

An Information Theoretic Study of Spatial Diversity Techniques for
Wireless Relay Networks

by

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This is to certify that I have examined this copy of a master's thesis by

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To my parents.

ABSTRACT

The main characteristic of wireless communication is the randomness that can be seen both in users transmission channels and in users geographical locations. These uncertainties lead to random signal attenuation independently across users and across transmission channels of individual users. This randomness can be used as a tool to enhance performance through *diversity*. We broadly define diversity as the method of conveying information through multiple *independent* instantiations of these random attenuations. In wireless communication systems to increase system reliability various diversity techniques have been proposed which can be collected under three domains, namely, *temporal diversity*, *frequency diversity* and *spatial diversity* domains. Due to its efficiency in terms of system resource usage, the diversity tool we utilize in the whole dissertation is *spatial diversity* which is attained through multiple spatially separate antennas and multiple geographically separate (independent) users.

It has been shown that diversity techniques, when properly administered, may provide tremendous gains to the performance of wireless systems. In this thesis, using tools provided by information theory, we propose a comparative study of the gains obtained by various spatial diversity techniques. Under different channel condition and power allocation schemes these diversity techniques will be compared and ones that give best results under different constraints will be clearly specified. Furthermore, impact of simultaneous use of multiple diversity techniques will be investigated.

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NOMENCLATURE

AWGN	Additive White Gaussian Noise
CCSCG	Complex Circularly Symmetric Complex Gaussian
BF	Beamforming
DT	Direct Transmission
SISO	Single-Input Single-Output
SIMO	Single-Input Multiple-Output
MISO	Multiple-Input Single-Output
MIMO	Multiple-Input Multiple-Output
CSI	Channel Side Information
NCSI	No Channel Side Information
MRC	Maximum Ratio Combining
EGC	Equal Gain Combining
SC	Selection Combining
AF	Amplify-and-Forward
DF	Decode-and-Forward
BS-DF	Best-Selection Decode-and-Forward
AP-DF	All-Participate Decode-and-Forward
CF	Compress-and-Forward
EF	Estimate-and-Forward
MU	Multi-User
MAC	Multiple Access Channel
BC	Broadcast Channel
SNR	Signal-to-Noise Ratio
STBC	Space-Time Block Codes
SLR	Signal-to-Leakage Ratio
SSLR	Sum of Signal-to-Leakage Ratio
RCPC	Rate Compatible Punctured Convolutional Codes

Chapter 1

INTRODUCTION

Because of the time-varying nature of the wireless channel, and the presence of limited resources for transmission, a transmitted signal faces various detrimental effects such as interference, propagation path loss, delay spread, doppler spread, shadowing and fading which make it difficult to achieve high data rates. By increasing the transmission bandwidth or using higher transmit power, it is possible to increase the transmission data rates. But, from a practical point of view, increasing the transmission bandwidth or the transmit power is not always feasible due to high system deployment costs. Moreover, increasing the transmit power results in an increased interference to other transmissions and also reduces the battery life-time of mobile transmitters.

To mitigate the effects of multi-path fading in wireless communications, a mechanism, known as diversity, was introduced as early as 1927 [3]. To date, various diversity techniques have been proposed in the literature. These can be classified under three domains, namely, *temporal diversity*, *frequency diversity* and *spatial diversity*. Among these, due to its efficiency in terms of system resource usage (no extra power and bandwidth utilization necessary), the diversity tool we utilize in this thesis is *spatial diversity* which is attained via multiple spatially separate antennas at the transmitters and multiple geographically separate users. Spatial diversity provides a more robust and reliable communication, and by introducing additional degrees of freedom to the system, it provides a higher system capacity without requiring any additional power or bandwidth.

1.1 Prior Work on Spatial Diversity

For point-to-point wireless communications, to mitigate the detrimental effects of deep fades of the wireless channel, various diversity techniques have been proposed in the literature [4–8]. Our focus is on spatial diversity techniques in this thesis. We now summarize some

of the related works from the literature.

Spatial diversity can be exploited via multiple spatially separate antennas and multiple geographically separate users. The very first spatial diversity technique proposed in the literature was the receiver antenna diversity where the receiver site is equipped with multiple antennas [4, 9]. In receiver diversity, in order to have *independently* faded copies of the transmitted signal for processing, the receiver antennas should be separated far enough, i.e., by an amount of $1/10$ of the wavelength of the carrier. Various combining techniques, i.e., maximum-ratio combining (MRC), equal gain combining (EGC) and selection combining (SC), may be used at the receiver. Practically, due to the size limitation of mobile users, it should be difficult to equip them with more than two multiple antennas. To overcome this constraint, transmit diversity algorithms have been developed. An easy and intelligent transmit diversity scheme is proposed by Alamouti [5] where no channel feedback is required to achieve diversity gains. Alamouti's transmit diversity scheme for two transmit antennas is one possible implementation of Space-Time Block Coding (STBC). A generalized study of STBC is given in [10, 11].

While multiple antennas can be used to provide diversity gain, they can also be used to provide additional multiplexing gain or higher channel capacity. The pioneering works of Foschini and Gans [12], and Telatar [13] ignited much interest in this area. When the channel exhibits rich scattering and its variations can be accurately tracked, it has been shown that with multiple transmit and/or multiple receiver antennas, and without utilizing any extra system resources (power and/or bandwidth), the channel capacity may be increased proportional to maximum of the number of antennas at the transmitter and receiver. Since then, there have been exhaustive research performed on this topic due to its great potential. In [14, 15], a practical implementation for the use of multiple antennas at the transmitter and receiver towards a system capacity increase is introduced.

In multi-user wireless systems, by allowing transmission to only the user with the best instantaneous channel conditions, Knopp and Humblet [16] proposed a kind of spatial diversity technique referred to as multi-user diversity. Although the mobile units do not have multiple antennas in their study, with efficient scheduling of geographically separated users' transmissions, significant diversity gains are observed. Such gains are proportional to the number of active users in the system.

Due to size and complexity constraints, full-fledged spatial diversity methods are not applicable to many wireless systems. A conventional example is the uplink of cellular systems, where the size of the mobile is the limiting factor. A more interesting example is ad-hoc networks and sensor networks, where size, complexity, and power are all limiting issues that preclude a large number of transmit antennas. In such wireless networks, geographically separated users can facilitate the transmission by assisting their partners. This kind of cooperation provides significant diversity gains when efficient signaling schemes are used. If, in the network, only one user has its own information to transmit via the assistance of others, such networks are called *relay networks* [17–23]. On the other hand if all users have both their own data and partners' data to transmit, such transmissions are called *cooperative communications*. It has been shown that cooperative communications provide significant gains over direct transmission of information in ad-hoc networks [1, 2, 24–26].

