	5746	5312	5136	3982	6344	6350	4621	1992	7179	6585
	h_3	5685	5309	5096	3977	6280	6342	4575	1983	7117	6578
P1	Th	-256		-1661		806		-3805		1212	
	h_1	2439	2010	1643	-14	2744	2778	531	-3514	2732	1391
	h_2	2364	2203	1572	843	2670	2773	617	-1559	2796	2224
	h_3	2303	2194	1444	848	2620	2767	526	-1559	2738	2223
P2	Th	-145		-1581		884		-3776		1202	
	h_1	2256	1844	1445	-161	2528	2602	490	-3805	2627	1270
	h_2	2196	2023	1349	658	2473	2606	521	-1827	2666	2064
	h_3	2136	2021	1290	653	2410	2600	456	-1836	2627	2056
P3	Th	-4747		-5435		-4292		-6578		-4312	
	h_1	-2615	-3101	-3409	-5907	-3096	-3104	-4782	-11364	-4242	-6487
	h_2	-2697	-2704	-3443	-3938	-3174	-3023	-4204	-6345	-3913	-4676
	h_3	-2785	-2696	-3554	-3853	-3279	-3023	-4234	-5945	-3953	-4538
P4	Th	2334		490		3654		-2068		3933	
	h_1	5464	5002	4852	3585	6045	6031	4252	1428	6820	6133
	h_2	5427	5006	4796	3638	5994	6021	4203	1552	6767	6180
	h_3	5371	5008	4737	3650	5943	6013	4163	1579	6717	6187
P5	Th	2266		458		3611		-2074		3914	
	h_1	5508	5031	4876	3622	6075	6057	4286	1455	6856	6118
	h_2	5445	5046	4812	3697	6004	6046	4252	1586	6780	6182
	h_3	5375	5043	4744	3702	5956	6039	4200	1610	6721	6191

Table C.9: Mean CVaR f2 high P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th	-4748		-5413		-4358		-6592		-4432	
	h_1	-2976	-3433	-3663	-6166	-3514	-3472	-4745	-11759	-4422	-6905
	h_2	-3062	-3040	-3720	-4270	-3580	-3404	-4345	-6725	-4170	-5087
	h_3	-3141	-3035	-3820	-4203	-3667	-3408	-4341	-6377	-4240	-4906
P7	Th	-4786		-5385		-4481		-6479		-4671	
	h_1	-2880	-3377	-3837	-6237	-3479	-3465	-5926	-11527	-5428	-6855
	h_2	-3043	-2916	-3876	-4006	-3631	-3373	-4752	-5921	-4701	-4901
	h_3	-3120	-2920	-3927	-3978	-3758	-3379	-4810	-5822	-4669	-4863
P8	Th	-5886		-6325		-5727		-7280		-6017	
	h_1	-3967	-4476	-4705	-7347	-4741	-4739	-6031	-13108	-6150	-8463
	h_2	-4066	-4021	-4708	-5105	-4872	-4649	-5193	-7065	-5706	-6301
	h_3	-4124	-4028	-4821	-5075	-4973	-4654	-5222	-6963	-5755	-6266
P9	Th	697		-882		1843		-3211		2292	
	h_1	3616	3172	2851	1383	4045	4068	1993	-1684	4405	3221
	h_2	3568	3295	2788	1918	3975	4059	1977	-484	4389	3702
	h_3	3502	3299	2717	1945	3920	4053	1901	-398	4321	3729
P10	Th	874		-753		2054		-3091		2388	
	h_1	3528	3114	2777	1335	3975	4011	1945	-1743	4340	3208
	h_2	3479	3218	2700	1810	3908	4000	1927	-615	4340	3690
	h_3	3431	3218	2632	1842	3850	3996	1872	-534	4261	3734

Table C.10: Mean CVaR f2 low IID-P5

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2508		663		3860		-1918		4135	
	h_1	4667	4069	4347	2570	5203	5266	4366	510	6479	5683
	h_2	4606	4066	4271	2567	5134	5257	4318	506	6417	5670
	h_3	4539	4056	4226	2558	5084	5247	4261	503	6363	5665
P1	Th	-256		-1661		806		-3805		1212	
	h_1	1374	814	918	-1544	1761	1868	392	-5020	2412	694
	h_2	1290	1069	863	-137	1720	1908	541	-2132	2602	2069
	h_3	1214	1057	781	-153	1635	1896	463	-2160	2541	2054
P2	Th	-145		-1581		884		-3776		1202	
	h_1	1214	661	783	-1670	1618	1718	361	-5169	2348	576
	h_2	1142	909	699	-328	1596	1760	469	-2262	2485	1946
	h_3	1073	893	636	-335	1545	1752	390	-2260	2437	1938
P3	Th	-4747		-5435		-4292		-6578		-4312	
	h_1	-3596	-4247	-3864	-7623	-3723	-3784	-4807	-12587	-4255	-6867
	h_2	-3735	-3711	-3878	-4500	-3763	-3560	-4168	-6287	-3904	-4391
	h_3	-3788	-3696	-3968	-4387	-3836	-3563	-4212	-5831	-3934	-4224
P4	Th	2334		490		3654		-2068		3933	
	h_1	4350	3760	4006	2169	4886	4965	3962	-58	6121	5223
	h_2	4287	3777	3952	2279	4809	4956	3922	161	6084	5312
	h_3	4219	3770	3899	2294	4756	4944	3880	194	6036	5332
P5	Th	2266		458		3611		-2074		3914	
	h_1	4371	3788	4030	2190	4916	4980	3997	-23	6144	5203
	h_2	4296	3810	3978	2308	4841	4969	3957	188	6100	5333
	h_3	4227	3796	3901	2329	4767	4957	3899	235	6010	5353

Table C.11: Mean CVaR f2 low P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th	-4748		-5413		-4358		-6592		-4432	
	h_1	-3921	-4552	-4128	-7829	-4076	-4118	-4785	-12881	-4402	-7219
	h_2	-4018	-4034	-4161	-4803	-4121	-3910	-4309	-6549	-4121	-4746
	h_3	-4096	-4031	-4244	-4703	-4210	-3915	-4305	-6147	-4178	-4564
P7	Th	-4786		-5385		-4481		-6479		-4671	
	h_1	-3873	-4542	-4355	-8048	-4116	-4137	-5982	-12974	-5305	-7302
	h_2	-4062	-3919	-4313	-4528	-4197	-3898	-4709	-5894	-4587	-4657
	h_3	-4167	-3921	-4413	-4504	-4314	-3903	-4745	-5802	-4605	-4616
P8	Th	-5886		-6325		-5727		-7280		-6017	
	h_1	-4959	-5602	-5107	-9082	-5264	-5357	-6057	-14247	-6139	-8732
	h_2	-5071	-5004	-5077	-5542	-5371	-5102	-5138	-6839	-5668	-5963
	h_3	-5143	-5005	-5145	-5507	-5432	-5111	-5172	-6721	-5709	-5913
P9	Th	697		-882		1843		-3211		2292	
	h_1	2517	1959	2107	-118	2996	3100	1810	-3151	3973	2440
	h_2	2449	2110	2034	760	2940	3103	1841	-1371	4032	3323
	h_3	2363	2111	1968	804	2871	3089	1776	-1227	3953	3380
P10	Th	874		-753		2054		-3091		2388	
	h_1	2461	1915	2048	-120	2959	3063	1759	-3184	3912	2436
	h_2	2414	2059	1993	686	2924	3070	1781	-1429	3975	3294
	h_3	2362	2058	1917	732	2873	3061	1734	-1296	3909	3362

Table C.12: Mean CVaR f1 high f2 high IID-P5

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2508		663		3860		-1918		4135	
	h_1	7445	7369	6879	6618	8052	8124	6253	5645	8902	8891
	h_2	7396	7367	6822	6617	7997	8121	6200	5643	8860	8884
	h_3	7360	7362	6774	6616	7955	8117	6157	5633	8809	8880
P1	Th	-256		-1661		806		-3805		1212	
	h_1	3899	3808	3362	3006	4323	4440	2735	2056	4856	4779
	h_2	3850	3913	3311	3261	4277	4431	2711	2353	4839	4866
	h_3	3816	3910	3253	3255	4217	4427	2667	2349	4783	4865
P2	Th	-145		-1581		884		-3776		1202	
	h_1	3712	3619	3185	2824	4140	4244	2584	1868	4672	4564
	h_2	3663	3725	3121	3066	4099	4239	2547	2140	4627	4657
	h_3	3598	3719	3084	3065	4056	4235	2496	2137	4594	4653
P3	Th	-4747		-5435		-4292		-6578		-4312	
	h_1	-1324	-1491	-1465	-1986	-1450	-1343	-1540	-2216	-1629	-1742
	h_2	-1404	-1267	-1541	-1492	-1530	-1352	-1529	-1775	-1648	-1609
	h_3	-1445	-1258	-1594	-1465	-1609	-1355	-1560	-1734	-1684	-1594
P4	Th	2334		490		3654		-2068		3933	
	h_1	7092	7017	6517	6248	7680	7771	5867	5244	8522	8492
	h_2	7057	7021	6476	6264	7635	7769	5828	5280	8477	8493
	h_3	7014	7023	6437	6271	7592	7764	5786	5284	8434	8495
P5	Th	2266		458		3611		-2074		3914	
	h_1	7127	7055	6554	6284	7725	7805	5915	5292	8556	8531
	h_2	7088	7065	6521	6307	7679	7800	5871	5315	8518	8540
	h_3	7048	7063	6473	6310	7626	7799	5814	5319	8476	8538

