



REPUBLIC OF TURKEY
ALTINBAŞ UNIVERSITY
Institute of Graduate Studies
Civil Engineering

**STATISTICAL ANALYSIS AND MODELING FOR
THE STRUCTURAL PERFORMANCE OF
CONTINUOUS BEAM SYSTEM**

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Master's Thesis

Supervisor

Prof. Dr. Tuncer ÇELİK

Istanbul, 2022

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2022

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ABSTRACT

STATISTICAL ANALYSIS AND MODELING FOR THE STRUCTURAL PERFORMANCE OF CONTINUES BEAM SYSTEM

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Date: December/2022

Pages: 68

The continuous beam system is important form that appeared in various types of construction projects like ordinary and tall buildings. The analysis of the moments in the system spans cannot reached easily as the system have many unknown according to simple equilibrium analysis. The study aimed to build effective analysis for the continues beam system starting from the analysis of the moments at joints based on slope deflection method. The study the link the analytical pathway with structural analysis by using ultimate design method. The study made many linearization process on the form of drawing to extract the best fitting of each parameter with magnitudes of the main moments at joints. The study converted the equation into one equation for each moment at the external and internal joints based on the uniform loading at each span and the spa length that designed for the system. The study analyzed the effect of the structural parameters on the safety factor including the effective depth and the width of the members. The study built new equations that link the magnitude of the applicable safety of factor and the magnitude of the uniform loading. The two stages of the study was made to improve the design of the system as the designers can identify the magnitude of the moments and also the required geometry and strength of the members to be considered. The study made simulation of the cases using ANSYS program by building the

geometry of the system under various uniform loadings. The study showed the bending moments, the shear forces, and directional deformation for each case of loading. The study showed all items in related figures, tables and images to explain the outline toward the results.

Keywords: Continues Beam, Reinforcement, Structural Performance.



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ABBREVIATIONS

FEA	:	Finite Element Analysis
FRP	:	Fiber-Reinforced Polymer
RC	:	Reinforced Concrete
As	:	Area of Steel



LIST OF SYMBOLS

M : Moment on beam

f'_c : Compressive strength of concrete

I : Moment of Inertia

W : Weight of beam

d : Depth of beam

h : Height of beam

b : Width of beam

L : Length of beam

1. INTRODUCTION

1.1 OVERVIEW

The continuous beam is indeed a systemic structure that provides bending strength once subjected to a load as well as force. Such beams are commonly used in bridge design. This element has much more and over two points of support running the length of that too. The stretches between the human immunodeficiency virus (hiv are usually in a singular straight line within the same flat direction. A constant element seems to be much more rigid and durable than a beam, which seems to have aids at every end as well as a load distributed along any route it along length. A simple beam is a structure made up of beams which thus span always between supports. Once two or even more studs have been safely linked around each other atop different supports, the tower becomes constant[1]. The type of load and the strength properties of the element used to form the beam are the two key parameters to consider in the design of a continuous element. The reactions that occur at the assists of an easily supported beam can be determined by analyzing merely the forces applied to the beam. As a result, simple beams are recognized as statically determinate. A continuous element has more aids than are required to maintain equilibrium, and the deformation behavior under loading is also taken into account when selecting the support reactions. As a consequence, this element is referred to as statically indeterminate. Whenever a force is applied, this same future building of a beam would then affect its binding as well as deflection. Depending on the use, beams are built in the same way using a range of materials such as wood, aluminum, and concrete [2]. Constant steel beams have been constructed from various beams which are fastened together now and supported by many other beams to create a stable but flexible element for construction bodies. Connections, cross buildings, complicated roof, and many other building elements, for example, use constant beams. Continuous steel beams have both advantages and downsides[3].

1.2 ADVANTAGES

Here are some of the most prevalent reasons why continuous steel beams are used in construction [4]:

- a) A higher vertical maximum load.
- b) Lower this same deflection of mid-span

- c) Minimize the number of main deck as well as bearings required in bridge design.
- d) Require fewer tendon anchors.
- e) Create superfluous load paths.
- f) Enhance the framework while lightening it.
- g) They are much less costly than other substances, so they are financially viable.
- h) They have minimal vibration sensitivity, making them appropriate for load-reversal systems in earthquake-prone areas.

1.3 DISADVANTAGES

Unfortunately, continuous steel girders aren't always the best option. Here are some of the drawbacks of this building material:

Cause an increase in internal forces above columns.

Enhance the lateral stresses on supporting beams.

They complicate the procedure and necessitate the employment of expert professionals to ensure effective application [5].

1.4 BEAM DEFLECTION

Deflection is indeed the dislocation of an aspect or node out of its original positions caused by powers as well as force exerted to the element in having to build technical education terms. This is also referred to as dislocation and can be caused by external loads or the mass of the framework itself, and the gravity in that particular instance. Deflection can occur in beams, trusses, frames, and pretty much any other construction [6]. To describe deflection, take into account the deflection of the simple slatted beam with a person of mass standing at the end. In the long term, the pressure of this person standing for the finish will cause the component to bend as well as divert attention of its proper position[7] [8]:

In general, four key variables impact how much the beam deflects:

- a) Weight is above the structure
- b) Unsupported member's length
- c) Substance, specifically the Young's Modulus
- d) Cross sectional area, and more specifically the moment of inertia (I)

1.5 CONCRETE BEAMS IN COMPRESSION AND TENSION

A beam is indeed a longitudinal structural part endorsed both at ends that spans open space. After that, this same beam could indeed support some person due above it halfway between the finished supports. A beam is a type of ground joist. Tangible cabinetries also function as beams. When a weight is placed on top of an element, the weight causes the element to deflect (bend). Small weights placed above stiff beams cause essentially minimal deflection, whereas heavy weights placed on flexible beams cause tremendous deflection [7]. The deflection inside the beam leads two things to happen: the above level surface of the beam is compressed thus tries to shorten, while the bottom surface is in tension and wants to lengthen. Something significant happens between the two. Compression is the inverse of tension in that as one proceeds down the element from the top to the bottom, initial compression stress gradually reduces to zero and then the applied stresses reverse, go out of tension, and gradually increase towards the bottom of the beam. This same stress switch takes place in the middle of an unstrengthen girder with a geometrical cross-section, for instance a rectangle[8].

1.6 PROBLEM STATEMENT

The construction of continuous beams been made in large building in the way of increase the rigidity and strength of these type of buildings. The analysis of these type of member must been implemented in effective way for assign the real deflection ranges the predicted carefully as well as the stresses that formed along the long span inside the beams. The errors in such types of elements can lead to dangerous cases as the loading carried in sharing by the whole system.

1.7 CONTRIBUTION

Applying techniques of analysis for continuous beams can lead to more confidence in assigning the suitable design output including the cross section sizes and the maximum span lengths. The study move inside that approach for supporting the evaluation procedures of the continuous beam strength under various loading. The cost effective matter as well can be main goal when the study outline been achieved as better design of these non-small amounts of constructed members reflect positive impact on the overall cost of the project.

1.8 OBJECTIVE OF STUDY

- a) Assigning the properties of continuous beams cases including the specifying of the loading and geometrical properties
- b) Building analysis steps using the techniques of slope deflection method
- c) Linking the data with statistical analysis using SPSS program and explaining the results in figures and tables
- d) Building simulation views using the finite element applications under ANSYS workbench program
- e) Collecting the results toward applicable model that cover the output design properties under the loading conditions

2. LITERATURE REVIEW

2.1 OVERVIEW

Continuous beams are defined as beams that have more than one span. Continuous beams are extremely common in bridge and building structures. As a consequence, when trying to design, constant beams exposed to lateral loading and support settlers must be commonly analyzed. Whenever a light is constant over multiple supports as well as the point in time of air resistance action in various spans differs, the force analysis approach becomes really complex if lateral parts of interactions are handled as superfluous responses [9]. Nonetheless, the compel analysis approach for this specific state (constant beam) could be streamlined further by choosing the unnamed flexural capacity for the unidentified endorses. One interoperability equation is given in aspects of the lots and lots just on adjoining span as well as the maximum deflection for the left, core (the assistance in which the interoperability formula is authored), and strict continues to support at each transitional assistance of a beam[10]. Two consecutive bandwidths of the beam are regarded at the very same time. The interoperability equation is recognized as the calculation of 3 instances as it is written through three-moment gestures. Each time frame of a simple evaluated girder with applied stress and also more for one end endorse point in time is thus kept separate[11]. One interoperability equation can be written in aspects of which one of 3 separate support instants. As a result, there are many equations with unknowns. In the future, each equation will only have three unknowns [2].

2.2 DESIGN DATA

A beam is indeed a lateral component used to absorb gravity load. While it may be intended as an important contributor of in state shot to counter lateral load, the deformation, M , as well as base shear are the same pervasive design influences for a beam[12]. The fundamental pitches for designing which R C beams are [13] [37]:

- a) Establish the design loads
- b) Calculate the derived forces
- c) Determine the reinforcement.

d) Check deflection [38]

e) Specification

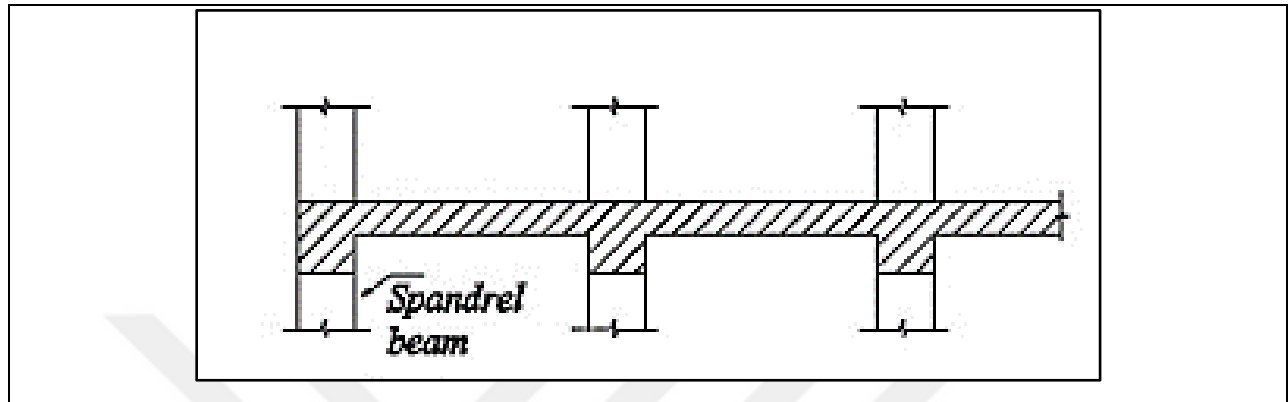


Figure 2.1: General Appearance Of Continuous Beam Systems [14]

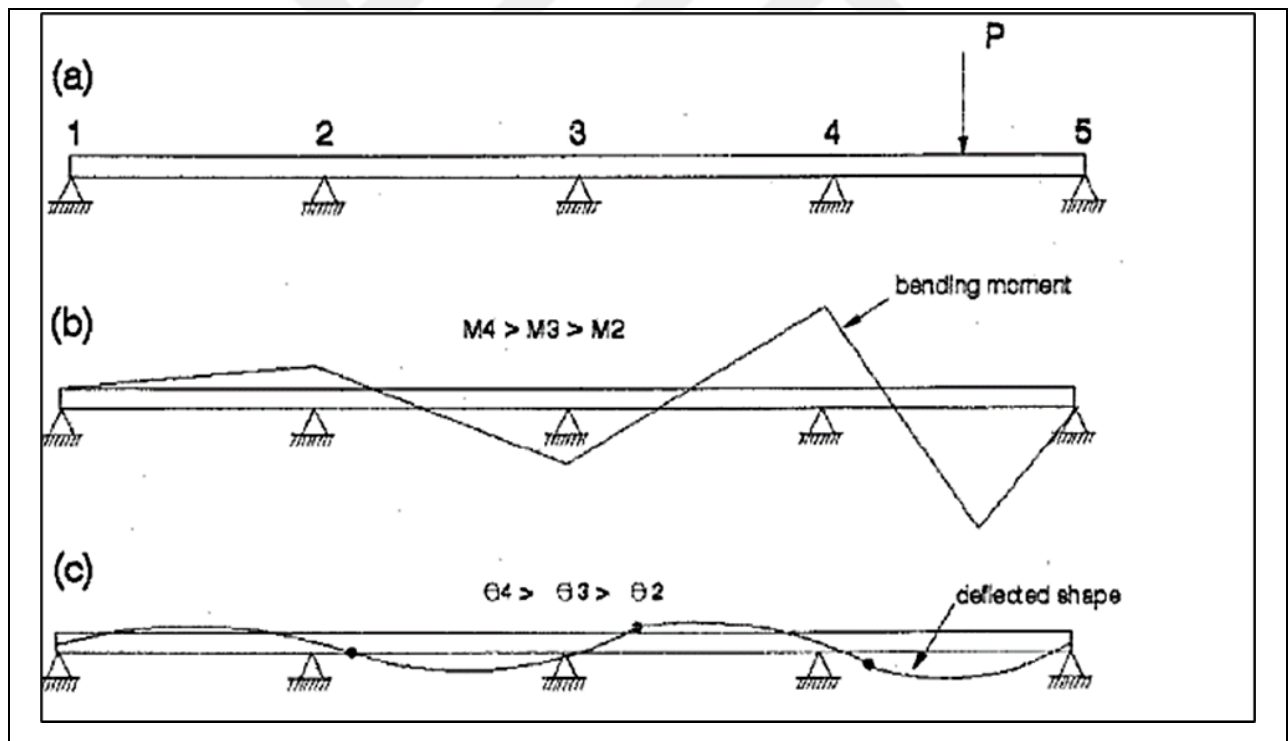


Figure 2.2: Explanation For A Case Of Continuous Beams System: (a) The Supporting Behavior (b) the Diagram Of Bending Moment And (c) the deflection shape[35]

In any of the preceding steps. Some information may have to be assumed at the outset and then verified once the outcome is available. Throughout the process, iterations are always required. For example, in order to calculate the required steel area, the impressive depth of the section must be

assumed[39] [40] .It is critical to identify the data needed or design considerations for inputting into appropriate formulae to arrive at the desired and then confirm for conformance with the sourced code[15] [41].

2.3 CONCEPTS OF FIXED CONNECTION BEAMS

The developer presumed that espouses exert no limitation on beam wanting to join in all the studies of the sustained load and deflections of steel girders to multiple people who support. The axis of a factor is free to play catch - up whatever tendency to the path of supports [10]. So because person did not comprehend how to cope with deflections of the beam's axial direction, he wasn't able to discover the though between above a beam during the period of the support systems restricting the orientation of the axial direction [11].Whenever the beam's edges are repaired in path, the laser's axis should keep its original orientation at the points that provide support. Take into account a straight beam having to carry structural load and backed by two empowers A with B. When there is no limitation on the laser's axis [12]. Again, inside the case of dynamic loading passing over viaducts, the prevalent variations in elastic deformation and sensations will soon make the college education of repairing at the ends extremely unclear [13]. There seem to be numerous drawbacks to crafted beams: initially, a small seduction from the one of the supports inside the long term will tend to help fix large strain, and, during erection, the endorses must be allied with best accuracy; changes in warm air will also often seem to fix up massive stresses [14]. There seem to be points in which the ductility is zero so because bending memories at the end points of a constructed component are of negative polarity to those from the center of the beam. Since there is no flexing action of instant to resist, a detachable joint inside the girder at those locations could be decided to make, with both the seam axis comparing to the flexural axis[16]. If it is done at each point of inflexion, the beam would then appear as a central girder supported by two end cantilevers; the bending moment curve as well as deflection curve will be exactly the same if the beam were solid and built in. With this design, the element can adjust to changes in heat or the subsistence of the endorses [17] [42].

2.4 STRENGTHEN STRATEGY OF CONTINUOUS BEAMS

A continuous beam is defined as a close to sufficient beam that runs over three or more supports. The properties of continuous beam system must be taken for each element to satisfy better design under the applied load case [18] [43]. Modern technology developments cause frequent distinctions inside the elements of building as well as structure space-planning choices. In most instances, this necessitates not only a shift in spatial solutions, as well as a increase in packing on main load-carrying structures. In overall, all methods of reinforcing bolstered component structures are based on increasing the pass area. [19]. Thus according architecture and design as well as room designs, even though changes inside the design, in most instances, lead to significant deterioration which of room solutions in each of building structures, given the presence of supplementary people who are part - divided up frames, rate of stream images, or component lower legs, and also bracing beams. Constructing reassurance of sequence of instructions bodies of concrete is typically performed in such a way that the entire beam chair zone and girder span are reinforced[20] [44]. As load increases, so do the action of moment of an assist and the element span moment. Regardless, an increase in the cross section of the beam above the beam seat generally causes difficulties, which can sometimes become insurmountable [21]. It is related to the fact that connecting two adjacent parts of a continuous beam above a beam seat is extremely difficult (or on a column, on the main beam in which of a one-way ribbed floor, and so on.). In the case of monolith flooring with main and secondary beams, it becomes practically impossible to strengthen the secondary beam where it meets the main beam [22]. In the case of strengthening above the column, literature more generally recommends fastening to columns. On a higher level, when reinforcing the element seat zone, it becomes necessary to knock down the floor systems. The totality of these objects not only complicate the execution of work, but also raise the cost of reconstruction[45] [46].. It might be interesting to consider the possibility of reinforcing continuous reinforced-concrete beams in such a way that the reinforcement can be given a wide berth in which of the beam seat zone, and only focus on reinforcing the element span[47].. However, this raises the issue of the calculation procedure for such a beam, in which all of the support and beam span moment increase, but only the element span is strengthened [23].

2.5 CURRENT WORK

The study used the slope deflection method to investigate the influence of different parameters on the final strength of the beams system under various loading (concentrated and uniform). The influence of parameters was examined statistically for evaluating the degree of effectiveness for each one on the design outline. The parameters including the geometrical shape of the beams in the system and the properties of concrete and steel. The simulation was part of the study to support the result with valuable 3-d and 2-d figures. The study modelling the data to improve ease the design steps for each case of loading.

2.6 LITERATURE SURVEY

The study presented a comparison of statically indeterminate beams using the slope and deflection method. The first model ignores shear deformations (taking into account only bending deformation), which is the common style analysis for continuous beams, and the second version assumes permanent shear deformation (taking into account flexural and shear deformations). An analysis of three different concrete sections is performed for both models[49]. The first section is a rectangular section, the second is an "I" section with a larger web thickness, and the third is an "I" section with a thinner web thickness. Every section is checked for three different lights at distances of 10.00m, 5.00m and 3.00m. This paper describes the differences in approach between the two trouble modeling under consideration. The obtained results show that there is a larger variant in section "I" with a thinner mesh thickness and in light of 3.00 m. As a consequence, the standard practice of not taking into account shear deformations is not a recommended solution in the particular case of short beams and thin web-thick 'I' sections. Then he suggests taking into account the distortions of the story and attaching more to reality [24]. The study showed that self-compacting concrete is a type of concrete that is placed in molds under its own weight. Although there are many studies demonstrating the behavior of SCC packets, the majority of them are concerned with the performance of estimated simple packets. In contrast to these, this is a study of continuous beams made of SCC with the aim of analyzing the bending performance as well as confirming the feasibility of using nonlinear finite element analysis (FEA) to derive such in construction case elements[50]. An experimental study was performed on three continuous girders with two spans with a total length of 3400 mm, a span between 1600 mm, and a cross-section of

150/200 mm constructed from an SCC subjected to short-term loading. The variable parameter is the percentage of rebar in the amounts of 0.65, 0.86 and 0.94 in the selected case. Since all analyzed beams showed sufficient load bearing capacity and stiffness, the study confirmed the feasibility of using SCC in continuous beams in civil design practice. A nonlinear numerical model has been proposed using the Abaqus/Standard software, which has been validated and confirmed against the experiment performed, as there is only 5% discrimination in the numerically calculated final load especially when compared to the values of the experimental measurements [2].