1.2 Contributions of the Thesis

This thesis is concerned with information theoretic analyses of multiple relay MIMO systems in fading channels. The novel contributions contained in this study can be classified into two broad categories:

1. We investigate the system capacity of MIMO multi-relay networks where beamforming is employed at the source node. This scheme assumes channel state information (CSI) of all channels between the multiple antennas of the source and the relays as well as the relays and the destination at the source end.
2. We investigate the system capacity of MIMO multi-relay networks where the source multiplexes the information across the relays using beamforming such that the Sum of Signal-to-Leakage Ratios (SSLR) are maximized.

Thus, in this thesis, we first propose a simple source node beamforming algorithm for fading Multiple-Relay MIMO channels. Here, each data stream, from a total of $K + 1$ data streams, is allowed to be relayed, if possible, by all relay nodes to the destination node, where K is the number of available relays for the transmission. We propose to use the conventional best eigen-vector beamforming algorithm for the construction of the transmit

covariance matrix. We examine amplify and forward (AF) and two different decode and forward (DF) relaying protocols. The first DF protocol, referred in this thesis as the Best-Selection DF (BS-DF) protocol, transmission is established such that only the best relaying receiver is able to decode the signal. In the second DF protocol, referred in this thesis as the All-Participate DF (AP-DF), source transmission is established such that all available relay nodes are able to decode the transmission. We show that the proposed source node beamforming algorithm provides significant increases in the channel capacity in the low-SNR regime.

Next, we extend on the proposed source node beamforming algorithm for multiplexed transmission via multiple relays. Here beamforming is established so that the Sum of the Signal-to-Leakage ratios (SSLR) observed at the relay sets is maximized. Specifically, we assume that each of the multiplexed data streams is intended for transmission via a pre-determined subset of the available relay nodes. Each available relay node is utilized for the transmission of only one of the multiplexed data streams. The transmission streams are beamed such that the SSLR is minimized. In other words, we build the beamforming vectors so that, for each data stream, the ratio of the sum of that data stream's received power at the intended receivers to sum of that at the non-intended receivers is maximum. This clearly results in preventing interference propagation. We focus our channel capacity analysis to the amplify and forward (AF) protocol for this problem and observe significant gains from the SSLR maximizing source node beamforming algorithm.

1.3 Outline of the Thesis

Since spatial diversity forms the backbone of this thesis, we present detailed analyses of various spatial diversity techniques in Chapter 2, where analyses are conducted in terms of ergodic capacity and outage probability. We investigate the information theoretic performances of receiver diversity (SIMO) and transmit diversity (MISO) algorithms. We observe that both algorithms provide only a single degree of freedom (or multiplexing gain) while achieving full diversity gain as well as array gain for receiver diversity. Moreover, we derive both ergodic capacities and outage probabilities for systems employing transmit beamforming and space-time block codes (STBCs) and compare their performance to a system employing multi-user diversity. Finally, in Chapter 2, we present various relaying protocols

that have been studied in the literature: amplify-and-forward (AF), decode-and-forward (DF), adaptive decode-and-forward (AdDF), estimate-and-forward (EF) and compress-and-forward (CF). We show that with the exception of the DF protocol, all other relaying modes provide second order diversity.

In Chapter 3, we study the performance of relay networks where the source, the destination and multiple relay nodes are all equipped with multiple antennas. We study the ergodic capacities of multiple relay networks based on two of the most prominent relaying strategies, namely AF and DF relaying protocols. Two different DF protocols are investigated, namely, *Best-Selection DF* (BS-DF) and *All-Participate DF* (AP-DF). We derive the system capacities based on both no CSI as well as full-CSI at all transmitter sides: source node as well as relay nodes. We assume that all receiving parts also have their associated CSI. When full-CSI is available at all transmitter nodes, we introduce a simple but efficient transmit beamforming algorithm for the source node and study its impact on the capacity of the MIMO multi-relay channel. The proposed approach is based on two fundamental concepts: transmit beamforming and power allocation over multiple beams (waterfilling [27]). As conventional with beamforming, we direct our transmission power to the best eigen-channel. With waterfilling, using the CSI knowledge, we allocate more power on better beams and less power to worse beams.

In Chapter 4, we study a MIMO wireless relay network scenario where a designated source and destination node communicate via the cooperation of multiple relays. We consider the coherent AF transmission proposed in [23], where each relay node is assigned to one of the multiplexed data streams. the source and destination nodes are assumed to have the same number of antennas and each transmitter-receiver antenna is virtually paired during the course of transmission. Additionally, we assume that the source node has full CSI. To mitigate the interference observed by the non-assigned relay nodes from an individual data stream, we propose a multi-relay source node beamforming algorithm by which the system capacity is increased. The proposed beamforming algorithm is based on maximizing the *Sum of Signal-to-Leakage Ratios*(SSLR) for all data streams. With this method, the sum of signal strengths at the set of relay nodes that are assigned to a corresponding data stream is maximized relative to the interference terms caused by the transmission of that specific stream at the non-assigned relay nodes. We show that with the use of beamforming

at the source node, the coherent AF relay transmission benefits from *transmitter array gain*. We derive capacity expressions as a function of the number of relay nodes relay for fixed number of antennas to observe the effect of increase in available relay numbers. We show that an increase in the the number of available in relays results in *array gains* since each data stream is relayed over an increased set of relays in this case, resulting in an increase in the average SINR at the destination node.

For the sake of completeness in the coverage of spatial diversity, we briefly discuss user cooperative diversity in Chapter 5. We summarize some of the information theoretic results presented in [1]. As a practical implementation of cooperative diversity, we discuss the coded cooperation concept that was first proposed in [2]. We observe that cooperative communications provide significant gains over direct transmission.

Finally, in Chapter 6, we provide concluding remarks and discuss possible future work based on the ideas developed in this thesis.

1.4 Notation

In the thesis, we use following notations. ^{T} , $*$ and ^{H} stand for transposition, element-wise conjugate and conjugate transposition, respectively. Bold uppercase letters describe matrices while bold lowercase letters describe column vectors of appropriate dimensions. $\text{tr}(\mathbf{A})$ and $\det(\mathbf{A})$ stand for trace and determinant of the matrix $\mathbf{A}$. $\|\mathbf{a}\|$ denotes the Euclidean norm of the vector $\mathbf{a}$. We denote the expectation operator as $\mathcal{E}[\cdot]$. $\mathbf{I}_N$ is the $N \times N$ identity matrix, and $\mathbf{0}$ stands for an all zeros matrix of appropriate dimensions. $|\mathcal{X}|$ is the cardinality of the set $\mathcal{X}$.

We represent a circularly symmetric complex Gaussian (CSCG) random variable (RV) as a RV $Z = X + jY \sim \mathcal{CN}(0, \sigma^2)$, where X and Y are independent, identically distributed (i.i.d.) Gaussian random variables that are zero-mean and have variances of σ^2 , $\mathcal{N}(0, \sigma^2/2)$.

$A \rightarrow B$ describes a link between A and B nodes in the wireless relay networks studied.

Throughout the thesis all logarithms, unless specified otherwise, are of base 2.

