USING EQUITABLE OPTIMIZATION FOR THE HAZMAT TRANSPORT NETWORK DESIGN PROBLEM

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By

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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August 2024

The shipment of hazardous materials is challenging due to the risk the population centers face during transportation. The policy makers often have concerns for ensuring a balanced allocation of the risks to the population centers. We structure this problem as an equitable optimization problem with multiple objectives aiming to minimize the risk exposure of different neighborhoods. The resulting multiobjective mixed integer linear programming problem is solved to provide equitably nondominated solutions, each with different levels of efficiency (total risk) and fairness (allocation of risk). Moreover, a robust programming extension is discussed and shown to be valuable under uncertainty.

Keywords: Fairness, multiobjective optimization, equitable optimization, robust efficiency, hazmat network design, risk allocation.

ÖZET

TEHLİKELİ MADDE TAŞIMA AĞI TASARIM PROBLEMİ İÇİN EŞİTLİKÇİ ENİYİLEME YAKLAŞIMI

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Tehlikeli maddelerin taşınması, nüfus merkezlerinin karşılaştığı risk nedeniyle zorlu bir problemdir. Nüfus merkezlerine yönelik risklerin adil bir şekilde dağıtılması karar vericiler için de önem arz etmektedir. Bu problem çok amaçlı eşitlikçi eniyileme problemi şeklinde ele alınarak, farklı nüfus merkezlerinin maruz kaldığı risk değerleri enazlanmaktadır. Ortaya çıkan çok amaçlı karma tam sayılı doğrusal programlama modeli, her biri farklı verimlilik (toplam risk) ve adalet (risk dağılımı) seviyelerine sahip eşitlikçi baskın çözümler bulmak için çözülmektedir. Ayrıca, problemin gürbüz programlama uzantısı tartışılıp, bu yaklaşımın belirsizlik altında faydalı olduğu gösterilmiştir.

Anahtar sözcükler: Adillik, çok amaçlı eniyileme, eşitlikçi eniyileme, gürbüz baskınlık, tehlikeli madde taşıma ağı tasarımı, risk dağıtımı.

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Chapter 1

Introduction

Hazardous materials transportation an area that Operational Research is widely used to make the systems efficient, yet fair for the groups exposed to the resulting risks. A solution that focuses on minimizing the total risk may not be acceptable on the grounds of fairness, if disproportional amounts of risk are borne by some population groups. In line with this, there have been various attempts in the literature, focusing on different aspects of the problem such as exploring alternative methods for quantifying the risk and incorporating equity in the risk distribution.

Most of these, however, utilize a single type of fairness function to trade efficiency off-against fairness. In this thesis we address this problem using a multiobjective, specifically, equitable optimization perspective and present an approach that presents the decision maker with a set of alternatives, each with different degrees of inequity aversion. We do so by tractable linear programming formulations as long as the operational constraints of the network design problem is linear.

Our focus in this work is on providing structured decision support for hazardous materials network design that takes fairness of risk allocation to different population groups into account. From the methodological side we propose an equitable optimization approach for this problem, which is based on formulating and solving special types of multiobjective optimization problems. We further extend our methodology so as to find robust solutions hence utilize concepts from robust multiobjective programming. From the application side, we focus on hazardous material network design.

The rest of the thesis is organized as follows. In Chapter 2 we provide a review of the related literature as well as the our contributions. In Chapter 3, we provide some preliminaries on equitable optimization. In Chapter 4, we provide the mathematical modelling formulations of the deterministic and robust hazmat network design problems, respectively. We discuss the results of our computational analysis in Chapter 6 , where we demonstrate that a spectrum of solutions can be obtained with our approach, which includes solutions obtained by well-known metrics such as the Rawlsian. Our results also underscore the relevance of a robust programming approach as the best plan with respect to an expected scenario may lead to high levels of imbalance under some scenarios.

Chapter 2

Literature Review

In line with our scope, we discuss the relevant literature in three sections. We first provide a review of attempts in the literature to incorporate fairness concerns into operational research applications. We then discuss studies that have considered hazmat network design and clarify our contribution to this domain. We finally briefly discuss the solution methodologies used in multiobjective programming in general as well as the ones used in multiobjective equitable and robust optimization.

2.1 Inequity aversion in optimization

One of the approaches to handling the concerns of fairness in optimization is incorporating some fairness metrics into the optimization model either in the constraints or as another objective function [1]. In literature many examples can be found where inequality indexes are utilized in the constraints, we just name a few. Luan et al. [2] worked on a train rescheduling problem and incorporated fairness concerns by using Gini Index in the constraints to ensure equity among operating companies. Chang et al. [3] focused on optimal time slot and channel allocation problems, where system capacity is maximized while ensuring fairness among cells. To establish fairness, they used a specific type of range function where the ratio of the minimum capacity allocated to the maximum capacity allocated is greater than a threshold. Besides Gini and range, there are some other commonly used fairness metrics in literature, such as mean deviation and variance, that can be incorporated into the optimization model [1]. Ogryczak [4] worked on the facility allocation problem, in which fairness concerns are incorporated in the form of objective functions. Mean deviation is used as an objective besides efficiency, and a bi-criteria problem is proposed. Martin et al. [5] studied the rostering of the nurses where minimum staff requirements are met in the observance of nurses' contracts. They used four different metrics, and by using them, they formulated bi-criteria problems.

Another approach to handle the fairness-efficiency trade-off is incorporating outcome vector's aggregation function in the optimization model. The aggregation functions yielding fair outcomes are called Inequity-Averse Aggregation Functions. These functions can take different forms based on how they capture the trade-off between equity and efficiency.However, there are some core properties that any inequity-averse function is expected to satisfy. For a function to be inequity-averse it has to be symmetric to regard anonymity [1]. The collection of these symmetric functions that respect Pigou-Dalton principle of transfers and monotonicity are called equitable aggregation functions. Kostreva et al. [6] showed that all Equitable Aggregation Functions are strictly Schur-concave (Schur-convex in the minimization settings). Ordered Weighted Averaging (OWA) functions proposed by Yager [7] are examples of Schur-concave functions. They are equitable by nature, when the weights are chosen in a non-decreasing manner so that worst off (resp. best off) entity in the allocation vector will have the highest(resp. lowest) weight.

There are various studies in literature that maximize (minimizes) a specific Schur-concave (Schur-convex) function to capture the equity-efficiency trade-off. For instance, Marín et al. [8] worked on a reformulation of a discrete ordered median problem (DOMP), which minimizes the weighted sum of the sorted allocation costs.

Our aim in this thesis is to find the so-called equitably efficient solutions (and robust counterparts) for hazmat network design problem so that the policy makers will be informed on the implications of alternative network designs on risk allocation across population centers. To achieve this, problem is formulated as an equitable (multiobjective) optimization problem and the corresponding fair Pareto solutions are found through ordered weighted averaging based scalarizations. That is, as opposed to most of the studies in the literature, we do not obtain a single solution by optimizing a specific equitable aggregation function but find a set of Pareto solutions. As we will demonstrate later in the case study, the set of solutions returned by our approach will include the ones found by using an inequality index in the model but will also present other solutions that are also equitable to the decision makers.

2.2 Hazmat Network Design

Erkut et al. [9] examined the hazardous materials transportation literature in four main categories that are risk assessment, routing, combined facility location and routing, and network design. In this part we briefly mention the literature on network design. Then we focus on the risk assessment and the efforts for distributing this risk in equitable manner.

In hazmat transportation, the challenge is transporting hazardous material between populated areas. The nature of hazardous goods necessitates some precautions to be taken such as closing some road segments while connecting these populated areas. Considering these, Kara and Verter [10] characterized the hazmat network design problem (htndp) in two levels. After defining the htndp, they formulated the problem as a bi-level programming problem. In this nested programme, inner level is based on the transportation decisions of the carriers to minimize their costs. The second level is selecting the road segments to be closed by the authority to minimize their assessed risk. Erkut and Gzara [11] demonstrated that these problems cannot be solved independently and bi-level formulation is needed. The literature revolves around the single-level mixed integer programming problem, which is the reformulation of bi-level problem derived using Karush-Kuhn-Tucker (KKT) conditions. Using the network flow structure of the problems, Erkut and Gzara [12] solved the bi-level problem by developing a heuristic approach without requiring single-level reformulation and they report reduction in the execution times. Fontaine et al. [13] proposed a multicut Benders decomposition approach to solve the htndp while minimizing the risk defined. Bianco et al. [14] proposed a game-theoretical approach in toll setting. The utilizations of the arcs of the network by the other carriers determine the price of a toll. Problem of the carriers in this setting is a Nash equilibrium problem and the whole problem is a mathematical programming problem with equilibrium constraints. In order to solve the corresponding problem local search heuristis is developed.

For risk assessment in hazmat transportation literature, it is common to use a parameter indicating the consequence of a hazmat accident on the arc of the network (influence parameter, inf_{ij}). This parameter is defined based on various factors in the literature such as hazardous material type, amount of population impacted in the stretch of an arc [15, 16]. Using this notation various risk measures are defined to be minimized by the authority [17]. One such measure is the so called traditional risk [18], which corresponds to the expected risk outcome over the given path (r). This risk is calculated as the cumulative sum of the accident probability (p_{ij}) of an arc multiplied by the influence parameter $Trad(r) = \sum_{(i,j)\in A} inf_{ij}p_{ij}$. Erkut and Ingolfsson [19] used mean-variance objective function to assess the risk while Abkowitz et al. [20] considered that risk preference of the decision makers may vary and proposed the perceived risk of the path as follows: $Perceived(r) = \sum_{(i,j)\in A} inf_{ij}^{k}p_{ij}$, where k denotes the risk preference and lower values of k corresponds to risk averse behaviour.

When it comes to equilibrating the risk, Bianco et al. [14] minimized the maximum risk on an arc. Carotenuto et al. [21] imposed a constraint on each traversed arc to make sure that the risk incurred is below a determined threshold while minimizing the total risk. Besides evaluating and equating the risk for specific paths, Gopalan et al. [22] proposed a population (zone) based approach for evaluating the risk in the hazmat network where several trips are needed. The pairwise risk difference between population areas is kept under a threshold value. To solve the problem they developed a heuristic solving the single-trip problems iteratively. Fontaine et al. [23] considered several risk assessment functions from the literature (developed for evaluating the risk of a path) and using population zones, proposed their counterparts for the population based approach. They solved the multi-mode hazmat network design problem (mHTNDP) using alternative risk assessment functions and compared their performance.

In this thesis we propose a way to find a set of equitably nondominated points, each corresponding to a different degree of inequity aversion. As we demonstrate the implementation and computational feasibility of our approach on mHTNDP, the closest work in the literature to ours is [23]. We, however do not utilize a specific social welfare function, as the parameters of such a function may not be easy to determine, but provide a set of solutions for further investigation by the decision makers.

2.3 Multiobjective, equitable and robust optimization

In multiobjective optimization (MOP) literature there are various approaches to find the solutions of interest. Such approaches can be classified into two main categories as the decision space based approaches and criterion space based approaches. Since the number of objectives is typically much smaller than the number of decision variables, criterion space based approaches tend to be more scalable. We also use a criterion space based approach hence we mention some existing algorithms in this domain.

Cohon [24] proposed an algorithm for finding the extreme supported nondominated (esn) points of bi-objective mixed integer programming problems. In this algorithm, new solutions are found by a search through the normal vector passing from already-found pairs. If a new solution is found through this search, new pairs are generated and the algorithm terminates when no pair to be searched remains. Przybylski et al. [25] showed that algorithm presented by [24] is not directly applicable to problems having more than two objectives and proposed an algorithm to find esn points of multi-objective mixed integer problems. In this algorithm, they decomposed the weight space for each esn point and searched the corresponding weight space at the boundaries, utilizing the algorithm proposed by [24] earlier. Although theoretically it is possible to generate all esn points through this algorithm, due to the computational burden they were able to present the solutions for only the three objective case. Özpeynirci and Köksalan [26] proposed an exact algorithm (ExA) that searches the criterion space in a systematic manner to generate the esn points for any number of objectives. ExA is discussed in Chapter 5 in detail.

The main assumption in classical multiple criteria optimization is that the criteria are incomparable. However, in equitable settings, the criteria correspond to the outcomes enjoyed by multiple entities, hence they are uniform and comparable [27]. Kostreva and Ogryczak [27] proposed the concepts of equitable rational preference and equitable efficiency and discussed the use of ordered weighted averaging (OWA) aggregation functions for finding the equitably efficient solutions. The ordering operator makes the corresponding optimization problem nonlinear. Yager [28] proposed the constrained OWA aggregation and showed how to formulate it as a mixed integer linear programming problem. Note that in OWA aggregation, strictly monotonic weights should be used to ensure the principle of transfers. Ogryczak et al. [29] proposed a linear programming formulation for OWA optimization using the idea of cumulative ordered achievement vectors.