The study provided numerical parameters of the performance of continuous prestressed concrete beams with coherent fiber-reinforced polymer (FRP) and steel tendons. A finite element model was created and validated against the available test data. In concrete girders of the previous stage, the numerical condition test is carried out on two connected stages pressed into concrete girders. For a comparative study, three types of tendons are taken into consideration: aramid FRP (AFRP), carbon FRP (CFRP), and prestressed steel. There are several levels of x_p (indicator of prestressed tendon reinforcement) used. The results show that with the increase of x_p , the stressful FRP failure mode of the concrete girders in the previous stage can shift from the tendon rupture to the concrete cracking body, while the fracturing failure always occurs in the stressed steel of the concrete girders of the previous stage. Furthermore, when compared to steel tendons, FRP tendons move with significantly different behavior in terms of cracking pattern, deformation characteristics, and actual axis depth. The ultimate instantaneous redistribution in the structure of AFRP prestressed concrete beams is comparable to that in the prestressed steel body of concrete beams, while the CFRP strings clearly register less redistribution than the steel stringers at low x_p [25]. The study included many tests on steel roof links, four exams on fiberglass linkage, as well as filled samples. The two naked steel joint tests revealed combined actions during slab erect penis in addition to concrete. A arithmetical model was created to calculate the roster qualities of a combined answer moment based on the physical mutual properties and empirical setup. In the prototype formula, the basic mechanism of load distribution inside the modules of capacity of national was used. Designers are recommended simple and powerful methods based on the model but also validated by experiment findings. Four fiberglass link samples were then evaluated to assess the influence of floor characteristics on heat transfer actions in terms of the type of reassurance utilized in the concrete surface, the strain ratio, and the strength of concrete. The original study goal was to learn more about of the geometrical features of mid composite joints as well as apply this information

to the construction of structures frequently used in Europe. A quasi concept is used in a highly innovations in-case layout that helps connect a slender based on these factors to a steel frame reinforced concrete columns filled with a tangible body. The subset was put in place and the building was tested in a thorough experiment conducted[26].

the study analyzed concepts for the linear evaluation of a fixed composite concrete girder with incomplete contact. Threaded shear connectors join the bodies of steel and concrete parts. The equilibrium and compatibility equations for forces and displacement at the assumed element are derived. As a result, the designer have one nonlinear synchronous mathematical problem in sliding expressions. The accepted role of the concrete layer in negative moment movement is limited to being a medium surrounding the steel bars and not related to the strength of the composite beam. To implement the proposed theoretical model, a computer program is written in Visual Basic. A conceptual approach has been introduced to analyze continuous complex beams with interlayer slip, where the functional equilibrium and conformational equations are reduced to a single second-order differential state in interlayer slip expressions rather than axial forces. The proposed approach provides reasonable prediction and can be used for any kind of loading and boundary conditions. By expressing the derivatives in the form of a finite difference, the solutions of the basic equations can be obtained. The numerical solution obtained through this path closely corresponds to the original analytical solution, which assumes the behavior of a linear element and a shear conductor and that the assumption of ignoring the strength of concrete in the region of negative moment is true [27]. The study proves that the elastic reinforcement of composite beams by external bonding of composite materials is a reliable and practical technique. The study evaluated the bending action of continuous reinforced concrete beams accompanied by three spans that were fixed and reinforced using CFRP or GFRP. The trial program includes multiple groups. Group 1 consists of nine beams, while Group 2 consists of seven items, each group containing a reference beam. All beams have the same length of 9 meters and a section of 15 x 25 (cm). Tensile stress experiments were performed using a 1000 kN piston. The effect of reinforcement with CFRP and GFRP, optimum thickness of reinforcement, repair efficiency by CFRP and GFRP over damaged elements, mechanical behavior, and failure mechanisms of reinforced concrete girders were investigated. In the numerical case simulation, empirical tests and calculations were performed. Packages fail in five different ways. The peak strength of the element can be increased by 14.8 to 33.0 percent for beams reinforced with CFRP bonding and by 7.2 to 11.8 percent for

beams reinforced by GFRP bonding. There is optimum reinforcing thickness and optimum thickness ratio between the three girder spans. The damaged component can be completely repaired completely. The maximum ductility of the damaged and repaired item may be greater than the ductility of the package. In addition to strengthening the continuous concrete beams[28].

The goal of the research was to look into the nonlinear deformation behaviour of constant RC element specimens reinforced to stem - cell cementitious matrix resources in both experiments conducted and numerical modeling. Eight two-span RC element samples were constructed and tested. The sort of FRCM used for tests , as well as the amount of FRCM layers (three and four layers) all were recorded. Bolstering the sag considerably enhanced by 17–29%, while bolstering the screw enhanced load carrying capacity by 9–17%. The application of resulted in a larger strength development than when using Doped composites , in contrast hand, is more ductile and deformable than C-FRCM-enhanced equivalents. The rank trebling of layers resulted in a decrease in girder ductility rather than an increase in loading capacity. Immediate redistributing wealth ratios were found in samples bolstered in sag regions. Stagnation as well as starting to sag areas ranged from age of 14 to 26 percent. There was really no significant allocation moment in this case. Three-dimensional (3D) digital government simulation models have been developed, with and without interstitial-coupon law at the texture-matrix interface. Incorporating the bond coupon law into the model had no effect on the expected response. Despite the fact that the reproductions tended to undervalue the skew value, the expected load dimensions were within the 12 percent error range. The arithmetical results are consistent with those obtained from the pilot test [29]. The study showed that the static beams can redistribute the internal forces because they are not specified in the building case features, which leads to the improvement of the beam deformation, the reduction of reinforcement crowding, and the use of the cross-section capacitance more effectively. As a result, continuous reinforced concrete (RC) girders are popular structural members. However, RC may degrade on it in corrosive environments due to corrosion of the armature. In this study, degraded beams are repaired using a recently summarized dual-functional intervention method, forced current cathodic protection and structural strengthening (ICCP-SS). In the intervention method, carbon fabric reinforced cement matrix (C-FRCM) serves two functions. It is necessary to investigate the effects of the erosion of the reinforcement, cathodic protection, and C-FRCM strengthening system on the behavior of the continuous beams. The objectives of this study often provide experimental data for continuous RC beams rehabilitated in

atmospheric erosion using ICCP-SS techniques, as well as to investigate structural responses, interfering momentary redistribution, and derivation of bases for those beams. This document contains a pilot program, a discussion of the results, and a design proposal. Electrochemical monitoring results showed the success of steel reinforcements in continuous beams in corrosive environments [30]. The study divided four objectives which are listed below. The first objective was to study the effect of the natural compressive strength of concrete on the structural behavior of continuous beams. The second objective is to study the effect of the concrete type on the condition behavior during construction of continuous beams. The third objective was to study the effect of steel fiber ratios ranging from 0 to 2.5 percent on the structural performance of incessant RPC beams. That fourth objective was to examine the effectiveness of longitudinal reinforcing bars on individual behavior during construction of continuous RPC beams using (CFRP and GFRP). As a result, nine continuous two-period beams were tested within a single-point loading for each period. The seven all beams have the same total length of 2700mm, pictorial span distance of 1250mm per span, and a suitable width of approximately 150mm. The test results showed that the continuous element made of RPC has a higher final load than the continuous NC beam and that the final load increased with the increase in the proportion of steel fibers. It was also found that the final load of the continuous beam increased when CFRP tape with longitudinal stiffening ratio was used. Furthermore, the final continuous beam loading was determined when GFRP tape was used as the longitudinal strengthening ratio [31]. The study showed that continuous beams are frequently used in all structures such as car parks and bridges that may be exposed to harsh climatic conditions and the application of de-icing salt. As a result, the use of non-corrosive fiber-reinforced polymer (FRP) rods in these types of structures is advantageous. However, the ability of these materials to redistribute forces and moments in continuous beams is questionable to cause the linear-elastic behavior of FRP materials until failure. In contrast to the simply evaluated structures, very few experimental studies have been conducted to verify the behavior of continuous concrete beams reinforced with FRP bars. Due to a lack of research, recently published design guidelines for FRP-reinforced structures provided only limited provisions for continuous beams. The objectives of this study are to verify the performance of continuous concrete beams for a structure reinforced with carbon and glass FRP bars and to provide design guidance for the prediction of failure load and location.. The paper describes the experimental results of two reinforced concrete beams with rectangular cross-sections of 200,300 mm continuous over two periods of 2,800 mm

each. . One of the elements was reinforced with longitudinal bars of carbon fibre-reinforced plastic, the other was not reinforced with GFRP bars. Both girders are reinforced with steel saddles and provided with alternative stiffening fittings for critical sections. The beams are tested within concentrated monotonous forces applied at the midpoint of any span. FRP reinforced concrete beams were found to be able to redistribute the moment of contact on the transitional support. Furthermore, while the Canadian CSA-S806-02 system could reasonably predict the failure load of tested packages, it failed to predict the failure site [32].the study examined the case of continuous reinforced concrete deep beams with fixed and hinged support conditions which are frequently found in structures, little research has been conducted on their performance. This can be attributed in part to the demanding nature of ensuring fixed assists in experimentally testing such members. Nonetheless, conducting numerical analysis in conjunction with experimental research has been a common method for producing a reliable in numerical state model as an alternative to destructive tests. This file is intended to investigate numerically the impact of different support conditions on the performance of two-span continuous reinforced concrete deep beams. A numerical model of three experimentally tested beams was created, along with two exterior rollers and interior hinge support conditions. With the help of tuning fewer parameters in the numerical model, such as element type, mesh size, and element constitutive relations, a good comparison with an acceptable variation between in numerical state and experimental results was achieved.. The refined numerical modeling was used as an alternative to destructive tests to conduct a parametric study to further investigate the influence of different sets of support states in the act of two-span continuous reinforced concrete deep beams. This analysis emphasizes the critical serious consequences of the support states in the act of such deep beams [29]. The study worked with the concept of durability, performing probability nature analysis of deflection in construction courses of continuous component and establishing it and according to building controlling requirements, and analyzing deflection reliability in different building conditions[1]. In the construction process, it is more likely to cause psychological panic among construction workers where the actual deflection is greater than the theoretical deflection. Complied with the construction in which the stage of change, deflection, and force of the continuous element is changing. Diversion observation rather than force were observed being more intuitive in construction procedures, but through the linear controlling load is also an important concept which of monitoring. As a result, during the construction troubles stage, structure troubles and trouble

volume can be measured using deflection. According to the findings of the study Based on the construction stress procedures, particularly the construction of continuous beam linear manage, which plays an adopted role in determining the characteristics, the deflection above reliability safety risk identification method is presented into the deflection reliability index. Provide a calculating formula and an example to demonstrate the method's applicability [33].The study investigated the technological plastic the method is used, which is based on current standards for steel structures. This theory and standard calculate procedures assumed the plastic redistribution of internal forces through the emergence and development of plastic hinges and the failure of the construction's plastic mechanism. The plastic redistribution of internal forces allows the structure's forces of action to increase. The plastic redistribution and increase in force action cause also differs in the assist reactions that are generally required to consider as structural support actions. This same distribution of wealth of internal forces and the changes in assist responses are affected by the variable elastic-plastic properties of constructive steels. As a result, plastic redistribution of internal forces may not be absolute or may not occur at all. The effect of elastic-plastic redistribution in which internal forces above the load-carrying ability and the differences in support reactions for selected continuous beams are investigated in this paper [34].

3. METHODOLOGY

3.1 ANALYTICAL CONCEPTS

Continuous beam let us first understand what is meant by the term beam and bending moments in structural engineering. A beam is a horizontal structural element that is capable of withstanding load primarily by resisting bending, bending moments are the bending forces induced to the material as a result of external load/s and due to its own weight over the span.

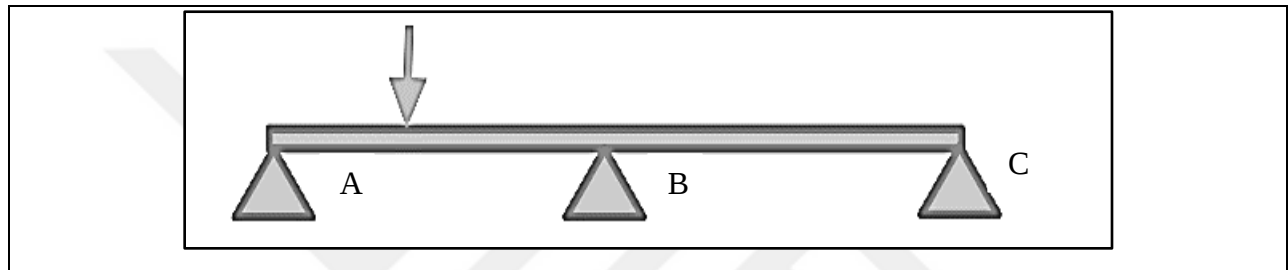


Figure 3.1 : Simple Sketch Of Continues Beam System

For two spans with uniform loading, the general equilibrium system was show in figure

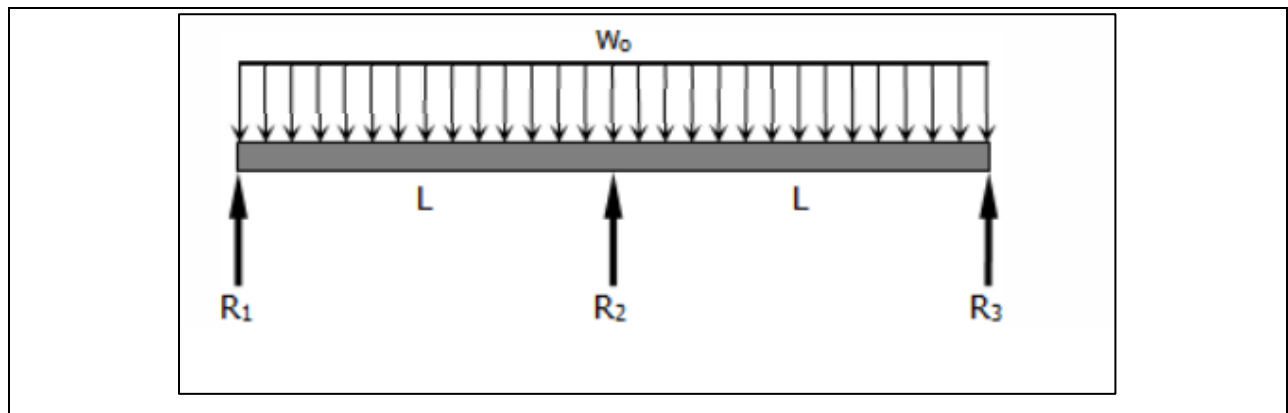


Figure 3.2: Continues Beams System under Uniform Loading

For continuous beams, the load acting at a particular span is not necessarily the same as that acting at other spans. Therefore, the bending moments and shear forces at the supports and near the middle of the span will always vary according to the load they actually carry. This aspect should be taken into account when designing, by assigning the most severe loading case at any given section to give the maximum bending moment and shear force possible.

3.2 APPLYING THE TECHNIQUE OF SLOPE DEFLECTION METHOD

Slope-deflection method is the second of the two classical methods presented in this course. This method considers the deflection as the primary unknowns, while the redundant forces were used in the force method. The basic idea of the slope deflection method is to write the equilibrium equations for each node in terms of the deflections and rotations. Solve for the generalized displacements. Using moment displacement relations, moments are then known. The structure is thus reduced to a determinate structure.

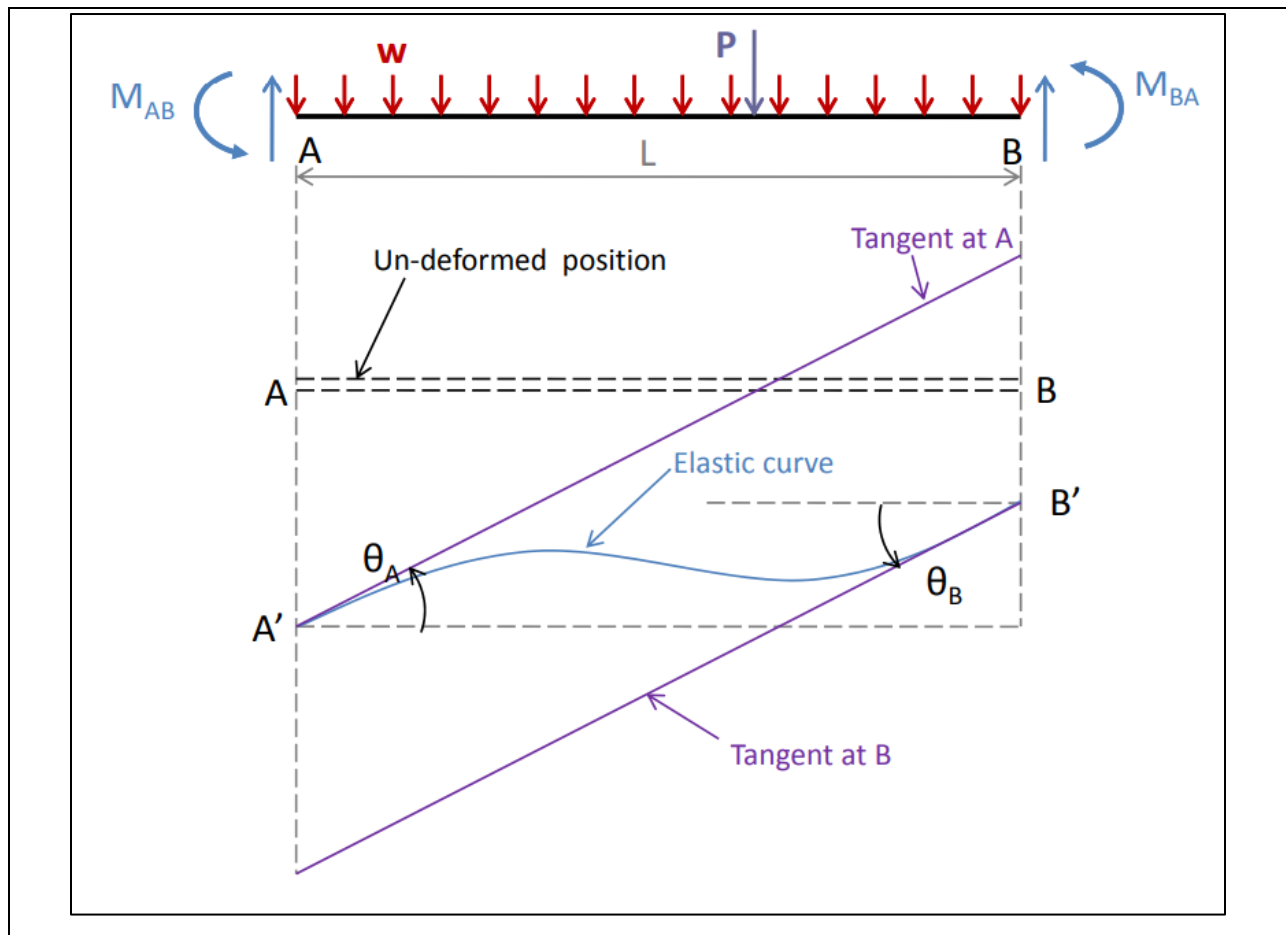


Figure 3.3: The General Beam Deformation Sketch Of Slope Deflection Method

The main equations of the method are based on fixed end moments for each case of loading:

$$M_{AB} = M_{AB}(fixed) + \frac{2EI}{L}(2\theta_A + \theta_B - \frac{3\Delta}{L}) \quad (3.1)$$

$$M_{BA} = M_{AB}(fixed) + \frac{2EI}{L}(2\theta_B + \theta_A - \frac{3\Delta}{L}) \quad (3.2)$$