Chapter 2

AN INFORMATION THEORETIC APPROACH TO DIVERSITY
TECHNIQUES

In this chapter we investigate some of the fundamental issues that arise in communication over fading channels. We focus our attention on coherent reception. In general, the error probability observed in fading channels is much higher than that observed in a non-faded, AWGN channel. The reason is there is a significant probability that the channel is in a deep fade in fading channels. This motivates us to investigate various diversity techniques which improve the system performance in such scenarios.

In wireless communications, occurrences of deep fades in the propagation channel make reliable communication difficult. In general, diversity techniques increase reliability by sending the same information through multiple independently faded paths so that the probability of successful transmission is higher. Diversity can be exploited over temporal (time), spectral (frequency) and spatial (space) domains. Diversity over temporal domain can be obtained via *coding* and/or *interleaving*: information is coded and/or the symbols are dispersed over time in different coherence periods so that different parts of the codewords experience independent fades. Analogously, one can also exploit diversity over the spectral domain provided that the channel is frequency-selective [28, 29]. Due to its efficiency in terms of system resource usage, the diversity domain we focus on in this thesis is *spatial diversity* which is attained through multiple spatially separate antennas and/or multiple geographically separate users.

Throughout the thesis, we frequently use the terms *degrees of freedom*, *diversity gain* and *array gain*. Thus, it is worthy to provide formal definitions for these terms:

Degrees of Freedom, also referred to as the multiplexing gain, is defined as the coefficient of $\log \rho$ occurring in the expression for *capacity* for high values of the signal-to-noise ratio, ρ . The degrees of freedom can be considered as the maximum number of independent channels available for communication in a particular communication system.

Thus, the available degrees of freedom in MIMO systems yields a linear (in the minimum of the number of transmit and receive antennas) increase in the system capacity with no additional power or bandwidth requirements [12, 13]. The corresponding gain is realized by simultaneously transmitting independent data streams over the multiple antennas in the same frequency band.

Diversity is defined as the negative exponent of the signal-to-noise ratio, ρ , in the *probability of error* expression for high values of ρ . Diversity can be considered as the redundancy, or maximum number of independent copies of the transmitted signal in a particular communication system. It may be obtained by transmitting the data signal over multiple independently fading paths in time, frequency and/or space and performing proper combining at the receiver [30]. Diversity mitigates fading and thus significantly improves link quality.

Array gain is defined as the gain in the signal-to-noise-ratio via coherent combining of received information streams. This requires knowledge of the channel state information (CSI) at the part of the communication link where this gain is observed. Array gain provides improved coverage of the wireless system [3, 30].

In this chapter, we first provide information theoretic analyses of receiver diversity (SIMO) and transmit diversity (MISO). We discuss the observed degrees of freedom, diversity gain and array gain for these scenarios.

Using multiple antennas at both the transmitter and the receiver potentially provide additional degrees of freedom, diversity gain and array gain. We investigate the capacity of MIMO channels at both low and high SNR regimes.

Another spatial diversity technique that provide additional degrees of freedom to the whole system in multiple user scenarios is *multi-user diversity*. As the number of users in the system increases, at a given time instance, the probability that one of the users observes a very good channel condition becomes higher. A system exploiting this characteristic may achieve significantly higher spectrum efficiency. In fact, it has been shown that allowing transmission to/from only one user that has the best channel conditions at a given time achieves the system capacity for a frequency flat fading channel [16]. One other form of

spatial diversity through utilization of the multiple users in the system is *cooperative diversity*. Here, transmission between two users is aided through the relaying of carefully selected partners [24, 25] in an ad-hoc network setting. Both multi-user diversity and cooperative diversity are overviewed in this chapter.

2.1 Antenna Diversity

The utilization of time or frequency diversity incurs an expense of time or bandwidth, respectively. Thus, spatial diversity is an attractive alternative that provides an increased average received SNR without sacrificing time or bandwidth. However, traditional spatial diversity techniques require multiple antennas at the receiver and/or transmitter and corresponding RF and baseband electronics, thereby increasing the transceiver complexity. Recently, exploiting multiple antennas at both the transmitter and the receiver (MIMO systems) has attracted much interest since it has been shown that such systems provide additional degrees of freedom, diversity gain and array gain [12, 13].

Let us now investigate receiver (SIMO), transmitter (MISO) and transmitter-receiver (MIMO) diversity techniques and derive the relevant receiver SNR values and associated system capacities. Fig.2.1 depicts all of the afore mentioned techniques.

2.1.1 Receiver Diversity (SIMO Systems)

A communication system operating in a frequency-flat fading channel using 1 transmit and N receive antennas can be said to have a channel vector given by

$$\mathbf{h} = [h_1 \ h_2 \ \cdots \ h_N]^T \quad (2.1)$$

where all of the channel coefficients are modeled to be independent identically distributed (i.i.d.) random variables with $\mathcal{E}[|h_i|^2] = 1$ for $i = \{1, 2, \dots, N\}$. Also, we assume that the transmit symbol x is drawn from a scalar constellation with $\mathcal{E}[|x|^2] = E_s$. Then, the output of the channel can be expressed as

$$\mathbf{y} = \mathbf{h} x + \mathbf{n}, \quad (2.2)$$

where $\mathbf{y}$ is the received signal vector of size $N \times 1$ and $\mathbf{n}$ is the zero mean circularly symmetric complex gaussian (CSCG) noise with $\mathcal{E}[\mathbf{n}\mathbf{n}^*] = \sigma_n^2 \mathbf{I}_N$. Then, the ergodic capacity of this

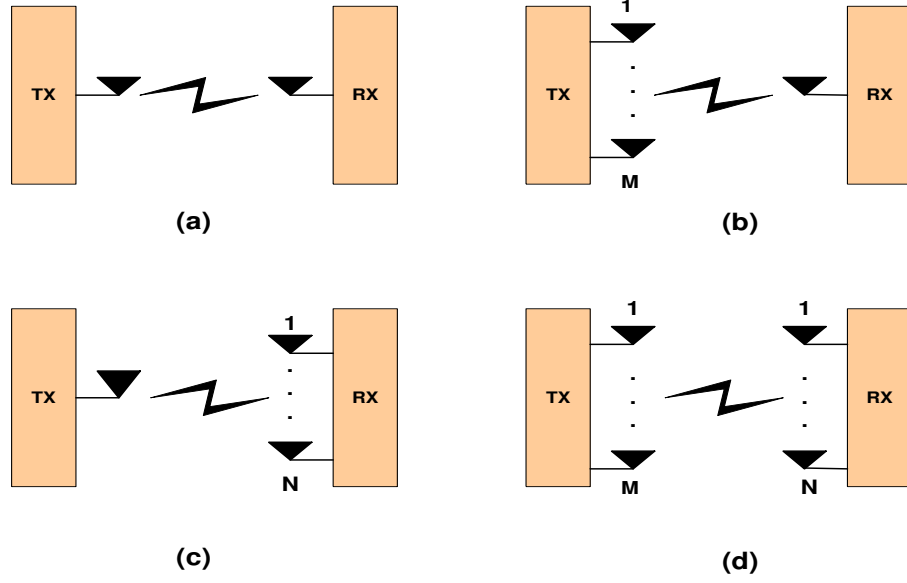


Figure 2.1: Wireless channel models: (a) SISO channel, (b) MISO channel, (c) SIMO channel and (d) MIMO channel

