A MATHEURISTIC APPROACH FOR THE LINER SHIP SCHEDULING AND CONTAINER ROUTING PROBLEM WITH TRANSIT TIME SENSITIVE DEMAND

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ABSTRACT

A MATHEURISTIC APPROACH FOR THE LINER SHIP SCHEDULING AND CONTAINER ROUTING PROBLEM WITH TRANSIT TIME SENSITIVE DEMAND

Along with globalization, due to the gradual growth in commercial cargo volumes in maritime transportation, the use of operations research based decision support systems has gained great importance. Employment of proper transportation models incorporating cost and service quality considerations ensures that the right decisions are made at operational, tactical, and strategic levels. In this context, for an available transportation fleet, tactical planning of ship schedules and the routing of containers depending on the operational needs constitute the main problems to be solved in maritime transportation. Simultaneous handling of these problems is of great importance for obtaining high quality solutions based on the revenue, operational costs, and service quality measures. Similarly, transit times of the schedule and customer demand pattern are also important issues, which should be considered within the solution process. The problem under consideration is described as the integrated Liner Ship Scheduling and Container Routing Problem (LSSCRP). Since the problem is of NP-hard complexity, its optimal solution in a reasonable time is not possible in general. In this thesis, a matheuristic solution approach for the integrated problem LSSCRP is developed, where the effect of the transit times on customer demand is taken into consideration. For this purpose, the liner ship scheduling and the container routing with transit time sensitive demand are solved iteratively in a matheuristic framework employing a variable neighborhood search (VNS) algorithm. The performance of the developed LSSCRP matheuristic is tested on a set of different problems.

ÖZET

TAŞIMA SÜRELERİNE DUYARLI TARİFELİ GEMİ ÇİZELGELEME VE KONTEYNER ROTALAMA PROBLEMİ İÇİN MATSEZGİSEL BİR YAKLAŞIM

Küreselleşmenin etkisiyle deniz taşımacılığı sektöründe giderek artan ticari yük taşımacılık hacmi çerçevesinde, gerek operasyonel, gerekse taktik ve stratejik düzeylerde doğru kararlar alınmasını sağlayacak yöneylem araştırması tabanlı karar destek sistemlerinin kullanımı büyük önem kazanmıştır. Bu bağlamda, deniz taşımacılığı alanında belirli bir gemi donanması ile taktik düzeydeki gemi tarifelerinin planlaması ve yük konteynerlarının operasyonel ihtiyaçlar doğrultusunda rotalanması, çözümlenmesine ihtiyaç duyulan temel problemdir. Bu problemlerin eş zamanlı olarak ele alınması, karlılık, operasyonal maliyetler ve servis kalitesi ölçütleri tabanlı kaliteli çözümlerin elde edilmesi açısından büyük bir önem taşımaktadır. Benzer şekilde, gemi çizelgesindeki taşıma süreleri ve müşteri talep yapısı da çözüm sürecinde göz önünde bulundurulması gereken önemli konulardır. Bu bütünleşik problem, tarifeli gemi çizelgeleme ve konteyner rotalama problemi TGÇKRP olarak tanımlanmıştır. NP-zor karmaşıklıkta olan TGÇKRP probleminin genelde makul bir sürede optimal çözümü mümkün değildir. Bu çalışmada, bütünleşik TGÇKRP problemi için, taşıma sürelerinin müşteri talebine etkisini de dikkate alan matsezgisel bir çözüm yaklaşımı geliştirilmiştir. Bu amaçla, taşıma sürelerine duyarlı tarifeli gemi çizelgeleme ve konteyner rotalama problemi matsezgisel bir çerçevede aşamalı olarak değişken komşuluk arama (DKA) algoritması yardımıyla çözülmüştür. Geliştirilen TGÇKRP matsezgiselinin başarımı örnek problemler üzerinde uygulanarak test edilmiştir.

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LIST OF SYMBOLS/ABBREVIATIONS

A	Set of arcs
A^t	Set of transshipment arcs
A_s	Set of arcs associated with services, $s \epsilon S$
A^{si}	Set of arcs in the space time network that represent leg i of service s
c_{mn}	Cost per unit flow on arc $(m, n) \in A$
Cap_s	The capacity of a ship assigned to service $s \in S$
c_s	The weekly cost of a service s
c_ρ	The revenue of one unit of container that sent along path ρ, $\rho \in \Omega_{od}$
$D^{od}(\tau_{u_j v_j})$	Demand for origin destination port pair, $(o, d) \in W$ according to transit time $\tau_{u_j v_j}$.
E	Set of fleet types, $e \in E$
$e(s)$	The vessel class assigned to service $s \in S$
f_{mn}^{uv}	Volume of containers in TEUs/week from node u to node v that flow on arc $(m, n) \in A$
g_{uv}	Net revenue from shipping one TEU on path $(u, v) \in W^{od}$ on the space time network
I_s	Set of port of calls on service s.
N	Set of nodes in the space time network, $m, n \in N$
$n_{e(s)}$	The number of vessels of class $e(s)$, $s \in S$
N_e	Number of vessels of fleet type $e \in E$ available for scheduling
n^{od}	Number of distinct transit times
S	Set of liner services, $s \in S$
$U_{e(s)}$	The capacity of the vessel class $e(s)$
W	Set of origin destination port pairs (o, d) with container shipment, $(o, d) \in W$
W^{od}	Set of container paths (u, v) in space time for the each$(o, d) \in W$, $(u, v) \in W^{od}$
x_ρ	Amount of containers to be sent between $(o, d) \in W$ along the path $\rho \in \Omega_{od}$

q_{od}	Container shipment demand for origin destination port pairs $(o,d) \in W$ (in TEUs) to be shipped
z_s	Binary value 1 if $s \in S$ is selected, or 0 if not seleceted
y^{uv}	The volume of containers in TEUs/week from node u to node v
$\tau_{u_j v_j}$	Transit time of each container path $(u,v) \in W^{od}$
$\hat{T}_{od}^{max}$	Threshold value for the maximum transit time between each origin destination pair.
τ_i^{od}	Transit times of each origin destination pair, $(o,d) \in W$
Ω	Set of feasible paths, $\rho \in \Omega$
$\Omega(a)$	Set of all paths using arc A, $a \in A$
Ω_{od}	Set of feasible paths for demand between each origin destination pair, $(o,d) \in W$ including forfeiting the container, $\rho \in \Omega_{od}$

BGVAR	Varna
CRP	Container Routing Problem
CYLMS	Limassol
DZALG	Algiers
DZORN	Oran
DZSKI	Skikda
EGDAM	Damietta
EGPSD	Port Said
ESAGP	Malaga
ESALG	Algeciras
ESBCN	Barcelona
ESTAR	Tarragona
GEPTI	Poti
HCUS	High capacity utilization services
ILASH	Ashdod
ILHFA	Haifa
ITGIT	Gioia Tauro
ITGOA	Genoa
ITSAL	Salerno

KRP	Konteyner Rotalama Problemi
LBBEY	Beirut
LCUS	Low capacity utilization services
LSSP	Liner Ship Scheduling Problem
LSSCRP	Liner Ship Scheduling and Container Routing Problem
MACAS	Casablanca
MAPTM	Tangier
TNTUN	Tunis
TRAMB	Ambarlı
TRIZM	Izmir
TRMER	Mersin
TGÇP	Tarifeli Gemi Çizelgeleme Problemi
UAODS	Odessa

1. INTRODUCTION

International cargo trade has shown significant growth over the past several decades with the effect of globalization. Maritime cargo transportation has become the dominating international transport sector in this expansion. Although ship transportation has long transit times, it is the least costly alternative for heavy and large cargo. Maritime transportation can be classified into three major types: industrial, tramp, and liner. In industrial shipping, the owner of the cargo employs a transporter trying to minimize the transportation cost. In tramp shipping, there is no fixed route and schedule of the carrier. In liner shipping, however, ships have a fixed schedule, which is decided by the carrier in advance. Among these alternatives, as far as the freight rates are considered, liner shipping is relatively the cheapest one with a higher safety level. Hence liner shipping has become the dominant mode of maritime transportation.

In the competitive liner shipping industry, companies need to improve their operational efficiency by focusing on the operational costs as well as the quality of shipping service levels. Liner shipping consists of three major planning levels: strategic, tactical, and operational [1]. Medium to long term planning decisions for time spans of more than two years are made at the strategic level. At this level, shipping companies make decisions on issues such as alliance strategies, network design, and optimum fleet size.

At the tactical level, medium term decisions are made, where the ship scheduling, fleet deployment, route frequency, and ship speeds are optimized. The key problem of schedule generation by appropriate ship type and speed assignments is referred to as the liner ship scheduling problem (LSSP). At the operational stage, depending on the current demand profile and operational needs, short term decisions are made where the optimum selection of container paths, allocation of containers to the paths and the cargo to be accepted or rejected are considered. This process is referred to as Container Routing Problem (CRP).

It should be noticed that, once the strategic level limits on the network and the fleet size are already set, the tactical level ship scheduling and operational level container routing processes have impacts on each other. Additionally, transshipment operations, transit time, and customer demand are also of great importance in the interaction of tactical and operational levels decisions. In particular, the effect of the transit times on customer demand is one of the most important factors affecting the quality of service since the customers typically prefer the carriers with shorter delivery times. In an independent treatment of LSSP and CRP, the effect of transit times on customer demand is not directly included at the tactical level. This results in a negative impact on the quality of service offered to customers. In this regard, the interaction of the operational and tactical level parameters is of great importance in making optimal decisions, based on the revenue, the operational cost, and the service quality measures as pointed out by Agarwal and Ergun [2]. In this context, cost, and service quality oriented decision support systems necessitate integrated handling of LSSP and CRP to identify better decisions on the scheduling and container routing processes.

In this thesis, integrated handling of the scheduling and the container routing problems is investigated. This integrated problem is described as the Liner Ship Scheduling and Container Routing Problem (LSSCRP). A mathematical model formulation for LSSCRP with a transit time sensitive demand pattern is introduced. Since the sub problem, LSSP is NP-hard as indicated by Karsten et al. [3], the integrated problem LSSCRP model is also of NP-hard complexity. Hence, its optimal solution in a reasonably finite time is not possible in general. For this reason, a matheuristic approach for the solution of liner ship scheduling and container routing problems is developed, where the effect of the transit times on customer demand is taken into consideration. Matheuristic approaches constitute an efficient complement of exact solution approaches in solving difficult optimization problems. They have been successfully employed in many complex optimization problems including scheduling and routing problems in the literature [3,4,6,7]. Matheuristic algorithms essentially involve the incorporation of mathematical programming strategies with appropriate heuristic search routines for an iterative solution. In this context, the liner ship scheduling and the container routing problems with transit time sensitive demand are solved iteratively in a matheuristic framework employing a variable neighborhood search (VNS) algorithm.

1.1. OBJECTIVE OF THE THESIS

In this thesis, an integrated treatment of the liner ship scheduling and the cargo allocation problems is addressed. By simultaneous handling of these problems, we aim to generate a higher quality schedule that enables an improved container routing solution. For this purpose, a transit time sensitive demand pattern is incorporated into the solution of the integrated LSSCRP based on the revenue, operational costs, and service quality measures. In this regard, the main objective of this work is to develop an integrated solution approach for the liner ship scheduling and container routing problems with transit time sensitive demand. Due to the highly nonlinear complex nature of the integrated problem LSSCRP, the focus is laid on the use of matheuristic techniques to obtain an improved solution with the help of proper feedback mechanisms.

1.2. OUTLINE OF THE THESIS

In Chapter 2, a literature review on the liner ship scheduling and container routing problems is presented. In Chapter 3, the integrated LSSCRP is introduced. Here, first, mathematical models of LSSP and CRP are presented. Then, the mathematical model of the integrated LSSCRP model with transit time sensitive demand is introduced. Chapter 4 is devoted to the development of a matheuristic approach for the solution of LSSCRP, where a Variable Neighborhood Search Algorithm (VNS) is employed. In Chapter 5, results of the computational experiments on the LSSCRP matheuristic are presented. Finally, in chapter 6, the outcomes of this thesis are summarized as conclusions.

2. LITERATURE REVIEW

The focus of this thesis is liner shipping, which is one of the dominant modes of global commercial maritime transportation. A systematic overview of the liner shipping problems can be found in the review papers of Christiansen et al. [8], Wang and Meng [9], and Meng et al. [10]. A fundamental classification of maritime liner shipping problems essentially involves critical decision making processes at strategic, tactical, and operational planning levels as discussed by Pesenti [1]. The objective of this thesis is to handle the liner ship problems incorporating the tactical and operational planning levels. We present in the following a brief discussion of the major contributions in the literature, which are closely related to the tactical and operational planning level problems.

Liner problems on a strategic level, involve long term strategic decisions for determining the optimal number and mix of ships in a fleet within a capital investment framework. Major issues and the related works in the literature can be listed as follows: fleet size [11,12,13,14], network design [2,4,10,11,12,13,14,16], and strategic alliance identification [21,22]. In the tactical stage, medium term planning issues associated with the liner ship problem for a time span of three to six months are considered. The major issues of this stage are ship scheduling, fleet deployment and route generation. Some key scheduling items and the related publications in the literature can be listed as follows: determination of route frequencies [23,24], fleet deployment [4,12,19,20,23,25,26,27,28,29,30], speed optimization [29,31,32,33], and creating ship schedules [2,27,34,35,36,37,38]. In the operational planning stage, mainly short term decisions on the container routing problem are considered. Major issues considered at this stage involve the cargo to be accepted (or rejected), the container paths and the associated container allocations [2,4,19,27,31,37,38,39,40,41,42,43], and operational modification of the schedule to overcome the emerging current needs, such as the problems in port traffic and weather conditions [44].

As indicated above, the studies on liner shipping considered in general cost minimization, demand fulfillment and service quality issues for the solution of schedule and container routing problems. Most of the works in the literature solely focus on problems involving a single decision level, i.e., strategic, tactical or operational level. In this orientation, the majority of the works consider developing solution methodologies to the tactical level liner ship scheduling problem (LSSP) and the operational level container routing problem (CRP) separately [19,29,30,45,46,47]. There exist also a limited number of works attempting to combine tactical and operational decision processes by solving LSSP and CRP simultaneously to improve the balance between operational cost and the service quality offered to the customers [2,3,5,27,38,48,49,50]. In this work, we focus on the development of an integrated model for liner ship scheduling and container routing problem which is referred to as LSSCRP. Major contributions to the problems LSSP, CRP and LSSCRP are summarized in the following.

In the LSSP and CRP, major critical factors of interest in the literature are observed to consist of cost, revenue, demand profile, transit time, transport frequency, transshipment, and port coverage [9]. In most of the works, while cost, transshipment and a constant demand pattern is addressed [2,3,5,27,29,38,48,49,50], the transit time is not taken into consideration at all, [3,9,27]. In the literature survey of Wang and Meng [27], the importance of the relationship between transit time and demand is identified to be the most critical factor affecting the quality of service. This relationship is defined as the transit time sensitive demand, which indicates the inversely proportional relationship between the customer demand and the transit time of the service [34]. In Wang et al. [30], a container ship schedule development approach is introduced to solve LSSP employing transit time sensitive demand function. A major assumption in the demand and transit time relationship is the customers' preference of the shortest transportation alternatives in general, as indicated by Wang et al. [42]. In this thesis, the transit times are incorporated into solution methodology by relating the demand to the service quality, hence to the transit times. In this way, by balancing the compromise between the operational cost and service quality, a high quality schedule is aimed to be generated. Zhang et al. [45] attempt to solve LSSP by proposing an approximation approach for vessel fleet deployment problems in liner shipping to control shipment demand overflow. A recent work, Trivella et al. [46], introduces a modeling approach for container routing as a multicommodity network flow problem. Here, transit time restrictions are modeled in the form of relaxed constraints, where transportation delays are penalized using penalty functions of transit time, while early arrivals are rewarded by a discount in the cost. An additional critical factor to be considered in the liner ship problems involves the container transfer between the container routes. The major works in this orientation involve the realization of transshipments at more than one ports of the routes [3,29,30].

As far as the simultaneous treatment of LSSP and CRP, one of the early works is proposed by Agarwal and Ergun [2]. They handle the ship scheduling and cargo routing simultaneously, considering transshipment operations and nonrestrictive transit times in liner shipping assuming constant demand. Wang et al. [47] solve LSSP using transit time sensitive demand. Similarly, Wang et al. [30] incorporate the transit time sensitive demand to solve CRP. Karsten et al. [3] propose a solution method for LSSCRP where they use fixed demand while employing a threshold transit time in the container routing model. Koza [48] suggests a model for combined tactical service scheduling problems and operational container allocation problems. The developed solution method for LSSCRP is employed for constant demand cases, considering the handling of cargo subject to transit time limits. A recent work by Ozcan et al. [5] presents a mixed integer linear programming model for the operational level cargo allocation and vessel scheduling problem. Their model includes flow dependent port stay lengths, transit times, and transshipment synchronizations. Koza et al. [55] introduce an integrated solution approach for liner network design and scheduling problem by incorporating a column generation matheuristic with linear programming techniques. Their work takes transit times, transshipment and constant demand into consideration. Another recent contribution by Jiang et al. [49] uses a simultaneous optimization approach for the liner schedule and route designs at the tactical level using a constant demand profile without any transshipment consideration. A traffic based time window assignment is utilized for determining the ports, the port of call sequence, ship arrival times, and sailing speeds.