There is a large body of literature exploring robustness concepts for single-objective optimization [30]. However, as the definition of worst-case is not clear in multi-objective optimization, robustness concepts for the single-objective optimization cannot be directly applicable to multi-objective settings [31]. Hence, various concepts for multi-objective settings are proposed [32]. Ide et al. propose the flimsily and highly robust efficiency concepts [32], assuming that the uncertainty set is known and consists of scenarios. A solution that is efficient for at least one scenario is called flimsily robust efficient while to be a highly robust efficient solution, it must be efficient in all scenarios. Kuroiwa and Lee [33] set the objective functions to their worst cases over the uncertainty set and solve the corresponding deterministic problem. Efficient solutions of this problem are said to be robust efficient. Note that, the robustness concept provided by [33] resembles the minmax robustness concept in single-objective problems proposed by [34]. Ehrgott et al. [31] extend the minmax robustness concept from [30] and propose the set-based minmax robust efficiency concept. In this concept the worst-case of the objective vector is considered as a set. More specifically, for a feasible solution of the problem they consider all possible objective value realizations under all scenarios. If this set does not contain any other set formed by other feasible solutions, the solution is called set-based robust efficient.

The minmax robustness concepts may be overly conservative; yet it is possible to control the degree of conservatism. To adjust the level of conservatism, Ide et al. [32] extend the concept of light robustness developed by [35] for the multi-objective settings. In this concept, a nominal scenario is determined. For a solution to be lightly robust efficient, it should perform good enough for the nominal case while minimizing the worst case over the uncertainty set.

Since hazmat transportation systems typically involve uncertainty in risk outcomes, we also propose an equitable robust programming approach to this problem. To the best of our knowledge this work is the first one considering an equitable robust programming in a hazmat network design setting, incorporating the two key features of fairness and robustness into optimization.

Chapter 3

Preliminaries

Consider a generic optimization problem with set C of objective functions and without loss of generality, we assume objective functions are to be minimized. The generic multiple-criteria optimization problem can be formulated as follows:

$$\begin{aligned} &\min(z_1(x), z_2(x), \ldots, z_{|C|}(x)) \\ &\text{s.t.} \quad x \in \mathcal{X} \end{aligned} \tag{3.1}$$

In the above model x denotes the vector of decision variables and $\mathcal{X}$ denotes the feasible set in the decision space. $z_c(x)$ is a function that maps the decision space into criterion space $\mathcal{Z}$. In multiple criteria decision analysis, preferences are based on these vectors of evaluation criteria. Although the exact preference model is decision maker specific, under the assumption of rationality the following properties hold:

1. Reflexivity: $z \preceq z, \forall z \in \mathcal{Z}$
2. Monotonicity: $z \preceq z - \varepsilon e_i$ where e_i denotes the i^{th} unit vector (less is better in minimization settings).
3. Transitivity: $(z^1 \preceq z^2$ and $z^2 \preceq z^3) \implies z^1 \preceq z^3, \forall z^i \in \mathcal{Z}$

Based on this rational preference model for two vectors in the criterion space z^1, z^2 we say that z^2 dominates z^1 if $z_c^2 \leq z_c^1$, $\forall c \in C$ and $z_c^2 < z_c^1$ for at least one $c \in C$. $z^2 \in \mathcal{Z}$ is called non-dominated point if there exists no $z^1 \in \mathcal{Z}$ such that $z_c^1 \leq z_c^2$, $\forall c \in C$ and $z_c^2 \neq z_c^1$ for at least one $c \in C$. A feasible solution $\bar{x} \in \mathcal{X}$ is called efficient (Pareto optimal) if there exists no other $x \in \mathcal{X}$ such that $z(x) \leq z(\bar{x})$. The set of non-dominated points is called the non-dominated set $(\mathcal{Z}_N)$. A point $z \in \mathcal{Z}_N$ is called a supported non-dominated point if it lies on the boundary of Conv$(\mathcal{Z}_N)$ and it is extreme if it is an extreme point of Conv$(\mathcal{Z}_N)$

Although the above properties are sufficient for the classical multiple-criteria problems where the criteria are incomparable, further properties must be satisfied in equitable settings. These properties are as follows:

4. Symmetry (Anonymity): $(z_{\tau_{(1)}}, z_{\tau_{(2)}}, \ldots, z_{\tau_{(C)}}) \cong (z_1, z_2, \ldots, z_{|C|})$ for any permutation of a vector. This property states that decision maker is indifferent between any two permutations of z. Consider the following vectors: $(9, 6, 1)$ and $(6, 1, 9)$. Under the anonymity principle, the decision maker is indifferent between these two.
5. Principle of Transfers (Pigou-Dalton): if $z_i > z_j$ then $z \preceq z - \varepsilon e_i + \varepsilon e_j$ for $0 < \varepsilon < z_i - z_j$. This property states that transfer of a small amount from better-off entity to worse-off entity results in a more preferred allocation. Consider the following benefit allocation vectors: $(9, 6, 1)$ and $(7, 7, 2)$. Under the principle of transfers, the decision maker prefers $(7, 7, 2)$.

Preference models satisfying the above axioms are called equitable rational preference relations [27]. A given vector equitably dominates another one, if it is preferred to the latter for all such equitable rational preference relations.

Equitable dominance can be easily checked through comparing the cumulative ordering vectors of the alternatives. In minimization settings, ordering map $\Theta : \mathbb{R}^{|C|} \mapsto \mathbb{R}^{|C|}$ is defined as $\Theta(z) = (\theta_1(z), \theta_2(z), \ldots, \theta_{|C|}(z))$, where $\theta_1(z) \geq \theta_2(z) \geq \ldots \geq \theta_{|C|}(z)$. Cumulative ordering map is defined as:

$\bar{\Theta} = (\bar{\theta}_1(z), \bar{\theta}_2(z), \ldots, \bar{\theta}_{|C|}(z))$ where $\bar{\theta}_c(z) = \sum_{j=1}^{c} \theta_j(z)$. Consider the vector $(4, 2, 3)$. Its cumulative ordering map is $\bar{\Theta}(4, 2, 3) = (4, 7, 9)$. We say that z^2 equitably dominates z^1 if $\bar{\Theta}(z^2) \leq \bar{\Theta}(z^1)$ and $\bar{\theta}_c(z^2) \leq \bar{\theta}_c(z^1)$ for at least one $c \in C$.

Corollary 1. *$z^2 \in \mathcal{Z}$ is equitably non-dominated $\iff$ there exists no $z^1 \in \mathcal{Z}$ such that $\bar{\theta}_c(z^1) \leq \bar{\theta}_c(z^2)$, $\forall c \in C$ and $\bar{\theta}_c(z^1) \neq \bar{\theta}_c(z^2)$ for at least one $c \in C$. A feasible solution $\bar{x} \in \mathcal{X}$ is called equitably efficient if $z(x)$ is equitable non-dominated point.*

Let $\mathcal{Z}_{EN}$ be the set of equitably non-dominated points. We focus on a specific subset of equitably non-dominated set, which is the set of extreme supported equitably non-dominated points. As in the classical setting, we will call a point $z \in \mathcal{Z}_N$ supported equitably non-dominated if it lies on the boundary of Conv($\mathcal{Z}_{EN}$) and extreme if it is an extreme point of Conv($\mathcal{Z}_{EN}$).

Proposition 1. *Let $z(x)$ denote the disbenefit (risk in our setting) allocation vector corresponding to decision x. A feasible solution $\bar{x} \in \mathcal{X}$ is an equitably efficient solution of the problem 3.1, iff it is an efficient solution for the following generic equitable model [6]*

$$\begin{aligned} & min(\bar{\theta}_1(z(x)), \bar{\theta}_2(z(x)), \ldots, \bar{\theta}_{|C|}(z(x)) \\ & s.t. \quad x \in \mathcal{X} \end{aligned} \tag{3.2}$$

Our objective is to develop a solution methodology to provide the decision maker with equitably non-dominated options for the equitable multiple-criteria mixed integer problems. Hence, we use the model (3.2) for solving formulation END and REND mentioned in sections 4.1 and 4.2 respectively with the solution methodology proposed.

Chapter 4

Equitable Hazmat Network Design Problem

Before presenting the formulation, we will explain the motivation behind seeking a set of equitably efficient solutions in the hazmat networks using a toy example. Consider the network given in Figure 4.1. Assume that the network is divided into three population centers. These could be based on municipalities that they belong to or any other attribute such as the socio-economic characteristics of their residents. Assume that nodes 0, 1 and 2 are in poulation center 1, 3 and 4 are in population center 2 and nodes 5 and 6 are in population center 3 as it can be seen from Table 4.1.

Table 4.1: Toy Example population centers used

Pop Center	Nodes
1	0, 1, 2
2	3, 4,
3	5, 6

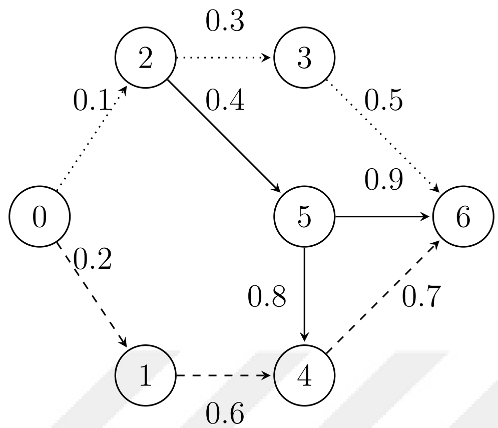


Figure 4.1: Toy Example Network

Assume that hazardous materials have to be sent from node 0 to node 6 and numbers over the arcs represent the accident rates. For simplicity, also assume that there is no uncertainty in the problem. So as long as an arc is traversed within the population center the corresponding population center will be exposed to a certain amoun of risk based on the accident rates and the population of the center. Similarly, whenever an arc is traversed between the population centers then the corresponding population centers will incur a certain amount of risk. This toy example is modeled using an equitable optimization, which is discussed in detail in Section 4.1. By discretizing the weight space, equitably nondominated solutions are obtained as follows:

- When total risk is minimized dotted path is obtained. Risk values incurred for each region is in the following order: $(51.5, 90, 18)$ and the cumulative map is $(90, 141.5, 159.5)$. This solution corresponds to the utilitarian solution.
- When maximum risk is minimized, dashed path is obtained. Risk values incurred for each region is in the following order: $(74.5, 70, 18)$ and the cumulative map is $(74.5, 144.5, 162.5)$. This solution corresponds to the Rawlsian solution.

The utilitarian solution minimizes the total risk, disregarding how it is allocated. This can be seen in the resulting allocation as region 2 bears a significantly higher risk than region 3. The Rawlsian solution, on the other hand, focuses on the worst-off entity and hence minimizes the maximum risk across regions, though mostly at the expense of total risk. In this solution the maximum risk reduces to 74.5 while the total risk increases to 162.5. Even in this small setting, one can see the trade-off between efficiency (total risk) and equity. In larger settings there may be more such equitable solutions, which would inform the policy makers on the underlying trade-offs. Our aim in this work is to propose an approach for finding such solutions and demonstrate its use. We first focus on establishing equity for a deterministic setting and then extend our analysis to cases where there is uncertainty.

4.1 Deterministic Model

Shipment of hazardous materials is significant for the economic development of countries and many industries [36]. Although the authorities take several precautions, accidents occur, posing harmful impacts on the environment and populations [37]. Therefore, mitigation of the hazmat risks in an equitable manner for populations is essential [38].

In the literature, definition of risk is related to inf_{ij} parameter for a given path of the network [10]. Using this definition of risk, Kara et al. [10] minimize the total (traditional) risk of the network while Bianco et al. [39] minimize the maximum arc risk. Fontaine et al. [23] argue that these objectives do not ensure the fair allocation of risk since they disregard that a population is impacted by the arcs crossing or bordering the population center [23]. They propose a population-based risk definition and use it in an equity measure with social bounds. In our approach we also use the same population based risk definition. However, as opposed to [23], we use in an equitable optimization framework that will guarantee finding a set of equitably efficient solutions. We demonstrate the computational feasibility and effectiveness of our approach on the multimode multicommodity network design setting addressed in [23].