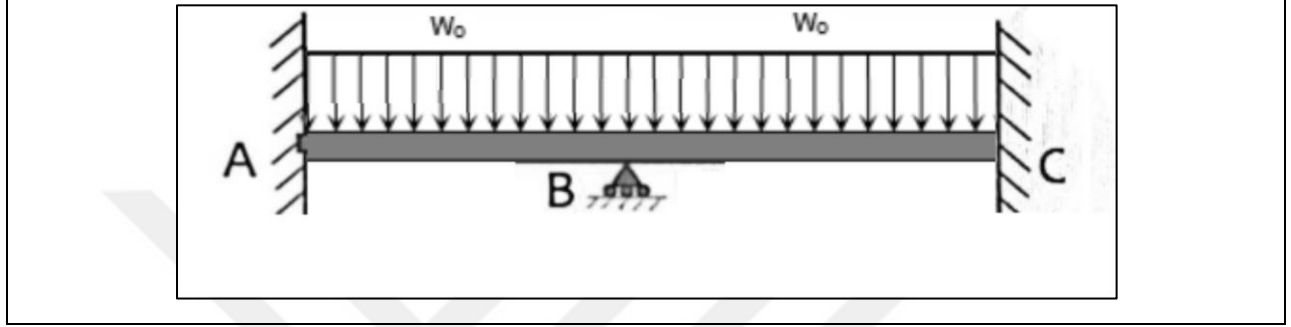


Figure 3.4: Two Beams with Fixed End Joints

For the case of two spans as explained in the figure, the slope of deformation at fixed ends is zero as well as the displacement. The set of equation then can be written and simplified as following:

$$\theta_A = \theta_C = 0 \quad (3.3)$$

$$M_{AB} = M_{AB}(fixed) + \frac{2EI}{L_1}(\theta_B) \quad (3.4)$$

$$M_{BA} = M_{BA}(fixed) + \frac{2EI}{L_1}(2\theta_B) \quad (3.4)$$

$$M_{BC} = M_{BC}(fixed) + \frac{2EI}{L_2}(2\theta_B) \quad (3.5)$$

$$M_{CB} = M_{CB}(fixed) + \frac{2EI}{L_2}(\theta_B) \quad (3.6)$$

By equilibrium state:

$$\sum M_{BA} + M_{BC} = 0 \quad (3.7)$$

$$M_{BA}(fixed) + \frac{2EI}{L_1}(2\theta_B) + M_{BC}(fixed) + \frac{2EI}{L_2}(2\theta_B) = 0 \quad (3.8)$$

$$\text{If } \beta = M_{BA}(fixed) + M_{BC}(fixed) \quad (3.9)$$

$$-\beta = \frac{2EI}{L_2}(2\theta B) + \frac{2EI}{L_1}(2\theta B) \quad (3.10)$$

$$-\beta = 4EI \theta B \left(\frac{1}{L_2} + \frac{1}{L_1} \right) \quad (3.11)$$

$$(\theta B) = -\left(\frac{1}{L_2} + \frac{1}{L_1} \right) \frac{\beta}{4EI} \quad (3.12)$$

$$\beta = \frac{L_1^2 W_1}{12} - \frac{L_2^2 W_2}{12} \quad (3.13)$$

$$\beta = \frac{L_1^2 W_1 - L_2^2 W_2}{12} \quad (3.14)$$

$$(\theta B) = -\frac{L_1 L_2}{L_1 + L_2} \frac{L_1^2 W_1 - L_2^2 W_2}{48EI} \quad (3.15)$$

$$MBA = MAB(fixed) + \frac{2EI}{L_1} 2 \left(-\frac{L_1 L_2}{L_1 + L_2} \frac{L_1^2 W_1 - L_2^2 W_2}{48EI} \right) \quad (3.16)$$

$$MBA = MAB(fixed) - \left(\frac{L_2}{L_1 + L_2} \frac{L_1^2 W_1 - L_2^2 W_2}{12} \right) \quad (3.17)$$

The other moment can be obtained from applying the equilibrium equation:

$$MAB = MAB(fixed) + \frac{2EI}{L_1} \left(-\frac{L_1 L_2}{L_1 + L_2} \frac{L_1^2 W_1 - L_2^2 W_2}{48EI} \right) \quad (3.18)$$

$$MAB = MAB(fixed) + \left(-\frac{L_2}{L_1 + L_2} \frac{L_1^2 W_1 - L_2^2 W_2}{24} \right) \quad (3.19)$$

$$MCB = MCB(fixed) + \left(-\frac{L_2}{L_1 + L_2} \frac{L_1^2 W_1 - L_2^2 W_2}{24} \right) \quad (3.20)$$

For more than three spans, the moments for the internal spans are:

If the spans indicated by n(i) from 1 to n, where n is the number of the spans.

$$M(i)2 = Fixed M(i)2 - \left(\frac{L(i+1)}{L(i) + L(i+1)} \frac{(L(i))^2 W(i) - (L(i+1))^2 W(i+1)}{k} \right) \quad (3.21)$$

K = 12 for internal spans and 24 for external spans:

3.3 STRUCTURAL ANALYSIS CONCEPTS

The ultimate strength design method indicated that the strength moment can be obtained by the following equation:

$$Mu = AS fy \left(d - \frac{a}{2}\right) \quad (3.22)$$

$$a = \frac{As fy}{0.85 b Fc} \quad (3.23)$$

The magnitude of the moments from the previous analysis can be joined with ultimate strength equation to achieve the better design state of the continuous beam members. The defined reinforcement and the dimensions for each beam in the system is based on positions of the span as well as the applied loading.

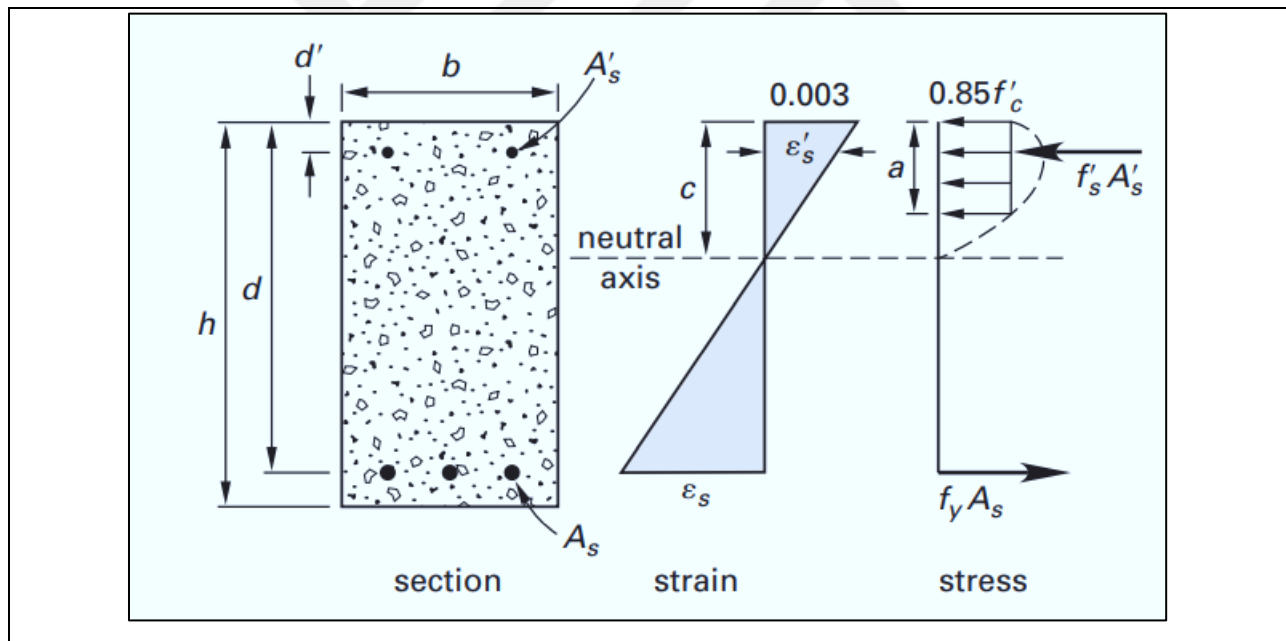


Figure 3.5: The Theoretical Analysis Sketch Of Ultimate Strength Method

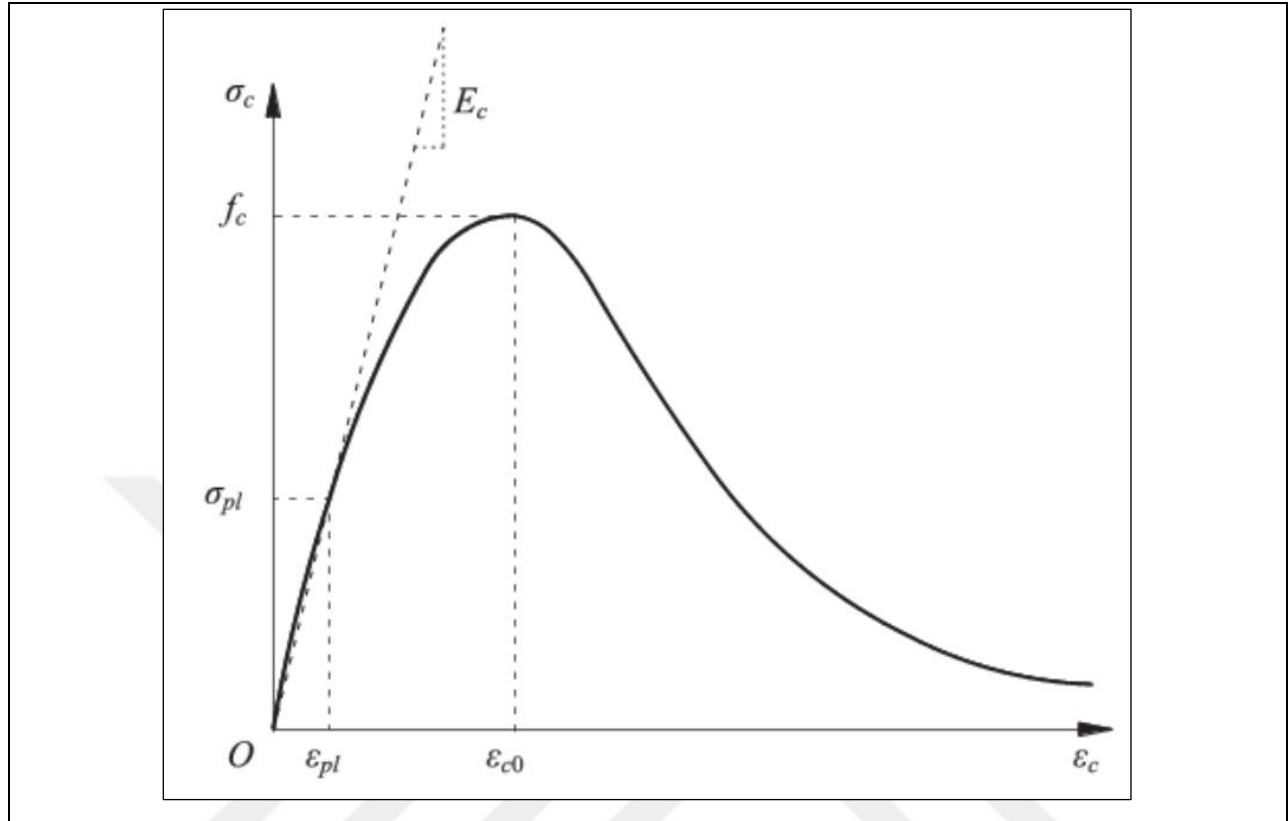


Figure 3.6: The Stress-Strain Shape for Concrete Beams Under Loading

3.4 ANSYS WORK

The ANSYS program was used to support pathway of the study by analyzing in visual images the continues beam systems under various uniform loading. Under the program the first step was the building the geometrical shape of two spans beams with the specific length and cross section. The length of each span was 4m and the cross section dimension was 300 * 300 mm. the following images showed the stages of building the model under the program to achieve the right ways of the results.

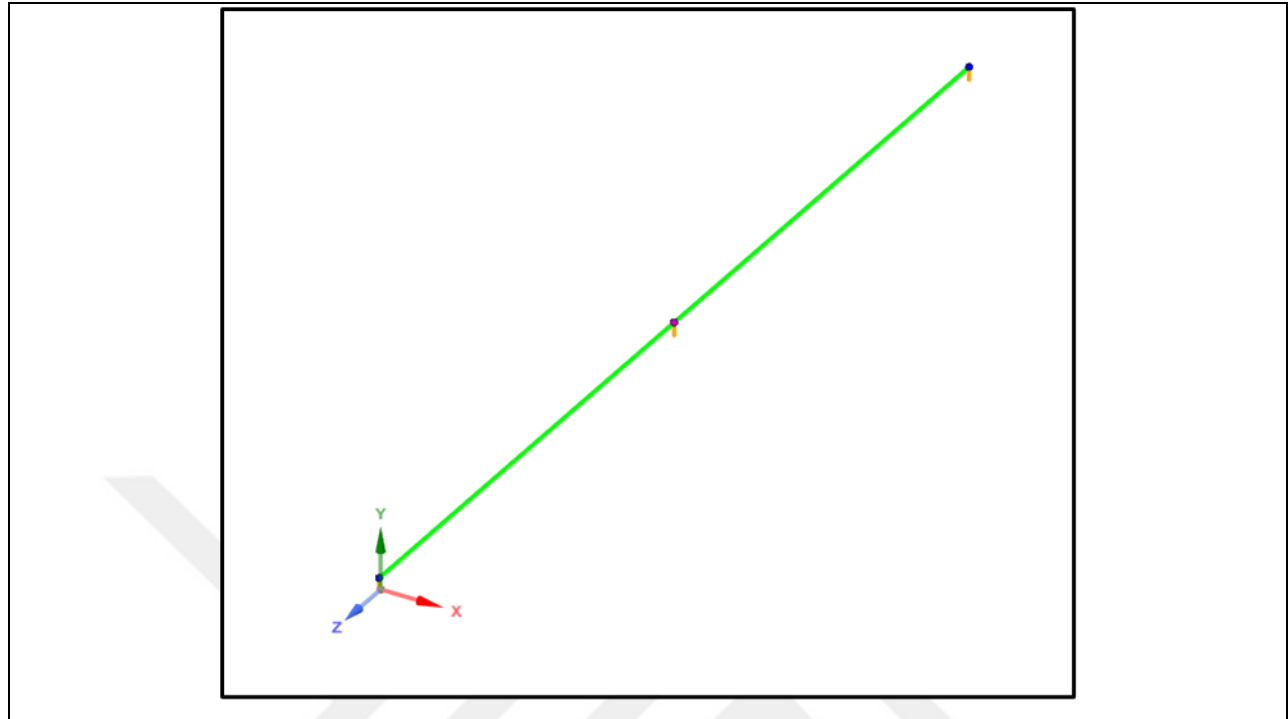


Figure 3.7: The Sketching Of the Beams Route In The Program Geometry Panel

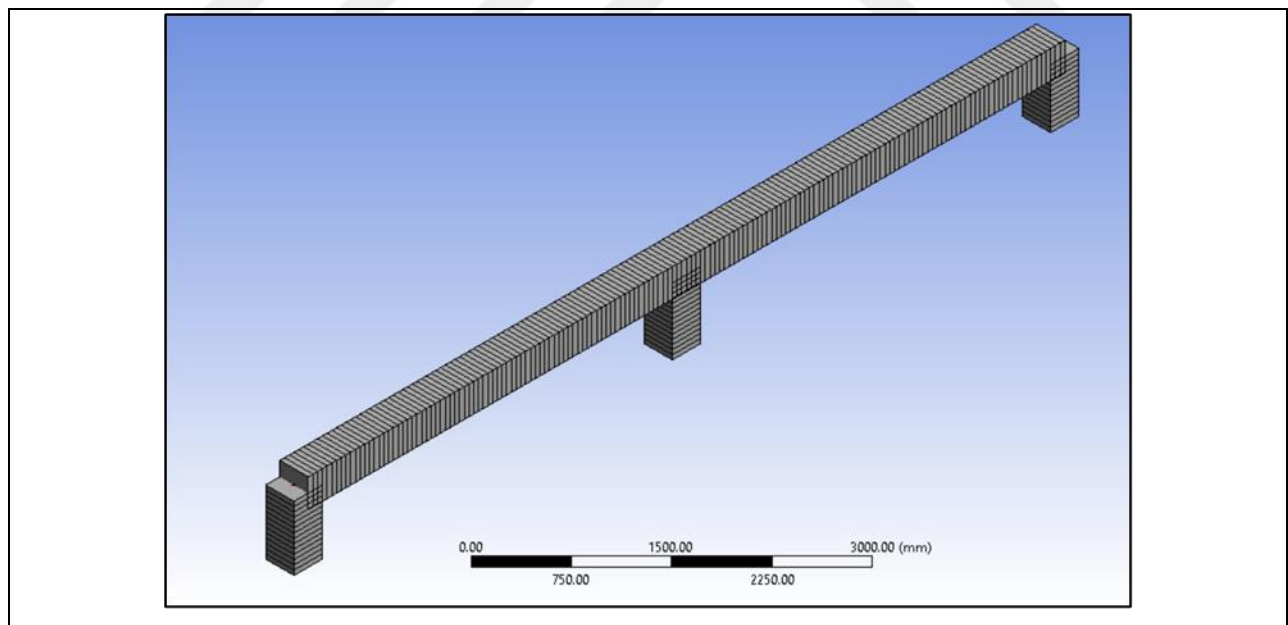


Figure 3.8: The Meshing Appearance Of The Beams In The Program Modeling Panel

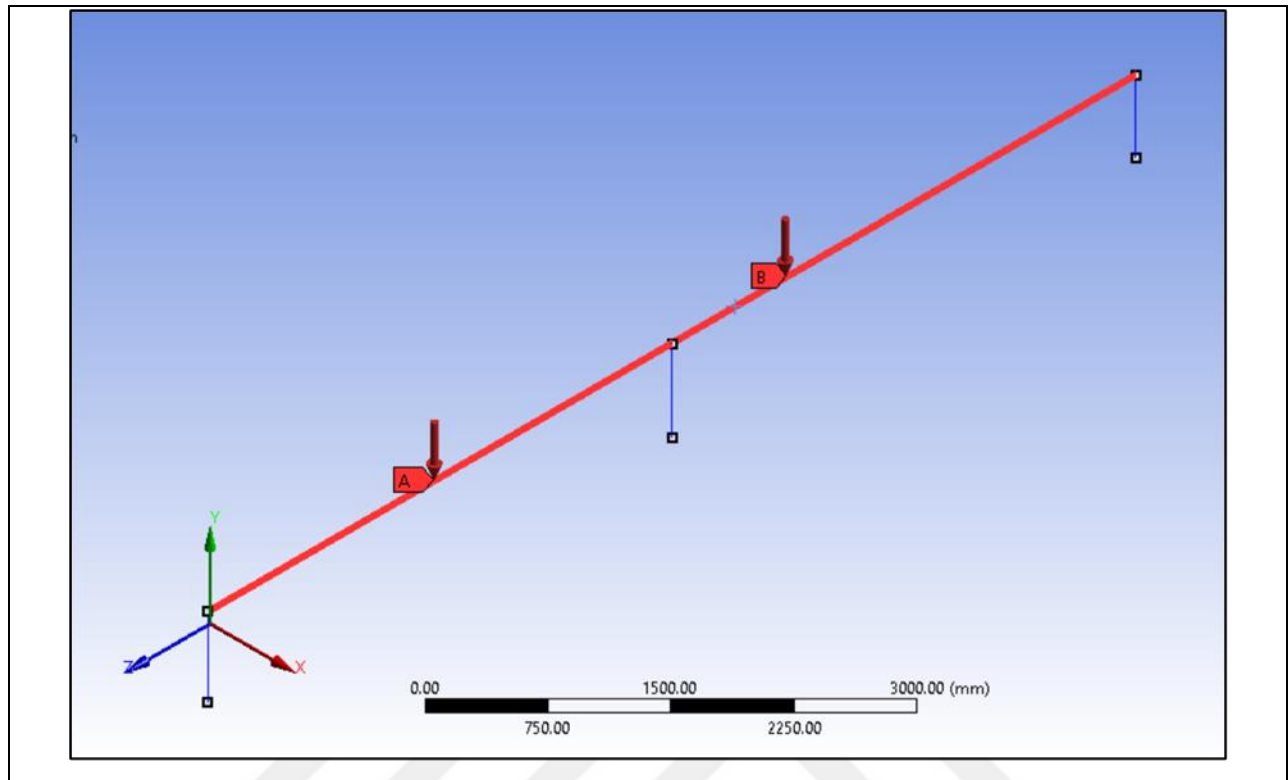


Figure 3.9: The Assigning Of the Uniform Loadings on the Beams Route in the Program Modeling Panel

4. RESULTS

4.1 CALCULATION AND ANALYSIS

The continuous beam system of tow spans were analyzed according to the contribution of each element on the overall strength. The output design of the system can be figured out easily according to improve the decision for the designer to assign the better conditions for that system. The impact of each parameter was investigated and tabled as following:

Table 4.1: The Impact of W1 on the Moments At Joints

L1	L2	W1	W2	MAB	MBA	MCB
4	4	20	20	-26.67	26.67	26.67
4	4	25	20	-35.00	30.00	25.00
4	4	30	20	-43.33	33.33	23.33
4	4	35	20	-51.67	36.67	21.67
4	4	40	20	-60.00	40.00	20.00
4	4	45	20	-68.33	43.33	18.33
4	4	50	20	-76.67	46.67	16.67
4	4	55	20	-85.00	50.00	15.00
4	4	60	20	-93.33	53.33	13.33
4	4	65	20	-101.67	56.67	11.67
4	4	70	20	-110.00	60.00	10.00
4	4	75	20	-118.33	63.33	8.33
4	4	80	20	-126.67	66.67	6.67
4	4	85	20	-135.00	70.00	5.00
4	4	90	20	-143.33	73.33	3.33
4	4	95	20	-151.67	76.67	1.67

