

INTEGRATED FLEXIBLE CAPACITY AND INVENTORY MANAGEMENT UNDER FLEXIBLE CAPACITY UNCERTAINTY

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MASTER OF SCIENCE

By

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August, 2006

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ABSTRACT

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In a manufacturing environment with volatile demand, inventory management can be coupled with dynamic capacity adjustments for handling the fluctuations more effectively. In this study we consider the integrated management of inventory and flexible capacity management under seasonal stochastic demand and uncertain labor supply. The capacity planning problem is investigated from the workforce planning perspective. We consider a manufacturing firm that can temporarily increase its production capacity by utilizing contingent workers from an external labor supply agency. The uncertainty of contingent capacity arises from the (un)availability of contingent workers, the pool size and the behavior of the labor supply agency. Using a dynamic programming approach, we formulate an infinite horizon model determining the optimal levels of permanent and contracted capacity, in order to minimize the total cost of operations. Within the dynamic program, we determine the optimal operational capacity decisions, namely the size of contracted and contingent capacity to be utilized in each period. The characteristics of the optimal policies are analyzed under an infinite horizon setting. In addition we consider temporary labor contracts, that diminish the effects of supply uncertainty on the manufacturer, at a specific contracting cost. Our analysis shows that as the supply uncertainty and/or the expected cost of utilizing contingent workers increase, the value of temporary labor contracts also increase. In contrast increasing demand variability reduces the incentive of the manufacturer to own capacity and increases the utilization of contingent resources.

Keywords: Flexible Capacity, Capacity Management, Inventory, Production Planning, Stochastic Dynamic Programming.

ÖZET

TEDARİK BELİRSİZLİĞİ ALTINDA EŞZAMANLI ESNEK KAPASİTE VE ÜRETİM PLANLAMA

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Üreticiler talep dalgalanmalarına, envanter yönetimi ve eşzamalı esnek kapasite yönetimi ile etkin bir şekilde karşılık verebilirler. Bu çalışmada eşzamanlı esnek kapasite ve envanter yönetimi, kapasite tedariği ve mevsimsel talep belirsizliği altında incelenmiştir. Kapasite planlama, işgücü bağlamında ele alınmıştır. Üretim kapasitesi işçi tedarik acentesinden geçici işçi alımıyla arttırılabilmektedir. Geçici işçi tedariğindeki belirsizlik, ortamdaki geçici işçi sayısından, işçi tedarik acentesinin elindeki işçi sayısından ve dönemsel kararlarından kaynaklanmaktadır. Operasyonel maliyetler, dinamik programlama kullanılarak sonsuz planlama çevreninde enaza indirgenmektedir. Bu çalışmada eniyi kapasite ve üretim politikalarının karakteristikleri incelenmiştir. Geçici işçi tedariğindeki belirsizlikten kaynaklanan etkileri azaltmak için tedarikçi acente ile işçi kontratları yapılması önerilmektedir. Geçici işçi kullanımının beklenen maliyetinin, belirsizlik ve/veya fiyattan dolayı artması durumunda işçi kontratlarının değerlendirildiği gözlemlenmiştir. Buna karşılık artan talep belirsizliğinin üreticiyi esnek kapasite kullanımına yönelttiği ve kontratların değerini azalttığı görülmüştür.

Anahtar sözcükler: Esnek Kapasite, Kapasite Yönetimi, Envanter, Üretim Planlama, Rassal Dinamik Programlama.

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Contents

1	Introduction	1
2	Literature Review	8
3	Model Formulation	20
3.1	Supply Uncertainty Structures	26
4	A Special Case: Deterministic Supply	29
4.1	Model in Operator Form	30
4.2	Conditions on the Convergence of Finite Horizon Results in the Infinite Horizon	32
4.2.1	Finite Horizon Conditions	33
4.2.2	Structured Attainment Condition	35
4.2.3	Infinite Horizon Conditions	35
4.2.4	Results	36
4.3	Value Iteration Algorithm	37
4.4	Analysis of the Infinite Horizon Optimal Policy	38

5	Analysis of the Model	46
5.1	All or Nothing Type Contingent Capacity Uncertainty	47
5.2	Partial Contingent Capacity Availability	61
5.2.1	Uniform Supply Uncertainty	61
5.2.2	Normal Supply Uncertainty	66
5.2.3	Binomial Supply Uncertainty with Increasing Probability of Success	67
5.2.4	Binomial Supply Uncertainty With Decreasing Probability of Success	70
5.2.5	Binomial Supply Uncertainty With High Probability of Success at Low and High Requests	71
5.2.6	Binomial Supply Uncertainty With High Probability of Success at Moderate Demand	72
6	Numerical Analysis	77
6.1	Effect of Labor Supply Uncertainty	78
6.2	Optimal Tactical Capacity Decisions	84
6.2.1	Optimal Contracted Capacity Level	84
6.2.2	Optimal Permanent and Contracted Capacity Decisions . .	88
6.3	Value of Contracting	91
6.3.1	Effect of Permanent Capacity	92
6.3.2	Effect of Contracting Cost c_i	92
6.3.3	Effect of Backordering Cost	93

6.3.4	Effect of Contingent Labor Cost (c_c) and Supply Uncertainty	94
6.3.5	Effect of Demand Variability	96
6.4	A Heuristic Policy	97
6.4.1	Effect of Labor Supply and Demand Variability	98
6.4.2	Effect of Permanent and Contracted Capacity	99
6.4.3	Effect of Contingent Labor Cost and Backordering Cost . .	100
7	Conclusions	103

List of Figures

5.1	Expected Inventory Level After Production-Uniform Uncertainty, $c_p = 2.5, c_c = 3.5, h = 1, b = 5 U = 10$, Poisson Demand	66
5.2	Optimal Capacity Position & Optimal Production vs Starting In- ventory Level - Uniform Uncertainty, $c_p = 2.5, c_c = 3.5, h = 1$, $b = 5 U = 10$, Poisson Demand	67
5.3	Expected Inventory Level After Production vs Starting Inventory- Normal Uncertainty, $c_p = 2.5, c_c = 3.5, h = 1, b = 5 U = 10$, Poisson Demand	68
5.4	Optimal Capacity Position vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success, $c_p = 2.5, c_c =$ $3.5, h = 1, b = 5 U = 10$, Poisson Demand	69
5.5	Optimal Capacity Position vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success, $c_p = 2.5, c_c =$ $3.5, h = 1, b = 5 U = 10$, Poisson Demand	70
5.6	Infinite Horizon Discounted Cost of Operations vs Capacity Position-Binomial Uncertainty with Decreasing Probability of Suc- cess, $c_p = 2.5, c_c = 3.5, h = 1, b = 5 U = 10$, Poisson Demand . . .	71
5.7	$H(w x = 0)$ vs Capacity Position-Binomial Uncertainty with In- creasing Probability of Success at High and Low Requests, $c_p =$ $2.5, c_c = 3.5, h = 1, b = 5 U = 10$, Poisson Demand	72

5.8	Optimal Capacity Position vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success at High and Low Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand	73
5.9	Expected Inventory Level After Production vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success at High and Low Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand	74
5.10	Expected Inventory Level After Production vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success at Moderate Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand	75
5.11	Optimal Production vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success at Moderate Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand	76
6.1	Periodic Production-Certain Supply vs Uniform Supply. $U = 10$, $V = 0$, $c_c = 3.5$, $b = 50$	83
6.2	Value of Contracting vs Permanent Capacity Level-Uniform Supply Uncertainty, Poisson Demand $b = 50$, $c_c = 3.5$	92
6.3	Relative Value of Contracting vs Permanent Capacity Level-Uniform Supply Uncertainty, Poisson Demand $b = 50$, $c_c = 3.5$	93
6.4	Value of Contracting vs Contracting Cost (c_i) - Uniform Supply Uncertainty, Poisson Demand $b = 50$, $c_c = 3.5$	94
6.5	Relative Value of Contracting vs Contracting Cost (c_i) - Uniform Supply Uncertainty, Poisson Demand $b = 50$, $c_c = 3.5$	95

List of Tables

3.1	Summary of Notation	21
4.1	Computational Analysis-Parameter Set	39
4.2	Effect of K_c on the optimal policy. $U = 10$, $b = 50$, $c_c = 3.5$, $c_p = 2.5$ and $h = 1$	41
4.3	Effect of K_p on the optimal policy. $b = 50$, $c_c = 3.5$, $c_p = 2.5$ and $h = 1$	42
4.4	Effect of U on the optimal policy. $b = 8$, $c_c = 3.5$, $c_p = 2.5$, $K_c = 0$, $K_p = 10$ and $h = 1$	43
4.5	Effect of contingent labor cost and backordering cost on the optimal policy. $U = 10$, $c_p = 2.5$, $K_c = 0$, $K_p = 0$ and $h = 1$	44
4.6	Effect of demand variability on the optimal policy. $U = 10$, $c_c =$ 5.5 , $c_p = 2.5$, $K_c = 0$, $K_p = 0$ and $h = 1$	45
6.1	Average Inventory Level and Average Production with Contingent Capacity. $U = 6$, $V = 0$	80
6.2	Average Inventory Level and Average Production with Contracted and Contingent Capacity. $U = 10$, $V = 2$, $c_i = 1.2$	82

6.3	Optimal Contracted Capacity Level (V^*)-Uniform Supply Uncertainty.	85
6.4	Optimal Contracted Capacity Level (V^*). $U = 6$, $c_c = 3.5$	86
6.5	Optimal Contracted Capacity Level (V^*). $b = 50$, $c_i = 0.6$	86
6.6	Effect of Backordering Cost on Optimal Contracted Capacity Level. Poisson Demand, $c_c = 3.5$	87
6.7	Effect of Demand Uncertainty on Optimal Contracted Capacity Level. Poisson Demand, $U = 6$, $c_c = 3.5$, $b = 50$	88
6.8	Effect of Contingent Labor Cost and Uncertainty on Owned Capacity (U^* , V^*). $c_p = 2.5$, $c_i = 0.6$, $b = 50$	90
6.9	Effect of Demand Variability on Owned Capacity. $c_c = 3.5$, $c_i = 0.6$	90
6.10	Effect of Backordering Cost on Owned Capacity. $c_p = 2.5$, $c_i = 0.6$	91
6.11	Effect of Backordering Cost on VCC and $\%VCC$. $c_c = 3.5$, $c_i = 0.6$	95
6.12	Effect of Contingent Labor Cost on Value of Contracting. $b = 50$, $c_i = 0.6$	96
6.13	Effect of Contingent Labor Uncertainty on Value of Contracting. $c_c = 3.5$, $c_i = 0.6$	96
6.14	Effect of Demand Variability on the Value of Contracting. $b = 50$, $c_i = 0.6$	97
6.15	Heuristic Performance and Supply Uncertainty. ($b = 5, 10, 15$), ($c_c = 1.5, 3.5, 5.5$)	99
6.16	Heuristic Performance (Average Gap) with Increasing Coefficient of Variation. Gamma Demand, $b = 15$, $c_c = 3.5$	99

6.17	Heuristic Performance with Increasing Permanent plus Contracted Capacity. Uniform Supply Uncertainty, $b = 15$, $c_c = 3.5$	100
6.18	Heuristic Performance with Increasing Permanent plus Contracted Capacity. Binomial Uncertainty with Increasing Probability of Success $k = 20$, $b = 15$, $c_c = 3.5$	101
6.19	Heuristic Performance under different Backordering and Contingent Labor Costs. Uniform Supply Uncertainty, $U = 6$	102

Chapter 1

Introduction

Demand volatilities have significant impacts on companies in the manufacturing industry. Seasonal fluctuations in demand influence the production decisions of manufacturing companies. In make-to-stock environments, a conventional approach is holding safety stocks for coping with uncertainties in demand. An alternative approach could be switching manufacturing lines between seasonal products prior to seasonal demand peaks. Under the existence of non-stationarities in demand, dynamic capacity adjustments arise as an effective tool in addition to these approaches. Manufacturing companies may increase their production capacity temporarily by acquiring external capacity resources (e.g. outsourcing, short term machinery leasing, hiring temporary workers). Effective utilization of such resources may result in increased demand responsiveness and reduced operational costs. However external capacity may not always be available at the desired amount and/or quality in the environment. Therefore the manufacturer has to consider the effect of uncertainty in the supply of external capacity, when making the production plan. In this study, we consider the integrated planning of production, inventory and flexible capacity under demand and external capacity supply uncertainties.

Capacity flexibility within the context of this study is the ability of the manufacturer to temporarily increase the production capacity with the usage of external resources. The problem under consideration focuses on labor intensive production environments, therefore capacity will be analyzed under the workforce context. Capacity can be defined as the maximum amount of production that can be achieved by utilizing internal and external resources. Production capacity of the firm can be classified under two main categories: Permanent capacity and temporary capacity. Permanent capacity can be considered as the maximum amount of production that can be achieved by the company's owned workforce, which is under the steady payroll. External capacity resources in our context are the contingent workers that are temporarily acquired from an external labor supply agency (ELSA). External labor supply agencies provide contingent workers to firms, from its own pool of workers or by finding temporary workers from the environment. Contingent worker requests of the manufacturer may not be met in full terms by the ELSA due to two main reasons. The first reason is that the ELSA may not be capable of meeting the manufacturer's request due to the lack of availability or skill of temporary workers. In the second case however, the ELSA may not be willing to meet the request of the manufacturer considering possible better alternatives in the market. In both cases the contingent labor request of the manufacturer may be totally unmet or partially fulfilled. A labor supply contract that is signed between a manufacturer and an ELSA is a possible way to alleviate the impact of labor supply uncertainty on the manufacturer. On the other hand, such a contract would bring guaranteed sales to the ELSA. There may exist a variety of labor contracts. In our setting the labor supply contract between the manufacturer and the ELSA is specified as follows. The manufacturer pays a specific fee each period per contracted worker and the ELSA provides contracted workers to the manufacturer with certainty with an additional fee per worker requested. Under this setting, the notion of contracting that we present becomes similar to *option contracting*. The cost of contracting is analogous to the reservation cost and the utilization cost is analogous to the exercise cost of option contracts. The manufacturer may still request contingent workers in addition to the contracted workers, but such workers are subject to supply uncertainty. Under this setting, we classify external capacity resources

under two categories: Contracted workers and contingent workers.

In this study we focus on designing labor supply contracts as well as the simultaneous management of capacity, production and inventory under a periodic review make-to-stock environment with non-stationary stochastic demand and uncertain labor supply. The design of the contract includes the determination of the number of contracted workers and the fees paid for those workers. Permanent capacity levels of companies are not adjusted frequently. In general a strategic decision on the permanent capacity level is given at the beginning of the planning horizon and it is not changed until a significant shift in the environment. In our problem permanent and contracted capacity levels are determined at the beginning and fixed throughout the planning horizon. Nevertheless, we analyze the problem under an infinite planning horizon setting in order to reflect the long term benefits of integrated capacity and production management.

Demand seasonality is one of the main driving forces behind dynamic capacity adjustments. Seasonality trends affect a major portion of the manufacturing and service industries. To give a brief description, seasonality can be defined as the source of predictable and uncontrollable variations in demand, which occur on a recurrent basis. Seasonality has many causes. Among the most general ones are natural causes such as weather, periodic annual government actions such as taxation; new product introductions especially in automotive and electronics industries; and seasonal patterns resulting in periodic peaks such as the Christmas season. As seen in the automotive and electronics examples sometimes industries create their own seasonality. Among the periodic trigger factors are annual trade shows and new product releases.

Companies facing seasonal fluctuations in demand generally use the approach of building up inventory for meeting peak demands, however, keeping excessive inventory, especially under the requirement of high service levels, may be too costly. On the other hand, under the presence of apparent seasonality, utilizing

flexible capacity may become a better approach. Determining the production capacity is generally considered as a strategic or tactical level decision. However, under the presence of capacity flexibility, it is a crucial decision at the operational level as well. After the determination of the permanent production capacity at the tactical level, the responsibility of the operational level is the management of capacity by utilizing external resources when necessary.

Dynamic adjustments of the permanent capacity, such as hiring or firing, are generally too costly. Besides the social and motivational effects, such adjustments, tend to have a negative effects on the efficiency of workers. When making the capacity decisions, a company must be aware of the long-run pattern of fluctuations and must set the permanent level of capacity accordingly. Setting the permanent capacity levels too high may result in excessive labor costs while low levels of permanent capacity may result in backordering or additional capacity acquisition costs. Under an unstable market, the strategic decision of determining the permanent capacity level is generally difficult when considering the long period of circulation. However, when the pattern of seasonality and the stability of the market is known, permanent capacity levels can be set more effectively considering flexible capacity management. While such downright planning of permanent and external capacity levels may result in major reductions in long-run operational costs, even the operational decision of determining the appropriate temporary capacity level alone yields significant benefits. Presence of a tight competition in the aforementioned environments increases the incentive of using flexible capacity to cut down operational costs. Especially when the inventory holding and/or backordering costs are high, the firm may rely on temporary capacity for meeting the demand.

Considering the capacity as the workforce level, we focus on the utilization of flexible workforce as contingent (contracted or temporary) capacity. Throughout the world, the usage of temporary workforce drastically increased from the late 80's through the mid 90's [6]. US Bureau of Labor Statistics indicates that in February 2005 there were 10.3 million independent contractors (7.4 percent of

total employment), 2.5 million on-call workers (1.8 percent of total employment), 1.2 million temporary help agency workers (0.9 percent of total employment), and 813,000 workers provided by contract firms (0.6 percent of total employment) in USA [24]. Aside from the workers with alternative work arrangements as indicated above, contingent workers (temporary jobs) accounted for 4.1 percent of the total US employment. As another example, 6.6 percent of the active labor force of the Netherlands was composed of flexible workers (temporary, standby, replacement and others) in 2003 [5]. In March 2006, 7.9 percent of the active labor force in Turkey was composed of contingent workers [32].

Temporary workers can be hired anytime and they are generally paid for labor hours. Experience shows that it takes as short as 1-2 days to receive temporary workers from external resources, such as labor supply agencies. The wage rate for temporary workers tend to be lower than that of their permanent counterparts however their costs to the hiring firm are generally higher. Nevertheless we do not confine ourselves to such constraint in our analysis. Productivity of temporary workers may vary for industries requiring different levels of skill. However for jobs with little or no skill requirement, temporary workers may not cause a significant reduction in productivity.

Availability of temporary workers is a major concern when the manufacturer relies on external capacity for production. A manufacturer may not always acquire contingent workers in the desired quantity and/or quality. The underlying reasons for availability uncertainty of temporary workers can be as follows. In a tightly competitive environment finding the desired quantity of temporary workers with the required set of skills may not be possible as the demand for skilled labor ascends. On the other hand, a labor supply agency may consider more than one alternative at a time, in contemplation of maximizing its benefits. In this case a manufacturer's request for temporary workers may be rejected if there exists a better alternative for the ELSA. Labor supplier's behavior and the available worker pool size may induce an uncertainty for the availability of temporary workers. While an ELSA with a large pool size may avoid small sized requests,

another ELSA with a limited pool size may motivate small sized requests by offering higher availability rates. Another important factor affecting the availability of temporary workers is the labor demand in the market. In a season with intense labor demand it is difficult to find temporary workers with the required skill level in the desired quantities. The skill level required by the manufacturer is also a factor causing uncertainty in temporary workforce availability. The labor supply agency may not be capable of finding workers with the required skill in the desired quantity. In addition, the delivery lead time and the timing of the request can be considered as a source of availability uncertainty. An ELSA may not always be quick enough in responding to a firm's request.

Contracting with the ELSA for temporary workers is a good solution for abrogating the effect of availability uncertainty. Contracted workers are available to the manufacturer whenever requested. Contracting may increase costs, as the manufacturer pays a specific fee to the ELSA even if the contracted workers are not utilized. But under appropriate cost rates, it is an attractive tool for diminishing the effects of availability uncertainty. In this study we investigate different settings to find out the cases where the manufacturer is better off with contracting. We also determine the optimal contract sizes under such cases.

In this study we analyze the problem of determining the capacity levels simultaneously with the production decision within the aforementioned environment. We find the optimal operational parameters with an infinite horizon dynamic program under discounted cost criterion. The problem is analyzed under different supply uncertainty structures. The structure of the availability uncertainty is dependent on the amount of contingent capacity requested by the manufacturer. The uncertainty structures include the cases where the availability rate is increasing or decreasing in the number of contingent workers requested. The characteristics of the optimal policy are discussed for different settings under different supply uncertainty types. We also consider a special case of the problem where there is no supply uncertainty. We show that the finite horizon results of the problem (as discussed by Tan and Alp [31]) extend to infinite horizon. An

important result that we provide in this study is the value of contracts. In cases where contracting temporary workers is beneficial for the manufacturer, we elaborate on the value of the contract by the savings gained. Computational analysis shows that contracting saves up-to 12 percent of the expected total cost in the experimental settings considered. Finally, a comparison of the cases with and without the presence of labor supply uncertainty is made. We illustrate the effect of supply uncertainty on the contingent capacity usage.

The rest of the thesis is organized as follows. In Chapter 2, we provide an extensive review of the related literature. In Chapter 3 we introduce our model and the supply uncertainty structures. In Chapter 4, a special case of the problem, where contingent labor is available with certainty is analyzed in an infinite horizon context. In Chapter 5, we analyze the problem and provide the characteristics of the optimal policy. Optimal permanent and contracted capacity levels, value of optimal contracts, comparison of the optimal policy with a heuristic policy and the effect of labor supply uncertainty is provided by the numerical analysis in Chapter 6. Finally a summary of results, conclusions and possible extensions are given in Chapter 7.

Chapter 2

Literature Review

Capacitated production/inventory problems and capacity/workforce planning problems have been extensively studied in literature separately. However a combined dynamic approach is not yet thoroughly analyzed. The problem under consideration focuses on the simultaneous planning of capacity and production under the presence of demand and capacity uncertainty. Therefore it has strong interactions with the following fields: (i) integrated production/capacity planning, (ii) workforce planning and flexibility, (iii) capacitated production/inventory planning and (iv) production/inventory planning with random capacity/yield. Here we provide a brief summary of the most related works in these fields and discuss their similarities and differences with our problem.

Capacity planning has been analyzed extensively in all levels of decision making. While the earlier works generally focus on strategic and tactical perspective, recent studies integrate an operational approach. An in depth review on strategic capacity management is provided by Van Mieghem [22]. The paper presents the fundamentals of the formulation and solution of major capacity planning problems, concerning the size, type and timing issues in an environment with demand and/or capacity uncertainty. The author also focuses on the recent studies focusing on multiple capacity types.

Atamturk and Hochbaum [4] focus on an integrated capacity and production planning problem under a non-stationary deterministic demand setting. Authors exploit the trade-offs between capacity expansions, subcontracting and inventory holding. The study is precursory in integrating the cross-impacts of decisions affected by capacity expansions. We investigate a similar problem under stochastic demand and with flexible capacity management instead of capacity expansions. Angelus and Porteus [2] concentrate on a simultaneous capacity and production planning problem for short life-cycle products, under stochastically increasing and then decreasing demand structure. The study investigates capacity expansions as well as contractions at exogenously determined prices. Target interval policy is shown to be optimal in both cases, with or without inventory carry-over. Target interval policy requires the smallest possible change in the capacity level in each period in order to bring the available level into the target interval of that period. In case of no inventory carry-over, authors provide a detailed characterization of the target intervals. For the case of inventory carry-over a general result is established and then capacity and inventory are shown to be economic substitutes. Authors indicate that capacity target intervals are decreasing in the initial inventory level and the optimal base stock levels decrease in the capacity level. The notion of dynamic capacity adjustments and integrated production planning provided in this study is comparable to that of our model. One major distinction between this study and our work is that, it considers periodic hiring and firing decisions for adjusting capacity, whereas we consider utilizing external capacity resources for a fixed permanent capacity level.

Dallaert and de Kok [8] investigate the integrated capacity and production planning problem under a more flexible capacity setting. In their study, capacity is composed of long-term contract workers and temporary workers. Authors analyze the capacity and production planning problem in a decoupled approach and an integrated approach. In the decoupled approach, separate resource and production decisions are made in the following order. First the order up-to levels and production decisions are made and then the levels of regular and temporary workforce are determined accordingly. In the integrated approach however, decisions

are made simultaneously. It is shown that the integrated approach outperforms the decoupled approach. The model of flexible capacity management used in this model is similar to our setting. However this study initially solves the inventory problem and accordingly determines the capacity levels, whereas we consider an integrated approach, considering the uncertainty in temporary production capacity. Hu et al. [18] investigate the problem of minimizing the long-run average cost of holding inventory and/or purchasing extra capacity under demand fluctuations. The study considers a fixed permanent production capacity, which can be temporarily increased by utilizing contingent capacity. Authors model the problem on a continuous time framework under a Markov modulated high-low demand rates. It is shown that the optimal policy is of hedging point type, where two hedging levels are associated with each discrete state of the system. The positive hedging level corresponds to the inventory target, while the negative hedging level corresponds to the backlog level below which extra capacity should be purchased. In a similar problem, Tan and Gershwin [30] study production and subcontracting strategies with limited production capacity and fluctuating demand. The problem under investigation considers lead time sensitive customers, who choose not to order for excessive lead times. The firm has a set of subcontractors with different capacity levels in order to supplement the production capacity. Authors derive a hedging type policy, namely the *feedback policy*, for determining the production rate and the rate at which subcontractors should be requested. The model presented in this study has similarities with the system that we consider, however we investigate the problem under periodic review in an environment with supply uncertainty. Yang et al. [36] deal with the optimal production-inventory-outsourcing policy for a firm with Markovian production capacity facing independent stochastic demand and the option of outsourcing. The model does not associate a fixed cost for production but outsourcing has a fixed cost. The decision maker in this model decides on the amount to be outsourced for the next period after observing the realized internal capacity and the demand. Therefore the problem does not consider a simultaneous planning of production/capacity. Authors derive simple optimal policy forms under reasonable assumptions. For stochastically monotone Markovian capacity, policy parameters are shown to be decreasing in the firm's current capacity level under

some basic conditions. The authors show that the results hold for infinite horizon and undiscounted-cost cases and derive the conditions under which outsourcing is more valuable. The model in this problem offers outsourcing as a tool to reduce the effect uncertain internal capacity, on the other hand in our model we present uncertain capacity outsourcing as a tool for coping with demand fluctuations.