In mHTNDP, the problem is structured as a leader-follower game with the two stakeholders: the authority (government) and the carriers (transportation agency). These two agencies have conflicting objectives. The authority tries to ensure an equitable distribution of risk among population centers while the carriers aim to minimize their operational costs [10]. Routing decisions of the carriers cannot be decided by the government. However, the government can ensure fair distribution of risk to the population centers by closing some arcs of the network for hazmat transportation. This decision is denoted by the decision variable $y_{ij}^m \in \{0,1\}$ in the model. $y_{ij}^m \in \{0,1\}$ refers whether an arc (i,j) of the network is allowed for hazmat transportation via mode m. As the authority has the lead to decide on the availability of arcs, this problem is called the leader problem (see Appendix A).

Based on the decision of the leader, carriers make their routing decisions to minimize their total operational costs [10].

$$\min \sum_{k \in K} \sum_{m \in M} \sum_{(ij) \in A} c_{ij}^{km} x_{ij}^{km} \tag{4.1}$$

where x_{ij}^{km} is the transportation percentage of commodity $k \in K$ carried on arc $(i,j) \in A$ with mode of transportation $m \in M$ and c_{ij}^{km} is the corresponding cost parameter (see Appendix B).

In addition to the above mentioned decision variables and parameters, we use other auxiliary variables and parameters that allow us to pose the model as an equitable optimization problem. We also have parameters that are set for the solution algorithm to be implemented. All notation used for modelling can be found at Table 4.2.

Table 4.2: Notation used

Index sets	
$G = (N, A)$	Graph
N	Nodes
A	Arcs
M	Transportation Modes
K	Commodities
C	Population Centers

Parameters	
o_k, d_k	Origin and Destination Pair for commodity $k \in K$ and $o_k, d_k \in N$
ϕ_k	Transport volume for commodity $k \in K$
c_{ij}^{km}	Transportation cost for carrying a unit of commodity $k \in K$ on arc $(i,j) \in A$ with mode $m \in M$
σ_{ij}^{km}	Incident probability on arc $(i,j) \in A$ with mode $m \in M$
P_c	Population of a population center $c \in C$
l_{ij}^{mkc}	Influence of an accident on arc $(i,j) \in A$ for commodity $k \in K$ with mode $m \in M$ on the population center $c \in C$
Λ	Existing weighted sum value of the facet
ϵ	Cut-off value
$\hat{M}$	Big Number

Decision variables	
x_{ij}^{km}	Transportation percentage of commodity k carried on arc (i,j) with mode of transportation m
y_{ij}^{m}	$= \begin{cases} 1, & \text{If the transportation mode } m \text{ of an arc } (i,j) \text{ is allowed} \\ & \text{for the transportation of hazardous materials} \\ 0, & \text{otherwise} \end{cases}$
f^{km}	percentage of commodity $k \in K$ shipped with mode m
z_c	Cumulatively ordered achievement variable
r_c	Ordering variable
d_{ic}	Ordering variable

Decision variables obtained from the dual of the follower problem	
$v^k \in \mathbb{R}$	Dual Variable
$w_{ij}^{km} \leq 0$	Dual Variable
$t_{ij}^{km} \leq 0$	Dual Variable
$u_j^{km} \in \mathbb{R}$	Dual Variable

Linearizing the bilevel program results in the following formulation, a single objective variant of which is given by Fontaine et. al. [23], using conditions provided in [40, 41, 42]. We re-structure it as a multiobjective equitable optimization problem, hence consider the following equitable multi-mode hazmat transport network design problem. We refer to this model as Model END: Equitable Network Design.

Formulation END

$$\min\ (z_1, \ldots, z_{|C|}) \tag{4.2}$$

s.t.

$$\sum_{(i,j)\in A} x_{ij}^{km} - \sum_{(j,l)\in A} x_{jl}^{km} = \begin{cases} 0, & \text{if } j \neq o_k, d_k \\ -f^{km}, & \text{if } j = o_k \\ f^{km}, & \text{if } j = d_k \end{cases} \qquad \forall j \in N, k \in K, m \in M \tag{4.3}$$

$$\sum_{m\in M} f^{km} = 1 \qquad \forall k \in K \tag{4.4}$$

$$x_{ij}^{km} \leq y_{ij}^{m} \qquad \forall (i,j) \in A, k \in K, m \in M \tag{4.5}$$

$$\sum_{k\in K}\sum_{m\in M}\sum_{(i,j)\in A} c_{ij}^{km} x_{ij}^{km} \leq \sum_{k\in K} v^k + \sum_{m\in M}\sum_{(i,j)\in A}\sum_{k\in K} w_{ij}^{km} \tag{4.6}$$

$$w_{ij}^{km} \leq t_{ij}^{km} + \hat{M}(1 - y_{ij}^{m}) \qquad \forall (i,j) \in A, m \in M, k \in K \tag{4.7}$$

$$w_{ij}^{km} \geq t_{ij}^{km} \qquad \forall (i,j) \in A, m \in M, k \in K \tag{4.8}$$

$$w_{ij}^{km} \geq -\hat{M} y_{ij}^{m} \qquad \forall (i,j) \in A, m \in M, k \in K \tag{4.9}$$

$$w_{ij}^{km} \leq 0 \qquad \forall (i,j) \in A, m \in M, k \in K \tag{4.10}$$

$$u_j^{km} - u_i^{km} + t_{ij}^{km} \leq c_{ij}^{km} \qquad \forall (i,j) \in A, k \in K, m \in M \tag{4.11}$$

$$u_{o_k}^{km} - u_{d_k}^{km} + v^k \leq 0 \qquad \forall k \in K, m \in M \tag{4.12}$$

$$z_c = cr_c + \sum_{i\in C} d_{ic} \qquad \forall c \in C \tag{4.13}$$

$$r_c + d_{ic} \geq P_c \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A} l_{ij}^{mkc} \sigma_{ij}^{km} \phi_k x_{ij}^{km} \qquad \forall i, c \in C \tag{4.14}$$

$$d_{ic} \geq 0 \qquad \forall i, c \in C \tag{4.15}$$

$$v^k \in \mathbb{R} \qquad \forall k \in K \tag{4.16}$$

$$u_j^{km} \in \mathbb{R} \qquad \forall j \in N, k \in K, m \in M \tag{4.17}$$

$$t_{ij}^{km} \leq 0 \qquad \forall (i,j) \in A, k \in K, m \in M \tag{4.18}$$

$$x_{ij}^{km} \geq 0 \qquad \forall (i,j) \in A, k \in K, m \in M \tag{4.19}$$

$$f^{km} \geq 0 \qquad \forall k \in K, m \in M \tag{4.20}$$

$$y_{ij}^m \in \{0,1\} \qquad \forall (i,j) \in A, m \in M \tag{4.21}$$

Constraint (4.3) ensures flow balance while constraint (4.4) guarantees that the demand is met via different transportation modes. (4.5) indicates that only the arcs allowed by the authority can be used.

In order to formulate the bi-level problem as single-level reformulation, follower problem is integrated into the model by the optimality conditions. Decision variables u_j^{km}, v^k and t_{ij}^{km} are the corresponding dual variables for the constraints (4.3), (4.4) and (4.5). Dual constraints (4.11) and (4.12) correspond to the primal variables x_{ij}^{km} and f^{km} respectively. For integrating the follower problem optimally, primal and the dual objectives have to satisfy the following inequality:

$$\sum_{k \in K} \sum_{m \in M} \sum_{(i,j) \in A} c_{ij}^{km} x_{ij}^{km} \leq \sum_{k \in K} v^k + \sum_{m \in M} \sum_{(i,j) \in A} \sum_{k \in K} t_{ij}^{km} y_{ij}^m$$

Note that the optimality condition is not linear. Cao and Chen [42] by proposing auxiliary variables w_{ij}^{km} and $\hat{M}$ linearize the optimality condition via the constraints (4.6)-(4.10).

(4.13) and (4.14) find the cumulative ordering of the population risk outcomes, which are variables z_c, $\forall c \in C$.

This is a multiobjective programming problem, the Pareto solutions of which are equitably nondominated due to the results of Kostreva et al [6]. To find such solutions, we adapt the solution algorithm proposed by Özpeynirci and Köksalan [26] for finding extreme supported points of multiobjective mixed integer programming problems. The algorithm is based on repetitively solving weighted sum scalarizations, each time setting the weights based on the previously found solutions so as to search over the facets of the Pareto frontier. Therefore, we solve a scalarization problem with the objective of $\min \sum_{c \in C} w_c z_c$ within the algorithm. Moreover, to alleviate the computational effort needed and avoid numerical issues, we use the constraint $\sum_{c \in C} w_c z_c \leq \Lambda - \epsilon$. This constraint prevents us from get stuck at a surface by ensuring that any new point found is sufficiently below the facet defined by the previously found solutions (corners) and hence would constitute a new corner, i.e. extreme supported point, rather than a nonextreme supported one. Λ denotes the weighted sum value of the extreme points, defining the facet. Note that Λ is different for each facet.

4.2 An Equitable Robust Programming Model

Many real world problems require ensuring equity in the outcomes under uncertainty. This makes the corresponding equitable optimization problems uncertain multi-objective problems, where the uncertainty is embedded in the objective functions. This uncertainty derives from an uncertainty set $\mathcal{U}$ and results in a generic problem as follows [43]:

$$\begin{pmatrix} \text{minimize} & (z_1(x,\xi), \ldots, z_{|C|}(x,\xi)) \\ \text{s.t.} & x \in \mathcal{X} \end{pmatrix}_{\xi \in \mathcal{U}} \tag{4.22}$$

In this study, we use the so called lightly robust programming to balance the degree of conservatism and efficiency. This method relies on determining a nominal scenario, which could be the most likely one. Then a set of efficient solutions is obtained by solving the deterministic counterpart of (4.22) for this nominal scenario. Let this set of efficient solutions be $\mathcal{X}^{nom}$. In Figure 4.2, we show an example, where $\hat{z}^i$ demonstrates the images of these solutions for a bi-objective setting.

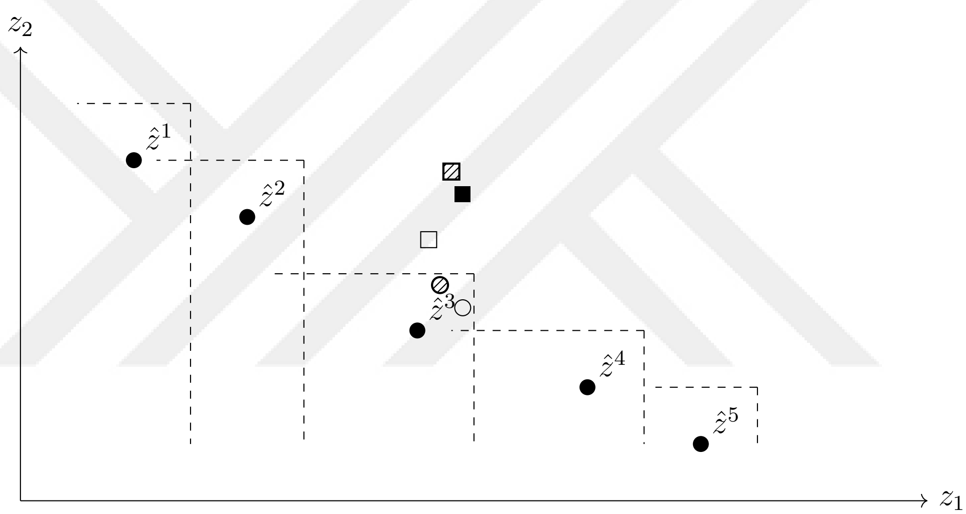


Figure 4.2: Lightly Robust Efficiency Concept

These solutions are solely based on a nominal scenario. To incorporate robustness, we try to find solutions that perform well over the whole uncertainty set, which is ensured by maximizing performance even under the worst case scenario while not sacrificing too much from the nominal performance, i.e. not deviating too much from the nominal outcome. Such solutions are called lightly robust efficient in the literature [32, 35]. To define the worst-case we utilize the idea of point-based minmax robustness [33, 44]. In order to find these robust solutions, model (4.23) is solved for each chosen $\hat{x} \in \mathcal{X}^{nom}$.

$$
\begin{aligned}
\text{minimize} \quad & \left(\sup_{\xi \in \mathcal{U}} z_1(x,\xi), \dots, \sup_{\xi \in \mathcal{U}} z_{|C|}(x,\xi) \right) \\
\text{s.t.} \quad & z^{nom}(x) \leq z^{nom}(\hat{x}) + \varepsilon \\
& x \in \mathcal{X}
\end{aligned}
\tag{4.23}
$$

Problem (4.23) optimizes decisions for the worst case while guaranteeing that this decision will perform sufficiently well under the nominal scenario $\hat{\xi}$. Specifically, the objective function vector under the nominal scenario, $z^{nom}(x) := (z_1(x,\hat{\xi}), \dots, z_{|C|}(x,\hat{\xi}))^\top$, is kept within a tolerable deviation, $\varepsilon = (\varepsilon_1, \dots, \varepsilon_{|C|})^\top$, from the objective vector of the nominal solution. In our application, the uncertainty set $\mathcal{U}$ is finite and $z_i : \mathcal{X} \times \mathcal{U} \to \mathbb{R}$ is continuous. Therefore, $\max_{\xi \in \mathcal{U}} z_c(x,\xi)$ exists for all $c \in C$, so (4.23) can be solved by achievement scalarizing function proposed by [45] as follows:

$$
\begin{aligned}
\text{minimize} \quad & \alpha + \rho \sum_{c \in C} w_c \big(\max_{\xi \in \mathcal{U}} (z_c(x,\xi) - \bar{z}_c) \big) \\
\text{s.t.} \quad & w_c \big(\max_{\xi \in \mathcal{U}} (z_c(x,\xi) - \bar{z}_c) \big) \leq \alpha && \forall c \in C \\
& z(x,\hat{\xi}) \leq z^{nom}(\hat{x}) + \varepsilon \\
& x \in \mathcal{X}
\end{aligned}
\tag{4.24}
$$

In this formulation α is utilized to replace the minmax term of the achievement scalarizing function and a scalar ρ is introduced to control the trade-offs between the objectives. Moreover, $\bar{z}$ in this model is a reference point.