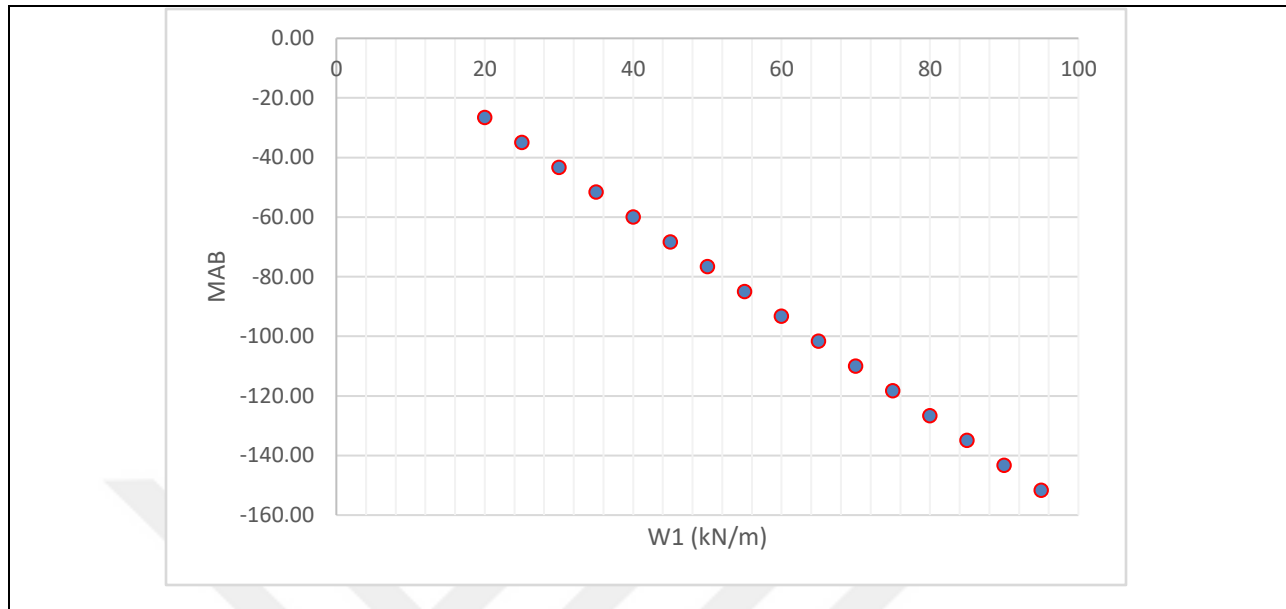


Figure 4.1: The Magnitudes Of Mab By The Change Of $W1$

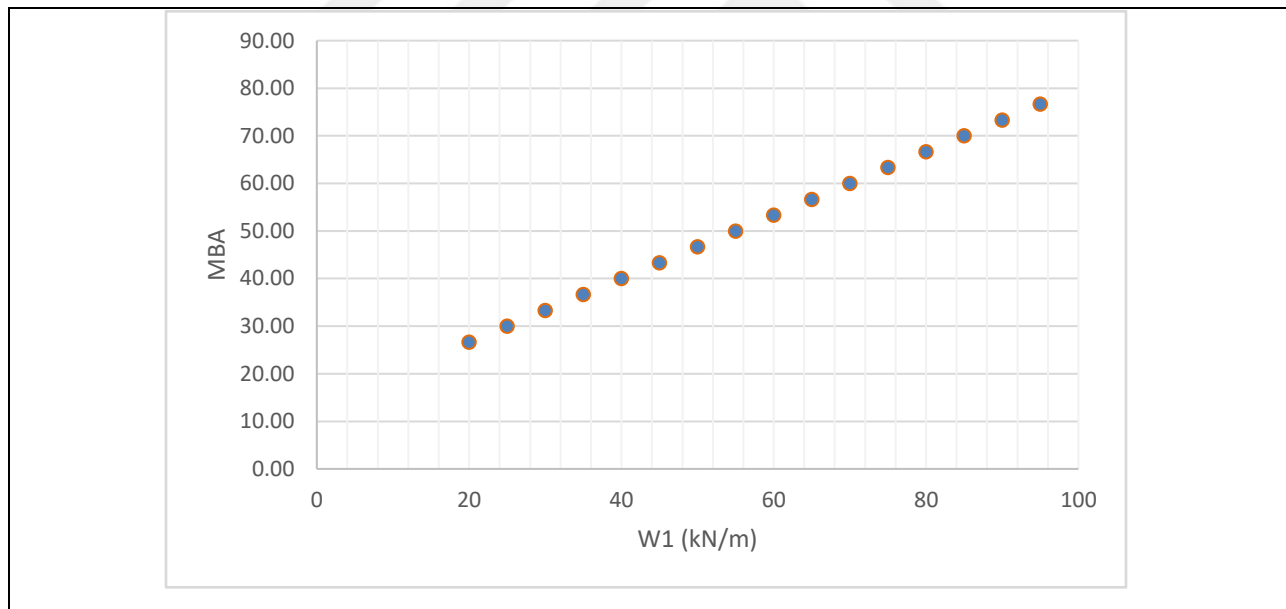


Figure 4.2: The Magnitudes of Mba By The Change Of $W1$

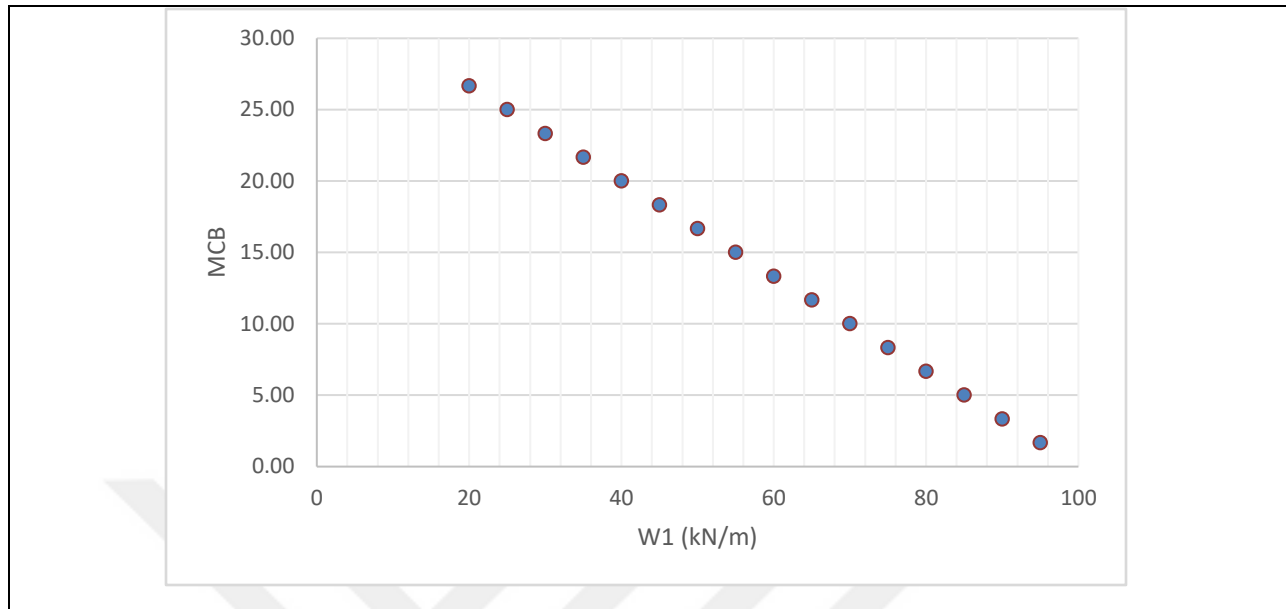


Figure 4.3: The Magnitudes of Mcb By The Change Of W1

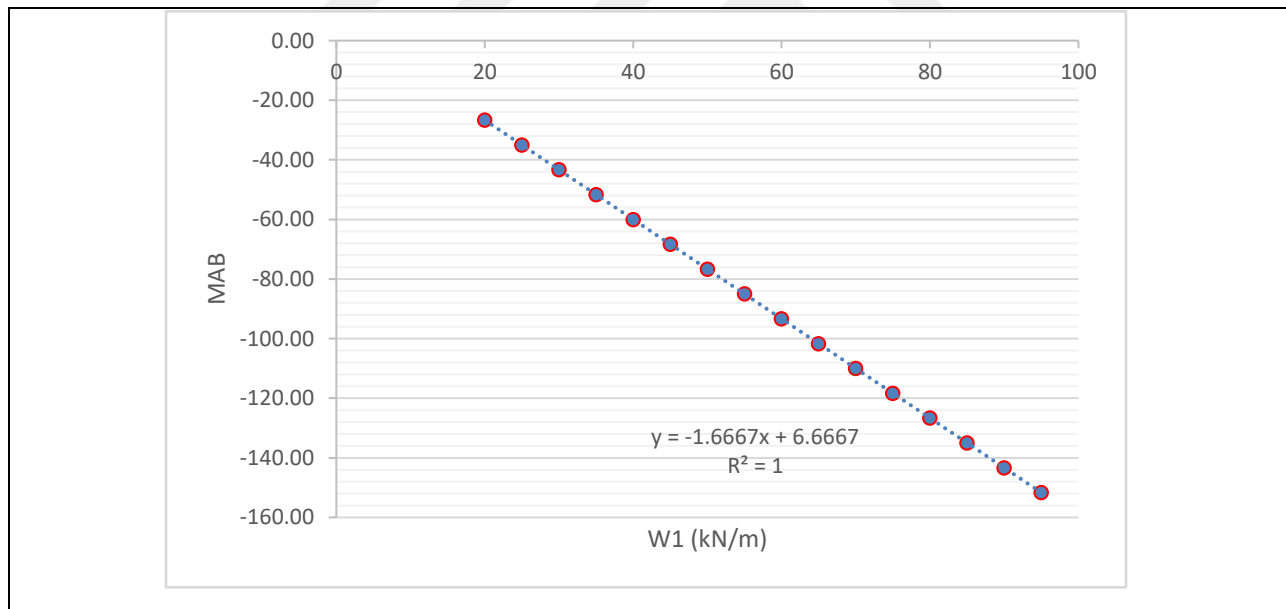


Figure 4.4: The Fitting Of Mab By The Change Of W1

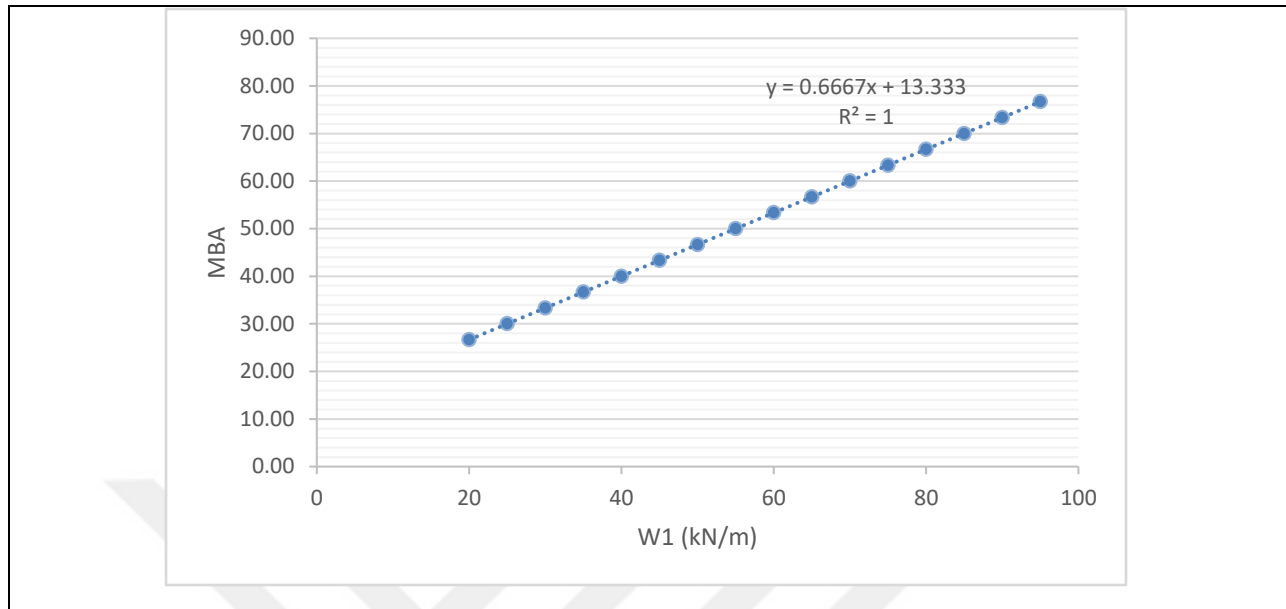


Figure 4.5: The Fitting Of Mba By The Change Of W1

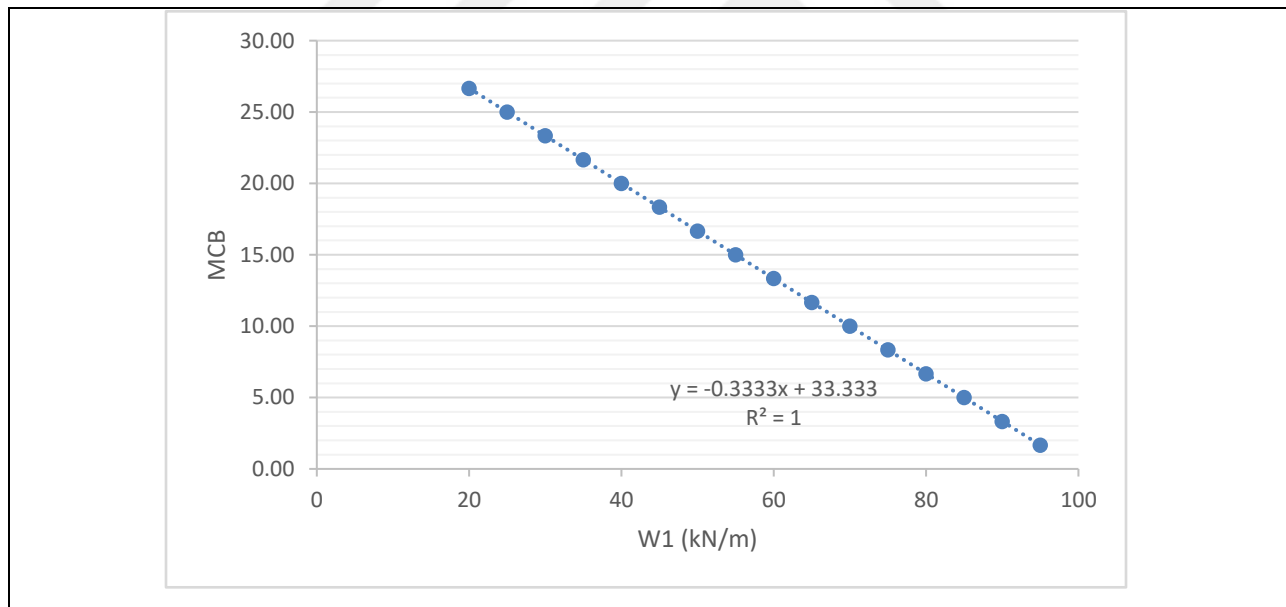


Figure 4.6: The Fitting of Mcb By The Change Of W1

From these figures the effect of the uniform load on the left side span on the moments at joints can be detected by:

Table 4.2: The Fitting Equations of W1 For The Moments At Joints

W1 (kN/m)	MAB eq (kN.m)	MBA eq (kN.m)	MCB eq (kN.m)
20-95	$y = -1.6667x + 6.6667$	$y = 0.6667x + 13.333$	$y = -0.3333x + 33.333$

W2= 30 kN/m

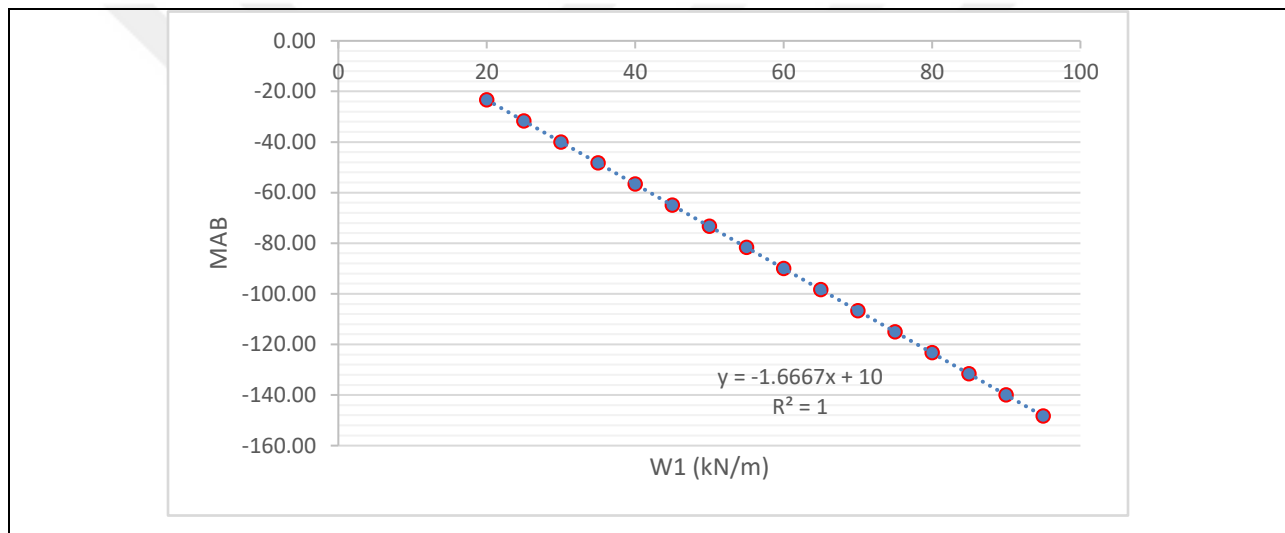


Figure 4.7: The Magnitudes Of Mab By The Change Of W1 (When W2 =30 Kn/M)

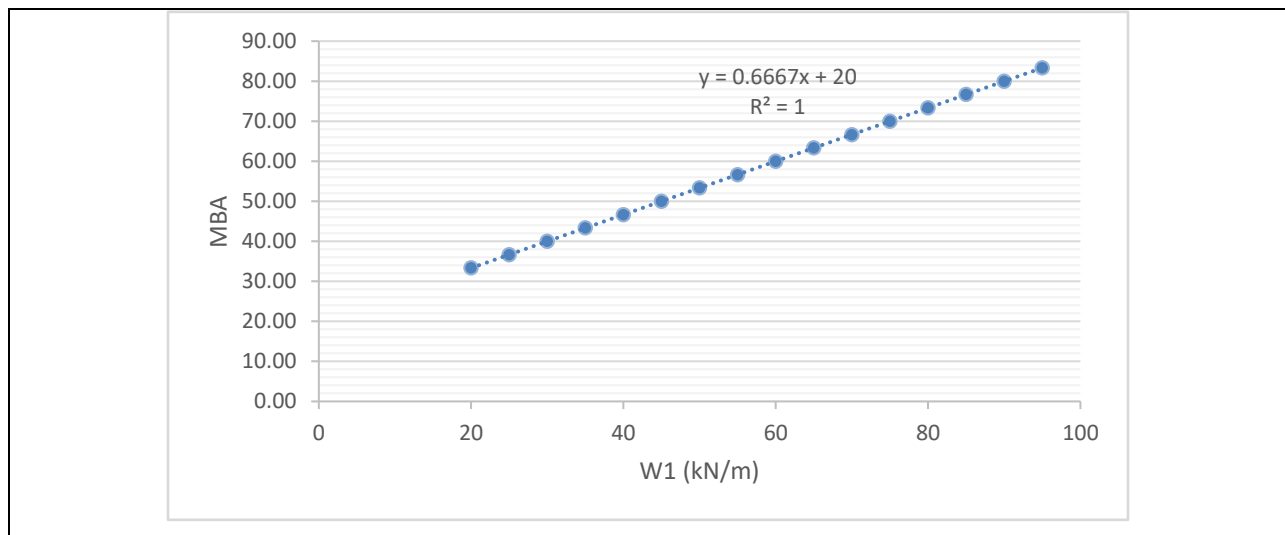


Figure 4.8: The Magnitudes Of Mba By The Change Of W1 (When W2 =30 Kn/M)