Tan and Alp [31] consider the management of flexible capacity, inventory and the determination of permanent capacity level under non-stationary stochastic demand in a finite planning horizon. This study constitutes the basis of our study. The model analyzed is based on production system where contingent workers are utilized on top of the permanent capacity, whenever required. The study considers cases with and without setup costs for initiating production and utilizing contingent capacity. Authors analyze the characteristics of the optimal policy for the integrated capacity/production planning problem and evaluate the value of utilizing flexible capacity under different settings. A state dependent order up-to type policy is shown to be optimal. Optimal order up-to points are shown to be non-decreasing in the initial inventory position. The study derives two critical points, one for production with permanent capacity and the other for production with permanent plus contingent capacity, which are shown to be non-increasing in the number of periods remaining. Our study is an extension of this study. We consider the availability uncertainty of contingent workers and introduce contracted workers as a source for diminishing the effects of uncertainty. Alp and Tan [1] consider the tactical planning of capacity under capacity flexibility in a periodic review make-to-stock environment with stochastic demand. Capacity flexibility in this model is provided by contingent workers which can be acquired whenever requested. The study derives optimal production parameters and the tactical capacity level and derives the value of flexibility under various conditions.

In the field of workforce planning and flexibility, Holt et al. [17] pioneer the research with the seminal work that presents models that analyze the trade-off between keeping large sized permanent workforce levels and frequent workforce level modifications for satisfying peak season demands under demand fluctuations.

This study is a milestone in introducing the notion of flexible capacity management. The model of flexible capacity management that we provide is an extension of this model with demand and supply uncertainty. Wild and Schneeweiss [35] analyze manpower capacity planning with a hierarchical approach. Optimization of the hierarchical model presented in the paper is done by using a stochastic dynamic program. The paper demonstrates the simultaneous application of variation of monthly working time, overtime, utilizing contingent workers and leasing of work force and compares their performances.

Milner and Pinker [23] analyze the design of labor supply contracts between firms and external labor supply agencies (ELSA). The contract under their study includes long term contracted workers and temporary workers, that will be provided to the firm which already has a specific level of permanent capacity. Authors analyze the problem in a single period environment, for both parties involved (firm and the ELSA). The study involves an uncertainty for temporary (non-contracted) workers provided by the ELSA under two different settings: Productivity uncertainty and availability uncertainty. Therefore the effect of uncertainty on contracts is analyzed under two different perspectives. In case of productivity uncertainty equilibrium contracts that coordinate both the firm and the ELSA in an optimal way exist. On the other hand under availability uncertainty, it is shown that it is not possible to construct optimal contracts. However there is still a wide region of parameters, where both parties are better-off with a contract. The paper investigates such set of parameters and trade-offs that should be considered in workforce contract negotiations. The model of flexible capacity management that we use is the same with the model introduced in this study. However we consider a multi-period setting where inventory carrying and backordering are allowed. Pinker and Larson [25], in a similar setting, focus on the determination of regular and contingent worker pool sizes for service industries from a more tactical perspective. The operational control parameters are determined with a stochastic dynamic programming approach, and the tactical level of permanent and temporary capacity are determined accordingly. In their study regular workers are defined as workers with fixed schedules, who can also

work overtime. Contingent workers on the other hand have flexible working hours specified by the contract over the finite planning horizon. The model represents a variety of contracts such as temporary workers, on-call workers with guaranteed minimum pay, and compensatory time arrangements. The model is formulated in order to minimize the expected labor and backlogging costs by optimal planning of regular and contingent worker levels. The problem includes the optimal planning of operational staffing decisions, such as overtime and capacity utilization. The study also reflects the effects of information timing on the benefits of flexibility. This study considers a multi-period flexible capacity planning problem from the service perspective. The decision of overtime and the number of contingent workers to be utilized is given according to the workload in the system. We study a similar flexible capacity management problem from the manufacturing perspective, where backordering and inventory carryover is allowed. The pool of contingent workers mentioned in this study is similar to the concept of contracted workers in our study. One major distinction is that we consider the availability of additional contingent workers with supply uncertainty.

In the field of capacitated inventory/production management, Federgruen and Zipkin [12] provide a fundamental analysis under stochastic demand. The study shows the optimality of base stock type policies when there exists no setup costs of production. Kapuscinski and Tayur [20] extend the results of Federgruen and Zipkin [12] by proving the optimality of the base stock policy under finite horizon, discounted infinite horizon and infinite horizon average cases for general Markovian, periodic demand. The authors use a simulation based technique to compute the optimal parameters and indicate several properties of the optimal policy. Gallego and Scheller-Wolf [13] consider the same problem under the existence of fixed order costs. By generalizing the K-convexity as CK-convexity, authors provide a partial characterization of the optimal policy. The optimal policy is shown to have an (s,S)-like structure. To characterize the optimal policy, the parameter space is divided into four regions. While in two of these regions the optimal policy is completely characterized, in the other two, it is partially specified. The study suggests the existence of a simpler optimal policy structure, however the

authors indicate that under specific parameter sets there still could be regions between the two critical points s and s' , where starting and stopping ordering can be optimal in succession. The problem considered in this study is a special case of our problem, where no contingent workers are available. In Chapter 5, we analyze the special case of our problem where the contingent workers are available with certainty. Our analysis shows that the optimal policy changes for different regions of starting inventory level as in Gallego and Scheller-Wolf [13], however the characteristics of the optimal policy are different.

When we consider the field of production/inventory planning under random capacity/yield, Silver [29] pioneers the research with the analysis of optimal order quantities when the arriving quantities are random under deterministic demand. The study extends the economic order quantity formulation with supply uncertainty in two different modes; in the initial case the variation of the quantity received is independent of the quantity ordered Q , whereas in the second it is dependent on Q . The study indicates that the optimal order quantity depends only on the standard deviation of the amount received. Yano and Lee [37] review the literature of lot sizing under random production or procurement yields. The study presents various methods in modeling of cost, yield uncertainty and performance in the context of systems with random yields. The literature investigated in this study considers a single source for production/procurement which is subject to a random upper limit (capacity), where the decision under question is the order quantity. Authors investigate the structural results of the previous works, indicate their shortcomings and provide directions for future research.

Ehrhardt and Taube [9] focus on the single period inventory model with random replenishment quantities and random demand. The replenishment quantity is considered as a random function of the amount ordered. The study characterizes the structure of optimal policies for linear ordering costs. For the case of no setup costs the optimal policy is shown to be of base stock type. Under the existence of setup costs, authors indicate that the optimal policy is of (s, S) type. The uncertainty model that we develop for the contingent workers is analogous to

the uncertainty model used for the replenishment quantities in this study. Wang and Gerchak [33] consider continuous review inventory control under variable capacity. The study analyzes the effects of variable capacity on optimal lot sizing in continuous review environments. Authors consider two models; the basic EOQ model and the order quantity/reorder point model with backlogging. For both cases, the optimality conditions for generally distributed variable capacity are presented. The analysis indicates that the optimal order quantities for both models and the optimal reorder point for the second model, are greater than those without variable capacity. The study also provides effective procedures for finding optimal solution for exponentially distributed variable capacity. Erdem and Ozekici [11] consider a single item inventory model in a randomly changing environment under periodic review. Model parameters are specified by the state of the environment which is determined by a time-homogenous Markov chain. Random yield is due to the random capacity of the vendor. The problem is analyzed in single period, multiple periods and infinite horizon. In all cases it is shown that the optimal policy is an order up-to level policy, which is dependent on the state of the environment. Their analysis show that the order up-to levels are identical with the results of the certain yield (infinite supply) case in the single period. However the multi-period and infinite horizon results show that the presence of an uncertain yield results in order up-to levels greater than or equal to that of the model with certainty.

Ciarallo et. al. [7] analyze the optimality of extended myopic policies under uncertain capacity and uncertain demand in a periodic review production planning system. In the single period analysis, it is shown that the random capacity does not affect the optimal policy but results in a unimodal, non-convex cost function. Optimal policies for single period are drawn by classical convexity arguments due to the unimodality. In the multi-period case despite the non-convex cost function optimal policies are shown to be of order up-to type for finite and infinite horizon settings. The results indicate that the optimal policy and the critical numbers depend on the distribution of capacity uncertainty. Authors indicate the benefit of extended myopic policies under the consideration of review

period of uncertain length. The uncertainty of production capacity specified in this study is analogous to that of our model. However in our study, production capacity is also considered as a decision variable, and it can be increased by utilizing contracted and contingent workers. On the other hand this study considers the production level as a decision variable which is constrained by the uncertain production capacity. Another important distinction is that we consider the availability uncertainty only for contingent workers and assume that the owned production capacity is always fully available, whereas this study considers the uncertainty of the whole production capacity. Iida [19] studies a non-stationary periodic review dynamic production inventory model with uncertain production capacity and uncertain demand. The author derives upper and lower bounds for the optimal order up-to levels and shows that the upper and lower bounds of the finite horizon problem converge in the infinite horizon problem. This study is analogous to our work as it considers production planning under uncertain capacity and uncertain non-stationary demand. However in our study we consider production capacity as a decision variable, whereas this study considers the uncertainty within the owned capacity of the firm and decides only on the production level.

Wang and Gerchak [34] combine the concepts of variable capacity and random yield in a periodic review production planning model with uncertain demand. The cost minimization problem under investigation is solved by a stochastic dynamic program. Authors prove that the finite horizon objective function is quasi-convex and the resulting optimal policy is a single critical point policy. The model provided in their study is a generalization of a previous model by Henig and Gerchak [16] which only considers random yields. Results of this study indicate that the finite horizon solutions converge to infinite horizon results. The study indicates that when both random yield and variable capacity are present at the system the optimal policy is of reorder point type, whereas with the presence of variable capacity alone it is of order-up-to type. Erdem et. al. [10] focus on a continuous review inventory system with deterministic demand and multiple suppliers with variable capacities. In this model the quantities received from suppliers are

equal to the ordered quantities if the realized capacities are sufficient, otherwise the firm receives the maximum amount that the realized capacity provides. The study aims to specify the optimal order quantities while diversifying the risk of shortage, by utilizing n suppliers. Each supplier in this model has a specific lead time and a continuous distribution of capacity. Gullu [14] focuses on base stock policies for production/inventory problems with uncertain capacity levels. The periodic review model differs from previous research in the sense that it gives a discrete time environment. Post production inventory level is either the quantity ordered (if the capacity permits) or the inventory level before production plus the full available capacity. The system is analyzed under expected average cost per period criterion. The system operates under a stationary modified base stock policy. The main contribution of this study is that it establishes an analogy between the class of base stock production/inventory policies that operate under demand/capacity uncertainty and G/G/1 queues and their associated random walks. The problem investigated in this study has a strong resemblance with our model in essence, however the definitions of capacity result in a fundamental distinction. In our study we define the production capacity as the total of permanent, contracted and contingent capacity, and introduce the number of contracted and contingent workers to be utilized as a decision variable. The uncertainty in the production capacity in our study arises from the uncertainty of the contingent capacity. On the other hand Gullu [14] considers the production level under variable production capacity as the decision.

Kouvelis and Milner [21] analyze the joint effects of demand and supply uncertainty on capacity and outsourcing decisions in multi-stage supply chains. Authors analyze the investment of a firm on the two stages of a supply chain, in which the first stage is responsible of the core activities of the firm while the second is responsible for the non-core activities. The objective is maximizing the multi-period discounted profit. The study discusses the effect of non-stationary stochastic demand on the outsourcing decisions and the effect of non-stationary supply, due to the market demand, on the capacity decisions. Characterization of the single period and the multi-period optimal capacity investment decisions

have been provided and the effects of changes in supply and demand uncertainty on the extent of outsourcing have been analyzed. The analysis shows that as the supply uncertainty increases capacity investments increase. On the other hand increasing demand uncertainty results in an increase on the reliance to outsourcing. Results indicate that if the demand is independent and identically distributed for all periods then it is not optimal to modify capacity after the first period. The model considered in this study has similarities with our model, however the capacity investments considered in this study are less dynamic, considering that the changes are made in the beginning of critical periods as expansions with a fixed cost paid only at the time of investment. Also the outsourcing option presented in this model differs from our decision of contingent capacity.

Gullu et. al. [15] analyze a periodic review, single-item inventory model under supply uncertainty. The authors model the uncertainty in supply using a three-point probability mass function; supply is either fully available, partially available or unavailable. The study shows the optimality of order up-to type policies by using a stochastic dynamic programming formulation. Schmitt and Snyder [28] examine a model with supply uncertainty from the outsourcing perspective. In their model supply is subject to regular variance as well as complete disruptions. Authors analyze the model under two different cases. In the initial case the unreliable supplier is the only option, whereas in the second case there exists a second reliable supplier with a higher cost rate. The study determines optimal order and reserve quantities. The case with a second reliable supplier is similar to our model, where we have uncertain contingent workers and certain owned capacity (permanent and contracted workers). Both studies provide important information on the trade-off between supply uncertainty and the cost of capacity/supply certainty. However our work differs from their study, as we plan the flexible workforce and determine the production after realizing the capacity. Arikan et. al. [3] provide an analysis of a periodic review capacitated inventory system with supply uncertainty where advance demand information is available. Authors assume three different supply processes, namely all or nothing type, partial availability and binomial availability. The all or nothing type

contingent labor supply uncertainty model that we analyze reduces to the all or nothing supply uncertainty problem modeled in this study, as capacity is fully utilized in our optimal production policy. A major distinction of our work from this study is that we determine the optimal production quantity after realizing the capacity, therefore requested capacity is not always fully utilized, whereas in this study the realized amount of supply is always used. In addition we consider the utilization of contracted workers for reducing the effects of supply uncertainty.

All in all, the problem that we analyze is closely related with the aforementioned fields. We analyze the integrated capacity and production planning problem from a workforce planning perspective. The studies in the field of flexible capacity management up to now have assumed that there is an ample supply of external capacity that can be acquired at a specific cost. There exists recent studies considering uncertainty of contingent workers, however the problem of flexible capacity management has not yet been analyzed with labor supply uncertainty in a production environment, where inventory carryover is allowed. Our major contribution to the literature is that we introduce the supply uncertainty of contingent workers in a production environment and illustrate the trade-off between holding inventory and utilizing external capacity. Another key contribution of our study is that we introduce temporary worker contracts for diminishing the effects of uncertainty. Under the presence of contracts, we analyze the cost of contingent capacity uncertainty and determine the amount that the manufacturer is willing to pay to abrogate it. On the other hand, in the literature of production/inventory management under random yield there is a single decision, which is the production/order quantity. We introduce capacity and production decisions separately but plan the levels simultaneously.

Chapter 3

Model Formulation

In this section, we provide a dynamic programming formulation that can be used to solve the integrated capacity and inventory management problem under consideration. We first present our basic definitions, assumptions and settings.

We define capacity position w as the total amount of capacity requested by the manufacturer. Capacity level is defined as the production capacity observed after the uncertainty is resolved, i.e. after the realization of the contingent capacity (if any) is observed. If the capacity position is less than or equal to the permanent production capacity plus the contracted capacity ($w \leq U + V$), the capacity level is equal to the capacity position. On the other hand, if $w > U + V$, then the capacity level is a number between $U + V$ and w . The permanent and contracted capacity levels are fixed and fully available throughout the planning horizon, but are decision variables to be determined at the beginning of the planning horizon. The unmet demand is fully backlogged. The costs under consideration are inventory holding and backordering costs, and unit costs of permanent, contracted and contingent capacity, which are all non-negative. We assume that there are no shortages of raw material and the lead time of production and acquiring external capacity can be neglected. There are no fixed costs for initiating production and no material related costs are considered in the model. The notation is summarized in Table 3.1. Further explanation of notation will be provided as need arises.

Table 3.1: Summary of Notation

T	: Number of periods in the planning horizon
U	: Size of available permanent capacity
V	: Size of available contracted capacity
c_p	: Unit cost of permanent capacity per period
c_i	: Unit preliminary cost (reservation cost) of contracted capacity per period
c_f	: Unit cost of utilizing contracted capacity (exercise cost) per period
c_c	: Unit cost of contingent capacity per period
h	: Inventory holding cost per unit per period
b	: Penalty cost per unit of backorder per period
α	: Discounting factor ($0 < \alpha \leq 1$)
w_t	: Capacity position in period t
N_t	: Contingent capacity called upon in period t
m_t	: Contingent capacity realized in period t
Q_t	: Number of items produced in period t
Z_t	: Random variable denoting the demand in period t
$G_t(w)$	: Distribution function of Z_t
$P_t(m, N)$	: Distribution function of m_t , dependent on the number called upon N_t
x_t	: Inventory position at the beginning of period t before production
y_t	: Inventory position in period t after production
$f_t(x_t)$	: Minimum total expected cost of operating the system in periods $t, t+1, \dots, T$, given the system state x_t

Throughout our analysis, we consider linear costs of capacity. The cost of permanent capacity is independent of the production quantity and paid each period even if there is no production. The unit cost of permanent capacity is c_p per period. Therefore the total permanent capacity cost for a workforce of size U is $c_p * U$ per period. In the particular contract type that we consider the unit cost of contracted workers (c'_w) is composed of two components; c_i and c_f , where $c'_i + c'_f = c'_w$. c'_i is the amount of money paid for each contracted worker at the start of every period, independent of their utilization. This cost component corresponds to the cost of keeping contracted workers. There is also an additional cost component c'_f , which is paid for each contracted worker utilized in production. We define unit production as the average number of products produced

by a permanent worker in a period in order to bring production and capacity into a line. The initial cost of contracted workers and the cost of production for contracted capacity and contingent capacity are also defined in the same unit basis. Considering that productivity rates of permanent, contracted and contingent workers differ in nature, let λ_c and λ_v be the productivity rates of a contingent worker and a contracted worker, respectively. The model works for all values of λ_c and λ_v , however we consider $0 < \lambda_c \leq \lambda_v \leq \lambda_p = 1$ as permanent workers are expected to be the most productive, whereas contingent workers are expected to be the least. Recalling that c'_i and c'_f are the initial cost and the utilization cost of a contracted worker respectively, the unit initial and utilization costs of a contracted worker can be written as $c_i = c'_i/\lambda_v$ and $c_f = c'_f/\lambda_v$, respectively, assuming that the productivity remains identical among the contracted workers. Similarly the unit cost of production for a contingent worker can be written as $c_c = c'_c/\lambda_c$, where c'_c is the unit cost of a contingent worker. The total initial cost of contracted workers of contracted capacity is equal to $c_i * V$ if the total contracted capacity is equivalent to V productive units. From now on permanent equivalent units will be considered for contracted and temporary workers. In particular N contracted/temporary workers will produce N units of product in a period.

The amount of contingent capacity received, m is distributed by $P(m, N)$, which depends on the number of contingent workers requested N . The firm can receive at most N and at least zero contingent workers depending on the type of the corresponding probability distribution. The total cost of contingent capacity is $m * c_c$ if the firm observes m contingent workers, regardless of whether they are utilized or not. Demand in period t , Z_t is distributed with a distribution function $G_t(z)$. The units left on hand after the realization of demand are held in inventory at a cost of h per unit per period. Unmet demand is backordered at a unit cost of b per period.

The capacity decision in period t , defined as the capacity position w_t , is made according to the starting inventory position x_t , and the distributions of demand

and contingent capacity availability, aiming the minimum total expected cost of operations for periods t and on. The production decision within the capacity position decision is bounded above by the observed capacity level. If the capacity position w , is set to a number less than $U + V$ (permanent and contracted) then there is a single corresponding production decision which is specified by minimizing the operating costs from period t and on. If the capacity position is set to a number above $U + V$, then an optimal production decision is to be given for each realization of the capacity level.

The order of events is as follows. At the beginning of each period t , the inventory level x_t is observed, and the capacity position decision is made as w_t . If $w_t \leq U + V$, then a production decision $Q_t \leq U + V$ is made to raise the inventory level to $x_t + Q_t = y_t \leq x_t + U + V$. If the capacity position $w_t > U + V$, then a production decision $Q_t \leq U + V + m_t$ is made to raise the inventory level to $x_t + Q_t = y_t \leq x_t + U + V + m_t$, where m_t is the observed contingent capacity. At the end of the period, the demand z_t is realized. If the demand is below the level on hand, then it is fully met and the leftover units are carried to the next period at a cost of h per unit. If the demand exceeds y_t , then it is backordered at a unit cost of b . The beginning inventory level for the next period is $x_{t+1} = y_t - z_t$. Accordingly the minimum cost of operating the system for periods t and on is denoted as $f_t(x_t)$. We assume that the the ending condition is $f_{T+1} = 0$. We solve our integrated capacity and inventory management problem using the following dynamic program:

$$f_t(x_t, U, V) = Uc_p + Vc_i + \min_{w_t: 0 \leq w_t} \{H_t(w_t|x_t)\} \text{ for } t = 1, 2, \dots, T$$

$$\text{and } f_0(U, V) = \min_{U \geq 0, V \geq 0} \{f_1(x_1, U, V)\}$$

where $H_t(w_t|x_t, U, V)$ is the minimum expected total cost of operation for periods t and on, for the capacity position w_t and starting inventory level x_t , and is given as follows:

$$H_t(w_t|x_t) = \begin{cases} \varphi_t(w_t|x_t) & \text{if } 0 \leq w_t \leq U \\ (w_t - U)c_f + \varphi_t(w_t|x_t) & \text{if } U < w_t \leq U + V \\ \gamma_t(w_t - U - V|x_t) & \text{if } U + V < w_t \end{cases} .$$

In function $H_t(w|x)$; $\varphi_t(\eta_t|x_t) = \min_{y_t: x_t \leq y_t \leq x_t + \eta_t} \{L_t(y_t) + \alpha E[f_{t+1}(y_t - z)]\}$ is the production decision function that attains the minimum total expected cost of operations, excluding the immediate labor costs, for periods $t, t+1, \dots, T$, where the observed capacity level is η_t and the starting inventory level is x_t and $\gamma_t(N|x) = Vc_f + \int_0^N (c_c m + \varphi(x + U + V + m))p(m, N)dm$ is the expected minimum cost of operations for periods t and on when N temporary workers are called upon.

In the above equations $L_t(y_t) = h \int_0^{y_t} (y_t - z_t)dG_t(z) + b \int_{y_t}^{\infty} (z_t - y_t)dG_t(z)$ is the regular convex loss function. Note that the optimal capacity position is independent of c_p and c_i since the permanent capacity and contracted capacity levels are determined at the beginning and fixed throughout the planning horizon.

Let $J_t(y_t) = L_t(y_t) + \alpha E[f_{t+1}(y_t - w_t)]$ be the total expected cost of operations, excluding the immediate labor costs, for periods t and on, when the inventory level after production is y_t and the starting inventory level is x_t . Note that $J_t(y_t)$ is independent of the starting inventory level x_t . If $J_t(y_t)$ is convex in y_t , then the optimal solution of the production problem $\varphi_t(\cdot)$ is known. Let $\hat{y}_t$ be the minimizer of $J_t(y_t)$ and y_t^* be the optimal inventory level after production under a realized capacity of η . Then it is well established in the literature that (Federgruen and Zipkin 1986):

$$y_t^* = \begin{cases} x + \eta & \text{if } x + \eta \leq \hat{y}_t \\ \hat{y}_t & \text{if } x \leq \hat{y}_t \leq x + \eta \\ x & \text{if } \hat{y}_t < x \end{cases} .$$

Recall that $f_{T+1} = 0$. Therefore the last period's (T) minimum cost of operations, excluding the immediate labor costs, for a capacity level of η_T can be written as,

$$\varphi_T(\eta_T|x_T) = \begin{cases} J_T(x_T + \eta_T) & \text{if } x_T + \eta_T \leq \hat{y} \\ J_T(\hat{y}) & \text{if } x_T \leq \hat{y} \leq x_T + \eta_T \\ J_T(x) & \text{if } \hat{y} < x_T \end{cases} \quad (3.1)$$

In such a case we can rewrite the cost function for utilizing temporary capacity in period T as follows:

$$\begin{aligned} \gamma_T(N|x_T) = & Vc_f + \int_0^N mc_p(m, N)dm + \int_0^{\hat{y}-U-V} L(x + U + V + m)p(m, N)dm \\ & + \int_{\hat{y}-U-V}^N L(\hat{y}_T)p(m, N)dm. \end{aligned}$$

This implies that, if the observed capacity level $U + V + m_T$ is not sufficient to produce up-to the optimal point $\hat{y}$ then it is fully utilized, Q_T is set to $U + V + m_T$ to bring the inventory level as close to $\hat{y}$ as possible, if $U + V + m \geq \hat{y}$ then the optimal production decision is producing up-to $\hat{y}$, not necessarily fully utilizing the observed capacity level.

Throughout our analysis we study the problem over an infinite planning horizon. The underlying reason for studying the problem over an infinite planning horizon is that tactical capacity decisions have a long validity unless an unpredictable shift in the market demand occurs. An infinite horizon analysis is also analytically convenient for this problem as it removes the ending condition. Another important fact supporting our infinite horizon analysis is that the finite horizon results converge to the infinite horizon results rather rapidly, in a time as short as 20 periods, as indicated in Chapter 4.

3.1 Supply Uncertainty Structures

The impact of supply uncertainty is investigated under different structures to reflect the effect of different behaviors of labor suppliers. Observations show that the behaviors of labor supply agencies differ according to the following factors.

- Size of available temporary worker pool
- Capability of finding skilled workers
- Competition in the environment
- Demand structure of different customers
- Opportunities in alternative options
- Information on the customer behavior

While an ELSA with a limited worker pool size in a competitive market may prefer moderate demands, in order to satisfy the demand from all customers, another ELSA in the same environment, having a relatively larger pool size, may prefer lower and higher demands and avert moderate demands by providing low availability rates at moderate requests, for maximizing its profit.

The fact that should be considered here is that the models of uncertainty that we provide are representative for different ELSAs and different environments, while they are not intended to reflect the rational decisions of ELSAs in such environment to the full extent. We use the following structures for modeling the supply uncertainty.

All or nothing type labor supply uncertainty: In the case of an all or nothing type uncertainty the firm receives the N contingent workers fully with probability p and does not receive any with probability $(1 - p)$. This is usually the case where the ELSA has better alternatives existing in the environments and

therefore rejects the offer of the firm for higher benefits from other alternatives. Here $1 - p$ can be considered as the probability of ELSA having better alternatives, and hence rejecting the offer.

Uniform labor supply uncertainty: Uniform labor supply uncertainty models an ELSA with a large pool size. However the capability of the ELSA in finding skilled workers is variable. In this type of uncertainty if N workers are requested, acquiring each additional worker has a probability $\frac{1}{N+1}$. Note that the expected number of workers acquired increases as the number requested increases. On the other hand the probability of acquiring each additional worker decreases as $\frac{1}{N}$ gets smaller. Under this model the firm has equally likely chance of acquiring 0 to N workers.

Normal labor supply uncertainty: We use normal distribution for an ELSA with a large pool size and a specific capability of finding workers with the required skill. If the firm requests N workers from the ELSA, the number of workers to be received is distributed with a normal distribution with mean $\mu \leq N$ and a standard deviation σ . As the number of workers requested (N) increases, expected number of workers received and the standard deviation increases linearly.

Binomial supply uncertainty with a decreasing probability of finding a worker: In this case the ELSA has a limited pool size and multiple customers with consistent demand rates. As the number requested from the ELSA increases, the probability of finding each worker decreases linearly. If the firm requests N workers, the number of workers acquired with a binomial distribution with rate of success equal to $\frac{K-N}{K}$, where K is the size of the ELSA's worker pool.