To solve the formulation (4.24) efficiently Zhou-Kangas and Miettinen [43] proposed the following reformulation of (4.24) by introducing a scalar decision variable γ and proved this formulation is also valid for uncertainty sets $\mathcal{U}$ that contain a finite number of scenarios.

$$
\begin{aligned}
\text{minimize } & \alpha + \rho \sum_{c \in C} w_c(\gamma - \bar{z}_c) \\
\text{s.t. } & w_c(\gamma - \bar{z}_c) \leq \alpha && \forall c \in C \\
& z(x, \hat{\xi}) \leq z^{nom}(\hat{x}) + \varepsilon \\
& z_c(x, \xi) \leq \gamma && \forall c \in C, \text{ and } \xi \in \mathcal{U} \\
& x \in \mathcal{X}
\end{aligned}
\tag{4.25}
$$

Consider Figure 4.2 for bi-objective programming problem and let the $\hat{z}^i$ (black circles) be the efficient solutions of the nominal scenario as denoted earlier. The dashed areas around the $\hat{z}^i$s represent the area where lightly robust efficient solutions can be found i.e. solutions can be found within tolerable degradation (ε) from the nominal scenario. Moreover, let the worst-case (z_1, z_2) values be denoted by the corresponding squares.

For example, for solution $\hat{z}^3$, there exists multiple solutions (denoted by the hatched and the empty circles) within the ε tolerance. Note that since we minimize the worst-case scenario performance, among those solutions only the empty circle is lightly robust efficient.

For each $\hat{x} \in \mathcal{X}^{nom}$ one can obtain many lightly robust efficient solutions, i.e. there exists a Pareto set for each $\hat{x}$. To reduce the number of solutions presented to the decision maker, a set of representative solutions can be obtained. In line with this, we assume that the decision maker chooses a point on the nominal Pareto frontier and we find a robust solution by solving (4.25) for a given weight w. To find robust solutions to our END model, the following model can be solved for a given $\hat{x} \in \mathcal{X}^{nom}$. For brevity we denote this finite set of scenarios Ω, indexing its elements with ω, where $\hat{\omega}$ is the nominal scenario:

$$
\text{minimize } \alpha + \rho \sum_{c \in C} w_c(\gamma - \bar{z}_c) \tag{4.26}
$$

$$
\text{s.t. } w_c(\gamma - \bar{z}_c) \leq \alpha \qquad \forall c \in C \tag{4.27}
$$

$$z_{c\hat{\omega}} \leq z_c^{nom}(\hat{x}) + \varepsilon_c \qquad \forall c \in C \tag{4.28}$$

$$z_{|C|\omega} \leq \gamma \qquad \omega \in \Omega \tag{4.29}$$

$$z_{c\omega} = cr_{c\omega} + \sum_{i \in C} d_{ic\omega} \qquad \forall i, c \in C \text{ and } \omega \in \Omega \tag{4.30}$$

$$r_{c\omega} + d_{ic\omega} \geq P_c \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A} l_{ij}^{mkc} \sigma_{ij}^{km\omega} \phi_k x_{ij}^{km} \quad \forall i, c \in C \text{ and } \omega \in \Omega \tag{4.31}$$

As a special case of this formulation we set the reference point ($\bar{z}_i = 0$, $\forall i \in C$). Once the reference point set to be 0 above model reduces the following:

Formulation REND

$$\text{minimize } \gamma \tag{4.32}$$

$$\text{s.t. } z_{c\hat{\omega}} \leq z_c^{nom}(\hat{x}) + \varepsilon_c \qquad \forall c \in C \tag{4.33}$$

$$z_{|C|\omega} \leq \gamma \qquad \forall \omega \in \Omega \tag{4.34}$$

$$z_{c\omega} = cr_{c\omega} + \sum_{i \in C} d_{ic\omega} \qquad \forall i, c \in C \text{ and } \omega \in \Omega \tag{4.35}$$

$$r_{c\omega} + d_{ic\omega} \geq P_c \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A} l_{ij}^{mkc} \sigma_{ij}^{km\omega} \phi_k x_{ij}^{km} \quad \forall i, c \in C \text{ and } \omega \in \Omega \tag{4.36}$$

$$x \in \mathcal{X} \tag{4.37}$$

Eventually, we solve problem REND (Robust END) to find lightly robust solutions for a given $\hat{x} \in \mathcal{X}^{nom}$.

Chapter 5

Algorithm

In order to find the equitably non-dominated points of the formulation END and REND discussed in Sections 4.1 and 4.2 an exact algorithm (ExA) proposed by Özpeynirci and Köksalan [26] is utilized. ExA works in the criterion space for classical multiobjective optimization problems to find all extreme supported non-dominated points. The algorithm is designed to solve the weighted sum scalarizations in a systematical manner. In this chapter details of this algorithm is provided.

Let $SF(q)$ to be the union of all q-dimensional non-dominated faces of Conv$\mathcal{Z}$. Then the non-dominated frontier (NDF) in criterion space is defined as follows:

$$\text{NDF} = \bigcup_{q=0}^{|C|-1} SF(q)$$

For the multiobjective problem having $|C|$ criteria, NDF can be characterized by $|C|-1$ dimensional facets or lower dimensional faces. However, the algorithm is designed to search the facets defined by the convex hull of the solution vectors in the objective space through weighted sum scalarizations. Dummy vectors are used to avoid lower dimensional faces.

In each iteration of the algorithm a facet defined by $|C|$ points is searched through its normal vector to find a new extreme supported non-dominated point. The facet formed by the $|C|$ points is called as stage and denoted by $\mathcal{R} = z^1, \ldots, z^{|C|}$. Throughout the algorithm, stages that have not yet been searched are kept in a set $\mathcal{L}$ and the algorithm terminates when no stage is left to search in $\mathcal{L}$. The algorithm is initialized with $|C|$ dummy vectors ($m^i = Me_i$) where e_i denotes the ith unit vector and M is a big number. A set consisting of dummy points is defined as Z_M.

In an arbitrary iteration, a stage $\mathcal{R} = z^1, \ldots, z^{|C|}$ is selected from $\mathcal{L}$ and the normal vector w of the stage formed is calculated by model WF as follows:

Formulation WF

$$
\begin{aligned}
\min\ & 0 \\
\text{s.t.}\ & \\
& \sum_{i=1}^{|C|} w_i = 1 \\
& w^\top z^i = w^\top z^{i+1} && \forall i = 1, \ldots, |C| - 1 \\
& w_i \geq 0 && \forall i = 1, \ldots, |C|
\end{aligned}
$$

This normal vector is used in the corresponding scalarization problem and the current stage is searched through this direction in criterion space. Once searched, the stage is removed from the set $\mathcal{L}$ and if a new solution z^* is obtained, new stages are added to $\mathcal{L}$. The new solution z^* is also added to the set of known extreme supported non-dominated points Z_E.

Note that in problems having three or more criteria normal vector w may contain negative components. In this case, the corresponding stage cannot be searched through w to avoid obtaining dominated point. However, there can be solutions that lie under the facet defined by the current stage. In such a case possible neighbouring facets need to be defined and added to $\mathcal{L}$. When a normal vector w contains a negative component, the algorithm checks whether a solution z from the set $Z_{EM} = Z_E \bigcup Z_M$ satisfies the inequality $w^\top z \leq w^\top r$, $\forall r \in \mathcal{R}$. A solution z satisfying the inequality is treated as z^*; neighbouring stages are generated and the algorithm continues. Model PF(z) is solved for points $z \in \mathrm{Z}_{EM} \setminus \mathcal{R}$ until finding one. Model PF(z) as follows:

Formulation PF(z)

$$
\begin{aligned}
\min\ & 0 \\
\text{s.t.}\ & \\
& \sum_{i=1}^{|C|} w_i = 1 \\
& w^\top z \leq w^\top r && \forall r \in \mathcal{R} \\
& w_i \geq 0 && \forall i = 1, \ldots, |C|
\end{aligned}
$$

If no such solution z can be found as a result of this procedure, $\mathcal{R}$ is removed from $\mathcal{L}$ and the algorithm continues.

ExA algorithm is adapted to find extreme supported equitably non-dominated points. In the adaptation of the ExA algorithm, instead of a classical multiobjective problem formulation, END is used, which has the elements of the cumulative ordered vector of the risk allocation as objective functions. Pseudo code of the adaptation can be seen in Appendix C.

Chapter 6

Computational Analysis

In this chapter we present our computational results to demonstrate the computational feasibility of our approach. We also provide insights into trade-off between equity-efficiency in hazmat transportation. As a case study, we implemented the Sioux Falls Network given in Figure 6.1. In the network, there are 24 nodes and 76 arcs. Length of each arc (km) is calculated using Mapbox API. There are six population centers in the network which can be seen in Table 6.1.

Table 6.1: Population centers used

Pop Center	Nodes	P_c
1	1, 3, 4, 11, 12	122.16
2	13, 14, 23, 24	146.59
3	2, 5, 6	114.02
4	8, 9, 10, 16, 17	195.46
5	15, 19, 20, 21, 22	122.16
6	7, 18	114.02

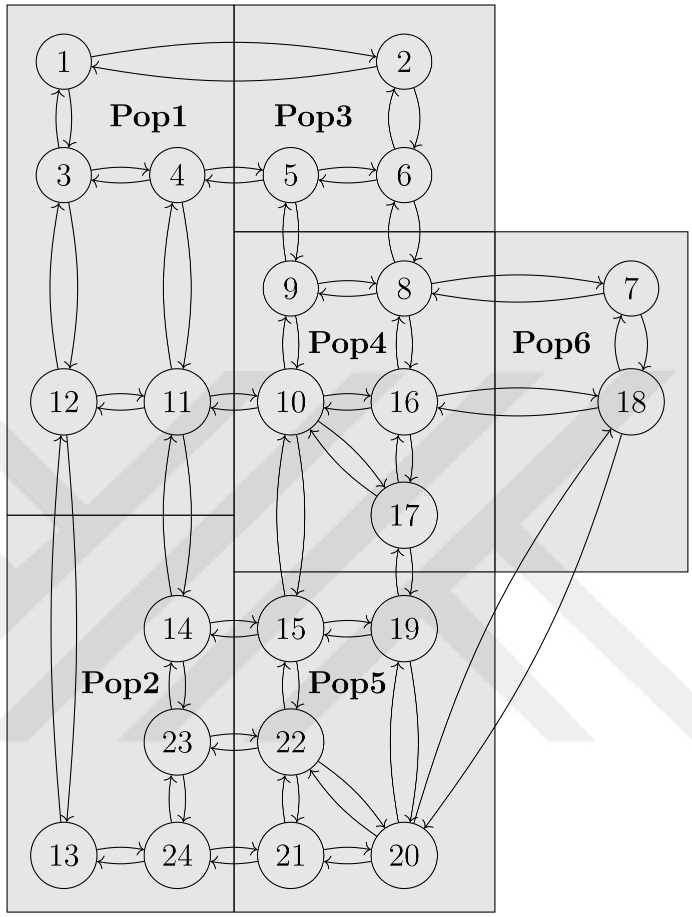


Figure 6.1: Sioux Falls Network

Influence of an accident parameter on the population center l_{ij}^{mkc} is set to be 1 if arc is contained in the same population and set to be 0.5 otherwise. Two transportation modes are considered corresponding to using small and large vehicles, respectively. For the first mode (smaller vehicle), transportation cost for carrying a unit of commodity c_{ij}^{km} is 1.1 while for the second mode (larger vehicle) it is 0.9. The accident probability is set to be %3 higher for the second mode. σ_{ij}^{km} is defined as the length of an arc multiplied by the accident rate. Only one type of commodity is considered and the accident rate for this commodity is generated between 9.56×10^{-9} and 1.08×10^{-7} [12]. We defined 43 origin-destination pairs with a random selection between nodes 1, 2, 13 and 20. In 13 of these shipments, transport volume ϕ_k is generated between 100 and 1000 while for the remaining 30, it is generated between 50 and 150. $\hat{M}$ for each commodity is set based on

$\sum_{i,j,m} c_{ij}^{km}$ and in our application it is set to be 20305.