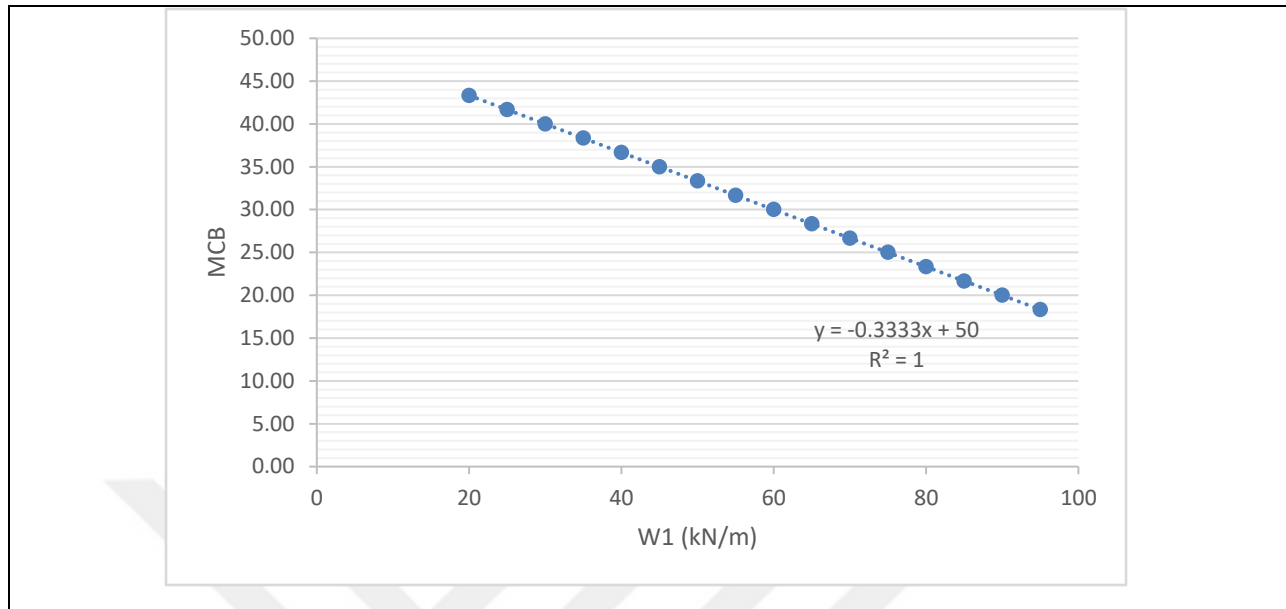


Figure 4.9: The Magnitudes Of Mcb By The Change Of $W1$ (When $W2 = 30$ Kn/M)

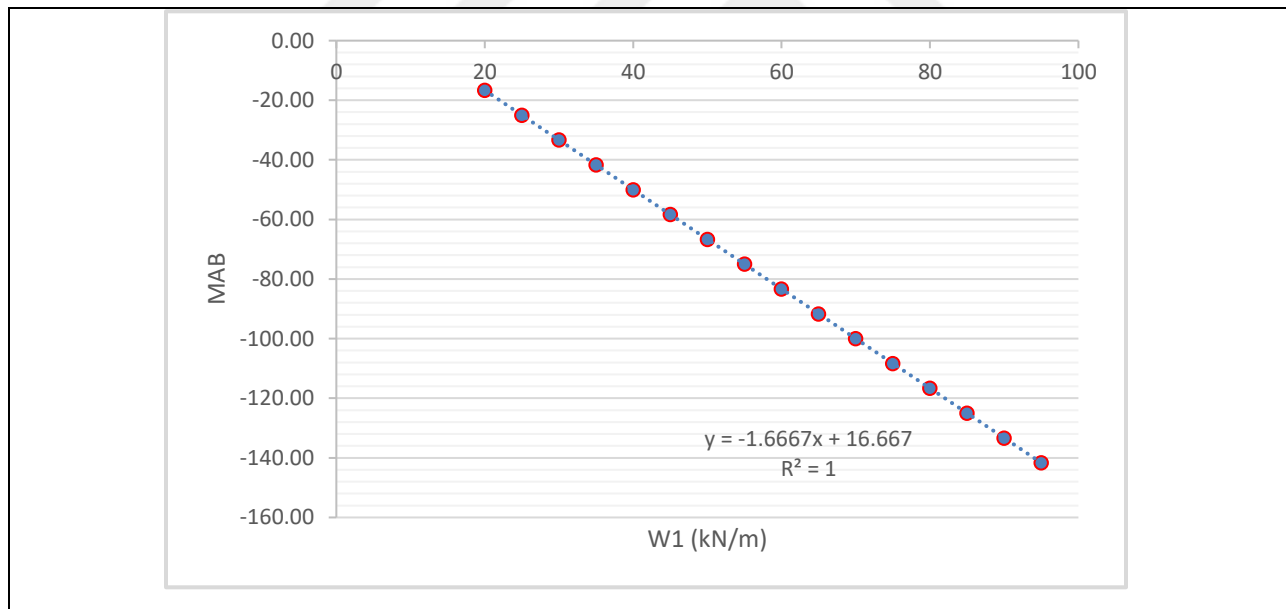


Figure 4.10: The Magnitudes Of Mab By The Change Of $W1$ (When $W2 = 50$ Kn/M)

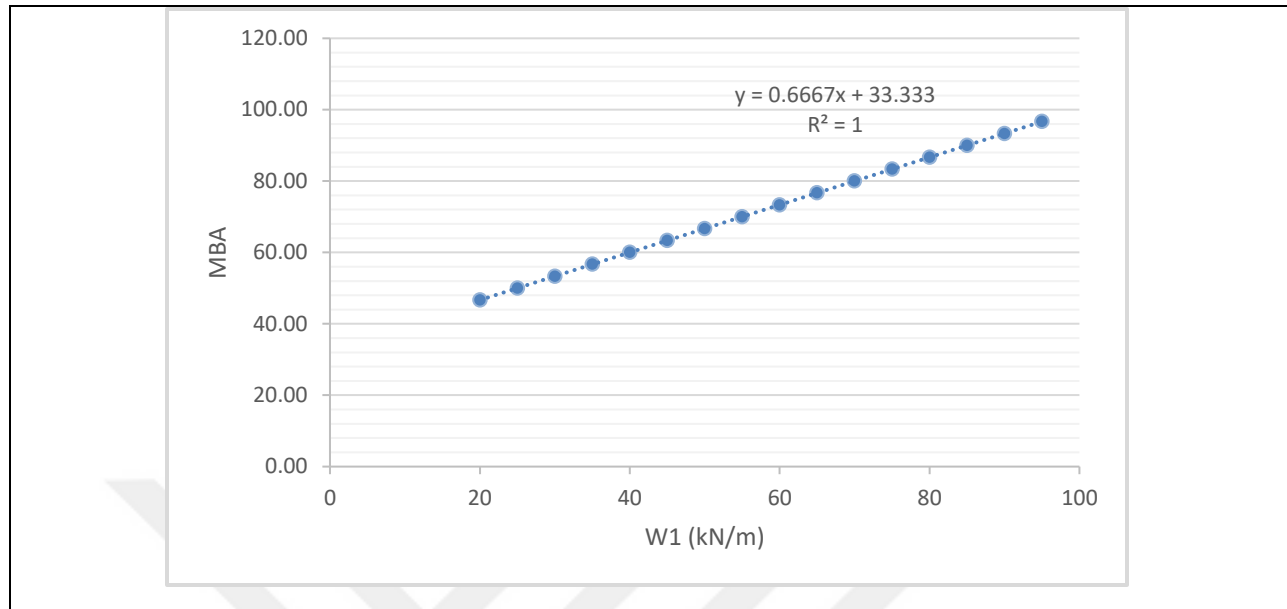


Figure 4.11: The Magnitudes of M_{ba} by the Change of $W1$ (When $W2 = 50$ Kn/M)

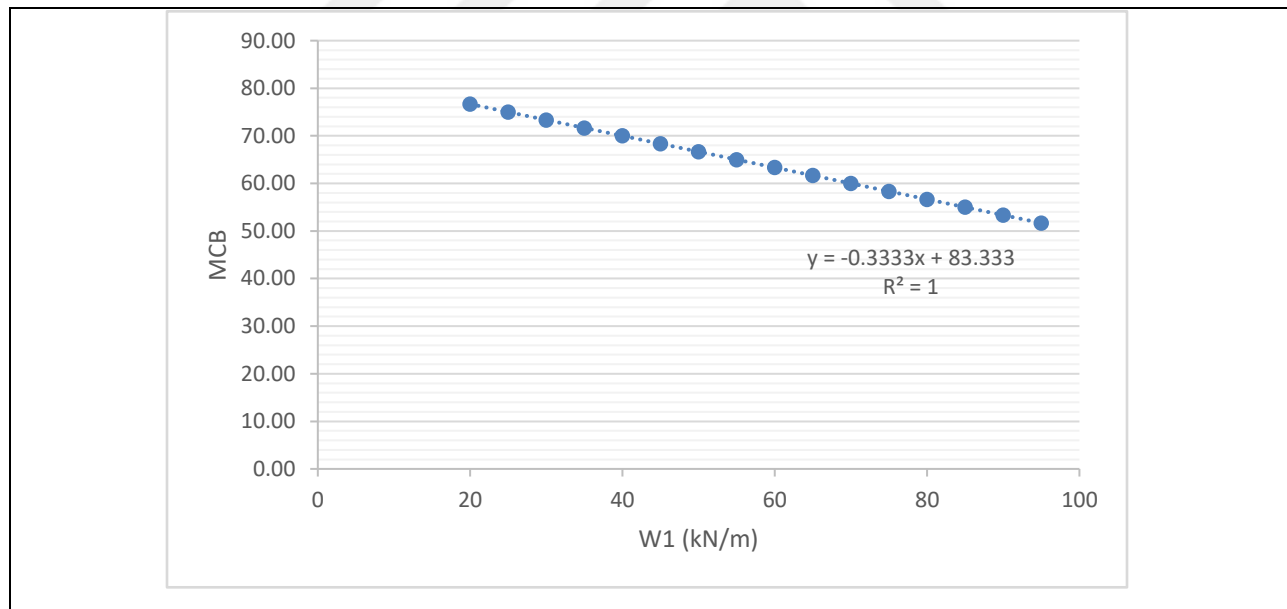


Figure 4.12: The Magnitudes of M_{cb} By The Change Of $W1$ (When $W2 = 50$ Kn/M)

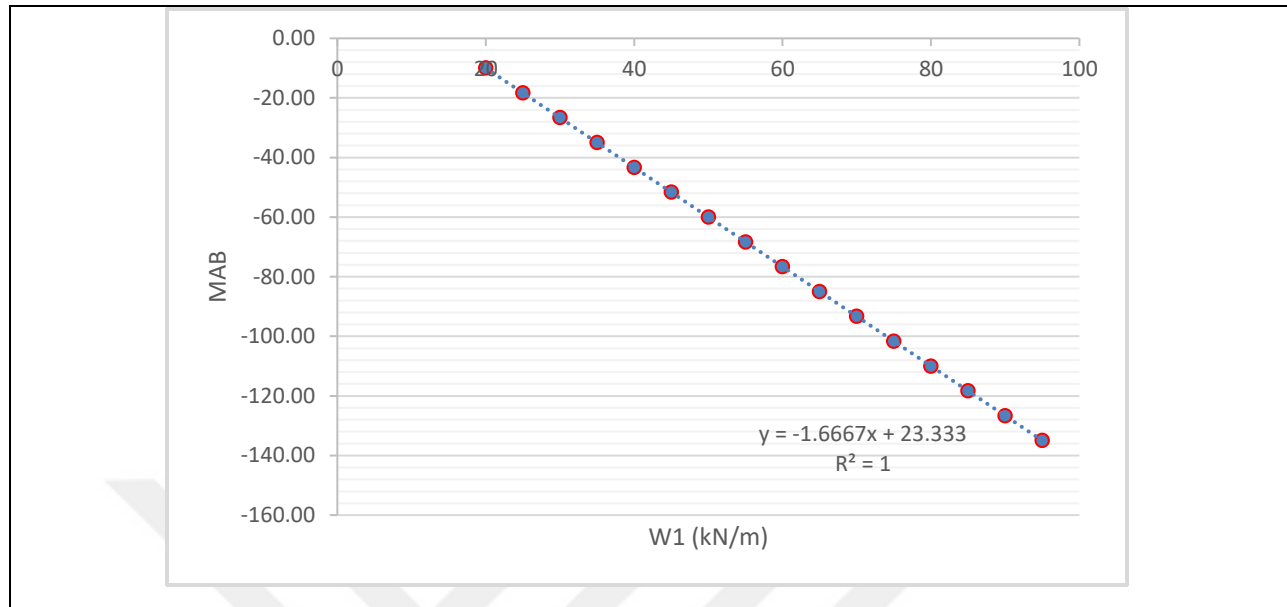


Figure 4.13: The Magnitudes of Mab by the Change Of W1 (When W2 =70 Kn/M)

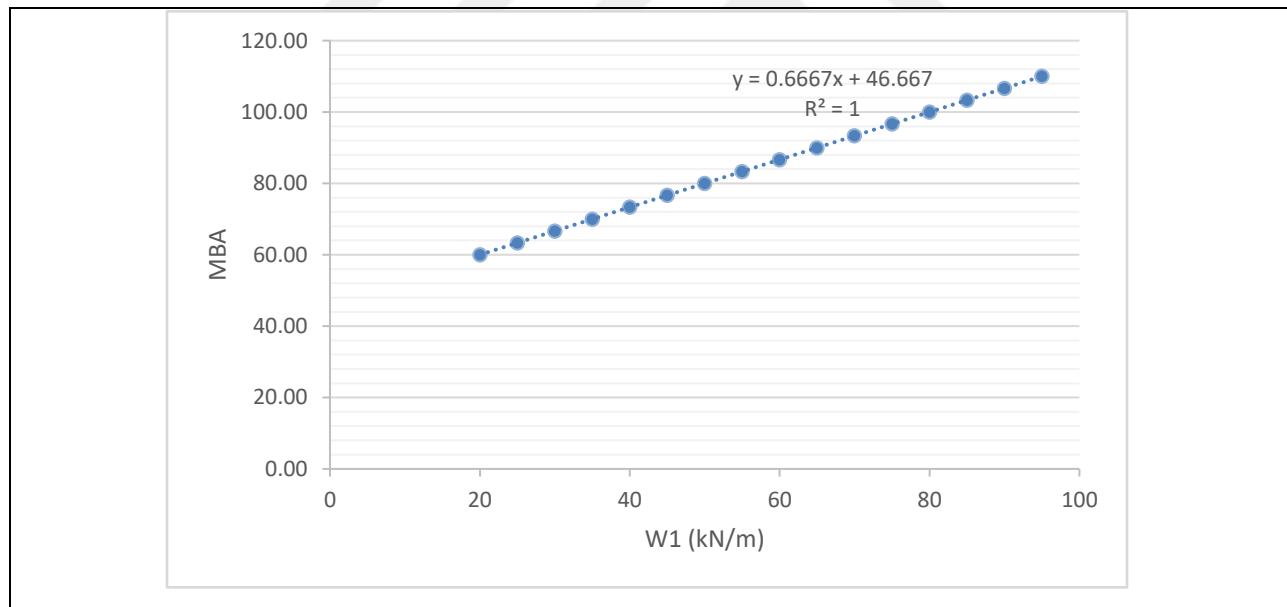


Figure 4.14: The Magnitudes Of Mba By The Change Of W1 (When W2 =70 Kn/M)

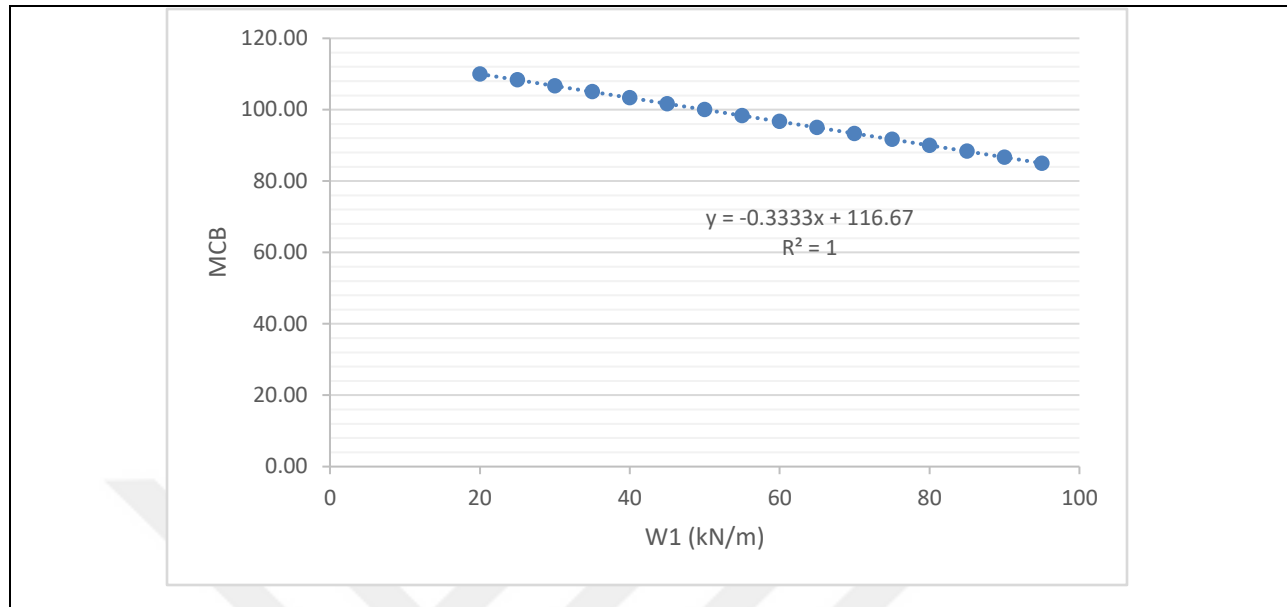


Figure 4.15: The Magnitudes Of MCB By The Change Of W1 (when W2 =70 kN/m)

From the curve fitting that used to examine the effect of W2 on the derived equation, the following table contained the linear equations when W2 changed from 20 to 70 kN/m

Table 4.3: The Impact of W1 and W2 On The Moments At Joints

W1 (kN/m)	W2 (kN/m)	MAB eq (kN.m)	MBA eq (kN.m)	MCB eq (kN.m)
20-95	20	$y = -1.6667x + 6.6667$	$y = 0.6667x + 13.333$	$y = -0.3333x + 33.333$
20-95	30	$y = -1.6667x + 10$	$y = 0.6667x + 20$	$y = -0.3333x + 50$
20-95	50	$y = -1.6667x + 16.667$	$y = 0.6667x + 33.333$	$y = -0.3333x + 83.333$
20-95	70	$y = -1.6667x + 23.333$	$y = 0.6667x + 46.667$	$y = -0.3333x + 116.67$

From the table, the effect of increasing the magnitude of W2 appeared in the second parts of the equation:

Table 4.4: The Moments Equations Second Parts After Investigation the Impact Of W1 Ad W2

W1 (kN/m)	W2 (kN/m)	Second part for MAB eq	Second part for MBA eq	Second part for MCB eq
20-95	20	6.6667	13.333	33.333
20-95	30	10	20	50
20-95	50	16.667	33.333	83.333
20-95	70	23.333	46.667	116.67

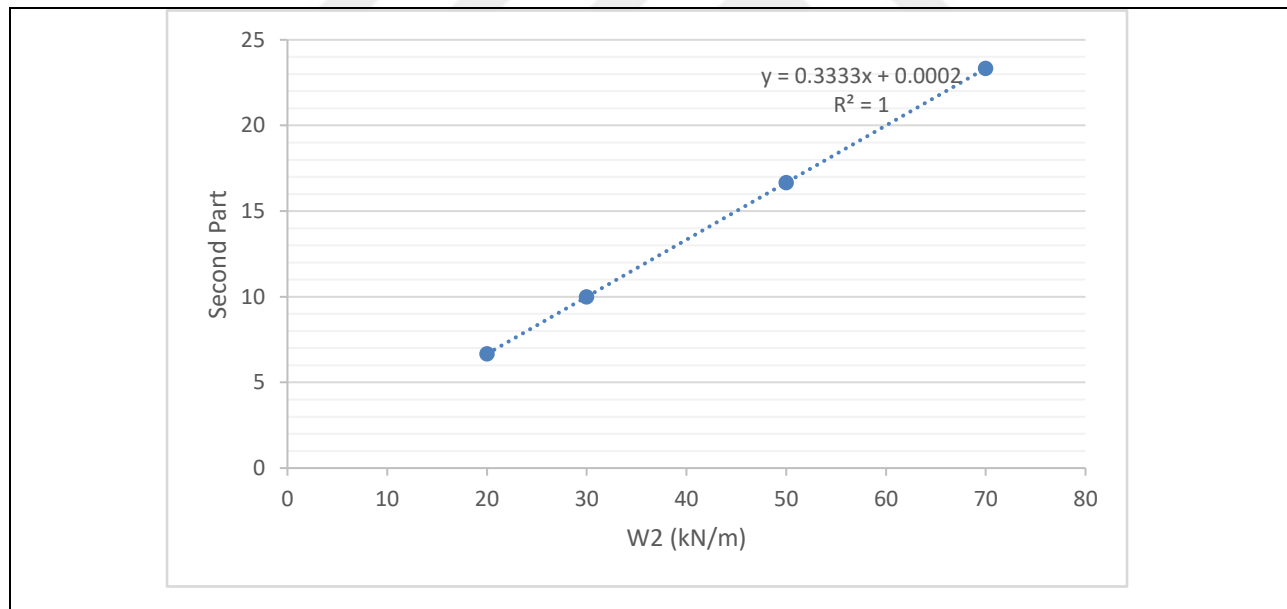


Figure 4.16: The Linear Fitting For W2 Effect On The Second Part Of The Equation Of Mab