Table C.13: Mean CVaR f1 high f2 high P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th		-4748		-5413		-4358		-6592		-4432
	h_1	-1710	-1849	-1784	-2306	-1855	-1735	-1776	-2494	-2034	-2187
	h_2	-1767	-1627	-1832	-1833	-1904	-1741	-1795	-2097	-2028	-2049
	h_3	-1812	-1624	-1876	-1807	-1959	-1743	-1851	-2057	-2078	-2040
P7	Th		-4786		-5385		-4481		-6479		-4671
	h_1	-1571	-1737	-1650	-2142	-1802	-1669	-1744	-2168	-2402	-2174
	h_2	-1676	-1482	-1699	-1584	-1881	-1680	-1643	-1699	-2251	-2034
	h_3	-1766	-1483	-1736	-1582	-1979	-1683	-1720	-1687	-2270	-2034
P8	Th		-5886		-6325		-5727		-7280		-6017
	h_1	-2735	-2900	-2692	-3232	-3053	-2937	-2534	-3165	-3510	-3577
	h_2	-2835	-2641	-2744	-2675	-3132	-2944	-2491	-2698	-3503	-3426
	h_3	-2890	-2642	-2801	-2669	-3201	-2947	-2542	-2688	-3545	-3422
P9	Th		697		-882		1843		-3211		2292
	h_1	5112	5034	4546	4214	5621	5732	3884	3207	6319	6218
	h_2	5071	5106	4515	4384	5582	5726	3866	3413	6282	6277
	h_3	5029	5109	4462	4394	5522	5724	3821	3423	6235	6283
P10	Th		874		-753		2054		-3091		2388
	h_1	5040	4966	4493	4152	5549	5664	3830	3123	6242	6154
	h_2	4998	5030	4428	4312	5489	5658	3776	3313	6189	6208
	h_3	4956	5030	4392	4319	5441	5654	3730	3331	6137	6206

Table C.14: Mean CVaR f1 high f2 low IID-P5

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2508		663		3860		-1918		4135	
	h_1	7287	7207	6773	6486	7880	7957	6241	5622	8838	8831
	h_2	7248	7204	6733	6482	7830	7952	6190	5616	8785	8823
	h_3	7202	7199	6679	6477	7787	7946	6132	5617	8749	8823
P1	Th	-256		-1661		806		-3805		1212	
	h_1	3748	3636	3282	2877	4203	4318	2739	2122	4856	4842
	h_2	3706	3748	3226	3175	4152	4306	2710	2426	4822	4937
	h_3	3639	3740	3195	3167	4100	4302	2666	2419	4765	4933
P2	Th	-145		-1581		884		-3776		1202	
	h_1	3542	3451	3102	2696	4011	4130	2592	1934	4694	4635
	h_2	3491	3561	3053	2986	3969	4124	2546	2235	4644	4738
	h_3	3447	3555	3008	2986	3929	4119	2500	2233	4603	4737
P3	Th	-4747		-5435		-4292		-6578		-4312	
	h_1	-1463	-1664	-1511	-2096	-1502	-1389	-1554	-2092	-1615	-1622
	h_2	-1563	-1428	-1551	-1517	-1577	-1399	-1545	-1655	-1643	-1494
	h_3	-1626	-1425	-1606	-1498	-1645	-1405	-1572	-1626	-1695	-1487
P4	Th	2334		490		3654		-2068		3933	
	h_1	6944	6854	6423	6116	7523	7609	5872	5239	8456	8450
	h_2	6905	6859	6381	6137	7479	7605	5833	5271	8421	8451
	h_3	6846	6855	6325	6138	7440	7598	5795	5267	8375	8452
P5	Th	2266		458		3611		-2074		3914	
	h_1	6972	6890	6456	6152	7559	7644	5914	5281	8487	8487
	h_2	6921	6897	6408	6176	7514	7638	5875	5308	8445	8493
	h_3	6880	6895	6365	6179	7454	7632	5818	5309	8399	8492

Table C.15: Mean CVaR f1 high f2 low P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th		-4748		-5413		-4358		-6592		-4432
	h_1	-1874	-2020	-1837	-2422	-1928	-1778	-1795	-2373	-2025	-2053
	h_2	-1932	-1780	-1888	-1855	-1984	-1784	-1815	-1977	-2013	-1924
	h_3	-1981	-1776	-1941	-1831	-2038	-1789	-1859	-1935	-2070	-1914
P7	Th		-4786		-5385		-4481		-6479		-4671
	h_1	-1738	-1915	-1685	-2247	-1838	-1712	-1745	-2089	-2174	-2072
	h_2	-1832	-1640	-1752	-1609	-1940	-1721	-1651	-1599	-2176	-1931
	h_3	-1910	-1641	-1795	-1606	-2011	-1726	-1727	-1588	-2223	-1932
P8	Th		-5886		-6325		-5727		-7280		-6017
	h_1	-2906	-3073	-2742	-3325	-3108	-2968	-2532	-3045	-3485	-3453
	h_2	-2991	-2795	-2799	-2686	-3185	-2978	-2492	-2576	-3496	-3315
	h_3	-3041	-2797	-2850	-2682	-3242	-2983	-2543	-2569	-3533	-3316
P9	Th		697		-882		1843		-3211		2292
	h_1	4966	4866	4471	4095	5479	5592	3900	3229	6278	6252
	h_2	4919	4939	4427	4280	5432	5584	3876	3458	6245	6317
	h_3	4863	4935	4374	4291	5382	5578	3839	3470	6202	6319
P10	Th		874		-753		2054		-3091		2388
	h_1	4872	4799	4385	4031	5419	5532	3842	3167	6212	6177
	h_2	4822	4866	4325	4216	5369	5525	3786	3374	6155	6245
	h_3	4781	4867	4283	4225	5309	5518	3741	3396	6101	6248

Table C.16: Mean CVaR f1 low f2 high IID-P5

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2508		663		3860		-1918		4135	
	h_1	6240	5942	5612	4810	6834	6882	5053	3149	7650	7282
	h_2	6185	5938	5565	4809	6781	6876	5004	3138	7594	7279
	h_3	6138	5936	5504	4808	6725	6871	4930	3141	7539	7276
P1	Th	-256		-1661		806		-3805		1212	
	h_1	2792	2544	2072	970	3164	3280	1137	-1469	3352	2564
	h_2	2736	2712	2000	1614	3106	3270	1220	-254	3397	3076
	h_3	2676	2709	1903	1601	3046	3263	1157	-237	3345	3064
P2	Th	-145		-1581		884		-3776		1202	
	h_1	2621	2370	1895	821	2947	3094	1052	-1655	3220	2435
	h_2	2578	2530	1813	1429	2893	3086	1061	-487	3239	2915
	h_3	2512	2525	1744	1417	2841	3080	1014	-494	3190	2908
P3	Th	-4747		-5435		-4292		-6578		-4312	
	h_1	-2337	-2636	-2889	-4544	-2640	-2549	-3681	-7136	-3337	-4563
	h_2	-2407	-2284	-2957	-3119	-2725	-2530	-3329	-4628	-3193	-3595
	h_3	-2474	-2272	-3060	-3065	-2792	-2532	-3389	-4396	-3265	-3529
P4	Th	2334		490		3654		-2068		3933	
	h_1	5906	5615	5262	4425	6477	6547	4653	2679	7247	6848
	h_2	5850	5620	5189	4478	6420	6542	4599	2750	7208	6867
	h_3	5794	5626	5127	4488	6359	6533	4555	2774	7161	6872
P5	Th	2266		458		3611		-2074		3914	
	h_1	5943	5648	5295	4455	6499	6578	4667	2692	7268	6862
	h_2	5889	5658	5239	4509	6423	6568	4629	2777	7225	6888
	h_3	5819	5659	5168	4522	6387	6563	4589	2797	7167	6904

Table C.17: Mean CVaR f1 low f2 high P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th		-4748		-5413		-4358		-6592		-4432
	h_1	-2687	-2977	-3154	-4806	-3077	-2933	-3723	-7432	-3651	-4963
	h_2	-2760	-2635	-3220	-3468	-3134	-2913	-3542	-4967	-3535	-3971
	h_3	-2833	-2628	-3337	-3421	-3225	-2915	-3585	-4785	-3573	-3889
P7	Th		-4786		-5385		-4481		-6479		-4671
	h_1	-2571	-2903	-3214	-4789	-3011	-2893	-4431	-7069	-4309	-4925
	h_2	-2725	-2505	-3220	-3217	-3131	-2867	-3710	-4358	-3910	-3896
	h_3	-2831	-2506	-3319	-3194	-3276	-2869	-3744	-4309	-3925	-3879
P8	Th		-5886		-6325		-5727		-7280		-6017
	h_1	-3676	-4022	-4144	-5869	-4312	-4161	-4763	-8251	-5242	-6389
	h_2	-3775	-3622	-4197	-4304	-4403	-4132	-4296	-5418	-5004	-5286
	h_3	-3822	-3624	-4279	-4286	-4497	-4134	-4352	-5357	-5016	-5257
P9	Th		697		-882		1843		-3211		2292
	h_1	3982	3733	3262	2303	4439	4569	2480	47	4903	4201
	h_2	3939	3842	3207	2695	4401	4556	2476	794	4888	4511
	h_3	3886	3845	3141	2718	4336	4551	2418	876	4826	4527
P10	Th		874		-753		2054		-3091		2388
	h_1	3938	3669	3210	2242	4383	4509	2422	-14	4861	4187
	h_2	3881	3765	3128	2608	4313	4496	2385	695	4846	4484
	h_3	3844	3770	3044	2626	4264	4490	2320	764	4775	4514