system can be calculated as:

$$\begin{aligned}
 C_{simo} &\leq \max_{p(x)} I(x; \mathbf{y}) \\
 &\stackrel{(a)}{=} \mathcal{E}_{\mathbf{h}} [I(x; \mathbf{y} | \mathbf{h})] \\
 &= \mathcal{E}_{\mathbf{h}} \left[\log \left(\mathbf{I}_N + \frac{E_s}{\sigma_n^2} \mathbf{h} \mathbf{h}^H \right) \right] \\
 &\stackrel{(b)}{=} \mathcal{E}_{\mathbf{h}} \left[\log \left(1 + \frac{E_s}{\sigma_n^2} \mathbf{h}^H \mathbf{h} \right) \right] \\
 &= \mathcal{E}_{\mathbf{h}} \left[\log \left(1 + \frac{E_s}{\sigma_n^2} \|\mathbf{h}\|^2 \right) \right]
 \end{aligned} \tag{2.3}$$

where equality (a) is thanks to the fact that the circularly symmetric complex gaussian input maximizes the mutual information and equality (b) is true for all proper matrix dimensions where $\det(\mathbf{I}_N + \mathbf{A}\mathbf{B}) = \det(\mathbf{I}_M + \mathbf{B}\mathbf{A})$.

This capacity calculation inherently assumes coherent detection at the receiver with matched filters. Now, we investigate different receiver diversity combining techniques and compare each with maximum achievable rate of the SIMO system derived in (2.3). We perform capacity calculations for Maximum Ratio Combining (MRC) which is the optimum diversity combining technique, Equal Gain Combining (EGC) and Selection Combining (SC).

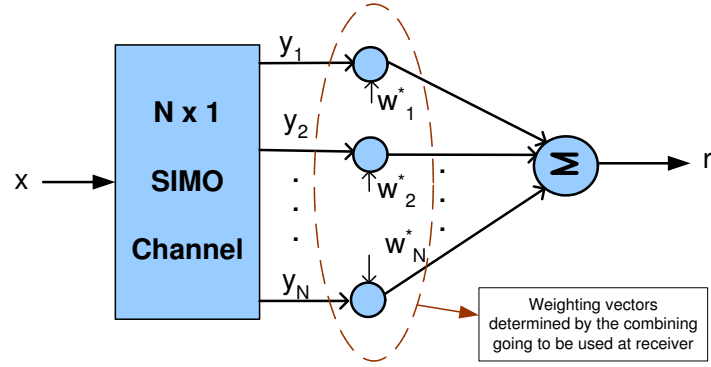


Figure 2.2: SIMO system with generalized post-detection diversity combining scheme

Combining Techniques

There have been numerous methods proposed in the literature for combining the signals that are received on the disparate diversity branches, and several ways of categorizing these methods [9]. In this thesis, we focus our attention on **post-detection diversity combining** techniques that take place at the baseband. We assume that all of the spatial branches are uncorrelated. We discuss Maximum-Ratio Combining (MRC), Equal Gain Combining (EGC) and Selection Combining (SC). We make use of the generalized MIMO system and derive the instantaneous SNR and average SNR values post-combining, and then derive the capacity expressions. A general framework for these diversity techniques is depicted in Fig.2.3.

The general input-output relation of the SIMO system is given in Eq.2.2. Post detection at the receiver, the received signal vector is multiplied by a weighting matrix with entry values based on the specific diversity combining technique in use. Then, we can express the signal at the output of the diversity combiner as follows:

$$\begin{aligned}
 r &= \mathbf{w}^H \mathbf{y} \\
 &= [w_1^* \ w_2^* \ \dots \ w_N^*] \mathbf{y} \\
 &= \sum_{i=1}^N w_i^* y_i.
 \end{aligned} \tag{2.4}$$

Now let us investigate at the behaviour of (2.4) when MRC, EGC and SC are used and derive the achievable capacities for these schemes.

For MRC, the weighting vector is selected proportional to channel vector, $\mathbf{w} = \alpha \mathbf{h}$. As

will be proved shortly, this combining method is optimal, maximizing the system capacity. It requires and uses both channel phase and gains. The corresponding combined output is given by

$$\begin{aligned} r &= \mathbf{w}^H \mathbf{y} \\ &= \alpha \mathbf{h}^H (\mathbf{h}x + \mathbf{n}) \\ &= \alpha \|\mathbf{h}\|^2 x + \alpha \mathbf{h}^H \mathbf{n}. \end{aligned} \quad (2.5)$$

Then, the corresponding SNR is given by

$$\gamma_{mrc} = \frac{\|\mathbf{h}\|^2 E_s}{\sigma_n^2} \quad (2.6)$$

which is independent of α . So the ergodic capacity corresponding to MRC is given by

$$C_{mrc} = \mathcal{E}_h \left[\log \left(1 + \frac{\|\mathbf{h}\|^2 E_s}{\sigma_n^2} \right) \right]. \quad (2.7)$$

From (2.6) and (2.7), we observe that MRC provides both an N order diversity gain and an array gain of $10 \log_{10}(N)$ compared to the SISO channel. We also will observe shortly that it achieves the capacity of the general SIMO channel.

For EGC, the combining weights are all set to be equal and unit gain. In general, the EGC weight selections are as follows:

$$\mathbf{w} = e^{j\angle \mathbf{h}}. \quad (2.8)$$

The corresponding EGC post-combining signal is thus given by

$$\begin{aligned} r &= \mathbf{w}^H \mathbf{y} \\ &= e^{-j\angle \mathbf{h}^T} (\mathbf{h} x + \mathbf{n}) \\ &= \left(\sum_{i=1}^N |h_i| \right) x + e^{-j\angle \mathbf{h}^T} \mathbf{n}. \end{aligned} \quad (2.9)$$

Then, the corresponding SNR is given by

$$\gamma_{egc} = \frac{\left(\sum_{i=1}^N |h_i| \right)^2 E_s}{N \sigma_n^2}. \quad (2.10)$$

So, the ergodic capacity corresponding to EGC is given by

$$C_{egc} = \mathcal{E}_h \left[\log \left(1 + \frac{\left(\sum_{i=1}^N |h_i| \right)^2 E_s}{N \sigma_n^2} \right) \right]. \quad (2.11)$$

From (2.10), we can see that the EGC scheme provides an N order diversity and an array gain of $10 \log_{10}(1 + \pi(N - 1)/4)$ [9], which is less than the array gain provided by MRC. Hence, EGC scheme can not achieve the general SIMO capacity, and thus is sub-optimal.