All models are solved using Gurobi 10.0.1 on Apple M1 with 8GB RAM computer.

6.1 Results of the Deterministic Model

In this chapter results of the END problem discussed in Section 4.1 are presented. Our aim is to provide a set of good quality solutions with respect to the efficiency and equity trade-off.

In the literature there are several solution methodologies to solve and provide such solutions. One way of doing this is solving the model iteratively through different scalarization techniques such as weighted sum and ϵ-constraint [46]. We modified an existing algorithm proposed by Özpeynirci and Köksalan [26] for finding the extreme supported non-dominated points of classical multiple criteria optimization problems (see Appendix C). In our modified implementation of the algorithm, we used our equitable scalarization model END in step 5 and set a time limit of 60 seconds for this model. Moreover, to present a set of good quality solutions (subset of the Pareto front) and prevent numerical issues, we used a tolerance for the equality of two numbers (ψ) in steps 6 and 10. Recall that we also use a cutoff value (ϵ) to avoid finding nonextreme supported points due to numerical issues. We set values of ψ and ϵ as $\{0.03, 0.05\}$, and $\{0.50, 0.75\}$, respectively. For each combination of (ψ, ϵ), time limit is set to be 3600 seconds. We refer the term setting as the combinations of parameters (ψ, ϵ).

The results can be seen at Table 6.3. All the settings are solved within the time limit. The average optimality gaps of the settings are below 1% while the highest optimality gap obtained throughout the settings is 7.10%. Across all settings, model END is solved 213 times and 34% of the time they are solved to optimality.

Table 6.2: Fairness metrics

Utilitarian (Trad)	$\sum_{c \in C} f_c(x)$
Rawlsian (Max)	$\max_{c \in C} f_c(x)$
Average deviation to mean (AdM)	$\frac{1}{\lvert C \rvert} \sum_{c \in C} \left\lvert f_c(x) - \frac{1}{\lvert C \rvert} \sum_{c' \in C} f_{c'}(x) \right\rvert$
Maximum deviation to mean (MdM)	$\max_{c \in C} \left\lvert f_c(x) - \frac{1}{\lvert C \rvert} \sum_{c' \in C} f_{c'}(x) \right\rvert$
Average pairwise difference (AdA)	$\frac{1}{\lvert C \rvert(\lvert C \rvert - 1)} \sum_{c,c' \in C \mid c \neq c'} \lvert f_c(x) - f_{c'}(x) \rvert$
Maximum pairwise difference (MdA)	$\max_{c,c' \in C \mid c \neq c'} \lvert f_c(x) - f_{c'}(x) \rvert$

Table 6.3: Summary of the settings

Setting (ψ, ϵ)	#Solutions Found	Max Opt Gap (%)	Avg Opt Gap (%)	#Optimal	#Models Solved	CPU (sec)
1 (0.05, 0.75)	9	4.90	0.92	15	41	1785.22
2 (0.05, 0.50)	10	7.10	0.97	14	46	2119.59
3 (0.03, 0.75)	18	4.90	0.76	22	67	2998.12
4 (0.03, 0.50)	16	5.70	0.73	21	59	2556.22

The solutions we present correspond to various levels of inequity aversion, each one demonstrating a different trade-off between efficiency and equity. In that sense, the set is expected to include solutions found by only considering a specific metric, as is the case in the current studies in the literature. Different fairness measures from the literature [23] can be seen in Table 6.2 where $f_c(x)$ denotes the risk that population center c bears as a result of decision x. In our application $f_c(x)$ is given by the following expression:

$$f_c(x) = P_c \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A} l_{ij}^{mkc} \sigma_{ij}^{km} \phi_k x_{ij}^{km} \quad \forall i, c \in C \tag{6.1}$$

Table 6.4 shows the performances of Setting 1 solutions for different fairness metrics from the literature, as well as the risk incurred for the population centers. Solution 5 with the lowest maximum risk value corresponds to the Rawlsian solution, which distributes the risks to the population centers in a fair manner while incurring the highest total risk. Solution 2 with the lowest total risk value corresponds to the utilitarian solution which minimizes the total risk, but does not consider equity. The lowest MdM, AdA and MdA values are incurred for the solution 5 (Rawlsian) while the lowest AdM is incurred for the solution 4. Corresponding fairness evaluation tables for other settings can be seen in Appendix D, Appendix E and F.

Table 6.4: Fairness evaluations for solutions of setting 1 (0.05, 0.75)

	Fairness measures						Risk of population centers					
Solution	Trad	Max	AdM	MdM	AdA	MdA	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.905	0.253	0.013	0.103	0.088	0.183	0.253	0.241	0.123	0.108	0.108	0.071
2	**0.834**	0.381	0.022	0.242	0.158	0.371	0.381	0.143	0.191	0.047	0.062	0.010
3	0.872	0.350	0.014	0.205	0.116	0.286	0.350	0.137	0.153	0.064	0.103	0.064
4	1.100	0.220	**0.004**	0.083	0.049	0.120	0.220	0.206	0.100	0.206	0.206	0.160
5	1.194	**0.209**	0.006	**0.034**	**0.018**	**0.044**	0.195	0.209	0.209	0.207	0.209	0.165
6	0.847	0.282	0.022	0.140	0.141	0.272	0.282	0.255	0.191	0.047	0.062	0.010
7	0.949	0.240	0.019	0.115	0.094	0.197	0.240	0.215	0.138	0.098	0.215	0.043
8	0.927	0.347	0.012	0.193	0.090	0.264	0.347	0.131	0.122	0.122	0.122	0.083
9	0.958	0.241	0.010	0.081	0.073	0.142	0.241	0.236	0.118	0.118	0.146	0.099

To provide further details on the network designs corresponding to these solutions, we show arc use frequencies for example solutions 2, 5 and 9 in Figures 6.2, 6.3 and 6.4 respectively. In solution 2, population center 6 incurs the risk of 0.010 while it increases to 0.099 in solution 9. This change can be observed from Figures 6.2 and 6.3. In Figure 6.2 some arcs in population center 6 (e.g. between nodes 7 and 18) are not frequently used while in Figure 6.3 transportation over these arcs become more frequent.

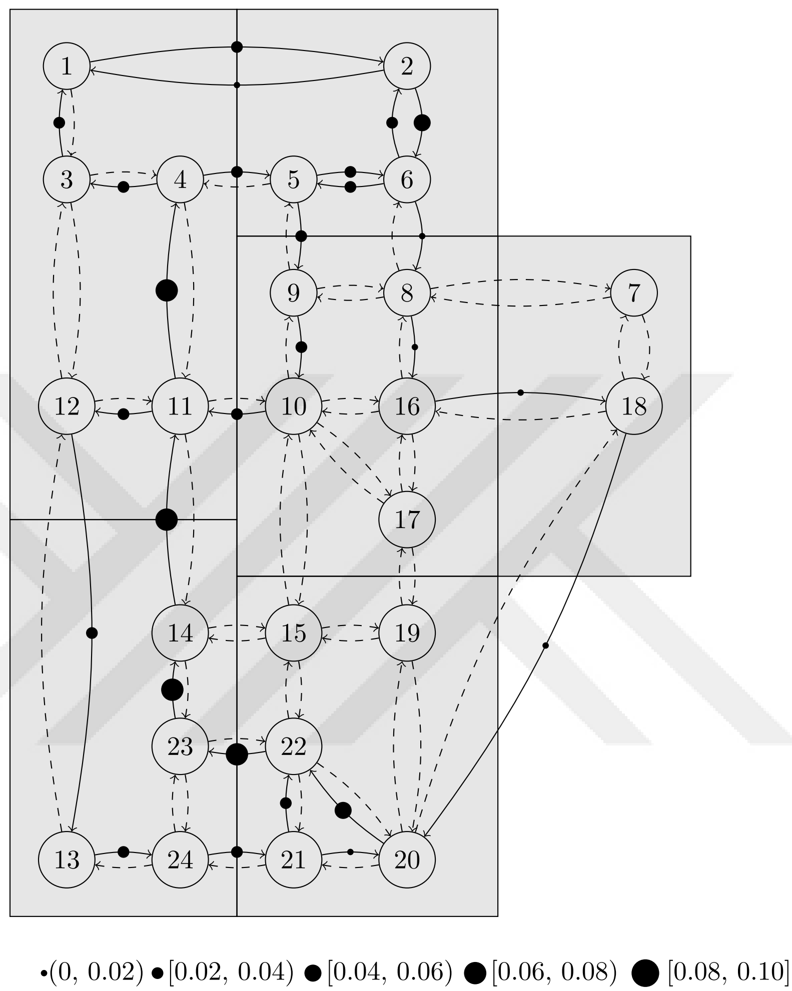
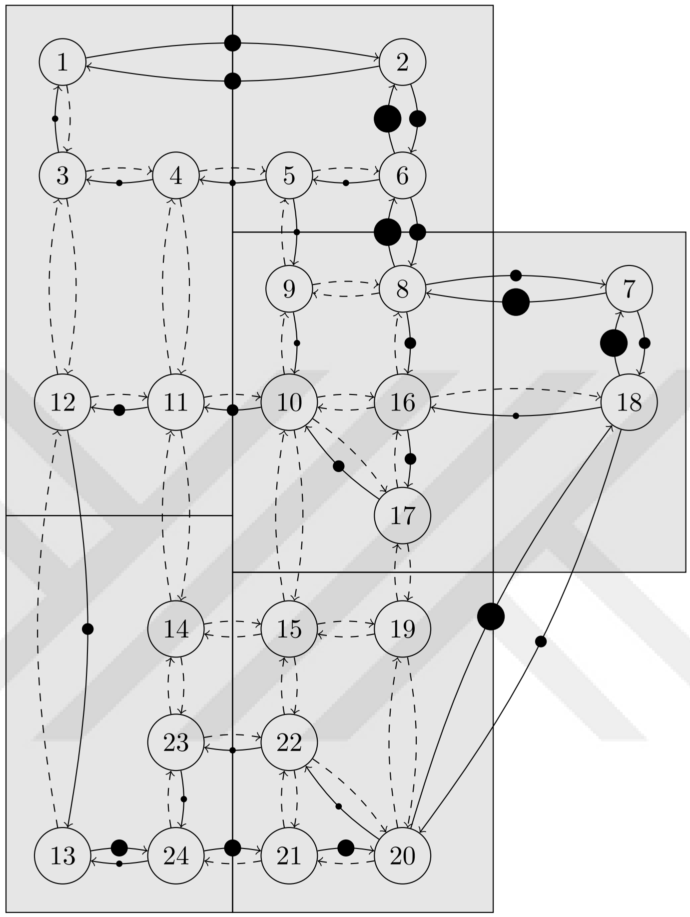


Figure 6.2: Setting 1 Utilitarian Solution (2)



•(0, 0.02) • [0.02, 0.04) ● [0.04, 0.06) ● [0.06, 0.08) ● [0.08, 0.10]

Figure 6.3: Setting 1 Rawlsian Solution (9)

