

**EVALUATION APPROACHES FOR SUSTAINABLE TRANSPORTATION
SYSTEMS AND THEIR APPLICATIONS**

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LIST OF SYMBOLS

AHP	: Analytic Hierarchy Process
CFS	: Cognitive Fuzzy Sets
CI	: Consistency Index
CINFUS	: Continuous Intuitionistic Fuzzy Sets
C-IFS	: Circular Intuitionistic Fuzzy Sets
DqROFS	: Decomposed q-Rung Orthopair Fuzzy Sets
DqROFN	: Decomposed q-Rung Orthopair Fuzzy Number
DqROFPIS	: Decomposed q-Rung Orthopair Fuzzy Positive Ideal Solution
DqROFNIS	: Decomposed q-Rung Orthopair Fuzzy Negative Ideal Solution
DM	: Decision-Maker
DqROWGM	: Decomposed q-Rung Orthopair Weighted Geometric Mean
EDAS	: Evaluation based on Distance from Average Solution
FFS	: Fermatean Fuzzy Sets
HFS	: Hesitant Fuzzy Sets
IFS	: Intuitionistic Fuzzy Sets
IVFS	: Interval Valued Fuzzy Sets
MAIRCA	: The Multi Attributive Ideal-Real Comparative Analysis
NS	: Neutrosophic Sets
PyFS	: Pythagorean Fuzzy Sets
PFS	: Picture Fuzzy Sets
q-ROFS	: Q-rung Orthopair Fuzzy Sets
SFS	: Spherical Fuzzy Sets
PIFS	: Proportional Intuitionistic Fuzzy Sets
PPFS	: Proportional Picture Fuzzy Sets
VIKOR	: Vlse Kriterijumsa Optimizacija I Kompromisno Resenje
WASPAS	: Weighted Aggregated Product Assessment
WSM	: Weighted Sum Model
WPM	: Weighted Product Model
SI	: Score Index

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ABSTRACT

Sustainable Transportation is one of the key factors for societies aiming to build a future based on Sustainable Development. Both the United Nations and the European Union are focusing on projects, plans and studies on Sustainable Transportation to achieve the Sustainability goal by 2030. The main reason for this is that as a result of a significant amount of research, it has been determined that Transportation in the broad sense contributes approximately 30% of the total harmful gas emissions released into our world. This study, as a result of a wide literature review, has observed that one of the most important methodologies in evaluating the criteria for sustainability of any sector or environment is Multi-Criteria Decision Making and Analysis techniques. Similarly, in evaluating sustainable transportation systems, MCDM methodologies have once again become one of the most frequently used techniques to make comparisons between alternatives. It has been observed that Fuzzy Set Theory is also used together with MCDM methods to eliminate uncertainty. Decomposed fuzzy sets are a fuzzy set extension that is newly introduced to the literature and derived from intuitive fuzzy sets. Its basic logic is based on taking the evaluations of experts from different perspectives when making decisions in situations involving uncertainty. This approach aims to enable experts to express their opinions by asking the same topic in different ways. By using functional and dysfunctional questions based on optimistic and pessimistic perspectives, decision makers are helped to reveal their knowledge, professional experience and personal opinions more clearly. In addition, decomposed fuzzy sets aim to provide the best solution under uncertainty by considering the situations where decision makers experience indecision or give inconsistent answers. Within the scope of this thesis, decomposed fuzzy sets are integrated with q-Rung Orthopair fuzzy number set and expanded with EDAS and MAIRCA from multi-criteria decision making methods and developed as DqROF EDAS and DqROF MAIRCA methods, respectively.

These new methods are explained with case studies for the aviation sector. Then, in order to test their validity and make comparisons, q-Rung Orthopair, which is the basis of decomposed fuzzy sets, was compared with fuzzy set-based approaches and it was seen that the new methods gave quite successful results. Therefore, this study aims to contribute to the field of Sustainable Transportation with a new MCDM technique. A case study of the techniques developed were conducted in one of the world's largest airline companies. In this way, the three main goals of sustainability related to transportation, social, economic and environmental, were evaluated in a quite complex environment with a new methodology. As a result, this study aims to contribute to both society and literature.

ÖZET

Sürdürülebilir Ulaşım, Sürdürülebilir Kalkınmaya dayalı bir gelecek inşa etmeyi hedefleyen toplumlar için kilit faktörlerden biridir. Hem Birleşmiş Milletler hem de Avrupa Birliği, 2030 yılına kadar Sürdürülebilirlik hedefine ulaşmak için Sürdürülebilir Ulaşım konusundaki proje, plan ve çalışmaları merkeze alıyor. Bunun başlıca sebebi önemli miktar araştırmanın sonucunda, geniş anlamda Ulaşımın toplam dünyamıza salınan toplam zararlı gaz emisyonuna yaklaşık %30 katkıda bulunduğu tespit edilmiştir. Bu çalışma, geniş bir literatür taraması sonucunda, herhangi bir sektör veya ortamın sürdürülebilirliğine ilişkin kriterlerin değerlendirilmesinde en önemli metodolojilerden birinin Çok Kriterli Karar Verme ve Analiz teknikleri olduğunu gözlemlemiştir. Benzer şekilde, sürdürülebilir ulaşım sistemlerini değerlendirmede, alternatifler arasında karşılaştırma yapmak için ÇKKV metodolojileri bir kez daha en sık kullanılan tekniklerden biri olmuştur. Genellikle belirsizliği gidermek için ÇKKV yöntemleriyle birlikte Bulanık Küme Teorisinin de kullanıldığı gözlemlenmiştir. Ayırıştırılmış bulanık kümeler, literatüre yeni kazandırılan ve sezgisel bulanık kümelerden türetilen bir bulanık küme uzantısıdır. Temel mantığı, belirsizlik içeren durumlarda karar alırken uzmanların değerlendirmelerini farklı açılardan almak üzerine kuruludur. Bu yaklaşım, aynı konuyu farklı biçimlerde sorarak uzmanların düşüncelerini ifade etmelerini sağlamayı amaçlar. Özellikle iyimser ve kötümser bakış açılarını temel alan işlevsel ve işlevsiz sorular kullanılarak, karar vericilerin bilgi birikimleri, mesleki deneyimleri ve kişisel görüşlerini daha net bir şekilde ortaya koymalarına yardımcı olunur. Ayrıca, ayırıştırılmış bulanık kümeler, karar vericilerin kararsızlık yaşadığı veya tutarsız cevaplar verdiği durumları da göz önünde bulundurarak, belirsizlik altında en iyi çözümü sunmayı hedefler. Bu tez kapsamında, ayırıştırılmış bulanık kümeler q-Rung Orthopair bulanık sayı kümesiyle entegre edilmiş ve çok kriterli karar verme yöntemlerinden EDAS ve MAIRCA ile

geniřletilerek sırasıyla DqROF EDAS ve DqROF MAIRCA yöntemleri olarak geliřtirilmiřtir.

Geliřtirilen bu yeni yöntemler, havacılık sektörüne yönelik vaka çalıřmaları ile açıklanmıřtır. Ardından, geçerliliklerini test etmek ve kıyaslama yapmak amacıyla, ayrıřtırılmıř bulanık kümelerin temelini oluřturan q-Rung Orthopair bulanık küme tabanlı yaklařımlar ile karřılařtırılmıř ve yeni yöntemlerin oldukça bařarılı sonuçlar verdiđi görölmüřtür. Dolayısıyla bu çalıřma, yeni bir ÇKKV tekniđi ile Sürdürülebilir Ulařım alanına katkı sađlamayı amaçlamaktadır. Geliřtirilen olan tekniđin dünyanın en büyük havayolu řirketlerinden birinde vaka çalıřması yapılmıřtır. Bu sayede sosyal, ekonomik ve çevresel; Sürdürülebilirliđin ulařımla ilgili üç ana hedefi, yeni bir metodoloji ile oldukça karmařık bir ortamda deđerlendirilmiřtir. Sonuç olarak bu çalıřma hem topluma hem de literatüre katkı sađlamayı amaçlamaktadır.

1. INTRODUCTION

1.1 Sustainable Transportation

According to Cambridge Dictionary, the meaning of « Transportation » is explained as the movement of people or goods from one place to another. The word etymology of Transportation is also guiding us to the same meaning, since the word is constructed by two different Latin words, namely “Trans” and “Portare” which are standing for the words of Across and Carry in English respectively (Cambridge Dictionary, n.d.).

While examining the roots of transportation historically then it can be seen that the history of transportation is strongly related to the focus of the study about the mode of transportation. For instance, if we are talking about the transportation mode of water, then the observations of first boats to cross the sea is directing us to 40,000 to 60,000 thousand years ago, the era which Australia was colonized first. However, further findings are also indicating that trade, which is using transportation mode of water, is going back to 900,000 years ago (Divall and Revill, 2005). Therefore, it is important for us to clearly define the modes of transportation which can be listed as follows (Williams, 2003):

- Air
- Land (Rail & Road)
- Water
- Cable
- Pipeline
- Space

One other important definition is crucial in terms of describing the fields of transportation. The fields of transportation can be described by three main categories: Vehicles, Infrastructure, and Operations. Thus, while transportation studies may focus on one of these fields or they may put concentration on a combination of more than just one field (Igathinathane et al., 2016 and Ryu et al., 2006).

The vehicles are highly dependent on whether the transported being is a material or a human-being. However, the vehicles are wagons, automobiles, bicycles, buses, trains, trucks, helicopter, watercraft, spacecraft, and aircraft. Likewise, the infrastructure preference that will be benefited is changing according to the chosen transportation vehicles. The most common vehicles are roads, railways, airways, waterways, canals, pipelines, and terminals (like airports, airports, bus and train stations, warehouses, and fuel stations).

The word Sustainability has also the roots coming from a Latin word « Sustinere » which basically means “Hold Up” in English. When it comes to Sustainability, there are two main definitions for this terminology in Cambridge Dictionary as follows:

¹the quality of being able to continue over a period of time.

²the quality of causing little or no damage to the environment and therefore able to continue for a long time.

Although both definitions are both valid and in use, throughout this thesis, the second one will be referred in a frequent way since Sustainable Development is going to be one of the primary goals of this study.

The Sustainable Development Goals (SDGs) are defined by United Nations General Assembly (UN-GA) to be achieved by the year of 2030. There are 17 SDGs as follows: The 17 SDGs are: (1) No Poverty, (2) Zero Hunger, (3) Good Health and Well-being, (4) Quality Education, (5) Gender Equality, (6) Clean Water and Sanitation, (7) Affordable and Clean Energy, (8) Decent Work and Economic Growth, (9) Industry, Innovation and Infrastructure, (10) Reduced Inequality, (11) Sustainable Cities and Communities, (12) Responsible Consumption and Production, (13) Climate Action, (14) Life Below Water, (15) Life On Land, (16) Peace, Justice, and Strong Institutions, (17) Partnerships for the Goals (Sharmina et al., 2020).

To clearly describe the main objectives of SDGs we may use the following categorization of them:

1. Economic factors

2. Social factors
3. Environmental factors

After understanding transportation and sustainability, then we are able to easily define Sustainable Transportation (or Sustainable Transport) which is the main objective regarding the topic of this study. Sustainable Transportation considers and evaluates all fields of transportation, which are Infrastructure, Vehicles and Operations, in a way that main sustainable development factors are satisfied.

The first moment that transportation was regarded as one of the key elements of sustainable development was in 1992 by United Nations during Earth summit. Later in 2012, sustainable transportation is put on the center to accomplish the sustainability goals since the studies showed that one fourth of greenhouse gas¹ emissions are caused by transportation (Chan et al. 2020). In 2016, the United Nations Secretary-General's High-level Advisory Group on Sustainable Transport collaborated with governments, transport providers, businesses, financial institutions, public and other contributors to support sustainable transportations systems and to include them into improvement policies (Kamga et al., 2023).

1. Prepare transportation planning, related regulations and investment choices according to three sustainable development indicators, namely as, social improvement, environmental effects and financial enhancements, plus a comprehensive life cycle study.
2. Bring every sustainable transportation planning initiative with and adequately balanced enhancement of transportation modes: stakeholders should be brought together vertically among the hierarchy of government and horizontally through modes, physical regions and business areas.
3. Build supporting organizational, lawful, and regulatory government structures to boost successful sustainable transportation.
4. Endorse technical abilities of transportation administrators and implementers, specifically in countries which are currently developing, by agreements with

¹ Greenhouse gases refer to gases in the atmosphere, both naturally occurring and human made, that absorb and release radiation at particular wavelengths within the thermal infrared spectrum emitted by the Earth's surface, the atmosphere and clouds.

global institutions, multilateral development banks, and governmental authorities at every level, to make sure equality about reaching to market, job vacancies, training opportunities and other necessary resources.

5. Encourage resolutions for avoiding fatalities and injuries on road traffic.
6. Stimulate informed and proactive public as an important stakeholder in improving sustainable transportation perspective.
7. Introduce follow-up and evaluation approaches for sustainable transportation and create capacity to act as a group which is formed by different stakeholders to analyze reputable and trustworthy information and data analysis.
8. Encourage a variety of funding options and consistent financial structures to support the progress of sustainable transportation systems, programs and endeavors.
9. Improve financial structures of global development and climate for sustainable transportation.
10. Stimulate technological infrastructure of sustainable transport utilizing the result-based financial initiatives and regulations of government which promote private sector inclusion by several motivational arrangements.

Transportation has been shaped by technological and social developments throughout human history and has been one of the cornerstones of economic growth and global integration. From ancient times to the present, transportation vehicles and systems have evolved depending on needs and discoveries. Today, the transportation sector is entering a new era shaped by issues such as energy efficiency, environmental sustainability and digitalization (Hong et al., 2011).

Transportation initially consisted of simple walking and animal-backed transportation. During the ancient Roman Empire, highways and sea routes were developed. Rome built a 250,000 km road network, and these roads were of great importance for war and trade (Dempsey, 2002). In the Middle Ages, transportation was mostly limited to transportation by sea. However, with the industrial revolution in the 19th century, revolutionary developments in transportation took place.

The discovery of steam power allowed the emergence of railways. In 1825, George Stephenson laid the foundation of the railway by operating his first steam train. This development accelerated sea and air transportation as well as land transportation (Dempsey, 2002). The production of motor vehicles gained momentum in the early 20th century. Henry Ford's production of the Model T in 1908 led to the individualization of transportation and the development of the automobile industry (Ford, 1922).

Today, transportation has become a major economic sector worldwide. The World Bank states that the transportation sector's contribution to global GDP in 2020 was around 10% (Gudmundsson, 2021). While land transportation is the most common form of transportation worldwide, sea, air and rail transportation also have a significant share. Land transportation is still the most common form of transportation. According to 2022 data, there are approximately 1.4 billion motor vehicles worldwide, and these vehicles constitute the majority of transportation (Ntshingila, 2022). However, these vehicles are a significant source of greenhouse gas emissions. Road transport accounts for 74% of total carbon dioxide emissions in the transportation sector worldwide (Mannattuparambil et al., 2022).

Railways are also an important form of transportation. As of 2020, China has the largest railway network in the world, operating 150,000 kilometers of railways and many transportation systems, including high-speed trains (He et al., 2021). Railway transportation is growing rapidly, especially in Asia and Europe. Air transportation has shown a major recovery after the COVID-19 pandemic. According to IATA, 4 billion passengers participated in air transportation worldwide in 2022, indicating a significant increase in the sector (IATA, 2022). However, air transportation creates environmental impacts due to high carbon emissions. Global air transportation's greenhouse gas emissions account for 2.5% of total transportation emissions (ICAO, 2021).

Figure 1.1 shows that after electricity and heat, transport sector comes second as a sector which contributes to the greenhouse gas emissions (GHG) in the world. This proves the

need for sustainable transportation research studies to decrease those emissions from environmental perspective.

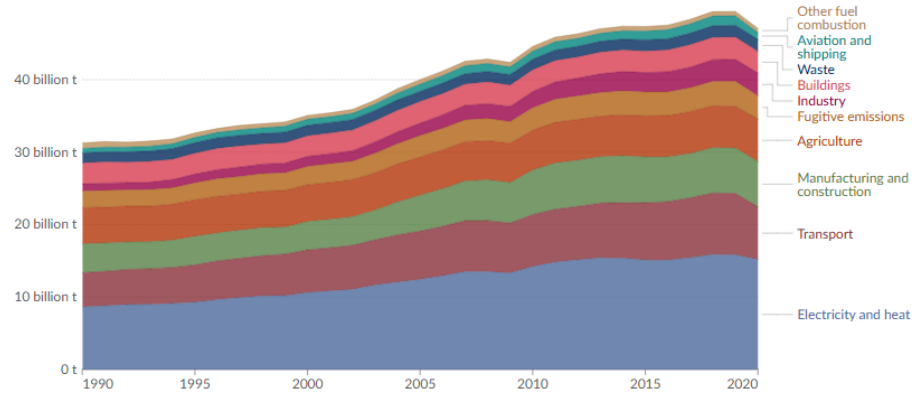


Figure 1.1 Greenhouse gas emissions by sector (World Climate Watch, 2023)

When we specifically examine the sectors that cause GHG emissions in Türkiye, we understand that the picture is the same with the world. Figure 1.2 shows that the transport sector comes second, right after electricity and heat and is followed by the manufacturing and construction sector. Thus, scientific research with a focus on sustainable transportation is critical in our country as well to lower those emissions and create an environmentally sustainable future.

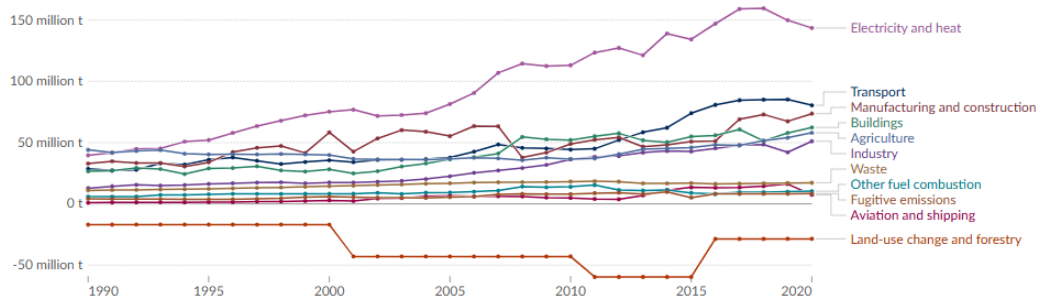


Figure 1.2 Greenhouse gas emissions by sector in Türkiye (Climate Watch, 2023)

Figure 1.3 depicts the top emitters (as countries) of carbon-dioxide (CO₂) in the aviation sector. United States leads the list with a great margin while China follows, however Türkiye is also one of the with 11.93 million tons of CO₂ emissions in 2018. This also adds up to the reasons why the aviation sector is chosen as the application area of the proposed methods in this thesis.

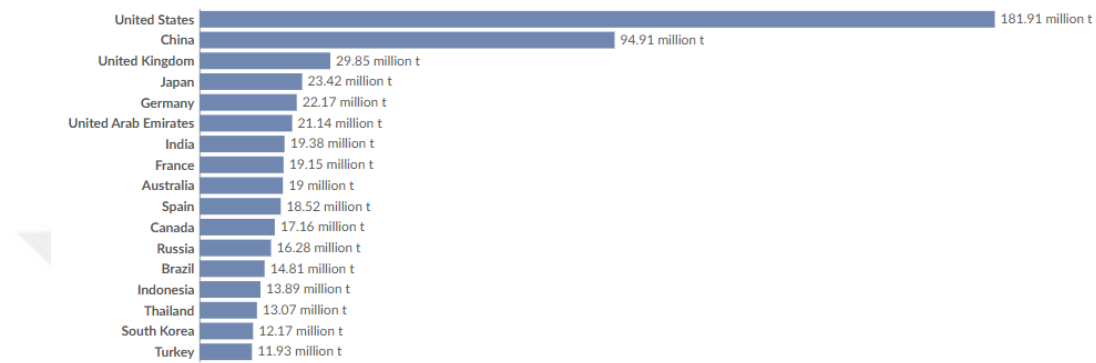


Figure 1.3 CO₂ Emissions from Aviation in 2018 (Graver et al., 2019)

The future of transportation will be shaped by environmentally friendly technologies. Electric vehicles, autonomous vehicles and sustainable air transportation are among the transportation trends of the future. Electric vehicles (EVs) offer significant environmental advantages compared to fossil fuel vehicles. In 2022, electric vehicle sales worldwide increased by 60% and exceeded 26 million. These figures are expected to reach 30-40% by 2030. Electric vehicles reduce carbon emissions in road transport by 40% (IEA, 2023).

Autonomous vehicles (driverless vehicles) are another technology that is set to revolutionize transportation. Autonomous vehicles have the potential to reduce traffic accidents, increase road safety and make journeys more efficient. By 2025, autonomous vehicles are expected to become widespread worldwide (Deloitte, 2020).

Sustainable air transportation will also play an important role in the future. Electric aircraft and hybrid aircraft technologies are technologies that are expected to become

widespread in commercial air transportation by 2035. NASA states that such aircraft will be used in commercial airlines by 2035 (Staack et al., 2021).

In addition, rail transportation will become even more important in the future, especially thanks to high-speed trains and high-speed railways. In Europe, it is planned to build 10,000 kilometers of high-speed train lines by 2030 (Baumeister, 2019). These developments aim to transform road transportation into an environmentally friendly way.

The transportation sector has historically undergone a major evolution and has become one of the key drivers of economic growth and global integration. Today, transportation is becoming more environmentally friendly with technological innovations and sustainability-focused solutions, despite being a major source of carbon emissions. Developments such as electric vehicles, autonomous transportation systems and sustainable air transportation will be key factors shaping the future of transportation.

The transportation sector has extensive environmental, social and economic impacts on society and therefore it is imperative to embrace the concept of sustainable development (Zhao et al., 2020). Transportation plays a critical role in the economic development of cities and countries and therefore the efficiency and effectiveness of transportation systems are of great importance (Roman, 2022). However, transportation systems often fall short in terms of environmental and social sustainability (Bamwesigye and Hlaváčková, 2019). Concerns about the consequences of transportation technology date back much further, although the term sustainability did not become widespread until the 1980s (Sultana et al., 2019). The sustainability of transportation systems must be addressed in environmental, economic and social dimensions (Bamwesigye and Hlaváčková, 2019).

Sustainable transportation should meet current transportation and mobility needs without compromising the ability of future generations to meet these needs (Satoh and Lan, 2007). Sustainable transportation policies are generally based on two main approaches: improving current transportation models with technologies that consume less resources and produce less waste and adopting a holistic approach that encourages more sustainable travel options (Bamwesigye and Hlaváčková, 2019). Innovative solutions such as new fuel types, electric vehicles and smart transportation systems are important for sustainable

transportation (Mehar et al., 2014). In addition, for sustainable transportation policies to be successful, the broader systems in which transportation activities are located should not be ignored (Goldman and Gorham, 2006).

In recent years, sustainable transportation research has increased significantly. These studies cover topics such as sustainable transportation indicators and performance models, sustainable transportation policies, stakeholder participation, supply chain and logistics management, environmental impact, travel behavior, new fuels for vehicles, transportation strategic planning and cycling and public transportation. These topics address different dimensions of sustainable transportation and shed light on future research directions (Bamwesigye and Hlaváčková, 2019).

Thus, nowadays sustainable transport is important to accomplish economic, social, and environmental sustainability. Problems, for instance, traffic congestion, air pollution and global warming are vital elements that create danger for the sustainability of conventional transportation operations.

The transportation industry is forecasted to be accounted for closely 24% of GHG emissions globally (Badassa et al., 2020). This necessitates to reevaluate transportation systems and adjust to the novel sustainable options.

For instance, as in line with the United Nations Sustainable Development Goals, it is targeted to improve the sustainability of transportation operations up to 2030 (Cledou et al., 2018).

These goals show the vitality of and the need for sustainable transport. Certain strategies are sought to get applied to build sustainable transport. This covers empowering public transportation, constructing bicycle roads and supporting electric vehicles.

The usage of electric bicycles appears to be an essential option in urban transport. The BiciMAD project applied in Madrid ensures a sustainable plan for urban transport through implementing 2028 electric bicycles to 165 points (Salmerón-Manzano & Manzano-Agugliaro, 2018).

Moreover, usage of electric vehicles shows 50% less emissions than the traditional engine vehicles (Sałabun et al., 2019).

The aviation industry is a vital part of sustainable transportation. The world's aviation sector emitted nearly 915 million tonnes of carbon dioxide in 2019 (Sharmina et al., 2020). Subsequently, sustainability studies in aviation are important to decreasing environmental effects.

Sustainable aviation frameworks incorporate the utilization of biofuels, the improvement of technologies that enhance energy efficiency and carbon compensation programs. For instance, certain airway companies set targets for decreasing their emissions by 80% through the utilization of biofuel in their planes (Letnik, 2023).

Furthermore, the improvement of electric and autonomous (driverless) aircraft is regarded as a vital step to provide sustainability in the aviation sector (Kovačić et al., 2022). City planning and policy-making procedures stand out as critical stakeholders of ensuring sustainable transport.

Approaches that evaluate the sustainability index of public transportation operations can be utilized to set ideal transport infrastructure of urban areas (Karjalainen and Juhola, 2019). For instance, to accomplish the transport aims, transport infrastructure in urban areas are required to get enhanced and public transport operations combined.

In this manner, intelligent urban implementations are essential to improve the efficiency of transportation systems (Bamwesigye and Hlaváčková, 2019). Eventually, sustainable transport is one of the most important stakeholders to provide economic, social and environmental sustainability.

Problems faced in the transportation industry can be handled with novel resolutions and sustainable routines. The aviation sector plays a critical role for this revolution, and the goal is to decrease environmental effects by sustainable aviation procedures. Related to this, research on sustainable transportation and aviation will improve the establishment of a more sustainable transport system in the years to come.

The implementation of fuzzy multi-criteria decision-making (MCDM) methods in sustainable transportation and aviation sectors obtained important focus due to their capability to overcome uncertainty and imprecision in decision-making procedures. In the context of sustainable transportation, fuzzy MCDM techniques, such as the Weighted Aggregated Sum Product Assessment (WASPAS) and Analytic Hierarchy Process (AHP), are applied to assess several transportation alternatives based on multiple

conflicting criteria, including environmental effect, cost, and efficiency (Turskis et al., 2019; Hamurcu & Eren, 2020). These techniques enable decision-makers to include qualitative evaluations and linguistic variables, which are significant when assessing sustainability measures that are usually subjective in its essence (Kahraman et al., 2015).

In the aviation industry, fuzzy MCDM solution frameworks are used to evaluate airport operations and performance under uncertain circumstances. For example, Ahmad et al. showed the strength of fuzzy MCDM in assessing airport performance along with multiple criteria, underlining how fuzzy logic can improve decision-making by dealing with the ambiguity inherent in performance measures (Ahmad et al., 2019). Likewise, the inclusion of fuzzy methods in supplier selection within the aviation supply chain is proved to improve the agility of decisions by taking into account numerous qualitative and quantitative criteria (Polat et al., 2017). In general, applying fuzzy MCDM in both sustainable transportation and aviation not only supports more informed decision-making but also aligns with the increasing focus on sustainability in these industries.

In the next section fuzzy sets and their extensions are provided with brief explanations.

1.2 Fuzzy Sets and Fuzzy Set Extensions

1.2.1 Fuzzy Sets

Classical logic, also known as Boolean logic (1 or 0), is a discipline whose principles and foundations were laid by Aristotle (384-322 BC). Since the events experienced or likely to be experienced in life have a degree of occurrence, two-valued logic was inadequate in evaluating the functioning of events and the concept of multivalence was needed. Based on this need, fuzzy logic, which is based on the idea of multivalence, was put forward by Dr. Lotfi Zadeh in the 1960s (Zadeh, 1965).

The basic idea that fuzzy logic is based on is that, while transforming human ideas and evaluations into a more specific and concrete structure, instead of classifying them with

definite labels, it matches them with values between $[0,1]$ (Organ and Kenger, 2012). “0” represents absolute wrongness, and “1” represents absolute truth. In an event or situation that is desired to be evaluated, it is a great advantage for individuals whose ideas are needed to be able to express their preferences, feelings or experiences when they do not have numerical data such as profit, cost, volume, weight, etc. Linguistic evaluations are used when transferring in these ways (Gül, 2021). Linguistic evaluations are the data used to model fuzziness and are therefore very important. When studies involving fuzzy logic are examined, it is seen that it is generally used in the fields of engineering, computer science, mathematics, decision-making, health sciences, social and human sciences, physical sciences and life sciences (Kahraman et al., 2016).

In fuzzy sets, the degree of uncertainty is defined by the “membership degree” and is set to a value between $[0,1]$. The membership degree of any x to the set A is expressed by $\mu_{\tilde{A}}(x)$.

The membership degree is calculated using the membership function. In classical sets, the membership function is as in Eq. (1.1):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \notin A \\ 1, & x \in A \end{cases} \quad (1.1)$$

In fuzzy sets, this situation is shown together with the cluster element and membership degree as in (1.2):

$$\tilde{A} = \{x, \mu_{\tilde{A}}(x), x \in X\} \quad (1.2)$$

Fuzzy sets are represented in different ways, depending on whether they are in a finite or infinite universal set. The representation of the fuzzy set A in a finite universal set X is as in (1.3):

$$\tilde{A} = \frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \dots + \frac{\mu_{\tilde{A}}(x_n)}{x_n} \quad (1.3)$$

In an infinite universal set X , the notation is as in (1.4):

$$\int_x \frac{\mu_{\tilde{A}}(x)}{x} \quad (1.4)$$

Although fuzzy sets can be of different types, the most well-known and frequently used fuzzy sets are triangular and trapezoidal fuzzy sets. Triangular fuzzy sets are shown as in (1.5):

$$\tilde{A}_{(a,b,c)}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & \text{other} \end{cases} \quad (1.5)$$

The graphical representation of this membership function is given in Figure 1.4:

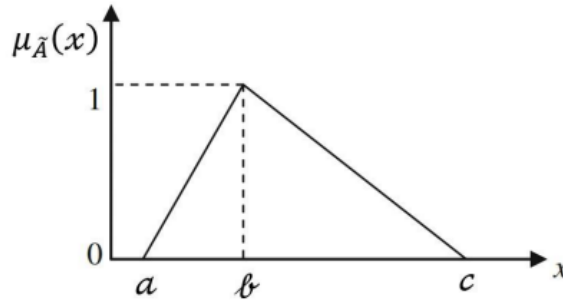


Figure 1.4 Triangular fuzzy set

The representation of trapezoidal fuzzy sets is as in equation (1.6):

$$\tilde{A}_{(a,b,c,d)}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & \text{other} \end{cases} \quad (1.6)$$

The graphical representation of this membership function is given in Figure 1.5.

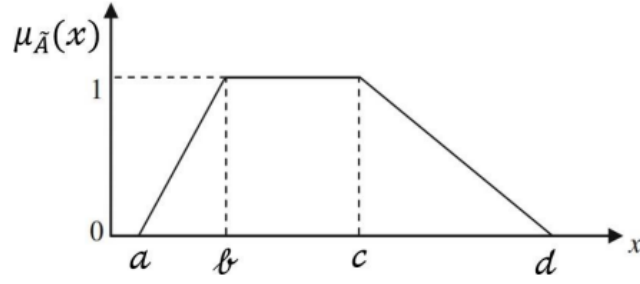


Figure 1.5 Trapezoidal fuzzy set

1.2.2 Fuzzy Set Extensions

Although fuzzy sets were introduced to cope with uncertainty, they may be inadequate in different types and degrees of uncertainty. For this reason, many extensions of traditional fuzzy sets have been developed to be used for different uncertainty situations. Each of these extensions has its own advantages and disadvantages, adequacy and inadequacy. The historical order of fuzzy set extensions is given in Figure 1.6.

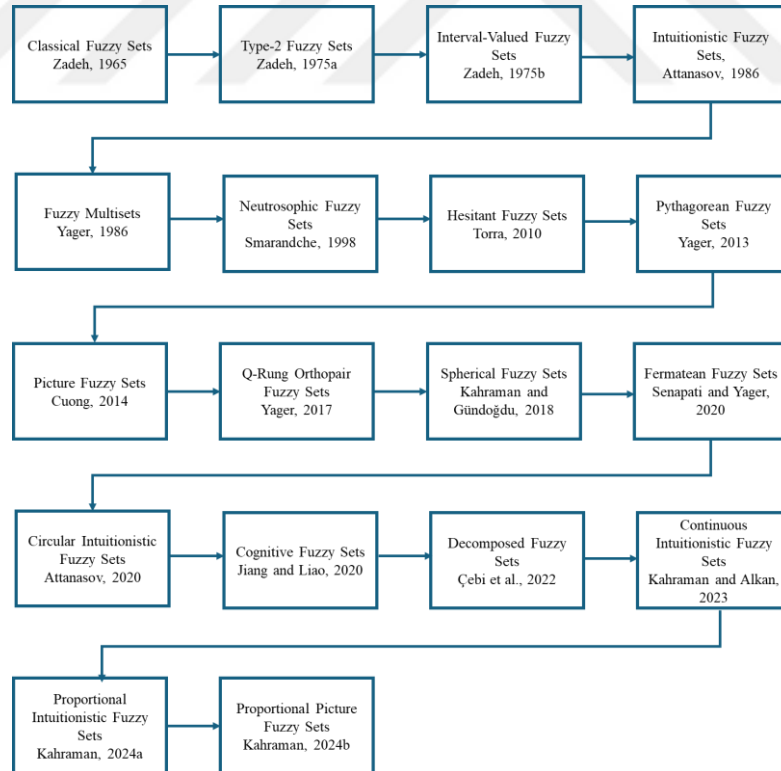


Figure 1.6 Chronological order of fuzzy set extensions.

1.2.2.1 Type-2 Fuzzy Sets

With the emergence of traditional (Type-1) fuzzy sets, it has become possible to model the uncertainties encountered. However, since there may be cases where this modeling is not at the desired level, type-2 fuzzy sets have been developed in order to increase the success of the modeling. In type-1 fuzzy sets, the fact that the parameters are definite values can limit the modeling of uncertainty. Because in some cases, the uncertainty is too advanced to be determined with deterministic parameters. For this reason, type-2 fuzzy sets have been developed in which these parameters can also be expressed with fuzzy sets [5]. The basic characteristic feature is that the membership functions, which are 2-dimensional in type-1 fuzzy sets, are 3-dimensional in type-2 fuzzy sets. In other words, in a type-2 fuzzy set, each $x \in X$ value has a primary and secondary membership function. The representation of any type-2 fuzzy set is as in (1.7):

$$\tilde{A} = \{(x, u), \mu_{\tilde{A}}(x, u) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{A}}(x, u) \leq 1 \quad (1.7)$$

Depending on whether the membership functions are discrete or fuzzy, other representations are as in (1.8) and (1.9):

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad (1.8)$$

$$\tilde{A} = \sum_{x \in X} \sum_{u \in J_x} \mu_{\tilde{A}}(x, u) / (x, u) \quad (1.9)$$

$J_x \subseteq [0,1]$ and $\iint$ mean the union over all valid x and u .

1.2.2.2 Interval-Valued Fuzzy Sets

Interval-Valued Fuzzy Sets (IVFS) are a special case of type-2 fuzzy sets where the lower and upper degrees of membership are determined. In other words, interval-valued fuzzy sets allow the membership degree to be defined as an interval. It was proposed by Zadeh in 1975 (Zadeh, 1975). Its general representation is as in (1.10). In (1.11), the range of membership degrees of the lower and upper limits is given.

$$\tilde{A} = \{x, [\mu_{\tilde{A}}^L(x), \mu_{\tilde{A}}^U(x)]\}, \quad (1.10)$$

$$\mu_{\tilde{A}}^L(x), \mu_{\tilde{A}}^U(x): X \rightarrow [0,1], \forall x \in X, \mu_{\tilde{A}}^L(x) < \mu_{\tilde{A}}^U(x) \quad (1.11)$$

In this representation, $\mu_{\tilde{A}}^L(x)$, represents the lower limit of the interval-valued membership degree, $\mu_{\tilde{A}}^U(x)$ represents the upper limit.

1.2.2.3 Intuitionistic Fuzzy Sets

Intuitive fuzzy sets (IFS) are fuzzy set extensions that add the concept of "uncertainty" to the "membership" and "non-membership" conditions found in type-1 fuzzy sets. They were proposed by Attanasov in 1986 (Atanassov, 1986). The inadequacy of type-1 fuzzy sets in modeling uncertainty can be compensated for by intuitionistic fuzzy sets. The general representation of intuitionistic fuzzy sets is given in (1.12). In such sets, the sum of the membership and non-membership conditions is less than 1, as shown in (1.13), and this sum is equal to 1 with the parameter expressing "uncertainty", as stated in (1.14).

$$A = \{\langle x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x) \rangle | x \in X\} \quad (1.12)$$

$$0 \leq \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) \leq 1 \quad (1.13)$$

$$\pi_{\tilde{A}}(x) = 1 - (\mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x)) \quad (1.14)$$

1.2.2.4 Fuzzy Multisets

Fuzzy multisets were proposed by Yager in 1986 (Yager, 1986). It takes into account the repetitive information and prevents them from being ignored, thus preventing data loss. In this way, the modeling of uncertainty is achieved in a more realistic way. Its general representation is given in (1.15):

$$A = \{\langle x_j, [\mu_A^1(x_j), \mu_A^2(x_j), \dots, \mu_A^{n_j}(x_j)] \rangle : j = 1, \dots, m\} \quad (1.15)$$

For example, let $X = \{x_1, x_2\}$ be a finite set and let the fuzzy multiset A on X be written as $A = \{\langle x_1, (0.4, 0.8, 0.8) \rangle, \langle x_2, (0.3, 0.1) \rangle\}$. The set A is actually $A = \{\langle x_1, 0.4 \rangle, \langle x_1, 0.8 \rangle, \langle x_1, 0.8 \rangle, \langle x_2, 0.3 \rangle, \langle x_2, 0.1 \rangle\}$. That is, x_1 and x_2 have more than one degree of membership.