As indicated in the above summary, some studies in the literature address an integrated treatment of LSSP and CRP, focusing on different aspects of liner ship scheduling and container routing [2,27,38,43,48,50]. In these works, it is recognized that one of the most important factors in the improvement of service quality is the incorporation of the customer demand profile into the model properly. This requires the incorporation of the transit times and the demand pattern into the solution method. In a recent work Kavas [51], an iterative sequential solution approach for LSSP and CRP models is developed. In this approach, transit times are taken into consideration in the ship scheduling stage, while a transit time sensitive demand pattern is incorporated into the container routing model. Following the solution of the LSSP mathematical model, the associated transit time sensitive CRP mathematical model solution is obtained. Then, based on the feedback obtained from the

CRP solution, the LSSP solution is updated to improve the scheduling solution. In this way, a gradual improvement of service quality is aimed to be achieved by employing a manual iterative approach.

Our focus in this thesis is the development of a model and solution methodology for the integrated LSSCRP. In the thesis, we introduce a systematically automated matheuristic algorithm for solving the integrated LSSCRP, which inherently includes transit time and transshipments along with transit time sensitive demand. To indicate the relative focus of this thesis within related literature, a comparative list of the major literature works is given in Table 2.1. In the table, the contributions of the works on LSSP, CRP and LSSCRP are compared according to the critical scheduling and routing factors, which are demand profile, transit time and transshipment. As seen in the table, this thesis involves the integrated solution of LSSCRP along with the considerations on transit time, transshipment as well as transit time demand features. A similar focus is observed in the work of Kavas [51], where LSSP and CRP stages are sequentially iterated based on observations on meeting the demands at CRP solution. In this way, LSSP solution is iterated with an increased or modified service set to reach a better solution. This iterative manual two stage approach depends on service modification decisions in discrete steps and does not offer a systematic modification scheme. Hence the approach does not ensure an effective integration of LSSP and CRP. However, in this thesis, we develop an integrated LSSCRP mathematical model to obtain the optimum solution to the problem. Furthermore, instead of a manual feedback iteration, a matheuristic algorithm for the solution of LSSCRP is developed. The two-stage approach is utilized to initialize the LSSCRP matheuristic designed in this thesis.

Table 2.1. Major works on liner shipping problems

	Problem Classification			**Some Major Operational Factors**			
	LSSP	**CRP**	**LSSCRP**	**Constant Demand**	**Transit Time Sensitive Demand**	**Transit Time**	**Trans-shipment**
Agarwal and Ergun [2]			●	●			●
Wang and Meng [27]			●	●		●	●
Wang et al. [47]	●				●	●	●
Brouer at al. [19]	●			●		●	●
Mulder and Dekker [38]			●	●			●
Wang et al. [50]			●	●		●	
Akyüz and Lee [29]		●		●		●	●
Wang et al. [30]		●			●	●	●
Karsten et al. [3]			●	●		●	●
Koza [48]			●	●		●	●
Özcan et al. [5]			●	●		●	●
Zhang et al. [45]	●			●		●	
Jiang et al. [49]	●			●		●	
Koza [55]			●	●		●	●
Trivella et al. [46]		●		●		●	●
Kavas [51]			●		●	●	●
This Thesis			●		●	●	●

3. THE LINER SHIP SCHEDULING AND CONTAINER ROUTING PROBLEM

The objective of this thesis is to develop a solution method for the Liner Ship Scheduling and Container Routing Problem (LSSCRP) with transit time sensitive demand. LSSCRP requires an integrated solution of the Liner Ship Scheduling Problem (LSSP) and the Container Routing Problem (CRP). In this chapter, first, these sub problems are introduced, then the mathematical model for the integrated LSSCRP is presented.

3.1. LINER SHIP SCHEDULING PROBLEM

Liner ship schedules are planned to cover a time span of 3 to 6 months by the liner companies. Services associated with the announced schedule are carried out without major changes during this time span. In the schedule announced by the liner company, each service is identified with the following attributes:

- The type and capacity of the sailing vessel
- The sequence of ports visited by the vessel
- The arrival and departure times at the ports

Liner ship scheduling is a tactical level problem involving the design of ship routes to be operated for a medium term time span. In general, ship routes are designed as simple routes (each port is visited once) or butterfly routes (one or more ports are visited twice). Butterfly routes allow shorter transportation times if transshipments are employed at the revisited ports [20]. Ship schedules are constructed in periodic form, such that each port is visited in regular time intervals. In general, the frequency of port visits is set as once a week for operational efficiency. Since ship routes may take in general more than one week, the visit frequency is assured by operating more than one vessel over each route. For example, to ensure a weekly visit of the ports in a route of three weeks, three vessels should start sailing from different ports, which are one week distance apart from each other. In the schedule planning, vessels are generally assumed to be operated at an optimum speed providing minimum fuel consumption. On the other hand, due to various cost and efficiency based reasons, it might be necessary to operate a vessel at a lower/higher speed than its optimal speed. For example, in the case of routes taking a duration of 2.8 weeks with three vessels, a slower speed selection would be appropriate to match the weekly cyclic schedule. Also, in case of load transfers between the vessels of two different routes at some ports, arrival, and departure times of the vessels to the transshipment port are required to be coordinated by appropriate speed adjustments. For these reasons, vessel speeds should also be determined within the ship scheduling problem in addition to the number of a particular vessel type.

The liner schedules are constructed to satisfy the transportation demand between the ports, which are served by the liner company. In the schedule planning, the amount of the demand is mainly estimated based on the historical data available for the service area in the previous years. At this stage, a directional demand pattern between all the port pairs within the service network is required to be estimated. However, the planning goal is generally set as the revenue maximization over the transported load without forcing the fulfillment of the total demand. The net revenue for a port pair is computed by taking the difference of the distance based unit profit per container and the operational cost, which includes fixed and variable vessel costs, fuel, port usage, loading, unloading and transshipment costs. In liner ship scheduling studies, demand is presumed to be constant in general, and the sensitivity of demand on transportation time is ignored. However, the transportation time offered to the customer for an origin destination port pair may differ in different company schedules. The transit time may even be different depending on the ship routes to be used on the same schedule. Schedules with long transit times obviously affect the liner company choice of the customers. For this reason, liner companies employing constant demand assumption in the construction of their schedules are prone to lose customers for competitive routes. The approach developed in this work focuses on the employment of a transit time sensitive demand pattern to balance out the operational cost and customer service quality for improving the schedule efficiency of the liner companies.

Based on the above mentioned criteria, we use the space time network representation of Wang et al. [30] to model the ship routes and schedule for a given service area. An example space time representation for a weekly cyclic schedule with three services is shown in Figure 3.1. In the space time network representation, the space axis (y-axis) corresponds to the ports of call, while the time axis (x-axis) represents the arrival time of vessels at each port of call. In the network, the nodes indicate port visits of the vessels of a service at a particular time instant, while the arcs between the nodes represent the flow of containers or transshipments.

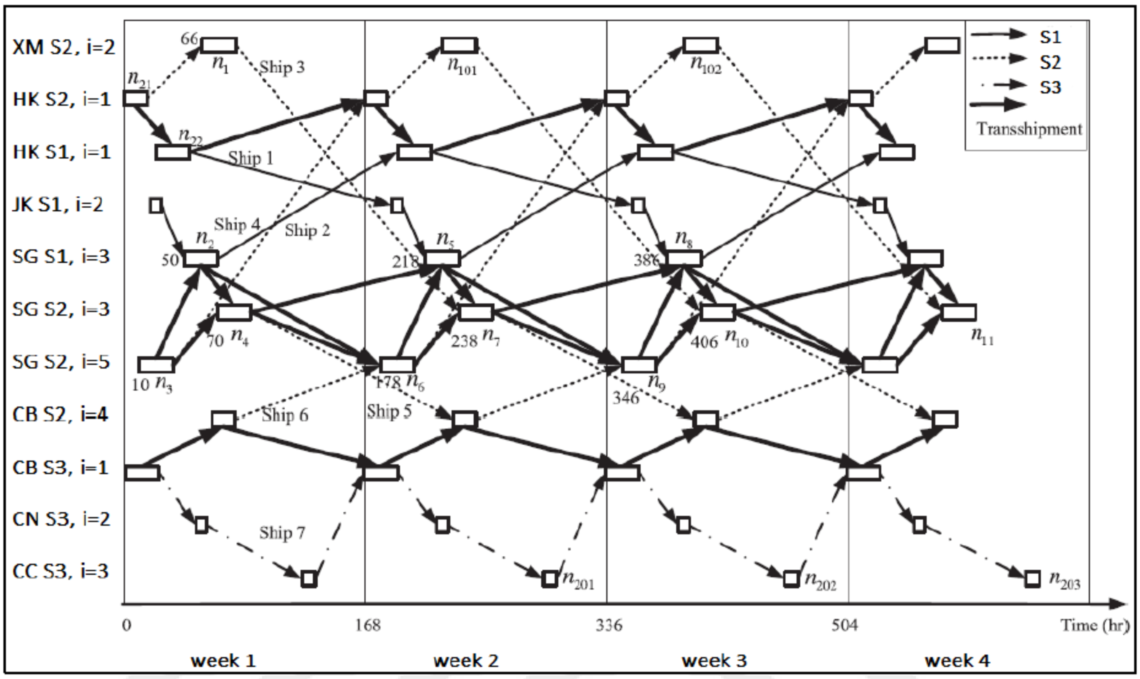

Figure 3.1. Space time network representation for three services [30]

Once the space time network is obtained, possible container transportation routes for any source destination port pair are generated. During the solution process, first, a limited number of candidate ship services are determined, and a subset of services is selected to maximize the net revenue. As indicated before, a shipping service consists of three major components, which are route, vessel type and sailing speed. These components also identify the maximum load transportation capacity of the service, and the arrival departure times to the ports. For a ship route, many alternative services can be generated depending on the type and sailing speed of the vessels assigned to that route. In this context, LSSP integer linear model presented in Karsten et al. [3] is solved to determine the services, which maximize the net revenue. Note that in the formulation of LSSP a fixed container shipment demand is assumed for all origin destination port pairs independent of the transit times. The mathematical model of LSSP is given below.

Mathematical Model of LSSP

Sets:

E: set of fleet types, $e \in E$

S: set of liner services, $s \in S$

W: set of origin destination port pairs (o, d) with container shipment, $(o, d) \in W$

A: set of arcs in the space-time network, $a \in A$

Ω : set of feasible paths, $\rho \in \Omega$

Ω_{od}: Set of feasible paths for demand between each origin destination pair, $(o, d) \in W$ including forfeiting the container, $\rho \in \Omega_{od}$

$\Omega(a)$: set of all paths using arc $a \in A$

A_s: set of arcs on service $s \in S$ in the space time network

Parameters:

$e(s)$: the vessel class assigned to service $s \in S$

$U_{e(s)}$: the capacity of the vessel class $e(s)$

$n_{e(s)}$: the number of vessels of class $e(s)$, $s \in S$

N_e : number of vessels of fleet type $e \in E$ available for scheduling

c_s: the weekly cost of a service $s \in S$ (total weekly cost of operating service $s \in S$ including the fixed cost of deploying a vessel of type $e(s)$, fuel cost at the operating speed of service s, and the port call costs for all ports visited by service s)

c_ρ : the revenue of one unit of container that sent along path ρ, $\rho \in \Omega_{od}$

q_{od} : container shipment demand for origin destination port pairs $(o, d) \in W$ (in TEUs) to be shipped

Decision variables:

z_s: binary value 1 if $s \in S$ is selected, or 0 if not seleceted

x_ρ: amount of containers to be sent between $(o, d) \in W$ along the path $\rho \in \Omega_{od}$

$$\text{Max} \quad \sum_{(o,d)\in W} \sum_{\rho\in\Omega_{od}} c_\rho x_\rho - \sum_{s\in S} c_s z_s \tag{3.1}$$

$$\text{s.t.} \quad \sum_{\rho\in\Omega_{od}} x_\rho = q_{od} \quad , \ \forall (o, d) \in W \tag{3.2}$$

$$\sum_{\rho\in\Omega(a)} x_\rho - U_{e(s)} z_s \leq 0 \qquad , \forall s \in S, \forall (m,n) \in A_s \qquad (3.3)$$

$$\sum_{s\in S:e(s)=e} n_{e(s)} z_s \leq N_e \qquad , \forall e \in E \qquad (3.4)$$

$$x_\rho \in R^+ \;\; \rho \in \Omega_{od}, \qquad , \forall (o,d) \in W \qquad (3.5)$$

$$z_s \in \{0,1\} \qquad , \forall s \in S \qquad (3.6)$$

The objective Function (3.1) maximizes the net profit of the liner shipping company calculated by subtracting the cost of operating the selected services. Constraints (3.2) ensure that all shipment demand is either met by using available container paths or forfeited. Note that this constraint also assumes a fixed shipment demand for each origin destination port pair (q_{od}) regardless of the transit time. Constraints (3.3) limit the number of containers shipped on each voyage arc of a service by the capacity of the vessel assigned to that service. Constraints (3.4) ensure that the number of vessels of each type assigned to selected services does not exceed the number of vessels available for that fleet type. Finally, the last two constraints are the domain restrictions for the decision variables. Constraints (3.5) check the non-negativity of x_ρ decision variable which indicates the number of TEUs of demand for origin destination port pair $(o,d) \in W$ shipped along container path $\rho \in \Omega_{od}$ in the space time network. Constraints (3.6) set the binary service selection variables. Binary variable z_s takes the value 1 if service $s \in S$ is selected and 0 otherwise.

3.2. CONTAINER ROUTING PROBLEM

After the construction of the liner schedule, container routing problem CRP is solved to meet the container transport demands between origin-destination port-pairs using the services offered in the schedule. In this process, container routes and the amount of the containers to be transported for each origin destination port pair are determined. Some containers can be transported directly from an origin port to a destination port via a single ship service, while some others are required to be transported to their destinations by employing more than one transshipment between the scheduled services. It is also probable that different loads related with the same source destination pair are transported through different container routes. Transit times of the loads are determined by adding the loading, unloading, transshipment and waiting times to the sailing times of carrier services along the container routes followed. Over the same schedule, there exist different transit times for each container route through which the loads related to the same origin destination port pair can be transported. In this work, sensitivity of the customer demand to the transit times is taken into consideration [30]. All container routes related with an origin destination port pair and the transit times of these routes are considered and a demand pattern, which is inversely related to the transit times of the routes is employed. Before introducing the mathematical model of CRP, the transit time sensitive demand structure proposed by Wang et al. [30] is briefly discussed below.

In the CRP solution, container paths are generated by using a set of liner ship services over the available space time network. First, let us define each transport demand, as a load to be transported from an origin port (o) to a destination port (d). Over an existing liner schedule, there might exist n^{od} number of different transit time path alternatives for each origin destination port pair (od). The transit times τ_i^{od} associated with a port pair can be sorted as $\tau_1^{od} < \tau_2^{od} < \cdots < \tau_{n^{od}}^{od} \leq \hat{T}_{od}^{max}$, where $\hat{T}_{od}^{max}$ identifies the maximum transit time. Let, $y_i^{od}$and τ_i^{od} denote the amount of the load transported between the port pair (od) and the associated transportation time for a given path alternative i respectively. It has been shown by Wang et al. [30] that, the following condition should be satisfied for a feasible container assignment.

$$\sum_{j=i}^{n^{od}} y_j^{od} \leq D^{od}\left(\tau_i^{od}\right) \tag{3.7}$$

Constraints (3.7) defines the relationship between the transit time sensitive demand and the transported demand. The left side of the inequality indicates the total amount of transport demand, which should lie below the total demand $D^{od}(x)$ as represented by the demand function shown in Figure 3.1. In this pattern, the demand is inversely related to the transit time.

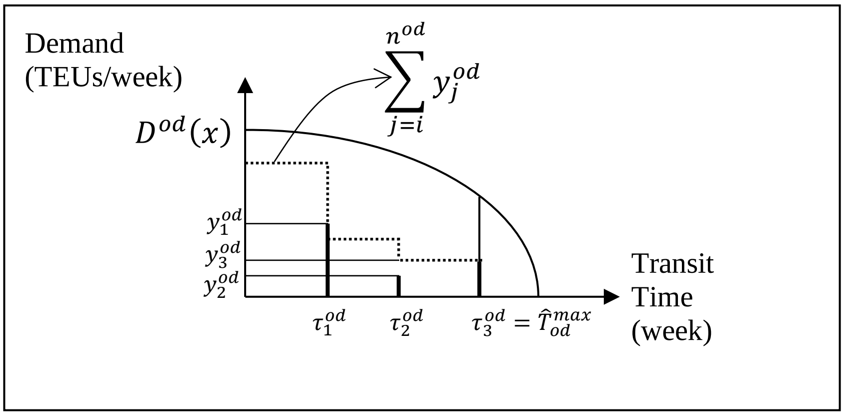

Figure 3.2. Transit time sensitive demand pattern [30]

In CRP model, first, a space time network is constructed for the ship services in the liner schedule considering any possible transshipments. In this network, ports, and the visit times of the ports for each ship route are displayed by nodes, and the voyage paths and transshipments between the vessels are shown by arcs. Over the constructed space time network, for each transport demand indicated by (o, d) origin destination port pair, all possible feasible paths and their transit times are computed. In this way, over the network, all direct or transshipment based alternative container paths W^{od} are determined. The mathematical model given below solves the container routing problem employing the transit time sensitive demand structure given in Figure 3.1.