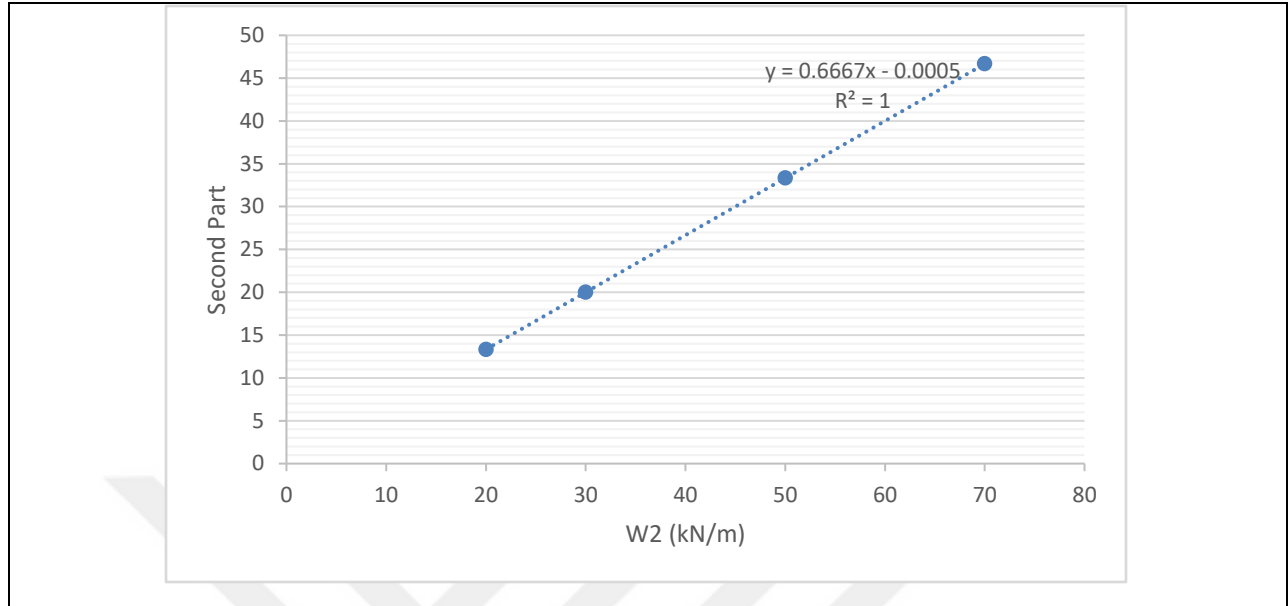


Figure 4.17: The Linear Fitting For W2 Effect On The Second Part Of The Equation Of Mba

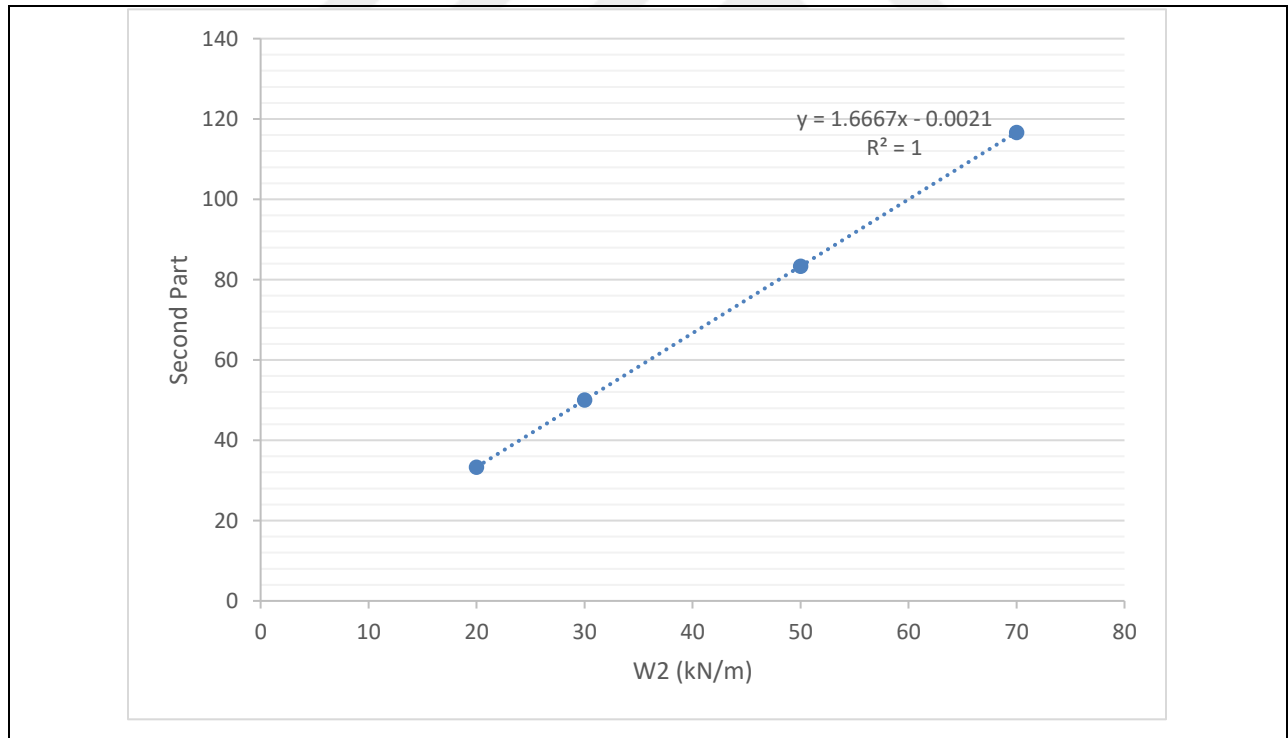


Figure 4.18: The Linear Fitting for W2 Effect On The Second Part Of The Equation Of Mcb

From these analysis steps the magnitude of the moments at joints (A, B, and C) can be obtained according to the following equations:

$$MAB = -1.6667 W1 + 0.3333 W2 + 0.0002 \quad (4.1)$$

$$MBA = 0.6667 W1 + 0.6667 W2 - 0.0005 \quad (4.2)$$

$$MBC = -0.3333 W1 + 1.6667 W2 - 0.0021 \quad (4.3)$$

The feasibility of obtaining this equation was examined by identifying the magnitude of the moments by analytical and derived equations:

Table 4.5: Checking The Derived Equations For Moments At Joints

W1	W2	MAB	MBA	MCB	eq MAB	eq MBA	eq MCB
20	20	-26.67	26.67	26.67	-26.67	26.668	26.672
30	25	-41.67	36.67	31.67	-41.67	36.668	31.675
40	30	-56.67	46.67	36.67	-56.67	46.669	36.679
50	35	-71.67	56.67	41.67	-71.67	56.669	41.682
60	40	-86.67	66.67	46.67	-86.67	66.67	46.686
70	45	-101.67	76.67	51.67	-101.7	76.67	51.689
80	50	-116.67	86.67	56.67	-116.7	86.671	56.693
90	55	-131.67	96.67	61.67	-131.7	96.671	61.696
100	60	-146.67	106.67	66.67	-146.7	106.67	66.7
110	65	-161.67	116.67	71.67	-161.7	116.67	71.703
120	70	-176.67	126.67	76.67	-176.7	126.67	76.707
130	75	-191.67	136.67	81.67	-191.7	136.67	81.71
140	80	-206.67	146.67	86.67	-206.7	146.67	86.714
150	85	-221.67	156.67	91.67	-221.7	156.67	91.717
160	90	-236.67	166.67	96.67	-236.7	166.67	96.721
170	95	-251.67	176.67	101.67	-251.7	176.68	101.72

From the table, the calculated magnitudes of the moments MAB, MBA, and MCB were so closed to the original calculated by the slope deflection equations.

4.2 INVESTIGATION THE EFFECT OF SPAN LENGTHS

The derived equation was made for $L_1=L_2= 4$ m, for generalizing the equation to be used for variable lengths of span, the span lengths were changed from 3 to 7 m, the resultant of the three moments were showed in table:

Table 4.6: The Impact of Span Lengths on the Moments At Joints

L1	L2	MAB	eq MAB	MAB/eqMAB
3	3	-15.00	-26.67	0.5625
3.25	3.25	-27.51	-41.67	0.6601
3.5	3.5	-43.39	-56.67	0.7656
3.75	3.75	-62.99	-71.67	0.8789
4	4	-86.67	-86.67	1
4.25	4.25	-114.77	-101.7	1.1289
4.5	4.5	-147.66	-116.7	1.2656
4.75	4.75	-185.67	-131.7	1.4101
5	5	-229.17	-146.7	1.5624
5.25	5.25	-278.50	-161.7	1.7226
5.5	5.5	-334.01	-176.7	1.8906
5.75	5.75	-396.06	-191.7	2.0663
6	6	-465.00	-206.7	2.2499
6.25	6.25	-541.18	-221.7	2.4413
6.5	6.5	-624.95	-236.7	2.6405
7	7	-770.73	-251.7	3.0624

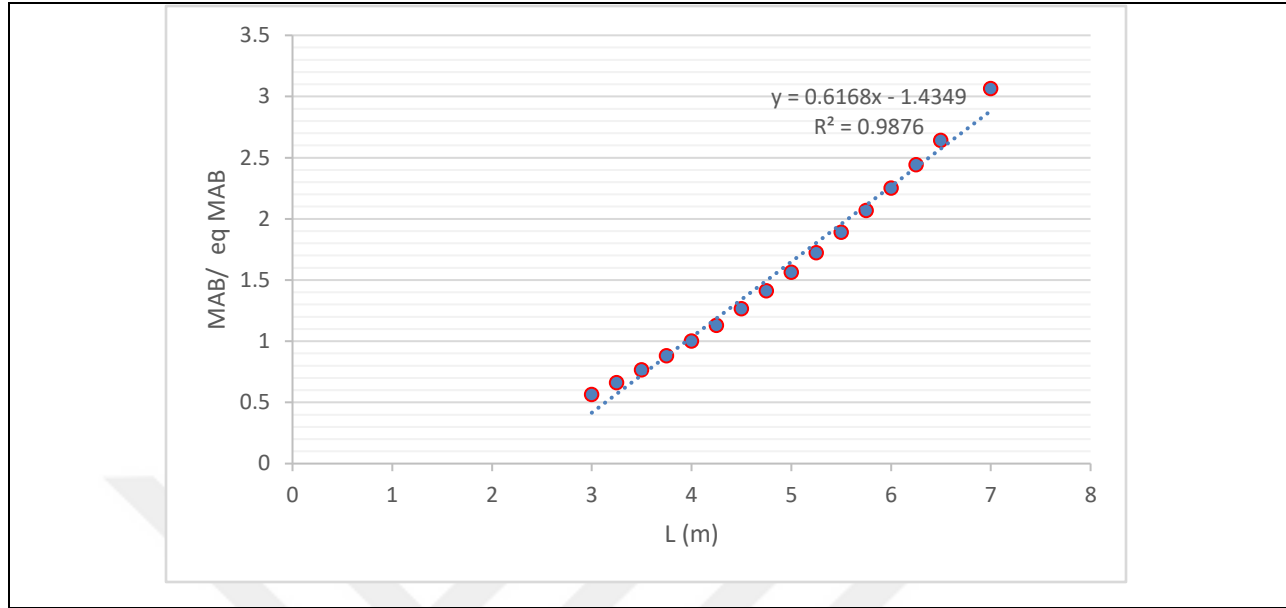


Figure 4.19: Checking the Derived Equation by the Effect Of Length Change

The equation then can be modified to get the effect of $L = L_1 = L_2$, the correlation term from the figure is:

$$\text{modification term} = 0.6168 L - 1.4349$$

The modified equations are:

$$MAB = (0.6168 L - 1.4349)(-1.6667 W_1 + 0.3333 W_2 + 0.0002) \quad (4.4)$$

$$MBA = (0.6168 L - 1.4349)(0.6667 W_1 + 0.6667 W_2 - 0.0005) \quad (4.5)$$

$$MBC = (0.6168 L - 1.4349)(-0.3333 W_1 + 1.6667 W_2 - 0.0021) \quad (4.6)$$

Table 4.7: Checking the Modified Equation For Mab

L1	L2	MAB	eq MAB	MAB/eqMAB
3	3	-15.00	-11.17	1.3424
3.25	3.25	-27.51	-23.9	1.1511
3.5	3.5	-43.39	-41.25	1.0516
3.75	3.75	-62.99	-63.25	0.9959
4	4	-86.67	-89.88	0.9643
4.25	4.25	-114.77	-121.1	0.9474
4.5	4.5	-147.66	-157	0.9403
4.75	4.75	-185.67	-197.6	0.9398
5	5	-229.17	-242.7	0.9441
5.25	5.25	-278.50	-292.5	0.952
5.5	5.5	-334.01	-347	0.9626
5.75	5.75	-396.06	-406.1	0.9754
6	6	-465.00	-469.8	0.9898
6.25	6.25	-541.18	-538.1	1.0057
6.5	6.5	-624.95	-611.1	1.0227
7	7	-770.73	-727.6	1.0593

4.3 STRUCTURAL DESIGN

By using the ultimate strength method, the design of the continuous beam can be obtained according to the reinforcement, the effective depth, the width, and the compressive strength of concrete and steel. The joining between the derived equations for the moments and the structural equation lead to the following:

For MAB:

$$(0.6168 L - 1.4349) (-1.6667 W1 + 0.3333 W2 + 0.0002) = AS f_y \left(d - \frac{a}{2}\right) \quad (4.7)$$

For MBA:

$$(0.6168 L - 1.4349)(0.6667 W1 + 0.6667 W2 - 0.0005) = AS f_y \left(d - \frac{a}{2}\right) \quad (4.8)$$

For MCB:

$$(0.6168 L - 1.4349)(-0.3333 W1 + 1.6667 W2 - 0.0021) = AS f_y \left(d - \frac{a}{2}\right) \quad (4.9)$$

By taking the properties of the member as:

Table 4.8: The General Properties of the Beams System

Item	Magnitude
Fy	400 MPa
Fc	20 MPa
b	300 mm

The minimum effective depth needed and the minimum steel area can be obtained using the derived equations

Table 4.9 : The Magnitudes of s.f. For the Three Moments When $w_1=30$ and $w_2=20$

Effective depth (mm)	MAB S.F.	MBA S.F.	MCB S.F.
250	1.56	2.11	3.01
260	1.64	2.20	3.15
270	1.71	2.30	3.29
280	1.78	2.40	3.42
290	1.85	2.49	3.56
300	1.92	2.59	3.70
310	1.99	2.69	3.84
320	2.07	2.78	3.98
330	2.14	2.88	4.11
340	2.21	2.98	4.25
350	2.28	3.07	4.39
360	2.35	3.17	4.53
370	2.42	3.27	4.66
380	2.49	3.36	4.80
390	2.57	3.46	4.94
400	2.64	3.56	5.08

Table 4.10: The Magnitudes of s.f. for the Three Moments When $W_1=60$ and $W_2=40$

Effective depth (mm)	MAB S.F.	MBA S.F.	MCB S.F.
250	0.78	1.05	1.51
260	0.82	1.10	1.57
270	0.85	1.15	1.64
280	0.89	1.20	1.71
290	0.93	1.25	1.78
300	0.96	1.30	1.85
310	1.00	1.34	1.92
320	1.03	1.39	1.99
330	1.07	1.44	2.06
340	1.10	1.49	2.13
350	1.14	1.54	2.19
360	1.18	1.58	2.26
370	1.21	1.63	2.33
380	1.25	1.68	2.40
390	1.28	1.73	2.47
400	1.32	1.78	2.54

Table 4.11: The Magnitudes of S.F. For the three moments when W1=90 and W2=70

Effective depth (mm)	MAB S.F.	MBA S.F.	MCB S.F.
250	0.54	0.66	0.81
260	0.56	0.69	0.85
270	0.58	0.72	0.88
280	0.61	0.75	0.92
290	0.63	0.78	0.96
300	0.66	0.81	1.00
310	0.68	0.84	1.03
320	0.71	0.87	1.07
330	0.73	0.90	1.11
340	0.76	0.93	1.14
350	0.78	0.96	1.18
360	0.80	0.99	1.22
370	0.83	1.02	1.26
380	0.85	1.05	1.29
390	0.88	1.08	1.33
400	0.90	1.11	1.37