Table C.18: Mean CVaR f1 low f2 low IID-P5

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
IID	Th	2508		663		3860		-1918		4135	
	h_1	5513	5162	5092	3978	6073	6186	4907	2349	7223	6775
	h_2	5448	5162	5025	3971	6017	6179	4858	2355	7146	6766
	h_3	5412	5160	4980	3962	5957	6171	4795	2345	7102	6757
P1	Th	-256		-1661		806		-3805		1212	
	h_1	2114	1788	1668	103	2547	2728	1109	-2014	3217	2376
	h_2	2052	2002	1618	1059	2502	2729	1199	-403	3295	3107
	h_3	1986	1999	1547	1056	2444	2719	1130	-407	3235	3089
P2	Th	-145		-1581		884		-3776		1202	
	h_1	1940	1619	1511	-33	2390	2555	1051	-2183	3123	2248
	h_2	1875	1833	1438	876	2364	2562	1072	-588	3155	2934
	h_3	1820	1819	1386	872	2290	2555	1015	-604	3105	2935
P3	Th	-4747		-5435		-4292		-6578		-4312	
	h_1	-2954	-3372	-3155	-5417	-3015	-2894	-3658	-7284	-3315	-4407
	h_2	-3057	-2930	-3165	-3422	-3072	-2828	-3312	-4393	-3181	-3303
	h_3	-3136	-2919	-3257	-3346	-3149	-2835	-3358	-4202	-3240	-3218
P4	Th	2334		490		3654		-2068		3933	
	h_1	5191	4834	4768	3584	5759	5872	4518	1869	6860	6379
	h_2	5128	4847	4689	3653	5703	5861	4466	1994	6824	6415
	h_3	5059	4849	4619	3662	5645	5855	4414	2031	6768	6422
P5	Th	2266		458		3611		-2074		3914	
	h_1	5218	4871	4781	3624	5778	5897	4543	1924	6867	6378
	h_2	5159	4891	4742	3699	5711	5889	4496	2052	6819	6449
	h_3	5112	4890	4692	3707	5666	5878	4440	2076	6777	6454

Table C.19: Mean CVaR f1 low f2 low P6-P10

		IID		l+		l-		h+		h-	
		CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP	CVaR	EP
P6	Th		-4748		-5413		-4358		-6592		-4432
	h_1	-3315	-3702	-3394	-5670	-3419	-3260	-3695	-7535	-3569	-4774
	h_2	-3378	-3265	-3459	-3741	-3452	-3198	-3508	-4733	-3450	-3696
	h_3	-3474	-3262	-3542	-3665	-3548	-3202	-3547	-4509	-3516	-3587
P7	Th		-4786		-5385		-4481		-6479		-4671
	h_1	-3240	-3652	-3481	-5712	-3403	-3239	-4406	-7356	-4116	-4823
	h_2	-3406	-3151	-3463	-3471	-3478	-3160	-3687	-4207	-3783	-3647
	h_3	-3495	-3148	-3527	-3461	-3620	-3164	-3715	-4140	-3839	-3631
P8	Th		-5886		-6325		-5727		-7280		-6017
	h_1	-4307	-4748	-4355	-6722	-4588	-4456	-4724	-8385	-5175	-6169
	h_2	-4437	-4251	-4369	-4499	-4681	-4379	-4259	-5156	-4954	-4960
	h_3	-4505	-4255	-4446	-4481	-4768	-4384	-4311	-5104	-4986	-4945
P9	Th		697		-882		1843		-3211		2292
	h_1	3295	2968	2823	1446	3806	3964	2425	-600	4689	3932
	h_2	3224	3107	2738	2039	3764	3956	2441	448	4705	4399
	h_3	3175	3108	2680	2070	3675	3945	2380	536	4663	4432
P10	Th		874		-753		2054		-3091		2388
	h_1	3226	2912	2773	1404	3755	3916	2365	-653	4664	3909
	h_2	3166	3043	2693	1972	3701	3909	2340	357	4657	4339
	h_3	3114	3041	2623	1998	3639	3901	2289	432	4584	4376

Table C.20: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means ($f^{(1)} = low$).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	4.13	-0.48	3.00	9.43	20.54
Very Low	P4-P5	4.13 - 4.07	-0.48 - -0.56	3.00 - 2.90	9.43 - 9.13	20.54 - 19.84
Low	P9-P10	4.08 - 3.04	-0.57 - -0.78	2.85 - 2.01	9.01 - 6.87	19.77 - 15.51
Moderate	P1-P2	2.90 - 2.34	-0.77 - -0.83	1.99 - 1.65	6.98 - 5.75	15.62 - 13.39
High	P3-P6	2.45 - 1.04	-0.86 - -0.93	1.47 - 0.47	5.70 - 2.50	13.54 - 7.33
Very High	P7-P8	1.39 - -0.65	-0.99 - -1.21	0.41 - 0.29	2.89 - 1.32	7.71 - 4.54

Table C.21: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means ($f^{(1)} = high$).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	0.21	-0.28	0.65	1.99	4.04
Very Low	P4-P5	0.21 - 0.23	-0.28 - -0.30	0.65 - 0.65	1.99 - 1.90	4.04 - 3.85
Low	P9-P10	0.20 - 0.06	-0.30 - -0.42	0.66 - 0.33	1.93 - 1.35	3.92 - 2.80
Moderate	P1-P2	0.02 - -0.20	-0.48 - -0.50	0.30 - 0.19	1.30 - 0.99	2.75 - 2.12
High	P3-P6	-0.05 - -0.39	-0.44 - -0.40	0.20 - -0.02	1.06 - 0.32	2.22 - 0.70
Very High	P7-P8	-0.37 - -1.09	-0.64 - -0.59	-0.20 - -0.21	0.33 - -0.05	0.87 - -0.10

Table C.22: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means ($f^{(2)} = low$).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	4.59	-0.36	3.42	10.10	21.78
Very Low	P4-P5	4.59 - 4.57	-0.36 - -0.45	3.42 - 3.27	10.10 - 9.73	21.78 - 21.32
Low	P9-P10	4.58 - 3.85	-0.37 - -0.61	3.21 - 2.32	9.67 - 7.42	21.29 - 17.51
Moderate	P1-P2	3.62 - 3.19	-0.64 - -0.85	2.30 - 1.74	7.48 - 6.01	17.55 - 15.35
High	P3-P6	3.22 - 2.05	-0.83 - -0.96	1.74 - 0.57	6.32 - 2.98	15.65 - 9.75
Very High	P7-P8	2.83 - 0.19	-0.98 - -1.30	0.63 - 0.26	3.27 - 1.09	10.78 - 6.45

Table C.23: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means ($f^{(2)} = high$).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	3.70	0.22	2.67	6.96	15.19
Very Low	P4-P5	3.70 - 3.66	0.22 - 0.08	2.67 - 2.61	6.96 - 6.83	15.19 - 15.13
Low	P9-P10	3.84 - 3.99	0.11 - -0.13	2.64 - 1.81	6.67 - 5.15	15.15 - 13.65
Moderate	P1-P2	3.57 - 3.43	-0.21 - -0.20	1.77 - 1.35	5.32 - 4.53	14.14 - 12.50
High	P3-P6	3.60 - 3.99	-0.45 - -0.43	1.33 - 0.46	4.47 - 2.53	13.47 - 10.21
Very High	P7-P8	4.70 - 1.27	-0.64 - -0.62	0.33 - 0.21	3.07 - 0.80	11.92 - 6.32

Table C.24: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means ($f^{(1)} = high$, $f^{(2)} = high$).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	0.06	-0.39	0.41	1.41	3.25
Very Low	P4-P5	0.06 - 0.15	-0.39 - -0.50	0.41 - 0.38	1.41 - 1.33	3.25 - 3.10
Low	P9-P10	0.09 - 0.20	-0.44 - -0.61	0.34 - 0.02	1.31 - 0.82	3.17 - 2.44
Moderate	P1-P2	0.19 - -0.06	-0.63 - -0.64	0.06 - -0.07	0.93 - 0.54	2.64 - 2.01
High	P3-P6	0.09 - -0.21	-0.58 - -0.59	-0.07 - -0.36	0.63 - 0.00	2.34 - 1.05
Very High	P7-P8	0.07 - -1.28	-0.67 - -0.74	-0.46 - -0.48	0.12 - -0.36	1.44 - 0.22