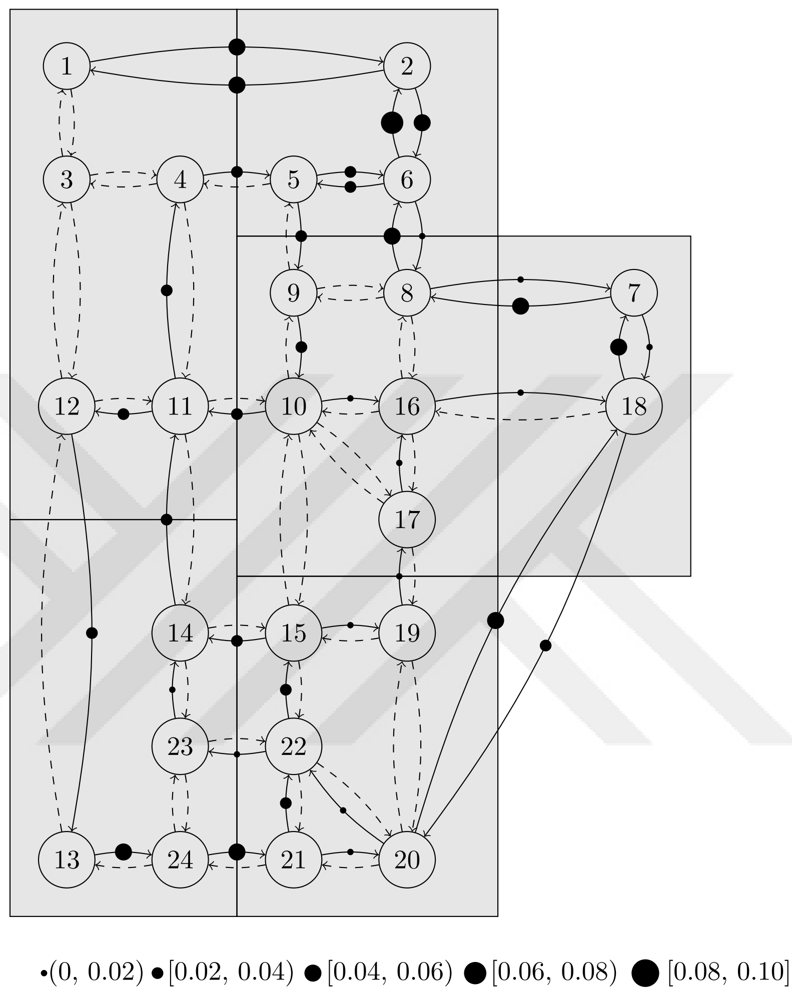


Figure 6.4: Setting 1 Midst Solution (5)

6.2 Results of the Robust Model

In this section results and insights regarding the REND problem discussed in Section 4.2 are presented. Our aim is to provide solutions that are robust to the uncertainty in the hazmat network while their performance under the nominal scenario is controlled.

For the numerical experiments, we used 6 different scenarios for the uncertainty set (Ω), one of which is the nominal scenario. The uncertain parameter (accident rate) for the scenarios is generated between 9.56×10^{-9} and 1.08×10^{-7} [12]. For each REND model a time limit of 180 seconds is used. For the robustness control parameter ε, we used the values of 0.05 and 0.10. Summary of the results can be seen at Table 6.5. In this table #Nominal Pareto denotes the number of solutions found in each setting through Formulation END, #Robust Pareto denotes the number of solutions found through solving Formulation Rend for each element of nominal pareto. Moreover, #Optimal refers to a count of Formulation Rend that are solved to optimality. We observed that as the allowed deviation from the nominal (ε) increases, optimality gaps decrease and we obtained different solutions in robust Pareto for each nominal solution.

Table 6.5: Summary of the robust programming results

(ψ, ϵ)	ε	Max Opt Gap (%)	Avg Opt Gap (%)	#Nominal Pareto	#Robust Pareto	#Optimal	CPU (sec)
(0.05, 0.75)	5	1.68	0.58	9	9	3	1178.48
	10	0.37	0.12	9	9	5	955.61
(0.05, 0.50)	5	3.18	0.78	10	10	4	1224.87
	10	0.92	0.19	10	10	4	1296.33
(0.03, 0.75)	5	9.80	1.06	18	18	6	2484.87
	10	0.99	0.17	18	18	10	1952.78
(0.03, 0.50)	5	2.09	0.70	16	16	5	2375.46
	10	0.45	0.10	16	16	11	1538.18

To see the value of robust programming, consider the utilitarian solution (2) in setting 1, whose details are provided in Table 6.6. The cumulative map of these risk distributions are given at Table 6.7. The risk distributions for the nominal solution under the nominal scenario ($z^{nom}(\hat{x})$) are shown in the last row of the left-hand side of the Table 6.7. Note that $z^{nom}(\hat{x}) + \varepsilon = (0.431, 0.622, 0.765, 0.827, 0.874, 0.884)$ and risk values incurred for the robust solution (last row of the right-hand side of the table) lie in the boundaries. When it comes to the performances of the nominal and the robust solutions under the uncertainty set we observe that robust solution equitably dominates the nominal solution under Scenario 3, while not being equitably dominated in any other scenarios. Specifically, implementing the robust solution would avoid the undesirable consequences in Scenario 3, in which a significantly higher total risk is observed (1.341). This result indicates that robust solution performs better under uncertainty, with a controllable degree of sacrifice from the best nominal performance.

Similar observations can be made for the effect of the ε. For instance consider the cumulative map of the risk distributions for utilitarian solution (2) when $\varepsilon = 0.05$ and $\varepsilon = 0.10$ in Table 6.7 and 6.8 respectively. In every scenario total risk incurred is smaller for $\varepsilon = 0.10$ except the nominal. The performance of the solution in the nominal scenario is worse, which is an expected result since higher ε gives more flexibility in terms of the sacrifice from the nominal performance.

Table 6.6: Risk values incurred for utilitarian solution (2) for $(\psi, \epsilon) = (0.05, 0.75)$, $\varepsilon = 0.05$

	Risk of population centers (nominal)						Risk of population centers (robust)					
Scenario	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.368	0.295	0.326	0.082	0.086	0.012	0.381	0.202	0.312	0.151	0.123	0.012
2	0.374	0.211	0.353	0.112	0.081	0.014	0.341	0.142	0.309	0.118	0.103	0.014
3	0.546	0.307	0.362	0.050	0.064	0.012	0.450	0.198	0.334	0.058	0.132	0.012
4	0.328	0.241	0.387	0.091	0.058	0.022	0.387	0.164	0.332	0.160	0.119	0.022
5	0.424	0.313	0.269	0.135	0.087	0.005	0.489	0.219	0.216	0.133	0.117	0.005
Nominal	0.381	0.143	0.191	0.047	0.062	0.010	0.427	0.100	0.160	0.079	0.107	0.010

Table 6.7: Cumulatively ordered risk values incurred for utilitarian solution (2) for $(\psi, \epsilon) = (0.05, 0.75)$, $\varepsilon = 0.05$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.368	0.694	0.989	1.075	1.157	1.169	0.381	0.693	0.895	1.046	1.169	1.181
2	0.374	0.727	0.938	1.050	1.131	1.145	0.341	0.650	0.792	0.910	1.013	1.027
3	0.546	0.908	1.215	1.279	1.329	1.341	0.450	0.784	0.982	1.114	1.172	1.184
4	0.387	0.715	0.956	1.047	1.105	1.127	0.387	0.719	0.883	1.043	1.162	1.184
5	0.424	0.737	1.006	1.141	1.228	1.233	0.489	0.708	0.924	1.057	1.174	1.179
Nominal	0.381	0.572	0.715	0.777	0.824	0.834	0.427	0.587	0.694	0.794	0.873	0.883

Table 6.8: Cumulatively ordered risk values incurred for utilitarian solution (2) for $(\psi, \epsilon) = (0.05, 0.75)$, $\varepsilon = 0.10$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.368	0.694	0.989	1.075	1.157	1.169	0.410	0.723	0.923	1.073	1.171	1.171
2	0.374	0.727	0.938	1.050	1.131	1.145	0.347	0.660	0.808	0.934	1.019	1.019
3	0.546	0.908	1.215	1.279	1.329	1.341	0.478	0.815	1.019	1.122	1.179	1.179
4	0.387	0.715	0.956	1.047	1.105	1.127	0.408	0.745	0.910	1.071	1.177	1.177
5	0.424	0.737	1.006	1.141	1.228	1.233	0.507	0.728	0.943	1.089	1.177	1.177
Nominal	0.381	0.572	0.715	0.777	0.824	0.834	0.455	0.618	0.723	0.812	0.895	0.895

Note that constraint (4.34) in formulation REND ensures that the total risk incurred in each scenario is bounded by γ and yields lightly robust solutions in utilitarian manner. Changing this constraint as $z_{1\omega} \leq \gamma$, $\forall \omega \in \Omega$ ensures that the maximum risk incurred among the population centers is bounded by γ in every scenario and yields lightly robust solutions in Rawlsian manner. To see the effect of this choice, we re-ran the experiments for this version, the summary of which can be seen in Table 6.9. Although we observe a decreasing trend in the optimality gaps as ε increases, optimality gaps in general are higher compared to the first approach which can be seen in Table 6.5.

Table 6.9: Summary of the robust programming results (Rawlsian approach)

(ψ, ϵ)	ε	Max Opt Gap (%)	Avg Opt Gap (%)	#Nominal Pareto	#Robust Pareto	#Optimal	CPU (sec)
(0.05, 0.75)	5	12.59	6.73	9	9	0	1707.33
	10	4.93	3.25	9	9	0	1699.07
(0.05, 0.50)	5	9.13	3.55	10	10	0	1890.3
	10	4.19	2.11	10	10	0	1886.86
(0.03, 0.75)	5	13.30	5.04	18	18	0	3405.66
	10	5.74	2.99	18	18	0	3396.97
(0.03, 0.50)	5	96.49	10.83	16	16	0	3022.62
	10	4.64	2.52	16	16	0	3020.35

In order to observe the effect of the Rawlsian approach consider the same utilitarian solution (2) and the cumulative map of the risk distributions in Table 6.10. Maximum risk incurred by the population centers in each scenario which can be seen in the right hand-side of the table under the $\bar{\theta}_1(z)$ column. As expected, this time we prevent undesirable consequences in terms of the maximum risk and avoid the case where the nominal scenario solution leads to a risk of 0.546 for a population center. Moreover, when these values are compared with the corresponding values in Table 6.7 we observe that Rawlsian approach incurs lower maximum risk values in every scenario compared to the approach that controls the total risk. Similarly, when we examine the total risk incurred by the population centers in each scenario ($\bar{\theta}_6(z)$ column) we observe that Rawlsian approach incurs higher total risk values in every scenario. Tables 6.7 and 6.10 demonstrate the efficiency-fairness trade-off.

Table 6.10: Cumulatively ordered risk values incurred for utilitarian solution (2) for $(\psi, \epsilon) = (0.05, 0.75)$, $\varepsilon = 0.05$ (Rawlsian approach)

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.368	0.694	0.989	1.075	1.157	1.169	0.311	0.620	0.927	1.083	1.204	1.285
2	0.374	0.727	0.938	1.050	1.131	1.145	0.291	0.572	0.852	0.956	1.056	1.099
3	0.546	0.908	1.215	1.279	1.329	1.341	0.400	0.800	1.124	1.286	1.382	1.450
4	0.387	0.715	0.956	1.047	1.105	1.127	0.315	0.626	0.936	1.081	1.188	1.258
5	0.424	0.737	1.006	1.141	1.228	1.233	0.397	0.710	0.906	1.076	1.230	1.332
Nominal	0.381	0.572	0.715	0.777	0.824	0.834	0.285	0.507	0.656	0.775	0.841	0.884

Across all settings, the same utilitarian solution was obtained so we do not provide its results for other (ψ, ϵ) combinations. In Appendices G, H, I and J we provide the results for Rawlsian solutions of the nominal Pareto. Theoretically, this (Rawlsian) approach is expected to result in lower maximum risk values. However, in practice, higher optimality gaps incurred in the Rawlsian approach may lead to increased maximum risk values in some settings.

Chapter 7

Conclusion

In this study we approach the hazmat transport network design problem from an equitable optimization perspective and propose an approach that returns a spectrum of solutions to the policy makers, making the trade-off between equity and efficiency explicit. Unlike existing studies that often propose a single solution based on a specific metric, our spectrum of solutions can encompass these single solutions while providing a broader view of potential outcomes. Furthermore we suggest using multiobjective robust programming extension so as to address the uncertainty that is intricate in these settings. This allows making decisions that do not only perform well in an expected scenario, but also prevent undesirable consequences in extreme cases.

We demonstrate the value of incorporating equity and robustness on set of settings, based on a well-known network. Our results show that the approach has high potential to reveal the gains and losses in alternative designs with different degrees of fairness while enabling protection from highly undesirable results of the uncertainty through robust solutions.

We used a specific problem formulation to illustrate the benefits of our approach. Future research can focus on implementing this approach on alternative problems defined for hazmat network transportation. Moreover, being a multiobjective programming approach, our method deals with problems that are significantly more difficult to solve than their single objective counterparts and robust programming extension exacerbates this difficulty. We believe that addressing these challenges to increase scalability would constitute promising future research directions for transportation researchers.