Mathematical Model of CRP

Sets:

W^{od}: set of container paths (u, v) in space time for the each$(o, d) \in W$, $(u, v) \in W^{od}$

A^t : set of transshipment arcs in the space time network

A^{si} : set of arcs for the i^{th} port of call of service $s \in S$ in the space time network

A: set of arcs in the space time network, $(m, n) \in A$

I_s : set of port of calls on service s.

N: set of nodes in the space time network, $m, n \in N$

Parameters:

g_{uv} : net revenue from shipping one TEU on path $(u, v) \in W^{od}$ on the space time network

c_{mn}: cost per unit flow on arc $(m, n) \in A$

Cap_s: the capacity of a ship in service $s \in S$

$\tau_{u_j v_j}$: transit time of each container path $(u, v) \in W^{od}$

$D^{od}(\tau_{u_j v_j})$: demand for origin destination pair, $(o, d) \in W$ according to transit time $\tau_{u_j v_j}$

Decision variables:

y^{uv}: volume of containers in TEUs/week from node u to node v

f_{mn}^{uv}: volume of containers in TEUs/week from node u to node v that flow on arc $(m, n) \in A$

$$\text{Max} \quad \sum_{(o,d)\in W} \sum_{(u,v)\in W^{od}} g_{uv} y^{uv} - \sum_{(m,n)\in A^t} c_{mn} \sum_{(o,d)\in W} \sum_{(u,v)\in W^{od}} f_{mn}^{uv} \tag{3.8}$$

$$\text{s.t.} \quad \sum_{n:(m,n)\in A} f_{mn}^{uv} - \sum_{n:(n,m)\in A} f_{nm}^{uv} = \begin{cases} y_{uv}, m = u \\ -y_{uv}, m = v, \\ 0, otherwise \end{cases} \quad , \forall m \in N, \forall (u, v) \in W^{od}, \forall (o, d) \in W \tag{3.9}$$

$$\sum_{(m,n)\in A^{si}} \sum_{(o,d)\in W} f_{mn}^{uv} \leq Cap_s \quad , \forall s \in S, \forall i \in I_s \tag{3.10}$$

$$\sum_{i=j}^{|W^{od}|} y^{u_i v_i} \leq D^{od}\left(\tau_{u_j v_j}\right) \quad , \forall j = 1,2 \ldots |W^{od}|, \forall (o, d) \in W \tag{3.11}$$

$$f_{mn}^{uv} \geq 0 \quad , \forall (m, n) \in A, \forall (o, d) \in W, \forall (u, v) \in W^{od} \tag{3.12}$$

$$y^{uv} \geq 0 \quad , \forall (o, d) \in W, \forall (u, v) \in W^{od} \tag{3.13}$$

The objective function (3.8) maximizes weekly net revenue to be obtained from the container shipments excluding the transshipment costs. The first term in the objective function represents the total revenue, while the second term represents the total transshipment cost. Here, the revenue parameter g_{uv} defines the net revenue of transporting one unit of the container between an origin destination port pair excluding the loading $(\hat{c}_u)$ and discharge costs $(\tilde{c}_v)$ as,

$$g_{uv} = g'_{uv} - \hat{c}_u - \tilde{c}_v \tag{3.14}$$

Constraints (3.9) describe the flow conservation at the nodes in the network. Equations in (3.10) are the service capacity constraints preventing the vessel capacities to be exceeded. Constraints (3.11) define the container amount to be shipped over paths depending on their transit time. In other words, a transit time sensitive demand structure is imposed by Constraints (3.11). Constraints (3.12) and (3.13) control the non-negativity of the decision variables.

3.3. MATHEMATICAL MODEL OF THE INTEGRATED LINER SHIP SCHEDULING AND CONTAINER ROUTING PROBLEM

An integrated mathematical model to solve LSSP and CRP problems is developed. The integrated model LSSCRP combines ship scheduling and container routing to incorporate the transit time sensitive demand structure to the tactical planning stage. This leads to higher quality liner schedules, which improve the profitability and service quality of the liner company. In the following, the mathematical model of integrated LSSCRP is defined.

Mathematical Model of LSSCRP

$$\text{Max} \quad \sum_{(o,d)\in W^{od}} \sum_{(u,v)\in W^{od}} g_{uv} y^{uv} - \sum_{s\in S} c_s z_s \tag{3.15}$$

$$\text{s.t.} \quad \sum_{n:(m,n)\in A} f_{mn}^{uv} - \sum_{n:(n,m)\in A} f_{nm}^{uv} = \begin{cases} y_{uv}, m = u \\ -y_{uv}, m = v, \\ 0, otherwise \end{cases} \quad , \forall m \in N, \forall (o,d) \in W, \forall (u,v) \in W^{od} \tag{3.16}$$

$$\sum_{(m,n)\in A^{si}} \sum_{(o,d)\in W} f_{mn}^{uv} \leq U_{e(s)} z_s \quad , \forall s \in S, \forall i \in I_s \tag{3.17}$$

$$\sum_{i=j}^{|W^{od}|} y^{u_i v_i} \leq D^{od}\left(\tau_{u_j v_j}\right) \quad , j = 1,2 \ldots |W^{od}|, \forall (o,d) \in W \tag{3.18}$$

$$\sum_{s \in S: e(s) = e} n_{e(s)} z_s \leq N_e \quad , e \in E \tag{3.19}$$

$$f_{mn}^{uv} \geq 0 \quad , \forall (m,n) \in A, \forall (o,d) \in W, \forall (u,v) \in W^{od} \tag{3.20}$$

$$y^{uv} \geq 0 \quad , \forall (o,d) \in W, \forall (u,v) \in W^{od} \tag{3.21}$$

$$z_s \in \{0,1\} \quad , \forall s \in S \tag{3.22}$$

The objective function (3.15) maximizes the total net revenue from the container shipments minus the cost of operating the selected services, including fixed costs, fuel costs and mooring fees. Constraint (3.16) and (3.18) are the same flow conservation and transit time sensitive demand constraints as in CRP. Constraint (3.17) and (3.19) impose vessel capacity and fleet size restrictions on the services selected for operation as in the LSSP model. Finally, Equations (3.20)-(3.22) give the domain restrictions on the decision variables. The LSSCRP model is similar to CRP in that it also accounts for the transit time sensitive nature of demand. However, unlike the CRP model, it also includes the scheduling decisions made by the LSSP model. Therefore, this model has the advantage of considering the transit time sensitive demand when it selects the services to be operated by the liner company.

4. A MATHEURISTIC APPROACH FOR THE SOLUTION OF LSSCRP

The integrated problem LSSCRP is of NP-hard complexity and its optimal solution is in general not possible within a reasonable time for large scale scheduling problems. In this chapter, we introduce a matheuristic approach for solving the integrated LSSCRP problem in a reasonable time. The proposed approach involves the incorporation of mathematical programming strategies and a metaheuristic algorithm for the iterative improvement of the scheduling and container routing solution.

4.1. LSSCRP MATHEURISTIC

The iterative solution methodology developed for the integrated scheduling and cargo routing problem is represented in Figure 4.1.

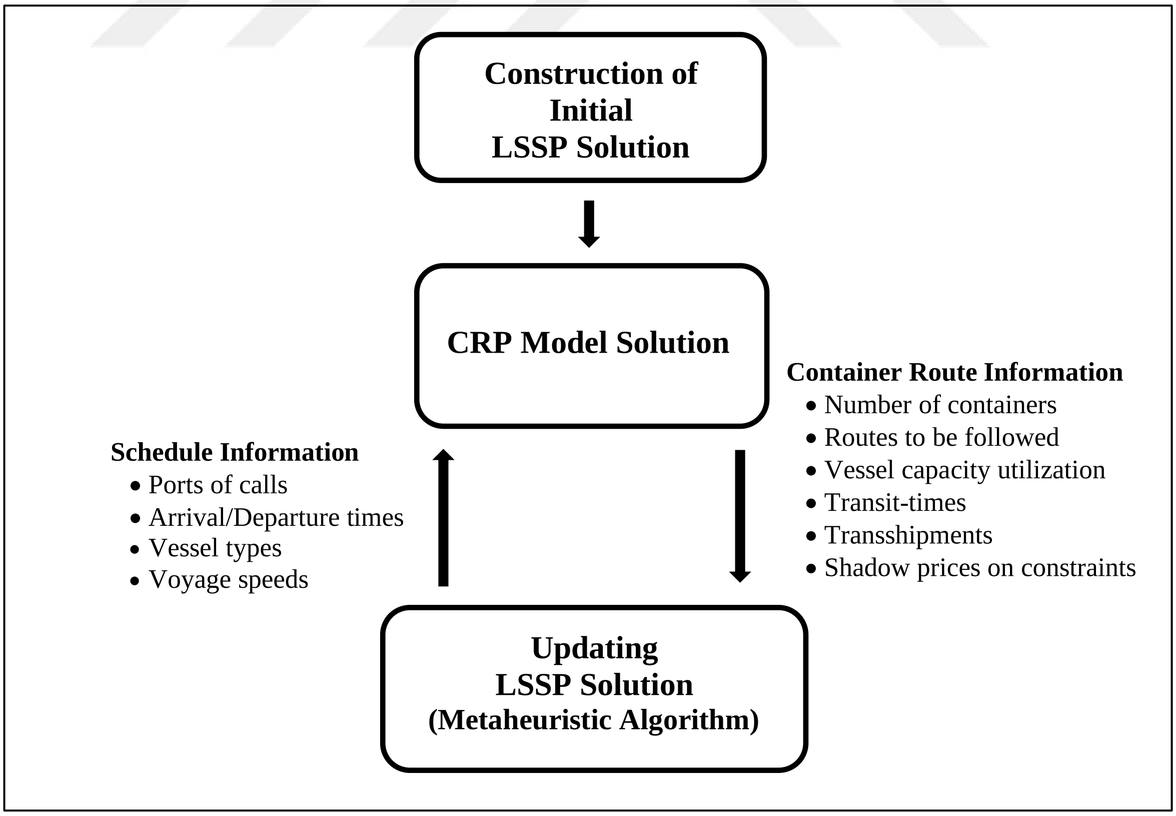

Figure 4.1. A matheuristic framework for the solution of LSSCRP

The developed matheuristic algorithm consists of three major steps. In the first step, initial schedule information for LSSP is constructed. In the second step, the routes of the containers to be transported are determined using the available schedule. These routes are obtained by solving the CRP mathematical model based on the schedule generated by LSSP. As demonstrated by Wang et al. in [30], CRP is of polynomial complexity, and it allows realistic size problems to be solved in a reasonable time. For this reason, the container routing step is solved to optimality by linear programming at each iteration of the matheuristic. The results obtained are then transferred to the scheduling metaheuristic in the third step for updating the current schedule. Since CRP is solved to optimality, the information on the amount of accepted/rejected transport demand, capacity utilization of ships and transit time data as well as the shadow prices associated with CRP constraints are all taken into consideration in updating the ship schedule. Based on these feedbacks, the LSSP metaheuristic is run in the third step to update the current ship schedule in an iterative manner. Since the schedule is updated in line with CRP results, the service quality is taken into account at a tactical level. The three steps of the matheuristic are detailed in the following.

Step 1. Construction of Initial LSSP Solution

In this step, an initial solution for LSSP is constructed. For this purpose, first, a limited number of candidate ship services are determined. Among these services, a service set that maximizes the net revenue is selected by the LSSP model. As introduced before, shipping service is characterized by three attributes: route, vessel type and vessel speed. These components also define the maximum load capacity and the arrival/departure times of the vessels to the ports. For a particular ship route, depending on the vessel types employed and the possible voyage speeds, many alternative services can be generated. To construct the LSSP initial solution, first a randomly selected candidate service set is obtained. Then, the linear mathematical LSSP model presented in chapter 3.1 is solved to identify the services maximizing the net revenue. As indicated by Karsten et al [3], the LSSP model is NP-hard, and it is not possible to come up with a feasible solution if a high number of service alternatives are involved. Therefore, in the construction of the initial solution for the matheuristic, the LSSP model is solved for a limited number of routes along with a limited number of services for each route. Additionally, although alternative paths are considered for realizing transport demands, a constant demand pattern is presumed. In the LSSP model, transport demand is assumed to be constant up to a maximum transit time threshold. Demand is assumed to be zero if this transit time limit is exceeded. As a result, initial schedule information is constructed based on a constant demand structure.

Step 2. CRP Model Solution

Once the initial solution is constructed, the second step involves the solution of the container routing problem based on the initial ship schedule. The LSSP model used in the construction of the initial solution also generates the container paths under a constant transportation demand assumption. However, as discussed in Chapter 3.2, the CRP model employs a transit time sensitive demand structure. As it has been indicated by Wang et al. [30], the CRP mathematical model denoted by the Equations (3.8)-(3.13) in Chapter 3.2 can be solved in finite polynomial time. Therefore, the CRP model is solved to optimality at the second step of the matheuristic and then the results of the model solution are utilized in the next step for updating the shipping schedule. For this purpose, the container route information obtained from the CRP solution is transferred to the metaheuristic for updating LSSP solution. These data consist of the number of transported containers, the routes and transit times used to transport these containers, the vessel capacity utilization, the number of transshipments and the shadow prices on the constraints.

Step 3. Updating LSSP Solution

In this step, based on the container route information obtained from the CRP solution of the previous step, the available ship schedule is updated by making use of a Variable Neighborhood Search (VNS) metaheuristic. VNS is one of the efficient search algorithms, and it has been successfully employed in scheduling and routing problems in the literature [52,53,54]. In the search for an improved schedule, an iterative search of the solution space of LSSP is realized based on the current container route information obtained from the CRP solution. For this purpose, the VNS algorithm is adopted to the LSSCRP matheuristic. A detailed discussion on the developed VNS heuristic is presented in the next section.

4.2. VARIABLE NEIGHBORHOOD SEARCH FOR LSSCRP

Variable neighborhood search (VNS) algorithm introduced by Hansen and Mladenovic [52] is an efficient heuristic for solving difficult optimization problems. The basic idea of VNS is to systematically explore the solution space using a set of pre-determined target neighborhoods. The basic steps of VNS can be presented as follows:

Basic VNS Scheme:

i. Initialization:

- Define the neighborhood set $\{N_k, k = 1,2, \dots, k_{max}\}$
- Set iteration index $k \leftarrow 1$
- Generate an initial solution x.
- Define a termination condition and repeat the following steps until the termination condition is satisfied, (t_{max}).

ii. Shaking:

- Randomly generate a solution $x' \in N_k(x)$

iii. Local Search:

- Perform a local search by starting with x' to find a local optimum x''.
- Move or Not:
  If the solution x″ is better than x, then $x \leftarrow x''$ and $k \leftarrow 1$,
  If not, then for $k < k_{max}$, set $k \leftarrow k + 1$, otherwise for $k = k_{max}$, set $k \leftarrow 1$ and go back to shaking (step ii).

In this thesis, the principles of the basic VNS scheme are adopted to update the LSSP solution in the last step of the LSSCRP matheuristic. In this process, a feedback mechanism is also developed to incorporate the information obtained from the solution of the CRP mathematical model into the VNS scheme. The flow chart of the newly developed VNS algorithm is given in Figure 4.2, and the adaptation of its steps to the LSSCRP matheuristic is discussed in the following part in detail.