1.2.2.5 Neutrosophic Sets

Neutrosophic sets (NS) are sets developed based on intuitionistic fuzzy sets. They were developed by Smarandche in 1999 (Smarandche, 1999). In intuitionistic fuzzy sets, the states such as “membership”, “non-membership” and “uncertainty” are interconnected and their sum is equal to 1. However, neutrosophic sets eliminate the requirement for the sum of these parameters to be equal to 1, making it possible to model uncertainty even in the presence of inconsistent data. The basic parameters of a neutrosophic set are “correctness (T)”, “incorrectness (F)” and “uncertainty (I)”. The relationship between them is as in (1.16) and (1.17):

$$(t, i, f) = (\text{truth}, \text{interminacy}, \text{falsehood}) \quad (1.16)$$

$$0 \leq t + i + f \leq 1, t, i, f \in [0, 1] \quad (1.17)$$

The neutrosophic set A in any universal set X is expressed by the membership functions $T_A(x)$ for truth, $I_A(x)$ for uncertainty, and $F_A(x)$ for falsity. These functions are in the range $] -0, 1 +[$, but no restriction can be stated for their sum. The general representation of the neutrosophic set X is given in (1.18).

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X\} \quad (1.18)$$

1.2.2.6 Hesitant Fuzzy Sets

In fuzzy sets, the membership degree is a value between $[0, 1]$ and this value is mostly determined by the decision maker. In this case, the decision maker's evaluation of the issue is of great importance and a mistake that the decision maker can make can change the entire course of the problem. For this reason, in order to minimize the effect of any

wrong evaluation, hesitant fuzzy sets were developed by Torra in 2010 (Torra, 2010). With hesitant fuzzy sets, the membership degree of an element to a set can be defined with more than one value between $[0,1]$. Its mathematical representation is given in (1.19):

$$\tilde{A} = \{\langle x, h_{\tilde{A}}(x) \rangle | x \in X\} \quad (1.19)$$

The bounds of h are as shown in (1.20) and (1.21):

$$h^-(x) = \min h(x) \quad (1.20)$$

$$h^+(x) = \max h(x) \quad (1.21)$$

$h^-(x)$ represents the lower bound and $h^+(x)$ represents the upper bound.

1.2.2.7 Pythagorean Fuzzy Sets

Pythagorean fuzzy sets (PyFS) are basically derived from intuitionistic fuzzy sets but also cover a wider range of situations than those covered by intuitionistic fuzzy sets. They allow the condition that the sum of the degrees of membership and non-membership in intuitionistic fuzzy sets does not exceed 1. In other words, the sum of the degrees of membership and non-membership in Pythagorean fuzzy sets can exceed 1. However, the sum of the squares of the degrees of membership cannot be greater than 1. It was put forward by Yager in 2013 (Yager, 2013). The definition of a Pythagorean fuzzy set is as in (1.22):

$$\tilde{P} = \{\langle x, \mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x) \rangle; x \in X\} \quad (1.22)$$

$\mu_{\tilde{P}}: X \rightarrow [0,1]$ is the membership degree, $\nu_{\tilde{P}}: X \rightarrow [0,1]$ is the non-membership degree and $0 \leq (\mu_{\tilde{P}}(x))^2 + (\nu_{\tilde{P}}(x))^2 \leq 1$ for $\forall x \in X$.

The index π , which indicates the degree of hesitation of whether x belongs to the Pythagorean fuzzy set $\tilde{P}$, is defined as in (1.23):

$$\pi_{\tilde{P}}(x) = \sqrt{1 - \mu_{\tilde{P}}(x)^2 - \nu_{\tilde{P}}(x)^2} \quad (1.23)$$

1.2.2.8 Picture Fuzzy Sets

Picture fuzzy sets (PFS) were introduced in the literature by Cuong and Krenovich in 2013 (Cuong and Krenovich, 2013). Accordingly, the representation of a descriptive fuzzy set in a set X is given in (1.24):

$$A = \{(x_i, \mu_A(x_i), \eta_A(x_i), \vartheta_A(x_i)) : x_i \in X\} \quad (1.24)$$

$\mu_A(x_i) \in [0,1]$ indicates the positive membership degree of x to A , $\eta_A(x_i) \in [0,1]$ indicates the neutral membership degree of x to A , $\vartheta_A(x_i) \in [0,1]$ indicates the negative membership degree of x to A . On the other hand, there is a relation between these membership degrees, $0 \leq \mu_A(x_i) + \eta_A(x_i) + \vartheta_A(x_i) \leq 1$. $\varepsilon_A(x_i) = 1 - (\mu_A(x_i) + \eta_A(x_i) + \vartheta_A(x_i))$ is called the rejection degree of x . In this context, descriptive fuzzy sets are essentially considered as an extension of traditional fuzzy sets and intuitionistic fuzzy sets because when $\varepsilon_A(x_i) = 0$ the descriptive fuzzy set becomes an intuitionistic fuzzy set, and when, $\varepsilon_A(x_i) = \eta_A(x_i) = 0$ it becomes a traditional fuzzy set.

1.1.2.9 Q-Rung Orthopair Fuzzy Sets

Q-rung orthopair fuzzy sets (q-ROFS) were introduced by Yager in 2017 as a generalization of Pythagorean and intuitionistic fuzzy sets (Yager, 2017). Its general representation is as in (1.25):

$$A = \{\langle x, \mu_A(x), \vartheta_A(x) \rangle | x \in X\} \quad (1.25)$$

Again, in accordance with the basic philosophy of fuzzy logic, $\mu_A: X \rightarrow [0,1]$ indicates the degree of membership and $\vartheta_A(x): X \rightarrow [0,1]$ indicates the degree of non-membership, and the condition specified in (1.26) is satisfied for $x \in X$:

$$(\mu_A(x))^q + (\vartheta_A(x))^q \leq 1 \quad (1.26)$$

In this case, when $q = 1$, the obtained intuitionistic fuzzy sets are the first q -rung fuzzy sets; and when $q = 2$, the obtained Pythagorean fuzzy sets are the second q -rung fuzzy sets.

In Figure 1.7, horizontal axis represents the values that membership degree ($\mu_A(x)$) may get, and vertical axis represents the values that non-membership ($\vartheta_A(x)$) degree may get. As q value enlarges, it is seen that, the value that membership and non-membership degrees can get, is getting larger, thus q -ROFN provides flexibility for evaluations in decision-making processes.

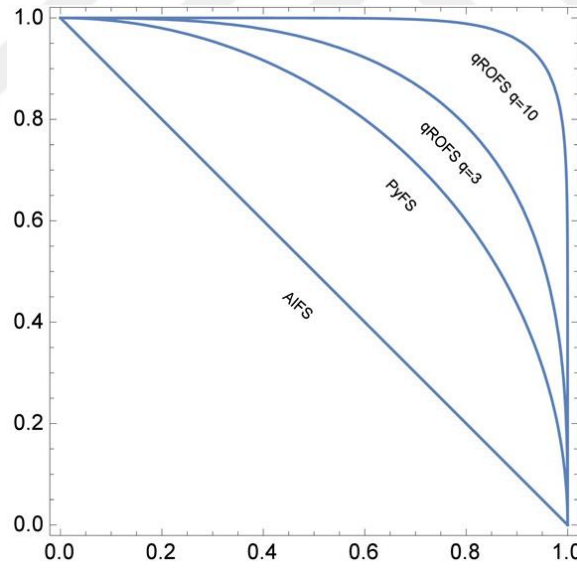


Figure 1.7 q -Rung orthopair fuzzy numbers with different q levels

The degree of hesitation in q -rung fuzzy sets is defined in (1.27):

$$\pi_A = (1 - ((\mu_A(x))^q + (\vartheta_A(x))^q))^{1/q} \quad (1.27)$$

1.2.2.10 Spherical Fuzzy Sets

Spherical fuzzy sets (SFS) were introduced to the literature by Kutlu and Kahraman in 2019 (Kutlu and Kahraman, 2019). It is basically based on the idea that the distance between membership degrees is a more realistic measure as spherical distance rather than linear distance. Based on this information, the membership degree of a spherical fuzzy set is defined as $x^2 = \mu_{\tilde{A}}(x)$, the non-membership degree as $y^2 = \nu_{\tilde{A}}(x)$ and the hesitation degree as $z^2 = \pi_{\tilde{A}}(x)$. The condition $0 \leq \mu_{\tilde{A}}^2(x) + \nu_{\tilde{A}}^2(x) + \pi_{\tilde{A}}^2(x) \leq 1$ must be satisfied for $\forall x \in A$.

The general representation of the spherical fuzzy set $\tilde{A}_S$ defined in the universal set X is as in (1.28):

$$\tilde{A}_S = \{x, (\mu_{\tilde{A}_S}(x), \nu_{\tilde{A}_S}(x), \pi_{\tilde{A}_S}(x)) | x \in X\} \quad (1.28)$$

The membership, non-membership and hesitation degree intervals are as in (1.29):

$$\mu_{\tilde{A}_S}(x): X \rightarrow [0,1], \nu_{\tilde{A}_S}(x): X \rightarrow [0,1], \pi_{\tilde{A}_S}(x)(x): X \rightarrow [0,1] \quad (1.29)$$

1.2.2.11 Fermatean Fuzzy Sets

Fermatean fuzzy sets (FFS) were defined by Senapati and Yager in 2020 for cases where the degrees of membership and non-membership are greater than 1 (Senapati and Yager, 2020). It is derived from intuitionistic fuzzy sets and the mathematical space it covers is larger than intuitionistic fuzzy sets. Its general representation is given in (1.30). Its boundaries are as in (1.31) and (1.32).

$$FFS = \{\langle x, \mu_F(x), \nu_F(x) \rangle : x \in X\} \quad (1.30)$$

$$\mu_F(x): X \in [0,1], \nu_F(x): X \in [0,1] \quad (1.31)$$

$$0 \leq (\mu_F(x))^3 + (\nu_F(x))^3 \leq 1 \quad (1.32)$$

The calculation of the degree of hesitancy is given in (1.33):

$$\pi_F(x) = \sqrt[3]{(\mu_F(x))^3 + (\vartheta_F(x))^3} \quad (1.33)$$

1.2.2.12 Circular Intuitionistic Fuzzy Sets

Circular intuitionistic fuzzy sets (CIFS) were introduced by Attanasov as an extension of intuitionistic fuzzy sets (Attanasov, 2020). The presence of a number circle consisting of membership and non-membership degrees distinguishes circular intuitionistic fuzzy sets from intuitionistic fuzzy sets (Kahraman and Alkan, 2021).

A circular intuitionistic fuzzy set $\tilde{C}_r$ in a fixed universe defined as E is defined as shown in (1.34). Its boundaries are given in (1.35) and (1.36).

$$\tilde{C}_r = \{\langle x, \mu_{\tilde{C}}(x), \vartheta_{\tilde{C}}(x); r \rangle \mid x \in E\} \quad (1.34)$$

$$0 \leq \mu_{\tilde{C}}(x) + \vartheta_{\tilde{C}}(x) \leq 1 \quad (1.35)$$

$$r \in [0,1], \mu_{\tilde{C}}: E \rightarrow [0,1] \text{ ve } \vartheta_{\tilde{C}}: E \rightarrow [0,1] \quad (1.36)$$

The value of r given in the equations is the radius of the circle around each $x \in E$. The calculation of the uncertainty function is as shown in (1.37):

$$\pi_{\tilde{C}}(x) = 1 - \mu_{\tilde{C}}(x) - \vartheta_{\tilde{C}}(x) \quad (1.37)$$

1.2.2.13 Cognitive Fuzzy Sets

Cognitive fuzzy sets (CFS) were introduced in the literature by Jiang and Liao (Jiang and Liao, 2020), again as an extension of intuitionistic fuzzy sets. There is a general consensus that in intuitionistic fuzzy sets, the values of $\mu_A(x)$ and $\vartheta_A(x)$ are independent. However, when determining the degrees of membership and non-membership, evaluators may have

supported both membership and non-membership degrees due to their emotions, experiences or knowledge. In other words, cognitive fuzzy sets are based on the idea that the information used for decision-making cannot be completely divided into two opposite parts. A cognitive fuzzy set is in the form specified in (1.38). The boundaries of the set elements are shown in (1.39) and (1.40).

$$A = \{\langle x, \mu_A(x), \vartheta_A(x), J_A^*(x) \rangle \mid x \in X\} \quad (1.38)$$

$$\mu_A(x), \vartheta_A(x) \text{ and } J_A^*(x) \in [0,1] \quad (1.39)$$

$$0 \leq \mu_A(x) + \vartheta_A(x) - J_A^*(x) \leq 1 \quad (1.40)$$

The value of $J_A^*(x)$ is the degree of combination of the values $\mu_A(x)$ and $\vartheta_A(x)$. Basically, it measures the cognitive overlap while giving information. In Equation (1.41), the equation measuring the degree of uncertainty is given.

$$\pi_A^*(x) = 1 - (\mu_A(x) + \vartheta_A(x) - J_A^*(x)) \quad (1.41)$$

1.2.2.14 Decomposed Fuzzy Sets

Decomposed fuzzy sets (DFS) were introduced to the literature by Çebi et al. in 2022 as an extension of intuitionistic fuzzy sets (Cebi et al., 2022). Its general philosophy is based on modeling human thoughts and perceptions in a more realistic and detailed way through functional and dysfunctional membership functions. Decision makers are asked questions from two different perspectives, functional and dysfunctional, that means the same thing about the subject to be evaluated. The aim is to take into account the situations in which decision makers may be indecisive or the inconsistent or non-overlapping situations they may face when making a decision on the subject being evaluated. Thus, a more accurate evaluation is ensured. The general representation of decomposed fuzzy sets is given in (1.42).

$$\tilde{A} = \left\{ \left(O \left(\mu_{\tilde{A}}^O(x), \vartheta_{\tilde{A}}^O(x) \right), P \left(\mu_{\tilde{A}}^P(x), \vartheta_{\tilde{A}}^P(x) \right) \mid x \in X \right) \right\} \quad (1.42)$$

$\mu_{\bar{A}}(x): X \rightarrow [0,1]$ and $\vartheta_{\bar{A}}(x): X \rightarrow [0,1]$ represent the degree of membership and non-membership of x in O and P , respectively. The rules regarding these values given in (1.43) and (1.44) are satisfied:

$$0 \leq \mu_{\bar{A}}^O(x) + \vartheta_{\bar{A}}^O(x) \leq 1 \quad (1.43)$$

$$0 \leq \mu_{\bar{A}}^P(x) + \vartheta_{\bar{A}}^P(x) \leq 1 \quad (1.44)$$

The inconsistency index is defined in (1.45).

$$I^A = \sqrt{\frac{(\mu_{\bar{A}}^O(x) - \vartheta_{\bar{A}}^P(x))^2 + (\vartheta_{\bar{A}}^O(x) - \mu_{\bar{A}}^P(x))^2 + (1 - \mu_{\bar{A}}^O(x) - \vartheta_{\bar{A}}^O(x))^2 + (1 - \mu_{\bar{A}}^P(x) - \vartheta_{\bar{A}}^P(x))^2}{2}} \quad (1.45)$$

Figure 1.8 illustrates the logic of DFSs. While IFSs only concentrate on how to explain the fullness of the bottle by giving membership and non-membership degrees in terms of fullness, DFSs also obtain membership and non-membership degrees in terms of emptiness as well.

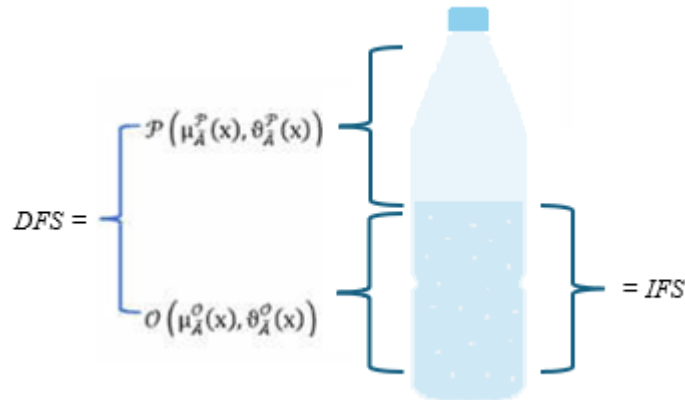


Figure 1.8 Depiction of the DFS logic.

1.2.2.15 Continuous Intuitionistic Fuzzy Sets

Continuous intuitionistic fuzzy sets (CINFUS) are a new fuzzy set extension derived from intuitionistic fuzzy sets by Kahraman and Alkan in 2023 (Kahraman and Alkan, 2023). In intuitionistic fuzzy sets, decision makers specify the degree of membership and non-membership of elements in the set. In continuous intuitionistic fuzzy sets, the degrees of membership and non-membership are represented by second-order non-linear functions. Thus, continuous intuitionistic fuzzy sets have been added to the discrete intuitionistic fuzzy sets and linear intuitionistic fuzzy sets that exist in the literature. Its general notation is shown in (1.46), and its boundaries are shown in (1.47):

$$C = \{\langle x, F(\mu_C(x), \tau), F(\vartheta_C(x), \tau) \rangle \mid x \in X\} \quad (1.46)$$

$$F(\mu_C(x), \tau): X \rightarrow [0,1] \text{ ve } F(\vartheta_C(x), \tau) \rightarrow [0,1] \quad (1.47)$$

Since $F(\mu_C(x), \tau) = \alpha\tau - \alpha\tau^2$ and $F(\vartheta_C(x), \tau) = (4 - \alpha)\tau - (4 - \alpha)\tau^2$, the conditions shown in (1.48) and (1.49) are satisfied:

$$0 \leq F(\mu_C(x), \tau) + F(\vartheta_C(x), \tau) \leq 1 \quad (1.48)$$

$$0 \leq 4\tau - 4\tau^2 \leq 1 \quad \forall x \in X \quad (1.49)$$

The degree of hesitancy is calculated as shown in (1.50):

$$F(\pi, \tau) = 1 - 4(\tau - \tau^2) \quad (1.50)$$

1.2.2.16 Proportional Intuitionistic Fuzzy Sets

Proportional intuitionistic fuzzy sets (PIFS) were introduced to the literature by Kahraman in 2024 as an extension of intuitionistic fuzzy sets (Kahraman, 2024). It was developed because it was not found sufficient and practical to assign decimal numbers directly for the degrees of membership and non-membership of an element in

intuitionistic fuzzy sets. Instead of assigning numbers directly, it is more appropriate to determine relational values for these values.

Consider an intuitionistic fuzzy set of $A = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \rangle | x \in X\}$ shape. Suppose that the decision maker expresses the ratios between $\mu(x)$ and $\nu(x)$ with respect to the degree of hesitation $\pi(x)$ as in (1.51) and (1.52).

$$\mu(x) = k_1 \pi_{\tilde{A}}(x) \quad (1.51)$$

$$\nu(x) = k_2 \pi_{\tilde{A}}(x) \quad (1.52)$$

When the conditions in (1.53) and (1.54) are satisfied, each element in the set A is shown as specified in (1.55).

$$\pi_{\tilde{A}}(x) + k_1 \pi_{\tilde{A}}(x) + k_2 \pi_{\tilde{A}}(x) = 1 \quad (1.53)$$

$$\pi_{\tilde{A}}(x) = \frac{1}{1+k_1+k_2} \quad (1.54)$$

$$A = \{\langle x; k_1 \frac{1}{1+k_1+k_2}, k_2 \frac{1}{1+k_1+k_2} \rangle | x \in X \} \quad (1.55)$$

In this case, the degree of hesitation is $\frac{1}{1+k_1+k_2}$. For example, consider the proposition “ P_0 : Electric vehicles cause problems in cold weather”. The decision maker believes that the degree of membership (truth) of the proposition is twice as large as the degree of hesitation (hesitation) and the degree of non-membership (incorrectness) is five times as large as the degree of indecision. In this case, this intuitionistic fuzzy set can be written as $\langle P_0; 0.250, 0.625 \rangle$ based on (1.55) (Kahraman, 2024). In summary, the general representation in proportional intuitionistic fuzzy sets is as in (1.56).

$$\tilde{A}_P = \{\langle x; k_{\pi 1}, k_{\pi 2} \rangle | x \in X\} \quad (1.56)$$

1.1.2.17 Proportional Picture Fuzzy Sets

Proportional picture fuzzy sets (PPFs) were introduced to the literature by Kahraman in 2024 and are the most up-to-date fuzzy set extension (Kahraman, 2024). In this fuzzy set extension, there is a ratio between the membership, non-membership and indecision degree values in classical picture fuzzy sets. For example, let a decision maker say that the membership degree of element x_0 is four times the indecision degree and equal to the non-membership degree. Let the rejection degree be zero. In this case, this picture fuzzy set is written as $\langle x_0; 0.667, 0.167, 0.167 \rangle$. Thus, the general representation of proportional picture fuzzy set can be expressed as in (1.57):

$$\tilde{A} = \{ \langle x; k_{\pi_1}, k_{\pi_2} \rangle \mid x \in X \} \quad (1.57)$$

On the other hand, $\pi(x) + k_1\pi(x) + k_2\pi(x) \leq 1$ and $\pi(x) \leq \frac{1}{1+k_1+k_2}$. The rejection degree is calculated as shown in (1.58):

$$r(x) = 1 - (\pi(x) + k_1\pi(x) + k_2\pi(x)) \quad (1.58)$$

1.3 Objectives of the Study

1.3.1 Contribution to the Academical Literature

In order to solve real life problems in accordance with the desired conditions and situations, the uncertainty factor must be taken into consideration. For this reason, the traditional fuzzy logic and fuzzy logic extensions developed have been used in solving many real life problems. The aim of this thesis consists of two main parts. In the first part a novel q-rung orthopair fuzzy set extension of decomposed fuzzy sets is introduced with the corresponding arithmetic rules. In the second part, the well-known MCDM methods, EDAS and MAIRCA methods are extended with the novel decomposed q-rung orthopair fuzzy set (DqROFS), which can be considered as a new fuzzy set in the literature.

Therefore, this newly created decomposed q-rung orthopair fuzzy set-based multi-criteria decision-making methods are implemented for aircraft selection problem in the aviation industry from sustainable transportation perspective.

1.3.2 Contribution to Sustainable Development Goals

The evaluation of aircraft models in the aviation sector guides decision makers in the field of “sustainable infrastructure and innovation”. Advanced multi-criteria decision-making methods such as the Decomposed Q-Rung Orthopair Fuzzy Set approach and Bayesian BWM contribute to R&D processes and industrial innovations, aiming to improve environmental and economic performance in the sector.

Air transportation has a significant impact on the development of cities, tourism and trade. The selection of more sustainable aircraft models increases the quality of urban life by reducing noise and air pollution. Considering criteria such as noise pollution, carbon emissions and community impact provides strategic data that supports making cities more livable; thus, it is aimed to contribute to SDG 11.

It is aimed at leading airlines to develop more responsible production and business practices by questioning the efficiency of resource use. It is emphasized that by integrating economic, environmental and social criteria, companies should also consider responsible consumption and production principles instead of focusing only on profit.

Finally, evaluating carbon emissions and fuel efficiency directly increases awareness of combating climate change. This approach encourages the development of low-carbon or carbon-neutral technologies in the airline industry.

1.4 Original Contribution

In the basic logic of traditional fuzzy logic and fuzzy logic extensions, the basic data required to solve decision-making problems are obtained from decision makers linguistically. Questions about the problem to be solved are directed at expert decision

makers. Decision makers respond to these questions with linguistic evaluations. The only available data to reach a conclusion is the decision makers' evaluations. In this respect, obtaining the most accurate evaluation from decision makers is very important for solving the problem. In decomposed fuzzy sets, which are an intuitionistic fuzzy set extension, unlike other fuzzy set extensions, the questions directed to the decision maker are two-way. In fact, two questions with the same meaning are asked in two different ways from functional and dysfunctional perspectives. Thus, it also aims to take into account the inconsistent answers or indecisions that decision makers may give. In this respect, decomposed fuzzy sets are distinguished from other fuzzy sets and are original.

In this thesis, for the first time, decomposed fuzzy set is extended with q-rung orthopair fuzzy set and novel decomposed q-rung orthopair fuzzy set is first integrated with MCDM methods. Their applications are carried out on the aviation industry from sustainability perspective. The results obtained were compared with the intuitionistic fuzzy set-based counterparts of the methods considered (EDAS and MAIRCA) by re-applying them to the same problem. In all these comparisons, it was concluded that the results obtained with decomposed q-rung orthopair fuzzy sets yield effective results and are close to reality. The gap arose due to the fact that the studies on aviation industry in literature were quite few, especially on the aircraft selection problem and the methods were first extended and applied on decomposed q-rung orthopair fuzzy sets, this thesis study provided a new perspective to researchers and studies working in this field. The rough depiction of both methods is provided in Figure 1.7.

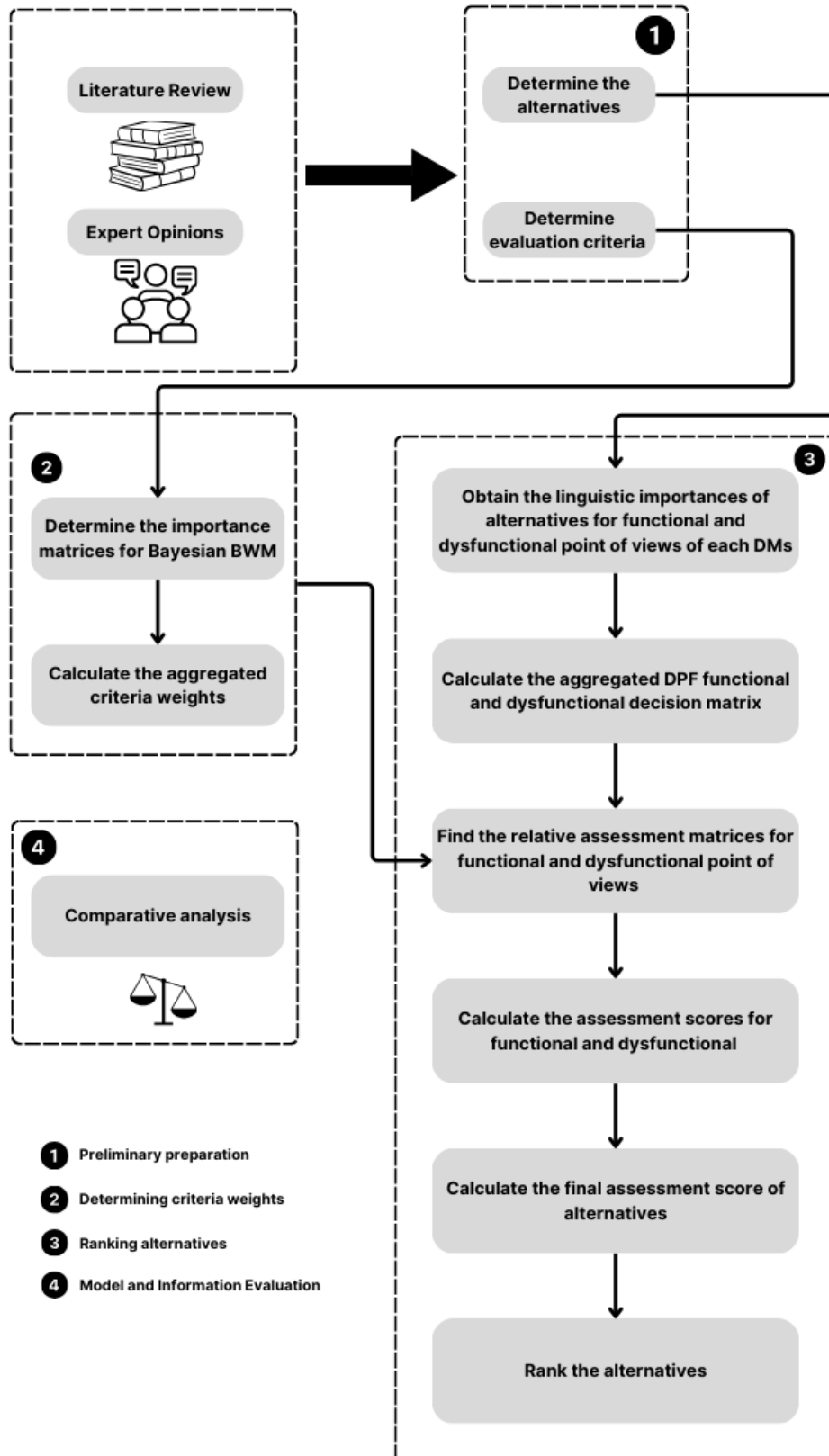


Figure 1.9. Framework of the developed models.

2. LITERATURE REVIEW

2.1 Literature Review on Sustainable Transportation

In the literature, first “sustainable transportation” topic is search on Web of Science portal and the query generates 3,162 different results including articles, conference papers and book chapters. Then those 3,162 research items are analyzed on VOSviewer software which is a tool designed for constructing and visualizing bibliometric networks. These networks can be based on various types of bibliometric data, such as citation networks, co-authorship networks, co-citation networks, and keyword co-occurrence networks.

Figure 1.1 shows the relationship between frequently used terms in sustainable transportation which are interconnected in mainly three different clusters. Two of the clusters (Figure 2.2) show the relationship between decision-making, MCDM and Vehicles which corresponds the three mainlines of this thesis study respectively: selected problem area, methodology used and field of application.

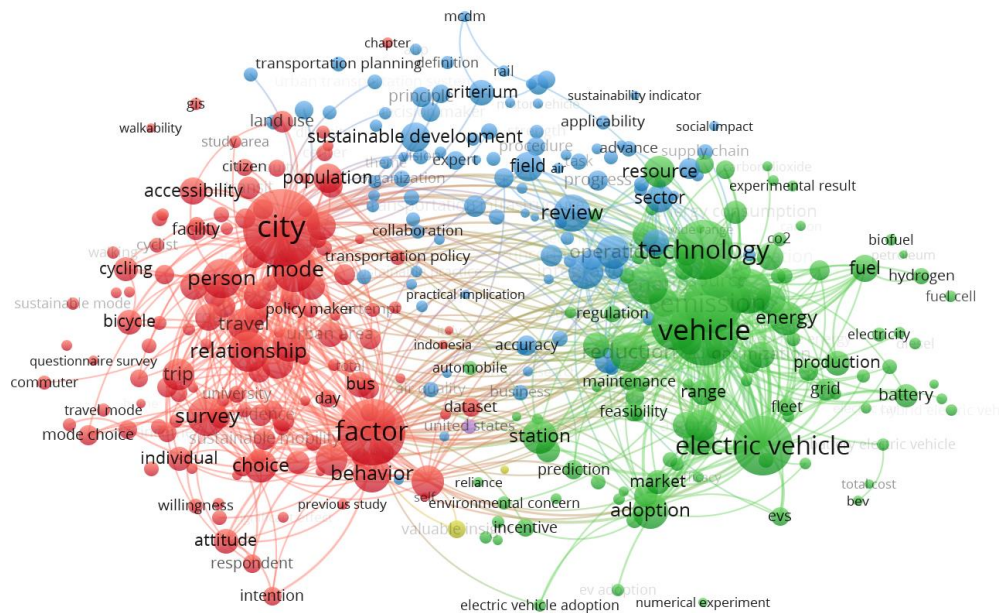


Figure 2.1 – Classification of the literature on VOSviewer

area gained momentum in 2010s and made peak between 2020 and 2025 thanks to the United Nations SDGs which are set in 2016.

The reviewed articles are described briefly with their problem areas and techniques they use with researcher's perspectives.

2.1.1 Infrastructure

Rybarczyk et al. (2010) came up with a MCDA approach to bring supply and demand-oriented factors to plan facilities of bikes. Analysis is conducted to two different geographic hierarchies: network and district, and Geographic Information System (GIS) oriented exploratory spatial data analysis (ESDA) technique is utilized to delve into spatial motives of bike facilities at the district level.

A Multiple Criteria Decision Making (MCDM) methodology to decide between sustainable transportation system alternatives such as car-sharing, ride-sharing and park-and-ride with vagueness on information processed is introduced by Awasthi et al. (2011), where the data is not complete. At step 1, they describe the factors for sustainability evaluation of transport. In step 2, professionals give linguistic scores to the possible alternatives with given criteria. Fuzzy Technique for Order-Preference by Similarity to Ideal Solution (TOPSIS) is utilized to compute aggregate rating for sustainability evaluation and determination of the ideal alternative. In step 3, sensitivity analysis is conducted to decide the effect of factor weights on the decision procedure.

Pryn et al. (2015) introduced a framework to sustainability with a nested model that searches for an alternative method for sustainable transport evaluation, namely as SUSTAIN decision support system (DSS) model. This framework is constructed using MCDA and multiplicative adaptation of Analytic Hierarchy Process (AHP).

Rajak et al. (2016) developed a method to assess the performance of urban transport sustainability using a proposed index with fuzzy extension. The results show that the

model is able to assess transportation sustainability efficiently and it is practically related to the aim of the study.

To reach the goal of sustainable transportation, Wey et al. (2016) introduced the transit-oriented development (TOD) assessment factors and modeled the TOD station determination standard, which satisfies the foundation of sustainable transport. Hence, a solution procedure to problems linked to city life is presented, which might provide a baseline for researchers dealing with related processes. The fuzzy Delphi method was adopted to select TOD assessment factors fulfills the foundation of sustainable transport, and the fuzzy analytic network process (FANP) was conducted to decide the weights of related criteria.

Mohagheghi et al. (2017) developed a novel technique for investing on the determination of sustainable transportation system. Their proposed technique utilizes Tomada de Decisão Interativa Multicritério (TODIM) with interval-valued fuzzy sets (IVFSs) extension to handle the vagueness in data. They employed relative preference relationship to clearly show the significance of factors. Opinions of professionals are assigned an importance rate according to each decision and the experience level of the decision makers.

Wey et al. (2018) proposed an article which studied the implementation of big data in sustainable transport strategies in Taipei City, Taiwan. Fundamental indexes were determined through the Fuzzy Delphi Method (FDM) by preparing a survey for six professionals from industry, government and academics. The study outcomes were utilized to propose decision-making strategies for answering to the environmental prospects of Taipei City's transport infrastructure system by utilizing the Dynamic Network Process (DNP) integration with a data-mining method. Hence, big data associated to city transport were examined to estimate the future research tendency of the fundamental indexes and give prioritization to the sustainable transport initiatives for a sustainable urban life with regards to ongoing physical and geographical developments.

Lima et al. (2019) proposed an assessment strategy for accessibility that undertakes multiple attributes, fuzzy sets, and spatial analysis mechanisms. Their proposed model

decides about indexes that is regarded as attractiveness levels for every possible destination (refers to location of interest—LI). The proposed model is applied in two stages: first they assess cost–distance measures, and subsequently ground friction factors are included in the solution procedure.

Mahmoudi et al. (2019) proposed assessment factors for the evaluation of the sustainability of urban transportation network (UTN). They also utilized Best Worst Method to assess and prioritize sustainability aspects and assessment factors.

Abdel Warith et al. (2019) demonstrates the usage of a novel approach, namely as the dynamic index for national advancement (DINA), for making sustainable development numerical. Their study is also focused on a real-world scenario that assesses two different transport alternatives. Those alternatives are evaluated utilizing the DINA approach and a conventional MCDA method. Moreover, an example is shown and computed using a simulation model which is dynamic.

Zhang et al. (2020) demonstrated the implementation tool for community transport financial assistance regulation from the point of view of carbon emission to explore the transformative stableness tactic of game which consists three parts in the part of various circumstances, by that making clear the fundamental criteria influencing the transformative road of the stableness tactic by simulation.

Trivedi et al. (2021) focused on the burdens to the application of Inland Waterway Transports in India through introducing and assessing the complicated relationship between them. DEMATEL and Interpretive Structural Modeling (ISM) methods are conducted to assess the cause-effect relationship among the burdens and further describe the fundamental impactful burdens from a determined set of criteria.

Alnoor et al. (2022) used linear Diophantine fuzzy rough sets (LDFRSs) and they combined it with MCDM methods to be able to create a benchmarking strategy for the international oil companies (IOCs) which they regard those companies as one of the important stakeholders of the sustainable transportation cycle globally. Their approach consists of two stages as most of the MCDM techniques. In the first stage of their approach, they weigh the criteria using LDFRSs integrated fuzzy-weighted with zero

inconsistency method. In the second stage, LDFRSs integrated MULTIMOORA method is conducted to rank the alternatives which are IOCs. According to their results, ‘cost leadership’ is the most important criterion whereas ‘priority of other external matters’ and ‘insufficient supply’ are the least important criteria.

