SINGLE AND PAIR PRODUCTION OF HEAVY LEPTONS BEYOND THE STANDARD MODEL

by

KUBİLAY BOYRAZ

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Prof. Dr. Nihat ÇELEBİ
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Kıvılcım KILIÇ
Head of Physics Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Nurettin KARAGÖZ
Supervisor

Examining Committee Members

1. Assoc. Prof. Dr. Nurettin KARAGÖZ ....
2. Assoc. Prof. Dr. Cabir TERZİOĞLU ....
3. Assoc. Prof. Dr. Cenap ÖZEL ....

ABSTRACT

SINGLE AND PAIR PRODUCTION OF HEAVY LEPTONS BEYOND THE STANDARD MODEL

Boyraz, Kubilay

Master of Science, Department of Physics

Supervisor: Assoc. Prof. Dr. Nurettin Karagöz

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In this thesis, general information on the Standard Model is presented and deficiencies of this model are expressed. Single and pair production of heavy leptons by means of string inspired E_6 model are investigated. Firstly, for each production analytical expressions of differential cross-sections are derived and numerical results are presented for linear colliders ILC ($\sqrt{s} = 0.5TeV$), ILC ($\sqrt{s} = 1.0TeV$), and CLIC ($\sqrt{s} = 3.0TeV$). Afterwards, probabilities of heavy leptons being observed in other words, being produced are analyzed. At the end, why the model can not be applied for production of ordinary heavy leptons is explained.

Keywords: Standard Model, heavy leptons, string inspired E_6 model, single and pair production.

ÖZET

STANDART MODEL ÖTESİNDE AĞIR LEPTONLARIN TEK VE ÇİFT ÜRETİMİ

Boyraz, Kubilay

Yüksek Lisans, Fizik Bölümü

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Bu tezde, standart model hakkında genel bilgi sunuldu ve bu modelin eksiklikleri anlatıldı. Ağır leptonların sicim esinli E_6 model ile tek ve çift üretimi incelendi. İlk olarak, her bir üretim için diferansiyel tesir kesitlerin analitik ifadeleri türetilmiş ve lineer çarpıştırıcıları ILC ($\sqrt{s} = 0.5TeV$), ILC ($\sqrt{s} = 1.0TeV$), ve CLIC ($\sqrt{s} = 3.0TeV$) için nümerik değerler sunulmuştur. Daha sonra, bu nümerik değerler yardımıyla ağır leptonların gözlemlenme başka bir deyişle üretilme olasılıkları analiz edilmiştir. En sonunda, bu modelin neden normal ağır leptonların üretimi için uygulanamıyacağı açıklanmıştır.

Anahtar Kelimeler: Standart model, ağır leptonlar, sicim esinli E_6 model, tek ve çift üretim.

To My Family . . .

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CHAPTER 1

INTRODUCTION

1.1 STANDARD MODEL

The model, developed in order to define all matter and forces except gravity in the universe, to classify all known particles and to examine interactions of these particles, is called the Standard Model. According to the Standard Model (SM)[1,6], matter is formed of 24 fundamental fermions including 6 quarks, 6 leptons and their antiparticles. Elementary particles of the Standard Model are displayed in Figure 1.1.

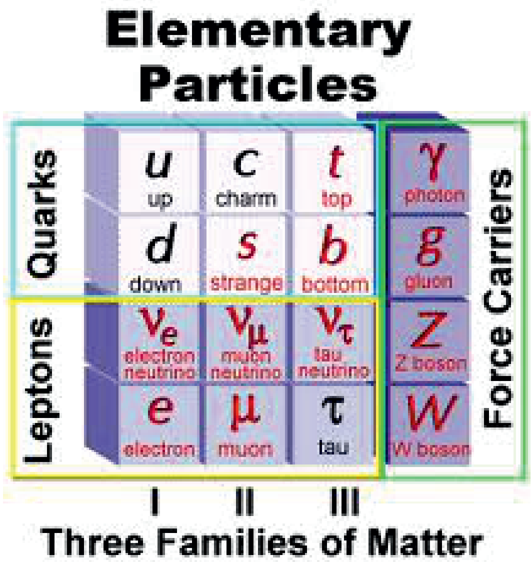

Figure 1.1: Standard Model.

1.1.1 Fermionic Sector

Fermions are the particles of the matter with spin -1/2 and interact by means of spin - 1 gauge bosons. Fermionic sector defines quarks and leptons.

$1^{st}Gen.$	$2^{nd}Gen.$	$3^{rd}Gen.$	Q	Spin
e^-	μ^-	τ^-	-1	1/2
ν_e	ν_μ	ν_τ	0	1/2

Table 1.1: Leptons in SM.

Flavor	Mass (MeV/c^2)
e^-	0.51
ν_e	$<3\times10^{-6}$
μ^-	105.658
ν_μ	0.19
τ^-	1777.03
ν_τ	<18.2

Table 1.2: Mass of Leptons.

According to the SM, there are three generations of leptons. Their main properties are listed in Table 1.1 and Table 1.2.

Muon and tau are kind of heavy leptons with short mean lifetimes (τ_μ=2.2 $\times$ 10^{-6}s and $\tau_\tau = 290.6 \pm 1.1\times 10^{-15}$s) and decay into more light leptons or quarks. On the other hand, electron and neutrino are only in steady structure. Quarks are another kind of matter particle. In the SM, there are three generations of quarks. Their main properties are listed in Table 1.3 and 1.4.

$1^{st}Gen.$	$2^{nd}Gen.$	$3^{rd}Gen.$	Q	Color	Spin
u	c	t	2/3	r,b,g	1/2
d	s	b	-1/3	r,b,g	1/2

Table 1.3: Quarks in SM.

Quarks do not act like free particle. They are embedded in hadrons. Furthermore, quark being a fermion has QCD color charge. Although lepton is also a fermion, it does not carry color charge. The hadrons which are formed by quarks are also colorless.

Flavor	Mass (MeV/c^2)
u	1 to 5
d	3 to 9
c	1.15 to 1.35
s	75 to 175
t	(174.3 $\pm$ 5.1)$\times$ 10^3
b	4.4$\times$ 10^3

Table 1.4: Mass of Quarks.

1.1.2 Gauge Sector

Particles and antiparticles are entirely identical except electrical charges. For instance, electron is negative, but positron is positive and they have both the same mass, so gravitation interactions on them are equal.

When a particle and an antiparticle meet, they disappear and neutral force mediators such as photon, Z boson or gluon form. Interactions between particles are represented by means of gauge groups in the Standard Model. All interactions except gravity appear with interchange of gauge bosons.

Photons are the mediator of the electromagnetic force between electrically charged particles, the gauge bosons ($W^{\pm}$, Z^0) are the mediator of the weak interactions and the gluons mediate the strong interactions between quarks.

Fundamental bosons and their main properties are listed in Table 1.5.

Bosons	Force	Q	Spin	Mass(GeV)/c^2
Gluons	Strong	0	1	0
Photon(γ)	Electromagnetic	0	1	0
$W^{\pm}$	Weak	$\pm$1	1	80.403
Z^0	Weak	0	1	91.187

Table 1.5: Fundamental Gauge bosons in SM.

According to the SM, in the nature, known four interactions exist. They are strong, electromagnetic, weak and gravity[7,8].

Electromagnetic interaction, appears between electrically charged particles with photon being neutral and massless, is in charge of connecting atoms and molecules.Intensity of this interaction is inversely proportional to the square of the distance between particles. Photons being neutral can not affect each other.

Strong interaction can be expressed by Quantum Chromo Dynamics (QCD) and the intensity of the interaction is defined by means of g_s coupling constant. The force restrains protons and neutrons in the nucleus. Although gluons mediating strong interaction are neutral like photons, they affect each other owing to the reality of carrying color charge.

Weak interaction which is mediated by gauge bosons as W^-, W^+ and Z^0 tends to propagate radioactivity in the nucleus.

Gravitational interaction restrains planets, stars and galaxies together and is inversely proportional to the square of the distance between them. Gauge boson of this interaction is called graviton, but graviton is not accepted to be a part of the SM owing to the fact that experimental evidence which explains the existence of graviton has not been found yet.

Unifying strong, weak and electromagnetic interaction under framework called Grand Unified Theory [12] is an important aim of particle physicists. Datas and theory of today anticipate that these forces unite at a point with high energy being about 10^{18} GeV.

1.1.3 Fields of Gauge Bosons

There are three main symmetry groups in the SM. These are SU (3) (Color), SU (2) (Weak isospin) and U (1) (hypercharge) symmetry groups. The SM is invariant under the following group:

$$SU(3)_{color} \times SU(2)_{left} \times U(1)_{hypercharge}. \tag{1.1}$$

Which describes the electromagnetic, weak and strong interactions of elementary particles. The weak and electromagnetic interactions are unified in the $SU(3)_L \times U(1)_Y$ group.