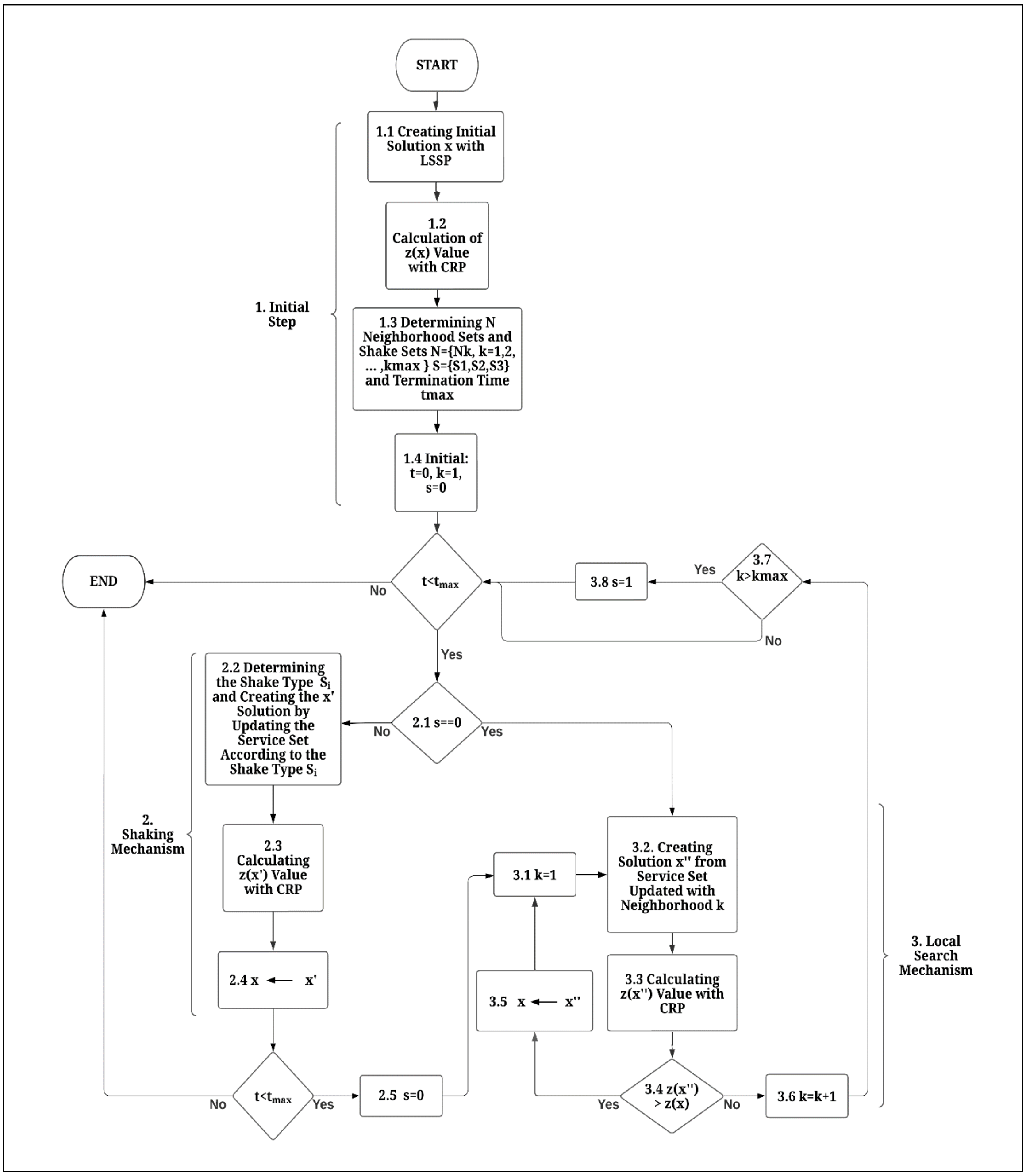

Figure 4.2. VNS flow chart for LSSCRP

i. <u>Initialization</u>

In this step, the neighborhood set N is defined and an initial solution x for starting the VNS algorithm is determined. In the VNS algorithm, the neighborhood set , consists of vessel type change, vessel speed change, port deletion and port insertion neighborhoods. The initial solution x is generated by solving the LSSP model with a limited number of service alternatives and then by running the CRP model with a selected set of services from the LSSP solution. The objective function value of the initial solution is hence calculated by using the selected services of the LSSP solution and the resulting container routes of the CRP solution.

ii. <u>Shaking</u>

The purpose of the shaking step is to escape from a current local minima by employing random changes on the solution obtained from the neighborhoods in the set $\boldsymbol{N}$. In other words, the aim is to generate a good starting point for a new local search process. The key point in shaking is to be able to create a solution point, x', which is sufficiently different than the local search solutions. In the shaking mechanism developed, the shaking type to be used at each shaking step is chosen randomly. In this work, three shaking types are developed, and they are employed with 50 percent probability as type 1 while with 25 percent probability each as type 2 and type 3. A detailed description of the developed shaking types is presented below.

Shaking Type 1:

Shaking Type 1 involves modification of the current service set by increasing the number of services in order to construct a new candidate service set for creating a new solution. The shaking is processed based on the following scheme:

- Half of the services from the current solution, x, are randomly kept in the candidate service set. Then, three times the selected service number is further picked up randomly from the pool of all services created within the algorithm so far and they are added to the candidate service set. In determining the services to be added, the routes of the selected services are ensured to be different than the routes of the services selected from the current solution, x.

- Using the candidate service set, the LSSP model is solved and a new solution, x', is obtained. Then, the CRP model is solved and the value of the objective function $z(x')$ is computed. The new solution x' obtained as a result of the shaking step is updated as the current incumbent solution x.

Shaking Type 2:

This shaking type involves the splitting of an existing service into two new services. Among the services of the current solution x, the service with the maximum number of the port of calls is chosen to be split into two services. Procedural details for this shaking type are presented as follows:

- Firstly, a port from the selected service is randomly chosen. Starting from this port, the first new route is constructed by adding a random number of ports to the route they are visited according to the present route. In order to have at least two ports for the second route, the number of randomly selected ports can be at most two less than the total number of ports in the route. The second route also starts with the initially selected port, which is followed by the remaining ports sequenced in the original order. Since the initially selected port takes place in both of the routes, as a result of Type 2 shaking, two connected routes are generated.
- If both of the generated routes are absent in the route set of the current solution, two new services are created from these routes without exceeding the available vessel number in the fleet. In the new services, the vessel type of the split route is used, and the vessel speed is set close to the optimal speed of the vessel used. The service whose route is split is removed and the two newly generated services are added to the set of services.
- By using the new set of services, a new solution, x', is obtained. Then, the CRP model is solved and the value of the objective function $z(x')$ is computed. The new solution x' is obtained as a result of the shaking step is updated as the current solution x.
- If at least one of the two generated routes belongs to the routes of the services in the current solution x, or if a feasible service cannot be constructed for any one of the generated routes, all the procedural steps of splitting is repeated with a randomly selected different starting port, until two different routes, both leading to feasible services are generated.

- After five trials, if a feasible service cannot be constructed from one of the routes, the type 2 shaking process is terminated without changing the current solution x. In this case, either type 1 or type 3 shaking is activated with a 50 percent probability.

***Shaking Type 3*:**

This shaking type involves merging two existing services. For this purpose, a set of services with the minimum number of port of calls are identified in the current solution. Routes of two randomly chosen services in this service set are then merged. The merging procedure consists of the following steps:

- To merge routes of the two selected services, a port is selected from each route. This pair is chosen as the one having the shortest distance between its ports among all possible port pair alternatives.
- In the new route to be constructed by merging, ports of the selected pair are visited sequentially. If the resulting route does not exist in the route set of the current solution, this route is added to the route set. Then it is used to construct new services for each vessel type. In this process, the speed of a vessel type is taken close to the optimal speed of that vessel.
- Two services used in merging are removed from the service set, and a randomly selected one of the newly constructed services is added to the set of services.
- By using the new set of services, a new solution x' is obtained. Then, the CRP model is solved and the value of the objective function $z(x')$ is computed. The new solution x' obtained as a result of the shaking step is updated as the current solution x.
- If the generated route is already included within the current solution, type 3 shaking process is terminated without changing the current solution x. In this case, either type 1 or type 2 shaking is activated with a 50 percent probability.

iii. Local Search

Local search is a key step of the VNS algorithm in searching for the optimum solution. In this step, the neighborhood processes for the services with high capacity utilization are treated differently than those of the services with low capacity utilization. For this reason, before the neighborhood search is initiated, the capacity utilization of the services in the current solution is calculated.

To increase the capacity usage of a service with low capacity utilization, the service speed is decreased in the speed change neighborhood, while the vessel type is changed to a lower capacity vessel in the vessel type change neighborhood. On the other hand, to increase the capacity of a service with high utilization, the service speed is increased in the speed change neighborhood, while the vessel type is changed to a higher capacity vessel in the vessel type change neighborhood. In the port insertion neighborhood, the service to which a port is to be added to its route is selected among the services with low capacity utilization. The capacity evaluation and the local search neighborhoods employed within the matheuristic are further detailed in the following.

Service Capacity Utilization:

In order to evaluate the capacity utilization of a service set for a current solution, *x*, the CRP model is solved first. In the obtained solution, the services that have at least one leg with full capacity utilization are evaluated as “High Capacity Utilization Services” (HCUS). Services in the list of HCUS are sorted in descending order concerning their total shadow prices, which is the sum of the shadow prices for all legs of the service. Services with higher shadow price values placed at the top of the list are considered to be the most promising candidates for a capacity increase.

Services that don’t include any arc with full capacity utilization, on the other hand, are classified as “Low capacity utilization services” (LCUS). For each of these services, the utilized capacity is calculated by taking the sum of all individual leg capacity utilizations weighted with the related leg lengths. Then, the list of LCUS is sorted in ascending order with respect to the capacity utilization ratio, which is defined as the ratio of the utilized capacity to the total capacity of a service. Services with lower capacity usage ratios placed at the top of the list are considered to be the most promising candidates for capacity reduction.

Vessel Type Neighborhood:

This neighborhood is applied to high capacity and low capacity utilization services in different ways. In applying the vessel type change neighborhood, the vessel types of the services with low capacity utilization are replaced by vessels with lower capacity. For the services with high capacity utilization, current vessels are replaced by vessels with higher capacity. This neighborhood is initiated by starting from the service list with low capacity utilization services following the steps given below.

i) Application of vessel type neighborhood for services with low capacity utilization

- The first service with the minimum capacity utilization ratio in the list of LCUS is selected.
- The vessel type of this service is reduced such that the replaced vessel type has the lowest possible capacity to transport the available load.
- If the number of available vessels in the selected type is not sufficient for the service to be generated or if the speed of the resulting service to be generated does not lie within the speed range of the selected vessel type, then the service is not modified. In this case, if the vessel type capacity lies below the capacity of the current type, the process is repeated by gradually trying a higher capacity vessel type. This process continues until a feasible service is determined or all vessels with a lower capacity have been tried.
- If a feasible service can be generated, the service set is updated by replacing the selected service with the new one.
- If the service set is updated, yield a new solution x''. Then the CRP model is solved, and the value of the objective function related to this case $z(x'')$ is computed. If the objective function value $z(x'')$ is better than that of the current one $z(x)$, $z(x'') > z(x)$, then the current solution and the associated objective function value are updated $x \leftarrow x'', z(x) \leftarrow z(x'')$.
- LCUS list is generated again according to the updated solution. Starting with the top entries of the new list, all the steps of the process are repeated for all entries of the list.

Upon completion of the LCUS list, the neighborhood procedure for the services with high capacity utilization is activated.

ii) Application of vessel type neighborhood for services with high capacity utilization

- The first service with maximum capacity utilization in the list of HCUS is selected.
- The vessel type of this service is changed with the vessel type having the next higher capacity than the current one.
- If the number of available vessels of the selected type is not sufficient for the service to be generated, or if the speed of the resulting service to be generated in this way does not lie within the speed range of the vessel type selected, then the service is not modified. In this case, the vessel type selection is repeated by gradually trying a higher capacity vessel type. The trial is repeated until a feasible service is found or all vessel types are considered.
- If a feasible service can be generated, the service set is updated by replacing the selected service with the new service, whose vessel type is modified with a higher capacity one.
- The service set is updated, to yield a new solution x''. Then the CRP model is solved, the value of the objective function $z(x'')$ is computed. If the objective function value $z(x'')$ is better than that of the starting one $z(x)$, $z(x'') > z(x)$, then the current solution and the associated objective function value are updated $x \leftarrow x''$, $z(x) \leftarrow z(x'')$.
- HCUS list is generated again according to the updated solution. Starting with the top entries of the regenerated list, all the steps of the process are repeated for all entries of the list.
- When the end of the HCUS list is reached, the update status of the objective function is checked. If there exists at least one update in the value of the objective function $z(x)$ throughout the vessel type neighborhood process, the search of the neighborhood is restarted from the beginning. Otherwise, the vessel type neighborhood search is terminated and then the vessel speed neighborhood is activated.

Vessel Speed Neighborhood:

In this neighborhood, vessel speed increase and vessel speed decrease neighborhoods are activated sequentially. The procedure is initiated, first by the vessel speed increase neighborhood.

i) Vessel speed increase neighborhood:

- The first service with maximum capacity utilization in the list of HCUS is selected.
- The number of vessels is increased by one and a new service is constructed by considering the speed range and availability of that vessel type.
- If a feasible service can be generated, then the service set is updated by replacing the selected service, with the new service. Once the service set is updated, a new solution x'' is obtained. Then, the CRP model is solved and the value of the objective function $z(x'')$ is computed. Comparing the objective function value $z(x'')$ with that of the starting one $z(x)$, if $z(x'') > z(x)$, then the current solution and the associated objective function value are updated $x \leftarrow x'', z(x) \leftarrow z(x'')$.
- Based on the updated solution, the HCUS list is generated. Starting with the top entries of the regenerated list, all the steps of the procedure are repeated for all entries of the list. When the end of the HCUS list is reached, then the vessel speed decrease neighborhood process is activated.

ii) Vessel speed decrease neighborhood

- The first service with minimum capacity utilization ratio in the list of LCUS is selected.
- The number of vessels is decreased by one and a new service is constructed by considering the speed range and the availability information of vessel type.
- If a feasible service can be generated, then the service set is updated by replacing the selected service with the new service. Once the service set is updated, a new solution x'' is obtained. Then, the CRP model is solved and the value of the objective function $z(x'')$ is computed. After comparing the objective function value $z(x'')$ with that of the starting one $z(x)$, if $z(x'') > z(x)$, the current solution and the associated objective function value is updated $x \leftarrow x''$, $z(x) \leftarrow z(x'')$.
- LCUS list is generated according to the updated solution. Starting with the top entries of the generated list, all the steps of the process is repeated for all entries of the list.
- When the end of the LCUS list is reached, the update status of the objective function is checked. If there exist an improvement in the value of the objective function z(x) in at least one of the vessel speed increase or vessel speed decrease neighborhood processes, the speed iterations of the neighborhood are terminated, and the search is returned to

the Vessel Type Neighborhood. On the other hand, if no improvement is obtained, the search process continues with the Port Insertion and Port Removal Neighborhood.

Port Insertion and Port Removal Neighborhood:

This neighborhood involves the generation of alternative service routes by adding or deleting certain ports of the current services. Procedural description of the port insertion and port removal neighborhood is given below:

i) Port Insertion

- Firstly, the ports that are not visited by the routes of any service within the current solution are determined.
- For each of the unvisited ports, the amount of the total unmet demand is determined. The unmet port demand is the demand from the unvisited port to the other ports as well as from others to the unvisited one.
- The unvisited ports are sorted in an insertion candidates list in descending order of the unmet demand amount. Starting from the top of the list, the ports are picked up one by one for insertion into the routes.
- A chosen port is tried to be inserted into the services within the service set. First, a list of neighbor ports for each port is generated. The neighbor list of a port is formed with the chosen closest five neighbor ports. For a currently chosen port, if any one of the ports in the neighbor list of that port appears in the route of a service, the port is inserted to that route of service at a position, which leads a minimum increase in the route length.
- Using the modified route, a new service is generated. The vessel speed of the newly generated service is taken close to the optimal speed of the vessel type of the route.
- If a feasible service can be generated, the service set is updated by replacing the selected service with the new service. After updating a service set, a new solution x'' is obtained. Then, the CRP model is solved and the value of the objective function $z(x'')$ is computed. After comparing the objective function value $z(x'')$ with that of the starting one z(x), if $z(x'') > z(x)$, the current solution and the associated objective function value are updated $\leftarrow x''$, $z(x) \leftarrow z(x'')$.

- For a currently chosen port, if any one of the ports in the neighbor list of that port does not appear in the route of a service, then the port is tried to be inserted to the route of the next service in the service set.
- The insertion process is repeated for all entries of the unvisited port list. After all the ports are iterated for insertion, then the port removal process is initiated.

ii) Port Removal

- Initially, ports that are visited in the routes of the services of the current solution are determined.
- For each of the visited ports, total number of arriving and departing containers are determined.
- The visited ports are sorted with respect to the total number of containers in an ascending order to construct a port removal candidate list.
- To reduce computational time, for the removal of non-profitable ports in a route, we pick up the ports three by three starting from the top of the list. The chosen three ports are removed from the routes of the available services.
- The services with modified routes are replaced with the new services generated. At this stage, the number and the type of vessels are preserved in the current service. If a feasible service is not possible with the same number of vessels, then the service with a vessel speed, which is closest to the optimal speed value of that vessel type is selected among the possible service alternatives.
- Using the updated service set, a new solution x'' is obtained by solving the CRP model. Thereafter, the value of the objective function $z(x'')$ is computed. If the objective function value $z(x'')$ is better than the starting one z(x), then the current solution and the associated objective function value are updated $x \leftarrow x''$, $z(x) \leftarrow z(x'')$.
- The process is repeated for the next three port selection from the port removal candidate list. Upon completion of the list, if no improvement is obtained in the objective function value, then the port removal process is reiterated by considering double port selection. Finally, the process is repeated by employing a single port selection scheme.

- The neighborhood search is terminated if the candidate port list is completely handled. Also, as a termination criterion, the port removal process is limited by five sequentially unsuccessful removal iterations.

As long as at least one improvement is achieved during the port insertion and port removal process, neighborhood search is restarted from the vessel type neighborhood, otherwise the local search is finalized, and the iteration process is continued by returning to the shaking step of the VNS algorithm.

5. COMPUTATIONAL RESULTS

In this chapter, results of the computational experiments for measuring the performance of the matheuristic developed in the thesis are presented. In the first part, construction methodology for the problem instances used in the experiments is introduced and the data related with the problem instances are described. In the second part, the results of the realized experiments for these problem instances are reported.