For the evaluation of up-and-running railways systems of West Africa, Bouraima et al. (2023) proposed a new technique. The new technique is called interval rough step-wise weight assessment ratio analysis-combined compromise solution (IR-SWARA-CoCoSo) model. The main purpose of the study is to undertake interval rough numbers (IRNs) to handle the vague group decision making process. To score the railway systems and criteria used, new interval rough CoCoSo and interval rough SWARA techniques are utilized respectively. The case study showed that the most important criterion for the evaluation is information systems and the best operating railway system is from Nigeria. They validated their results through sensitivity analysis. The first part of their analysis included comparison of their results with other interval rough methods. In the second part, they used Spearman's correlation coefficient and WS coefficient for all compared rankings.

Ismael et al. (2022) introduced a hybrid method to achieve the pavement strategy goal for sustainable transportation. They used a fuzzy MCDM approach for this purpose. They described their novel methodology in two phases. In the first phase, they establish a decision matrix for their evaluations on criteria and pavement options. For the evaluation of weights, they conduct multilayer dual hesitant fuzzy weighted zero inconsistency (DH-FWZIC) and for the evaluation of alternatives they undertake dual hesitant fuzzy decision by opinion score method (DH-FDOSM). After they run both algorithms, the results have proven that region's environment criterion is the most important criterion for the study and the most desirable pavement type is flexible/asphalt pavements.

Zagorskis and Turskis (2024) put the industrial zones under the spotlight of their late research in terms of sustainable transportation. Their aim is to support the transition of industrial areas' pathways from car-centered infrastructure to bicycle road and pedestrian way centered infrastructure. While doing so, they utilize the ARAS-G methodology as an MCDM technique to enhance the bicycle road network through the perspective of

prioritizing alternative actions. Connectivity, safety, accessibility, efficiency, integration within the urban fabric, and cost-effectiveness are among the criteria that they use during the decision-making process.

Table 2.1 shows the summary of the reviewed articles on infrastructure filed of sustainable transportation. The methods that are utilized in this field is diverse. Those methods are applied on different problem area and this shows the flexibility of MCDM methods in terms of applicability on different industries.

Figure 2.1 proves the utilization of any fuzzy set type in the Infrastructure field of sustainable transportation. 61% of the reviewed studies utilized one of the fuzzy set type, meanwhile 39% of them didn't use a fuzzy set. Since scientists use fuzzy sets in case of an uncertainty, this ratio proves the need of fuzzy set for most of the problems.

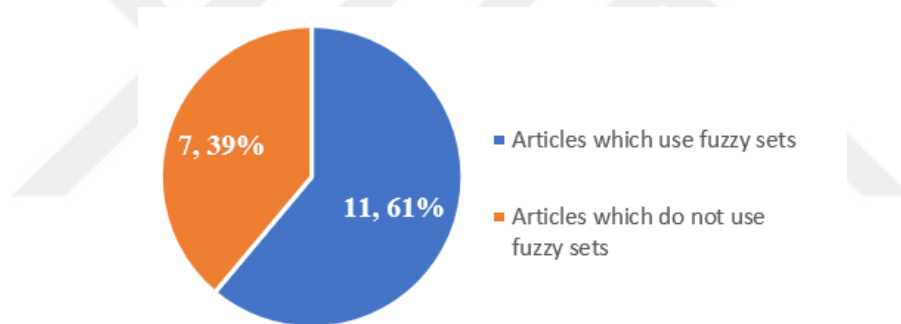


Figure 2.3 – Utilization ratio of fuzzy sets in Infrastructure field

Table 2.1 – Summary of the Literature Review on Infrastructure Field of Sustainable Transportation

Reference	Approach	Technique	Perspective
Rybarczyk and Wu (2010)	Model proposition & Framework	GIS-based ESDA and Multi-criteria analysis	Generation of optimal bicycle facilities plan for Milwaukee
Awasthi et al. (2011)	Framework	TOPSIS & Sensitivity analysis	Evaluation of sustainable transportation alternatives
Pryn et al. (2015)	Model proposition & Framework	SUSTAIN DSS and AHP	Evaluation of crossing alternatives in a city of Denmark
Rajak et al. (2016)	Model proposition & Framework	Transport Sustainability Index with TFN	Assessing the performance of urban transport sustainability
Wey et al. (2016)	Model proposition & Framework	Fuzzy Delphi, FANP and GIS	Assessment of transit-oriented development plans
Mohagheghi et al. (2017)	Model proposition & Framework	TODIM with IVFS	Evaluation of sustainable transportation investment plans
Wey et al. (2018)	Model proposition & Framework	FDM and DNP	Examination of sustainable transport strategies and forecast of future plans
Abdel Warith et al. (2019)	Model proposition & Framework	DINA	Evaluation of alternative sustainable transportation plans (light railway and highway) in Atlanta
Lima et al. (2019)	Model proposition	FMAM and Spatial Analysis	Accessibility assessment in a city of Brazil

Table 2.1 – Summary of the Literature Review on Infrastructure Field of Sustainable Transportation

Reference	Approach	Technique	Perspective
Mahmoudi et al. (2019)	Discussion & Framework	BWM	Establishing factors and their weights for urban transportation network sustainability
Zhang et al. (2020)	Framework	Evolutionary game method	Identification of key factors influencing evolutionary path to ensure stableness
Deveci and Torkayesh (2021)	Model proposition & Framework	Shannon's entropy and MACONT	Selection of the ideal charging strategy for the electrical buses.
Trivedi et al. (2021)	Framework	DEMATEL and Interpretive Structural Modeling	Assessment of the cause-effect relationship among the burdens for inland waterway transportation in India
Alnoor et al. (2022)	Model proposition & Framework	Linear Diophantine fuzzy rough sets extension of MULTIMOORA	Creation a benchmarking strategy for the international oil companies (IOCs)
Ismael et al. (2022)	Model proposition & Framework	Multilayer dual hesitant fuzzy weighted zero inconsistency and dual hesitant fuzzy decision by opinion score method	Selection of the pavement strategy goal for sustainable transportation
Bouraima et al. (2023)	Model proposition & Framework	Interval rough numbers based SWARA and CoCoSo	Evaluation of up-and-running railways systems of West Africa
Zagorskas and Turskis (2024)	Framework	ARAS-G	Prioritization of alternative action for transition from car-based infrastructure to bicycle-based one

2.1.1 Operations

Hatzopoulou et al. (2008) evaluated the extension of organizational inclusion in the opinions, financials, and applications of transportation regulations. With this aim, a survey was prepared with decision makers and planners connecting to the three hierarchies of government (city, state, and federal governance).

Awasthi and Chauhan (2011) introduced an integrated method which brings Analytical Hierarchy Process (AHP) and Dempstere-Shafer theory together to evaluate the effect of eco-friendly transportation metric such as mode-sharing, multi-modal transportation alternatives, intelligent transportation alternatives, etc. on municipal sustainability.

Paz et al. (2013) introduced a system of systems (SOS) and a modeling technique based on fuzzy logic. The SOS consists of Transport, Activity and Environment sections. The fuzzy logic-oriented modeling technique makes it possible to treat the uncertainty related with certain part of the given data. Performance Indices (PIs) are calculated upon every system utilizing a bunch of performance metrics. The PIs demonstrate the aggregated performance of every system and the connection between them as well. The introduced technique provides the prediction of a Composite Sustainability Index for putting brief outcome about the aggregated performance of the SOS in overall.

Validi et al. (2014) established a robust solution framework to design a capacitated distribution network for two-layered supply chain focused to distribute milk in Ireland. Specifically, the green multi-objective model that decreases CO₂ emissions which are caused by transport and costs of the logistics. This distribution network is assessed in detail to make sure there is no domination for results which are spread along the Pareto fronts. The TOPSIS method is conducted for ranking the feasible transport routes as a result of a balance between costs and carbon dioxide emissions.

Chen et al. (2017) presented a novel technique for agrarian transport management. They also proposed a heuristic process using data envelopment analysis for the definition and analysis of the route planning options on the components of costs and multi-objective

metrics of fairness among citizens. A GIS oriented decision support system is developed to handle comprehensive data which is necessary in their study.

Jasti et al. (2018) presented an evaluation framework that examines the ongoing case of current community transportation system alongside the PIs which have crucial importance on the financial, environmental, and social factors. Moreover, an adequate procedure is developed as well for comparison to achieve custom performance evaluation of city bus system with sustainability as fundamental point.

Fathi et al. (2018), proposed novel network data envelopment analysis (NDEA) model to handle the bidirectioned connections, taking into account the bounded connectivity to assess sustainable supply chains.

Sun et al. (2019) unsound items for routing area in the road and rail multi-modal transportation system and modeled the vague populace risk factors from fuzzy programming approach by utilizing triangular fuzzy numbers (TFNs). Furthermore, the CO₂ emission limitation in model to notice the sustainable transport of risky items. The problem is undertaken for optimizing this approach by the authors and they introduce a bi-objective fuzzy mixed integer programming model which is nonlinear, subsequently they present a three-level exact solution methodology which integrates fuzzy credibility chance limitation, linearization tool and the normalized weighting technique.

Semanjski et al. (2019) utilized the AHP, to combine route sustainability into the procedure for city logistics routing. Firstly, they used the database of traffic sign to derive sustainability substance. Secondly, this sustainability substance is incorporated into the cost computations of the routes using the MCDM method. With the help of the Dijkstra technique, the proposed method is assessed on a real-world problem and its applicability and effectiveness are proven.

Gupta et al. (2021), conducts a broad approach which formulates a combined multi-objective optimization model for an extended capacitated sustainable transportation problem in a coal mining industry using AHP and DEA methods. First, AHP method is utilized to determine the weights of various alternative vehicles which are listed as

alternative for transport. (2) The DEA method is utilized for computing efficiency rates of vehicles on several routes through inputs and outputs observed as important especially for the mining sector. Moreover, they decrease dependency on carbon-oriented vehicles for transport which improves reduction in GHG. (3) A fuzzy interactive optimization framework is proposed to obtain compromise transportation solutions which also consists of the optimal number of vehicles selected for sustainable transport. (4) The case study focuses on a mining sector in India. They explore if they can apply the proposed optimization framework and solution procedure. Furthermore, certain comparisons are shown with current transportation models to establish the advantages of their model.

Shu et al. (2021) presented a data-centered technique to create shuttle services to enhance the effectiveness of and reliability of the last mile transport. In specific, an integrated approach is identified to describe the last mile transport demands from several information points. Using the gathered demand data as an input, the exact locations of bus stop are determined by an reinforced clustering method, and the route and schedule of the shuttle services are prepared utilizing a data-centered technique. They undertook a simulation-oriented risk analysis to assess the performances of shuttle services in various areas.

Rajak et al. (2021) explored the burdens of achieving sustainable transportation goal in India. Their proposed methodology consists of two well-known techniques. The first one is the Grey theory-based method and it handles the ambiguity of data which is obtained by the decision-makers. The second one is DEMATEL and they undertake this method to design cause and effect relationship between twenty-two determined burdens.

Martins et al. (2021) reviewed the existing work in the field of electric vehicle routing problems which are obtained by researchers from Computer Science, Artificial Intelligence, and Operations Research communities who have been developing optimization, simulation, and machine learning approaches that aim at generating efficient and sustainable routing plans for hybrid fleets, including both electric and internal combustion engine vehicles.

Dwivedi et al. (2021) described and formulated the readjustment metrics of sustainable freight transport (SFT) after COVID-19 pandemic. In their work, 13 crucial readjustment metrics are decided with the help of the professional team. A combined Grey Decision-Making Trial and Evaluation Laboratory (DEMATEL) is utilized to prioritize and introduce the causal nexus among the described crucial readjustment metrics. They explored sensitivity analysis to evaluate the validity of the presented model.

Arroyo-López et al. (2021) undertook a complete statistical analysis on information gathered by 55 drivers in 155 trips, utilizing GPS data (e.g., geographic location, pace, frequency of rotations for a minute) and direct examinations, for one of the biggest Mexican retail companies that handles the transportation of its own business. The outcomes demonstrate that ecological driving behavior consumes 27.8% less gas which also means average 13 kg less CO₂ emission per route.

Karuppiah et al. (2021) utilized the Pareto diagram method, the research establishes 20 enablers through the list of 25 enablers for location decision and assesses the enablers for sustainable transport. The ultimate enablers are put first, and their dependencies are unfolded utilizing the Grey DEMATEL technique. They used sensitivity analysis to decide the impact of professionals' weight in the outcomes of the study.

Sadrani et al. (2023) studied the EBs' charging strategy. They establish economic, environmental, social, operational, and quality-of-service as their criteria. These criteria are given weights using Fuzzy Best-Worst Method. The alternative strategies are evaluated using Fuzzy Ranking of Alternatives through Functional mapping of criterion subintervals into a Single Interval (FRAFSI) approach. Their case study is centered on the EBs of Munich city of Germany. As a results, they found the most important criterion as the economic one. The evaluation of alternatives shows that overnight charging is the best option as a charging strategy. The robustness of the results is assessed using a comparative analysis with fuzzy TOPSIS and fuzzy EDAS methods.

Since public transportation holds a high proportion in urban transportation, it is important delve into the conversion strategy from conventional vehicles to electrical buses because of their proven benefit to sustainable transportation. With this purpose, Deveci and

Torkayesh (2021) proposed a methodology for the selection of the ideal charging strategy for the electrical buses. The proposed methodology brings Shannon's Entropy and mixed aggregation by comprehensive normalization technique (MACONT) together with interval-valued neutrosophic sets. MACONT method with tri-normalization property is specifically preferred to handle the subjectivity of using single normalization method. The study concludes that depot charging is the best alternative for charging the public buses.

Hmamed et al. (2023) focused their study on waste transportation. The proposed framework includes two major methodologies, namely Analytical Hierarchy Process (AHP) and Data Envelopment Analysis (DEA). With AHP, they first identify the evaluation criteria for route selection of waste transportation company. The efficiencies of the routes are computed through DEA method using inputs and outputs of the system.

Boz et al. (2023) proposed a framework to assess air cargo company selection problem. In determining the criteria weights, one of the new methods, Bayesian BWM, is applied, while the WASPAS method is used in ranking the air cargo companies. Thus, these two methods are integrated, and at the same time, the ranking results are compared and analyzed with TOPSIS and COPRAS methods. As a result of these evaluations, it is seen that the most important criterion for air cargo company selection in a chaotic environment is economic criteria.

Senapati et al. (2022) established a novel model for prioritization of transportation mode-sharing problem. They assessed the criteria with Shannon entropy-based power weighted approach. Aczel-Alsina operations are used for IF numbers for aggregation of the decision information. These aggregation operators are utilized to develop the IF power Aczel-Alsina model. Their case study in Novi Sad city recommended that sharing e-bicycle fleet is the prioritized mode.

From accessibility of the sustainable transportation systems perspective, Hezam et al. (2022) undertook a study to explore the burdens. They place the focus on digital aspects of those burdens since they discuss digital technology as a supporting tool for accessibility of the transportation systems for persons with disabilities (PWDs). In their article, they use a novel approach to weight the decision criteria. They establish aggregation operators

for Fermatean fuzzy sets. An integrated criteria weight-determining approach is proposed. The proposed approach brought inter-criteria correlation and stepwise weight assessment ratio analysis models together to handle the incompetency of individual objective and subjective model. Using them, a hybrid double normalization based multi-aggregation approach is offered. The study proved that administrative restrictions is the most effective barrier for PWDs and the most preferred alternative as the digital technology is found to be artificial intelligence and machine learning.

Kundu et al. (2024) established a framework to assess urban transportation systems. The fuzzy BWM technique is employed to identify the weights of the criteria. The fuzzy MAIRCA technique is utilized for ranking the alternatives using the calculated weights of the criteria.

Another integrated methodology is introduced by Akram et al. (2024). They proposed the Pythagorean fuzzy rough Entropy-MARCOS method (PFR-Entropy-MARCOS) to evaluate the criteria and alternatives of shared mobility models, namely as electric vehicle (EV), autonomous electric vehicle (AEV), electric scooter (ES) and electric bike (EB). They explain that while Pythagorean fuzzy sets provide way to overcome uncertainty, it fails to eliminate the subjective evaluations, and rough sets are presenting strong mathematical approach in this manner. They undertake the Entropy method to assign the weight values and MARCOS technique to rank the alternatives. The case study focuses on sustainable transportation solutions of the city Belgrade.

Görçün et al. (2024) introduced a mathematical model to select the ideal transportation type and therefore the route to enhance the performance of the freight sector. They used Neutrosophic Number-based Delphi (NN-Delphi) to set the criteria, m-Generalized q-Neutrosophic Sets (mGqNSs)-based Stepwise Weight Assessment Ratio Analysis (MGqNS-SWARA) to calculate the weights of the criteria and mGqNSs-based Additive Ratio Assessment (mGqNS-ARAS) to score the sustainability performance of the freight mode. According to their results, cargo security and accident rates are the most important criteria and maritime transport is the most sustainable transport mode for the freight industry.

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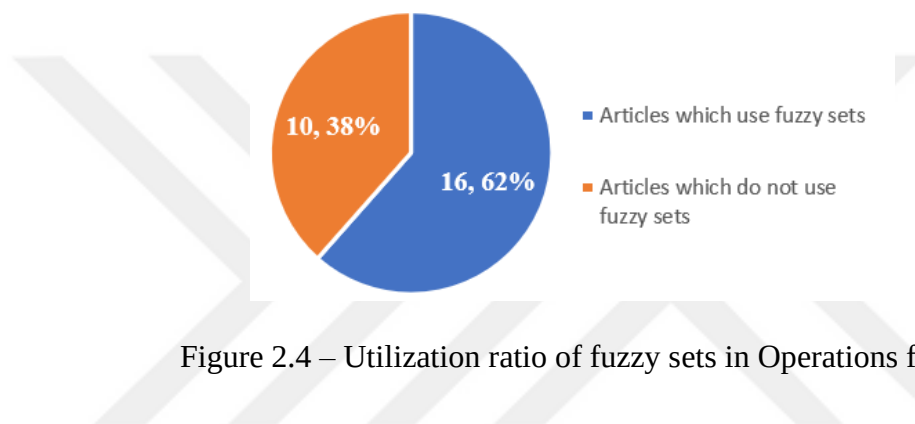


Figure 2.4 – Utilization ratio of fuzzy sets in Operations field

Table 2.2 – Summary of the Literature Review on Operations Field of Sustainable Transportation

Reference	Approach	Technique	Perspective
Hatzopoulou and Miller (2008)	Discussion	Survey	Institutional integration evaluation in Canada
Awasthi et al. (2011)	Framework	Demspter–Shafer theory and AHP	Evaluation of municipal sustainable transportation
Paz et al. (2013)	Model proposition & Framework	SOS	Performance index calculation for sustainable transportaion with a case study in entire USA
Validi et al. (2014)	Model proposition & Framework	Genetic Algorithm and TOPSIS	Route generation and their ranking in a distribution network in Ireland
Chen et al. (2017)	Model proposition & Framework	DEA-based heuristic	Construction of vehicle routes and their rankings
Fathi et al. (2018)	Model proposition	NDEA	Evaluation of bidirectioned sustainable supply chains
Jasti et al. (2018)	Framework	Weight based ranking system with TFN	Performance evaluation of city bus system
Semanjski et al. (2019)	Framework	AHP & Dijkstra	Evaluation of route sustainability
Sun et al. (2019)	Model proposition & Framework	Fuzzy Programming	Routing of hazardous materials

Table 2.2 – Summary of the Literature Review on Operations Field of Sustainable Transportation

Reference	Approach	Technique	Perspective
Tsang et al. (2020)	Model proposition & Framework	FBLPF (MRTM and ANFIS)	Management of PEVBPS in the market
Arroyo-López et al. (2021)	Discussion	Statistical analysis	Understanding the effect of eco-driving practices
Dwivedi et al. (2021)	Model proposition	Grey DEMATEL	Formulation of the readjustment metrics of sustainable freight transport (SFT) after COVID-19 pandemic
Günay et al. (2021)	Model proposition & Framework	Multi-Objective RPP*	Design of sustainable transportation network
Karuppiah et al. (2021)	Discussion & Framework	Pareto Analysis and Grey DEMATEL	Exploring the enablers of sustainable transportation and their relationships
Martins et al. (2021)	Discussion	NA	Review of the electric vehicle routing problems
Rajak et al. (2021)	Framework	Grey DEMATEL	Investigation of burdens towards sustainable transportations
Shu et al. (2021)	Framework	Clustering and Cost-Benefit Analysis	Designing shuttle services to improve the efficiency and convenience of last mile transportation
Hezam et al. (2022)	Model proposition	Aggregation operators for Fermatean fuzzy sets	Exploring the burdens of accessibility of sustainable transportation

Table 2.2 – Summary of the Literature Review on Operations Field of Sustainable Transportation

Reference	Approach	Technique	Perspective
Senapati et al. (2022)	Model proposition & Framework	Shannon entropy-based power weighted approach and IF power Aczel-Alsina model	Prioritization of transportation mode-sharing problem
Boz et al. (2023)	Framework	Bayesian BWM and WASPAS	Evaluation of air cargo companies in chaotic environment
Hmamed et al. (2023)	Framework	AHP and DEA	Assessment of efficiencies of the routes for waste transportation
Qadir et al. (2023)	Model proposition	Three-way decision-making (TWD)	Evaluation of logistic service providers and sustainable transportation projects
Qahtan et al. (2023)	Model proposition & Framework	Q-rung orthopair fuzzy rough set-based fuzzy-weighted zero-inconsistency and fuzzy decision by the opinion score method (FDOSM)	Assessment of the performance of ship energy systems
Sadrani et al. (2023)	Model proposition & Framework	Fuzzy BWM and Fuzzy Ranking of Alternatives through Functional based Single Interval	Evaluation of electrical battery charging strategy
Görçün et al. (2024)	Model proposition & Framework	Neutrosophic Number-based Delphi and m-Generalized q-Neutrosophic Sets (mGqNSs)-based SWARA	Selection of the ideal transportation type
Kundu et al. (2024)	Framework	Fuzzy BWM and Fuzzy MAIRCA	Assessment of urban transportation systems

Table 2.2 – Summary of the Literature Review on Operations Field of Sustainable Transportation

Reference	Approach	Technique	Perspective
Aslantas and Gündoğdu (2024)	Framework	Decomposed Fuzzy Set (DFS) based AHP	Evaluating risks of autonomous vehicles
Farid et al. (2024)	Model proposition & Framework	q-rung orthopair fuzzy CRITIC-EDAS method	Prioritization of sustainable approaches for smart waste management of automotive fuel cells (AFCs)

2.1.3 Vehicles

Jeon et al. (2010) exhibited an implementation of Multiple Criteria Decision Analysis (MCDA) technique to assess the determined transport and land usage policies in the Atlanta area utilizing various sustainability parameters. A synthesized index for sustainability is established as a decision support system for transportation regulations, where the index of sustainability evaluates the multiple-dimensioned intersecting factors in the transport planning procedure.

Onat et al. (2016) presented an input-output-oriented integrated life cycle sustainability evaluation model utilizing various macro-perspectives namely as social, financial, and environmental factors. Consequently, the findings of an integrated life cycles sustainability evaluation for distinct traditional and alternative vehicles (battery electric vehicles, plug-in-hybrid vehicles, hybrid electric vehicles, and internal combustion electric vehicles) are listed as inputs to the TOPSIS and Intuitionistic Fuzzy (IF) Sets utilizing two policy cases.

Büyüközkan et al. (2018) assessed various technologies for community buses as intercity transport options. They presented an aggregation technique with Intuitionistic Fuzzy Choquet Integral (IFCI) and Group Decision Making (GDM) approaches. A fuzzy technique which identifies pairwise-comparison metric is utilized in IFCI to describe its parameters. TOPSIS technique as a method that disregards the interactions, and 2-additive Choquet integral are also conducted. The results of IFCI, 2-additive Choquet integral and TOPSIS approaches are compared with each other.

Liang et al. (2019) conducted a linear goal programming priority oriented Fuzzy AHP (LGPPFAHP) method to decide about the weights of the assessment factor for sustainability evaluation of alternative fuel vehicles (AFVs). They also used Fuzzy Group Multi-Attribute Decision Analysis (FGMADA) to propose a fuzzy group decision support system for prioritizing the sustainability of AFVs. They verified the outcomes by fuzzy TOPSIS technique and to evaluate the impacts of the evaluation criteria weights a sensitivity analysis is applied.

Fuzzy-based battery life-cycle prediction framework (FBLPF) is presented by Tsang et al. (2020). Their proposed method is to designate ideal plug-in electric vehicle battery pack standardization management (PEVBPS) in the sector. To do so, they combine the multi responses Taguchi method (MRTM) and the adaptive neuro-fuzzy inference system (ANFIS) for decision-making procedure. MRTM is modeled for the selection of the most related and important input variables from professional's point of view, then ANFIS plays role in time-series forecasting of the specialized product life cycle on demand and electricity usage.

Ecer (2021) focused their study on the performance evaluation of the battery electric buses. Eleven criteria were utilized to evaluate ten different BEV models. To determine the criteria weights, SECA method is conducted. Afterwards, for the ranking of the alternative BEV models, SECA, MARCOS, MAIRCA, CoCoSo, ARAS and COPRAS techniques are undertaken. The results show that the most important factors to decide between different BEV models are price, permitted load and energy consumption. Tesla Model S is the ideal alternative. The results are validated with several other MCDM approaches.

A multi-objective robust possibilistic programming approach (RPP*) is presented by utilizing and reciprocal fuzzy programming technique because of the lack of information on the real value of various traveler discomfort degrees and vagueness in demand by Günay et al. (2021). Other than other robust possibilistic methods, RPP* optimizes the average of the objective function and chance limitation contraventions and the risk score obtained by vague metrics by taking into account the absolute deviation of the objective function.

Pamucar et al. (2021) developed an MCDM technique to prioritize the several alternative fuel vehicles (AFVs) for sustainable transportation. The evaluation of the AFVs might be regarded as an MCDM formulation because of the inclusion of various intersected factors. They proposed the model on fuzzy Full Consistency Method (FUCOM-F) neutrosophic fuzzy Measurement Alternatives and Ranking according to the COMpromise Solution (MARCOS) framework for the assessment of the AFVs.

Ziembra (2021) introduced a novel hybrid methodology for the evaluation of alternative electric cars in Poland. Their methodology is based on the integration of new easy approach to fuzzy PROMETHEE (NEAT F-PROMETHEE), Monte Carlo method and Stochastic Multicriteria Acceptability Analysis (SMAA). During the decision making process, while NEAT F-PROMETHEE method considers the score of a single decision maker, stochastic methods make it possible for them to simulate the opinion of the Polish community.

Aboushaqrah et al. (2022) proposed a methodology to evaluate alternative fuel taxis. They set their alternatives as gasoline vehicles, natural gas vehicles, hybrid vehicles, and battery electric vehicles. They use hybrid life cycle sustainability evaluation model to better identify the effects of those differently fueled vehicles. Subsequently, they utilize interval-valued neutrosophic sets -based analytic hierarchy process approach to determine the weights of the criteria and rank the alternatives. Their application results prove that, when all factors are examined, solar-powered battery electric vehicles are the best among the alternatives, followed by natural gas vehicles.

Alnoor et al. (2022) used linear Diophantine fuzzy rough sets (LDFRSs) and they combined it with MCDM methods to be able to create a benchmarking strategy for the international oil companies (IOCs) which they regard those companies as one of the important stakeholders of the sustainable transportation cycle globally. Their approach consists of two stages as most of the MCDM techniques. In the first stage of their approach, they weigh the criteria using LDFRSs integrated fuzzy-weighted with zero inconsistency method. In the second stage, LDFRSs integrated MULTIMOORA method is conducted to rank the alternatives which are IOCs. According to their results, ‘cost leadership’ is the most important criterion whereas ‘priority of other external matters’ and ‘insufficient supply’ are the least important criteria.

To come up with a solution procedure on the trade-off of pros and cons of different alternative fuel vehicles, Onat (2022) suggested an integrated framework which brings hybrid multi-regional input–output based life cycle sustainability assessment (LCSA) and AHP-based combinative distance-based assessment (CODAS) methods together with newly introduced Spherical fuzzy sets. The case study for the application of the novel

model is selected in Qatar, which evaluates internal combustion, hybrid, plug-in hybrid, and full battery electric vehicles under fourteen criteria. The results demonstrate that battery electric vehicles are ideal among the alternatives.

Biswas et al. (2023) proposed a novel integrated method to compare currently used EVs. Their solution approach included three methods. First, with q-rung orthopair fuzzy (qROF) numbers and Einstein aggregation, Entropy technique is used to determine the weights of the criteria with full consistency method (FUCOM). Then the alternatives are ranked utilizing the alternative ranking order method with two-step normalization method (AROMAN). The study proved that electricity consumption is the most important criterion.

For the evaluation of three different fuel alternatives (compressed natural gas, electric buses, and diesel buses), Elagouz et al. (2023) proposed a new model named life cycle sustainability assessment (LCSA) integrated multi region input-output analysis with novel multi-criteria decision-making techniques. The Interval-Valued Neutrosophic Fuzzy (IVNF)-Analytic Hierarchy Process with the Combined Compromise Solution (CoCoSo) technique is used for the evaluation of alternatives from the sustainability perspective. Their case study is focused on global supply chains in Qatar for the application of the proposed methods.

Ghoushchi et al. (2023) presented a new method using spherical fuzzy set (SFS). Their case study is focused on the evaluation of different alternative vehicles in Tehran, Iran. The proposed methodology uses SWARA method that incorporates SFS to determine the criteria weights. The alternative vehicles are evaluated afterwards with the help of the MARCOS technique which also utilizes SFS. The results show that autonomous vehicles are the best alternative to reduce air pollutants and the crucial criteria set is the environmental one.

For urban planning purposes, Gulcimen et al. (2023) proposed a methodology to evaluate urban sustainable transportation alternatives. Their methodology consists of hesitant fuzzy AHP (HF-AHP) and multiple attribute utility model (MAUT). In their results, they

emphasized that ‘number of fatalities or injuries’ is the most important factor in the selection process. Low-motorized vehicles (low-MVs) came out to be the best among the sustainable urban transportation alternatives according to its utility score.

Qadir et al. (2023) proposed a new methodology to the linguistic three-way decision-making (TWDs) problem for the evaluation of logistic service providers and sustainable transportation projects. The name of the new technique is the Pythagorean double hierarchy linguistic term sets (PyDHLTSs). They also identify certain aggregation operators and basic operational laws for this set. The proposed method uses this new set which has its roots from decision-theoretic rough sets (DTRSs) to deal with three-way decision-making problems. They also calculate conditional probability utilizing grey relational analysis to enhance the decision-making process. The results are computed by the loss function that uses proposed aggregation operators, the minimum-loss is chosen as the decision.

Qahtan et al. (2023) established a new method that uses the extension of q-rung orthopair fuzzy rough sets (q-ROFRS) to assess the performance of ship energy systems (SEs). First they identify a decision matrix between SEs and sustainability. Then they develop q-ROFRS and fuzzy-weighted zero-inconsistency (q-ROFRS–FWZIC) method to decide about the criteria weights. A combination of the q-ROFRS and fuzzy decision by the opinion score method (q-ROFRS-FDOSM) is proposed as a new technique to evaluate SEs. As a result, decision support methods appear to be the most crucial criterion for their problem and they also obtain the most appropriate SE as their decision.

Alternative fuel vehicles (AFVs) are generally considered among the sustainable transportation alternatives. In this area, Raj et al. (2023) proposed a combined DEMATEL and TOPSIS approach to evaluate the feasibility of the EVs. Fourteen major factors, such as economic sustainability and end-of-life options, are considered during the decision-making process. Those factors weights are calculated using DEMATEL method and the alternatives are ranked through the TOPSIS method. The case study in India showed that electric vehicles (EVs) are accepted as the mostly preferred among AFV alternatives to take the place of gasoline vehicles.

Table 2.3 shows the summary of the reviewed articles on Vehicles filed of sustainable transportation. The methods that are utilized in this field is diverse. Those methods are applied on different problem area and this shows the flexibility of MCDM methods in terms of applicability on different industries.

Figure 2.3 proves the utilization of any fuzzy set type in the Vehicles field of sustainable transportation. An overwhelming majority (94%) of the reviewed studies utilized one of the fuzzy set types, meanwhile only 6% of them didn't use a fuzzy set. Since scientists use fuzzy sets in case of an uncertainty, this ratio proves the need of fuzzy set for most of the problems.

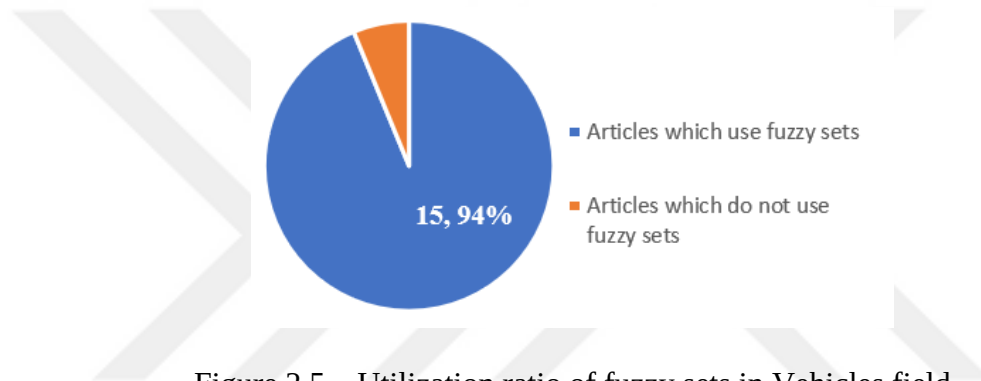


Figure 2.5 – Utilization ratio of fuzzy sets in Vehicles field

Table 2.3 – Summary of the Literature Review on Vehicles Field of Sustainable Transportation

Reference	Approach	Technique	Perspective
Jeon et al. (2010)	Model proposition & Framework	Composite Sustainability Index & Weighted Sum Model	Evaluation of alternative sustainable transportation plans in Atlanta
Onat et al. (2016)	Model proposition & Framework	TOPSIS and IF	Evaluation of different sustainable vehicle types in USA
Büyüközkan et al. (2018)	Framework	IFCI	Assessment of various technologies for community buses
Liang et al. (2019)	Model proposition & Framework	LGPPFAHP & FGMADA	Prioritization of AFVs in China
Ecer (2021)	Model proposition & Framework	SECA, MARCOS, MAIRCA, CoCoSo, ARAS and COPRAS	Evaluation of the battery electric buses
Gupta et al. (2021)	Model proposition & Framework	Multi-Objective Optimization, AHP, DEA	Evaluation of alternative carrier vehicles in mining industry
Pamucar et al. (2021)	Discussion & Framework	FUCOM with TFN & MARCOS with neutrosophic FN	Assessment of the several alternative fuel vehicles in USA
Ziemba (2021)	Model proposition & Framework	NEAT F-PROMETHEE, Monte Carlo Method, Stochastic Multicriteria Acceptability Analysis	Evaluation of alternative electric cars in Poland
Aboushaqrah et al. (2022)	Model proposition & Framework	Interval-valued neutrosophic sets extension of AHP	Evaluation of alternative fuel taxis

Table 2.3 – Summary of the Literature Review on Vehicles Field of Sustainable Transportation

Reference	Approach	Technique	Perspective
Onat (2022)	Model proposition & Framework	Life cycle sustainability assessment (LCSA) and AHP-based CODAS	Trade-off of pros and cons of different alternative fuel vehicles in Qatar
Biswas et al. (2023)	Framework	Q-rung Orthopair Fuzzy Set based FUCOM and AROMAN	Comparison of currently used EVs
Elagouz et al. (2023)	Model proposition & Framework	LCSA integrated multi region input-output analysis and IVFN-based CoCoSo	Evaluation of three different fuel alternatives for supply chains in Qatar
Ghoushchi et al. (2023)	Framework	Spherical fuzzy set-based SWARA and MARCOS	Evaluation of different alternative vehicles in Tehran, Iran
Gulcimen et al. (2023)	Model proposition & Framework	Hesitant fuzzy AHP (HF-AHP) and multiple attribute utility model (MAUT).	Evaluation of urban sustainable transportation alternatives
Raj et al. (2023)	Framework	DEMATEL and TOPSIS	Evaluation of the feasibility of the Electric Vehicles (Evs)
Akram et al. (2024)	Model proposition & Framework	Pythagorean fuzzy rough Entropy-MARCOS method	Evaluation the criteria and alternatives of shared mobility models

2.1.1 Summary of Sustainable Transportation Literature Review

Literature review proves that the number of studies which focuses on aviation industry is limited. The reviewed literature shows that in the sustainable transportation area there is not a study which proposes a solution framework to airplane model (aircraft) selection problem. This has been one of the motivations of problem definition of this thesis.

Another outcome that is obtained from literature review is about the complexity of the problem definitions of the studies. As more researchers put interest in sustainable transportation, the methodologies introduced, and problem types are studied with increasing complexity.

