

A STOCHASTIC DISASSEMBLY LINE BALANCING PROBLEM WITH
HAZARDOUS TASKS

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HAZARDOUS TASKS**

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ABSTRACT

A STOCHASTIC DISASSEMBLY LINE BALANCING PROBLEM WITH HAZARDOUS TASKS

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In this thesis, we study a Stochastic Disassembly Line Balancing Problem (SDLBP) with hazardous tasks. We define success and failure events for each hazardous task and describe several scenarios over all hazardous tasks. Our aim is to maximize expected profit over all scenarios. We construct mathematical model with one, two and three hazardous tasks. Then, general formulation both in nonlinear and linear form is studied. We test the performance of the model on randomly generated data using the networks taken from the literature. Moreover, we study the Value of Stochastic Solution (VSS) which is the added value of considering uncertainty in a decision. The limits of the SDLBP models are observed by the increase of number of tasks in a network and by the increase in number of hazardous tasks. We propose some solution approaches and bounding procedures, and show their satisfactory performances

Keywords: Stochastic Disassembly Lines, Disassembly Line Balancing, Hazardous Tasks

ÖZ

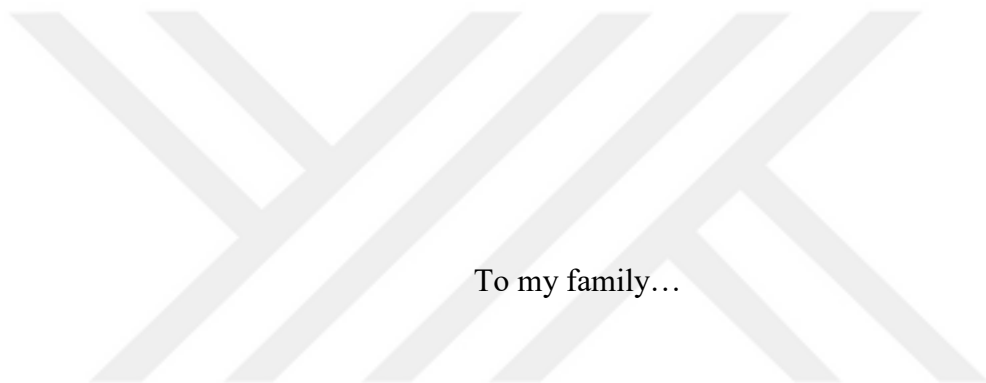
TEHLİKELİ İŞLER İÇEREN STOKASTİK DEMONTAJ HATTI DENGELEME PROBLEMİ

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Bu tezde tehlikeli aktiviteler içeren Stokastik Demontaj Hattı Dengeleme Problemi'ni ele alıyoruz. Her bir tehlikeli aktivite için başarılı ve başarısız durumları, böylece bütün zararlı işler için senaryoları tanımlıyoruz. Amacımız tüm senaryolar üzerinden toplam karı ençoklamaktır. Önce bir, iki ve üç tane tehlikeli aktivitesi olan matematiksel modeller kurulmuştur. Sonra doğrusal olmayan ve doğrusal genel modeller üzerine çalışılmıştır. Modelin performansı literatürden alınan ağlar ve rassal üretilen veriler ile test edilmiştir. Ayrıca karar verirken belirsizliği dahil etmenin katma degerini gosteren Stokastik Çözümün Değeri çalışılmıştır. Artan aktivite sayısı ve zararlı aktivite sayısı ile Stokastik Demontaj Hattı Dengeleme Probleminin limitleri gözlemlenmiştir. Çözüm yöntemleri ve sınırlama yöntemleri önerip, başarılı performanslarını gösterdik.

Anahtar Kelimeler: Stokastik Demontaj Hatları, Demontaj Hat Dengeleme, Zararlı Aktiviteler



To my family...

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CHAPTER 1

INTRODUCTION

The recognized importance of environmental issues and end of product life cycle together give rise to different concerns about the alternative evaluations of the used products. Landfill, reuse, recycle, remanufacture are the main alternative evaluations for any product which is in its end of life. Recycling preserves the material content of the used products via some disassembly operations and reprocessed into a form that can be used as a product. (a remolded car tire can be used as a conveyor belt). Reused means that the product is used “as is” with minor operations (refilled milk bottle). Remanufacturing, on the other hand, keeps the functional content of the used products and improves their quality up to a desired usable level via some manufacturing and disassembly operations. Products with no value result in landfills. These evaluations and recognitions are considered not only by customer awareness towards environmental issues but also by the new and more rigid environmental regulations. Moreover, the recent advances in technology have made it possible to manufacture the products that meet the environmental standards and are easy to reuse after consumed by the customer.

For any valuable discarded product, to avoid from landfill, one needs to physically separate the product into its useful parts. Thus, disassembly is the first important step of product recovery activities (McGovern and Gupta, 2011) and it is the methodical extraction of valuable parts, through a series of operations that involve the separation of the reusable parts from the discarded products. The parts are either subject to some re-manufacturing process or sold to suppliers.

A disassembly product can include some hazardous parts as well as non-hazardous ones. For example, electrical and electronic equipment consist of many different

substances some of which may contain hazardous components, like static charges and mercury. In automotive industry we see battery acids and gasoline vapors as hazardous components. Printed Circuit Boards of the computer drives which are found in Hard Disk Drive, CD-ROM drive and Floppy Drive contain hazardous materials like mercury, cadmium, chromium and lead, and toxic substances like antimony, arsenic, barium and beryllium. For many products, all toxic chemicals, even sharp edge and broken glass are in the category of hazardous products. Hazardous parts are crucial for disassembly lines both for the environmental impact and cost analysis. These parts are undesirable since they may damage the demanded parts or can even result in the stoppage of the line. They all need special handling if resulted in any disassembly operation. Disassembly operations are usually performed on a disassembly line that consists of a number of serial workstations. The Disassembly Line Balancing Problem assigns the set of tasks to each workstation for each product to be disassembled.

Gungor and Gupta (2001) discuss some complications in a disassembly line that are encountered due to the hazardous tasks. Below we list some noteworthy of those complications:

- *Early leaving units:* Some tasks cannot be accomplished due to the hazardous tasks; thus, the unit leaves the workstation early.
- *Self skipping units:* The hazardous tasks can cause the inability in the accomplishment of all tasks of that unit for the current workstation, then unit leaves the workstation early.
- *Disappearing units:* If hazardous tasks of a unit cause the discontinuation of other tasks on that unit, the unit is removed from the line.
- *Normal flow:* The hazardous tasks do not cause any problem in the flow of the unit.

Disassembly process has two main categories, such as *partial disassembly* and *complete disassembly*, according to the retrieved parts. In *partial disassembly*, the

product is not fully disassembled and in *complete disassembly* the product is fully disassembled so as to obtain all available parts. Partial disassembly can be favored. Due to economic and environmental aspects, we consider partial disassembly in order to minimize excess materials.

In this thesis, we study Disassembly Line Balancing Problem (DLBP) with fixed number of workstations and hazardous tasks. We consider “Disappearing Unit” complication versus “Normal Flow” case for each hazardous task. When any hazardous task fails, it causes damages of the tasks on the workstation that it is performed on and all remaining tasks that would be performed in the succeeding workstations. In other words, the disassembly unit disappears. The successful completion of the unit refers to the normal flow case. We consider partial disassembly and attribute probabilities for the successful completion and failure of the disassembly unit and aim to maximize the total expected net revenue.

To the best of our knowledge, the previous stochastic disassembly line balancing studies consider the uncertainty of the task times and our Disassembly Line Balancing Problem study is the first attempt that considers stochasticity of the hazardous tasks in.

In this study we aim to explore the stochastic behavior of hazardous tasks and investigate the answers of questions regarding the parameters attributed to them as well as other problem parameters. The research questions considering the hazardous task locations and the success probabilities are inquired. For example, for a given workstation design where hazardous tasks should be better assigned or what if the probability of hazardous task is close to or far from a non-hazardous task. Other problem parameters are also examined regarding the expected net revenue and complexity of DLBP. The reason why a stochastic programming is preferred rather than a deterministic case when hazardous tasks are included is illustrated by the value of stochastic solution calculations. It is a good approach to appraise the constructed problem, thus several computations performed accordingly. Finally, the solution

methods that can be used to solve large-sized problem instances besides the exact solution approaches are investigated.

The rest of the study is organized as follows. Chapter 2 reviews the literature on the deterministic and stochastic disassembly line balancing problems. In Chapter 3 we define and formulate our problem. We present our solution method in Chapter 4 and computational experiments in Chapter 5. The conclusion and future research directions are presented in Chapter 6.



CHAPTER 2

LITERATURE REVIEW

The DLBP was first introduced in Güngör and Gupta (2002). McGovern and Gupta (2007a) showed that the decision version of the DLBP is NP-complete. We review the studies based on their deterministic and stochastic nature.

2.1 Deterministic DLBP Studies

McGovern and Gupta (2007b) considered a DLBP to minimize the number of workstations while balancing the idle times between the workstations. They proposed an exhaustive search method and a genetic algorithm. Güngör and Gupta (2002) proposed a heuristic procedure for the DLBP under complete disassembly. They assumed an infinite supply of a single product and considered the efficient utilization of the resources while satisfying the minimum release quantity. Koc et al. (2009) considered a complete disassembly DLBP so as to minimize the number of workstations. They proposed Integer and Dynamic Programming formulations. Altekin et al. (2008) studied the DLBP under partial disassembly and formulate the net revenue maximization model as a mixed-integer linear program. They used the model relaxations to find lower and upper bounds. Kalaycilar et al. (2016) studied the partial DLBP with fixed number of workstations and finite supply so as to maximize the total value of the recovered parts. They developed a heuristic procedure using the optimal solutions of the LP relaxations and proposed several upper bounds. Ilgin et al (2017) proposed a linear physical programming-based disassembly line balancing method with four objectives: minimizing the cycle time,

minimizing the number of stations, minimizing the difference between the slack times and minimizing the total idle time.

Güngör and Gupta (2001), Güngör and Gupta (2002), McGovern and Gupta (2007a) studied deterministic assembly line problem with hazardous tasks. Güngör and Gupta (2001) mentioned several complications and to reduce the effects of the complications related to the hazardous parts, they proposed a shortest path based solution algorithm. Güngör and Gupta (2002) considered the complete disassembly line balancing problem so as to minimize number of workstations. They proposed a heuristic approach that considers a priority function that gives highest priority to the hazardous tasks to prevent other useful parts and reduce hazardous material spill. McGovern and Gupta (2007a) studied complete disassembly line balancing problem so as minimize the number of workstations and propose exhaustive search procedure. They aimed to remove hazardous tasks early in the disassembly sequence and to minimize the number of part removal direction changes required for disassembly.

2.2 Stochastic DLBP Studies

The stochastic disassembly line balancing studies considered the uncertainty of the task times with known probability distributions. Bentaha et al. (2014a) considered complete disassembly line balancing problem and proposed a sample average approximation method that treats the uncertainty using resource costs. Bentaha et al. (2014b) considered the disassembly line balancing and sequencing problems simultaneously for the complete disassembly problem. Bentaha et al.(2014c) gave the maximum total profit model for the complete disassembly problem and proposed a lagrangean relaxation approach for its solution. Bentaha et al. (2014d) considered complete disassembly line balancing to maintain workload balancing problem with fixed number of workstations. They developed a stochastic binary program. Altekin (2017) addressed minimum number of workstations problem under complete disassembly and developed a stochastic Mixed Integer model using a chance constraint.

Bentaha et al. (2015a, 2015b) considered the problem of minimizing the workstation operation costs and additional costs for handling the hazardous tasks. All studies assume that the task times are stochastic with known probability distributions. They developed several lower and upper bounding mechanisms, including mixed integer programs with chance constraints and Monte Carlo simulation techniques.



CHAPTER 3

PROBLEM DEFINITION AND FORMULATION

We consider a disassembly line balancing model with N tasks some of which are hazardous. Task i is specified by its cost c_i and processing time t_i . Task i , if releases a part, brings a revenue r_i that includes its market value and recycled material value. R_i is the net revenue of task i , $R_i = r_i - c_i$. Negative revenue for a part releasing task represents a disposal cost.

We make the following additional assumptions.

- The workstations are already set up and there are K workstations.
- All workstations are identically equipped and are capable of performing any one of the tasks at the same pace.
- The tasks are not divisible, i.e., they should be assigned to exactly one workstation.
- The cycle time, CT , is known with certainty and is not subject to change.
- All parameters, such that task times, precedencies are deterministic.
- A partial disassembly is done, i.e., a subset of -but not- all parts is released.
- There exist hazardous task(s) inside the discarded product that has(have) a success probability.

During the disassembly operations, two events may occur for each hazardous task:

- *Normal Flow*: The hazardous task does not fail and the line flows as planned. In other words, the hazardous task behaves like a non-hazardous one.
- *Disappearing units*: The hazardous task causes the discontinuation of other tasks on that unit; the unit is removed from disassembly operations. In other words, if a hazardous task fails, it will cause the harm to its own workstation and to all succeeding workstations.

We let $1 - \pi_i$ denote the probability that the hazardous task i fails. Accordingly, π_i is the probability that the hazardous task behaves like a non-hazardous one.

Our problem is to assign the tasks to the workstations of the disassembly line so as to maximize the total expected net revenue.

The network comprises *AND* and *OR* type relations.

- Predecessor *AND* relations imply that a task cannot start before all of its predecessor tasks are finished. We use $AND(i)$ to denote the set of all *AND* predecessors of task i and $PAND(i)$ to denote the set of all *AND* immediate predecessors of task i .
- Predecessor *OR* relations imply that at least one task in a specified set should be complete before the successor task may begin. We let $POR(i)$ denote the set of *OR* predecessors of task i .

We formulate the DLBP under uncertainty in the successful completion of hazardous tasks by using two-stage stochastic programming. We first introduce stochastic programming and then present our problem formulation.

3.1 Background Information on Stochastic Programming (SP)

A stochastic program is a mathematical program where the problem parameters are not known with certainty. Instead, the probability distributions of problem parameters are available. The resolution of uncertainty over time is considered through decision stages in stochastic programming. A two-stage stochastic program is composed of first and second decision stages that take place before and after the resolution of uncertainty, respectively. The first-stage decisions ($x \in \mathbf{X}$) are made before the realization of the random scenario (i.e., a set of specific values for the problem parameters). Given the first-stage decisions, the second-stage decisions ($y_s \in \mathbf{Y}$) are made after the random scenario (indexed by $s \in S$) is realized and the values of random parameters become known. The objective function includes a first-stage term and an expected second-stage term. Letting p_s be the realization

probability of scenario s , the extensive form of a two-stage stochastic program where the objective is of maximization type can be represented as follows:

$$\begin{aligned}
& \text{Max } z = cx + \sum_{s \in S} p_s q_s y_s \\
& \text{s.t.} \\
& Ax = b, \\
& T_s x_s + W_s y_s = h_s \quad \forall s \in S, \\
& x \in X, \\
& y_s \in Y \quad \forall s \in S,
\end{aligned}$$

where c, A, B, q_s, T_s, W_s and h_s are real-valued vectors and matrices of compatible sizes. When there are a few scenarios (i.e., when the size of S is reasonably small), any solution approach for linear programs or integer programs can be used to solve the extensive form of a stochastic program. However, as the size of the stochastic program grows with the increasing number of scenarios, the computational efficiency of the standard solution methods decreases. For solving such large-size problems, solution methods exploiting the special structure (separability of the second stage problem across scenarios) of stochastic programs can be used.

3.2 Two-Stage Stochastic Integer Program for DLBP

In the DLBP defined in Section 3.1, we consider event-based uncertainty where the completion of each hazardous task follows a Bernoulli distribution. Although the probability of successful completion (which is the distribution parameter) is known for each hazardous task, the scenario to be realized is uncertain. If all hazardous tasks are successfully completed, the unit is disassembled as planned. However, if there is failure related to any of the hazardous tasks, the workpiece disappears from the line as no more tasks can be performed on it. Accordingly, the revenues from the tasks

that are assigned to the same workstation as or to a later workstation than the failed hazardous task cannot be generated for that workpiece.

We first present our formulation for the case with one hazardous task. Then we extend our formulation for the general case with multiple hazardous tasks after explicitly presenting the models for the cases with two and three hazardous tasks. Note that, these models are referred as SP models.

3.2.1 Single Hazardous Task Case

For one hazardous task case, we have two scenarios. Scenarios 1 and 2 represent the successful completion and failure of the hazardous task, respectively. We use the following notation in our formulation.

Indices and Sets:

i, b, z : task indices

k : workstation index

$PAND_i$: index set of AND predecessors of task i

POR_i : index set of OR predecessors of task i

H : index set of hazardous tasks

NH : index set of non-hazardous tasks

Parameters:

N : number of tasks

K : number of workstations

CT : maximum cycle time allowed for any of the workstations

π : probability of success for the hazardous task

t_i : processing time of task i

rev_i : net revenue of task i ($rev_i = r_i - c_i$ where r_i is the revenue due to task i and c_i is the cost of task i . $r_i = 0$ if task i does not release a part.)

Decision Variables:

$$x_{ik} = \begin{cases} 1, & \text{if task } i \text{ is assigned to workstation } k \\ 0, & \text{otherwise} \end{cases}$$

$$a_i = \begin{cases} 1, & \text{if a hazardous task is assigned and task } i \text{ is performed on the ws before the ws of} \\ & \text{the hazardous task, or if a hazardous task is not assigned at all} \\ 0, & \text{otherwise} \end{cases}$$

Based on this notation, our mathematical formulation is as follows:

$$\text{Max } z = \pi * \sum_i \sum_k rev_i * x_{ik} + (1 - \pi) * \sum_{i \in NH} rev_i * a_i \quad (1)$$

subject to

$$\sum_i t_i x_{ik} \leq CT \quad \forall k \quad (2)$$

$$x_{ik} \leq \sum_{j=1}^k x_{lj} \quad \forall i, k, l \in PAND(i) \quad (3)$$

$$x_{ik} \leq \sum_{j=1}^k \sum_{b \in POR(i)} x_{bj} \quad \forall i, k \quad (4)$$

$$\sum_k x_{ik} \leq 1 \quad \forall i \quad (5)$$

$$(\sum_k x_{ik} - \sum_k x_{zk}) + 1 \leq (K + 1) (1 - a_i + 1 - \sum_k x_{zk}) \quad \forall i \in NH, \forall z \in H \quad (6)$$

$$(\sum_k x_{zk} - \sum_k x_{ik}) \leq (K) (a_i + 1 - \sum_k x_{ik}) \quad \forall i \in NH \text{ and } \forall z \in H \quad (7)$$

$$a_i \leq \sum_k x_{ik} \quad \forall i \in NH \quad (8)$$

$$\sum_k x_{zk} \geq \sum_k x_{ik} - a_i \quad \forall i \in NH \text{ (for } rev_i \leq 0 \text{)} \text{ and } \forall z \in H \quad (9)$$

$$x_{ik} \in \{0,1\} \quad \forall i, k \quad (10)$$

$$a_i \in \{0,1\} \quad \forall i \quad (11)$$

The objective function (1) represents the expected revenue. If scenario 1 is realized (i.e., the hazardous task is completed successfully), all of the assigned tasks are

completed successfully and the resulting revenue is considered in the first part of the objective function. If scenario 2 is realized (i.e., the hazardous task is not completed successfully), only the tasks which are assigned to earlier workstations are completed successfully and the resulting revenue is considered in the second part of the objective function. Note that, the realization probability is $p_1 = \pi$ for scenario 1 and $p_2 = 1 - \pi$ for scenario 2.

Constraint (2) ensures that cycle time limit is not exceeded. Constraints (3) and (4) are related to AND and OR type precedence relations, respectively. Accordingly, constraint (3) allows the assignment of task i to workstation k if and only if all of its AND type predecessors are assigned to stations 1 through k ; and constraint (4) allows the assignment of task i to workstation k if and only if at least one of its POR type predecessors is assigned to stations 1 through k . Constraint (5) guarantees that a task can be assigned to at most one workstation. Constraints (6)-(9) are related to the assignment of hazardous tasks. According to constraint (6), if a hazardous task is assigned to a workstation and a non-hazardous task is performed at the same or a later workstation, the corresponding a_i value should be 0. According to constraint (7), if a hazardous task is assigned to a workstation and a non-hazardous task is performed at an earlier workstation, the corresponding a_i value should be 1. Constraint (8) guarantees that $a_i = 0$ if non-hazardous task i is not assigned to any workstation. Constraint (9) ensures that $a_i = 1$ for any non-hazardous task assigned if the hazardous task is not assigned to any workstation. Finally, constraints (10) & (11) represent the binary set restrictions for the variables.

3.2.2 Two Hazardous Tasks Case

When there is more than one hazardous task, the relative position of the tasks with respect to each hazardous task determines whether the corresponding task can be performed and the related revenue can be generated under each scenario. To accommodate this structure, we extend the parameter π and the decision variable a_i as follows:

π_z : probability of success for hazardous task z (where z is used to represent the task index of hazardous tasks)

$$a_{iz} = \begin{cases} 1, & \text{if a hazardous task } z \text{ is assigned and task } i \text{ is performed on the ws before the ws of} \\ & \text{the hazardous task, or if a hazardous task is not assigned at all} \\ 0, & \text{otherwise} \end{cases}$$

The number of scenarios in the problem is determined by the number of hazardous tasks. For each hazardous task, there are two scenarios: successful completion (with probability π_z) or failure (with probability $1 - \pi_z$). Therefore, the number of scenarios for the problem is $2^{|H|}$, which is $2^2 = 4$ for two hazardous tasks case. Letting z_1 and z_2 be the indices of hazardous tasks and p_s be the realization probability of scenario s , these four scenarios and their probabilities are listed below.

- Both hazardous tasks are performed successfully with probability
 $p_1 = \pi_{z_1} * \pi_{z_2}$.
- Task z_1 is performed successfully and task z_2 fails with probability
 $p_2 = \pi_{z_1} * (1 - \pi_{z_2})$.
- Task z_1 fails and task z_2 is performed successfully with probability
 $p_3 = (1 - \pi_{z_1}) * \pi_{z_2}$.
- Both hazardous tasks fail with probability
 $p_4 = (1 - \pi_{z_1}) * (1 - \pi_{z_2})$.

Based on the extended notation and the presented scenario structure, the mathematical formulation for the two hazardous tasks is as follows:

$$\begin{aligned} \text{Max } z = & p_1 * \sum_i \sum_k rev_i * x_{ik} \\ & + p_2 * \sum_i rev_i * a_{iz_2} + p_3 * \sum_i rev_i * a_{iz_1} + p_4 * \sum_i rev_i * a_{iz_1} * a_{iz_2} \end{aligned} \quad (12)$$

subject to

$$(2)-(5), (10)$$

$$(\sum_k k x_{ik} - \sum_k k x_{zk}) + 1 \leq (K + 1) (1 - a_{iz} + 1 - \sum_k x_{zk}) \forall i, \forall z \in H \quad (13)$$

$$(\sum_k k x_{zk} - \sum_k k x_{ik}) \leq (K) (a_{iz} + 1 - \sum_k x_{ik}) \quad \forall i \text{ and } \forall z \in H \quad (14)$$

$$a_{iz} \leq \sum_k x_{ik} \quad \forall i \text{ and } \forall z \in H \quad (15)$$

$$\sum_k x_{zk} \geq \sum_k x_{ik} - a_{iz} \quad \forall i \text{ (for } rev_i \leq 0 \text{) and } \forall z \in H \quad (16)$$

$$a_{iz} \in \{0,1\} \quad \forall i, \forall z \in H \quad (17)$$

The objective function (12) represents the expected revenue. If scenario 1 is realized, all of the assigned tasks are completed successfully and the resulting revenue is considered in the first part of the objective function. If scenario 2 is realized, only the tasks which are assigned to earlier workstations than that of z_2 are completed successfully and the resulting revenue is considered in the second part of the objective function. If scenario 3 is realized, only the tasks which are assigned to earlier workstations than that of z_1 are completed successfully and the resulting revenue is considered in the third part of the objective function. And finally, if scenario 4 is realized, only the tasks which are assigned to earlier workstations than that of z_1 and z_2 are completed successfully and the resulting revenue is considered in the fourth part of the objective function.

Constraints (13)-(16), which are modified versions of constraints (6)-(9), are related to the assignment of hazardous tasks and their relation to the other tasks. Constraint (17) represents the binary set restrictions for a_{iz} variables.

The last component of the objective function includes a nonlinear term ($a_{iz_1} * a_{iz_2}$) to represent the case where both hazardous tasks fail. To linearize this part, we define C_i as a new set of decision variables whose value is 1 if both a_{iz_1} and a_{iz_2} are equal to 1, 0 otherwise. Based on this definition, the linearized formulation is as follows:

$$\begin{aligned} \text{Max } z = & p_1 * \sum_i \sum_k rev_i * x_{ik} + p_2 * \sum_i rev_i * a_{iz_2} + p_3 * \sum_i rev_i * a_{iz_1} \\ & + p_4 * \sum_i rev_i * C_i \end{aligned} \quad (18)$$

subject to

(2)-(5), (10), (13)-(17)

$$C_i \geq a_{iz_1} + a_{iz_2} - 1 \quad \forall i \quad (19)$$

$$C_i \leq a_{iz_1} \quad \forall i \text{ such that } rev_i \leq 0 \quad (20)$$

$$C_i \leq a_{iz_2} \quad \forall i \text{ such that } rev_i \leq 0 \quad (21)$$

$$C_i \geq 0 \quad \forall i \quad (22)$$

If both a_{iz_1} and a_{iz_2} are equal to 1, $C_i = 1$ due to constraint (19). If any of these variables are 0, $C_i = 0$ due to constraints (20) and (21). Note that, we need to write constraints (20) and (21) only for the tasks with negative net revenue since this requirement is taken care of the sense of the objective function (maximization) for the tasks with positive net revenues.

3.2.3 Three Hazardous Tasks Case

The number of scenarios is $2^3 = 8$ for three hazardous tasks case. Letting z_1, z_2 and z_3 be the indices of hazardous tasks and p_s be the realization probability of scenario s , these eight scenarios and their probabilities are listed below.

- All hazardous tasks are performed successfully with probability
 $p_1 = \pi_{z_1} * \pi_{z_2} * \pi_{z_3}$.
- Tasks z_1 and z_2 are performed successfully and task z_3 fails with probability
 $p_2 = \pi_{z_1} * \pi_{z_2} * (1 - \pi_{z_3})$.
- Tasks z_1 and z_3 are performed successfully and task z_2 fails with probability
 $p_3 = \pi_{z_1} * (1 - \pi_{z_2}) * \pi_{z_3}$.
- Tasks z_2 and z_3 are performed successfully and task z_1 fails with probability
 $p_4 = (1 - \pi_{z_1}) * \pi_{z_2} * \pi_{z_3}$.
- Task z_1 is removed successfully, tasks z_2 and z_3 fail with probability

$$p_5 = \pi_{z_1} * (1 - \pi_{z_2}) * (1 - \pi_{z_3}).$$

- Task z_2 is removed successfully, tasks z_1 and z_3 fail with probability

$$p_6 = (1 - \pi_{z_1}) * \pi_{z_2} * (1 - \pi_{z_3}).$$

- Task z_3 is removed successfully, tasks z_1 and z_2 fail with probability

$$p_7 = (1 - \pi_{z_1}) * (1 - \pi_{z_2}) * \pi_{z_3}.$$

- All three hazardous tasks fail with probability

$$p_8 = (1 - \pi_{z_1}) * (1 - \pi_{z_2}) * (1 - \pi_{z_3}).$$

Based on this scenario structure, the mathematical formulation for the three hazardous tasks case is as follows:

$$\begin{aligned} \text{Max } z = & p_1 * \sum_i \sum_k \text{rev}_i * x_{ik} + p_2 * \sum_i \text{rev}_i * a_{iz_3} + p_3 * \sum_i \text{rev}_i * a_{iz_2} \\ & + p_4 * \sum_i \text{rev}_i * a_{iz_1} + p_5 * \sum_i \text{rev}_i * a_{iz_2} * a_{iz_3} + p_6 * \sum_i \text{rev}_i * a_{iz_1} * a_{iz_3} \\ & + p_7 * \sum_i \text{rev}_i * a_{iz_1} * a_{iz_2} + p_8 * \sum_i \text{rev}_i * a_{iz_1} * a_{iz_2} * a_{iz_3} \end{aligned} \quad (23)$$

subject to

(2)-(5), (10), (13)-(17).

As in the formulation for the two hazardous tasks case, the nonlinear terms in the objective function are related to the scenarios with failure in multiple hazardous tasks, which are scenarios 5, 6, 7, and 8. To linearize these terms, we define and use C_{is} (an extended version of C_i) as a new set of decision variables whose value is 1 if task i is assigned and successfully performed (i.e., it is assigned to an earlier workstation than that of the failed hazardous task) in the case of a hazardous task failure, 0 otherwise. We define C_{is} only for $s \geq 5$. The linearized formulation is as follows:

$$\begin{aligned} \text{Max } z = & p_1 * \sum_i \sum_k \text{rev}_i * x_{ik} + p_2 * \sum_i \text{rev}_i * a_{iz_3} + p_3 * \sum_i \text{rev}_i * a_{iz_2} \\ & + p_4 * \sum_i \text{rev}_i * a_{iz_1} + p_5 * \sum_i \text{rev}_i * C_{i5} + p_6 * \sum_i \text{rev}_i * C_{i6} \\ & + p_7 * \sum_i \text{rev}_i * C_{i7} + p_8 * \sum_i \text{rev}_i * C_{i8} \end{aligned} \quad (24)$$

subject to

(2)-(5), (10), (13)-(17)

$$C_{i5} \geq a_{iz_2} + a_{iz_3} - 1 \quad \forall i \quad (25)$$

$$C_{i5} \leq a_{iz_2} \quad \forall i \text{ such that } rev_i \leq 0 \quad (26)$$

$$C_{i5} \leq a_{iz_3} \quad \forall i \text{ such that } rev_i \leq 0 \quad (27)$$

$$C_{i6} \geq a_{iz_1} + a_{iz_3} - 1 \quad \forall i \quad (28)$$

$$C_{i6} \leq a_{iz_1} \quad \forall i \text{ such that } rev_i \leq 0 \quad (29)$$

$$C_{i6} \leq a_{iz_3} \quad \forall i \text{ such that } rev_i \leq 0 \quad (30)$$

$$C_{i7} \geq a_{iz_1} + a_{iz_2} - 1 \quad \forall i \quad (31)$$

$$C_{i7} \leq a_{iz_1} \quad \forall i \text{ such that } rev_i \leq 0 \quad (32)$$

$$C_{i7} \leq a_{iz_2} \quad \forall i \text{ such that } rev_i \leq 0 \quad (33)$$

$$C_{i8} \geq a_{iz_1} + a_{iz_2} + a_{iz_3} - 2 \quad \forall i \quad (34)$$

$$C_{i8} \leq a_{iz_1} \quad \forall i \text{ such that } rev_i \leq 0 \quad (35)$$

$$C_{i8} \leq a_{iz_2} \quad \forall i \text{ such that } rev_i \leq 0 \quad (36)$$

$$C_{i8} \leq a_{iz_3} \quad \forall i \text{ such that } rev_i \leq 0 \quad (37)$$

$$C_{is} \geq 0 \quad \forall i \text{ and } \forall s \geq 5 \quad (38)$$

3.2.4 General Case

To extend our mathematical formulation for the general case, we define $Z(s)$ as the set of failed hazardous tasks under scenario s . The $Z(s)$ sets are the subsets of the set of hazardous tasks, H . The realization probability for scenario s is $p_s = \prod_{z \in Z(s)} (1 - \pi_z) * \prod_{z \in H \setminus Z(s)} \pi_z$. Based on the definition of set $Z(s)$ and the

realization probabilities, the linearized mathematical formulation for the general case is as follows:

$$\text{Max } z = p_1 * \sum_i \sum_k \text{rev}_i * x_{ik} + \sum_{s=2}^{|S|} p_s * \sum_i \text{rev}_i * C_{is} \quad (39)$$

subject to

(2)-(5), (10), (13)-(17)

$$C_{is} \geq \sum_{z \in Z(s)} a_{iz} - (|Z(s)| - 1) \quad \forall i, \forall s \geq 2 \quad (40)$$

$$C_{is} \leq a_{iz} \quad \forall i \text{ such that } \text{rev}_i \leq 0, \forall s \geq 2, \forall z \in Z(s) \quad (41)$$

$$C_{is} \geq 0 \quad \forall i \text{ and } \forall s \geq 2 \quad (42)$$

Note that C_{is} is defined for $s \geq 2$ (instead of $s \geq 5$ as in three hazardous tasks case) to simplify the representation.

3.2.5 Example

We illustrate the decisions and objective function value of the model for 12-task, 3 workstation problem. Two tasks, task 5 and task 8, are hazardous with respective success probabilities of $\pi_{z_1} = 0.8$ and $\pi_{z_2} = 0.2$.

Assume a feasible solution with the following workstation assignments:

Workstation	Tasks Assigned	First Stage Decision Variables
1	1, 2, 5	$x_{11} = x_{21} = x_{51} = 1$
2	3, 7	$x_{32} = x_{72} = 1$
3	8, 11, 12	$x_{83} = x_{11,3} = x_{12,3} = 1$

Unassigned tasks are tasks 4, 6, 9, 10. Note that both hazardous tasks (task 5 and task 8) are assigned.

There are four scenarios. Let R_s be expected net revenue of each scenario s .

Scenario 1: Both hazardous tasks are performed successfully.

$$p_1 = \pi_{z_1} * \pi_{z_2} = 0.8 * 0.2 = 0.16$$

$$R_1 = 0.16 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7 + rev_8 + rev_{11} + rev_{12})$$

Scenario 2: Hazardous task 5 is performed successfully and hazardous task 8 fails.

$$p_2 = \pi_{z_1} * (1 - \pi_{z_2}) = 0.8 * 0.8 = 0.64$$

Tasks 11 and 12 are in the same workstation with task 8, hence they cannot be performed.

$$R_2 = 0.64 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7)$$

Scenario 3: Hazardous task 8 is performed successfully and hazardous task 5 fails.

$$p_3 = (1 - \pi_{z_1}) * \pi_{z_2} = 0.2 * 0.2 = 0.04$$

Hazardous task 5 is in the first workstation, hence no task in any workstation can be performed.

$$R_3 = 0$$

Scenario 4: Both hazardous tasks fail.

$$p_4 = (1 - \pi_{z_1}) * (1 - \pi_{z_2}) = 0.2 * 0.8 = 0.16$$

Hazardous task 5 is in the first workstation, hence no task in any workstation can be performed.

$$R_4 = 0$$

$$\text{Expected Profit } z = R_1 + R_2 + R_3 + R_4$$

$$\begin{aligned} &= 0.16 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7 + rev_8 + \\ &rev_{11} + rev_{12}) + 0.064 * (rev_1 + rev_2 + rev_5 + rev_3 + \\ &rev_7) \end{aligned}$$

$$= 0.8 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7) + 0.16 * (rev_8 + rev_{11} + rev_{12})$$

Now assume another feasible solution with the following assignments:

<u>Workstation</u>	<u>Tasks Assigned</u>	<u>First Stage Decision Variables</u>
1	1, 2, 5	$x_{11} = x_{21} = x_{51} = 1$
2	3, 7	$x_{32} = x_{72} = 1$

Unassigned tasks are tasks 4, 6, 8, 9, 10, 11, 12. Note that hazardous task 8 is not assigned.

Scenarios 1 & 2: Hazardous task 5 is performed successfully.

$$p_1 = 0.16 \quad p_2 = 0.64$$

$$R_1 = 0.16 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7)$$

$$R_2 = 0.64 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7)$$

Scenarios 3 & 4: Hazardous task 5 is not performed successfully.

$$p_3 = 0.04 \quad p_4 = 0.16$$

Hazardous task 5 is in the first workstation, hence no task in any workstation can be performed.

$$R_3 = R_4 = 0$$

$$\text{Expected Profit } z = R_1 + R_2 + R_3 + R_4$$

$$\begin{aligned}
&= 0.16 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7) \\
&\quad + 0.64 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7) + 0.04 * 0 \\
&\quad + 0.16 * 0 \\
&= 0.8 * (rev_1 + rev_2 + rev_5 + rev_3 + rev_7)
\end{aligned}$$

3.3 Expected Value Problem and the Value of the Stochastic Solution

Since stochastic programs are not as computationally tractable as their deterministic counterparts, solving the *expected value problem (EV)*, which is obtained by replacing the random parameters with their expected values, is a commonly used approach in practice. In other words, *EV* is a stochastic program with a single scenario where the problem parameters are equal to their expected values and hence known as the *expected value scenario*. The *value of the stochastic solution (VSS)* is an important measure in stochastic programming and is defined as the difference between the *expected result of using the EV solution (EEV)* and the expected result of the using the SP solution (i.e., the optimal objective value of the SP). *EEV* can be obtained by solving a modified version of *SP* where the optimal first-stage decision variable values of *EV* is considered as a set of constraints on the variables. It is the solution where the variables are assigned according to *EV* and the optimal solution is obtained with the original parameters. It reveals the situation that the decisions are made according to *EV* but the results of these decisions in real life are realized with original random parameters. *VSS* is the value of considering uncertainty and it represents the amount by which the expected objective function value could be improved by formulating and solving the problem as a stochastic program rather than a deterministic mathematical program. Thus, *VSS* measures the difference between the objective function value obtained by *SP* and *EEV*. Simply we can show as;

$$VSS = \text{Objective Function Value of } SP - EEV$$

As our problem involves event-based uncertainty, constructing the expected value scenario is not straightforward. We adapt the threshold-based approach proposed by Bayram and Yaman (2018) and define the expected value scenario as the one where the hazardous tasks with $\pi_z \geq 0.5$ are successfully completed and the remaining hazardous tasks (i.e., the ones with $\pi_z < 0.5$) fail. To represent these two different sets of hazardous tasks, we define π'_z as a binary parameter such that

$$\pi'_z = 1 \text{ if } \pi_z \geq 0.5,$$

$$\pi'_z = 0 \text{ otherwise}$$

Using π'_z values, we define and compute the probability of scenario s in the EV as $p'_s = \prod_{z \in Z(s)} (1 - \pi'_z) * \prod_{z \in H \setminus Z(s)} \pi'_z$. Note that, $p'_s = 1$ for only one scenario, which is the expected value scenario.

3.3.1 VSS Calculation for Single Hazardous Task

The expected value problem for single hazardous task is illustrated below. Note that all indices, sets, parameters and decision variables are same with the previous stochastic programming model mentioned in section 3.2.1, only the new added parameters and variables are shown below.

Parameters:

π' : threshold-based success probability for the hazardous task

Decision Variables:

Z_3 : expected result of using the EV solution (EEV)

The mathematical formulation for EV problem and EEV is follows:

$$\text{Max } Z_2 = \pi' * \sum_i \sum_k \text{rev}_i * x_{ik} + (1 - \pi') * \sum_{i \in NH} \text{rev}_i * a_i \quad (43)$$

subject to

(2)-(11)

$$Z_3 = \pi * \sum_i \sum_k \text{rev}_i * x_{ik} + (1 - \pi) * \sum_{i \in NH} \text{rev}_i * a_i \quad (44)$$

The objective function (43), which we refer to as Z_2 , represents the revenue in the expected value scenario. We explicitly represent the corresponding expected revenue as Z_3 (EEV) in (44) by using the original realization probabilities of scenarios.

Letting Z_1 represent the objective function value attained by the SP solution given in Section 3.2.1, the value of the stochastic solution is $VSS = Z_1 - Z_3$ for our problem.

3.3.2 VSS Calculation for Two Hazardous Task

Similar to the single hazardous task case, the mathematical formulation for EV problem and EEV with two hazardous tasks is illustrated below.

$$\begin{aligned} \text{Max } Z_2 = & p_1' * \sum_i \sum_k rev_i * x_{ik} \\ & + p_2' * \sum_i rev_i * a_{iz_2} + p_3' * \sum_i rev_i * a_{iz_1} + p_4' * \sum_i rev_i * C_i \end{aligned} \quad (45)$$

subject to

(2)-(5), (10), (13)-(17), (19)-(22)

$$\begin{aligned} Z_3 = & p_1 * \sum_i \sum_k rev_i * x_{ik} \\ & + p_2 * \sum_i rev_i * a_{iz_2} + p_3 * \sum_i rev_i * a_{iz_1} + p_4 * \sum_i rev_i * C_i \end{aligned} \quad (46)$$

Note that;

- $p_1' = \pi_{z_1}' * \pi_{z_2}'$.
- $p_2' = \pi_{z_1}' * (1 - \pi_{z_2}')$.
- $p_3' = (1 - \pi_{z_1}') * \pi_{z_2}'$.
- $p_4' = (1 - \pi_{z_1}') * (1 - \pi_{z_2}')$.

The objective function (45), which we refer to as Z_2 , represents the revenue in the expected value scenario. We explicitly represent the corresponding expected revenue as Z_3 (EEV) in (46) by using the original realization probabilities of scenarios.

Letting Z_1 represent the objective function value attained by the SP solution given in Section 3.2.2, the value of the stochastic solution is $VSS = Z_1 - Z_3$ for our problem.

3.3.3 VSS Calculation for General Case

The linearized mathematical formulation for the general case is as follows:

$$\text{Max } z = p_1' * \sum_i \sum_k rev_i * x_{ik} + \sum_{s=2}^{|S|} p_s' * \sum_i rev_i * C_{is} \quad (47)$$

subject to

$$(2)-(5), (10), (13)-(17), (40)-(42)$$

$$Z_3 = p_1 * \sum_i \sum_k rev_i * x_{ik} + \sum_{s=2}^{|S|} p_s * \sum_i rev_i * C_{is} \quad (48)$$

As mentioned before, to extend our mathematical formulation for the general case, the realization probability of scenario s in EV is $p_s' = \prod_{z \in Z(s)} (1 - \pi_z') * \prod_{z \in H \setminus Z(s)} \pi_z'$.

The objective function (47), which we refer to as Z_2 , represents the revenue in the expected value scenario. We explicitly represent the corresponding expected revenue as Z_3 (EEV) in (48) by using the original realization probabilities of scenarios. Letting Z_1 represent the objective function value attained by the SP solution given in Section 3.2.4, the value of the stochastic solution is $VSS = Z_1 - Z_3$ for our problem.

An Example Problem 1

To clarify the differences between the SP , EV , EEV and VSS , the results of these problems are compared and illustrated. For $N=34$, $K=4$ and CT2 combination, with single hazardous task, objective function values of SP (Z_1), EV (Z_2) and EEV (Z_3) are shown in the Table 3.1.

Average objective function value with original $\pi=0.8$ and POS1 is $Z_1 = 591.2$. When we solve the EV problem, the solution with the success probability $\pi'=1$ (since $\pi=0.8 \geq 0.5$ threshold value) yields an objective function value of $Z_2 = 601.7$ on average. However, this is the objective function value of the EV problem, not the expected result of using the EV solution, i.e. we give decisions according to the EV (Z_2) and these decisions are applied to reality by EEV (Z_3) with original success probabilities. Thus, the expected revenue that could be attained by using the EV solution is $Z_3 = 502.9$ on average. Finally, we can calculate the added value of using stochastic programming (Value of stochastic solution, VSS); i.e. considering uncertainty in a decision.

$$\text{VSS} = \text{Objective Function Value of SP} - \text{EEV}$$

$$\text{VSS} = 88.3$$

The VSS value for this setting is 88.3 on average, which is the average of the VSS values ($Z_1 - Z_3$) computed for each instance.

Thus, while disassembling a product with given network, CT, number of workstations, considering uncertainty in the success probability redounds to expected net revenue. It would be simpler to use the EV problem but it will be costly since it causes the loss of the revenue that can be earned via SP.

Note that, as it can be observed from the Table 3.1, when $\pi'=1$ after considering the threshold values, average objective function values of EV problem at each combination with $\pi'=1$ are all equal to each other as expected. Moreover, it is expected to observe $\text{VSS}=0$ when original success probability $\pi=1$ for SP Model, since the corresponding EV and EEV values result exactly same with the same success probability, hence no uncertainty.

Table 3.1 Average Objective Function Values for SP, EV, EEV and VSS

		$\pi=1$ ($\pi'=1$)	$\pi=0.8$ ($\pi'=1$)			$\pi=0.5$ ($\pi'=1$)			$\pi=0.2$ ($\pi'=0$)		
		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
SP (Z1)	AVG	601.7	591.2	597.7	582.4	576.4	591.7	554.0	561.8	585.7	525.9
	MAX	669.0	661.6	664.4	658.0	659.5	657.5	641.5	657.4	651.8	625.0
EV(Z2)	AVG	601.7	601.7	601.7	601.7	601.7	601.7	601.7	552.0	581.7	507.1
	MAX	669.0	669.0	669.0	669.0	669.0	669.0	669.0	656.0	649.0	614.0
EEV(Z3)	AVG	601.7	502.9	521.0	485.2	354.6	400.0	310.4	554.7	581.7	507.1
	MAX	669.0	573.6	607.2	568.4	511.5	544.0	426.5	656.0	649.0	614.0
VSS (SP-EEV)	AVG	0.0	88.3	76.7	97.2	221.8	191.8	243.7	7.1	4.0	18.8
	MAX	0.0	131.2	121.4	122.8	328.0	303.5	307.0	20.2	5.6	127.0

An Example Problem 2

To discuss the meaning of VSS, advantage of SP over EV problem and identify the realization of EEV, we go over a two hazardous task case via presenting the assignments of tasks to workstations.

The result of an instance obtained by combination $N=22$, $K=2$, CT1 and POS1 is illustrated in the Table 3.2 both for SP and EV problem. The assignments of tasks differ according to the success probabilities. Note that hazardous tasks are *task 7* and *task 10* for this instance and success probabilities are $\pi_7 = 0.8$ and $\pi_{10} = 0.8$ respectively.

The objective function for *SP Model*:

$$\begin{aligned} \text{Max } Z_1 = & (0.8 * 0.8) * \sum_i \sum_k rev_i * x_{ik} + (0.8 * 0.2) * \sum_i rev_i * a_{i10} \\ & + (0.2 * 0.8) * \sum_i rev_i * a_{i7} + (0.2 * 0.2) * \sum_i rev_i * C_i \end{aligned}$$

The results show that for SP Model C_i variables are as follows:

$$C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_8 = C_{12} = C_{13} = 1$$

These are the tasks that are assigned to earlier workstations of both hazardous tasks to maximize the expected net revenue, since we have success probabilities smaller than 1.

Objective function value with original π_7 and π_{10} is 301.24.

Table 3.2 Assignments of tasks according to SP and EV Model

Stations	1 st	2 nd	Objective
SP (Z_1)	1,2,3,4,5,6,8,12,13	7,9,10,11,14,15, 17,18,19,21,22	301.24
EV (Z_2)	1,2,3,4,5,6,7,10, 13,17,19,21,22	8,9,11, 12,14,15,18	379
EEV (Z_3)	1,2,3,4,5,6,7,10, 13,17,19,21,22	8,9,11, 12,14,15,18	242.56

Since $\pi=0.8 \geq 0.5$ threshold value, for EV problem success probabilities are utilized as $\pi'_7=1$ and $\pi'_{10}=1$.

The objective function value of EV problem :

$$\begin{aligned}
 \text{Max } Z_2 &= (1 * 1) * \sum_i \sum_k rev_i * x_{ik} + (1 * 0) * \sum_i rev_i * a_{i10} \\
 &\quad + (0 * 1) * \sum_i rev_i * a_{i7} + (0 * 0) * \sum_i rev_i * C_i \\
 &= \sum_i \sum_k rev_i * x_{ik}
 \end{aligned}$$

The results show that for EV problem $C_i = 0 \forall i$.

Since we have success probabilities equal to 1, the hazardous tasks can be assigned to the 1st workstation. Thus, via using the threshold-based success probabilities instead of original success probabilities, the decision of assignment for hazardous tasks changed from 2nd workstation to the 1st workstation.

Average objective function value with success probabilities π'_7 and π'_{10} is 379.

Last but not the least measure is $EEV (Z_3)$ which is obtained from the constraint in the EV problem via using the original success probabilities. When we solve the EV problem, the solution is obtained by threshold-based success probabilities and decisions are given accordingly. $EEV (Z_3)$ measures the realization of these decisions by original success probabilities. The value of $EEV (Z_3)$ is calculated as follows;

$$Z_3 = (0.8 * 0.8) * \sum_i \sum_k rev_i * x_{ik} + (0.8 * 0.2) * \sum_i rev_i * a_{i10} \\ + (0.2 * 0.8) * \sum_i rev_i * a_{i7} + (0.2 * 0.2) * \sum_i rev_i * C_i$$

where X_{ik} , a_{iz} , C_i variables are obtained by EV problem. Thus, the same decision variables used for calculation of $EEV (Z_3)$.

Expected revenue, $EEV (Z_3)$, with original success probabilities π_7 and π_{10} is 242.56.

Finally, we can calculate the added value of using stochastic programming (Value of stochastic solution, VSS); i.e. considering uncertainty in a decision.

$$VSS = \text{Objective Function Value of SP} - EEV.$$

$$VSS = 58.68$$

VSS gives the amount by which the expected objective function value could be improved by formulating and solving the problem as a stochastic program rather than a deterministic mathematical program.

CHAPTER 4

SOLUTION METHODS

In this section we discuss the exact solution method and bounding procedures. Three upper bounds, *Upper Bound 1* (UB_1), *Upper Bound 2* (UB_2), *Upper Bound 3* (UB_3) and two lower bounds *Lower Bound 1* (LB_1) and *Lower Bound 2* (LB_2) are developed on the optimal value of the Stochastic DLBP.

4.1 Exact Solution Method

All SP models which are formulated as a two-stage stochastic mixed-integer program containing different types of hazardous tasks are solved by CPLEX Solver.

After an extensive experiment, we observe that the size of the model increases drastically with the increase in the number of tasks (N). Therefore, the optimal solutions to large-sized problem instances are not likely to be obtained within reasonable computational times. The unsolved instances with one, two and three hazardous task(s) are observed especially for large networks. Thus, the bounding procedures are considered and presented in the succeeding sections.

4.2 Properties of Optimal Solution and Upper Bounds

Three upper bounds are studied. UB_1 is obtained by relaxing the integrality constraints of Two Stage Stochastic Integer Model presented in Section 3.2. UB_1 is improved by UB_2 through exact valid cuts which are formed by using the properties of optimal solution. UB_2 is improved by UB_3 through heuristic cuts.

4.2.1 Upper Bound 1 (UB_I)

An optimal solution to any relaxation provides an upper bound on the optimal objective function value of our maximization problem. Two Stage Stochastic Integer Model is solved to optimality by relaxing the integrality constraints on the binary x_{ik} and a_{iz} variables. The resulting optimal objective function value provides an upper bound on the optimal total net revenue.

The Linear Programming Relaxation (LPR) of one hazardous task case is illustrated below:

$$\text{Max } Z_{LP} = \pi * \sum_i \sum_k \text{rev}_i * x_{ik} + (1 - \pi) * \sum_{i \in NH} \text{rev}_i * a_i \quad (1)$$

subject to

$$\sum_i t_i x_{ik} \leq CT \quad \forall k \quad (2)$$

$$x_{ik} \leq \sum_{j=1}^k x_{lj} \quad \forall i, k, l \in PAND(i) \quad (3)$$

$$x_{ik} \leq \sum_{j=1}^k \sum_{b \in POR(i)} x_{bj} \quad \forall i, k \quad (4)$$

$$\sum_k x_{ik} \leq 1 \quad \forall i \quad (5)$$

$$(\sum_k k x_{ik} - \sum_k k x_{zk}) + 1 \leq (K + 1) (1 - a_i + 1 - \sum_k x_{zk}) \quad \forall i \in NH, \forall z \in H \quad (6)$$

$$(\sum_k k x_{zk} - \sum_k k x_{ik}) \leq (K) (a_i + 1 - \sum_k x_{ik}) \quad \forall i \in NH, \forall z \in H \quad (7)$$

$$a_i \leq \sum_k x_{ik} \quad \forall i \in NH \quad (8)$$

$$\sum_k x_{zk} \geq \sum_k x_{ik} - a_i \quad \forall i \in NH \text{ (for } \text{rev}_i \leq 0 \text{) and } \forall z \in H \quad (9)$$

$$x_{ik} \geq 0 \quad \forall i, k \quad (10)$$

$$a_i \geq 0 \quad \forall i \quad (11)$$

Z_{LP} , optimal solution for LP Relaxation, is upper bound on the optimal solution of original SP Model z^* . Only difference between these models are integrality constraints regarding x_{ik} and a_{iz} variables. LPR of two and three hazardous task cases are obtained with the same method, i.e. relaxing the integrality constraints.