Table C.25: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means ($f^{(1)} = high$, $f^{(2)} = low$).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	0.04	-0.42	0.43	1.55	3.32
Very Low	P4-P5	0.04 - 0.02	-0.42 - -0.47	0.43 - 0.46	1.55 - 1.54	3.32 - 3.22
Low	P9-P10	-0.03 - -0.23	-0.46 - -0.62	0.41 - 0.15	1.50 - 0.97	3.24 - 2.30
Moderate	P1-P2	-0.20 - -0.45	-0.62 - -0.63	0.03 - -0.00	0.86 - 0.58	2.38 - 1.66
High	P3-P6	-0.25 - -0.76	-0.66 - -0.64	-0.11 - -0.21	0.63 - -0.07	1.89 - 0.41
Very High	P7-P8	-0.58 - -1.44	-0.84 - -0.71	-0.54 - -0.54	-0.03 - -0.42	0.72 - -0.32

Table C.26: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means ($f^{(1)} = low$, $f^{(2)} = high$).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	2.10	-0.27	1.68	4.49	10.60
Very Low	P4-P5	2.10 - 2.14	-0.27 - -0.39	1.68 - 1.58	4.49 - 4.34	10.60 - 10.46
Low	P9-P10	2.09 - 2.19	-0.45 - -0.74	1.60 - 0.77	4.33 - 3.05	10.41 - 8.96
Moderate	P1-P2	2.02 - 1.89	-0.72 - -0.66	0.95 - 0.45	3.27 - 2.56	9.25 - 8.15
High	P3-P6	1.91 - 2.08	-0.84 - -0.64	0.51 - -0.37	2.61 - 0.98	8.65 - 5.94
Very High	P7-P8	2.20 - -0.20	-0.95 - -0.84	-0.33 - -0.37	1.49 - -0.11	6.90 - 3.31

Table C.27: The percentage of the difference between out-sample CVaR values of CVAR-LP and EP-LP for different scenarios to the theoretical out-sample means ($f^{(1)} = low$, $f^{(2)} = low$).

Variance of Price	Corresponding Price Process	Correlation of Price and Demand				
		Very Low (h-)	Negative (l-)	Independent (i.i.d.)	Positive (l+)	Very High (h+)
Very Low	IID	2.62	-0.66	2.03	6.42	14.62
Very Low	P4-P5	2.62 - 2.57	-0.66 - -0.66	2.03 - 1.98	6.42 - 6.37	14.62 - 14.26
Low	P9-P10	2.42 - 1.63	-0.69 - -0.93	1.89 - 1.08	6.19 - 4.34	14.15 - 10.95
Moderate	P1-P2	1.71 - 1.13	-0.94 - -1.06	1.06 - 0.65	4.46 - 3.51	11.10 - 9.22
High	P3-P6	1.32 - 0.24	-1.01 - -1.11	0.62 - -0.20	3.65 - 1.10	9.54 - 5.06
Very High	P7-P8	0.87 - -0.98	-1.31 - -1.45	-0.31 - -0.53	1.56 - -0.01	5.69 - 2.56

Table C.28: Dispersion of simulation results of CVaR for h – Demand (CVAR-LP vs. Theoretical EP)

Price	Mean CVaR		St.Dev. CVaR		Max. CVaR		Min. CVaR	
	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP
IID	5573.06	4183.75	1075.35	1371.73	8934.85	8897.86	67.34	-1607.26
P1	2109.00	1178.41	2167.49	2376.84	8286.98	8936.03	-9826.26	-10966.10
P2	2048.66	1006.53	1660.25	1851.67	6838.20	7157.05	-6135.95	-7956.61
P3	-3941.89	-4443.63	3805.74	3949.01	5971.97	6732.36	-25999.09	-25912.62
P4	5277.35	3926.56	1114.76	1400.32	8675.97	9152.52	151.68	-1915.19
P5	5265.84	3950.36	1197.78	1498.23	9423.47	8896.20	-664.62	-1835.99
P6	-4175.90	-4846.24	2711.84	2820.61	3123.89	3553.01	-19374.06	-19951.69
P7	-4629.77	-4870.63	6060.25	6248.15	11150.01	12338.92	-53956.77	-53812.08
P8	-5662.89	-6180.48	4439.36	4443.14	4784.85	5539.99	-36278.87	-34731.69
P9	3446.64	2300.00	1553.37	1810.03	8298.15	8513.59	-4882.03	-6611.49
P10	3433.00	2248.31	1318.62	1561.31	7098.64	7588.79	-3087.73	-4946.55

Table C.29: Dispersion of simulation results of CVaR for l - Demand (CVAR-LP vs. Theoretical EP)

Price	Mean CVaR		St.Dev. CVaR		Max. CVaR		Min. CVaR	
	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP
IID	4003.19	3987.84	1102.43	1095.79	7659.85	7914.68	-514.79	-931.69
P1	830.81	866.32	2012.20	2001.84	7527.78	7566.15	-8861.33	-8691.17
P2	717.25	708.32	1548.87	1544.78	5585.13	5417.45	-6479.60	-6843.43
P3	-4396.42	-4345.33	3248.82	3256.12	5379.14	5755.05	-20931.08	-21053.82
P4	3717.28	3702.45	1128.82	1123.56	7404.52	7438.93	-564.53	-941.10
P5	3734.10	3729.10	1216.34	1210.26	8130.89	8286.85	-1657.40	-1294.46
P6	-4692.65	-4689.29	2323.89	2330.83	2070.13	2395.15	-15794.06	-15599.00
P7	-4764.84	-4681.84	5248.74	5274.01	12459.33	12266.64	-42722.80	-42667.61
P8	-5874.25	-5848.70	3666.96	3674.75	3938.45	4010.59	-27996.66	-27711.12
P9	1997.57	1994.68	1493.67	1481.88	7545.33	7451.21	-4892.68	-4876.04
P10	1962.57	1941.84	1286.70	1284.53	6088.32	5999.14	-3595.79	-3536.25

Table C.30: Dispersion of simulation results of CVaR for *i.i.d.* Demand (CVAR-LP vs. Theoretical EP)

Price	Mean CVaR		St.Dev. CVaR		Max. CVaR		Min. CVaR	
	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP
IID	3496.88	2557.22	1132.35	1298.80	7208.93	7566.48	-1798.60	-3247.48
P1	364.14	-197.65	1955.86	1977.26	6855.66	7040.25	-8306.29	-8721.05
P2	212.18	-352.93	1505.96	1558.90	5053.81	4512.07	-7656.14	-8884.45
P3	-4454.76	-4742.18	2924.39	2899.07	5226.55	5173.71	-17579.92	-17574.67
P4	3196.60	2302.80	1152.98	1306.07	6827.11	7019.90	-2060.17	-3554.50
P5	3220.70	2331.90	1246.17	1393.14	7724.22	7759.89	-1850.41	-4040.00
P6	-4770.38	-5059.08	2088.49	2086.85	1682.56	1533.23	-14539.37	-15477.11
P7	-4749.84	-4980.30	4726.38	4646.60	12899.68	12806.53	-33913.58	-33920.51
P8	-5770.74	-6037.16	3206.68	3155.74	3450.57	2941.82	-22184.01	-22497.06
P9	1471.15	798.24	1484.65	1559.70	7269.69	6501.30	-5384.26	-6337.18
P10	1421.08	742.99	1279.82	1383.63	5497.77	4788.22	-4751.18	-5910.76

Table C.31: Dispersion of simulation results of CVaR for $l+$ Demand (CVAR-LP vs. Theoretical EP)

Price	Mean CVaR		St.Dev. CVaR		Max. CVaR		Min. CVaR	
	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP
IID	3362.38	627.65	1168.67	1549.11	7122.02	6843.54	-2443.03	-6738.53
P1	147.65	-1649.75	1935.33	2118.88	6377.70	6233.96	-8891.58	-10385.42
P2	-4.90	-1806.57	1496.91	1712.57	4786.33	3950.91	-8271.11	-10219.54
P3	-4442.01	-5419.50	2689.95	2724.73	4692.98	4294.40	-17320.98	-16308.95
P4	3046.73	413.96	1180.74	1551.22	6659.10	6301.99	-2686.00	-6934.26
P5	3070.33	447.06	1284.04	1647.73	7139.90	7263.33	-2919.62	-7401.27
P6	-4707.41	-5722.84	1935.16	1988.39	1505.57	1139.42	-15321.62	-15650.11
P7	-4797.50	-5549.27	4351.20	4190.38	10796.02	13322.92	-32763.94	-24989.81
P8	-5609.88	-6498.72	2860.91	2771.60	2157.28	1785.03	-21054.35	-18248.38
P9	1249.23	-830.06	1490.72	1775.35	6450.61	5451.10	-6453.30	-8844.17
P10	1191.75	-886.51	1288.15	1596.36	5057.23	4551.75	-4889.87	-7903.41

Table C.32: Dispersion of simulation results of CVaR for $h+$ Demand (CVAR-LP vs. Theoretical EP)