Figure 2.4 shows the split of the transportation fields with respect to their frequencies and percentages in the literature review. It shows even though Operations is leading field, there are no overwhelming gaps between research statistics on transportation fields.

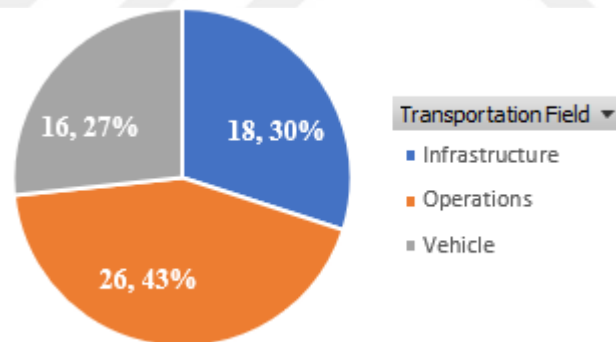


Figure 2.6 - Field split of sustainable transportation in reviewed literature

For the reviewed articles, it is observed that majority of them use fuzzy logic with different types (70%). Since fuzzy logic is mostly preferred in case of uncertainties within MCDM problems, the graphic proves the uncertainty level of the sustainable transportation problem areas.

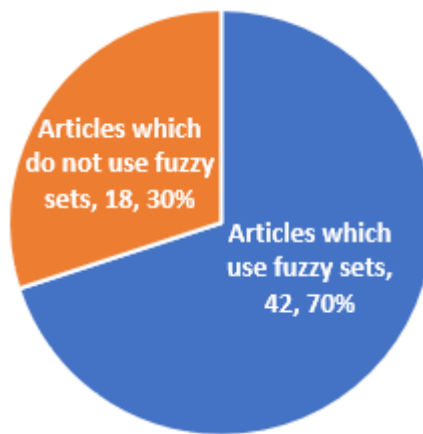


Figure 2.7 – Fuzzy logic utilization ratios in the reviewed articles

On the other hand, literature review on sustainable transportation proved that EDAS and MAIRCA methods are frequently addressed in MCDM problems.

There are several important reasons to choose to use the EDAS (Distance-Based Assessment from Average Solution) method in the field of sustainable transportation. First, the EDAS method offers the ability to successfully handle multi-criteria decision-making (MCDM) scenarios. Transportation decisions usually evaluate multiple, often conflicting criteria; therefore, it is important to use a method that can include variables such as environmental impacts, costs, social acceptability. EDAS can easily manage this diversity and allow balanced evaluations in decision-making processes.

In addition, the EDAS method stands out with its ability to deal with uncertainties and incomplete information. When evaluating the feasibility of sustainable transportation solutions, it is possible to encounter scenarios with a large number of uncertainties. The EDAS method contributes to obtaining solid results even in cases where there is less information, by evaluating how far the alternatives are from an average solution.

The transparency of EDAS is also a great advantage. Being able to clearly explain how the decision-making process works creates trust between different stakeholders. In the transportation sector, this transparency is of critical importance in terms of ensuring stakeholder participation and social acceptance of decisions.

Finally, the rapid analysis opportunity provided by the EDAS method provides great convenience for decision makers. Especially considering the rapidly developing nature of sustainable transportation solutions, there is an increasing need for timely decision making. The EDAS method supports decision processes in this area by providing fast and effective results.

For all these reasons, the EDAS method stands out as an effective and efficient tool in decision-making processes in the field of sustainable transportation.

On the other hand, the MAIRCA method allows the comparison of ideal and real situations of alternatives in multi-criteria decision-making (MCDM) processes. This allows various factors to be taken into consideration in sustainable transportation projects; for example, it offers an integrated approach for criteria such as environmental impacts, economic costs and social acceptability.

In addition, the MAIRCA method offers considerable flexibility in determining the weights of certain criteria, helping decision makers to clearly understand which criteria are more prioritized. This facilitates the selection of the most effective and sustainable options in transportation projects.

In addition, the capacity of the MAIRCA method to cope with uncertainty and complexity in the decision-making process is an important advantage. Under variable conditions that may be encountered in the transportation field, the flexible structure provided by MAIRCA can facilitate making the right decisions.

Finally, in addition to the fact that MAIRCA has been tested in various application areas, seeing that it also yields successful results in sustainability-focused projects has been effective in my preference for this method. In this context, MAIRCA stands out as a powerful tool for developing sustainable transportation solutions.

For these reasons, EDAS and MAIRCA methods are selected to be integrated with the proposed novel Decomposed Q-Rung Orthopair Fuzzy Sets. In the next two sections,

fundamental descriptions, advantages, and disadvantages are given for both methods. Moreover, to understand the role of both methods in sustainable transportation area, and the selected application field of aviation industry, related works in literature are given. The core steps of both methods are also described in detail in these sections.

2.2 EDAS Method and Related Works

The Evaluation based on Distance from Average Solution (EDAS) technique is a notable MCDM method which has obtained attention in several areas as a result of its novel framework to assess alternatives built on their distances to an average solution. The technique is established by Keshavarz Ghorabae et al. in 2015 (Ghorabae, 2015). The EDAS technique works with computing both positive and negative distances to an average solution, enabling decision-makers to evaluate the analysis of alternatives in a more sophisticated way in comparison with traditional approaches (Stanujkić et al., 2017; Ghorabae et al., 2015). This method condenses the decision-making procedure and improves the robustness of assessments by integrating both quantitative and qualitative criteria, which is crucial in complicated decision-making scenarios (Ghorabae et al., 2015).

The calculation steps of the EDAS technique is based on introduced guidelines from other MCDM methods, such as the Simple Additive Weighting (SAW) technique, which promotes the combination of several criteria into a united assessment approach (Stanujkić et al., 2017; Ghorabae et al., 2015). The techniques's power is illustrated through its implementation in various real-life cases, involving stock categorization and supplier selection, where it proved to surpass conventional techniques by supplying deeper understanding into the cross-analysis performances of alternatives (Ghorabae et al., 2015; Ulutaş et al., 2021). The EDAS technique's flexibility to diverse environments, including its extension with interval grey numbers, proves its adaptability and capacity for wider implementations in decision-making processes (Stanujkić et al., 2017; Hani, 2024).

Furthermore, the EDAS technique is also utilized with several modifications to improve its applicability and to confront special obstacles faced in real cases. For example, latest

articles introduced modifications on the EDAS technique to tackle specific challenges in a better path, thus enhancing its credibility and efficiency in real-life implementations (Ghorabae, 2022). The technique's capability to include fuzzy logic theory is also examined, enabling for a more dynamic handling of ambiguity and vagueness in decision-making scenarios. This versatility is critical in areas such as sustainable development and e-commerce, where decision-making usually includes intersecting criteria set and fast-paced transforming settings (Hani, 2024).

2.2.1 Computational Steps

One of the biggest differences of the EDAS method is the normalization process. Traditional multi-criteria decision-making methods, TOPSIS and VIKOR, focus on determining the best alternative according to ideal and anti-ideal solutions. However, in real-life decision problems, a solution being closer to the ideal or farther from the anti-ideal does not always guarantee the best result. This is where EDAS adopts a different approach: It bases the normalization process on the average solution when determining the best alternative.

EDAS uses two basic criteria to evaluate and rank the performance of each alternative: PDA (positive distance from the average value) and NDA (negative distance from the average value). If we have a complex and multi-faceted decision-making problem with N criteria and M alternatives, the EDAS method can be applied by following the steps below.

Step 1. The initial decision matrix is created by decision makers or experts based on real data or a qualitative scale. For a decision-making problem involving n criteria ($n = 1, 2, \dots, j$) and m alternatives ($m = 1, 2, \dots, i$), this matrix forms the basis of the decision process and enables the evaluation of alternatives according to the determined criteria. In this context, the initial decision matrix is created as follows.

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1m} \\ x_{21} & x_{22} & \dots & x_{2m} \\ \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \dots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nm} \end{bmatrix} \quad (2.1)$$

Step 2. In the normalization process, an average solution is determined by considering all criteria. The average value of each criterion j is calculated by Equation (2.2).

$$AV_j = \frac{\sum_{i=1}^n x_{ij}}{n} \quad (2.2)$$

Step 3. PDA and NDA used in the EDAS method are calculated according to the nature of the conflicting criteria. While PDA and NDA values for benefit criteria are determined by Equations (2.3)–(2.4), these values for cost criteria are calculated using Equations (2.5)–(2.6).

$$PDA_{ij} = \frac{\max(0, x_{ij} - AV_j)}{AV_j} \quad (2.3)$$

$$NDA_{ij} = \frac{\max(0, AV_j - x_{ij})}{AV_j} \quad (2.4)$$

$$PDA_{ij} = \frac{\max(0, AV_j - x_{ij})}{AV_j} \quad (2.5)$$

$$NDA_{ij} = \frac{\max(0, x_{ij} - AV_j)}{AV_j} \quad (2.6)$$

Step 4. For each criterion, the weighted sum of PDA and NDA values is calculated according to Equations (2.7)–(2.8), using the weight coefficient (w_j) determined by experts or other methods.

$$SP_i = \sum_j^n w_j PDA_{ij} \quad (2.7)$$

$$SN_i = \sum_j^n w_j NDA_{ij} \quad (2.8)$$

Step 5. The weighted sum values of PDA and NDA are normalized using Equations (2.9)–(2.10).

$$NSP_i = \frac{SP_i}{\max_i(SP_i)} \quad (2.9)$$

$$NSN_i = 1 - \frac{SN_i}{\max_i(SN_i)} \quad (2.10)$$

Step 6. Finally, the compromise score of each alternative is calculated using Equation (2.11). The alternatives are ranked from high to low based on their compromise scores, and the alternative with the highest score is considered the best option.

$$AS_i = \frac{1}{2}(NSP_i + NSN_i) \quad (2.11)$$

2.2.2 Advantages and Disadvantages

The EDAS technique introduces various advantages and disadvantages which are crucial for decision-makers to review when implementing it in several scenarios as well as transportation. Below is a nuanced explanation of these features.

2.2.2.1 Advantages

One of the outstanding advantages of the EDAS technique is its simple calculation procedure. The technique needs only basic mathematical computation, enabling it for utilizers without comprehensive background in complicated mathematical modeling (Li & Wang, 2020). This straightforwardness makes its implementation in several areas, also in transportation, where rapid decision-making is usually significant.

The EDAS technique is flexible in facilitating both quantitative and qualitative criteria. This versatility enables decision-makers to integrate a large range of variables, such as environmental effect, cost and safety in their assessments (Yan et al., 2019). For example,

in transportation, it can be utilized to evaluate various routes or modes of transportation focusing on multiple criteria, improving the resilience of the decision-making procedure.

The EDAS technique is tailored to MCDM problems, enabling it appropriate for cases where multiple intersecting criteria should be assessed synchronously. This factor is specifically useful in transportation, where decision usually include comparisons between environmental sustainability, safety, cost and time (Ghorabae et al., 2015).

The technique proved robustness in handling the ambiguity and vagueness in input data. By concentrating on distances to an average solution, it may efficiently control deviations in input data, which is widespread in real-life transportation cases (Huang et al., 2022). This resilience improves the consistency of output obtained from the EDAS technique.

The EDAS technique enables a straightforward trade-off between alternatives though making a coherent ranking focusing on their distances to the average solution. This property is essentially helpful in transportation planning, where experts are required to assess several alternatives, such as vehicle types or routes, on the basis of multiple criteria (Barauskas et al., 2018).

2.2.2.2 Disadvantages

One of the important shortcomings of the EDAS technique is its vulnerability to the weights given to numerous criteria. If the weights are not properly assigned, the output may misguide, with a high possibility of non-ideal results in transportation planning (Wang, 2023). This sensitivity requires considerate evaluation and verification of the weighting procedure.

While the EDAS technique is useful for independent criteria, it can face hardship with cases if criteria are interdependent. In transportation, criteria such as time and cost can affect one another, and the EDAS technique does not specifically consider these relationships, which could result in deficient evaluations (Xu, 2024).

2.2.3 Related Works in Sustainable Transportation

The EDAS technique is more and more applied in many studies associated with transportation, especially with the sustainable transportation environment. This technique gives a structured framework to MCDM, enabling experts to assess several transportation alternatives on the basis of multiple intersecting criteria, such as service quality, environmental effect, cost and safety.

One outstanding implementation of the EDAS technique in transportation is in evaluating sustainable passenger transport systems. (Ghoushchi et al., 2023) applied MCDM techniques, also EDAS, to assess and to assign priority to sustainable vehicles for transport systems, pointing out the attention-gathering issue of climate change. This study underlines the technique's capability to integrate numerous criteria, such as GHG emissions and fuel efficiency, which are important for putting forward sustainable applications in the transport industry.

One other crucial implementation is in the assessment of urban public transportation systems. Wang (2023) used the EDAS technique to evaluate service quality in urban public transportation, showing its efficiency in establishing key performance indicators that adds value to overall service effectiveness. This implementation is essentially relevant since urban regions confront incrementing obligations to improve public transportation systems while they are reducing environmental effects.

Moreover, the EDAS technique is employed in the area of park-and-ride points. Barauskas et al. (2018) utilized the EDAS technique to order prospective location alternatives for park-and-ride parking places, evaluating several environmental, economic and social criteria. This study emphasizes the technique's flexibility in dealing with complicated decision-making cases in transport planning, where multiple shareholder stakes should be compensated.

The technique is applied in assessing alternative propulsion systems for urban public transportation. Petrović (2024) employed the EDAS technique to evaluate various propulsion systems, putting emphasis on their environmental and economic sustainability. This study is crucial as cities struggle to switch to greener transportation

alternatives, underlining the significance of systematic assessment techniques like EDAS in navigating policy decisions.

Additionally, the EDAS technique is combined with other MCDM methods to improve its usefulness. For instance, in the area of hazardous materials (hazmat) transport, a study integrated the Best-Worst Method (BWM) with EDAS to analyze routes for hazmat transport, highlighting the technique's versatility to various transport problems (Kumar, 2019).

2.2.4 Related Works in Aviation Industry

There is an acceleration in the studies of the aviation industry using the EDAS technique, especially on the sustainable aviation applications. This technique is specifically helpful for MCDM environment, where several intersecting criteria should be assessed synchronously. The EDAS technique enables decision-makers to evaluate alternatives on the basis of their distances to an average solution, accommodating a structured approach for analyzing alternatives in complicated scenarios such as aviation industry.

One important implementation of the EDAS technique in aviation industry is the assessment of sustainable aviation fuels. A study by Rajamanickam et al. (2023) concentrated on selecting fuel options of aviation utilizing the EDAS technique. The study is designed to make decision-makers assess the sustainability of numerous fuel options by taking into account multiple criteria, such as economic potential, environmental effect and performance effectiveness. This implementation shows how the EDAS technique may accommodate knowledge-based decision procedures in the aviation industry, especially as the sector looks for ways to decrease its carbon output and improve sustainability.

One other significant field where the EDAS technique is implemented is in airport performance assessment. Research by Ersoy (2021) used the EDAS technique to evaluate airport performance within the COVID19 pandemic period. Research had a target to assess several airports based on multiple criteria like health safety metrics, passenger satisfaction and operational effectiveness. Using the EDAS technique the study proposed

a structured framework to rank airports, which is critical for shareholders trying to enhance service quality and operational robustness amidst unprecedented challenges.

Furthermore, the EDAS technique is combined into a decision-making approach for aviation safety assessments. For example, research by Su et al. (2023) introduced the utilization of the EDAS technique in association with other decision-making methods to improve safety estimation in the aviation sector. The study drew attention to the significance of precisely estimating safety standards to apply efficient risk management plans. The EDAS technique's capability to manage multiple factors and to ensure well-established rankings of options makes it a beneficial technique for aviation safety management.

Additionally, the EDAS technique is used in assessing the sustainability of aviation operations. Research by Singh et al. (2022) analyzed the criteria impacting aviation sustainability and employed the EDAS technique to rank several sustainability solutions. This research highlighted the assets of the EDAS technique in leading decision-makers to apply efficient plans that are in line with sustainability goals in the aviation sector.

2.3 MAIRCA Method and Related Works

The Multi-Attribute Ideal-Real Comparative Analysis (MAIRCA) technique is a well-designed multi-criteria decision-making (MCDM) method that has obtained focus since its establishment in 2014 by a research team from the University of Defense's Center for Logistics Research in Belgrade, Serbia. It is introduced to construct a procedure for the assessment and selection of alternatives based on multiple criteria, specifically in complicated decision-making cases where comparisons between conflicting aims are needed (Vu, 2024; Pamučar et al., 2018). The MAIRCA technique works on the foundation of examining the ideal and real attributes of alternatives, enabling decision-makers to determine the most appropriate alternative based on a set of pre-established criteria (Muravev et al., 2020; Nguyen et al., 2022).

One of the important properties of the MAIRCA technique is its capability to overcome imprecision and uncertainty in decision-making. This is accomplished through the integration of fuzzy logic and interval-valued fuzzy sets, which improve the robustness of the evaluation by facilitating vagueness in the data input (Mishra, 2023; Yenilmez, 2024). For example, the incorporation of interval-valued intuitionistic fuzzy sets is shown to enhance the decision-making procedure by ensuring a more sophisticated representation of professional judgements and priorities (Ecer, 2022; Hezam et al., 2023). Moreover, the technique may be altered to many contexts, consisting of financial performance analysis, technology selection processes, and sustainability assessments, showing its adaptability.

The MAIRCA technique basically includes numerous phases: identifying the criteria and alternatives, defining the weights of criteria, and applying the trade-off analysis to rank the alternatives. The weight analysis can be achieved utilizing several techniques, such as the Best-Worst Method (BWM) or the Full Consistency Method (FUCOM), which sustain that the weights represent the relative significance of each criterion precisely (Zolfani et al., 2020; Pamučar et al., 2018). Once the weights are determined, the MAIRCA technique assesses the alternatives against the ideal solution, which is defined by the best possible performance throughout all criteria (Muravev et al., 2020; Pamučar et al., 2018). This assessment is usually illustrated through distance metrics, enabling a plain reflection of how each alternative weighs up against the ideal scenario (Wang et al., 2023).

Additionally, the MAIRCA technique is implemented with success in various fields, involving agriculture, energy technology development and logistics, proving its functional relevance across different areas (Pamučar et al., 2018; Esangbedo & Tang, 2023). For example, regarding sustainable electricity technology development, the MAIRCA technique efficiently assessed multiple technologies based on sustainable criteria, resulting in conscious decision-making (Tran, 2024). Likewise, in the agriculture industry, the technique is used to evaluate the financial analysis of insurance plans, emphasizing its applicability in real-world scenarios (Burhan, 2023; Kavas, 2024).

2.3.1 Computational Steps

The MAIRCA method is conducted in six steps:

Step 1. The formation of the initial decision matrix (X). The criteria values (x_{ij} , $i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$) are determined in the initial decision Equation (2.12) for each of the observed alternatives.

$$X = \begin{matrix} & \begin{matrix} C_1 & C_2 & \dots & C_n \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ \dots \\ A_m \end{matrix} & \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \end{matrix} \quad (2.12)$$

Equation (2.12) are obtained according to the personal preferences of the DM or by aggregation of experts' decisions.

Step 2. Determining the preference according to the alternative selection P_{A_i} . During the alternative selection, DM is neutral towards the process. In fact, it has no preference towards any of the proposed alternatives. The main assumption is that DM does not take into account the probability of choosing any alternative. DM perceives the alternatives as any of them can have an equal probability of appearing, so the preference to choose one among m possible alternatives is as follows:

$$P_{A_i} = \frac{1}{m}; \sum_{i=1}^m P_{A_i} = 1, i = 1, 2, \dots, m \quad (2.13)$$

where m denotes the total number of choices.

Step 3. Calculation of the elements of the theoretical evaluation (T_p). The theoretical evaluation matrix is constructed in the form of $n \times m$ (n is the total number of criteria, m is the total number of alternatives). The elements of the theoretical evaluation matrix (t_{pij}) are computed as the product of the preferences over the alternatives P_{A_i} and the weights of the criteria (w_i , $i = 1, 2, \dots, n$)

$$T_p = \begin{matrix} & w_1 & w_2 & \dots & w_n \\ \begin{matrix} P_{A_1} \\ P_{A_2} \\ \dots \\ P_{A_m} \end{matrix} & \begin{bmatrix} t_{p11} & t_{p12} & \dots & t_{p1n} \\ t_{p21} & t_{p22} & & t_{p2n} \\ \dots & \dots & \dots & \dots \\ t_{pm1} & t_{pm2} & \dots & t_{pmn} \end{bmatrix} \end{matrix} = \begin{matrix} & w_1 & w_2 & \dots & w_n \\ \begin{matrix} P_{A_1} \\ P_{A_2} \\ \dots \\ P_{A_m} \end{matrix} & \begin{bmatrix} P_{A_1}w_1 & P_{A_1}w_2 & \dots & P_{A_1}w_n \\ P_{A_2}w_1 & P_{A_2}w_2 & & P_{A_2}w_n \\ \dots & \dots & \dots & \dots \\ P_{A_m}w_1 & P_{A_m}w_2 & \dots & P_{A_m}w_n \end{bmatrix} \end{matrix} \quad (2.14)$$

Since DM is neutral towards the initial selection of alternatives, all preferences (P_{A_i}) are equal for all alternatives.

$$T_p = \begin{matrix} & w_1 & w_2 & \dots & w_n \\ P_{A_i} & [t_{p1} & t_{p2} & \dots & t_{pn}] \end{matrix} = \begin{matrix} & w_1 & w_2 & \dots & w_n \\ P_{A_i} & [P_{A_i}w_1 & P_{A_i}w_2 & \dots & P_{A_i}w_n] \end{matrix} \quad (2.15)$$

where n is the total number of criteria, t_{pi} is the theoretical evaluation.

Step 4. Determine the actual evaluation Equation (2.16).

$$T_r = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ \begin{matrix} A_1 \\ A_2 \\ \dots \\ A_m \end{matrix} & \begin{bmatrix} t_{r11} & t_{r12} & \dots & t_{r1n} \\ t_{r21} & t_{r22} & & t_{r2n} \\ \dots & \dots & \dots & \dots \\ t_{rm1} & t_{rm2} & \dots & t_{rmn} \end{bmatrix} \end{matrix} \quad (2.16)$$

where n is the total number of criteria, m is the total number of alternatives.

The calculation of the elements of the real evaluation matrix (T_r) is performed by multiplying the elements of the real evaluation matrix (T_p) and the elements of the initial decision matrix (X) according to the following equation:

- (a) For “Benefit” type criteria (larger criterion value is preferred).

$$t_{rij} = t_{pij} \left(\frac{x_{ij} - x_i^-}{x_i^+ - x_i^-} \right) \quad (2.17)$$

(b) For “Cost” type criteria (smaller criterion value is preferred).

$$t_{rij} = t_{pij} \left(\frac{x_{ij} - x_i^+}{x_i^- - x_i^+} \right) \quad (2.18)$$

where x_{ij} , x_i^+ and x_i^- denote the elements of the initial decision matrix (X), x_i^+ and x_i^- are defined as follows:

$x_i^+ = \max(x_1, x_2, \dots, x_m)$: are the maximum values of the marked criterion with respect to its alternatives.

$x_i^- = \min(x_1, x_2, \dots, x_m)$: are the minimum values of the marked criterion with respect to its alternatives.

Calculation of the total gap matrix (G). The elements of the matrix G are calculated as the difference (gap) between the theoretical (t_{pij}) and real evaluations (t_{rij}), or by subtracting the elements of the theoretical evaluation matrix (T_p) from the elements of the actual evaluation matrix (T_r).

$$G = T_p - T_r = \begin{bmatrix} g_{11} & g_{12} & \dots & g_{1n} \\ g_{21} & g_{22} & \dots & g_{2n} \\ \dots & \dots & \dots & \dots \\ g_{m1} & g_{m2} & \dots & g_{mn} \end{bmatrix} = \begin{bmatrix} t_{p11} - t_{r11} & t_{p12} - t_{r12} & \dots & t_{p1n} - t_{r1n} \\ t_{p21} - t_{r21} & t_{p22} - t_{r22} & \dots & t_{p2n} - t_{r2n} \\ \dots & \dots & \dots & \dots \\ t_{pm1} - t_{rm1} & t_{pm2} - t_{rm2} & \dots & t_{pmn} - t_{rmn} \end{bmatrix} \quad (2.19)$$

where n is the total number of criteria, m is the total number of alternatives.

Gap g_{ij} takes values from the range $g_{ij} \in [0, (t_{pij} - t_{rij})]$, according to Equation (2.20)

$$g_{ij} = \begin{cases} 0, & \text{if } t_{pij} = t_{rij} \\ t_{pij} - t_{rij}, & \text{if } t_{pij} > t_{rij} \end{cases} \quad (2.20)$$

It is desirable for g_{ij} to go to zero ($g_{ij} \rightarrow 0$) because the alternative with the smallest difference between the theoretical (t_{pij}) and real evaluation (t_{rij}) is selected. If the theoretical evaluation value of alternative A_i for criterion C_i is equal to the actual evaluation value ($t_{pij} = t_{rij}$), then the gap of alternative A_i for criterion C_i is $g_{ij} = 0$. In fact, alternative A_i is the best (ideal) alternative for criterion C_i (A_i^+).

If the alternative A_i for the criterion C_i has a theoretical evaluation value t_{pij} and the real ponder value $t_{rij} = 0$, then the gap for the alternative A_i for the criterion C_i is $g_{ij} = t_{pij}$. In fact, the alternative A_i is the worst (anti-ideal) alternative (A_i^-) for the criterion C_i .

Step 6. The calculation of the final values of the criteria functions (Q_i) for the alternatives. The values of the criteria functions are gathered from the sum of gaps (g_{ij}) for the alternatives, or simply from the sum of the matrix elements (G) in columns, utilizing Equation (2.21)

$$Q_i = \sum_{j=1}^n g_{ij}, i = 1, 2, \dots, m \quad (21)$$

2.3.2 Advantages and Disadvantages

As in any quantitative solution approach, MAIRCA has its advantages and disadvantages, which are important for decision-makers to take into account when determining a suitable MCDM tool.

2.3.2.1 Advantages

One of the important advantages of the MAIRCA technique is its capability to integrate imprecision and uncertainty through fuzzy logic and interval-valued fuzzy sets. This property enables for a more multifaceted representation of decision-makers evaluations, making the decision-making procedure more robust when faced with data vagueness (Yenilmez, 2024; Mishra, 2023). The MAIRCA technique is praised for its simple analytical calculations, which improve its applicability in comparison with other MCDM techniques. This straightforwardness enables experts to implement the technique without intensive training in complicated analytical approaches (Gökgöz & Yalçın, 2022; Ayadi et al., 2021). Moreover, the technique's dependence on an aggregating function to compute the proximity of alternative to an ideal solution refines the assessment process (Gökgöz & Yalçın, 2022). MAIRCA proved adaptability in several implementations, varying from sustainable technology assessments to financial performance analysis. This versatility makes it appropriate for a diverse collection of decision-making applications (Vu, 2024; Hezam et al., 2023). For example, it is efficiently implemented in assessing sustainable materials for biofuel production and evaluating the financial performance of institutions (Hezam et al., 2023). The MAIRCA technique can be incorporated with other MCDM methods, such as DEMATEL and TOPSIS, to improve its effectiveness. This amalgamation enables for a more extensive evaluation by utilizing the power muscles of multiple techniques (Pamučar et al., 2018; Esangbedo & Tang, 2023).

2.3.2.2 Disadvantages

An important shortcoming of the MAIRCA technique is its dependence on the precise identification of criteria weights. The performance of the technique is reliance upon effectively these weights represent the comparative importance of each criterion. If the weights are imprecisely determined, the results can be deceptive (Biswas et al., 2022; Albahri et al., 2020). The MAIRCA technique's results can be vulnerable to the data input, especially in cases where criteria are interrelated or include interdependencies. This vulnerability can result in fluctuation in outcomes, making it important for decision-makers to undertake comprehensive sensitivity analysis (Biswas et al., 2022; Muravev et al., 2020). Even though the technique's straightforwardness is an advantage, it can also result in over-simplified interpretation of complicated decision-making cases. Essential distinctions and connections across criteria may be neglected, possibly leading to distant solutions to ideal decisions (Krishnan et al., 2021; Mulliner et al., 2016). Despite the fact that MAIRCA can integrate fuzzy logic, its principle emphasis on analytical measures can constraint its efficiency in cases where qualitative evaluations are preeminent. This constraint requires mindful examination of how qualitative components are incorporated into the decision-making procedure (Yenilmezel, 2024; Mishra, 2023).

2.3.3 Related Works in Sustainable Transportation

The MAIRCA technique is efficiently used in several studies in sustainable transportation. This technique involves a well-defined framework for assessing multiple criteria, making it notably appropriate for indicating the fundamental challenges in transportation projects and policies.

MAIRCA is implemented in the context of sustainable urban transport development, where it helps in evaluating numerous transport alternatives based on sustainability criteria. For example, (Duleba & Moslem, 2018) used a synthesis of analytic hierarchy process (AHP) and MAIRCA to include stakeholders in the decision-making process for sustainable transport resolutions in Mersin, Türkiye. This framework enabled for an

extensive evaluation of transportation options, taking into account social and environmental effects (Duleba & Moslem, 2018).

Hezam et al. (2022) leveraged the MAIRCA technique to evaluate sustainable transport investment projects. By incorporating trapezoidal neutrosophic fuzzy numbers with MAIRCA, the study sustained a robust approach for the assessment of the sustainability of several transport projects, hence expediting knowledgeable decision-making procedure in investment determination. This implementation underlines MAIRCA's ability to integrate subjective judgments and vagueness, which are usually existing in transportation assessments (Hezam et al., 2022).

The MAIRCA technique is also effective in determining ideal locations for transportation infrastructure, i.e. tram lines and logistic centers. For example, Pamučar et al. (2018) introduced a combined DEMATEL-MAIRCA framework to determine a location for a multimodal logistics center, taking into account several sustainability criteria (Pamučar et al., 2018). Likewise, Jakimavičius and Burinskienė (2013) utilized MCDM tools, consisting of also MAIRCA, to evaluate cases for ultimate tram line improvements in Vilnius, proving its adaptability in urban transport planning (Jakimavičius & Burinskienė, 2013).

With regards to sustainable transportation, MAIRCA is used to assess alternative fuel vehicles as well. Hezam et al. (2022) developed a hybrid intuitionistic fuzzy MEREC-RS-DNMA framework, which included MAIRCA, to evaluate the sustainability of several alternative fuel vehicles. This study highlights the significance of considering numerous sustainability factors in the assessment procedure (Hezam et al., 2022).

The MAIRCA technique is applied to the selection of sustainable materials for transport projects. For instance, Haq et al. (2023) implemented an interval-valued neutrosophic MAIRCA technique to evaluate materials for their sustainability in transportation services, proving the technique's adaptability to deal with material selection difficulties within the transportation industry (Haq et al., 2023).

2.3.4 Related Works in Aviation Industry

MAIRCA is used to assess the sustainability of numerous aviation fuels. For example, Baroutaji et al. (2019) carried out a thorough investigation for fuel cell and hydrogen technology in the aviation industry, underlining the capabilities of these technologies as green energy sources. The MAIRCA technique can be used to evaluate the environmental effect, financial feasibility, and technical viability of distinct sustainable fuel alternatives, consequently helping decision-makers in deciding the most suitable fuel for sustainable aviation.

The MAIRCA technique is applied in safety management evaluations in the aviation sector. For instance, Ouyang (2017) explored safety management and official safety evaluation approaches, which can be improved by combining MAIRCA to assess numerous safety metrics and their impacts. By analyzing the best safety standards with real-life applications, MAIRCA can support defining gaps and emphasize safety protocol improvements.

The technique is also utilized to evaluate new technologies targeting to improve aviation safety and effectiveness. For example, Franke et al. (2023) examined metal-hydride-based hydrogen sensors for forthcoming sustainable aviation implementations. MAIRCA technique can introduce the assessment of these technologies by analyzing their performance against ideal standards, thus helping to select technologies that are in line with sustainability goals.

MAIRCA has been utilized to evaluate air traffic management systems, focusing on their efficiency and sustainability. By assessing various criteria such as operational efficiency, environmental impact, and safety, decision-makers can use MAIRCA to identify the most effective air traffic management solutions that minimize delays and reduce emissions. However, the references Basora et al. (2019) and Karimi et al. (2019) do not specifically address MAIRCA's application in this context and should be removed.

The aviation sector puts an important focus on human factors and maintenance, which are crucial for establishing operational effectiveness and safety. The MAIRCA technique can be implemented to assess training plans for maintenance employees, along with the efficiency of human factors measures in error reduction. For instance, Chai (2024)

explores human error in aviation maintenance, which can leverage MAIRCA by analyzing the efficiencies of alternative training plans against ideal benchmarks for performance. Moreover, Clare and Kourousis Clare & Kourousis (2021) bring understanding into deriving lessons from incidents in aviation maintenance that can provide expanded information on implementation of MAIRCA in this topic.

2.2.5 Summary of Literature Review

MAIRCA (The Multi Attributive Ideal-Real Comparative Analysis) method is one of the MCDM methods. It was introduced to the literature by Pamucar et al. (2014) and is a relatively new method compared to other MCDM methods. In order to provide solutions to decision-making problems involving uncertainty, the MAIRCA method has been extended with traditional fuzzy sets or fuzzy set extensions. When the studies in the literature since 2020 are examined, it is seen that the studies in which the method is extended with traditional or fuzzy set extensions have generally increased. In this section of the thesis study, when the "topic" tag in the Web of Science database was written as "fuzzy MAIRCA" and the studies of the last 5 years were filtered, a total of 52 articles emerged. 9 of these 52 articles were removed because they did not contain fuzziness or the MAIRCA method and one article is retracted.

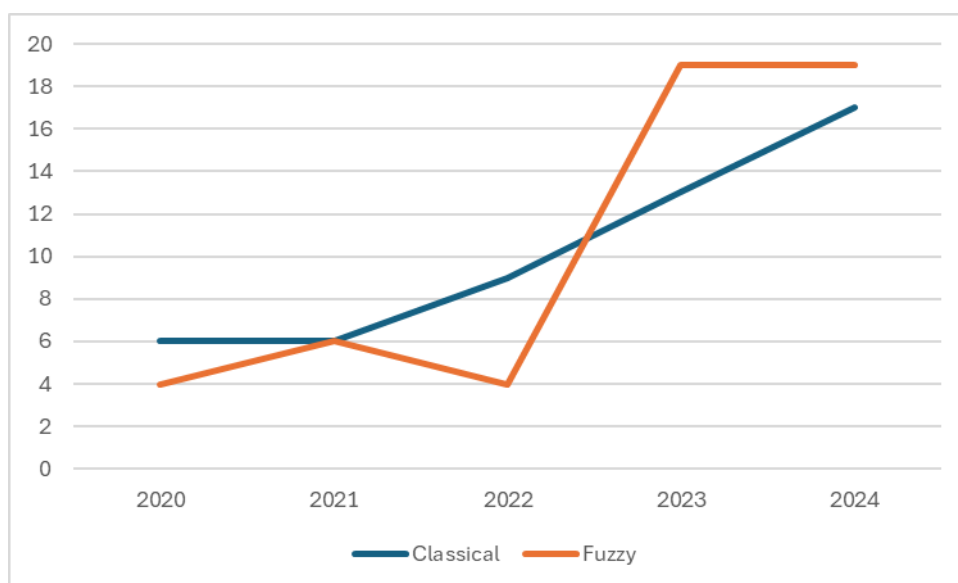


Figure 2.8 – MAIRCA method (2020-2024)

As seen in Figure 2.8, the number of studies, which was 4 in 2020, increased to 6 in 2021, and decreased slightly to 4 again in 2022. 2023 and 2024 show significant increase, total 19 articles were published both years.



3 PROPOSED NOVEL DECOMPOSED Q-RUNG ORTHOPAIR FUZZY SET EXTENSION

In this thesis, a novel fuzzy set named Decomposed q-Rung Orthopair Fuzzy Set (DqROFS) is introduced. Moreover, EDAS and MAIRCA methodologies are extended with DqROFSs to propose new evaluation techniques for the purpose of introducing new multiple criteria decision-making techniques in transportation industry from sustainability perspective. In the case study, these proposed evaluation methods with novel decomposed q-rung orthopair fuzzy sets extensions are applied in sustainable transportation for the alternative aircraft models in aviation industry.

Q-rung orthopair fuzzy set has broader representation strength when it is compared with classical and intuitionistic fuzzy sets. It enables a flexible establishment of membership and non-membership levels. Parameter q makes it possible to adjust membership and non-membership levels and eases complicated and conflicting expert decisions. It has more robust structure against conflicting and outlier evaluations from decision makers and produces more reliable results during decision processes.

Decomposed fuzzy set structure obtains functional (or optimistic) and dysfunctional (or pessimistic) evaluations from decision makers. It eliminates the impact of positively and negatively constructed survey questions. The decomposed approach easily reveals which elements are strong or weak in particular by allowing positive and negative inputs to be considered in separate dimensions. In this way, a much more in-depth and target-oriented evaluation is possible in the decision process. At the same time, instead of using a single membership value, it provides a multi-dimensional information structure, helping experts interpret the results more clearly. In this way, the processing and understanding of complex data becomes much more transparent and useful.