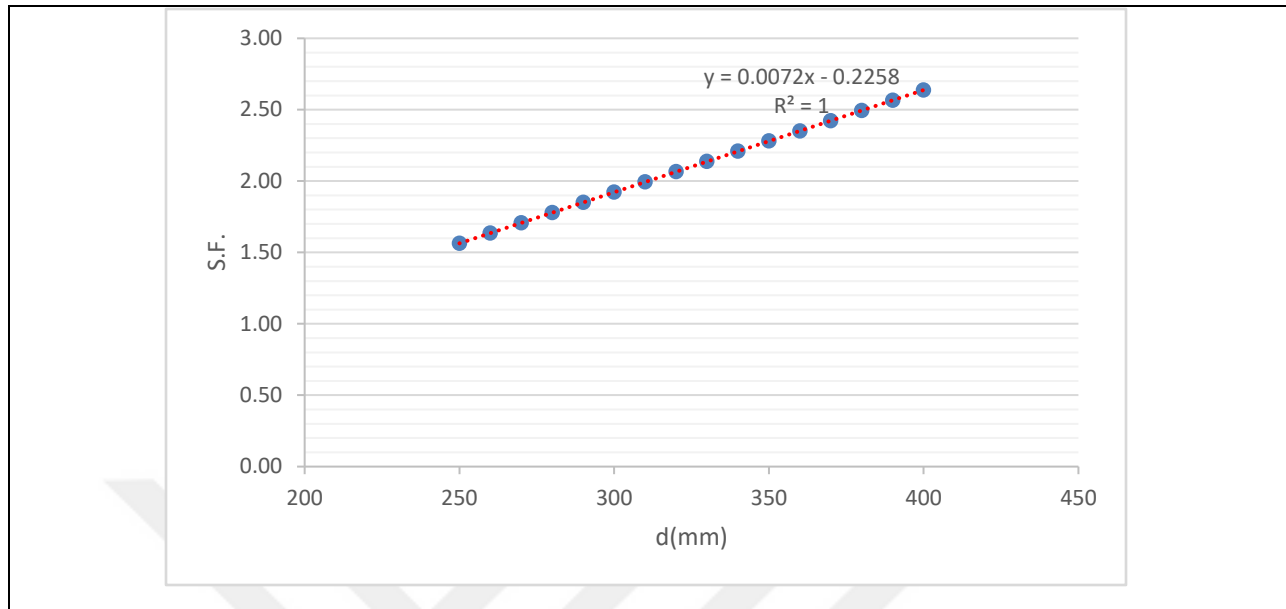


Figure 4.20: The Impact Of The Effective Depth on S.F. when $W1=30$ and $W2=20$

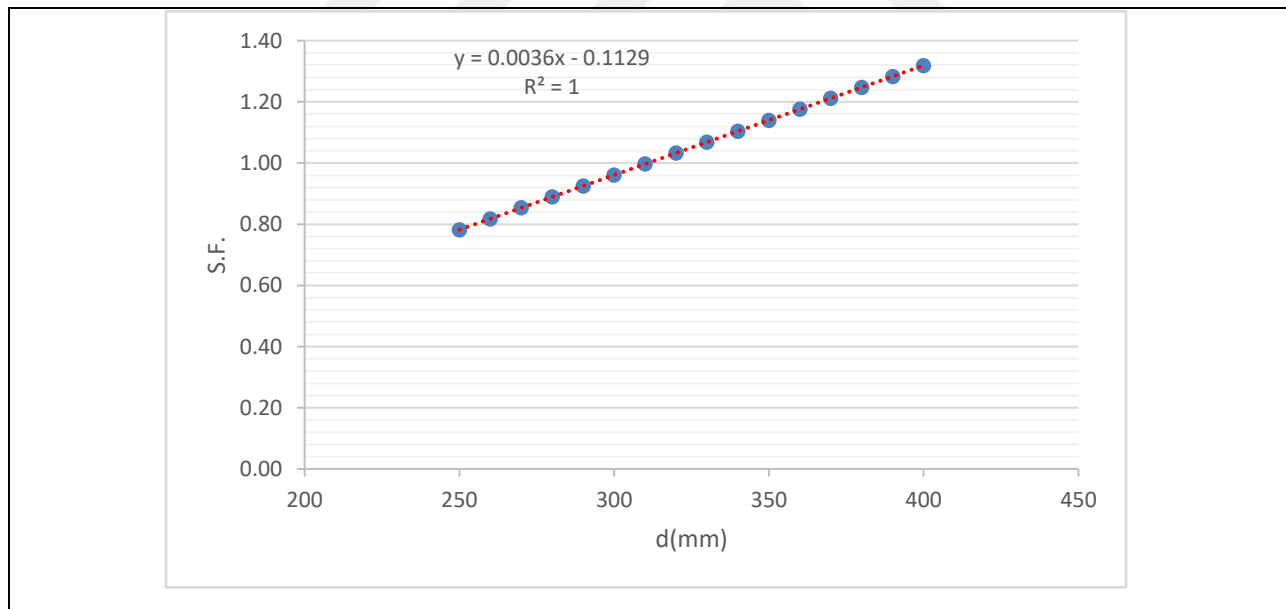


Figure 4.21: The Impact Of The Effective Depth On S.F. when $W1=60$ and $W2=40$

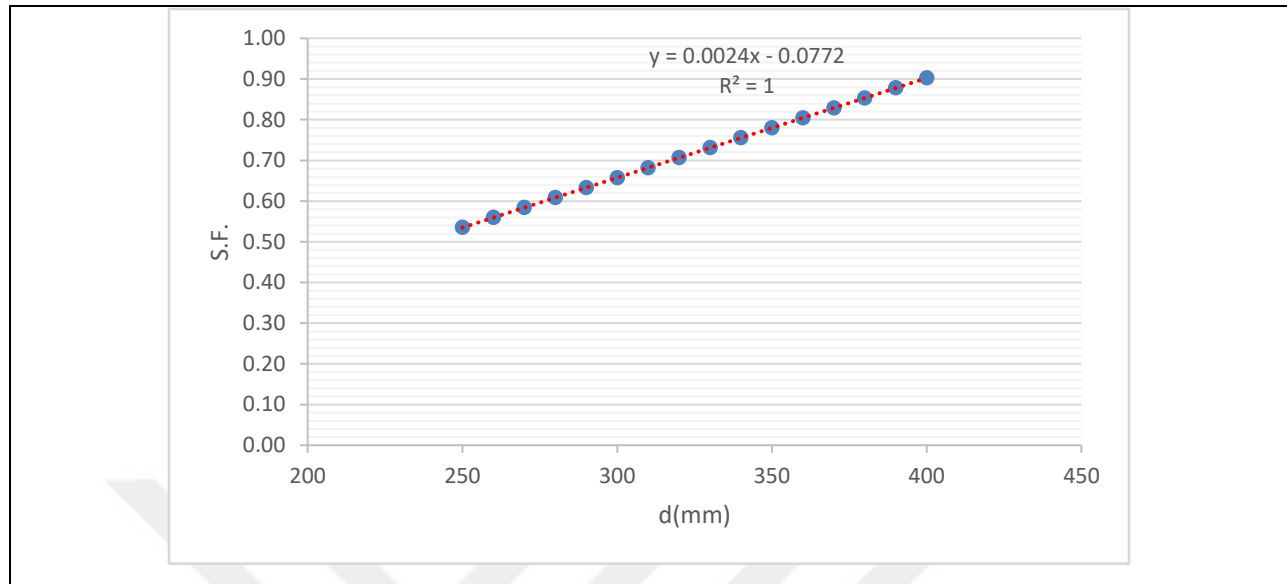


Figure 4.22: The Impact Of The Effective Depth On S.F. when W1=90 and W2=70

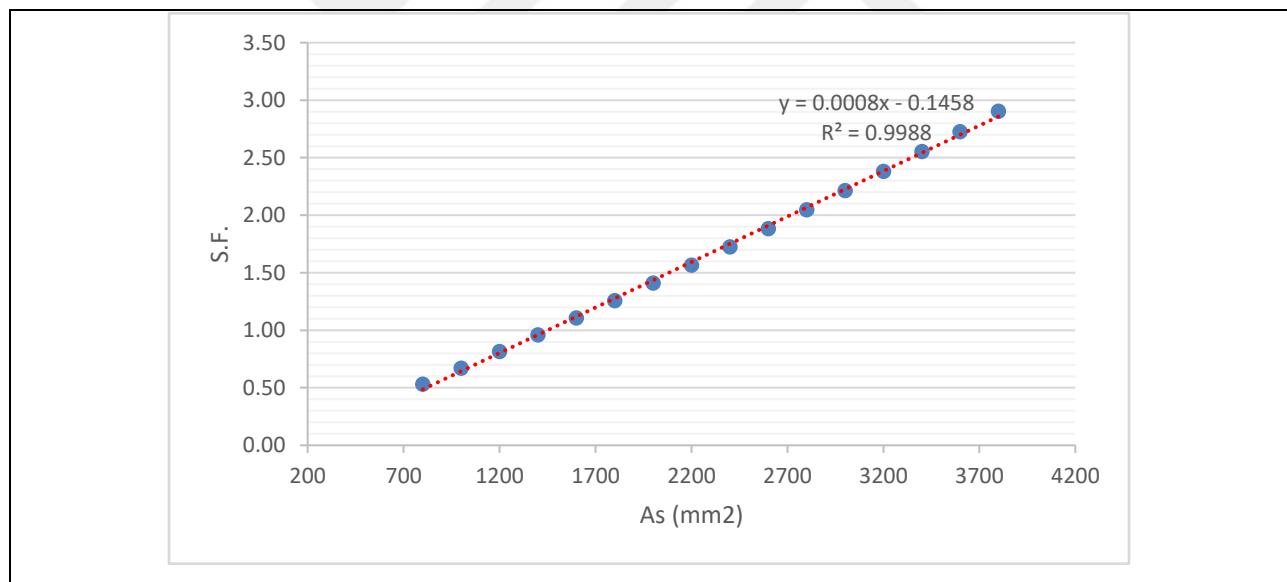


Figure 4.23: The Impact of the Reinforcement Steel Area on S.F.

Table 4.12: The Examination of the Impact of the effective depth on S.F.

d (mm)	W1 (kN/m)	Eq
250-400	30	$y = 0.0072x - 0.2258$
250-400	60	$y = 0.0036x - 0.1129$
250-400	90	$y = 0.0024x - 0.0772$

Where W1 taken as the more effective in the increasing of the moment MAB

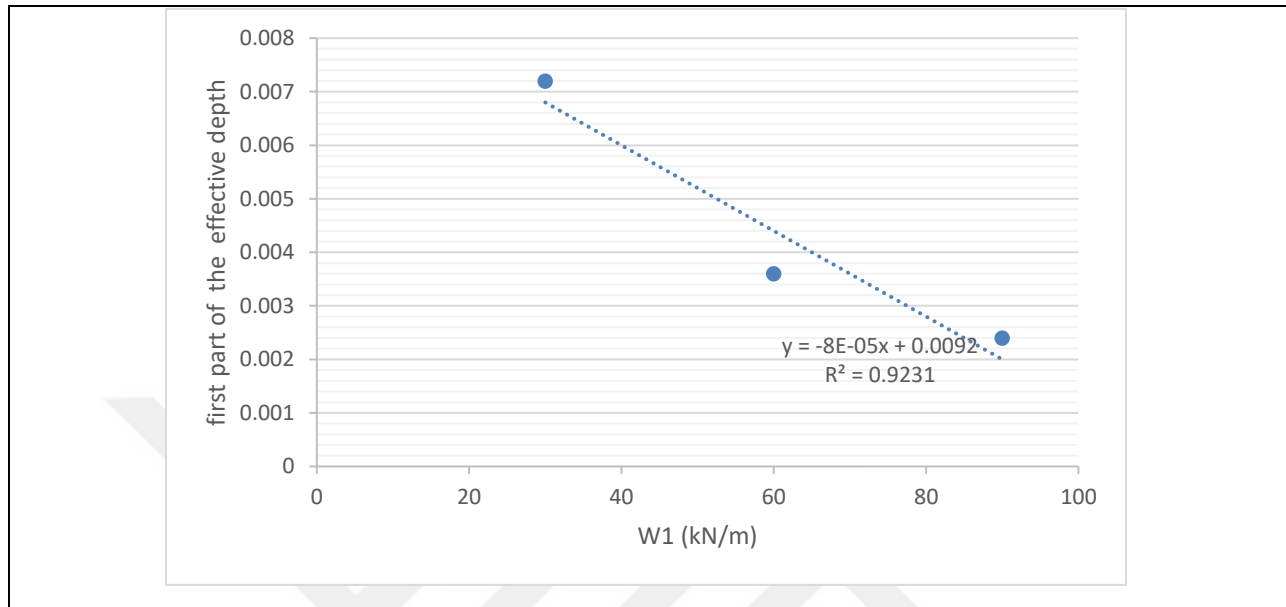


Figure 4.24: The Examination Of The Impact Of The W1 On The First Part Of The Effective Depth S.F. equation

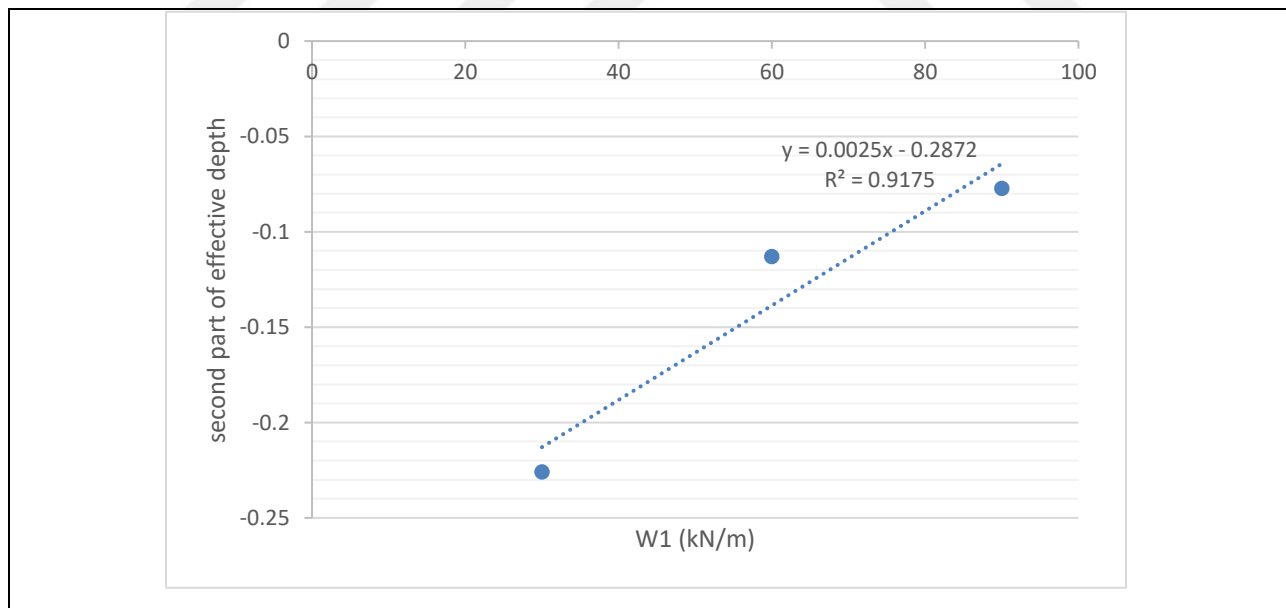


Figure 4.25: The Examination Of The Impact Of The W1 On The Second Part Of Effective Depth S.F. equation

$$S.F. = -0.00008 W1 d + 0.0092 d + 0.0025 W1 - 0.2872$$

Table 4.13: Checking the Effective Depth Impact for the Derived Equations

Effective depth (mm)	Derived MAB S.F.	Ultimate strength MAB S.F.
250	0.9628	0.7819
260	1.0068	0.8177
270	1.0508	0.8535
280	1.0948	0.8893
290	1.1388	0.9251
300	1.1828	0.9609
310	1.2268	0.9967
320	1.2708	1.0325
330	1.3148	1.0683
340	1.3588	1.1041
350	1.4028	1.1399
360	0.80	0.99
370	0.83	1.02
380	0.85	1.05
390	0.88	1.08
400	0.90	1.11

For eliminating the difference that caused by the fitting error distances, a factor of 0.8 can be multiplied by the derived formula of S.F., the results became:

Table 4.14: Correlation the Derived Equations with the Impact on the Effective Depth

Effective depth (mm)	0.8 * Derived MAB S.F.	Ultimate strength MAB S.F.
250	0.7702	0.7819
260	0.8054	0.8177
270	0.8406	0.8535
280	0.8758	0.8893
290	0.911	0.9251
300	0.9462	0.9609
310	0.9814	0.9967
320	1.0166	1.0325
330	1.0518	1.0683
340	1.087	1.1041
350	1.1222	1.1399
360	1.1574	0.99
370	1.1926	1.02
380	1.2278	1.05
390	1.263	1.08
400	1.2982	1.11

$$S.F. = 0.8 * (-0.00008 W1 d + 0.0092 d + 0.0025 W1 - 0.2872) \quad (4.10)$$

4.4 ANSYS OUTPUT

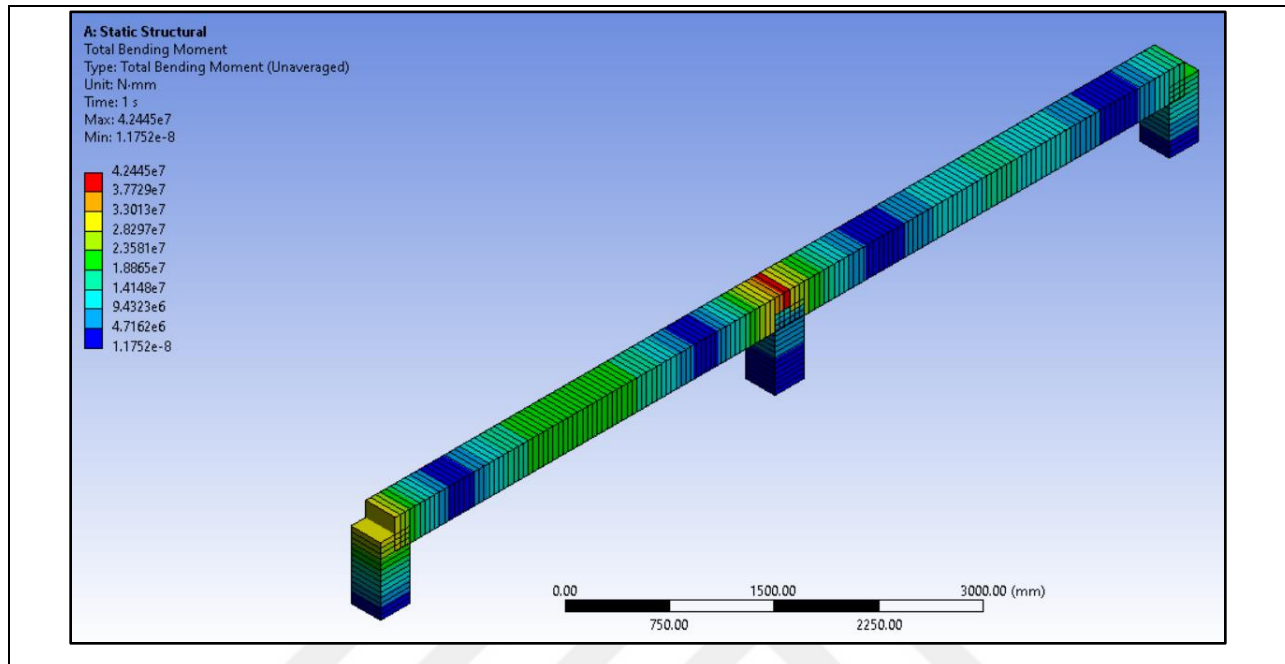


Figure 4.26: Bending Moments Shown By ANSYS program ($W1= 30 \text{ kN/m}$ and $W2= 20 \text{ kN/m}$)

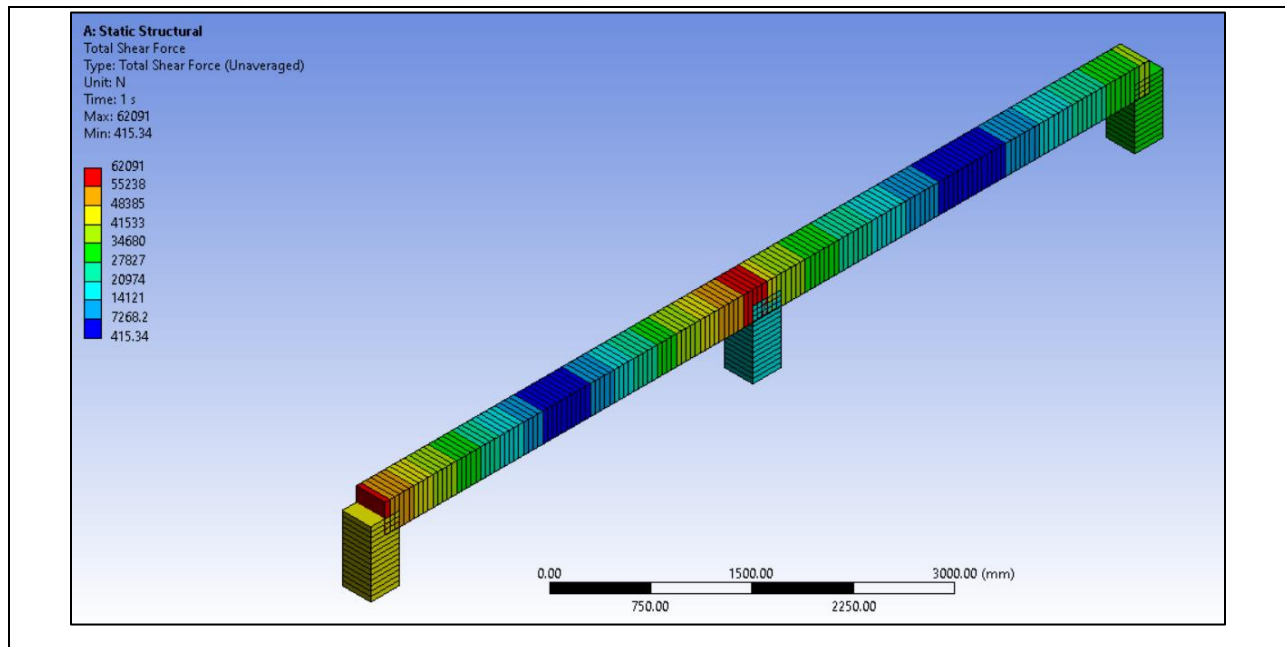


Figure 4.27: Shear Force Shown By ANSYS Program ($W1= 30 \text{ kN/m}$ and $W2= 20 \text{ kN/m}$)