Price	Mean CVaR		St.Dev. CVaR		Max. CVaR		Min. CVaR	
	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP	CVAR-LP	TH-EP
IID	3831.97	-1951.90	1206.44	1772.41	7430.96	5371.59	-1894.66	-9739.29
P1	245.63	-3836.70	2019.96	2345.98	6430.33	4929.96	-10199.34	-13217.02
P2	173.01	-3996.33	1539.69	1928.60	4625.54	3488.65	-8564.82	-12411.13
P3	-4218.55	-6627.60	2594.28	2683.11	3481.35	2899.62	-19353.44	-15972.66
P4	3457.38	-2148.89	1235.61	1775.97	7131.21	4785.74	-2987.75	-9915.01
P5	3492.70	-2108.55	1338.04	1887.88	7635.01	5403.39	-3359.26	-10502.19
P6	-4359.05	-6910.26	1826.88	2021.97	1214.60	315.14	-14342.79	-15373.94
P7	-4717.82	-6610.58	4150.17	3862.54	8490.47	13305.68	-32898.82	-18651.05
P8	-5171.30	-7443.28	2625.36	2482.36	1779.41	903.13	-25467.98	-17918.73
P9	1474.00	-3198.47	1568.31	2027.72	5919.83	4134.40	-5739.55	-11495.60
P10	1433.89	-3264.02	1336.08	1826.61	5209.70	3555.69	-6569.21	-11874.24

Figure C.1: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P1 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

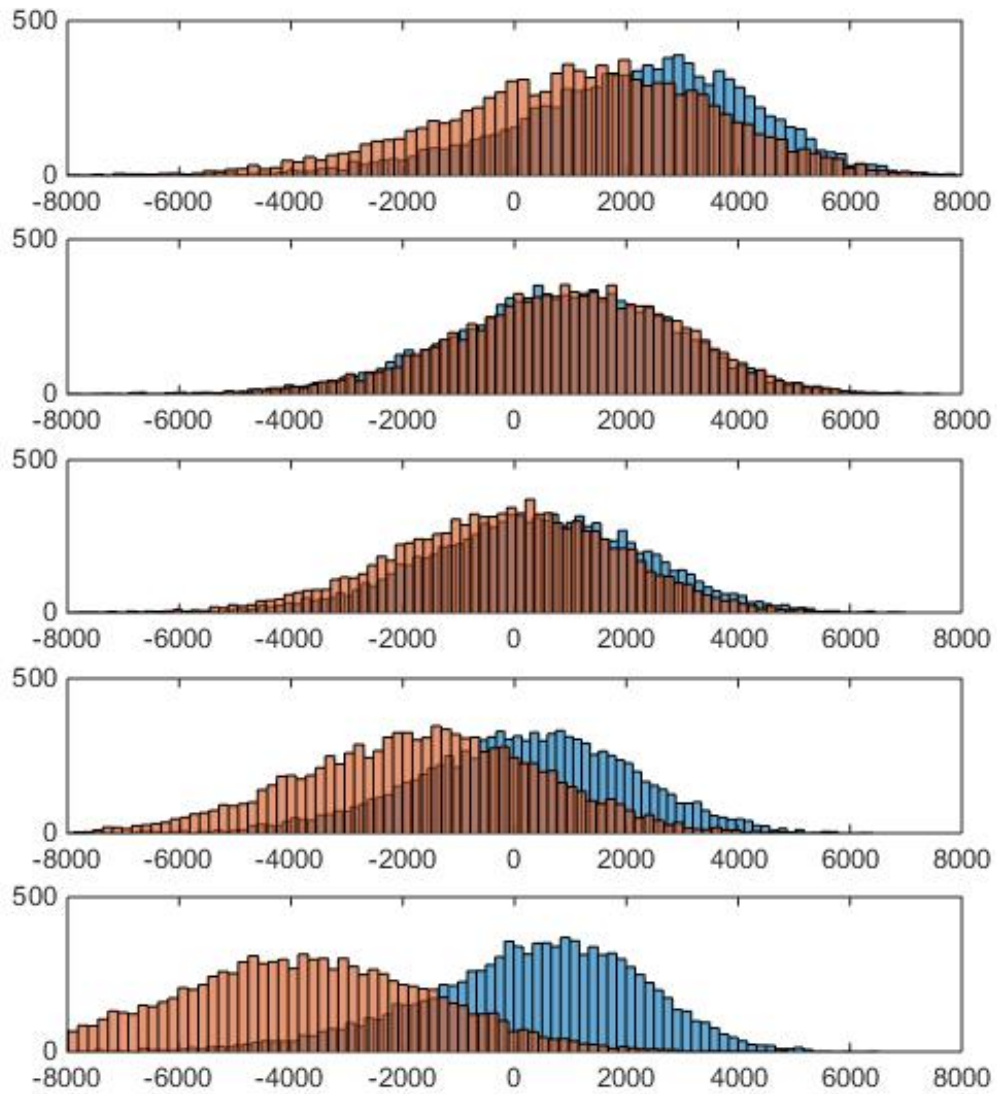


Figure C.2: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P2 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

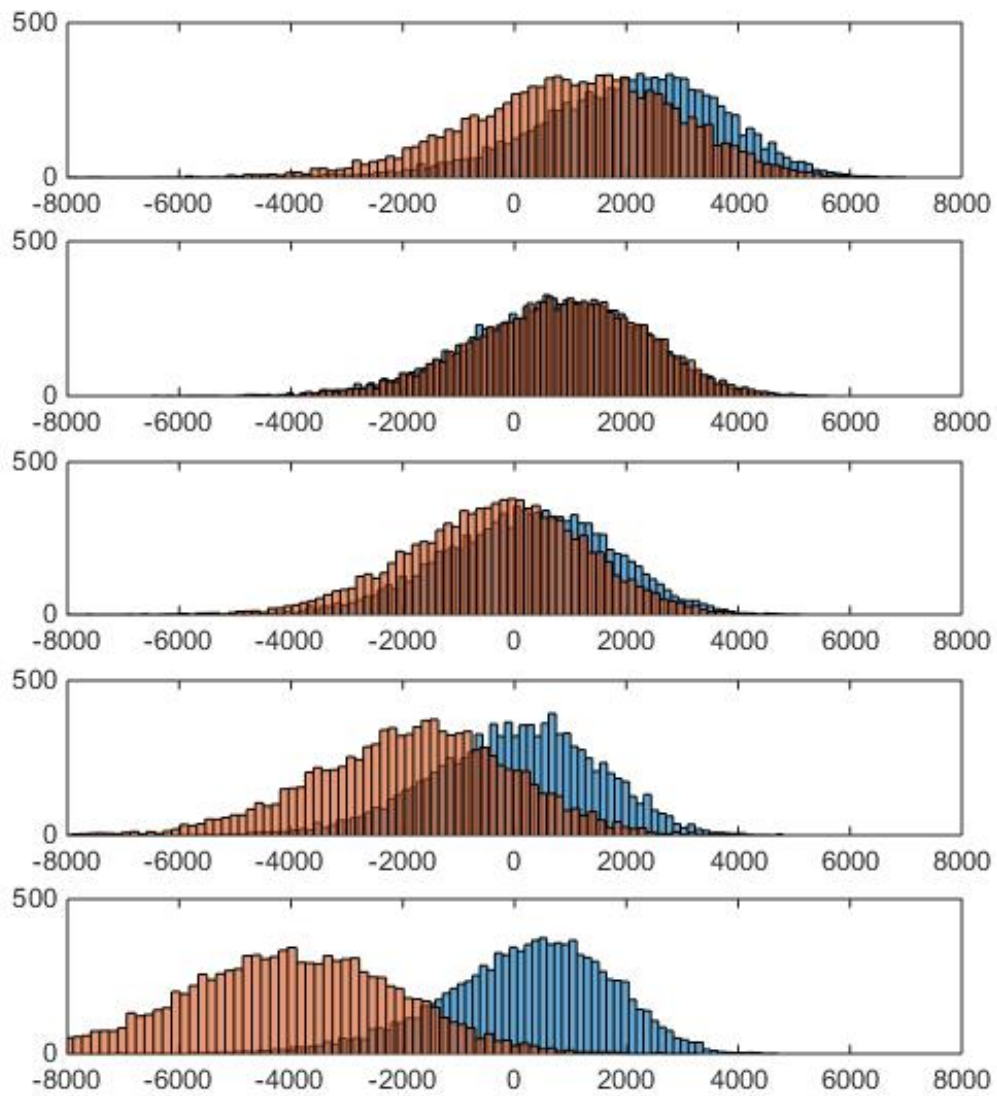


Figure C.3: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P3 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