Binomial supply uncertainty with an increasing probability of finding a worker: Under this setting we model an ELSA, preferring larger sized demands. The ELSA avoids the division of its workforce by providing low availability rates for small number of workers requested. As the number requested

increases, the probability of acquiring each additional worker increases linearly. If the firm requests N workers, the number of workers acquired is distributed with binomial distribution with rate of success being equal to $\frac{N}{K}$, where K is the threshold limit specified by the ELSA. If the firm requests more than K workers the ELSA fully meets the demand.

Binomial supply uncertainty with higher success probability at low and high demands: This structure of uncertainty models an ELSA preferring demand levels that are either low or high enough. The underlying reason for such preference may be the competitive environment and the large pool size. The ELSA may deter from committing a moderate size of its workers to a firm, considering the chance of a larger sized demands from other customers. On the other hand small or sufficiently large demands are supplied at high rates. The number of workers acquired by the firm is distributed binomially with probability of acquiring each worker being $\frac{\sin(2\Pi N/(K+\Pi/2))+1}{2}$ where K is the threshold level over which the ELSA supplies the demand of the firm to the full extent.

Binomial supply uncertainty with higher success probability at moderate demands: Under this uncertainty structure we model an ELSA with a limited pool size preferring moderate demand. The ELSA motivates moderate demand by providing high availability rates at the specified interval. Lower demands are not preferred by the ELSA due to attractive benefits in a secondary market, while higher demands have a lower chance of being met due to the scarcity of supply. The number of workers acquired is distributed binomially with probability of finding each worker being $\frac{\cos(2\Pi N/(K-\Pi))+1}{2}$, where K is the pool size of the ELSA.

Chapter 4

A Special Case: Deterministic Supply

In this chapter we analyze the special case of the integrated flexible capacity and inventory management problem, in which the contingent workers are available with certainty. In this setting we also include non-negative setup costs for initiating production K_p and utilizing contingent workers K_c . Throughout this chapter we assume that contracting temporary workers is not allowed, $V = 0$. Our main purpose is to investigate the structure of the optimal policy and to show that the finite horizon results presented by Tan and Alp [31] hold in the infinite horizon. We use the *Markov Decision Process* [27] approach for solving the infinite horizon problem. In obtaining the solution, we use a dynamic programming based *value iteration algorithm*, which stops in finite number of iterations. The algorithm is presented in detail in the following section of this chapter.

To prove the convergence of finite horizon optimal policies in the infinite horizon, we present the necessary conditions and show that they hold under our model. The method and the necessary conditions are based on the work of Porteus [26] on conditions for characterizing optimal strategies in infinite horizon dynamic programs.

When the contingent workers are available with certainty, the capacity decision becomes equivalent to the production decision. Therefore the formulation of the problem changes. We use the following formulation for solving the special case of the problem with supply certainty and non-negative setup costs:

$$f_t(x_t) = U c_p + \min_{y_t: x_t \leq y_t} \{H_t(y_t|x_t)\} \text{ for } t = 1, 2, \dots, T$$

where

$$H_t(y_t|x_t) = \begin{cases} L(x_t) + E[f_{t+1}(y_t - z)] & \text{if } y_t = x_t \\ K_p + L(y_t) + E[f_{t+1}(y_t - z)] & \text{if } x_t < y_t < x_t + U \\ K_p + K_c + c_c(y_t - U - x) + L(y_t) \\ \quad + E[f_{t+1}(y_t - z)] & \text{if } x_t + U < y_t \end{cases}$$

Note that K_p is paid as far as there is production, and K_c is only paid when the number of contingent workers utilized is greater than 0. In the rest of this chapter we prove the convergence of finite horizon optimal policy structure in the infinite horizon and analyze the optimal policy under different settings.

4.1 Model in Operator Form

In this section we present the *operator approach* that we use in proving the convergence of finite horizon results in the infinite horizon. We begin by transforming the elements of our dynamic program into operators and vectors.

For theoretical considerations, we consider the seasonality as an element of the state space, therefore the operators are constructed in a stationary fashion. The state space is assumed to be finite by using lower and upper bounds on the inventory level. This assumption does not result in any structural changes in the optimal policy. The operators are designed such that a transition can occur from

a state in the current season only into a state in the next season. Let Ω_t be our two dimensional state space, with one dimension being the inventory level and the other being the season of the system. We present our analysis on a seasonal problem with a cycle of 4 periods. Results can be extended to any given seasonal problem easily. We denote the state of the system with a two dimensional vector (s, x) , where $s \in \{1, 2, 3, 4\}$ and x being the integer inventory level.

We denote the admissible policy space for period t as Δ_t . An admissible policy in our setting is such that it brings the inventory level to $y_t \geq x_t$, before the realization of that period's demand. Consequently the admissible strategy space is denoted as $\Pi = \Delta_1 \times \Delta_2 \times \dots \times \Delta_T$. Note that a decision in period t , $\delta_t \in \Delta_t$, only considers order up-to levels for corresponding states in the current season. Therefore a decision δ_t can be considered as a vector containing order up-to levels (not necessarily optimal) for all starting inventory levels.

Let v_t^n be the value vector of all states at period t through period n . For operational simplicity we consider v as a vector of length $4 \times I_x$, where 4 is the number of seasons and I_x is the number of states (inventory levels). Using the value vector we can define our policy operator for a specific policy δ as $H_t(\delta)$ such that:

$$v_t(\delta) = H_t(\delta)v_{t+1} = r_\delta + \lambda P_\delta v_{t+1}$$

In the above equation, r_δ is the vector of immediate costs resulting from decision δ and P_δ is the corresponding transition matrix, e.g. if the decision δ raises the inventory level to $y > x + U$, then the corresponding entry in the vector is $K_p + K_c + c_c(y - U - x) + L(y)$. The vector r_d has 0 entries for all states in the other seasons, since no cost is incurred for those states. The transition matrix $P_{\delta(t)}$ contains the probabilities of transition from the order up-to levels to the next seasons starting inventory level. The matrix is composed of the current season's demand probabilities for the states in the current season to the next season. For

the states in the other seasons the values are preserved.

The operation $v_t = H_t(d)v_{t+1}$ can be illustrated in the following form, for the current season being 1:

$$r_d + \lambda P_{d(t)} v_{t+1} = \begin{bmatrix} r_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \alpha & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & P'_{d(t)} & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}.$$

λ is the discounting vector, in the above equation. Note that only the values of the inventory levels in the current season are discounted. The matrix $P'_{d(t)}$ in $P_{d(t)}$ is the transition matrix consisting of demand probabilities for the current season. Note that the matrix $P'_{d(t)}$ multiplies the values of inventory levels that can be reached from the current order up-to level. Therefore the above operation only changes the value of the states in season 1 and preserves the value of states in the other seasons.

4.2 Conditions on the Convergence of Finite Horizon Results in the Infinite Horizon

In this section we provide the necessary conditions for the convergence of the finite horizon optimality results in the infinite horizon. We present the conditions under three main categories: Finite horizon conditions, structured attainment condition and infinite horizon conditions.

4.2.1 Finite Horizon Conditions

(a) $H_t(\pi)$ is isotone on V_{t+1} for each π and t ; i.e., $u, v \in V_{t+1}$, and $u \leq v$ implies $H_t(\pi)u \leq H_t(\pi)v$.

This assumption holds for our model since the transition matrix $P_{\delta(t)}$ and the cost vector r_δ are non-negative (our policy operator $H_t(\delta)$ adds up identical costs for corresponding states and linearly discounts the value of the state to be entered), for all policies $\delta \in \Delta$. Therefore $u \leq v$ implies $Hu \leq Hv$ for all value vectors in V_t for all t .

(b) There exists a real number α_t (not necessarily less than one) for each t such that $H_t(\pi)$ is α_t -Lipschitzian on V_{t+1} ; i.e., $\|H_t(\pi)u, H_t(\pi)v\|_{l_\infty} \leq \alpha_t\|u, v\|_{l_\infty}$, for $u, v \in V_{t+1}$.

$\|\cdot\|_{l_\infty}$ in the above inequality is the l_∞ -norm, which is equal to the absolute value of the greatest difference between the elements of the two vectors.

Condition (b) holds for our model as our operator adds finite costs and linearly discounts the next periods value v_{t+1} . Using the properties of extended real numbers and extended real functions it is trivial to see that H is α_t -Lipschitzian.

If $\|u, v\|_{l_\infty} \neq \infty \rightarrow \|H_t(u), H_t(v)\|_{l_\infty} \neq \infty$

Since $\|\cdot\|_{l_\infty}$ is an extended real function.

(c) For each $t, \varepsilon > 0$, and $v \in V_{t+1}$, there exists $\delta \in \Delta_t$ such that

$$\|H_{t\delta}v, A_tv\|_{l_\infty} \leq \varepsilon.$$

where A_t is the optimal policy operator.

The condition holds for our model since the optimal policy operator A_t is an element of the set of all policy operators in period t , in other words there exists an operator $H_t(\delta^*)$ such that $H_t(\delta^*)v \leq H_t(\delta)v$ for all $\delta \in \Delta$. This implies from the solution of the dynamic program:

$$f_t^{(i)}(x_t) = U c_p + \min_{y_t: x_t \leq y_t} \{H_t^{(i)}(y_t|x_t)\}.$$

where

$$H_t^{(i)}(y_t|x_t) = \begin{cases} L(x_t) + E[v_{t+1}(y_t - z)] & \text{if } y_t = x_t \\ K_p + L(y_t) + E[v_{t+1}(y_t - z)] & \text{if } x_t < y_t < x_t + U \\ K_p + K_c + c_c(y_t - U - x) + L(y_t) \\ \quad + E[v_{t+1}(y_t - z)] & \text{if } x_t + U < y_t \end{cases}$$

Note that we replaced the optimal value function $f_{t+1}(\cdot)$ with an arbitrary value function v_{t+1} . The solution of $f_t^{(i)}$ exists and attained by a policy $\delta \in \Delta$ for all t and all finite v . This implies that the condition is satisfied for all $\varepsilon \geq 0$.

(d) The optimal policy operator A_t is a mapping from V_{t+1} to V_t , i.e. $A_t : V_{t+1} \rightarrow V_t$

This condition holds from the definition of our policy operator H_t . H_t maps values in V_{t+1} to V_t for all $\delta \in \Delta$, and the optimal policy operator A_t is an element of the set of all policy operators in period t .

(e) There exist sequences $\{V_t^0\}$ and $\{u_t\}$ such that

$$\phi \neq V_t^* \subset V_t, u_t \in V_t^0, \text{ and} \\ v_{n+1}^n(\pi) = u_{n+1}, \text{ i.e., } v_t^n(\pi) = H_t^n(\pi)u_{n+1}.$$

Condition (e) holds as the value in period t under the strategy π is equal to value obtained by applying the policy operator $H_t(\pi)$ to a finite ending value u_{n+1} , n times. In our case with seasonality $H_t(\pi)$ differs for distinct seasons, however this does not contradict with the above definition.

(f) There exists a sequence $\{V_t^*\}$ such that

$$\begin{aligned} \phi &\neq V_t^* \subset V_t^0; \\ \text{for each } v \in V_{t+1}^* \text{ and } \varepsilon > 0, \text{ there exists } \delta \in \Delta_t^* \text{ such that} \\ ||H_{t\delta}v, A_tv||_{l_\infty} &\leq \varepsilon; \end{aligned}$$

and for each t there exists N such that $n > N$ implies: $A_t^n : V_{n+1}^0 \rightarrow V_t^*$

4.2.2 Structured Attainment Condition

For each t and $v \in V_{t+1}^*$ there exists $\delta \in \Delta_t^*$ such that $H_{t\delta}v = A_tv$.

Finite horizon condition (e) holds, therefore the finite horizon value functions v_t are equal to the operators applied to terminal-value functions v_{t+1}^* . From the finite horizon condition (c), we know that there exists a decision δ in period t such that the value obtained by applying the optimal operator is attained. Therefore there exists a structured policy δ in the set of structured policies Δ^* that attains the value obtained by applying the optimal policy operator A_t for all $v \in V_{t+1}$.

4.2.3 Infinite Horizon Conditions

For each t the following conditions hold:

(a) V_t^* is complete.

According to our definition of H_t this assumption holds since an optimal policy operator should be from V_{t+1}^* to V_t^* so every limit of the Cauchy sequence is within the set.

(b) There exists real numbers M_t and $N (= N_t)$ such that $\|H_t^n(\pi)u, H_t^n(\pi)v\|_{l_\infty} \leq M_t$ for all $n \geq N$, $\pi \in \Pi_t^*$, and $u, v \in V_{n+1}^0$.

This condition hold for all finite values u and v as our policy operator is a contraction mapping for all $\alpha < 1$. Then there exist finite M_t and N such that the l_∞ -norm converges to M_t for different ending values $u, v \in V_{n+1}^0$.

(c) There exists a sequence $\{\alpha_t^n; n \geq t\}$ of real numbers such that $H_t^n(\pi)$ is $\alpha_t - Lipschitzian$ on V_{n+1} for all $\pi \in \Pi_t^*$ and $n \geq t$, and

$$\alpha_t^n M_{n+1} \rightarrow 0, \text{ as } n \rightarrow \infty.$$

Our finite horizon assumption (b) and the optimal policy operator being a contraction mapping for $\alpha < 1$ implies that this condition holds.

4.2.4 Results

If finite horizon and infinite horizon assumptions hold then the following hold:

(a) For each t and $\varepsilon > 0$, there exists N and a strategy $\pi \in \Pi^*$ that is $\varepsilon - optimal$ for stages k thorough n and also satisfies

$$\rho(f_k^\infty, v_k^n(\pi)) \leq \varepsilon,$$

Simultaneously for all $k \leq t$ and $n \geq N$.

(b) $f_t = f_t^\infty = \sup_{\pi \in \Pi^*} \nabla_t(\pi)$, for each t .

(c) f_t is structured and independent of the sequence of regular terminal value functions for each t .

(d) The optimality equations hold:

$$f_t = A_t f_{t+1}, \text{ for each } t.$$

- (e) f_t is the unique sequence of regular value functions that satisfies (d).
 (f) The optimality criterion holds: If a strategy $\pi \in \Pi^*$ satisfies

$$H_t(\pi)f_{t+1} = A_tf_{t+1}, \text{ for each } t, \text{ then } \pi \text{ is optimal.}$$

- (g) If structured attainment holds, then there exists an optimal structured strategy.

The above results imply that the optimal strategy in the finite horizon converges in the infinite horizon. In other words there exists an attainable structured optimal policy in the infinite horizon case.

4.3 Value Iteration Algorithm

An important consequence of the convergence of the optimal structured policies in the infinite horizon is the convergence of the value iteration algorithm for solving the optimality equation. The value iteration algorithm finds a stationary ε optimal policy. ε -optimality occurs if one of the following occur:

- The minimum is not attained.
- The algorithm is terminated in finite number of iterations.

In our case the minimum is attained, however for computational purposes, we stop the algorithm in finite number of iterations assuring the ε -optimality. Under this assumption, the policy operator T satisfies $Tv = \min_{d \in D} \{r_d + \lambda P_d v\}$. Therefore the stopping condition in step 3 of the algorithm holds. The main step of the algorithm is step 2, where the recursion $v^{n+1} = Tv^n$ occurs.

Value Iteration Algorithm

Step 1 Select $v^0 \in V$, specify $\varepsilon > 0$ and set $n = 0$.

Step 2 For each $s \in S$, compute $v^{n+1}(s)$ by

$$v^{n+1}(s) = \min_{a \in A_s} \{r(s, a) + \sum_{j \in S} \alpha p(j|s, a) v^n(j)\}.$$

Step 3 If $\|v^{n+1} - v^n\| < \varepsilon(1 - \alpha)/2\alpha$ go to step 4. Otherwise increment n by 1 and return to step 2.

Step 4 For each $s \in S$, set

$$d^\varepsilon(s) = \operatorname{argmin}_{a \in A_s} \{r(s, a) + \sum_{j \in S} \alpha p(j|s, a) v^{n+1}(j)\}.$$

and stop.

If the argmin is not unique, any action achieving this minimum can be selected.

Note that the algorithm is equivalent to applying an operator which is optimal for each period t . $v^n(s)$ in the above algorithm is the value of state s when there are n periods remaining in the planning horizon, $r(s, a)$ is the cost of decision a for state s , S is the set of states that can be reached after decision a and $p(j|s, a)$ is the probability of transition to state j after decision a in state s .

4.4 Analysis of the Infinite Horizon Optimal Policy

In this section we analyze the structure of the optimal policy under stationary demand. The underlying reason for using stationary demand is to abrogate the effects of demand fluctuations on the structure of optimal policy, and to illustrate the characteristics that arise from the nature of the problem. For the sensitivity analysis of the optimal policy under different cost parameters we use a poisson demand distribution with mean 10. For the analysis of the sensitivity in demand variability we use gamma distributed demands with mean 10 and coefficient of variation 0.5, 1.0 and 1.5. We have conducted our computational analysis on the set of parameters described in Table 4.1.

Table 4.1: Computational Analysis-Parameter Set

K_p	0, 10, 20, 30, 40
K_c	0, 10, 20, 30, 40
U	2, 4, 6, 8, 10, 12, 14, 16
b	2, 5, 8, 11, 50
c_c	1.5, 3.5, 5.5
α	0.95, 0.99

Setup costs have an important impact on the structure of the optimal policy. When both of the setup costs are equal to zero the optimal policy has three critical points, namely y_c^* , $x + U$ and y_p^* , which refer to production with permanent plus contingent capacity, production with full permanent capacity and production with permanent capacity, respectively. However, as the fixed cost for initiating production becomes positive, the number of critical points vary depending on both of the fixed costs K_p and K_c . For example in a setting, with parameters $K_p = 10$, $K_c = 0$, $b = 8$, $c_c = 3.5$ and $U = 14$, the optimal policy has 5 critical points. The optimal production level in this case is non-monotonic due to the setup cost for initiating production, K_p . In this case for $x < -4$ the optimal order up-to level is equal to $y_c^* = 9$ and the optimal production level is decreasing in x . For $-4 \leq x < 2$ the optimal production level is U and the corresponding order up-to level is equal to $y = x + U$. If $2 \leq x < 8$ the optimal order up-to level is $y_{p1}^* = 15$ and the optimal production is decreasing in x . At $x = 8$ the optimal policy is producing at full permanent capacity up-to $y = x + U$, at this point the optimal production level and the optimal order up-to level increase together. The manufacturer utilizes the permanent capacity fully in order to compensate for the positive setup cost, K_p . For $x \geq 9$ the optimal order up-to level becomes $y_{p2}^* = 22$ and the optimal production level decreases in the starting inventory level. This example is a good illustration of the non-monotonicity in the optimal production level, arising from positive setup costs for production and utilizing contingent capacity. Analogously the setup cost of utilizing contingent capacity also results in multiple critical points for production with contingent plus permanent capacity. When there is no setup cost for initiating production, critical numbers y_c^* and y_p^* are increasing in the setup cost for utilizing contingent workers, K_c .

For positive and small values of K_p (≤ 10), if the permanent capacity level (U) is below the expected demand (10), then y_c^* is increasing and $y_p^* = x + U$ is decreasing in K_c , if $U \geq 10$, then both y_c^* and y_p^* are increasing in K_c . For $K_p \geq 20$, y_c^* is increasing and $y_p^* = x + U$ is decreasing in K_c . Table 4.2 illustrates the impact of K_p and K_c on the optimal policy. The table indicates the optimal order up-to levels y for the corresponding intervals of starting inventory level x . Our analysis indicates that the number of optimal critical points are not directly sensitive to K_c . Note that for larger values of K_p the manufacturer produces at full capacity and only when the inventory level is below a threshold level in order to compensate the cost of K_p (Table 4.3). The table depicts the optimal order up-to levels and the starting y^* and the starting inventory level x that y^* is valid from and on to the next critical point.

The optimal policy structure is significantly affected by the setup cost of production, K_p . As the setup cost of production increases, the number of optimal critical numbers decrease. However there is an exception; if the permanent capacity level is equal to the average demand then the number of optimal critical numbers increase as K_p becomes positive and then decreases with increasing K_p . Optimal parameter y_c^* is increasing in K_p , while y_p^* decreases as K_p becomes positive and is superseded by $x + U$ for $K_p \geq 20$ (see Table 4.3).

The level of permanent capacity is an important factor effecting optimal policy parameters. As the level of permanent capacity increases the need for holding inventory and/or utilizing contingent capacity decreases. Therefore increasing permanent capacity, U , decreases the levels of optimal control parameters y_c^* and y_p^* (see Table 4.4). The level of permanent capacity does affect the number of optimal critical points, however there does not exist an increasing or decreasing relationship. Nevertheless the number of optimal critical points are the most when the capacity is significantly above the average demand ($U \geq 14$) and there exists setup costs for production. For moderate levels of backordering cost, there

Table 4.2: Effect of K_c on the optimal policy. $U = 10$, $b = 50$, $c_c = 3.5$, $c_p = 2.5$ and $h = 1$.

K_p	K_c	y_c^*	for $x \in$	$x + U$	for $x \in$	y_p^*	for $x \in$	—	
0	0	16	$[-50, 7)$	$x + U$	$[7, 10)$	19	$[10, 19)$	—	
	10	17	$[-50, 4)$	$x + U$	$[4, 14]$	21	$(14, 21)$		
	20	17	$[-50, 3)$	$x + U$	$[3, 13)$	22	$[13, 22)$		
	30	18	$[-50, 3)$	$x + U$	$[3, 13)$	22	$[13, 22)$		
	40	19	$[-50, 3)$	$x + U$	$[3, 14)$	23	$[14, 23)$		
	K_c	y_c^*	for $x \in$	$x + U$	for $x \in$	y_p^*	for $x \in$	$x + U$	for $x \in$
10	0	16	$[-50, 7)$	$x + U$	$[7, 11)$	20	$[11, 20)$	—	—
	10	17	$[-50, 4)$	$x + U$	$[4, 13)$	22	$[13, 15)$	$x + U$	$[15, 20)$
	20	17	$[-50, 3)$	$x + U$	$[3, 14)$	23	$[14, 16)$	$x + U$	$[16, 20)$
	30	18	$[-50, 3)$	$x + U$	$[3, 15)$	24	$[15, 16)$	$x + U$	$[16, 21)$
	40	19	$[-50, 3)$	$x + U$	$[3, 15)$	24	$[15, 17)$	$x + U$	$[17, 22)$
	K_c	y_c^*	for $x \in$	y_p^*	for $x \in$	—			
20	0	16	$[-50, 7)$	17	$[7, 17)$	—			
	10	17	$[-50, 4)$	14	$[4, 14)$				
	20	18	$[-50, 3)$	13	$[3, 13)$				
	30	18	$[-50, 3)$	13	$[3, 13)$				
	40	19	$[-50, 3)$	13	$[3, 13)$				
30	0	16	$[-50, 7)$	17	$[7, 17)$	—			
	10	17	$[-50, 4)$	14	$[4, 14)$				
	20	18	$[-50, 3)$	13	$[3, 13)$				
	30	19	$[-50, 3)$	13	$[3, 13)$				
	40	20	$[-50, 3)$	13	$[3, 13)$				

Table 4.3: Effect of K_p on the optimal policy. $b = 50$, $c_c = 3.5$, $c_p = 2.5$ and $h = 1$.

	K_p	y_c^*	for $x \in$	$x + U$	for $x \in$	y_p^*	for $x \in$	$x + U$	for $x \in$
$K_c = 0$ $U = 8$	0	17	[-50,10)	$x + U$	[10,16)	23	[16,23)	—	—
	10	17	[-50,10)	$x + U$	[10,12)	—	—	—	—
	20	17	[-50,10)	$x + U$	[10,13)	—	—	—	—
	30	17	[-50,10)	$x + U$	[10,13)	—	—	—	—
	40	26	[-50,14)	—	—	—	—	—	—
$K_c = 10$ $U = 8$	0	19	[-50,6)	$x + U$	[6,20)	27	[20,27)	—	—
	10	19	[-50,6)	$x + U$	[6,10)	—	—	—	—
	20	19	[-50,6)	$x + U$	[6,12)	—	—	—	—
	30	19	[-50,6)	$x + U$	[6,12)	—	—	—	—
	40	28	[-50,7)	$x + U$	[7,13)	—	—	—	—
$K_c = 20$ $U = 8$	0	21	[-50,6)	$x + U$	[6,22)	29	[22,29)	—	—
	10	21	[-50,6)	$x + U$	[6,10)	—	—	—	—
	20	21	[-50,6)	$x + U$	[6,11)	—	—	—	—
	30	22	[-50,6)	$x + U$	[6,12)	—	—	—	—
	40	29	[-50,6)	$x + U$	[6,12)	—	—	—	—
$K_c = 30$ $U = 8$	0	23	[-50,5)	$x + U$	[5,24)	31	[24,31)	—	—
	10	23	[-50,5)	$x + U$	[5,9)	—	—	—	—
	20	23	[-50,5)	$x + U$	[5,10)	—	—	—	—
	30	23	[-50,5)	$x + U$	[5,11)	—	—	—	—
	40	31	[-50,6)	$x + U$	[6,12)	—	—	—	—
$K_c = 0$ $U = 10$	0	16	[-50,7)	$x + U$	[7,10)	19	[10,13)	—	—
	10	16	[-50,7)	$x + U$	[7,11)	20	[11,13)	—	—
	20	16	[-50,7)	$x + U$	[7,12)	—	—	—	—
	30	16	[-50,7)	$x + U$	[7,12)	—	—	—	—
	40	17	[-50,8)	$x + U$	[8,12)	—	—	—	—
$K_c = 10$ $U = 10$	0	17	[-50,4)	$x + U$	[4,12)	21	[12,14)	—	—
	10	17	[-50,4)	$x + U$	[4,13)	22	[13,14)	$x + U$	[14,16)
	20	17	[-50,4)	$x + U$	[4,12)	—	—	—	—
	30	17	[-50,4)	$x + U$	[4,12)	—	—	—	—
	40	18	[-50,4)	$x + U$	[4,13)	—	—	—	—
$K_c = 20$ $U = 10$	0	17	[-50,4)	$x + U$	[4,13)	22	[13,15)	—	—
	10	17	[-50,4)	$x + U$	[4,14)	23	[14,15)	$x + U$	[15,17)
	20	18	[-50,4)	$x + U$	[4,14)	—	—	—	—
	30	18	[-50,4)	$x + U$	[4,15)	—	—	—	—
	40	19	[-50,4)	$x + U$	[4,15)	—	—	—	—
$K_c = 30$ $U = 10$	0	18	[-50,4)	$x + U$	[4,13)	22	[13,15)	—	—
	10	18	[-50,4)	$x + U$	[4,15)	24	[15,16)	$x + U$	[16,18)
	20	18	[-50,4)	$x + U$	[4,15)	—	—	—	—
	30	19	[-50,4)	$x + U$	[4,15)	—	—	—	—
	40	20	[-50,4)	$x + U$	[4,15)	—	—	—	—

Table 4.4: Effect of U on the optimal policy. $b = 8$, $c_c = 3.5$, $c_p = 2.5$, $K_c = 0$, $K_p = 10$ and $h = 1$.