For SC, the diversity branch with the greatest SNR is chosen for further processing and the other branches are all discarded. Weights in Fig.2.3 are set in the following fashion:

$$w_i = \begin{cases} 1 & i|\gamma_i = \max_n \{\gamma_n\}, \quad n = \{1, 2, \dots, N\} \\ 0 & \text{otherwise} \end{cases} \quad (2.12)$$

Since the element chosen is the one with the maximum SNR, the output SNR of the selection diversity scheme is $\gamma_{sc} = \max_n \{\gamma_n\}$. Such a scheme would need only a measurement of signal power, phase shifters or channel gains are not required. We can show that the selection combiner output average SNR is in the following form [9],

$$\gamma_{sc}^{av} = \mathcal{E}[\gamma_{sc}] = \frac{E_s}{\sigma_n^2} \sum_{n=1}^N \frac{1}{n}. \quad (2.13)$$

The ergodic capacity for SC scheme is found as follows

$$\begin{aligned} C_{sc} &= \mathcal{E}_h \left[\log \left(1 + \max_n \{\gamma_n\} \right) \right] \\ &= \mathcal{E}_h \left[\log \left(1 + \max_n \left\{ \frac{E_s}{N_0} |h_n|^2 \right\} \right) \right]. \end{aligned} \quad (2.14)$$

In [9], it is shown that SC, like MRC and EGC, provides an N -order diversity gain and from (2.13), we can see that SC provides an array gain of $10 \log_{10}(\sum_{n=1}^N \frac{1}{n})$ which is less than both of the array gains provided by MRC and EGC. Hence, SC scheme can not achieve the general SIMO capacity and is sub-optimal.

In Fig.2.3-(a), the derived ergodic capacities for the three SIMO combining techniques (MRC, EGC and SC) are graphed for $M = 1$ and $N = 2, 4$. As a benchmark, the capacity of the SISO channel is also graphed. It can be seen that since all SIMO systems have only a single degree of freedom, they only enjoy array gains over the SISO channel in terms of capacity. The performance degradation in EGC and SC schemes are due to not utilizing the full CSI at receiver. We see that MRC is capable of extracting the full N order *diversity gain* and *array gain*. In Fig.2.3-(b), the outage probabilities of SISO, MISO, MIMO and SIMO (with combining techniques of MRC, EGC and SC at the receiver) for a transmission rate of $R = 2$ [bit/sec/Hz] with $M = 1, 2$ and $N = 1, 2$ are graphed. It can be seen that

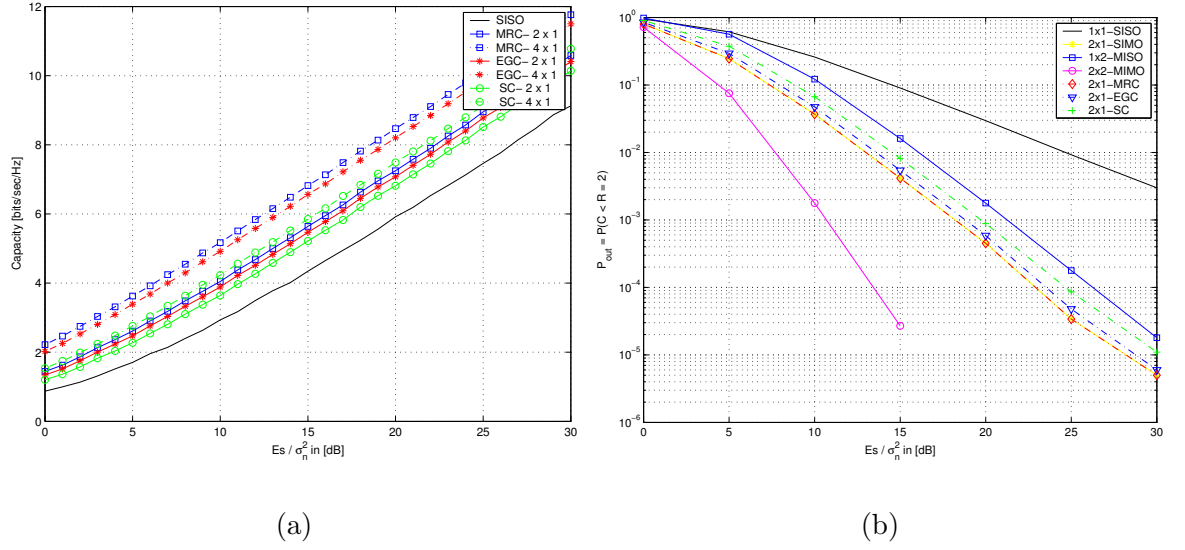


Figure 2.3: (a) Ergodic capacities of MRC, EGC and SC schemes for $M = 1$ and $N = 2, 4$; (b) Outage probabilities of SISO, MISO, MIMO and SIMO (MRC, EGC and SC at receiver) for a transmission rate of $R = 2$ [bit/sec/Hz] with $M = 1, 2$ and $N = 1, 2$.

a second order diversity is achieved for the 2×1 channel. Moreover, it can be seen that 2×2 MIMO channel has fourth order diversity potential if only a single degree of freedom is utilized.

The performance improvement for SIMO systems is proportional to the number of antennas at the receiver. However, equipping receiver terminals with multiple antennas is not often feasible due to cost and space limitations. In these cases, multiple antennas may be furnished at the transmitter and associated transmit diversity techniques may be used. Now, we let us investigate the performance of MISO systems.

2.1.2 Transmit Diversity and Beamforming (MISO Systems)

In this section, we consider a system equipped with M transmitter and 1 receiver antennas (MISO system). To exploit transmit antenna diversity, the transmitter requires to send pre-coded or pre-processed signal components from the antennas. The need for pre-coding is due to the reason that simply transmitting the same symbol over all antennas at the same time does not provide any *diversity gain* at the receiver [28, 29]. Repetition coding may be used, where over a course of M time instants, only one of the transmit antennas is used at each time instant; but this usage of system resources is not efficient due to the transmission

rate decrease by a factor of M .

We represent the communication channel for the general MISO system as

$$y = \mathbf{h}^T \mathbf{x} + n \quad (2.15)$$

where $\mathbf{h} \in \mathbf{C}^{M \times 1}$ channel vector, $\mathbf{x} \in \mathbf{C}^{M \times 1}$ transmit signal vector with $\mathcal{E}[\mathbf{x}\mathbf{x}^H] = (E_s/M)\mathbf{I}_M$ and n is zero mean CSCG noise term with σ_n^2 variance at receiver end. The ergodic capacity of this channel is given by [13]

$$\begin{aligned} C_{miso} &= \max_{p(\mathbf{x})} I(\mathbf{x}; y) \\ &= \mathcal{E}_{\mathbf{h}}[I(\mathbf{x}; y|\mathbf{h})] \\ &= \mathcal{E}_{\mathbf{h}} \left[\log \left(1 + \frac{E_s}{M\sigma_n^2} \|\mathbf{h}\|^2 \right) \right]. \end{aligned} \quad (2.16)$$

Compared to SIMO capacity, we observe that without CSI at the transmitter, the MISO system capacity is less due to the absence of transmitter array gain. But with CSI information available at the transmitter, all of the available power can be directed to a single path (beamforming) or MRT, and consequently the MISO system capacity becomes equal to that of the SIMO system given in equation (2.3).