4.2.2 Upper Bound 2 (UB_2)

By analyzing the properties of optimal solutions that may not be fulfilled via LP Relaxation, valid cuts are formed and added to UB_1 in order to obtain a better solution such as UB_2 . UB_2 is strengthened LPR with exact properties (cuts 1 through 6)

4.2.2.1 Earliest Workstation Position (CUT1)

A lower bound on the workstation number can be obtained via Cut1 that can reside earliest for any defined task. In all optimal solutions, task i cannot be assigned to workstations $1, 2, \dots, E_i - 1$ where $E_i = \left\lceil \frac{(\sum_{j \in P_i} t_j + t_i)}{CT} \right\rceil$ and P_i is the set of “AND” predecessors. (Kalaycilar et al., 2016)

For this purpose, task i is not assigned to workstations $1, 2, \dots, E_i - 1$ with the following constraint set that is utilized with AND precedence relations.

$$\sum_{k=1}^{E_i-1} X_{ik} = 0 \quad \forall i$$

Since we have “POR” relations in our model, we modify the earliest workstation considering these relations. “POR” relations are replaced by “AND” relations, where the minimum task time of “POR” relation tasks are used as task time of the “AND” relation task. As the minimum times are used in place of exact times, the sum of the processing time $\sum_{j \in POR_i} t_j \geq \sum_{j \in \overline{POR_i}} t_j$. The modification of the constraint set including “POR” relations, where $\overline{POR_j}$ is the POR_j relation replaced by “AND” relation using minimum task time, is shown below:

$$E_i = \left\lceil \frac{(\sum_{j \in (P_i \cup \overline{POR_i})} t_j + t_i)}{CT} \right\rceil$$

CUT1 and related computations are illustrated via an example:

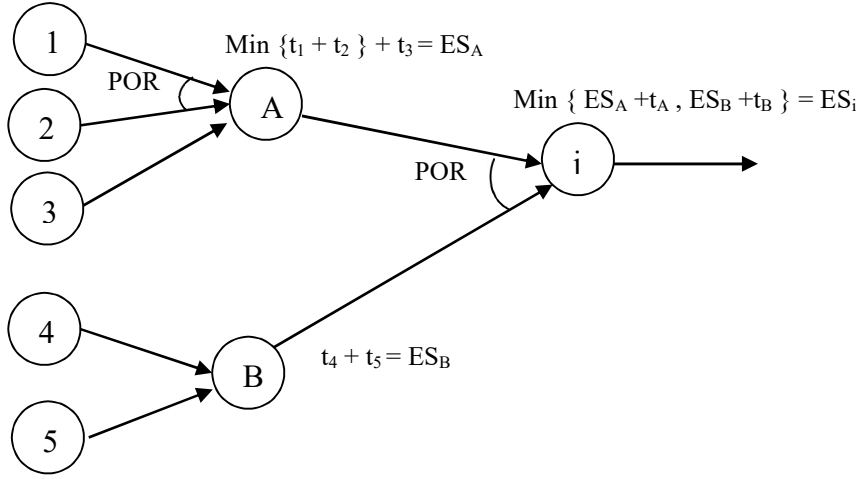


Figure 4.1. An Example Network illustrating CUT1

$\overline{POR}_A$ is obtained by replacing task times $\{1, 2\}$ with task time $\text{Min} \{t_1, t_2\}$. $\text{Min} \{t_1, t_2\} + t_3$ is the earliest time needed to start task A (ES_A) and $(t_4 + t_5)$ is the earliest time needed to start task B (ES_B). To sum up, the earliest workstation for task j is calculated as follows;

$$E_i = \left\lceil \frac{\text{Earliest time to start task } i + t_i}{CT} \right\rceil = \left\lceil \frac{\text{Min} \{(\text{min} \{t_1, t_2\} + t_3 + t_A), (t_4 + t_5 + t_B)\} + t_i}{CT} \right\rceil$$

$$\text{Then, } E_i = \left\lceil \frac{\text{Min} \{(\text{min} \{t_1, t_2\} + t_3 + t_A), (t_4 + t_5 + t_B)\} + t_i}{CT} \right\rceil$$

We use the below network to illustrate Cut1 with $K=4$, $CT=63$. E_i value is calculated as follows for task 14;

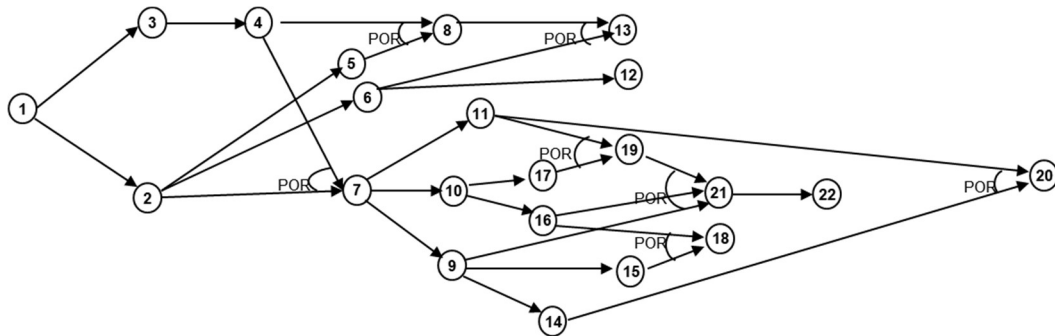


Figure 4.2. Precedence diagram modified by Kalaycilar et al. (22 tasks)

$$E_{14} = \left\lceil \frac{\text{Min} \{(t_1 + t_3 + t_4), (t_1 + t_2)\} + t_7 + t_9 + t_{14}}{CT} \right\rceil$$

$$E_{14} = \left\lceil \frac{\text{Min} \{(13 + 10 + 6), (15 + 7)\} + 19 + 15 + 17}{63} \right\rceil$$

$$E_{14} = \left\lceil \frac{73}{63} \right\rceil = 2$$

Hence, *task 14* cannot be assigned to the first workstation and the constraint $X_{14_1} = 0$ can be added as Cut1.

4.2.2.2 Latest Workstation Position (CUT2)

Cut2 provides an upper bound on the workstation number that can reside latest for any defined task. This cut gets its motivation by considering the path related with two tasks that are attached to each other with a successor relationship. Only the tasks that are found on the way of this path should be considered.

Note that successor relationship need not to be an immediate one.

To state *Theorem 1* we need the following notation,

S_i : set of all successors of *task i*

$PATH_{ij}$: set of successors of *task i* that are predecessor of *task j*.

$$L_{ij} = K - \left\lceil \frac{(\sum_{d \in PATH_{ij}} t_d + t_i + t_j)}{CT} \right\rceil + 2$$

Theorem 1: In all feasible solutions in which *task j* is processed, *task i* cannot be assigned to workstations $L_{ij}, L_{ij}+1, L_{ij}+2 \dots K$.

Proof: If *task j* is performed then after performing *task i*, *task j* cannot start before its predecessors, i.e all tasks between task i and task j should be performed. Hence *task j* should wait at least $t_i + \sum_{d \in PATH_{ij}} t_d$ units to start and $t_i + \sum_{d \in PATH_{ij}} t_d + t_j$ units to complete. When task splitting is allowed and all other

tasks are ignored, $t_i + \sum_{d \in PATH_{ij}} t_d + t_j$ units require $\left\lceil \frac{(\sum_{d \in PATH_{ij}} t_d + t_i + t_j)}{CT} \right\rceil$ workstations. When task splitting is not allowed and other tasks are considered $K - \left\lceil \frac{(\sum_{d \in PATH_{ij}} t_d + t_i + t_j)}{CT} \right\rceil + 2$ becomes an upper bound on the latest workstation for *task i*. $\square$

The theorem gives an upper bound on the station number that resides *task i*. The following constraint set is used to support Theorem 1.

$$\sum_{k=L_{ij}}^K X_{ik} + \sum_k^K X_{jk} \leq 1 \quad \forall i, j \in S_i$$

Task i is not assigned to workstations $L_{ij}, L_{ij}+1, L_{ij}+2, \dots, K$ depending on the condition of *task j* completion with the given constraint set.

Since we have “POR” relations in our model, we need to modify the latest workstation considering these relations. As we mentioned before, the task having “POR” relation can be performed if at least one of the predecessor tasks is performed. Thus, the cut that is discussed cannot be applied directly to the network having “POR” relations. Consider tasks 3 and 13 that are given in Figure 4.3 and the path between these tasks regarding Cut2. To apply Cut2, we have a condition that is the completion of task 13 which can also be performed by executing a different path including tasks 1, 2 and 6. At that point, the latest workstation cannot be obtained via summation of task 3 or via taking the minimum time of any path. Because task 3 can be performed even in the last workstation since the precedence of task 13 already satisfied by executing tasks 1,2 and 6. Hence, to apply Cut2, the tasks having “POR” relations such as tasks 8,7,13 are not considered. The minimized network having only “AND” precedence relations is illustrated in the Figure below and the paths that are valid for Cut2 is scratched:

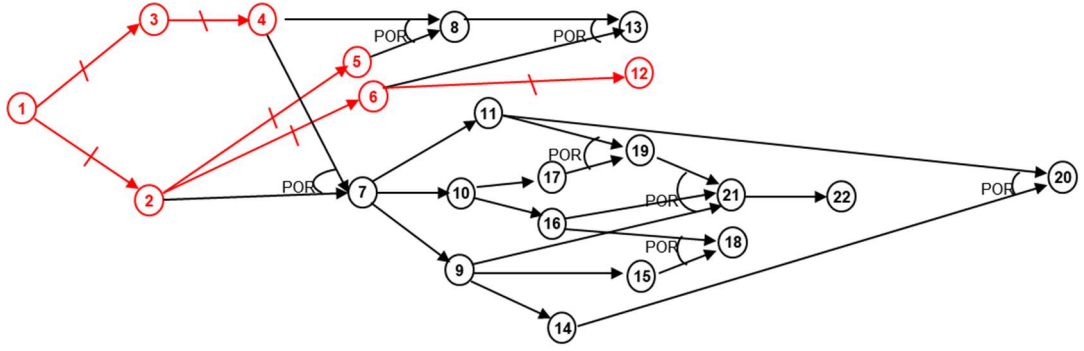


Figure 4.3. Precedence Diagram of 22 Tasks

We use the above network to illustrate Cut2 with $K=4$ and $CT=51$. L_{ij} value is calculated as follows for tasks 1 and 12;

$$L_{1,12} = 4 - \left\lceil \frac{(t_1 + t_2 + t_6 + t_{12})}{CT} \right\rceil + 2$$

$$L_{1,12} = 4 - \left\lceil \frac{(11 + 9 + 15 + 17)}{51} \right\rceil + 2$$

$$L_{1,12} = 4$$

Hence, *task 1* cannot be assigned to the last workstation if *task 12* is performed and the constraint presented below can be added as Cut2.

$$X_{1,4} + \sum_k X_{12k} \leq 1$$

4.2.2.3 Eliminating Tasks in POR relation (CUT3)

Cut3 eliminates the tasks in POR relation that have negative net revenues.

Theorem 2: Assume *task i* and *task j* are POR predecessors of *task n*. If $rev_j < 0$ and *task j* has no other immediate successor then in all optimal solutions *task i* and *task j* are not processed together.

Proof: *Task n* having POR relation, cannot start before the completion of at least one of the *tasks i* and *j*. All options are, performing *task i* or performing *task j* or performing both of them to start for *task n*. For *task j*, $rev_j < 0$ and has one immediate successor (Let IS_j is the set of immediate successors of any *task j* then $|IS_{j \in (PORn)}| = 1$). For *task i* we do not have any constraint regarding immediate successors, hence we can have a successor such that *task m* for *task i*.

Suppose we already performed all the tasks to start for *task i* or *task j*. The selection of any task or selecting both to perform *task n* requires the evaluation of net revenues for these tasks and the path that exist after these tasks. Let the tasks that can be performed after *task i* is *i'* and tasks that can be performed after *task j* is *j'*. The selection process requires the evaluation of two options for revenue generation:

- 1) $rev_i + rev_{i'}$
- 2) $rev_j + rev_{j'}$

Note that $rev_{i'}$ covers $rev_{j'}$, since *task j* has one immediate successor *task n* which is already successor of *task i*. Task *i* also has other successors such as *task m*.

Case 1: $rev_i \geq 0$ given that $rev_j < 0$

Then, $rev_{option1} \geq rev_{option2}$ at all time, since $rev_{i'}$ covers $rev_{j'}$, and $rev_i \geq rev_j$. Note that selection of *task i* is enough to start *task n*. In an optimal solution where *task i* is performed and only immediate successor for *task j* is *task n*, then *task j* is not selected to start *task n* since $rev_j < 0$. ($rev_i + rev_j + rev_n + rev_{n'} \leq rev_i + rev_n + rev_{n'}$)

Case 2: $rev_i \leq 0$ and given that $rev_j < 0$

- If $rev_{option1} \geq rev_{option2}$, it implies that option 1 preferred to option 2. This is similar with the first case where *task i* is performed. In an optimal solution where *task i* is performed and only immediate successor for *task j* is *task n*, then *task j* is not selected to start *task n* since $rev_j < 0$. ($rev_j + rev_n + rev_{n'} \leq rev_n + rev_{n'}$)

- If $rev_{optio} \geq rev_{option1}$, it implies that option 2 preferred to option 1, then *task j* will be performed. In an optimal solution where *task j* is performed then *task i* is not selected to start *task n* since $rev_i \leq 0$. ($rev_i + rev_j + rev_n + rev_{n'} \leq rev_j + rev_n + rev_{n'}$)

If one of the POR predecessors of a given task has negative net revenue and has no other immediate successors, it satisfies the condition such that in an optimal solution two POR predecessors are not processed together. $\square$

Suppose that *task i* and *task j* is the predecessor of *task n* with POR relation as shown in the Figure below. Moreover, *task j* has negative net revenue such that, $rev_j \leq 0$ and has one immediate successor *n*. Then, the optimal solution guarantees that only one of the tasks is performed. However, the optimal LP relaxation may not satisfy this property due to the allowed partial assignments.

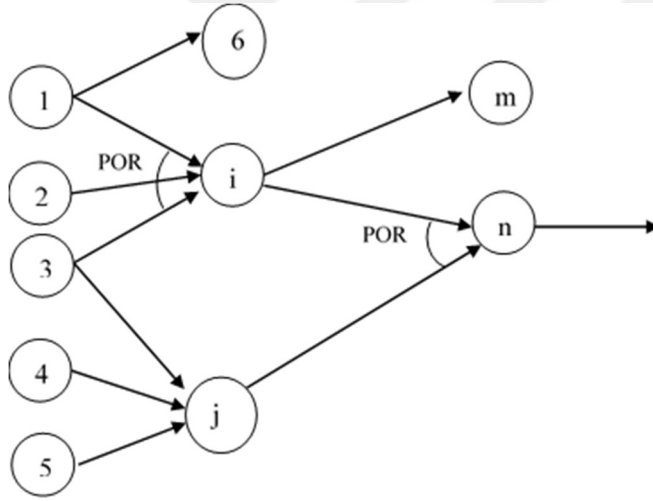


Figure 4.4. An Example Network illustrating CUT3

To clarify the valid cut with an example, we can use the Figure above. To perform *task n*, one should perform *task i* or *task j* or both of them. Note that $j \in \text{POR}_n$, $|IS_j| = 1$ and $rev_j \leq 0$. Then in an optimal solution only one of the tasks (*task i* or *task j*) should be performed. For *task n*, this relation is imposed by the following constraint:

$$\sum_k X_{ik} + \sum_k X_{jk} \leq 1 \quad i, j \in POR_n, \quad |IS_j| = 1, \quad rev_j \leq 0.$$

For the tasks having more than two predecessors in POR relation, we can extend the theorem. For instance, *task 1, 2 and 3* preceding *task i*, we can write the constraint such that ;

$$\sum_k X_{1k} + \sum_k X_{2k} + \sum_k X_{3k} \leq 2 \quad \text{task } 1,2,3 \in POR_i, \quad |IS_2| = 1 \text{ and } rev_2 \leq 0.$$

Hence, in general considering different number of tasks in POR relation, for any *task i* the following constraint set is used to support Theorem 2.

$$\sum_{i \in (POR_j)} X_{ik} \leq |POR_j| - 1 \quad \forall i \text{ s.t } i \in POR_j, \quad rev_i \leq 0, \quad |IS_{i \in (POR_j)}| = 1$$

We can use the 22 task network, illustrated in Figure 4.2, to show Cut3. Consider *tasks 8 and 21* that has predecessors in POR relations. Suppose that we have $rev_5 \leq 0$ and $rev_9 \leq 0$, where *task 5 and task 9* both have only one immediate successor. Then, *task 5* cannot be performed together with *task 4*, and *task 9* cannot be performed together with *task 16* and *task 19*. The constraints presented below can be added as Cut3.

$$\sum_k X_{4k} + \sum_k X_{5k} \leq 1$$

$$\sum_k X_{9k} + \sum_k X_{16} + \sum_k X_{19k} \leq 2$$

4.2.2.4 Bound on Immediate Successors (CUT4)

Cut4 considers the tasks having negative net revenues. If a task that has negative net revenue is performed, then one of its immediate successors should be performed.

Theorem 3: In all optimal solutions, if net revenue of *task i* is negative such that $rev_i < 0$ for any *task i*, then *task i* is performed only when at least one of its immediate successors is performed.

Proof: Assume a solution in which *task i*, such that $rev_i < 0$ if performed, however none of its immediate successors is performed. Such a solution contradicts with the

condition stated by the theorem and cannot be optimal. It cannot be optimal as it can be improved by not performing *task i* while keeping all assignments the same. $\square$

In our mathematical models, we include the condition stated by Theorem 3 through the following constraint.

$$\sum_k x_{ik} \leq \sum_k \sum_{j \in IS_i} x_{jk} \text{ for all } i \mid rev_i < 0$$

To present Cut4 via an example, we can use the 22 task network illustrated in Figure 4.2. Consider *task 6* which has immediate successors such as *task 12* and *task 13*. Suppose that we have $rev_6 < 0$. Then, *task 6* is performed only when at least one of the immediate successors, such as *task 12* and *task 13*, is performed. The constraints presented below can be added as Cut4.

$$\sum_k X_{6k} \leq \sum_k X_{12k} + \sum_k X_{13k} \quad rev_6 < 0$$

4.2.2.5 Bound on All Successors (CUT5)

Cut5 provides a method for the tasks having negative net revenues. If a task that has negative net revenue is performed, then one of its successors should be performed.

Theorem 4: In all optimal solutions, if $rev_i < 0$ for any *task i* then *task i* is performed only when one of its successors with positive rev_j is performed.

Proof: Assume a solution in which *task i*, such that $rev_i < 0$ if performed, however none of its successors with positive net revenue is performed. Such a solution contradicts with the condition of the theorem and can be improved by removing *task i* and its successors that are already processed hence cannot be optimal. $\square$

In our mathematical models, we include the following constraint for Theorem 4.

$$\sum_k x_{ik} \leq \sum_k \sum_{j \in SI \mid rev_j > 0} x_{jk} \text{ for all } i \mid rev_i < 0$$

We can use the 22 task network illustrated in Figure 4.2, to show Cut5. Consider *task 9* that has successors *14, 15, 18, 20, 21, 22*. Suppose that we have $rev_9 < 0$ and let other revenues are all positive except *task 15* and *task 22* such that $rev_{15} < 0$ and $rev_{22} < 0$. Then, *task 9* is performed only when one of its successors with positive revenue is performed. The constraints presented below can be added as Cut5.

$$\sum_k X_{9k} \leq \sum_k X_{14k} + \sum_k X_{18} + \sum_k X_{20k} + \sum_k X_{21k} \quad rev_{9, 15, 22} < 0, rev_{14, 18, 20, 21} > 0,$$

4.2.2.6 Assignment According to the Contribution of Tasks (CUT6)

Cut6 provides a method as comparing the contribution of the tasks with the best objective function value. If the contribution of tasks with positive revenues that are not successors of any defined task is less than the best objective function value, then this task should be performed.

To state Theorem 5 we need the following notation,

Z_j : contribution of *task j* to Z and $Z = \sum_j Z_j$

Theorem 5: If the contribution of tasks with positive revenues that are not successors of *task i* is less than the best objective function value such that $\sum_{j \notin S_i | rev_j > 0} Z_j < Z_{BEST}$, then *task i* should be performed.

Proof: If *task i* is not performed then no task in S_i can be performed. This follows the maximum contribution for the remaining tasks is $\sum_{j \notin S_i | rev_j > 0} Z_j$.

If the maximum contribution is less than the best objective function value known so far then there cannot be any optimal solution in which *task i* is not performed. $\square$

The condition stated by Theorem 5 is included in our mathematical models as follows:

$$\sum_k x_{ik} = 1 \mid \text{for all } i \text{ such that } \sum_{j \notin S_i | rev_j > 0} Z_j < Z_{BEST}$$

We can use the 22 task network illustrated in Figure 4.2 to show Cut6. Consider *task* 3. Suppose for the tasks that are not successors of *task* 3, we have

$$rev_1 > 0, rev_2 > 0, rev_5 > 0, rev_6 > 0, rev_{12} > 0 \text{ and}$$

$$rev_1 + rev_2 + rev_5 + rev_6 + rev_{12} < Z_{BEST}$$

Then, *task* 3 should be performed and the constraint presented below can be added as Cut6.

$$\sum_k x_{3k} = 1$$

4.2.3 Upper Bound 3 (UB_3)

UB_2 is improved by UB_3 through heuristic cuts. We presented two heuristic cuts to improve UB_2 .

We introduce two cuts that may not be satisfied by the optimal solutions, however might decrease the objective function values, hence enhance upper bound estimations. UB_3 is strengthened LPR with exact properties (cuts 1 through 6) and heuristic properties.

We do call those cuts as heuristic cuts and the model that uses them produces neither lower bounds, nor upper bounds.

4.2.3.1 Heuristic Cut 1

This heuristic cut implies a rule such that if the total processing time of any *task* i and all its predecessors does not exceed the cycle time, $t_i + \sum_{j \in P_i} t_j \leq CT$, then *task* i is not processed in workstations 2 through k . Note that P_i is the set of “AND” predecessors.

The constraint set that supports heuristic cut 1 is stated below.

$$\sum_{k=2}^K x_{ik} = 0 \text{ for any task } i \text{ such that } t_i + \sum_{j \in P_i} t_j \leq CT$$

This rule may not be satisfied by the optimal solutions, hence cannot be used as an exact cut. Assume that we have three *tasks a, b, c* where *tasks a* and *b* do not have any predecessors and *tasks a* and *b* precede *task c*. Consider two workstations and $t_a = 5, t_b=4, t_c=4, CT=8$. Then we have an optimal solution where all tasks can be performed, such that *task a* in the first workstation and *tasks b, c* in the second workstation. Suppose we use *task b* to apply the rule as it has no predecessors. Accordingly, *task b* cannot be assigned to the second workstation and also cannot be assigned together with *task a* due to the cycle time. The only solution that applies the rule is to assign *task b* to the first workstation and *task a* to the second one. Only two task assignment can be performed with this rule whereas in the optimal solution we can perform all tasks.

To apply this rule, we need to modify the “POR” relations in our model. “POR” relations are replaced by “AND” relations, where the total task time of all predecessors can be obtained as if the task has “AND” relation with its predecessors. Thus, the set P_i includes all predecessor tasks that have “AND” or “POR” type precedencies. Moreover, we select *task i* as the most divided task on the workstations.

We now illustrate the cut via an example. Assume *task 13* is the most divided task of the LPR solution, thus it is selected to apply the heuristic cut 1. Figure 4.2 shows that *task 13* has “POR” relation. We treat this relation as “AND” type and check the condition such that;

$$t_{13} + t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_8 \leq CT \text{ where } 1,2,3,4,5,6,8 \in P_{13}$$

If the condition is satisfied, then we do not check other tasks since we can apply this heuristic cut to only one task. Therefore, we can include the constraint set presented below as heuristic cut 1 in our models:

$$\sum_{k=2}^K x_{13,k} = 0$$

4.2.3.2 Heuristic Cut 2

This heuristic cut implies a rule such that if the total processing time of any *task i* and all its successors does not exceed the cycle time, $t_i + \sum_{j \in S_i} t_j \leq CT$, then *task i* is not processed in workstations 1 through k-1. Note that S_i is the set of all successors.,

The constraint set that supports heuristic cut 2 is stated below

$$\sum_{k=1}^{K-1} x_{ik} = 0 \text{ for any task } i \text{ such that } t_i + \sum_{j \in S_i} t_j \leq CT$$

As heuristic cut 1, heuristic cut 2 may not be satisfied by the optimal solutions, hence cannot be used as an exact cut. Assume that we have three *tasks a, b, c* where *tasks b* and *c* do not have any successors and *task a* precedes *task b* and *task c*. Consider two workstations and $t_a = 4$, $t_b=5$, $t_c=4$, $CT=8$. Then we have an optimal solution where all tasks can be performed, such that *task a, c* in the first workstation and *task b* in the second workstation. Suppose we select *task c* to apply the rule and it satisfies the rule since it does not have any successors. Hence *task c* cannot be assigned to first workstation and also cannot be assigned together with *task b* due to the cycle time. The only solution that applies the rule is to assign *task a* to the first workstation and *task b or task c* to the second one. Only two task assignment can be performed with this rule whereas in the optimal solution we can perform all tasks.

To apply this heuristic cut 2, we select *task i* as the most divided task on the workstations but different than the task that is selected previously for heuristic cut 1 to prevent infeasibility.

We illustrate this cut via an example. Let *task 21* be the task that is divided most among all the tasks in LPR solution and different than the task previously selected for heuristic cut 1. As illustrated in Figure 4.2 *task 21* has only one successor that is *task 22*. If it satisfies the condition $t_{21} + t_{22} \leq CT$ where $\text{task } 22 \in S_{21}$ then we do not check other tasks since we can apply this heuristic cut to only one task. Therefore, we can include the constraint set presented below as heuristic cut 2 in our models:

$$\sum_{k=1}^{K-1} x_{21,k} = 0$$

Heuristic Cuts 1 and 2 might produce infeasible solutions when the same task is used for both cases or more than two tasks satisfy the condition of any one of the cuts. To prevent those cases, we use heuristic cut 1 only for one task and heuristic 2 again only for one and different task. Moreover, those cuts may produce infeasible solutions once used together with Cut 6. As Cut 6 may force more than one task that satisfy the conditions of the heuristic cuts.

For example, assume *task a* and *task b* that should be performed according to Cut 6 and that should be processed only in Workstation 1 according to heuristic cut 1. If $t_a + t_b + \sum_{i \in Pa \cup Pb} t_i > CT$ then they cannot fit to Workstation 1. Hence Cut 6 once used with heuristic cut 1 delivers an infeasible solution.

4.3 Lower Bounds (Heuristic Methods)

Two lower bounds are studied as *Lower Bound 1 (LB₁)* and *Lower Bound 2 (LB₂)*. *LB₁* is obtained by utilizing the LP Relaxation solutions with cuts and obtaining the best integral solution through forward pass. *LB₁* is improved by *LB₂* through backward pass, which compares other solutions and gathers a better integral solution if exists.

Algorithms regarding lower bounds are presented at Figures 4.5-4.7. We use the following notation in our figures.

r : subproblem index

$t(r)$: selected task at each subproblem r

$w(r)$: selected task's workstation at each subproblem r

Z_r : optimal solution of each subproblem r

Z_{INT-m} : integer solution of subproblem r (if exists)

Note that from the above notations, $x_{t(r),w(r)}$ represents the selected x_{ik} variable at each subproblem.

4.3.1 Lower Bound 1 (LB_I)

LB_I is obtained by utilizing the LP Relaxation solutions with cuts and obtaining the best integral solution through forward pass.

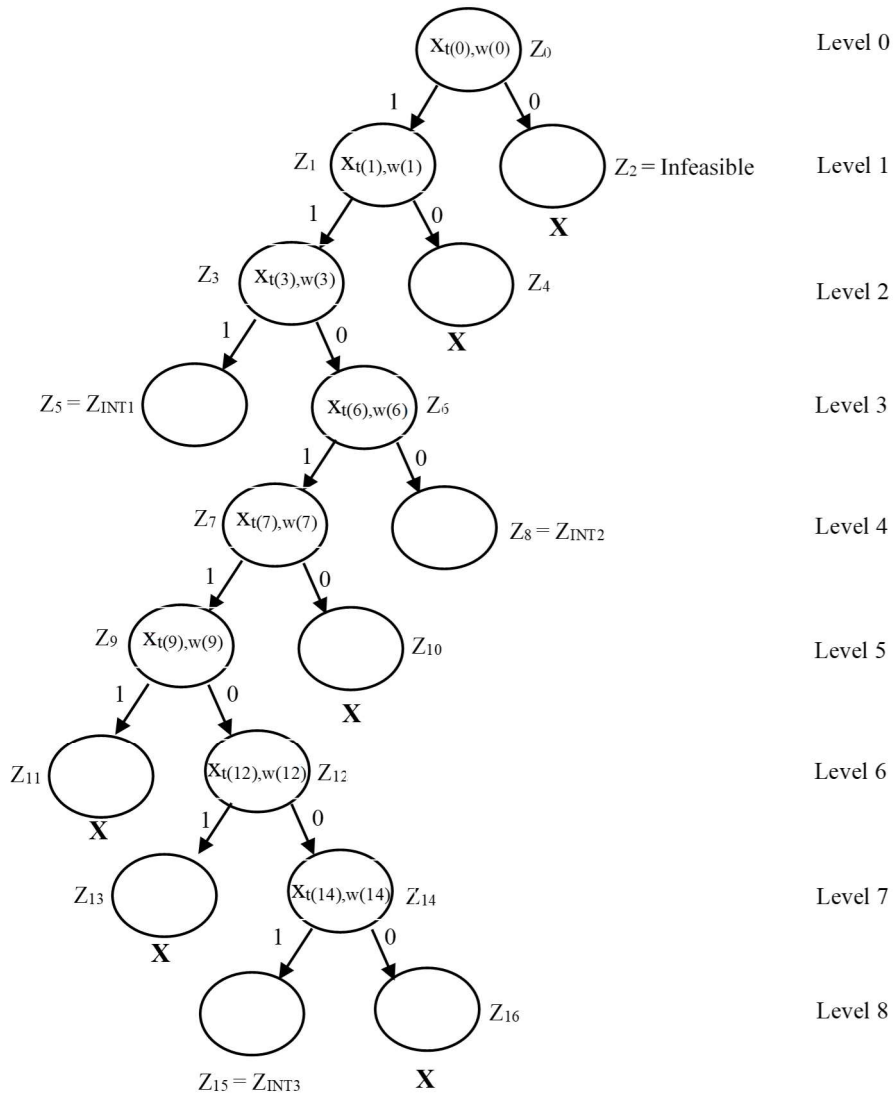


Figure 4.5. Forward Pass Scheme

The forward pass algorithm is based on proceeding with the better optimal solution at each level, while using the mathematical model LPR with exact cuts i.e. upper bounds. The explanation of our algorithm by using the scheme presented at Figure 4.5 is discussed below.

Forward Pass Algorithm

Step 0: Find an initial solution Z_0 at Level 0 using the LPR solution with exact cuts i.e. upper bounds. If Z_0 produces all integer x_{ik} variables, then solution is optimal, Stop. Else go to Step 1.

Step 1: Find the largest fractional variable x_{ik} in the solution to proceed such that;

$$x_{ik} = \text{Max } \{x_{ik} \mid 0 < x_{ik} \leq 1\}$$

Two subproblems are created at this level via using selected x_{ik}

Subproblem 1: $x_{ik} = 1$

Subproblem 2: $x_{ik} = 0$

The optimal solutions for each subproblem (Z_r) are obtained. Set Level = Level+1.

The subproblem is fathomed if

- produces integer solutions for all x_{ik} variables (Z_5, Z_8, Z_{15})
- infeasible (Z_2)
- delivers an optimal solution worse than the other subproblem at each level, i.e. not promising ($Z_4 < Z_3, Z_{10} < Z_9, Z_{11} < Z_{12}, Z_{13} < Z_{14}$)

The integer solutions are saved as Z_{INT-m} where m is the number of integer solutions obtained. Note that we can proceed with the subproblem which has promising objective function value than the subproblem having integer solutions for all x_{ik} variables. ($Z_5 < Z_6, Z_{16} < Z_{15}$)

Step 2: If one of the subproblems remain, go to Step 1. Otherwise go to Step 3.

Step 3: All subproblems are eliminated, Stop. Select the best solution among the solutions that have all integer x_{ik} variables (Z_{INT-m}) and set as Z_{BEST-F} for forward pass.

For the example at Figure 4.5, we stop at the last level since one of the subproblems is integer (Z_{15}) and the other optimal solution is not promising ($Z_{16} < Z_{15}$). Through forward pass three optimal solutions with integer x_{ik} variables are delivered. Let $Z_{15} = Z_{INT3} > Z_8 = Z_{INT2} > Z_5 = Z_{INT1}$.

Note that only integrality of x_{ik} variables are guaranteed with this method. However, the original objective function value may not be equal to these solutions since hazardous tasks can be assigned and other variables may not all be integral. Thus, best objective function value (Z_{BEST-F}) may not be equal to LB_I . The value of LB_I can easily be calculated since all x_{ik} variables are integral and assignments of tasks to the workstations are known. LB_I will be delivered from the best solution corresponding to the original objective function value.

Z_{BEST-F} is the LPR based objective function value that corresponds to the LB_I . The details of the calculation of these values are clearly defined in the Example Problem.

4.3.2 Lower Bound 2 (LB_2)

LB_I is improved by LB_2 through backward pass considering both hazardous and non-hazardous tasks. The solutions obtained by forward pass are compared with the solutions obtained by backtracking. This method gathers a better integral solution if exist.

Note that we have two types of backtracking. Backtracking for hazardous and non-hazardous tasks. After the completion of backtracking with hazardous tasks and non-hazardous tasks we compare these solutions with LB_I , which is delivered by forward pass, then assign the best of all as LB_2 .

i. Backward Pass with Hazardous Tasks

In our mathematical model, optimal solution is highly affected from the assignment of hazardous tasks due to the success probabilities to remove them. If an assignment

of hazardous task occurs during forward pass, the main idea for the backtracking related with hazardous tasks is turning back to the level that hazardous task is assigned. All constraints between these levels which are added during forward pass and also the constraint related with the hazardous task assignment, i.e. $x_{zk} = 1$ are deleted.

The backward pass algorithm uses the base solution obtained via forward pass, starts from the level that hazardous task is assigned and continues with the deletion of constraint $x_{zk} = 1$ and addition of $x_{zk} = 0$ instead. The procedure continues similarly as forward pass and LPR model with exact cuts are utilized, i.e. upper bounds. The explanation of our algorithm by using the scheme presented at Figure 4.6 is discussed below.

Backward Pass Algorithm for Hazardous Tasks

Step 0: Check if hazardous task is assigned during the assignments of forward pass. If there exists no assignment, Stop. Let the best solution for backward pass of hazardous tasks $Z_{BEST-B} = 0$.

If there exists a hazardous task assignment, go to the level that hazardous task is assigned; $Level = Level_{HZ}$. From the Figure 4.6, let the largest fractional task is *hazardous task* z for solution Z_7 and assigned to a workstation previously at forward pass. Thus, $Level_{HZ} = Level\ 4$ at Figure 4.6. Keep only the constraints of forward pass up to Level 4 and delete others. Moreover, unassign the hazardous task.

- Delete $x_{zk} = 1$
- Add $x_{zk} = 0$

Solve the problem with new constraint set. If we deliver all integer x_{ik} variables, then solution is optimal, Stop. Set the solution as Z_{BEST-B} . Otherwise go to Step1.

Step 1: Same as forward pass, proceeds with the promising subproblem if exists.

$Level = Level + 1$.

The integer solutions are saved as Z_{INT-m} .

Step 2: If one of the subproblems remain, go to Step 1. Otherwise go to Step 3.

Step 3: All subproblems are eliminated, Stop. Select the best solution among the solutions that have all integer x_{ik} variables (Z_{INT-m}) and set as Z_{BEST-B} for backward pass.

As mentioned before, only integrality of x_{ik} variables are guaranteed with this method. The original objective function value should be calculated for the selection of best integer solution. The original objective function value of each Z_{INT-m} can easily be calculated since all x_{ik} variables are integral and assignments of tasks to the workstations are known. The best original objective function value will be set as $Z_{BEST-B-Original}$. The best LPR based objective function value Z_{BEST-B} is the Z_{INT-m} value corresponds to the best original objective function value ($Z_{BEST-B-Original}$). The details of the calculation of these values are clearly defined in the Example Problem.

Note that if there is more than one hazardous task, the algorithm is applied to each one. Then the best solution of backward pass is selected among all hazardous tasks.

In Figure 4.6, we stop at the last level 7 since one of the subproblems is integer (Z_{19}) and the other optimal solution is not promising ($Z_{20} < Z_{19}$). Through backward pass for hazardous task one new optimal solution with integer x_{ik} variables is delivered. Let $Z_{19} = Z_{INT4}$, then we calculate the original objective function value $Z_{BEST-B-Original}$ regarding the assignments in the Z_{INT4} solution. In this case we have only one hazardous task and one integer solution (Z_{INT4}), hence we conclude with best LPR based objective function value $Z_{BEST-B} = Z_{19}$.

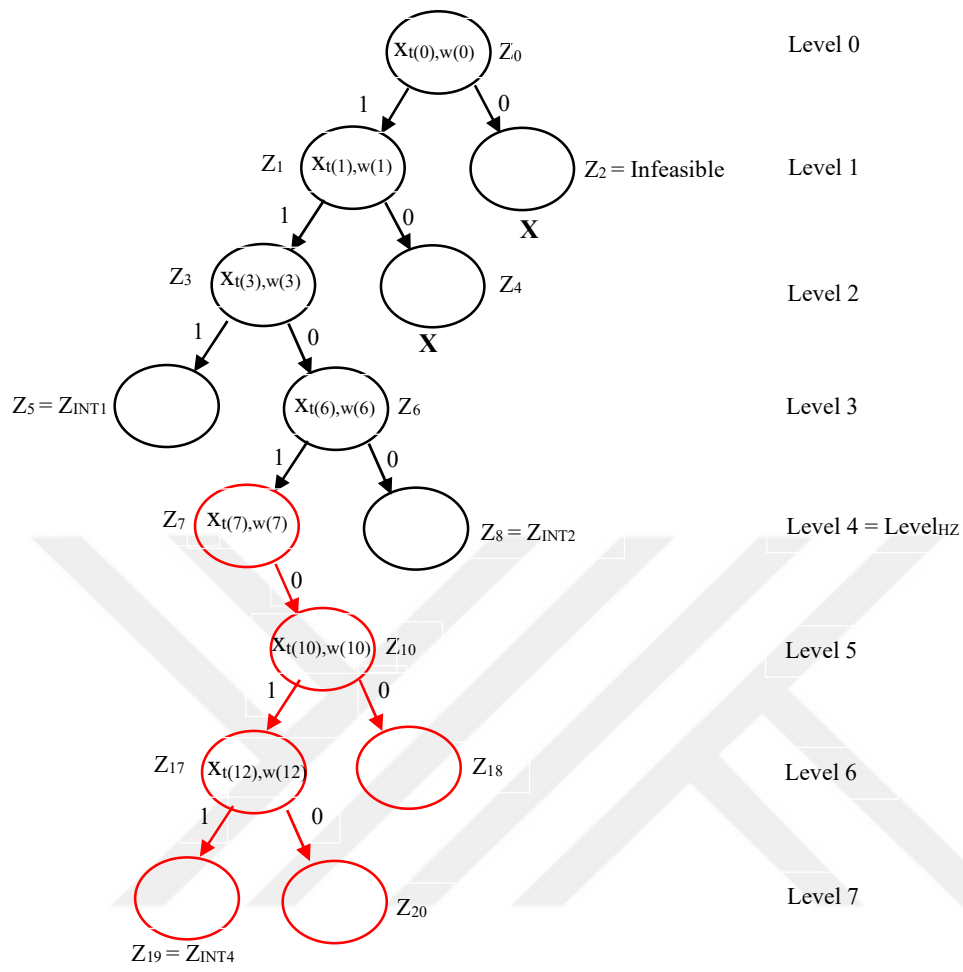


Figure 4.6. Backward Pass Scheme for Hazardous Task

ii. Backward Pass with Non-Hazardous Tasks

As previously mentioned, the hazardous tasks are considered to obtain a better feasible solution. While backtracking for hazardous tasks we use the base solution as forward pass solution and turn back accordingly. However, instead of using forward pass as base solution for non-hazardous tasks, the solution that gives the best original objective function value between forward pass and backward pass is selected as base solution for non-hazardous backtracking.

The backward pass algorithm for non-hazardous tasks starts with the selected base solution. We set a predefined limit level such as two levels to consider backtracking for non-hazardous tasks. The solutions obtained previously but fathomed are considered for this case. We turn back one by one to each level and compare the solutions. The explanation of our algorithm by using the scheme presented at Figure 4.7 is discussed below.

Backward Pass Algorithm for Non-Hazardous Tasks

Step 0: The best selected solution obtained via forward/backward pass for hazardous tasks, Z_{BEST-F} / Z_{BEST-B} , is compared with the solutions that are fathomed in the previous levels.

Stop if Level=1, otherwise Level=Level-1 and Number of Backtrack= Number of Backtrack +1.

Select the solution that is fathomed (Z_r) in this level and compare with Z_{BEST-F} / Z_{BEST-B} .

If $Z > Z_{BEST-F} / Z_{BEST-B}$ then delete the assignment constraints skipped by backtracking and go to Step 1.

If $Z < Z_{BEST-F} / Z_{BEST-B}$ and Number of Backtrack < 2 repeat Step 0.

If $Z < Z_{BEST-F} / Z_{BEST-B}$ and Number of Backtrack=2, hence no promising solution is delivered. Set the best objective function value for the backward pass of non-hazardous tasks as $Z_{BEST-NHZ} = 0$.

Step 1: Procedure is same as forward pass, proceeds with the promising subproblem if exists. Level = Level+1.

The integer solutions are saved as Z_{INT-m} .

Step 2: If one of the subproblems remain, go to Step 1. Otherwise go to Step 3.

Step 3: All subproblems are eliminated, Stop. Select the best solution among the solutions that have all integer x_{ik} variables (Z_{INT-m}) and set as $Z_{BEST-NHZ}$ for the backward pass of non-hazardous tasks.

Suppose that the forward pass algorithm illustrated at Figure 4.5 gives a better objective function value than backward pass which is illustrated at Figure 4.6. ($Z_{BEST-F-Original} > Z_{BEST-B-Original}$). Hence, backtracking for non-hazardous tasks are applied to the forward pass scheme illustrated at Figure 4.5 up to a predefined level.

From the Figure 4.5, we first compare $Z_{BEST-F} = Z_{15}$ at Level 8 with the previously fathomed node Z_{15} at Level 7. Suppose that $Z_{13} < Z_{15}$, thus we continue backtracking one more time and compare $Z_{BEST-F} = Z_{15}$ with Z_{11} at Level 6. Let $Z_{11} > Z_{15}$ and we deliver a promising solution from that node. Then we continue the procedure and we obtain a solution that is presented at Figure 4.7 . Note that, we stop at the last Level 8 since one of the subproblems is integer (Z_{23}) and the other optimal solution is not promising ($Z_{24} < Z_{23}$). Through backward pass for non-hazardous task, one new optimal solution with integer x_{ik} variables is delivered. Let $Z_{23} = Z_{INT5}$, then we calculate the original objective function value $Z_{BEST-NHZ-Original}$ regarding the assignments in the Z_{INT5} solution. In this case we have only one integer solution (Z_{INT5}), hence we conclude with best LPR based objective function value for the backward pass of non-hazardous tasks as $Z_{BEST-NHZ} = Z_{23}$.

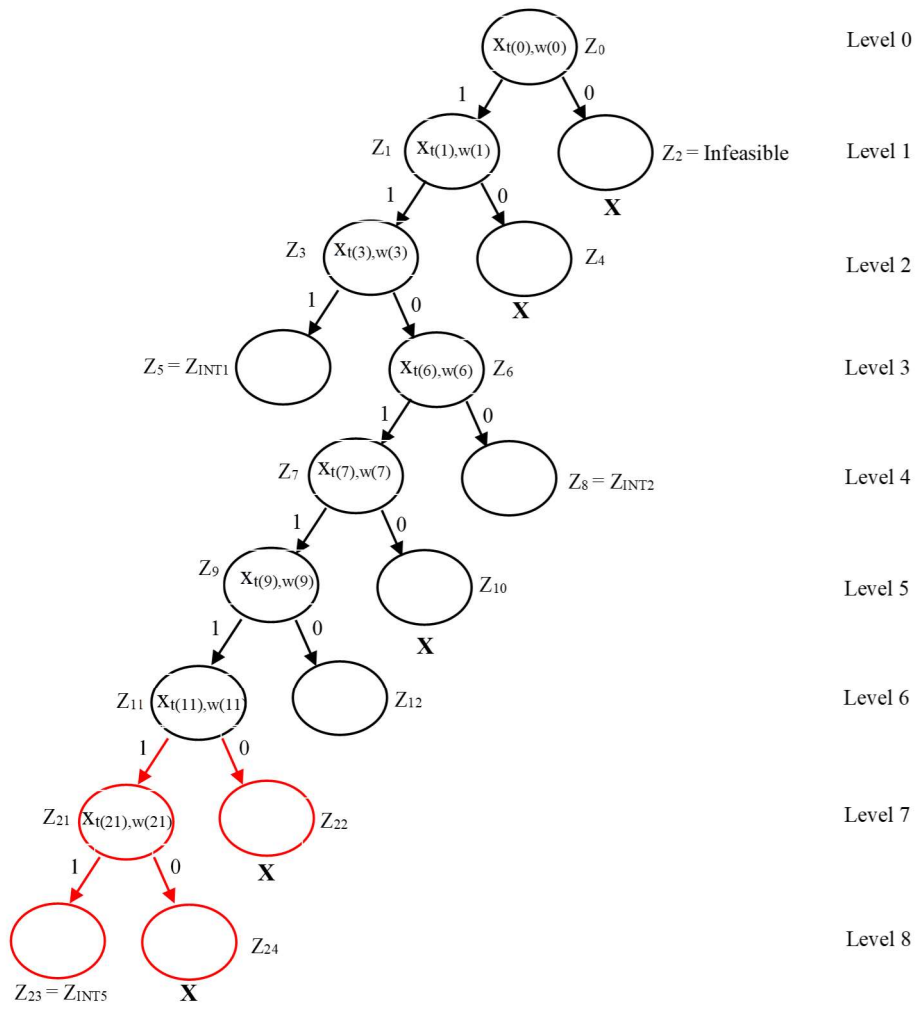


Figure 4.7. Backward Pass Scheme for Non-Hazardous Tasks

iii. Determination of LB_2

As explained in the previous sections, we have two types of backtracking, performed for hazardous and non-hazardous tasks. After the completion of backtracking with hazardous tasks (*section i*) and non-hazardous tasks (*section ii*), we need to compare these solutions together with LB_1 , which is delivered by forward pass, then assign the best of all as LB_2 .

The original objective function values stated below are compared and best of all is set as LB_2 .

- Forward Pass Solution (LB_1)
- Backward Pass for Hazardous Tasks Solution ($Z_{BEST-B-Original}$)
- Backward Pass for Non-Hazardous Tasks Solution ($Z_{BEST-NHZ-Original}$)

Hence, $LB_2 = \text{Max } \{LB_1, Z_{BEST-B-Original}, Z_{BEST-NHZ-Original}\}$

4.3.3 An Example Problem

To illustrate the forward pass and backward pass algorithms more clearly, we go through an example problem where the solution is obtained by using an instance with $N=22$, $K=2$. Note that there are two hazardous tasks which are task 19 and task 20, success probabilities are $\pi_{z_1} = 0.2$ and $\pi_{z_2} = 0.2$ respectively.

Forward Pass Algorithm

Forward pass algorithm of the example problem is illustrated at Figure 4.8.

Step 0 : An initial solution $Z_0 = 315.4$ at Level 0 is found by LPR with exact cuts i.e. upper bound.

Step 1 : The largest fractional variable in the solution is found as $x_{2,2}$. Two subproblems are created with $x_{2,2} = 1$ and $x_{2,2} = 0$. The optimal solutions for each subproblem are obtained and set Level =1. Then we select the promising solution $Z_1=316.8 > Z_2=314.8$ and fathom Z_2 .

Step 2: With the selected subproblem Z_1 we repeat Step 1.

We proceed with the promising subproblem at each level and we save the integer solution Z_{46} as Z_{INT-1} .

Step 3: All subproblems are eliminated at last Level 24, since all the subproblems at this level delivered integer solution. Hence these solutions are saved as $Z_{INT-2} = Z_{47}$ and $Z_{INT-3} = Z_{48}$

The assignments regarding each Z_{INT-m} solution are illustrated below:

Solution	Workstation 1	Workstation 2
$Z_{INT-1} = 269.0$	1,3,4,7,8,9,10,14,15,16,18	2,5,6,11,12,13,17,19,20
$Z_{INT-2} = 285.3$	1,3,4,7,8,9,10,14,15,16,18	2,5,6,11,12,13,17,19,20,21,22
$Z_{INT-3} = 271.7$	1,3,4,7,8,9,10,14,15,16,18	2,5,6,11,12,13,17,19,20,21

As we mentioned before, the x_{ik} variables are guaranteed to be integral however the original objective function value may not be equal to these solutions. By the help of assignments, we can easily calculate original objective function values O_{INT-m} and LB_I will be delivered with the best solution.

Both hazardous tasks are assigned to the second workstation in all solutions. Remember that for two hazardous task case we calculate the objective function value for the original problem such that;

$$\begin{aligned} \text{Max } z = & p_1 * \sum_i \sum_k rev_i * x_{ik} + p_2 * \sum_i rev_i * a_{iz_2} + p_3 * \sum_i rev_i * a_{iz_1} \\ & + p_4 * \sum_i rev_i * C_i \end{aligned}$$

Then, we can calculate the original objective function values(O_{INT-m}) for each Z_{INT-m} obtained via forward pass as;

$$O_{INT-1} = 0.2*0.2 (297) + 0.8*0.2 (174) + 0.2*0.8 (174) + 0.8*0.8 (174) = 178.9$$

$$O_{INT-2} = 0.2*0.2 (321) + 0.8*0.2 (174) + 0.2*0.8 (174) + 0.8*0.8 (174) = 179.8$$

$$O_{INT-3} = 0.2*0.2 (301) + 0.8*0.2 (174) + 0.2*0.8 (174) + 0.8*0.8 (174) = 179.1$$

$$\text{Hence } LB_I = \text{Max } \{178.9, 179.8, 179.1\} = 179.8$$

Z_{BEST-F} is the LPR based objective function value (Z_{INT-m}) that corresponds to the LB_I , such that $Z_{BEST-F} = 285.3$.