Q-rung's flexible uncertainty management ability, integrated with the multi-dimensional analysis abilities of the decomposed nature, provides effective outcomes in more complicated decision-making scenarios. The different perspectives of decision makers are gathered in a disaggregated way, and uncertainty is balanced with the Q-rung parameter. Thus, both detailed and holistic assessment is ensured.

After the development of intuitionistic fuzzy sets with an innovation brought by Atanassov to membership functions, different extensions of IFSs have been introduced by many researchers. Decomposed fuzzy sets developed by Cebi et al. (2022), the latest new extension of intuitionistic fuzzy sets in the literature deal with membership and non-membership degrees from two different perspectives, the functional point of view and dysfunctional point of view, in order to measure the inconsistency in expert judgments.

Definition 3.1

A DFS $\tilde{A}$ in a non-empty set X is an object having the form (Cebi et al., 2022)

$$\tilde{A} = \left\{ \langle x, \mathcal{F}(\mu_{\tilde{A}}^{\mathcal{F}}(x), \vartheta_{\tilde{A}}^{\mathcal{F}}(x)), \mathfrak{D}(\mu_{\tilde{A}}^{\mathfrak{D}}(x), \vartheta_{\tilde{A}}^{\mathfrak{D}}(x)) \rangle \mid x \in X \right\} \quad (3.1)$$

where the function $\mu_{\tilde{A}}^{\mathcal{F}}(x): X \rightarrow [0, 1]$, $\vartheta_{\tilde{A}}^{\mathcal{F}}(x): X \rightarrow [0, 1]$, $\mu_{\tilde{A}}^{\mathfrak{D}}(x): X \rightarrow [0, 1]$ and $\vartheta_{\tilde{A}}^{\mathfrak{D}}(x): X \rightarrow [0, 1]$ are the degrees of membership, non-membership of x to $\mathcal{F}$ and $\mathfrak{D}$, respectively where $\mathcal{F}$ and $\mathfrak{D}$ are functional point of view and dysfunctional point of view. DFSs must satisfy the following conditions: $\tilde{A}$

$$0 \leq \mu_{\tilde{A}}^{\mathcal{F}}(x) + \vartheta_{\tilde{A}}^{\mathcal{F}}(x) \leq 1 \quad (3.2)$$

$$0 \leq \mu_{\tilde{A}}^{\mathfrak{D}}(x) + \vartheta_{\tilde{A}}^{\mathfrak{D}}(x) \leq 1 \quad (3.3)$$

In this section, the basic principles and mathematical operations of single-valued Decomposed q-Rung Orthopair Fuzzy Sets (SV-Dq-ROFS), which consider both the properties of DFSs developed by Cebi et al. (2022) and the properties of q-Rung Orthopair fuzzy sets developed by Yager (2017), are introduced.

Definition 3.2

Let X be a universe of discourse. A decomposed q -Rung Orthopair fuzzy set (Dq-ROFS) $\tilde{A}$ in X is given by

$$\tilde{A} = \left\{ \langle x, \mathcal{F}(\mu_A^{\mathcal{F}}(x), \vartheta_A^{\mathcal{F}}(x)), \mathfrak{D}(\mu_A^{\mathfrak{D}}(x), \vartheta_A^{\mathfrak{D}}(x)) \rangle \mid x \in X \right\} \quad (3.4)$$

where the function $\mu_A^{\mathcal{F}}(x): X \rightarrow [0, 1]$, $\vartheta_A^{\mathcal{F}}(x): X \rightarrow [0, 1]$, $\mu_A^{\mathfrak{D}}(x): X \rightarrow [0, 1]$ and $\vartheta_A^{\mathfrak{D}}(x): X \rightarrow [0, 1]$ are the degrees of membership, non-membership of x to $\mathcal{F}$ and $\mathfrak{D}$, respectively where $\mathcal{F}$ and $\mathfrak{D}$ are functional point of view and dysfunctional point of view. DqROFS must satisfy the following conditions

$$0 \leq \left(\mu_A^{\mathcal{F}}(x)\right)^q + \left(\vartheta_A^{\mathcal{F}}(x)\right)^q \leq 1 \quad (3.5)$$

$$0 \leq \left(\mu_A^{\mathfrak{D}}(x)\right)^q + \left(\vartheta_A^{\mathfrak{D}}(x)\right)^q \leq 1 \quad (3.6)$$

Definition 3.3

Let $\tilde{A} = \{\mathcal{F}(\mu_A^{\mathcal{F}}, \vartheta_A^{\mathcal{F}}), \mathfrak{D}(\mu_A^{\mathfrak{D}}, \vartheta_A^{\mathfrak{D}})\}$, $\tilde{A}_1 = \{\mathcal{F}(\mu_{A_1}^{\mathcal{F}}, \vartheta_{A_1}^{\mathcal{F}}), \mathfrak{D}(\mu_{A_1}^{\mathfrak{D}}, \vartheta_{A_1}^{\mathfrak{D}})\}$ and $\tilde{A}_2 = \{\mathcal{F}(\mu_{A_2}^{\mathcal{F}}, \vartheta_{A_2}^{\mathcal{F}}), \mathfrak{D}(\mu_{A_2}^{\mathfrak{D}}, \vartheta_{A_2}^{\mathfrak{D}})\}$ be three Dq-ROFSs. The basic operators of Dq-ROFSs are defined as follows:

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 = & \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{\left(\mu_{A_1}^{\mathcal{F}}(x)\right)^q + \left(\mu_{A_2}^{\mathcal{F}}(x)\right)^q - \left(\mu_{A_1}^{\mathcal{F}}(x)\right)^q \left(\mu_{A_2}^{\mathcal{F}}(x)\right)^q}, \vartheta_{A_1}^{\mathcal{F}}(x) \vartheta_{A_2}^{\mathcal{F}}(x) \right), \\ & \mathfrak{D} \left(\sqrt[q]{\left(\mu_{A_1}^{\mathfrak{D}}(x)\right)^q + \left(\mu_{A_2}^{\mathfrak{D}}(x)\right)^q - \left(\mu_{A_1}^{\mathfrak{D}}(x)\right)^q \left(\mu_{A_2}^{\mathfrak{D}}(x)\right)^q}, \vartheta_{A_1}^{\mathfrak{D}}(x) \vartheta_{A_2}^{\mathfrak{D}}(x) \right) \end{aligned} \right\} \quad (3.7) \end{aligned}$$

$$\tilde{A}_1 \otimes \tilde{A}_2 = \left\{ \begin{aligned} &\mathcal{F} \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x) \mu_{\tilde{A}_2}^{\mathcal{F}}(x), \sqrt[q]{\left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^q + \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^q - \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^q \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^q} \right), \\ &\mathfrak{D} \left(\mu_{\tilde{A}_1}^{\mathfrak{D}}(x) \mu_{\tilde{A}_2}^{\mathfrak{D}}(x), \sqrt[q]{\left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}}(x) \right)^q + \left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}}(x) \right)^q - \left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}}(x) \right)^q \left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}}(x) \right)^q} \right) \end{aligned} \right\} \quad (3.8)$$

$$\lambda \tilde{A} = \left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}}^{\mathcal{F}})^q)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}}^{\mathfrak{D}})^q)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathfrak{D}})^{\lambda} \right) \right\} \quad (3.9)$$

$$\tilde{A}^{\lambda} = \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^q)^{\lambda}} \right), \mathfrak{D} \left((\mu_{\tilde{A}}^{\mathfrak{D}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_{\tilde{A}}^{\mathfrak{D}})^q)^{\lambda}} \right) \right\} \quad (3.10)$$

Remark 3.1

The following equations present the special cases of λ and $\tilde{A}$.

a) If $\tilde{A} = \{\mathcal{F}(\mathbf{1}, \mathbf{0}), \mathfrak{D}(\mathbf{0}, \mathbf{1})\}$ then,

$$\lambda \tilde{A} = \left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}}^{\mathcal{F}})^q)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}}^{\mathfrak{D}})^q)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathfrak{D}})^{\lambda} \right) \right\} = \{\mathcal{F}(\mathbf{1}, \mathbf{0}), \mathfrak{D}(\mathbf{0}, \mathbf{1})\} \quad (3.11)$$

$$\tilde{A}^{\lambda} = \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^q)^{\lambda}} \right), \mathfrak{D} \left((\mu_{\tilde{A}}^{\mathfrak{D}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_{\tilde{A}}^{\mathfrak{D}})^q)^{\lambda}} \right) \right\} = \{\mathcal{F}(\mathbf{1}, \mathbf{0}), \mathfrak{D}(\mathbf{0}, \mathbf{1})\} \quad (3.12)$$

b) If $\tilde{A} = \{\mathcal{F}(\mathbf{0}, \mathbf{1}), \mathfrak{D}(\mathbf{1}, \mathbf{0})\}$ then,

$$\lambda \tilde{A} = \left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}}^{\mathcal{F}})^q)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}}^{\mathfrak{D}})^q)^{\lambda}}, (\vartheta_{\tilde{A}}^{\mathfrak{D}})^{\lambda} \right) \right\} = \{\mathcal{F}(\mathbf{0}, \mathbf{1}), \mathfrak{D}(\mathbf{1}, \mathbf{0})\} \quad (3.13)$$

$$\tilde{A}^{\lambda} = \left\{ \mathcal{F} \left((\mu_{\tilde{A}}^{\mathcal{F}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_{\tilde{A}}^{\mathcal{F}})^q)^{\lambda}} \right), \mathfrak{D} \left((\mu_{\tilde{A}}^{\mathfrak{D}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_{\tilde{A}}^{\mathfrak{D}})^q)^{\lambda}} \right) \right\} = \{\mathcal{F}(\mathbf{0}, \mathbf{1}), \mathfrak{D}(\mathbf{1}, \mathbf{0})\} \quad (3.14)$$

c) If $\lambda \rightarrow \mathbf{0}$ and $\mathbf{0} < \mu_{\tilde{A}}^{\mathcal{F}}, \vartheta_{\tilde{A}}^{\mathcal{F}}, \mu_{\tilde{A}}^{\mathfrak{D}}, \vartheta_{\tilde{A}}^{\mathfrak{D}} < \mathbf{1}$ then,

$$\lambda \tilde{A} = \left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_A^{\mathcal{F}})^q)^{\lambda}}, (\vartheta_A^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_A^{\mathfrak{D}})^q)^{\lambda}}, (\vartheta_A^{\mathfrak{D}})^{\lambda} \right) \right\} = \{\mathcal{F}(\mathbf{0}, \mathbf{1}), \mathfrak{D}(\mathbf{0}, \mathbf{1})\} \quad (3.15)$$

$$\tilde{A}^{\lambda} = \left\{ \mathcal{F} \left((\mu_A^{\mathcal{F}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_A^{\mathcal{F}})^q)^{\lambda}} \right), \mathfrak{D} \left((\mu_A^{\mathfrak{D}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_A^{\mathfrak{D}})^q)^{\lambda}} \right) \right\} = \{\mathcal{F}(\mathbf{1}, \mathbf{0}), \mathfrak{D}(\mathbf{1}, \mathbf{0})\} \quad (3.16)$$

d) If $\lambda \rightarrow \infty$ and $\mathbf{0} < \mu_A^{\mathcal{F}}, \vartheta_A^{\mathcal{F}}, \mu_A^{\mathfrak{D}}, \vartheta_A^{\mathfrak{D}} < \mathbf{1}$ then,

$$\lambda \tilde{A} = \left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_A^{\mathcal{F}})^q)^{\lambda}}, (\vartheta_A^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_A^{\mathfrak{D}})^q)^{\lambda}}, (\vartheta_A^{\mathfrak{D}})^{\lambda} \right) \right\} = \{\mathcal{F}(\mathbf{1}, \mathbf{0}), \mathfrak{D}(\mathbf{1}, \mathbf{0})\} \quad (3.17)$$

$$\tilde{A}^{\lambda} = \left\{ \mathcal{F} \left((\mu_A^{\mathcal{F}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_A^{\mathcal{F}})^q)^{\lambda}} \right), \mathfrak{D} \left((\mu_A^{\mathfrak{D}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_A^{\mathfrak{D}})^q)^{\lambda}} \right) \right\} = \{\mathcal{F}(\mathbf{0}, \mathbf{1}), \mathfrak{D}(\mathbf{0}, \mathbf{1})\} \quad (3.18)$$

e) If $\lambda = \mathbf{1}$ then,

$$\lambda \tilde{A} = \left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_A^{\mathcal{F}})^q)^{\lambda}}, (\vartheta_A^{\mathcal{F}})^{\lambda} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_A^{\mathfrak{D}})^q)^{\lambda}}, (\vartheta_A^{\mathfrak{D}})^{\lambda} \right) \right\} = \{\mathcal{F}(\mu_A^{\mathcal{F}}, \vartheta_A^{\mathcal{F}}), \mathfrak{D}(\mu_A^{\mathfrak{D}}, \vartheta_A^{\mathfrak{D}})\} \quad (3.19)$$

$$\tilde{A}^{\lambda} = \left\{ \mathcal{F} \left((\mu_A^{\mathcal{F}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_A^{\mathcal{F}})^q)^{\lambda}} \right), \mathfrak{D} \left((\mu_A^{\mathfrak{D}})^{\lambda}, \sqrt[q]{1 - (1 - (\vartheta_A^{\mathfrak{D}})^q)^{\lambda}} \right) \right\} = \{\mathcal{F}(\mu_A^{\mathcal{F}}, \vartheta_A^{\mathcal{F}}), \mathfrak{D}(\mu_A^{\mathfrak{D}}, \vartheta_A^{\mathfrak{D}})\} \quad (3.20)$$

i.e.

$$\lambda \tilde{A} = \tilde{A} \quad (3.21)$$

$$\tilde{A}^{\lambda} = \tilde{A} \quad (3.22)$$

Definition 3.4

Let $\lambda, \lambda_1, \lambda_2 \geq \mathbf{0}$, then

$$\text{i.} \quad \tilde{A}_1 \oplus \tilde{A}_2 = \tilde{A}_2 \oplus \tilde{A}_1 \quad (3.23)$$

$$\text{ii.} \quad \tilde{A}_1 \otimes \tilde{A}_2 = \tilde{A}_2 \otimes \tilde{A}_1 \quad (3.24)$$

$$\text{iii.} \quad \lambda(\tilde{A}_1 \oplus \tilde{A}_2) = \lambda\tilde{A}_1 \oplus \lambda\tilde{A}_2 \quad (3.25)$$

$$\text{iv.} \quad (\tilde{A}_1 \otimes \tilde{A}_2)^\lambda = \tilde{A}_1^\lambda \otimes \tilde{A}_2^\lambda \quad (3.26)$$

$$\text{v.} \quad \lambda_1\tilde{A} \oplus \lambda_2\tilde{A} = (\lambda_1 + \lambda_2)\tilde{A} \quad (3.27)$$

$$\text{vi.} \quad \tilde{A}^{\lambda_1} \otimes \tilde{A}^{\lambda_2} = \tilde{A}^{\lambda_1 + \lambda_2} \quad (3.28)$$

Proof 3.4

(i) by Definition 3.3, we have

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 = & \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{\left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x)\right)^q + \left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x)\right)^q - \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x)\right)^q \left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x)\right)^q}, \vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right), \\ & \mathcal{D} \left(\sqrt[q]{\left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x)\right)^q + \left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x)\right)^q - \left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x)\right)^q \left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x)\right)^q}, \vartheta_{\tilde{A}_1}^{\mathcal{D}}(x) \vartheta_{\tilde{A}_2}^{\mathcal{D}}(x) \right) \end{aligned} \right\} \quad (3.29) \end{aligned}$$

$$\begin{aligned} \tilde{A}_2 \oplus \tilde{A}_1 = & \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{\left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x)\right)^q + \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x)\right)^q - \left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x)\right)^q \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x)\right)^q}, \vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right), \\ & \mathcal{D} \left(\sqrt[q]{\left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x)\right)^q + \left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x)\right)^q - \left(\mu_{\tilde{A}_2}^{\mathcal{D}}(x)\right)^q \left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x)\right)^q}, \vartheta_{\tilde{A}_2}^{\mathcal{D}}(x) \vartheta_{\tilde{A}_1}^{\mathcal{D}}(x) \right) \end{aligned} \right\} \quad (3.30) \end{aligned}$$

and so, $\tilde{A}_1 \oplus \tilde{A}_2 = \tilde{A}_2 \oplus \tilde{A}_1$.

(ii) by Definition 3.3, we have

$$\begin{aligned} \tilde{A}_1 \otimes \tilde{A}_2 = & \left\{ \begin{aligned} & \mathcal{F} \left(\mu_{\tilde{A}_1}^{\mathcal{F}}(x) \mu_{\tilde{A}_2}^{\mathcal{F}}(x), \sqrt[q]{\left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x)\right)^q + \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x)\right)^q - \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x)\right)^q \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x)\right)^q} \right), \\ & \mathcal{D} \left(\mu_{\tilde{A}_1}^{\mathcal{D}}(x) \mu_{\tilde{A}_2}^{\mathcal{D}}(x), \sqrt[q]{\left(\vartheta_{\tilde{A}_1}^{\mathcal{D}}(x)\right)^q + \left(\vartheta_{\tilde{A}_2}^{\mathcal{D}}(x)\right)^q - \left(\vartheta_{\tilde{A}_1}^{\mathcal{D}}(x)\right)^q \left(\vartheta_{\tilde{A}_2}^{\mathcal{D}}(x)\right)^q} \right) \end{aligned} \right\} \quad (3.31) \end{aligned}$$

$$\begin{aligned} \tilde{A}_2 \otimes \tilde{A}_1 = & \left\{ \begin{aligned} & \mathcal{F} \left(\mu_{\tilde{A}_2}^{\mathcal{F}}(x) \mu_{\tilde{A}_1}^{\mathcal{F}}(x), \sqrt[q]{\left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^q + \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^q - \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}}(x) \right)^q \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}}(x) \right)^q} \right), \\ & \mathfrak{D} \left(\mu_{\tilde{A}_2}^{\mathfrak{D}}(x) \mu_{\tilde{A}_1}^{\mathfrak{D}}(x), \sqrt[q]{\left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}}(x) \right)^q + \left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}}(x) \right)^q - \left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}}(x) \right)^q \left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}}(x) \right)^q} \right) \end{aligned} \right\} \quad (3.32) \end{aligned}$$

and so, $\tilde{A}_1 \otimes \tilde{A}_2 = \tilde{A}_2 \otimes \tilde{A}_1$.

(iii) by Definition 3.3, we have

$$\begin{aligned} \lambda(\tilde{A}_1 \oplus \tilde{A}_2) &= \lambda \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{\left(\mu_{\tilde{A}_1}^{\mathcal{F}} \right)^q + \left(\mu_{\tilde{A}_2}^{\mathcal{F}} \right)^q - \left(\mu_{\tilde{A}_1}^{\mathcal{F}} \right)^q \left(\mu_{\tilde{A}_2}^{\mathcal{F}} \right)^q}, \vartheta_{\tilde{A}_1}^{\mathcal{F}} \vartheta_{\tilde{A}_2}^{\mathcal{F}} \right), \\ & \mathfrak{D} \left(\sqrt[q]{\left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \right)^q + \left(\mu_{\tilde{A}_2}^{\mathfrak{D}} \right)^q - \left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \right)^q \left(\mu_{\tilde{A}_2}^{\mathfrak{D}} \right)^q}, \vartheta_{\tilde{A}_1}^{\mathfrak{D}} \vartheta_{\tilde{A}_2}^{\mathfrak{D}} \right) \end{aligned} \right\} = \\ & \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{1 - \left(1 - \left(\left(\mu_{\tilde{A}_1}^{\mathcal{F}} \right)^q + \left(\mu_{\tilde{A}_2}^{\mathcal{F}} \right)^q - \left(\mu_{\tilde{A}_1}^{\mathcal{F}} \right)^q \left(\mu_{\tilde{A}_2}^{\mathcal{F}} \right)^q \right)^{\lambda}}, \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}} \right)^{\lambda} \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}} \right)^{\lambda} \right), \\ & \mathfrak{D} \left(\sqrt[q]{1 - \left(1 - \left(\left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \right)^q + \left(\mu_{\tilde{A}_2}^{\mathfrak{D}} \right)^q - \left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \right)^q \left(\mu_{\tilde{A}_2}^{\mathfrak{D}} \right)^q \right)^{\lambda}}, \left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}} \right)^{\lambda} \left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}} \right)^{\lambda} \right) \end{aligned} \right\} \quad (3.33) \\ \lambda \tilde{A}_1 \oplus \lambda \tilde{A}_2 &= \\ & \left\{ \left(\mathcal{F} \left(\sqrt[q]{1 - \left(1 - \left(\mu_{\tilde{A}_1}^{\mathcal{F}} \right)^q \right)^{\lambda}}, \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}} \right)^{\lambda} \right), \mathfrak{D} \left(\sqrt[q]{1 - \left(1 - \left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \right)^q \right)^{\lambda}}, \left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}} \right)^{\lambda} \right) \right) \oplus \right. \\ & \left. \left(\mathcal{F} \left(\sqrt[q]{1 - \left(1 - \left(\mu_{\tilde{A}_2}^{\mathcal{F}} \right)^q \right)^{\lambda}}, \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}} \right)^{\lambda} \right), \mathfrak{D} \left(\sqrt[q]{1 - \left(1 - \left(\mu_{\tilde{A}_2}^{\mathfrak{D}} \right)^q \right)^{\lambda}}, \left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}} \right)^{\lambda} \right) \right) \right\} = \\ & \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{1 - \left(1 - \left(\mu_{\tilde{A}_1}^{\mathcal{F}} \right)^q \right)^{\lambda} + 1 - \left(1 - \left(\mu_{\tilde{A}_2}^{\mathcal{F}} \right)^q \right)^{\lambda} - \left(1 - \left(1 - \left(\mu_{\tilde{A}_1}^{\mathcal{F}} \right)^q \right)^{\lambda} \right) \left(1 - \left(1 - \left(\mu_{\tilde{A}_2}^{\mathcal{F}} \right)^q \right)^{\lambda} \right)}, \left(\vartheta_{\tilde{A}_1}^{\mathcal{F}} \right)^{\lambda} \left(\vartheta_{\tilde{A}_2}^{\mathcal{F}} \right)^{\lambda} \right), \\ & \mathfrak{D} \left(\sqrt[q]{1 - \left(1 - \left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \right)^q \right)^{\lambda} + 1 - \left(1 - \left(\mu_{\tilde{A}_2}^{\mathfrak{D}} \right)^q \right)^{\lambda} - \left(1 - \left(1 - \left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \right)^q \right)^{\lambda} \right) \left(1 - \left(1 - \left(\mu_{\tilde{A}_2}^{\mathfrak{D}} \right)^q \right)^{\lambda} \right)}, \left(\vartheta_{\tilde{A}_1}^{\mathfrak{D}} \right)^{\lambda} \left(\vartheta_{\tilde{A}_2}^{\mathfrak{D}} \right)^{\lambda} \right) \end{aligned} \right\} = \end{aligned}$$

$$\begin{aligned}
& \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^q)^\lambda (1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^q)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^\lambda \right), \\ & \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^q)^\lambda (1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^q)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^\lambda \right) \end{aligned} \right\} = \\
& \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{1 - \left(1 - ((\mu_{\tilde{A}_1}^{\mathcal{F}})^q + (\mu_{\tilde{A}_2}^{\mathcal{F}})^q - (\mu_{\tilde{A}_1}^{\mathcal{F}})^q (\mu_{\tilde{A}_2}^{\mathcal{F}})^q)\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^\lambda \right), \\ & \mathfrak{D} \left(\sqrt[q]{1 - \left(1 - ((\mu_{\tilde{A}_1}^{\mathfrak{D}})^q + (\mu_{\tilde{A}_2}^{\mathfrak{D}})^q - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^q (\mu_{\tilde{A}_2}^{\mathfrak{D}})^q)\right)^\lambda}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^\lambda \right) \end{aligned} \right\} \quad (3.34)
\end{aligned}$$

and so, $\lambda(\tilde{A}_1 \oplus \tilde{A}_2) = \lambda\tilde{A}_1 \oplus \lambda\tilde{A}_2$.

(iv) By Definition 3.3, we have

$$\begin{aligned}
(\tilde{A}_1 \otimes \tilde{A}_2)^\lambda &= \left\{ \begin{aligned} & \mathcal{F} \left(\mu_{\tilde{A}_1}^{\mathcal{F}} \mu_{\tilde{A}_2}^{\mathcal{F}}, \sqrt[q]{(\vartheta_{\tilde{A}_1}^{\mathcal{F}})^q + (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^q - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^q (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^q} \right), \\ & \mathfrak{D} \left(\mu_{\tilde{A}_1}^{\mathfrak{D}} \mu_{\tilde{A}_2}^{\mathfrak{D}}, \sqrt[q]{(\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^q + (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^q - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^q (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^q} \right) \end{aligned} \right\} = \\
& \left\{ \begin{aligned} & \mathcal{F} \left((\mu_{\tilde{A}_1}^{\mathcal{F}})^\lambda (\mu_{\tilde{A}_2}^{\mathcal{F}})^\lambda, \sqrt[q]{1 - \left(1 - ((\vartheta_{\tilde{A}_1}^{\mathcal{F}})^q + (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^q - (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^q (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^q)\right)^\lambda} \right), \\ & \mathfrak{D} \left((\mu_{\tilde{A}_1}^{\mathfrak{D}})^\lambda (\mu_{\tilde{A}_2}^{\mathfrak{D}})^\lambda, \sqrt[q]{1 - \left(1 - ((\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^q + (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^q - (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^q (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^q)\right)^\lambda} \right) \end{aligned} \right\} \quad (3.35)
\end{aligned}$$

and so, $(\tilde{A}_1 \otimes \tilde{A}_2)^\lambda = \tilde{A}_1^\lambda \otimes \tilde{A}_2^\lambda$.

The other proofs of the equations in Definition 2.4 are omitted here since their proofs are trivial.

Definition 3.5

Let $\tilde{A}_i = \{\mathcal{F}(\mu_{\tilde{A}_i}^{\mathcal{F}}, \vartheta_{\tilde{A}_i}^{\mathcal{F}}), \mathfrak{D}(\mu_{\tilde{A}_i}^{\mathfrak{D}}, \vartheta_{\tilde{A}_i}^{\mathfrak{D}})\}$, ($i = 1, 2, \dots, n$) be a set of DqROFSs and $\lambda_i = (\lambda_1, \lambda_2, \dots, \lambda_n)$ be weight vector of $\tilde{A}_i$ and with $\sum_{i=1}^n \lambda_i = 1$ and $\lambda_i \in [0, 1]$, then a Decomposed q-Rung Orthopair Weighted Arithmetic Mean (**DqROWAM**) operator is defined as

$$\begin{aligned}
DqROWAM(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) &= \lambda_1 \tilde{A}_1 \oplus \lambda_2 \tilde{A}_2 \oplus \dots \oplus \lambda_n \tilde{A}_n \\
&= \left\{ \mathcal{F} \left(\sqrt[q]{1 - \prod_{i=1}^n (1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^q)^{\lambda_i}}, \prod_{i=1}^n (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^{\lambda_i} \right), \mathfrak{D} \left(\sqrt[q]{1 - \prod_{i=1}^n (1 - (\mu_{\tilde{A}_i}^{\mathfrak{D}})^q)^{\lambda_i}}, \prod_{i=1}^n (\vartheta_{\tilde{A}_i}^{\mathfrak{D}})^{\lambda_i} \right) \right\}
\end{aligned} \tag{3.36}$$

Proof 3.5

We can prove Eq. (3.37) by using mathematical induction on n:

when n = 2,

$$DqROWAM(\tilde{A}_1, \tilde{A}_2) = \lambda_1 \tilde{A}_1 \oplus \lambda_2 \tilde{A}_2 \tag{3.37}$$

According to Definition 3.3, we can see that both $\lambda_1 \tilde{A}_1$ and $\lambda_2 \tilde{A}_2$ are DqROFSs, and the value of $\lambda_1 \tilde{A}_1 + \lambda_2 \tilde{A}_2$ is also a DqROFSs. By the operational law (iv) in Definition 2.4, we have

$$\lambda_1 \tilde{A}_1 = \left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^q)^{\lambda_1}}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^{\lambda_1} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^q)^{\lambda_1}}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^{\lambda_1} \right) \right\} \tag{3.38}$$

$$\lambda_2 \tilde{A}_2 = \left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^q)^{\lambda_2}}, (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^{\lambda_2} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^q)^{\lambda_2}}, (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^{\lambda_2} \right) \right\} \tag{3.39}$$

$$\begin{aligned}
\lambda_1 \tilde{A}_1 \oplus \lambda_2 \tilde{A}_2 &= \\
&\left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^q)^{\lambda_1}}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^{\lambda_1} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathfrak{D}})^q)^{\lambda_1}}, (\vartheta_{\tilde{A}_1}^{\mathfrak{D}})^{\lambda_1} \right) \right\} \oplus \\
&\left\{ \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^q)^{\lambda_2}}, (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^{\lambda_2} \right), \mathfrak{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_2}^{\mathfrak{D}})^q)^{\lambda_2}}, (\vartheta_{\tilde{A}_2}^{\mathfrak{D}})^{\lambda_2} \right) \right\} =
\end{aligned}$$

$$\begin{aligned}
& \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^q)^{\lambda_1} + 1 - (1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^q)^{\lambda_2}} - \right. \\ & \left. (1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^q)^{\lambda_1}) (1 - (1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^q)^{\lambda_2}), (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^{\lambda_1} (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^{\lambda_2} \right), \\ & \mathcal{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{D}})^q)^{\lambda_1} + 1 - (1 - (\mu_{\tilde{A}_2}^{\mathcal{D}})^q)^{\lambda_2}} - \right. \\ & \left. (1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{D}})^q)^{\lambda_1}) (1 - (1 - (\mu_{\tilde{A}_2}^{\mathcal{D}})^q)^{\lambda_2}), (\vartheta_{\tilde{A}_1}^{\mathcal{D}})^{\lambda_1} (\vartheta_{\tilde{A}_2}^{\mathcal{D}})^{\lambda_2} \right) \end{aligned} \right\} = \\
& \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{F}})^q)^{\lambda_1} (1 - (\mu_{\tilde{A}_2}^{\mathcal{F}})^q)^{\lambda_2}}, (\vartheta_{\tilde{A}_1}^{\mathcal{F}})^{\lambda_1} (\vartheta_{\tilde{A}_2}^{\mathcal{F}})^{\lambda_2} \right), \\ & \mathcal{D} \left(\sqrt[q]{1 - (1 - (\mu_{\tilde{A}_1}^{\mathcal{D}})^q)^{\lambda_1} (1 - (\mu_{\tilde{A}_2}^{\mathcal{D}})^q)^{\lambda_2}}, (\vartheta_{\tilde{A}_1}^{\mathcal{D}})^{\lambda_1} (\vartheta_{\tilde{A}_2}^{\mathcal{D}})^{\lambda_2} \right) \end{aligned} \right\} \quad (3.40)
\end{aligned}$$

Thus, this result is true for $\mathbf{n} = \mathbf{2}$. Assume that the result is true for $\mathbf{n} = \mathbf{k}$, Equation **DqROWAM** holds, i.e.

$$\begin{aligned}
& \mathbf{DqROWAM}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_k) = \lambda_1 \tilde{A}_1 \oplus \lambda_2 \tilde{A}_2 \oplus \dots \oplus \lambda_k \tilde{A} = \\
& \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{1 - \prod_{i=1}^k (1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^q)^{w_i}}, \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^{w_i} \right), \\ & \mathcal{D} \left(\sqrt[q]{1 - \prod_{i=1}^k (1 - (\mu_{\tilde{A}_i}^{\mathcal{D}})^q)^{w_i}}, \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathcal{D}})^{w_i} \right) \end{aligned} \right\} \quad (3.41)
\end{aligned}$$

Then, when $\mathbf{n} = \mathbf{k} + \mathbf{1}$,

$$\begin{aligned}
& \mathbf{DqROWAM}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_{k+1}) = \mathbf{DqROWAM}(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_k) \oplus \lambda_{k+1} \tilde{A}_{k+1} = \\
& \left\{ \begin{aligned} & \mathcal{F} \left(\sqrt[q]{1 - \prod_{i=1}^k (1 - (\mu_{\tilde{A}_i}^{\mathcal{F}})^q)^{\lambda_i}}, \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^{\lambda_i} \right), \\ & \mathcal{D} \left(\sqrt[q]{1 - \prod_{i=1}^k (1 - (\mu_{\tilde{A}_i}^{\mathcal{D}})^q)^{\lambda_i}}, \prod_{i=1}^k (\vartheta_{\tilde{A}_i}^{\mathcal{D}})^{\lambda_i} \right) \end{aligned} \right\} \oplus
\end{aligned}$$

$$\begin{aligned}
& \left\{ \mathcal{F} \left(\sqrt[q]{1 - \left(1 - \left(\mu_{\tilde{A}_{k+1}}^{\mathcal{F}}\right)^q\right)^{\lambda_{k+1}}}, \left(\vartheta_{\tilde{A}_{k+1}}^{\mathcal{F}}\right)^{\lambda_{k+1}} \right), \right. \\
& \left. \mathcal{D} \left(\sqrt[q]{1 - \left(1 - \left(\mu_{\tilde{A}_{k+1}}^{\mathcal{D}}\right)^q\right)^{\lambda_{k+1}}}, \left(\vartheta_{\tilde{A}_{k+1}}^{\mathcal{D}}\right)^{\lambda_{k+1}} \right) \right\} = \\
& \left\{ \mathcal{F} \left(\sqrt[q]{1 - \prod_{i=1}^{k+1} \left(1 - \left(\mu_{\tilde{A}_i}^{\mathcal{F}}\right)^q\right)^{\lambda_i}}, \prod_{i=1}^{k+1} \left(\vartheta_{\tilde{A}_i}^{\mathcal{F}}\right)^{\lambda_i} \right), \right. \\
& \left. \mathcal{D} \left(\sqrt[q]{1 - \prod_{i=1}^{k+1} \left(1 - \left(\mu_{\tilde{A}_i}^{\mathcal{D}}\right)^q\right)^{\lambda_i}}, \prod_{i=1}^{k+1} \left(\vartheta_{\tilde{A}_i}^{\mathcal{D}}\right)^{\lambda_i} \right) \right\} \quad (3.42)
\end{aligned}$$

For $n = k + 1$, **DqROWAM** operator also holds. **DqROWAM** operator must be shown as a number of DqROFS. As $\tilde{A}_i = \left\{ \mathcal{F} \left(\mu_{\tilde{A}_i}^{\mathcal{F}}, \vartheta_{\tilde{A}_i}^{\mathcal{F}} \right), \mathcal{D} \left(\mu_{\tilde{A}_i}^{\mathcal{D}}, \vartheta_{\tilde{A}_i}^{\mathcal{D}} \right) \right\}$ for all i is an DqROFN, thus $0 \leq \mu_{\tilde{A}_i}^{\mathcal{F}}, \vartheta_{\tilde{A}_i}^{\mathcal{F}}, \mu_{\tilde{A}_i}^{\mathcal{D}}, \vartheta_{\tilde{A}_i}^{\mathcal{D}} \leq 1$ and $0 \leq \left(\mu_{\tilde{A}_i}^{\mathcal{F}}\right)^2 + \left(\vartheta_{\tilde{A}_i}^{\mathcal{F}}\right)^2 \leq 1, 0 \leq \left(\mu_{\tilde{A}_i}^{\mathcal{D}}\right)^2 + \left(\vartheta_{\tilde{A}_i}^{\mathcal{D}}\right)^2 \leq$

1. Thus, $0 \leq \prod_{i=1}^n \left(1 - \left(\mu_{\tilde{A}_i}^{\mathcal{F}}\right)^q\right)^{\lambda_i} \leq 1$ and so, $0 \leq$

$$\sqrt[q]{1 - \prod_{i=1}^k \left(1 - \left(\mu_{\tilde{A}_i}^{\mathcal{F}}\right)^q\right)^{\lambda_i}} \leq 1 \quad \text{and} \quad 0 \leq \prod_{i=1}^k \left(\vartheta_{\tilde{A}_i}^{\mathcal{F}}\right)^{\lambda_i} \leq 1. \quad \text{Similarly, } 0 \leq$$

$$\sqrt[q]{1 - \prod_{i=1}^k \left(1 - \left(\mu_{\tilde{A}_i}^{\mathcal{D}}\right)^q\right)^{\lambda_i}} \leq 1 \quad \text{and} \quad 0 \leq \prod_{i=1}^k \left(\vartheta_{\tilde{A}_i}^{\mathcal{D}}\right)^{\lambda_i} \leq 1.$$

Definition 3.6

Let $\tilde{A}_i = \left\{ \mathcal{F} \left(\mu_{\tilde{A}_i}^{\mathcal{F}}, \vartheta_{\tilde{A}_i}^{\mathcal{F}} \right), \mathcal{D} \left(\mu_{\tilde{A}_i}^{\mathcal{D}}, \vartheta_{\tilde{A}_i}^{\mathcal{D}} \right) \right\}$, ($i = 1, 2, \dots, n$) be a set of DqROFSs and $\lambda_i = (\lambda_1, \lambda_2, \dots, \lambda_n)$ be weight vector of $\tilde{A}_i$ with $\sum_{i=1}^n \lambda_i = 1$ and $\lambda_i \in [0, 1]$, then a Decomposed q-Rung Orthopair Weighted Geometric Mean (**DqROWGM**) the operator is defined as

$$\begin{aligned}
DqROWGM(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) &= \lambda_1 \tilde{A}_1 \otimes \lambda_2 \tilde{A}_2 \otimes \dots \otimes \lambda_n \tilde{A}_n = \\
&\left(\mathcal{F} \left(\prod_{i=1}^n (\mu_{\tilde{A}_i}^{\mathcal{F}})^{\lambda_i}, \sqrt[q]{1 - \prod_{i=1}^n (1 - (\vartheta_{\tilde{A}_i}^{\mathcal{F}})^q)^{\lambda_i}} \right), \right. \\
&\left. \mathfrak{D} \left(\prod_{i=1}^n (\mu_{\tilde{A}_i}^{\mathfrak{D}})^{\lambda_i}, \sqrt[q]{1 - \prod_{i=1}^n (1 - (\vartheta_{\tilde{A}_i}^{\mathfrak{D}})^q)^{\lambda_i}} \right) \right)
\end{aligned} \tag{3.43}$$

Proof 3.6

The proof corresponding to this definition closely follows the same reasoning and structure as the one provided for Definition 3.5. Therefore, to avoid redundancy and maintain the conciseness of the exposition, the detailed proof is not presented for this definition.