We consider two cases of transmit diversity systems in this thesis: with CSI available at the transmitter, transmit beamforming or transmit-MRC [31]; and with CSI unavailable at the transmitter, Orthogonal Transmit Diversity(OTD) or Space-Time Coding (STC) [5,7,32]. Throughout the thesis, we will refer to the transmit-MRC, namely Maximum Ratio Transmission (MRT) [31], as transmit beamforming where we all channel coefficients as assumed to be i.i.d. RVs with the appropriate level of separation between transmit antennas guaranteeing independence. When the antenna array elements are located closer than a distance of $\lambda/10$, the channel coefficients become correlated. In this case, *transmit array gain* may be achieved at average received *SNR* using conventional beamforming [30,32].

Transmit Diversity

To achieve transmit diversity without any CSI knowledge at the transmitter, one should perform pre-processing and pre-coding of information streams at the transmitter. Recently, it has been shown in [5] that a certain class of spatio-temporal codes provide full diversity and achieve higher data rates than simple repetition based codes. Such codes are called

Space-Time Block Codes (STBC) which is a subset of the general Space-Time Codes (STC). These codes require simple receiver and transmitter structures. While Alamouti in [5] has considered only 2 transmit antennas for STBCs, generalized STBC structures have also been derived in [10].

In a system employing orthogonal space time block codes, each $L \leq T$ complex input symbols $\{s_1, s_2, \dots, s_L\}$, belonging to a given set $\mathcal{S}$ are mapped into M orthogonal sequences of length T and are simultaneously transmitted through the M transmit antennas [33]. Since L input symbols from the signal set $\mathcal{S}$ are transmitted within T symbol durations (a frame period), the information code rate of the STBC is defined as $R_c = L/T$. With this set up, the ergodic capacity of general MIMO STBC system is given by [33, 34]

$$C_{stbc} = \mathcal{E}_{\mathbf{H}} \left[R_c \log \left(1 + \frac{E_s}{R_c M \sigma_n^2} \|\mathbf{H}\|_F^2 \right) \right]. \quad (2.17)$$

where $\mathbf{H} \in \mathbf{C}^{N \times M}$ is general MIMO channel with each component i.i.d. complex circular Gaussian RVs with the distribution, $\mathcal{CN}(0, 1)$. By decoupling the signals transmitted by different antennas, STBC converts the matrix channel into a scalar channel and consequently the second term inside the logarithm in (2.17) represents the effective SNR per symbol at the output of the STBC decoder. Hence, it can be seen that with STBC no multiplexing gain but a diversity gain of MN , and only a receiver array gain of N with a coding gain of $1/R_c$ are achieved. The STBC scheme achieves the general MISO channel capacity given by (2.16) using $M = 2$ and $N = 1$ antennas at the transmitter and the receiver, respectively.

Transmit Beamforming

Throughout the thesis, we use the term transmit beamforming to refer to a system that is the transmitter equivalent of the MRC scheme employed at the receiver. Naturally, unlike conventional beamforming, the transmit beamforming scheme does not have the means for steering the antenna array beams to a specific direction so as to boost the mean SNR at the receiver located at that direction. Our concerned beamforming is like MRT scheme considered in [31] and we assume channel coefficients are i.i.d. and the beam-steering (conventional beamforming) does not work well under this assumption [35].

Consider a MISO system with M transmit and 1 receiver antennas and assume full CSI knowledge at the transmitter. To exploit transmit diversity, the signal is transmitted from

each transmit antenna after having been weighted appropriately so that the signals arrive in phase at the receiver antenna and can be added coherently. In this case, the signal at receiver antenna is given by

$$y = \mathbf{h}^T \mathbf{w} s + n \quad (2.18)$$

where y is the received signal, $\mathbf{h} \in \mathbf{C}^{M \times 1}$ is channel coefficient vector, $\mathbf{w} \in \mathbf{C}^{M \times 1}$ is the weighting vector, s is the transmitted data with $\mathcal{E}[|s|^2] = E_s$ and n is the zero mean CSCG noise. The weight vector must be chosen so that $\|\mathbf{w}\|^2 = 1$ to ensure that the average transmitted total energy is E_s .

The weighting vector that maximizes the SNR at the receiver is given by

$$\mathbf{w} = \frac{\mathbf{h}^*}{\|\mathbf{h}\|}. \quad (2.19)$$

This scheme is known as transmit-MRC [31], referred to in this thesis as *transmit beamforming*. The SNR at the receiver is given by

$$\gamma = \|\mathbf{h}\|^2 \rho. \quad (2.20)$$

where $\rho = E_s/\sigma_n^2$. We observe that average SNR, $\mathcal{E}[\gamma] = M\rho$, is improved by a factor of M when compared to the SISO scheme. It can be concluded that when channel knowledge is available at the transmitter, transmit beamforming will provide both *array gain* and *diversity gain*.

We observe an increase in the system capacity due to the achieved *array gain*. Since, both MISO and SIMO schemes have only a single *degree of freedom* (in both of the cases, the received signal is 1-D) there is no additional capacity increase relative to the SISO fading channel for the same average received SNR. To observe this behavior, we can observe the single receiver antenna system as a function of the number of transmit antennas. It can be seen that for a specific SNR value, the system capacity saturates which means that we do not gain in capacity anymore with increasing the number of receiver antennas. This is due to the single degree of freedom. To have additional capacity increase we should introduce multiple degrees of freedom to the channel which is accomplished by adding more antennas at both transmitter and receiver.