Level 0

Level 1

Level 2

Level 3

Level 4

Level 5

Level 6

Level 7

Level 8

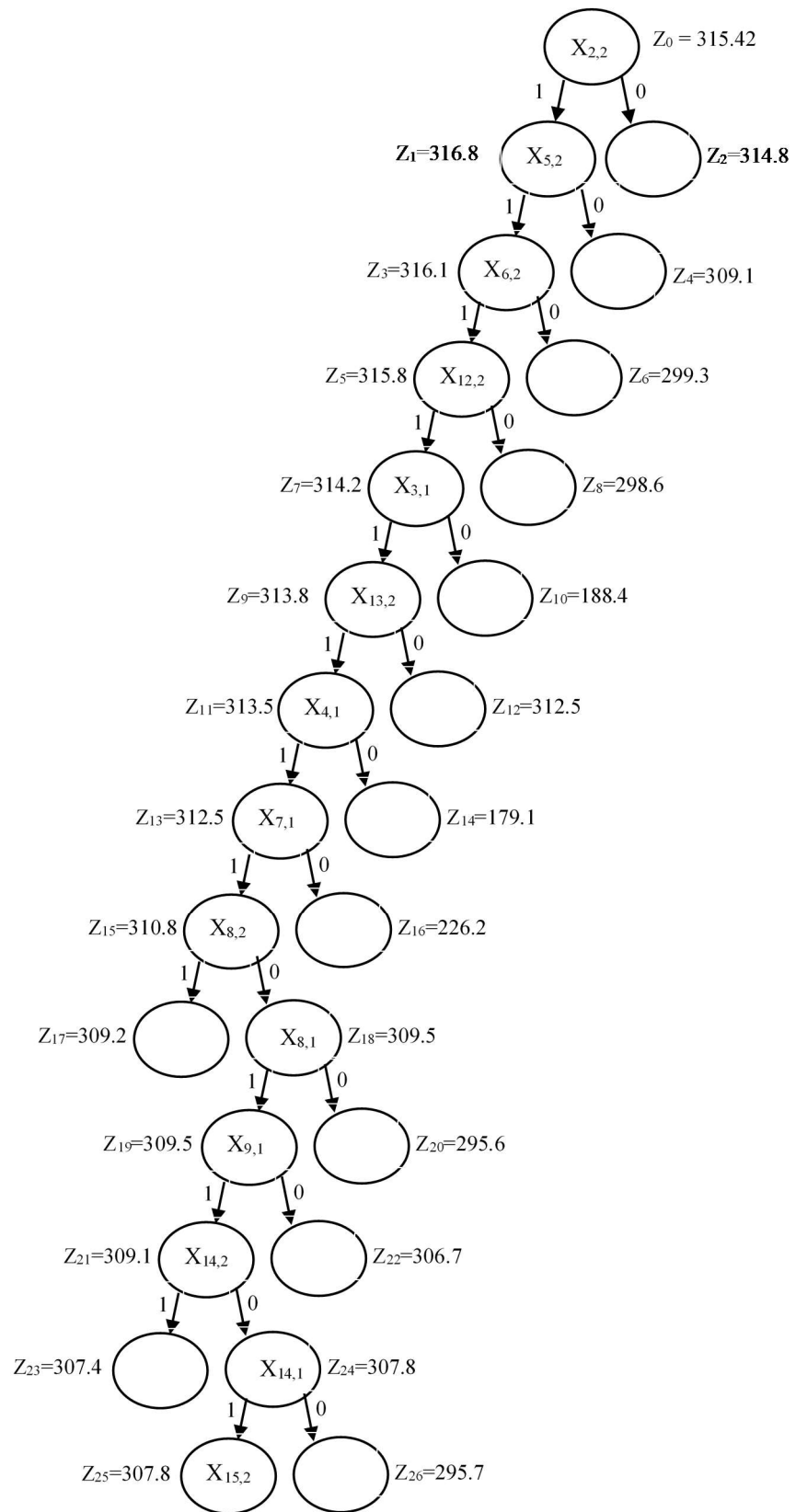
Level 9

Level 10

Level 11

Level 12

Level 13



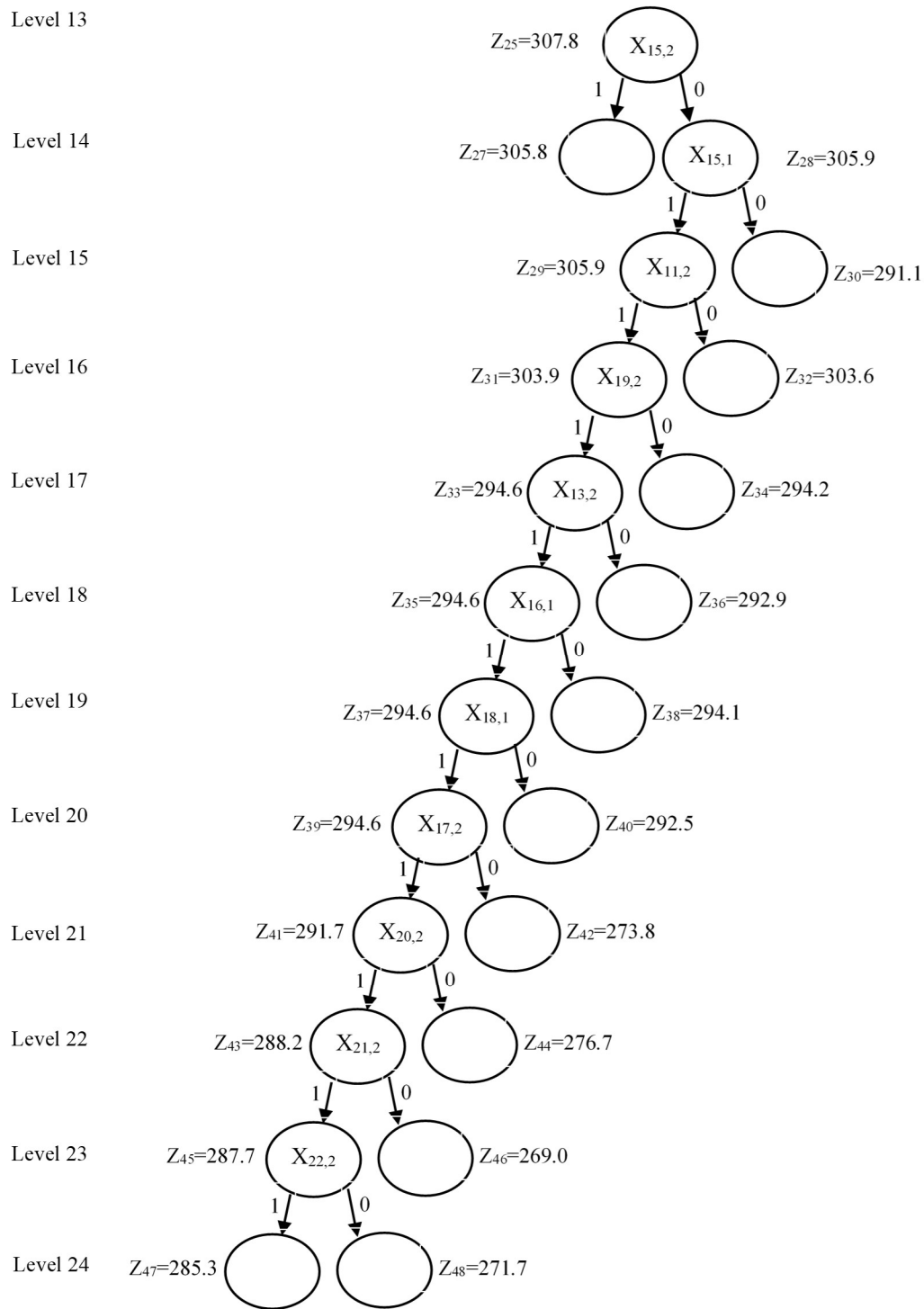


Figure 4.8. Forward Pass Scheme

Backward Pass Algorithm for First Hazardous Task

Backward pass algorithm of the example problem for First Hazardous Task is illustrated at Figure 4.9.

Step 0: We checked the assignments of hazardous task. First hazardous task is assigned. Go to the level that hazardous task is assigned, Level=16. Delete constraints between Level 16-24 and add $X_{19,2} = 0$. We solve the problem with new constraint set at Level 17.

Step 1: The largest fractional variable in the solution is found as $x_{21,2}$. Two subproblems are created with $x_{21,2} = 1$ and $x_{21,2} = 0$. The optimal solutions for each subproblem are obtained at Level =18. Then we select the promising solution $Z_{49}=293.8 > Z_{50}=289.6$ and fathom Z_{50} .

Step 2: With the selected subproblem Z_{49} we repeat Step 1.

We proceed with the promising subproblem at each level and we save the integer solution Z_{54} as Z_{INT-4} .

Step 3: All subproblems are eliminated at last Level 24, since one of the subproblem is infeasible and the other (Z_{62}) delivered integer solution. Hence this last integer solution Z_{64} is saved as Z_{INT-5} .

The assignments regarding each Z_{INT-m} solution are illustrated below:

Solution	Workstation 1	Workstation 2
$Z_{INT-4} = 280.0$	1,3,4,7,8,9,14,15,18	2,5,6,10,11,12,13,16,17,21,22
$Z_{INT-5} = 280.0$	1,3,4,7,8,9,10,14,15,16	2,5,6,11,12,13,17,18,21,22

None of the hazardous tasks is assigned. We can calculate the original objective function values (O_{INT-m}) for each Z_{INT-m} obtained via backward pass for first hazardous task as;

$$O_{INT-4} = 0.2*0.2 (280) + 0.8*0.2 (280) + 0.2*0.8 (280) + 0.8*0.8 (280) = 280.0$$

$$O_{INT-5} = 0.2*0.2 (280) + 0.8*0.2 (280) + 0.2*0.8 (280) + 0.8*0.8 (280) = 280.0$$

$$\text{Hence } Z_{BEST-BI_Original} = \text{Max } \{280.0, 280.0\} = 280.0$$

$Z_{BEST-BI}$ is the LPR based objective function value (Z_{INT-m}) that corresponds to the $Z_{BEST-BI_Original}$, such that $Z_{BEST-BI} = 280.0$.



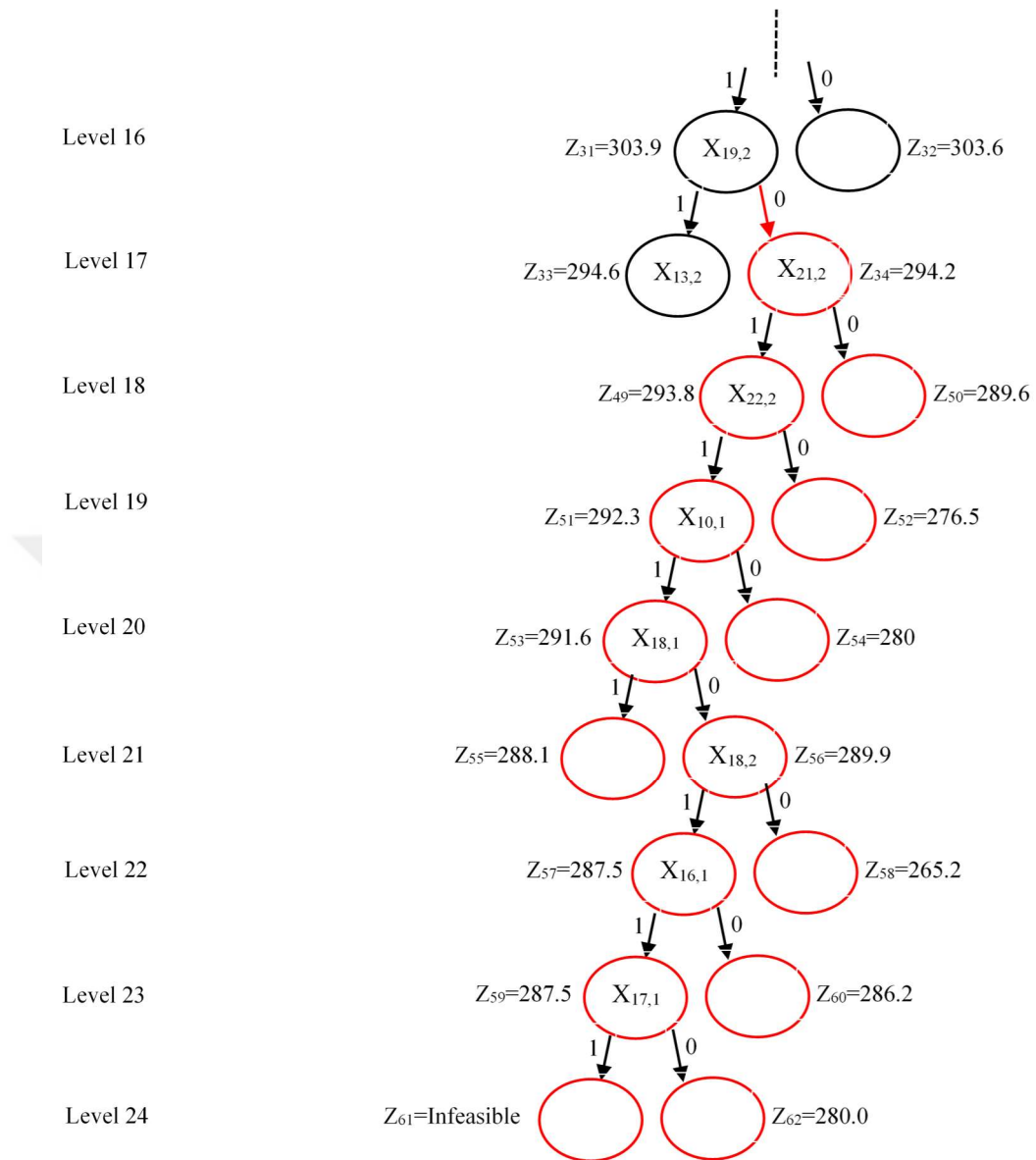


Figure 4.9. Backward Pass Scheme for First Hazardous Task

Backward Pass Algorithm for Second Hazardous Task

Backward pass algorithm of the example problem for second hazardous task is illustrated at Figure 4.10.

Step 0: We checked the assignments of hazardous task. Second hazardous task is assigned. Go to the level that hazardous task is assigned, Level=21. Delete constraints between Level 21-24 and add $X_{20,2}=0$. We solve the problem with new constraint set at Level 22.

Step 1: The largest fractional variable in the solution is found as $x_{21,2}$. Two subproblems are created with $x_{21,2} = 1$ and $x_{21,2} = 0$. The optimal solutions for each subproblem are obtained at Level =23. Then we select the promising solution $Z_{63}=276.5 > Z_{64}=257.5$. Fathom and save Z_{64} as Z_{INT-6} , since it is an integer solution

Step 2: With the selected subproblem Z_{63} we repeat Step 1.

We proceed with the promising subproblem at each level.

Step 3: All subproblems are eliminated at last Level 24, since they deliver integer solutions. Hence these last integer solutions are saved as $Z_{INT-7} = Z_{65}$.and $Z_{INT-8} = Z_{66}$

The assignments regarding each Z_{INT-m} solution are illustrated below:

Solution	Workstation 1	Workstation 2
$Z_{INT-6} = 257.5$	1,3,4,7,8,9,10,14,15,16,18	2,5,6,11,12,13,17,19
$Z_{INT-7} = 275.1$	1,3,4,7,8,9,10,14,15,16,18	2,5,6,11,12,13,17,19,21,22
$Z_{INT-8} = 260.5$	1,3,4,7,8,9,10,14,15,16,18	2,5,6,11,12,13,17,19,21

Hazardous task 19 is assigned to workstation 2. We can calculate the original objective function values (O_{INT-m}) for each Z_{INT-m} obtained via backward pass for second hazardous task as;

$$O_{INT-6} = 0.2*0.2 (279) + 0.8*0.2 (174) + 0.2*0.8 (279) + 0.8*0.8 (174) = 195.0$$

$$O_{INT-7} = 0.2*0.2 (303) + 0.8*0.2 (174) + 0.2*0.8 (303) + 0.8*0.8 (174) = 199.8$$

$$O_{INT-8} = 0.2*0.2 (283) + 0.8*0.2 (174) + 0.2*0.8 (283) + 0.8*0.8 (174) = 195.8$$

$$\text{Hence } Z_{BEST-B2-Original} = \text{Max } \{195.0, 199.8, 195.8\} = 199.8$$

$Z_{BEST-B2}$ is the LPR based objective function value (Z_{INT-m}) that corresponds to the $Z_{BEST-B2_Original}$, such that $Z_{BEST-B2} = 275.1$

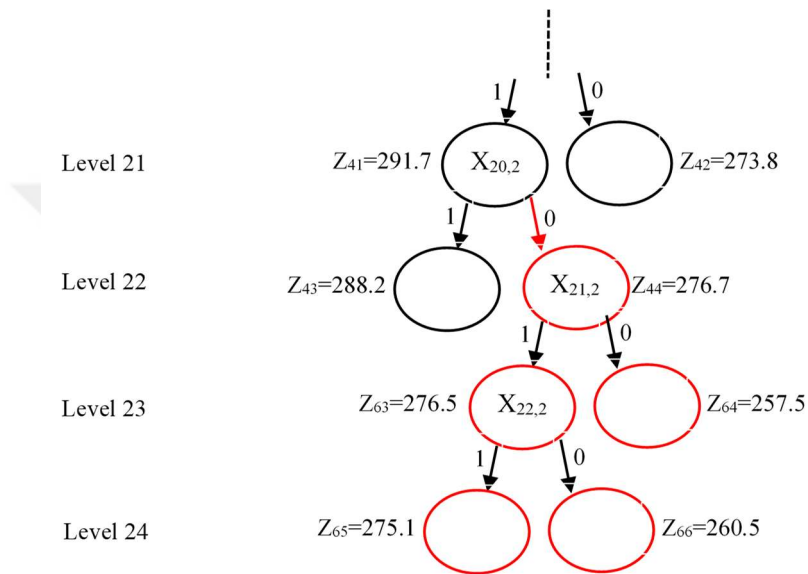


Figure 4.10. Backward Pass Scheme for Second Hazardous Task

Backward Pass Algorithm for Non-Hazardous Tasks

The solution that gives the best objective function value between forward pass and backward pass for hazardous tasks is selected as base solution for non-hazardous backtracking case.

Note that $LB_I = 179.8$, $Z_{BEST-B1-Original} = 280.0$, $Z_{BEST-B2-Original} = 199.8$

Then we select the backward pass for first hazardous task scheme as base solution of backward pass for non-hazardous tasks.

Backward pass algorithm of the example problem for non-hazardous tasks is illustrated at Figure 4.11.

Step 0: The best selected solution obtained via backward pass for first hazardous task $Z_{BEST-B1}$, is compared with the solutions that are fathomed in the previous levels.

Note that best solution obtained at Level 23. We compare the solution that is fathomed (Z_{60}) in this level with $Z_{BEST-B1}$. Since $Z_{60} > Z_{BEST-B1}$ the constraints of of skipped level is deleted.

Step 1: The largest fractional value of Z_{60} is considered and same procedure follows for subproblems. proceeds with the promising subproblem if exists.

Step 2: After solving the problems none of the subproblems remained since all of them result with integer solution.

Step 3: All subproblems are eliminated at last Level 24, since they deliver integer solutions. Hence these last integer solutions are saved as $Z_{INT-9} = Z_{67}$.and $Z_{INT-10} = Z_{68}$

The assignments regarding each Z_{INT-m} solution are illustrated below:

Solution	Workstation 1	Workstation 2
$Z_{INT-9} = 280.0$	1,3,4,7,8,9,10,14,15,17	2,5,6,11,12,13,16,18,21,22
$Z_{INT-10} = 280.0$	1,3,4,7,8,9,10,14,15	2,5,6,11,12,13,16,17,18,21,22

There is not any hazardous task assigned. We can calculate the original objective function values (O_{INT-m}) for each Z_{INT-m} obtained via backward pass for non-hazardous task as;

$$O_{INT-9} = 0.2*0.2 (280) + 0.8*0.2 (280) + 0.2*0.8 (280) + 0.8*0.8 (280) = 280.0$$

$$O_{INT-10} = 0.2*0.2 (303) + 0.8*0.2 (280) + 0.2*0.8 (280) + 0.8*0.8 (280) = 280.0$$

$$\text{Hence } Z_{BEST-NHZ-Original} = \text{Max } \{280.0, 280.0\} = 280.0$$

$Z_{BEST-NHZ}$ is the LPR based objective function value (Z_{INT-m}) that corresponds to the $Z_{BEST-NHZ_Original}$, such that $Z_{BEST-NHZ} = 280.0$ which is also equal to $Z_{BEST-NHZ_Original}$

Determination of LB_2

The original objective function values stated below are compared and best of all is set as LB_2 .

- Forward Pass Solution (LB_1)
- Backward Pass for Hazardous Tasks Solution ($Z_{BEST-B-Original}$)
- Backward Pass for Non-Hazardous Tasks Solution ($Z_{BEST-NHZ-Original}$)

$$\text{Hence, } LB_2 = \text{Max } \{LB_1, Z_{BEST-B-Original}, Z_{BEST-NHZ-Original}\}$$

For the problems having more than one hazardous task:

$$Z_{BEST-B-Original} = \text{Max } \{Z_{BEST-B1-Original}, Z_{BEST-B2-Original}\}$$

We already calculated values of the example problem discussed such that;

$$Z_{BEST-B-Original} = \text{Max } \{280.0, 199.8\} = 280.0$$

$$LB_2 = \text{Max } \{179.8, 280.0, 280.0\} = 280.0$$

Note that, the original optimal *SP Problem* also resulted with an objective function value 280.

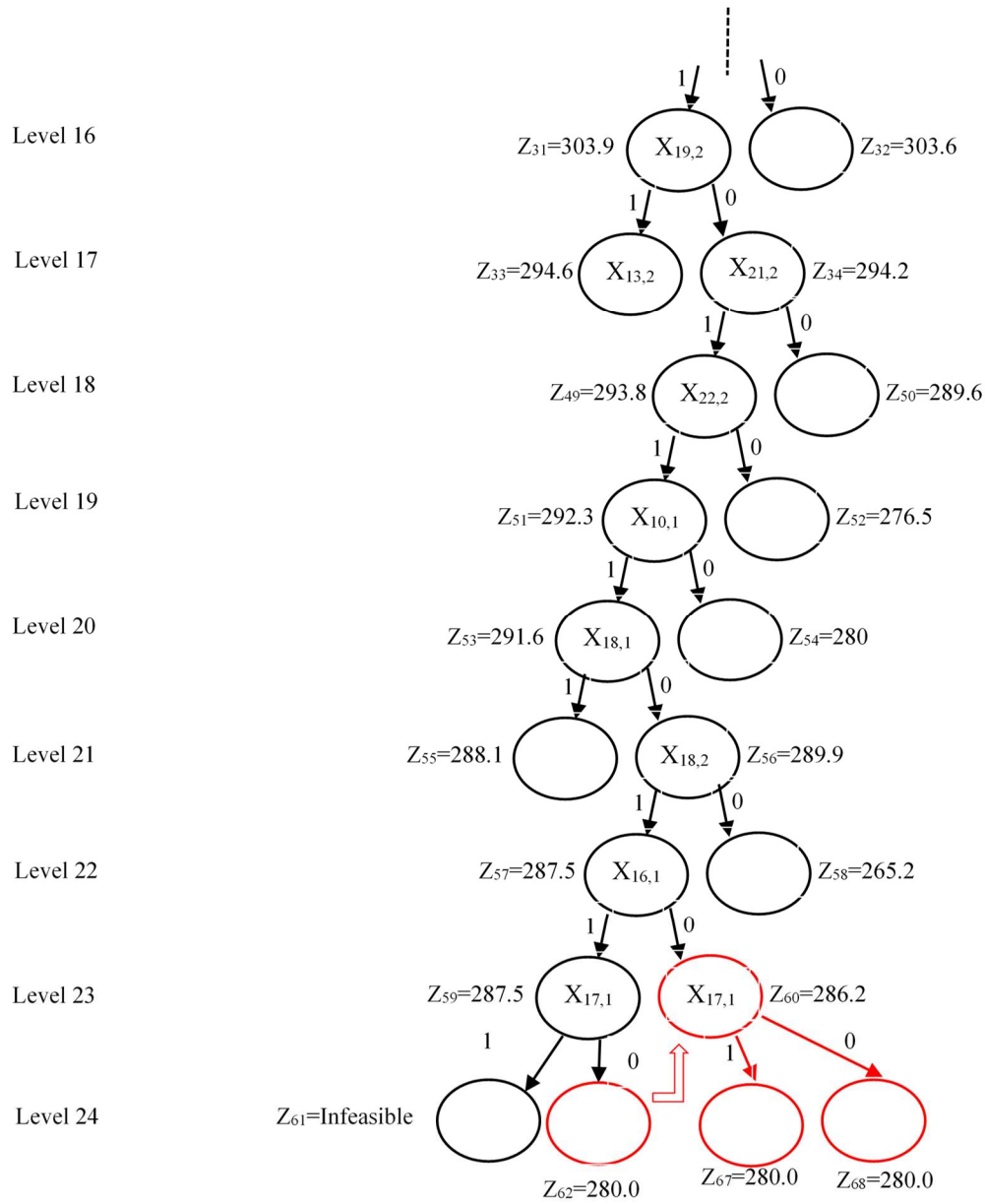


Figure 4.11. Backward Pass Scheme for Non-Hazardous Tasks

CHAPTER 5

COMPUTATIONAL EXPERIMENT

In this chapter, we first present our data generation scheme and illustrate the results of our experiment. In succeeding parts of the study, performance measures and evaluation of computational results are discussed.

5.1 Data Generation

Five networks are taken from Kalaycilar et al. (2016).

1st Network (22 tasks) : The network illustrated in Figure 5.1 is first presented in Bourjault (1984) and modified by Kalaycilar et al. (2016).

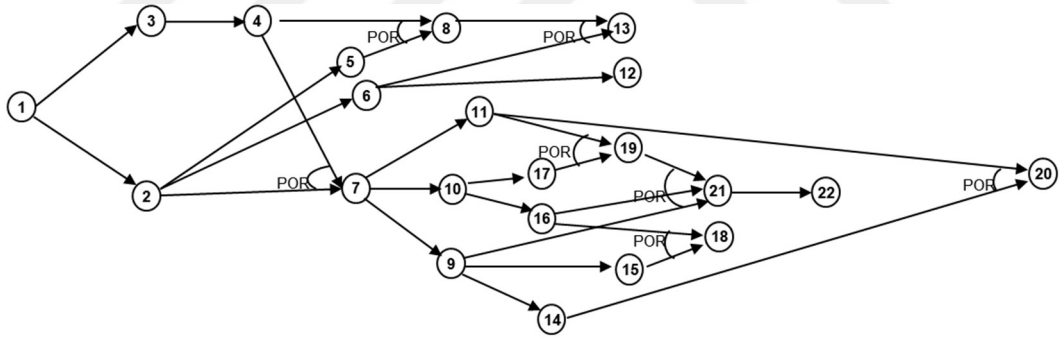


Figure 5.1. Precedence diagram modified by Kalaycilar et al. (22 tasks)

2nd Network (34 tasks) : The network illustrated in Figure 5.2 is first presented in Lambert (1997) and modified by Kalaycilar et al. (2016).

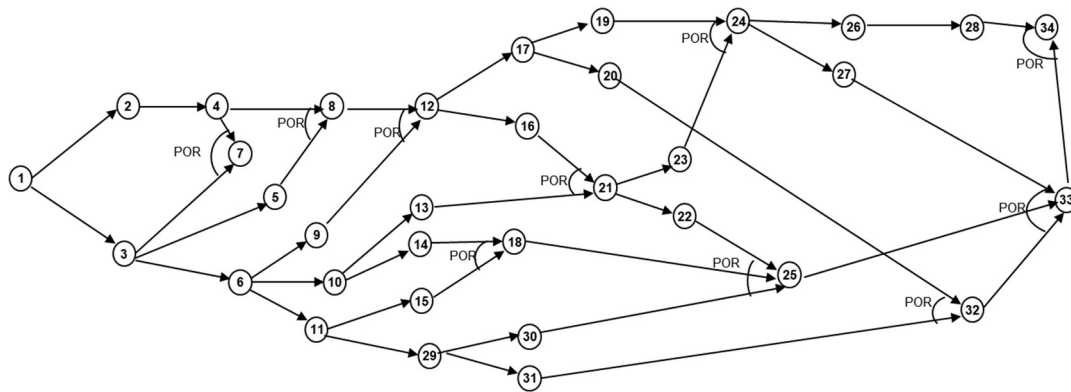


Figure 5.2. Precedence diagram modified by Kalaycilar et al. (34 tasks)

3rd Network (47 tasks) : 22 task and 34 task networks are combined by some arcs.

Precedence diagram of this network is shown at Appendix A

4th Network (60 tasks) : 22 task and 47 task networks are combined by some arcs.

Precedence diagram of this network is shown at Appendix A

5th Network (73 tasks) : 22 task and 60 task networks are combined by some arcs.

Precedence diagram of this network is shown at Appendix A

We set the number of workstations, K, for each network as follows:

Table 5.1 The Number of Workstations (K)

Network Size	K values
20, 34	2, 4
47	2, 4, 6
60, 73	4, 6, 8

Note that we use more workstations when the number of tasks is higher.

The cycle times, processing times, costs and revenues are generated according to the schemes used in Kalaycilar et al. (2016) as follows:

Cycle times are set to $\left\lceil \frac{\sum t_i}{K} \right\rceil$ and $1,5 * \left\lceil \frac{\sum t_i}{K} \right\rceil$; and are referred as *Set CT1* and *Set CT2*, respectively.

Processing times, costs and revenues are generated as follows:

Table 5.2 Distribution of Processing Times, Costs and Revenues

Parameter	Distribution
Processing Time	U(1,20)
Cost	U(5,20)
Revenue	U(10,50)

For revenues, we generate a random number in $(0,1)$; if the generated number is below 0.3, we set the revenue to zero, otherwise the revenue of a part is generated from the distribution illustrated in the above table.

We run our mathematical model for one, two and three hazardous task cases. We first solve the problem with $\pi = 1$ and select the hazardous task from the assigned task set for other values of π . For each stochastic programming case we have cases I-III, and for the purpose of VSS calculation mathematical model is run with the threshold-based success probabilities, then we have case IV.

5.1.1 Case I : One Hazardous Task Case

We set the success probabilities (π) to four different values: 1, 0.8, 0.5, 0.2. Note that $\pi = 1$ refers to the case with no hazardous task.

Three types of positions are considered.

- *Position-1 (POS1)*: The hazardous task is selected randomly from the first half of the line. (Workstation 1 if $K=2$; workstations 1 or 2 if $K=4$)

- *Position-2 (POS2)*: The hazardous task is selected randomly from the second half of the line. (Workstation 2 if K=2; workstations 3 or 4 if K=4)
- *Position-3 (POS2)*: The hazardous task is selected randomly from the line. (Without considering the workstation)

5.1.2 Case II : Two Hazardous Tasks Case

We set the success probabilities (π) of the tasks as follows:

Table 5.3 Success probabilities (π) of the tasks for two hazardous task case

Task 1	Task2
1	1
0.8	0.8
0.8	0.2
0.2	0.2

First success probability pair ($\pi_{z_1} = \pi_{z_2} = 1$) refers to the case with no hazardous task.

Three different positions are considered.

- *Position-1 (POS1)*: Both hazardous tasks are selected randomly from the first half of the line.
- *Position-2 (POS2)*: Both hazardous tasks are selected randomly from the second half of the line.
- *Position-3 (POS3)*: Both hazardous tasks are selected randomly from the line without considering the workstation assignment.

5.1.3 Case III : Three Hazardous Tasks Case

We set the success probabilities (π) of the tasks as follows:

Table 5.4 Success probabilities (π) of the tasks for three hazardous task case

Task 1	Task2	Task3
1	1	1
0.8	0.8	0.8
0.8	0.2	0.8
0.8	0.2	0.2
0.2	0.2	0.2

First success probability pair ($\pi_{z_1} = \pi_{z_2} = \pi_{z_3} = 1$) refers to the case with no hazardous task.

Three different positions are considered.

- *Position-1 (POS1)*: All hazardous tasks are selected randomly from the first half of the line.
- *Position-2 (POS2)*: All hazardous tasks are selected randomly from the second half of the line.
- *Position-3 (POS3)*: All hazardous tasks are selected randomly from the line without considering the workstation assignment.

We have 5 networks, two K values for 2 networks, three K values for 3 networks. For each (N, K) combination there are two cycle times. This makes 26 combinations of N, K and CT values.

For one hazardous task case, there are four π values. For $\pi = 1$, there is no position defined, there are three positions for the other three π values. Hence there are 10 probability, position combinations.

For two hazardous task case, there are again four π value pairs. For $\pi_{z_1} = \pi_{z_2} = 1$, there is no position defined. For the π value pair, three positions are defined. Hence a total of 10 probability, position combinations exist.

For three hazardous task case, there are five π value pairs. For $\pi_{z_1} = \pi_{z_2} = \pi_{z_3} = 1$, there is no position defined. For the π value pair, three positions are defined. Hence a total of 12 probability, position combinations exist.

These altogether follow that our experiment set resides $26 \times 32 = 832$ combinations. 10 problem instances are generated for each combination; hence a total of 8320 problem instances are included in our experiments. But especially for 60 task and 73 task network instances, some of them could not be solved in our pre-set termination limit of 1 hour.

5.1.4 Case IV : VSS Calculation

With the model discussed in section 3.3, the experiments are executed for all combinations of N, K, POS, CT that are same with the combinations utilized for Stochastic Programming. To accurately measure the VSS, same instances are used for SP and EV. Only the difference is related with our stochastic event, i.e. success probability. For EV problem, as mentioned before a threshold value (0.5) is determined to classify the probabilities. If the probability (π) is greater than or equal to the threshold value (≥ 0.5), the corresponding threshold-based probability values used in the EV problem (π') will be equal to 1, otherwise zero. We set the success probabilities (π') of the tasks for single hazardous task model as follows. The corresponding original (π) probability values are also illustrated.

Table 5.5 One Hazardous Task Case_The threshold-based π' vs original π

π	π'
1 (≥ 0.5)	1
0.8 (≥ 0.5)	
0.5	
0.2	1

Note that with success probability $\pi=1$, the success probability $\pi'=1$. There is no uncertainty in the problem and the same objective function value both for SP, EV and EEV will be obtained, hence VSS=0.

The threshold-based success probabilities (π') of the tasks and the corresponding original (π) probabilities for two hazardous task model as follows.

Table 5.6 Two Hazardous Task Case_The threshold-based π' vs original π

π_1	π_2	π'_1	π'_2
1 (≥ 0.5)	1 (≥ 0.5)	1	1
0.8 (≥ 0.5)	0.8 (≥ 0.5)	1	1
0.8 (≥ 0.5)	0.2 (< 0.5)	1	0
0.2 (< 0.5)	0.2 (< 0.5)	0	0

The threshold-based success probabilities (π') of the tasks and the corresponding original (π) probabilities for three hazardous task model as follows.

Table 5.7 Three Hazardous Task Case_The threshold-based π' vs original π

π_1	π_2	π_3	π'_1	π'_2	π'_3
1 (≥ 0.5)	1 (≥ 0.5)	1 (≥ 0.5)	1	1	1
0.8 (≥ 0.5)	0.8 (≥ 0.5)	0.8 (≥ 0.5)	1	1	1
0.8 (≥ 0.5)	0.2 (< 0.5)	0.8 (≥ 0.5)	1	0	1
0.8 (≥ 0.5)	0.2 (< 0.5)	0.2 (< 0.5)	1	0	0
0.2 (< 0.5)	0.2 (< 0.5)	0.2 (< 0.5)	0	0	0

We have 5 networks, $K=4$ and small cycle time (CT1) is considered in VSS related experiments. This makes 5 combinations of N, K and CT values.

For one, two, three hazardous task cases, there are 10, 10, 12 probability-position combinations respectively.

These altogether follow that our experiment set resides $5 \times 32 = 160$ combinations. 10 problem instances are generated for each combination; hence a total of 1600 problem instances are included in this part of our experiments. It should be noted that some of the larger network problem instances could not be solved in our pre-set termination limit of 1 hour.

We solve our model by CPLEX 10.1 and implement on Intel Core i7, 2.40 gigahertz, 8 gigabyte RAM.

5.2 Discussion of Results

In this section, we discuss the results of our computational experiments. In our computational experiments;

- SP models that contain one hazardous task, two hazardous tasks and three hazardous tasks are executed separately for all combinations of N , K , CT , π and POS .
- In one hazardous task, two hazardous task and three hazardous task cases, VSS calculation is performed for $K=4$ and $CT1$ combinations of all networks, POS and π values.
- Note that we set a threshold limit $3/10$ for number of unsolved instances such that if the threshold limit is reached, then the related combination will not be solved for the succeeding harder problem. A harder problem could be a bigger network or any parameter that makes the problem hard to be solved regarding the CPU time.
- We first solve SP model for each instance. If the instance could not be solved within our pre-set termination limit of 1 hour, the EV model is not executed since it is not possible to calculate the related VSS value.
- We discuss the bounding procedures for our original SP Model and execute experiments both for upper and lower bounds.

5.2.1 Discussion of SP Results with One Hazardous Task

As mentioned before for our computational analysis we utilized two types of networks (small, large according to N), two types of cycle times ($CT1$ & $CT2$), different types of number of workstations (K) for each cycle time and network combination, four types of success probabilities ($\pi = 1, 0.8, 0.5$ and 0.2) and three types of hazardous task position (POS1, POS2, POS3) for each instance. We solve 10 problem instances for each combination, thus a total of 2600 instances utilized.

The average objective function values across the instances in each problem setting are reported in Tables 5.8–5.11. The computational performance of the proposed model is summarized in Tables 5.12-5.15.

Table 5.8 Objective Function Values for $K=2$

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	378.3	354.9	361.7	359.0	334.2	351.7	350.8	324.1	348.4	350.8
	MAX	482.0	456.0	446.8	456.0	443.0	406.0	456.0	443.0	406.0	456.0
34	AVG	600.3	564.1	574.3	565.5	527.3	568.2	542.7	492.7	568.2	525.0
	MAX	746.0	641.0	716.0	691.6	641.0	716.0	680.0	641.0	716.0	680.0
47	AVG	831.2	792.2	796.3	779.4	755.7	790.4	737.7	714.6	786.1	697.6
	MAX	947.6	815.8	909.9	867.9	842.2	926.7	851.3	844.0	926.7	851.3

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	401.6	375.9	393.2	386.1	338.0	382.4	364.0	304.7	378.2	344.9
	MAX	497.0	489.8	489.4	490.0	479.0	478.0	479.5	468.2	466.6	472.0
34	AVG	586.5	570.5	576.6	566.1	546.7	567.4	539.0	531.1	562.2	518.2
	MAX	716.0	696.0	699.8	696.4	666.0	698.0	689.0	665.0	698.0	689.0
47	AVG	829.6	780.3	816.6	816.1	706.4	805.0	800.8	643.9	799.2	795.1
	MAX	1048.8	1030.7	1023.6	1027.7	989.2	1018.6	1031.6	990.5	1014.3	1047.3

Table 5.9 Objective Function Values for K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	375.9	354.6	365.4	364.6	326.0	352.3	350.3	300.6	343.9	342.6
	MAX	459.0	434.0	443.8	444.8	425.0	433.0	443.0	425.0	433.0	443.0
34	AVG	618.5	582.6	607.4	601.7	534.6	597.7	584.5	497.4	597.6	576.6
	MAX	766.0	748.2	750.2	758.0	721.5	734.0	758.0	720.0	734.0	758.0
47	AVG	856.4	818.2	842.2	829.4	766.2	831.4	794.6	721.3	826.8	766.2
	MAX	973.0	952.2	953.4	951.2	948.0	950.0	949.0	948.0	950.0	949.0
60	AVG	1045.7	1005.1	1032.5	1019.1	949.1	1024.1	991.9	900.5	1024.1	974.1
	MAX	1135.0	1114.4	1133.0	1112.6	1107.0	1133.0	1088.0	1107.0	1133.0	1088.0
73	AVG	1274.5	1241.9	1253.0	1255.2	1198.7	1233.5	1244.6	1168.9	1230.0	1244.4
	MAX	1433.0	1372.0	1405.0	1409.4	1307.0	1365.0	1405.0	1307.0	1365.0	1405.0

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	404.3	379.0	399.9	392.4	341.4	393.3	374.7	304.2	386.9	356.9
	MAX	470.0	463.0	464.8	456.8	452.5	457.0	437.0	442.0	449.2	417.2
34	AVG	601.7	591.2	597.7	582.4	576.4	591.7	554.0	561.8	585.7	525.9
	MAX	669.0	661.6	664.4	658.0	659.5	657.5	641.5	657.4	651.8	625.0
47	AVG	851.1	808.6	846.5	839.7	744.8	839.6	823.1	681.0	832.6	807.0
	MAX	980.0	979.8	971.8	971.0	979.5	959.5	960.5	979.2	947.2	950.0
60	AVG	1070.2	1038.5	1065.0	1053.5	991.6	1057.3	1030.3	944.6	1049.5	1007.0
	MAX	1287.0	1287.0	1282.6	1285.2	1287.0	1276.0	1282.5	1287.0	1269.4	1279.8
73	AVG	1281.4	1242.0	1277.2	1237.3	1183.1	1271.0	1171.4	1124.2	1264.7	1105.5
	MAX	1436.0	1393.2	1434.0	1418.8	1347.0	1431.0	1411.0	1300.8	1428.0	1403.2

Table 5.10 Objective Function Values for K=6

		CT1									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	801.9	783.4	794.0	781.5	756.4	783.0	751.8	734.0	774.2	724.6
	MAX	986.0	974.6	983.0	973.4	960.0	983.0	957.0	960.0	983.0	957.0
60	AVG	1078.1	1050.1	1065.6	1054.2	1011.8	1050.3	1022.1	975.9	1040.7	996.3
	MAX	1185.0	1183.0	1168.2	1176.8	1183.0	1143.0	1166.0	1183.0	1123.0	1166.0
73	AVG	1343.7	1286.0	1333.1	1329.6	1201.7	1321.0	1313.0	1120.6	1318.5	1301.6
	MAX	1503.0	1429.6	1495.0	1495.0	1421.0	1486.0	1494.0	1421.0	1486.0	1494.0

		CT2									
π / POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	788.1	746.4	783.7	781.3	683.8	777.1	771.1	621.2	770.5	760.9
	MAX	911.0	906.0	909.6	903.8	898.5	907.5	893.0	891.0	905.4	882.2
60	AVG	1120.6	1077.7	1110.8	1108.5	1013.3	1097.2	1091.7	948.9	1083.5	1074.9
	MAX	1217.0	1196.6	1213.0	1210.6	1166.0	1207.0	1201.0	1135.4	1201.0	1191.4
73	AVG	1307.7	1276.7	1299.9	1299.2	1231.1	1288.3	1286.7	1185.4	1276.6	1274.1
	MAX	1569.0	1562.0	1550.0	1553.8	1551.5	1521.5	1531.0	1541.0	1493.0	1508.2

Table 5.11 Objective Function Values for K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	1103.9	1075.8	1097.3	1096.2	1035.7	1088.9	1087.4	997.3	1082.3	1081.0
	MAX	1253.0	1200.8	1242.4	1241.2	1181.0	1238.0	1232.0	1179.0	1238.0	1232.0
73	AVG	1254.4	1212.3	1240.6	1231.4	1149.4	1221.0	1198.5	1089.8	1206.5	1171.6
	MAX	1410.0	1401.8	1403.8	1393.4	1389.5	1394.5	1382.0	1377.2	1391.0	1379.0

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	1041.5	1018.9	1034.5	1008.5	985.1	1024.0	959.1	951.3	1013.4	909.6
	MAX	1136.0	1123.6	1131.4	1103.4	1105.0	1124.5	1101.0	1086.4	1117.6	1098.6
73	AVG	1327.9	1279.5	1316.7	1309.6	1207.3	1299.9	1282.2	1135.0	1283.0	1254.7
	MAX	1438.0	1430.4	1433.0	1435.0	1419.0	1425.5	1433.5	1414.8	1418.0	1432.0

Table 5.12 CPU Times (sec.) for K=2

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.03	0.06	0.05	0.05	0.05	0.03	0.04	0.04	0.04	0.03
	MAX	0.05	0.08	0.06	0.08	0.06	0.06	0.06	0.06	0.06	0.04
34	AVG	0.03	0.04	0.06	0.06	0.03	0.04	0.04	0.03	0.03	0.03
	MAX	0.03	0.06	0.09	0.11	0.09	0.08	0.05	0.08	0.06	0.06
47	AVG	0.03	0.24	0.17	1.59	0.00	0.12	0.29	0.03	0.12	61.19
	MAX	0.04	1.75	0.30	10.59	0.00	0.47	0.67	0.08	0.89	604.81

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.03	0.05	0.05	0.03	0.04	0.05	0.04	0.04	0.04	0.04
	MAX	0.03	0.11	0.06	0.05	0.06	0.11	0.06	0.06	0.06	0.05
34	AVG	0.02	0.04	0.05	0.06	0.05	0.04	0.04	0.04	0.03	0.03
	MAX	0.03	0.06	0.08	0.11	0.08	0.06	0.06	0.08	0.08	0.06
47	AVG	0.02	188.53	0.05	0.01	73.89	0.10	0.01	1.25	0.06	0.01
	MAX	0.03	1187.33	0.07	0.01	325.78	0.18	0.01	6.90	0.20	0.01

Table 5.13 CPU Times (sec.) for K=4

		CT1									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.06	0.29	0.24	0.18	0.40	0.15	0.14	1.31	0.15	0.10
	MAX	0.13	0.70	0.66	0.42	2.70	0.33	0.28	11.47	0.39	0.21
34	AVG	0.03	0.40	0.21	0.49	7.86	0.21	0.34	360.26	0.15	0.18
	MAX	0.05	0.66	0.49	1.19	75.49	0.69	1.34	3600.0	0.24	0.38
47	AVG	0.04	2.42	0.62	11.99	1.02	0.62	2.62	360.67	0.61	360.30
	MAX	0.06	18.20	1.56	114.30	3.83	4.18	19.24	3601.1	3.31	3600.0
60	AVG	0.05	365.18	3.87	1.45	362.44	1.27	361.77	361.01	0.62	0.85
	MAX	0.09	3622.78	29.50	2.59	3613.4	4.31	3600.0	3600.0	0.97	1.67
73	AVG	0.08	5.15	12.28	7.27	2.07	2.09	3.47	3.57	1.13	0.90
	MAX	0.13	21.35	50.58	30.32	6.30	5.94	17.30	18.18	1.95	1.17

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.05	0.40	0.06	0.10	0.18	0.07	0.10	0.32	0.07	0.11
	MAX	0.06	2.16	0.11	0.17	0.50	0.16	0.19	1.50	0.11	0.25
34	AVG	0.02	0.09	0.06	21.29	0.07	0.04	57.47	0.08	0.04	4.32
	MAX	0.03	0.19	0.19	212.35	0.14	0.13	574.01	0.14	0.09	42.57
47	AVG	0.02	455.77	0.06	0.11	103.14	0.10	0.14	2.55	0.09	0.10
	MAX	0.03	3600.29	0.17	0.34	588.90	0.36	0.39	12.48	0.23	0.41
60	AVG	0.02	362.29	0.20	0.28	332.08	0.13	0.28	361.97	0.18	0.28
	MAX	0.05	3620.93	0.49	0.81	3318.9	0.28	0.48	3617.6	0.42	0.69
73	AVG	0.06	0.57	0.32	354.01	0.50	0.28	360.41	0.60	0.30	309.09
	MAX	0.14	1.06	1.09	3535.8	0.88	0.81	3600.0	1.27	0.83	3087.1

Table 5.14 CPU Times (sec.) for K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.06	7.51	449.26	23.95	3.74	722.60	6.58	7.53	6.44	8.49
	MAX	0.16	37.46	3600.0	195.26	16.28	3600.0	33.42	51.39	41.49	63.93
60	AVG	0.12	15.34	21.25	57.78	21.68	363.59	364.48	53.92	3.73	21.01
	MAX	0.27	100.55	163.07	315.86	110.52	3600.0	3600.1	427.45	8.86	122.49
73	AVG	0.33	834.99	10.12	9.69	445.23	11.47	8.99	830.86	368.03	74.12
	MAX	0.70	3600.02	35.41	51.94	3600.0	52.27	41.98	3767.7	3613.9	604.30

		CT2									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.03	15.81	0.52	0.52	53.79	0.55	0.62	16.19	0.75	0.62
	MAX	0.05	137.60	1.06	0.95	395.80	1.34	1.45	141.24	2.30	1.56
60	AVG	0.07	1.72	1.21	1.14	3.02	2.39	1.40	2.63	208.59	361.45
	MAX	0.20	3.06	2.14	2.11	8.97	15.41	3.73	9.81	2073.5	3600.1
73	AVG	0.07	2.86	1.30	1.53	2.74	1.70	1.05	5.96	2.39	2.66
	MAX	0.11	13.77	2.50	3.44	6.41	4.61	2.20	43.67	6.86	5.67

Table 5.15 CPU Times (sec.) for K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.30	1162.99	395.73	984.93	1110.5	1104.6	374.21	90.10	33.08	760.33
	MAX	0.56	3600.02	3600.0	3600.0	3600.3	3600.0	3600.0	651.54	178.70	3604.8
73	AVG	1.06	1989.66	1032.9	1374.8	2033.3	1288.2	815.26	1616.4	1195.9	1113.0
	MAX	1.80	3600.03	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0

		CT2									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.08	14.67	5.48	18.30	16.64	186.20	12.55	389.98	12.35	76.83
	MAX	0.17	78.55	20.49	142.18	54.88	1804.3	60.82	3600.0	77.71	705.63
73	AVG	0.13	378.17	369.50	10.78	39.59	6.58	10.42	58.01	8.24	14.16
	MAX	0.22	3668.26	3600.0	49.92	310.30	21.58	32.82	424.06	38.93	88.62

The impact of problem parameters (K , N , CT , POS and π) on the optimal objective function value and the solution time is analyzed by considering one parameter (and keeping the others constant) at a time. Our findings are discussed below.

Effect of the Number of Tasks in the Network (N)

The effect of N on average objective function value will not provide any meaningful insight since different N values correspond to different network structures and hence a different set of data. Assignment of more tasks implies more chances to increase the profit due to selection of profitable tasks in a wider set. More rooms for assignments are available when there are more tasks (higher N),

As mentioned before, we utilized different number of workstations for different networks in the computational experiments. Solutions for all networks are obtained only when $K=4$. We analyzed the solutions of problem instances via $K=4$ for both small ($N=22, 34, 47$) and large ($N=60, 73$) networks, when all other parameters constant. The CPU time required to solve the problem instances for small networks is much smaller than large networks. For large networks, in some combinations average computational time reaches more than 300 seconds. Moreover, we analyze the solutions with $K=6$ and $N=47, N=60, N=73$. For instance, with combination $CT1$, $\pi=0.2$ and $POS1$, the average CPU time are “7,53”, “53,92” and “830,86” for $N=47$, $N=60$ and $N=73$ respectively. In general, the solutions imply that the computational time required to solve the instances with bigger networks is more than the time required for small networks. It is not directly proportional to the task numbers but there is an obvious increment especially for networks having task numbers 60 and 73 with some combinations. The increase in number of variables, such that task numbers of networks, increases the problem size while increasing the average CPU time accordingly.

As networks get larger, the number of unsolved instances increases. The solutions reveal that there are almost no unsolved instances for small networks $N=22, 34, 47$. However, unsolved instances can be seen for large ones. ($N=60, 73$).

Note that, the impact of problem parameters on solution time and optimal objective function value is more apparent in large networks, especially for $N=60$ and $N=73$. Thus, large networks are considered for the remaining experiments and analysis related with parameters.

Effect of Number of Workstations (K)

The effect of K on average objective function value does not give a consistent pattern. Generally, the values are close to each other, there is not much variation between the solutions.

For the analysis of the effect of K on average CPU, we cannot see a consistent effect related with the workstation number. Although in most of the cases CPU time increases with the increase in K , especially for CT1, we cannot generalize this behavior. It increases when $K=6$ and $K=8$ is compared, however there exist a decrease when $K=4$ and $K=6$ is compared. Since there are many factors effecting the CPU time, such as random selection of hazardous tasks, the solutions do not provide any inferences.

When we checked the number of unsolved instances, a similar behavior as in the case of average CPU time is observed. In most of the cases, there exist an increase in number of unsolved instances especially when CT1 and $K=8$ combination is considered together. For example, when $N=60$, the number of unsolved instances for combination $K=6$, CT2, $\pi=0.2$, POS1 is “0”, where the number of unsolved instances for combination $K=8$, CT1, $\pi=0.2$, POS1 is “4”. Probably, total effect of decrease in cycle time and increase in variables of K affect the solutions and due to this effect, number of unsolved instances increase drastically both for $N=60$ and $N=73$. However, this pattern cannot be generalized when only cycle time is decreased or K is increased.

Effect of Cycle Time (CT)

In most of the combinations, average objective function values are slightly larger in the results when CT2 is utilized. However, this inference cannot be generalized. CT2

does not always give a better objective function value with respect to CT1. Probably randomly selected hazardous tasks, different hazardous task selection/position and their assignment choices dominates the cycle time effect.

Especially for higher probabilities (higher chance of success), the average CPU time increases for small cycle time values (CT1). However, this cannot be generalized since for small success probabilities the observation fails at some combinations. In other words, the effect of cycle time to CPU time and number of unsolved instances is not monotonic.

Effect of Success Probabilities (π)

The results show that when success probabilities (π) are decreasing from 1 to 0.2, the objective function value does not improve. As success probability increases, the chance that the hazardous task affects its succeeding workstations decreases; hence many more tasks can be completed. Recall that more task completions lead to higher profits. This result is true for all number of hazardous tasks and all problem combinations. For example, for $K=2$ & $N=22$ with combination CT1 and POS1, the average objective function value decreases from “378.3” to “354.9”, then to “334.2” and finally decreases to “324.1” as success probability decreases “ $\pi=1$ ” through “ $\pi=0.2$ ”. We see the same situation for all instances within the same combinations of network, cycle time and hazardous task position. It is reasonable that due to the decrease of success probability, the expected net revenue decreases since we lost the revenues that are assigned after the hazardous task with a higher probability.