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Appendix A

Leader Problem

$$\min\ (z_1, \ldots, z_{|C|}) \tag{A.1}$$

$$\text{s.t.} \tag{A.2}$$

$$z_c = cr_c + \sum_{i=1}^{C} d_{ic} \qquad \forall c \in C \tag{A.3}$$

$$r_c + d_{ic} \geq P_c \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A} l_{ij}^{mkc} \sigma_{ij}^{km} \phi_k x_{ij}^{km} \qquad \forall i, c \in C \tag{A.4}$$

$$d_{ic} \geq 0 \qquad \forall i, c \in C \tag{A.5}$$

$$y_{ij}^{m} \in \{0, 1\} \qquad \forall (i, j) \in A, m \in M \tag{A.6}$$

Appendix B

Follower Problem

$$\min \sum_{k\in K}\sum_{m\in M}\sum_{(i,j)\in A} c_{ij}^{km} x_{ij}^{km} \tag{B.1}$$

s.t. (B.2)

$$\sum_{(i,j)\in A} x_{ij}^{km} - \sum_{(j,l)\in A} x_{jl}^{km} = \begin{cases} 0, & \text{if } j \neq o_k, d_k \\ -f^{km}, & \text{if } j = o_k \\ f^{km}, & \text{if } j = d_k \end{cases} \qquad \forall j \in N, k \in K, m \in M$$

(B.3)

$$\sum_{m\in M} f^{km} = 1 \qquad \forall k \in K$$

(B.4)

$$x_{ij}^{km} \leq y_{ij}^{m} \qquad \forall (i,j) \in A, k \in K, m \in M$$

(B.5)

$$x_{ij}^{km} \geq 0 \qquad \forall (i,j) \in A, k \in K, m \in M$$

(B.6)

$$f^{km} \geq 0 \qquad \forall k \in K, m \in M$$

(B.7)

(B.8)

Appendix C

Pseudo Code of the Algorithm

Algorithm 1 ExA Algorithm for Equitable Settings

$Z_E = \emptyset$, $\mathcal{L} = \{\{m^1, ..., m^{|C|}\}\}$.
2: Select a stage $\mathcal{R} = \{z^1, z^2, \ldots, z^{|C|}\} \in \mathcal{L}$
Calculate w such that $w^\top z^1 = w^\top z^2 = \ldots = w^\top z^{|C|}$.
4: **if** $w \in \mathbb{R}^m_>$ **then**
  Solve problem REND(w) and let the optimal point be $z^* = (z^*_1, z^*_2, \ldots, z^*_{|C|})$.
6: **if** $z^* \notin \mathcal{R}$ **then**
    $\mathcal{L} = \mathcal{L} \cup \{\{z^1, \ldots, z^*\}, \{z^1, \ldots, z^*, z^{|C|}\}, \ldots, \{z^*, \ldots, z^{|C|}\}\}$.
8: **end if**
end if
10: **if** $r^* \notin Z_E$ **then**
  $Z_E = Z_E \cup \{r^*\}$.
12: **end if**
if $w \notin \mathbb{R}^m_>$ **then**
14: Find $z \in Z_{EM} \setminus \{\mathcal{R}\}$ and $w' \in \mathbb{R}^m_>$ such that $(w')^\top z \leq (w')^\top r$, $\forall r \in \mathcal{R}$. Set $r^* = z$ and go to Step 10.
end if
16: $\mathcal{L} = \mathcal{L} \setminus \{\mathcal{R}\}$.
if $\mathcal{L} = \emptyset$ **then**
18: report Z_E and stop, otherwise go to Step 2.
end if

Appendix D

Setting 2 Risk Table

Table D.1: Fairness evaluations for solutions of setting 2 (0.05, 0.50)

	Fairness measures						Risk of population centers					
Solution	Trad	Max	AdM	MdM	AdA	MdA	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.905	0.253	0.013	0.103	0.088	0.183	0.253	0.241	0.123	0.108	0.108	0.071
2	**0.834**	0.381	0.022	0.242	0.158	0.371	0.381	0.143	0.191	0.047	0.062	0.010
3	0.872	0.350	0.014	0.205	0.116	0.286	0.350	0.137	0.153	0.064	0.103	0.064
4	0.944	0.335	0.010	0.178	0.083	0.235	0.335	0.144	0.123	0.123	0.120	0.100
5	1.081	0.221	**0.005**	0.085	0.054	0.126	0.221	0.206	0.096	0.204	0.206	0.147
6	0.847	0.282	0.022	0.140	0.141	0.272	0.282	0.255	0.191	0.047	0.062	0.010
7	1.177	**0.215**	**0.005**	**0.030**	**0.019**	**0.049**	0.196	0.215	0.196	0.207	0.196	0.166
8	0.925	0.246	0.019	0.111	0.098	0.204	0.246	0.219	0.138	0.085	0.194	0.043
9	1.083	0.229	0.006	0.048	0.039	0.086	0.218	0.229	0.165	0.165	0.165	0.142
10	0.984	0.224	0.017	0.103	0.083	0.162	0.222	0.224	0.101	0.155	0.222	0.061

Appendix E

Setting 3 Risk Table

Table E.1: Fairness evaluations for solutions of setting 3 (0.03, 0.75)

	Fairness measures						Risk of population centers					
Solution	Trad	Max	AdM	MdM	AdA	MdA	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.905	0.253	0.013	0.103	0.088	0.183	0.253	0.241	0.123	0.108	0.108	0.071
2	**0.834**	0.381	0.022	0.242	0.158	0.371	0.381	0.143	0.191	0.047	0.062	0.010
3	0.872	0.350	0.014	0.205	0.116	0.286	0.350	0.137	0.153	0.064	0.103	0.064
4	0.918	0.339	0.012	0.186	0.090	0.255	0.339	0.142	0.118	0.118	0.118	0.084
5	1.100	0.220	**0.004**	0.083	0.049	0.120	0.220	0.206	0.100	0.206	0.206	0.160
6	1.193	**0.209**	0.006	**0.034**	**0.018**	**0.044**	0.194	0.209	0.209	0.207	0.209	0.165
7	0.883	0.356	0.018	0.209	0.110	0.319	0.356	0.133	0.121	0.121	0.115	0.037
8	0.847	0.282	0.022	0.140	0.141	0.272	0.282	0.255	0.191	0.047	0.062	0.010
9	0.949	0.240	0.019	0.115	0.094	0.197	0.240	0.215	0.138	0.098	0.215	0.043
10	0.953	0.245	0.013	0.086	0.081	0.163	0.245	0.218	0.128	0.102	0.179	0.081
11	1.125	0.224	0.005	0.036	0.031	0.064	0.204	0.224	0.167	0.204	0.167	0.160
12	1.039	0.219	0.015	0.088	0.070	0.134	0.218	0.219	0.095	0.204	0.219	0.085
13	0.878	0.253	0.017	0.107	0.107	0.211	0.253	0.241	0.149	0.066	0.126	0.043
14	1.151	0.216	0.009	0.054	0.026	0.078	0.199	0.216	0.199	0.199	0.199	0.138
15	0.956	0.329	0.010	0.170	0.078	0.230	0.329	0.138	0.130	0.130	0.130	0.099
16	0.991	0.234	0.012	0.069	0.071	0.138	0.234	0.212	0.119	0.119	0.212	0.096
17	0.944	0.280	0.014	0.122	0.085	0.209	0.280	0.177	0.119	0.119	0.177	0.071
18	0.983	0.225	0.017	0.104	0.084	0.166	0.220	0.225	0.099	0.159	0.220	0.060

Appendix F

Setting 4 Risk Table

Table F.1: Fairness evaluations for solutions of setting 4 (0.03, 0.50)

	Fairness measures						Risk of population centers					
Solution	Trad	Max	AdM	MdM	AdA	MdA	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.905	0.253	0.013	0.103	0.088	0.183	0.253	0.241	0.123	0.108	0.108	0.071
2	**0.834**	0.381	0.022	0.242	0.158	0.371	0.381	0.143	0.191	0.047	0.062	0.010
3	0.872	0.350	0.014	0.205	0.116	0.286	0.350	0.137	0.153	0.064	0.103	0.064
4	0.918	0.338	0.011	0.185	0.089	0.250	0.338	0.146	0.115	0.115	0.115	0.088
5	1.082	0.221	**0.005**	0.086	0.054	0.127	0.221	0.206	0.094	0.206	0.206	0.148
6	0.883	0.356	0.018	0.209	0.110	0.319	0.356	0.133	0.121	0.121	0.115	0.037
7	0.847	0.282	0.022	0.140	0.141	0.272	0.282	0.255	0.191	0.047	0.062	0.010
8	1.120	0.208	0.007	0.039	0.032	0.060	0.208	0.207	0.147	0.204	0.207	0.147
9	1.188	**0.207**	0.006	**0.035**	**0.015**	**0.044**	0.204	0.204	0.204	0.207	0.204	0.163
10	0.885	0.327	0.017	0.179	0.111	0.284	0.327	0.149	0.149	0.066	0.149	0.043
11	1.018	0.229	0.010	0.059	0.063	0.117	0.229	0.210	0.112	0.145	0.210	0.112
12	1.155	0.211	0.008	0.045	0.022	0.064	0.198	0.211	0.198	0.204	0.198	0.147
13	0.878	0.253	0.017	0.107	0.107	0.211	0.253	0.241	0.149	0.066	0.126	0.043
14	0.916	0.253	0.014	0.101	0.093	0.186	0.253	0.222	0.148	0.067	0.155	0.071
15	0.963	0.309	0.008	0.148	0.074	0.196	0.309	0.161	0.130	0.119	0.130	0.113
16	0.984	0.233	0.016	0.093	0.079	0.162	0.233	0.213	0.147	0.107	0.213	0.071

Appendix G

Setting 1 Lightly Robust Results

Table G.1: Risk values incurred for Rawlsian solution (5) for $(\psi, \epsilon) = (0.05, 0.75)$, $\varepsilon = 0.05$

	Risk of population centers (nominal)						Risk of population centers (robust)					
Scenario	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.211	0.301	0.421	0.259	0.304	0.224	0.292	0.343	0.295	0.209	0.231	0.007
2	0.135	0.242	0.353	0.251	0.221	0.150	0.199	0.247	0.251	0.138	0.171	0.008
3	0.267	0.330	0.391	0.235	0.384	0.259	0.353	0.354	0.300	0.107	0.257	0.007
4	0.227	0.302	0.298	0.165	0.294	0.255	0.296	0.307	0.261	0.187	0.226	0.012
5	0.157	0.345	0.157	0.441	0.290	0.335	0.256	0.397	0.148	0.147	0.214	0.002
Nominal	0.195	0.209	0.209	0.207	0.209	0.165	0.259	0.209	0.121	0.155	0.209	0.005

Table G.2: Cumulatively ordered risk values incurred for Rawlsian solution (5) for $(\psi, \epsilon) = (0.05, 0.75)$, $\varepsilon = 0.05$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.421	0.725	1.026	1.285	1.509	1.720	0.343	0.638	0.930	1.161	1.370	1.377
2	0.353	0.604	0.846	1.067	1.217	1.352	0.251	0.498	0.697	0.868	1.006	1.014
3	0.391	0.775	1.105	1.372	1.631	1.866	0.354	0.707	1.007	1.264	1.371	1.378
4	0.302	0.600	0.894	1.149	1.376	1.541	0.307	0.603	0.864	1.090	1.277	1.289
5	0.441	0.786	1.121	1.411	1.568	1.725	0.397	0.653	0.867	1.015	1.162	1.164
Nominal	0.209	0.418	0.627	0.834	1.029	1.194	0.259	0.468	0.677	0.832	0.953	0.958

Table G.3: Cumulatively ordered risk values incurred for Rawlsian solution (5) for $(\psi, \epsilon) = (0.05, 0.75)$, $\varepsilon = 0.10$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.421	0.725	1.026	1.285	1.509	1.720	0.292	0.584	0.838	1.067	1.295	1.295
2	0.353	0.604	0.846	1.067	1.217	1.352	0.257	0.498	0.696	0.864	1.008	1.008
3	0.391	0.775	1.105	1.372	1.631	1.866	0.364	0.658	0.934	1.194	1.295	1.295
4	0.302	0.600	0.894	1.149	1.376	1.541	0.302	0.550	0.789	1.025	1.238	1.238
5	0.441	0.786	1.121	1.411	1.568	1.725	0.313	0.624	0.831	0.984	1.119	1.119
Nominal	0.209	0.418	0.627	0.834	1.029	1.194	0.309	0.518	0.689	0.855	0.969	0.969

Table G.4: Cumulatively ordered risk values incurred for Rawlsian solution (5) for $(\psi, \epsilon) = (0.05, 0.75)$, $\varepsilon = 0.05$ (Rawlsian approach)