Gauge sector defines gauge bosons. In the SM, gauge sector consists of G_μ^a being gauge boson of $SU(3)$ and having g_s coupling constant, (W_μ^i, B_μ) being four gauge bosons of $SU(2)_L \times U(1)_Y$ and having respectively g and $g^{'}$ coupling constant [9].

Charged vector bosons are expressed as the following:

$$W_\mu{}^{\pm} = \frac{1}{\sqrt{2}}(W_\mu{}^1 \pm iW_\mu{}^2) \tag{1.2}$$

Fields of γ and Z bosons are represented as the following:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} \tag{1.3}$$

Here $W_\mu{}^i$ and B_μ are massless bosons and have not been observed. θ_w is called Weinberg angle.

Coupling constants of $SU(2)_L$ and $U(1)_Y$ are the following:

$$g\sin\theta_w = g^{'}\cos\theta_w = g_e \tag{1.4}$$

Here g_e is charge of electron.

Fields of γ and Z bosons become as the following:

$$\begin{aligned} A_\mu &= \frac{gB_\mu + g'W_\mu^3}{\sqrt{g^2+g'^2}} \\ Z_\mu &= \frac{-g^{'}B_\mu + gW_\mu^3}{\sqrt{g^2+g'^2}} \end{aligned} \tag{1.5}$$

1.2 DEFICIENCIES OF STANDARD MODEL

Most of the Standard Model foresight was confirmed experimentally. For instance, τ lepton was observed in 1975. Bottom (b) quark was observed in 1977 at Fermilab and gluons were discovered in 1979 by PETRA experiment at the DESY. In 1995 t quark was discovered by the CDF [10] and D0 [11] experiments at Fermilab Tevatron. Discovery of W, Z bosons in 1983 at CERN is the biggest success of SM.

In SM, six quarks, leptons and also gauge bosons (γ, Z^0, $W^{\pm}$, g) have been observed experimentally so far. The only particle which SM anticipated, but has not been discovered yet is Higgs.

Although the SM can explain a lot of events successfully in high energy physics, the SM is not the ultimate theory of the fundamental particles and their interactions. Some of fundamental problems that the SM can not answer are:

• Why are there three generations of quark and lepton? Is there the fourth generation?

• Are quarks and leptons fundamental or were they formed of other particles?

• How will gravity be included in the SM?

• Although the SM has particle and antiparticle symmetry, why is it being observed that the universe consisted of particle completely?

• What is invisible dark matter?

One of the most important deficiencies of the SM is not be able to unify three fundamental forces (EM, weak and strong) at only a value in high energy level. Supersymmetrical Grand Unified Theory [13] can provide the unification between three fundamental forces.

Another problem that the SM can not explain is Hierarchy problem [14]. There is a large difference between energy scale which electroweak interactions unite at and Planck scale which is important for quantum gravitation. This situation is called hierarchy problem.

As known, there are three important constants in physics. These are velocity of light (c), Planck constant (h) and Newton constant(G). These constants form Planck mass.

$$m_P = \sqrt{hc/G} \cong 10^{19} GeV \tag{1.6}$$

In quantum gravitational interactions, this scale has a big importance. On the other hand, mass scale of electroweak symmetry breaking is approximately 100 GeV. The difference between mass scales can not be explained by the SM.

In the SM there are lots of free parameters. For instance, although masses of all quarks except t quark was known, mass of top quark could not be estimated correctly without experimental evidence, on the grounds that there is not a mathematical model being useful in order to find a pattern for masses of particles in the SM.

In nature, there are particles from the first generation (electron, electron neutrino, u quark and d quark). Why does not the nature need two generations?

Moreover, Higgs boson has not been observed yet and it is not clear whether it is fundamental or composite particle.

As a result, on account of these problems, some non-standard models have facility to go beyond the SM. There are two possible ways of going beyond the SM. The first one is to consider new interactions with same fields, which leads supersymmetry, grand unification, string theory, etc. The other way is to consider new interactions with new fields.

CHAPTER 2

SINGLE PRODUCTION OF HEAVY LEPTONS

In this chapter we study the possible production of new single heavy leptons suggested by string inspired E_6 model [15] at linear colliders International Linear Collider (ILC) ($\sqrt{s} = 0.5TeV$), ILC ($\sqrt{s} = 1.0TeV$) and Compact Linear Collider (CLIC) ($\sqrt{s} = 3.0TeV$) [17,19].

The single production of heavy leptons L, in e^- e^+ collisions occur through the s and t channel processes e^- e^+ $\longrightarrow$ L^-e^+ caused by the flavor changing neutral current (FCNC) Lagrangian [16]:

$$L_{nc} = g_Z \sin\theta_{mix}\psi_L\gamma^\mu(1+\gamma^5)\psi_e Z^\mu + h\cdot c \tag{2.1}$$

where θ_{mix} are the mixing angles between right handed components of the ordinary and new heavy charged leptons. The Feynman diagrams of the s and t channel processes e^- e^+ $\longrightarrow$ L^-e^+ are given in Fig.2.1. We use the parameter b_{lLZ} to denote $sin\theta_{mix}$ and take 0.1 as an upper value in the numerical calculations.

2.1 Definition of Differential Cross Section

The total differential cross section [21] for the process e^- e^+ $\longrightarrow$ L^-e^+ is obtained as

$$\frac{d\sigma_{tot}}{dt} = \frac{1}{16\pi s^2} < |\mathcal{M}_{tot}|^2 > \tag{2.2}$$

where

$$< |\mathcal{M}_{tot}|^2 >=< |\mathcal{M}_s|^2 > + < |\mathcal{M}_t|^2 > + < \overline{\mathcal{M}}_s\mathcal{M}_t > + < \mathcal{M}_s\overline{\mathcal{M}}_t > \tag{2.3}$$

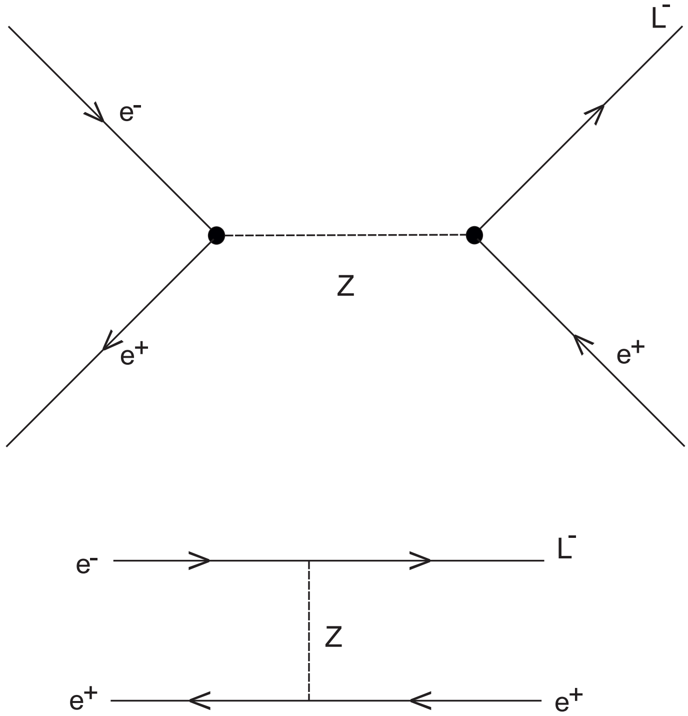

Figure 2.1: The Feynman diagrams of the s and t channel Z exchange processes in ep collision.

s channel matrix elements [20] are,

$$-i\mathcal{M}_s = [\overline{v}(p_2)(\frac{-ig_z}{2})\gamma^\mu(c_V - c_A\gamma^5)u(p_1) \times [\frac{-i(g_{\mu\nu} - q_\mu q_\nu/{M_Z}^2)}{[(s - {M_Z}^2) + i\Gamma_Z M_Z]}]$$

$$\times[\overline{u}(p_3)g_Z b_{lLZ}\gamma^\nu(1+\gamma^5)v(p_4)] \quad (2.4)$$

When using Dirac equations for the massless particles, matrix elements become as,

$$\mathcal{M}_s = -\frac{ig_Z^2 b_{lLZ}}{2}\frac{1}{[(s - M_Z^2) + i\Gamma_Z M_Z]} \times [\overline{v}(p_2)\gamma^\mu(c_V - c_A\gamma^5)u(p_1)]$$

$$\times[\overline{u}(p_3)\gamma_\mu(1+\gamma^5)v(p_4)] \quad (2.5)$$

and

$$\overline{\mathcal{M}}_s = \frac{ig_Z^2 b_{lLZ}}{2}\frac{1}{[(s - M_Z^2) - i\Gamma_Z M_Z]} \times [\overline{u}(p_1)\gamma^\nu(c_V - c_A\gamma^5)v(p_2)]$$