5.1. EXPERIMENT SET USED TO TEST THE PERFORMANCE OF THE MATHEURISTIC

For testing the performance of the developed matheuristic, an experiment set with seven problem instances is designed by employing the data summarized in Table 5.1 to Table 5.5. To construct the initial solution to be used in these problem instances, three different size candidate service sets are employed. For a small size problem, (S), the candidate service number is 6, while there exist 12 and 18 candidate services in the medium size (M) and the large size (L) problems respectively. In the experiment set, there exist one large size problem and three groups of problem instances, which are constructed by the subsets of the services of this problem. Each problem instance here consists of one small size and one medium size problems. Thus, the experiment set consists of seven problem instances.

In the construction of the experiment set, first, a large size problem with 18 candidate services is generated. The middle size problems in each problem group are then constructed by a random selection of 12 services out of these 18 candidate services. These subsets are selected as different as possible from each other and assigned as the candidate services of the three groups of problem instances. In a similar manner, the small size problems within a problem group are constructed by randomly selecting 6 services from the medium size services of that group. In this way, a total of seven problem instances are constructed: three problems with 6 starting services, three problems with 12 starting services and one large scale problem with 18 starting services.

In all of the problem instances, a common set of ports is used, and a common container transportation demand data is assumed for each origin destination port pairs obtained among these ports. Recall that, container routing model is based on a transit time sensitive demand pattern. For this case, the transportation demand associated with transit times are inversely related. That is, for any source destination pair, the lowest transit time and the highest transit time values correspond to maximum demand and minimum demand respectively. For this reason, to be able to use the same demand values in all problems, min/max transit time values of each origin destination pair, and the corresponding min/max demand is assigned as identical values.

The data common to all problem instances in the experiment set are given in the Tables 5.1-5.4. The port list used in the problems are given in Table 5.1. and the features of the vessel types to be used for transportation are given in Table 5.2. Table 5.3 shows the routes used to generate 18 candidate services, which will be employed in the generation of initial solutions. The set of 18 candidate services is listed in Table 5.4.

Table 5.5 displays the data differences used in the seven problem instances, along with the naming convention of problem groups and codes. Here, the problem group codes are defined to include a letter labelling. Letter labelling identify the type of the problem to be in small, medium, and large size.

Table 5.1. Names and codes of the ports used in the problem instances

Port	Port Code
Varna	BGVAR
Limassol	CYLMS
Algiers	DZALG
Oran	DZORN
Skikda	DZSKI
Damietta	EGDAM
Port Said	EGPSD
Malaga	ESAGP
Algeciras	ESALG
Barcelona	ESBCN
Tarragona	ESTAR
Poti	GEPTI
Ashdod	ILASH
Haifa	ILHFA
Gioia Tauro	ITGIT
Genoa	ITGOA
Salerno	ITSAL
Beirut	LBBEY
Casablanca	MACAS
Tangier	MAPTM
Tunis	TNTUN
Ambarlı	TRAMB
Izmir	TRIZM
Mersin	TRMER
Odessa	UAODS

Table 5.2. Features of vessel types used in problem instances

Vessel Type	Vessel Capacity (TEU)	Minimum Speed (knot)	Maximum Speed (Knot)	Charter Rate/day ($)
A	450	7	14	5,000.00
B	800	8	17	8,000.00
C	1200	9	19	11,000.00

Table 5.3. Routes used in the initial solutions obtained for the problem instances

Route ID	Route
1	ITSAL, ITGIT, TNTUN, DZALG, ESBCN, ESTAR
2	CYLMS, EGDAM, EGPSD, LBBEY
3	BGVAR, TRMER, ILHFA, EGPSD, TRIZM, TRAMB, GEPTI, UAODS,BGVAR
4	ESALG, ITGIT, MACAS
5	ESALG, DZORN, DZALG, DZSKI, MACAS
6	ESTAR, ITSAL, ITGIT, TNTUN, DZALG, ESBCN
7	ESALG, MAPTM, ESALG, ITGOA, ITGIT, LBBEY, ILASH, EGPSD
8	MACAS, ESALG, DZORN
9	ESAGP, DZALG, DZSKI, TRIZM, ITSAL, ITGOA, ESALG
10	ESALG, MAPTM, ESALG, ITGIT, LBBEY, EGPSD
11	ESAGP, DZSKI, TRIZM, ITSAL, ESALG
12	TNTUN, DZALG, ESBCN, ESTAR, ITSAL

Table 5.4. Services used in initial solutions obtained for the problem instances

Service ID	Number of Vessels	Vessel Speed (Knot)	Vessel Type	Route ID
1	2	8.51	A	1
2	1	9.10	B	2
3	3	11.52	B	3
4	2	8.83	A	4
5	1	18.26	C	12
6	3	12.31	C	10
7	2	7.18	A	5
8	3	10.75	A	9
9	1	8.85	B	8
10	4	9.72	B	7
11	2	16.06	C	11
12	3	11.52	C	3
13	2	8.51	B	6
14	2	8.51	B	1
15	1	9.10	C	2
16	3	14.95	C	7
17	1	8.85	A	8
18	3	9.03	A	11

Table 5.5. Data differences in the problem instances

Problem Group	Problem Code	Problem Size	Number of Vessels (A/B/C Type)	Candidate Services of LSSP for Creation of Initial Solution
PG-1	1S	Small	4-4-4	1,2,3,4,5,6
	1M	Medium	9-9-9	1,2,3,4,5,6,7,8,9,10,11,12
PG-2	2S	Small	4-4-4	7,8,9,10,11,12
	2M	Medium	9-9-9	4,5,6,7,8,9,10,11,12,13,14,15
PG-3	3S	Small	4-4-4	13,14,15,16,17,18
	3M	Medium	9-9-9	4,5,6,7,8,9,13,14,15,16,17,18
PG-4	L	Large	13-13-13	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18

5.2. PERFORMANCE TEST RESULTS OF MATHEURISTIC

In this section, performance test results of matheuristic based on the problem instances introduced in the previous section are presented. First, the improvement performance of the matheuristic with respect to the initial solution is examined. Next, a comparative analysis of the matheuristic results with respect to computational results obtained by mathematical models is presented. Since the solution of the integrated LSSCRP model is difficult for large size problems, the comparison of matheuristic solution with that of the integrated model is performed in an indirect manner. For this purpose, the results of matheuristic are first compared with the solution of the sequential two-stage LSSP-CRP approach. Next, the two-stage solution is compared with the integrated LSSCRP model solution.

5.2.1. Performance of Matheuristic on Problem Instances

Since LSSCRP matheuristic involves randomness, for a proper performance test each example problem is solved five times, with a time limit of 120 minutes for each run. The results of five replications are summarized in Tables 5.6 - 5.8.

Table 5.6. Performance of the matheuristic approach with 5 replications on small size problem instances initialized with 6 services

Problem Code	Replication No	Profit ($) (Initial Solution)	Profit ($) (Matheuristic Solution)	Improvement Ratio (%)
1S	1	1,803,583.61	4,757,774.75	163.80
	2		4,063,115.36	125.28
	3		4,933,600.89	173.54
	4		4,504,682.47	149.76
	5		4,410,103.34	144.52
	AVERAGE	**1,803,583.61**	**4,533,855.36**	**151.38**
2S	1	2,762,183.82	4,640,677.57	68.01
	2		4,994,472.13	80.82
	3		4,812,493.79	74.23
	4		4,618,044.99	67.19
	5		4,591,253.80	66.22
	AVERAGE	**2,762,183.82**	**4,731,388.46**	**71.29**
3S	1	2,173,391.69	5,152,584.69	137.08
	2		4,651,445.68	114.02
	3		4,811,005.08	121.36
	4		4,714,115.12	116.90
	5		4,324,384.91	98.97
	AVERAGE	**2,173,391.69**	**4,730,707.10**	**117.66**
OVERALL AVERAGE		**2,246,386.37**	**4,665,316.97**	**113.45**

Table 5.7. Performance of the matheuristic approach with 5 replications on medium size problem instances initialized with 12 services

Problem Code	Replication No	Profit ($) (Initial Solution)	Profit ($) (Matheuristic Solution)	Improvement Ratio (%)
1M	1	3,646,037.21	5,637,805.73	54.63
	2		4,937,948.06	35.43
	3		5,678,754.62	55.75
	4		5,449,034.88	49.45
	5		4,855,401.74	33.17
	AVERAGE	**3,646,037.21**	**5,311,789.01**	**45.69**
2M	1	3,520,994.60	5,183,548.22	47.22
	2		5,121,327.75	45.45
	3		4,950,253.42	40.59
	4		5,293,095.86	50.33
	5		5,390,224.90	53.09
	AVERAGE	**3,520,994.60**	**5,187,690.03**	**47.34**
3M	1	3,504,787.66	5,110,239.88	45.81
	2		5,534,323.31	57.91
	3		5,444,414.28	55.34
	4		5,232,678.00	49.30
	5		5,723,126.79	63.29
	AVERAGE	**3,504,787.66**	**5,408,956.45**	**54.33**
OVERALL AVERAGE		**3,557,273.16**	**5,302,811.83**	**49.12**

Table 5.8. Performance of the matheuristic approach with 5 replications on large size problem instances initialized with 18 services

Problem Code	Replication No	Profit ($) (Initial Solution)	Profit ($) (Matheuristic Solution)	Improvement Ratio (%)
L	1	3,826,228.54	5,271,605.42	37.78
	2		5,352,818.48	39.90
	3		5,407,215.71	41.32
	4		5,127,905.94	34.02
	5		5,520,839.67	44.29
	AVERAGE	**3,826,228.54**	**5,336,077.04**	**39.46**

When the results are examined, it is observed that matheuristic method provides significant improvements with respect to the initial solution. The average improvement ratios for the small, medium, and large size problems are over 113 percent, 49 percent, and 39 percent respectively. Higher improvement ratios are observed in small size problems. This is due to the lower quality of initial solutions generated with small number of initial services. Even in these cases, high quality solutions can be obtained in a short time by using the developed matheuristic. Tables 5.6-5.8 show that the number of candidate services used in the initial solution is effective on the quality of the solution. That is to say, the objective function value obtained from the instance initialized with 6 candidate services is considerably smaller than the solutions initialized with 18 and 12 candidate services. There does not exist however any remarkable quality difference between the solutions obtained with 12 and 18 initial candidate services.

In the graphs of Figures 5.1-5.4, the time convergence performance of the matheuristic approach with 5 replications on the problem instances are shown. The horizontal axis in the graphs indicates the run time of the matheuristic and the vertical axis indicates the objective function value of the best solution obtained so far. For a time-based performance analysis of the matheuristic, a run time of 6 hours for all problems are reported in these figures. As it is seen in the figures, matheuristic runs provide major improvement on the initial solution within the first 120 minutes in all the problems. Further improvements are also observed for run durations longer than 120 minutes.

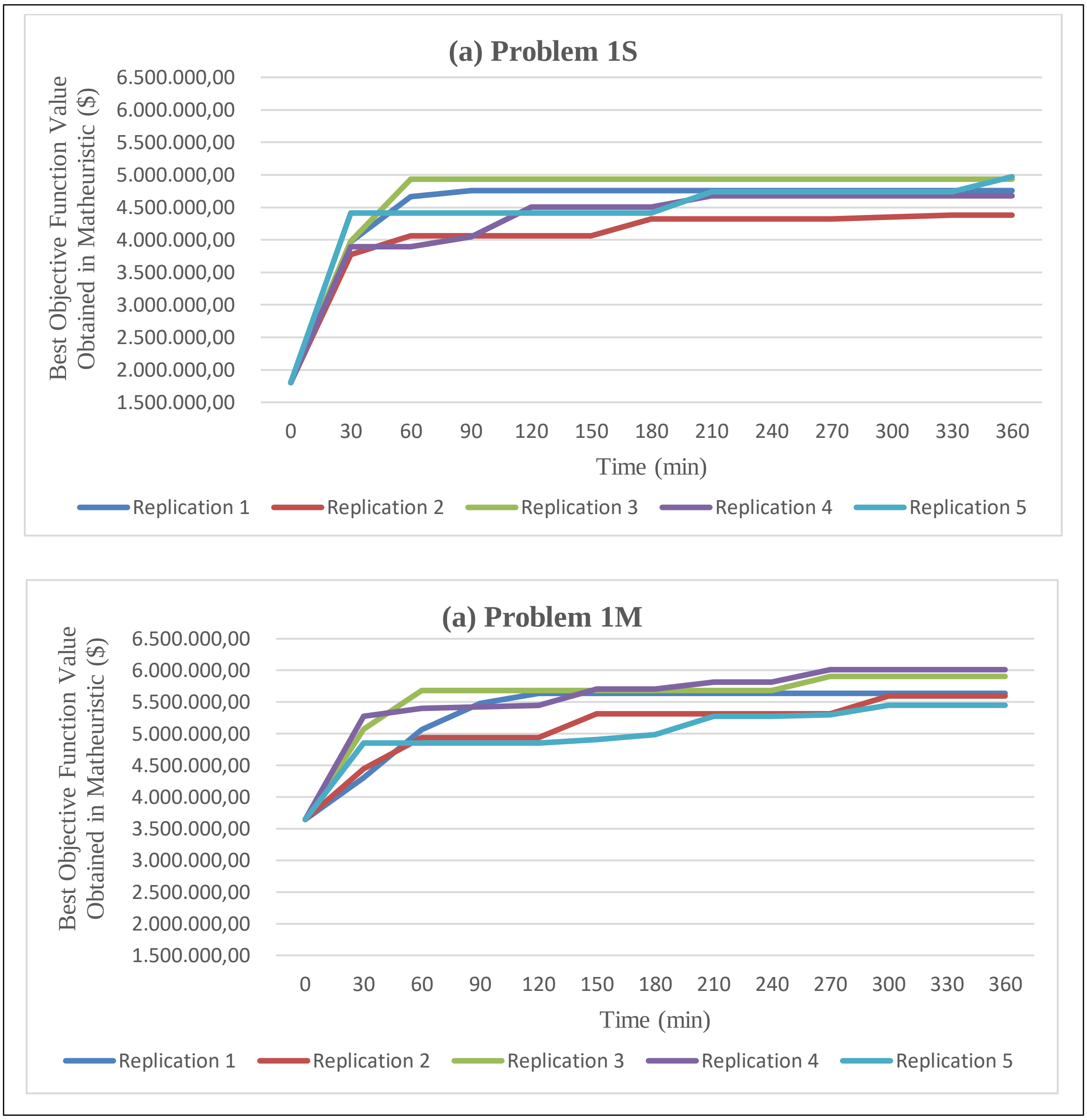

Figure 5.1. Convergence performance of the matheuristic on problem group PG-1: (a) Problem 1S with 6 initial services, (b) Problem 1M with 12 initial services

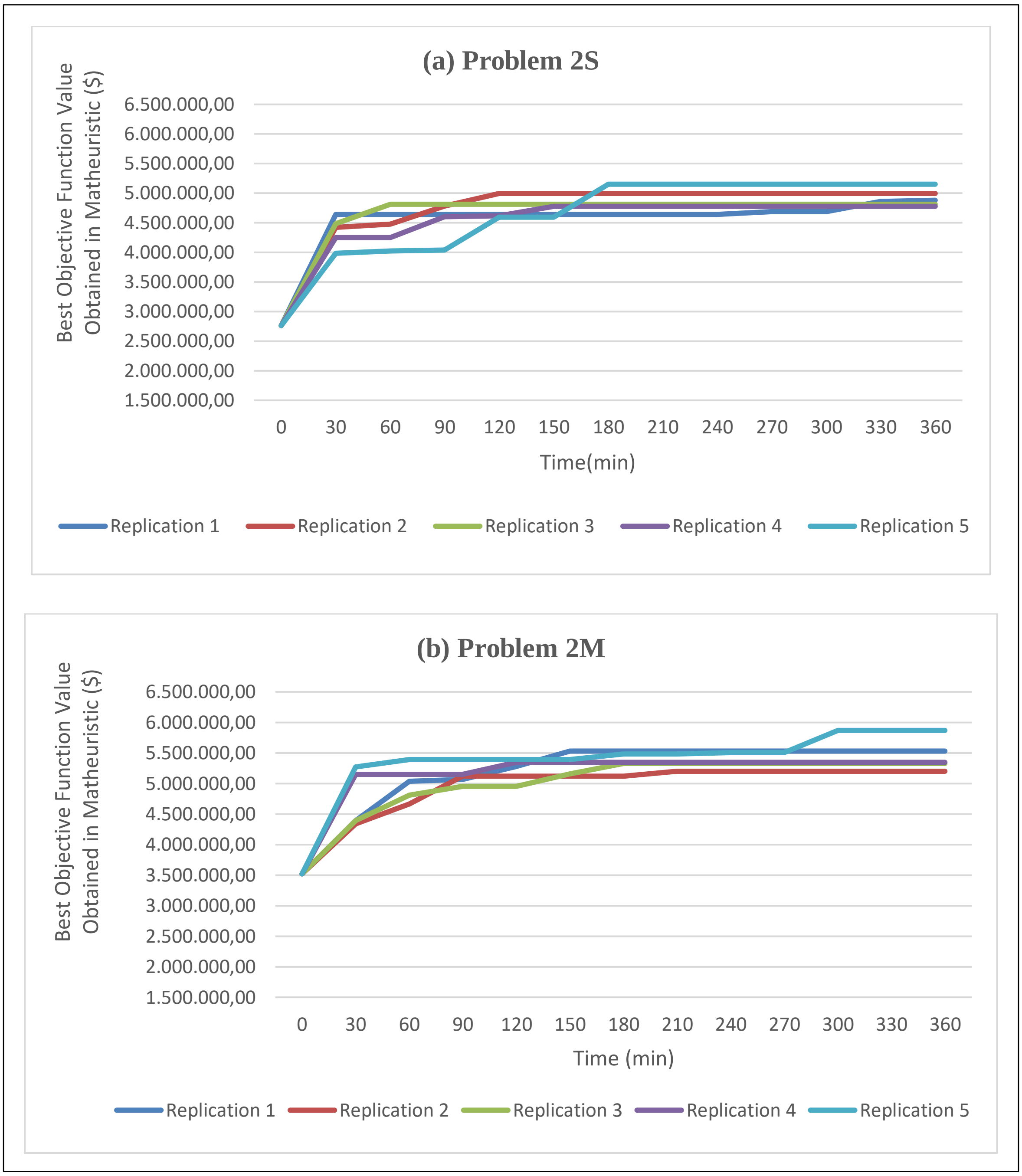

Figure 5.2. Convergence performance of the matheuristic on problem group PG-2: (a) Problem 2S with 6 initial services, (b) Problem 2M with 12 initial services