U	y_c^*	for $x \in$	$x + U$	for $x \in$	y_{p1}^*	for $x \in$	$x + U$	for $x \in$	y_{p2}^*	for $x \in$
2	14	[-50,13)	$x + U$	[13,17)	—	—	—	—	—	—
4	14	[-50,11)	$x + U$	[11,16)	—	—	—	—	—	—
6	14	[-50,9)	$x + U$	[9,15)	—	—	—	—	—	—
8	13	[-50,9)	$x + U$	[9,15)	—	—	—	—	—	—
10	12	[-50,3)	$x + U$	[3,10)	19	[10,12)	$x + U$	[12,14)	—	—
12	10	[-50,-1)	$x + U$	[-1,5)	16	[5,10)	—	—	—	—
14	9	[-50,-4)	$x + U$	[-4,2)	15	[2,8)	$x + U$	[8,9)	22	[9,22)
16	9	[-50,-6)	$x + U$	[-6,0)	15	[0,5)	$x + U$	[5,7)	22	[7,22)

exists up to 5 optimal critical points. In that case, the manufacturer utilizes the permanent capacity fully in two distinct regions. The first region is the transition from production with permanent and contingent capacity to production with permanent capacity, which is common for all settings with $c_c > 0$. In the second region the manufacturer changes the order up-to level from a $y_p^* < x + U$ to $x + U$ in order to utilize the capacity fully. This is a direct effect of the production setup cost, K_p as the manufacturer wants to utilize the capacity more efficiently. As K_p increases this effect becomes more evident. Note that in this case there are two distinct critical points y_{p1}^* and y_{p2}^* for production with permanent capacity. Our analysis on stationary demand indicates that there does not exist multiple optimal critical points for production with contingent plus permanent capacity, however in the case with demand seasonality, observations show that there exist more than one critical point for production with contingent capacity.

Backordering cost is another factor having an important effect on the values and the structure of the optimal policy. Optimal parameters y_c^* and y_p^* are increasing in the backordering cost as expected. The number of optimal critical points is neither decreasing nor increasing in the backordering cost. However it is the highest when the backordering cost is at moderate levels ($5 \leq b \leq 11$), and there exists a positive setup cost for production. The cost of contingent capacity affects the incentive of the manufacturer to utilize contingent capacity. As the

Table 4.5: Effect of contingent labor cost and backordering cost on the optimal policy. $U = 10$, $c_p = 2.5$, $K_c = 0$, $K_p = 0$ and $h = 1$.

b	c_c	y_c^*	for $x \in$	$x + U$	for $x \in$	y_p^*	for $x \in$
8	1.5	13	$[-50, 4)$	$x + U$	$[4, 6)$	15	$[6, 15)$
	3.5	12	$[-50, 3)$	$x + U$	$[3, 8)$	17	$[8, 17)$
	5.5	12	$[-50, 3)$	$x + U$	$[3, 9)$	18	$[9, 18)$
11	1.5	14	$[-50, 5)$	$x + U$	$[5, 7)$	16	$[7, 16)$
	3.5	13	$[-50, 4)$	$x + U$	$[4, 8)$	17	$[8, 17)$
	5.5	13	$[-50, 4)$	$x + U$	$[4, 8)$	18	$[9, 18)$
50	1.5	16	$[-50, 7)$	$x + U$	$[7, 9)$	18	$[9, 18)$
	3.5	16	$[-50, 7)$	$x + U$	$[7, 10)$	19	$[10, 19)$
	5.5	15	$[-50, 6)$	$x + U$	$[6, 12)$	21	$[12, 21)$

level of c_c increases, the manufacturer relies less on contingent capacity and more on permanent capacity. Consequently, y_c^* declines and y_p^* increases. Table 4.5 depicts the effect of contingent labor cost and backordering cost on the optimal policy.

Demand variability has significant effect on the values of optimal critical numbers. As the variability increases the manufacturer tends to increase the inventory levels by increasing the optimal order up-to levels y_p^* and y_c^* . One exception is the case with considerably low backordering cost ($b = 2$). In that particular case y_p^* is still increasing in demand variability, however y_c^* is decreasing. The underlying reason is that the manufacturer prefers backordering rather than utilizing contingent capacity, which is more costly, as a response to increasing demand variability. Table 4.6 illustrates the effect of demand variability under different levels of the backordering cost.

Table 4.6: Effect of demand variability on the optimal policy. $U = 10$, $c_c = 5.5$, $c_p = 2.5$, $K_c = 0$, $K_p = 0$ and $h = 1$.

b	Demand	y_c^*	for $x \in$	$x + U$	for $x \in$	y_p^*	for $x \in$
2	Gamma(0.5)	8	[-50,-1)	9	[-1,12)	21	[12,21)
	Gamma(1.0)	3	[-50,-6)	4	[-6,16)	25	[16,25)
	Gamma(1.5)	0	[-50,-9)	1	[-9,16)	25	[16,25)
8	Gamma(0.5)	14	[-50,5)	15	[5,18)	27	[18,27)
	Gamma(1.0)	15	[-50,6)	16	[6,28)	37	[28,37)
	Gamma(1.5)	12	[-50,3)	13	[3,34)	43	[34,43)
50	Gamma(0.5)	22	[-50,13)	23	[13,27)	36	[27,36)
	Gamma(1.0)	34	[-50,25)	35	[25,49)	58	[49,58)
	Gamma(1.5)	44	[-50,35)	45	[35,66)	75	[66,75)

Chapter 5

Analysis of the Model

The problem characteristics demonstrate significant differences under distinct supply uncertainties. The structure of the optimal policy and the relation between the capacity and production decisions vary as the type of uncertainty changes. In this chapter, we provide analytical results on the problem for the all or nothing and the uniform supply uncertainty cases. Structure of the optimal policy is analyzed and numerically interpreted for the cases that are analytically intractable. Results are supported with related graphics. We show that under an all or nothing type uncertainty if the specified condition is satisfied, the resulting cost function is convex and consequently the optimal policy is of state dependent order up-to type. In the case of uniform supply uncertainty, we show that the single period cost function is convex in the capacity position w , while the multi-period cost function is observed to be non-convex. Under the other uncertainty types we observe that the problem is non-convex both in single period and multi-period cases. However our observations indicate that the minimum expected total cost function is unimodal under all types of supply uncertainty. Throughout the analysis in this chapter, we do not consider the sunk costs, namely the cost of permanent capacity (Uc_p) and the preliminary cost of contracted capacity (Vc_i), without loss of generality as they do not have any impact on the structural analysis. The effect of those cost parameters will be mentioned in the analysis of tactical capacity decisions.

5.1 All or Nothing Type Contingent Capacity Uncertainty

In the case of an all or nothing type labor supply uncertainty, we show that our problem reduces to the problem of determining production levels and the corresponding capacity levels. Recall that the firm receives the contingent workers requested fully with probability p and does not receive any contingent worker with probability $(1 - p)$. If the expected cost of a marginal increase in the capacity position with a contingent worker is greater than or equal to the cost of a marginal increase by utilizing contracted workers then we show that the problem is convex in single and multi-period cases.

Theorem 1 *If the expected marginal cost of increasing the capacity position by an additional contingent worker is higher than the marginal cost of utilizing a contracted worker $pc_c \geq c_f$, then (i) the multi-period decision function $H_t(w|x)$ is convex in w , (ii) a state dependent order up-to type policy is optimal and (iii) optimal order up-to levels are independent of p . Moreover the optimal order up-to levels can be stated as:*

$$y_t^*(x_t) = \begin{cases} y_{tc}^* & \text{if } x_t < y_{tc}^* - U - V \\ x_t + U + V & \text{if } y_{tc}^* - U - V < x_t \leq y_{tv}^* - U - V \\ y_{tv}^* & \text{if } y_{tv}^* - U - V < x_t \leq y_{tv}^* - U \\ x_t + U & \text{if } y_{tv}^* - U < x_t \leq y_{tp}^* - U \\ y_{tp}^* & \text{if } y_{tp}^* - U < x_t \leq y_{tp}^* \\ x_t & \text{if } y_{tp}^* < x_t \end{cases}$$

where y_{tp}^* , y_{tv}^* and y_{tc}^* are three critical numbers that are fixed for each period t , and refer to production with permanent capacity only, production with permanent and contracted capacity only, and production with permanent, contracted and contingent capacity, respectively.

Proof

We begin by proving the convexity of the single period cost function.

$H(w|x)$ =: single period minimum expected total cost of operations when the capacity position is w and the starting inventory position is x .

$$H(w|x) = \begin{cases} \varphi(w|x) & \text{if } 0 \leq w \leq U \\ (w - U)c_f + \varphi(w|x) & \text{if } U < w \leq U + V \\ \gamma(w - U - V|x) & \text{if } U + V < w \end{cases} \quad (5.1)$$

where $\varphi(w|x) = \min_{y: x \leq y \leq x+w} \{L(y)\}$ for $w \geq 0$ and

$$\gamma(N|x) = Vc_f + p(c_c N + \varphi(U + V + N|x)) + (1 - p)\varphi(U + V|x) \text{ for } N \geq 0 \quad (5.2)$$

Note that the same results hold for the last period T , under the assumption $f_{T+1}(\cdot) = 0$. We begin our analysis of the single period problem by analyzing the three regions ($w \leq U$, $U < w \leq U + V$, $U + V < w$).

I. ($0 \leq w \leq U$)

Let $\hat{y}$ be the minimizer of the convex function $L(y)$. Then by using equations (3.1) and (5.1) we can write $H(w|x)$ as,

$$H(w|x) = \begin{cases} L(x + w) & \text{if } 0 \leq w \leq \hat{y} - x \\ L(\hat{y}) & \text{if } \hat{y} - x < w \leq U \\ L(x) & \text{if } \hat{y} < x \end{cases}.$$

The convexity of $H(w|x)$ in this region follows from the convexity of $L(y)$. Note that the resulting function is a piecewise linear convex function.

II. ($U < w \leq U + V$)

We can rewrite $H(w|x)$ for this region as: $H(w|x) = (w - U)c_f + \min_{y: x \leq y \leq x+w} \{L(y)\}$, then by equation (3.1)

$$H(w|x) = \begin{cases} (w - U)c_f + L(x + w) & \text{if } U < w \leq \hat{y} - x \\ (w - U)c_f + L(\hat{y}) & \text{if } \hat{y} - x < w \leq U + V \end{cases}.$$

Note that the above function is a linear function $((w - U)c_f)$ plus a convex function $(L(x + w))$ for $U \leq w \leq \hat{y} - x$ and a linear function $((w - U)c_f)$ plus a constant $(L(\hat{y}))$ for $\hat{y} - x < w \leq U + V$ which are both convex. Now we show that the convexity is preserved at the transition point $w = \hat{y} - x$. To do so, we check the first derivative of $H(w|x)$:

$$\frac{dH(w|x)}{dw} = \begin{cases} c_f + L'(x + w) & \text{if } U \leq w \leq \hat{y} - x \\ c_f & \text{if } \hat{y} - x < w \leq U + V \end{cases}$$

Note that $L'(x + w)$ is negative for $w \leq \hat{y} - x$ and is equal to 0 at $w = \hat{y} - x$. Therefore the first derivative of $H(w|x)$ in $U \leq w \leq \hat{y} - x$ is always smaller than in $\hat{y} - x < w \leq U + V$, thus the convexity is preserved within this region.

III. ($U + V < w$)

By using equations (3.1), (5.1) and (5.2) we have,

$$H(w|x) = \begin{cases} Vc_f + p((w - U - V)c_c + L(x + w)) \\ \quad + (1 - p)L(x + U + V) & \text{if } U + V < w \leq \hat{y} - x \\ Vc_f + p((w - U - V)c_c + L(\hat{y})) \\ \quad + (1 - p)L(\psi(x)) & \text{if } \hat{y} - x < w \end{cases}$$

where

$$\psi(x) = \begin{cases} x + U + V & \text{if } x \leq \hat{y} - U - V \\ \hat{y} & \text{otherwise} \end{cases}.$$

Note that $\psi(x)$ is constant in w , therefore it does not have an effect on the convexity. We analyze the first and second derivatives of $H(w|x)$ to show the convexity of $H(w|x)$ within this region.

$$\frac{dH(w|x)}{dw} = \begin{cases} pc_c + pL'(x + w) & \text{if } U + V \leq w \leq \hat{y} - x \\ pc_c & \text{if } \hat{y} - x < w \end{cases}$$

$$\frac{d^2H(w|x)}{dw^2} = \begin{cases} pL''(x + w) & \text{if } U + V \leq w \leq \hat{y} - x \\ 0 & \text{if } \hat{y} - x < w \end{cases}$$

The first derivative condition is satisfied as $pc_c + pL'(x + w) \leq pc_c$ since the derivative of the convex function $L'(x + w)$ is negative for $x + w < \hat{y}$. The second

derivative is non-negative throughout the region, therefore $H(w|x)$ is convex in this region. To conclude the convexity of $H(w|x)$, we must prove that the convexity is preserved at transitions from permanent capacity to contracted capacity and from contracted capacity to contingent capacity, for all starting inventory levels x .

If we call regions $w \leq U$, $U < w \leq U+V$, $U+V < w$ as *I*, *II* and *III* respectively and denote $H(w|x)$ in the corresponding region with a subscript, the following first order condition is sufficient for concluding the convexity of $H(w|x)$.

$$\frac{dH_I(w|x)}{dw} \leq \frac{dH_{II}(w|x)}{dw} \leq \frac{dH_{III}(w|x)}{dw}$$

$H(w|x)$ is convex in all three regions. Therefore we only need to look at transition points $w = U$ and $w = U + V$. Since $H(w|x)$ is dependent on the starting inventory level, the condition must be satisfied for all values of x .

For the transition between region I and region II it is sufficient to analyze the first derivative of $H(w|x)$ for values of x below and above $\hat{y} - U$:

if $x \leq \hat{y} - U$

$$\frac{dH(w|x)}{dw} = \begin{cases} L'(x+w) & \text{if } w \leq U \text{ (I)} \\ c_f + L'(x+w) & \text{if } U \leq w \leq \hat{y} - x \text{ (II)} \end{cases}$$

if $x > \hat{y} - U$

$$\frac{dH(w|x)}{dw} = \begin{cases} 0 & \text{if } \hat{y} - x < w \leq U \text{ (I)} \\ c_f & \text{if } U \leq w \leq U + V \text{ (II)} \end{cases}$$

The above equations show that the condition holds for non-negative c_f .

If we analyze the transition between region II and region III:

if $x \leq \hat{y} - U - V$

$$\frac{dH(w|x)}{dw} = \begin{cases} c_f + L'(x+w) & \text{if } U < w \leq U + V \text{ (II)} \\ pc_c + pL'(x+w) & \text{if } U + V < w \leq \hat{y} - x \text{ (III)} \end{cases}$$

if $x > \hat{y} - U - V$

$$\frac{dH(w|x)}{dw} = \begin{cases} c_f & \text{if } \hat{y} - x < w \leq U + V \text{ (II)} \\ pc_c & \text{if } U + V < w \leq \hat{y} - x \text{ (III)} \end{cases}$$

Note that for the transition condition to hold, pc_c must be greater than c_f . Therefore $H(w|x)$ is convex for all x if $pc_c \geq c_f$.

After proving the convexity of the decision function, we now characterize the optimal policy of the single period problem. Recall that,

$$H(w|x) = \begin{cases} \varphi(w|x) & \text{if } 0 \leq w \leq U \\ (w - U)c_f + \varphi(w|x) & \text{if } U \leq w \leq U + V \\ Vc_f + pc_c((w - U - V) + \varphi(w|x)) \\ \quad + (1 - p)\varphi(U + V|x) & \text{if } U + V < w \end{cases}.$$

If $\hat{y} - U \leq x < \hat{y}$ then the value of w minimizing $H(w|x)$ is in region (I) and it is the minimizer of $L(x + w)$. From the classical newsboy solution we derive the optimal capacity position as:

$$w^*(x) = \hat{y} - x = G^{-1}\left(\frac{b}{h+b}\right) - x = y_p^* - x.$$

From now on we call the minimizing order up-to point in this region as y_p^* .

If $y_p^* - U - V < x < y_p^* - U$, then the minimizer of the function $H(w|x)$ is in region (II). Particularly the minimizer of $H(w|x)$ is the value of w which minimizes $(w - U)c_f + L(x + w)$. Using the optimality condition we get:

$$0 = c_f + L'(x + w).$$

Note that the optimality equation may not be satisfied even if x is in the above region. In that case the marginal cost of utilizing a contracted worker is higher than the benefit gained by marginal production by the contracted worker at $w = U$, therefore it is optimal to produce at full permanent capacity, $w^* = U$, the resulting order up-to level is $x + U$. On the other hand, if the marginal cost of utilizing a contracted worker is smaller than the benefit gained by marginal production then the minimizer of $H(w|x)$ is the minimizer of $(w - U)c_f + L(x + w)$. From the solution of the optimality equation the optimal capacity position is $w^*(x) = G^{-1}\left(\frac{b-c_f}{b+h}\right) - x$ and the corresponding order up-to level is $y_v^* = G^{-1}\left(\frac{b-c_f}{b+h}\right)$. Note that for non-negative c_f , $y_v^* \geq y_p^*$. The optimal capacity policy for this particular region can be summarized as:

$$w^*(x) = \begin{cases} y_v^* - x & \text{if } x \leq y_v^* - U \\ U & \text{if } y_v^* - U \leq x < y_p^* - U \end{cases}.$$

If $x < y_v^* - U - V$ then the minimizer of $H(w|x)$ is in region *III*. The value minimizing $H(w|x)$ is the value of w minimizing $Vc_f + p(c_c(w - U - V) + L(x + w)) + (1 - p)L(x + U + V)$. Using the optimality condition we get:

$$0 = pc_c + pL'(x + w).$$

Note that the optimality may not be satisfied in this region. The reason is that the expected marginal cost of utilizing a contingent worker pc_c is higher than the expected benefit of marginal production at $w = U + V$. Therefore in such a case it is optimal to produce with full permanent and contracted capacity, $w^* = U + V$. The resulting optimal order up-to level is $x + U + V$. If the expected marginal cost of utilizing a contingent worker is smaller than the marginal benefit, then the optimal capacity position is the minimizer of $Vc_f + p(c_c(w - U - V) + L(x + w)) + (1 - p)L(x + U + V)$. From the optimality equation we derive the optimal capacity position as $w^*(x) = G^{-1}(\frac{b-c_c}{b+h}) - x$ and the corresponding order up-to level as $y_c^* = G^{-1}(\frac{b-c_c}{b+h})$. Note that for $c_c > c_f$, $y_c^* < y_v^*$. The optimal capacity policy for this region is:

$$w^*(x) = \begin{cases} y_c^* - x & \text{if } x \leq y_c^* - U - V \\ U + V & \text{if } y_c^* - U - V \leq x < y_v^* - U - V \end{cases}.$$

for $0 < c_f < c_c$, the optimal values for the above functions have the following relation:

$$y_p^* > y_v^* > y_c^* \quad (5.3)$$

Using the above property we characterize the single period optimal policy. The resulting policy is of state dependent order up-to type. Optimal order up-to

level $y^*(x)$, is in the following form:

$$y^*(x) = \begin{cases} y_c^* & \text{if } x < y_c^* - U - V \\ x + U + V & \text{if } y_c^* - U - V < x \leq y_v^* - U - V \\ y_v^* & \text{if } y_v^* - U - V < x \leq y_v^* - U \\ x + U & \text{if } y_v^* - U < x \leq y_p^* - U \\ y_p^* & \text{if } y_p^* - U < x \leq y_p^* \\ x & \text{if } y_p^* < x \end{cases}.$$

Note that the order up-to level y_c^* is only realized if the ordered contingent workers are acquired.

After proving the convexity of the single period cost function, we prove the multi-period convexity of the problem by showing that the 2-period problem is convex and the optimal policy is of state dependent order up-to type. In order to prove the convexity of the multi-period problem, we begin by showing that the minimum expected cost of operations for starting inventory level x , $f(x)$, is convex in x . As stated earlier we can take the single period function as of the last period, if we assume that $f_{T+1} = 0$.

$$f(x) = \begin{cases} Uc_p + Vc_i + Vc_f \\ \quad + p(c_c(y_c^* - U - V - x) + L(y_c^*)) \\ \quad + (1 - p)L(x + U + V) & \text{if } x < y_c^* - U - V \\ Uc_p + Vc_i + Vc_f + L(x + U + V) & \text{if } y_c^* - U - V < x \leq y_v^* - U - V \\ Uc_p + Vc_i + (y_v^* - U - x)c_f + L(y_v^*) & \text{if } y_v^* - U - V < x \leq y_v^* - U \\ Uc_p + Vc_i + L(x + U) & \text{if } y_v^* - U < x \leq y_p^* - U \\ Uc_p + Vc_i + L(y_p^*) & \text{if } y_p^* - U < x \leq y_p^* \\ Uc_p + Vc_i + L(x) & \text{if } y_p^* < x \end{cases}$$

If we take the first derivative we get:

$$\frac{df(x)}{dx} = \begin{cases} -pc_c + (1-p)L'(x+U+V) & \text{if } x < y_c^* - U - V \\ L'(x+U+V) & \text{if } y_c^* - U - V < x \leq y_v^* - U - V \\ -c_f & \text{if } y_v^* - U - V < x \leq y_v^* - U \\ L'(x+U) & \text{if } y_v^* - U < x \leq y_p^* - U \\ 0 & \text{if } y_p^* - U < x \leq y_p^* \\ L'(x) & \text{if } y_p^* < x \end{cases}$$

If we numerate the regions defined above from 1 to 6 and indicate $f(x)$ in a specific region with the corresponding subscript, the first order condition for $f(x)$ can be written as:

$$\frac{df_1(x)}{dx} \leq \frac{df_2(x)}{dx} \leq \frac{df_3(x)}{dx} \leq \frac{df_4(x)}{dx} \leq \frac{df_5(x)}{dx} \leq \frac{df_6(x)}{dx}$$

The first order condition is satisfied as we provide the following consequences using the optimality conditions of y_c^* , y_v^* and y_p^* .

- the first inequality holds as $c_c + L'(x+U+V) \geq 0$ for $x \geq y_c^* - U - V$.
- the second inequality holds as $c_f + L'(x+U+V) \leq 0$ for $x \leq y_v^* - U - V$
- the third inequality holds as $c_f + L'(x+U) \geq 0$ for $x \geq y_v^* - U$
- the fourth inequality holds as $L'(x+U) \leq 0$ for $x \leq y_p^* - U$
- the last inequality holds as $L'(x) \geq 0$ for $x \geq y_p^*$

$$\frac{d^2f(x)}{dx^2} = \begin{cases} (1-p)L''(x+U+V) & \text{if } x < y_c^* - U - V \\ L''(x+U+V) & \text{if } y_c^* - U - V < x \leq y_v^* - U - V \\ 0 & \text{if } y_v^* - U - V < x \leq y_v^* - U \\ L''(x+U) & \text{if } y_v^* - U < x \leq y_p^* - U \\ 0 & \text{if } y_p^* - U < x \leq y_p^* \\ L''(x) & \text{if } y_p^* < x \end{cases}$$

The second order condition for $f(x)$ is satisfied for all starting inventory levels, thus $f(x)$ is convex in x . If $f(x)$ is convex in x then we know that the expectation $E[f(y - z)]$ is convex in y .

The two period problem can be written as:

$$H_2(w|x) = \begin{cases} \varphi_2(w|x) & \text{if } 0 \leq w \leq U \\ (w - U)c_f + \varphi_2(w|x) & \text{if } U < w \leq U + V \\ \gamma_2(w - U - V|x) & \text{if } 0 \leq w \leq U \end{cases}$$

where $\varphi_2(w|x) = \min_{y:x \leq y \leq x+w} \{L(y) + \alpha E[f(y - z)]\}$ and

$$\gamma_2(N|x) = Vc_f + p(c_c N + \varphi_2(U + V + N|x)) + (1 - p)\varphi(U + V|x)$$

Let $J_2(y) = L(y) + \alpha E[f(y - z)]$ be the two period production cost function excluding the immediate labor costs. As stated earlier, if $J_2(y)$ is convex then the resulting policy is of order up-to type. In our case $J_2(y)$ is convex, since both $L(y)$ and $E[f(y - z)]$ are known to be convex. Let y_{2p}^* be the minimizer of $J_2(y)$. We can analyze the convexity of the two period cost function $H_2(w|x)$, using the same procedure as in the single period case. Once more we divide the domain into three regions, namely $0 \leq w \leq U$ (I), $U < w \leq U + V$ (II) and $U + V < w$ (III).

I. ($0 \leq w \leq U$)

In this region $H_2(w|x)$ reduces to the convex minimization of $J_2(y)$ between the starting inventory level x and the highest possible inventory level after production $x + w$.

$$H_2(w|x) = \min_{y:x \leq y \leq x+w} \{J_2(y)\}.$$

Then we can rewrite $H_2(w|x)$, for $w \leq U$, in the following form:

$$H_2(w|x) = \begin{cases} J_2(x + w) & \text{if } 0 \leq w \leq y_{2p}^* - x \\ J_2(y_{2p}^*) & \text{if } y_{2p}^* - x < w \leq U \end{cases}$$

The convexity of $H_2(w|x)$ in the above region follows from the convexity of $J_2(\cdot)$.

II. ($U < w \leq U + V$)

We can rewrite $H_2(w|x)$ for this region as:

$$H_2(w|x) = (w - U)c_f + \min_{y: x \leq y \leq x+w}$$

From the convexity of $J_2(\cdot)$, the function reduces to:

$$H_2(w|x) = \begin{cases} (w - U)c_f + J_2(x + w) & \text{if } U < w \leq y_{2p}^* - x \\ (w - U)c_f + J_2(y_{2p}^*) & \text{if } y_{2p}^* - x < w \leq U + V \end{cases}$$

The above function is a linear function $((w - U)c_f)$ plus a convex function $(J_2(x + w))$ for $U \leq w \leq y_{2p}^* - x$ and a linear function $((w - U)c_f)$ plus a constant $(J_2(y_{2p}^*))$ for $y_{2p}^* - x < w \leq U + V$. The only point of concern here is that the convexity at the transition point $w = y_{2p}^* - x$. We check the first derivative of $H_2(w|x)$ at the transition point to ensure the first order condition.

$$\frac{dH_2(w|x)}{dw} = \begin{cases} c_f + J_2'(x + w) & \text{if } U \leq w \leq y_{2p}^* - x \\ c_f & \text{if } y_{2p}^* - x < w \leq U + V \end{cases}$$

Note that y_{2p}^* is the minimizer of $J_2(y)$, so $J_2'(x + w)$ is negative for $w \leq y_{2p}^* - x$. Therefore the first derivative of $H_2(w|x)$ in $U \leq w \leq y_{2p}^* - x$ is always smaller than the derivative in $y_{2p}^* - x < w \leq U + V$, thus the convexity is preserved within this region.