The CPU time and number of unsolved instances do not follow a consistent pattern with the change in success probability when all other parameters are constant. Lack of monotonicity in the pattern for the success probability changes can be observed clearly with the combination $N=60$, $K=8$, POS2, CT1. The CPU time when “ $\pi=0.8$ ” is “395.73” and increases to “1104.6” for “ $\pi=0.5$ ”, but then decreases to “33.08” for “ $\pi=0.2$ ”

Effect of Hazardous Task Position (POS)

We also investigate the change in average objective function value related with hazardous task position. POS1 and POS2 refer to the instances where the hazardous tasks are selected from the first half and second half of the disassembly line when $\pi=1$, respectively. The better objective function is obtained when the hazardous task is selected further than the start points. The regression in objective function value with respect to decrease in success probability from $\pi=1$ is higher when the hazardous task is selected randomly from the tasks that are assigned to the first half of workstations (POS1). In other words when all other factors are same, the average objective function value with POS2 is greater than the average objective function value with POS1. This is expected as the adverse effects of the hazardous tasks are less when they are located at later workstations. In case of a failure less tasks are affected. Thus, objective function value related with expected net revenue improves as hazardous task position gets closer to the end. For example, with $K=8$, $N=60$, CT1 and $\pi=0.2$, the average objective function value increases (improves) from “997.3” to “1082.3” as we move from “POS1” to “POS2” and again increases (improves) from “706.4” to “805” as we move from “POS1” to “POS2” when $K=2$, $N=47$, CT2 and $\pi=0.5$.

As mentioned before, POS3 is obtained via selecting the hazardous task randomly from the assigned task set with $\pi = 1$, without considering workstation assignment. The average objective function value of POS3 is also compared with the average objective function value of POS1 and POS2. Solutions for different hazardous task positions are illustrated at Figure 5.3 and it is obvious that average objective function values are larger in the results when POS2 is utilized. For instance, if we check the results with $N=73$, $K=8$ and CT1& CT2, at each success probability the average objective function value for POS2 is bigger than average objective function value for POS3 and POS1. Mainly, the average objective function value of POS3 lies between the average objective function value of POS1 and POS2. Similar results can be obtained via $N=60$. The results are reasonable since POS3 is selected without considering workstation position of the line, whereas POS1 is related with first half

of the line and POS2 is related with the second half of the workstation. One may expect that for some instances POS3 act as good as POS2 and for some instances act as poor as POS1.

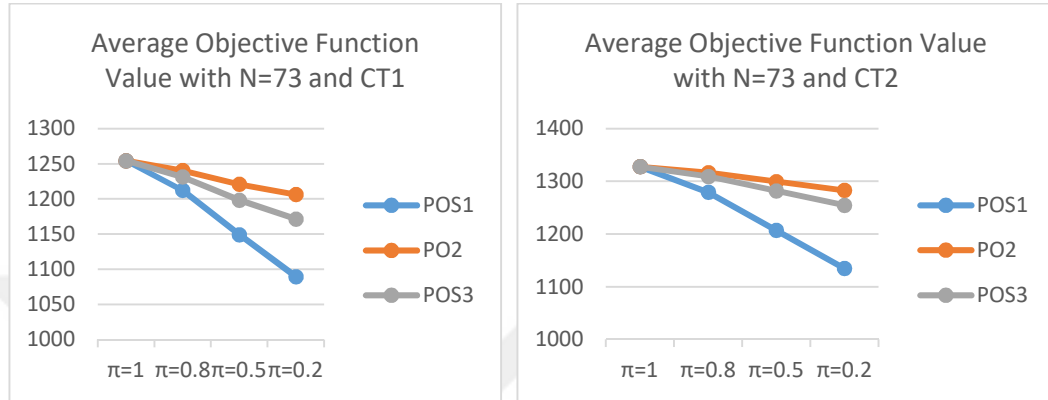


Figure 5.3. Average Objective Function Values, POS1& POS2 &POS3

Unlike the average objective function value, the average CPU time does not follow a consistent pattern with the change in the position of hazardous tasks when all other parameters are constant.

5.2.2 Discussion of SP Results with Two Hazardous Task

For two hazardous task computational analysis we utilized two types of networks, two types of cycle times, different types of number of workstations (K) for each cycle time and network combination, four types of success probabilities ($\pi_{z_1}, \pi_{z_2}=1, \pi_{z_1}=0.8 \pi_{z_2}=0.8, \pi_{z_1}=0.8 \pi_{z_2}=0.2, \pi_{z_1}=0.2 \pi_{z_2}=0.2$) and three types of hazardous task position (POS1, POS2, POS3) for each instance. We solve 10 problem instances for each combination, thus a total of 2600 instances utilized.

The average objective function values across the instances in each problem setting are reported in Tables 5.16-5.19. The computational performance of the proposed model is summarized in Tables 5.20-5.23.

Table 5.16 Objective Function Values for K=2

		CT1									
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8$			$\pi z1= 0.8, \pi z2= 0.2$			$\pi z1= 0.2, \pi z2= 0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	395.6	340.3	362.2	357.3	294.5	356.8	342.3	258.7	350.0	321.4
	MAX	503.0	452.2	436.8	455.5	388.6	429.4	447.8	382.0	387.0	437.0
34	AVG	635.0	566.9	588.9	566.0	546.3	585.7	551.9	494.4	580.3	499.0
	MAX	703.0	639.8	649.8	647.2	621.0	649.8	647.2	621.0	627.0	636.0
47	AVG	821.0	745.4	771.9	752.1	650.1	768.1	744.8	639.4	758.6	699.5
	MAX	863.0	819.0	843.0	817.0	819.0	843.0	817.0	819.0	843.0	817.0

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	349.5	296.5	328.6	323.9	231.9	314.1	309.4	177.2	298.4	289.3
	MAX	433.0	338.0	397.0	408.9	336.6	353.8	398.8	308.4	347.8	381.0
34	AVG	606.6	541.1	584.5	576.2	478.2	570.9	549.2	403.9	556.2	535.0
	MAX	681.0	645.4	667.6	646.1	638.8	667.6	637.0	603.0	657.0	622.0
47	AVG	865.3	809.2	834.9	824.9	771.9	829.3	809.0	732.7	816.8	787.0
	MAX	1037.0	960.4	1013.0	989.8	921.4	1013.0	988.2	882.0	1013.0	987.0

Table 5.17 Objective Function Values for K=4

		CT1									
π & POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	408.6	367.7	390.0	384.8	346.6	377.8	373.2	290.6	368.3	353.2
	MAX	512.0	465.8	489.4	501.0	455.0	489.4	501.0	422.0	457.6	471.0
34	AVG	611.7	553.4	582.9	577.4	459.2	572.7	570.8	435.8	559.5	518.3
	MAX	686.0	663.4	651.0	655.8	645.0	637.4	645.2	639.0	637.0	629.0
47	AVG	827.1	749.4	798.8	761.7	604.6	784.0	741.6	576.4	777.0	630.0
	MAX	1003.0	810.2	960.2	951.2	808.6	956.0	941.4	808.0	956.0	928.0
60	AVG	1057.6	956.7	1017.9	1018.9	834.3	1006.9	1001.1	-	991.6	986.3
	MAX	1144.0	1106.4	1117.4	1098.4	1097.0	1105.0	1098.4	-	1105.0	1077.0
73	AVG	1287.9	1214.0	1245.7	1233.5	1117.8	1224.9	1224.9	-	1203.2	1151.5
	MAX	1476.0	1437.2	1412.3	1423.0	1357.0	1405.4	1423.0	-	1396.0	1423.0

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	380.9	342.8	366.4	349.5	300.2	353.0	309.6	244.0	340.3	274.0
	MAX	445.0	431.3	425.6	421.6	423.8	406.0	405.3	411.2	406.0	398.4
34	AVG	624.8	542.4	607.8	570.3	432.1	594.9	537.8	331.1	575.4	441.8
	MAX	721.0	653.4	700.5	680.8	651.4	688.8	667.4	631.2	670.2	650.6
47	AVG	846.0	794.2	832.1	824.1	687.1	817.6	814.8	665.6	810.4	781.9
	MAX	927.0	864.2	922.7	911.2	830.6	918.2	911.2	818.0	916.6	872.8
60	AVG	1062.4	987.9	1036.8	1024.4	822.7	1017.0	997.4	785.6	988.8	941.9
	MAX	1185.0	1125.8	1140.5	1145.0	1092.2	1127.1	1132.4	1092.2	1102.0	1084.0
73	AVG	1287.4	1223.1	1260.4	1222.6	1178.3	1241.8	1122.2	1070.4	1220.0	1050.6
	MAX	1434.0	1406.3	1400.9	1389.6	1373.8	1398.8	1389.6	1368.4	1356.2	1379.4

Table 5.18 Objective Function Values for K=6

		CT1									
π & POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	829.2	763.1	809.8	790.7	681.9	799.3	728.6	612.4	777.4	712.7
	MAX	905.0	864.3	875.1	880.6	855.2	867.6	880.6	842.0	840.0	854.0
60	AVG	1045.0	976.9	1018.4	1012.5	907.4	996.8	990.7	832.1	985.5	956.7
	MAX	1124.0	1094.2	1109.8	1089.4	1066.2	1085.0	1089.4	1063.0	1085.0	1069.0
73	AVG	1286.1	1228.8	1253.0	1213.1	1186.0	1233.6	1132.6	1112.4	1217.0	1040.7
	MAX	1434.0	1354.4	1386.5	1357.8	1296.8	1374.8	1355.8	1295.6	1358.0	1335.0

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	865.8	798.4	850.6	812.8	697.9	831.4	691.4	619.4	814.3	665.1
	MAX	1075.0	1051.8	1028.4	1034.6	998.7	954.8	1015.4	990.0	918.0	932.6
60	AVG	1069.9	964.6	1056.2	1028.4	926.2	1042.2	960.9	666.1	1022.3	918.1
	MAX	1208.0	1173.8	1200.9	1185.5	1162.2	1199.9	1161.3	1082.6	1180.5	1150.1
73	AVG	1290.8	-	1273.3	1237.1	-	1254.9	1164.5	-	1233.6	1088.5
	MAX	1451.0	-	1425.5	1378.1	-	1407.3	1371.9	-	1367.3	1339.7

Table 5.19 Objective Function Values for K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	1063.4	1006.8	1039.8	1020.6	962.7	1023.0	992.6	878.8	1005.0	940.9
	MAX	1201.0	1158.0	1177.1	1179.0	1123.2	1175.6	1170.4	1084.8	1144.4	1147.6
73	AVG	1285.4	-	-	1236.1	-	1244.7	1217.7	-	1205.1	1117.5
	MAX	1450.0	-	-	1379.4	-	1408.6	1379.4	-	1401.0	1336.6

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	1050.7	947.0	1033.5	1018.7	833.9	1013.8	974.5	681.2	992.7	939.6
	MAX	1153.0	1057.2	1138.3	1096.8	1038.0	1121.5	1078.8	1012.2	1105.3	1077.0
73	AVG	1290.8	1233.7	1275.9	1237.1	-	1263.7	1164.5	1082.9	1241.4	1088.5
	MAX	1451.0	1390.3	1445.9	1378.1	-	1442.5	1371.9	1271.7	1433.9	1339.7

Table 5.20 CPU Times (sec.) for K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.03	0.09	0.08	0.08	0.08	0.06	0.07	0.08	0.04	0.05
	MAX	0.03	0.11	0.11	0.11	0.13	0.09	0.09	0.23	0.08	0.08
34	AVG	0.04	0.17	0.15	0.17	0.14	0.11	0.14	0.12	0.07	0.11
	MAX	0.05	0.34	0.20	0.27	0.22	0.17	0.23	0.20	0.13	0.31
47	AVG	0.04	0.28	0.23	0.25	0.26	0.30	0.20	0.22	0.11	0.13
	MAX	0.09	1.11	0.45	0.41	0.51	1.36	0.33	0.38	0.28	0.25

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.03	0.05	0.06	0.05	0.06	0.06	0.04	0.06	0.05	0.06
	MAX	0.05	0.06	0.09	0.09	0.09	0.08	0.08	0.09	0.08	0.09
34	AVG	0.03	0.12	0.10	0.11	0.11	0.09	0.10	0.11	0.09	0.08
	MAX	0.05	0.20	0.19	0.24	0.22	0.13	0.19	0.24	0.19	0.13
47	AVG	0.04	0.19	0.22	0.26	0.12	0.18	0.24	0.10	0.11	0.13
	MAX	0.05	0.53	0.38	0.80	0.34	0.31	0.47	0.25	0.28	0.27

Table 5.21 CPU Times (sec.) for K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.06	0.42	0.67	0.29	0.40	0.24	0.28	0.48	0.20	0.26
	MAX	0.20	0.61	3.72	0.42	0.91	0.52	0.52	1.95	0.41	0.52
34	AVG	0.06	7.48	1.03	1.41	360.71	0.90	1.29	7.43	0.45	9.75
	MAX	0.23	61.39	2.19	8.07	3600.00	1.92	6.08	62.13	0.89	92.12
47	AVG	0.08	364.70	2.54	49.59	722.15	2.12	367.55	1080.87	2.68	546.86
	MAX	0.16	3600.07	6.54	402.50	3600.02	8.72	3600.1	3600.02	15.36	3600.0
60	AVG	0.10	723.78	116.81	13.79	363.92	6.70	53.75	-	4.40	7.19
	MAX	0.23	3600.00	1075.2	75.71	3600.01	23.84	282.75	-	12.70	32.66
73	AVG	0.22	655.03	378.98	388.39	440.46	960.20	387.32	-	427.79	392.97
	MAX	0.66	3600.10	3600.0	3600.4	3600.01	3600.2	3600.0	-	3600.0	3524.7

		CT2									
π & POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.03	0.26	0.15	0.21	0.24	0.12	1.35	1.13	0.11	3.06
	MAX	0.05	0.94	0.25	0.52	1.00	0.20	11.63	8.34	0.20	28.81
34	AVG	0.02	7.05	0.29	2.68	96.36	0.19	1.43	59.02	0.18	12.26
	MAX	0.03	48.61	0.58	22.66	482.85	0.53	9.46	447.84	0.28	112.29
47	AVG	0.03	360.66	0.50	0.54	159.46	0.42	0.44	360.53	0.31	0.43
	MAX	0.03	3600.29	0.78	0.86	1587.10	0.80	0.80	3600.02	0.70	0.78
60	AVG	0.06	722.80	2.17	14.88	777.19	2.40	1.37	721.90	0.76	3.16
	MAX	0.09	3600.05	12.94	117.91	3616.05	15.89	4.02	3607.73	1.68	21.04
73	AVG	0.08	701.05	2.23	347.21	789.32	2.73	361.41	362.55	1.44	362.84
	MAX	0.14	3600.03	5.85	3417.8	3600.18	12.70	3600.0	3600.02	2.89	3600.0

Table 5.22 CPU Times (sec.) for K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.13	298.78	304.64	412.62	379.85	11.24	372.94	726.19	20.82	523.44
	MAX	0.33	2235.66	2913.8	3600.0	3600.00	50.16	3600.0	3600.00	114.95	3600.0
60	AVG	0.35	146.73	355.84	253.43	389.27	212.06	83.05	393.52	384.92	787.91
	MAX	0.66	933.83	3351.9	1598.6	3600.00	1879.4	671.88	2729.51	3600.0	3609.9
73	AVG	0.96	436.90	855.57	1605.2	934.92	1396.5	1397.9	987.55	1122.6	877.34
	MAX	1.88	3396.50	3600.0	3600.0	3600.01	3601.3	3600.0	3600.03	3600.1	3600.0

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.04	374.36	0.90	729.45	309.56	0.81	720.79	724.33	0.99	626.83
	MAX	0.06	3609.12	1.80	3688.5	3051.62	1.74	3600.2	3600.01	2.22	3600.0
60	AVG	0.09	1111.84	2.10	9.01	1158.93	3.67	7.55	1120.01	2.93	18.71
	MAX	0.17	3843.47	5.17	53.77	3659.93	18.30	29.38	3813.28	5.80	80.32
73	AVG	0.28	-	6.91	9.41	-	5.18	12.32	-	12.20	19.89
	MAX	1.61	-	19.83	15.50	-	16.11	32.55	-	48.40	52.87

Table 5.23 CPU Times (sec.) for K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.69	1304.23	1390.0	897.94	1410.51	1022.6	602.69	1300.40	1041.6	888.09
	MAX	1.19	3600.02	3600.0	3593.4	3600.02	3600.0	3600.0	3600.01	3600.0	3600.0
73	AVG	1.36	-	-	2124.9	-	1564.1	1770.3	-	1584.4	2015.7
	MAX	2.33	-	-	3600.0	-	3600.0	3600.0	-	3600.0	3600.0

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.13	632.14	16.49	82.24	1245.57	30.86	45.37	1221.22	381.95	158.55
	MAX	0.25	3600.01	59.57	277.53	3600.05	136.12	228.39	3600.02	3600.0	1295.4
73	AVG	0.29	422.22	75.58	437.68	-	394.74	705.14	1263.93	455.31	499.41
	MAX	0.83	3134.33	363.75	3600.1	-	3600.0	3600.2	3600.02	1729.2	3600.0

The impact of problem parameters (K, N, CT, POS and π) on the optimal objective function value and the solution time is analyzed by considering one parameter (and keeping the others constant) at a time.

As mentioned before, the problem parameter set for two hazardous task case differs in success probabilities. Experiments are executed considering all problem parameters, however in order to avoid repetition only the results that are different from one hazardous task case are interpreted. For instance, effect of task number in a network is not discussed since the change of this problem parameter on average objective function value and average computational time results is in similar behavior with one hazardous task case. Note that, as discussed before, we set a threshold limit 3/10 for number of unsolved instances such that if the threshold limit is reached, then the related combination will not be solved for the succeeding harder problem. For instance, for N=60, with K=8, CT1, P2, POS1&POS2 the number of unsolved instances is “3”. Thus, the limit 3/10 is achieved and the combination is not solved for a larger network N=73 since we know that our model is hard to be solved when the network gets larger.

As observed in one hazardous task case, the impact of problem parameters on solution time and optimal objective function value is more apparent in large networks. Thus, large networks are considered for the remaining experiments and for the analysis related with parameters. Our findings are discussed below.

Effect of Workstation Number (K)

There is not a direct relationship with K, but there exists an obvious increase in number of unsolved instances when K increases. When we analyze the large networks N=60 and N=73, there is a remarkable difference in number of unsolved instances between K=6 and K=8, in general. For instance, for N=60, with K=6, CT1, P4 the number of unsolved instances is “0”, “1”, “2” for POS1, POS2 and POS3. For N=60, the number of unsolved instances with K=8, CT1, P4 is “3”, “2”, “2” for POS1, POS2 and POS3 respectively. There is an increase in general, except some instances.

The average CPU time behavior is rational with respect to the number of unsolved instances. Except some instances, there exist an increase in average CPU time when K increases. For instance, for N=60, the average CPU time with K=4, CT1, P3, POS1 is “363,92”. With every other factor are fixed only K increased to K=6 and K=8, the average CPU time becomes “389,27” and “1410,5” respectively.

Effect of Success Probabilities (π)

We analyze the effect of success probabilities to average objective function values. The results show that within the same cycle time, same network, with same hazardous task position and same workstation number when success probabilities are decreasing the objective function value does not improve. Let the pairs of success probabilities for two hazardous tasks,

$$P1: \pi_{z_1} = 1 \quad \pi_{z_2} = 1$$

$$P2: \pi_{z_1} = 0.8 \quad \pi_{z_2} = 0.8,$$

$$P3: \pi_{z_1} = 0.8 \quad \pi_{z_2} = 0.2,$$

$$P4: \pi_{z_1} = 0.2 \quad \pi_{z_2} = 0.2.$$

Henceforth pairs are referred as to P1, P2, P3 and P4. Note that the success probabilities decrease from P1 to P4 since at each step one of the success probability worsens. The corresponding average objective function values related with success probabilities are lined up as $O_{P1} > O_{P2} > O_{P3} > O_{P4}$. For example, for K=2 & N=34 with combination CT1 and POS1, the average objective function value decreases from “635.0” to “566.9”, then to “546.3” and finally decreases to “494.4” as success probability decreases “P1” through “P4”. The average objective function value decreases from “1290.8” to “1275.9”, then to “1263.7” and finally decreases to “1241.4” as success probability decreases “P1” through “P4”, for K=8 & N=73 with combination CT2 and POS2. It is reasonable that due to the decrease of success probability, the expected net revenue decreases since we lost the revenues that are assigned after the hazardous task with a higher probability. We see the same situation

for all instances within the same combinations of network, cycle time and hazardous task position. It is reasonable that due to the decrease of success probability, the expected net revenue decreases since we lost the revenues that are assigned after the hazardous task with a higher probability.

The CPU time and number of unsolved instances do not follow a consistent pattern with the change in success probability when all other factors are fixed.

Effect of Hazardous Task Position (POS)

We investigate the change in average objective function value related with the hazardous task positions. The average objective function value when both of the hazardous tasks are selected randomly from the tasks that are assigned to the second half of workstations (POS2) is greater than the average objective function value when both of the hazardous tasks are selected randomly from the tasks that are assigned to the first half of workstations (POS1). The inferences resemble the results of one hazardous task case. Objective function value related with expected net revenue improves as all hazardous tasks' positions are located at later workstations. For example, with $K=8$, $N=73$, CT2 and P1, the average objective function value increases (improves) from "1233.7" to "1275.9" as we move from "POS1" to "POS2" and again increases (improves) from "639.4" to "758.6" as we move from "POS1" to "POS2" when $K=2$, $N=47$, CT1 and P4. The similar results are observed when N , K , CT and success probabilities are fixed and the change in objective function value related with hazardous task position analyzed.

We compare the average objective function value of POS3 with the average objective function value of POS1 and POS2. Solutions for different hazardous task positions are illustrated at Figure 5.4-5.5 and it is obvious that average objective function values are larger in the results when POS2 is utilized. When we analyze the results with $N=60$, $K=8$ and CT1 & CT2, at each success probability the average objective function value for POS2 is bigger than average objective function value for POS3 and POS1. Mainly, the average objective function value of POS3 lies between the average objective function value of POS1 and POS2. The results are reasonable since

POS3 is selected without considering workstation position of the line, whereas POS1 is related with first half of the line and POS2 is related with the second half of the workstation.

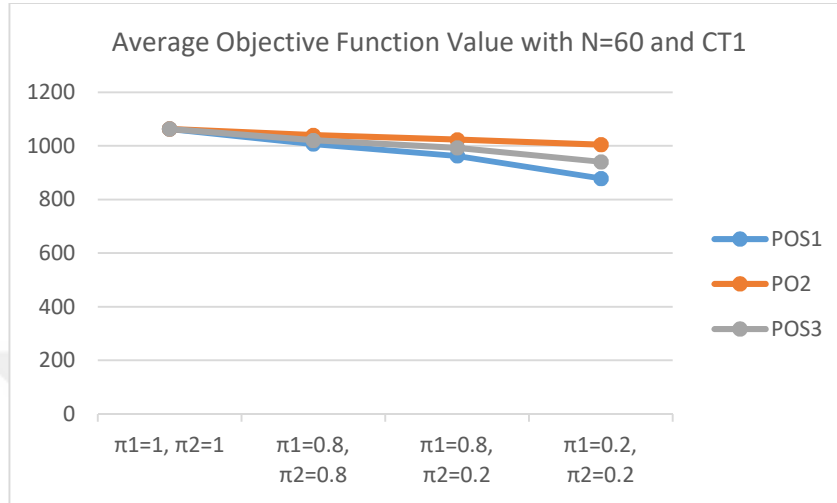


Figure 5.4. Comparison of POS1& POS2 &POS3 for N=60, CT1

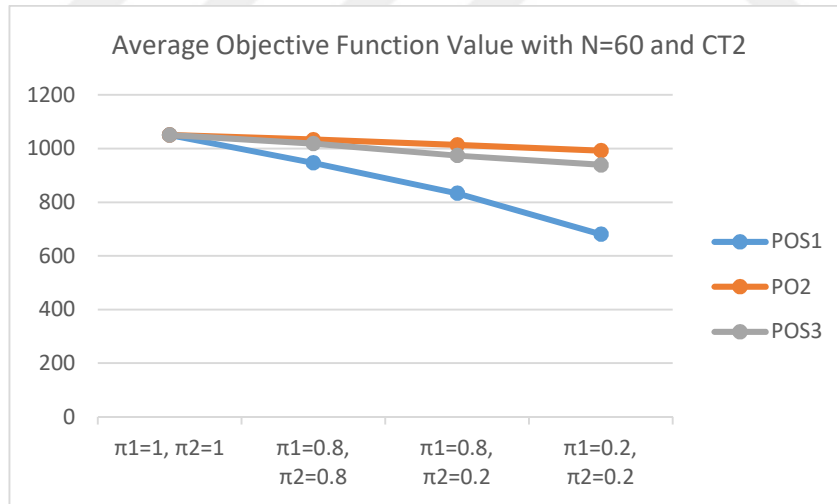


Figure 5.5. Comparison of POS1& POS2 &POS3 for N=60, CT2

Note that, the average CPU time and number of unsolved instances do not follow a consistent pattern with the change in the position of hazardous tasks when all other factors are fixed.

5.2.3 Discussion of SP Results with Three Hazardous Task

For three hazardous task computational analysis we utilized two types of networks, two types of cycle times, different types of number of workstations (K) for each cycle time and network combination, five types of success probabilities ($\pi_{z_1}, \pi_{z_2}, \pi_{z_3}=1$, $\pi_{z_1}=0.8 \ \pi_{z_2}=0.8 \ \pi_{z_3}=0.8$, $\pi_{z_1}=0.8 \ \pi_{z_2}=0.2 \ \pi_{z_3}=0.8$, $\pi_{z_1}=0.8 \ \pi_{z_2}=0.2 \ \pi_{z_3}=0.2$, $\pi_{z_1}=0.2 \ \pi_{z_2}=0.2 \ \pi_{z_3}=0.2$) and three types of hazardous task position (POS1, POS2, POS3) for each instance. We solve 10 problem instances for each combination, thus a total of 3120 instances utilized.

The average objective function values across the instances in each problem setting are reported in Tables 5.24-5.27. The computational performance of the proposed model is summarized in Tables 5.28-5.31.

Table 5.24 Objective Function Values for K=2

		CT1												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	386.0	303.7	338.5	326.1	283.0	331.2	322.1	253.6	328.7	300.4	180.2	316.4	272.0
	MAX	463.0	364.2	431.0	407.4	364.2	416.0	407.4	364.2	416.0	407.4	327.0	416.0	396.0
34	AVG	575.4	476.1	514.7	502.1	453.0	512.4	472.5	382.9	512.4	465.3	345.9	490.4	458.8
	MAX	634.0	560.6	576.0	554.6	560.6	576.0	548.2	560.6	576.0	548.2	536.0	576.0	544.0
47	AVG	839.2	691.1	770.9	725.2	655.7	767.7	692.0	519.9	767.7	662.4	507.2	763.8	635.8
	MAX	887.0	816.0	847.0	790.8	816.0	847.0	790.8	816.0	847.0	790.8	816.0	847.0	769.0

		CT2												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	379.6	300.5	345.6	316.2	269.7	336.4	268.7	189.7	327.7	236.0	183.9	313.8	202.6
	MAX	464.0	358.7	426.6	360.2	358.7	407.6	354.9	349.0	398.4	350.0	319.0	398.4	339.0
34	AVG	612.2	531.3	573.0	558.3	497.8	564.7	545.8	465.1	554.9	530.3	444.1	541.9	505.7
	MAX	720.0	613.4	673.9	658.8	604.8	673.9	654.6	599.0	652.0	605.4	583.2	625.0	605.4
47	AVG	848.0	753.5	800.7	765.1	726.5	793.0	703.9	677.7	780.3	699.3	634.8	770.9	651.9
	MAX	956.0	884.5	894.8	881.8	880.8	862.0	846.0	823.2	850.0	846.0	820.2	850.0	819.0

Table 5.25 Objective Function Values for K=4

		CT1												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	388.0	311.8	361.1	337.4	267.7	356.9	308.9	223.4	347.1	284.8	175.7	335.1	272.5
	MAX	458.0	391.8	421.1	411.8	363.8	418.1	411.8	358.8	406.6	372.4	316.6	401.4	372.2
34	AVG	568.7	500.3	530.1	520.9	458.0	527.2	505.5	437.5	515.1	475.2	387.6	495.1	463.0
	MAX	634.0	604.8	587.3	581.9	604.8	587.3	567.2	604.8	574.0	565.4	585.0	567.0	544.0
47	AVG	862.6	717.1	806.3	784.1	629.4	801.7	780.0	606.6	786.4	699.1	427.1	769.0	664.4
	MAX	903.0	824.1	863.8	851.4	797.4	859.2	851.4	782.2	859.2	833.8	781.0	841.0	818.0
60	AVG	1091.7	921.6	1044.7	961.4	794.7	1036.9	922.5	740.4	1022.4	823.6	611.5	1006.7	754.4
	MAX	1188.0	1049.5	1160.4	1100.6	1040.6	1160.4	1100.6	1040.6	1160.4	1046.8	1010.0	1146.0	1008.0
73	AVG	1303.9	1147.0	1245.7	1209.7	1078.6	1228.7	1148.2	1023.5	1213.0	1112.6	-	1199.4	-
	MAX	1417.0	1312.2	1368.4	1314.7	1312.2	1368.4	1310.9	1258.4	1333.0	1304.0	-	1333.0	-

		CT2												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	414.0	326.4	388.9	370.1	253.2	371.2	325.4	174.4	363.9	298.9	138.9	349.4	282.9
	MAX	498.0	417.9	455.0	435.9	416.3	422.4	428.6	393.7	415.2	385.6	331.7	408.6	379.2
34	AVG	646.9	557.6	617.5	590.1	510.8	607.7	548.2	459.4	595.7	500.2	398.4	582.6	477.0
	MAX	739.0	692.4	710.7	713.6	684.8	706.9	711.0	648.9	696.6	711.0	637.4	674.4	696.0
47	AVG	835.3	761.1	804.5	782.4	728.2	795.1	747.8	639.6	778.7	718.0	605.1	756.9	689.2
	MAX	916.0	870.6	890.5	889.1	866.5	879.5	873.3	844.4	879.5	847.2	824.6	858.6	847.2
60	AVG	1073.2	931.7	1035.2	994.3	827.5	1023.7	950.6	707.0	1004.4	868.5	618.4	990.8	844.1
	MAX	1252.0	1057.2	1203.6	1185.1	1029.8	1197.6	1182.6	1029.8	1173.6	1153.4	1026.0	1164.6	1079.0
73	AVG	1257.2	1179.5	1217.5	1191.8	1108.1	1190.9	1175.8	1059.7	1168.2	1111.4	-	1159.5	1058.8
	MAX	1402.0	1283.0	1387.3	1326.3	1265.7	1384.7	1326.3	1251.3	1374.0	1308.1	-	1373.6	1193.2

Table 5.26 Objective Function Values for K=6

		CT1													
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$			
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
47	AVG	822.7	721.2	792.9	753.1	653.2	784.9	728.7	526.6	776.1	709.3	511.0	761.1	629.8	
	MAX	922.0	864.9	893.4	894.2	850.0	886.6	876.4	850.0	886.6	876.4	805.0	871.0	862.0	
60	AVG	1082.4	956.3	1039.5	998.0	926.9	1024.0	916.8	849.3	1002.8	880.0	716.0	993.5	822.9	
	MAX	1270.0	1062.3	1218.0	1242.0	1023.5	1198.1	1242.0	1017.6	1191.4	1238.6	955.0	1179.0	1235.0	
73	AVG	1248.0	1111.6	1207.2	1171.8	1014.8	1199.4	1154.2	870.4	1184.7	1137.0	-	1166.5	1029.2	
	MAX	1368.0	1265.8	1346.0	1278.4	1246.6	1346.0	1265.0	1214.6	1318.0	1256.8	-	1318.0	1252.0	

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	812.0	725.7	786.8	751.9	657.2	776.1	687.7	605.6	762.4	671.6	537.0	743.5	609.5
	MAX	943.0	869.7	914.8	915.5	852.6	899.8	908.1	804.0	891.2	895.1	753.1	874.2	879.3
60	AVG	1033.6	912.3	1002.3	935.9	860.3	989.1	865.2	788.4	974.4	747.8	642.8	950.8	705.1
	MAX	1147.0	1018.0	1126.6	1052.8	1005.9	1118.9	1025.1	1000.6	1102.2	1012.1	909.4	1094.5	962.6
73	AVG	1267.0	1148.6	1230.9	1225.2	1007.6	1214.9	1212.5	924.4	1199.7	1150.4	-	1166.5	1138.0
	MAX	1376.0	1258.5	1354.3	1350.9	1258.5	1349.9	1350.9	1225.5	1331.7	1322.1	-	1319.3	1299.9

Table 5.27 Objective Function Values for K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	1061.3	888.7	1023.2	987.6	633.8	1012.8	891.4	577.2	1002.8	866.8	527.0	983.6	850.2
	MAX	1265.0	1161.7	1214.8	1230.3	1022.0	1211.1	1222.6	1015.6	1196.4	1222.6	1011.2	1173.4	1203.4
73	AVG	1291.0	-	1249.9	-	-	1236.1	-	-	-	-	-	-	-
	MAX	1514.0	-	1481.0	-	-	1452.2	-	-	-	-	-	-	-

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	1079.9	995.2	1047.5	998.6	955.1	1018.2	974.8	877.2	1001.0	922.7	792.8	984.5	816.8
	MAX	1305.0	1265.3	1235.6	1162.0	1248.6	1182.9	1146.2	1234.1	1130.2	1116.4	1189.8	1122.1	1014.2
73	AVG	1308.7	1204.5	1287.1	1239.2	1156.1	1270.1	1154.1	1024.3	1257.2	1131.2	978.3	-	-
	MAX	1491.0	1372.7	1459.7	1470.8	1342.3	1424.6	1462.2	1323.9	1411.5	1451.9	1305.2	-	-

Table 5.28 CPU Times (sec.) for K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.10	0.22	0.21	0.20	0.18	0.19	0.18	0.18	0.15	0.18	0.21	0.17	0.24
	MAX	0.06	0.33	0.29	0.31	0.33	0.40	0.33	0.63	0.24	0.31	0.56	0.29	0.77
34	AVG	0.08	0.33	0.42	0.44	0.31	0.44	0.43	0.36	0.39	0.41	0.41	0.34	0.37
	MAX	0.10	0.55	0.57	0.61	0.52	0.54	0.55	0.67	0.65	0.54	0.71	0.65	0.64
47	AVG	0.08	0.52	0.84	1.13	0.50	1.03	1.47	0.65	0.81	1.00	0.84	0.49	0.57
	MAX	0.13	0.97	2.49	2.99	1.03	5.01	8.93	1.24	2.38	4.55	3.83	0.63	1.01

		CT2												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.03	0.14	0.10	0.12	0.12	0.08	0.12	0.14	0.09	0.09	0.11	0.09	0.12
	MAX	0.05	0.25	0.20	0.19	0.17	0.19	0.26	0.21	0.18	0.17	0.18	0.15	0.20
34	AVG	0.08	0.27	0.26	0.25	0.26	0.26	0.22	0.26	0.25	0.21	0.40	0.25	0.27
	MAX	0.12	0.59	0.37	0.45	0.52	0.42	0.52	0.63	0.38	0.42	1.68	0.64	0.53
47	AVG	0.08	0.36	0.62	0.36	0.32	0.53	0.39	0.32	0.41	0.25	0.34	0.38	0.33
	MAX	0.12	0.64	0.93	0.66	0.75	0.91	0.55	0.55	0.62	0.45	0.71	0.48	0.58

Table 5.29 CPU Times (sec.) for K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.1	1.3	0.5	0.8	4.0	0.4	6.9	16.9	0.4	1.0	73.8	0.4	13.2
	MAX	0.3	3.1	0.9	2.0	23.0	0.7	64.2	152.2	0.7	5.5	680.1	0.8	127.6
34	AVG	0.1	7.1	6.1	2.1	361.7	1.9	2.1	362.1	48.9	7.6	237.9	6.9	39.8
	MAX	0.3	5.8	33.9	3.3	3600.1	4.0	4.8	3600.1	471.1	58.8	2143.3	20.5	186.7
47	AVG	0.2	203.8	168.5	23.6	399.7	14.6	113.1	777.5	727.4	367.5	1460.0	87.1	397.0
	MAX	0.4	785.8	3600.0	102.2	3600.0	9.9	1050.1	3600.0	3600.0	3600.0	3605.5	577.4	3626.4
60	AVG	0.5	270.9	183.0	46.2	850.1	130.4	22.0	860.4	386.3	395.1	1755.8	869.9	1485.7
	MAX	1.5	290.0	255.8	282.2	3601.3	207.9	147.1	3600.0	3600.0	3600.0	3600.1	3600.2	3600.1
73	AVG	0.5	635.0	552.9	114.5	1191.4	1113.1	149.2	1162.1	432.7	448.5	-	2148.0	-
	MAX	2.2	3600.0	3600.0	436.8	3600.0	3600.0	1359.7	3600.2	3600.1	3600.0	-	3600.0	-

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.0	1.1	0.3	0.4	6.0	0.3	0.5	4.9	0.3	1.0	363.9	0.3	0.6
	MAX	0.0	4.6	0.5	1.0	52.3	0.5	1.6	17.7	0.4	6.9	3600.1	0.5	2.3
34	AVG	0.1	2.0	0.9	0.9	1.9	0.9	1.0	2.6	0.7	23.3	94.0	0.8	77.2
	MAX	0.1	6.6	2.6	2.0	5.5	2.4	2.2	12.2	1.2	225.6	599.5	1.4	762.9
47	AVG	0.1	13.9	8.0	30.0	303.2	25.8	56.8	365.5	1.6	6.5	138.0	1.9	1.8
	MAX	0.2	126.5	67.4	288.3	3013.9	244.2	556.8	3600.0	2.1	51.6	892.1	3.9	3.1
60	AVG	0.2	375.0	113.3	45.1	506.6	142.9	65.3	448.4	103.0	83.6	1107.6	84.5	89.0
	MAX	1.3	3705.0	486.1	126.0	3600.0	512.5	717.8	3600.0	806.4	300.5	3600.0	316.1	410.4
73	AVG	0.3	382.8	199.0	97.3	641.7	328.3	163.8	512.3	326.4	117.0	-	409.3	317.1
	MAX	1.7	3600.0	951.2	934.3	3600.0	943.6	857.6	3600.0	1013.5	517.3	-	920.1	1110.9

Table 5.30 CPU Times (sec.) for K=6

		CT1												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.4	303.0	21.0	464.3	801.7	9.3	412.8	1535.2	52.5	654.1	1100.7	169.7	778.2
	MAX	1.6	1241.9	101.3	3600.0	3605.6	21.0	3600.1	3600.0	338.3	3600.2	3600.0	912.3	3600.0
60	AVG	1.4	932.8	216.6	1119.8	914.2	756.4	476.1	1257.5	747.8	512.7	1824.7	1338.6	1149.8
	MAX	2.6	3600.1	1298.1	3600.2	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.1	3600.0
73	AVG	1.6	1217.2	678.9	1199.5	1835.0	1217.1	515.4	2161.2	1158.8	2090.9	-	2273.2	2501.3
	MAX	3.6	3600.0	1501.2	3600.1	3600.1	2089.5	3600.1	3600.0	3600.2	3600.0	-	3605.0	3600.1

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.1	378.2	3.3	724.2	73.7	4.7	723.3	484.6	8.7	410.5	788.5	5.6	733.1
	MAX	0.3	3600.0	7.9	3603.9	322.1	10.1	3600.0	3600.0	61.4	3600.1	3600.0	12.4	3600.0
60	AVG	0.2	404.1	23.2	436.2	398.0	12.6	580.7	630.5	13.0	492.7	1208.0	45.6	878.4
	MAX	0.7	3600.1	50.5	3602.3	3600.2	33.9	3602.6	3600.0	36.9	3838.2	3601.1	211.0	3600.0
73	AVG	1.1	583.0	65.9	386.4	709.0	90.2	38.3	1129.1	24.8	440.9	-	91.1	257.6
	MAX	3.9	3600.1	239.8	3613.3	3600.0	657.4	166.8	3600.0	80.0	3600.2	-	345.5	1874.4

Table 5.31 CPU Times (sec.) for K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	1.5	3247.7	1472.1	1750.6	3266.8	541.1	1810.3	2416.7	1610.3	1980.6	2934.2	1622.8	2696.1
	MAX	3.0	3600.6	3600.1	3601.1	3600.1	3395.4	3600.1	3600.0	3600.1	3600.0	3600.0	3600.0	3600.0
73	AVG	1.6	-	1488.2	-	-	1120.5	-	-	-	-	-	-	-
	MAX	6.5	-	3600.1	-	-	3600.0	-	-	-	-	-	-	-

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.6	792.7	124.1	620.9	831.8	147.3	1053.0	934.3	156.0	198.1	1446.6	1401.3	1873.4
	MAX	3.7	3600.1	807.6	3600.3	3600.1	476.1	3600.0	3600.0	439.0	768.6	3600.0	3600.0	3600.1
73	AVG	0.3	1194.8	449.1	1205.3	941.2	194.5	658.4	1777.8	444.0	1284.9	1527.3	-	-
	MAX	0.6	3600.0	2568.1	3600.0	3600.0	714.3	3600.1	3600.4	3600.1	3600.0	3600.0	-	-

The impact of problem parameters (K , N , CT , POS and π) on the optimal objective function value and the solution time is analyzed by considering one parameter (and keeping the others constant) at a time.

As mentioned before, the problem parameter set for three hazardous task case differs in success probabilities. Experiments are executed considering all problem parameters, however in order to avoid repetition only the results that are different from two hazardous task case are interpreted. Effect of task number in a network is not discussed since the change of this problem parameter on average objective function value and average computational time results is in similar behavior with one/two hazardous task cases. Moreover, there is an increment in number of unsolved instances and the average CPU time as the number of workstations increases. These results are common with two hazardous task case.

As mentioned before, the impact of problem parameters on solution time and optimal objective function value is more apparent in large networks. Thus, large networks are considered for the remaining experiments and for the analysis related with parameters. A threshold limit 3/10 for number of unsolved instances also used for three hazardous task case, in order to decide continuing experiments with a harder problem, i.e. larger network.

Our findings are discussed below.

Effect of Success Probabilities (π)

As we expected, it is observed that when success probabilities are decreasing the average objective function value does not improve. The results procreate similar inferences as one and two hazardous task cases; however, the success probabilities contain different elements in each case. Thus, the inferences regarding three hazardous task case examined according to the changes of success probabilities.

Let the pairs of success probabilities for three hazardous tasks,

$$P5: \pi_{z_1} = 1 \quad \pi_{z_2} = 1 \quad \pi_{z_3} = 1$$

$$P6: \pi_{z_1} = 0.8 \quad \pi_{z_2} = 0.8 \quad \pi_{z_3} = 0.8,$$

$$P7: \pi_{z_1} = 0.8 \quad \pi_{z_2} = 0.2 \quad \pi_{z_3} = 0.8,$$

$$P8: \pi_{z_1} = 0.8 \quad \pi_{z_2} = 0.2 \quad \pi_{z_3} = 0.2,$$

$$P9: \pi_{z_1} = 0.2 \quad \pi_{z_2} = 0.2 \quad \pi_{z_3} = 0.2.$$

Henceforth pairs are referred as to P5, P6, P7, P8 and P9. Note that the success probabilities decrease from P5 to P9 since at each step one of the success probability worsens. The corresponding average objective function values related with success probabilities are lined up as $O_{P5} > O_{P6} > O_{P7} > O_{P8} > O_{P9}$. Solutions for different hazardous task positions at each success probability are illustrated at Figure 10, for $K=8$ & $N=60$ with combination CT1. For POS1, the average objective function value decreases from “1061.3” to “888.7”, then to “633.8”, “577.2” and “527” respectively as success probability decreases from “P5” through “P9”. Similar results can be obtained with different hazardous task positions, or different combinations of N , K , CT or π .

The average CPU time and number of unsolved instances do not follow a consistent pattern with the change in success probability when all other factors are fixed.

Effect of Hazardous Task Position (POS)

We investigate the change in average objective function value related with hazardous task positions. Although the inferences resemble the results of one and two hazardous task case, the contents of positions are different. The average objective function value with POS2 is greater than the average objective function value with POS1. The average objective function value, i.e. expected net revenue, improves as all hazardous tasks' positions are located at later workstations. The average objective function value of POS3 is also compared with the average objective function value of POS1 and POS2. To illustrate the position effect on objective function value, large network $N=60$ with $K=8$, CT1 is utilized and solutions for different hazardous task

positions are given at Figure 5.6. It is obvious that average objective function values are larger in the results when POS2 is utilized. For example, with combination K=8, N=60, CT1; the average objective function value increases (improves) from “888.7” to “1023.2” as we move from “POS1” to “POS2” when success probability is fixed at P6. The average objective function value increases (improves) from “633.8” to “1012.8” as we move from “POS1” to “POS2” at success probability P7. Similar results are observed with different combinations. Mainly, the average objective function value of POS3 lies between the average objective function value of POS1 and POS2.

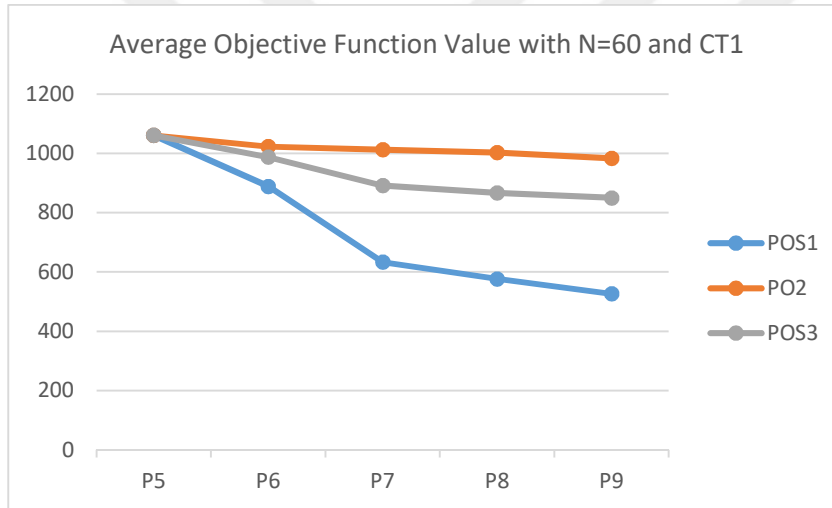


Figure 5.6. Comparison of POS1& POS2 &POS3 for N=60, CT1

We analyze the solutions with respect to the average CPU time. The average CPU time with POS1 is more than the average CPU time with POS2. For instance, for N=60 with combination K=4, CT1, P7 the average CPU time decreases from “850.1” to “130.4” as we move from “POS1” to “POS2”. Similar results can be obtained with different network and combinations between POS1 and POS2. Note that, the average CPU time and number of unsolved instances do not follow a consistent pattern with the change in the position of hazardous tasks when one/two hazardous tasks are considered in the model. However, with three hazardous task case we observe a

monotonic pattern between POS1 and POS2. Probably the number of hazardous tasks become significant to affect the CPU time of solutions regarding the hazardous task positions. Actually, the hazardous tasks are selected randomly from the assigned task set with $\pi = 1$. Thus, this selection does not guarantee a hazardous task being assigned to any workstation or exactly to the former workstations. As number of these hazardous tasks increases, the assignment of them becomes more probable affecting the average CPU time regarding POS1-POS2 selection. As expected, POS3 does not behave monotonic regarding the changes of hazardous task positions since it is obtained via selecting the hazardous tasks randomly from the assigned task set with $\pi = 1$, without considering workstation assignment.

5.2.4 Discussion of VSS Results with One Hazardous Task

As mentioned before, 5 networks, $K=4$ and small cycle time (CT1) is considered in VSS experiments. This makes 5 combinations of N , K and CT values. For one hazardous task case, there are 10 probability-position combinations. 10 problem instances are generated for each combination; hence a total of 500 problem instances are included in our experiments.

To analyze the VSS and to observe the combinations for which SP results are significantly better than EEV, the average and maximum SP/EEV values are reported at Table 5.32. Note that $SP/EEV - 1$ represents the relative VSS value (with respect to EEV).

Table 5.32 Ratio of SP/EEV for K=4, CT1, One Hazardous Task

π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.00	1.13	1.19	1.13	1.52	1.69	1.48	1.19	1.00	1.01
	MAX	1.00	1.24	1.23	1.22	1.98	1.94	1.94	2.65	1.03	1.14
34	AVG	1.00	1.13	1.10	1.17	1.52	1.36	1.66	1.06	1.00	1.01
	MAX	1.00	1.22	1.23	1.24	1.92	1.93	1.98	1.41	1.00	1.08
47	AVG	1.00	1.15	1.12	1.14	1.58	1.39	1.52	1.05	1.00	1.07
	MAX	1.00	1.24	1.23	1.22	1.95	1.94	1.95	1.30	1.00	1.73
60	AVG	1.00	1.16	1.12	1.16	1.64	1.41	1.61	1.01	1.00	1.01
	MAX	1.00	1.25	1.23	1.24	1.99	1.93	1.97	1.04	1.00	1.08
73	AVG	1.00	1.18	1.17	1.16	1.69	1.62	1.61	1.01	1.00	1.00
	MAX	1.00	1.23	1.23	1.23	1.94	1.95	1.95	1.04	1.00	1.00

The impact of problem parameters (N, POS and π) on the optimal objective function value is analyzed by considering one parameter (and keeping the others constant) at a time. Our findings are discussed below.

Effect of the Number of Tasks in the Network (N)

When we analyze the ratio SP/EEV to observe the combinations that SP results are better than EEV, the results show that there is not a monotonic pattern regarding the increase of N. Although VSS (SP-EEV) is increasing at each problem setting, the ratio is not monotonic. The ratio is increasing especially when success probabilities are higher and POS1 is utilized, which means that the increase in SP is higher than EEV. However, in general the behavior of SP/EEV is not monotonic, that is EEV gets closer to SP at some problem settings and opens the gap at some others. For example, when $\pi=0.5$ and POS2 is utilized, the ratio SP/EEV decreases from “1.69” to “1.36”, then increases to “1.39” when N changes such that N=22 to N=34, then N=47 respectively, in an increasing way.

Effect of Success Probabilities (π)

We analyze the effect of success probabilities to the ratio, SP/EEV. We can say that it is increasing when we decrease success probability from $\pi=0.8$ to $\pi=0.5$, which means that SP results are better than EEV. However, the ratio is decreasing when we decrease success probability from $\pi=0.5$ to $\pi=0.2$ which means that EEV results are getting closer to SP results. SP/EEV ratio, and hence VSS, is observed to be higher in the setting where $\pi=0.5$, which represents a higher level of uncertainty compared to other probability values. For the combination with $N=73$ and POS1, the SP/EEV ratio is 1.69 (i.e., relative VSS is 69%) when $\pi=0.5$ whereas it is only 1.18 and 1.01 for $\pi=0.8$ and $\pi=0.2$, respectively. As also discussed earlier, $\pi=1$ represents a deterministic setting and accordingly $VSS = 0$ for such instances.

The peak point obtained via combination $N=73$, $K=4$, CT1 and POS1 for one hazardous task case ($\pi=0.5$) can be seen from the Figure below. In other words, the added value of considering uncertainty in a decision is higher when we have success probability $\pi=0.5$ for a hazardous task.

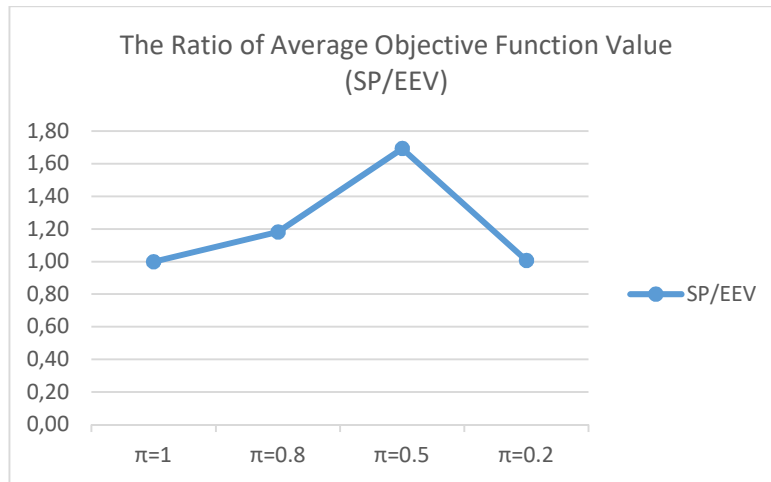


Figure 5.7. SP/EEV with $N=73$, $K=4$, CT1 and POS1

Effect of Hazardous Task Position (POS)

For most of the N - π combinations, SP/EEV is higher when the hazardous task is selected from one of the earlier workstations (POS1) rather than the later workstations (POS2), which is expected since the potential impact of a hazardous task is higher when it is followed by more tasks to be completed on the line. For example, with $N=47$ and $\pi=0.5$, SP/EEV ratio decreases from 1.58 to 1.39 as we move from POS1 setting to POS2 setting. As POS3 setting represents random selection of the position of the hazardous task, its impact on the SP/EEV ratio is not consistent across the problem combinations.