4 CRITERIA EVALUATION WITH BAYESIAN BEST-WORST METHOD

4.1 Sustainable Transportation and Determination of Criteria in Aircraft Selection Problem

In the literature, there is not a study that directly focuses on aircraft selection from a sustainability perspective. However, there are several studies that concentrate either on aircraft selection without sustainability considerations or on other aviation operations such as evaluation of airport operations, assessment of environment-friendly fuel alternatives and technological or digital systems selection from sustainability perspectives. To define criteria for sustainable aircraft selection problem, both group of studies are examined from the literature, and the most appropriate criteria set are defined with consensus among expert decision makers.

Table 4.1 Selected criteria with respective notation and references

Criterion Name	Notation	Reference studies
Economic Criteria		
Acquisition Cost	C_1	Goldberg et al. (2017), Dhara et al. (2021), Chen et al. (2019), Fera et al. (2020), Güntut (2023)
Operational Cost	C_2	Wang et al. (2023), Engholm et al. (2020), Jacyna and Wasiak (2015), Fera et al. (2020), Dhara et al. (2021), Güntut (2023), Chen et al. (2019), Zhang et al. (2023)
Fuel Efficiency	C_3	Singh et al. (2016), Liu et al. (2021), Kramer et al. (2022), Zhou et al. (2023),
Return on Investment	C_4	Grewe et al. (2021), Karagiannis et al. (2019), Vysikantsev et al. (2023), Qiu et al. (2021), Bahman (2023), Fera et al. (2020), Markatos, N., et al. (2023), Dožić, I. (2019)
Social Criteria		
Passenger Comfort	C_5	Liu, Y., Wang, Y., & Zhang, Y. (2020), Wang, Y., et al. (2021), Vledder, A. (2023), Kaitaro, T. (2024), Liu, Y., et al. (2020), Lin, Y., et al. (2009)
Safety Record	C_6	Şenol, S. (2020), Fleischer, A., et al. (2015), Kammüller, F., & Kerber, F. (2020), Molin, E., et al. (2017), Ballard, J. (2014), Takami, K., & Obayashi, S. (2022)

Accessibility	C_7	Kimeria, J. (2022), Gadziński, M., and Bul, D. (2017), Michniak, A., and Székely, B. (2019), Antipova, A., et al. (2020), Pourramazani, N., and Miralles-Garcia, J. (2022), Zhou, Y., et al. (2020), Karam, A., et al. (2022), Caetano, F., et al. (2022)
Community Impact	C_8	Aldahmashi, A. (2023), Kafalı, H., & Altuntaş, H. (2020), Gritta, R., & Lippman, S. (2012), Dube, K., et al. (2021), Molin, E., et al. (2017), Zhang, Y., et al. (2021)
Environmental Criteria		
Carbon Emissions	C_9	Orhan (2021), Baxter (2022), Liu et al. (2023), Kim and Son (2021), Tuladhar et al. (2021), Markatos et al. (2023), Ibrahim (2024), Dožić (2019), Ren et al. (2015), Wang et al. (2019)
Noise Pollution	C_{10}	Toriya and Self (2018), Sadr et al. (2020), Camara et al. (2018), Matthes et al. (2017), Kiracı, A., and Akan, P. (2020), Başar et al. (2020), Fu et al. (2020), Caetano et al. (2022)
Lifecycle Assessment and Recyclability	C_{11}	Vivalda (2024), Zhao et al. (2021), Rashid and Pagone (2023), Hawkins et al. (2012), Nealer and Hendrickson (2015), Kim et al. (2015)

Acquisition Cost (C_1): It addresses not only the initial purchase price but also the implications of financing. A thorough evaluation of this cost is essential for ensuring that aircraft selections contribute positively to the financial sustainability of aviation operations.

Operational Cost (C_2): This includes costs related to fuel, maintenance, crew salaries, airport fees, and other operational expenditures. Understanding and managing these costs are crucial for airlines and operators to maintain profitability and ensure long-term sustainability in a highly competitive industry.

Fuel Efficiency (C_3): This refers to the effectiveness with which an aircraft converts fuel into usable energy for flight. High fuel efficiency can lead to significant cost savings over the aircraft's operational life, making it a key factor in decision-making processes.

Return on Investment (C_4): This encompasses not only the assessment of financial returns relative to the costs associated with acquiring and operating an aircraft but

also the revenue generation potential and resale value. By prioritizing ROI, airlines can make informed decisions that enhance their economic viability while contributing to the overall sustainability of the aviation industry.

Passenger Comfort (C₅): This refers to the overall experience and satisfaction of passengers during their flight, encompassing various factors such as seating, cabin environment, noise levels, and service quality. This criterion is increasingly recognized as essential for enhancing travel experience, promoting customer loyalty, and ensuring that air travel remains a viable option compared to other modes of transportation.

Safety Record (C₆): This refers to the historical performance of an aircraft type or model in terms of safety incidents, accidents, and overall reliability. This criterion is crucial for ensuring the well-being of passengers and crew, as well as for maintaining public trust in air travel. A strong safety record not only reflects the design and operational integrity of the aircraft but also influences airline reputation, regulatory compliance, and market competitiveness.

Accessibility (C₇): This criterion addresses the equitable provision of air travel opportunities, the integration of social needs into decision-making processes, and the potential for aircraft to serve diverse communities effectively. This criterion is crucial for fostering social equity and enhancing the overall sustainability of the aviation sector.

Community Impact (C₈): This criterion refers to a wide range of considerations, such as air quality, job creation, and the overall relationship between airlines, service providers and the communities they serve. Understanding and addressing community impact is essential for fostering positive relationships with stakeholders and ensuring that aviation contributes to sustainable development.

Carbon Emissions (C₉): This refers to the amount of carbon dioxide (CO₂) and other greenhouse gases emitted by an aircraft during its operation. This criterion is critical for assessing the environmental impact of aviation and plays a significant role in the

industry's efforts to mitigate climate change. Understanding and managing carbon emissions is essential for airlines aiming to enhance their sustainability profiles and comply with increasingly stringent environmental regulations.

Noise Pollution (C_{10}): This addresses the level of noise generated by aircraft during various phases of flight, including takeoff, landing, and cruising. This criterion is critical for assessing the environmental impact of aviation on surrounding communities, as excessive noise can lead to various health and social issues, including sleep disturbances, stress, and reduced quality of life for residents living near airports.

Lifecycle Assessment and Recyclability (C_{11}): This involves evaluating the environmental impacts of aircraft throughout their entire lifecycle and ensuring that materials used are recyclable. This comprehensive approach is essential for promoting sustainable practices in the aviation industry and minimizing the environmental footprint of aircraft.

In this thesis, a relatively novel methodology called Bayesian BWM which is proved by Rezai is used to determine the weight of the criteria because of its strength to assess the individual opinions of the group decision making process. First, to better understand the Bayesian BWM and the logic behind the methodology, it is important to visit the original Best Worst Method (BWM).

4.2 Classical Best-Worst Method

In the original BWM, when performing pairwise comparisons between n criteria, a scale of 1/9 to 9 is typically employed. Within the pairwise comparison matrix A , the entry a_{ij} signifies the comparative preference of criterion i to criterion j . A value of $a_{ij} = 1$ denotes equal importance between i and j . For $a_{ij} > 1$, i is deemed more significant than j , with $a_{ij} = 9$ signifying utmost importance of i over j . The significance of j to i is reflected by a_{ij} . To maintain the reciprocity of matrix A , it is essential that $a_{ij} = 1/a_{ji}$, and $a_{ii} = 1$ for all i and j .

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \quad (4.1)$$

To construct a complete matrix A , $n(n-1)/2$ pairwise comparisons are needed, considering the reciprocal property. Matrix A is considered perfectly consistent if it satisfies the following condition:

$$a_{ik} \times a_{kj} = a_{ij}, \forall i, j \quad (4.2)$$

Rezai aims to enhance the understanding of pairwise comparisons, which form the basis of their proposed method known as BWM. They categorize pairwise comparisons into two main types: (1) reference comparisons and (2) secondary comparisons.

(1) A reference comparison is defined as comparison a_{ij} when element i is considered the best and/or element j is considered the worst.

(2) A secondary comparison is defined as comparison a_{ij} when neither element i nor element j are considered the best or the worst elements.

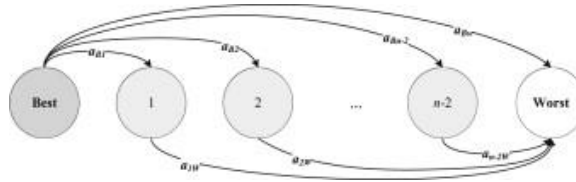


Figure 4.1 Reference comparisons (Rezaei, 2015)

The BWM approach involves the following steps to determine the weights of the criteria:
Step 1. Define the set of criteria $\{c_1, c_2, \dots, c_n\}$ that are relevant to the decision-making process and need to be prioritized.

Step 2. Identify the best (e.g., most desirable, most important) and worst (e.g., least desirable, least important) criteria. Note that if there are multiple criteria considered to be the best or the worst, one can be chosen arbitrarily.

Step 3. Assign a preference value to indicate the superiority of the best criterion over each of the other criteria. Use a numerical scale ranging from 1 to 9, where 1 represents equal

importance and 9 signifies extreme superiority. The resulting vector, known as the Best-to-Others vector, illustrates the preference values assigned to the best criterion in relation to each of the remaining criteria:

$$\mathbf{A}_B = (\mathbf{a}_{B1}, \mathbf{a}_{B2}, \dots, \mathbf{a}_{Bn}), \quad (4.3)$$

where $\mathbf{a}_{Bj}$, represents the preference of the best criterion B over criterion j . It is important to note that $\mathbf{a}_{BB} = 1$, as the best criterion is considered equal to itself.

Step 4. Assign a preference value, denoted as $\mathbf{a}_{jW}$, to represent the preference of criterion j over the worst criterion W . This preference value ranges between 1 and 9, indicating the relative importance or desirability of each criterion compared to the worst criterion. The resulting vector, known as the Others-to-Worst vector, captures these preference values for each criterion with respect to the worst criterion:

$$\mathbf{A}_W = (\mathbf{a}_{1W}, \mathbf{a}_{2W}, \dots, \mathbf{a}_{nW})^T, \quad (4.4)$$

where $\mathbf{a}_{jW}$, represents the preference of criterion j over the worst criterion W . This preference value ranges between 1 and 9, indicating the relative importance or desirability of each criterion compared to the worst criterion. The preference value for the worst criterion itself, $\mathbf{a}_{WW} = 1$, reflecting its equal importance or desirability to itself.

Step 5. Calculate the optimal weights of the criteria using the information obtained from the previous steps $(\mathbf{w}_1^*, \mathbf{w}_2^*, \dots, \mathbf{w}_n^*)$.

To determine the optimal weights for the criteria, we aim to find a solution that satisfies the conditions $\mathbf{w}_B/\mathbf{w}_j = \mathbf{a}_{Bj}$ and $\mathbf{w}_j/\mathbf{w}_W = \mathbf{a}_{jW}$ for all j . This can be achieved by minimizing the maximum absolute differences $|\frac{\mathbf{w}_B}{\mathbf{w}_j} - \mathbf{a}_{Bj}|$ and $|\frac{\mathbf{w}_j}{\mathbf{w}_W} - \mathbf{a}_{jW}|$ for all j .

To formulate this as a problem, taking into account the non-negativity and sum conditions for the weights, we can express it as follows:

$$\begin{aligned} & \min_{\mathbf{w}} \max_j \{ |\frac{\mathbf{w}_B}{\mathbf{w}_j} - \mathbf{a}_{Bj}|, |\frac{\mathbf{w}_j}{\mathbf{w}_W} - \mathbf{a}_{jW}| \} \\ & \text{subject to} \end{aligned}$$

$$\begin{aligned} \sum_j w_j &= 1 \\ w_j &\geq 0, \text{ for all } j \end{aligned} \quad (4.5)$$

The aforementioned problem can be transformed into the following formulation:

$$\begin{aligned} &\min \xi \\ &\text{subject to} \\ &\left| \frac{w_B}{w_j} - a_{Bj} \right| \leq \xi, \text{ for all } j \\ &\left| \frac{w_j}{w_W} - a_{jW} \right| \leq \xi, \text{ for all } j \\ &\sum_j w_j = 1 \\ &w_j \geq 0, \text{ for all } j \end{aligned} \quad (4.6)$$

By solving this problem, the optimal weights $(w_1^*, w_2^*, \dots, w_n^*)$ and ξ^* are determined. Solving for different values of $a_{BW} \in \{1, 2, \dots, 9\}$, one can find the maximum possible ξ ($\max \xi$).

4.3 Bayesian Best-Worst Method

The typical outcome of the MCDM methods is the weight vector $\mathbf{w} = [w_1, \dots, w_n]$ such that $w_j \geq 0$, $\sum_{j=1}^n w_j = 1$. The magnitude of each w_j indicates the importance of the corresponding criteria c_j .

Taking a probabilistic viewpoint, the criteria are viewed as random events, and their weights correspond to their likelihood of occurrence. In mathematical terms, this interpretation aligns with MCDM principles, as $w_j \geq 0$ and $\sum_{j=1}^n w_j = 1$, in accordance with probability theory. It is crucial to emphasize that a probabilistic modeling approach is meaningful in the context of decision-making.

To apply probabilistic reasoning, it is necessary to represent both the inputs and outputs as probability distributions. Initially, A_B and A_W which serve as inputs for the BWM. Mathematically, multinomial distribution can model these vectors because all their components are integers. The probability mass function (PMF) of the multinomial distribution for a given A_W is expressed as:

$$P(A_W|w) = \frac{(\sum_{j=1}^n a_{jW})!}{\prod_{j=1}^n a_{jW}!} \prod_{j=1}^n w_j^{a_{jW}} \quad (4.7)$$

with w is the probability distribution.

$$\frac{w_j}{w_W} \propto a_{jW}, \forall j = 1, \dots, n, \quad (4.8)$$

Likewise, A_B can be characterized by multinomial distribution. However, A_B differs from A_W in that A_B reflects the preferences for the best option over the other criteria, while A_W represents the preferences for the other options over the worst. Consequently, A_B produces the inverse of the weight, which is:

$$A_B \sim \text{multinomial}(1/w) \quad (4.9)$$

where w is the probability distribution, and $/$ is the element-wise division operator. Similar to the worst criterion, one can express:

$$\begin{aligned} \frac{1}{w_j} &\propto \frac{a_{Bj}}{\sum_{i=1}^n a_{Bi}}, \frac{1}{w_B} \propto \frac{a_{BB}}{\sum_{i=1}^n a_{Bi}} = \frac{1}{\sum_{i=1}^n a_{Bi}} \\ &\Rightarrow \frac{w_B}{w_j} \propto a_{Bj}, \forall j = 1, \dots, n, \end{aligned} \quad (4.10)$$

this is once more the precise relationship which is aimed at being established in the BWM.

For an MCDM weight vector, it must adhere to the properties of non-negativity and summing to one. Hence, a suitable distribution for modeling these weights is the Dirichlet distribution. Defined by a parameter $\alpha \in \mathbb{R}^n$, the Dirichlet distribution of the weights w is expressed as:

$$Dir(w|\alpha) = \frac{1}{B(\alpha)} \prod_{j=1}^n w_j^{\alpha_j-1}. \quad (4.11)$$

The distribution has only a vector parameter α , and w meets the constraints of an optimal weight vector of MCDM since it is a probability distribution. Thus w^* matrix is represented as: $(w^*) \sim Dir(\gamma x w^*)$.

The distribution is characterized solely by a vector parameter α , and w satisfies the criteria of being an optimal weight vector for MCDM because it functions as a probability distribution.

The independence and conditional independence of variables must be established before building a Bayesian model. The suggested approach's graphical model is depicted in Fig. 2. The variables are represented by the graph's nodes, and the observed variables—which were used as inputs for the initial BWM—are shown as rectangles. Circular nodes represent the variables that need to be estimated. Arrows indicate dependency, meaning that the value of w^k depends on A_W^k and A_B^k , and the value of w_{agg} depends on w_k .

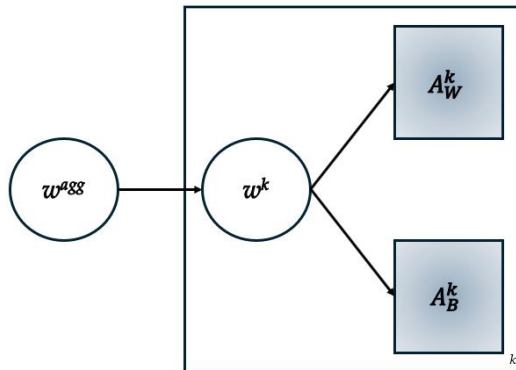


Figure 4.2 The probabilistic graphical model of the Bayesian BWM

In the Bayesian deduction, the Dirichlet distribution is utilized as the prior to the multinomial distribution. It can be undertaken as a weight set thanks to the properties of non-negativity and sum-to-one.

When Dirichlet is utilized as the prior and multinomial as the likelihood, the posterior distribution is Dirichlet as well with the posterior parameter $\alpha_{post} = \alpha + A_W$.

Then the prior distribution must be established for w^{agg} by employing a non-informative Dirichlet distribution with the parameter $\alpha=1$, represented as:

$$w^{agg} \sim Dir(\alpha). \quad (4.12)$$

The proposed model does not offer a solution in closed form. Consequently, Markov-chain Monte Carlo (MCMC) methods are required to calculate posterior distribution. Specifically, the "just another Gibbs sampler" (JAGS) is used for MCMC sampling purposes. The valuable result of the model is the posterior distribution of weights for each individual decision maker and the aggregated weights, w^{agg} .

The Bayesian model will fill in for Step 5 of the original BWM. Essentially, the optimization problem is replaced with a probabilistic model, although the inputs for both methods remain the same.

Advantages

Bayesian BWM offers significant advantages in multi-criteria decision making (MCDM) processes due to its probabilistic structure and robustness in managing uncertainties. Compared to traditional methods such as the Analytical Hierarchy Process (AHP), one of the most striking advantages of Bayesian BWM is that it can obtain more reliable results with fewer pairwise comparisons ($2n-2$ comparisons for n criteria), which generally simplifies the decision-making process and increases clarity among stakeholders (Shakur et al., 2024; Mohammadi and Rezaei, 2020). This efficiency is extremely valuable, especially in cases where expert consensus is critical, because Bayesian BWM allows preferences to be broadly aggregated by taking into account differences in expert opinions (Yang et al., 2020; Ahmed et al., 2022). In addition, Bayesian BWM allows the

incorporation of prior knowledge into the analysis, thus providing a framework for updating beliefs based on new evidence (Debnath et al., 2023; McCann and Schwab, 2023). Such an approach plays a vital role in complex areas such as pharmaceuticals or supply chains, where uncertainties are high and informed decision-making is critical (Afghah et al., 2023; Chauhan et al., 2022). The probabilistic nature of Bayesian BWM allows it to effectively manage outliers and inconsistencies in expert judgments, thus producing more stable and robust decision results (Ahmed et al., 2022; Tusher et al., 2022).

Furthermore, the Bayesian approach increases the interpretability of the results by providing confidence intervals that reflect the uncertainty around the estimates made. Thus, a deeper understanding of the importance of the evaluated metrics is obtained (Mohammadi and Rezaei, 2020; Flor et al., 2020). This competence is increasingly important, especially in areas where rigorous risk assessments are needed, such as healthcare and environmental management (Shakur et al., 2024; Woldehellasse and Tesfamariam, 2024). In summary, the advantages of Bayesian BWM in terms of efficiency, reliability, adaptability, and transparency make it a primary choice for modern decision-making challenges.

4.4 Case Study

As described in Section 4.1, eleven criteria are determined to evaluate the airplane models in the case study and those are: C_1 : Acquisition Cost, C_2 : Operational Cost, C_3 : Fuel Efficiency, C_4 : Return on Investment, C_5 : Passenger Comfort, C_6 : Safety Record, C_7 : Accessibility, C_8 : Community Impact, C_9 : Carbon Emissions, C_{10} : Noise Pollution and C_{11} : Lifecycle Assessment and Recyclability criteria.

While applying the Bayesian BWM method for determining the weights of the criteria and while applying proposed DqROF EDAS and DqROF MAIRCA methods to the case study, the evaluations of three decision makers were taken. The opinion weights of all decision makers and criteria were assumed to be equal. The profile information of all three decision makers is given below.

Decision-Maker 1 (DM_1): 33 years old, holds a bachelor's and master's degree in aeronautical engineering. He works as Aircraft Systems Chief Engineer at technical branch of one of the commercial airline companies in Türkiye. His area of expertise mainly focuses on aircraft structural and system design, maintenance/procedure development, and technical fault diagnosis.

Decision-Maker 2 (DM_2): 45 years old, holds a bachelor's degree in aeronautical engineering and an MBA. He works as an Engineering Coordination Supervisor at technical branch of one of the commercial airline companies in Türkiye. Since he has both an engineering and business (management) perspective, he stands out in areas such as cost/operation management, project coordination, and engineering team leadership.

Decision-Maker 3 (DM_3): 35 years old, holds a bachelor's degree in industrial engineering. He works as a manager in the Sustainability unit at technical branch of one of the commercial airline companies in Türkiye. His work mainly focuses on environmental impacts, carbon emission reduction strategies, fuel efficiency, and green innovation projects.

First, to apply the Bayesian BWM to determine criteria's weights, the decisions of those three DMs are taken. Initially they determined the best and the worst criteria with consensus and they proceeded with criteria evaluations individually. Fuel Efficiency (C_3) is determined to be the best criterion and Accessibility (C_7) is decided to be the worst criterion, respectively.

Then, each DM made evaluations for Best-to-Others and Others-To-Worst for the remaining criteria. Thus, A_B and A_W matrices are obtained as in Table 4.2 and Table 4.3, respectively.

Table 4.2 – Best-to-Others matrix (A_B) for three DMs

Criteria	DM1	DM2	DM3
Acquisition Cost (C_1)	3	3	3
Operational Cost (C_2)	3	3	3
Fuel Efficiency ($C_3, Best$)	1	1	1
Return on Investment (C_4)	3	3	3
Passenger Comfort (C_5)	3	3	3
Safety Record (C_6)	1	1	1
Accessibility ($C_7, Worst$)	9	9	9
Community Impact (C_8)	3	3	1
Carbon Emissions (C_9)	1	1	1
Noise Pollution (C_{10})	3	3	3
Lifecycle Assessment and Recyclability (C_{11})	3	3	3

Table 4.3 – Best-to-Others matrix (A_W) for three DMs

Criteria	DM_1	DM_2	DM_3
Acquisition Cost (C_1)	7	7	5
Operational Cost (C_2)	7	7	5
Fuel Efficiency ($C_3, Best$)	9	9	9
Return on Investment (C_4)	5	7	5
Passenger Comfort (C_5)	5	5	5
Safety Record (C_6)	9	9	9
Accessibility ($C_7, Worst$)	1	1	1
Community Impact (C_8)	5	5	9
Carbon Emissions (C_9)	9	9	9
Noise Pollution (C_{10})	5	5	7
Lifecycle Assessment and Recyclability (C_{11})	5	5	7

In the computation of the Bayesian BWM, which is described in detail in Section 4.3, the required γ parameter is taken as 0.1 ($Gamma(0.1,0.1)$). A gamma value of 0.1 often presents an adequate degree of uncertainty and fuzziness in the experts' opinions and it also reflects a conservative estimation as a hyperparameter that one assumes a certain variability with avoiding overestimation.

Similarly, in Bayesian BWM, the α parameter of the Dirichlet distribution is taken as 1 ($w^{agg} \sim Dir(1)$) because this choice ensures that the weights are distributed with equal probability without relying on any initial assumptions. $\alpha = 1$ transforms the Dirichlet distribution into a uniform distribution, allowing the decision maker to determine the weights based on the comparisons he makes, and does not introduce a priori bias into the model. It also provides the most unbiased distribution in accordance with the maximum entropy principle and makes Bayesian updating mathematically simpler. This choice provides consistency with classical BWM, while allowing the model to operate entirely based on the decision maker's inputs. Larger α values may cause the weights to be distributed more evenly, while smaller α values may cause extreme weightings; however, $\alpha = 1$ offers the most unbiased and flexible approach.

When the Bayesian BWM is computed with the steps explained in Section 4.3, the weights are calculated as in Table 4.4.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
w^{agg}	0.051	0.051	0.210	0.051	0.048	0.210	0.039	0.048	0.196	0.048	0.048

4.5 Results

The criteria taken into consideration during the aircraft selection process require balanced consideration of economic, environmental and operational factors. During the evaluation process, fuel efficiency (21%) and safety record (21%) have the highest weight on the decision and stand out as critical elements that directly affect long-term costs and flight safety. Carbon emissions (19.6%), on the other hand, have become a criterion increasingly taken into consideration in the modern aviation industry, as they are of great importance in terms of environmental sustainability. Financial factors such as Acquisition cost, Operational cost and Return on investment are evaluated with equal weight (5.1%) and contribute to ensuring economic feasibility. While the weights of elements such as Passenger comfort, Accessibility and Community impact related to passenger experience vary between 3.9% and 4.8%, sustainability criteria such as Noise pollution and Life-

cycle-assessment and recyclability are also considered at a similar level of importance (4.8%). This multi-dimensional approach offers a balanced decision-making process that considers not only economic efficiency but also environmental sustainability and social acceptability.



5 DECOMPOSED Q-RUNG ORTHOPAIR FUZZY NUMBER BASED EDAS METHOD

5.1 Description and Steps of the Proposed Method

The EDAS method is a known and used MCDM method in literature and its basic logic is based on distances to average solution. The method considers positive and negative distances to an average solution rather than ideal solutions. The method's capability to integrate fuzzy logic theory also enables for a more dynamic handling of ambiguity and vagueness in decision-making scenarios. In this section of the thesis, the EDAS method is extended for the first time with decomposed q-rung orthopair fuzzy sets and presented as DqROF EDAS. The steps of the DqROF MAIRCA method are as follows:

Step 1. To structure the developed method, determine the decision criteria, alternatives and the set of decision makers (DMs). The set of alternatives is denoted by $A_i = \{A_1, A_2, \dots, A_m\}$ and is defined as $i = 1, 2, \dots, m$. Each alternative is evaluated according to the set of criteria $C_j = \{C_1, C_2, \dots, C_n\}$. The criterion weights are expressed by $w_j = (w_1, w_2, \dots, w_n)$, where $w_j > 0$ and are also defined as $j = 1, 2, \dots, n$. The set of DMs who are competent to make judgments based on their areas of expertise in the decision process are denoted by $D_1, D_2, \dots, D_K$.

Step 2. Employ the Bayesian BWM in Section 4 to compute the criteria weights.

Step 3. Obtain ratings for criteria from Bayesian BWM and decision matrices from each decision maker (DM). These ratings and decision matrices are completed using the linguistic terms specified in Table 5.1. One linguistic term is assigned to functional questions and a different linguistic term to nonfunctional questions, allowing the criteria to be weighted and the decision matrices to be constructed. For example, (HI, MHI) is one possible assignment.

Table 5.1 – Linguistic terms

Importance level	Linguistic score
Absolutely high	AH
Very high	VH
High	H
Medium high	MH
Average	A
Medium lo	ML
Low	L
Very low	VL
Absolutely low	AL

Step 4. Convert the linguistic ratings into DqRONs using Table 5.2.

Table 5.2 – Linguistic terms and their DqROF conversions (Kahraman and Alkan, 2021)

Linguistic term	μ	ϑ
Absolutely high (AH)	0.99	0.11
Very high (VH)	0.88	0.22
High (H)	0.77	0.33
Medium high (MH)	0.66	0.44
Average (A)	0.55	0.55
Medium low (ML)	0.44	0.66
Low (L)	0.33	0.77
Very low (VL)	0.22	0.88
Absolutely low (AL)	0.11	0.99

Step 5. After the linguistic ratings are determined and converted to DqROFS values for alternatives, DqROF criteria weight matrix is constructed as presented in Table 5.3.

Table 5.3 - DqROF criteria weight matrix

Criteria	Alternatives			
	C_1	C_2	...	C_n
A_1	$\langle (\mu_{A_{11}}^{\mathcal{F}}, \vartheta_{A_{11}}^{\mathcal{F}}), (\mu_{A_{11}}^{\mathcal{D}}, \vartheta_{A_{11}}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_{12}}^{\mathcal{F}}, \vartheta_{A_{12}}^{\mathcal{F}}), (\mu_{A_{12}}^{\mathcal{D}}, \vartheta_{A_{12}}^{\mathcal{D}}) \rangle$	...	$\langle (\mu_{A_{1n}}^{\mathcal{F}}, \vartheta_{A_{1n}}^{\mathcal{F}}), (\mu_{A_{1n}}^{\mathcal{D}}, \vartheta_{A_{1n}}^{\mathcal{D}}) \rangle$
A_2	$\langle (\mu_{A_{21}}^{\mathcal{F}}, \vartheta_{A_{21}}^{\mathcal{F}}), (\mu_{A_{21}}^{\mathcal{D}}, \vartheta_{A_{21}}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_{22}}^{\mathcal{F}}, \vartheta_{A_{22}}^{\mathcal{F}}), (\mu_{A_{22}}^{\mathcal{D}}, \vartheta_{A_{22}}^{\mathcal{D}}) \rangle$	...	$\langle (\mu_{A_{2n}}^{\mathcal{F}}, \vartheta_{A_{2n}}^{\mathcal{F}}), (\mu_{A_{2n}}^{\mathcal{D}}, \vartheta_{A_{2n}}^{\mathcal{D}}) \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$\langle (\mu_{A_{m1}}^{\mathcal{F}}, \vartheta_{A_{m1}}^{\mathcal{F}}), (\mu_{A_{m1}}^{\mathcal{D}}, \vartheta_{A_{m1}}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_{m2}}^{\mathcal{F}}, \vartheta_{A_{m2}}^{\mathcal{F}}), (\mu_{A_{m2}}^{\mathcal{D}}, \vartheta_{A_{m2}}^{\mathcal{D}}) \rangle$	...	$\langle (\mu_{A_{mn}}^{\mathcal{F}}, \vartheta_{A_{mn}}^{\mathcal{F}}), (\mu_{A_{mn}}^{\mathcal{D}}, \vartheta_{A_{mn}}^{\mathcal{D}}) \rangle$

Step 6. Obtain the aggregated weights for criteria by using Bayesian BWM then construct the aggregated decision matrix by using DqROFWG operator.