$$\times[\overline{v}(p_4)\gamma_\nu(1+\gamma^5)u(p_3)] \quad (2.6)$$

ignoring the ordinary lepton masses we find,

$$< |\mathcal{M}_s|^2 >= \frac{1}{4} \sum_{spins} |\mathcal{M}_s|^2 \tag{2.7}$$

$$< |\mathcal{M}_s|^2 >= \frac{g_Z^4 b_{lLZ}^2}{16[(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2}] \times \mathrm{Tr}[\gamma^\mu (c_V - c_A\gamma^5) \not{p}_1 \gamma^\nu (c_V - c_A\gamma^5) \not{p}_2]$$
$$\times \mathrm{Tr}[\gamma_\mu (1+\gamma^5) \not{p}_4 \gamma_\nu (1+\gamma^5)(\not{p}_3 + m_3)] \tag{2.8}$$

t channel matrix elements are,

$$-i\mathcal{M}_t = [\overline{u}(p_3) g_Z b_{lLZ} \gamma^\mu (1+\gamma^5) u(p_1)] \times \Big[- \frac{i(g_{\mu\nu} - q_\mu q_\nu / M_Z^2)}{[(t-M_Z^2) + i\Gamma_Z M_Z]} \Big]$$
$$\times [\overline{v}(p_2)(-\frac{ig_Z}{2}) \gamma^\nu (c_V - c_A\gamma^5) v(p_4)] \tag{2.9}$$

when using Dirac equations for the massless particles, matrix elements become as,

$$\mathcal{M}_t = -\frac{ig_Z^2 b_{lLZ}}{2} \frac{1}{[(t-M_Z^2) + i\Gamma_Z M_Z]} \times [\overline{u}(p_3)\gamma^\mu (1+\gamma^5) u(p_1)]$$
$$\times [\overline{v}(p_2)\gamma_\mu (c_V - c_A\gamma^5) v(p_4)] \tag{2.10}$$

and

$$\overline{\mathcal{M}}_t = \frac{ig_Z^2 b_{lLZ}}{2} \frac{1}{[(t-M_Z^2) - i\Gamma_Z M_Z]} \times [\overline{u}(p_1)\gamma^\nu (1+\gamma^5) u(p_3)]$$
$$\times [\overline{v}(p_4)\gamma_\nu (c_V - c_A\gamma_5) v(p_2)] \tag{2.11}$$

ignoring the ordinary lepton masses we find,

$$< |\mathcal{M}_t|^2 >= \frac{g_Z^4 b_{lLZ}^2}{16[(t-M_Z^2)^2 + \Gamma_Z^2 M_Z^2}] \times \mathrm{Tr}[\gamma^\mu (1+\gamma^5) \not{p}_1 \gamma^\nu (1+\gamma^5)(\not{p}_3 + m_3)]$$
$$\times \mathrm{Tr}[\gamma_\mu (c_V - c_A\gamma^5) \not{p}_4 \gamma_\nu (c_V - c_A\gamma^5) \not{p}_2]. \tag{2.12}$$

The interference terms of the s and t channel processes are given by,

$$< \mathcal{M}_s \overline{\mathcal{M}}_t >= \frac{g_Z^4 b_{lLZ}^2}{16[(s-M_Z^2) + i\Gamma_Z M_Z][(t-M_Z^2) - i\Gamma_Z M_Z]}$$

$$\times \mathrm{Tr}[\gamma^\mu(c_V - c_A\gamma^5)\not{p}_1\gamma^\nu(1+\gamma^5)(\not{p}_3+m_3)$$

$$\times\gamma_\mu(1+\gamma^5)\not{p}_4\gamma_\nu(c_V - c_A\gamma^5)\not{p}_2]. \tag{2.13}$$

and

$$< \overline{\mathcal{M}}_s\mathcal{M}_t >= \frac{g_Z^4 b_{lLZ}^2}{16[(s-M_Z^2)-i\Gamma_Z M_Z][(t-M_Z^2)+i\Gamma_Z M_Z]}$$

$$\times \mathrm{Tr}[\gamma^\nu(c_V - c_A\gamma^5)\not{p}_2\gamma^\mu(1+\gamma^5)\not{p}_1$$

$$\times\gamma_\nu(1+\gamma^5)(\not{p}_3+m_3)\gamma_\mu(c_V - c_A\gamma^5)\not{p}_4]. \tag{2.14}$$

$$p_1\cdot p_2 = \frac{s}{2},$$

$$p_3\cdot p_4 = \frac{s-m_L^2}{2},$$

$$p_1\cdot p_3 = \frac{m_L^2-t}{2},$$

$$p_2\cdot p_4 = -\frac{t}{2},$$

$$p_1\cdot p_4 = -\frac{u}{2},$$

$$p_2\cdot p_3 = \frac{m_L^2-u}{2},$$

$$u = m_L^2 - s - t, \tag{2.15}$$

using the trace theorems and Mandelstam variables in equation (2.15), the differential cross section takes the form,

$$\frac{d\sigma_{tot}}{dt} = \frac{1}{16\pi s^2}[< |\mathcal{M}_s|^2 > + < |\mathcal{M}_t|^2 > + < \mathcal{M}_s\overline{\mathcal{M}}_t > + < \overline{\mathcal{M}}_s\mathcal{M}_t >] \tag{2.16}$$

$$\frac{d\sigma_{tot}}{dt} = \frac{g_Z^4 b_{lLZ}^2}{16\pi s^2}\Big\{\frac{(c_A-c_V)^2(t+s)(t+s-m^2)+(c_A+c_V)^2 t(t-m^2)}{[(s-M_Z^2)^2+\Gamma_Z^2M_Z^2]}$$

$$+\frac{(c_A-c_V)^2(t+s)(t+s-m^2)+(c_A+c_V)^2 s(s-m^2)}{[(t-M_Z^2)^2+\Gamma_Z^2M_Z^2]}$$

$$+\frac{2(c_A-c_V)^2(t+s)(m^2-s-t)((s-M_Z^2)(t-M_Z^2)+\Gamma_Z^2M_Z^2)}{[(s-M_Z^2)^2+\Gamma_Z^2M_Z^2][(t-M_Z^2)^2+\Gamma_Z^2M_Z^2]}\Big\} \tag{2.17}$$

where $c_A = -0.5$, $c_V = -0.03$, m is the mass of heavy lepton, Γ_Z is the decay width and M_Z is the mass of Z boson, g_Z is the coupling constant for weak interaction, s and t are Mandelstam variables.

2.2 Numerical Results

The total cross section can be obtained by integrating the differential cross section as the following equation:

$$\sigma_{tot} = \int_{t_{min}}^{t_{max}} \frac{d\sigma_{tot}}{dt} dt \tag{2.18}$$

Boundaries of the integral are $t_{min} = -(s - m_L^2)$ and $t_{max} = 0$ [22]. After this integration, relation between the total cross sections and heavy lepton masses, for linear colliders ILC ($\sqrt{s} = 0.5$ and $1.0TeV$) and CLIC ($\sqrt{s} = 3.0TeV$) is obtained. In Tables 2.1, 2.2 and 2.3 numerical results are presented.

m_L(GeV)	σ(pb)
100	1.43
200	1.24
300	0.93
400	0.49

Table 2.1: The total cross sections depending on the heavy lepton masses for ILC with $\sqrt{s} = 0.5TeV$.

m_L(GeV)	σ(pb)
100	1.50
300	1.38
500	1.32
700	0.76
900	0.26

Table 2.2: The total cross sections depending on the heavy lepton masses for ILC with $\sqrt{s} = 1.0TeV$.

In Fig.2.2, we display the total cross sections as function of the heavy lepton masses, for the three center of mass energies.

m_L(GeV)	σ(pb)
250	1.51
750	1.43
1250	1.26
1750	1.00
2250	0.66
2750	0.24

Table 2.3: The total cross sections depending on the heavy lepton masses for CLIC with $\sqrt{s} = 3.0TeV$.