5.2.5 Discussion of VSS Results with Two Hazardous Task

As mentioned before, 5 networks, $K=4$ and small cycle time (CT1) is considered in VSS experiments. This makes 5 combinations of N , K and CT values. For one hazardous task case, there are 10 probability-position combinations. 10 problem instances are generated for each combination; hence a total of 500 problem instances are included in our experiments.

The average and maximum SP/EEV values for the setting with two hazardous tasks are reported at Table 5.33.

Table 5.33 Ratio of SP/EEV for K=4, CT1, Two Hazardous Task

		CT1									
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8$			$\pi z1= 0.8, \pi z2= 0.2$			$\pi z1= 0.2, \pi z2= 0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.00	1.35	1.28	1.35	1.14	1.15	1.16	1.12	1.01	1.01
	MAX	1.00	1.50	1.44	1.52	1.27	1.23	1.24	1.79	1.03	1.05
34	AVG	1.00	1.30	1.29	1.25	1.24	1.18	1.16	1.09	1.00	1.09
	MAX	1.00	1.51	1.40	1.42	1.74	1.23	1.23	1.59	1.02	1.93
47	AVG	1.00	1.40	1.39	1.30	1.18	1.16	1.17	1.03	-	1.07
	MAX	1.00	1.51	1.53	1.46	1.23	1.24	1.30	1.08	-	1.55
60	AVG	1.00	1.62	1.41	1.35	1.18	1.17	1.18	1.09	-	1.00
	MAX	1.00	1.54	1.51	1.53	1.25	1.24	1.24	1.18	-	1.03
73	AVG	1.00	1.42	1.35	1.32	1.19	1.20	1.19	1.03	-	1.36
	MAX	1.00	1.55	1.49	1.46	1.33	1.24	1.24	1.07	-	4.63

In order to avoid repetition only the results that are different from one hazardous task case are interpreted, which is the success probability in that case.

As in the one hazardous task case, SP/EEV is higher when the hazardous tasks are performed at earlier workstations (POS1) rather than later workstations (POS2) for most of the N- π combinations.

Effect of Success Probabilities (π)

Unlike in the case with one hazardous task, all probability settings considered in two hazardous task case represent a similar level of uncertainty as π_z values are 0.2 or 0.8, both of which are equally distant from the deterministic extremes (0 and 1). For all N-position combinations and at this level of uncertainty, SP/EEV ratio decreases with the decreasing probability of success for the hazardous tasks. For instance, with N=60 and POS1 the ratio decreases from “1.62” to “1.18” then decreases to “1.09” when the success probability decreases from P2 to P3, then to P4 respectively. In other words, value of modeling uncertainty is higher when the probability of successfully completing the hazardous tasks is higher.

5.2.6 Discussion of VSS Results with Three Hazardous Task

There are 5 combinations of N, K and CT values and 12 probability-position combinations. 10 problem instances are generated for each combination; hence a total of 600 problem instances are included in our experiments.

The average and maximum SP/EEV values for the setting with three hazardous tasks are reported at Table 5.34.

Table 5.34 Ratio of SP/EEV for K=4, CT1, Three Hazardous Task

π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.00	1.37	1.36	1.42	1.32	1.31	1.34	1.28	1.16	1.18	1.16	1.01	1.06
	MAX	1.00	1.64	1.64	1.65	1.65	1.50	2.08	2.37	1.23	1.24	1.56	1.03	1.29
34	AVG	1.00	1.44	1.40	1.48	1.37	1.33	1.31	1.18	1.17	1.20	1.03	1.01	1.01
	MAX	1.00	1.69	1.68	1.73	1.52	1.54	1.49	1.24	1.24	1.70	1.10	1.06	1.02
47	AVG	1.00	1.48	1.45	1.57	1.31	1.30	1.32	1.17	1.16	1.17	1.06	1.00	1.02
	MAX	1.00	1.71	1.69	1.75	1.54	1.51	1.51	1.31	1.23	1.28	1.16	1.01	1.09
60	AVG	1.00	1.43	1.40	1.53	1.31	1.29	1.33	1.17	1.13	1.17	1.03	1.00	1.08
	MAX	1.00	1.72	1.69	1.81	1.41	1.50	1.40	1.23	1.23	1.31	1.07	1.01	1.26
73	AVG	1.00	1.58	1.38	1.47	1.29	1.27	1.37	1.13	1.11	1.20	-	1.00	-
	MAX	1.00	1.76	1.75	1.78	1.45	1.43	1.51	1.28	1.24	1.34	-	1.01	-

The problem parameter set for three hazardous task case differs in success probabilities. Our observations for three hazardous task case are consistent with our earlier findings. For instance, with N=60 and POS1 the ratio decreases from “1.43” to “1.31”, “1.17” and finally to “1.03” when the success probability decreases from P6 through P9 respectively. At this level of uncertainty, SP/EEV is higher and hence using a stochastic model rather than a deterministic one is more valuable when the probability of successfully completing the hazardous tasks is higher. Moreover, this

value tends to be even higher when the hazardous tasks are among the tasks performed at earlier workstations.

5.2.7 Discussion of Upper Bound Results

We already discussed the Upper Bounds (UB) studied in our experiments. Three Upper Bounds are presented such that;

- Upper Bound 1 (UB_1): Pure LPR Solutions
- Upper Bound 2 (UB_2): Strengthened LPR with exact properties (Cuts 1-6)
- Upper Bound 3 (UB_3): Strengthened LPR with exact properties (Cuts 1-6) and heuristic properties (Heuristic Cuts 1-2)

The analysis of upper bounds in our experiments are based on the performance measures presented below.

- The average and maximum percent deviations of each Upper Bound from Optimal SP solution

$$\%Dev = \frac{(UB_i - Z^*)}{Z^*} * 100 \quad (Only\ for\ known\ optimal\ solutions)$$

Z^* : Optimal Objective Function value of SP Problem (Expected Net Revenue)

UB_i : Objective Function Value of Upper Bound i (Expected Net Revenue delivered by Upper Bound i)

- The average and maximum CPU Times (sec.)

The performance of upper bounds for one hazardous, two hazardous and three hazardous task cases are investigated according to the defined performance measures and reported for each case. Note that, the impact of problem parameters on solution time and optimal objective function value is more apparent in large networks, especially for $N=60$ and $N=73$. Thus, percent deviations of upper bounds for large

networks are reported in Tables 5.35-5.52 for one, two and three hazardous tasks respectively. The solutions of small networks are presented at Appendix B as well.



Table 5.35 Percent Deviation of UBs, One Hazardous Task, N=60, K=4

		CT1									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.38	4.87	2.22	3.70	6.10	2.42	4.19	9.64	3.10	4.68
	MAX	0.82	6.33	4.50	7.29	7.67	4.40	7.18	12.06	5.23	7.40
UB2	AVG	0.36	4.67	1.99	3.07	5.08	2.31	3.60	7.30	2.64	4.11
	MAX	0.80	5.18	3.68	6.31	6.08	4.23	7.00	8.94	5.11	6.87
UB3	AVG	0.03	3.37	1.16	2.47	4.21	1.18	2.50	7.02	1.22	2.53
	MAX	0.14	3.88	3.27	6.22	5.43	2.22	6.35	8.38	3.13	4.57

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.09	4.66	2.12	2.63	10.89	3.30	4.98	19.26	5.32	8.52
	MAX	0.19	6.07	4.58	5.47	14.15	6.33	8.31	23.44	8.60	12.36
UB2	AVG	0.08	3.98	1.23	1.63	6.15	3.02	3.09	16.03	3.05	5.24
	MAX	0.14	4.77	3.15	4.52	8.37	4.58	4.26	19.55	5.77	7.45
UB3	AVG	0.06	3.47	1.16	1.24	4.37	1.18	1.71	7.12	1.25	3.39
	MAX	0.10	4.44	3.04	3.92	6.81	2.05	3.61	9.76	2.70	5.70

Table 5.36 Percent Deviation of UBs, One Hazardous Task, N=60, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.44	5.60	2.56	4.26	7.02	2.78	4.82	11.08	3.56	5.38
	MAX	0.60	7.47	6.06	7.33	10.59	5.07	8.42	13.58	7.50	7.95
UB2	AVG	0.43	5.37	2.29	3.53	5.84	2.66	4.14	8.39	3.03	4.73
	MAX	0.58	5.58	3.75	7.23	10.02	5.03	7.67	10.02	5.84	5.78
UB3	AVG	0.04	3.87	1.16	2.84	4.84	1.18	2.87	7.02	1.40	2.91
	MAX	0.14	5.51	2.32	6.47	8.37	2.40	7.02	8.77	2.48	4.46

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.10	5.36	2.44	3.03	12.52	3.79	5.73	22.15	6.12	9.80
	MAX	0.14	9.39	4.14	6.38	18.21	6.51	11.38	28.84	12.97	14.01
UB2	AVG	0.09	4.57	1.41	1.88	11.67	3.47	3.56	18.43	3.51	6.02
	MAX	0.13	8.29	2.68	4.66	18.14	6.13	6.86	22.37	8.78	9.52
UB3	AVG	0.03	3.99	1.32	1.43	8.30	1.39	1.96	11.29	1.44	3.90
	MAX	0.10	7.74	1.32	4.39	14.63	2.03	4.05	17.70	3.41	6.10

Table 5.37 Percent Deviation of UBs, One Hazardous Task, N=60, K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.49	6.27	2.86	4.77	7.86	3.12	5.40	12.42	3.99	6.03
	MAX	1.39	9.71	7.97	10.82	9.40	4.69	9.03	14.50	7.03	8.59
UB2	AVG	0.49	6.02	2.56	3.95	6.54	2.98	4.63	9.40	3.39	5.30
	MAX	1.47	7.26	4.94	10.97	8.89	5.21	8.22	10.70	5.47	6.24
UB3	AVG	0.04	4.34	1.30	3.18	5.42	1.32	3.22	7.86	1.57	3.26
	MAX	0.32	7.29	3.05	9.55	7.42	2.22	7.52	9.36	2.33	4.82

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.11	6.00	2.73	3.39	14.03	4.25	6.42	24.81	6.85	10.97
	MAX	0.32	9.47	5.06	7.34	18.87	8.92	10.40	27.42	11.94	16.18
UB2	AVG	0.11	5.12	1.58	2.10	13.07	3.89	3.98	20.64	3.93	6.74
	MAX	0.30	8.37	3.27	5.36	18.90	9.37	6.27	24.95	8.08	11.00
UB3	AVG	0.00	4.46	1.48	1.60	9.30	1.55	2.20	12.64	1.62	4.37
	MAX	0.00	7.81	1.61	5.05	15.16	2.78	3.70	19.74	3.14	7.04

Table 5.38 Percent Deviation of UBs, One Hazardous Task, N=73, K=4

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.32	3.41	1.56	2.59	4.27	1.69	2.93	6.75	2.17	3.27
	MAX	0.65	4.75	2.82	5.13	5.44	3.10	5.20	8.51	3.13	5.97
UB2	AVG	0.25	2.34	0.99	1.53	3.55	1.62	2.52	5.11	1.84	2.88
	MAX	0.54	3.59	2.00	3.34	4.55	2.99	4.12	6.22	3.10	4.28
UB3	AVG	0.03	1.35	0.58	1.15	2.95	0.83	1.75	4.91	0.85	1.77
	MAX	0.11	2.15	1.77	3.33	4.07	1.91	3.76	6.18	2.30	2.85

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.06	3.26	1.49	1.84	7.62	2.31	3.49	13.48	3.72	5.96
	MAX	0.10	5.43	3.28	2.82	13.81	5.00	4.07	15.96	7.53	7.10
UB2	AVG	0.05	2.78	0.86	1.14	4.31	2.11	2.16	11.22	2.13	3.67
	MAX	0.07	5.25	2.61	2.70	8.00	3.77	4.02	15.20	5.05	4.36
UB3	AVG	0.04	2.08	0.81	0.87	3.06	0.83	1.19	4.05	0.88	2.37
	MAX	0.06	4.69	1.34	1.41	5.58	1.69	2.67	6.98	2.36	3.33

Table 5.39 Percent Deviation of UBs, One Hazardous Task, N=73, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.29	3.07	1.40	2.33	3.84	1.52	2.64	6.07	1.95	2.95
	MAX	0.59	5.53	3.08	4.99	5.22	3.41	3.43	7.62	3.47	3.45
UB2	AVG	0.23	2.10	0.89	1.38	3.20	1.46	2.27	4.60	1.66	2.59
	MAX	0.49	2.95	1.36	4.37	4.89	3.36	3.39	6.25	2.69	3.37
UB3	AVG	0.04	1.21	0.52	1.04	2.65	0.74	1.57	4.42	0.77	1.59
	MAX	0.19	2.37	0.97	4.30	4.08	1.64	2.48	5.58	1.14	3.22

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.05	2.94	1.34	1.66	6.86	2.08	3.14	11.13	3.35	4.04
	MAX	0.09	5.50	3.19	2.96	8.76	4.68	4.33	14.40	6.81	6.52
UB2	AVG	0.05	2.51	0.77	1.03	3.87	1.90	1.95	9.10	1.92	2.30
	MAX	0.08	4.85	2.07	2.92	5.32	4.57	3.36	11.94	4.59	4.23
UB3	AVG	0.04	1.80	0.63	0.78	2.76	0.74	1.07	3.05	0.79	1.14
	MAX	0.07	2.74	0.89	2.87	3.27	1.43	2.03	4.57	1.78	3.18

Table 5.40 Percent Deviation of UBs, One Hazardous Task, N=73, K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.35	3.68	1.68	2.80	4.61	1.83	3.17	7.29	2.34	3.54
	MAX	0.71	6.64	3.70	5.23	6.27	4.09	4.56	9.15	4.17	4.15
UB2	AVG	0.27	2.52	1.07	1.66	3.84	1.75	2.72	5.52	1.99	3.11
	MAX	0.59	3.54	1.64	3.13	5.93	4.04	4.15	6.75	3.24	3.76
UB3	AVG	0.05	1.46	0.63	1.24	3.18	0.89	1.89	4.53	0.92	1.91
	MAX	0.27	2.85	1.16	2.54	4.54	2.22	3.80	6.09	2.58	3.71

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.06	3.38	1.40	1.99	8.23	2.30	3.77	13.36	3.47	4.84
	MAX	0.10	6.32	3.34	3.55	10.51	5.18	5.19	16.96	8.17	5.88
UB2	AVG	0.06	3.01	0.93	1.24	4.65	1.78	2.34	10.92	2.30	2.76
	MAX	0.09	5.82	2.48	2.59	5.66	4.60	3.13	14.15	5.53	3.70
UB3	AVG	0.05	2.16	0.61	0.94	3.31	0.83	1.29	3.65	0.95	1.36
	MAX	0.08	4.48	0.86	2.44	4.54	1.60	2.35	5.56	2.15	1.81

Table 5.41 Percent Deviation of UBs, Two Hazardous Task, N=60, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.52	10.82	4.16	4.11	27.14	5.31	5.98	-	6.93	7.57
	MAX	1.11	14.07	8.42	8.09	34.09	9.65	10.23	-	11.70	11.98
UB2	AVG	0.43	8.76	3.06	3.64	23.22	4.17	4.86	-	5.55	5.58
	MAX	0.97	9.71	5.67	7.48	27.80	8.35	9.46	-	11.19	9.31
UB3	AVG	0.04	4.55	1.99	2.21	14.54	2.66	2.68	-	1.57	3.16
	MAX	0.18	5.24	5.60	6.46	18.76	5.02	6.82	-	4.03	5.71

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.51	7.96	2.73	4.05	29.68	4.75	6.89	35.79	7.73	13.21
	MAX	1.11	10.37	5.89	8.41	38.55	9.12	11.49	43.56	12.49	19.17
UB2	AVG	0.45	6.78	2.29	3.04	25.62	3.67	5.41	31.43	5.52	11.10
	MAX	0.73	8.12	5.89	8.41	34.90	5.57	7.46	38.33	10.46	15.80
UB3	AVG	0.22	4.69	1.67	1.85	15.42	2.83	1.80	20.17	4.19	5.44
	MAX	0.53	7.35	5.40	5.83	24.00	4.93	3.81	28.05	9.01	9.15

Table 5.42 Percent Deviation of UBs, Two Hazardous Task, N=60, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.68	7.56	3.19	3.80	15.79	5.44	6.10	26.27	6.63	9.87
	MAX	0.93	10.09	7.55	6.54	23.83	9.91	10.67	32.18	13.95	14.59
UB2	AVG	0.59	5.94	2.67	3.11	12.20	4.08	4.80	21.03	5.63	7.45
	MAX	0.85	6.18	4.39	6.54	20.94	8.64	8.91	25.12	10.85	9.10
UB3	AVG	0.20	3.65	2.15	2.07	9.10	2.59	2.99	16.09	4.57	4.61
	MAX	0.75	5.28	4.30	4.71	15.72	5.25	7.31	20.10	8.09	7.08

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.67	4.78	1.77	3.74	18.10	4.88	7.02	19.68	7.42	15.62
	MAX	0.93	8.38	3.01	7.88	26.31	8.37	13.94	25.63	15.72	22.34
UB2	AVG	0.57	4.24	1.58	3.18	14.29	3.68	5.06	16.62	6.04	12.45
	MAX	0.73	7.68	3.01	7.88	22.33	7.25	9.76	20.17	15.12	19.68
UB3	AVG	0.15	2.45	0.78	2.22	10.23	1.76	3.43	11.18	3.03	9.96
	MAX	0.24	4.77	0.78	6.85	18.04	2.58	7.08	17.53	7.15	15.57

Table 5.43 Percent Deviation of UBs, Two Hazardous Task, N=60, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8$			$\pi z1= 0.8, \pi z2= 0.2$			$\pi z1= 0.2, \pi z2= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.29	8.68	3.36	4.09	20.40	4.72	6.39	34.28	6.43	9.69
	MAX	0.83	13.45	9.34	9.28	24.40	7.11	10.68	40.04	11.32	13.81
UB2	AVG	0.25	6.83	2.81	3.34	15.77	3.55	5.02	27.44	5.46	7.32
	MAX	0.76	8.23	5.43	9.28	21.45	6.20	8.92	31.25	8.80	8.61
UB3	AVG	0.09	4.19	2.27	2.22	11.76	2.24	3.13	21.00	4.43	4.53
	MAX	0.68	7.04	5.33	6.68	16.10	3.77	7.32	25.00	6.56	6.70

		CT2									
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8$			$\pi z1= 0.8, \pi z2= 0.2$			$\pi z1= 0.2, \pi z2= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.29	8.17	3.33	4.08	17.67	4.68	6.27	26.57	6.35	9.67
	MAX	0.83	12.89	6.17	8.84	23.76	9.83	10.16	29.37	11.07	14.27
UB2	AVG	0.25	7.24	2.98	3.47	13.95	3.53	4.52	22.44	5.17	7.71
	MAX	0.66	11.82	6.17	8.84	20.17	8.51	7.11	27.12	10.65	12.57
UB3	AVG	0.06	4.19	1.47	2.43	9.99	1.69	3.06	15.10	2.59	6.16
	MAX	0.22	7.33	1.60	7.68	16.29	3.03	5.16	23.57	5.04	9.94

Table 5.44 Percent Deviation of UBs, Two Hazardous Task, N=73, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8$			$\pi z1= 0.8, \pi z2= 0.2$			$\pi z1= 0.2, \pi z2= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.34	5.90	3.57	4.36	14.91	5.21	5.06	-	7.02	11.62
	MAX	0.70	8.23	6.46	8.63	18.97	8.32	8.98	-	10.13	14.10
UB2	AVG	0.28	2.80	1.16	1.97	10.16	2.57	2.19	-	4.13	6.85
	MAX	0.60	4.30	2.33	4.28	13.02	6.32	5.53	-	8.73	10.18
UB3	AVG	0.03	1.45	0.75	1.19	6.36	1.64	1.21	-	1.17	3.88
	MAX	0.11	2.32	2.30	3.70	8.79	3.80	3.98	-	3.14	6.24

		CT2									
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8$			$\pi z1= 0.8, \pi z2= 0.2$			$\pi z1= 0.2, \pi z2= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.27	4.80	1.75	4.77	8.75	3.24	14.10	19.64	5.05	21.85
	MAX	0.45	7.99	3.85	7.29	15.85	7.00	16.26	23.25	10.21	26.02
UB2	AVG	0.24	4.18	1.27	3.09	7.72	2.40	5.00	15.10	3.61	11.11
	MAX	0.30	5.47	3.85	7.29	14.35	4.28	10.55	20.46	8.54	13.20
UB3	AVG	0.12	2.20	0.93	1.88	5.41	1.85	1.97	8.69	2.74	5.44
	MAX	0.21	4.96	1.53	3.06	9.87	3.79	4.39	14.97	7.37	7.65

Table 5.45 Percent Deviation of UBs, Two Hazardous Task, N=73, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.57	4.89	2.94	6.09	8.61	4.54	13.60	15.77	5.94	23.59
	MAX	1.16	8.82	6.46	13.04	11.69	10.15	17.68	19.79	10.56	27.65
UB2	AVG	0.49	3.84	2.46	4.12	6.72	3.41	9.87	12.73	5.07	15.04
	MAX	1.06	5.40	3.76	13.04	10.28	7.85	14.76	15.45	8.21	21.25
UB3	AVG	0.17	2.36	1.98	2.26	5.01	2.16	5.77	8.74	4.12	8.17
	MAX	0.94	4.61	3.68	9.39	7.71	4.77	9.11	12.36	6.12	16.53

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.45	-	1.57	4.52	-	3.04	11.02	-	4.81	18.74
	MAX	0.74	-	3.75	8.05	-	6.86	15.18	-	9.77	22.75
UB2	AVG	0.38	-	1.40	2.84	-	2.30	7.96	-	3.93	10.95
	MAX	0.59	-	3.75	8.05	-	5.94	13.73	-	9.39	14.04
UB3	AVG	0.10	-	0.69	1.68	-	1.10	4.39	-	1.97	7.96
	MAX	0.20	-	0.97	6.19	-	2.11	8.31	-	4.44	11.86

Table 5.46 Percent Deviation of UBs, Two Hazardous Task, N=73, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.80	-	-	4.63	-	3.97	6.22	-	7.36	15.70
	MAX	1.63	-	-	8.64	-	8.88	8.96	-	13.09	18.41
UB2	AVG	0.69	-	-	3.78	-	2.98	4.90	-	6.25	11.86
	MAX	1.48	-	-	7.15	-	7.74	7.48	-	10.18	14.34
UB3	AVG	0.23	-	-	2.52	-	1.89	3.05	-	5.07	7.34
	MAX	1.32	-	-	5.15	-	4.70	6.14	-	7.59	11.15

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.46	4.94	1.44	4.59	-	2.39	11.04	19.47	4.20	18.77
	MAX	0.77	9.25	3.44	8.18	-	5.38	15.21	24.71	8.53	22.79
UB2	AVG	0.39	4.38	1.29	3.91	-	1.80	7.95	14.44	3.42	14.95
	MAX	0.60	8.48	3.44	8.18	-	4.66	10.65	18.72	8.20	20.07
UB3	AVG	0.10	2.54	0.63	2.73	-	0.86	5.39	8.72	1.71	11.96
	MAX	0.20	5.26	0.89	7.11	-	1.66	9.81	13.27	3.88	15.88

Table 5.47 Percent Deviation of UBs, Three Hazardous Task, N=60, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.54	11.14	4.28	4.23	21.71	5.47	6.16	24.71	6.30	6.98	27.71	7.14	7.80
	MAX	1.14	14.49	8.67	8.33	27.27	9.94	10.54	30.97	11.00	11.44	34.66	12.05	12.34
UB2	AVG	0.44	9.02	3.15	3.75	18.58	4.30	5.01	20.79	5.01	5.38	23.00	5.72	5.75
	MAX	1.00	10.00	5.84	7.70	22.24	8.60	9.74	25.21	10.06	9.67	28.17	11.53	9.59
UB3	AVG	0.04	4.69	2.05	2.28	11.63	2.18	2.76	13.28	2.41	3.01	14.93	2.62	3.25
	MAX	0.19	5.40	5.77	6.65	15.01	5.17	7.02	18.54	4.66	6.45	22.07	4.15	5.88

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.53	8.20	2.81	4.17	23.74	4.89	7.10	26.19	6.43	10.35	28.63	7.96	13.61
	MAX	1.14	10.68	6.07	8.66	30.84	9.39	11.83	32.84	11.13	15.79	34.85	12.86	19.75
UB2	AVG	0.46	6.98	2.36	3.13	20.50	3.78	5.57	22.82	4.73	8.50	25.14	5.69	11.43
	MAX	0.75	8.36	6.07	8.66	27.92	5.74	7.68	29.29	8.26	11.98	30.66	10.77	16.27
UB3	AVG	0.23	4.83	1.72	1.86	12.34	2.91	1.91	14.24	3.62	3.73	16.14	4.32	5.60
	MAX	0.55	7.57	5.56	6.00	19.20	5.08	3.92	20.82	7.18	6.67	22.44	9.28	9.42

Table 5.48 Percent Deviation of UBs, Three Hazardous Task, N=60, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.70	7.79	3.29	3.91	12.63	5.60	6.28	16.82	6.22	8.22	21.02	6.83	10.17
	MAX	0.96	10.39	7.78	6.74	19.06	10.21	10.99	22.40	12.29	13.01	25.74	14.37	15.03
UB2	AVG	0.61	6.12	2.75	3.20	9.76	4.20	4.94	13.29	5.00	6.31	16.82	5.80	7.67
	MAX	0.88	6.37	4.52	6.74	16.75	8.90	9.18	18.42	10.04	9.28	20.10	11.18	9.37
UB3	AVG	0.21	3.76	2.21	2.13	7.28	2.67	3.08	10.08	3.69	3.91	12.87	4.71	4.75
	MAX	0.77	5.44	4.43	4.85	12.58	5.41	7.53	14.33	6.87	7.41	16.08	8.33	7.29

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.69	4.92	1.82	3.85	14.48	5.03	7.23	15.11	6.33	11.66	15.74	7.64	14.09
	MAX	0.96	8.63	3.10	8.12	21.05	8.62	14.36	20.78	12.41	18.68	20.50	16.19	23.01
UB2	AVG	0.59	4.37	1.63	3.28	11.43	3.79	5.21	12.36	5.01	9.02	13.30	6.22	12.82
	MAX	0.75	7.91	3.10	8.12	17.86	7.47	10.05	17.00	11.52	15.16	16.14	15.57	20.27
UB3	AVG	0.15	2.52	0.80	2.29	8.18	1.81	3.53	8.56	2.47	6.90	8.94	3.12	7.26
	MAX	0.25	7.61	0.80	7.06	14.43	2.66	7.29	14.23	5.01	11.66	14.02	7.36	16.04

Table 5.49 Percent Deviation of UBs, Three Hazardous Task, N=60, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.30	8.94	3.46	4.21	16.32	4.86	6.58	21.87	5.74	8.28	27.42	6.62	9.98
	MAX	0.85	13.85	9.62	9.56	19.52	7.32	11.00	25.78	9.49	12.61	32.03	11.66	14.22
UB2	AVG	0.26	7.03	2.89	3.44	12.62	3.66	5.17	17.28	4.64	6.36	21.95	5.62	7.54
	MAX	0.78	8.48	5.59	7.56	17.16	6.39	9.19	21.08	7.73	9.03	25.00	9.06	8.87
UB3	AVG	0.09	4.32	2.34	2.29	9.41	2.39	3.22	13.10	3.44	3.94	16.80	4.56	4.67
	MAX	0.70	7.25	5.49	4.88	12.88	3.88	5.54	16.44	5.32	6.22	20.00	6.76	6.90

		CT2												
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.2$			$\pi z1= 0.2, \pi z2= 0.2, \pi z3= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.30	8.42	3.43	4.20	14.14	4.82	6.46	17.70	5.68	8.21	21.26	6.54	9.96
	MAX	0.85	13.28	6.36	9.11	19.01	10.12	10.46	21.25	10.76	12.58	23.50	11.40	14.70
UB2	AVG	0.26	7.46	3.07	3.57	11.16	3.64	4.66	14.56	4.48	6.30	17.95	5.33	7.94
	MAX	0.68	12.17	6.15	9.01	16.14	8.77	7.32	18.92	9.87	10.14	21.70	10.97	12.95
UB3	AVG	0.06	4.32	1.51	2.50	7.99	1.74	3.15	10.04	2.20	4.75	12.08	2.67	6.34
	MAX	0.23	7.55	1.65	7.91	13.03	3.12	5.31	15.94	4.16	7.78	18.86	5.19	10.24

Table 5.50 Percent Deviation of UBs, Three Hazardous Task, N=73, K=4

		CT1												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.35	6.08	3.68	4.49	15.36	5.11	5.21	17.13	6.17	8.59	-	7.23	-
	MAX	0.72	8.48	6.65	8.89	19.54	8.16	9.25	21.68	9.30	11.89	-	10.43	-
UB2	AVG	0.29	2.88	1.19	2.03	10.46	2.18	2.26	11.80	3.22	4.66	-	4.25	-
	MAX	0.62	4.43	2.40	4.41	13.41	5.36	5.70	14.71	7.18	8.09	-	8.99	-
UB3	AVG	0.03	1.49	0.77	1.23	6.55	1.19	1.25	6.59	1.20	2.62	-	1.21	-
	MAX	0.11	2.39	2.37	3.81	9.05	2.76	4.10	9.40	3.00	5.26	-	3.23	-

		CT2												
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.2$			$\pi z1= 0.2, \pi z2= 0.2, \pi z3= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.28	4.94	1.80	4.91	9.01	3.34	8.52	14.62	4.27	14.58	-	5.20	18.51
	MAX	0.46	8.23	3.97	7.51	16.33	7.21	9.83	20.14	8.86	19.40	-	10.52	22.04
UB2	AVG	0.25	4.31	1.31	3.18	7.95	2.47	5.15	11.75	3.10	8.30	-	3.72	11.44
	MAX	0.31	5.63	3.97	7.51	14.78	4.41	8.87	17.93	6.60	12.23	-	8.80	13.60
UB3	AVG	0.12	2.27	0.96	1.94	5.57	1.91	2.03	6.26	2.36	3.82	-	2.82	5.60
	MAX	0.22	5.11	1.58	3.15	10.17	3.90	4.52	11.07	5.75	6.20	-	7.59	7.88

Table 5.51 Percent Deviation of UBs, Three Hazardous Task, N=73, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.59	5.04	3.03	4.97	8.87	4.68	7.68	12.56	5.40	10.99	-	6.12	14.30
	MAX	1.19	9.08	6.65	7.64	12.04	10.45	10.78	16.21	10.67	13.37	-	10.88	16.76
UB2	AVG	0.50	3.96	2.53	3.64	6.92	3.51	6.87	10.02	4.37	9.68	-	5.22	12.49
	MAX	1.09	5.56	3.87	5.52	10.59	8.09	10.27	13.25	8.27	13.16	-	8.46	15.85
UB3	AVG	0.18	2.43	2.04	2.33	5.16	2.22	4.12	6.08	3.23	5.27	-	4.24	6.42
	MAX	0.97	4.75	3.79	4.67	7.94	4.91	6.50	8.91	5.61	9.75	-	6.30	12.99

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.46	3.66	1.62	3.58	7.39	3.13	7.35	11.24	4.04	10.83	-	4.95	14.30
	MAX	0.76	6.84	3.86	6.38	9.44	7.07	10.12	14.29	8.56	13.74	-	10.06	17.36
UB2	AVG	0.39	3.24	1.44	2.93	5.84	2.37	5.20	9.29	3.21	8.24	-	4.05	11.28
	MAX	0.61	6.27	3.86	6.21	8.01	6.12	8.97	11.54	7.89	11.72	-	9.67	14.46
UB3	AVG	0.10	1.88	0.71	1.73	4.18	1.13	3.52	5.42	1.58	4.86	-	2.03	5.20
	MAX	0.21	3.89	1.00	3.38	6.47	2.17	6.66	8.32	3.37	7.95	-	4.57	9.24

Table 5.52 Percent Deviation of UBs, Three Hazardous Task, N=73, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.82	-	1.47	-	-	4.09	-	-	-	-	-	-	-
	MAX	1.68	-	3.22	-	-	9.15	-	-	-	-	-	-	-
UB2	AVG	0.71	-	1.23	-	-	3.07	-	-	-	-	-	-	-
	MAX	1.52	-	1.87	-	-	7.97	-	-	-	-	-	-	-
UB3	AVG	0.24	-	0.99	-	-	1.95	-	-	-	-	-	-	-
	MAX	1.36	-	1.84	-	-	4.84	-	-	-	-	-	-	-

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.47	5.09	1.48	4.73	15.61	2.46	11.37	17.83	3.39	15.35	20.05	-	-
	MAX	0.79	9.53	3.54	8.43	19.92	5.54	15.67	22.69	7.16	19.57	25.45	-	-
UB2	AVG	0.40	4.51	1.33	4.03	12.32	1.85	8.19	13.60	2.69	11.79	14.87	-	-
	MAX	0.62	8.73	3.54	8.43	14.99	4.80	10.97	17.13	6.62	15.82	19.28	-	-
UB3	AVG	0.10	2.62	0.65	2.51	6.85	0.89	5.55	6.91	1.32	5.94	6.98	-	-
	MAX	0.21	5.42	0.92	6.54	9.40	1.71	10.10	10.01	2.85	9.25	10.62	-	-

For one hazardous task case, over all problem combinations the deviations of UB_1 is slightly higher. The average deviations are *below 24.81%* when $N=60$ and *below 13.36%* when $N=73$. The maximum deviations are *below 28.84%* and *below 16.96%* for $N=60$ and $N=73$ respectively. UB_2 produces smaller deviations, and UB_3 produces the smallest one. As cuts are applied to the relaxed problem, we observe the power of cuts and the decrease in the deviations. For instance, for one hazardous task the average deviation of UB_1 for $N=73$, $K=8$, $\pi=0.2$ and POS1 is *13.36 %* and it decreases to *10.92%* for UB_2 , then *3.65%* for UB_3 . Similar observations hold for two and three hazardous tasks cases.

We analyze the effects of the number of tasks (N), number of workstations (K) and cycle time (CT) on the deviations of UB_1 , UB_2 and UB_3 . The increase in number of tasks do not deliver a deterioration in deviations. Contrary, in some combinations it slightly gets better when networks get larger. For one hazardous task case, in general the effect of cuts seems to be more efficient with an increase in the number of tasks, especially for large networks. For example, the average deviation of UB_3 with $K=6$, $\pi=0.8$, POS1, CT1 decreases from *3.87 %* to *1.21%* when we move from $N=60$ to $N=73$. However, results do not show a monotonous behavior for two or three hazardous task cases regarding this observation.

We could not observe a consistent behavior of the cycle time and the number of workstations on deviations of upper bounds when there are one or two or three hazardous tasks. Although the increase in K inflates the number of variables in the problem, the deviations do not show a consistent increase. Cuts seems to be effective to decrease the impact of inflation in some combinations. Hence, the increase in K itself, when the other parameters are fixed, do not cause a monotone increase in deviation.

The results in Tables 5.35–5.52 support our observations according to the effects of the number of tasks (N), number of workstations (K) and cycle time (CT).

We observe that the success probabilities (π) and positions of the hazardous tasks (POS values) are two important parameters that significantly affect the deviations of upper bounds.

Recall that as success probability increases, the chance that the hazardous task affects its succeeding workstations decreases; hence many more tasks can be completed. However, the decrease in success probability deteriorates the solutions. Hence, a decrease in success probabilities triggers an increase in the deviations of upper bounds. This result is true for all number of hazardous tasks and all problem combinations of each upper bound. Note that for one hazardous tasks case and UB_3 , when $K=6$, $N=73$, CT1 and POS1, the average deviation increases from 0.04% to 1.21%, then to 2.65% and finally increases to 4.42% as success probability decreases 1 through $\pi=0.2$. For two hazardous tasks case and UB_3 , when $K=6$, $N=73$, CT1 and POS2 the average deviation increases from 0.17% to 1.98%, then to 2.16%, and finally increases to 4.12% as success probability decreases from P1 through P4. For three hazardous tasks case and UB_3 , when $K=4$, $N=60$, CT2 and POS3, the average deviation increases from 0.23% to 1.86%, then to 1.91%, 3.73% and 5.60% respectively as success probability decreases from P5 through P9.

We next analyze the effect of hazardous tasks' positions on the deviation of upper bounds. POS1 and POS2 refer to the instances where the hazardous tasks are selected from the first half and second half of the disassembly line with $\pi=1$, respectively. The average deviation of upper bounds from optimal solutions when all of the hazardous tasks are selected randomly from the tasks that are assigned to the second half of workstations (POS2) is smaller than the average deviation with POS1. This is expected as the adverse effects of the hazardous tasks is less when they are located at later workstations. Note from Table 5.35 that for one hazardous task case when $K=4$, $N=60$, CT1 and $\pi=0.8$, the average deviation decreases from 3.37% to 1.16% as we move from POS1 to POS2. Table 5.43 shows that for two hazardous tasks case $K=8$, $N=60$, CT2 and P2, the average deviation decreases from 4.19% to 1.47% as we move from POS1 to POS2. We see from Table 5.51 that for three hazardous tasks

case when $K=6$, $N=73$, CT2; the average deviation decreases from 4.18% to 1.13% as we move from POS1 to POS2 when success probability is P7.

POS3 refers to the instances where the hazardous tasks are selected randomly from any half of the line with $\pi=1$. The average deviation of upper bounds regarding POS3 does not show a consistent behavior, it has randomness in selection and deviations of upper bounds illustrates non monotonous pattern.

The computational performance of the upper bounds UB_1 , UB_2 and UB_3 for large networks and greater number of workstations is presented in Tables 5.53-5.70 for one, two and three hazardous tasks respectively.



Table 5.53 CPU Times (sec.) of UB₁, One Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.02	0.02	0.02	0.01	0.02	0.02	0.01	0.01	0.01
	MAX	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02	0.02
60	AVG	0.03	0.05	0.05	0.06	0.04	0.05	0.04	0.04	0.04	0.04
	MAX	0.07	0.08	0.06	0.08	0.07	0.09	0.05	0.07	0.06	0.07
73	AVG	0.10	0.15	0.12	0.16	0.12	0.13	0.07	0.13	0.09	0.10
	MAX	0.23	0.20	0.12	0.25	0.13	0.26	0.08	0.23	0.18	0.23

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02
60	AVG	0.03	0.03	0.04	0.04	0.03	0.04	0.03	0.02	0.02	0.02
	MAX	0.09	0.04	0.05	0.05	0.07	0.08	0.03	0.04	0.03	0.07
73	AVG	0.10	0.07	0.10	0.09	0.07	0.13	0.11	0.05	0.04	0.06
	MAX	0.39	0.10	0.13	0.09	0.22	0.30	0.06	0.11	0.04	0.23

Table 5.54 CPU Times (sec.) of UB₁, One Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.12	0.12	0.11	0.16	0.09	0.12	0.08	0.07	0.06	0.08
	MAX	0.43	0.17	0.13	0.23	0.15	0.23	0.10	0.15	0.10	0.19
73	AVG	0.34	0.33	0.27	0.43	0.26	0.31	0.16	0.25	0.16	0.22
	MAX	1.47	0.46	0.32	0.70	0.31	0.71	0.19	0.55	0.32	0.66

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.11	0.07	0.08	0.10	0.06	0.09	0.07	0.04	0.03	0.06
	MAX	0.55	0.10	0.10	0.14	0.15	0.20	0.08	0.11	0.05	0.19
73	AVG	0.34	0.16	0.23	0.24	0.16	0.31	0.24	0.10	0.06	0.13
	MAX	2.54	0.22	0.28	0.26	0.52	0.81	0.29	0.25	0.07	0.66

Table 5.55 CPU Times (sec.) of UB₂, One Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
60	AVG	0.04	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04	0.03
	MAX	0.08	0.06	0.08	0.08	0.07	0.10	0.08	0.08	0.12	0.04
73	AVG	0.15	0.17	0.16	0.27	0.12	0.13	0.16	0.15	0.14	0.08
	MAX	0.47	0.18	0.28	0.38	0.23	0.48	0.37	0.30	0.77	0.09

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.01	0.02	0.01	0.02	0.02	0.01	0.01	0.01
60	AVG	0.03	0.03	0.04	0.03	0.03	0.03	0.04	0.02	0.03	0.02
	MAX	0.07	0.03	0.06	0.05	0.05	0.08	0.08	0.04	0.12	0.04
73	AVG	0.10	0.09	0.19	0.08	0.09	0.10	0.14	0.07	0.12	0.06
	MAX	0.29	0.16	0.28	0.14	0.21	0.31	0.31	0.13	1.45	0.11

Table 5.56 CPU Times (sec.) of UB₂, One Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.10	0.11	0.12	0.13	0.10	0.09	0.11	0.08	0.11	0.05
	MAX	0.25	0.18	0.22	0.19	0.16	0.33	0.29	0.15	0.77	0.06
73	AVG	0.42	0.39	0.42	0.64	0.28	0.29	0.41	0.30	0.36	0.13
	MAX	1.54	0.47	0.78	0.86	0.56	1.63	1.25	0.60	2.90	0.16

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.08	0.07	0.10	0.07	0.07	0.07	0.09	0.05	0.09	0.04
	MAX	0.24	0.16	0.16	0.12	0.11	0.26	0.26	0.07	0.77	0.06
73	AVG	0.28	0.22	0.48	0.19	0.21	0.22	0.35	0.13	0.32	0.09
	MAX	0.94	0.31	0.77	0.32	0.52	1.07	1.05	0.26	1.23	0.18

Table 5.57 CPU Times (sec.) of UB₃, One Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.02
60	AVG	0.03	0.05	0.04	0.05	0.05	0.05	0.04	0.05	0.04	0.04
	MAX	0.06	0.06	0.08	0.08	0.09	0.10	0.09	0.08	0.08	0.05
73	AVG	0.18	0.22	0.16	0.19	0.15	0.17	0.15	0.17	0.13	0.13
	MAX	0.28	0.37	0.32	0.37	0.39	0.53	0.83	0.37	0.38	0.12

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
60	AVG	0.03	0.04	0.03	0.04	0.03	0.03	0.04	0.04	0.02	0.04
	MAX	0.06	0.08	0.05	0.07	0.06	0.07	0.14	0.07	0.06	0.04
73	AVG	0.09	0.14	0.09	0.16	0.08	0.09	0.14	0.14	0.05	0.11
	MAX	0.43	0.59	0.28	0.52	0.33	0.48	1.50	0.38	0.28	0.11

Table 5.58 CPU Times (sec.) of UB₃, One Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.13	0.13	0.11	0.16	0.11	0.13	0.11	0.14	0.10	0.10
	MAX	0.35	0.17	0.25	0.33	0.30	0.36	0.45	0.31	0.31	0.10
73	AVG	0.68	0.63	0.45	0.66	0.37	0.50	0.44	0.53	0.36	0.35
	MAX	1.66	0.99	1.01	1.46	1.33	1.89	4.08	1.38	1.40	0.42

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.10	0.11	0.08	0.14	0.08	0.08	0.12	0.13	0.06	0.10
	MAX	0.35	0.21	0.17	0.29	0.22	0.24	0.67	0.26	0.20	0.08
73	AVG	0.35	0.40	0.24	0.54	0.20	0.25	0.41	0.41	0.13	0.29
	MAX	2.59	1.59	0.88	2.05	1.12	1.71	2.41	1.42	1.03	0.35

Table 5.59 CPU Times (sec.) of UB₁, Two Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.02	0.03	0.03	0.04	0.02	0.03	0.03	0.02	0.02	0.02
	MAX	0.03	0.05	0.05	0.05	0.05	0.05	0.05	0.03	0.03	0.03
60	AVG	0.06	0.08	0.08	0.10	0.07	0.08	0.06	0.06	0.06	0.06
	MAX	0.11	0.13	0.10	0.14	0.11	0.14	0.08	0.11	0.10	0.11
73	AVG	0.16	0.24	0.19	0.26	0.20	0.21	0.12	0.21	0.15	0.17
	MAX	0.38	0.33	0.20	0.42	0.22	0.44	0.14	0.39	0.29	0.38

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MAX	0.03	0.03	0.03	0.05	0.03	0.03	0.03	0.03	0.03	0.03
60	AVG	0.05	0.05	0.06	0.06	0.05	0.06	0.05	0.04	0.03	0.04
	MAX	0.14	0.07	0.08	0.09	0.11	0.13	0.06	0.07	0.05	0.11
73	AVG	0.16	0.12	0.16	0.15	0.12	0.22	0.18	0.09	0.06	0.10
	MAX	0.65	0.16	0.22	0.18	0.37	0.51	0.22	0.18	0.07	0.12

Table 5.60 CPU Times (sec.) of UB₁, Two Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.19	0.19	0.18	0.26	0.15	0.20	0.14	0.12	0.10	0.14
	MAX	0.71	0.29	0.22	0.39	0.25	0.38	0.17	0.26	0.17	0.31
73	AVG	0.57	0.55	0.45	0.71	0.43	0.51	0.27	0.41	0.26	0.37
	MAX	2.45	0.77	0.54	1.17	0.52	1.18	0.32	0.91	0.53	1.10

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.18	0.11	0.14	0.16	0.10	0.15	0.12	0.07	0.05	0.09
	MAX	0.92	0.17	0.19	0.24	0.25	0.33	0.14	0.18	0.08	0.31
73	AVG	0.56	0.26	0.39	0.41	0.27	0.51	0.40	0.17	0.10	0.22
	MAX	4.23	0.37	0.47	0.43	0.86	1.35	0.48	0.42	0.12	1.10

Table 5.61 CPU Times (sec.) of UB₂, Two Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MAX	0.02	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03
60	AVG	0.06	0.08	0.08	0.09	0.07	0.07	0.07	0.07	0.07	0.05
	MAX	0.13	0.10	0.13	0.14	0.11	0.16	0.14	0.13	0.20	0.06
73	AVG	0.25	0.28	0.27	0.45	0.20	0.22	0.27	0.26	0.23	0.13
	MAX	0.78	0.30	0.47	0.63	0.38	0.80	0.61	0.50	1.29	0.16

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.02	0.01	0.02	0.02	0.02	0.02	0.01	0.02	0.01
	MAX	0.03	0.03	0.02	0.03	0.02	0.03	0.03	0.02	0.02	0.02
60	AVG	0.05	0.05	0.07	0.05	0.05	0.05	0.06	0.04	0.06	0.04
	MAX	0.12	0.06	0.09	0.09	0.08	0.13	0.13	0.06	0.20	0.06
73	AVG	0.17	0.16	0.32	0.13	0.15	0.17	0.24	0.11	0.21	0.09
	MAX	0.48	0.19	0.47	0.24	0.36	0.52	0.51	0.22	2.42	0.18

Table 5.62 CPU Times (sec.) of UB₂, Two Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.17	0.18	0.20	0.22	0.17	0.16	0.18	0.14	0.19	0.08
	MAX	0.41	0.22	0.36	0.32	0.27	0.54	0.48	0.25	1.29	0.10
73	AVG	0.70	0.65	0.70	1.06	0.47	0.49	0.68	0.49	0.60	0.21
	MAX	2.57	0.78	1.30	1.43	0.93	2.72	2.09	1.00	8.17	0.21

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.13	0.12	0.17	0.11	0.11	0.11	0.15	0.08	0.15	0.06
	MAX	0.40	0.14	0.26	0.19	0.19	0.44	0.43	0.12	1.29	0.10
73	AVG	0.47	0.36	0.81	0.31	0.34	0.37	0.58	0.21	0.54	0.15
	MAX	1.57	0.43	1.29	0.54	0.87	1.78	1.76	0.44	15.38	0.30

Table 5.63 CPU Times (sec.) of UB₃, Two Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MAX	0.02	0.02	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.03
60	AVG	0.06	0.08	0.07	0.08	0.08	0.08	0.06	0.08	0.06	0.06
	MAX	0.10	0.11	0.13	0.14	0.14	0.17	0.15	0.14	0.14	0.08
73	AVG	0.30	0.37	0.26	0.32	0.24	0.29	0.25	0.29	0.21	0.22
	MAX	0.46	0.61	0.53	0.62	0.65	0.88	1.38	0.62	0.63	0.26

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MAX	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
60	AVG	0.04	0.06	0.05	0.07	0.05	0.05	0.07	0.07	0.04	0.06
	MAX	0.10	0.13	0.09	0.12	0.11	0.11	0.23	0.12	0.09	0.06
73	AVG	0.16	0.24	0.14	0.26	0.13	0.14	0.24	0.23	0.08	0.18
	MAX	0.72	0.98	0.46	0.87	0.54	0.80	0.29	0.64	0.47	0.22

Table 5.64 CPU Times (sec.) of UB₃, Two Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.22	0.22	0.19	0.27	0.19	0.22	0.18	0.23	0.17	0.17
	MAX	0.58	0.29	0.41	0.55	0.49	0.60	0.76	0.52	0.52	0.20
73	AVG	1.13	1.06	0.74	1.11	0.62	0.83	0.73	0.88	0.59	0.58
	MAX	2.76	1.66	1.69	2.43	2.22	3.15	0.88	2.31	2.33	0.70

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.17	0.18	0.14	0.23	0.13	0.14	0.20	0.21	0.11	0.17
	MAX	0.58	0.34	0.28	0.48	0.36	0.40	1.12	0.44	0.34	0.23
73	AVG	0.58	0.67	0.40	0.91	0.33	0.42	0.69	0.68	0.22	0.48
	MAX	4.32	2.65	1.47	3.42	1.86	2.85	0.83	2.37	1.72	0.58

Table 5.65 CPU Times (sec.) of UB₁, Three Hazardous Task, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.03	0.04	0.04	0.05	0.03	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.04	0.07	0.07	0.07	0.07	0.07	0.07	0.05	0.05	0.05	0.04	0.04	0.04
60	AVG	0.08	0.10	0.10	0.13	0.09	0.10	0.08	0.08	0.09	0.08	0.08	0.08	0.08
	MAX	0.14	0.17	0.13	0.18	0.14	0.18	0.10	0.14	0.16	0.12	0.14	0.13	0.14
73	AVG	0.21	0.31	0.25	0.34	0.26	0.27	0.16	0.27	0.23	0.19	0.27	0.20	0.22
	MAX	0.49	0.43	0.26	0.55	0.29	0.57	0.18	0.40	0.47	0.34	0.51	0.38	0.49

		CT2												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.04	0.04	0.04	0.07	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
60	AVG	0.07	0.07	0.08	0.08	0.07	0.08	0.07	0.06	0.06	0.06	0.05	0.04	0.05
	MAX	0.18	0.09	0.10	0.12	0.14	0.17	0.08	0.12	0.12	0.11	0.09	0.07	0.14
73	AVG	0.21	0.16	0.21	0.20	0.16	0.29	0.23	0.14	0.18	0.18	0.12	0.08	0.13
	MAX	0.85	0.21	0.29	0.21	0.48	0.66	0.14	0.36	0.38	0.32	0.23	0.09	0.49

Table 5.66 CPU Times (sec.) of UB₁, Three Hazardous Task, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.25	0.25	0.23	0.34	0.20	0.26	0.18	0.18	0.20	0.18	0.16	0.13	0.18
	MAX	0.92	0.38	0.28	0.51	0.33	0.49	0.22	0.33	0.36	0.29	0.34	0.22	0.40
73	AVG	0.74	0.72	0.59	0.92	0.56	0.66	0.35	0.55	0.50	0.42	0.53	0.34	0.48
	MAX	3.19	1.00	0.71	1.52	0.68	1.53	0.42	0.93	1.11	0.86	1.18	0.69	1.43

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.23	0.14	0.18	0.21	0.13	0.20	0.16	0.11	0.13	0.14	0.09	0.07	0.12
	MAX	1.20	0.22	0.22	0.31	0.33	0.43	0.19	0.28	0.27	0.27	0.23	0.10	0.40
73	AVG	0.73	0.34	0.51	0.53	0.35	0.66	0.52	0.29	0.40	0.40	0.22	0.13	0.29
	MAX	5.50	0.48	0.61	0.56	1.12	1.76	0.62	0.83	0.96	0.83	0.55	0.16	1.43

Table 5.67 CPU Times (sec.) of UB₂, Three Hazardous Task, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.03	0.04	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
60	AVG	0.08	0.10	0.10	0.12	0.09	0.09	0.09	0.09	0.09	0.08	0.09	0.09	0.07
	MAX	0.17	0.13	0.17	0.18	0.14	0.21	0.18	0.16	0.23	0.13	0.17	0.26	0.08
73	AVG	0.33	0.36	0.35	0.59	0.26	0.29	0.35	0.30	0.29	0.26	0.34	0.30	0.17
	MAX	1.01	0.39	0.61	0.82	0.49	1.04	0.79	0.57	1.36	0.47	0.65	1.68	0.16

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.03	0.01	0.03	0.03	0.03	0.03	0.02	0.03	0.02	0.01	0.03	0.01
	MAX	0.04	0.04	0.03	0.04	0.03	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03
60	AVG	0.07	0.07	0.09	0.07	0.07	0.07	0.08	0.06	0.07	0.07	0.05	0.08	0.05
	MAX	0.16	0.08	0.12	0.12	0.10	0.17	0.17	0.09	0.21	0.12	0.08	0.26	0.08
73	AVG	0.22	0.21	0.42	0.17	0.20	0.22	0.31	0.17	0.25	0.21	0.14	0.27	0.12
	MAX	0.62	0.13	0.61	0.31	0.47	0.68	0.66	0.38	1.91	0.45	0.29	3.15	0.23

Table 5.68 CPU Times (sec.) of UB₂, Three Hazardous Task, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.22	0.23	0.26	0.29	0.22	0.21	0.23	0.20	0.23	0.17	0.18	0.25	0.10
	MAX	0.53	0.28	0.47	0.42	0.35	0.70	0.62	0.34	1.19	0.38	0.33	1.68	0.13
73	AVG	0.91	0.85	0.91	1.38	0.61	0.64	0.88	0.62	0.71	0.58	0.64	0.78	0.27
	MAX	3.34	1.02	1.69	1.86	1.21	3.54	2.72	1.25	1.85	1.50	1.30	1.94	0.32

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.17	0.16	0.22	0.14	0.14	0.14	0.20	0.12	0.17	0.14	0.10	0.20	0.08
	MAX	0.52	0.19	0.34	0.25	0.25	0.57	0.56	0.20	1.12	0.34	0.16	1.68	0.13
73	AVG	0.61	0.47	1.05	0.40	0.44	0.48	0.75	0.36	0.59	0.47	0.27	0.70	0.20
	MAX	2.04	0.56	1.68	0.70	1.13	2.31	2.29	0.85	1.71	1.34	0.57	1.84	0.39

Table 5.69 CPU Times (sec.) of UB₃, Three Hazardous Task, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.03	0.03	0.04	0.04	0.04	0.04	0.03	0.04	0.04	0.03	0.04	0.04	0.04
60	AVG	0.08	0.10	0.09	0.10	0.10	0.10	0.08	0.10	0.09	0.08	0.10	0.08	0.08
	MAX	0.13	0.14	0.17	0.18	0.18	0.22	0.20	0.18	0.20	0.15	0.18	0.18	0.10
73	AVG	0.39	0.48	0.34	0.42	0.31	0.38	0.33	0.34	0.33	0.31	0.38	0.27	0.29
	MAX	0.60	0.79	0.69	0.81	0.85	1.14	1.79	0.83	0.98	1.03	0.81	0.82	0.35

		CT2												
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.2$			$\pi z1= 0.2, \pi z2= 0.2, \pi z3= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
60	AVG	0.05	0.08	0.07	0.09	0.07	0.07	0.09	0.08	0.06	0.08	0.09	0.05	0.08
	MAX	0.13	0.17	0.12	0.16	0.14	0.14	0.30	0.15	0.13	0.19	0.16	0.12	0.08
73	AVG	0.21	0.31	0.18	0.34	0.17	0.18	0.31	0.23	0.14	0.27	0.30	0.10	0.23
	MAX	0.94	1.27	0.60	1.13	0.70	1.04	3.25	0.77	0.83	1.74	0.83	0.61	0.27

Table 5.70 CPU Times (sec.) of UB₃, Three Hazardous Task, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.29	0.29	0.25	0.35	0.25	0.29	0.23	0.27	0.25	0.23	0.30	0.22	0.22
	MAX	0.75	0.38	0.53	0.72	0.64	0.78	0.99	0.66	0.73	0.60	0.68	0.68	0.21
73	AVG	1.47	1.38	0.96	1.44	0.81	1.08	0.95	0.98	0.92	0.85	1.14	0.77	0.75
	MAX	3.59	2.16	2.20	3.16	2.89	4.10	1.14	2.94	3.56	1.68	3.00	3.03	0.90

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	0.22	0.23	0.18	0.30	0.17	0.18	0.26	0.22	0.16	0.24	0.27	0.14	0.22
	MAX	0.75	0.44	0.36	0.62	0.47	0.52	1.46	0.52	0.48	0.81	0.57	0.44	0.17
73	AVG	0.75	0.87	0.52	1.18	0.43	0.55	0.90	0.66	0.42	0.76	0.88	0.29	0.62
	MAX	5.62	3.45	1.91	4.45	2.42	3.71	1.08	2.75	2.97	1.27	3.08	2.24	0.74

We observe that the computational times delivered from upper bounds are short. For small networks UB_1 , UB_2 , UB_3 run almost at same speed and in negligible times such that the average is *below 0.01 seconds* and maximum is *below 0.03 seconds* over all problem combinations in one hazardous task case. For large networks CPU time increases slightly such that the average is *below 0.68 seconds* and maximum is *below 4.08 seconds* over all problem combinations in one hazardous task case. Note that for large networks UB_1 and UB_2 run almost at same speed however CPU time increases very slightly in UB_3 . Similar observations hold for two and three hazardous task cases. Hence, the problem parameters affecting the CPU time performance are mostly apparent in large networks especially when workstation numbers are high. The tables of processing times with large networks ($N=60$ and $N=73$) and greater number of workstations ($K=6$ and $K=8$) are presented at Tables 5.53-5.70. The data for small networks and few numbers of workstations are presented at Appendix D

The number of tasks and number of workstations are the most significant parameters that affect the complexity of the mathematical model solutions. This is due to the fact that as N or K values increase the number of integer variables increase, hence problem size increases.