III. ($U + V < w$)

Using the convexity of $J_2(\cdot)$, the function $H_2(w|x)$ can be rewritten for this region as:

$$H_2(w|x) = \begin{cases} Vc_f + p((w - U - V)c_c + J_2(x + w)) + (1 - p)J_2(x + U + V) & \text{if } U + V < w \leq y_{2p}^* - x \\ Vc_f + p((w - U - V)c_c + J_2(y_{2p}^*)) + (1 - p)J_2(\psi_2(x)) & \text{if } y_{2p}^* - x < w \end{cases}$$

where,

$$\psi_2(x) = \begin{cases} x + U + V & \text{if } x \leq y_{2p}^* - U - V \\ y_{2p}^* & \text{otherwise} \end{cases}$$

$\psi_2(x)$ is constant in w , thus it does not our analysis. We check the first and second order conditions to show the convexity of $H_2(w|x)$ in this region.

$$\frac{dH_2(w|x)}{dw} = \begin{cases} pc_c + pJ_2'(x + w) & \text{if } U + V \leq w \leq y_{2p}^* - x \\ pc_c & \text{if } y_{2p}^* - x < w \end{cases}$$

$$\frac{d^2 H_2(w|x)}{dw^2} = \begin{cases} pJ_2''(x+w) & \text{if } U+V \leq w \leq y_{2p}^* - x \\ 0 & \text{if } y_{2p}^* - x < w \end{cases}$$

The first order condition is satisfied as $pc_c + pJ_2'(x+w) \leq pc_c$ since the derivative of the convex function $J_2'(x+w)$ takes on negative values for $x+w < y_{2p}^*$. The second order condition is satisfied as well, since the second derivative takes on non-negative values throughout the region. To complete the convexity proof, we must show that the convexity is preserved at transitions from permanent capacity to contracted capacity and from contracted capacity to contingent capacity, for all starting inventory levels x .

The necessary first order condition for concluding the convexity of $H(w|x)$ is as follows:

$$\frac{dH_{2I}(w|x)}{dw} \leq \frac{dH_{2II}(w|x)}{dw} \leq \frac{dH_{2III}(w|x)}{dw}$$

We have shown the convexity of $H_2(w|x)$ in all three regions. Therefore it is sufficient to check the transition points; $w = U$ and $w = U + V$.

For the transition between region I and region II it is sufficient to analyze the first derivative of $H_2(w|x)$ for $x \leq y_{2p}^*$ and $x > y_{2p}^* - U$.

if $x \leq y_{2p}^* - U$

$$\frac{dH_2(w|x)}{dw} = \begin{cases} J_2'(x+w) & \text{if } w \leq U \\ c_f + J_2'(x+w) & \text{if } U \leq w \leq y_{2p}^* - x \end{cases}$$

if $x > y_{2p}^* - U$

$$\frac{dH_2(w|x)}{dw} = \begin{cases} 0 & \text{if } y_{2p}^* - x < w \leq U \\ c_f & \text{if } U \leq w \leq U + V \end{cases}$$

The above equations show that the condition holds for non-negative c_f .

If we analyze the transition between region II and region III:

if $x \leq y_{2p}^* - U - V$

$$\frac{dH_2(w|x)}{dw} = \begin{cases} c_f + J_2'(x+w) & \text{if } U < w \leq U + V \\ pc_c + pJ_2'(x+w) & \text{if } U + V < w \leq y_{2p}^* - x \end{cases}$$

if $x > y_{2p}^* - U - V$

$$\frac{dH_2(w|x)}{dw} = \begin{cases} c_f & \text{if } y_{2p}^* - x < w \leq U + V \\ pc_c & \text{if } U + V < w \leq y_{2p}^* - x \end{cases}$$

Note that once more, the first order condition holds if pc_c is greater than c_f . Therefore under the specified condition the multi-period cost function is convex in w and the resulting policy is of state dependent order up-to type.

If we denote the minimizers of the convex functions $J_2(y)$, $(y - U - x)c_f + J_2(y)$, and $Vc_f - p((y - U - V - x)c_c + J_2(y)) + (1 - p)J_2(x + U + V)$ as y_{2p}^* , y_{2v}^* , and y_{2c}^* respectively, then the relation $y_{2p}^* > y_{2v}^* > y_{2c}^*$ holds as:

$$0 = J'_2(y) < c_f + J'_2(y) < c_c + J'_2(y)$$

for $0 < c_f < c_c$. Hence we can write the optimal state dependent order up-to type policy for the two period problem in the following form:

$$y_2^*(x) = \begin{cases} y_{2c}^* & \text{if } x < y_{2c}^* - U - V \\ x + U + V & \text{if } y_{2c}^* - U - V < x \leq y_{2v}^* - U - V \\ y_{2v}^* & \text{if } y_{2v}^* - U - V < x \leq y_{2v}^* - U \\ x + U & \text{if } y_{2v}^* - U < x \leq y_{2p}^* - U \\ y_{2p}^* & \text{if } y_{2p}^* - U < x \leq y_{2p}^* \\ x & \text{if } y_{2p}^* < x \end{cases}$$

Note that the order up-to level y_{2c}^* is only realized if the ordered contingent workers are acquired, as in the single period case. From $y_2^*(x)$ we can write the two period function of minimum expected total cost of operations for the starting

inventory level x as:

$$f_2(x) = \begin{cases} Uc_p + Vc_i + Vc_f \\ \quad + p(c_c(y_{2c}^* - U - V - x) + J_2(y_{2c}^*)) \\ \quad + (1-p)J_2(x + U + V) & \text{if } x < y_{2c}^* - U - V \\ Uc_p + Vc_i + Vc_f + J_2(x + U + V) & \text{if } y_{2c}^* - U - V < x \leq y_{2v}^* - U - V \\ Uc_p + Vc_i + (y_{2v}^* - U - x)c_f + J_2(y_{2v}^*) & \text{if } y_{2v}^* - U - V < x \leq y_{2v}^* - U \\ Uc_p + Vc_i + J_2(x + U) & \text{if } y_{2v}^* - U < x \leq y_{2p}^* - U \\ Uc_p + Vc_i + J_2(y_{2p}^*) & \text{if } y_{2p}^* - U < x \leq y_{2p}^* \\ Uc_p + Vc_i + J_2(x) & \text{if } y_{2p}^* < x \end{cases}$$

$f_2(x)$ is convex, which can be shown by a similar analysis as in the single period case. The convexity of the two period minimum expected total cost function $f_2(x)$ implies that the multi-period function $f_t(x)$ is convex, which can be easily shown by induction.

Corollary 1 *In the special case of the problem involving no contracted workers ($V = 0$), $H(w|x)$ and the multi-period cost function $H_t(w|x)$ are convex in w , for all x , for all non-negative cost parameters.*

Theorem 2 *Optimal order up-to points of the two period problem, y_{2c}^* , y_{2v}^* and y_{2p}^* , are greater than their single period counterparts.*

Proof

Recall that convex single period cost functions and their minimizers (order up-to levels) are as follows.

$$\begin{aligned} & L(y) \quad \text{at} \quad y_p^* \\ & (y - U - x)c_f + L(y) \quad \text{at} \quad y_v^* \\ & Vc_f + p((y - U - V)c_c + L(y)) + (1-p)L(x + U + V) \quad \text{at} \quad y_c^* \end{aligned}$$

Two period counterparts of the above functions are:

$$J_2(y) \quad \text{at} \quad y_{2p}^*$$

$$(y - U - x)c_f + J_2(y) \quad \text{at} \quad y_{2v}^*$$

$$Vc_f + p((y - U - V)c_c + J_2(y)) + (1 - p)J_2(x + U + V) \quad \text{at} \quad y_{2c}^*$$

where $J_2(y) = L(y) + \alpha E[f(y - z)]$. If we write $E[f(y - z)]$ explicitly the equation becomes:

$$\begin{aligned} J_2(y) = & L(y) + \alpha [\int_0^{y-y_p^*} L(y-z)g(z)dz + \int_{y-y_p^*}^{y-y_p^*+U} L(y_p^*)g(z)dz \\ & + \int_{y-y_p^*+U}^{y-y_p^*+U+V} L(y-z+U)g(z)dz + \int_{y-y_v^*+U}^{y-y_v^*+U+V} (c_f(y_v^* - U - y + z) \\ & + L(y_v^*))g(z)dz + \int_{y-y_c^*+U+V}^{y-y_c^*+U+V} (c_fV + L(y-z+U+V))g(z)dz \\ & + \int_{y-y_c^*+U+V}^{\infty} (c_fV + p(c_c(y_c^* - U - V - y + z) + L(y_c^*)) + (1-p)L(y-z+U+V))g(z)dz] \end{aligned}$$

$J_2(y)$ attains its minimum at the point where $\frac{dJ_2(y)}{dy} = 0$.

$$\begin{aligned} 0 = & L'(y) + \alpha [L(y_p^*)g(y - y_p^*) + \int_0^{y-y_p^*} L'(y-z)g(z)dz + L(y_p^*)g(y - y_p^* + U) \\ & - L(y_p^*)g(y - y_p^*) + L(y_v^*)g(y - y_v^* + U) - L(y_p^*)g(y - y_p^* + U) \\ & + \int_{y-y_p^*+U}^{y-y_p^*+U+V} L'(y-z+U)g(z)dz + (c_fV + L(y_v^*))g(y - y_v^* + U + V) \\ & - L(y_v^*)g(y - y_v^* + U) - \int_{y-y_v^*+U}^{y-y_v^*+U+V} L'(y-z+U+V)g(z)dz \\ & - (c_fV + L(y_c^*))g(y - y_c^* + U + V) + \int_{y-y_c^*+U+V}^{\infty} (-pc_c + (1-p)L'(y-z+U+V))g(z)dz] \\ \Rightarrow 0 = & L'(y) - \alpha [c_f \int_{y-y_v^*+U}^{y-y_v^*+U+V} g(z)dz + pc_c \int_{y-y_c^*+U+V}^{\infty} g(z)dz] \\ & + \alpha [\int_0^{y-y_p^*} L'(y-z)g(z)dz + \int_{y-y_p^*}^{y-y_p^*+U} L'(y-z)g(z+U)dz \\ & + \int_{y-y_v^*}^{y-y_v^*+U} L'(y-z)g(z+U+V)dz + (1-p) \int_{y-y_c^*}^{\infty} L'(y-z)g(z+U+V)dz] \end{aligned}$$

We prove that $y_{2p}^* > y_p^*$ by contradiction. Note that the last three terms in the last bracket are always negative, since $L'(\cdot)$ is negative in the specified region. By contradiction, if the y satisfying the above equality was smaller than y_p^* then all the terms in the right hand side of the equality would be negative, since $L'(y) < 0$ for $y < y_p^*$ and the second bracket has positive terms inside but a negative sign in front. If y_{2p}^* was equal to y_p^* then the first term would be zero and all the remaining terms would be negative. Therefore for the above equality to hold y_{2p}^* must be greater than y_p^* . Note that the result holds for y_v^* and y_c^* as we only add constants, c_f and c_c , to the right hand side of the above equation.

5.2 Partial Contingent Capacity Availability

In this section we consider a continuous distribution $P(m, N)$ for the number of contingent workers acquired. The distribution depends on the number of contingent workers requested. Note that if no contingent workers are requested ($N = 0$), then no contingent workers will be received as expected. The distribution limits the number of contingent workers acquired by N , in other words the firm receives no more than the number ordered.

Our computational analysis shows that in the infinite horizon, the minimum expected cost of operations $f_t(x_t)$ is non-convex in the starting inventory level x_t . However from the computational results we conjecture that it is quasi-convex.

5.2.1 Uniform Supply Uncertainty

In this subsection we show that the single period decision function $H(\cdot)$ is convex under uniform labor supply uncertainty. However the multi-period cost function is observed to be non-convex. We provide the structure of the multi-period problem and the optimal policy according to the results of the computational analysis.

Theorem 3 *Under uniform labor supply uncertainty the single period cost function $H(w|x)$ is convex in the capacity position w for all starting inventory levels x , if the expected marginal cost of increasing capacity position with contingent capacity is greater than the expected marginal cost of increasing capacity position with contracted capacity $c_f \leq \frac{c_c}{2}$.*

Proof

Single period cost function for the uniform contingent labor uncertainty case can be written as:

$$H(w|x) = \begin{cases} \varphi(w|x) & \text{if } 0 \leq w \leq U \\ (w - U)c_f + \varphi(w|x) & \text{if } U < w \leq U + V \\ Vc_f + \int_0^N (mc_c + \varphi(U + V + m|x)) \frac{1}{N} dm & \text{if } U + V < w \end{cases}$$

To prove that $H(w|x)$ is convex it is sufficient to analyze the case with contingent capacity region and the corresponding transition point, since for $w \leq U + V$ the function remains identical for all labor supply uncertainty types.

For $w > U + V$

$$H(w|x) = \begin{cases} Vc_f + c_c \frac{(w-U-V)}{2} + \int_0^{w-U-V} \frac{L(x+U+V+m)}{(w-U-V)} dm & \text{if } U + V < w \leq y_p^* - x \\ Vc_f + c_c \frac{(w-U-V)}{2} + \int_0^{y_p^*-x-U-V} \frac{L(x+U+V+m)}{(w-U-V)} dm + \int_{y_p^*-x-U-V}^{w-U-V} \frac{L(y_p^*)}{(w-U-V)} dm & \text{if } y_p^* - x < w \end{cases}$$

We take the first derivative of the function to check the first order condition:

$$\frac{dH(w|x)}{dw} = \begin{cases} \frac{c_c}{2} + \frac{L(x+w)}{(w-U-V)} - \int_0^{w-U-V} \frac{L(x+U+V+m)}{(w-U-V)^2} dm & \text{if } U + V < w \leq y_p^* - x \\ \frac{c_c}{2} + \frac{L(y_p^*)(y_p^*-x-U-V)}{(w-U-V)^2} - \int_0^{y_p^*-x-U-V} \frac{L(x+U+V+m)}{(w-U-V)^2} dm & \text{if } y_p^* - x < w \end{cases}$$

At the transition point $w = y_p^* - x$ the first derivatives are equal, therefore if the second derivative is non-negative at both sides of the transition point, the first

order condition will be satisfied.

$$\frac{d^2 H(w|x)}{dw^2} = \begin{cases} \frac{L'(x+w)(w-U-V)^2 - 2L(x+w)(w-U-V)}{(w-U-V)^3} + \frac{2 \int_0^{w-U-V} L(x+U+V+m) dm}{(w-U-V)^3} & \text{if } U+V < w \leq y_p^* - x \\ \frac{2 \int_0^{y_p^* - x - U - V} L(x+U+V+m) dm}{(w-U-V)^3} - \frac{2L(y_p^*)(y_p^* - x - U - V)}{(w-U-V)^3} & \text{if } y_p^* - x < w \end{cases}$$

It is evident that $\frac{d^2 H(w|x)}{dw^2}$ is positive for $w > y_p^* - x$, because the first term in the nominator is greater than the second term, since it integrates $L(x+U+V+m)$ over a region where the values are greater than the optimal $L(y_p^*)$, whereas the second term is equivalent to the integration of $L(y_p^*)$ over the same region. For $U+V < w \leq y_p^* - x$ we take the limit of the second derivative as $w \rightarrow U+V$ and show that it is positive, and remains positive throughout the whole domain.

$$\lim_{w \rightarrow U+V} \frac{L'(x+w)(w-U-V)^2 - 2L(x+w)(w-U-V) + 2 \int_0^{w-U-V} L(x+U+V+m) dm}{(w-U-V)^3} = \frac{0}{0}$$

By using L'Hopital's Rule we get:

$$\frac{L''(x+w)(w-U-V)^2 + 2L'(x+w)(w-U-V) - 2L'(x+w)(w-U-V) - 2L(x+w) + 2L(x+w)}{3(w-U-V)^2} = \frac{L''(x+w)}{3} > 0$$

The second derivative is positive at $U+V$, we have to ensure that it remains positive for $w > U+V$. To do so we check the numerator of the second derivative, since the denominator is always positive for $w > U+V$. We take the derivative of the numerator and check if it is positive. Let us denote the numerator by $\varpi(w)$, then $\frac{d\varpi(w)}{dw} = L''(x+w)(w-U-V)^2$, which is positive for all w , hence the function is convex for $w > U+V$.

To conclude the convexity of $H(w|x)$, we need to show that the convexity is preserved at the transition point $w = U+V$. Since $H(w|x)$ is dependent on the starting inventory level x , the first order condition should be satisfied for all x . It is sufficient to analyze the transition point for $x < y_p^* - U - V$ and $x \geq y_p^* - U - V$. Note that we analyze the derivative of the function on both sides of the transition

point. For the initial case $H(w|x)$ takes the following form near the transition point.

$$H(w|x) = \begin{cases} c_f(w - U) + L(x + w) & \text{if } U < w \leq U + V, \\ Vc_f + c_c \frac{w-U-V}{2} + \int_0^{w-U-V} \frac{L(x+U+V+m)}{w-U-V} dm & \text{if } U + V < w \leq y_p^* - x. \end{cases}$$

The first derivative for this region is:

$$\frac{dH(w|x)}{dw} = \begin{cases} c_f + L'(x + w) & \text{if } U < w \leq U + V, \\ \frac{c_c}{2} + \frac{L(x+w)}{w-U-V} + \int_0^{w-U-V} \frac{L'(x+U+V+m)}{(w-U-V)^2} dm & \text{if } U + V < w \leq y_p^* - x. \end{cases}$$

The first order condition for the above region is:

$$c_f + L'(x + w) \leq \frac{c_c}{2} + \frac{L(x+w)(w-U-V) - \int_0^{w-U-V} L(x+U+V+m) dm}{(w-U-V)^2}.$$

Taking the limit as $w \rightarrow U + V$ we get

$$c_f + L'(x + w) \leq \frac{c_c}{2} + \frac{L'(x+w)}{2}$$

Note that the above inequality holds if $c_f \leq \frac{c_c}{2}$.

For the second case ($x \geq y_p^* - U - V$) the cost function takes the following form:

$$H(w|x) = \begin{cases} c_f(w - U) + L(y_p^*) & \text{if } y_p^* - x < w \leq U + V \\ Vc_f + c_c \frac{w-U-V}{2} + L(y_p^*) & \text{if } U + V < w \end{cases}$$

The first derivative for this region is in the following form:

$$\frac{dH(w|x)}{dw} = \begin{cases} c_f & \text{if } y_p^* - x < w \leq U + V \\ \frac{c_c}{2} & \text{if } U + V < w \end{cases}$$

For the first order condition to hold c_f must be less than or equal to $\frac{c_c}{2}$. Therefore if $c_f \leq \frac{c_c}{2}$ then the single period cost function is convex in w for all starting inventory levels x .

Corollary 2 *In the special case of the problem involving no contracted workers ($V = 0$), the single period cost function $H(w|x)$ is convex in w , for all x , for non-negative cost parameters.*

The remaining characteristics of the problem is provided from computational analysis due to the analytical intractability of the problem. Throughout the rest of the chapter we assume that there are no contracted workers $V = 0$. For the numerical analysis of the optimal control parameters we use stationary Poisson demand with mean 10. Our computational results show that the multi-period function $H(w|x)$ is non-convex for all supply uncertainty structures except the all or nothing case. Figure 5.1 shows the expected level of inventory position after production, $E[y^*(x)]$ as a function of the starting inventory level x for the infinite horizon problem. An important fact that should be noted here is that capacity and production decisions does not necessarily match, namely the realized capacity is not fully utilized at all times. This is also the characteristic presented in y , for a convex function $J(\cdot)$. In the infinite horizon problem, under the presence of uniform contingent supply uncertainty, one of the important results is that the expected inventory level after production, $E[y^*(x)]$, is strictly increasing in x for $x < x_c$ as in Figure 5.1. Here x_c is the threshold level, under which the firm prefers to utilize contingent workers on top of the permanent and contracted capacity. In contrast, under the presence of other uncertainty types analyzed, this behavior is not observed. Note that $E[y^*(x)]$ is linearly increasing in x , for $x_c < x \leq y_p^*$. In that region the manufacturer produces at full permanent capacity to raise the inventory level to $x + U$.

The decision maker under this setting, sets the capacity position considering the expected outcome, consequently the production decision is made after realizing the capacity in order to minimize the cost with the available resource. Under this setting the optimal capacity position exceeds the optimal production quantity by a significant margin. In other words if all or almost all of the contingent workers requested are acquired then there will be a significant capacity surplus (Fig 5.2). The difference between the optimal production quantity and the optimal capacity position reduces as the usage of contingent capacity declines.

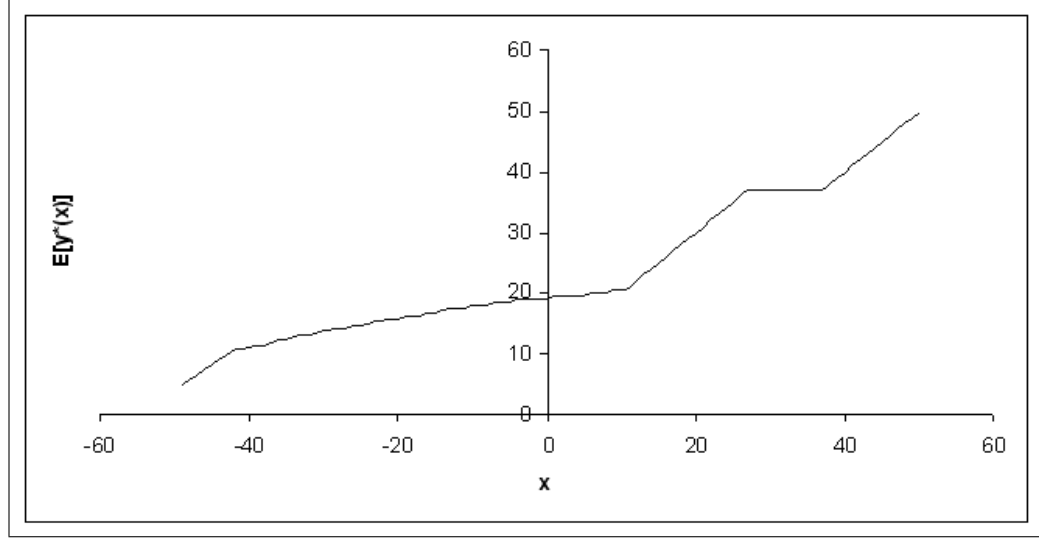


Figure 5.1: Expected Inventory Level After Production-Uniform Uncertainty, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$, $U = 10$, Poisson Demand

5.2.2 Normal Supply Uncertainty

Under Normal uncertainty, the structure of the infinite horizon cost function is similar to that of uniform uncertainty. The infinite horizon cost function is not necessarily convex. In our computational analysis for this subsection we used a normal labor supply distribution with mean $N/2$ and standard deviation $N/8$, where N is the number of contingent workers requested. Note that the normal distribution that we used has a lower coefficient of variation than the uniform distribution. Consequently the optimal capacity positions are set higher and the resulting total discounted cost is lower. However as the number of contingent workers requested increase, the variance increases as well and the gap between the optimal production level and the capacity position widens. Even though the gap between the optimal capacity position and the optimal production is higher than under uniform uncertainty, the rate of unutilized capacity is lower, since the optimal capacity position is set considering the mean of the normal distribution (under lower coefficient of variation). In particular, within a specific setting the total expected capacity surplus under uniform distribution is 1.9 percent of the total capacity utilized whereas this number is equal to 0.2 percent under normal

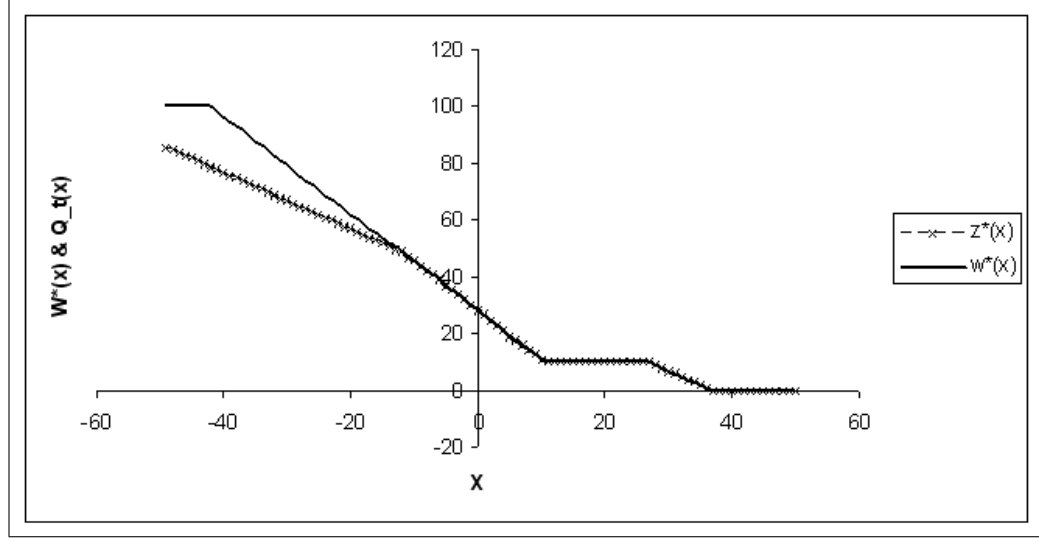


Figure 5.2: Optimal Capacity Position & Optimal Production vs Starting Inventory Level - Uniform Uncertainty, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$, $U = 10$, Poisson Demand

supply uncertainty with coefficient of variation equal to 0.25. Under normal supply uncertainty, expected inventory level after production is non-decreasing in the starting inventory level x . In the single period problem the expected inventory level after production is increasing in the starting inventory level x for $x < x_c$, as in the case of uniform supply uncertainty. However this effect diminishes in the infinite horizon setting and the expected inventory level after production remains constant in x for low starting inventory levels, where contingent workers are required for production. Figure 5.3 illustrates the expected inventory level after production.