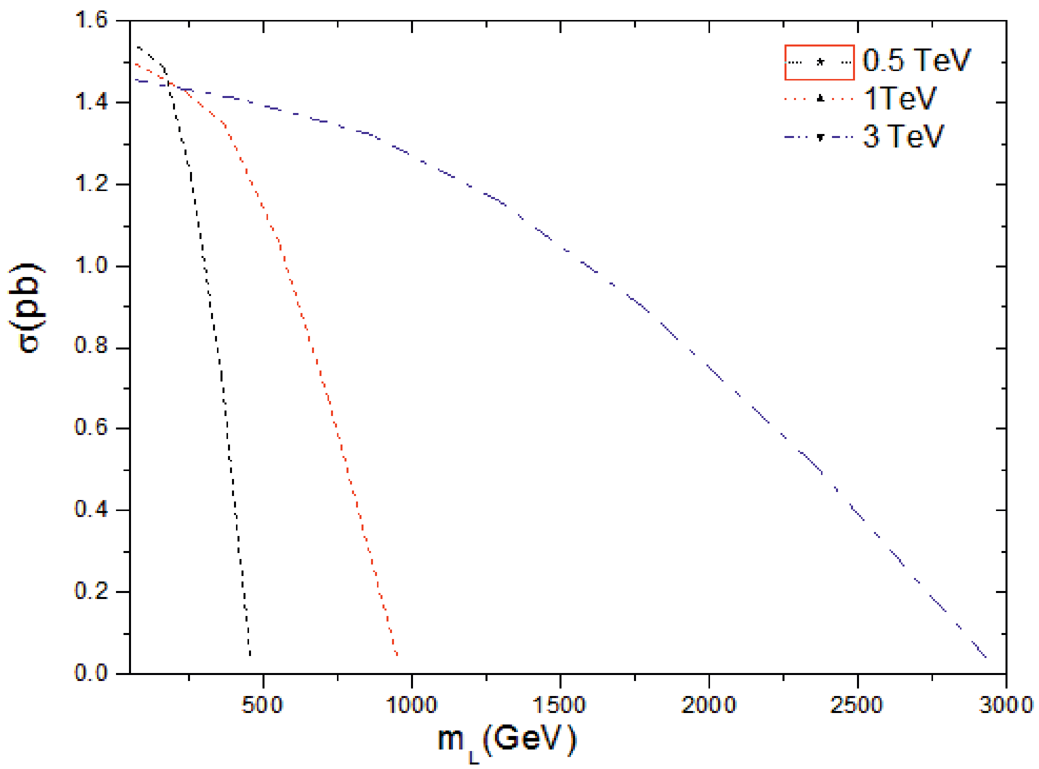

Figure 2.2: The total cross sections for the process $e^-e^+ \to L^-e^+$, as function of the heavy lepton masses, for linear colliders ILC ($\sqrt{s} = 0.5$ and $1.0TeV$) and CLIC ($\sqrt{s} = 3.0TeV$).

CHAPTER 3

PAIR PRODUCTION OF HEAVY LEPTONS

In this chapter we study the possible production of new pair heavy leptons by using the same model in Chapter 2.

Pair production of heavy leptons in E_6 occur through the t-channel flavor changing neutral current process $e^-e^+ \to L^-L^+$. The Feynman diagram of the t-channel process $e^-e^+ \to L^-L^+$ is given in Fig. 3.1.

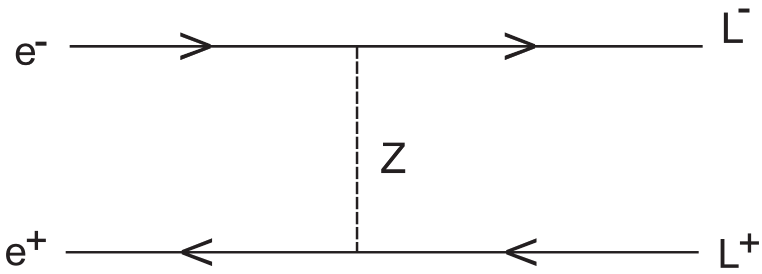

Figure 3.1: The Feynman diagram of the t-channel process $e^-e^+ \to L^-L^+$.

3.1 Definition of Differential Cross Section

We can write the matrix elements from the FCNC Lagrangian given in Eq. (2.1) as,

$$\begin{aligned} -i\mathcal{M}_t &= [\overline{u}(p_3) g_Z b_{lLZ} \gamma^\mu (1+\gamma^5) u(p_1)] \\ &\times \Big[-\frac{i(g_{\mu\nu} - q_\mu q_\nu / M_Z^2)}{[(t-M_Z^2) + i\Gamma_Z M_Z]} \Big] \\ &\times [\overline{v}(p_2) g_Z b_{lLZ} \gamma^\nu (1+\gamma^5) v(p_4)] \end{aligned} \tag{3.1}$$

When using Dirac equations for the massless particles, matrix elements become as,

$$\begin{aligned} \mathcal{M}_t &= \frac{g_Z^2 b_{lLZ}^2}{[(t-M_Z^2) + i\Gamma_Z M_Z]} \Big(1 - \frac{p_3 p_4}{M_Z^2}\Big) \\ &\times [\overline{u}(p_3)\gamma^\mu (1+\gamma^5) u(p_1)] \times [\overline{v}(p_2)\gamma_\mu (1+\gamma^5) v(p_4)] \end{aligned} \tag{3.2}$$

and

$$\overline{\mathcal{M}}_t = \frac{g_Z^2 b_{lLZ}^2}{[(t - M_Z^2) - i\Gamma_Z M_Z]}\Big(1 - \frac{p_3 p_4}{M_Z^2}\Big)$$

$$\times[\overline{u}(p_1)\gamma^\nu(1+\gamma^5)u(p_3)] \times [\overline{v}(p_4)\gamma_\nu(1+\gamma^5)v(p_2)] \tag{3.3}$$

ignoring the ordinary lepton masses,

$$< |\mathcal{M}_t|^2 >= \frac{g_Z^4 b_{lLZ}^4}{4[(t - M_Z^2)^2 + \Gamma_Z^2 M_Z^2]}\Big(1 - \frac{p_3 p_4}{M_Z^2}\Big)^2$$

$$\times \mathrm{Tr}[\gamma^\mu(1+\gamma^5)\not{p}_1\gamma^\nu(1+\gamma^5)(\not{p}_3 + m_3)]$$

$$\times \mathrm{Tr}[\gamma_\mu(1+\gamma^5)(\not{p}_4 - m_4)\gamma_\nu(1+\gamma^5)\not{p}_2] \tag{3.4}$$

Mandelstam variables for the pair production of heavy lepton are,

$$p_1 \cdot p_2 = \frac{s}{2},$$

$$p_3 \cdot p_4 = \frac{s - 2m_L^2}{2},$$

$$p_1 \cdot p_3 = \frac{m_L^2 - t}{2},$$

$$p_2 \cdot p_4 = \frac{m_L^2 - t}{2},$$

$$p_1 \cdot p_4 = \frac{m_L^2 - u}{2},$$

$$p_2 \cdot p_3 = \frac{m_L^2 - u}{2},$$

$$u = 2m_L^2 - s - t. \tag{3.5}$$

Using the trace theorems and the Mandelstam variables in Eq.(3.5), the differential cross section for this process is obtained as,

$$\frac{d\sigma}{dt} = \frac{g_Z^4 b_{lLZ}^4}{4\pi s^2 M_Z^4[(t - M_Z^2)^2 + \Gamma_Z^2 M_Z^2]}$$

$$\times[(m^2 - t)^2 m^4 + 4M_Z^2 m^4 s + 4(s + t - m^2)^2 M_Z^4] \tag{3.6}$$

3.2 Numerical Results

The cross section can be obtained by integrating the differential cross section as the following equation:

$$\sigma = \int_{t_{min}}^{t_{max}} \frac{d\sigma}{dt} dt \qquad (3.7)$$

Boundaries of the integral are $t_{min} = -\left[\sqrt{s/4} + \sqrt{(s^2 + 4m^4)/4s}\right]^2$ and $t_{max} = -\left[\sqrt{s/4} - \sqrt{(s^2 + 4m^4)/4s}\right]^2$ [22].

After this integration, relation between cross sections and heavy lepton masses, for linear colliders ILC ($\sqrt{s} = 0.5$ and $1.0TeV$) and CLIC ($\sqrt{s} = 3.0TeV$) is obtained. In Tables 3.1, 3.2, and 3.3 numerical results are presented.

m_L(GeV)	σ(pb)
100	0.36
150	0.35
200	0.37
250	0.27

Table 3.1: The total cross sections depending on the heavy lepton masses for ILC with $\sqrt{s} = 0.5TeV$.

m_L(GeV)	σ(pb)
100	0.43
200	0.44
300	0.56
400	0.94

Table 3.2: The total cross sections depending on the heavy lepton masses for ILC with $\sqrt{s} = 1.0TeV$.

m_L(GeV)	σ(pb)
250	0.47
500	0.61
750	1.51
1000	3.96
1250	7.79

Table 3.3: The total cross sections depending on the heavy lepton masses for CLIC with $\sqrt{s} = 3.0TeV$.

In Fig.3.2, we display the total cross sections as function of the heavy lepton masses, for the three center of mass energies.