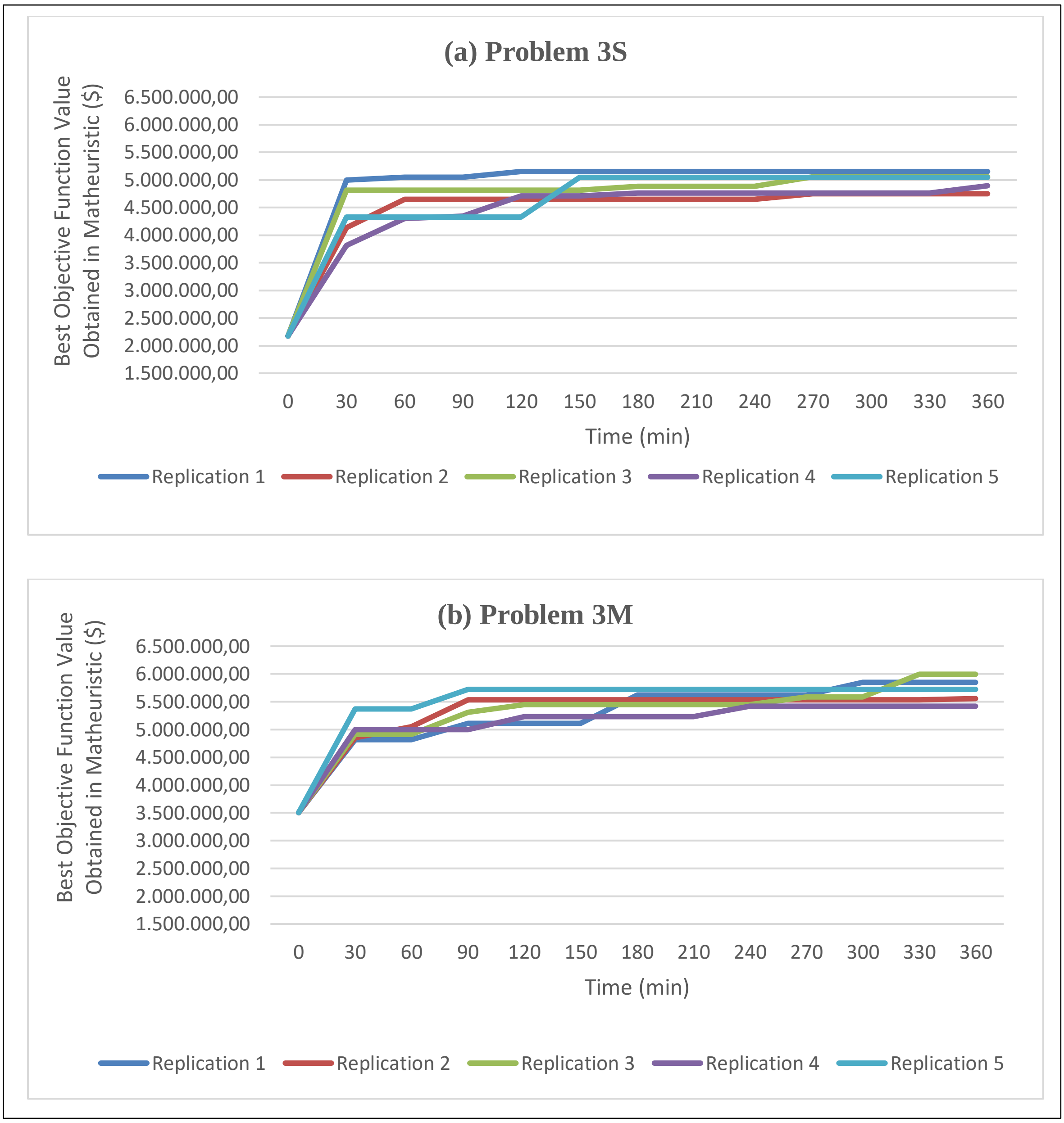

Figure 5.3. Convergence performance of the matheuristic on problem group PG-3: (a) Problem 3S with 6 initial services, (b) Problem 3M with 12 initial services

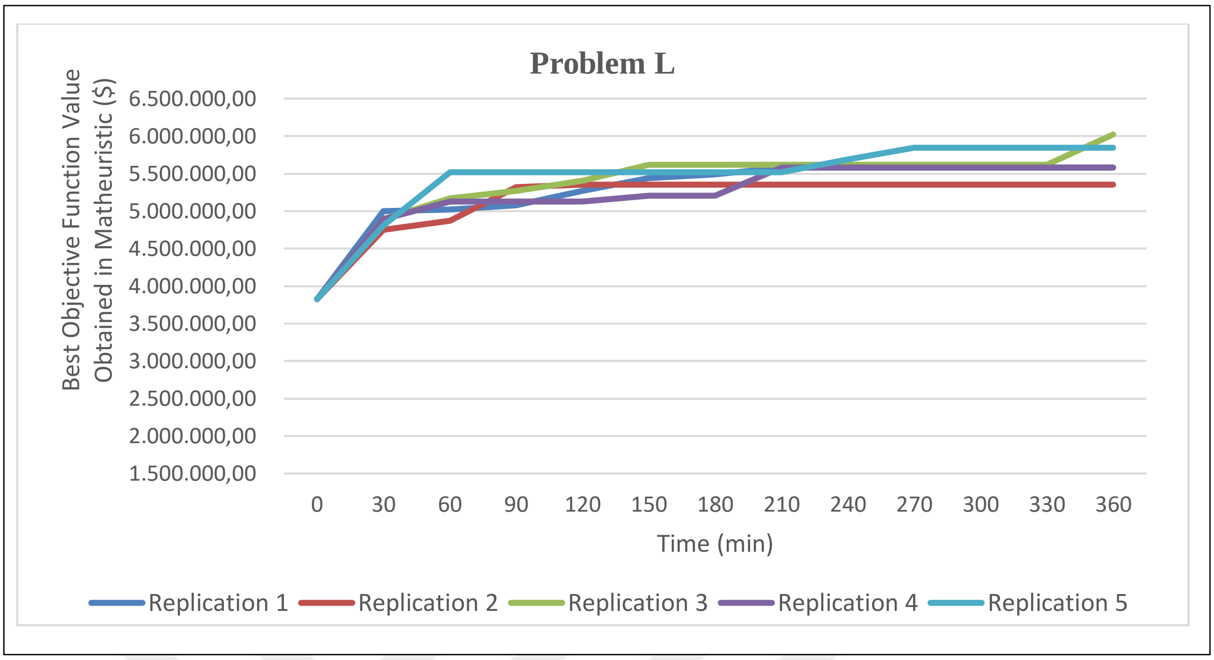

Figure 5.4. Convergence performance of the matheuristic on problem group PG-4 with 18 initial services

Another important issue to be investigated related to the matheuristic performance, is the variation of solution quality throughout the search process, the effects of shaking and local neighborhood search mechanism on searching the solution space. For these purposes, the performance of matheuristic on three different size problem instances, namely 1S, 1M and 1L are investigated. In Figures 5.5-5.7, the iteration based solutions generated for the 5th replication of these problems for 6 hours of running time are presented. Plots in these figures reflect the variation of objective function value with respect to the number of iterations within the algorithm. The yellow, green and red colored points indicate, respectively, the steps of shaking types 1, 2, and 3 of the algorithm. Blue colored lines indicate the objective function value associated with an iteration. The orange colored line represents the objective function value associated with the best solution obtained until that iteration. A close investigation of these figures reveals that the matheuristic can explore different regions of the solution space with the help of the shaking mechanism and consequently, better quality solutions are obtained as the number of iterations increases.

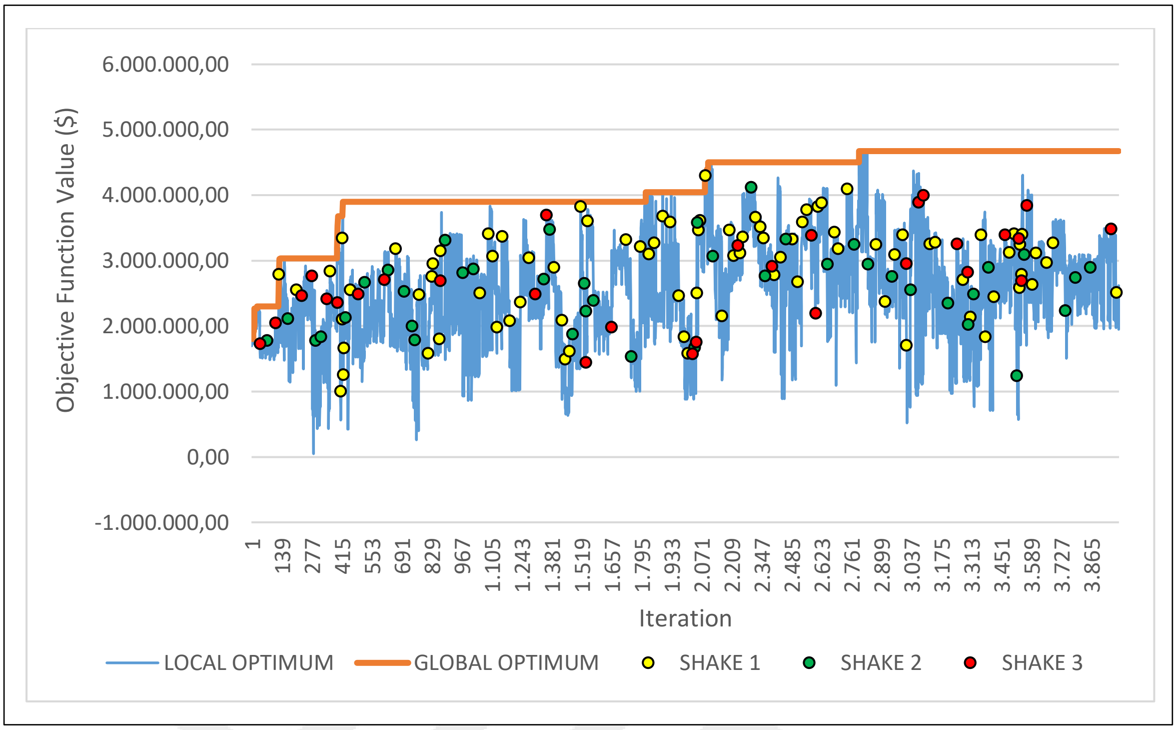

Figure 5.5. Performance of the matheuristic over time on problem 1S in the 5th replication with 6 initial services

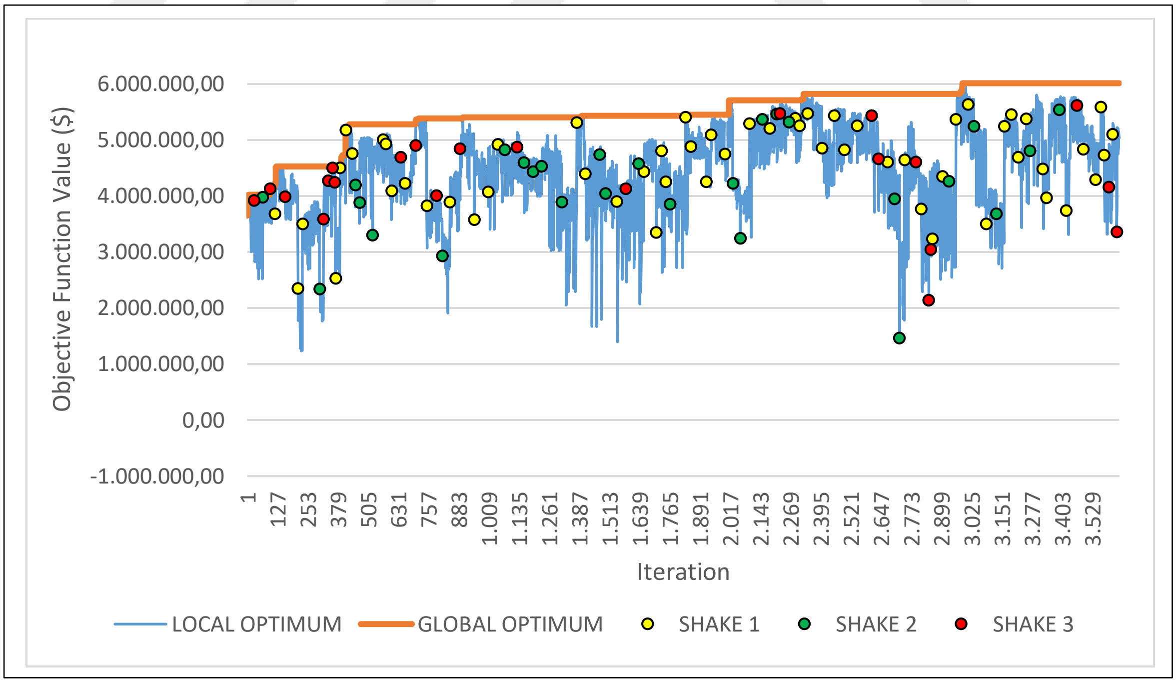

Figure 5.6. Performance of the matheuristic over time on problem 1M in the 5th replication with 12 initial services

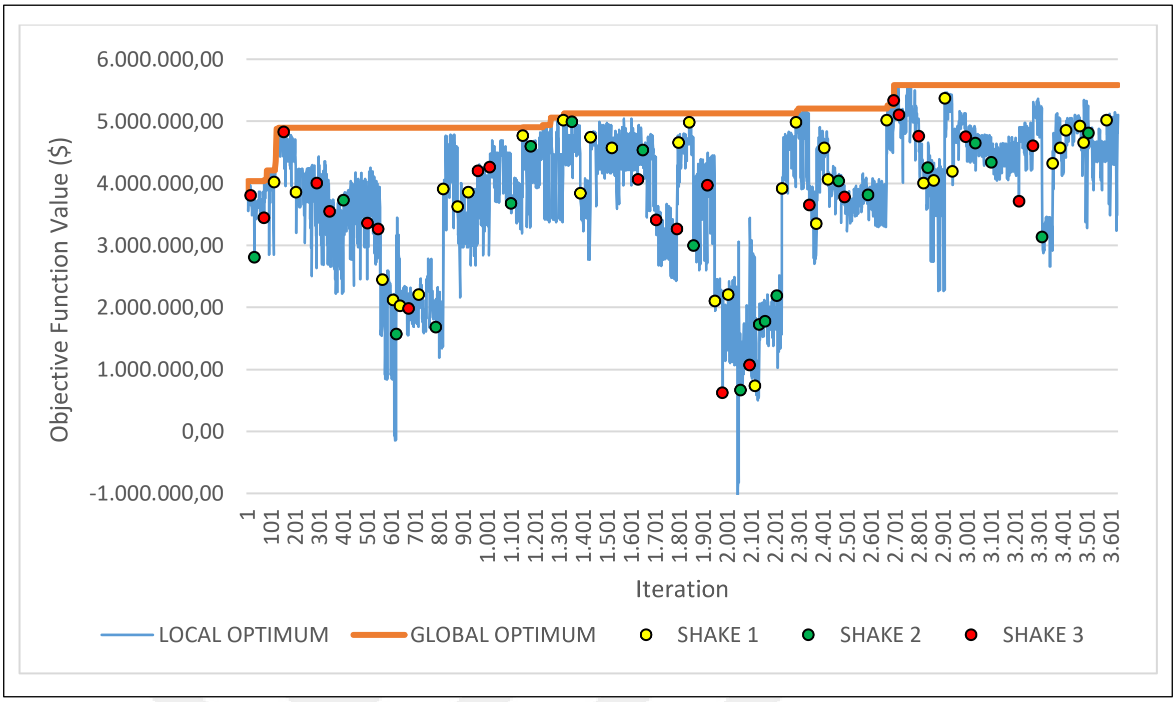

Figure 5.7. Performance of the matheuristic over time on problem L in the 5th replication with 18 initial services

Regarding the performance analysis of the search mechanisms on the problem instances, the iteration based improvements obtained in the objective function values by the employed local neighborhoods and shaking types are given in Tables 5.9-5.11 and in Tables 5.12-5.14. respectively. As it is observed from the tables, all of the neighborhoods and shakings are implemented a significant number of times and they all contribute to the improvement of the objective function value at different levels throughout the search process. The instances initialized with a low number of services result in a higher number of iterations, and a relatively lower successful iteration rate. Problem instances with a higher number of initial services result in a higher improvement by the matheuristic within fewer iterations, each providing a relatively smaller amount objective function value increase. As far as the relative performances of the neighborhood searches are considered, in all of the instances Port Removal, Vessel Type (Low Capacity Utilization) and Vessel Speed Increase neighborhood searches are the first three runners offering higher successful iteration rates. Similarly, the same triple is within the highest contributors providing the greatest increase in objective function value for the instances under consideration. Therefore, these neighborhoods can be considered as the most effective ones in the local search.