5.2.3 Binomial Supply Uncertainty with Increasing Probability of Success

In this setting the supply uncertainty is binomial and the probability of success p , is increasing in the number of workers requested. After a threshold, the supplier provides the contingent workers with certainty. The probability of receiving each

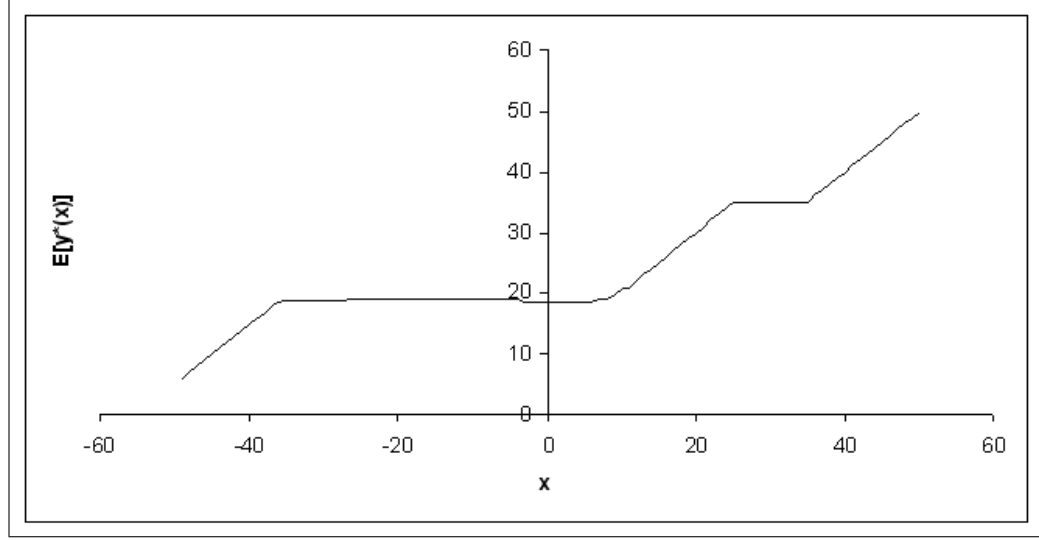


Figure 5.3: Expected Inventory Level After Production vs Starting Inventory-Normal Uncertainty, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand

additional contingent worker is $p = \frac{N}{k}$ with k being this threshold value. In our computational analysis we used $k = 20$. Under this setting the manufacturer is motivated to request higher number of contingent workers depending on the threshold value set by the ELSA. The multi-period cost function $H_t(w|x)$ is observed to be non-convex. Optimal capacity position and optimal production is concavely decreasing in the starting inventory level, when the inventory level is such that the manufacturer requests contingent workers below the threshold set by the ELSA (see Figure 5.4).

In this region as the starting inventory level increases and the need for production decreases, the manufacturers incentive to request contingent workers declines faster as the probability of acquiring contingent workers decrease with decreasing request size N . An important result under this type of supply uncertainty is that the expected inventory level after production is no more non-decreasing in the starting inventory level x (see Figure 5.5). In the region where the manufacturer requests contingent capacity of relatively small size, the expected inventory level after production slightly falls with increasing starting inventory. This is an immediate implication of the concave decline in the number of contingent workers

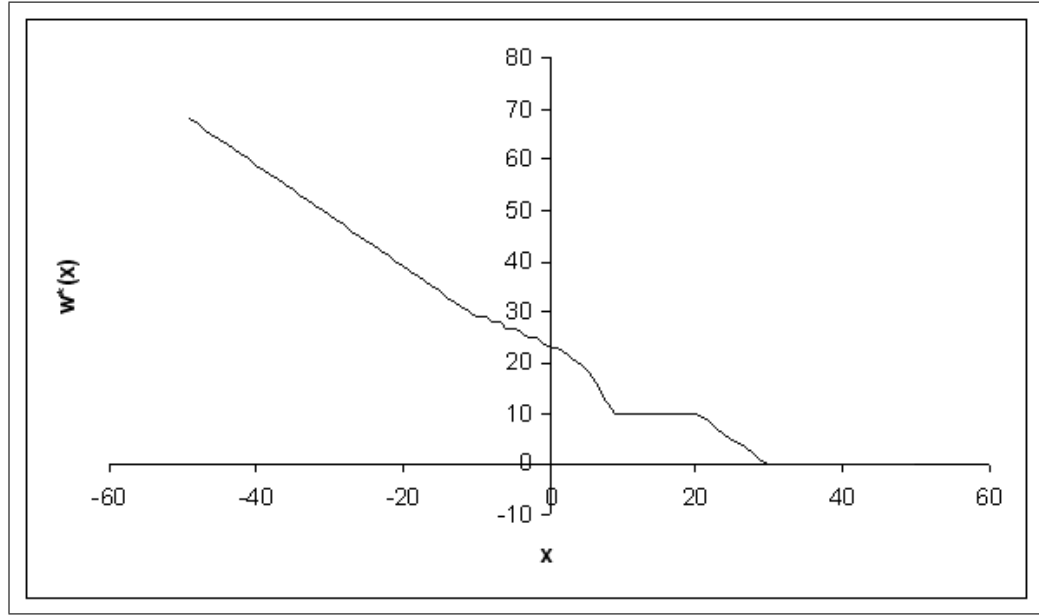


Figure 5.4: Optimal Capacity Position vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand

requested. The realized capacity level goes under a sharp fall and the manufacturer relies on its owned capacity rather than requesting external capacity with low availability. Under this setting optimal capacity and production decisions are identical. Therefore capacity is always fully utilized.

The optimal production level under the binomial supply uncertainty structures considered in this study, the optimal production level is generally equal to the observed capacity level, when contingent workers are requested. Therefore the contingent capacity surplus under these supply uncertainty structures is almost equal to zero.

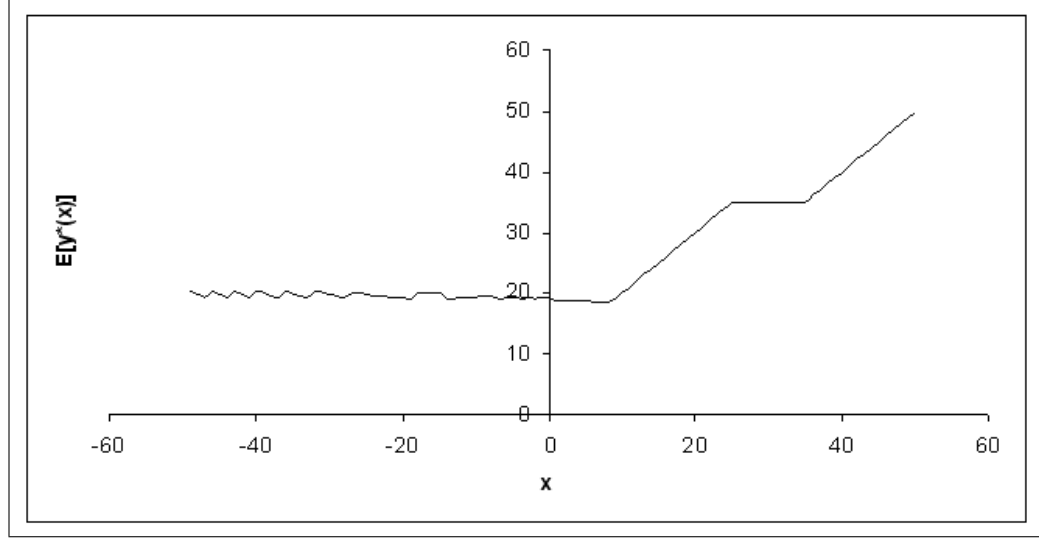


Figure 5.5: Optimal Capacity Position vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand

5.2.4 Binomial Supply Uncertainty With Decreasing Probability of Success

In this setting the availability of contingent workers decrease as the need increases. The probability of receiving each additional contingent worker is $p = \frac{k-N}{k}$ with k being the pool size of the ELSA. In this case the ELSA can not provide more than k contingent workers. In our computational analysis we used $k = 20$. The ELSA motivates the manufacturer to request small sizes of contingent workforce. As the inventory level increases and the need for external capacity reduces, manufacturers incentive for requesting contingent workers increase. The underlying reason here is that the probability of acquiring contingent workers is high if the number requested is small. Therefore as the starting inventory level increases and gets closer to x_c , number of contingent workers requested decrease convexly. Under this type of supply uncertainty optimal capacity and production decisions are again identical therefore capacity is fully utilized. Figure 5.6 shows the infinite horizon total discounted cost as a function of the capacity position. It is evident that the function is non-convex. Note that contingent workers are requested at

the concavely increasing region.

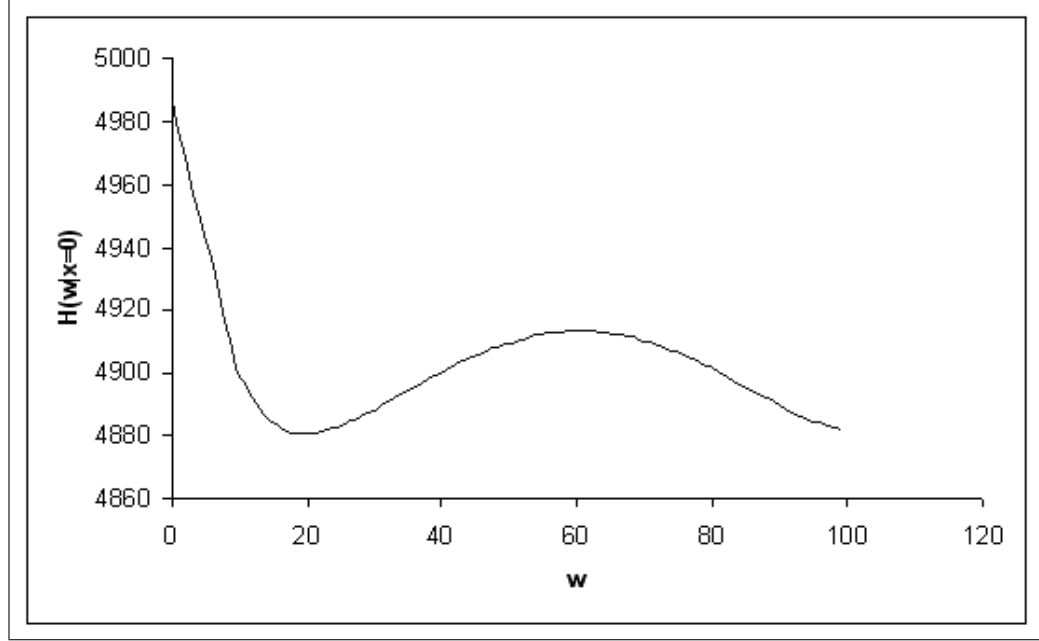


Figure 5.6: Infinite Horizon Discounted Cost of Operations vs Capacity Position-Binomial Uncertainty with Decreasing Probability of Success, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$, $U = 10$, Poisson Demand

5.2.5 Binomial Supply Uncertainty With High Probability of Success at Low and High Requests

This type of uncertainty models an ELSA avoiding a region of moderate demand by providing low availability rates. The manufacturer is motivated to make either small sized or large requests. Note that above a threshold level ($k = 20$) the ELSA supplies contingent workers with certainty. $H(w|x)$ is not unimodal for this type of labor supply uncertainty. Figure 5.7 illustrates the total cost of operations from period t and on as a function of the capacity position for $x = 0$. The optimal capacity position is non-increasing in the starting inventory level x (see Figure 5.8). Due to low availability rates the manufacturer avoids moderate sized

orders, therefore the optimal capacity position faces a steep fall as the number of contingent workers requested passes from the high end to the low end.

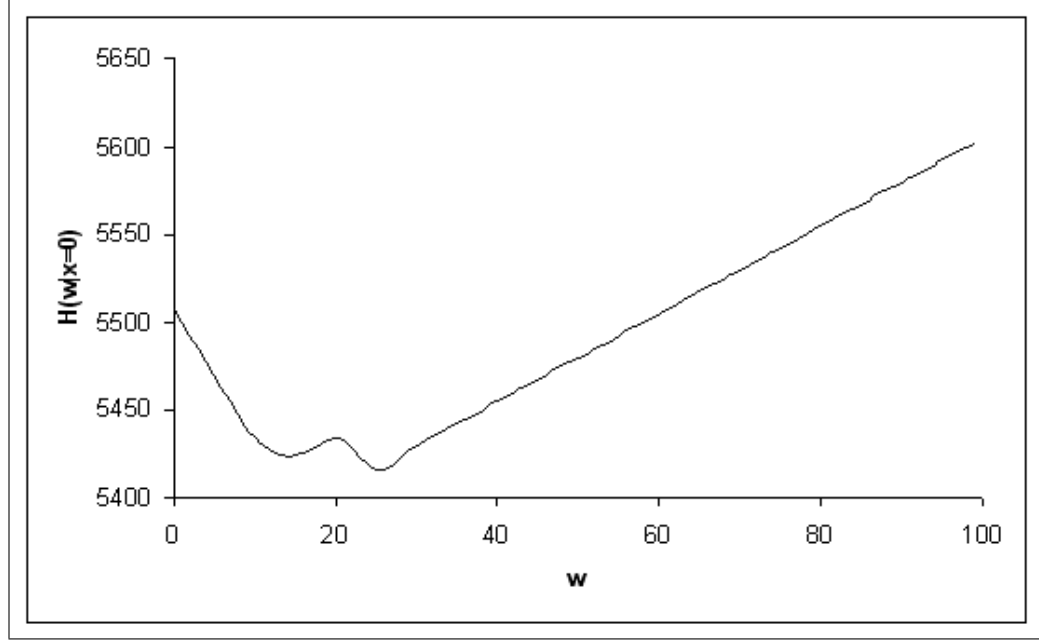


Figure 5.7: $H(w|x=0)$ vs Capacity Position-Binomial Uncertainty with Increasing Probability of Success at High and Low Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand

Under this setting the production decision and the capacity are equal thus realized capacity is fully utilized. The expected inventory level after production fluctuates around a constant critical level when $x < x_c$ (Fig 5.9). This is due to the structure of the sinusoidal binomial distribution.

5.2.6 Binomial Supply Uncertainty With High Probability of Success at Moderate Demand

Under this setting, the ELSA avoids high and low requests and concentrates the demand around the desired region where the availability rate is high. In our computational analysis we considered the pool size of the ELSA as 20 ($k = 20$). The

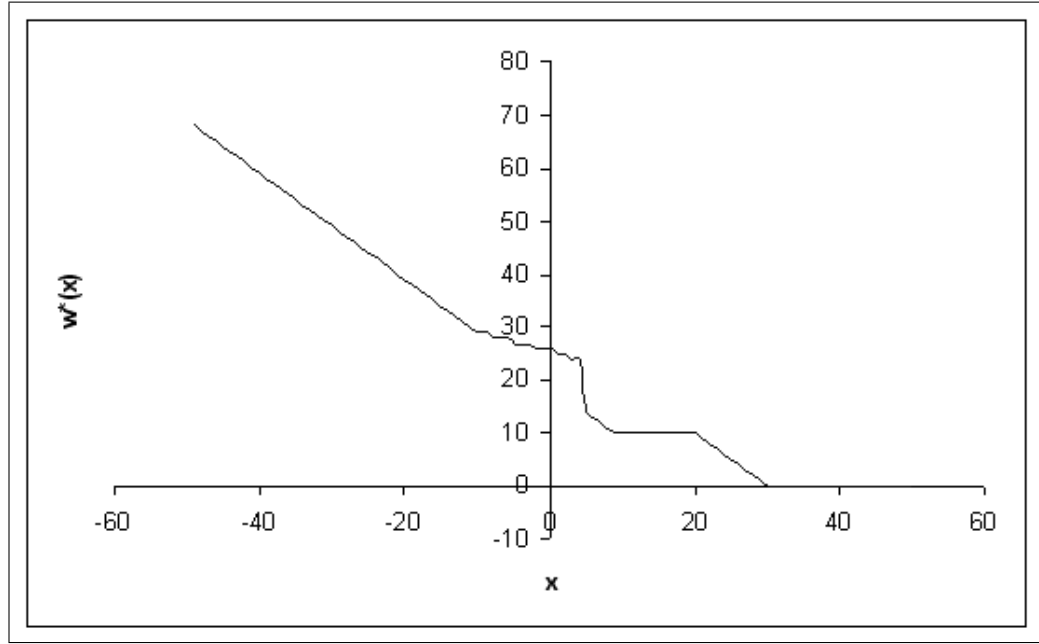


Figure 5.8: Optimal Capacity Position vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success at High and Low Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand

manufacturer does not request contingent workers above or below the specified moderate region. The optimal capacity position faces a steep fall as the manufacturer decides not to use contingent capacity, bypassing low levels of contingent capacity. For very low inventory levels the expected inventory level after production is increasing in the starting inventory level, as the manufacturer constantly requests the specific "moderate" level (in this setting 10) where the ELSA supplies the contingent workers with a high probability. However as the starting inventory level gets above the threshold the expected level after production is stabilized with decreasing contingent capacity usage. Figure 5.10 displays the expected inventory level after production as a function of the starting inventory level. In the previous cases with binomial supply uncertainty and unlimited pool size we have stated that the expected inventory position after production fluctuated around a constant critical value when contingent workers are utilized. However in this case the expected inventory level is strictly increasing for $x < -4$. The underlying reason is that the inventory level is too low and the manufacturer requests the

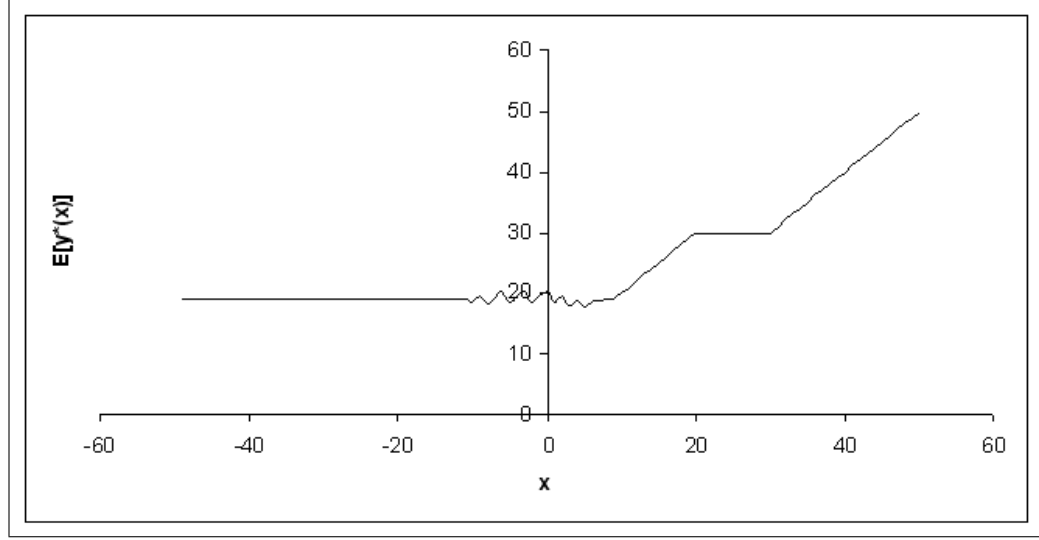


Figure 5.9: Expected Inventory Level After Production vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success at High and Low Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$, $U = 10$, Poisson Demand

number of contingent workers yielding the highest expected capacity in order to raise the inventory level. In our computational setting the moderate request level yielding the highest expected number of contingent workers acquired is equal to one half of the ELSA's pool size. Therefore in this case the manufacturer always requests 10 contingent workers for $x < -4$. Note that the manufacturer still utilizes contingent workers for $x > -4$ and the expected inventory level after production fluctuates around a critical value as in the previous cases.

Under this setting capacity and production decisions are again equivalent. Figure 5.11 illustrates the optimal production as a function of the starting inventory level.

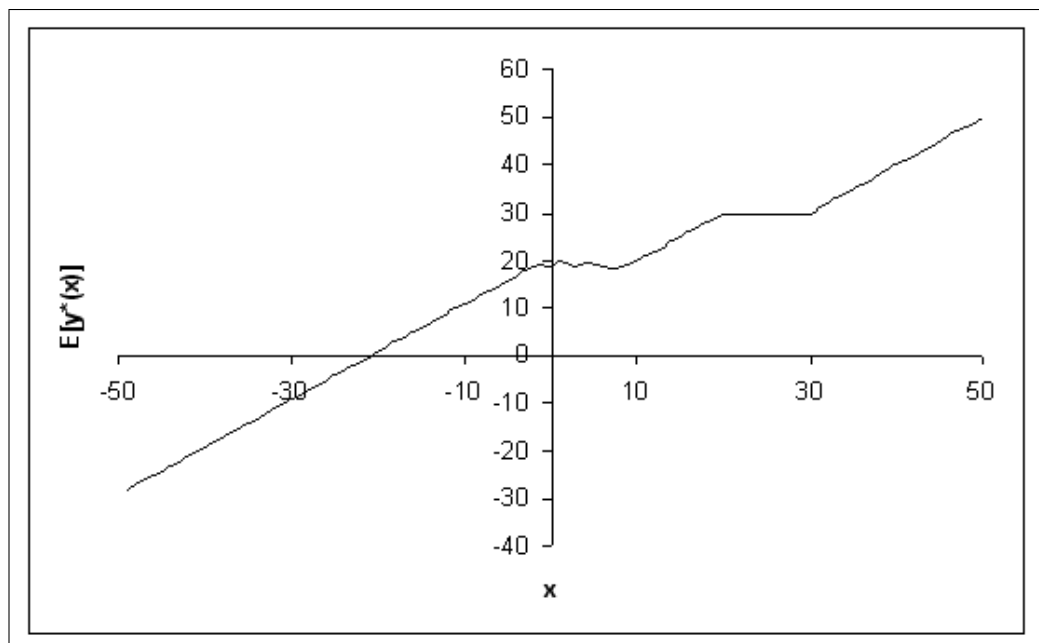


Figure 5.10: Expected Inventory Level After Production vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success at Moderate Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand

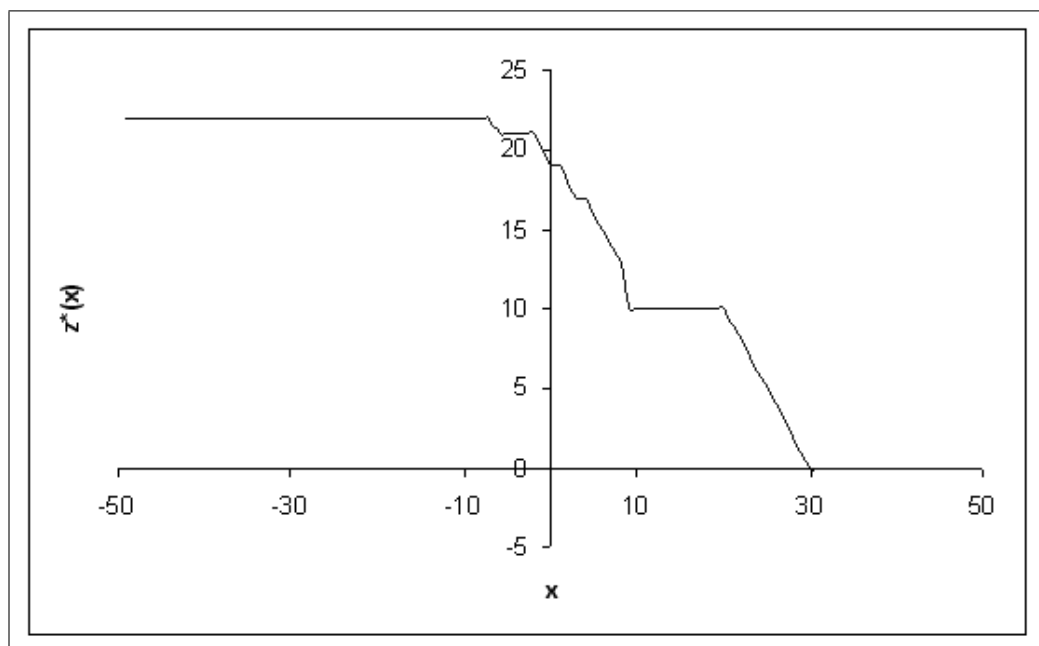


Figure 5.11: Optimal Production vs Starting Inventory Level-Binomial Uncertainty with Increasing Probability of Success at Moderate Requests, $c_p = 2.5$, $c_c = 3.5$, $h = 1$, $b = 5$ $U = 10$, Poisson Demand

Chapter 6

Numerical Analysis

In this chapter, we present the results of our numerical analysis for the infinite horizon problem. The intention of our numerical study is to measure the impact of various factors on flexible capacity management. We investigate the topics in the upcoming sections under the light of our computational experiments. Throughout the analysis, we consider an experimental setting where the demand pattern follows a seasonal cycle of 4 periods, with the expected demands being 10, 15, 10 and 5, respectively. We solve the integrated flexible capacity and inventory management problem for the following set of input parameters unless otherwise noted: $h = 1$, $c_i + c_f = 3$, $c_p = 2.5$, $\alpha = 0.99$ and $x_1 = 0$. In our computational analysis we use the following distributions for contingent labor supply uncertainty:

- Normal distribution with mean $N/2$ and coefficient of variations, (CV), 0.1, 0.15, 0.2 and 0.25
- Uniform distribution with mean $N/2$, $CV = 0.57$
- Binomial distribution with increasing probability of success, $p = \frac{N}{k'}$
- Binomial distribution with decreasing probability of success, $p = \frac{k-N}{k}$
- Binomial distribution with higher probability of success at moderate requests, $p = \frac{\cos(2N\Pi/k-\Pi)+1}{2}$

- Binomial distribution with higher probability of success at low and high request, $p = \frac{\sin(2N\Pi/k' + \Pi/2) + 1}{2}$

where N is the number of contingent workers requested from the ELSA, k is the pool size of the ELSA, and k' is the threshold level after which the ELSA supplies contingent labors with certainty. In the upcoming sections of the analysis, we will use k for both the pool size and the threshold level since the distributions are sufficient for specifying the distinction.

The rest of this chapter is organized as follows. In Section 6.1, we analyze the effect of uncertainty on flexible capacity management. In Section 6.2, we investigate the optimal tactical capacity decisions. In Section 6.3, we investigate the value of contracting temporary workers. In Section 6.4, we provide a heuristic and test its performance under various settings.

6.1 Effect of Labor Supply Uncertainty

In this section we investigate the effects of labor supply uncertainty on flexible capacity and production management. The integrated capacity and production management problem is analyzed under contingent labor certainty and several labor uncertainty structures. Results in this section are obtained by simulating the system by using the optimal policy. The averages presented in the analysis are per period averages of simulation replications. Analysis shows that the manufacturer responds to labor supply uncertainty in different ways under different settings. The operational tools used by the manufacturer for reducing the effect of labor supply uncertainty are:

- adjusting the production level,
- adjusting the levels of inventory and backorder,
- requesting surplus capacity, and

- adjusting the amount of contingent capacity utilized in production.

Under the presence of labor supply uncertainty, a manufacturer may select one or more than one of the above approaches to minimize the total operational costs. Manufacturer's preference strongly depends on the structure of supply uncertainty and the cost parameters. Our purpose in this section is to compare the behavior of the manufacturer under certain and uncertain contingent labor supply and to investigate the situation as the variability of labor supply increases.