We can observe the effect of the increase in number of tasks especially when K is higher. For instance, for one hazardous task case and UB_2 , the average CPU time increases from *0.11 seconds* to *0.39 seconds* as we increase $N=60$ to $N=73$ with $K=8$, CT1, POS1, $\pi=0.8$ combination. For two hazardous task case and UB_2 , when $K=8$, CT1, POS1, P2 the average CPU times with $N=60$ and $N=73$ is *0.18 and 0.65 seconds*, respectively. For three hazardous task case and UB_2 , when $K=8$, CT1, POS1, P6 the average CPU times with $N=60$ and $N=73$ is *0.23 and 0.85 seconds*, respectively. Similar observations hold for the other problem combinations.

The increase in CPU time can be observed with the increase in K , such that for one hazardous task case and UB_1 , when $N=73$, CT1, $\pi=0.5$, POS2 the average CPU time increases from *0.13* to *0.31 seconds*, as K increases from $K=6$ to $K=8$. For two hazardous task case and UB_2 , when $N=60$, CT1, P4, POS2 the average CPU time

with $K=6$ and $K=8$, are *0.07 and 0.19 seconds*, respectively. For three hazardous task case and UB_2 , the average CPU times with $K=6$ and $K=8$ are *0.10 and 0.23 seconds* when $N=60$, CT1, P6, POS1.

The next parameter that we analyze is the cycle time and we could not observe a consistent effect of the cycle time on the processing time of the upper bounds.

Moreover, the results reveal that the success probability is not a significant parameter for the CPU time changes. Finally, we analyze the effect of the positions of the hazardous tasks on the solution times. We could not see a consistent behavior with the change in the position of hazardous tasks when there are one or two or three hazardous tasks.

5.2.8 Discussion of Lower Bound Results

The Lower Bounds (LB) studied in our experiments are presented such that;

- Lower Bound 1 (LB_1): Lower Bound delivered by forward pass
- Lower Bound 2 (LB_2): Lower Bound delivered by forward and backward passes

The analysis of lower bounds in our experiments are based on the performance measures presented below.

- The average and maximum percent deviations of each Lower Bound from Optimal SP solution

$$\%Dev = \frac{(Z^* - LB_i)}{Z^*} * 100 \quad (Only\ for\ known\ optimal\ solutions)$$

Z^* : Optimal Objective Function value of SP Problem (Expected Net Revenue)

LB_i : Objective Function Value of Lower Bound i (Expected Net Revenue delivered by Lower Bound i)

- The average and maximum CPU Times (sec.)

The performance of lower bounds for one hazardous, two hazardous and three hazardous task cases are discussed according to the defined performance measures. Discussions are based on the percent deviations of lower bounds for large networks that are reported in Tables 5.71-5.88 for one, two and three hazardous tasks respectively. The solutions of small networks are presented at Appendix C.

Table 5.71 Percent Deviation of LBs, One Hazardous Task, N=60, K=4

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.97	6.78	6.69	5.20	9.67	7.15	5.90	12.00	9.43	8.78
	MAX	1.10	9.37	10.63	7.13	11.47	10.18	8.20	15.18	10.86	11.26
LB2	AVG	0.41	2.33	1.85	1.66	2.28	2.09	2.15	2.46	2.30	1.79
	MAX	0.60	5.01	3.23	4.18	4.52	4.33	4.35	4.47	4.11	3.97

		CT2									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.06	3.41	2.22	2.44	4.78	4.50	5.59	6.57	5.90	5.78
	MAX	0.16	4.93	3.93	4.40	6.45	9.02	8.14	9.81	10.00	8.26
LB2	AVG	0.05	1.92	1.34	1.31	2.24	1.82	1.61	2.43	2.09	1.69
	MAX	0.10	4.49	2.67	3.89	3.30	2.77	3.22	4.14	3.70	3.72

Table 5.72 Percent Deviation of LBs, One Hazardous Task, N=60, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.96	6.66	6.20	4.67	8.89	7.71	5.45	11.55	8.39	8.86
	MAX	1.20	10.06	9.82	7.64	10.43	11.29	7.38	14.43	10.43	11.16
LB2	AVG	0.60	2.18	1.94	1.84	2.37	2.18	2.02	2.51	2.34	2.10
	MAX	0.63	4.84	3.70	4.45	4.95	4.82	4.48	4.88	4.89	4.12

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.17	4.46	2.86	3.46	6.95	4.84	5.93	7.38	6.73	5.99
	MAX	0.32	7.20	5.28	5.50	9.94	11.48	7.99	9.53	8.69	8.20
LB2	AVG	0.16	2.13	1.37	1.83	2.51	2.09	1.80	2.67	2.58	2.16
	MAX	0.26	4.56	2.94	4.36	3.90	4.46	3.71	4.70	5.21	4.13

Table 5.73 Percent Deviation of LBs, One Hazardous Task, N=60, K=8

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.19	8.37	7.79	5.85	11.17	9.67	6.82	13.00	10.51	11.10
	MAX	1.49	12.00	11.69	10.20	13.70	14.14	9.83	16.18	13.65	14.55
LB2	AVG	0.74	2.73	2.42	2.29	2.95	2.70	2.59	3.10	2.67	2.58
	MAX	0.77	6.06	4.61	5.54	6.16	5.98	5.54	6.04	6.04	5.05

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 60		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.19	5.12	3.20	4.21	7.15	5.43	6.22	7.64	7.54	7.46
	MAX	0.35	8.07	5.91	6.16	11.13	12.85	8.95	10.73	9.73	9.82
LB2	AVG	0.17	2.39	2.16	2.05	2.81	2.34	2.02	3.00	2.60	2.42
	MAX	0.29	5.10	3.29	4.88	4.37	4.99	4.15	4.63	5.83	4.62

Table 5.74 Percent Deviation of LBs, One Hazardous Task, N=73, K=4

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.91	5.39	4.83	3.75	6.96	5.95	4.43	9.01	7.08	6.59
	MAX	1.32	7.66	8.69	5.83	9.37	8.32	6.70	12.41	8.87	9.21
LB2	AVG	0.52	1.76	1.39	1.26	1.98	1.82	1.49	2.02	1.89	1.64
	MAX	0.86	7.66	4.94	6.39	4.18	4.00	4.03	4.24	3.45	3.61

		CT2									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.04	2.97	1.93	2.13	4.74	3.84	4.38	5.56	5.00	4.90
	MAX	0.07	5.60	4.46	4.99	6.62	7.32	6.60	7.71	7.87	6.50
LB2	AVG	0.03	1.46	1.02	0.99	1.96	1.60	1.41	1.97	1.70	1.38
	MAX	0.55	3.40	2.02	2.94	3.66	3.07	3.58	4.01	3.43	3.45

Table 5.75 Percent Deviation of LBs, One Hazardous Task, N=73, K=6

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.94	5.56	4.84	3.82	7.12	6.17	4.49	9.28	7.12	6.74
	MAX	1.56	7.89	7.70	6.00	9.83	9.62	7.00	12.78	9.23	9.39
LB2	AVG	0.66	1.81	1.61	1.53	2.04	1.87	1.59	2.08	2.22	1.67
	MAX	0.94	7.89	6.02	7.25	4.31	4.19	3.90	4.37	4.38	3.68

		CT2									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.24	3.06	1.96	2.38	4.88	3.91	4.43	5.73	5.22	4.99
	MAX	0.45	5.77	4.59	4.77	6.82	7.88	5.49	7.94	7.24	6.84
LB2	AVG	0.18	1.50	1.03	1.17	2.02	1.68	1.45	2.03	1.96	1.64
	MAX	0.42	3.50	2.26	3.35	3.77	4.30	3.58	4.13	4.57	3.63

Table 5.76 Percent Deviation of LBs, One Hazardous Task, N=73, K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.97	5.72	4.93	4.21	7.33	6.35	4.53	9.56	7.73	8.16
	MAX	1.56	8.13	7.91	6.90	10.13	8.45	7.26	13.16	11.10	11.84
LB2	AVG	0.66	1.87	1.66	1.56	2.11	1.92	1.85	2.15	1.56	1.71
	MAX	0.95	4.13	5.17	5.12	4.44	4.31	3.99	4.50	4.50	3.76

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 73		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.24	3.16	1.97	2.59	5.03	3.94	4.52	5.90	5.82	5.76
	MAX	1.56	5.94	4.36	4.53	7.03	8.12	5.65	8.18	7.42	7.49
LB2	AVG	0.18	1.54	1.40	1.33	2.08	1.73	1.50	2.11	1.48	1.67
	MAX	0.43	3.61	2.33	3.45	3.88	4.43	3.69	4.25	5.35	4.24

Table 5.77 Percent Deviation of LBs, Two Hazardous Task, N=60, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.38	9.69	9.55	7.42	13.82	10.21	8.43	-	13.47	12.54
	MAX	1.57	13.38	15.19	10.19	16.38	14.54	11.71	-	15.51	16.09
LB2	AVG	0.58	3.33	2.64	2.38	3.26	2.98	3.08	-	3.28	2.55
	MAX	0.86	7.16	4.62	5.97	6.46	6.18	6.22	-	5.87	5.67

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.09	4.87	3.17	3.49	6.83	6.44	7.98	9.38	8.43	8.26
	MAX	0.23	7.05	5.62	6.28	9.21	12.89	11.62	14.01	14.29	11.81
LB2	AVG	0.07	2.75	1.92	1.87	3.20	2.60	2.30	3.47	2.98	2.42
	MAX	0.14	6.42	3.82	5.56	4.71	3.95	4.61	5.91	5.29	5.32

Table 5.78 Percent Deviation of LBs, Two Hazardous Task, N=60, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.37	9.52	8.86	6.67	12.70	11.01	7.78	16.50	11.99	12.66
	MAX	1.72	14.36	14.02	10.92	14.91	16.13	10.54	20.62	14.90	15.94
LB2	AVG	0.85	3.12	2.78	2.63	3.39	3.11	2.89	3.58	3.34	3.00
	MAX	0.90	6.92	5.28	6.35	7.07	6.88	6.40	6.97	6.99	5.88

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.24	6.37	4.08	4.95	9.93	6.92	8.48	10.55	9.62	8.55
	MAX	0.45	10.29	7.54	7.85	14.19	16.39	11.41	13.62	12.41	11.72
LB2	AVG	0.22	3.04	1.95	2.62	3.58	2.98	2.57	3.82	3.69	3.09
	MAX	0.37	6.51	4.20	6.22	5.57	6.36	5.30	6.71	7.44	5.89

Table 5.79 Percent Deviation of LBs, Two Hazardous Task, N=60, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.70	11.96	11.12	8.36	15.95	13.81	9.74	18.58	15.01	15.86
	MAX	2.13	17.15	16.70	14.57	19.57	20.19	14.04	23.12	19.49	20.79
LB2	AVG	1.06	3.90	3.46	3.27	4.22	3.86	3.70	4.43	3.82	3.68
	MAX	1.10	8.66	6.58	7.91	8.81	8.55	7.92	8.62	8.63	7.21

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.27	7.32	4.57	6.01	10.22	7.75	8.88	10.92	10.78	10.66
	MAX	0.50	11.53	8.45	8.80	15.90	18.36	12.78	15.33	13.90	14.02
LB2	AVG	0.25	3.41	3.09	2.93	4.01	3.34	2.88	4.28	3.71	3.46
	MAX	0.41	7.29	4.70	6.97	6.24	7.13	5.93	6.62	8.33	6.60

Table 5.80 Percent Deviation of LBs, Two Hazardous Task, N=73, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.30	7.71	6.89	5.36	9.94	8.51	6.32	-	10.11	9.41
	MAX	1.89	10.94	12.42	8.33	13.39	11.89	9.57	-	12.68	13.16
LB2	AVG	0.74	2.51	1.99	1.79	2.84	2.60	2.13	-	2.70	2.34
	MAX	1.23	10.94	7.06	9.12	5.98	5.72	5.75	-	4.93	5.15

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.06	4.25	2.76	3.04	6.77	5.49	6.25	7.95	7.14	6.99
	MAX	0.11	8.00	6.38	7.14	9.46	10.45	9.42	11.02	11.24	9.28
LB2	AVG	0.04	2.08	1.45	1.41	2.81	2.29	2.02	2.82	2.42	1.97
	MAX	0.79	4.86	2.89	4.21	5.22	4.38	5.11	5.73	4.91	4.93

Table 5.81 Percent Deviation of LBs, Two Hazardous Task, N=73, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.34	7.94	6.91	5.45	10.17	8.82	6.42	13.26	10.18	9.63
	MAX	2.23	11.27	11.00	8.57	14.05	13.74	10.00	18.26	13.19	13.41
LB2	AVG	0.94	2.59	2.31	2.18	2.92	2.68	2.27	2.98	3.17	2.39
	MAX	1.35	11.27	8.60	10.35	6.16	5.99	5.57	6.24	6.26	5.26

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.34	-	2.80	3.40	-	5.58	6.33	-	7.46	7.13
	MAX	0.64	-	6.55	6.82	-	11.26	7.84	-	10.34	9.77
LB2	AVG	0.25	-	1.48	1.67	-	2.41	2.08	-	2.81	2.35
	MAX	0.60	-	3.22	4.78	-	6.15	5.12	-	6.53	5.18

Table 5.82 Percent Deviation of LBs, Two Hazardous Task, N=73, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.38	-	-	6.01	-	9.07	6.47	-	11.04	11.66
	MAX	2.23	-	-	9.86	-	14.93	10.38	-	15.86	16.91
LB2	AVG	0.94	-	-	2.23	-	2.75	2.64	-	2.23	2.44
	MAX	1.36	-	-	10.61	-	6.16	5.70	-	6.43	5.37

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.35	4.51	2.81	3.70	-	5.64	6.46	8.43	8.32	8.23
	MAX	2.23	8.49	6.22	6.48	-	11.60	8.07	11.69	10.60	10.69
LB2	AVG	0.25	2.21	2.00	1.90	-	2.48	2.14	3.02	2.11	2.38
	MAX	0.62	5.15	3.32	4.93	-	6.33	5.27	6.07	7.65	6.06

Table 5.83 Percent Deviation of LBs, Three Hazardous Task, N=60, K=4

		CT1												
π & POS		π=1	πz1= 0.8, πz2= 0.8, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.8			πz1= 0.8, πz2= 0.2, πz3= 0.2			πz1= 0.2, πz2= 0.2, πz3= 0.2		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.46	10.27	7.87	10.12	14.65	8.94	10.82	16.41	11.11	12.55	18.17	13.29	14.28
	MAX	1.66	14.18	12.51	13.90	17.36	12.73	15.03	20.17	14.02	16.68	22.98	15.31	18.32
LB2	AVG	0.61	3.50	2.50	2.77	3.42	3.13	3.23	3.55	2.90	3.34	3.68	2.68	3.44
	MAX	0.90	7.52	4.37	6.95	6.78	6.49	6.53	6.74	5.64	7.09	6.70	4.79	7.66

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.10	5.16	3.36	3.70	7.24	6.83	8.46	8.59	7.88	8.61	9.94	8.94	8.76
	MAX	0.24	7.47	5.96	6.66	9.76	13.66	12.32	12.31	14.41	12.42	14.85	15.15	12.52
LB2	AVG	0.07	2.89	2.02	1.96	3.36	2.73	2.42	3.50	2.93	2.48	3.64	3.13	2.54
	MAX	0.15	6.74	4.01	5.84	4.95	4.15	4.84	5.58	4.85	5.21	6.21	5.55	5.59

Table 5.84 Percent Deviation of LBs, Three Hazardous Task, N=60, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.45	10.09	7.07	9.39	13.46	8.25	11.67	15.48	10.48	12.54	17.49	12.71	13.42
	MAX	1.82	15.22	11.19	15.37	15.80	12.09	15.81	18.83	13.94	16.35	21.86	15.79	16.90
LB2	AVG	0.89	3.28	2.92	2.76	3.56	3.27	3.03	3.66	3.39	3.09	3.76	3.51	3.15
	MAX	0.95	7.27	5.54	6.67	7.42	7.22	6.72	7.37	7.28	6.45	7.32	7.34	6.17

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.25	6.75	4.32	5.25	10.53	7.34	8.99	10.85	8.77	9.03	11.18	10.20	9.06
	MAX	0.48	10.91	7.99	8.32	15.04	12.37	12.09	14.74	12.76	12.26	14.44	13.15	12.42
LB2	AVG	0.23	3.19	2.05	2.75	3.76	3.13	2.70	3.89	3.50	2.97	4.01	3.87	3.24
	MAX	0.39	6.84	4.41	6.53	5.85	6.68	5.57	6.45	7.25	5.87	7.05	7.81	6.18

Table 5.85 Percent Deviation of LBs, Three Hazardous Task, N=60, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.80	12.68	8.86	11.79	16.91	10.32	14.64	18.30	13.12	15.73	19.69	15.91	16.81
	MAX	2.26	18.18	13.31	20.54	20.74	15.09	21.10	22.63	17.88	21.57	24.51	20.66	22.04
LB2	AVG	1.11	4.10	3.43	3.63	4.43	3.85	4.05	4.54	3.86	4.03	4.65	3.89	4.01
	MAX	1.16	9.09	6.53	8.79	9.25	8.53	8.68	9.15	8.66	8.27	9.05	8.79	7.86

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.29	7.76	4.84	6.37	10.83	8.22	9.41	11.20	9.76	10.42	11.58	11.30	11.43
	MAX	0.53	12.22	8.96	9.33	16.85	14.46	13.55	16.55	14.52	14.29	16.25	14.57	15.03
LB2	AVG	0.26	3.58	3.02	3.24	4.21	3.08	3.51	4.35	3.36	3.70	4.49	3.63	3.90
	MAX	0.43	7.65	4.59	7.72	6.55	6.57	7.22	6.75	7.37	7.33	6.95	8.16	7.43

Table 5.86 Percent Deviation of LBs, Three Hazardous Task, N=73, K=4

		CT1													
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$				$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
LB1	AVG	1.38	8.17	5.68	7.30	10.54	6.70	9.02	12.09	8.34	9.87	-	9.97	-	
	MAX	2.00	11.60	9.24	11.35	14.19	9.36	13.66	16.49	10.94	14.32	-	12.51	-	
LB2	AVG	0.78	2.64	1.88	2.09	2.98	2.24	2.73	3.01	2.35	2.78	-	2.46	-	
	MAX	1.29	5.49	4.67	6.65	6.28	4.92	7.37	6.32	4.70	6.80	-	4.49	-	

		CT2													
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$				$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
LB1	AVG	0.06	4.51	2.93	3.22	7.18	5.82	6.63	7.80	6.61	7.10	-	7.41	7.57	
	MAX	0.12	8.48	6.76	7.57	10.03	11.08	9.99	10.85	11.37	10.02	-	11.67	10.05	
LB2	AVG	0.04	2.18	1.48	1.52	2.95	2.12	2.40	2.96	2.14	2.47	-	2.15	2.54	
	MAX	0.10	5.10	2.95	4.55	5.48	4.06	6.08	5.75	4.21	6.22	-	4.36	6.36	

Table 5.87 Percent Deviation of LBs, Three Hazardous Task, N=73, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.42	8.42	5.78	7.32	10.78	6.81	9.35	12.42	8.51	10.07	-	10.21	10.79
	MAX	2.36	11.95	9.20	11.52	14.89	10.60	14.56	17.12	11.91	14.79	-	13.23	15.03
LB2	AVG	0.99	2.72	2.29	2.43	3.07	2.31	2.81	3.24	2.45	3.07	-	2.51	3.33
	MAX	1.42	5.83	5.52	6.52	6.47	5.38	6.90	6.80	5.34	7.12	-	5.45	7.33

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.36	4.64	2.97	3.60	7.39	5.91	6.71	8.03	6.74	7.31	-	7.56	7.91
	MAX	0.68	8.74	6.94	7.23	10.33	7.94	8.31	11.18	10.21	11.57	-	10.48	11.84
LB2	AVG	0.26	2.25	1.55	1.75	3.04	2.18	2.53	3.04	2.33	2.74	-	2.47	2.95
	MAX	0.63	5.25	3.38	5.02	5.65	5.57	6.23	5.92	5.65	6.37	-	5.73	6.50

Table 5.88 Percent Deviation of LBs, Three Hazardous Task, N=73, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.46	-	6.37	-	-	6.86	-	-	-	-	-	-	-
	MAX	2.36	-	10.23	-	-	11.29	-	-	-	-	-	-	-
LB2	AVG	0.99	-	2.34	-	-	2.38	-	-	-	-	-	-	-
	MAX	1.43	-	5.72	-	-	5.33	-	-	-	-	-	-	-

		CT2												
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.2$			$\pi z1= 0.2, \pi z2= 0.2, \pi z3= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.37	4.78	2.98	3.92	7.61	5.98	6.85	8.27	7.40	7.79	8.94	-	-
	MAX	2.36	9.00	6.59	6.87	10.64	8.30	8.55	11.52	10.77	9.94	12.39	-	-
LB2	AVG	0.26	2.32	2.00	2.10	3.13	2.25	2.60	3.15	2.37	2.41	3.17	-	-
	MAX	0.65	5.41	3.31	5.45	5.82	5.74	6.41	6.10	6.40	7.03	6.37	-	-

For one hazardous task case, over all problem combinations the average deviations of LB_2 is better than average deviations of LB_1 . This is an expected result since LB_1 refers to the objective function value of forward pass whereas LB_2 refers to the best objective function value of forward and backward pass. LB_1 is improved by LB_2 , hence returns smaller and quite satisfactory deviations than LB_1 . For instance, when $N=73$ and there is one hazardous task, with $K=8$, CT_2 , $\pi=0.5$ and $POS1$ the average deviation decreases from 5.03% to 2.08% as LB_1 is improved by LB_2 . For two hazardous task case, with the same problem combination when success probability is $P2$ the average deviation decreases from 4.51% to 2.21% as we move from LB_1 to LB_2 . For three hazardous task case, with the same problem combination when success probability is $P6$ the average deviation decreases from 4.78% to 2.32% as LB_1 is improved by LB_2 . The improvement in deviations are observed in all problem combinations.

The increase in number of tasks (N), do not deliver a deterioration in average deviations. Contrary, the heuristic works better in large networks. Nevertheless, the increase in number of workstation (K) which also leads the increase in number of variables, causes a slight increase in the lower bound deviations. For example, for one hazardous task case and LB_2 , when $N=73$, CT_1 , $\pi=0.5$ and $POS2$, the average deviation increases from 1.87% to 1.92% as number of workstations increases from $K=6$ to $K=8$. With the same problem combination; when there are two hazardous tasks and success probability is $P3$ the average deviation of LB_2 increase from 2.68% to 2.75%, when there are three hazardous tasks and success probability is $P6$, it increases from 2.29% to 2.34% as we increase number of workstations from $K=6$ to $K=8$.

We observe that an increase in cycle time (CT) decreases the average deviations of lower bounds. The increase in cycle time increases the opportunity of task selection and assignment, hence affects positively the improvement of lower bounds while decreasing the average deviations. For instance, when $N=60$ and there is one hazardous task the average deviation of LB_2 decreases from 2.59% to 2.02% as we increase cycle time from CT_1 to CT_2 with $K=8$, $\pi=0.5$, $POS3$ combination. With

the same problem combination; when there are two hazardous tasks and success probability is P3 the average deviation of LB_2 decreases from 3.70% to 2.88%, when there are three hazardous tasks and success probability is P6 it decreases from 3.63% to 3.24% as we increase cycle time from CT1 to CT2.

We observe that deviations of lower bounds are significantly affected by the success probabilities (π values). As success probability increases the deviations are decreasing. We already know that hazardous task affects its succeeding workstations and completion of other tasks, hence the efficiency of bounds affected from the success probabilities as well. For one hazardous task case and LB_2 , when $K=6$, $N=73$, CT2 and POS2, the average deviation decreases from 1.96% to 1.68%, then to 1.03% and finally decreases to 0.18% as success probability increases $\pi=0.2$ through $\pi=1$. For two hazardous task case, with the same problem combination and LB_2 , the average deviation decreases from 2.81% to 2.41%, then to 1.48% and finally decreases to 0.25% as success probability increases from P4 through P1. For three hazardous task case, with the same problem combination and LB_2 , the average deviation decreases from 2.47% to 2.33%, then to 2.18%, 1.55% and 0.26% respectively as success probability increases from P9 through P5. Similar observations hold for the other problem combinations.

Position of the hazardous tasks (POS values) is other important parameter that affects the deviations of lower bounds. Recall that POS1 and POS2 refer to the instances where the hazardous tasks are selected from the first half and second half of the disassembly line with $\pi=1$, respectively. The average deviation of lower bounds from optimal solutions with POS2 is smaller than the average deviation with POS1. This is expected as the adverse effects of the hazardous tasks is less when they are located at later workstations. For one hazardous task case when $K=6$, $N=73$, CT1 and $\pi=0.8$, the average deviation decreases from 1.81% to 1.61% as we move from POS1 to POS2. With the same problem combination; when there are two hazardous tasks and success probability is P2 the average deviation of LB_2 decreases from 2.59% to 2.31%, when there are three hazardous tasks and success probability is P6 it decreases from 2.72% to 2.29% as we move from POS1 to POS2.

POS3 refers to the instances where the hazardous tasks are selected randomly from any half of the line with $\pi=1$. The average deviation of lower bounds does not show a consistent behavior with POS3. The observations regarding hazardous task positions are valid for all problem combinations of each lower bound.

The computational performance of the lower bounds LB_1 and LB_2 for large networks and greater number of workstations is summarized in Tables 5.89-5.100 for one, two and three hazardous tasks respectively.



Table 5.89 CPU Times (sec.) of LB₁, One Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	4.46	5.00	5.13	5.46	4.99	4.47	5.46	4.40	4.73	5.02
	MAX	5.75	6.27	7.26	6.91	6.32	6.02	6.43	6.76	7.49	7.15
60	AVG	8.66	10.62	9.86	9.51	10.19	10.80	10.22	8.44	10.32	8.88
	MAX	11.68	15.44	11.48	13.44	12.36	17.33	13.55	14.02	13.84	11.51
73	AVG	16.79	22.55	18.97	16.57	20.80	26.11	19.15	16.19	22.49	15.73
	MAX	23.74	38.05	28.17	26.17	24.15	49.86	28.54	29.10	25.58	18.53

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	3.67	4.00	4.09	4.35	3.93	4.05	4.31	4.23	4.10	4.06
	MAX	4.86	4.80	6.10	5.99	4.80	4.83	5.89	5.70	5.92	5.99
60	AVG	7.41	7.45	7.17	6.11	7.52	7.32	6.89	5.24	7.61	5.71
	MAX	6.74	12.47	9.16	5.18	7.27	8.64	6.38	6.00	9.01	5.59
73	AVG	14.96	13.89	12.59	8.57	14.39	13.21	11.01	6.49	14.11	8.02
	MAX	9.36	32.40	13.75	4.48	11.03	15.48	6.90	6.32	13.73	5.22

Table 5.90 CPU Times (sec.) of LB₁, One Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	13.70	15.72	12.78	12.06	16.49	16.76	14.53	10.12	16.00	11.00
	MAX	19.18	22.01	17.89	16.88	23.09	23.46	20.34	14.17	22.40	15.40
73	AVG	26.58	33.38	24.59	21.01	33.65	40.50	27.23	19.43	34.87	19.48
	MAX	37.21	46.73	34.43	29.41	47.11	56.70	38.12	27.20	48.82	27.27

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	11.73	11.04	9.30	7.74	12.17	11.35	9.79	6.29	11.79	7.07
	MAX	17.94	19.93	17.76	14.73	19.27	13.88	16.96	12.45	14.19	15.60
73	AVG	23.69	20.56	16.32	10.87	23.28	20.49	15.65	7.79	21.87	9.94
	MAX	31.02	51.78	21.64	14.08	34.05	24.85	19.54	16.79	25.61	15.23

Table 5.91 CPU Times (sec.) of LB₂, One Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	10.81	7.76	6.88	8.63	7.70	6.82	8.37	6.37	6.80	7.76
	MAX	15.02	11.43	12.61	11.27	13.50	10.47	12.12	10.19	13.02	10.23
60	AVG	17.97	16.24	16.49	14.19	15.37	16.83	15.71	11.79	15.25	11.74
	MAX	32.07	22.94	29.69	20.78	23.62	28.71	24.23	30.18	21.25	19.76
73	AVG	29.86	34.01	39.51	23.35	30.69	41.58	29.48	21.84	34.19	17.75
	MAX	68.47	46.06	69.90	38.30	41.32	78.74	48.47	89.37	41.67	38.16

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	8.32	6.78	7.21	7.72	6.35	6.40	7.43	6.81	6.47	6.06
	MAX	11.11	10.17	10.09	14.01	10.28	8.46	14.41	11.42	12.37	9.36
60	AVG	15.61	13.23	17.02	8.56	16.87	12.10	11.11	6.99	9.47	6.52
	MAX	21.85	18.52	23.83	11.98	23.62	16.94	15.55	9.79	13.26	9.13
73	AVG	29.27	25.81	20.04	9.48	22.18	22.86	16.60	7.18	17.86	10.01
	MAX	40.98	36.13	28.06	13.27	31.05	32.00	23.24	10.05	25.00	12.81

Table 5.92 CPU Times (sec.) of LB₂, One Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	23.16	21.22	18.25	15.72	24.36	22.90	21.16	13.21	18.87	12.28
	MAX	47.52	22.87	42.04	18.15	28.82	33.17	30.88	48.27	26.14	16.66
73	AVG	38.49	44.44	43.72	25.86	48.63	56.55	39.70	24.47	42.32	20.56
	MAX	101.46	62.21	98.99	33.46	50.41	90.98	61.76	142.97	56.33	32.18

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	20.12	17.28	19.30	9.48	18.81	16.46	14.96	7.83	14.15	8.48
	MAX	32.46	22.33	38.63	18.85	26.17	17.08	21.23	30.11	16.44	16.49
73	AVG	37.73	33.72	22.18	11.50	35.14	31.09	22.35	8.05	26.24	11.93
	MAX	64.00	53.95	104.43	21.40	42.87	39.87	24.54	49.63	31.49	17.82

Table 5.93 CPU Times (sec.) of LB₁, Two Hazardous Task, K=6

		CT1										
π & POS		π=1	πz1= 0.8, πz2= 0.8				πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
47	AVG	8.11	9.09	9.32	9.93	9.08	8.13	9.92	7.99	8.61	9.12	
	MAX	10.46	11.40	13.19	12.56	11.50	10.95	11.69	12.29	13.62	13.01	
60	AVG	15.74	19.31	17.93	17.29	18.53	19.64	18.59	15.34	18.76	16.15	
	MAX	21.24	28.08	20.88	24.44	22.47	31.51	24.64	25.50	25.17	20.93	
73	AVG	30.52	41.00	34.49	30.12	37.82	47.48	34.82	29.44	40.90	28.60	
	MAX	43.16	69.19	41.39	47.58	43.91	90.65	51.90	52.91	46.51	33.68	

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	6.67	7.28	7.43	7.91	7.15	7.37	7.83	7.70	7.46	7.38
	MAX	8.83	8.72	11.09	10.90	8.73	8.78	10.71	10.37	10.76	10.88
60	AVG	13.47	13.55	13.05	11.10	13.68	13.30	12.52	9.53	13.83	10.38
	MAX	16.16	22.67	16.65	13.32	16.42	15.72	15.02	11.44	16.60	12.46
73	AVG	27.20	25.25	22.89	15.58	26.17	24.02	20.01	11.80	25.65	14.59
	MAX	32.64	58.91	25.00	18.70	31.40	28.14	24.01	14.16	30.78	17.51

Table 5.94 CPU Times (sec.) of LB₁, Two Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	24.91	28.58	23.24	21.93	29.98	30.46	26.42	18.40	29.08	20.00
	MAX	25.01	44.88	27.89	22.30	35.98	50.60	26.89	27.39	39.62	24.00
73	AVG	48.33	60.69	44.70	38.20	61.18	73.63	49.50	35.32	63.39	35.42
	MAX	50.81	110.58	53.64	43.41	73.42	145.57	56.65	56.85	73.22	42.50

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	21.32	20.07	16.91	14.08	22.13	20.63	17.80	11.43	21.45	12.85
	MAX	25.58	36.23	20.29	16.90	26.56	25.24	21.36	13.72	25.79	15.42
73	AVG	43.07	37.38	29.67	19.76	42.33	37.25	28.45	14.16	39.76	18.07
	MAX	51.68	94.15	35.60	23.71	50.80	45.19	34.14	16.99	47.71	21.68

Table 5.95 CPU Times (sec.) of LB₂, Two Hazardous Task, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	19.65	14.10	12.52	15.68	14.00	12.39	15.23	11.58	12.37	14.11
	MAX	27.31	20.78	22.92	20.49	24.55	19.03	22.03	18.52	23.68	18.59
60	AVG	32.66	29.53	29.98	25.80	27.95	30.61	28.57	21.44	27.73	21.34
	MAX	58.30	41.72	53.97	37.78	42.95	52.20	44.06	54.86	38.63	35.92
73	AVG	54.28	61.84	71.83	42.45	55.79	75.60	53.61	39.72	62.17	32.27
	MAX	124.49	83.75	127.10	69.63	75.13	143.17	88.13	162.50	63.03	69.39

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	15.13	12.32	13.11	14.04	11.55	11.64	13.51	12.38	11.76	11.03
	MAX	20.19	18.49	18.35	25.48	18.68	15.38	26.20	20.76	22.50	17.02
60	AVG	28.38	24.05	21.85	15.56	21.58	22.00	20.19	12.71	17.21	11.86
	MAX	39.82	40.74	49.60	18.42	24.11	26.89	30.29	34.22	20.65	14.23
73	AVG	53.21	46.93	36.44	17.23	40.32	41.56	30.18	13.06	30.19	16.75
	MAX	78.53	89.75	134.09	20.68	48.38	47.00	35.01	56.41	39.25	23.45

Table 5.96 CPU Times (sec.) of LB₂, Two Hazardous Task, K=8

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	42.11	38.58	33.18	28.58	44.29	41.63	38.47	24.02	34.31	22.32
	MAX	86.40	46.30	76.43	34.30	52.39	60.31	56.14	87.77	41.17	30.29
73	AVG	69.98	80.79	79.49	47.02	88.42	102.82	72.18	44.49	76.94	37.75
	MAX	184.47	113.11	179.97	60.83	106.10	165.41	112.29	259.95	92.33	58.52

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	36.59	31.42	24.18	30.23	34.20	29.92	27.19	14.24	27.30	31.40
	MAX	59.01	40.60	70.24	36.09	44.46	38.90	38.59	54.74	35.49	39.99
73	AVG	68.60	61.31	40.33	52.08	63.90	56.53	40.64	14.63	41.18	37.33
	MAX	116.37	98.10	189.87	65.63	83.07	73.49	44.61	90.24	53.53	59.95

Table 5.97 CPU Times (sec.) of LB₁, Three Hazardous Task, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	9.98	11.18	11.46	12.21	11.17	10.00	12.20	10.50	10.30	11.71	9.83	10.59	11.22
	MAX	12.87	14.02	16.22	15.45	14.15	13.47	14.38	14.63	15.11	15.19	15.12	16.75	16.00
60	AVG	19.36	23.75	22.05	21.27	22.79	24.16	22.87	20.83	23.62	21.37	18.87	23.07	19.86
	MAX	26.13	34.54	25.68	30.06	27.64	38.76	30.31	29.50	34.86	28.03	31.37	30.96	25.74
73	AVG	37.54	50.43	42.42	37.05	46.52	58.40	42.83	41.36	54.35	39.00	36.21	50.31	35.18
	MAX	53.09	85.10	50.90	58.52	54.01	111.50	63.84	59.54	84.35	52.63	65.08	57.21	41.43

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	8.20	8.95	9.14	9.73	8.79	9.07	9.63	9.13	9.12	9.35	9.47	9.18	9.08
	MAX	10.86	10.73	13.64	13.41	10.74	10.80	13.17	11.75	12.02	13.28	12.76	13.23	13.38
60	AVG	16.57	16.67	16.05	13.65	16.83	16.36	15.40	14.27	16.68	14.08	11.72	17.01	12.77
	MAX	19.88	27.88	20.48	16.38	20.20	19.34	18.48	17.12	19.75	16.90	14.06	20.16	15.32
73	AVG	33.46	31.06	28.15	19.16	32.19	29.54	24.61	23.35	30.55	21.28	14.51	31.55	17.95
	MAX	40.15	72.46	30.75	22.99	38.63	34.61	29.53	28.02	32.66	25.54	17.41	37.86	21.54

Table 5.98 CPU Times (sec.) of LB₁, Three Hazardous Task, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	30.64	35.15	28.59	26.97	36.88	37.47	32.50	29.75	36.62	28.55	22.63	35.77	24.60
	MAX	30.76	55.20	34.31	27.43	44.26	62.24	33.07	34.45	55.49	34.26	33.69	48.73	29.52
73	AVG	59.45	74.65	54.98	46.99	75.25	90.56	60.89	59.35	84.27	52.23	43.44	77.97	43.57
	MAX	62.50	136.01	65.98	53.39	90.30	179.05	69.68	69.37	134.56	62.68	69.93	90.06	52.28

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	26.22	24.69	20.80	17.32	27.22	25.37	21.89	20.64	25.88	18.85	14.06	26.38	15.81
	MAX	31.46	44.56	24.96	20.78	32.66	31.05	26.27	24.77	31.38	22.62	16.87	31.72	18.97
73	AVG	52.98	45.98	36.49	24.30	52.07	45.82	34.99	34.74	47.36	28.61	17.42	48.90	22.23
	MAX	63.58	115.80	43.79	29.16	62.48	55.58	41.99	41.69	51.96	34.33	20.90	58.68	26.68

Table 5.99 CPU Times (sec.) of LB₂, Three Hazardous Task, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	24.2	17.3	15.4	19.3	17.2	15.2	18.7	15.7	15.2	18.0	14.2	15.2	17.4
	MAX	33.6	25.6	28.2	25.2	30.2	23.4	27.1	26.5	26.3	25.0	22.8	29.1	22.9
60	AVG	40.2	36.3	36.9	31.7	34.4	37.7	35.1	30.4	35.9	30.7	26.4	34.1	26.3
	MAX	71.7	51.3	66.4	46.5	52.8	64.2	54.2	60.2	55.9	49.2	67.5	47.5	44.2
73	AVG	66.8	76.1	88.4	52.2	68.6	93.0	65.9	58.7	84.7	52.8	48.9	76.5	39.7
	MAX	153.1	103.0	156.3	85.6	92.4	176.1	108.4	146.1	126.8	96.9	199.9	91.8	85.4

		CT2												
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.2$			$\pi z1= 0.2, \pi z2= 0.2, \pi z3= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
47	AVG	18.6	15.2	16.1	17.3	14.2	14.3	16.6	14.7	14.4	15.1	15.2	14.5	13.6
	MAX	24.8	22.7	22.6	31.3	23.0	18.9	32.2	24.3	23.3	26.6	25.5	27.7	20.9
60	AVG	34.9	29.6	26.9	19.1	26.5	27.1	24.8	21.1	24.1	19.7	15.6	21.2	14.6
	MAX	49.0	50.1	61.0	22.7	29.7	33.1	37.3	35.9	26.0	25.9	42.1	25.4	17.5
73	AVG	65.5	57.7	44.8	21.2	49.6	51.1	37.1	32.8	41.1	26.4	16.1	37.9	21.5
	MAX	96.6	110.4	164.9	25.4	59.5	57.8	43.1	53.8	49.3	31.7	69.4	45.4	25.8

Table 5.100 CPU Times (sec.) of LB₂, Three Hazardous Task, K=8

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	51.8	47.5	40.8	35.2	54.5	51.2	47.3	42.0	46.7	37.4	29.5	42.2	27.5
	MAX	106.3	56.9	94.0	40.6	64.4	74.2	69.1	86.2	60.7	53.2	108.0	50.6	37.3
73	AVG	86.1	99.4	97.8	57.8	108.8	126.5	88.8	81.7	110.6	65.2	54.7	94.6	45.5
	MAX	226.9	152.7	221.4	74.8	112.7	203.5	138.1	216.2	141.2	105.1	319.7	113.6	72.0

		CT2												
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.2$			$\pi z1= 0.2, \pi z2= 0.2, \pi z3= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
60	AVG	45.0	38.7	29.7	21.2	42.1	36.8	33.4	29.8	31.5	24.4	17.5	29.2	15.3
	MAX	72.6	49.9	86.4	25.4	50.5	44.2	40.1	51.8	37.8	29.9	67.3	38.0	42.3
73	AVG	84.4	75.4	49.6	23.5	78.6	69.5	50.0	48.3	53.9	33.2	18.0	58.7	26.7
	MAX	143.1	121.0	233.5	35.7	94.3	83.4	60.0	78.8	64.7	39.8	111.0	70.4	32.0

We observe that the computational times delivered for LB_2 are greater than the computational times delivered for LB_1 . This is an expected result as LB_1 runs for only forward pass whereas LB_2 runs for both forward and backward passes. For small networks and few numbers of workstations, lower bounds run much faster than the large networks with greater number of workstations. The average CPU time is mainly below 6 seconds over all problem combinations for small networks with $K=2$ and $K=4$ in one hazardous task case. For large networks with greater number of workstations CPU time increases such that the average is below 56 seconds over all problem combinations in one hazardous task case. Similar observations hold for two and three hazardous task cases. Hence, the problem parameters affecting the CPU time performance mostly apparent in large networks especially when workstation numbers are high. The tables of processing times regarding large networks ($N=60$ and $N=73$) and greater number of workstations ($K=6$ and $K=8$) are presented at Tables 5.89-5.100. The data for small networks and few numbers of workstations are presented at Appendix E.

The number of tasks and number of workstations are the most significant parameters that affect the complexity of the mathematical model solutions. This is the result of inflation in number of variables, hence the increase in problem size. We can observe the difference regarding the increase in number of tasks easily with large networks. For instance, for one hazardous task case and LB_2 , the average CPU time increases from *13.23 seconds* to *25.81 seconds* as we increase $N=60$ to $N=73$ with $K=6$, CT2, POS1, $\pi=0.8$ combination. For two hazardous task case and LB_2 , when $K=6$, CT1, POS1, P4 the average CPU times with $N=60$ and $N=73$ is *21.44* and *39.72 seconds*, respectively. For three hazardous task case and LB_2 , when $K=8$, CT2, POS2, P6 the average CPU times with $N=60$ and $N=73$ is *29.74* and *49.61 seconds*, respectively. Similar observations hold for the other problem combinations.

As we mentioned, the increase in CPU time can be observed with the increase in K . For one hazardous task case and LB_2 , when $N=60$, the average CPU time increases from *12.10* to *16.46 seconds* as K increases from $K=6$ to $K=8$, with combination CT2, $\pi=0.5$, POS2. For two hazardous task case and LB_2 when $N=60$, the average

CPU time increases from *24.05 to 31.42 seconds* as K increases from $K=6$ to $K=8$, with combination CT2, P2, POS1. For three hazardous task case and LB_2 the average CPU times with $K=6$ and $K=8$ is *29.58 and 38.65 seconds* respectively when $N=60$, CT2, P6, POS1.

We also analyze the effect of cycle time on lower bounds speed. We could not observe a notable effect of the cycle time on the complexity of the solutions.

Moreover, we analyze the effect of success probability on the solution times. We could not see a monotonous behavior with the change in the success probability when there are one or two or three hazardous tasks. Similar observations are also valid for the effect of the positions of the hazardous tasks. The results reveal that position of hazardous tasks is not a significant parameter for the CPU time changes.

CHAPTER 6

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this study, we consider a disassembly line balancing problem with hazardous tasks whose successful completions are uncertain. The successful completion of these tasks results in the normal flow of the line whereas their failure prevents the processing of the remaining tasks on that unit.

Our problem is to assign the tasks to the workstations so as to maximize the total expected net revenue. We formulate the problem as a two-stage stochastic mixed-integer program where the assignment of the tasks to the workstations is decided in the first-stage, before the resolution of the uncertainty.

An extensive experiment is designed to see the effects of the parameters on the difficulty of attaining optimal solutions. We find that the size of the model increases drastically with an increase in the number of tasks. Therefore, the optimal solutions to large-sized problem instances are not likely to be obtained within reasonable computational times. The number of workstations and cycle time do not affect the objective function values, computational times and number of unsolved instances in a monotonic pattern for one hazardous task case. However, the computational time and number of unsolved instances increase in two and three hazardous task cases with an increase in the number of workstations. We have dissimilarity in success probabilities and hazardous task positions for one, two and three hazardous task cases which makes the results incomparable. Only when success probabilities are listed in a decreasing way for both cases, the results show similar behaviors for the SP Model. By the decrease in success probabilities, the objective function values decrease in all cases. For the hazardous task position factor, in general the objective function values are greater when hazardous task(s) is (are) selected close to the end part of the networks, since less effect is spent the assigned non-hazardous tasks. Both

for one hazardous and two hazardous cases, there is a third position (POS3) where the hazardous task(s) is (are) selected randomly from the assigned tasks without considering workstation assignments. As expected, the average objective function value related with the third position lies between the other positions, in general.

We also investigate the value of the stochastic solution (VSS), which is the benefit that can be obtained by solving the stochastic model rather than its deterministic counterpart. For this purpose, the expected result of using the expected value solution (EEV) is obtained and compared with the SP solution through the use of the SP/EEV ratio. We observe from the results of one hazardous task experiments that SP/EEV is higher and hence VSS is higher when the level of uncertainty in the problem is higher (i.e., $\pi=0.5$ as opposed to $\pi=0.2$ or $\pi=0.8$ for this case). Note that, one may consider a relationship between the problem complexity and level of uncertainty. However, the results of SP illustrate that there is not an increase in the complexity of the problem by an increase in the level of uncertainty. The level of uncertainty presents insight regarding value of stochastic solution, but does not increase any variables or it does not cause any change in problem parameters regarding the complexity. We observe from two and three hazardous task experiments that, at a given level of uncertainty, SP/EEV ratio decreases with the decreasing probability of success for the hazardous tasks. Moreover, the results of our VSS experiments (for all number of hazardous tasks) indicate that SP/EEV is higher when the hazardous task is performed at earlier workstations (POS1) rather than later workstations (POS2) for most of the problem settings.