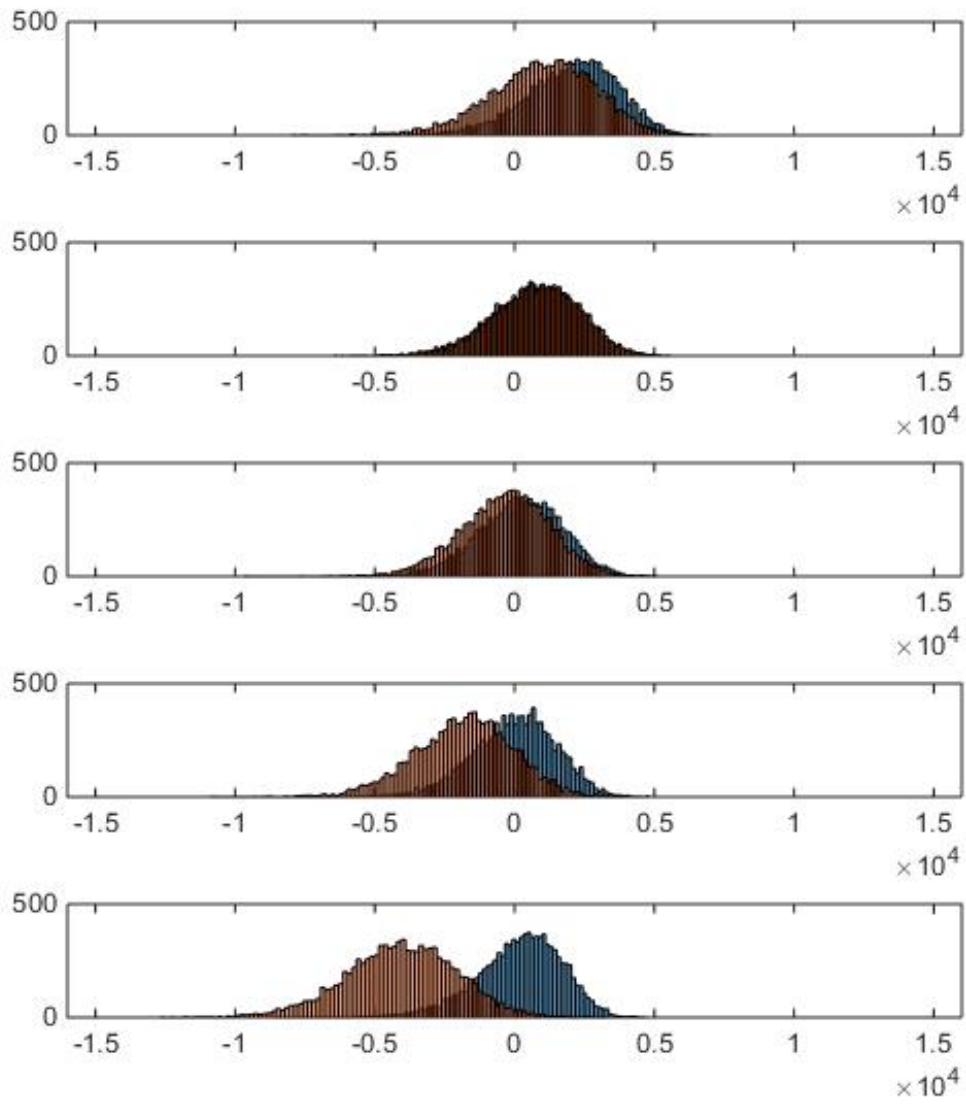


Figure C.4: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P4 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

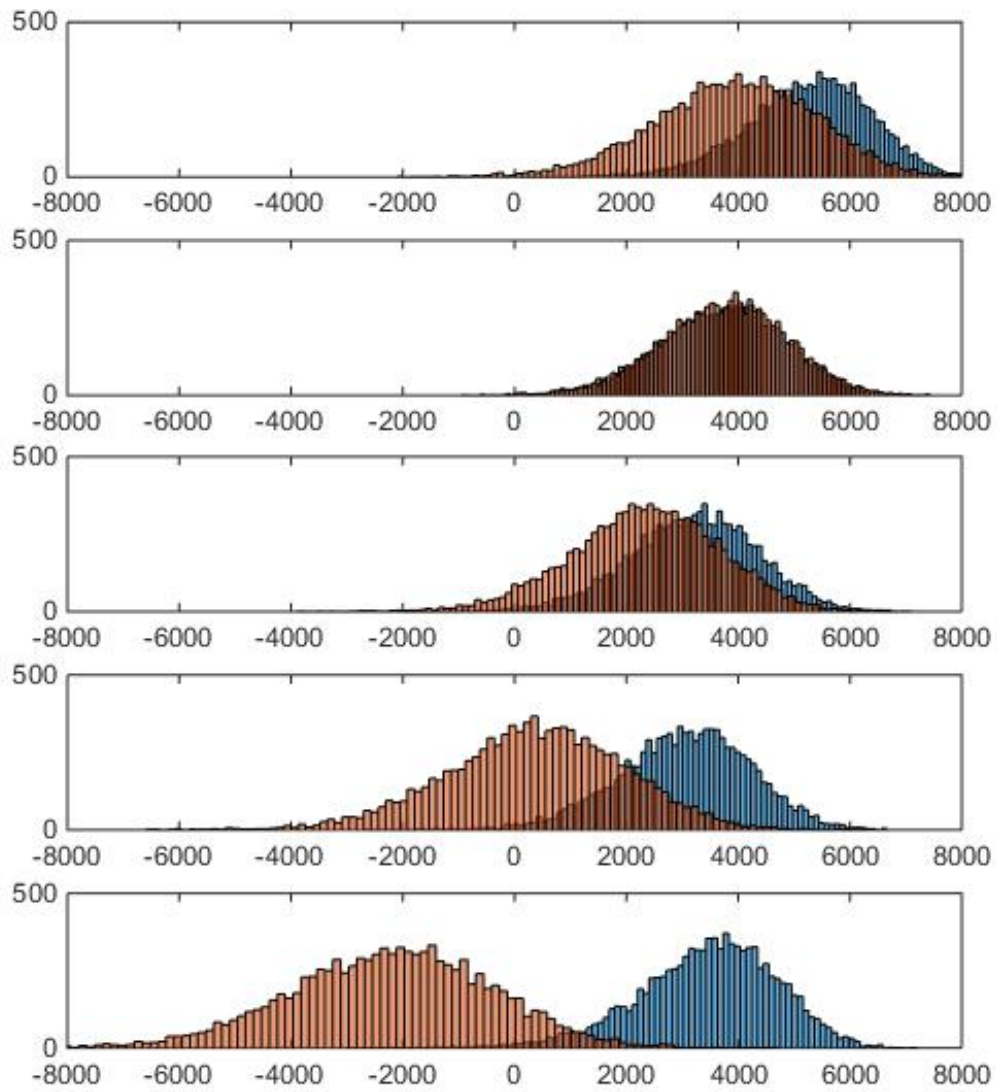


Figure C.5: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P5 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

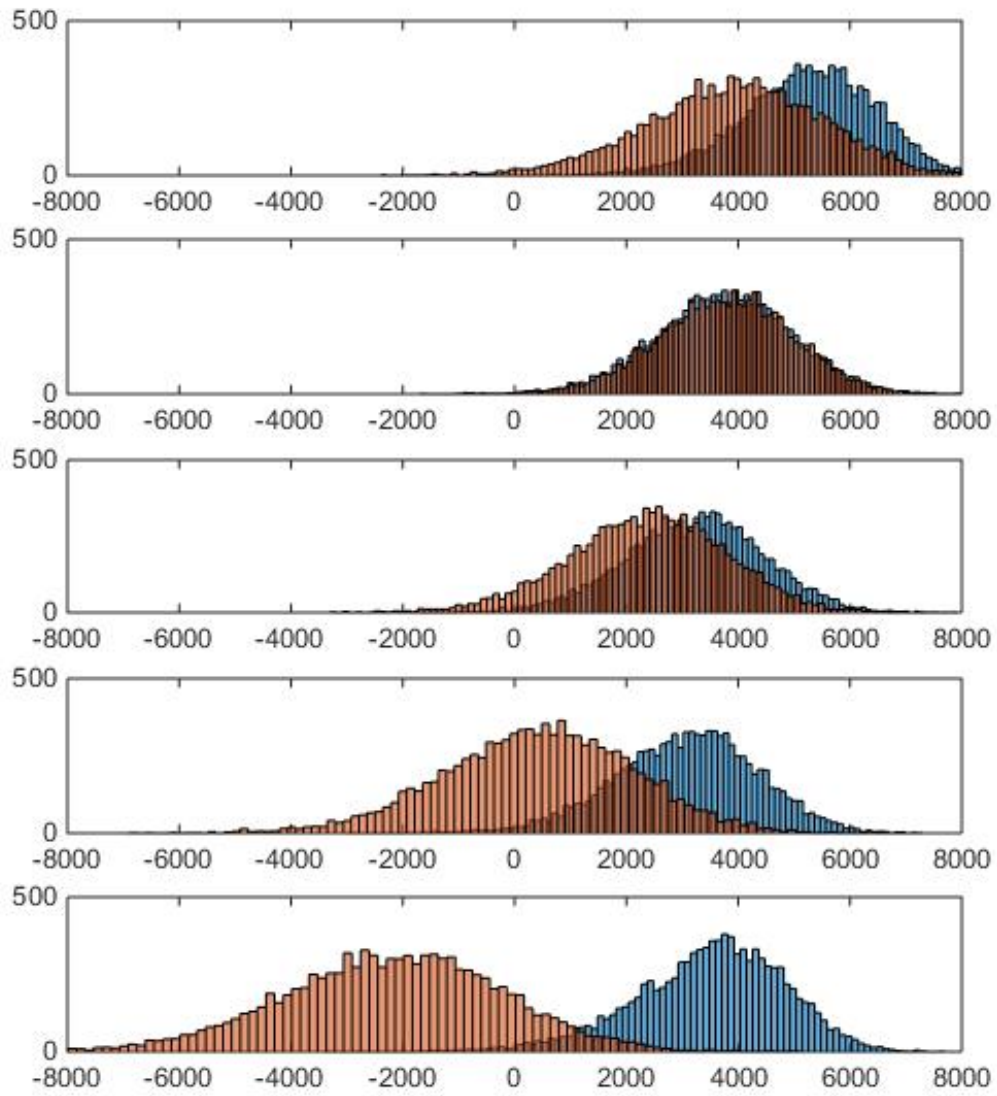


Figure C.6: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P6 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