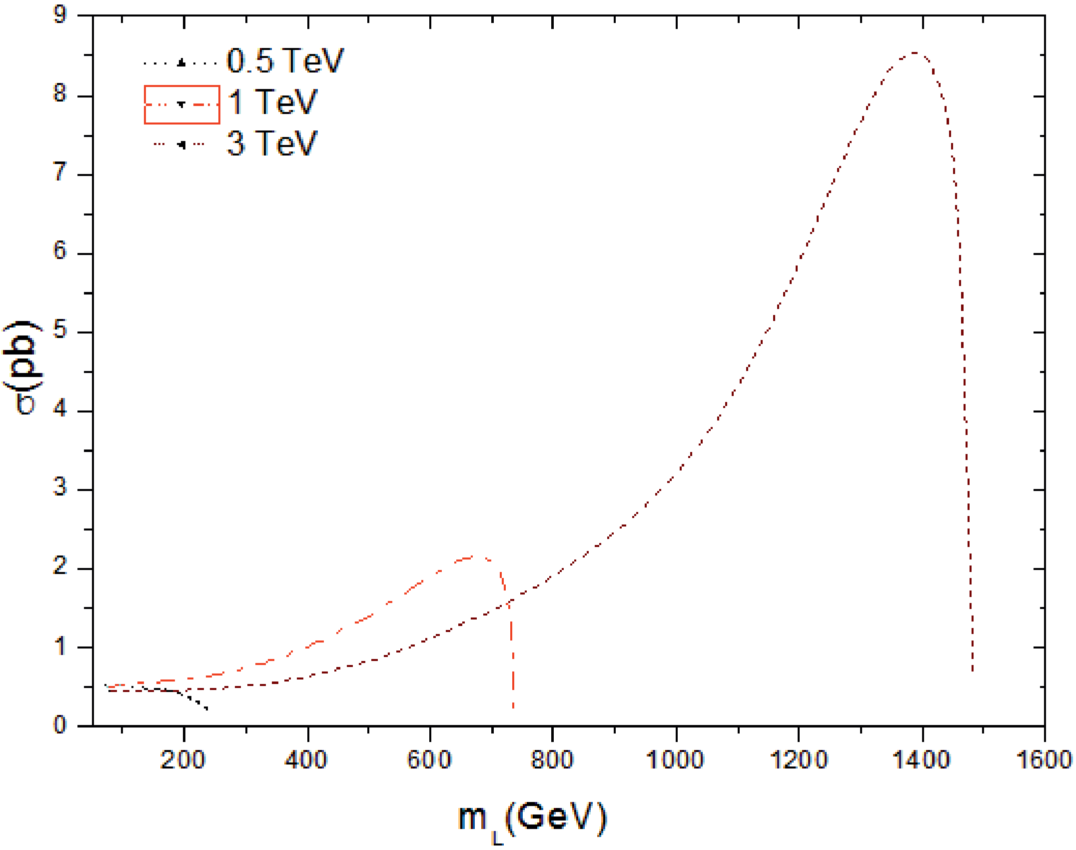

Figure 3.2: The total cross sections for the process $e^-e^+ \to L^-L^+$, as function of the heavy lepton masses, for linear colliders ILC ($\sqrt{s} = 0.5$ and $1.0 TeV$) and CLIC ($\sqrt{s} = 3.0 TeV$).

CHAPTER 4

RESULT AND DISCUSSION

We obtain cross sections by using the FCNC Lagrangian given in Eq.(2.1) for single and pair production of heavy leptons. We investigate the relation between cross sections and heavy lepton masses, for linear colliders ILC ($\sqrt{s} = 0.5$ and $1.0TeV$) and CLIC ($\sqrt{s} = 3.0TeV$) and then we obtain numerical results which are displayed in Tables 2.1, 2.2, 2.3, 3.1, 3.2, and 3.3. We draw Fig.2.2 and 3.2 which express the cross sections as function of the heavy lepton masses, for the three center of mass energies of the proposed options by using these numerical results.

Cross sections explain the probabilities of single and pair production of heavy leptons occurring through $e^-e^+ \to L^-e^+$ and $e^-e^+ \to L^-L^+$ processes.

According to the Fig.2.2, minimum values of heavy lepton masses for ILC ($\sqrt{s} = 0.5$ and $1.0TeV$) and CLIC ($\sqrt{s} = 3.0TeV$) provide maximum values of cross sections. It means that probability of single production is more relevant at these values. When we increase masses for three linear colliders, cross sections begin diminishing and for each collider after some approximate values of heavy lepton masses, cross sections approach to zero. These values may be called critical values of heavy lepton masses.

It means that probability of single production approaches to zero. In other words, increase in heavy lepton mass causes the probability of single production to drop.

The cross sections as function of heavy lepton masses m_L for pair production are displayed in Fig.3.2. According to the Fig.3.2, when we increase values of masses, cross sections begin increasing too expect ILC ($\sqrt{s} = 0.5TeV$), but after some approximate values dropping in cross sections begin being observed and then probabilities, that is cross sections approaches to zero. These approximate values may be defined as critical values of heavy lepton masses. In other words, increase in lepton masses is proportional to increase in the probability of pair production up to these critical values. This condition is valid except ILC ($\sqrt{s} = 0.5TeV\sqrt{}$). Now that for ILC ($\sqrt{s} = 0.5TeV$) droppings in probability is permanent. By means of this condition it can be said that pair production is not harmonius for three center of mass energies.

As a result, we can not apply string inspired E_6 model to the productions of ordinary heavy lepton μ^- or τ^- owing to a reason. Namely, we use the FCNC Lagrangian given in Eq.(2.1) in order to write matrix elements, that is amplitudes. The Lagrangian includes $\sin\theta_{mix}$ factor which is denoted with b_{lLZ}. θ_{mix} are the mixing angles between right handed components of the ordinary (e, μ, τ) and new heavy charged leptons. We use b_{lLZ} factor while writing matrix elements for each non-standard vertex from Feynman diagrams through single and pair production. After their production, the heavy leptons will decay via the neutral current process $L \to lZ$, where $l = e, \mu, \tau$. $\sin\theta_{mix}$ or b_{lLZ} represents the decay. On the other hand, if we investigate the productions of ordinary lepton μ or τ, we can not write b_{lLZ} factor while obtaining matrix elements. On the grounds that ordinary heavy lepton can not decay into Z in stationary state.

In brief, string inspired E_6 model is not valid for productions of ordinary heavy leptons μ and τ.

CHAPTER 5

CONCLUSION

In this chapter we briefly present the summaries of other chapters. In chapter 1, we give some information of the Standard Model, list and express deficiencies of this model.

In chapter 2, single production of heavy leptons is investigated and numerical results are presented.

In chapter 3, pair production of heavy leptons is investigated and numerical results are presented.

In chapter 4, we discuss our numerical results and explain the defect of string inspired E_6 model for production of ordinary heavy leptons μ and τ. While drawing Fig. 2.2 for single production, we use data of cross sections entirely and our result is as we have expected. For pair production, cross sections approach to zero after same values which we anticipated. Namely, heavy leptons are not observed after these values.

In conclusion, single production is more relevant than pair production.

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APPENDIX A

DIRAC MATRICES AND TRACE THEOREMS

We first prove some useful properties of Dirac matrices. We start with the definitions of the γ matrices:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \qquad i = 1, 2, 3. \tag{A.1}$$

These matrices satisfy the basic anti-commutation relation

$$\left\{ \gamma^\mu, \gamma^\mu \right\} = 2g^{\mu\nu}, \tag{A.2}$$

In terms of the metric

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{A.3}$$

note that $g^{\mu\nu} g_{\mu\nu} = 4$, we have,

$$\gamma_\mu \gamma^\mu = 4$$

$$\gamma_\mu \gamma^\nu \gamma^\mu = -2\gamma^\nu$$

$$\gamma_\mu \gamma^\nu \gamma^\lambda \gamma^\mu = 4g^{\nu\lambda}$$

$$\gamma_\mu \gamma^\nu \gamma^\lambda \gamma^\sigma \gamma^\mu = -2\gamma^\sigma \gamma^\lambda \gamma^\mu \tag{A.4}$$

$$\gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \tag{A.5}$$

so that the following relations are satisfied;

$$(\gamma_5)^2 = I \qquad \gamma_5\gamma_\mu = -\gamma_\mu\gamma_5 \tag{A.6}$$

The trace theorems are (using the notation $a = \gamma_\mu a^\mu$:

$$\begin{aligned}
\mathrm{Tr}[1] &= 4 \\
\mathrm{Tr}[\not{a}\not{b}] &= 4a\cdot b \\
\mathrm{Tr}[\not{1}\not{b}\not{c}\not{d}] &= 4\Big[(a\cdot b)(c\cdot d) - (a\cdot c)(b\cdot d) + (a\cdot d)(b\cdot c)\Big] \\
\mathrm{Tr}[\gamma^5] &= 0 \\
\mathrm{Tr}[\gamma^5\not{a}\not{b}] &= 0 \\
\mathrm{Tr}[\gamma^5\not{a}\not{b}\not{c}\not{d}] &= 4i\epsilon_{\mu\nu\lambda\sigma}a^\mu b^\nu c^\lambda d^\sigma
\end{aligned}$$

where $\epsilon_{\mu\nu\lambda\sigma} = +1(-1)$ for $\mu, \nu, \lambda, \sigma$ an even (odd) permutation of $0, 1, 2, 3$; and 0 if two indices are the same.