In all the problem instances with different number of initial services, it is also observed that the shaking type 1 results in the highest change of the objective function value by exploring different areas of the solution space. As the number of candidate services is increased, shaking type 1 becomes the most effective shaking type leading to higher number of successful iterations. Shaking type 2 is the next best performer in searching the solution space for the change of the objective function value, except for the case initialized with 6 services in Table 5.12.

Table 5.9. Performance analysis of local search neighborhoods used in matheuristic for problem 1S initialized with 6 services

Neighborhoods	Number of Iterations	Number of Iterations with Improvement	Successful Iterations (%)	Increase in Objective Function Value		
				Minimum	Maximum	Average
Vessel Type	2704	63	2.33	569.00	1,060,592.93	275,954.11
Vessel Type High Capacity Utilization	509	2	0.39	9,861.21	82,865.92	46,363.57
Vessel Type Low Capacity Utilization	2195	61	2.77	569.00	1,060,592.93	283,481.67
Vessel Speed	4264	116	2.72	901.37	1,250,404.07	231,476.98
Vessel Speed Increase	2184	77	3.52	901.37	1,250,404.07	294,095.70
Vessel Speed Decrease	2080	39	1.87	2,988.91	525,589.61	107,845.15
Port Removal & Port Insertion	11888	242	2.03	17.47	1,027,485.44	130,602.00
Port Removal	1954	107	5.47	17.47	1,027,485.44	114,298.31
Port Insertion	9934	135	1.35	415.22	785,610.46	143,524.18

Table 5.10. Performance analysis of local search neighborhoods used in matheuristic for problem 1M initialized with 12 services

Neighborhood	Number of Iterations	Number of Iterations with Improvement	Successful Iterations (%)	Increase in Objective Function Value		
				Minimum	Maximum	Average
Vessel Type	1788	83	4.64	569.00	2,411,921.98	337,778.14
Vessel Type High Capacity Utilization	376	9	2.39	4,521.25	529,455.48	116,626.16
Vessel Type Low Capacity Utilization	1412	74	5.24	569.00	2,411,921.98	364,675.00
Vessel Speed	2734	90	3.29	2,448.15	956,480.50	122,807.51
Vessel Speed Increase	1415	60	4.24	6,676.77	956,480.50	133,147.89
Vessel Speed Decrease	1319	30	2.27	2,448.15	425,372.75	102,126.75
Port Removal & Port Insertion	5164	125	2.42	647.88	1,257,806.47	104,447.03
Port Removal	1226	67	5.46	647.88	1,257,806.47	106,593.59
Port Insertion	3938	58	1.47	1,991.12	365,493.32	101,967.38

Table 5.11. Performance analysis of local search neighborhoods used in matheuristic for problem L initialized with 18 services

Neighborhood	Number of Iterations	Number of Iterations with Improvement	Successful Iterations (%)	Increase in Objective Function Value		
				Minimum	Maximum	Average
Vessel Type	945	42	4.44	165.32	544,832.34	83,516.94
Vessel Type High Capacity Utilization	152	6	3.94	910.66	150,177.93	37,145.89
Vessel Type Low Capacity Utilization	793	36	4.54	165.32	544,832.34	91,245.45
Vessel Speed	1323	39	2.94	6,725.64	499,440.98	114,284.87
Vessel Speed Increase	687	27	3.93	7,753.66	499,440.98	116,144.24
Vessel Speed Decrease	636	12	1.88	6,725.64	319,619.98	110,101.29
Port Removal & Port Insertion	2041	48	2.35	2,104.14	658,581.69	90,241.76
Port Removal	474	21	4.43	2,104.14	658,581.69	97,654.20
Port Insertion	1567	27	1.72	2,908.20	315,878.36	84,476.53

Table 5.12. Performance analysis of shaking types used in matheuristic for problem 1S initialized with 6 services

Shaking	Number of Iterations	Number of Iterations with Improvement	Successful Iterations (%)	Amount of Objective Function Change		
				Minimum	Maximum	Average
Shake Type 1	110	43	39.09	7,784.84	2,310,302.22	748,969.33
Shake Type 2	62	18	29.03	3,013.79	1,670,605.93	540,604.49
Shake Type3	44	15	34.09	15,446.74	2,018,157.39	672,019.71

Table 5.13. Performance analysis of shaking types used in matheuristic for problem 1M initialized with 12 services

Shaking	Number of Iterations	Number of Iterations with Improvement	Successful Iterations (%)	Amount of Objective Function Change		
				Minimum	Maximum	Average
Shake Type 1	44	18	40.90	121,226.05	2,371,231.84	788,230.24
Shake Type 2	26	8	30.76	44,109.72	2,875,013.59	756,864.52
Shake Type3	39	8	20.51	44,109.72	2,875,013.59	756,864.52

Table 5.14. Performance analysis of shaking types used in matheuristic for the problem L initialized with 18 services

Shaking	Number of Iterations	Number of Iterations with Improvement	Successful Iterations (%)	Amount of Objective Function Change		
				Minimum	Maximum	Average
Shake Type 1	21	5	23.80	24,050.08	2,263,839.15	439,375.68
Shake Type 2	6	0	0	2,489.11	1,165,546.45	366,614.32
Shake Type3	11	3	27.27	53,031.85	474,334.50	255,858.13

Services and the associated routes obtained from the matheuristic solution of different size problem instances are presented in Tables 5.15-5.20. Graphical representations of all routes are also provided in Figure 5.8-5.23.

Table 5.15. Services included in the solution of problem 1S in the 5th replication with 6 initial services

Service ID	Number of Vessels	Vessel Speed (knot)	Vessel Type	Route ID
768	4	14.85	B	1508
1298	2	17.14	C	2170
335	3	13.41	D	485
439	3	12.70	A	713

Table 5.16. Routes included in the solution of problem 1S in the 5th replication with 6 initial services

Route ID	Route
1508	ITSAL, ITGOA, ESALG, EGPSD, LBBEY, ESALG, MAPTM, MACAS, ESAGP
2170	BGVAR, TRAMB, TRIZM, ITSAL, ITGIT, ESALG, ESAGP, TNTUN, TRMER, EGPSD, UAODS
485	LBBEY, EGPSD, TNTUN, ESALG, MAPTM, ESALG, ESTAR
713	BGVAR, TRMER, LBBEY, ILHFA, EGPSD, TRIZM, GEPTI, UAODS

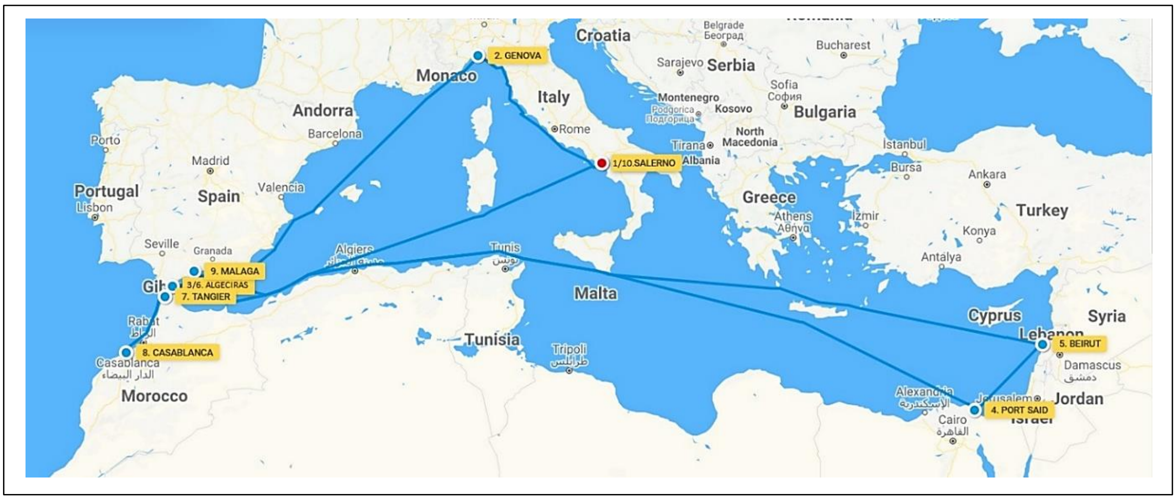

Figure 5.8. Graphical representation of route 1508 in the solution of problem 1S in the 5th replication with 6 initial services

Figure 5.9. Graphical representation of route 2170 in the solution of problem 1S in the 5th replication with 6 initial services

Figure 5.10. Graphical representation of route 485 in the solution of problem 1S in the 5th replication with 6 initial services

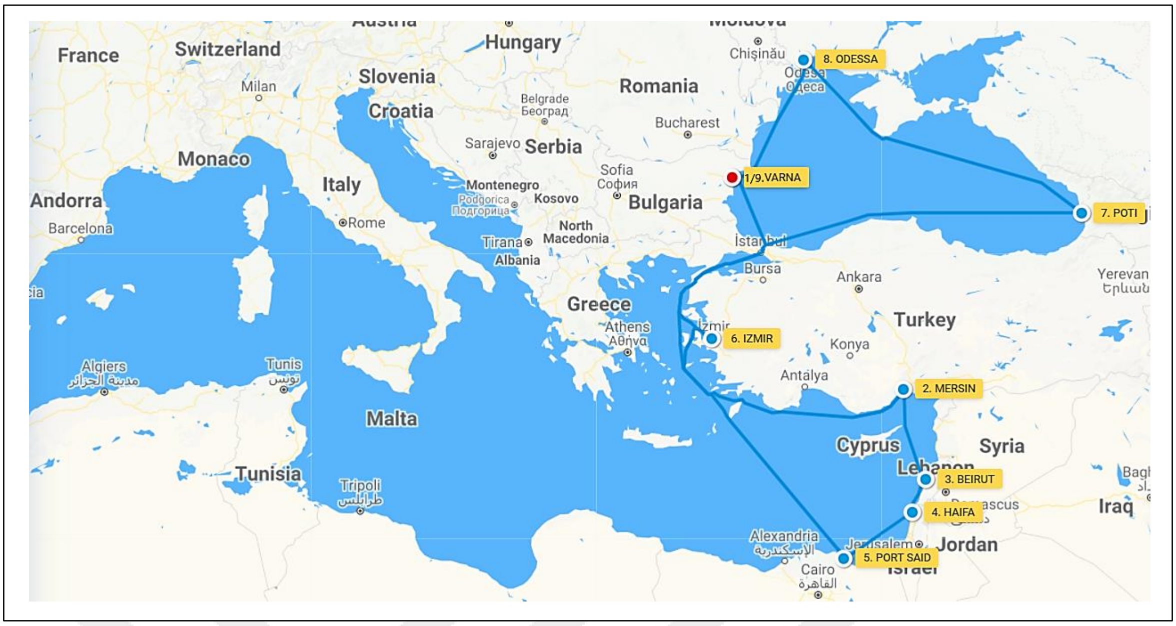

Figure 5.11. Graphical representation of route 713 in the solution of problem 1S in the 5th replication with 6 initial services

Table 5.17. Services included in the solution of problem 1M in the 5th replication with 12 initial services

Service ID	Number of Vessels	Vessel Speed (knot)	Vessel Type	Route ID
788	4	16.05	B	827
806	2	12.85	A	705
51	2	14.40	B	14
813	2	12.85	A	876
634	5	7.40	A	634
302	3	16.16	B	84

Table 5.18. Routes included in the solution of problem 1M in the 5th replication with 12 initial services

Route ID	Route
827	ESAGP, ESALG, BGVAR, TRAMB, UAODS, BGVAR, ESAGP, ITGOA, ITGIT, ITGOA
705	EGPSD, ILASH, LBBEY, ITGIT, ESTAR, ESALG, MACAS, MAPTM, ESALG, ESAGP
14	ESAGP, TRIZM, ITSAL, ESALG, ESAGP
876	BGVAR, TRMER, CYLMS, ILHFA, EGDAM, EGPSD, TRIZM, TRAMB, GEPTI, UAODS
634	BGVAR, ITGIT, ITSAL, ESTAR, ESBCN, TRMER, ILHFA
84	DZORN, ESALG, MAPTM, ESALG, ITGOA, ITGIT, LBBEY, ILASH, EGPSD

Figure 5.12. Graphical representation of route 827 in the solution of problem 1M in the 5th replication with 12 initial services

Figure 5.13. Graphical representation of route 705 in the solution of problem 1M in the 5th replication with 12 initial services

Figure 5.14. Graphical representation of route 14 in the solution of problem 1M in the 5th replication with 12 initial services

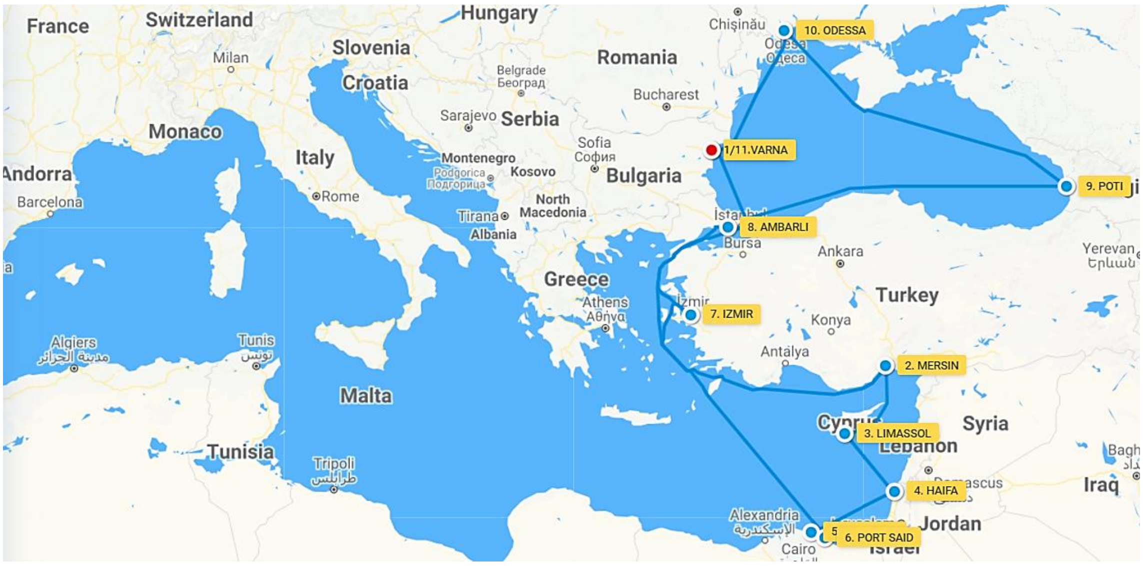

Figure 5.15. Graphical representation of route 876 in the solution of problem 1M in the 5th replication with 12 initial services

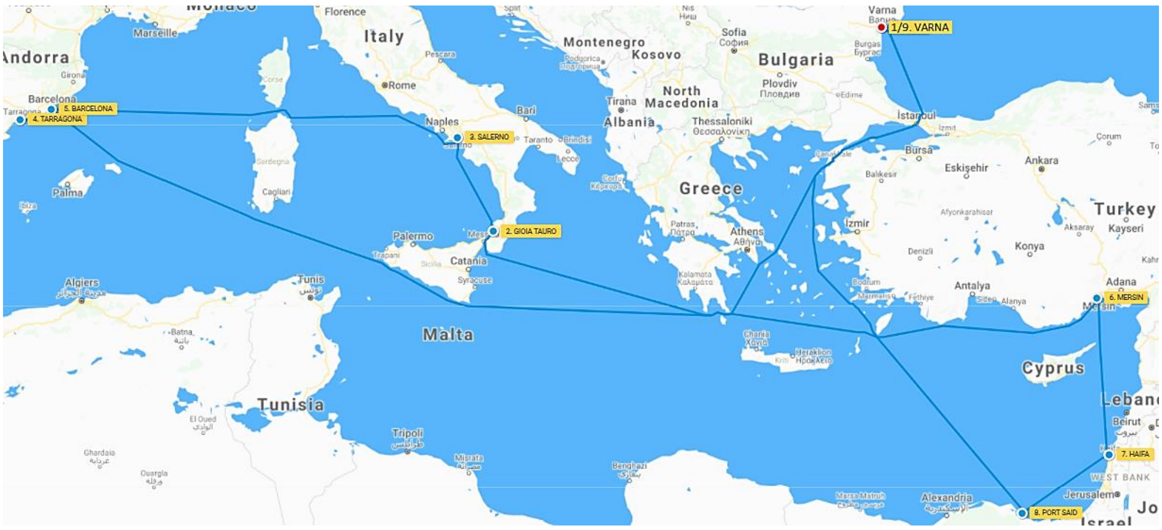

Figure 5.16. Graphical representation of route 634 in the solution of problem 1M in the 5th replication with 12 initial services