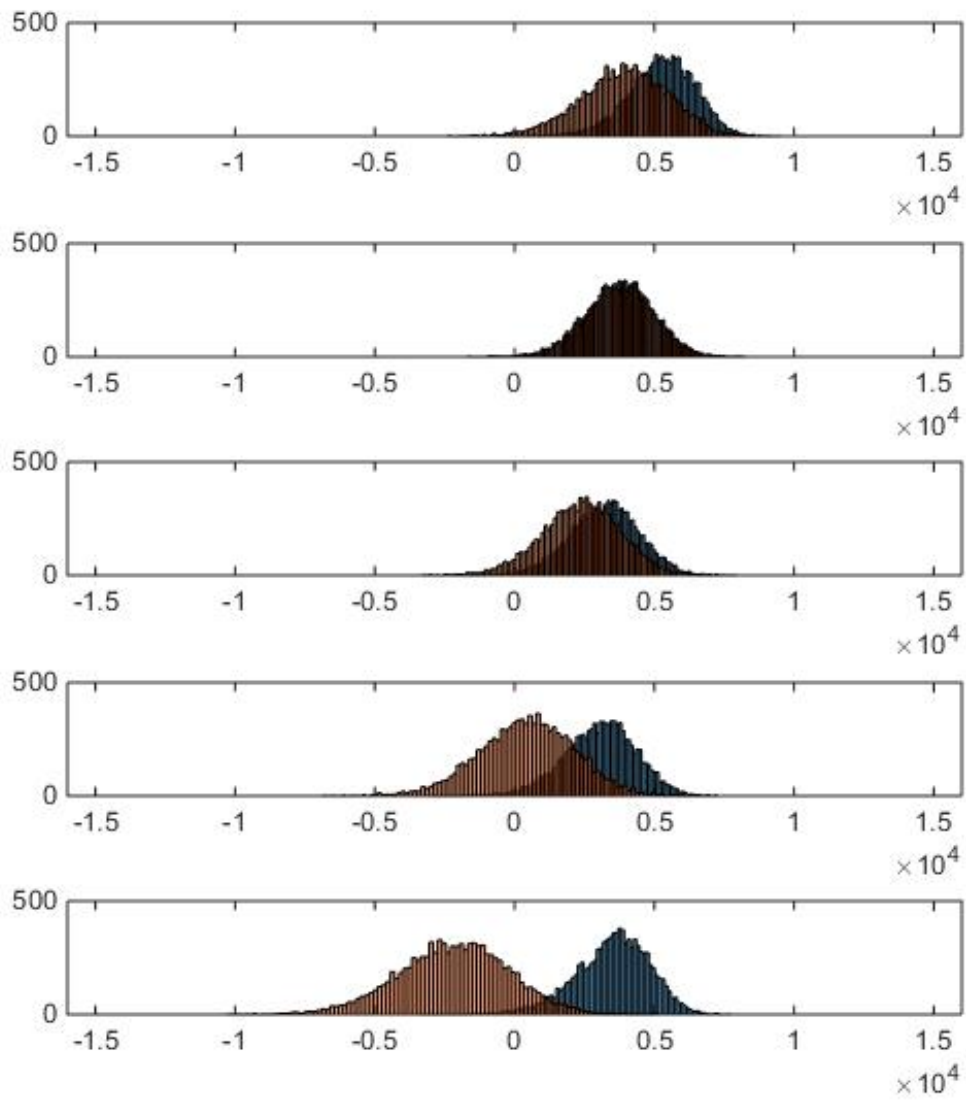


Figure C.7: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P7 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

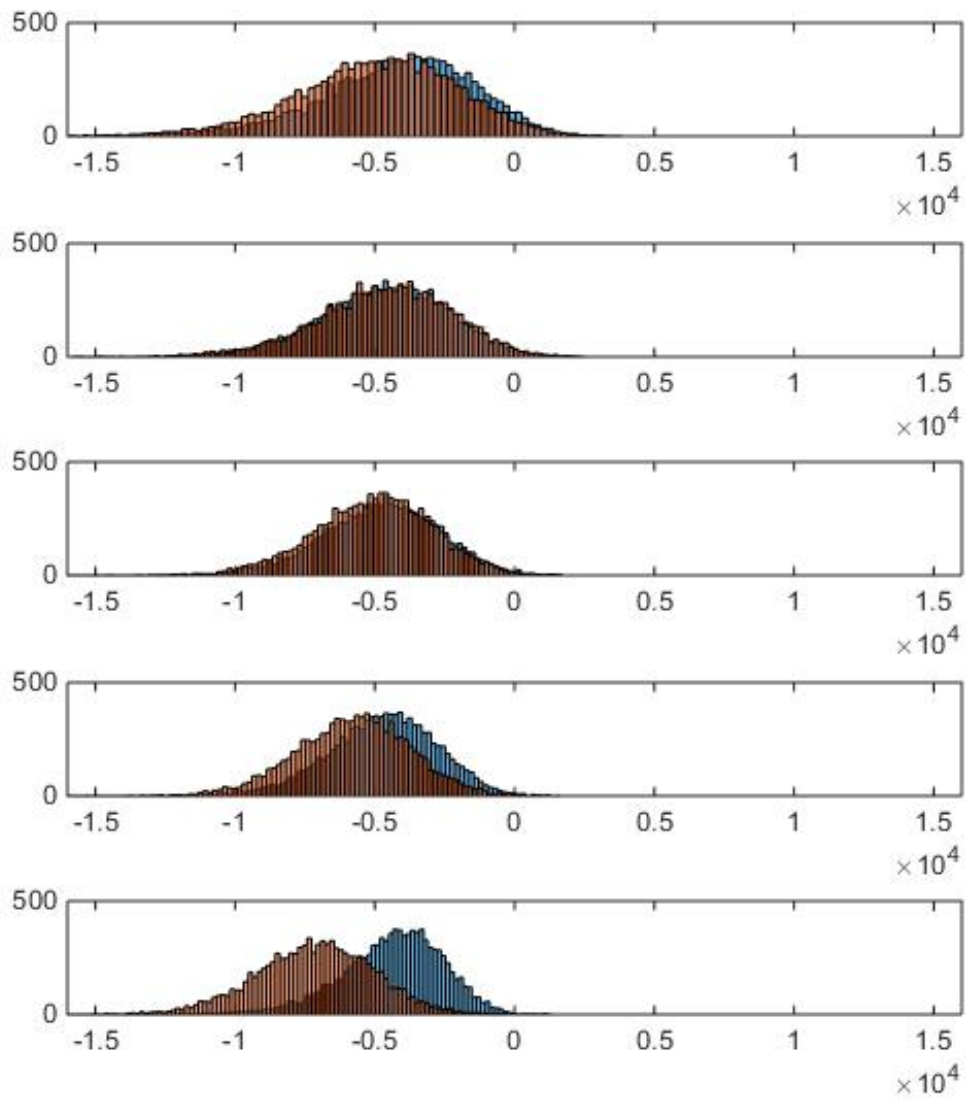


Figure C.8: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P8 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