If A and B are any two matrices and α is any number,

$$\begin{aligned}
\mathrm{Tr}(A+B) &= \mathrm{Tr}A + \mathrm{Tr}B \\
\mathrm{Tr}(\alpha A) &= \alpha\mathrm{Tr}A \\
\mathrm{Tr}(AB) &= \mathrm{Tr}(BA) \\
\mathrm{Tr}(ABC) &= \mathrm{Tr}(CBA) = \mathrm{Tr}(BCA)
\end{aligned} \tag{A.7}$$

APPENDIX B

MATHEMATICA OUTPUT

```
(****************** S CHANNEL *******************)

G1 = G[l, {mu}] ** (cv - ca ** G[l, G5])
```

G[l, {mu}] c_v – G[l, {mu}, G5] c_a

```
G2 = G1 ** G[l, p1]
```

G[l, {mu}, p1, G5] c_a + G[l, {mu}, p1] c_v

```
G3 = G[l, {nu}] ** (cv - ca ** G[l, G5])
```

G[l, {nu}] c_v – G[l, {nu}, G5] c_a

```
G4 = G3 ** G[l, p2]
```

G[l, {nu}, p2, G5] c_a + G[l, {nu}, p2] c_v

```
G2G4 = G2 ** G4 /. l → ls // Simplify
```

4 (–2 i Eps[p1,p2,{mu},{nu}] c_a c_v + g_{mu nu} (– p1.p2) c_a^2 + p1_{mu} p2_{nu} ($c_a^2 + c_v^2$) + p1_{nu} p2_{mu} ($c_a^2 + c_v^2$) – g_{mu nu} p1.p2 c_v^2)

```
G55 = G[l, {mu}] ** (1 + G[l, G5])
```

G[l, {mu}] + G[l, {mu}, G5]

```
G6 = G55 ** G[l, p4]
```

G[l, {mu}, p4] – G[l, {mu}, p4, G5]

```
G7 = G[l, {nu}] ** (1 + G[l, G5])
```

G[l, {nu}] + G[l, {nu}, G5]

```
G8 = G7 ** (G[l, p3] + mL)
```

G[l, {nu}] m_L + G[l, {nu}, G5] m_L + G[l, {nu}, p3] − G[l, {nu}, p3, G5]

```
G6G8 = G6 ** G8 /. l → ls // Simplify
```

8 (−i Eps[p3,p4,{mu},{nu}] − g_{mu nu} p3.p4 + p3_{mu} p4_{nu} + p3_{nu} p4_{mu})

```
G2G4G6G8 = G2G4 G6G8 // Simplify
```

$$64\left(\text{p1.p3 p2.p4}\,(c_a+c_v)^2+\text{p1.p4 p2.p3}\,(c_a-c_v)^2\right)$$

```
U1 = G2G4G6G8 /. p1.p3 → (mL^2 - t) / 2
```

$$64\left(\frac{1}{2}\,\text{p2.p4}\,(c_a+c_v)^2\left(m_L^2-t\right)+\text{p1.p4 p2.p3}\,(c_a-c_v)^2\right)$$

```
U2 = U1 /. p2.p4 → -t / 2
```

$$64\left(\text{p1.p4 p2.p3}\,(c_a-c_v)^2-\frac{1}{4}\,t\,(c_a+c_v)^2\left(m_L^2-t\right)\right)$$

```
U3 = U2 /. p1.p4 → (t + s - mL^2) / 2
```

$$64\left(\frac{1}{2}\,\text{p2.p3}\,(c_a-c_v)^2\left(-m_L^2+s+t\right)-\frac{1}{4}\,t\,(c_a+c_v)^2\left(m_L^2-t\right)\right)$$

```
U4 = U3 /. p2.p3 → (t + s) / 2
```

$$64\left(\frac{1}{4}(s+t)(c_a-c_v)^2\left(-m_L^2+s+t\right)-\frac{1}{4}t(c_a+c_v)^2\left(m_L^2-t\right)\right)$$

```
U5 = U4 gZ^4 b1LZ^2 / (16 ((s - MZ^2)^2 + ΓZ^2 MZ^2)) // Simplify
```

$$\frac{b_{1LZ}^2 g_Z^4\left((s+t)(c_a-c_v)^2\left(-m_L^2+s+t\right)+t(c_a+c_v)^2\left(t-m_L^2\right)\right)}{\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2}$$

```
(************T CHANNEL ************)

A1 = G[l, {mu}] ** (1 + G[l, G5])
```

G[1, {mu}] + G[1, {mu}, G5]

```
A2 = A1 ** G[l, p1]
```

G[1, {mu}, p1] – G[1, {mu}, p1, G5]

```
A3 = G[l, {nu}] ** (1 + G[l, G5])
```

G[1, {nu}] + G[1, {nu}, G5]

```
A4 = A3 ** (G[l, p3] + mL)
```

G[1, {nu}] m_L + G[1, {nu}, G5] m_L + G[1, {nu}, p3] – G[1, {nu}, p3, G5]

```
A2A4 = A2 ** A4 /. l → ls // Simplify
```

$$8\,(i\,\mathrm{Eps[p1,p3,\{mu\},\{nu\}]} - \mathrm{g\_\{mu\ nu\}\ p1.p3} + \mathrm{p1\_\{mu\}\ p3\_\{nu\}} + \mathrm{p1\_\{nu\}\ p3\_\{mu\}})$$

```
A5 = G[l, {mu}] ** (cv - ca ** G[l, G5])
```

$$\mathrm{G[\,l,\{mu\}\,]}\,c_v - \mathrm{G[\,l,\{mu\},G5\,]}\,c_a$$

```
A6 = A5 ** G[l, p4]
```

$$\mathrm{G[\,l,\{mu\},p4,G5\,]}\,c_a + \mathrm{G[\,l,\{mu\},p4\,]}\,c_v$$

```
A7 = G[l, {nu}] ** (cv - ca ** G[l, G5])
```

$$\mathrm{G[\,l,\{nu\}\,]}\,c_v - \mathrm{G[\,l,\{nu\},G5\,]}\,c_a$$

```
A8 = A7 ** G[l, p2]
```

$$\mathrm{G[\,l,\{nu\},p2,G5\,]}\,c_a + \mathrm{G[\,l,\{nu\},p2\,]}\,c_v$$

```
A6A8 = A6 ** A8 /. l → ls // Simplify
```

$$4\left(2\,i\,\mathrm{Eps[p2,p4,\{mu\},\{nu\}]}\,c_a\,c_v + \mathrm{g\_\{mu\ nu\}}\,(-\mathrm{p2.p4})\,c_a^2 + \mathrm{p2\_\{mu\}\ p4\_\{nu\}}\left(c_a^2 + c_v^2\right) + \mathrm{p2\_\{nu\}\ p4\_\{mu\}}\left(c_a^2 + c_v^2\right) - \mathrm{g\_\{mu\ nu\}\ p2.p4}\,c_v^2\right)$$

```
A2A4A6A8 = A2A4 A6A8 // Simplify
```

$$64\left(\mathrm{p1.p2\ p3.p4}\,(c_a + c_v)^2 + \mathrm{p1.p4\ p2.p3}\,(c_a - c_v)^2\right)$$

```
T1 = A2A4A6A8 /. p1.p2 → s / 2
```

$$64\left(\mathrm{p1.p4\ p2.p3}\,(c_a - c_v)^2 + \frac{1}{2}\,\mathrm{p3.p4}\,s\,(c_a + c_v)^2\right)$$

```
T2 = T1 /. p3.p4 → (s - mL^2) / 2
```

$$64\left(\frac{1}{4}\,s\,(c_a+c_v)^2\left(s-m_L^2\right)+\text{p1.p4 p2.p3}\,(c_a-c_v)^2\right)$$

```
T3 = T2 /. p1.p4 → (t + s - mL^2) / 2
```

$$64\left(\frac{1}{2}\,\text{p2.p3}\,(c_a-c_v)^2\left(-m_L^2+s+t\right)+\frac{1}{4}\,s\,(c_a+c_v)^2\left(s-m_L^2\right)\right)$$

```
T4 = T3 /. p2.p3 → (t + s) / 2
```

$$64\left(\frac{1}{4}\,(s+t)\,(c_a-c_v)^2\left(-m_L^2+s+t\right)+\frac{1}{4}\,s\,(c_a+c_v)^2\left(s-m_L^2\right)\right)$$

$$\texttt{T5 = T4}\ \frac{g_Z\texttt{\^{}4}\ b_{1LZ}\texttt{\^{}2}}{\texttt{16 ((t - }M_Z\texttt{\^{}2)\^{}2 + }\Gamma_Z\texttt{\^{}2 }M_Z\texttt{\^{}2)}}\ \texttt{// Simplify}$$

$$\frac{b_{1LZ}^2\,g_Z^4\left((s+t)\,(c_a-c_v)^2\left(-m_L^2+s+t\right)+s\,(c_a+c_v)^2\left(s-m_L^2\right)\right)}{\left(t-M_Z^2\right)^2+M_Z^2\,\Gamma_Z^2}$$

```
(*********************** INTERFERENCE 1 *******************)

B1 = G[l, {mu}] ** (cv - ca ** G[l, G5])
```