A manufacturer facing labor supply uncertainty may adjust the production in two ways; either it can spread the total yearly production into periods flatly or it can build up inventory before the peak season and reduce the production in the following periods. The solution depends on the cost of contingent labor, c_c and the cost of backordering, b . If the cost of contingent labor is high, the manufacturer may want to reduce the amount utilized by spreading the production to periods and utilizing the permanent capacity fully. On the other hand, if the backordering cost is high the manufacturer may build up inventory before the peak season and may use the excess inventory in the following periods.

We have observed that increasing the level of inventory is an immediate response of the manufacturer when the uncertainty in labor supply is relatively low. Our analysis shows that as the labor supply becomes uncertain, the initial approach of the manufacturer is to increase the inventory level, however as the labor supply uncertainty increases to moderate levels, the manufacturer may reduce the inventory level and try to cope with the increasing supply uncertainty by requesting excess contingent workers, resulting in a potential capacity surplus. As the cost of contingent capacity increases, the incentive of the manufacturer to request excess contingent capacity does not necessarily fall. However if the supply uncertainty becomes considerably high, the manufacturer decreases the amount of contingent capacity requested and relies on inventory. This behavior is observed more evidently when the cost of contingent capacity is high.

Table 6.1: Average Inventory Level and Average Production with Contingent Capacity. $U = 6, V = 0$

Parameters	Criteria	Certain	Norm 0.1	Norm 0.15	Norm 0.2	Norm 0.25	$Cos(k = 20)$	Unif
cc=1.5 b=2.5	Tot.Cost	3125.89	3135.56	3142.82	3153.13	3171.11	3150.01	3345.23
	Avg.Inv.	7.42	9.26	9.47	9.51	7.46	7.36	9.07
	%Contingent	40.26	40.33	40.38	40.42	40.38	40.23	40.36
	Cont Surplus	0	0.00025	0.00496	0.023	0.0419	0	0.2678
cc=1.5 b=4.5	Tot.Cost	3224.4	3240.68	3248.18	3260.57	3283.55	3259.14	3515.37
	Avg.Inv.	12.67	14.43	14.33	14.41	13.07	12.71	16.79
	%Contingent	40.41	40.45	40.49	40.55	40.51	40.3	40.3
	Cont Surplus	0	0.00075	0.01016	0.0415	0.0895	0	0.5276
cc=1.5 b=50	Tot.Cost	3618.71	3639.56	3660.39	3687.09	3738.06	3704.2	4225
	Avg.Inv.	29.54	31.33	31.97	32.61	32.05	31.87	45.37
	%Contingent	40.88	41.27	41.22	41.28	41.22	40.82	40.3
	Cont Surplus	0	0.013	0.0881	0.2545	0.4714	0	1.4133
cc=4.5 b=2.5	Tot.Cost	4339.04	4349.6	4354.25	4364.94	4385.78	4365.61	4559.25
	Avg.Inv.	8.41	8.43	8.44	8.46	8.50	8.46	8.84
	%Contingent	40.02	40.02	40.02	40.02	40.02	40.02	40.01
	Cont Surplus	0	0	0	0	0.00002	0	0
cc=4.5 b=4.5	Tot.Cost	4443.37	4455.47	4466.55	4477.23	4505.83	4475.01	4744.06
	Avg.Inv.	12.99	13.97	14.04	14.21	12.99	12.58	17.14
	%Contingent	40.03	40.03	40.04	40.02	40.03	40.02	40.03
	Cont Surplus	0	0	0	0	0.00002	0	0.00004
cc=4.5 b=50	Tot.Cost	4853.59	4876.41	4899.08	4921.82	4983.26	4934.73	5495.85
	Avg.Inv.	30.32	32.36	32.88	33.81	33.57	32.17	48.05
	%Contingent	40.04	40.05	40.05	40.04	40.04	40.03	40.05
	Cont Surplus	0	0	0	0.00002	0.00034	0	0.03015

Labor supply uncertainty does not effect the amount of contingent capacity used significantly, when the permanent plus contracted capacity of the manufacturer is low (below the average demand). In this setting the manufacturer copes with the supply uncertainty with holding inventory and/or requesting excess contingent capacity, which results in significant capacity surplus. Table 6.1 illustrates the change in total cost, average inventory level and the contribution of contingent capacity in production under labor supply certainty and several different uncertainty structures. $Cos(k = 20)$ denotes the labor supply uncertainty where the availability rate is high at moderate requests and the pool size is 20 workers.

Note that the average production done with the contingent capacity does not change significantly, since the permanent capacity level is low. The manufacturer increases the inventory level significantly, when the supply uncertainty is low, however as can be seen from Table 6.1 when the variation is sufficiently high

the manufacturer decreases the inventory level and prefers backordering. As the variability of labor supply increases the manufacturer requests excess contingent capacity, which results in contingent capacity surplus. However when the variability and/or the cost of contingent labor is high, requesting excess contingent capacity and not utilizing the acquired amount (contingent capacity surplus) is a costly approach. In that case the manufacturer once more prefers increasing the inventory level. Note that the contingent capacity surplus increases as the variability of supply increases, except the case of binomial supply uncertainty, where ($Cos(k = 20)$) there is no contingent capacity surplus as mentioned in Chapter 5. When the cost of contingent labor is high the capacity surplus diminishes, as the manufacturer utilizes the valuable resource whenever realized.

As the level of owned capacity (permanent plus contacted) increases, the need for contingent capacity decreases and the effect of labor supply uncertainty reduces. In this setting as the labor supply uncertainty increases, the manufacturer may respond by decreasing the inventory level and the contingent capacity requested simultaneously. With this approach the manufacturer holds higher inventory in low demand seasons and backorders at the peak demand season. Note that the inventory levels tend to increase with the increasing variation, but are not necessarily higher than the case with contingent capacity certainty. On the other hand the contribution of contingent capacity in total production increases for low uncertainty levels and does not decrease until the supply uncertainty becomes sufficiently high. Table 6.2 depicts the total cost, average inventory level and the average contribution of contracted and contingent workers in total production under labor supply certainty and several different uncertainty structures.

When the level of owned capacity is sufficient to produce the average demand and the contingent labor supply has high variability, it is observed that the manufacturer avoids using too much contingent capacity and spreads the total production among periods, rather than utilizing flexible capacity against the demand seasonality (see Figure 6.1).

Table 6.2: Average Inventory Level and Average Production with Contracted and Contingent Capacity. $U = 10$, $V = 2$, $c_i = 1.2$

Parameters	Criteria	Certain	Norm 0.1	Norm 0.15	Norm 0.2	Norm 0.25	$Cos(k = 20)$	Unif
b=2.5 cc=1.5	Tot.Cost	4350.86	4353	4353.53	4354.3	4355.45	4355.94	4368.48
	Avg.Inv.	7.91	8.36	8.35	8.64	7.84	8.19	8.05
	%Contracted	4.17	4.02	4.03	4.46	4.23	4.74	5.19
	%Contingent	4.62	5.18	5.17	5.00	4.60	4.36	3.68
	Cont. Surplus	0	0	0.0004	0.0017	0.0021	0	0.0038
b=4.5 cc=1.5	Tot.Cost	4463.57	4468.43	4467.43	4468.99	4471.3	4469.53	4491.86
	Avg.Inv.	11.85	13.64	13.54	13.44	12.92	12.92	13.13
	%Contracted	4.82	4.17	4.20	4.23	4.41	4.42	5.34
	%Contingent	6.58	6.21	6.06	5.94	5.29	5.29	4.31
	Cont. Surplus	0	0	0.0005	0.0020	0.0046	0	0.0227
b=50 cc=1.5	Tot.Cost	4889.73	4896.04	4895.05	4900.81	4911.47	4907.06	4977.23
	Avg.Inv.	29.77	30.33	30.34	30.33	29.95	30.06	33.39
	%Contracted	6.05	5.90	5.89	5.90	6.01	5.97	5.77
	%Contingent	7.52	8.46	8.49	8.47	7.78	8.03	5.07
	Cont. Surplus	0.0000	0.0053	0.0240	0.0544	0.0756	0	0.2971
b=2.5 cc=4.5	Tot.Cost	4422.31	4421.43	4422	4420.45	4422.46	4422.27	4424.49
	Avg.Inv.	8.08	8.23	8.22	8.21	8.11	8.02	7.87
	%Contracted	5.92	5.84	5.84	5.85	5.90	5.94	6.00
	%Contingent	0.92	1.05	1.05	1.04	0.97	0.90	0.82
	Cont. Surplus	0	0	0	0	0	0	0
b=4.5 cc=4.5	Tot.Cost	4553.17	4557.16	4556.22	4557.01	4558.03	4559.44	4563.05
	Avg.Inv.	14.14	14.35	14.35	14.35	14.22	14.02	13.82
	%Contracted	6.41	6.30	6.30	6.30	6.37	6.47	6.56
	%Contingent	1.44	1.65	1.65	1.65	1.52	1.34	1.19
	Cont. Surplus	0	0	0	0	0	0	0
b=50 cc=4.5	Tot.Cost	5040.69	5042.75	5047.81	5045.03	5049.26	5051.19	5094.05
	Avg.Inv.	34.93	35.21	35.21	36.38	36.11	36.09	36.78
	%Contracted	5.85	5.74	5.74	5.22	5.33	5.34	6.28
	%Contingent	2.59	2.91	2.90	2.57	2.32	2.30	1.88
	Cont. Surplus	0	0	0	0.0003	0.0009	0	0.0139

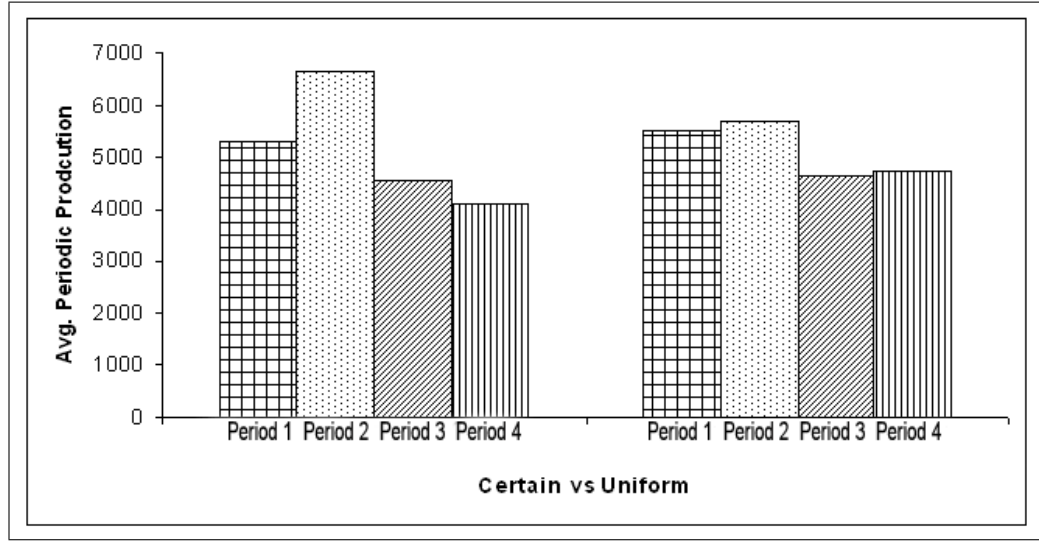


Figure 6.1: Periodic Production-Certain Supply vs Uniform Supply. $U = 10$, $V = 0$, $c_c = 3.5$, $b = 50$

Labor supply uncertainty affects the capacity and production decisions of the manufacturer significantly. However there is little or no monotonicity in the effects, as the data indicate. When the owned capacity is lower than the average demand, the amount of contingent capacity utilized does not vary much with increasing labor supply uncertainty. However the manufacturer plans the production in order to minimize the cost due to the variability. When the permanent capacity is greater than or equal to the average demand, the level of contingent capacity utilized becomes more sensitive to the supply uncertainty. As the variability increases, contingent workers become less attractive. However as mentioned before for high backordering costs or low contingent labor costs, the manufacturer prefers leaving contingent capacity surplus by ordering excess contingent capacity. In such cases, the contribution of contingent capacity in the total production increases as well.

6.2 Optimal Tactical Capacity Decisions

In this section we focus on the tactical decision of determining the levels of permanent and contracted capacity. As mentioned earlier, the decisions on the levels of permanent and contracted capacity are given at the beginning of the planning horizon and not modified throughout the planning horizon. The aim of the analysis is to investigate the optimal levels of permanent and contracted capacity under different settings. Initially we analyze the optimal contracted capacity level for given permanent capacity levels. Afterwards we focus on the optimal permanent and contracted capacity levels under various environments.

6.2.1 Optimal Contracted Capacity Level

Contracting temporary workers is a useful approach for diminishing the effects of contingent labor uncertainty. However it also brings an additional cost of contracting to the manufacturer, regardless of the amount utilized. Therefore the manufacturer has to be careful in the trade off between capacity uncertainty and contracting costs. In this subsection we analyze the effects of contingent labor cost c_c , backordering cost b , contracting cost c_i , labor supply uncertainty and demand uncertainty on the optimal size of contracted capacity.

Contracting cost c_i is an important parameter affecting the manufacturer's capacity decision. Recall that the contracting cost is the amount paid to the ELSA each period for every contracted worker, regardless of the amount utilized. As c_i gets larger, manufacturer's incentive to contract temporary workers falls. Recall that, we assume the total cost of a contracted worker ($c_i + c_f = c_w$) to be fixed. Therefore if $c_i = c_w$ then contracted capacity becomes practically equivalent to permanent capacity, which is always on the payroll, regardless of the amount utilized. Our analysis shows that the level of contracted capacity tends to decrease in c_i . Table 6.3 illustrates the optimal contracted capacity levels for different levels of c_i .

Table 6.3: Optimal Contracted Capacity Level (V^*)-Uniform Supply Uncertainty.

Parameters	c_i	V^*
$c_c = 3.5$ $U = 6$ $b = 50$	0	11
	0.6	7
	1.2	5
	1.8	4
	2.4	3
	3	3
$c_c = 3.5$ $U = 8$ $b = 5.5$	0	11
	0.6	3
	1.2	2
	1.8	1
	2.4	1
	3	1

The optimal contracted capacity level does not decrease after a specific level of c_i . In those cases the manufacturer still benefits from contracting temporary workers despite the high contracting cost, as the total cost is still lower than the expected cost of contingent workers.

The number of temporary workers to be contracted strongly depends on the cost and the uncertainty of contingent workers. As the cost of contingent workers increases, the optimal level of contracted capacity also increases as expected. Contingent labor uncertainty has a similar effect. As the variability of contingent labor supply increases the incentive to contract temporary workers increase. In that case the manufacturer prefers to contract temporary workers as the expected cost of contingent workers increase with increasing uncertainty. Table 6.4 depicts the optimal contracted capacity sizes under different labor supply uncertainty structures. Note that as the coefficient of variation increases the size of contracted capacity increases. Note that under uniform supply uncertainty the optimal size of contracted capacity increases as the backordering cost increases. In this case the manufacturer does not risk the production by relying on variable supply, and assures production by increasing the level of contracted capacity.

Table 6.4: Optimal Contracted Capacity Level (V^*). $U = 6$, $c_c = 3.5$

b	c_i	Norm 0.1	Norm 0.15	Norm 0.20	Norm 0.25	UNIF
2.5	0.6	3	4	4	4	5
2.5	1.2	2	2	2	3	4
2.5	1.8	2	2	2	2	3
2.5	2.4	2	2	2	2	3
2.5	3	2	2	2	2	3
5.5	0.6	3	4	4	4	5
5.5	1.2	2	2	2	2	4
5.5	1.8	2	2	2	2	3
5.5	2.4	1	2	2	2	3
5.5	3	1	2	2	2	3
50	0.6	3	3	4	4	7
50	1.2	1	2	2	2	5
50	1.8	1	1	2	2	4
50	2.4	1	1	1	2	3
50	3	1	1	1	2	3

Table 6.5: Optimal Contracted Capacity Level (V^*). $b = 50$, $c_i = 0.6$

U	c_c	$V^*(Unif)$	$V^*(Cos(k = 10))$	$V^*(Bin(20 - N))$
6	1.5	0	4	3
	2.5	5	5	4
	3.5	7	6	5
8	1.5	0	2	1
	2.5	3	3	2
	3.5	4	3	3
10	1.5	0	0	0
	2.5	0	0	0
	3.5	1	1	0

Table 6.5 illustrates the effect of contingent capacity cost. The numbers correspond to uniform labor supply uncertainty, binomial supply uncertainty with high availability rates at moderate requests ($Cos(k = 10)$) and binomial supply uncertainty with decreasing availability rate ($Bin(k - N)$). The pool size in the second and third cases are 10 and 20 respectively, therefore the probability of finding more contingent workers above the pool size is equal to 0. The underlying reason for presenting the optimal contracted capacity numbers for those cases is illustrating the effect of contingent capacity cost more evidently.

Table 6.6: Effect of Backordering Cost on Optimal Contracted Capacity Level. Poisson Demand, $c_c = 3.5$.

Parameters	b	$V^*(Unif)$	$V^*(Norm\ 0.25)$
$c_i = 0.6$ $U = 6$	2.5	5	3
	5.5	5	2
	50	7	2
$c_i = 0.6$ $U = 8$	2.5	2	2
	5.5	3	1
	50	4	1
$c_i = 1.2$ $U = 6$	2.5	4	3
	5.5	4	2
	50	5	2
$c_i = 1.2$ $U = 8$	2.5	1	1
	5.5	2	0
	50	2	0

Backordering cost is another important factor affecting the contracted capacity decision. Our analysis shows that backordering cost has opposite effects on the level of contracted capacity for high and low labor supply uncertainty. If the contingent labor supply uncertainty is low, the optimal size of contracted capacity decreases with increasing backordering cost, as the manufacturer prefers to utilize contingent workers from the relatively more reliable ELSA. However if the labor supply uncertainty is high, then the size of contracted capacity increases with increasing backordering cost. In such a case the manufacturer does not want to take the risk of utilizing contingent workers, where backordering is considerably costly. Table 6.6 shows the effect of backordering cost on the optimal contracted capacity size under high and low labor supply uncertainty.

The effect of demand variability on the size of contracted capacity is analogous to that of backordering cost. If the labor supply uncertainty is high the manufacturer responds to increasing demand variability by contracting more temporary workers. On the other hand if the contingent labor supply is considerably reliable the manufacturer reduces the size of contracted capacity with increasing demand variation, and responds by requesting more contingent workers. If the ELSA is reliable the manufacturer wants to stay more flexible as the variability of demand

Table 6.7: Effect of Demand Uncertainty on Optimal Contracted Capacity Level. Poisson Demand, $U = 6$, $c_c = 3.5$, $b = 50$.

Uncertainty Structure		Demand Distribution		
$U = 4$	c_i	$V^*(Norm\ 0.1)$	$V^*(Norm\ 0.2)$	$V^*(Norm\ 0.3)$
Norm 0.25	0.6	8	8	7
	1.2	6	5	5
	1.8	6	5	4
Bin(N20)	0.6	8	9	9
	1.2	6	7	8
	1.8	6	6	7
$U = 4$	c_i	$V^*(Gamma\ 0.5)$	$V^*(Gamma\ 1.0)$	$V^*(Gamma\ 1.5)$
Norm 0.25	0.6	6	4	2
	1.2	4	2	0
	1.8	3	2	0
Bin(N20)	0.6	8	11	11
	1.2	7	10	10
	1.8	6	8	8

increases. This is done by decreasing the level of contracted capacity and utilizing more contingent workers. In this way the manufacturer avoids the constant cost of contracting workers (c_i). However, when the variability of contingent labor supply is high the manufacturer does not rely on contingent workers and assures the production by contracting more workers as the variability of demand increases. Table 6.7 shows the effect of demand variation on the optimal size of contracted capacity under Normal and Gamma distributed demand. The table depicts the effect of demand variability under low supply uncertainty (Normal $cv = 0.25$) and high supply uncertainty (Binomial uncertainty with decreasing probability of success, $k = 20$).

6.2.2 Optimal Permanent and Contracted Capacity Decisions

In this subsection, we investigate the optimal levels of permanent and contingent capacity under various settings. Recall that we use the term "owned capacity"

for the total of permanent and contracted capacity. Note that owned production capacity is fully available to the manufacturer whenever required. However owned capacity brings a constant cost to the manufacturer regardless of the utilization. Therefore the tactical decision of determining the levels of permanent and contracted capacity is crucial. Under the presence of flexible capacity management, manufacturers may not require to hold excessive amounts of permanent production capacity. Our aim is to explain the manufacturer's owned capacity decision under different cost parameters and uncertainty structures.

Availability and the cost of contingent workers are important factors affecting the tactical capacity decision of the firm. If contingent workers are favorable, in terms of cost and availability, the manufacturer may not even hire permanent workers. However increasing contingent labor cost and/or supply uncertainty causes the manufacturer to increase the level of owned capacity, and utilize less contingent capacity. Table 6.8 illustrates the effect of contingent labor cost and uncertainty on the optimal owned capacity level. Note that the uncertainty and the cost of contingent workers may affect the number of contracted workers negatively. In Table 6.8 for $c_c = 3.5$, the manufacturer reduces the number of contracted workers and increases the number of permanent workers as the total owned capacity remains the same as the uncertainty of contingent labor supply increases from low to moderate (Norm 0.2 to Norm 0.25). In this case the manufacturer prefers to utilize the permanent capacity and to build up inventory for the peak demand season, contingent capacity is utilized only if the inventory level is low. However as the uncertainty of contingent labor supply increases the manufacturer increases the level of owned capacity by contracting more workers. In this case the manufacturer also increases the flexibility of the system by reducing the number of permanent workers.

The variability of the demand is another factor significantly affecting the size and the composition of the owned capacity. As the demand variability increases the manufacturer wants to be more flexible, therefore contracted workers are preferred over permanent workers. However the effect of demand variability on the size of the owned capacity depends on the structure of labor supply uncertainty.

Table 6.8: Effect of Contingent Labor Cost and Uncertainty on Owned Capacity (U^*, V^*). $c_p = 2.5$, $c_i = 0.6$, $b = 50$

c_c	Norm(0.1)	Norm(0.15)	Norm(0.2)	Norm(0.25)	Uniform
1.5	(0,0)	(0,0)	(0,0)	(0,0)	(8,0)
2.5	(2,0)	(4,0)	(6,0)	(6,0)	(8,3)
3.5	(8,1)	(8,2)	(8,2)	(10,0)	(8,4)

Table 6.9: Effect of Demand Variability on Owned Capacity. $c_c = 3.5$, $c_i = 0.6$.

(U^*, V^*)	b	Gamma 0.5	Gamma 1.0	Gamma 1.5
$Cos(k = 20)$	2.5	(8,1)	(6,4)	(5,5)
	5.5	(8,1)	(6,5)	(4,7)
	50	(8,2)	(4,10)	(3,11)
$Norm\ 0.25$	2.5	(8,0)	(6,0)	(4,0)
	5.5	(8,0)	(6,0)	(4,1)
	50	(8,0)	(6,1)	(4,2)

If the contingent labor supplier is reliable (uncertainty is low), then the manufacturer reduces the total owned capacity but increases the amount of contracted workers. If the uncertainty in contingent labor supply is high (limited and/or variable supply), then the manufacturer increases the total owned capacity by increasing the number of contracted workers. The immediate conclusion that can be drawn from this result is that, the manufacturer responds to demand variability by increasing the flexibility of production capacity through increasing the amount of contracted workers. Table 6.9 illustrates the effect of demand variability under high ($Cos(k = 20)$) and low ($Norm\ 0.25$) contingent labor uncertainty cases.

Backordering cost also has a significant effect on the size of the owned production capacity. The effect is analogous to that of demand variability; if the contingent labor supplier is reliable ($Norm\ 0.1$), then the manufacturer reduces the level of owned capacity to respond to the increasing backorder cost. In contrary if the contingent labor supply is highly uncertain (uniform), then the manufacturer increases the owned capacity level, mainly by increasing the number of

Table 6.10: Effect of Backordering Cost on Owned Capacity. $c_p = 2.5$, $c_i = 0.6$.

Supply Uncertainty		Norm 0.1		Uniform	
Demand Uncertainty		Norm 0.2	Norm 0.3	Norm 0.2	Norm 0.3
c_c	b	(U^*, V^*)	(U^*, V^*)	(U^*, V^*)	(U^*, V^*)
2.5	2.5	(4,0)	(2,0)	(8,0)	(8,0)
2.5	5.5	(4,0)	(0,0)	(8,1)	(8,1)
2.5	50	(2,0)	(0,0)	(8,3)	(8,3)
3.5	2.5	(10,0)	(10,0)	(10,0)	(8,3)
3.5	5.5	(10,0)	(8,1)	(10,1)	(8,4)
3.5	50	(8,1)	(8,0)	(8,4)	(8,5)

contracted workers. Table 6.10 illustrates the effect of backordering cost on the optimal level and composition of owned capacity.

6.3 Value of Contracting

In this section we aim to investigate the value of contracting temporary workers under different settings. We compare a contract setting (CC) with a no contract setting (NC). In the contract setting there exists a non-zero number of optimal contracted workers, whereas in the no contract setting the manufacturer has the same permanent capacity size but contracting temporary workers is not allowed. In both cases the manufacturer can utilize additional contingent capacity whenever required.

We define the value of contracting, VCC , as the difference between the optimal expected cost of operating the no contract system, ETC_{NC} , and that of the contracted system, ETC_{CC} . Therefore the value of contracting is, $VCC = ETC_{NC} - ETC_{CC}$. To reflect the relative effect, we also present the relative value of contracting as VCC/ETC_{NC} . Note that the value of contracting is always non-negative.

6.3.1 Effect of Permanent Capacity

The value of contracting temporary workers falls as the permanent capacity level increases. The underlying reason is trivial; as the owned capacity level increases, the manufacturer's need for additional capacity declines, therefore both the value of contracting and the incentive to contract will fall. Figure 6.2 illustrates the effect of increasing permanent capacity on the value of contracting. Note that the relative value (Figure 6.3) is also decreasing in the permanent capacity level.