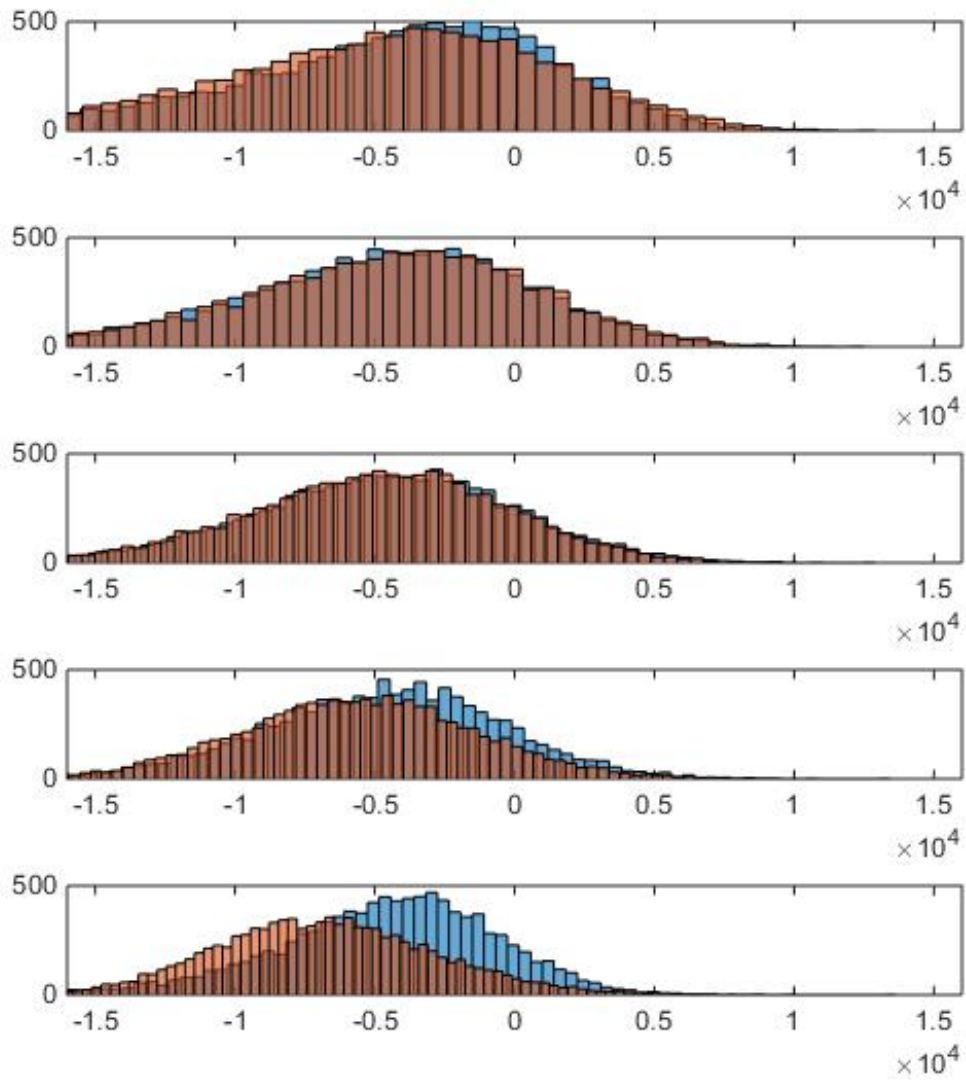


Figure C.9: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P9 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)

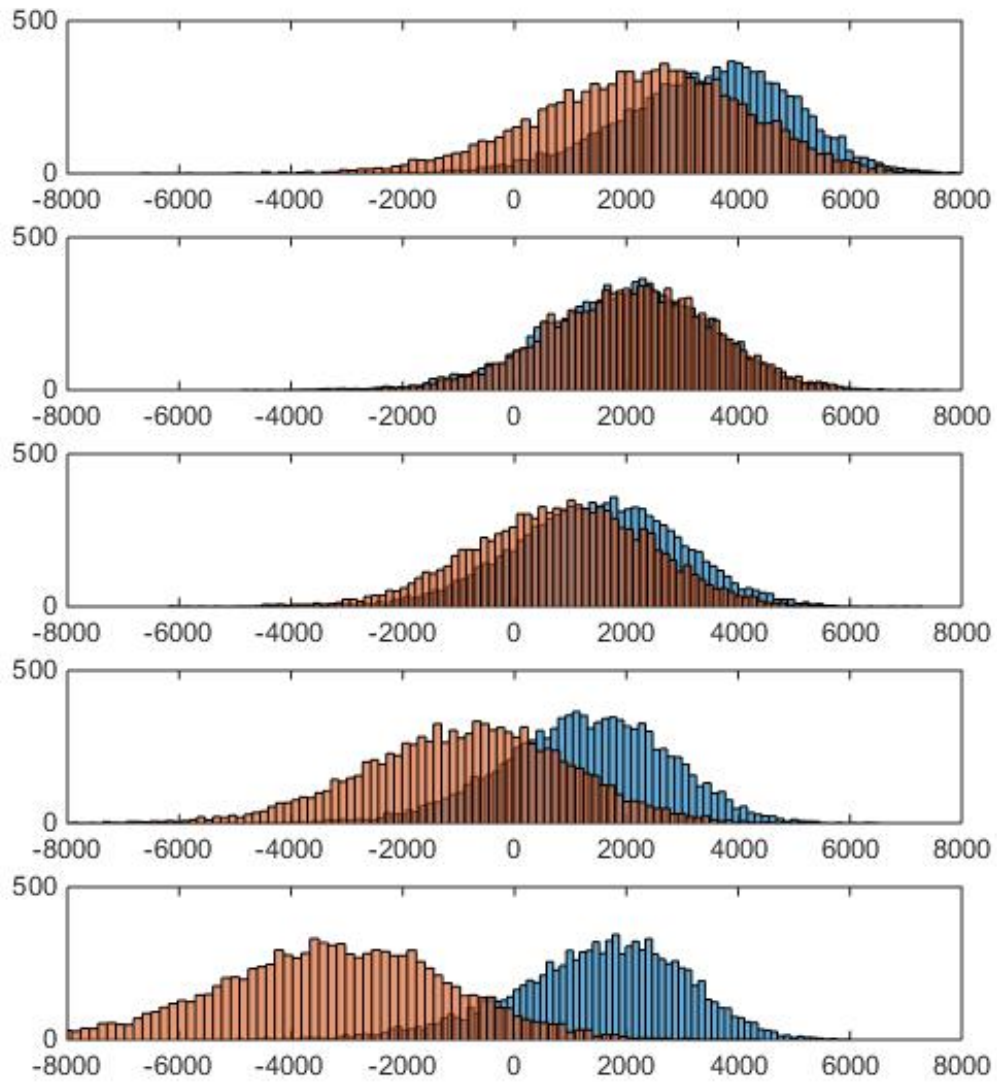
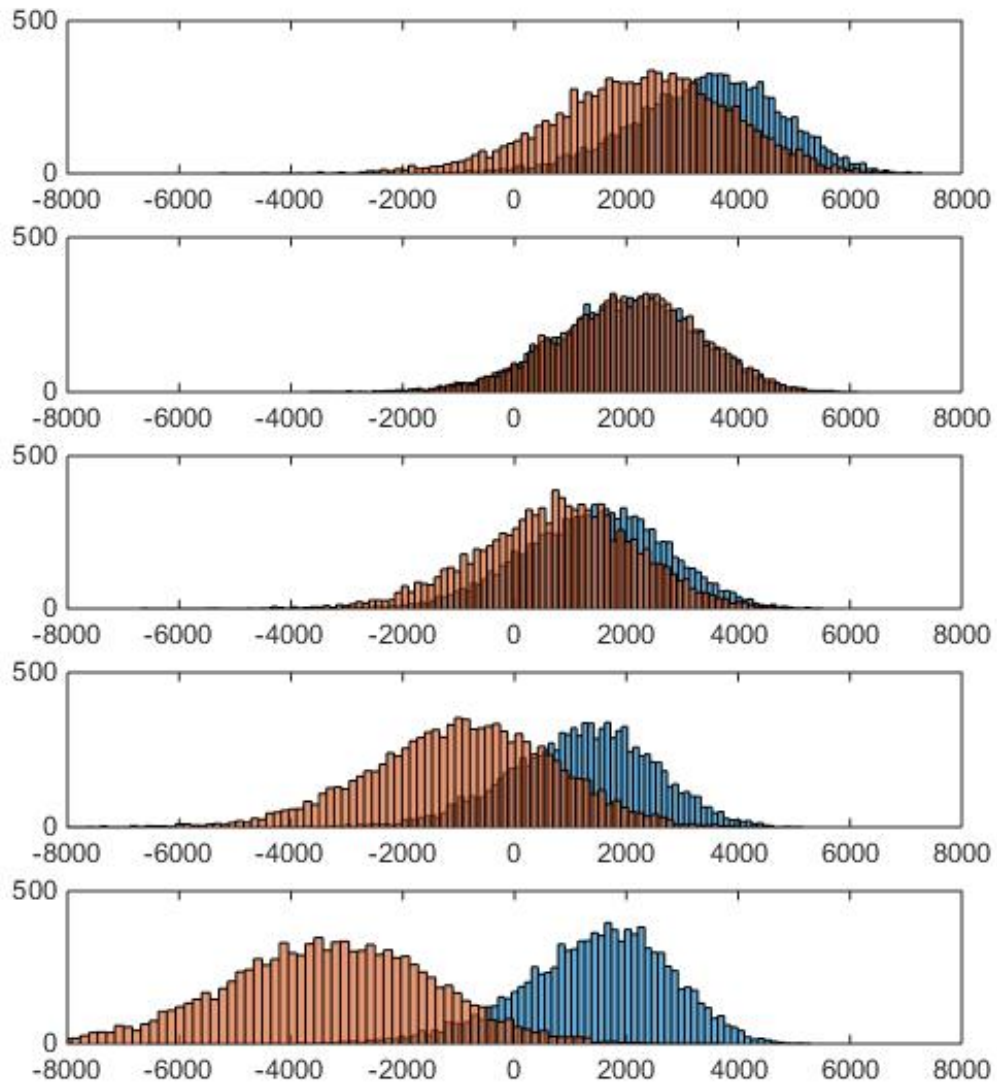


Figure C.10: Distributions of CVaR values of CVAR-LP (Blue) and TH-EP (Red) for P10 Price Process, and Demand Models $h-$, $l-$, $i.i.d.$, $l+$, and $h+$ (in descending order)



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