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.421	0.725	1.026	1.285	1.509	1.72	0.302	0.586	0.841	1.094	1.338	1.492
2	0.353	0.604	0.846	1.067	1.217	1.352	0.275	0.525	0.766	0.958	1.113	1.237
3	0.391	0.775	1.105	1.372	1.631	1.866	0.355	0.677	0.991	1.299	1.497	1.682
4	0.302	0.600	0.894	1.149	1.376	1.541	0.297	0.592	0.883	1.156	1.385	1.587
5	0.441	0.786	1.121	1.411	1.568	1.725	0.322	0.583	0.828	1.025	1.217	1.394
Nominal	0.209	0.418	0.627	0.834	1.029	1.194	0.220	0.438	0.643	0.835	0.973	1.111

Appendix H

Setting 2 Lightly Robust Results

Table H.1: Risk values incurred for Rawlsian solution (7) for $(\psi, \epsilon) = (0.05, 0.50)$, $\varepsilon = 0.05$

	Risk of population centers (nominal)						Risk of population centers (robust)					
Scenario	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.215	0.323	0.407	0.260	0.277	0.226	0.268	0.293	0.350	0.222	0.217	0.000
2	0.136	0.246	0.337	0.251	0.188	0.151	0.242	0.247	0.310	0.136	0.161	0.000
3	0.270	0.341	0.378	0.236	0.359	0.260	0.349	0.348	0.346	0.092	0.242	0.000
4	0.230	0.310	0.287	0.166	0.264	0.256	0.281	0.287	0.299	0.215	0.220	0.000
5	0.158	0.374	0.148	0.442	0.274	0.339	0.268	0.371	0.174	0.142	0.198	0.000
Nominal	0.196	0.215	0.196	0.207	0.196	0.166	0.265	0.207	0.168	0.155	0.196	0.000

Table H.2: Cumulatively ordered risk values incurred for Rawlsian solution (7) for $(\psi, \epsilon) = (0.05, 0.50)$, $\varepsilon = 0.05$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.407	0.730	1.007	1.267	1.493	1.708	0.350	0.643	0.911	1.133	1.350	1.350
2	0.337	0.588	0.834	1.022	1.173	1.309	0.310	0.557	0.799	0.960	1.096	1.096
3	0.378	0.737	1.078	1.348	1.608	1.844	0.349	0.697	1.043	1.285	1.377	1.377
4	0.310	0.597	0.861	1.117	1.347	1.513	0.299	0.586	0.867	1.087	1.302	1.302
5	0.442	0.816	1.155	1.429	1.587	1.735	0.371	0.639	0.837	1.011	1.153	1.153
Nominal	0.215	0.422	0.618	0.814	1.010	1.176	0.265	0.472	0.668	0.836	0.991	0.991

Table H.3: Cumulatively ordered risk values incurred for Rawlsian solution (7) for $(\psi, \epsilon) = (0.05, 0.50)$, $\varepsilon = 0.10$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.407	0.730	1.007	1.267	1.493	1.708	0.299	0.591	0.855	1.080	1.297	1.297
2	0.337	0.588	0.834	1.022	1.173	1.309	0.246	0.488	0.686	0.857	1.001	1.001
3	0.378	0.737	1.078	1.348	1.608	1.844	0.364	0.658	0.936	1.194	1.297	1.297
4	0.310	0.597	0.861	1.117	1.347	1.513	0.309	0.559	0.799	1.030	1.232	1.232
5	0.442	0.816	1.155	1.429	1.587	1.735	0.318	0.630	0.836	0.991	1.128	1.128
Nominal	0.215	0.422	0.618	0.814	1.010	1.176	0.315	0.522	0.690	0.856	0.971	0.971

Table H.4: Cumulatively ordered risk values incurred for Rawlsian solution (7) for $(\psi, \epsilon) = (0.05, 0.50)$, $\varepsilon = 0.05$ (Rawlsian approach)

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.407	0.730	1.007	1.267	1.493	1.708	0.302	0.587	0.866	1.130	1.383	1.498
2	0.337	0.588	0.834	1.022	1.173	1.309	0.275	0.516	0.752	0.966	1.121	1.202
3	0.378	0.737	1.078	1.348	1.608	1.844	0.334	0.657	0.971	1.279	1.468	1.577
4	0.310	0.597	0.861	1.117	1.347	1.513	0.313	0.610	0.901	1.174	1.395	1.528
5	0.442	0.816	1.155	1.429	1.587	1.735	0.324	0.599	0.867	1.064	1.241	1.364
Nominal	0.215	0.422	0.618	0.814	1.010	1.176	0.237	0.457	0.662	0.865	1.003	1.079

Appendix I

Setting 3 Lightly Robust Results

Table I.1: Risk values incurred for Rawlsian solution (6) for $(\psi, \epsilon) = (0.03, 0.75)$, $\varepsilon = 0.05$

	Risk of population centers (nominal)						Risk of population centers (robust)					
Scenario	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.210	0.300	0.421	0.259	0.304	0.224	0.222	0.264	0.400	0.214	0.244	0.074
2	0.134	0.243	0.353	0.251	0.222	0.150	0.218	0.205	0.323	0.127	0.158	0.034
3	0.265	0.330	0.391	0.235	0.384	0.259	0.304	0.285	0.394	0.179	0.282	0.067
4	0.227	0.302	0.299	0.165	0.294	0.255	0.243	0.251	0.274	0.141	0.250	0.059
5	0.155	0.344	0.157	0.441	0.290	0.335	0.237	0.329	0.193	0.211	0.227	0.107
Nominal	0.194	0.209	0.209	0.207	0.209	0.165	0.259	0.183	0.209	0.188	0.209	0.041

Table I.2: Cumulatively ordered risk values incurred for Rawlsian solution (6) for $(\psi, \epsilon) = (0.03, 0.75)$, $\varepsilon = 0.05$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.421	0.725	1.025	1.284	1.508	1.718	0.400	0.664	0.908	1.130	1.344	1.418
2	0.353	0.604	0.847	1.069	1.219	1.353	0.323	0.541	0.746	0.904	1.031	1.065
3	0.391	0.775	1.105	1.370	1.629	1.864	0.394	0.698	0.983	1.265	1.444	1.511
4	0.302	0.601	0.895	1.150	1.377	1.542	0.274	0.525	0.775	1.018	1.159	1.218
5	0.441	0.785	1.120	1.410	1.567	1.722	0.329	0.566	0.793	1.004	1.197	1.304
Nominal	0.209	0.418	0.627	0.834	1.028	1.193	0.259	0.468	0.677	0.865	1.048	1.089

Table I.3: Cumulatively ordered risk values incurred for Rawlsian solution (6) for $(\psi, \epsilon) = (0.03, 0.75)$, $\varepsilon = 0.10$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.421	0.725	1.025	1.284	1.508	1.718	0.293	0.586	0.839	1.068	1.296	1.296
2	0.353	0.604	0.847	1.069	1.219	1.353	0.257	0.498	0.696	0.864	1.008	1.008
3	0.391	0.775	1.105	1.370	1.629	1.864	0.364	0.658	0.933	1.194	1.295	1.295
4	0.302	0.601	0.895	1.150	1.377	1.542	0.302	0.551	0.789	1.026	1.239	1.239
5	0.441	0.785	1.120	1.410	1.567	1.722	0.312	0.623	0.830	0.983	1.118	1.118
Nominal	0.209	0.418	0.627	0.834	1.028	1.193	0.309	0.518	0.689	0.856	0.970	0.970

Table I.4: Cumulatively ordered risk values incurred for Rawlsian solution (6) for $(\psi, \epsilon) = (0.03, 0.75)$, $\varepsilon = 0.05$ (Rawlsian approach)

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.421	0.725	1.025	1.284	1.508	1.718	0.301	0.587	0.863	1.127	1.380	1.509
2	0.353	0.604	0.847	1.069	1.219	1.353	0.242	0.479	0.712	0.908	1.064	1.160
3	0.391	0.775	1.105	1.370	1.629	1.864	0.332	0.657	0.965	1.265	1.479	1.618
4	0.302	0.601	0.895	1.150	1.377	1.542	0.298	0.591	0.865	1.099	1.315	1.477
5	0.441	0.785	1.120	1.410	1.567	1.722	0.325	0.580	0.835	1.033	1.179	1.305
Nominal	0.209	0.418	0.627	0.834	1.028	1.193	0.232	0.457	0.678	0.884	0.998	1.095

Appendix J

Setting 4 Lightly Robust Results

Table J.1: Risk values incurred for Rawlsian solution (9) for $(\psi, \epsilon) = (0.03, 0.50)$, $\varepsilon = 0.05$

	Risk of population centers (nominal)						Risk of population centers (robust)					
Scenario	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6	Pop1	Pop2	Pop3	Pop4	Pop5	Pop6
1	0.217	0.304	0.416	0.260	0.297	0.224	0.279	0.339	0.309	0.240	0.223	0.012
2	0.146	0.237	0.347	0.251	0.211	0.149	0.210	0.242	0.222	0.134	0.167	0.014
3	0.280	0.325	0.386	0.235	0.374	0.256	0.366	0.346	0.301	0.124	0.253	0.012
4	0.229	0.298	0.294	0.165	0.285	0.252	0.275	0.302	0.211	0.206	0.220	0.022
5	0.167	0.346	0.154	0.441	0.284	0.334	0.244	0.390	0.109	0.143	0.209	0.005
Nominal	0.204	0.204	0.204	0.207	0.204	0.163	0.257	0.204	0.112	0.184	0.204	0.010

Table J.2: Cumulatively ordered risk values incurred for Rawlsian solution (9) for $(\psi, \epsilon) = (0.03, 0.50)$, $\varepsilon = 0.05$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.416	0.720	1.017	1.277	1.501	1.718	0.339	0.648	0.927	1.167	1.390	1.402
2	0.347	0.598	0.835	1.046	1.195	1.341	0.242	0.464	0.674	0.841	0.975	0.989
3	0.386	0.760	1.085	1.365	1.621	1.856	0.366	0.712	1.013	1.266	1.390	1.402
4	0.298	0.592	0.877	1.129	1.358	1.523	0.302	0.577	0.797	1.008	1.214	1.236
5	0.441	0.787	1.121	1.405	1.572	1.726	0.390	0.634	0.843	0.986	1.095	1.100
Nominal	0.207	0.411	0.615	0.819	1.023	1.186	0.257	0.461	0.665	0.849	0.961	0.971

Table J.3: Cumulatively ordered risk values incurred for Rawlsian solution (9) for $(\psi, \epsilon) = (0.03, 0.50)$, $\varepsilon = 0.10$

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.416	0.720	1.017	1.277	1.501	1.718	0.294	0.586	0.845	1.070	1.294	1.294
2	0.347	0.598	0.835	1.046	1.195	1.341	0.256	0.500	0.703	0.869	1.012	1.012
3	0.386	0.760	1.085	1.365	1.621	1.856	0.366	0.661	0.944	1.198	1.296	1.296
4	0.298	0.592	0.877	1.129	1.358	1.523	0.303	0.555	0.798	1.029	1.240	1.240
5	0.441	0.787	1.121	1.405	1.572	1.726	0.318	0.629	0.833	0.985	1.124	1.124
Nominal	0.207	0.411	0.615	0.819	1.023	1.186	0.307	0.511	0.684	0.845	0.961	0.961

Table J.4: Cumulatively ordered risk values incurred for Rawlsian solution (9) for $(\psi, \epsilon) = (0.03, 0.50)$, $\varepsilon = 0.05$ (Rawlsian approach)

	$\bar{\Theta}$: Risk of population centers (nominal)						$\bar{\Theta}$: Risk of population centers (robust)					
Scenario	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$	$\bar{\theta}_1(z)$	$\bar{\theta}_2(z)$	$\bar{\theta}_3(z)$	$\bar{\theta}_4(z)$	$\bar{\theta}_5(z)$	$\bar{\theta}_6(z)$
1	0.416	0.720	1.017	1.277	1.501	1.718	0.311	0.598	0.873	1.136	1.390	1.533
2	0.347	0.598	0.835	1.046	1.195	1.341	0.256	0.507	0.755	0.943	1.115	1.225
3	0.386	0.760	1.085	1.365	1.621	1.856	0.334	0.668	0.984	1.298	1.494	1.664
4	0.298	0.592	0.877	1.129	1.358	1.523	0.302	0.585	0.853	1.108	1.315	1.512
5	0.441	0.787	1.121	1.405	1.572	1.726	0.326	0.624	0.876	1.093	1.264	1.413
Nominal	0.207	0.411	0.615	0.819	1.023	1.186	0.229	0.457	0.666	0.870	0.999	1.117