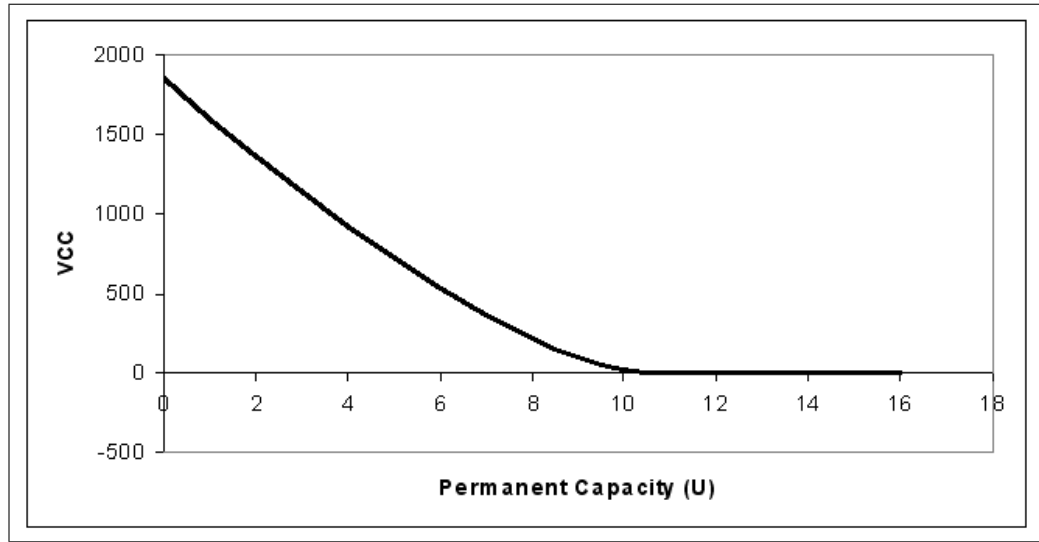


Figure 6.2: Value of Contracting vs Permanent Capacity Level-Uniform Supply Uncertainty, Poisson Demand $b = 50$, $c_c = 3.5$

6.3.2 Effect of Contracting Cost c_i

The value of the labor supply contract is strongly dependent on the cost of contracting. Recall that the contracting cost c_i is paid for each contracted worker every period, regardless of the number utilized in production. Therefore increasing contracting cost reduces the value of contracting as it increases the amount of the steady cost in the payroll for contracted workers. Figures 6.4 and 6.5 illustrate the effect of c_i on VCC and $\%VCC$ respectively.

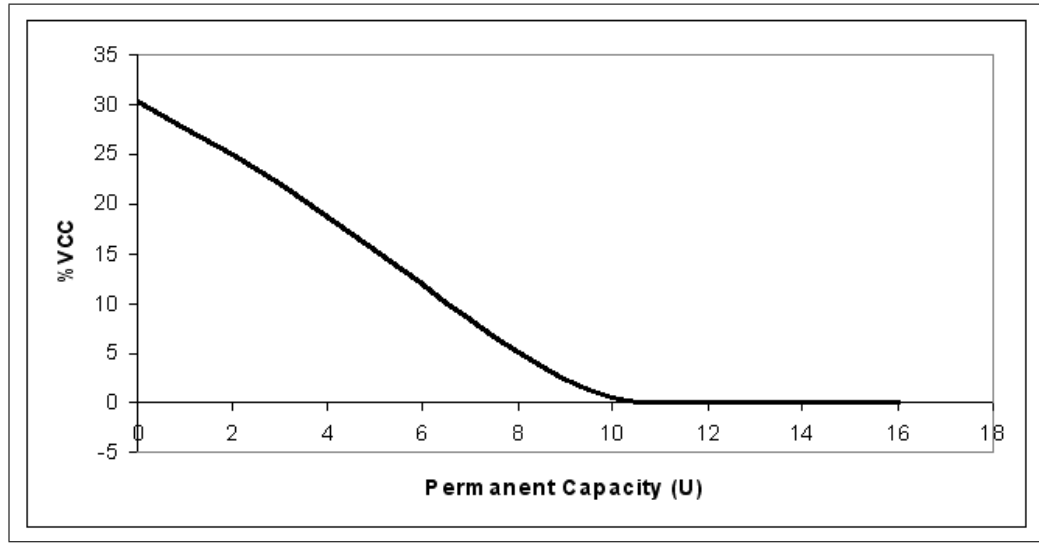


Figure 6.3: Relative Value of Contracting vs Permanent Capacity Level-Uniform Supply Uncertainty, Poisson Demand $b = 50$, $c_c = 3.5$

As the figures illustrate, contracting becomes more attractive as the cost of contracting, $c_i V$ becomes less costly. Recall that increasing c_i reduces the flexibility of contracted capacity and at $c_i = 3$ contracted workers become practically equivalent to permanent workers.

6.3.3 Effect of Backordering Cost

The effect of backordering cost on the value of contracting is dependent on the type of labor supply uncertainty. Under low and high supply uncertainty cases, the change in the backordering cost results in contrasting effects. If the contingent labor supply is highly uncertain, then the value and the relative value of contracting increases with increasing backordering cost. In this case the manufacturer increases the level of owned capacity for better response to demand, in order to avoid backordering. On the other hand, if the contingent labor supply is fairly reliable, then the value and the relative value of contracting decline

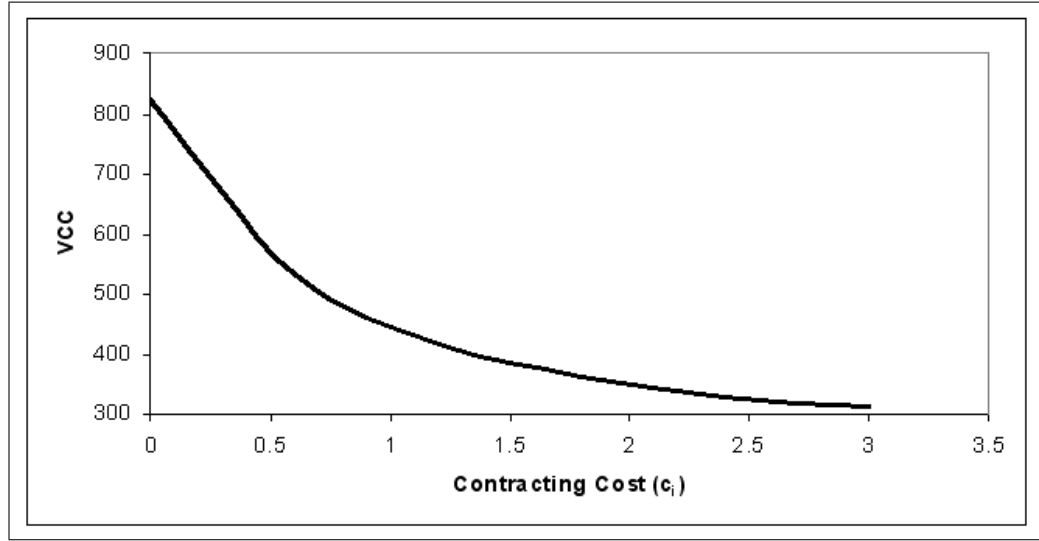


Figure 6.4: Value of Contracting vs Contracting Cost (c_i) - Uniform Supply Uncertainty, Poisson Demand $b = 50$, $c_c = 3.5$

with increasing backordering cost. As the uncertainty level increases the decreasing effect on the value of contracting diminishes, however the relative value of contracting remains decreasing in the backordering cost until the labor supply uncertainty becomes sufficiently high. Our analysis shows that for normal supply uncertainty, for $CV \geq 0.25$ the decreasing effect of backordering cost on the value and the relative value of contracting diminishes. Table 6.11 depicts the effect of backordering cost on the value of contracting under different supply uncertainty structures and permanent capacity levels. Note that for lower levels of permanent capacity the value of contracting is high and is not decreasing in the backordering cost unless the labor supply uncertainty is very low ($cv < 0.2$).

6.3.4 Effect of Contingent Labor Cost (c_c) and Supply Uncertainty

Contracting temporary workers is most valuable when it is costly to utilize contingent workers. The incentive to contract workers increases as the cost or the uncertainty of contingent labor increases. Contingent workers turn out to be more

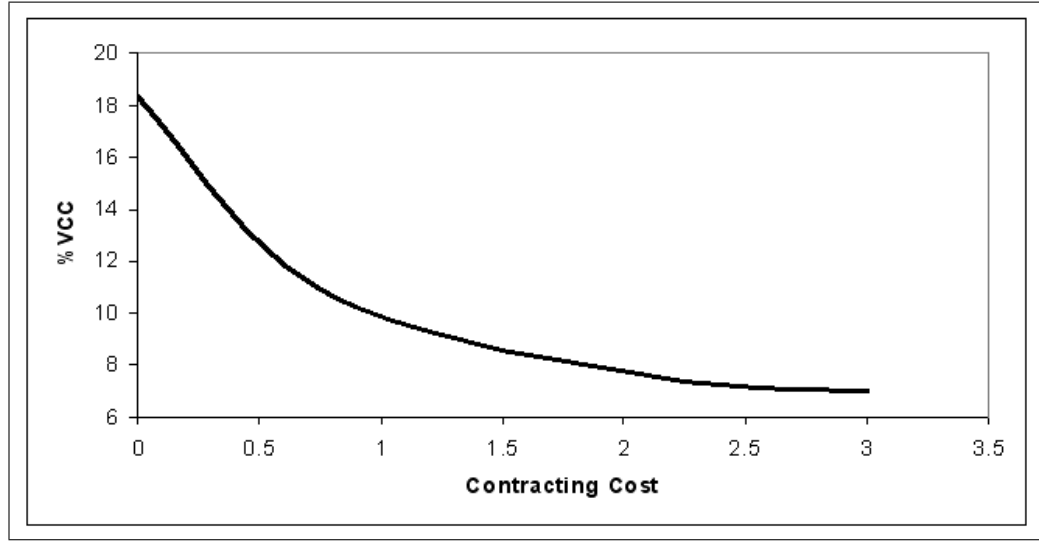


Figure 6.5: Relative Value of Contracting vs Contracting Cost (c_i) - Uniform Supply Uncertainty, Poisson Demand $b = 50$, $c_c = 3.5$

attractive as the expected cost of utilizing them becomes less costly. Therefore as the cost and/or the uncertainty of contingent labor increases, the manufacturer refrains from using contingent workers and the value of contracting increases. Note that, if the contingent labor cost is sufficiently low, then the manufacturer does not contract temporary workers, considering that excess contingent capacity requests and resulting capacity surplus will not be too costly. Also if the contingent labor supply is reliable (low uncertainty) then the value of contracting is

Table 6.11: Effect of Backordering Cost on VCC and $\%VCC$. $c_c = 3.5$, $c_i = 0.6$.

		<i>Norm 0.15</i>		<i>Norm 0.20</i>		<i>Norm 0.25</i>		<i>Uniform</i>	
U	b	VCC	$\%VCC$	VCC	$\%VCC$	VCC	$\%VCC$	VCC	$\%VCC$
4	2.5	163.96	4.64	181.29	5.10	86.66	2.58	451.18	11.70
4	5.5	157.04	4.27	179.5	4.84	87.71	2.49	560.35	13.55
4	50	145.17	3.58	183.34	4.46	100.67	2.55	922.73	18.67
6	2.5	72.03	2.15	82.28	2.44	191.99	5.39	242.77	6.82
6	5.5	64.71	1.85	78.42	2.23	197.48	5.31	309.47	8.15
6	50	54.29	1.40	76.89	1.96	224.54	5.41	534.14	11.91
8	2.5	7.74	0.24	10.2	0.32	10.64	0.33	67.36	2.03
8	5.5	5.76	0.17	8.64	0.26	10.03	0.29	97.15	2.75
8	50	1.75	0.05	7.35	0.19	12.52	0.33	211.04	5.12

Table 6.12: Effect of Contingent Labor Cost on Value of Contracting. $b = 50$, $c_i = 0.6$.

U	c_c	V^*	VCC	$\%VCC$
4	1.5	3	33.05	0.89
	2.5	7	386.13	8.93
	3.5	9	922.73	18.67
6	1.5	0	0	0
	2.5	5	190.97	4.70
	3.5	7	534.14	11.91
8	1.5	0	0	0
	2.5	3	52.49	1.34
	3.5	4	211.04	5.12

Table 6.13: Effect of Contingent Labor Uncertainty on Value of Contracting. $c_c = 3.5$, $c_i = 0.6$.

Labor Supply	<i>Norm 0.1</i>		<i>Norm 0.15</i>		<i>Norm 0.2</i>		<i>Norm 0.25</i>		<i>Uniform</i>	
b	VCC	$\%VCC$	VCC	$\%VCC$	VCC	$\%VCC$	VCC	$\%VCC$	VCC	$\%VCC$
2.5	64.64	1.93	72.03	2.14	82.28	2.44	86.66	2.57	242.77	6.82
5.5	56.22	1.61	64.71	1.85	78.42	2.23	87.71	2.49	309.47	8.15
50	40.81	1.06	54.29	1.39	76.89	1.96	100.67	2.55	534.14	11.91

less sensitive to the change in contingent labor cost. Tables 6.12 and 6.13 show the effects of contingent labor cost and uncertainty respectively.

6.3.5 Effect of Demand Variability

The variability of demand has a significant effect on the incentive to contract temporary workers. As the variability of demand increases the manufacturer wants to be more flexible. The value of contracting decreases with increasing demand variability as the manufacturer does not want to tie up a contracting cost for contracting temporary workers and rather prefers using contingent workers, which are more flexible. One exception is the case where $c_i = 0$. However in that case, contracted workers become equivalent to contingent workers with supply certainty, as the preliminary cost disappears. Table 6.14 depicts the effect of

Table 6.14: Effect of Demand Variability on the Value of Contracting. $b = 50$, $c_i = 0.6$.

Demand Uncertainty		Norm 0.1		Norm 0.2		Norm 0.3	
U	b	VCC	$\%VCC$	VCC	$\%VCC$	VCC	$\%VCC$
4	2.5	284.39	8.05	242.29	6.67	202.1	5.59
	5.5	308.01	8.50	251.66	6.70	203.79	5.40
	50	404.5	10.44	308.45	7.53	233.98	5.56
6	2.5	151.65	4.59	124.89	3.65	100.33	2.93
	5.5	167.9	4.95	127.85	3.61	97.32	2.71
	50	231.9	6.41	162.2	4.18	112.2	2.79
8	2.5	41.25	1.32	32.58	1.00	21.33	0.65
	5.5	48.47	1.51	31.9	0.94	19.13	0.55
	50	91.81	2.70	44.15	1.19	22.69	0.58

demand uncertainty on the value of contracting.

6.4 A Heuristic Policy

In this section we present the *Target Mean Heuristic* that we have developed, and compare its performance under various settings. The aim of the heuristic is to relatively ease the decision process of the manufacturer by disregarding the structure of the labor supply uncertainty. The heuristic only considers the mean of the contingent workers to be acquired μ_N , when N contingent workers are requested. Recall that w_t is the capacity position set by the decision maker and η is the observed capacity level available for production in period t . The procedure of the *Target Mean Heuristic* is as follows:

- Solve the infinite horizon integrated capacity and production problem for the case of no contingent capacity uncertainty. Let $y_t^*(x_t)$ be the optimal order up-to level for the inventory level x_t .
- If $y_t^*(x_t) \leq x_t + U + V$, then set the capacity position to $w_t^h(x) = y_t^*(x_t) - x_t$

If $y_t^*(x_t) > x_t + U + V$, then set the number of contingent workers to be requested N such that $x_t + U + V + \mu_N = y_t^*(x_t)$

- Produce at full observed capacity level and raise the inventory position to $y_t^h = x_t + \eta_t$.

Note that the heuristic allows no contingent capacity surplus, in other words if contingent workers are requested, the amount acquired is fully utilized regardless of the proposed order up-to level. We analyze the performance of the heuristic under different settings and determine the effect of different parameters. Our aim is to determine the regions, where the heuristic performs as a good decision tool and where it is necessary to consider the labor supply uncertainty. The conclusions obtained in this section are derived from the comparison of optimal results and the simulation results of the heuristic policy.

6.4.1 Effect of Labor Supply and Demand Variability

Supply variability has a major effect on the performance of the heuristic. The heuristic performs better as the variability in the supply decreases. This is an expected result as the heuristic uses the solution of the problem with no supply uncertainty. We analyze the effect of different supply uncertainty structures and determine under which the heuristic performs better. As the supply variability increases the performance of the heuristic becomes worse. As expected, the decision maker has to consider the uncertainty more carefully as the variability in labor supply increases. Table 6.15 illustrates the performance of the heuristic under Binomial with increasing probability of success ($\text{Bin}(p = \frac{k}{N})$), Binomial with high probability of success at low and high request ($\text{Bin}(k = 20)$), Uniform and Normal ($\text{CV} = 0.25$) uncertainty structures. The demand is Poisson distributed in this experimental setting.

Demand variability on the other hand has a relatively minor and an unpredictable effect on the performance of the heuristic. However for levels of CV higher than 0.5, we have observed that with increasing demand variation the

Table 6.15: Heuristic Performance and Supply Uncertainty. ($b = 5, 10, 15$), ($c_c = 1.5, 3.5, 5.5$)

Uncertainty Structure	Maximum Gap	Average Gap
$\text{Bin}(p = \frac{N}{20})$	%50.18	%12.01
$\text{Sin}(k=20)$	%30.26	%7.11
Uniform	%24.63	%2.78
Normal	%3.37	%0.51

Table 6.16: Heuristic Performance (Average Gap) with Increasing Coefficient of Variation. Gamma Demand, $b = 15$, $c_c = 3.5$

Supply Uncertainty	$CV = 0.5$	$CV = 1$	$CV = 1.5$
$\text{Bin}(p = \frac{N}{20})$	%4.9	%4.7	%4.3
$\text{Bin}(p = \frac{k-N}{20})$	%6.1	%5.2	%4.6
Uniform	%2.3	%1.32	%0.35
Normal	%0.92	0.65	%0.54

gap between the optimal and heuristic solution decreases significantly. Table 6.16 illustrates the effect of variability on the performance of the heuristic under Gamma distributed demand.

6.4.2 Effect of Permanent and Contracted Capacity

As the level of permanent capacity and contracted capacity, $U + V$ increases the heuristic performs better. There are two relevant reasons for this result. The first reason is that as the level of capacity increases the reliance on contingent capacity falls and the distinction between the heuristic decision and the optimal decision diminishes. The second reason is that as the number of contingent workers requested declines the variation in the observed capacity falls. One exception to this

Table 6.17: Heuristic Performance with Increasing Permanent plus Contracted Capacity. Uniform Supply Uncertainty, $b = 15$, $c_c = 3.5$

U+V	Optimal Cost	Heuristic Cost	Performance
0	5388.3	6171.37	%14.53
2	4927.81	5441.12	%10.42
4	4499.73	4806.52	%6.82
6	4111.98	4268.27	%3.8
8	3803.45	3865.16	%1.62
10	3721.33	3735.17	%0.37
12	3926.86	3935.98	%0.23
14	4273.07	4274.95	%0.04
16	4703.84	4704.17	%0.01

is the *binomial uncertainty with increasing probability of success*, where the variability decreases as the number of contingent workers ordered increases. However even including the aforementioned case, the effect of capacity increase dominates the effect of increasing variance and the heuristic performs better. Tables 6.17 and 6.18 show the performance of the heuristic with increasing capacity.

Note that under both cases the heuristic results get closer to the optimal as $U + V$ increases. However under low capacity levels the heuristic approach results in significantly higher costs. Especially when $U + V$ is lower than the average of seasonal demands (10 in this setting), it is better for the decision maker to use the optimal solution rather than the convenient heuristic.

6.4.3 Effect of Contingent Labor Cost and Backordering Cost

Contingent labor cost c_c and backordering cost b have important effects on the performance of the heuristic. For capacity levels lower than the average of the seasonal demands (10 in this setting), increasing backorder cost significantly reduces the performance of the heuristic, as the cost of backordering increases, and the order up-to levels tend to raise. In contrast, with increasing contingent labor

Table 6.18: Heuristic Performance with Increasing Permanent plus Contracted Capacity. Binomial Uncertainty with Increasing Probability of Success $k = 20$, $b = 15$, $c_c = 3.5$

U+V	Optimal Cost	Heuristic Cost	Performance
0	4250.23	5329.29	%25.38
2	4055.6	5101.61	%25.19
4	3868.77	4835.63	%24.99
6	3703.5	4580.47	%23.67
8	3593.67	4433	%23.35
10	3648.94	4248.26	%16.42
12	3905.44	4213.87	%7.89
14	4266.34	4432.64	%3.89
16	4702.04	4748.18	%0.98

cost the performance of the heuristic increases. There are two underlying reasons for this. The first reason is that, as the cost of contingent labor increases the manufacturer tends to utilize more of the contingent workers acquired. The second and the more important reason is that, the manufacturer relies less on contingent capacity as its cost increases. Table 6.19 illustrates the effects of backordering cost and contingent labor cost under Poisson distributed demand.

The heuristic presented in this section provides good solutions under particular cases. However as discussed above there are cases where the gap between the heuristic solution and the optimal becomes significantly large. In those cases the decision maker should consider the structure of the labor supply uncertainty more carefully and should decide on the capacity and production levels simultaneously but separately. The decision maker should use the integrated approach considering the supply uncertainty when

- labor supply uncertainty is high,
- permanent and contracted capacity levels are low,
- backordering is costly, and
- tendency to use contingent labor is high, i.e. c_c is low.

Table 6.19: Heuristic Performance under different Backordering and Contingent Labor Costs. Uniform Supply Uncertainty, $U = 6$

b	c_c	Performance
5	1.5	%1.84
5	3.5	%1.16
5	5.5	%0.93
10	1.5	%4.27
10	3.5	%2.55
10	5.5	%2.01
15	1.5	%5.97
15	3.5	%3.81
15	5.5	%3.15

Chapter 7

Conclusions

In this study we consider the problem of integrated flexible capacity and inventory management problem under contingent capacity uncertainty and seasonal stochastic demand. We investigate the problem under the workforce planning framework. The manufacturer aims to minimize its operational costs by planning capacity, production and inventory simultaneously. In this context the manufacturer has two external resources for temporarily increasing the production capacity; contracted workers and contingent workers. Temporary workers are contracted at a specified fee and utilized whenever required with an additional utilization fee. On the other hand contingent workers are requested after fully utilizing the permanent and contracted workers (if any). Contingent workers are acquired by the manufacturer according to the specific distribution and the number requested. The external labor supplier's pool size and the behavior is reflected by the probability distribution of acquiring contingent workers. The manufacturer plans to minimize its operational costs by optimizing the levels of production and external capacity resources to be utilized at each period. At the beginning of the planning horizon the manufacturer sets the optimal levels of permanent and contracted capacity levels, which remain unchanged throughout the planning horizon.

Initially we focus on the characteristics of the optimal policy under different

supply uncertainty structures. Under all-or-nothing type supply uncertainty the decision function $H_t(w_t|x_t)$ and the minimum cost function of operating the system for periods t and on $(f_t(x_t))$ are convex, the resulting optimal policy is of state dependent order up-to type and is independent of the probability of acquiring the requested contingent workers (p). The production and capacity position decisions are equivalent under this type of supply uncertainty. In this case the optimal order up-to levels are increasing in the number of periods remaining for the 2 period case. In the case of uniform supply uncertainty the decision function $H_t(w_t|x_t)$ is convex in the capacity position w . However the minimum cost of operating the system for periods t and on $(f_t(x_t))$ is observed to be non-convex. For the other supply uncertainty forms that we analyze, both $H_t(w)$ and $f_t(x_t)$ are non-convex. Our analysis indicates that $f_t(x_t)$ is uni-modal for all forms of supply uncertainty.

In the special case of the problem with deterministic contingent labor supply, we show that the finite horizon optimal policy structure presented by Tan and Alp [31] converges in the infinite horizon. The analysis of the optimal policy indicates that under the presence of positive setup costs for initiating production and utilizing contingent workers, the optimal production level is non-monotonic, in other words as the starting inventory level increases, the optimal production level may increase as well. Our computational analysis shows that under stationary demand there exists up to five critical points in the optimal policy. For seasonal demand the number of critical points go up to seven, depending on the demand distribution.

When we compare the case with deterministic contingent labor supply and uncertain labor supply, we observe that uncertainty of contingent labor does not necessarily reduce the amount of contingent workers utilized in production. This is evident when the owned production capacity (permanent plus contracted) of the manufacturer is below the average demand, but the result still holds when the owned capacity is sufficient for meeting the average demand and the contingent capacity uncertainty has small variation. If the variability of contingent workers become sufficiently high the manufacturer deters from utilizing contingent labor

and rather relies on inventory or backordering depending on the corresponding costs. As the variability of contingent workers increase, the production levels in distinct seasons become closer, in other words the manufacturer spreads the production to all seasons more evenly. Another outcome of increasing labor supply variability is the contingent capacity surplus. As the level of uncertainty increases the level of unutilized contingent capacity increases. The underlying reason is that the manufacturer requests in higher levels in order to guarantee production, and if the realized level of contingent capacity is higher than the optimal production level, then contingent workers are not fully utilized even though the fee is paid.

Temporary labor contracts is one of the most important concepts presented in this study. As the expected cost of utilizing contingent workers increase, due to increasing contingent labor cost and/or uncertainty, the manufacturer contracts for more temporary workers with the ELSA. The size of optimal contracted capacity falls as the contracting cost c_i increases. Our analysis shows that increasing backordering cost reduces the size of optimal contracted capacity if the ELSA is reliable. In this case the manufacturer prefers to be more flexible and avoids incurring costs for contracting temporary workers. On the other hand if the uncertainty in contingent labor supply is high, the manufacturer contracts more workers as the backordering cost increases. In this case the manufacturer abrogates the risk of backordering, that could arise from contingent labor uncertainty, by contracting more temporary workers. The size of contracted capacity is strongly affected by the variability of demand. The interaction between increasing demand variability and the optimal size of contracted capacity is similar to that in the case of backordering cost. If the contingent labor supply has small variability, then the manufacturer prefers to remain more flexible by contracting less workers, as the variability of demand increases. In contrast if the ELSA supplies contingent worker with a high uncertainty, then the manufacturer prefers to contract more workers in order to cope with the fluctuations in demand. When the levels of permanent and contracted capacity are planned simultaneously, our observation shows that when the contingent labor supply is reliable and cheap

the manufacturer tends to have a small sized permanent capacity. The size of optimal capacity increases with increasing contracting cost, contingent labor cost and contingent labor uncertainty.

The advantage of contracting for temporary workers is analyzed under the name of value of contracting. Contracting is most valuable when the expected cost of utilizing contingent workers is high and the demand variability is low. As the level of permanent capacity increases, the value of contracting declines as the manufacturer's need for external resources diminishes. The value of temporary workers fall as the contracting cost c_i increases. Increasing backordering cost decreases the value of contracts when the contingent labor supply is reliable. If the contingent labor supply has high variability, then the value of contracting increases with increasing backordering cost. Our analysis shows that the value of contracting falls with increasing demand variability as the manufacturer prefers flexibility against variability.

In this study we analyze the integrated flexible capacity and production management problem from the manufacturers perspective. An extension of this research can include the external labor agencies optimal capacity planning, and a game theoretic equilibrium contract design. In such a study, the ELSA may decide on the contracting cost and the optimal pool size, while the manufacturer decides on the size on the optimal level of contracted capacity.

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