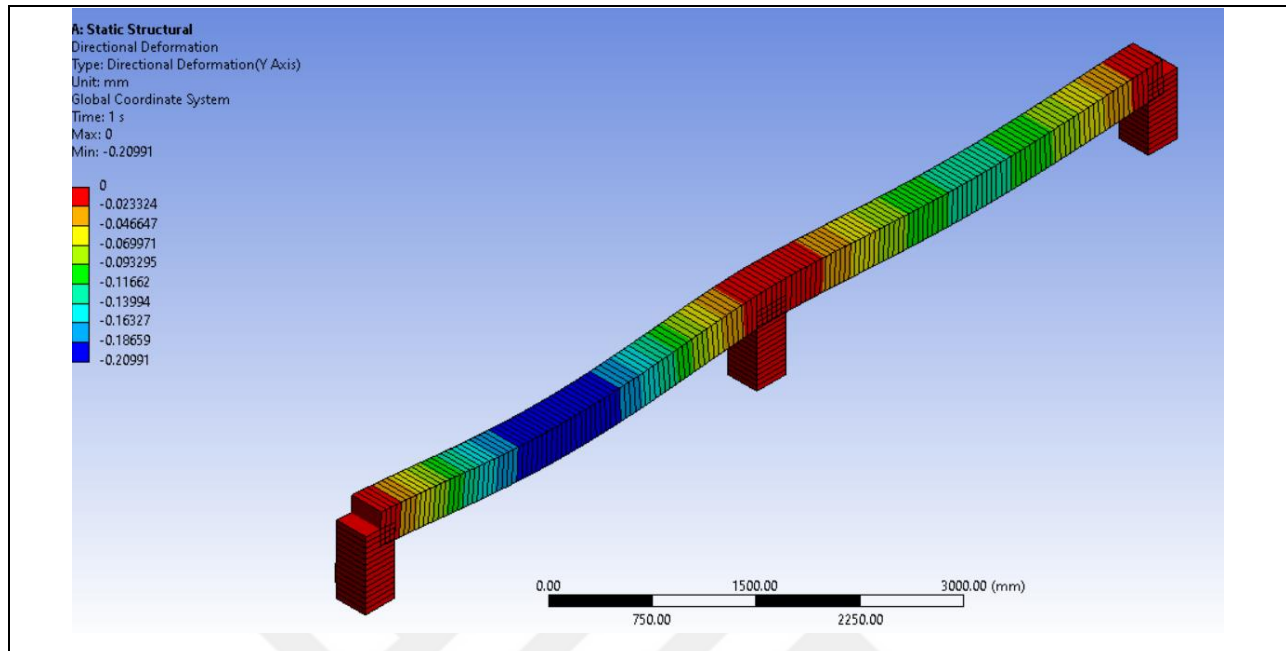


Figure 4.28: Directional Deformation Shown by ANSYS program ($W1 = 30 \text{ kN/m}$ and $W2 = 20 \text{ kN/m}$)

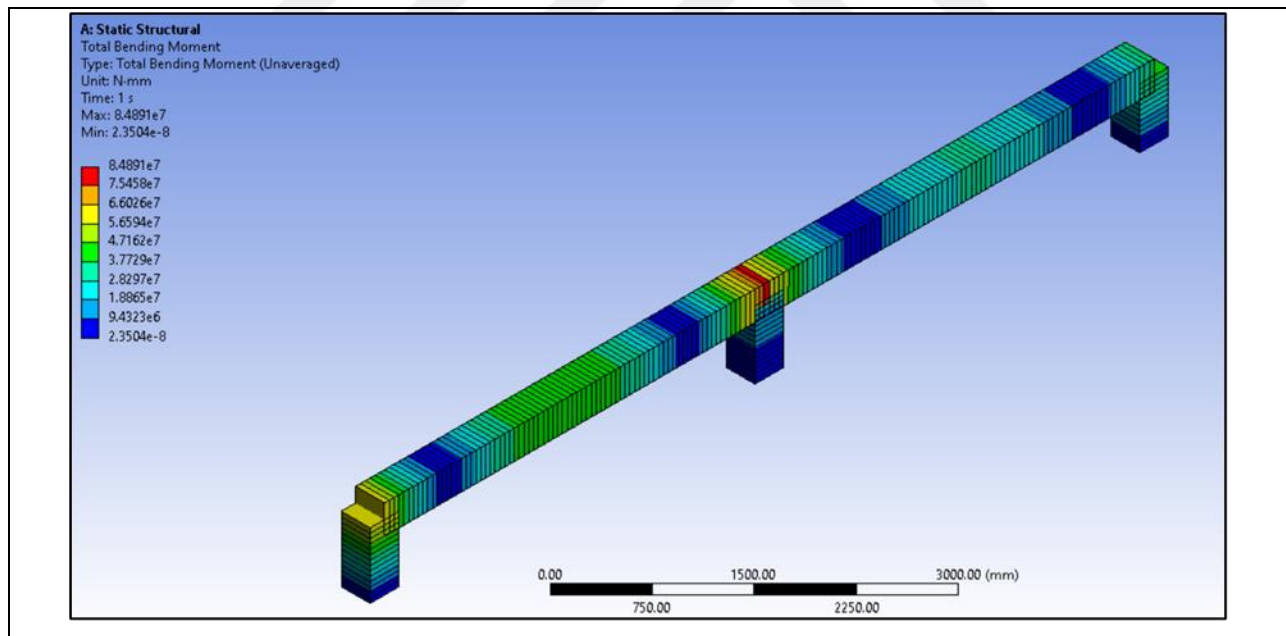


Figure 4.29 : Bending Moments Shown By ANSYS program ($W1 = 60 \text{ kN/m}$ and $W2 = 40 \text{ kN/m}$)

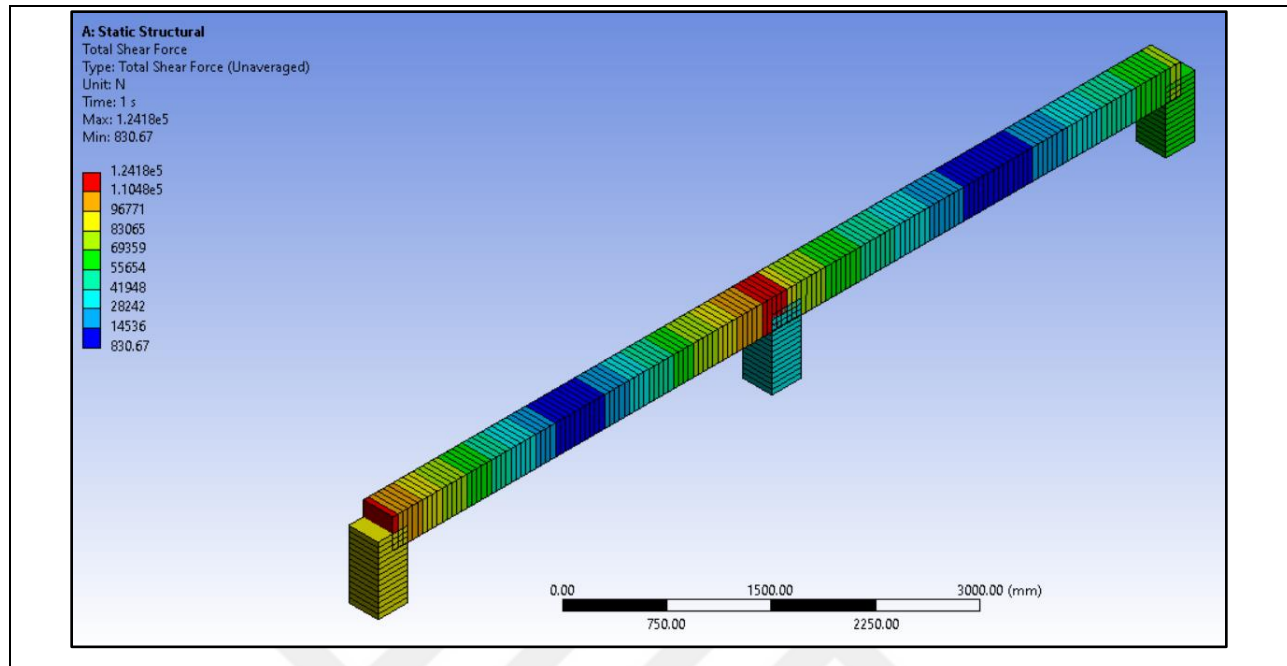


Figure 4.30: Shear Force Shown By ANSYS program ($W1= 60 \text{ kN/m}$ and $W2= 40 \text{ kN/m}$)

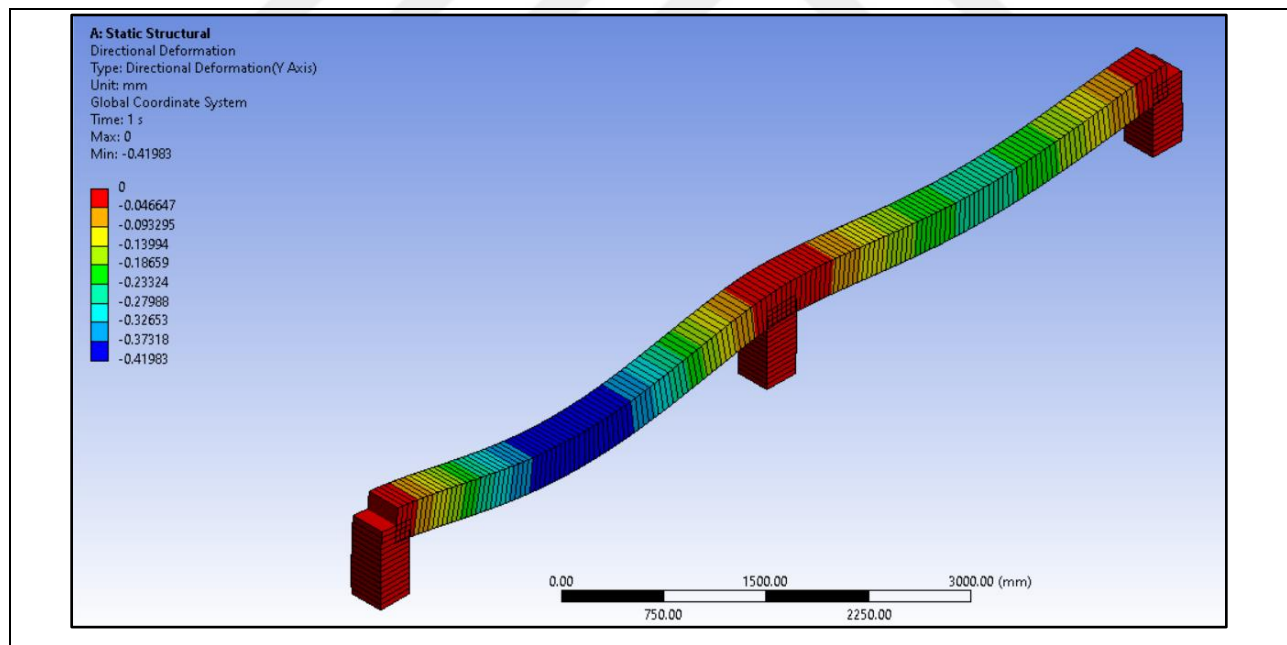


Figure 4.29: Directional Deformation Shown by ANSYS program ($W1= 60 \text{ kN/m}$ and $W2= 40 \text{ kN/m}$)

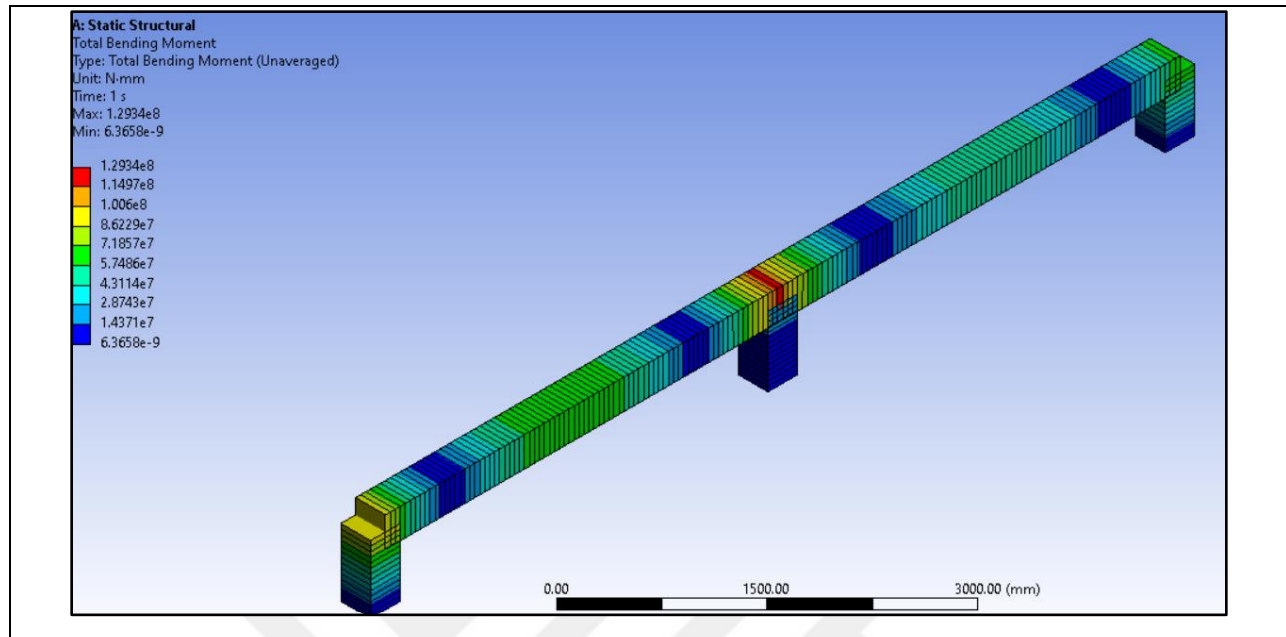


Figure 4.30: Bending Moments Shown by ANSYS program ($W1 = 90 \text{ kN/m}$ and $W2 = 70 \text{ kN/m}$)

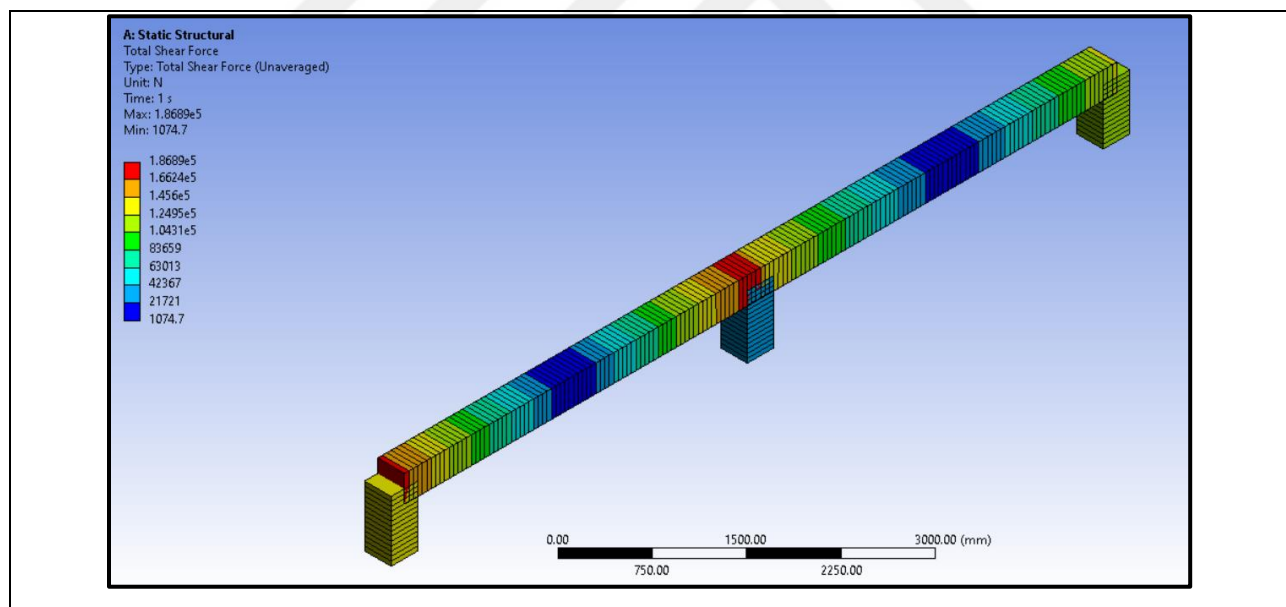


Figure 4.31: Shear Force Shown by ANSYS Program ($W1 = 90 \text{ kN/m}$ and $W2 = 70 \text{ kN/m}$)

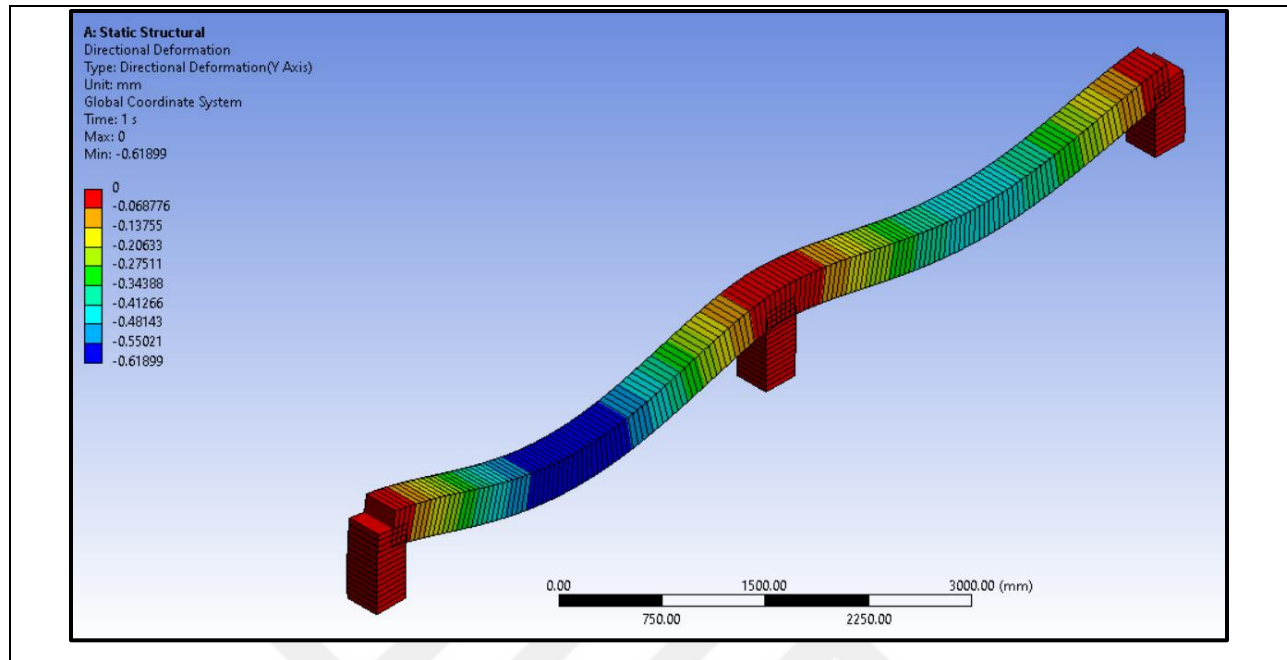


Figure 4.32: Directional Deformation Shown by ANSYS Program (W1= 90 kN/m and W2= 70)

5. DISCUSSION AND CONCLUSIONS

5.1 CONCLUSIONS

The study built two stage of analysis in the outline, the first stage was by using the theoretical concepts of the slope deflection method. The study set the effect of the uniform loadings W_1 and W_2 on the two span continuous beams system then deriving equation for each moment at the joints: M_{AB} , M_{BA} , and M_{CB} under the changing of the uniform loading and the span lengths. The study made curve fitting formulas for the impact of W_1 on the three moments with fixing of W_2 then changing the magnitudes of W_2 to build more comprehensive equation that can be used to identify the magnitude of each moments of M_{AB} , M_{BA} , and M_{CB} in simpler form then what showed in the slope deflection analysis path. The study took the effect of the span lengths as correlation for the value of the moments. The magnitude of W_1 was changed from 30 to 90 kN/m under the analysis and the magnitude of W_2 was changed from 20 to 70 kN/m. The study made these difference in the two magnitudes of the uniform loadings to examine the derived equation for unsymmetrical cases. The equation took the impact of the main parameters to give closer to actual solutions. The structural analysis was made as second stage to improve the structural design of the system. The ultimate strength method was used. The study joined the derived equation and the ultimate strength then analyzed the effect on the effective depth on the strength of the system. The safety factor for the system which referred to the division of the ultimate strength by the system geometry and reinforcement by the external maximum bending moment. The safety factor was calculated with the magnitude of d under various W_1 values. The study derived new equation to give rapid figure to the designers about the strength of the system under the assigned conditions. the study made simulations to support the analysis. The simulation was made using ANSYS program. the program showed in colored images the magnitudes of the bending moments, shear forces, and directional deformations in which the moments satisfied the analytical results.

5.2 FUTURE WORK

The coming studies can deal with modeling more cases of indeterminate systems in tall buildings. The future works can analysis and modeling the cases of columns in buildings with sways to improve the design of these construction elements. The effect of n-panels system on the structural analysis as quantities and cost effective matter can be taken for future ideas to enhance the decision making of these important and widely used construction elements.



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