As seen in Fig.2.4(a), we observe that the transmit beamforming (TB) scheme provides an additional array gain over the conventional MISO scheme with no CSI knowledge at the

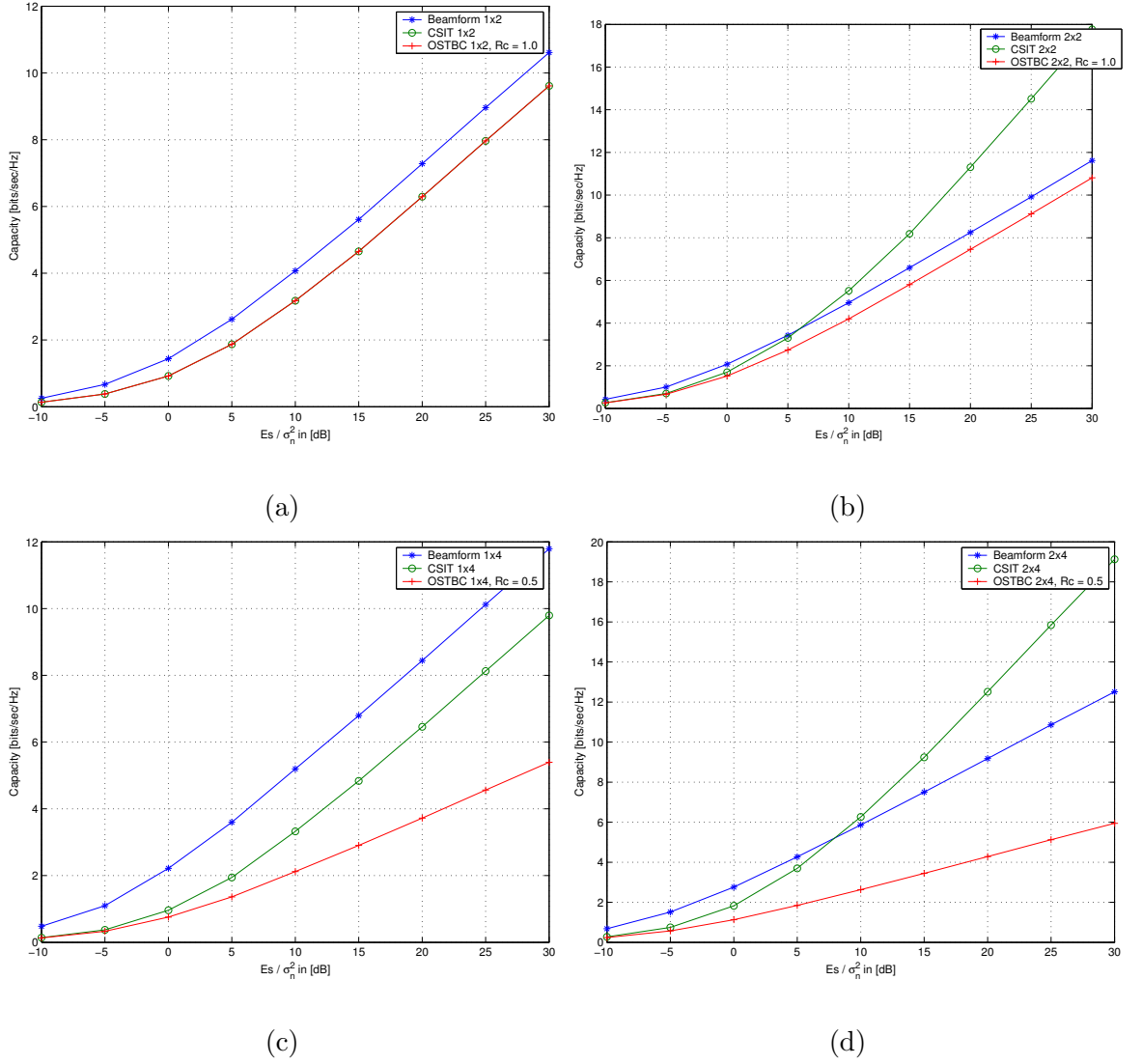


Figure 2.4: Ergodic capacities of 1×2 and 1×4 MISO as well as 2×2 and 2×4 MIMO with no CSI knowledge at the transmitter, transmit beamforming, Orthogonal STBC (OSTBC) with the coding rate R_c . (a) $M = 2$ and $N = 1$ and $R_c = 1$; (b) $M = 2$ and $N = 2$ and $R_c = 1$; (c) $M = 4$ and $N = 1$ and $R_c = 1/2$; (d) $M = 4$ and $N = 2$ and $R_c = 1/2$.

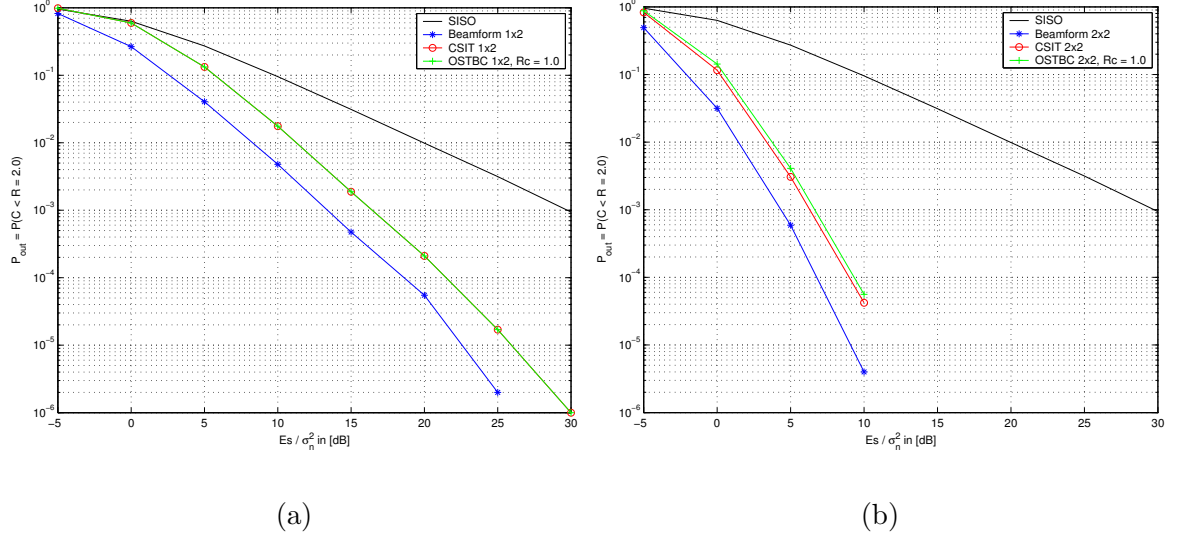


Figure 2.5: Outage probabilities of SISO channel, MISO channel with no CSI at transmitter, Transmit Beamforming, Orthogonal STBC with coding rate R_c . (a) $M = 2$ and $N = 1$ and $R_c = 1$; (b) $M = 2$ and $N = 2$ and $R_c = 1$. Results are given as functions of E_s / σ_n^2 .

transmitter as well as the system employing OSTBC for the 1×2 channel. As observed in Fig.2.4(c), as the coding rate is halved, $R_c = 1/2$, OSTBC results in a multiplexing loss when compared to the conventional MISO scheme. In Fig.2.4(b)-(d), since the TB scheme utilizes a single degree of freedom while there is 2 degrees of freedom available for the MISO scheme at high SNR regimes, the TB scheme can not perform as well; but at low SNR values, the power distribution will be more sensitive to the channel variations resulting in better performance than the MISO scheme. In Fig.2.4(b)-(d), we observe that the OSTBC scheme has less transmit diversity gain and multiplexing gain when compared to the MIMO schemes due to the availability of multiple degrees of freedom available to the MIMO schemes.