Table 5.4 – Aggregated Decision Matrix

Criteria	Alternatives			
	C_1	C_2	...	C_n
A_1	$\langle (\mu_{A_{11}}^{\mathcal{F}}, \vartheta_{A_{11}}^{\mathcal{F}}), (\mu_{A_{11}}^{\mathcal{D}}, \vartheta_{A_{11}}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_{12}}^{\mathcal{F}}, \vartheta_{A_{12}}^{\mathcal{F}}), (\mu_{A_{12}}^{\mathcal{D}}, \vartheta_{A_{12}}^{\mathcal{D}}) \rangle$		$\langle (\mu_{A_{1n}}^{\mathcal{F}}, \vartheta_{A_{1n}}^{\mathcal{F}}), (\mu_{A_{1n}}^{\mathcal{D}}, \vartheta_{A_{1n}}^{\mathcal{D}}) \rangle$
A_2	$\langle (\mu_{A_{21}}^{\mathcal{F}}, \vartheta_{A_{21}}^{\mathcal{F}}), (\mu_{A_{21}}^{\mathcal{D}}, \vartheta_{A_{21}}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_{22}}^{\mathcal{F}}, \vartheta_{A_{22}}^{\mathcal{F}}), (\mu_{A_{22}}^{\mathcal{D}}, \vartheta_{A_{22}}^{\mathcal{D}}) \rangle$		$\langle (\mu_{A_{2n}}^{\mathcal{F}}, \vartheta_{A_{2n}}^{\mathcal{F}}), (\mu_{A_{2n}}^{\mathcal{D}}, \vartheta_{A_{2n}}^{\mathcal{D}}) \rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$\langle (\mu_{A_{m1}}^{\mathcal{F}}, \vartheta_{A_{m1}}^{\mathcal{F}}), (\mu_{A_{m1}}^{\mathcal{D}}, \vartheta_{A_{m1}}^{\mathcal{D}}) \rangle$	$\langle (\mu_{A_{m2}}^{\mathcal{F}}, \vartheta_{A_{m2}}^{\mathcal{F}}), (\mu_{A_{m2}}^{\mathcal{D}}, \vartheta_{A_{m2}}^{\mathcal{D}}) \rangle$	...	$\langle (\mu_{A_{mn}}^{\mathcal{F}}, \vartheta_{A_{mn}}^{\mathcal{F}}), (\mu_{A_{mn}}^{\mathcal{D}}, \vartheta_{A_{mn}}^{\mathcal{D}}) \rangle$

Step 7. In the normalization process, average solutions are determined by considering all criteria for functional and dysfunctional ratings. The average values of each criterion j are calculated by Equation (5.1).

$$AV_j^{\mathcal{F}} = \frac{\sum_{i=1}^n x_{ij}}{n} \text{ and } AV_j^{\mathcal{D}} = \frac{\sum_{i=1}^n x_{ij}}{n} \quad (5.1)$$

Step 8. $PDA^{\mathcal{F}}$, $NDA^{\mathcal{F}}$, $PDA^{\mathcal{D}}$ and $NDA^{\mathcal{D}}$ used in the EDAS method are calculated according to the nature of the conflicting criteria. Values for benefit criteria are determined by Equations (5.2)–(5.3), these values for cost criteria are calculated using Equations (5.4)–(5.5) for.

$$PDA_{ij}^{\mathcal{F}} = \frac{\max(0, x_{ij}^{\mathcal{F}} - AV_j^{\mathcal{F}})}{AV_j^{\mathcal{F}}} \text{ and } PDA_{ij}^{\mathcal{D}} = \frac{\max(0, x_{ij}^{\mathcal{D}} - AV_j^{\mathcal{D}})}{AV_j^{\mathcal{D}}} \quad (5.2)$$

$$NDA_{ij}^{\mathcal{F}} = \frac{\max(0, AV_j^{\mathcal{F}} - x_{ij}^{\mathcal{F}})}{AV_j^{\mathcal{F}}} \text{ and } NDA_{ij}^{\mathcal{D}} = \frac{\max(0, AV_j^{\mathcal{D}} - x_{ij}^{\mathcal{D}})}{AV_j^{\mathcal{D}}} \quad (5.3)$$

$$PDA_{ij}^{\mathcal{F}} = \frac{\max(0, AV_j^{\mathcal{F}} - x_{ij}^{\mathcal{F}})}{AV_j^{\mathcal{F}}} \text{ and } PDA_{ij}^{\mathcal{D}} = \frac{\max(0, AV_j^{\mathcal{D}} - x_{ij}^{\mathcal{D}})}{AV_j^{\mathcal{D}}} \quad (5.4)$$

$$NDA_{ij}^{\mathcal{F}} = \frac{\max(0, x_{ij}^{\mathcal{F}} - AV_j^{\mathcal{F}})}{AV_j^{\mathcal{F}}} \text{ and } NDA_{ij}^{\mathcal{D}} = \frac{\max(0, x_{ij}^{\mathcal{D}} - AV_j^{\mathcal{D}})}{AV_j^{\mathcal{D}}} \quad (5.5)$$

Step 9. For each criterion, the weighted sum of PDA and NDA values is calculated according to Equations (5.6)–(5.7), using the weight coefficient (w_j) determined by Bayesian BWM.

$$SP_i^{\mathcal{F}} = \sum_j^n w_j PDA_{ij}^{\mathcal{F}} \text{ and } SP_i^{\mathcal{D}} = \sum_j^n w_j PDA_{ij}^{\mathcal{D}} \quad (5.6)$$

$$SN_i^{\mathcal{F}} = \sum_j^n w_j NDA_{ij}^{\mathcal{F}} \text{ and } SN_i^{\mathcal{D}} = \sum_j^n w_j NDA_{ij}^{\mathcal{D}} \quad (5.7)$$

Step 10. The weighted sum values of PDA and NDA are normalized using Equations (5.8)–(5.9).

$$NSP_i^{\mathcal{F}} = \frac{SP_i^{\mathcal{F}}}{\max_i(SP_i^{\mathcal{F}})} \text{ and } NSP_i^{\mathcal{D}} = \frac{SP_i^{\mathcal{D}}}{\max_i(SP_i^{\mathcal{D}})} \quad (5.8)$$

$$NSN_i^{\mathcal{F}} = 1 - \frac{SN_i^{\mathcal{F}}}{\max_i(SN_i^{\mathcal{F}})} \text{ and } NSN_i^{\mathcal{D}} = 1 - \frac{SN_i^{\mathcal{D}}}{\max_i(SN_i^{\mathcal{D}})} \quad (5.9)$$

Step 11. The compromise score of each alternative is calculated using Equation (5.10).

$$AS_i^{\mathcal{F}} = \frac{1}{2}(NSP_i^{\mathcal{F}} + NSP_i^{\mathcal{F}}) \text{ and } AS_i^{\mathcal{D}} = \frac{1}{2}(NSP_i^{\mathcal{D}} + NSP_i^{\mathcal{D}}) \quad (5.10)$$

Step 12. Create the aggregated compromise score with the following Equation (5.11). In this formula β represents the preference of decision makers on proportion of functional utility scores on the final aggregated utility score and $(1 - \beta)$ represents the proportion of dysfunctional utility scores on the aggregated one. β takes value between $[0,1]$. The alternatives are ranked from high to low based on their aggregated compromise scores, and the alternative with the highest score is considered the best option.

$$AS_i^{agg} = \beta \cdot AS_i^{\mathcal{F}} + (1 - \beta) \cdot AS_i^{\mathcal{D}} \quad (5.11)$$

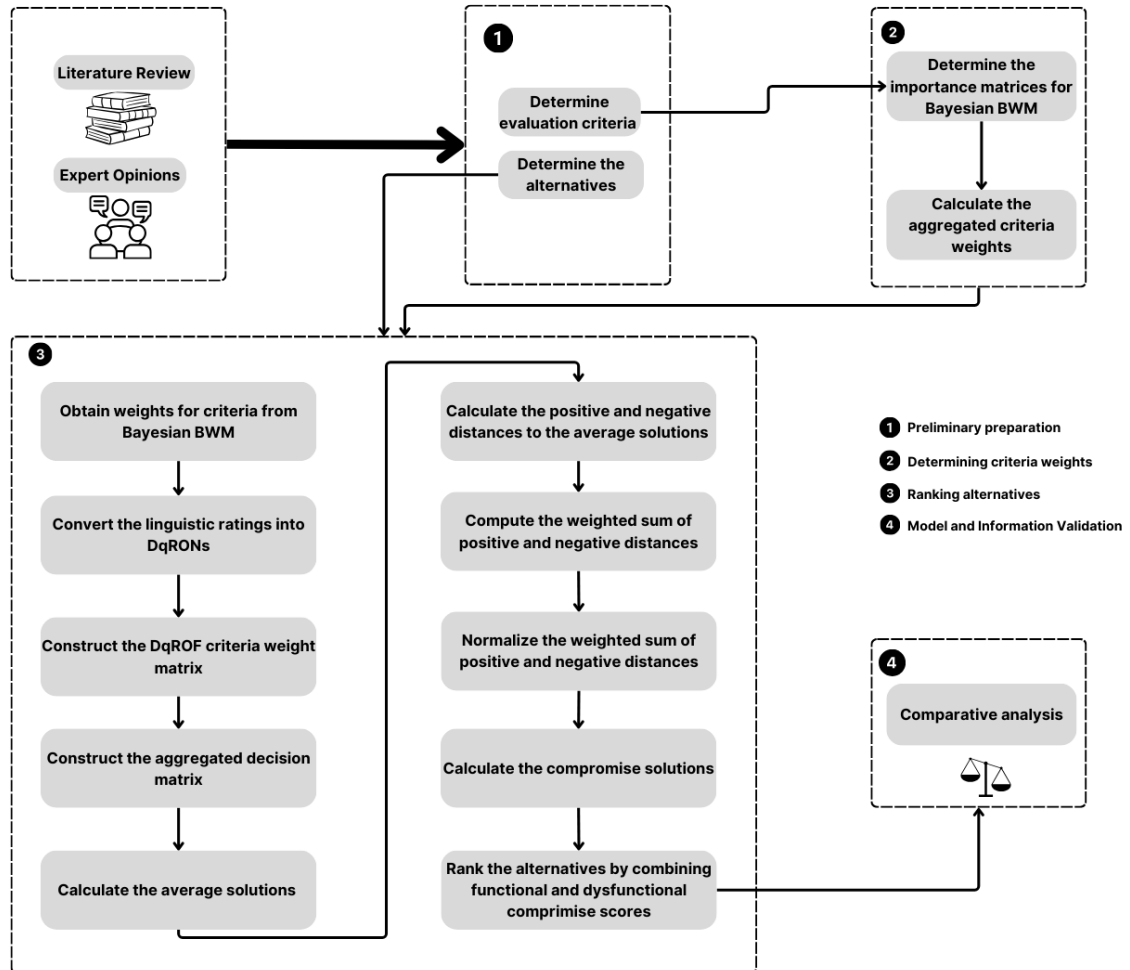


Figure 5.1 – Flowchart of the proposed DqROF EDAS Methodology

5.2 Case Study

In the field of multi-criteria decision making in operations research, it is quite common to rank alternatives by considering the criteria determined for a problem.

However, this section of this thesis is quite unique in that it presents the EDAS method, which has just begun to be recognized in the literature, by expanding it with DqROFS.

The DqROF EDAS method has been applied to the aircraft selection problem from a sustainability perspective. The aim is to rank the existing eight airplane model alternatives by considering the eleven criteria determined after the comprehensive literature review conducted in Section 2.

The determined criteria are: C_1 : Acquisition Cost, C_2 : Operational Cost, C_3 : Fuel Efficiency, C_4 : Return on Investment, C_5 : Passenger Comfort, C_6 : Safety Record, C_7 : Accessibility, C_8 : Community Impact, C_9 : Carbon Emissions, C_{10} : Noise Pollution and C_{11} : Lifecycle Assessment and Recyclability criteria. All the criteria in this case study are obtained as benefit criteria since DMs made evaluations for on importance levels of each criterion based on a linguistic scale. The alternative airplane models are again named A_1 , A_2 , A_3 , A_4 , A_5 , A_6 , A_7 and A_8 .

First, the linguistic evaluations of the decision makers were obtained. The evaluations are presented in Table 5.5.

Each DM is asked two sets of questions for their evaluations in terms of functional and dysfunctional perspectives. For example, for the evaluation of A_1 under criterion C_1 , these two questions were given to the DMs:

1. Functional question: “Would you rate the performance of Alternative 1 as good in terms of purchasing cost? Please answer according to the linguistic scale below.”
2. Dysfunctional question: “Would you rate the performance of Alternative 1 as poor in terms of acquisition cost? Please answer according to the linguistic scale below.”

As can be seen in Table 5.5, positive and negative structures of the question set directly affect the answering behavior of the DMs.

Table 5.5 – Linguistic evaluations of DMs

DM_1	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
C_1	H,MH	MH,H	L,ML	A,A	VH,VH	ML,ML	H,H	MH,MH
C_2	MH,H	H,MH	AL,L	MH,MH	VH,VH	ML,A	A,MH	H,H
C_3	VH,VH	MH,H	L,ML	AH,AH	VH,VH	ML,L	H,H	VH,VH
C_4	MH,MH	H,H	VL,VL	MH,A	VH,VH	ML,ML	MH,H	VH,MH
C_5	ML,ML	MH,A	A,L	VH,MH	ML,H	H,VH	VH,A	MH,ML
C_6	VH,VH	VH,H	AH,MH	AH,AH	VH,VH	VH,MH	AH,H	AH,VH
C_7	VH,MH	H,H	ML,ML	MH,A	VH,VH	L,ML	MH,H	H,MH
C_8	H,H	MH,MH	L,L	VH,MH	H,VH	L,A	MH,MH	VH,H
C_9	H,VH	MH,H	ML,ML	AH,AH	VH,VH	ML,L	MH,H	VH,VH
C_{10}	MH,MH	MH,H	VL,VL	VH,A	VH,VH	ML,ML	MH,H	VH,MH
C_{11}	MH,ML	MH,A	ML,L	VH,MH	MH,H	ML,VH	MH,A	VH,ML
DM_2	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
C_1	MH,H	H,MH	ML,L	MH,MH	H,H	A,A	MH,MH	H,H
C_2	H,MH	VH,H	VL,ML	H,A	VH,MH	L,ML	MH,H	MH,MH
C_3	H,H	A,VH	ML,L	VH,VH	H,H	L,ML	MH,MH	MH,H
C_4	H,H	VH,MH	AL,L	H,MH	H,H	A,A	H,MH	MH,H
C_5	MH,A	H,MH	MH,ML	H,H	L,MH	MH,H	MH,MH	H,A
C_6	H,H	VH,VH	VH,H	VH,VH	H,H	AH,VH	VH,MH	H,H
C_7	H,H	MH,MH	L,L	H,MH	H,H	VL,A	A,MH	MH,H
C_8	MH,MH	H,H	ML,ML	H,A	MH,MH	VL,ML	H,H	MH,MH
C_9	MH,H	H,VH	A,L	VH,VH	H,H	L,ML	H,MH	MH,H
C_{10}	H,H	A,MH	AL,L	H,MH	MH,H	L,A	H,MH	MH,H
C_{11}	H,A	H,MH	A,ML	H,H	A,MH	L,H	H,MH	MH,A
DM_3	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
C_1	A,MH	A,H	A,ML	H,A	MH,VH	L,ML	A,H	A,MH
C_2	A,H	MH,MH	L,L	A,MH	H,VH	A,A	H,MH	A,H
C_3	MH,VH	A,H	ML,ML	VH,AH	H,VH	L,L	MH,H	MH,VH
C_4	H,MH	VH,H	AL,VL	H,A	VH,VH	ML,ML	MH,H	VH,MH
C_5	A,ML	MH,A	A,L	VH,MH	ML,H	H,VH	VH,A	MH,ML
C_6	VH,VH	VH,H	VH,MH	VH,AH	VH,VH	VH,MH	VH,H	VH,VH

C_7	A,MH	H,H	ML,ML	A,A	VH,VH	AL,ML	MH,H	MH,MH
C_8	A,H	MH,MH	L,L	MH,MH	H,VH	L,A	MH,MH	AH,H
C_9	A,VH	MH,H	ML,ML	AH,AH	VH,VH	ML,L	MH,H	VH,VH
C_{10}	MH,MH	MH,H	AL,VL	VH,A	VH,VH	ML,ML	MH,H	VH,MH
C_{11}	MH,ML	MH,A	A,L	H,MH	A,H	ML,VH	H,A	MH,ML

After the linguistic evaluations of three different decision makers are obtained, they are converted to the corresponding DF numbers according to the scale specified in Table 3.1. The DqROF numbers are combined using the DqROWGM operator specified in Equation (3.43). The aggregated values are shown in Table 5.6.

Tablo 5.6 – Aggregated DqROF Decision Matrix

	A_1	A_2	A_3	A_4
C_1	(0.654,0.453),(0.695,0.408)	(0.654,0.453),(0.731,0.372)	(0.431,0.676),(0.400,0.703)	(0.654,0.453),(0.584,0.518)
C_2	(0.654,0.453),(0.731,0.372)	(0.765,0.345),(0.695,0.408)	(0.200,0.937),(0.363,0.739)	(0.654,0.453),(0.621,0.481)
C_3	(0.765,0.345),(0.842,0.263)	(0.584,0.518),(0.805,0.299)	(0.400,0.703),(0.400,0.703)	(0.915,0.191),(0.952,0.156)
C_4	(0.731,0.372),(0.695,0.408)	(0.842,0.263),(0.731,0.372)	(0.139,0.977),(0.252,0.852)	(0.731,0.372),(0.584,0.518)
C_5	(0.543,0.564),(0.474,0.628)	(0.695,0.408),(0.584,0.518)	(0.584,0.518),(0.363,0.739)	(0.842,0.263),(0.695,0.408)
C_6	(0.842,0.263),(0.842,0.263)	(0.880,0.220),(0.805,0.299)	(0.915,0.191),(0.695,0.408)	(0.915,0.191),(0.952,0.156)
C_7	(0.720,0.401),(0.695,0.408)	(0.731,0.372),(0.731,0.372)	(0.400,0.703),(0.400,0.703)	(0.654,0.453),(0.584,0.518)
C_8	(0.654,0.453),(0.731,0.372)	(0.695,0.408),(0.695,0.408)	(0.363,0.739),(0.363,0.739)	(0.765,0.345),(0.621,0.481)
C_9	(0.654,0.453),(0.842,0.263)	(0.695,0.408),(0.805,0.299)	(0.474,0.628),(0.400,0.703)	(0.952,0.156),(0.952,0.156)
C_{10}	(0.695,0.408),(0.695,0.408)	(0.621,0.481),(0.731,0.372)	(0.139,0.977),(0.252,0.852)	(0.842,0.263),(0.584,0.518)
C_{11}	(0.695,0.408),(0.474,0.628)	(0.695,0.408),(0.584,0.518)	(0.511,0.592),(0.363,0.739)	(0.805,0.299),(0.695,0.408)
	A_5	A_6	A_7	A_8
C_1	(0.765,0.345),(0.842,0.263)	(0.431,0.676),(0.474,0.628)	(0.654,0.453),(0.731,0.372)	(0.654,0.453),(0.695,0.408)
C_2	(0.842,0.263),(0.800,0.315)	(0.431,0.676),(0.511,0.592)	(0.654,0.453),(0.695,0.408)	(0.654,0.453),(0.731,0.372)
C_3	(0.805,0.299),(0.842,0.263)	(0.363,0.739),(0.363,0.739)	(0.695,0.408),(0.731,0.372)	(0.726,0.385),(0.842,0.263)
C_4	(0.842,0.263),(0.842,0.263)	(0.474,0.628),(0.474,0.628)	(0.695,0.408),(0.731,0.372)	(0.800,0.315),(0.695,0.408)
C_5	(0.400,0.703),(0.731,0.372)	(0.731,0.372),(0.842,0.263)	(0.800,0.315),(0.584,0.518)	(0.695,0.408),(0.474,0.628)
C_6	(0.842,0.263),(0.842,0.263)	(0.915,0.191),(0.726,0.385)	(0.915,0.191),(0.731,0.372)	(0.875,0.239),(0.842,0.263)
C_7	(0.842,0.263),(0.842,0.263)	(0.200,0.937),(0.474,0.628)	(0.621,0.481),(0.731,0.372)	(0.695,0.408),(0.695,0.408)
C_8	(0.731,0.372),(0.800,0.315)	(0.288,0.816),(0.511,0.592)	(0.695,0.408),(0.695,0.408)	(0.832,0.297),(0.731,0.372)
C_9	(0.842,0.263),(0.842,0.263)	(0.400,0.703),(0.363,0.739)	(0.695,0.408),(0.731,0.372)	(0.800,0.315),(0.842,0.263)
C_{10}	(0.800,0.315),(0.842,0.263)	(0.400,0.703),(0.474,0.628)	(0.695,0.408),(0.731,0.372)	(0.800,0.315),(0.695,0.408)
C_{11}	(0.584,0.518),(0.731,0.372)	(0.400,0.703),(0.842,0.263)	(0.731,0.372),(0.584,0.518)	(0.726,0.385),(0.474,0.628)

$AV_j^{\mathcal{F}}$ and $AV_j^{\mathcal{D}}$ values are calculated with Equation (5.1) and given in Table 5.7.

Table 5.7 – Average solutions from functional and dysfunctional perspectives.

	$AV_j^{\mathcal{F}}$		$AV_j^{\mathcal{D}}$	
	$\mu^{\mathcal{F}}$	$\vartheta^{\mathcal{F}}$	$\mu^{\mathcal{D}}$	$\vartheta^{\mathcal{D}}$
C_1	0.627	0.484	0.67	0.439
C_2	0.647	0.471	0.665	0.445
C_3	0.709	0.412	0.790	0.336
C_4	0.710	0.407	0.661	0.450
C_5	0.688	0.424	0.624	0.486
C_6	0.891	0.217	0.826	0.29
C_7	0.649	0.468	0.670	0.439
C_8	0.666	0.452	0.665	0.445
C_9	0.750	0.380	0.790	0.336
C_{10}	0.678	0.441	0.661	0.450
C_{11}	0.664	0.445	0.624	0.486

$PDA_{ij}^{\mathcal{F}}$ and $PDA^{\mathcal{D}}$ values are computed with Equation (5.2) and provided in Table 5.8 and Table 5.9, respectively.

Table 5.8 - $PDA_{ij}^{\mathcal{F}}$ matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.048	0.023	0.093	0.046	0	0	0.104	0	0	0.044	0.054
A_2	0.048	0.193	0	0.202	0.021	0	0.144	0.061	0	0	0.054
A_3	0	0	0	0	0	0.029	0	0	0	0	0
A_4	0.048	0.023	0.313	0.046	0.234	0.029	0.019	0.159	0.301	0.262	0.219
A_5	0.222	0.321	0.158	0.202	0	0	0.315	0.116	0.154	0.185	0
A_6	0	0	0	0	0.074	0.029	0	0	0	0	0
A_7	0.048	0.023	0.001	0	0.16	0.029	0	0.061	0	0.044	0.108
A_8	0.048	0.023	0.036	0.129	0.021	0	0.088	0.233	0.084	0.185	0.091

Table 5.9 - $PDA^{\mathcal{D}}$ matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.044	0.107	0.092	0.061	0	0.032	0.044	0.107	0.092	0.061	0
A_2	0.099	0.053	0.045	0.116	0	0	0.099	0.053	0.045	0.116	0
A_3	0	0	0	0	0	0	0	0	0	0	0
A_4	0	0	0.23	0	0.123	0.163	0	0	0.23	0	0.123
A_5	0.263	0.196	0.092	0.282	0.181	0.032	0.263	0.196	0.092	0.282	0.181
A_6	0	0	0	0	0.357	0	0	0	0	0	0.357
A_7	0.099	0.053	0	0.116	0	0	0.099	0.053	0	0.116	0
A_8	0.044	0.107	0.092	0.061	0	0.032	0.044	0.107	0.092	0.061	0

$NDA^{\mathcal{F}}$ and $NDA^{\mathcal{D}}$ values are computed with Equation (5.3) and provided in Table 5.10 and Table 5.11, respectively.

Table 5.10 - $NDA^{\mathcal{F}}$ matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0	0	0	0	0.206	0.052	0	0.006	0.105	0	0
A_2	0	0	0.158	0	0	0.004	0	0	0.045	0.066	0
A_3	0.309	0.698	0.424	0.804	0.141	0	0.374	0.445	0.348	0.794	0.225
A_4	0	0	0	0	0	0	0	0	0	0	0
A_5	0	0	0	0	0.412	0.052	0	0	0	0	0.114
A_6	0.309	0.326	0.477	0.321	0	0	0.699	0.561	0.45	0.399	0.394
A_7	0	0	0	0.005	0	0	0.027	0	0.045	0	0
A_8	0	0	0	0	0	0.025	0	0	0	0	0

Table 5.11 - $NDA^{\mathcal{D}}$ matrix

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0	0	0	0	0.234	0	0	0	0	0	0.234
A_2	0	0	0	0	0.056	0.013	0	0	0	0	0.056
A_3	0.399	0.449	0.48	0.616	0.413	0.147	0.399	0.449	0.48	0.616	0.413
A_4	0.121	0.058	0	0.108	0	0	0.121	0.058	0	0.108	0
A_5	0	0	0	0	0	0	0	0	0	0	0
A_6	0.287	0.226	0.528	0.276	0	0.116	0.287	0.226	0.528	0.276	0
A_7	0	0	0.05	0	0.056	0.102	0	0	0.05	0	0.056
A_8	0	0	0	0	0.234	0	0	0	0	0	0.234

$SP_i^{\mathcal{F}}$ and $SP_i^{\mathcal{D}}$ values are computed with Equation (5.6) and provided in Table 5.1; and $SN_i^{\mathcal{F}}$ and $SN_i^{\mathcal{D}}$ values are calculated through Equation (5.7) and given in Tables 5.123, respectively.

Table 5.12 - SP Positive values for functional and dysfunctional perspectives

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
$SP_i^{\mathcal{F}}$	0.034	0.035	0.006	0.18	0.128	0.01	0.028	0.063
$SP_i^{\mathcal{D}}$	0.065	0.044	0	0.14	0.132	0.034	0.026	0.065

Table 5.13 - SP Negative values for functional and dysfunctional perspectives

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
$SN_i^{\mathcal{F}}$	0.034	0.035	0.006	0.18	0.128	0.01	0.028	0.063
$SN_i^{\mathcal{D}}$	0.022	0.008	0.407	0.027	0	0.314	0.047	0.022

$NSP_i^{\mathcal{F}}$ and $NSP_i^{\mathcal{D}}$ values are computed with Equation (5.8) and provided in Table 5.14.

Table 5.14 - NSP values for functional and dysfunctional perspectives

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
$NSP_i^{\mathcal{F}}$	0.191	0.193	0.034	1	0.714	0.053	0.154	0.351
$NSP_i^{\mathcal{D}}$	0.463	0.314	0	1	0.946	0.245	0.183	0.463

$NSN_i^{\mathcal{F}}$ and $NSN_i^{\mathcal{D}}$ values are computed with Equation (5.9) and provided in Table 5.15.

Table 5.15 - NSN values for functional and dysfunctional perspectives

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
$NSN_i^{\mathcal{F}}$	0.877	0.865	0	1	0.894	0.035	0.97	0.985
$NSN_i^{\mathcal{D}}$	0.945	0.98	0	0.933	1	0.227	0.884	0.945

Table 5.16 - Compromise scores for functional and dysfunctional perspectives

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
$AS_i^{\mathcal{F}}$	0.534	0.529	0.017	1	0.804	0.044	0.562	0.668
$AS_i^{\mathcal{D}}$	0.704	0.647	0	0.966	0.973	0.236	0.533	0.704

To calculate the aggregated compromise scores (AS_i^{agg}), β is taken as 0.5 to make sure the final decision is not biased towards either functional score or dysfunctional score.

The aggregated compromise scores are calculated by Equation (5.11) and given in Table 5.17.

Table 5.17 - Aggregated Compromise Scores and Alternatives' Ranks

A_i	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
AS_i^{agg}	0.619	0.588	0.0085	0.983	0.8885	0.14	0.5475	0.686
Ranks	4	5	8	1	2	7	6	3

As the result, in the case study on sustainable aircraft model selection, the results found with the DqROF MAIRCA method applied to rank aircraft model alternatives are as follows: $A_4 > A_5 > A_8 > A_1 > A_2 > A_7 > A_6 > A_3$, as seen in Table 5.9.

5.3 Comparative Analysis

5.3.1 Comparison with q-Rung Orthopair Fuzzy Numbers

In this part of the study, the same problem is addressed with the q-ROF EDAS method to evaluate the accuracy and validity of the ranking of aircraft model alternatives obtained with the sustainability criteria by the DqROF EDAS method. DqROFS is a fuzzy set extension derived from qROF and therefore the q-ROF based EDAS method is selected to compare the presented DqROF EDAS method. The main distinction between these fuzzy sets is based on the data collection approach. In DqROFS, data is received from decision makers in the form of functional and dysfunctional data pairs, while in q-ROFS

this is single. Since the data received from decision makers are structurally different, it is important to be able to transform the decision maker evaluations obtained for DqROFS into q-ROFS.

For this comparison, two scenarios were considered. In the first scenario, the q-ROF EDAS method was applied by considering the answers given by the decision makers to the functional questions (Scenario 1). In the second scenario, the q-ROF EDAS method was applied by considering the answers given to the dysfunctional questions (Scenario 2). The results are shown in Table 5.18.

When the q-ROF EDAS method is applied, the order obtained by considering only the decision makers' responses to functional questions in Scenario 1 is $A_4 > A_5 > A_8 > A_7 > A_1 > A_2 > A_6 > A_3$. In Scenario 2, dysfunctional evaluations are taken as basis and the order is changed to $A_5 > A_4 > A_8 = A_1 > A_2 > A_7 > A_6 > A_3$. The order found with the DFS method is $A_4 > A_5 > A_8 > A_1 > A_2 > A_7 > A_6 > A_3$.

Table 5.18 – Comparison of compromise scores and rankings

	DqROF EDAS		q-ROF EDAS (Scenario 1)		q-ROF EDAS (Scenario 2)	
	Compromise Score	Rank	Compromise Score	Rank	Comprom ise Score	Rank
A_1	0.619	4	0.534	5	0.704	3
A_2	0.588	5	0.529	6	0.647	5
A_3	0.0085	8	0.017	8	0	8
A_4	0.983	1	1	1	0.966	2
A_5	0.8885	2	0.804	2	0.973	1
A_6	0.14	7	0.044	7	0.236	7
A_7	0.5475	6	0.562	4	0.533	6
A_8	0.686	3	0.668	3	0.704	3

The analysis of these rankings shows that there may be differences in determining the best alternative. While A5 is determined as the best alternative only with functional evaluations in Scenario 1, A4 stands out as the best alternative in Scenario 2 according to dysfunctional evaluations. Since the DqROFS method also takes into account the decision makers' indecisiveness and inconsistent responses, choosing A4 as the best alternative is consistent with Scenario 2.

The DqROFS method collects data through functional and dysfunctional questions. The decision maker's answers to both types of questions are expected to complement each other. If the answers are not complementary, the decision maker's inconsistency increases. This inconsistency is clearly observed when the q-ROF EDAS method is applied.

The q-ROF EDAS results obtained in the first scenario differ from those in the second scenario. If the answers given to functional questions and the answers given to dysfunctional questions were equally complementary, similar results would have been obtained in both scenarios using q-ROF EDAS. However, the fact that the results are not the same shows that the q-ROF MAIRCA method reveals the inconsistency of the data collected from decision makers.

Since DqROFS carries traces of both functional and dysfunctional answers, it has the ability to balance uncertainties and inconsistencies in the decision-making process. Thanks to this feature, it is successful in overcoming inconsistencies and offers a significant advantage in its application to such problems.

5.3.2 Comparison with Other Methods

In this part of the study, the same problem is addressed with the DqROF ARAS and DqROF WASPAS methods to evaluate the accuracy and validity of the ranking of aircraft model alternatives obtained with the sustainability criteria by the DqROF EDAS method.

The EDAS method is in the distance-based MCDM classification therefore the ARAS and the WASPAS methods are selected to compare the presented DqROF EDAS method since they are also in the distance-based MCDM classification. The main distinction between EDAS and ARAS methods is their evaluation approach. While EDAS bases the solution strategy on average solution, ARAS bases its solution strategy on ideal solution.

When the DqROF EDAS, DqROF ARAS and DqROF WASPAS methods are applied, the order obtained is $A_4 > A_5 > A_8 > A_1 > A_2 > A_7 > A_6 > A_3$ for all of the methods.

The fact that different MCDM methods give the same ranking indicates that the decision process is robust and reliable. Although the methods use different mathematical approaches, the fact that the results are consistent shows that the differences between the alternatives are clear and the criteria's weights are determined correctly. This means that the decision maker can trust the method when making a choice (Zavadskas et al., 2017).

Table 5.19 - Comparison of evaluation scores and rankings with different methods

	DqROF EDAS		DqROF ARAS		DqROF WASPAS	
	Compromise Score	Rank	Relative Utility	Rank	Score	Rank
A_1	0.619	4	0.921	4	0.688	4
A_2	0.588	5	0.919	5	0.681	5
A_3	0.0085	8	0.847	8	0.4255	8
A_4	0.983	1	0.976	1	0.7995	1
A_5	0.8885	2	0.948	2	0.74	2
A_6	0.14	7	0.86	7	0.4765	7
A_7	0.5475	6	0.914	6	0.6695	6
A_8	0.686	3	0.931	3	0.713	3

5.3.3 Comparison with Other q-levels

In this part of the study, the same problem is addressed with $q=3$ and $q=4$ levels within the same methods to evaluate the accuracy and validity of the ranking of aircraft model alternatives obtained with sustainability criteria by the DqROF EDAS method when q is

assumed to be two. In the case study section DqROF EDAS and DqROF MAIRCA methods were executed for $q=2$, so it is compulsory to assess the same proposed techniques with different q levels to observe and validate the decomposed q -rung orthopair set's q degree effect on the models. The utilization of the proposed DqROF EDAS and DqROF MAIRCA are the same except for the DqROWGM aggregator to create an aggregated decision matrix. The DqROWGM aggregator takes q parameter as two in the case study, however assessment of different q -levels is beneficial since as q increases the effect of membership and non-membership values also increase. Thus, the value of hesitation degree increases and the model becomes less dependent on the membership and non-membership values.

Table 5.20 - Comparison of evaluation scores and rankings with different q -levels

	DqROF EDAS ($q=2$)		DqROF EDAS ($q=3$)		DqROF EDAS ($q=4$)	
	Compromise Score	Rank	Compromise Score	Rank	Compromise Score	Rank
A_1	0.619	4	0.614	4	0.614	4
A_2	0.588	5	0.597	5	0.605	5
A_3	0.0085	8	0.0085	8	0.0085	8
A_4	0.983	1	0.9835	1	0.984	1
A_5	0.8885	2	0.8905	2	0.893	2
A_6	0.14	7	0.14	7	0.1405	7
A_7	0.5475	6	0.5525	6	0.5565	6
A_8	0.686	3	0.6805	3	0.677	3

The fact that the same alternative ranking is obtained at different q levels indicates that the decision model is stable and reliable, the differences between the alternatives are clear and strong, and the MCDM method used does not affect the variability introduced by the q -rung fuzzy system directly on the ranking. This is a sign that the proposed method is insensitive to q changes for the given case study and that the decision structure is robust.

5.4 Results

Multi-Criteria Decision Making (MCDM) methods are developed to provide solutions to decision-making problems and are applied in different types and varieties. These methods are based on the principle of ranking the alternatives from best to worst or selecting the best alternative according to the determined criteria. Although there are many applications for each method in the literature, fuzzy methods are widely preferred in decision problems involving uncertainty. In addition to traditional fuzzy approaches, MCDM methods extended with different fuzzy sets have also been developed depending on the nature of the problem and the degree of uncertainty.

In this context, the EDAS method is also an MCDM method and is considered relatively new in the literature. Therefore, studies on its application with different extensions are limited. In the current literature, it is seen that the EDAS method has not been extended with the Decomposed Fuzzy Set (DFS) based on q-rung orthopair fuzzy set (q-ROFS). In order to fill this gap, in this study, the EDAS method was first extended with the newly developed Dq-ROFS and the DqROF EDAS method was created. In order to demonstrate the applicability of the developed method, a case study on aircraft model selection from a sustainability perspective in the aviation sector was conducted.

Then, the criteria weights were determined by the Bayesian BWM method. After the criteria were determined, the answers given by three different decision makers (DMs) whose weights were considered equal to the functional and dysfunctional questions were taken to rank the alternatives using the DqROF EDAS method. The final ranking obtained as a result of the application of the method is as follows:

$$A_4 > A_5 > A_8 > A_7 > A_2 > A_1 > A_6 > A_3$$

In order to test the reliability of the ranking obtained, the EDAS method was extended with q-ROFS, from which DqROFS was derived, and two different scenarios were created:

- Scenario 1: The q-ROF EDAS method was applied by taking into account the functional answers given by decision makers to functional questions. Ranking of alternatives: $A_4 > A_7 > A_8 > A_5 > A_2 > A_1 > A_6 > A_3$
- Scenario 2: The q-ROF EDAS method was applied, taking into account the dysfunctional answers given by decision makers to dysfunctional questions. Ranking of alternatives: $A_4 > A_5 > A_1 = A_8 > A_2 > A_7 > A_6 > A_3$

The results show that the ranking obtained with DqROFS is between Scenario 1 and Scenario 2. This situation reveals that the DqROF EDAS method includes certain aspects of both functional and dysfunctional evaluations. In addition, considering the indecisiveness of decision makers and possible inconsistencies in their responses, the DqROF EDAS method seems to provide a reliable and realistic evaluation.

The proposed method is also compared to different DqROF methods and assessed with different q levels and yielded the same ranking for both which proves the decision model's reliability and stability.

6 DECOMPOSED Q-RUNG ORTHOPAIR FUZZY NUMBER BASED MAIRCA METHOD

6.1 Description and Steps of the Proposed Method

The MAIRCA method is a known and used MCDM method in literature and its basic logic is based on distances. On the other hand, it assigns equal probability to the selection of each of the available alternatives. Therefore, the method is considered reliable. In this section of the thesis, the MAIRCA method is extended for the first time with decomposed q-rung orthopair fuzzy sets and presented as DqROF MAIRCA. The steps of the DqROF MAIRCA method are as follows:

Considering the set of experts as $e = \{1, 2, \dots, k\}$, the set of alternatives as $i = \{1, 2, \dots, f\}$, the set of criteria as $j = \{1, 2, \dots, n\}$,

Step 1: Obtain the linguistic evaluations of the decision makers and translate these linguistic evaluations into the corresponding DqROF numbers using Table 5.2.

Step 2. Aggregate the obtained DqROF numbers using the DqROWGM operator specified in Equation (3.43).

Step 3. Obtain decomposed q-rung orthopair fuzzy positive ideal (DqROFPIS) and decomposed q-rung orthopair fuzzy negative ideal (DqROFNIS) solutions. Under each criterion, the largest DqROF number among the alternatives is DqROFPIS and the smallest DqROF number is DqROFNIS.

Step 4. Calculate the distance of each alternative from DqROFPIS and obtain $S_m^{+\mathcal{F}}$ and $S_m^{+\mathcal{D}}$. Calculate the distance of each alternative from DqROFNIS and obtain $S_m^{-\mathcal{F}}$ and $S_m^{-\mathcal{D}}$.

Step 5. Compute the closeness coefficient. The value of CW_m means the CC of the m^{th} criterion and this value is obtained by applying the equation stated in (6.1). CW_m values

must be calculated for functional and dysfunctional evaluations with $CW_m^{\mathcal{F}}$ and $CW_m^{\mathcal{D}}$, respectively.

$$CW_m^{\mathcal{F}} = \frac{S_m^{-\mathcal{F}}}{S_m^{-\mathcal{F}} + S_m^{+\mathcal{F}}} \text{ and } CW_m^{\mathcal{D}} = \frac{S_m^{-\mathcal{D}}}{S_m^{-\mathcal{D}} + S_m^{+\mathcal{D}}} \quad (6.1)$$

Step 6. Create the initial DqROF decision matrix. If normalization is necessary, perform normalization using the equations shown in (6.2) and (6.3), taking into account whether the criterion is a benefit or cost criterion.

$$n_{ij}^{\mathcal{F}} = \frac{x_{ij} - x_{min}}{x_{max} - x_{min}} \text{ and } n_{ij}^{\mathcal{D}} = \frac{x_{ij} - x_{min}}{x_{max} - x_{min}} \text{ if } xi \in B \quad (6.2)$$

$$n_{ij}^{\mathcal{F}} = \frac{x_{max} - x_{ij}}{x_{max} - x_{min}} \text{ and } n_{ij}^{\mathcal{D}} = \frac{x_{max} - x_{ij}}{x_{max} - x_{min}} \text{ if } xi \in C \quad (6.3)$$

Step 7. Determine the theoretical DqROF decision matrix. To do this, calculate the preference status of the possible alternative f using (6.4) and (6.6) and then multiply this value by the criterion weights using (6.5) and (6.7). Here n is the number of criteria and t_{pi} is the theoretical score. The alternatives are assumed to have equal probabilities of being selected, but multiplication by $\frac{1}{f}$ is still necessary for normalization and comparability of scores.

$$P_{A_i}^{\mathcal{F}} = \frac{1}{f} \text{ and } \sum_{i=1}^f P_{A_i}^{\mathcal{F}} = 1. \quad (6.4)$$

$$P_{A_i}^{\mathcal{F}} \cdot [t_{p1}^{\mathcal{F}} t_{p2}^{\mathcal{F}} \dots t_{pn}^{\mathcal{F}}] \quad (6.5)$$

$$P_{A_i}^{\mathcal{D}} = \frac{1}{f} \text{ and } \sum_{i=1}^f P_{A_i}^{\mathcal{D}} = 1. \quad (6.6)$$

$$P_{A_i}^{\mathcal{D}} \cdot [t_{p1}^{\mathcal{D}} t_{p2}^{\mathcal{D}} \dots t_{pn}^{\mathcal{D}}] \quad (6.7)$$

Step 8. Create the real DqROF evaluation matrix by multiplying the elements of the normalized DqROF decision matrix with the elements of the theoretical DqROF decision matrix. Equation (6.8) shows the formula required for this calculation.

$$r_{ij}^{\mathcal{F}} = n_{ij}^{\mathcal{F}} * t_{pi}^{\mathcal{F}} \text{ and } r_{ij}^{\mathcal{D}} = n_{ij}^{\mathcal{D}} * t_{pi}^{\mathcal{D}} \quad (6.8)$$

Step 9. Construct the DqROF gap matrix by subtracting the real DqROF evaluation matrix from the theoretical DqROF decision matrix as shown in (6.9).

$$g_{ij}^{\mathcal{F}} = t_{pi}^{\mathcal{F}} - r_{ij}^{\mathcal{F}} \text{ and } g_{ij}^{\mathcal{D}} = t_{pi}^{\mathcal{D}} - r_{ij}^{\mathcal{D}} \quad (6.9)$$

Step 10. Calculate the utility points for each alternative by adding the gaps obtained in Step 9. The gaps should be added row by row. The mathematical formulation of this sum is shown in (6.10) for functional and dysfunctional utility scores respectively.

$$q_i^{\mathcal{F}} = \sum_{j=1}^n g_{ij}^{\mathcal{F}}, i = 1, 2, \dots, f \text{ and } q_i^{\mathcal{D}} = \sum_{j=1}^n g_{ij}^{\mathcal{D}}, i = 1, 2, \dots, f \quad (6.10)$$

Step 10. Create aggregated q_i^{agg} score with the following formula (6.11). In this formula β represents the preference of decision makers on proportion of functional utility scores on the final aggregated utility score and $(1 - \beta)$ represents the proportion of dysfunctional utility scores on the aggregated one. β takes value between $[0,1]$.

$$q_i^{agg} = \beta \cdot q_i^{\mathcal{F}} + (1 - \beta) \cdot q_i^{\mathcal{D}} \quad (6.11)$$

Step 11. Rank the alternatives according to their aggregated utility scores. The basic logic of ranking is as follows: The best alternative is the one with the smallest aggregated utility score. If the difference between the obtained value and the ideal value is small, it means that the alternative is close to the ideal. The flowchart of the proposed DqROF MAIRCA method is given in Figure 6.1.