$G[\,1,\{\text{mu}\}\,]\,c_v - G[\,1,\{\text{mu}\},\text{G5}\,]\,c_a$

```
B2 = B1 ** G[l, p1]
```

G[1, {mu}, p1, G5] c_a + G[1, {mu}, p1] c_v

```
B3 = G[1, {nu}] ** (1 + G[1, G5])
```

G[1, {nu}] + G[1, {nu}, G5]

```
B4 = B3 ** (G[1, p3] + mL)
```

G[1, {nu}] m_L + G[1, {nu}, G5] m_L + G[1, {nu}, p3] − G[1, {nu}, p3, G5]

```
B5 = G[1, {mu}] ** (1 + G[1, G5])
```

G[1, {mu}] + G[1, {mu}, G5]

```
B6 = B5 ** G[1, p4]
```

G[1, {mu}, p4] − G[1, {mu}, p4, G5]

```
B7 = G[1, {nu}] ** (cv - ca ** G[1, G5])
```

G[1, {nu}] c_v − G[1, {nu}, G5] c_a

```
B8 = B7 ** G[1, p2]
```

G[1, {nu}, p2, G5] c_a + G[1, {nu}, p2] c_v

```
B2B4B6B8 = B2 ** B4 ** B6 ** B8 /. l → ls // Simplify
```

−64 p1.p4 p2.p3 $(c_a - c_v)^2$

```
W1 = B2B4B6B8  /. p1.p4 → (t + s - mL^2) / 2
```

$-32\,\text{p2.p3}\,(c_a - c_v)^2\left(-m_L^2 + s + t\right)$

```
W2 = W1 /. p2.p3 → (t + s) / 2
```

$-16\,(s+t)\,(c_a - c_v)^2\left(-m_L^2 + s + t\right)$

```
W3 = W2 gZ^4 b1LZ^2 / (16 ((s - MZ^2) + ⅈ MZ ΓZ) ((t - MZ^2) - ⅈ MZ ΓZ)) // Simplify
```

$$-\frac{b_{1LZ}^2\, g_Z^4\,(s+t)\,(c_a - c_v)^2\left(-m_L^2 + s + t\right)}{\left(i\, M_Z\, \Gamma_Z - M_Z^2 + s\right)\left(-i\, M_Z\, \Gamma_Z - M_Z^2 + t\right)}$$

```
(***************** INTERFERENCE 2*****************************)

C1 = G[l, {nu}] ** (cv - ca ** G[l, G5])
```

G[l, {nu}] c_v – G[l, {nu}, G5] c_a

```
C2 = C1 ** G[l, p2]
```

G[l, {nu}, p2, G5] c_a + G[l, {nu}, p2] c_v

```
C3 = G[l, {mu}] ** (1 + G[l, G5])
```

G[l, {mu}] + G[l, {mu}, G5]

```
C4 = C3 ** G[l, p1]
```

G[l, {mu}, p1] – G[l, {mu}, p1, G5]

```
C5 = G[l, {nu}] ** (1 + G[l, G5])
```

G[1, {nu}] + G[1, {nu}, G5]

```
C6 = C5 ** (G[l, p3] + mL)
```

G[1, {nu}] m_L + G[1, {nu}, G5] m_L + G[1, {nu}, p3] − G[1, {nu}, p3, G5]

```
C7 = G[l, {mu}] ** (cv - ca ** G[l, G5])
```

G[1, {mu}] c_v − G[1, {mu}, G5] c_a

```
C8 = C7 ** G[l, p4]
```

G[1, {mu}, p4, G5] c_a + G[1, {mu}, p4] c_v

```
C2C4C6C8 = C2 ** C4 ** C6 ** C8 /. l → ls // Simplify
```

$-64\ \mathrm{p1.p4\ p2.p3}\,(c_a - c_v)^2$

```
Y1 = C2C4C6C8 /. p1.p4 → (t + s - mL^2) / 2
```

$-32\ \mathrm{p2.p3}\,(c_a - c_v)^2\left(-m_L^2 + s + t\right)$

```
Y2 = Y1 /. p2.p3 → (t + s) / 2
```

$-16\,(s+t)\,(c_a - c_v)^2\left(-m_L^2 + s + t\right)$

$$\mathrm{Y3} = \mathrm{Y2}\,\frac{g_Z\verb|^|4\ b_{1LZ}\verb|^|2}{16\,((s - M_Z\verb|^|2) - i\,M_Z\,\Gamma_Z)\,((t - M_Z\verb|^|2) + i\,M_Z\,\Gamma_Z)}\ \verb|// Simplify|$$

$$-\frac{b_{1LZ}^2\,g_Z^4\,(s+t)\,(c_a - c_v)^2\left(-m_L^2 + s + t\right)}{\left(-i\,M_Z\,\Gamma_Z - M_Z^2 + s\right)\left(i\,M_Z\,\Gamma_Z - M_Z^2 + t\right)}$$

```
DCS1 = (U5 + T5 + W3 + Y3) / (16 × 22 / 7 s^2) // Simplify
```

$$\frac{1}{352\,s^2} 7\, b_{\mathrm{lLZ}}^2\, g_Z^4 \left(-\frac{(s+t)(c_a-c_v)^2\left(-m_L^2+s+t\right)}{\left(i M_Z \Gamma_Z - M_Z^2 + s\right)\left(-i M_Z \Gamma_Z - M_Z^2 + t\right)} - \frac{(s+t)(c_a-c_v)^2\left(-m_L^2+s+t\right)}{\left(-i M_Z \Gamma_Z - M_Z^2 + s\right)\left(i M_Z \Gamma_Z - M_Z^2 + t\right)} + \frac{(s+t)(c_a-c_v)^2\left(-m_L^2+s+t\right)+t(c_a+c_v)^2\left(t-m_L^2\right)}{\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2} + \frac{(s+t)(c_a-c_v)^2\left(-m_L^2+s+t\right)+s(c_a+c_v)^2\left(s-m_L^2\right)}{\left(t-M_Z^2\right)^2+M_Z^2\Gamma_Z^2} \right)$$

$$\mathrm{DCS2} = \frac{1}{22/7\,s\text{^}2} b_{\mathrm{lLZ}}\text{^}2\, g_Z\text{^}4 \left(-\frac{2\,(s+t)\,(c_a-c_v)\text{^}2\,(-m\text{^}2+s+t)\,((s-M_Z\text{^}2)\,(t-M_Z\text{^}2)+\Gamma_Z\text{^}2\,M_Z\text{^}2)}{16\,((s-M_Z\text{^}2)\text{^}2+M_Z\text{^}2\,\Gamma_Z\text{^}2)\,((t-M_Z\text{^}2)\text{^}2+M_Z\text{^}2\,\Gamma_Z\text{^}2)} + \frac{(s+t)\,(c_a-c_v)\text{^}2\,(-m\text{^}2+s+t)+s\,(c_a+c_v)\text{^}2\,(s-m\text{^}2)}{16\,((t-M_Z\text{^}2)\text{^}2+M_Z\text{^}2\,\Gamma_Z\text{^}2)} + \frac{(s+t)\,(c_a-c_v)\text{^}2\,(-m\text{^}2+s+t)+t\,(c_a+c_v)\text{^}2\,(t-m\text{^}2)}{16\,((s-M_Z\text{^}2)\text{^}2+M_Z\text{^}2\,\Gamma_Z\text{^}2)} \right)$$

$$\frac{7\, b_{\mathrm{lLZ}}^2\, g_Z^4 \left(-\frac{(s+t)(c_a-c_v)^2\left(-m^2+s+t\right)\left(\left(s-M_Z^2\right)\left(t-M_Z^2\right)+M_Z^2\Gamma_Z^2\right)}{8\left(\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right)\left(\left(t-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right)} + \frac{(s+t)(c_a-c_v)^2\left(-m^2+s+t\right)+t\left(t-m^2\right)(c_a+c_v)^2}{16\left(\left(s-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right)} + \frac{(s+t)(c_a-c_v)^2\left(-m^2+s+t\right)+s\left(s-m^2\right)(c_a+c_v)^2}{16\left(\left(t-M_Z^2\right)^2+M_Z^2\Gamma_Z^2\right)} \right)}{22\,s^2}$$

```
(****************PAIR PRODUCTION ************************)
```

```
D1 = (1 - p3.p4 / M_Z ^2) ^2 G[l, {mu}] ** (1 + G[l, G5])
```

$$(\mathrm{G}[\ 1, \{\mathrm{mu}\}\ ] + \mathrm{G}[\ 1, \{\mathrm{mu}\}, \mathrm{G5}\ ]) \left(1 - \frac{\mathrm{p3.p4}}{M_Z^2}\right)^2$$

```
D2 = D1 ** G[l, p1]
```