Finally, we propose a heuristic procedure that uses the optimal linear programming relaxations of our mixed integer linear programming model. Our aim is to solve large-sized instances that our model could not return solutions in reasonable times. We propose three upper bounds (UB₁, UB₂, UB₃) and two lower bounds (LB₁, LB₂). UB₁ is the Linear Programming Relaxation (LPR) that is strengthened by the exact and heuristic cuts (UB₂, UB₃). Lower Bounds are returned based on a heuristic method where the relaxed model with exact cuts are solved by assigning the largest fractional assignment variable to 1 or 0. The best objective function value is selected

at each step. The lower bound obtained by this method, forward pass, is named as LB_1 . LB_1 is improved by LB_2 via backtracking which is applied to both hazardous and non-hazardous tasks. The upper bounds and lower bounds returned satisfactory solutions especially for large networks. The percent deviations from the optimal solutions and CPU times of the bounds are analyzed according to the changing parameters. We observe that both upper and lower bounds are affected significantly from the success probabilities and hazardous task positions. The CPU times of the upper and lower bounds are increasing especially by the increases in number of tasks and number of workstations.

Using the results of our experiments, we derive some managerial insights for the potential real life applications of our findings. The hazardous tasks are the most significant part of the study over which some conclusions could be derived. Other things being equal, the maximum yield would be obtained once the hazardous tasks are assigned to the latest possible workstations to decrease their adverse effects on other tasks.

An important attribute related with the hazardous tasks is the success probability. As success probability increases, the hazardous tasks get closer to a non-hazardous task case which is a desirable attitude. Other things being equal, the maximum yield would be obtained at the highest attainable success probabilities.

For future research, several extensions of our study can be investigated some of which are discussed now. The objective function can be defined in different ways such as minimizing the expected cost over all scenarios instead of maximizing the expected net revenue. In the cost minimization problem, one could consider the effect of number of workstations since their increase would increase the fixed and operational costs. Another extension could be to define multi-criteria problems, like maximizing the expected net revenue with minimum number of workstations or generating all nondominated solutions with respect to the maximum expected net revenue and minimum number of workstations objectives.

The future research may also consider different assumptions regarding the hazardous tasks. We considered the complications in a disassembly line due to the hazardous tasks and the disappearing unit and normal flow scenarios. Early leaving unit is another option that can be considered instead of disappearing unit case, where some of the tasks cannot be accomplished due to the hazardous tasks and the unit leaves the workstation early. Some different failure structures may be handled such as the case where a failure of a task would only cause the failure of its successors and early leaving unit scenario can be applied. Moreover, the self skipping unit scenario where only the current workstation is skipped in case of a failure can be studied. In that scenario, the succeeding workstations will still function. However, the precedence structure should be reconsidered due to the failure in the current workstation and these complications will make the problem even more complex.

In our study, the discrete events (success or failure of hazardous task) are considered. Based on those events, the expected value problem cannot be defined in the standard way. Instead, we consider a threshold value (0.5) and construct the expected value scenario in a way that the hazardous tasks having success probability greater than the threshold value succeed, and the remaining ones fail. In the future studies, different strategies for constructing the expected value scenario can be considered.

Another noteworthy topic for future research might be to define the hazardous tasks using different parameters. The processing times of the hazardous tasks can be higher than the non-hazardous tasks due to their special handling requirements. Moreover, their cost and revenue structures could be different than those of the non-hazardous tasks. Removing the hazardous tasks might increase the costs. On the other hand, removing the hazardous tasks might have environmental benefits that could increase the total yield.

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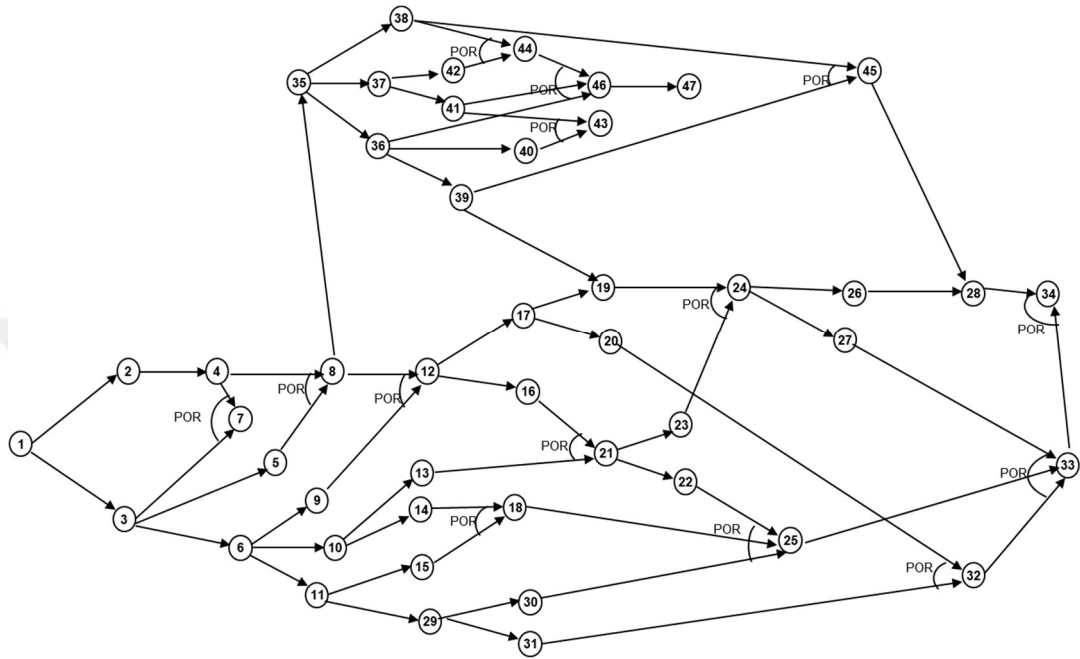
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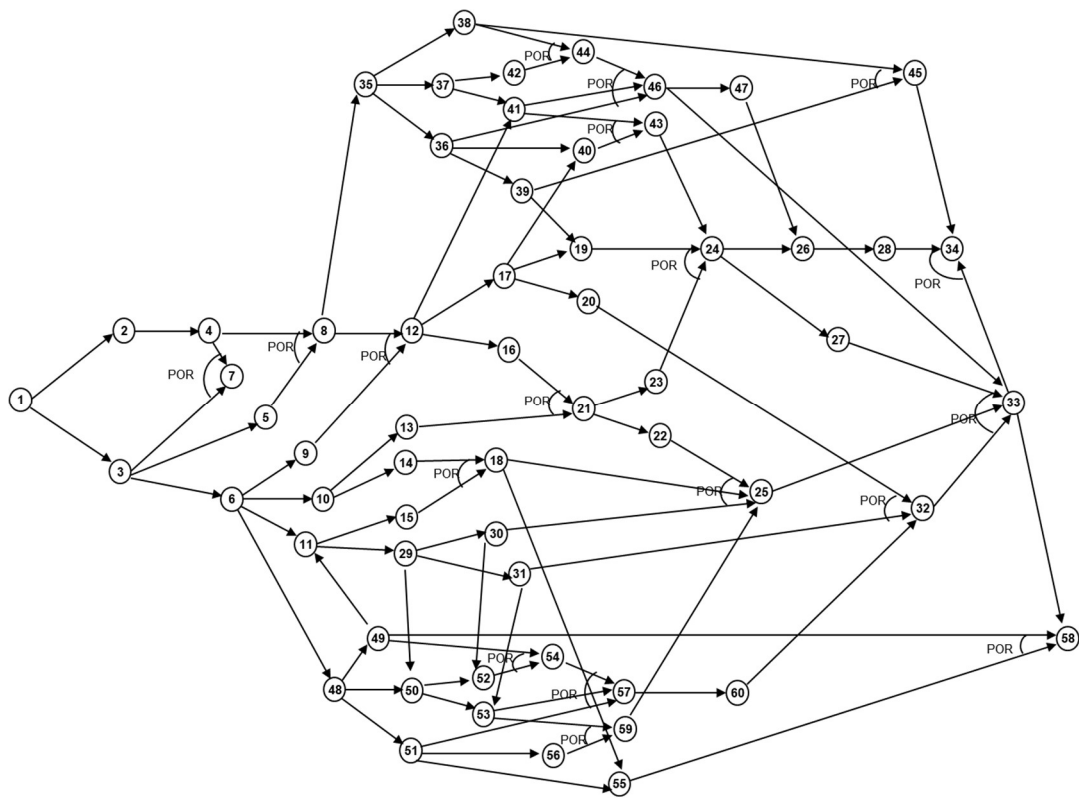
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APPENDICES

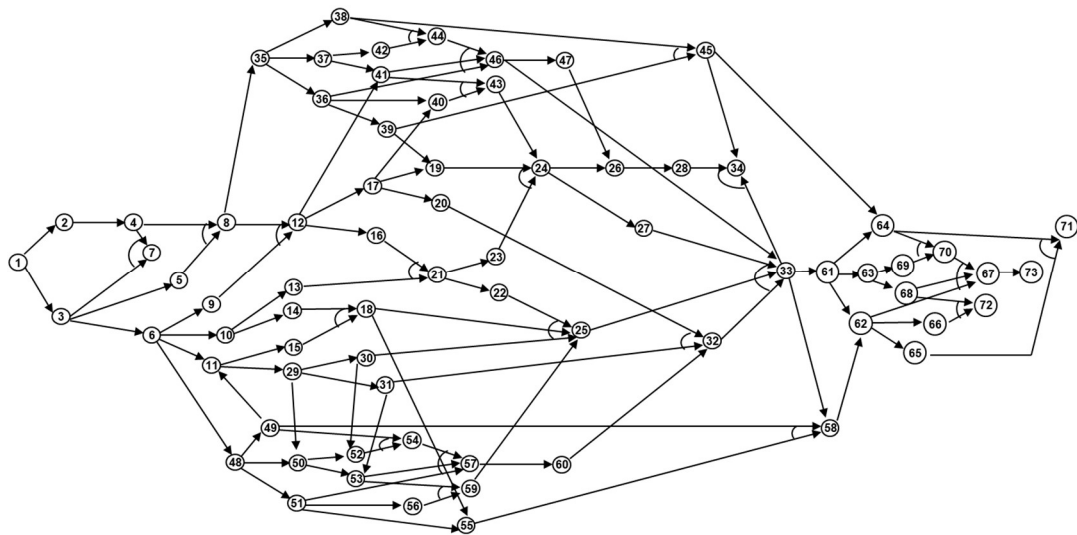
A. Networks



Precedence Diagram of Network (47 tasks)



Precedence Diagram of Network (60 tasks)



Precedence Diagram of Network (73 tasks)

B. Percent Deviation of Upper Bounds

For small Networks $N=22, 34, 47$ the Percent Deviation of Upper Bounds (UB) for one hazardous, two hazardous and three hazardous cases are presented in the Tables below

Table 6.1 Percent Deviation of UBs, One Hazardous Task, $N=22, K=2$

		CT1										
π & POS		$\pi=1$	$\pi=0.8$				$\pi=0.5$			$\pi=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
UB1	AVG	0.36	5.18	3.39	4.01	10.81	5.61	5.68	14.22	6.66	5.69	
	MAX	2.27	10.18	6.56	7.18	13.38	7.91	9.03	18.92	15.59	8.21	
UB2	AVG	0.36	4.80	3.03	3.91	8.20	3.54	4.35	9.65	3.74	4.42	
	MAX	2.27	9.07	5.50	7.40	11.87	5.27	8.47	13.47	10.60	7.20	
UB3	AVG	0.07	3.72	2.47	3.61	7.02	3.10	3.70	8.09	3.14	3.72	
	MAX	0.67	6.07	5.60	7.65	12.99	5.75	9.41	12.91	7.07	6.59	

		CT2										
π & POS		$\pi=1$	$\pi=0.8$				$\pi=0.5$			$\pi=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
UB1	AVG	0.21	5.44	1.65	3.16	16.29	4.20	8.86	28.94	5.43	14.83	
	MAX	0.30	10.34	2.41	4.19	21.39	8.69	14.89	31.03	7.65	24.16	
UB2	AVG	0.21	5.18	1.62	3.02	14.01	3.67	7.70	24.40	4.24	12.64	
	MAX	0.30	10.00	2.37	4.01	21.66	8.11	14.02	26.74	6.17	19.98	
UB3	AVG	0.06	4.00	0.83	0.78	9.19	2.75	2.14	15.45	2.47	7.35	
	MAX	0.13	8.14	1.69	1.23	13.33	7.80	5.81	18.49	4.02	13.89	

Table 6.2 Percent Deviation of UBs, One Hazardous Task, N=22, K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.16	5.97	2.73	3.06	15.03	6.15	6.99	24.76	8.74	9.39
	MAX	0.50	12.24	4.20	4.92	23.71	15.15	13.37	29.81	16.00	15.18
UB2	AVG	0.16	5.93	2.72	3.06	14.89	6.08	6.97	24.31	8.35	9.16
	MAX	0.48	11.85	4.11	4.87	23.40	14.73	12.70	25.08	11.58	14.92
UB3	AVG	0.00	2.89	2.40	2.71	10.52	5.40	6.31	18.27	7.49	8.27
	MAX	0.00	3.10	4.92	5.04	16.65	13.69	11.80	15.45	15.26	14.08

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.59	7.12	1.51	3.49	18.64	2.98	8.26	33.25	4.74	13.74
	MAX	1.41	14.51	2.39	5.98	30.97	4.51	13.79	38.40	8.78	16.99
UB2	AVG	0.59	6.12	1.11	3.19	17.62	2.46	7.24	31.89	4.22	11.48
	MAX	1.41	12.47	1.76	5.56	30.87	3.65	11.20	35.97	7.29	13.67
UB3	AVG	0.00	3.86	0.73	1.47	11.48	1.88	3.54	22.15	3.40	3.63
	MAX	0.00	9.96	3.77	2.95	15.48	4.53	5.27	25.91	3.80	5.10

Table 6.3 Percent Deviation of UBs, One Hazardous Task, N=34, K=2

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 34		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.49	5.16	2.88	4.80	6.47	3.14	5.43	10.22	4.02	6.06
	MAX	1.49	7.28	6.68	7.48	11.41	4.95	7.99	12.59	6.45	8.37
UB2	AVG	0.49	4.96	2.58	3.98	5.38	3.00	4.66	7.74	3.42	5.33
	MAX	1.49	6.80	4.57	6.27	10.13	4.82	7.40	11.14	6.41	7.39
UB3	AVG	0.04	3.57	1.13	3.20	4.47	1.33	3.24	7.39	1.58	3.28
	MAX	0.39	5.49	2.61	4.60	9.81	1.76	6.42	11.07	5.43	5.25

		CT2									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 34		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.11	4.94	2.75	3.41	11.55	4.28	6.46	20.43	6.90	11.04
	MAX	2.05	8.67	7.61	10.55	16.88	7.88	11.93	24.16	12.31	17.65
UB2	AVG	0.11	4.22	1.59	2.12	10.76	3.91	4.01	17.00	3.95	6.79
	MAX	0.53	7.50	3.72	6.32	13.96	7.58	6.13	23.21	8.74	13.29
UB3	AVG	0.00	3.68	1.49	1.61	7.66	1.56	2.21	13.18	1.63	4.39
	MAX	0.00	7.47	2.77	3.94	10.79	4.78	4.49	19.40	3.72	10.72

Table 6.4 Percent Deviation of UBs, One Hazardous Task, N=34, K=4

		CT1									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N = 34		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.59	6.20	3.46	5.76	7.77	3.76	6.52	12.27	4.82	7.28
	MAX	1.17	9.48	6.78	9.24	11.08	7.77	14.83	14.66	8.85	9.68
UB2	AVG	0.59	5.95	3.09	4.77	6.46	3.60	5.59	9.29	4.10	6.40
	MAX	1.19	7.97	6.72	7.67	9.42	6.35	9.56	11.10	7.28	8.85
UB3	AVG	0.05	4.29	1.36	3.84	5.36	1.60	3.88	8.87	1.90	3.93
	MAX	0.23	7.32	3.39	9.83	8.39	2.81	8.80	13.84	3.52	6.51

		CT2									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N = 34		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.13	5.93	3.30	4.09	13.86	5.13	7.75	24.51	8.28	13.25
	MAX	0.26	7.71	8.98	6.62	18.64	9.44	11.28	25.86	16.61	19.32
UB2	AVG	0.13	5.06	1.91	2.54	12.91	4.70	4.81	20.40	4.74	8.14
	MAX	0.27	6.58	6.61	4.12	18.12	14.02	7.06	25.74	9.67	13.24
UB3	AVG	0.00	4.41	1.78	1.93	9.19	1.87	2.65	15.82	1.95	5.27
	MAX	0.00	7.22	4.54	3.95	14.65	6.27	5.05	24.63	6.24	9.70

Table 6.5 Percent Deviation of UBs, One Hazardous Task, N=47, K=2

		CT1									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.35	3.69	2.06	3.43	4.62	2.24	3.88	7.30	2.87	4.33
	MAX	0.72	5.53	3.37	6.24	6.46	3.46	6.52	9.66	4.44	5.60
UB2	AVG	0.34	3.54	1.84	2.84	3.85	2.14	3.33	5.53	2.44	3.81
	MAX	0.52	5.33	3.17	5.73	5.72	3.27	6.23	7.38	3.49	4.87
UB3	AVG	0.03	2.55	0.81	2.28	3.19	0.95	2.31	5.28	1.13	2.34
	MAX	0.06	4.07	2.05	5.12	4.45	1.47	6.19	6.77	1.63	4.67

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.08	3.53	1.97	2.44	8.25	3.05	4.62	14.59	4.93	7.89
	MAX	0.16	5.34	4.50	5.15	13.54	7.16	8.08	21.28	8.89	10.93
UB2	AVG	0.08	3.01	1.14	1.51	7.69	2.79	2.86	12.14	2.82	4.85
	MAX	0.16	4.95	2.88	3.72	12.35	5.57	5.08	17.62	6.68	8.83
UB3	AVG	0.00	2.63	1.06	1.15	5.47	1.12	1.58	9.42	1.16	3.14
	MAX	0.00	4.40	1.67	2.94	10.61	2.61	2.77	12.50	3.44	6.35

Table 6.6 Percent Deviation of UBs, One Hazardous Task, N=47, K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.42	4.43	2.47	4.11	5.55	2.69	4.66	8.76	3.44	5.20
	MAX	0.50	8.07	3.94	5.58	6.48	4.38	6.28	10.68	5.32	6.65
UB2	AVG	0.42	4.25	2.21	3.41	4.61	2.57	4.00	6.63	2.93	4.57
	MAX	0.50	6.61	3.22	4.82	5.13	4.27	6.13	7.91	5.28	6.16
UB3	AVG	0.03	3.06	0.97	2.74	3.83	1.14	2.77	6.34	1.36	2.81
	MAX	0.08	4.95	2.16	4.52	4.59	1.92	5.56	7.82	3.19	4.10

		CT2									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.10	4.24	2.36	2.92	9.90	3.66	5.54	17.51	5.91	9.47
	MAX	0.19	5.87	4.36	4.79	13.43	7.95	11.91	21.39	10.48	14.62
UB2	AVG	0.10	3.62	1.36	1.82	9.22	3.35	3.44	14.57	3.39	5.82
	MAX	0.12	4.61	3.62	3.95	13.12	5.75	6.11	17.84	7.03	8.81
UB3	AVG	0.00	3.15	1.27	1.38	6.56	1.34	1.90	11.30	1.39	3.77
	MAX	0.00	4.26	3.53	3.43	10.66	2.63	5.17	15.77	3.29	6.74

Table 6.7 Percent Deviation of UBs, One Hazardous Task, N=47, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.49	5.09	2.84	4.73	6.38	3.09	5.36	10.08	3.96	5.98
	MAX	0.62	8.38	6.60	7.42	7.55	6.35	6.91	12.01	7.25	9.42
UB2	AVG	0.49	4.89	2.54	3.92	5.31	2.95	4.60	7.63	3.37	5.26
	MAX	0.60	6.27	4.09	7.41	7.14	5.55	6.29	8.86	5.64	6.84
UB3	AVG	0.04	3.52	1.12	3.15	4.40	1.31	3.19	7.29	1.56	3.23
	MAX	0.14	6.25	2.19	6.55	5.96	3.00	5.76	8.85	2.40	5.28

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.11	4.87	2.71	3.36	11.39	4.21	6.37	20.14	6.80	10.89
	MAX	0.14	6.68	5.63	4.78	12.76	7.95	8.37	24.24	11.07	12.47
UB2	AVG	0.11	4.16	1.57	2.09	10.61	3.86	3.95	16.76	3.90	6.69
	MAX	0.13	5.90	3.64	3.49	12.71	7.35	5.05	18.81	7.49	8.47
UB3	AVG	0.00	3.62	1.47	1.58	7.55	1.54	2.18	12.99	1.60	4.33
	MAX	0.00	5.51	1.79	3.29	10.26	2.48	2.98	18.73	2.91	5.43

Table 6.8 Percent Deviation of UBs, Two Hazardous Task, N=22, K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.37	12.04	6.52	7.45	28.54	7.76	11.72	46.19	9.88	18.92
	MAX	2.34	23.67	12.62	13.35	35.33	10.95	18.62	61.46	23.11	27.30
UB2	AVG	0.37	9.71	6.24	6.62	15.20	6.51	7.28	30.96	7.21	13.31
	MAX	2.34	18.34	11.34	12.51	21.99	9.70	14.18	43.23	20.44	21.69
UB3	AVG	0.21	8.89	4.81	4.60	9.16	4.88	5.12	23.95	5.45	10.85
	MAX	2.18	14.51	10.91	9.74	16.95	9.07	13.02	38.22	12.27	19.23

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.33	13.97	4.40	5.74	44.85	8.93	10.23	89.22	14.74	18.01
	MAX	0.46	26.57	6.41	7.60	58.90	18.48	17.20	95.65	20.76	29.34
UB2	AVG	0.33	10.33	4.37	5.66	33.05	7.90	8.49	67.09	13.19	16.05
	MAX	0.46	19.93	6.38	7.51	51.10	17.45	15.45	73.52	19.20	25.38
UB3	AVG	0.13	9.09	2.89	3.65	26.72	4.12	4.05	42.83	9.54	10.49
	MAX	0.26	18.49	5.91	5.70	38.76	11.66	11.02	51.25	15.56	19.82

Table 6.9 Percent Deviation of UBs, Two Hazardous Task, N=22, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.33	10.67	4.49	5.74	17.30	7.72	8.87	39.86	10.40	14.95
	MAX	1.04	21.89	6.91	9.22	27.29	19.00	16.98	47.99	19.06	24.16
UB2	AVG	0.31	10.50	4.45	5.64	16.51	7.35	8.36	27.65	9.90	14.07
	MAX	0.94	20.98	6.74	8.98	25.94	17.81	15.23	28.52	13.72	22.92
UB3	AVG	0.00	3.65	2.67	4.12	7.10	2.66	6.20	20.58	5.00	9.87
	MAX	0.00	3.93	5.46	7.66	11.24	6.76	11.60	23.41	10.19	16.80

		CT2									
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8$			$\pi z1= 0.8, \pi z2= 0.2$			$\pi z1= 0.2, \pi z2= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.63	11.05	3.93	9.10	26.63	7.70	23.05	55.69	11.68	39.04
	MAX	1.51	22.54	6.19	15.58	44.24	11.65	38.46	64.30	21.63	48.27
UB2	AVG	0.63	11.05	3.93	8.09	21.10	7.32	20.66	45.60	11.20	31.53
	MAX	1.51	22.54	6.19	14.07	36.97	10.87	31.94	51.45	19.37	37.55
UB3	AVG	0.00	4.74	0.86	4.11	17.27	2.93	15.63	25.70	6.16	19.26
	MAX	0.00	12.24	4.44	8.27	23.29	7.09	23.27	30.07	6.88	27.05

Table 6.10 Percent Deviation of UBs, Two Hazardous Task, N=34, K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.36	8.57	5.48	8.90	12.21	5.90	11.08	23.73	6.94	22.81
	MAX	1.09	12.08	12.70	13.86	17.75	9.31	16.30	29.22	11.13	31.50
UB2	AVG	0.36	7.57	4.67	7.72	7.13	1.64	6.26	17.74	2.11	16.47
	MAX	1.09	10.39	11.01	12.17	16.06	4.62	12.61	25.53	5.44	22.81
UB3	AVG	0.09	6.11	4.30	6.98	6.40	0.97	5.18	15.64	1.47	13.96
	MAX	0.89	9.39	9.90	10.04	14.06	1.29	10.27	25.53	5.04	22.37

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.12	9.07	2.65	3.74	22.72	5.00	8.53	44.98	7.78	11.47
	MAX	2.28	15.91	7.33	11.56	33.21	9.21	15.75	53.19	13.89	18.33
UB2	AVG	0.12	8.00	2.41	3.31	18.13	2.96	5.92	33.38	5.51	8.50
	MAX	0.59	14.22	5.64	9.87	23.52	7.52	9.06	49.50	12.20	16.64
UB3	AVG	0.00	5.96	1.65	1.80	15.90	1.80	1.96	26.11	4.00	4.54
	MAX	0.00	13.73	3.08	4.41	22.40	5.52	3.97	38.42	9.15	11.08

Table 6.11 Percent Deviation of UBs, Two Hazardous Task, N=34, K=4

		CT1										
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$				$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
UB1	AVG	0.32	10.25	4.71	5.84	32.73	6.45	6.99	39.86	8.93	17.77	
	MAX	0.63	15.68	9.24	9.36	46.68	13.31	15.91	47.63	16.39	23.63	
UB2	AVG	0.31	10.19	4.69	5.81	31.93	6.10	6.70	22.98	8.26	17.20	
	MAX	0.62	13.65	10.21	9.34	46.58	10.77	11.46	27.48	14.67	23.78	
UB3	AVG	0.02	5.45	3.55	2.71	15.37	4.74	3.48	16.64	6.61	6.89	
	MAX	0.12	9.30	8.85	6.94	24.05	8.34	7.88	25.96	12.28	11.40	

		CT2										
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$				$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
UB1	AVG	0.33	15.15	2.70	9.43	44.41	4.87	15.91	88.39	8.33	41.07	
	MAX	0.63	19.67	7.34	15.25	59.73	8.97	23.14	93.24	16.73	59.88	
UB2	AVG	0.33	15.14	2.70	9.41	30.70	4.59	15.53	65.66	7.82	30.46	
	MAX	0.66	19.67	9.34	15.25	43.09	13.71	22.78	82.87	15.95	49.52	
UB3	AVG	0.05	10.26	1.58	6.08	18.70	1.43	9.71	40.31	4.86	19.39	
	MAX	0.36	16.79	4.02	12.46	29.82	4.77	18.47	62.77	15.53	35.66	

Table 6.12 Percent Deviation of UBs, Two Hazardous Task, N=47, K=2

		CT1										
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$				$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
UB1	AVG	0.34	7.40	4.49	6.88	22.63	4.98	7.66	24.76	6.33	14.69	
	MAX	0.70	11.10	7.34	12.51	31.60	7.68	12.87	32.74	9.79	18.99	
UB2	AVG	0.32	5.65	3.38	5.75	17.39	1.15	3.54	19.11	1.65	9.82	
	MAX	0.46	8.98	6.74	11.60	25.84	4.81	5.46	25.52	3.72	16.92	
UB3	AVG	0.16	3.72	1.72	3.33	10.92	0.68	1.23	12.56	1.03	5.57	
	MAX	0.35	5.93	4.36	7.46	15.24	1.06	3.30	16.11	1.48	11.10	

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.27	5.38	3.00	3.71	10.20	3.78	5.71	16.18	5.46	8.75
	MAX	0.55	8.14	6.86	7.85	16.74	8.86	9.99	23.59	9.86	12.11
UB2	AVG	0.23	4.59	2.44	2.88	8.32	1.94	2.56	13.40	3.12	5.35
	MAX	0.51	7.54	6.19	7.07	13.37	3.87	4.54	19.45	7.37	9.75
UB3	AVG	0.13	3.72	1.51	1.63	5.77	0.12	0.61	10.60	1.31	3.53
	MAX	0.17	6.24	2.37	4.17	11.19	0.29	1.07	14.07	3.88	7.15

Table 6.13 Percent Deviation of UBs, Two Hazardous Task, N=47, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.57	10.61	3.84	8.83	37.12	5.79	11.79	43.86	6.75	31.61
	MAX	0.68	19.36	6.12	11.97	43.35	9.42	15.90	53.44	10.42	40.43
UB2	AVG	0.47	8.59	2.83	7.82	31.76	4.55	9.58	36.42	5.40	23.31
	MAX	0.59	13.36	4.12	11.07	35.35	8.15	14.70	43.44	9.97	31.43
UB3	AVG	0.05	4.46	1.84	4.74	19.89	2.91	5.29	23.64	1.53	13.20
	MAX	0.11	7.21	4.07	9.56	23.86	4.90	10.59	34.03	3.59	19.27

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.45	6.72	1.87	2.83	23.34	3.66	4.02	27.30	4.58	8.35
	MAX	0.88	9.31	3.47	4.63	31.66	7.93	8.63	33.35	8.11	12.90
UB2	AVG	0.45	5.72	1.57	2.13	20.16	2.82	3.15	23.97	3.27	7.02
	MAX	0.58	7.29	3.47	4.63	28.66	4.85	5.60	29.35	6.79	10.63
UB3	AVG	0.22	3.95	1.15	1.29	12.13	2.18	1.05	15.39	2.48	3.44
	MAX	0.42	6.60	3.18	3.21	19.71	4.29	2.86	21.47	5.85	6.16

Table 6.14 Percent Deviation of UBs, Two Hazardous Task, N=47, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.80	9.39	3.08	5.51	22.46	4.44	14.51	36.29	7.37	17.03
	MAX	1.01	15.47	7.17	8.64	26.58	9.12	18.72	43.25	13.49	26.82
UB2	AVG	0.69	7.38	2.59	4.50	17.36	3.33	11.42	29.05	6.26	12.86
	MAX	0.92	9.47	4.17	8.64	23.36	7.95	15.63	33.75	10.49	16.73
UB3	AVG	0.23	4.53	2.08	3.00	12.95	2.11	7.11	22.23	5.08	7.96
	MAX	0.82	8.10	4.09	6.22	17.53	4.83	12.83	27.00	7.82	13.01

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.46	8.76	2.05	6.77	24.39	4.38	25.44	40.10	6.55	30.38
	MAX	0.57	12.01	4.27	9.63	27.34	8.26	33.44	48.28	10.66	34.80
UB2	AVG	0.39	7.76	1.83	5.76	19.26	3.31	18.34	33.86	5.33	24.20
	MAX	0.45	11.01	4.27	9.63	23.21	7.15	23.42	38.00	10.25	30.65
UB3	AVG	0.10	4.50	0.90	4.02	13.79	1.58	12.42	22.78	2.67	19.36
	MAX	0.15	6.83	1.11	8.37	18.74	2.55	16.98	33.02	4.85	24.26

Table 6.15 Percent Deviation of UBs, Three Hazardous Task, N=22, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.39	12.76	6.91	7.90	30.25	8.23	12.42	33.60	8.36	13.78	36.95	8.90	15.14
	MAX	2.48	15.09	9.38	14.15	37.45	11.61	19.74	43.31	15.05	20.79	49.17	18.49	21.84
UB2	AVG	0.39	10.29	6.61	7.02	16.11	6.90	7.72	20.44	7.33	9.18	24.77	7.77	10.65
	MAX	2.48	14.44	9.02	13.26	23.31	10.28	15.03	28.95	13.32	16.19	34.58	16.35	17.35
UB3	AVG	0.22	9.42	5.10	4.88	9.71	5.17	5.43	14.43	5.77	7.05	19.16	5.96	8.68
	MAX	1.31	14.38	8.56	10.32	14.97	9.61	13.80	18.27	9.72	14.59	26.58	9.82	15.38

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.35	14.81	4.66	6.08	37.54	9.47	10.84	57.46	10.63	12.63	65.38	11.79	14.41
	MAX	0.49	28.16	6.79	12.06	46.43	19.59	16.23	65.48	18.10	18.85	71.52	16.61	21.47
UB2	AVG	0.35	10.95	4.63	6.00	35.03	8.37	9.00	44.35	9.46	10.92	53.67	10.55	12.84
	MAX	0.49	21.13	6.76	7.96	44.17	18.50	16.18	56.49	16.93	18.34	58.82	15.36	20.30
UB3	AVG	0.14	9.64	3.06	3.87	28.32	4.37	4.29	31.29	6.00	6.34	34.26	7.63	8.39
	MAX	0.28	19.60	6.26	6.04	41.09	12.36	11.68	41.04	12.40	13.77	41.00	12.45	15.86

Table 6.16 Percent Deviation of UBs, Three Hazardous Task, N=22, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.35	11.31	4.76	6.08	18.34	8.18	9.40	25.11	8.25	10.68	31.89	8.32	11.96
	MAX	1.10	23.20	7.32	9.77	28.93	20.14	18.00	33.66	17.69	18.66	38.39	15.25	19.33
UB2	AVG	0.33	11.13	4.72	5.98	17.50	7.79	8.86	19.81	7.86	10.06	22.12	7.92	11.26
	MAX	1.00	22.24	7.14	9.52	27.50	18.88	16.14	25.16	14.93	17.24	22.82	10.98	18.34
UB3	AVG	0.10	3.87	2.81	3.37	7.53	2.82	6.57	12.00	3.41	7.23	16.46	4.00	7.90
	MAX	0.21	4.17	5.79	4.12	11.91	7.17	10.30	13.92	7.66	12.87	19.93	8.15	13.44

		CT2												
π & POS		$\pi=1$	$\pi z1= 0.8, \pi z2= 0.8, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.8$			$\pi z1= 0.8, \pi z2= 0.2, \pi z3= 0.2$			$\pi z1= 0.2, \pi z2= 0.2, \pi z3= 0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.67	11.71	4.17	9.65	28.23	8.16	24.43	36.39	8.75	27.83	44.55	9.34	31.23
	MAX	1.60	16.89	6.56	16.51	36.89	12.35	30.77	43.17	14.83	33.69	51.44	17.30	38.62
UB2	AVG	0.67	10.71	4.15	8.58	22.37	7.76	21.90	29.42	8.36	23.56	36.48	8.96	25.22
	MAX	1.60	15.78	6.51	14.91	29.19	11.52	27.86	35.17	13.51	29.95	41.16	15.50	30.04
UB3	AVG	0.15	5.02	0.91	4.36	18.31	3.11	13.57	19.43	4.02	13.99	20.56	4.93	14.41
	MAX	0.23	12.97	4.71	8.77	23.69	7.52	19.67	23.87	6.51	19.75	24.06	5.50	19.84

Table 6.17 Percent Deviation of UBs, Three Hazardous Task, N=34, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.38	9.08	5.81	9.03	12.94	6.25	11.74	15.96	6.25	15.00	18.98	6.55	16.25
	MAX	1.16	12.80	9.46	14.69	18.82	9.87	17.28	20.10	9.87	21.24	23.38	8.90	25.20
UB2	AVG	0.27	8.02	1.95	7.65	9.56	2.17	7.88	10.87	2.29	9.91	14.19	2.69	13.18
	MAX	1.03	11.01	3.67	12.90	14.02	4.90	13.37	18.72	4.62	15.81	20.42	4.35	18.25
UB3	AVG	0.10	6.48	1.01	5.40	6.78	1.08	5.49	9.65	1.10	8.33	12.51	1.58	10.17
	MAX	0.94	9.95	2.49	10.64	10.90	1.37	10.89	17.66	2.70	14.39	20.42	4.03	17.90

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.17	9.61	2.81	3.96	24.08	5.30	9.04	30.03	5.76	9.11	35.98	6.22	9.18
	MAX	0.82	16.86	7.77	12.25	31.20	9.76	16.70	35.88	10.44	15.68	42.55	11.11	14.66
UB2	AVG	0.13	8.48	2.55	3.51	19.22	3.14	6.28	22.96	3.77	6.54	26.70	4.41	6.80
	MAX	0.63	15.07	5.98	10.46	24.93	7.97	9.60	32.27	8.87	11.46	39.60	9.76	13.31
UB3	AVG	0.08	6.32	1.75	1.91	16.85	1.91	2.08	18.87	2.55	2.85	20.89	3.20	3.63
	MAX	0.24	14.55	3.26	4.67	23.74	5.85	4.21	27.24	6.59	6.54	30.74	9.32	8.86

Table 6.18 Percent Deviation of UBs, Three Hazardous Task, N=34, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.34	8.20	3.77	4.67	22.18	5.16	5.59	24.54	6.15	9.90	26.89	7.14	14.22
	MAX	0.67	12.54	8.39	7.49	31.63	10.65	12.73	31.88	11.88	15.82	32.13	13.11	19.90
UB2	AVG	0.33	8.15	3.75	4.65	15.54	4.88	5.36	16.96	5.74	9.56	18.38	6.61	13.76
	MAX	0.66	10.92	8.17	7.47	22.67	8.62	9.17	22.33	10.18	14.10	21.98	11.74	19.02
UB3	AVG	0.02	4.36	2.84	2.17	12.30	3.79	2.78	12.80	4.54	4.15	13.31	5.29	5.51
	MAX	0.13	7.44	7.08	5.55	19.24	6.67	6.30	20.00	8.25	7.71	20.77	9.82	9.12

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.35	6.56	3.01	3.74	17.74	4.13	4.47	19.63	4.92	7.92	21.51	5.72	11.37
	MAX	0.67	8.52	7.19	6.04	23.87	9.60	6.51	23.28	11.31	11.54	22.69	12.48	16.58
UB2	AVG	0.33	6.52	3.00	3.72	12.43	3.90	4.29	13.57	4.60	7.65	14.71	5.29	11.01
	MAX	0.62	8.47	7.18	6.03	17.45	8.96	6.29	18.01	11.22	11.49	18.56	11.78	16.41
UB3	AVG	0.05	3.49	2.27	1.73	9.84	3.03	2.23	10.24	3.63	3.32	10.65	4.23	4.41
	MAX	0.28	5.71	5.78	3.55	15.69	8.12	4.24	16.13	11.12	6.17	16.58	11.02	8.11

Table 6.19 Percent Deviation of UBs, Three Hazardous Task, N=47, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.36	7.84	4.76	7.29	23.99	5.28	8.12	25.12	5.99	11.85	26.25	6.71	15.57
	MAX	0.74	11.77	7.78	13.26	33.50	8.14	13.64	34.10	9.26	16.89	34.70	10.38	20.13
UB2	AVG	0.32	6.05	1.18	4.97	18.43	1.22	5.18	19.35	1.48	7.08	20.26	1.75	10.41
	MAX	0.49	9.52	3.71	6.30	27.39	4.10	5.79	27.22	4.52	11.86	27.05	5.94	17.94
UB3	AVG	0.17	3.94	0.70	3.39	11.58	0.72	3.50	12.44	0.91	3.60	13.31	1.09	5.90
	MAX	0.37	6.29	2.62	6.21	16.15	1.12	7.35	16.62	1.35	7.63	17.08	1.57	11.77

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.29	5.70	3.18	3.93	10.81	4.01	6.05	13.98	4.90	7.66	17.15	5.79	9.28
	MAX	0.58	8.63	7.27	8.32	17.74	9.39	10.59	21.37	9.92	11.71	25.01	10.45	12.84
UB2	AVG	0.24	4.87	2.39	2.55	8.82	2.56	2.71	11.51	2.68	4.19	14.20	3.31	5.67
	MAX	0.54	7.99	6.56	7.49	14.17	7.10	7.61	17.39	7.70	7.57	20.62	7.81	10.34
UB3	AVG	0.14	3.94	0.60	1.23	6.12	0.73	1.65	8.68	0.76	2.19	11.24	1.39	3.74
	MAX	0.18	6.61	2.31	4.42	11.86	2.51	3.13	13.39	2.71	4.36	14.91	4.11	7.58

Table 6.20 Percent Deviation of UBs, Three Hazardous Task, N=47, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.60	15.49	3.07	7.06	29.70	4.63	9.43	32.39	5.02	17.36	35.09	5.40	22.29
	MAX	0.72	21.49	4.90	9.58	34.68	7.54	12.72	38.72	7.94	22.53	42.75	8.34	27.34
UB2	AVG	0.50	9.87	2.26	6.26	25.41	3.64	7.66	27.27	3.98	13.16	29.14	4.32	18.65
	MAX	0.63	13.69	3.30	8.86	28.28	6.52	11.76	31.52	7.25	18.45	34.75	7.98	25.14
UB3	AVG	0.05	4.57	1.47	3.49	15.91	1.83	4.23	17.41	1.98	7.40	18.91	2.22	10.56
	MAX	0.12	8.18	3.26	7.65	19.09	3.92	8.47	23.16	3.40	11.94	27.22	2.87	15.42

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.48	9.38	2.50	2.26	18.67	2.93	3.22	20.26	3.30	4.95	21.84	3.66	6.68
	MAX	0.93	13.45	5.78	3.70	25.33	6.34	6.90	26.00	6.42	8.61	26.68	6.49	10.32
UB2	AVG	0.42	7.58	1.96	1.70	16.13	2.26	2.52	17.65	2.44	4.07	19.18	2.62	5.62
	MAX	0.61	12.83	3.20	3.70	22.93	3.88	4.48	23.20	4.66	6.49	23.48	5.43	8.50
UB3	AVG	0.23	3.16	0.92	1.03	9.70	1.74	1.28	11.01	1.86	1.80	12.31	1.98	2.75
	MAX	0.45	5.28	2.54	2.57	15.77	3.43	2.29	16.47	4.06	3.61	17.18	4.68	4.93

Table 6.21 Percent Deviation of UBs, Three Hazardous Task, N=47, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.85	7.51	2.76	5.41	17.97	3.65	11.61	23.50	4.72	12.62	28.03	5.90	13.62
	MAX	1.07	12.38	5.74	6.91	21.26	7.30	14.98	27.93	9.04	18.22	34.60	10.79	21.46
UB2	AVG	0.73	5.90	2.07	3.60	13.89	2.66	9.14	18.56	3.84	9.71	23.24	5.01	10.29
	MAX	0.98	7.58	3.34	6.91	18.69	6.36	12.50	22.84	7.38	12.94	27.00	8.39	13.38
UB3	AVG	0.24	3.62	1.66	2.40	10.36	1.69	5.69	14.07	2.88	6.03	17.78	4.06	6.37
	MAX	0.87	6.48	3.27	4.98	14.02	3.86	10.26	17.81	5.06	10.34	21.60	6.26	10.41

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
UB1	AVG	0.49	7.01	1.64	5.42	19.51	3.50	18.35	25.80	4.37	22.33	32.08	5.24	24.30
	MAX	0.60	9.61	3.42	7.70	21.87	6.61	26.75	30.25	7.57	27.30	38.62	8.53	27.84
UB2	AVG	0.41	6.21	1.46	4.61	15.41	2.65	14.67	21.25	3.46	17.02	27.09	4.26	19.36
	MAX	0.48	8.81	3.31	7.40	18.57	5.72	18.74	24.48	6.96	21.63	30.40	8.20	24.52
UB3	AVG	0.11	3.60	0.72	3.22	11.03	1.26	9.94	14.63	1.70	12.71	18.22	2.14	15.49
	MAX	0.16	5.46	0.89	5.17	14.99	2.04	13.58	20.70	2.96	16.50	26.42	3.88	19.41

C. Percent Deviation of Lower Bounds

For small Networks $N=22, 34, 47$ the Percent Deviation of Lower Bounds for one hazardous, two hazardous and three hazardous cases are presented in the Tables below

Table 6.22 Percent Deviation of LBs, One Hazardous Task, $N=22, K=2$

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
$N = 22$		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.25	3.05	1.91	0.95	5.46	5.02	3.38	10.11	5.54	9.40
	MAX	0.28	3.94	2.88	1.45	7.03	5.87	7.31	14.71	6.77	10.88
LB2	AVG	0.20	1.05	0.31	0.75	0.90	0.83	0.89	3.29	2.34	1.89
	MAX	0.22	1.61	0.79	1.74	2.28	1.76	1.37	5.55	4.06	3.07

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
$N = 22$		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	0.55	0.54	0.34	2.60	1.00	1.80	3.94	1.80	2.18
	MAX	0.00	0.58	0.57	0.68	4.70	2.39	3.40	5.61	2.89	5.44
LB2	AVG	0.00	0.36	0.15	0.21	0.88	0.21	0.22	1.15	0.35	0.80
	MAX	0.00	0.43	0.37	0.68	1.74	0.64	0.80	2.27	0.74	2.71

Table 6.23 Percent Deviation of LBs, One Hazardous Task, $N=22, K=4$

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
$N = 22$		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.72	9.32	5.83	5.45	8.24	7.57	6.44	10.13	8.60	9.58
	MAX	3.72	14.49	10.62	7.33	10.78	7.84	11.27	16.51	13.07	13.15
LB2	AVG	0.76	2.74	2.25	2.66	2.81	2.46	2.73	3.37	2.37	2.52
	MAX	1.51	5.71	4.79	6.43	5.33	4.79	6.02	6.72	4.17	6.06

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
$N = 22$		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	8.05	6.55	3.14	8.20	7.39	6.52	10.07	7.90	8.18
	MAX	0.00	10.04	8.71	5.97	11.75	9.83	9.67	14.87	12.59	12.26
LB2	AVG	0.00	1.68	1.06	0.80	1.96	1.11	1.16	2.29	1.18	1.26
	MAX	0.00	2.91	2.15	1.77	4.76	2.54	2.20	7.31	3.96	3.08

Table 6.24 Percent Deviation of LBs, One Hazardous Task, N=34, K=2

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 34		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.10	3.42	2.14	1.07	6.13	5.64	3.80	11.35	6.22	10.56
	MAX	0.15	4.42	3.24	1.63	7.89	6.59	8.21	16.52	7.60	12.22
LB2	AVG	0.10	0.95	0.35	0.84	1.01	0.75	1.00	3.05	2.63	2.12
	MAX	0.15	1.80	0.89	1.96	2.56	1.98	1.53	6.23	4.56	3.44

		CT2									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 34		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	0.62	0.61	0.38	2.92	1.13	2.02	4.42	2.02	2.44
	MAX	0.00	0.65	0.64	0.76	5.28	2.68	3.82	6.30	3.25	6.11
LB2	AVG	0.00	0.29	0.13	0.17	0.72	0.17	0.18	0.94	0.29	0.66
	MAX	0.00	0.35	0.30	0.56	1.42	0.52	0.66	1.86	0.60	2.22

Table 6.25 Percent Deviation of LBs, One Hazardous Task, N=34, K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 34		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.81	10.47	6.55	6.12	9.25	8.51	7.23	11.37	9.65	10.76
	MAX	4.80	16.27	11.92	8.23	12.11	8.80	12.66	18.54	14.68	14.76
LB2	AVG	0.95	2.68	2.28	2.45	2.74	2.49	2.56	3.10	3.15	2.74
	MAX	1.11	4.97	5.35	4.91	6.69	5.28	7.55	6.06	4.54	7.01

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 34		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	9.38	1.10	3.21	6.80	8.04	7.15	8.85	7.12	8.97
	MAX	0.00	12.97	2.19	8.98	9.85	13.63	11.39	12.01	13.19	12.22
LB2	AVG	0.00	2.28	1.05	2.08	2.33	2.12	2.18	2.64	2.68	2.33
	MAX	0.00	4.22	4.54	4.17	5.69	4.49	6.42	5.15	3.86	5.96

Table 6.26 Percent Deviation of LBs, One Hazardous Task, N=47, K=2

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.50	1.75	1.17	2.34	1.34	0.91	3.61	5.15	3.68	3.14
	MAX	1.99	4.43	2.98	3.49	3.58	3.06	6.60	10.12	7.83	4.84
LB2	AVG	0.44	0.63	0.45	0.83	1.22	0.15	0.26	1.63	0.30	0.35
	MAX	1.34	3.13	2.30	2.21	2.88	2.82	2.14	4.42	0.37	0.41

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	0.68	0.36	0.59	2.73	1.34	4.22	2.00	0.79	1.62
	MAX	0.00	1.83	0.97	1.42	4.84	1.85	9.72	5.38	3.27	4.77
LB2	AVG	0.00	0.29	0.21	0.27	1.07	0.36	0.56	0.62	0.28	0.44
	MAX	0.00	0.73	0.77	0.49	2.84	1.12	1.15	1.25	0.79	0.81

Table 6.27 Percent Deviation of LBs, One Hazardous Task, N=47, K=4

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.89	4.36	5.13	6.85	8.71	10.16	11.17	7.54	9.12	11.23
	MAX	2.91	5.79	8.38	10.87	12.27	13.27	14.83	10.65	14.12	14.78
LB2	AVG	1.30	2.54	2.01	1.81	2.43	2.27	2.35	2.68	1.70	1.95
	MAX	2.31	4.23	3.09	2.28	3.70	3.53	3.74	4.44	3.24	4.14

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.01	3.36	2.30	1.87	8.16	3.12	4.98	6.90	2.73	5.54
	MAX	0.04	4.07	3.54	3.29	10.72	4.30	9.04	9.32	5.01	7.80
LB2	AVG	0.01	2.08	1.24	1.30	2.42	0.62	2.05	2.60	1.05	1.83
	MAX	0.04	3.37	2.71	2.59	3.83	0.73	3.54	4.27	2.63	3.95

Table 6.28 Percent Deviation of LBs, One Hazardous Task, N=47, K=6

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	2.13	5.93	5.77	6.97	9.45	13.54	11.67	9.74	9.15	8.58
	MAX	3.46	10.11	9.29	7.61	12.88	18.87	15.06	13.01	12.17	11.91
LB2	AVG	1.90	2.81	2.77	2.71	3.16	3.08	2.61	3.48	3.28	3.22
	MAX	2.55	5.21	5.15	4.11	6.02	5.96	5.26	6.71	6.35	5.61

		CT2									
π & POS		π=1	π= 0.8			π= 0.5			π= 0.2		
N = 47		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.24	5.69	3.72	4.13	8.29	6.72	9.08	7.28	6.03	6.00
	MAX	0.39	9.11	6.54	5.73	11.47	9.75	11.51	10.11	9.08	9.05
LB2	AVG	0.24	2.55	1.29	2.46	3.02	2.61	2.48	3.15	3.27	2.86
	MAX	0.39	3.51	3.30	3.63	5.17	4.48	4.62	5.52	5.96	4.85

Table 6.29 Percent Deviation of LBs, Two Hazardous Task, N=22, K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.48	1.41	2.73	4.36	4.82	7.18	7.80	7.92	14.45	13.86
	MAX	0.52	1.82	4.12	6.67	6.21	8.38	16.88	11.52	17.65	16.04
LB2	AVG	0.48	1.07	0.45	1.49	1.27	1.19	1.29	2.80	4.20	4.70
	MAX	0.52	1.63	1.13	3.47	3.21	2.52	1.97	4.72	7.29	7.62

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	0.77	0.77	0.48	3.71	1.43	2.58	5.62	2.57	3.11
	MAX	0.00	0.81	0.81	0.97	6.71	3.41	4.85	8.01	4.13	7.77
LB2	AVG	0.00	0.51	0.22	0.30	3.40	0.30	0.10	1.15	0.50	1.65
	MAX	0.00	0.61	0.53	0.97	6.71	0.91	0.35	2.26	1.05	5.56

Table 6.30 Percent Deviation of LBs, Two Hazardous Task, N=22, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	2.46	13.32	8.33	7.78	11.77	10.82	9.20	14.47	12.28	13.69
	MAX	5.31	20.70	15.17	10.47	15.40	11.20	16.10	23.58	18.67	18.78
LB2	AVG	1.08	3.80	3.22	3.91	3.90	3.52	4.01	4.82	3.38	3.60
	MAX	2.16	7.92	6.84	9.45	7.40	6.84	8.84	9.61	5.96	8.66

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	11.51	9.36	4.48	9.32	10.56	12.62	11.69	9.85	14.67
	MAX	0.00	14.34	12.44	8.53	13.35	14.04	18.72	17.26	15.70	21.98
LB2	AVG	0.00	2.40	1.52	1.14	1.66	1.58	2.80	1.80	1.68	3.27
	MAX	0.00	4.15	3.07	2.53	4.03	3.63	5.30	5.75	5.66	7.98

Table 6.31 Percent Deviation of LBs, Two Hazardous Task, N=34, K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	3.51	12.94	4.18	12.72	3.53	5.65	16.64	4.43	8.39
	MAX	0.00	8.32	26.48	13.45	19.06	11.56	11.75	21.03	7.36	15.82
LB2	AVG	0.00	0.79	0.80	1.80	2.99	0.87	0.50	0.82	2.58	0.21
	MAX	0.00	3.14	3.14	5.79	6.07	4.62	3.20	3.85	5.78	1.59