Figure 5.17. Graphical representation of route 84 in the solution of problem 1M in the 5th replication with 12 initial services

Table 5.19. Services included in the solution of problem L in the 5th replication with 18 initial services

Service ID	Number of Vessels	Vessel Speed (knot)	Vessel Type	Route ID
19	3	14.95	B	7
580	2	12.85	A	338
581	3	13.96	A	473
583	4	11.86	B	518
595	3	13.13	A	533
607	4	15.52	B	541

Table 5.20. Routes included in the solution of problem L in the 5th replication with 18 initial services

Route ID	Route
7	ESALG, MAPTM, ESALG, ITGOA, ITGIT, LBBEY, ILASH, EGPSD
338	TNTUN, DZALG, ESALG, MAPTM, ESALG, ESBCN, ITGOA, ITGIT, LBBEY, EGPSD
473	TNTUN, TRIZM, TRAMB, BGVAR, TRMER, EGPSD, ESBCN
518	UAODS, ESBCN, ITGOA, ESTAR, MAPTM, ESALG, TNTUN, TRIZM
533	EGPSD, TRIZM, GEPTI, UAODS, TRIZM, EGPSD, ILHFA, TRMER
541	ESALG, ESAGP, TNTUN, TRIZM, ITSAL, ITGIT, MACAS, ESBCN, ITGIT, ITSAL, DZALG, ESAGP

Figure 5.18. Graphical representation of route 7 in the solution of problem L in the 5th replication with 18 initial services

Figure 5.19. Graphical representation of route 338 in the solution of problem L in the 5th replication with 18 initial services

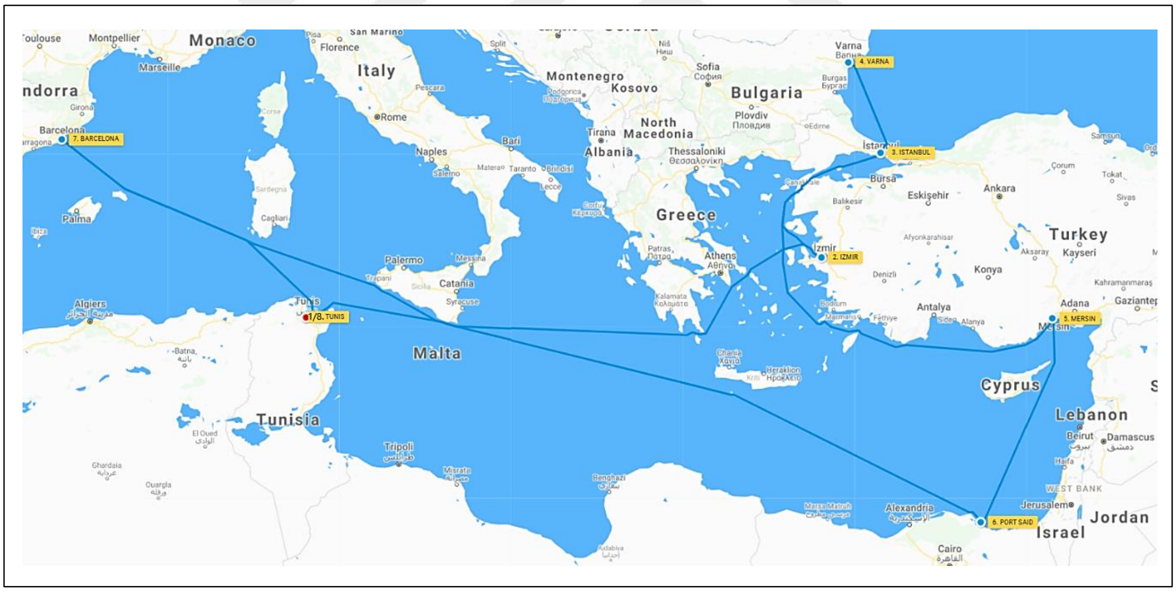

Figure 5.20. Graphical representation of route 473 in the solution of problem L in the 5th replication with 18 initial services

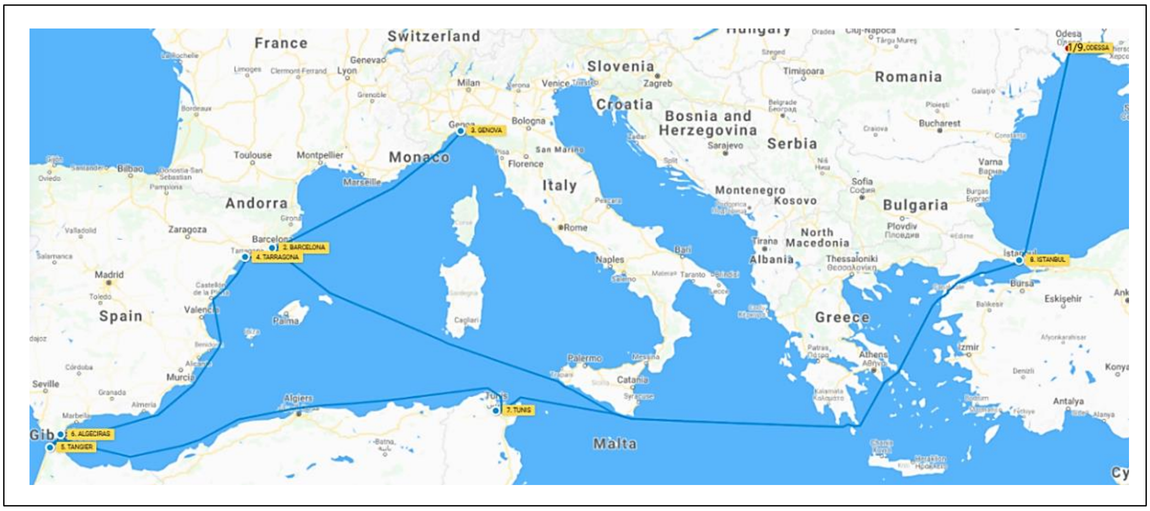

Figure 5.21. Graphical representation of route 518 in the solution of problem L in the 5th replication with 18 initial services

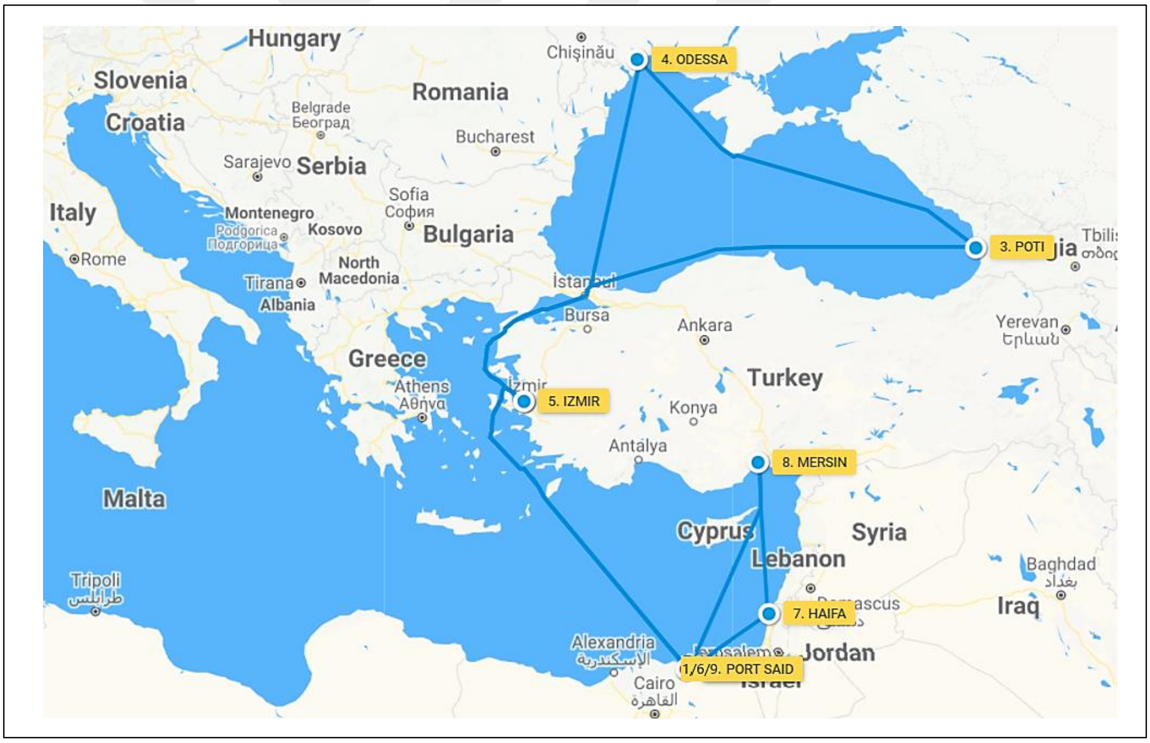

Figure 5.22. Graphical representation of route 533 in the solution of problem L in the 5th replication with 18 initial services

Figure 5.23. Graphical representation of route 541 in the solution of problem L in the 5th replication with 18 initial services

5.2.2. Performance Comparison of the Matheuristic and Two-Stage Solution Approaches

In this section, the performance of the LSSCRP matheuristic is compared with respect to the sequential two-stage solution of LSSP and CRP mathematical models. For all problem groups, the first 20, 30 and 40 feasible services generated in the 1st run of the matheuristic are used as the candidate service sets for the LSSP model of the two-stage technique. Since the solution of the LSSP model for more than 40 candidate services is not computationally possible, the results of matheuristic solution obtained after 120 minutes cannot be compared with the two-stage solution approach.

With the specified initial candidate service set, LSSP and CRP models are sequentially solved, and the solution durations and the resulting objective function values are noted to compare with LSSCRP matheuristic results. The results of both approaches for all the problem groups are summarized in Table 5.21. This table includes the iteration time, the objective function value of both approaches and the percentage difference of the objective function values between the two approaches. The time required to reach the optimum value in the two-stage approach is taken as the time limit for the matheuristic for a fair comparison. Review of the results in Table 5.21 reveals that, for all problem groups, matheuristic performs significantly better than the two-stage solution technique, providing a performance difference of up to 46.1 percent..

Table 5.21. Performance of LSSCRP matheuristic and the two-stage LSSP-CRP solutions on the problem instances in the 1st replication ('-' indicates that a solution was not obtained in 120 minutes)

Problem Code	Solution Time (Minutes)	Two-Stage Objective function value ($)	Matheuristic Objective function value ($)	Difference between Two-stage and Matheuristic (%)
1S	4.14	2,300,802.76 (20 service)	3,361,474.29	-46.10
	22.96	3,365,563.80 (30 service)	3,410,875.11	-1.35
	61.84	3,349,553.12 (40 service)	4,664,975.31	-39.27
	120	-	4,757,774.75	-
2S	3.55	3,004,452.37 (20 service)	3,019,551.86	-0.5
	12.76	3,437,559.90 (30 service)	3,859,129.50	-12.26
	51.71	3,442,608.65 (40 service)	4,640,677.57	-34.8
	120	-	4,640,677.57	-
3S	3.27	4,034,842.07 (20 service)	4,289,542.36	-6.31
	29.77	4,254,027.68 (30 service)	4,999,369.41	-17.52
	108	4,371,880.36 (40 service)	5,152,584.69	-17.86
	120	-	5,152,584.69	-
1M	20.33	4,123,164.29 (20 service)	4,307,922.33	-4.48
	33.96	4,089,575.43 (30 service)	4,307,922.33	-5.34
	86.56	4,640,391.76 (40 service)	5,479,834.04	-18.09
	120	-	5,637,805.73	-
2M	1.87	3,688,281.81 (20 service)	4,015,150.23	-8.86
	15.56	4,188,476.88 (30 service)	4,395,160.83	-4.93
	48.48	4,284,359.07 (40 service)	5,038,169.96	-17.59
	120	-	5,183,548.22	-
3M	12.98	3,697,033.96 (20 service)	4,047,168.91	-9.47
	20.66	3,837,018.47 (30 service)	4,540,705.83	-18.34
	59.14	3,936,644.86 (40 service)	4,819,505.67	-22.43
	120	-	5,110,239.88	-
L	3.38	3,828,684.86 (20 service)	4,039,770.97	-5.51
	13.57	4,114,571.66 (30 service)	4,653,149.45	-13.09
	47.8	4,211,267.35 (40 service)	4,996,152.98	-18.64
	120	-	5,271,605.42	-

5.2.3. Performance Comparison of the Integrated LSSCRP and Two-Stage LSSP-CRP Solution Approaches

In the previous section, it is seen that the matheuristic approach yields better results compared to the two-stage LSSP-CRP solution. In order to compare the matheuristic performance with that of the optimal solution, the two-stage LSSP-CRP should be compared with the performance of integrated LSSCRP model solution. Employing the initialization approach considered in the previous part for 20, 30 and 40 candidate services, both the two stage LSSP-CRP and the integrated LSSCRP models are solved for the same problem groups. All of the results are summarized in Table 5.22.

Investigation of the results reveals that the performance of both solution approaches is in general similar to each other. There exists, however, a significant computation time difference between the two approaches for most of the problems. In particular, the solution time of the integrated LSSCRP model is observed to be increasing exponentially as the number of candidate services increases. The percentage deviations of the objective function values between the two approaches in all problems are close to zero. This implies that the iterative two stage LSSP-CRP technique can be utilized to test the performance of matheuristic when a feasible solution of the integrated LSSCRP model cannot be achieved within a reasonable time for large scale problems.

Table 5.22. Performance comparison of the two-stage LSSP-CRP and the integrated LSSCRP solutions on the problem instances in the 1st replication ('-' indicates that a solution was not obtained in 1440 minutes)

Problem Code	Number of Services	LSSP-CRP Solution Time (Minutes)	LSSP-CRP Objective Function ($)	Integrated LSSCRP Solution Time (Minutes)	Integrated LSSCRP Objective Function ($)	Difference between Two-stage and Integrated LSSCRP (%)
1S	20	4.14	2,300,802.77	5.23	2,300,802.77	0
	30	22.97	3,365,563.80	57.36	3,365,563.80	0
	40	61.85	3,349,553.12	-	-	-
2S	20	3.55	3,004,452.37	21.92	3,029,530.03	-0.83
	30	12.76	3,437,559.90	722.33	3,437,559.90	0
	40	51.71	3,442,608.65	-	-	-
3S	20	3.27	4,034,842.07	110.2	4,048,083.91	-0.33
	30	29.77	4,254,027.68	-	-	-
	40	108	4,371,880.36	-	-	-
1M	20	20.33	4,123,164.29	11.04	4,124,103.89	-0.02
	30	33.96	4,089,575.43	66.01	4,163,459.00	-1.81
	40	86.56	4,640,391.76	-	-	-
2M	20	1.87	3,688,281.81	5.4	3,694,408.92	-0.17
	30	15.56	4,188,476.88	144.29	4,271,240.42	-1.98
	40	48.48	4,284,359.07	-	-	-
3M	20	12.98	3,697,033.96	29.45	3,697,033.96	0
	30	20.66	3,837,018.47	833	3,901,789.80	-1.69
	40	59.14	3,936,644.86	-	-	-
L	20	3.38	3,828,684.86	4.25	3,828,684.86	0
	30	13.57	4,114,571.66	221.59	4,114,571.66	0
	40	47.8	4,211,267.35	-	-	-

6. CONCLUSION

One of the major concerns of liner shipping companies in the competitive maritime transportation sector is to offer high quality service with shorter transit times. Liner companies should pay special attention to transit time sensitive customer demand, in order to come up with the right decisions in the tactical planning of ship schedules along with the routing of containers. In this regard, integrated handling of the liner ship scheduling and the container routing problems is of great importance in making optimal decisions based on the operational costs and the service quality measures. In this thesis, an integrated solution of the liner ship scheduling problem (LSSP) and the container routing problem (CRP) is investigated. After a brief introduction of the sub problems LSSP and CRP, the integrated problem LSSCRP with transit time sensitive demand characteristic is discussed.

A mathematical model formulation for the integrated problem LSSCRP with transit time sensitive demand is presented. Since the LSSCRP model is of NP hard complexity, its optimal solution in a reasonable time is not possible in general. For this reason, a matheuristic algorithm to solve the LSSCRP is developed. In this matheuristic algorithm, the LSSCRP is solved by improving the liner schedule utilizing the feedback from the container routes generated to meet the customer demand. In this matheuristic, the effect of transit times on customer demand is taken into consideration during the schedule generation and container routing stages. In this way, the interaction between the operational and tactical level parameters is considered to identify near optimal decisions based on the revenue, the operational cost and the service quality measures. The proposed matheuristic approach incorporates mathematical programming strategies with a variable neighborhood search algorithm adopted to LSSCRP for the iterative improvement of the schedule and container routing based on an initial solution.

The performance of the LSSCRP matheuristic and the quality of the solutions are evaluated by a set of computational experiments on selected problem instances. These experiments involve testing the improvement performance of the matheuristic with respect to an initial solution and the comparative analysis of the matheuristic solutions with that of the mathematical model solution of the integrated LSSCRP. The results of the computational experiments show that the developed matheuristic results in high quality solutions within reasonable time. Therefore, it is believed that the developed matheuristic can be utilized as an efficient solution technique that can be incorporated within a decision support system to solve the liner ship scheduling and container routing problems of the maritime transportation sector.

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