$$(\mathrm{G}[\ 1, \{\mathrm{mu}\}, \mathrm{p1}\ ] - \mathrm{G}[\ 1, \{\mathrm{mu}\}, \mathrm{p1}, \mathrm{G5}\ ]) \left(1 - \frac{\mathrm{p3.p4}}{M_Z^2}\right)^2$$

```
D3 = G[l, {nu}] ** (1 + G[l, G5])
```

$$\mathrm{G}[\ 1, \{\mathrm{nu}\}\ ] + \mathrm{G}[\ 1, \{\mathrm{nu}\}, \mathrm{G5}\ ]$$

```
D4 = D3 ** (G[l, p3] + m_L)
```

$$\mathrm{G}[\ 1, \{\mathrm{nu}\}\ ]\, m_L + \mathrm{G}[\ 1, \{\mathrm{nu}\}, \mathrm{G5}\ ]\, m_L + \mathrm{G}[\ 1, \{\mathrm{nu}\}, \mathrm{p3}\ ] - \mathrm{G}[\ 1, \{\mathrm{nu}\}, \mathrm{p3}, \mathrm{G5}\ ]$$

```
D2D4 = D2 ** D4 /. l → ls // Simplify
```

$$\frac{8\left(\mathrm{p3.p4} - M_Z^2\right)^2 \left(i\, \mathrm{Eps[p1,p3,\{mu\},\{nu\}]} - \mathrm{g\_\{mu\ nu\}\ p1.p3} + \mathrm{p1\_\{mu\}\ p3\_\{nu\}} + \mathrm{p1\_\{nu\}\ p3\_\{mu\}}\right)}{M_Z^4}$$

```
D5 = G[l, {mu}] ** (1 + G[l, G5])
```

$$\mathrm{G}[\ 1, \{\mathrm{mu}\}\ ] + \mathrm{G}[\ 1, \{\mathrm{mu}\}, \mathrm{G5}\ ]$$

```
D6 = D5 ** (G[l, p4] - mL)
```

G[l, {mu}] $(-m_L)$ – G[l, {mu}, G5] m_L + G[l, {mu}, p4] – G[l, {mu}, p4, G5]

```
D7 = G[l, {nu}] ** (1 + G[l, G5])
```

G[l, {nu}] + G[l, {nu}, G5]

```
D8 = D7 ** G[l, p2]
```

G[l, {nu}, p2] – G[l, {nu}, p2, G5]

```
D6D8 = D6 ** D8 /. l → ls // Simplify
```

8 ($-i$ Eps[p2,p4,{mu},{nu}] – g_{mu nu} p2.p4 + p2_{mu} p4_{nu} + p2_{nu} p4_{mu})

```
D2D4D6D8 = D2D4 D6D8 // Simplify
```

$$\frac{256\,\mathrm{p1.p4}\ \mathrm{p2.p3}\left(\mathrm{p3.p4}-M_Z^2\right)^2}{M_Z^4}$$

```
J1 = D2D4D6D8 /. p1.p4 → (t + s - m^2) / 2
```

$$\frac{128\,\mathrm{p2.p3}\left(-m^2+s+t\right)\left(\mathrm{p3.p4}-M_Z^2\right)^2}{M_Z^4}$$

```
J2 = J1 /. p2.p3 → (t + s - m^2) / 2
```

$$\frac{64\left(-m^2+s+t\right)^2\left(\mathrm{p3.p4}-M_Z^2\right)^2}{M_Z^4}$$

```
J3 = J2 /. p3.p4 → (s - 2 m^2) / 2
```

$$\frac{64\left(-m^2+s+t\right)^2\left(\frac{1}{2}\left(s-2m^2\right)-M_Z^2\right)^2}{M_Z^4}$$

```
J4 = J3 gZ^4 b1LZ^4 / (4 ((t - MZ^2)^2 + ΓZ^2 MZ^2)) // Simplify
```

$$\frac{4\, b_{1LZ}^4\, g_Z^4\left(-m^2+s+t\right)^2\left(-2m^2-2M_Z^2+s\right)^2}{M_Z^4\left(M_Z^2\left(\Gamma_Z^2-2t\right)+M_Z^4+t^2\right)}$$

```
DFCS2 = J4 / (16 × 22 / 7 s^2) // Simplify
```

$$\frac{7\, b_{1LZ}^4\, g_Z^4\left(-m^2+s+t\right)^2\left(-2m^2-2M_Z^2+s\right)^2}{88\, s^2 M_Z^4\left(M_Z^2\left(\Gamma_Z^2-2t\right)+M_Z^4+t^2\right)}$$

```
DFCS3 = gZ^4 b1LZ^4 / (4 × 22 / 7 s^2 MZ^4 ((t - MZ^2)^2 + MZ^2 ΓZ^2))
        ((m^2 - t)^2 m^4 + 4 m^4 s MZ^2 + 4 (s + t - m^2)^2 MZ^4)
```

$$\frac{7\, b_{1LZ}^4\, g_Z^4\left(4m^4\, s\, M_Z^2+4M_Z^4\left(-m^2+s+t\right)^2+m^4\left(m^2-t\right)^2\right)}{88\, s^2 M_Z^4\left(\left(t-M_Z^2\right)^2+M_Z^2\,\Gamma_Z^2\right)}$$

```
(******************** DATAS FOR SINGLE PRODUCTION ********************)

gZ = 0.7564;

cV = -0.03;
```

```
ca = -0.5;

b1LZ = 0.1;

MZ = 91.1882;

ΓZ = 2.4952;

s = 500^2;

K1 = Table[NIntegrate[DCS2, {t, -(s - m^2), 0}], {m, 100, 400, 100}]
```

$\{3.69986 \times 10^{-9}, 3.20804 \times 10^{-9}, 2.39083 \times 10^{-9}, 1.26108 \times 10^{-9}\}$

```
gZ = 0.7564;

cv = -0.03;

ca = -0.5;

b1LZ = 0.1;

MZ = 91.1882;

ΓZ = 2.4952;

s = 1000^2;

K2 = Table[NIntegrate[DCS2, {t, -(s - m^2), 0}], {m, 100, 900, 200}]
```

$\{3.86783 \times 10^{-9}, 3.54895 \times 10^{-9}, 2.91138 \times 10^{-9}, 1.95647 \times 10^{-9}, 6.91995 \times 10^{-10}\}$

```
gZ = 0.7564;

cv = -0.03;

ca = -0.5;

b1LZ = 0.1;

MZ = 91.1882;

ΓZ = 2.4952;

s = 3000^2;

K3 = Table[NIntegrate[DCS2, {t, -(s - m^2), 0}], {m, 250, 2750, 500}]
```

$\{3.89769\times10^{-9}, 3.67895\times10^{-9}, 3.24149\times10^{-9}, 2.58534\times10^{-9}, 1.71068\times10^{-9}, 6.18357\times10^{-10}\}$

```
(**************** DATAS FOR PAIR PRODUCTION *********************)

gZ = 0.7564;

cv = -0.03;

ca = -0.5;

b1LZ = 0.1;

MZ = 91.1882;

ΓZ = 2.4952;

s = 500^2;
```

```
K4 = Table[NIntegrate[DFCS3, {t, -(Sqrt[s/4] + Sqrt[(s^2 + 4 m^4)/(4 s)])^2, -(Sqrt[s/4] - Sqrt[(s^2 + 4 m^4)/(4 s)])^2}], {m, 100, 250, 50}]
```

$\{1.02702\times10^{-9}, 1.2571\times10^{-9}, 2.19158\times10^{-9}, 4.82261\times10^{-9}\}$

```
gZ = 0.7564;

cV = -0.03;

ca = -0.5;

b1LZ = 0.1;

MZ = 91.1882;

ΓZ = 2.4952;

s = 1000^2;

K5 = Table[NIntegrate[DFCS3, {t, -(Sqrt[s/4] + Sqrt[(s^2 + 4 m^4)/(4 s)])^2, -(Sqrt[s/4] - Sqrt[(s^2 + 4 m^4)/(4 s)])^2}], {m, 100, 400, 100}]
```

$\{1.16716\times10^{-9}, 1.40266\times10^{-9}, 2.94879\times10^{-9}, 9.18235\times10^{-9}\}$

```
gZ = 0.7564;

cV = -0.03;

ca = -0.5;



b1LZ = 0.1;

MZ = 91.1882;

ΓZ = 2.4952;

s = 3000^2;

K6 = Table[NIntegrate[DFCS3, {t, -(Sqrt[s/4] + Sqrt[(s^2 + 4 m^4)/(4 s)])^2, -(Sqrt[s/4] - Sqrt[(s^2 + 4 m^4)/(4 s)])^2}], {m, 250, 1250, 250}]
```

$\{1.30483\times10^{-9}, 2.77748\times10^{-9}, 1.40858\times10^{-8}, 7.31549\times10^{-8}, 2.25652\times10^{-7}\}$