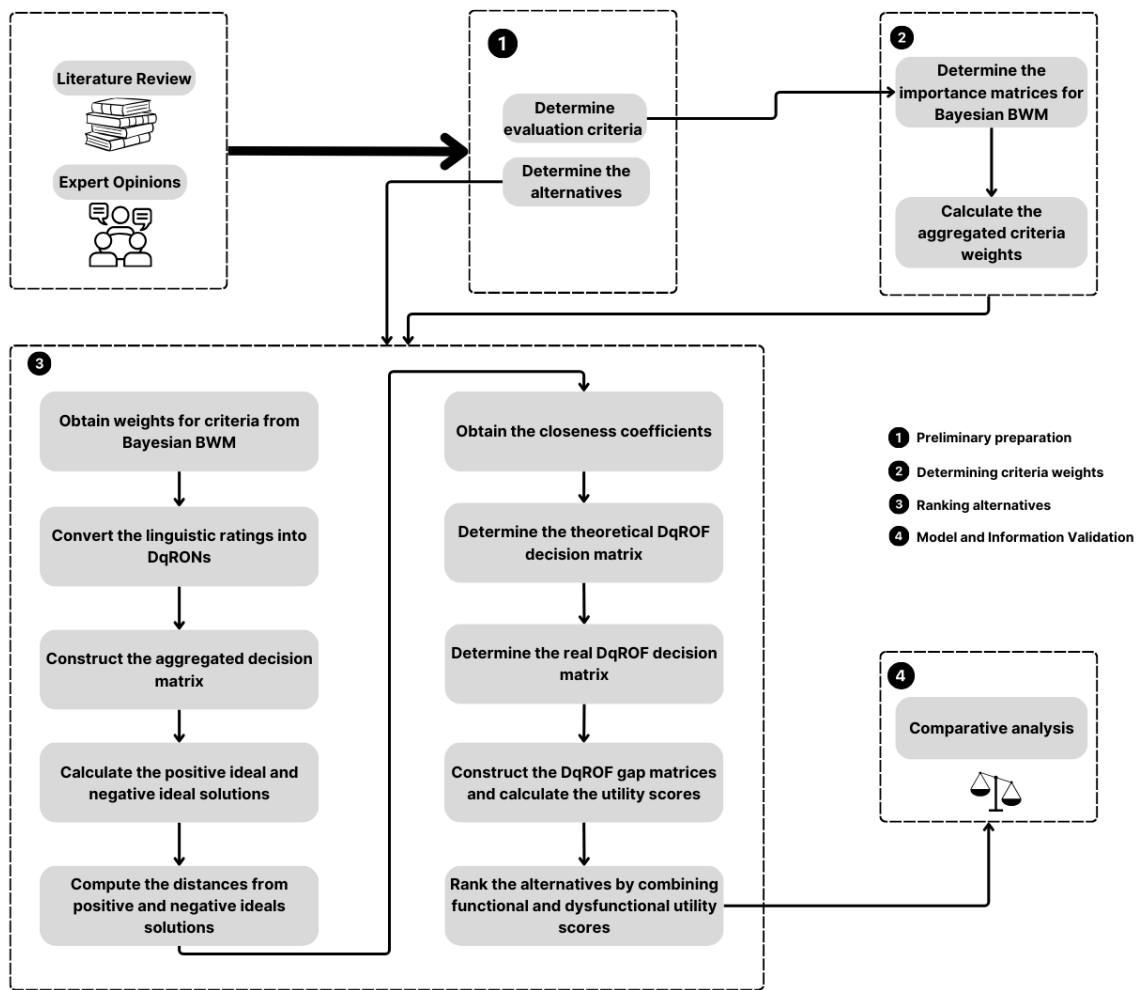


Figure 6.1 – Flowchart of the proposed DqROF MAIRCA Methodology

6.2 Case Study

In the field of multi-criteria decision making in operations research, it is quite common to rank alternatives by considering the criteria determined for a problem.

However, this section of this thesis is quite unique in that it presents the MAIRCA method, which has just begun to be recognized in the literature, by expanding it with DqROFS.

The DqROF MAIRCA method has been applied to the aircraft selection problem from sustainability perspective. The aim is to rank the existing eight airplane model alternatives by considering the eleven criteria determined after the comprehensive literature review conducted in Section 2.

The determined criteria are the same with the case study of DqROF EDAS method C_1 : Acquisition Cost, C_2 : Operational Cost, C_3 : Fuel Efficiency, C_4 : Return on Investment, C_5 : Passenger Comfort, C_6 : Safety Record, C_7 : Accessibility, C_8 : Community Impact, C_9 : Carbon Emissions, C_{10} : Noise Pollution and C_{11} : Lifecycle Assessment and Recyclability criteria. Again, all the criteria in this case study are also obtained as benefit criteria since DMs made evaluations for on importance levels of each criterion based on a linguistic scale. The alternative airplane models are again named $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ and A_8 .

The distance of each alternative from DqROFPIS, and the distance of each alternative from DqROFNIS are calculated for functional and dysfunctional perspectives and provided in Table 6.1 and Table 6.2.

Tablo 6.1 – Distances from positive ideal solution from functional perspective and their calculated closeness coefficients

	A_1			A_2			A_3			A_4		
	S_m^{+F}	S_m^{-F}	CW^F	S_m^{+F}	S_m^{-F}	CW^F	S_m^{+F}	S_m^{-F}	CW^F	S_m^{+F}	S_m^{-F}	CW^F
C_1	0.183	0.124	0.597	0.183	0.124	0.597	0.117	0.190	0.618	0.183	0.124	0.597
C_2	0.124	0.183	0.597	0.092	0.216	0.701	0.264	0.053	0.168	0.124	0.183	0.597
C_3	0.092	0.216	0.701	0.143	0.163	0.532	0.198	0.108	0.354	0.050	0.261	0.839
C_4	0.100	0.207	0.673	0.069	0.240	0.776	0.279	0.039	0.122	0.100	0.207	0.673
C_5	0.156	0.150	0.49	0.111	0.196	0.639	0.143	0.163	0.532	0.069	0.240	0.776
C_6	0.069	0.240	0.776	0.058	0.252	0.814	0.050	0.261	0.839	0.050	0.261	0.839
C_7	0.107	0.201	0.652	0.100	0.207	0.673	0.198	0.108	0.354	0.124	0.183	0.597
C_8	0.124	0.183	0.597	0.111	0.196	0.639	0.209	0.098	0.319	0.092	0.216	0.701
C_9	0.124	0.183	0.597	0.111	0.196	0.639	0.176	0.130	0.425	0.042	0.272	0.867
C_{10}	0.111	0.196	0.639	0.132	0.174	0.568	0.279	0.039	0.122	0.069	0.240	0.776
C_{11}	0.111	0.196	0.639	0.111	0.196	0.639	0.165	0.141	0.461	0.079	0.229	0.743
	A_5			A_6			A_7			A_8		
	S_m^{+F}	S_m^{-F}	CW^F	S_m^{+F}	S_m^{-F}	CW^F	S_m^{+F}	S_m^{-F}	CW^F	S_m^{+F}	S_m^{-F}	CW^F
C_1	0.216	0.092	0.299	0.117	0.190	0.618	0.183	0.124	0.403	0.183	0.124	0.403
C_2	0.069	0.240	0.776	0.190	0.117	0.382	0.124	0.183	0.597	0.124	0.183	0.597
C_3	0.079	0.229	0.743	0.209	0.098	0.319	0.111	0.196	0.639	0.103	0.204	0.664
C_4	0.069	0.240	0.776	0.176	0.130	0.425	0.111	0.196	0.639	0.083	0.226	0.731
C_5	0.198	0.108	0.354	0.100	0.207	0.673	0.083	0.226	0.731	0.111	0.196	0.639
C_6	0.069	0.240	0.776	0.050	0.261	0.839	0.050	0.261	0.839	0.062	0.248	0.799
C_7	0.069	0.240	0.776	0.264	0.053	0.168	0.132	0.174	0.568	0.111	0.196	0.639
C_8	0.100	0.207	0.673	0.232	0.076	0.247	0.111	0.196	0.639	0.078	0.234	0.75
C_9	0.069	0.240	0.776	0.198	0.108	0.354	0.111	0.196	0.639	0.083	0.226	0.731
C_{10}	0.083	0.226	0.731	0.198	0.108	0.354	0.111	0.196	0.639	0.083	0.226	0.731
C_{11}	0.143	0.163	0.532	0.198	0.108	0.354	0.100	0.207	0.673	0.103	0.204	0.664

Tablo 6.2 – Distances from negative ideal solution from functional perspective and their calculated closeness coefficients

	A_1			A_2			A_3			A_4		
	$S_m^{+\mathfrak{D}}$	$S_m^{-\mathfrak{D}}$	$CW^{\mathfrak{D}}$	$S_m^{+\mathfrak{D}}$	$S_m^{-\mathfrak{D}}$	$CW^{\mathfrak{D}}$	$S_m^{+\mathfrak{D}}$	$S_m^{-\mathfrak{D}}$	$CW^{\mathfrak{D}}$	$S_m^{+\mathfrak{D}}$	$S_m^{-\mathfrak{D}}$	$CW^{\mathfrak{D}}$
C_1	0.196	0.111	0.361	0.207	0.100	0.327	0.108	0.198	0.646	0.163	0.143	0.468
C_2	0.100	0.207	0.673	0.111	0.196	0.639	0.209	0.098	0.319	0.132	0.174	0.568
C_3	0.069	0.240	0.776	0.079	0.229	0.743	0.198	0.108	0.354	0.042	0.272	0.867
C_4	0.111	0.196	0.639	0.100	0.207	0.673	0.243	0.066	0.214	0.143	0.163	0.532
C_5	0.176	0.130	0.425	0.143	0.163	0.532	0.209	0.098	0.319	0.111	0.196	0.639
C_6	0.069	0.240	0.776	0.079	0.229	0.743	0.111	0.196	0.639	0.042	0.272	0.867
C_7	0.111	0.196	0.639	0.100	0.207	0.673	0.198	0.108	0.354	0.143	0.163	0.532
C_8	0.100	0.207	0.673	0.111	0.196	0.639	0.209	0.098	0.319	0.132	0.174	0.568
C_9	0.069	0.240	0.776	0.079	0.229	0.743	0.198	0.108	0.354	0.042	0.272	0.867
C_{10}	0.111	0.196	0.639	0.100	0.207	0.673	0.243	0.066	0.214	0.143	0.163	0.532
C_{11}	0.176	0.130	0.425	0.143	0.163	0.532	0.209	0.098	0.319	0.111	0.196	0.639
	A_5			A_6			A_7			A_8		
	$S_m^{+\mathfrak{D}}$	$S_m^{-\mathfrak{D}}$	$CW^{\mathfrak{D}}$	$S_m^{+\mathfrak{D}}$	$S_m^{-\mathfrak{D}}$	$CW^{\mathfrak{D}}$	$S_m^{+\mathfrak{D}}$	$S_m^{-\mathfrak{D}}$	$CW^{\mathfrak{D}}$	$S_m^{+\mathfrak{D}}$	$S_m^{-\mathfrak{D}}$	$CW^{\mathfrak{D}}$
C_1	0.240	0.069	0.224	0.130	0.176	0.575	0.207	0.100	0.327	0.196	0.111	0.361
C_2	0.083	0.226	0.731	0.165	0.141	0.461	0.111	0.196	0.639	0.100	0.207	0.673
C_3	0.069	0.240	0.776	0.209	0.098	0.319	0.100	0.207	0.673	0.069	0.240	0.776
C_4	0.069	0.240	0.776	0.176	0.130	0.425	0.100	0.207	0.673	0.111	0.196	0.639
C_5	0.100	0.207	0.673	0.069	0.240	0.776	0.143	0.163	0.532	0.176	0.130	0.425
C_6	0.069	0.240	0.776	0.103	0.204	0.664	0.100	0.207	0.673	0.069	0.240	0.776
C_7	0.069	0.240	0.776	0.176	0.130	0.425	0.100	0.207	0.673	0.111	0.196	0.639
C_8	0.083	0.226	0.731	0.165	0.141	0.461	0.111	0.196	0.639	0.100	0.207	0.673
C_9	0.069	0.240	0.776	0.209	0.098	0.319	0.100	0.207	0.673	0.069	0.240	0.776
C_{10}	0.069	0.240	0.776	0.176	0.130	0.425	0.100	0.207	0.673	0.111	0.196	0.639
C_{11}	0.100	0.207	0.673	0.069	0.240	0.776	0.143	0.163	0.532	0.176	0.130	0.425

Theoretical DqROF decision matrices are calculated and are provided for functional and dysfunctional perspectives in Table 6.3 and Table 6.4, respectively.

Table 6.3 - Theoretical DqROF decision matrix for functional perspective ($t_{pi}^{\mathcal{F}}$)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_2	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_3	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_4	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_5	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_6	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_7	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_8	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006

Table 6.4 - Theoretical DqROF decision matrix for dysfunctional perspective ($t_{pi}^{\mathfrak{D}}$)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_2	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_3	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_4	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_5	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_6	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_7	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006
A_8	0.006	0.006	0.026	0.006	0.006	0.026	0.005	0.006	0.025	0.006	0.006

Real DqROF decision matrices are calculated and provided for functional and dysfunctional perspectives in Table 6.5 and Table 6.6, respectively.

Table 6.5 - Real DqROF decision matrix for functional perspective ($r_{ij}^{\mathcal{F}}$)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.004	0.004	0.019	0.005	0.002	0	0.004	0.004	0.012	0.005	0.004
A_2	0.004	0.006	0.011	0.006	0.004	0.016	0.004	0.005	0.014	0.004	0.004
A_3	0	0	0.002	0	0.003	0.026	0.001	0.001	0.003	0	0.002
A_4	0.004	0.004	0.026	0.005	0.006	0.026	0.003	0.005	0.025	0.006	0.006
A_5	0.006	0.006	0.021	0.006	0	0	0.005	0.005	0.02	0.006	0.003
A_6	0	0.002	0	0.003	0.005	0.026	0	0	0	0.002	0
A_7	0.004	0.004	0.016	0.005	0.005	0.026	0.003	0.005	0.014	0.005	0.005
A_8	0.004	0.004	0.017	0.006	0.004	0.01	0.004	0.006	0.018	0.006	0.005

Table 6.6 - Real DqROF decision matrix for dysfunctional perspective ($r_{ij}^{\mathfrak{D}}$)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.004	0.005	0.022	0.005	0.001	0.016	0.003	0.005	0.021	0.005	0.001
A_2	0.005	0.005	0.02	0.005	0.003	0.012	0.004	0.005	0.019	0.005	0.003
A_3	0	0	0.002	0	0	0	0	0	0.002	0	0
A_4	0.003	0.004	0.026	0.004	0.004	0.026	0.002	0.004	0.025	0.003	0.004
A_5	0.006	0.006	0.022	0.006	0.005	0.016	0.005	0.006	0.021	0.006	0.005
A_6	0.001	0.002	0	0.002	0.006	0.003	0.001	0.002	0	0.002	0.006
A_7	0.005	0.005	0.017	0.005	0.003	0.004	0.004	0.005	0.016	0.005	0.003
A_8	0.004	0.005	0.022	0.005	0.001	0.016	0.003	0.005	0.021	0.005	0.001

Gap matrixes of each alternative are calculated and provided for functional and dysfunctional perspectives in Table 6.7 and Table 6.8, respectively.

Table 6.7 Gap matrix for functional perspective ($g_{ij}^{\mathcal{F}}$)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.002	0.002	0.007	0.001	0.004	0.026	0.001	0.002	0.013	0.001	0.002
A_2	0.002	0.001	0.015	0	0.002	0.011	0.001	0.001	0.011	0.002	0.002
A_3	0.006	0.006	0.024	0.006	0.003	0	0.003	0.005	0.021	0.006	0.004
A_4	0.002	0.002	0	0.001	0	0	0.001	0.001	0	0	0
A_5	0	0	0.005	0	0.006	0.026	0	0.001	0.004	0	0.003
A_6	0.006	0.004	0.026	0.003	0.001	0	0.005	0.006	0.025	0.004	0.006
A_7	0.002	0.002	0.01	0.001	0.001	0	0.002	0.001	0.011	0.001	0.001
A_8	0.002	0.002	0.009	0	0.002	0.017	0.001	0	0.007	0	0.001

Table 6.8 - Gap matrix for dysfunctional perspective ($g_{ij}^{\mathfrak{D}}$)

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.002	0.001	0.004	0.002	0.005	0.01	0.002	0.001	0.004	0.001	0.005
A_2	0.002	0.001	0.006	0.001	0.003	0.014	0.001	0.001	0.006	0.001	0.003
A_3	0.006	0.006	0.025	0.006	0.006	0.026	0.005	0.006	0.023	0.006	0.006
A_4	0.004	0.003	0	0.003	0.002	0	0.003	0.002	0	0.003	0.002
A_5	0	0	0.004	0	0.001	0.01	0	0	0.004	0	0.001
A_6	0.005	0.004	0.026	0.004	0	0.023	0.004	0.004	0.025	0.004	0
A_7	0.002	0.001	0.009	0.001	0.003	0.022	0.001	0.001	0.009	0.001	0.003
A_8	0.002	0.001	0.004	0.002	0.005	0.01	0.002	0.001	0.004	0.001	0.005

The utility scores of alternatives are calculated and provided for functional and dysfunctional perspectives in Table 6.9 and Table 6.10 respectively.

Table 6.9 - Utility Scores for functional perspective ($q_i^{\mathcal{F}}$)

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
0.061	0.047	0.087	0.007	0.046	0.087	0.032	0.041

Table 6.10 - Utility Score for dysfunctional perspective s ($q_i^{\mathcal{D}}$)

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
0.036	0.04	0.122	0.02	0.022	0.099	0.054	0.036

To calculate the aggregated utility scores (q_i^{agg}), β is taken as 0.5 to make sure the final decision is not biased towards either functional score or dysfunctional score. The aggregated utility scores are computed and provided in Table 6.11.

Table 6.11 - Aggregated Utility Scores and Alternatives' Ranks

A_i	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
q_i^{agg}	0.0485	0.0435	0.1045	0.0135	0.034	0.093	0.043	0.0385
Ranks	6	5	8	1	2	7	4	3

As a result, in the case study on sustainable aircraft model selection, the results found with the DqROF MAIRCA method applied to rank aircraft model alternatives are as follows: $A_4 > A_5 > A_8 > A_7 > A_2 > A_1 > A_6 > A_3$, as seen in Table 5.9.

6.3 Comparative Analysis

6.3.1 Comparison with q-Rung Orthopair Fuzzy Numbers

In this part of the study, the same problem is addressed with the q-ROF MAIRCA method to evaluate the accuracy and validity of the ranking of aircraft model alternatives obtained with the sustainability criteria by the DqROF MAIRCA method. DqROFS is a fuzzy set extension derived from qROF and therefore the q-ROF based MAIRCA method is selected to compare the presented DqROF MAIRCA method. The main distinction between these fuzzy sets is based on the data collection approach. In DqROFS, data is

received from decision makers in the form of functional and dysfunctional data pairs, while in q-ROFS this is single. Since the data received from decision makers are structurally different, it is important to be able to transform the decision maker evaluations obtained for DqROFS into q-ROFS.

For this comparison, two scenarios were considered. In the first scenario, the q-ROF MAIRCA method was applied by considering the answers given by the decision makers to the functional questions (Scenario 1). In the second scenario, the q-ROF MAIRCA method was applied by considering the answers given to the non-functional questions (Scenario 2). The results are shown in Table 6.12.

Table 6.12 - Comparison of utility scores and rankings

	DqROF MAIRCA		q-ROF MAIRCA (Scenario 1)		q-ROF MAIRCA (Scenario 2)	
	Utility Score	Rank	Utility Score	Rank	Utility Score	Rank
A_1	0.036	6	0.061	6	0.036	3
A_2	0.040	5	0.047	5	0.04	5
A_3	0.122	8	0.087	8	0.122	8
A_4	0.020	1	0.007	1	0.020	1
A_5	0.022	2	0.046	4	0.022	2
A_6	0.099	7	0.087	7	0.099	7
A_7	0.054	4	0.032	2	0.054	6
A_8	0.036	3	0.041	3	0.036	3

When the q-ROF MAIRCA method is applied, the order obtained from the decision makers' functional answers in Scenario 1 is $A_4 > A_7 > A_8 > A_5 > A_2 > A_1 > A_6 > A_3$. In the second scenario, it is different because it is obtained by considering dysfunctional evaluations and is as $A_4 > A_5 > A_1 = A_8 > A_2 > A_7 > A_6 > A_3$.

The ranking found with DqROFS is as $A_4 > A_5 > A_8 > A_7 > A_2 > A_1 > A_6 > A_3$. When the rankings are examined, it is seen that the best alternative is the same in all three methods. In the ranking of the second-best alternative, A_5 is in second place in q-ROF (Scenario 2) where DqROFS and dysfunctional scores are considered, and A_7 is in second place in q-ROF (Scenario 2) where functional scores are considered. In the DqROFS method, data is

collected by asking functional and dysfunctional questions. The decision maker's answers to functional and dysfunctional questions are expected to complement each other. The more the answers to functional and dysfunctional questions are far from complementing each other, the higher the decision maker's inconsistency. This inconsistency is clearly observed when the DqROFS MAIRCA method is applied to the data collected in the applied scenario. In addition, this method can be useful without eliminating the equivalence that may occur between the alternatives, as seen in q-ROF (Scenario 2).

6.3.2 Comparison with Other Methods

In this part of the study, the same problem is addressed with the DqROF ARAS and DqROF WASPAS methods to evaluate the accuracy and validity of the ranking of aircraft model alternatives obtained with the sustainability criteria by the DqROF MAIRCA method. The MAIRCA method is in the distance-based MCDM classification therefore the ARAS and the WASPAS methods are selected to compare the presented DqROF MAIRCA method since they are also in the distance-based MCDM classification. The main distinction between MAIRCA and ARAS methods is their evaluation approach. While MAIRCA is based on comparisons between ideal and realistic solutions, ARAS compares only with the ideal solution.

Table 6.13 - Comparison of evaluation scores and rankings with different methods

	DqROF MAIRCA		DqROF ARAS		DqROF WASPAS	
	Compromise Score	Rank	Relative Utility	Rank	Score	Rank
A ₁	0.0485	6	0.921	4	0.688	4
A ₂	0.0435	5	0.919	5	0.681	5
A ₃	0.1045	8	0.847	8	0.4255	8
A ₄	0.0135	1	0.976	1	0.7995	1
A ₅	0.034	2	0.948	2	0.74	2
A ₆	0.093	7	0.86	7	0.4765	7
A ₇	0.043	4	0.914	6	0.6695	6
A ₈	0.0385	3	0.931	3	0.713	3

When the DqROF MAIRCA method is applied, the ranking is $A_4 > A_5 > A_8 > A_7 > A_2 > A_1 > A_6 > A_3$. When DqROF ARAS and DqROF WASPAS methods are applied, the ranking calculated is $A_4 > A_5 > A_8 > A_1 > A_2 > A_7 > A_6 > A_3$ for both methods.

When the rankings are examined, it is seen that the best alternative is the same in all three methods and it is *Alternative 4*. In the ranking of the second and third-best alternative, the rankings yield the same results, and it is *Alternative 5* and *Alternative 8*, respectively. DqROF MAIRCA returns different results for the rankings of *Alternative 1* and *Alternative 7* when compared with DqROF ARAS and DqROF WASPAS. While *Alternative 1* is placed to the sixth ranking in DqROF MAIRCA, it is placed fifth for DqROF ARAS and DqROF WASPAS methods. This shows that although there may be minor changes in the ranking of certain alternatives due to the computational logic of different methods, the decision model is generally consistent and reliable.

6.3.3 Comparison with Other q -levels

In this part of the study, the same problem is addressed with $q=3$ and $q=4$ levels within the same methods to evaluate the accuracy and validity of the ranking of aircraft model alternatives obtained with sustainability criteria by the DqROF MAIRCA methods when q is assumed to be two. In the case study section DqROF EDAS and DqROF MAIRCA methods were executed for $q=2$, so it is compulsory to assess the same proposed techniques with different q levels to observe and validate the decomposed q -rung orthopair set's q degree effect on the models. The utilization of the proposed DqROF EDAS and DqROF MAIRCA are the same except for the DqROWGM aggregator to create an aggregated decision matrix. The DqROWGM aggregator takes q parameter as two in the case study, however assessment of different q -levels is beneficial since as q increases the effect of membership and non-membership values also increase. Thus, the value of hesitation degree increases, and the model becomes less dependent on the membership and non-membership values.

Table 6.14 - Comparison of evaluation scores and rankings with different q -levels

	DqROF MAIRCA ($q=2$)		DqROF MAIRCA ($q=3$)		DqROF MAIRCA ($q=4$)	
	Utility Score	Rank	Utility Score	Rank	Utility Score	Rank
A_1	0.0485	6	0.0485	6	0.0485	6
A_2	0.0435	5	0.0425	4	0.045	5
A_3	0.1045	8	0.1045	8	0.101	8
A_4	0.0135	1	0.0135	1	0.013	1
A_5	0.034	2	0.0335	2	0.0365	2
A_6	0.093	7	0.0935	7	0.0915	7
A_7	0.043	4	0.043	5	0.0445	4
A_8	0.0385	3	0.04	3	0.0425	3

When the DqROF MAIRCA method is applied with $q=2$ and $q=4$, the ranking is $A_4 > A_5 > A_8 > A_7 > A_2 > A_1 > A_6 > A_3$. When DqROF MAIRCA method is applied with $q=3$, the ranking calculated is as $A_4 > A_5 > A_8 > A_1 > A_2 > A_7 > A_6 > A_3$. This shows that although there are changes in the ranking of some alternatives when different q values are used, the overall ranking is largely preserved, and the decision model is stable. These results demonstrate that the DqROF MAIRCA method is partially sensitive to changes in q value but can provide reliable and consistent decision outputs even at different uncertainty levels, showing that the basic ranking structure is largely preserved.

6.4 Results

Multi-Criteria Decision Making (MCDM) methods have been developed to provide solutions to decision-making problems. These methods are of different types and varieties and are based on the principle of ranking the existing alternatives from the best to the worst according to the determined criteria or selecting the best alternative. There are many applications for each of these methods in the literature, and fuzzy methods are widely used in decision-making problems involving uncertainty. In addition to traditional fuzzy applications, MCDM methods extended with different fuzzy sets have also been developed depending on the change in the problem and the degree of uncertainty.

In this context, the MAIRCA method is also one of the MCDM methods and is relatively new in literature. Therefore, studies on its application with different extensions are relatively limited. In the current literature, the MAIRCA method has not been extended with the Decomposed Fuzzy Set (DFS) based on q-rung orthopair fuzzy set (q-ROFS). In this study, for the first time, the MAIRCA method was extended with the newly developed Dq-ROFS and the DqROF MAIRCA method was developed. A case study was conducted on the aircraft model selection problem from the sustainability perspective in the aviation field.

First of all, the criteria to be used were determined as a result of the detailed literature review conducted in Literature Review section. The determined criteria are as follows: C_1 : Acquisition Cost, C_2 : Operational Cost, C_3 : Fuel Efficiency, C_4 : Return on Investment, C_5 : Passenger Comfort, C_6 : Safety Record, C_7 : Accessibility, C_8 : Community Impact, C_9 : Carbon Emissions, C_{10} : Noise Pollution and C_{11} : Lifecycle Assessment and Recyclability. Then the criteria weights are determined with Bayesian BWM.

After determining the criteria, in order to rank the available alternatives using the DF MAIRCA method, the answers given to the functional and dysfunctional questions by three different decision makers (DM) whose weights were assumed to be equal were obtained. The final ranking obtained as a result of the application of the method is as follows: $A_4 > A_5 > A_8 > A_7 > A_2 > A_1 > A_6 > A_3$.

In order to test the reliability of the ranking obtained, the MAIRCA method was extended with the q-ROFS from which the DqROFS was derived, and two different scenarios were created.

Scenario 1: The q-ROF MAIRCA method was applied by considering the answers to the functional questions given by the decision makers to the functional questions. The ranking of alternatives is as follows: $A_4 > A_7 > A_8 > A_5 > A_2 > A_1 > A_6 > A_3$.

Scenario 2: The q-ROF MAIRCA method was applied using the dysfunctional answers given by decision makers to dysfunctional questions. The ranking of alternatives is as follows: $A_4 > A_5 > A_1 = A_8 > A_2 > A_7 > A_6 > A_3$.

The results show that the ranking found with DqROFS is between Scenario 1 and Scenario 2. This situation reveals that the DqROF MAIRCA method carries certain aspects of both functional and dysfunctional evaluations. In addition, considering the indecisiveness of decision makers and the possibility that their answers may be inconsistent, it is seen that the DqROF MAIRCA method provides a reliable and realistic evaluation.



7 CONCLUSIONS

In this thesis, new methods have been developed by integrating novel decomposed q-rung orthopair fuzzy sets with certain multi-criteria decision-making methods. The application of these new methods has been made on sustainable transportation, more specifically on sustainable aircraft model selection in the aviation field. The multi-criteria decision-making methods integrated with novel decomposed q-rung orthopair fuzzy sets are, respectively, EDAS and MAIRCA methods.

The purpose of extending the methods with DqROFS is to prove the validity of the extension of decomposed fuzzy sets, which has been presented in the literature in the recent past, with q-rung orthopair fuzzy number sets. For this reason, the new methods derived based on DqROFS have been extended with q-ROFS and their effectiveness has been tested. In other words, EDAS and MAIRCA methods were first presented as a new method by extending them with DqROFS and then applied to a problem determined on aircraft selection in the sustainable aviation field. Then, the same methods, EDAS and MAIRCA, have been extended with q-ROFS and applied again to the aircraft selection problem in the aviation field. Thus, the rankings obtained by extending the same methods with different extensions were evaluated.

The reason for choosing q-ROFS for the comparative analysis of the methods extended with DqROFS is that DqROFS is basically derived from q-ROFS. The main differences are that only linguistic evaluations are taken from decision makers, while the structure of the questions asked to decision makers is different and the flexibility of q-rung orthopair fuzzy number sets compared to intuitionistic fuzzy numbers that are utilized in the original DFS. While a pair of data, functional and non-functional, is collected from decision makers in DqROFS due to the two-way question asked to them, this data collected in q-ROFS is single. For this reason, DqROFS data was converted to q-ROFS under different scenarios for comparative analysis. When the comparisons are examined, it is seen that the methods extended with DqROFS yield quite successful and reliable

results. Questions directed to decision makers from two different perspectives provide them with a wider area to better express their knowledge, feelings and emotions gained from their experiences in their fields. On the other hand, when the comparisons made with q-ROFS are examined, it is seen that decision makers can sometimes be indecisive while conveying their linguistic evaluations and sometimes give inconsistent answers. The fact that DqROFS takes these inconsistencies and uncertainties into account increases the success and reliability rate obtained in the rankings.

When there are multi-layered and bidirectional dependencies between criteria, methods such as EDAS and MAIRCA may not be sufficient on their own. These methods may not capture the feedback loops that occur both directly and indirectly between “fuel efficiency” and “operational cost” or “return on investment” to the extent required. In this study, the lack of detailed analysis of the interaction and feedback mechanisms between the determined criteria stands out as an important limitation. The direct or indirect influence of different criteria on each other can significantly change the results in a complex decision-making process such as aircraft selection. Therefore, while the analysis is limited to evaluating and weighting each criterion separately, ignoring the interactive connections between these criteria creates the risk that the results will not fully reflect the real-life situation. This issue can be resolved by including multi-criteria decision-making methods (especially network-based approaches such as ANP or DEMATEL) in future studies.

ANP generalizes the hierarchical structure in AHP and enables modeling of bidirectional interactions (feedback loops) between criteria, sub-criteria and alternatives. For example, in aircraft selection, the “operational cost” criterion may affect the “fuel efficiency” criterion, while “fuel efficiency” may also be a determinant on “operational cost”. In addition, “return on investment” may be affected by various factors such as “carbon emissions” or “passenger comfort” simultaneously and with different weights. Another advanced method preferred before or in addition to ANP is the DEMATEL (Decision-Making Trial and Evaluation Laboratory) approach. DEMATEL analyzes the interaction network between criteria based on cause-effect relationships and determines which criteria are “driving/influencing” and which are “effected”. When this resulting cause-effect diagram is integrated into the ANP model, relationships such as “fuel efficiency”

directly seriously affecting “carbon emissions”, while “noise pollution” creates indirect effects through more societal reactions and operational restrictions, can be clearly demonstrated. Thus, a stronger idea is obtained about which criteria are more critical or in which improvements will provide the greatest benefit in the system.

As a result, ANP, DEMATEL or similar approaches are recommended to be integrated with DqROFS for problems where there are many criteria that directly and indirectly affect each other, such as aircraft selection. Thanks to these methods, it is possible to create a much more realistic and strategic decision mechanism that takes into account the feedback loops between criteria instead of the usual one-way comparisons.

Another limitation is that the proposed DqROF extended methods doubles the amount of data requested from decision makers. This may challenge the applicability of the method in cases where it is difficult to collect information from decision makers or where data access is limited. However, in scenarios where there are many decision makers, it is thought that the EDAS and MAIRCA methods with the DqROF extension offer a significant advantage as they facilitate the stable acquisition of evaluator scores. These methods have the potential to produce more reliable results by increasing the consistency of data in multi-stakeholder and intensively participating decision processes.

Moreover, in future studies, other MCDM and risk assessment methods can be expanded with DqROFS in order to shed light on decision-making problems and new methods and solution suggestions can be created. In addition, DqROFS-based decision support systems can be developed to facilitate the calculation steps. On the other hand, these methods can be applied by addressing real-life problems in other areas such as supply chain management, medicine, etc. outside the field of sustainable transportation.

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BIOGRAPHICAL SKETCH

He graduated from İzmir Atatürk Lisesi in 2009 then he started to Industrial Engineering Department of Izmir University of Economics. In 2014, he graduated from Izmir University of Economics with an honor degree. After working 4 years in FMCG companies, he is currently working as a research assistant at Istanbul Bilgi University and pursuing his doctoral's degree in Industrial Engineering under supervision of Assoc. Prof. Dr. Mehtap Dursun Karahüseyin at the Institute of Science and Engineering.

As part of his academic work, he has co-authored several publications, including the following:

Journal Article from This Thesis Study:

- Ari, E., & Dursun, M. (2024). An Integrated Bayesian Best–Worst Method and Consensus-Based Intuitionistic Fuzzy Evaluation Based on Distance from Average Solution Approach for Evaluating Alternative Aircraft Models from a Sustainability Perspective. *Symmetry*, 16(8), 1086. <https://doi.org/10.3390/sym16081086>

Other Publications:

- Şerbetçioğlu, C., Ari, E., Yetkin Ekren, B., & Özpeynirci, S. (2014). A scheduling optimization study in an assembly line of a company in Turkey. In CIE 2014-44th International Conference on Computers and Industrial Engineering and IMSS 2014-9th International Symposium on Intelligent Manufacturing and Service Systems, Joint International Symposium on "The Social Impacts of Developments in Information, Manufacturing and Service Systems"-Proceedings. Computers and Industrial Engineering.

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