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	0.91	0.28	0.34	3.59	2.08	3.50	4.16	3.38	1.77
	MAX	0.00	3.63	0.81	1.60	11.56	12.86	8.32	9.54	9.61	9.87
LB2	AVG	0.00	0.59	0.14	0.18	0.79	1.48	1.33	1.03	0.51	0.82
	MAX	0.00	3.25	0.64	0.61	3.91	6.86	4.32	4.57	2.71	4.26

Table 6.32 Percent Deviation of LBs, Two Hazardous Task, N=34, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	2.59	14.88	13.33	6.35	13.38	13.12	12.40	17.17	11.31	14.84
	MAX	6.85	18.69	16.91	10.05	25.31	19.89	18.57	25.26	17.64	22.68
LB2	AVG	1.35	3.82	3.26	3.95	3.92	3.56	3.66	4.08	4.50	3.92
	MAX	5.87	7.10	7.64	7.01	9.56	7.55	10.79	8.66	6.49	10.02

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	13.40	1.57	4.59	9.72	11.48	10.21	12.65	10.17	12.81
	MAX	0.00	18.53	3.13	12.83	14.07	19.47	16.28	17.15	18.84	17.46
LB2	AVG	0.00	2.80	1.48	4.29	3.42	2.42	2.87	3.08	3.57	3.45
	MAX	0.00	6.83	2.77	7.94	9.76	8.80	6.28	11.15	11.75	8.48

Table 6.33 Percent Deviation of LBs, Two Hazardous Task, N=47, K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.72	2.50	1.66	3.34	1.92	1.30	5.16	7.35	5.26	4.49
	MAX	2.84	6.34	4.26	4.99	5.12	4.37	9.43	14.46	11.18	6.92
LB2	AVG	0.63	0.90	0.64	1.19	1.74	0.21	0.37	2.33	0.00	0.00
	MAX	1.92	4.47	3.28	3.15	4.12	4.02	3.06	6.32	0.00	0.00

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.00	0.97	0.51	0.85	3.91	1.91	6.03	2.86	1.13	2.31
	MAX	0.00	2.61	1.39	2.02	6.91	2.64	13.89	7.69	4.67	6.81
LB2	AVG	0.00	0.41	0.29	0.38	2.43	0.51	0.80	0.88	0.40	0.64
	MAX	0.00	1.04	1.10	0.70	6.91	1.60	1.64	1.78	1.13	1.16

Table 6.34 Percent Deviation of LBs, Two Hazardous Task, N=47, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	2.70	6.22	7.33	9.79	12.44	14.51	15.96	10.77	13.03	16.05
	MAX	4.15	8.27	11.97	15.53	17.53	18.95	21.18	15.22	20.17	21.11
LB2	AVG	1.86	3.63	2.88	2.59	3.48	3.25	3.35	3.82	2.43	2.78
	MAX	3.30	6.05	4.42	3.25	5.28	5.04	5.34	6.34	4.63	5.91

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.01	4.79	3.29	2.66	11.65	4.46	7.11	9.86	3.90	7.92
	MAX	0.05	5.81	5.05	4.70	15.31	6.14	12.91	13.32	7.16	11.14
LB2	AVG	0.01	2.98	1.77	1.85	3.45	0.89	2.93	3.71	1.50	2.61
	MAX	0.05	4.81	3.88	3.70	5.47	1.04	5.06	6.10	3.75	5.65

Table 6.35 Percent Deviation of LBs, Two Hazardous Task, N=47, K=6

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	3.05	8.47	8.25	9.96	13.49	19.34	16.68	13.91	13.06	12.25
	MAX	4.95	14.44	13.27	10.88	18.40	26.96	21.51	18.59	17.38	17.01
LB2	AVG	2.71	4.02	3.95	3.88	4.52	4.40	3.73	4.98	4.68	4.60
	MAX	3.65	7.44	7.35	5.88	8.61	8.51	7.51	9.59	9.07	8.01

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.35	8.13	5.31	5.90	11.85	9.61	12.97	10.40	8.62	8.57
	MAX	0.55	13.02	9.34	8.18	16.38	13.92	16.45	14.44	12.98	12.93
LB2	AVG	0.35	3.64	1.84	3.51	4.31	3.73	3.54	4.50	4.67	4.09
	MAX	0.55	5.02	4.72	5.18	7.38	6.40	6.60	7.88	8.52	6.93

Table 6.36 Percent Deviation of LBs, Three Hazardous Task, N=22, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.51	1.49	2.89	4.62	5.11	7.61	8.27	6.75	11.46	11.48	8.40	15.32	14.69
	MAX	0.55	1.93	4.37	7.07	6.58	8.88	17.89	9.40	13.80	17.45	12.21	18.71	17.00
LB2	AVG	0.51	1.13	0.48	1.58	1.35	1.26	1.37	2.16	2.86	3.17	2.97	4.45	4.98
	MAX	0.55	1.73	1.20	3.68	3.40	2.67	2.09	4.20	5.20	5.08	5.00	7.73	8.08

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.10	0.82	0.82	0.51	3.93	1.52	2.73	4.94	2.12	3.02	5.96	2.72	3.30
	MAX	0.15	0.86	0.86	1.03	7.11	3.61	5.14	7.80	4.00	6.69	8.49	4.38	8.24
LB2	AVG	0.08	0.54	0.23	0.32	3.60	0.32	0.11	2.41	0.42	0.93	1.22	0.53	1.75
	MAX	0.11	0.65	0.56	1.03	7.11	0.96	0.37	4.75	1.04	3.13	2.40	1.11	5.89

Table 6.37 Percent Deviation of LBs, Three Hazardous Task, N=22, K=4

		CT1													
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$				$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
LB1	AVG	1.61	11.12	8.83	8.25	12.48	11.47	9.75	13.91	12.24	12.13	15.34	13.02	14.51	
	MAX	3.48	17.28	16.08	11.10	16.32	11.87	17.07	20.66	15.83	18.49	24.99	19.79	19.91	
LB2	AVG	0.84	4.03	3.41	4.14	4.13	3.73	4.25	4.62	3.66	4.03	5.11	3.58	3.82	
	MAX	1.68	8.40	7.25	10.02	7.84	7.25	9.37	9.02	6.78	9.28	10.19	6.32	9.18	

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	1.21	9.75	8.62	4.75	9.88	10.19	13.38	11.14	10.32	14.46	12.39	10.44	15.55
	MAX	1.89	12.15	11.46	9.04	14.15	13.55	19.84	16.22	15.10	21.57	18.30	16.64	23.30
LB2	AVG	0.95	1.54	1.51	1.53	1.76	1.67	2.97	1.83	1.73	3.22	1.91	1.78	3.47
	MAX	1.34	2.66	3.05	3.40	4.27	3.85	5.62	5.18	4.92	7.04	6.10	6.00	8.46

Table 6.38 Percent Deviation of LBs, Three Hazardous Task, N=34, K=2

		CT1													
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$				$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
LB1	AVG	0.62	3.72	2.89	4.43	10.48	3.74	5.99	11.06	4.22	7.44	12.64	4.70	8.89	
	MAX	0.93	8.82	5.91	14.26	15.70	12.25	12.46	15.84	10.03	14.61	15.97	7.80	16.77	
LB2	AVG	0.45	0.84	0.58	0.22	1.17	0.65	0.53	2.19	1.24	0.65	3.21	1.83	0.82	
	MAX	0.82	3.33	2.28	0.71	2.38	1.45	2.39	4.02	2.77	2.75	5.67	4.10	3.11	

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.49	0.96	0.30	0.36	3.81	2.20	3.71	4.11	2.89	2.79	4.41	3.58	1.88
	MAX	0.67	3.85	0.86	1.70	12.25	13.63	8.82	11.18	11.91	9.64	10.11	10.19	10.46
LB2	AVG	0.35	0.63	0.15	0.19	0.84	1.57	1.41	0.96	1.05	1.14	1.09	0.54	0.87
	MAX	0.56	3.45	0.68	0.65	4.14	7.27	4.58	4.49	5.07	4.55	4.84	2.87	4.52

Table 6.39 Percent Deviation of LBs, Three Hazardous Task, N=34, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	2.75	12.77	6.68	6.73	14.18	13.91	13.14	16.19	12.95	14.44	18.20	11.99	15.73
	MAX	7.26	16.04	8.47	10.65	26.83	21.08	19.68	26.80	19.89	21.86	26.78	18.70	24.04
LB2	AVG	1.43	4.05	3.46	4.19	4.16	3.77	3.88	4.24	3.98	4.02	4.32	4.07	4.16
	MAX	6.22	7.53	8.10	7.43	10.13	8.00	11.44	9.66	6.94	11.03	9.18	5.87	10.62

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.85	9.89	1.66	4.87	10.30	9.17	10.82	11.86	9.98	12.20	13.41	10.78	13.58
	MAX	1.23	13.68	3.32	13.60	14.91	15.55	17.26	16.55	17.76	17.88	18.18	19.97	18.51
LB2	AVG	0.64	2.97	1.57	3.01	3.63	2.57	3.04	3.65	2.92	3.35	3.68	3.28	3.66
	MAX	0.87	7.24	2.94	5.57	10.35	6.33	6.66	11.83	7.07	7.82	13.32	7.80	8.99

Table 6.40 Percent Deviation of LBs, Three Hazardous Task, N=47, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.76	2.65	1.76	3.54	2.04	1.38	5.47	4.91	3.48	5.11	7.79	5.58	4.76
	MAX	3.01	6.72	4.52	5.29	5.43	4.63	10.00	10.38	8.24	8.67	15.33	11.85	7.34
LB2	AVG	0.67	0.95	0.68	1.06	1.84	0.72	1.19	2.16	0.78	1.27	2.47	0.83	1.34
	MAX	2.04	4.74	3.48	2.81	4.37	3.78	3.84	5.53	3.82	3.99	6.70	3.86	4.14

		CT2													
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$				$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
LB1	AVG	0.53	1.03	0.54	0.90	4.14	2.02	6.39	3.59	1.61	4.42	3.03	1.20	2.45	
	MAX	1.12	2.77	1.47	2.14	7.32	2.80	14.72	7.74	3.87	10.97	8.15	4.95	7.22	
LB2	AVG	0.42	0.43	0.31	0.40	1.28	0.54	0.85	1.45	0.58	0.88	1.62	0.62	0.91	
	MAX	0.85	1.10	1.17	0.74	3.64	1.70	1.74	3.46	1.72	1.69	3.28	1.75	1.65	

Table 6.41 Percent Deviation of LBs, Three Hazardous Task, N=47, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	2.86	6.59	7.77	10.38	11.19	11.08	16.92	12.30	12.20	16.97	13.42	13.31	17.01
	MAX	4.40	8.77	12.69	16.46	15.77	14.47	22.45	17.37	17.54	22.41	18.96	20.60	22.38
LB2	AVG	1.97	3.85	3.05	2.75	3.69	3.45	3.55	3.87	3.01	3.25	4.05	2.58	2.95
	MAX	3.50	6.41	4.69	3.45	5.60	5.34	5.66	6.16	5.13	5.96	6.72	4.91	6.26

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.01	5.08	3.49	2.82	10.35	4.13	7.54	10.42	4.43	7.97	10.45	4.72	8.40
	MAX	0.05	6.16	5.35	4.98	13.60	5.69	13.68	13.86	7.18	12.75	14.12	8.67	11.81
LB2	AVG	0.01	3.16	0.88	1.96	3.66	0.94	2.81	3.79	1.27	2.99	3.93	1.59	3.17
	MAX	0.05	5.10	1.93	3.92	5.80	1.10	4.85	6.13	2.54	5.86	6.47	3.98	6.86

Table 6.42 Percent Deviation of LBs, Three Hazardous Task, N=47, K=6

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	3.23	8.98	8.75	10.56	12.30	12.15	12.33	13.52	13.14	12.65	14.74	13.84	12.97
	MAX	5.25	15.31	12.07	13.53	16.78	16.94	15.90	18.24	17.68	16.96	19.71	18.42	18.01
LB2	AVG	2.85	4.22	3.15	4.07	4.62	3.62	3.92	4.73	3.77	4.37	4.84	3.91	4.83
	MAX	3.83	7.81	5.86	6.17	8.80	7.00	7.89	9.06	7.29	8.15	9.32	7.58	8.41

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
LB1	AVG	0.37	8.62	5.63	6.25	10.56	8.19	9.75	10.79	8.66	9.82	11.02	9.14	9.89
	MAX	0.58	13.80	9.90	8.67	14.60	11.86	12.37	14.95	12.81	13.64	15.31	13.76	14.92
LB2	AVG	0.37	3.82	1.93	3.69	4.53	3.92	3.72	4.63	4.41	4.01	4.73	4.90	4.29
	MAX	0.58	5.27	4.96	5.44	7.75	6.72	6.93	8.01	7.83	7.10	8.27	8.95	7.28

D. CPU Times (sec.) of Upper Bounds

For small Networks $N=22, 34, 47$ and small number of workstations $K=2, K=4$ the CPU Times of Upper Bounds for one hazardous, two hazardous and three hazardous cases are presented in the Tables below.

Table 6.43 CPU Times (sec.) of UB_1 , One Hazardous Task, $K=2$

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.03	0.01	0.01	0.04	0.01	0.03

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01

Table 6.44 CPU Times (sec.) of UB₁, One Hazardous Task, K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01
34	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01
47	AVG	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01
	MAX	0.02	0.03	0.03	0.02	0.02	0.05	0.04	0.02	0.02	0.02
60	AVG	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MAX	0.01	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.02
73	AVG	0.01	0.03	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.02
	MAX	0.01	0.04	0.05	0.04	0.04	0.02	0.02	0.04	0.05	0.03

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.01	0.02	0.01	0.01	0.03	0.01	0.01	0.02
60	AVG	0.01	0.01	0.02	0.01	0.01	0.02	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.03	0.02	0.03	0.03	0.02	0.02	0.02	0.02
73	AVG	0.01	0.02	0.03	0.02	0.02	0.03	0.02	0.02	0.02	0.01
	MAX	0.01	0.02	0.09	0.02	0.08	0.08	0.01	0.04	0.02	0.03

Table 6.45 CPU Times (sec.) of UB₂, One Hazardous Task, K=2

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.01	0.01	0.02	0.02	0.01	0.02	0.01	0.01	0.01

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.03	0.01	0.01	0.01	0.01	0.02	0.01	0.01

Table 6.46 CPU Times (sec.) of UB₂, One Hazardous Task, K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.02
60	AVG	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MAX	0.02	0.03	0.03	0.04	0.03	0.03	0.02	0.04	0.02	0.02
73	AVG	0.02	0.03	0.03	0.04	0.03	0.03	0.03	0.04	0.02	0.03
	MAX	0.03	0.05	0.04	0.07	0.04	0.03	0.03	0.08	0.02	0.03

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.00	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.01	0.02	0.01	0.02	0.02	0.02	0.01	0.01	0.02
60	AVG	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
73	AVG	0.02	0.02	0.03	0.02	0.01	0.02	0.02	0.02	0.03	0.02
	MAX	0.03	0.04	0.02	0.06	0.02	0.03	0.03	0.03	0.03	0.03

Table 6.47 CPU Times (sec.) of UB₃, One Hazardous Task, K=2

		CT1									
π & POS		$\pi=1$	$\pi= 0.8$			$\pi= 0.5$			$\pi= 0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01
34	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.01	0.00	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.00	0.00	0.01	0.01	0.01	0.01	0.00	0.01	0.00	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01

Table 6.48 CPU Times (sec.) of UB₃, One Hazardous Task, K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.02	0.02	0.02	0.01	0.03	0.02	0.02	0.02
60	AVG	0.01	0.02	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.02	0.02	0.03	0.03	0.02	0.02	0.02	0.02
73	AVG	0.02	0.03	0.02	0.02	0.03	0.03	0.01	0.02	0.02	0.02
	MAX	0.01	0.03	0.03	0.03	0.03	0.07	0.01	0.03	0.03	0.03

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.01
60	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.03	0.02	0.02	0.02	0.02	0.03	0.02	0.02	0.02
73	AVG	0.01	0.02	0.02	0.01	0.02	0.01	0.02	0.02	0.01	0.02
	MAX	0.01	0.08	0.03	0.04	0.02	0.04	0.04	0.02	0.02	0.04

Table 6.49 CPU Times (sec.) of UB₁, Two Hazardous Task, K=2

		CT1									
π &POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
47	AVG	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.02	0.01	0.02
	MAX	0.02	0.02	0.02	0.02	0.05	0.02	0.02	0.06	0.02	0.04

		CT2									
π &POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.03	0.02

Table 6.50 CPU Times (sec.) of UB₁, Two Hazardous Task, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.01	0.01	0.01	0.01	0.02	0.01	0.02	0.01
34	AVG	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.02	0.01	0.01
	MAX	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.02
47	AVG	0.01	0.03	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.02
	MAX	0.03	0.05	0.05	0.04	0.03	0.08	0.06	0.03	0.03	0.03
60	AVG	0.02	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.02	0.05	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.04
73	AVG	0.02	0.05	0.04	0.06	0.06	0.04	0.03	0.05	0.06	0.04
	MAX	0.01	0.06	0.08	0.07	0.07	0.04	0.04	0.07	0.09	0.05

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
47	AVG	0.01	0.02	0.01	0.02	0.01	0.01	0.02	0.01	0.01	0.01
	MAX	0.03	0.03	0.02	0.03	0.02	0.02	0.05	0.02	0.02	0.03
60	AVG	0.01	0.02	0.03	0.02	0.02	0.03	0.02	0.02	0.02	0.02
	MAX	0.02	0.03	0.05	0.03	0.05	0.05	0.03	0.03	0.03	0.04
73	AVG	0.02	0.03	0.05	0.03	0.03	0.05	0.03	0.03	0.03	0.02
	MAX	0.01	0.03	0.15	0.03	0.14	0.14	0.02	0.06	0.04	0.05

Table 6.51 CPU Times (sec.) of UB₂, Two Hazardous Task, K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.02	0.02
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.04	0.02	0.02	0.03	0.03	0.02	0.03	0.02	0.02	0.02

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.04	0.02	0.02	0.02	0.02	0.03	0.02	0.02

Table 6.52 CPU Times (sec.) of UB₂, Two Hazardous Task, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.03	0.02	0.02	0.02	0.02	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01
	MAX	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.02
47	AVG	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MAX	0.03	0.04	0.03	0.03	0.03	0.05	0.03	0.03	0.03	0.03
60	AVG	0.02	0.03	0.03	0.04	0.03	0.03	0.03	0.04	0.03	0.03
	MAX	0.04	0.05	0.05	0.06	0.05	0.05	0.04	0.06	0.03	0.04
73	AVG	0.03	0.06	0.05	0.07	0.04	0.04	0.04	0.07	0.03	0.05
	MAX	0.05	0.08	0.07	0.12	0.07	0.05	0.05	0.13	0.03	0.05

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02
47	AVG	0.01	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.01	0.02
	MAX	0.03	0.02	0.03	0.02	0.04	0.03	0.03	0.02	0.02	0.03
60	AVG	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	MAX	0.04	0.03	0.03	0.04	0.03	0.04	0.04	0.03	0.03	0.04
73	AVG	0.03	0.04	0.05	0.03	0.02	0.03	0.03	0.03	0.05	0.03
	MAX	0.04	0.06	0.04	0.10	0.03	0.05	0.04	0.05	0.06	0.04

Table 6.53 CPU Times (sec.) of UB₃, Two Hazardous Task, K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
	MAX	0.01	0.02	0.01	0.02	0.02	0.02	0.02	0.02	0.01	0.02
47	AVG	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.01
	MAX	0.02	0.03	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.00	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.01	0.02	0.02	0.02	0.02	0.01	0.02
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.02

Table 6.54 CPU Times (sec.) of UB₃, Two Hazardous Task, K=4

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.00	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
	MAX	0.01	0.02	0.02	0.02	0.02	0.01	0.02	0.02	0.02	0.02
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
47	AVG	0.01	0.02	0.02	0.01	0.02	0.01	0.02	0.02	0.02	0.02
	MAX	0.02	0.03	0.03	0.03	0.03	0.02	0.05	0.03	0.03	0.03
60	AVG	0.02	0.03	0.02	0.02	0.03	0.03	0.02	0.02	0.02	0.02
	MAX	0.02	0.04	0.04	0.04	0.04	0.05	0.03	0.04	0.04	0.04
73	AVG	0.03	0.04	0.03	0.04	0.05	0.05	0.02	0.03	0.03	0.04
	MAX	0.01	0.05	0.05	0.04	0.06	0.12	0.02	0.04	0.05	0.05

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
47	AVG	0.01	0.02	0.01	0.02	0.02	0.01	0.01	0.01	0.01	0.01
	MAX	0.02	0.02	0.02	0.02	0.03	0.02	0.03	0.03	0.02	0.02
60	AVG	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.02
	MAX	0.02	0.05	0.03	0.03	0.03	0.03	0.05	0.03	0.03	0.03
73	AVG	0.01	0.03	0.03	0.02	0.03	0.02	0.04	0.04	0.02	0.04
	MAX	0.01	0.14	0.05	0.06	0.03	0.06	0.07	0.03	0.04	0.06

Table 6.55 CPU Times (sec.) of UB₁, Three Hazardous Task, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.03	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
47	AVG	0.01	0.01	0.01	0.03	0.03	0.01	0.01	0.03	0.01	0.02	0.03	0.01	0.03
	MAX	0.03	0.03	0.03	0.03	0.07	0.03	0.03	0.07	0.03	0.04	0.08	0.03	0.05

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.03

Table 6.56 CPU Times (sec.) of UB₁, Three Hazardous Task, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.03	0.01	0.01	0.01	0.01	0.03	0.01	0.02	0.02	0.01	0.03	0.01
34	AVG	0.01	0.03	0.03	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.03	0.01	0.01
	MAX	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03
47	AVG	0.01	0.04	0.03	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.04	0.07	0.07	0.05	0.04	0.10	0.08	0.04	0.07	0.06	0.04	0.04	0.04
60	AVG	0.03	0.05	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
	MAX	0.03	0.07	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.06	0.07	0.07	0.05
73	AVG	0.03	0.07	0.05	0.08	0.08	0.05	0.04	0.07	0.07	0.05	0.07	0.08	0.05
	MAX	0.01	0.08	0.10	0.09	0.09	0.05	0.05	0.09	0.08	0.06	0.09	0.12	0.07

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
47	AVG	0.01	0.03	0.01	0.03	0.01	0.01	0.03	0.01	0.01	0.02	0.01	0.01	0.01
	MAX	0.04	0.04	0.03	0.04	0.03	0.03	0.07	0.03	0.03	0.05	0.03	0.03	0.04
60	AVG	0.01	0.03	0.04	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.03	0.04	0.07	0.04	0.07	0.07	0.04	0.05	0.05	0.05	0.04	0.04	0.05
73	AVG	0.03	0.04	0.07	0.04	0.04	0.07	0.04	0.04	0.05	0.03	0.04	0.04	0.03
	MAX	0.01	0.04	0.20	0.04	0.18	0.18	0.03	0.13	0.12	0.05	0.08	0.05	0.07

Table 6.57 CPU Times (sec.) of UB₂, Three Hazardous Task, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.03	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.01	0.03	0.03	0.02	0.03	0.03	0.03	0.03
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.05	0.03	0.03	0.04	0.04	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.03	0.03
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.05	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03

Table 6.58 CPU Times (sec.) of UB₂, Three Hazardous Task, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.04	0.03	0.03	0.03	0.03	0.02	0.02	0.03	0.01	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.03	0.01	0.01
	MAX	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03
47	AVG	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.04	0.05	0.04	0.04	0.04	0.07	0.04	0.04	0.05	0.04	0.04	0.04	0.04
60	AVG	0.03	0.04	0.04	0.05	0.04	0.04	0.04	0.05	0.04	0.04	0.05	0.04	0.04
	MAX	0.05	0.07	0.07	0.08	0.07	0.07	0.05	0.07	0.05	0.05	0.08	0.04	0.05
73	AVG	0.04	0.08	0.07	0.09	0.05	0.05	0.05	0.07	0.05	0.06	0.09	0.04	0.07
	MAX	0.07	0.10	0.09	0.16	0.09	0.07	0.07	0.13	0.05	0.07	0.17	0.04	0.07

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.03	0.03	0.01
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.03	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
47	AVG	0.01	0.01	0.01	0.01	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.01	0.03
	MAX	0.04	0.03	0.04	0.03	0.05	0.04	0.04	0.04	0.03	0.04	0.03	0.03	0.04
60	AVG	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.05	0.04	0.04	0.05	0.04	0.05	0.05	0.04	0.05	0.05	0.04	0.04	0.05
73	AVG	0.04	0.05	0.07	0.04	0.03	0.04	0.04	0.03	0.05	0.04	0.04	0.07	0.04
	MAX	0.05	0.08	0.05	0.13	0.04	0.07	0.05	0.05	0.07	0.05	0.07	0.08	0.05

Table 6.59 CPU Times (sec.) of UB₃, Three Hazardous Task, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01
	MAX	0.01	0.03	0.01	0.03	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.01	0.03
47	AVG	0.01	0.03	0.01	0.01	0.01	0.03	0.01	0.01	0.02	0.01	0.01	0.01	0.01
	MAX	0.03	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.00	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.01	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.01	0.03
47	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03

Table 6.60 CPU Times (sec.) of UB₃, Three Hazardous Task, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.00	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.01	0.03	0.03	0.03	0.03	0.01	0.03	0.03	0.02	0.03	0.03	0.03	0.03
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01	0.03	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
47	AVG	0.01	0.03	0.03	0.01	0.03	0.01	0.03	0.03	0.02	0.03	0.03	0.03	0.03
	MAX	0.03	0.04	0.04	0.04	0.04	0.03	0.07	0.04	0.03	0.05	0.04	0.04	0.04
60	AVG	0.03	0.04	0.03	0.03	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.03
	MAX	0.03	0.05	0.05	0.05	0.05	0.07	0.04	0.05	0.06	0.05	0.05	0.05	0.05
73	AVG	0.04	0.05	0.04	0.05	0.07	0.07	0.03	0.05	0.05	0.04	0.04	0.04	0.05
	MAX	0.01	0.07	0.07	0.05	0.08	0.16	0.03	0.07	0.11	0.05	0.05	0.07	0.07

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
34	AVG	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
47	AVG	0.01	0.03	0.01	0.03	0.03	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01
	MAX	0.03	0.03	0.03	0.03	0.04	0.03	0.04	0.04	0.03	0.03	0.04	0.03	0.03
60	AVG	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.03	0.03	0.01	0.03
	MAX	0.03	0.07	0.04	0.04	0.04	0.04	0.07	0.04	0.04	0.05	0.04	0.04	0.04
73	AVG	0.01	0.04	0.04	0.03	0.04	0.03	0.05	0.05	0.03	0.05	0.05	0.03	0.05
	MAX	0.01	0.18	0.07	0.08	0.04	0.08	0.09	0.04	0.07	0.08	0.04	0.05	0.08

E. CPU Times (sec.) of Lower Bounds

For small Networks $N=22, 34, 47$ and small number of workstations $K=2, K=4$ the CPU Times of Lower Bounds for one hazardous, two hazardous and three hazardous cases are presented in the Tables below.

Table 6.61 CPU Times (sec.) of LB_1 , One Hazardous Task, $K=2$

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.69	0.84	0.84	0.78	0.83	0.83	0.79	0.86	0.83	0.87
	MAX	1.16	1.03	0.99	0.89	1.05	1.03	0.91	1.10	1.01	0.98
34	AVG	0.70	1.12	1.12	0.97	1.20	1.24	1.23	1.23	1.35	1.22
	MAX	1.18	1.33	1.32	1.17	1.80	1.53	1.72	1.41	1.52	1.48
47	AVG	1.42	2.31	2.21	2.13	2.38	2.66	2.34	2.29	2.39	2.46
	MAX	3.92	3.72	3.62	2.92	3.71	5.44	3.37	3.50	3.37	3.92

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.58	0.76	0.98	0.71	0.75	0.98	0.75	0.94	1.10	0.95
	MAX	1.48	1.36	1.26	1.12	1.17	1.28	1.20	1.37	1.22	1.21
34	AVG	0.50	1.23	1.19	1.22	1.32	1.34	1.40	1.35	1.43	1.38
	MAX	1.65	1.67	1.49	1.48	1.60	1.77	1.53	1.56	1.62	1.58
47	AVG	1.28	2.07	2.21	2.15	2.39	2.29	2.24	2.29	2.37	2.29
	MAX	2.81	2.67	3.01	2.76	3.78	3.16	3.45	3.11	3.31	3.06

Table 6.62 CPU Times (sec.) of LB₁, One Hazardous Task, K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.57	1.68	1.89	1.95	1.75	1.87	1.99	1.89	2.06	1.96
	MAX	2.09	2.20	2.41	2.30	2.14	2.21	2.51	2.32	3.03	2.54
34	AVG	1.73	2.09	1.98	2.13	1.88	2.31	2.21	1.78	2.07	2.27
	MAX	2.33	2.76	2.66	2.95	2.64	3.03	2.79	2.45	2.54	2.91
47	AVG	2.54	3.86	3.46	3.63	3.22	3.36	3.39	3.22	3.32	3.23
	MAX	3.22	7.37	4.38	4.90	3.78	3.81	4.57	4.26	4.09	3.70
60	AVG	5.47	7.17	7.61	7.50	6.30	6.97	7.19	7.03	6.66	7.17
	MAX	9.92	9.66	13.56	14.73	9.70	10.79	12.41	13.05	8.79	11.48
73	AVG	11.75	13.33	16.74	15.50	12.34	14.45	15.25	15.37	13.37	15.91
	MAX	30.60	12.67	41.97	44.29	24.90	30.61	33.71	39.96	18.91	35.61

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.39	1.60	1.96	1.84	1.63	2.00	1.68	1.50	1.89	1.76
	MAX	2.46	3.10	2.31	3.08	3.36	2.56	2.04	2.88	2.44	2.05
34	AVG	1.60	2.11	2.59	2.25	2.50	2.38	2.56	2.43	2.50	2.52
	MAX	3.31	2.97	3.15	2.75	3.20	2.87	3.16	2.90	2.94	3.35
47	AVG	2.74	2.86	2.56	2.95	2.71	2.79	3.49	2.85	2.92	3.08
	MAX	3.82	3.91	3.29	3.45	3.27	3.38	6.97	4.35	3.65	4.09
60	AVG	4.68	5.04	5.54	4.81	4.65	4.72	4.84	4.37	4.91	4.61
	MAX	5.73	7.80	10.81	5.68	5.71	5.38	5.84	5.59	5.72	5.57
73	AVG	8.00	8.88	11.98	7.86	7.97	7.99	6.72	6.71	8.25	6.89
	MAX	8.60	15.55	35.58	9.34	9.98	8.56	4.90	7.18	8.97	7.60

Table 6.63 CPU Times (sec.) of LB₂, One Hazardous Task, K=2

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.72	1.39	1.18	1.18	1.33	1.18	1.13	1.46	1.19	1.25
	MAX	1.37	1.74	1.83	1.46	1.70	1.63	1.47	2.01	1.90	1.63
34	AVG	1.65	1.95	2.14	1.65	2.10	1.64	1.78	1.88	1.50	1.61
	MAX	3.04	2.86	3.28	2.47	3.91	2.95	3.45	3.91	1.87	2.53
47	AVG	3.77	3.34	3.36	3.59	3.13	3.39	3.56	2.84	2.76	3.03
	MAX	10.22	7.31	4.96	5.78	7.29	6.06	6.97	6.51	4.13	5.76

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.81	1.24	1.67	1.14	1.20	1.59	1.04	1.56	1.77	1.34
	MAX	1.69	2.71	2.12	2.01	2.18	2.11	2.07	2.52	2.22	2.28
34	AVG	1.21	2.52	2.03	2.60	2.14	1.82	1.87	2.20	1.85	1.71
	MAX	4.20	3.68	2.94	4.07	2.73	2.86	3.11	2.91	2.89	2.25
47	AVG	3.33	3.44	3.99	3.90	3.23	3.00	2.90	2.88	2.59	2.89
	MAX	6.63	5.33	7.20	5.94	4.35	4.81	4.88	3.94	3.31	4.61

Table 6.64 CPU Times (sec.) of LB₂, One Hazardous Task, K=4

		CT1									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.80	2.33	2.29	2.63	2.57	2.34	2.30	3.10	2.88	2.37
	MAX	2.53	3.34	3.53	4.13	4.36	3.52	3.26	4.84	3.85	3.21
34	AVG	3.79	4.30	3.85	3.89	3.31	4.22	4.04	2.53	3.68	3.93
	MAX	5.21	6.75	5.32	5.51	6.86	6.41	5.48	4.55	4.95	4.97
47	AVG	6.34	6.81	6.34	7.09	5.82	5.34	6.27	5.25	6.16	5.83
	MAX	9.04	12.96	9.05	10.85	7.45	7.47	8.67	6.99	10.16	8.89
60	AVG	13.93	12.43	14.90	12.81	9.70	12.38	11.67	10.53	12.32	11.22
	MAX	21.64	23.02	20.96	23.79	19.36	24.85	19.02	18.86	27.97	23.43
73	AVG	30.63	22.70	35.05	23.15	16.18	28.67	21.71	21.13	24.64	21.59
	MAX	51.82	40.89	48.56	52.13	50.31	82.66	41.74	50.90	77.05	61.73

		CT2									
π & POS		$\pi=1$	$\pi=0.8$			$\pi=0.5$			$\pi=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.44	2.55	2.85	2.70	2.50	3.12	2.64	2.54	2.68	2.60
	MAX	2.63	4.31	4.66	3.65	3.88	5.17	3.50	3.98	4.06	3.76
34	AVG	3.64	4.91	4.89	5.24	4.67	4.22	5.06	4.94	4.15	4.94
	MAX	8.29	6.61	6.90	6.77	7.16	7.06	6.42	8.68	6.94	8.45
47	AVG	6.81	6.76	5.22	5.31	5.31	4.31	6.20	4.50	4.27	4.81
	MAX	10.17	9.09	6.95	6.83	6.69	5.73	14.05	7.57	6.50	7.60
60	AVG	12.11	10.12	10.86	7.72	7.49	8.90	8.25	6.24	7.65	6.23
	MAX	14.78	22.48	19.27	11.60	10.87	12.80	13.07	11.77	11.16	7.72
73	AVG	21.52	15.17	22.62	11.24	10.56	18.37	10.98	8.67	13.71	8.08
	MAX	21.49	55.56	53.43	19.70	17.65	28.60	12.16	18.28	19.15	7.85

Table 6.65 CPU Times (sec.) of LB₁, Two Hazardous Task, K=2

		CT1										
π & POS		π=1	πz1= 0.8, πz2= 0.8				πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	
22	AVG	0.84	1.41	1.27	1.41	1.42	1.39	1.47	1.44	1.47	1.55	
	MAX	1.84	1.79	1.75	1.66	1.78	1.71	1.74	1.65	1.91	2.02	
34	AVG	1.27	2.04	2.03	1.77	2.18	2.25	2.24	2.24	2.45	2.22	
	MAX	2.15	2.42	2.39	2.12	3.28	2.78	3.12	2.56	2.76	2.68	
47	AVG	2.58	4.21	4.01	3.88	4.33	4.84	4.26	4.16	4.34	4.48	
	MAX	7.13	6.76	6.59	5.31	6.75	9.89	6.13	6.37	6.12	7.13	

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	0.97	1.43	1.53	1.42	1.44	1.28	1.55	1.39	1.49	1.56
	MAX	2.21	2.61	1.93	2.08	2.27	2.38	1.98	1.63	1.90	1.79
34	AVG	0.91	2.23	2.16	2.22	2.41	2.44	2.54	2.46	2.60	2.52
	MAX	3.01	3.04	2.70	2.69	2.91	3.23	2.79	2.84	2.94	2.88
47	AVG	2.33	3.76	4.01	3.91	4.35	4.16	4.07	4.16	4.32	4.17
	MAX	5.11	4.85	5.47	5.01	6.87	5.74	6.28	5.65	6.01	5.56

Table 6.66 CPU Times (sec.) of LB₁, Two Hazardous Task, K=4

		CT1									
π & POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	2.18	2.36	2.84	2.86	2.88	2.78	2.89	2.89	2.89	3.05
	MAX	3.13	3.17	3.77	3.28	3.41	3.34	3.48	3.42	3.24	3.37
34	AVG	3.15	3.79	3.60	3.87	3.42	4.20	4.03	3.23	3.76	4.12
	MAX	4.23	5.01	4.84	5.37	4.80	5.52	5.07	4.45	4.62	5.29
47	AVG	4.62	7.02	6.28	6.60	5.85	6.11	6.16	5.85	6.03	5.88
	MAX	5.85	13.40	7.96	8.91	6.87	6.92	8.31	7.75	7.43	6.73
60	AVG	9.94	13.04	13.83	13.64	11.46	12.67	13.07	12.79	12.10	13.04
	MAX	18.04	17.57	24.65	26.79	17.64	19.62	22.57	23.73	15.99	20.88
73	AVG	21.37	24.24	30.44	28.19	22.43	26.28	27.74	27.94	24.31	28.93
	MAX	55.64	23.04	76.31	80.52	45.28	55.65	61.29	72.66	34.39	64.74

		CT2									
π & POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.91	2.34	2.37	2.34	2.50	2.91	2.60	2.36	2.73	2.64
	MAX	4.17	2.76	3.10	3.12	3.24	3.68	3.34	3.27	3.93	3.40
34	AVG	2.91	3.84	4.71	4.08	4.55	4.33	4.65	4.42	4.54	4.58
	MAX	6.02	5.40	5.72	5.00	5.83	5.22	5.74	5.27	5.35	6.09
47	AVG	4.97	5.19	4.65	5.36	4.94	5.07	6.34	5.17	5.31	5.61
	MAX	6.94	7.12	5.97	6.28	5.94	6.15	12.67	7.90	6.64	7.43
60	AVG	8.51	9.16	10.06	8.75	8.46	8.58	8.81	7.94	8.93	8.38
	MAX	10.41	14.18	19.66	10.32	10.38	9.79	10.62	10.16	10.41	10.13
73	AVG	14.55	16.14	21.78	14.29	14.50	14.52	12.23	12.20	14.99	12.52
	MAX	15.63	28.27	64.69	16.98	18.15	15.57	8.91	13.06	16.31	13.81

Table 6.67 CPU Times (sec.) of LB₂, Two Hazardous Task, K=2

		CT1									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.81	2.73	2.33	2.46	2.31	2.27	2.42	2.24	2.45	2.46
	MAX	3.70	4.15	4.12	3.02	4.17	3.18	3.20	3.66	3.82	3.28
34	AVG	2.99	3.54	3.90	3.00	3.82	2.98	3.23	3.42	2.72	2.92
	MAX	5.54	5.20	5.96	4.50	7.11	5.36	6.26	7.12	3.39	4.59
47	AVG	6.86	6.08	6.11	6.53	5.68	6.16	6.48	5.16	5.02	5.51
	MAX	18.59	13.30	9.02	10.51	13.25	11.01	12.66	11.84	7.50	10.48

		CT2									
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8$			$\pi z1=0.8, \pi z2=0.2$			$\pi z1=0.2, \pi z2=0.2$		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.94	3.66	3.08	2.59	2.55	2.00	2.38	2.49	2.10	2.19
	MAX	4.58	7.86	4.31	3.86	4.67	3.72	3.96	4.16	2.85	3.15
34	AVG	2.20	4.59	3.69	4.73	3.89	3.31	3.39	4.00	3.37	3.12
	MAX	7.64	6.69	5.35	7.39	4.97	5.19	5.66	5.29	5.25	4.09
47	AVG	6.06	6.26	7.26	7.10	5.88	5.46	5.28	5.23	4.70	5.25
	MAX	12.05	9.69	13.09	10.80	7.91	8.74	8.87	7.17	6.02	8.38

Table 6.68 CPU Times (sec.) of LB₂, Two Hazardous Task, K=4

		CT1									
π & POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	5.65	4.70	6.14	6.20	4.97	5.96	5.19	4.24	5.67	5.84
	MAX	6.72	7.48	8.01	7.75	7.37	8.29	6.75	6.95	8.31	7.41
34	AVG	6.89	7.83	7.01	7.07	6.03	7.67	7.34	4.60	6.68	7.14
	MAX	9.47	12.28	9.68	10.02	12.47	11.65	9.97	8.27	8.99	9.04
47	AVG	11.52	12.38	11.52	12.89	10.58	9.72	11.41	9.54	11.20	10.60
	MAX	16.43	23.56	16.46	19.73	13.55	13.58	15.76	12.71	18.47	16.16
60	AVG	25.34	22.60	27.10	23.30	17.64	22.50	21.22	19.14	22.40	20.40
	MAX	39.34	41.85	38.12	43.25	35.21	45.18	34.58	34.30	50.86	42.59
73	AVG	55.70	41.28	63.73	42.09	29.41	52.12	39.47	38.43	44.80	39.26
	MAX	94.22	74.35	88.28	94.79	91.48	150.30	75.89	92.55	140.10	112.24

		CT2									
π & POS		π=1	πz1= 0.8, πz2= 0.8			πz1= 0.8, πz2= 0.2			πz1= 0.2, πz2= 0.2		
N (#)		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	4.23	4.73	4.36	5.02	4.19	5.30	4.19	3.89	4.42	4.56
	MAX	7.62	7.24	7.05	7.86	8.92	7.91	6.64	7.50	6.06	7.48
34	AVG	6.62	8.93	8.89	9.52	8.50	7.66	9.20	8.98	7.55	8.98
	MAX	15.06	12.02	12.54	12.32	13.02	12.83	11.67	15.78	12.62	15.37
47	AVG	12.38	12.28	9.49	9.65	9.66	7.83	11.27	8.17	7.76	8.75
	MAX	18.48	16.53	12.63	12.41	12.17	10.41	25.55	13.77	11.82	13.81
60	AVG	22.01	18.41	19.75	14.04	13.62	16.17	15.00	11.35	13.91	11.34
	MAX	26.87	40.87	35.03	21.09	19.76	23.27	23.77	21.39	20.29	14.04
73	AVG	39.12	27.58	41.12	20.44	19.20	33.40	19.96	15.76	24.93	14.69
	MAX	39.07	101.02	97.14	35.82	32.09	52.00	22.11	33.24	34.83	14.28

Table 6.69 CPU Times (sec.) of LB₁, Three Hazardous Task, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.03	1.73	1.56	1.73	1.75	1.71	1.81	1.76	1.76	1.86	1.77	1.81	1.91
	MAX	2.26	2.20	2.15	2.04	2.19	2.10	2.14	2.11	2.23	2.31	2.03	2.35	2.48
34	AVG	1.56	2.51	2.50	2.18	2.68	2.77	2.76	2.72	2.89	2.74	2.76	3.01	2.73
	MAX	2.64	2.98	2.94	2.61	4.03	3.42	3.84	3.59	3.41	3.57	3.15	3.39	3.30
47	AVG	3.17	5.18	4.93	4.77	5.33	5.95	5.24	5.22	5.65	5.38	5.12	5.34	5.51
	MAX	8.77	8.31	8.11	6.53	8.30	12.16	7.54	8.07	9.85	8.15	7.84	7.53	8.77

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	1.19	1.76	1.88	1.75	1.77	1.57	1.91	1.74	1.70	1.91	1.71	1.83	1.92
	MAX	2.72	3.21	2.37	2.56	2.79	2.93	2.44	2.40	2.63	2.32	2.00	2.34	2.20
34	AVG	1.12	2.74	2.66	2.73	2.96	3.00	3.12	3.00	3.10	3.11	3.03	3.20	3.10
	MAX	3.70	3.74	3.32	3.31	3.58	3.97	3.43	3.54	3.79	3.49	3.49	3.62	3.54
47	AVG	2.87	4.62	4.93	4.81	5.35	5.12	5.01	5.23	5.22	5.07	5.12	5.31	5.13
	MAX	6.29	5.97	6.73	6.16	8.45	7.06	7.72	7.70	7.23	7.28	6.95	7.39	6.84

Table 6.70 CPU Times (sec.) of LB₁, Three Hazardous Task, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	2.68	2.90	3.49	3.52	3.54	3.42	3.55	3.55	3.49	3.65	3.55	3.55	3.75
	MAX	3.85	3.90	4.64	4.03	4.19	4.11	4.28	4.20	4.05	4.21	4.21	3.99	4.15
34	AVG	3.87	4.66	4.43	4.76	4.21	5.17	4.96	4.09	4.90	5.01	3.97	4.62	5.07
	MAX	5.20	6.16	5.95	6.61	5.90	6.79	6.24	5.69	6.24	6.37	5.47	5.68	6.51
47	AVG	5.68	8.63	7.72	8.12	7.20	7.52	7.58	7.20	7.47	7.40	7.20	7.42	7.23
	MAX	7.20	16.48	9.79	10.96	8.45	8.51	10.22	8.99	8.83	9.25	9.53	9.14	8.28
60	AVG	12.23	16.04	17.01	16.78	14.10	15.58	16.08	14.91	15.23	16.06	15.73	14.88	16.04
	MAX	22.19	21.61	30.32	32.95	21.70	24.13	27.76	25.44	21.90	26.72	29.19	19.67	25.68
73	AVG	26.29	29.82	37.44	34.67	27.59	32.32	34.12	30.98	31.11	34.85	34.37	29.90	35.58
	MAX	68.44	28.34	93.86	99.04	55.69	68.45	75.39	72.53	55.37	77.51	89.37	42.30	79.63

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	2.35	2.88	2.92	2.88	3.08	3.58	3.20	2.99	3.47	3.22	2.90	3.36	3.25
	MAX	5.13	3.39	3.81	3.84	3.99	4.53	4.11	4.00	4.68	4.15	4.02	4.83	4.18
34	AVG	3.58	4.72	5.79	5.02	5.60	5.33	5.72	5.52	5.46	5.68	5.44	5.58	5.63
	MAX	7.40	6.64	7.04	6.15	7.17	6.42	7.06	6.83	6.50	7.28	6.48	6.58	7.49
47	AVG	6.11	6.38	5.72	6.59	6.08	6.24	7.80	6.22	6.38	7.35	6.36	6.53	6.90
	MAX	8.54	8.76	7.34	7.72	7.31	7.56	15.58	8.51	7.87	12.36	9.72	8.17	9.14
60	AVG	10.47	11.27	12.37	10.76	10.41	10.55	10.84	10.09	10.77	10.57	9.77	10.98	10.31
	MAX	12.80	17.44	24.18	12.69	12.77	12.04	13.06	12.63	12.42	12.76	12.50	12.80	12.46
73	AVG	17.90	19.85	26.79	17.58	17.84	17.86	15.04	16.42	18.15	15.22	15.01	18.44	15.40
	MAX	19.22	34.77	79.57	20.89	22.32	19.15	10.96	19.19	19.61	13.97	16.06	20.06	16.99

Table 6.71 CPU Times (sec.) of LB₂, Three Hazardous Task, K=2

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	2.23	3.36	2.87	3.03	2.84	2.79	2.98	2.80	2.90	3.00	2.76	3.01	3.03
	MAX	4.55	5.10	5.07	3.71	5.13	3.91	3.94	4.82	4.31	3.99	4.50	4.70	4.03
34	AVG	3.68	4.35	4.80	3.69	4.70	3.67	3.97	4.45	3.51	3.78	4.21	3.35	3.59
	MAX	6.81	6.40	7.33	5.54	8.75	6.59	7.70	8.75	5.38	6.67	8.76	4.17	5.65
47	AVG	8.44	7.48	7.52	8.03	6.99	7.58	7.97	6.67	6.88	7.37	6.35	6.17	6.78
	MAX	22.87	16.36	11.09	12.93	16.30	13.54	15.57	15.43	11.38	14.23	14.56	9.23	12.89

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	2.39	4.50	3.79	3.19	3.14	2.46	2.93	3.10	2.52	2.81	3.06	2.58	2.69
	MAX	5.63	9.67	5.30	4.75	5.74	4.58	4.87	5.43	4.04	4.37	5.12	3.51	3.87
34	AVG	2.71	5.65	4.54	5.82	4.78	4.07	4.17	4.85	4.11	4.00	4.92	4.15	3.84
	MAX	9.40	8.23	6.58	9.09	6.11	6.38	6.96	6.31	6.42	6.00	6.51	6.46	5.03
47	AVG	7.45	7.70	8.93	8.73	7.23	6.72	6.49	6.83	6.25	6.48	6.43	5.78	6.46
	MAX	14.82	11.92	16.10	13.28	9.73	10.75	10.91	9.27	9.08	10.61	8.82	7.40	10.31

Table 6.72 CPU Times (sec.) of LB₂, Three Hazardous Task, K=4

		CT1												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	7.0	5.8	7.6	7.6	6.1	7.3	6.4	5.7	7.2	6.8	5.2	7.0	7.2
	MAX	8.3	9.2	9.9	9.5	9.1	10.2	8.3	8.8	10.2	8.7	8.6	10.2	9.1
34	AVG	8.5	9.6	8.6	8.7	7.4	9.4	9.0	6.5	8.8	8.9	5.7	8.2	8.8
	MAX	11.7	15.1	11.9	12.3	15.3	14.3	12.3	12.8	12.7	11.7	10.2	11.1	11.1
47	AVG	14.2	15.2	14.2	15.9	13.0	12.0	14.0	12.4	12.9	13.5	11.7	13.8	13.0
	MAX	20.2	29.0	20.3	24.3	16.7	16.7	19.4	16.2	19.7	19.6	15.6	22.7	19.9
60	AVG	31.2	27.8	33.3	28.7	21.7	27.7	26.1	22.6	27.6	25.6	23.5	27.6	25.1
	MAX	48.4	51.5	46.9	53.2	43.3	55.6	42.5	42.8	59.1	47.5	42.2	62.6	52.4
73	AVG	68.5	50.8	78.4	51.8	36.2	64.1	48.6	41.7	59.6	48.4	47.3	55.1	48.3
	MAX	115.9	91.5	108.6	116.6	112.5	184.9	93.3	113.2	178.6	115.7	113.8	172.3	138.1

		CT2												
π & POS		$\pi=1$	$\pi z1=0.8, \pi z2=0.8, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.8$			$\pi z1=0.8, \pi z2=0.2, \pi z3=0.2$			$\pi z1=0.2, \pi z2=0.2, \pi z3=0.2$		
N = 22		-	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3	POS1	POS2	POS3
22	AVG	5.2	5.8	5.4	6.2	5.2	6.5	5.2	5.0	6.0	5.4	4.8	5.4	5.6
	MAX	9.4	8.9	8.7	9.7	11.0	9.7	8.2	10.1	8.6	8.7	9.2	7.5	9.2
34	AVG	8.1	11.0	10.9	11.7	10.5	9.4	11.3	10.8	9.4	11.2	11.1	9.3	11.1
	MAX	18.5	14.8	15.4	15.2	16.0	15.8	14.4	17.7	15.7	16.6	19.4	15.5	18.9
47	AVG	15.2	15.1	11.7	11.9	11.9	9.6	13.9	11.0	9.6	12.3	10.1	9.5	10.8
	MAX	22.7	20.3	15.5	15.3	15.0	12.8	31.4	16.0	13.7	24.2	16.9	14.5	17.0
60	AVG	27.1	22.6	24.3	17.3	16.8	19.9	18.5	15.4	18.5	16.2	14.0	17.1	14.0
	MAX	33.1	50.3	43.1	25.9	24.3	28.6	29.2	25.3	26.8	23.3	26.3	25.0	17.3
73	AVG	48.1	33.9	50.6	25.1	23.6	41.1	24.6	21.5	35.9	21.3	19.4	30.7	18.1
	MAX	48.1	124.3	119.5	44.1	39.5	64.0	27.2	40.2	53.4	22.4	40.9	42.8	17.6

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Göksoy Kalaycılar, Eda
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EDUCATION

Degree	Institution	Year of Graduation
MS	METU Industrial Engineering	2010
BS	METU Industrial Engineering	2007
High School	Science High School, Aydın	2002

WORK EXPERIENCE

Year	Place	Enrollment
2014-Present	ASELSAN	Senior Project Manager
2010-2014	ASELSAN	Project Manager
2007-2010	ASELSAN	Project Engineer

FOREIGN LANGUAGES

Advanced English, Intermediate German

PUBLICATIONS

Kalaycilar, E.G., Azizoglu, M., and Yeralan, S. "A Disassembly line balancing problem with fixed number of workstations", European Journal of Operational Research, 249, 592-604.(2016)

Kalaycilar, E.G., Batun S., and Azizoglu, M. "A Stochastic disassembly line balancing problem with hazardous tasks", submitted to European Journal of Operational Research

HOBBIES

Basketball, Movies