In Fig.2.5(a), it can be seen that when compared to the SISO scheme both the MISO and the OSTBC schemes provide a diversity order of 2 for the 1×2 channel while for this channel the TB scheme provides an additional transmit array gain. In Fig.2.5(b), we observe that for the 2×2 channel, all techniques provide a 4th order diversity and the TB scheme provides an additional array gain over other schemes. In Fig.2.5 we consider the system with a transmission rate of $R = 2$ [bits/sec/Hz].

2.1.3 Transmit & Receiver Diversity (MIMO Systems)

In point-to-point wireless channels, MIMO systems improve spectral efficiency through spatial *multiplexing gain*, link reliability through *diversity gain* and coverage through *array gain* [3, 12, 13, 30].

The general MIMO channel model is given by the following equation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (2.21)$$

where $\mathbf{x} \in \mathbf{C}^{M \times 1}$, the input information vector, has a covariance matrix of $\mathbf{Q}_\mathbf{x} = \mathcal{E}[\mathbf{x}\mathbf{x}^H] = (E_s/M)\mathbf{I}_M$, $\mathbf{H} \in \mathbf{C}^{N \times M}$ is the channel matrix with i.i.d. complex Gaussian entries with variance 1 and $\mathbf{n} \in \mathbf{C}^{N \times 1}$ is the zero mean CSCG noise vector with the distribution $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$.

To assess the *multiplexing gain* provided by the MIMO system, we derive the capacity expression. The capacity of the MIMO scheme with the CSI knowledge available only at the receiver end is given by [13]

$$C_{mimo} = \mathcal{E}_\mathbf{H} \left[\log \det \left(\mathbf{I}_N + \frac{E_s}{M\sigma_n^2} \mathbf{H}\mathbf{H}^H \right) \right]. \quad (2.22)$$

Given the equality $\mathbf{H}\mathbf{H}^H = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$, (2.22) can be expressed as

$$C_{mimo} = \mathcal{E}_\mathbf{H} \left[\log \det \left(\mathbf{I}_N + \frac{E_s}{M\sigma_n^2} \mathbf{V}\mathbf{\Lambda}\mathbf{V}^H \right) \right]. \quad (2.23)$$

Making use of the identities $\det(\mathbf{I}_m + \mathbf{A}\mathbf{B}) = \det(\mathbf{I}_n + \mathbf{B}\mathbf{A})$ for matrices $\mathbf{A}$ ($m \times n$) and $\mathbf{B}$ ($n \times m$) and $\mathbf{V}^H\mathbf{V} = \mathbf{I}_N$, (2.23) simplifies to

$$C_{mimo} = \mathcal{E}_\mathbf{H} \left[\log \det \left(\mathbf{I}_N + \frac{E_s}{M\sigma_n^2} \mathbf{\Lambda} \right) \right] \quad (2.24)$$

or equivalently

$$C_{mimo} = \mathcal{E}_\mathbf{H} \left[\sum_{i=1}^r \log \left(1 + \frac{E_s}{M\sigma_n^2} \lambda_i \right) \right], \quad (2.25)$$

where r is the *rank of the channel* and λ_i for $i = 1, 2, \dots, r$ are the eigen-values of $\mathbf{H}\mathbf{H}^H$. Equation (2.25) expresses the capacity of the MIMO channel as the sum of the capacities of r SISO channels, each having a power gain of $\sqrt{\lambda_i}$ for $i = 1, 2, \dots, r$ and transmit power of E_s/M . Note that inferring from this result, orthogonal MIMO channels maximize capacity.

Next, we derive the capacity expression for the MIMO channel when CSI is available also at the transmitter. In this case, the capacity of the MIMO channel is the sum of individual

parallel SISO channel capacities and is given by

$$C = \mathcal{E}_{\mathbf{H}} \left[\sum_{i=1}^r \log \left(1 + \frac{E_i}{\sigma_n^2} \lambda_i \right) \right], \quad (2.26)$$

where E_i is the power of the signal transmitted from the i th sub-channel such that $\sum_{i=1}^r E_i = E_s$.

As the transmitter has perfect channel knowledge, it can allocate the available power to the individual sub-channels so that the system capacity is maximized. In this case, the capacity maximization problem can be formulated as

$$C = \mathcal{E}_{\mathbf{H}} \left[\max_{\sum_{i=1}^r E_i = E_s} \sum_{i=1}^r \log \left(1 + \frac{E_i}{\sigma_n^2} \lambda_i \right) \right]. \quad (2.27)$$

To maximize the mutual information under the power constraint, the Lagrangian constrained optimization method is used. Then, the optimal energy allocation policy should satisfy

$$\begin{aligned} E'_i &= \left(\mu - \frac{\sigma_n^2}{\lambda_i} \right)_+, \quad i = 1, 2, \dots, r, \\ \sum_{i=1}^r E'_i &= E_s, \end{aligned} \quad (2.28)$$

where μ is a constant and $(x)_+ = \max\{0, x\}$. The optimal energy allocation is found iteratively through the *Water-filling Algorithm* [13, 27]. Using (2.25) and (2.28) we get

$$C = \mathcal{E}_{\mathbf{H}} \left[\sum_{i=1}^r \log \left(1 + \frac{E'_i}{\sigma_n^2} \lambda_i \right) \right]. \quad (2.29)$$

The capacity of the MIMO channel with perfect CSI available at the transmitter should obviously be greater than (or equal to) the capacity when the CSI is unknown to the transmitter. To assess the multiplexing gain provided by MIMO system, let us consider the system where $M = N = L$ and $\mathbf{H}$ with i.i.d. components. Using the strong law of large numbers [36], we see that

$$\frac{1}{L} \mathbf{H} \mathbf{H}^H \rightarrow \mathbf{I}_L \quad \text{as } L \rightarrow \infty. \quad (2.30)$$

Hence, the capacity of this channel in the absence of CSI at transmitter approaches

$$C_{mimo} = L \log \left(1 + \frac{E_s}{\sigma_n^2} \right), \quad (2.31)$$

which shows that asymptotic in L , the capacity of the spatially white MIMO channel becomes deterministic and is proportional to L for a fixed SNR. For the general case, it

may be shown that a multiplexing gain of $\min\{M, N\}$ is achievable by the $N \times M$ MIMO channels [13, 29].

It has been shown that with no CSI knowledge available at the transmitter, using orthogonal STBCs provide a diversity order of MN with the cost of data and link capacity decrease. Recall that we have stated in Sec.2.1.2 orthogonal STBC converts a matrix channel into a scalar channel for each of the data streams.

When the channel is known to the transmitter, spatial diversity can be extracted using a technique known as *dominant eigen-mode* transmission or *beamforming* [31], [6, 29]. The transmission weight and the receiver combining vectors are chosen to be proportional to the eigenvector corresponding to the maximum eigenvalue of $\mathbf{H}^H \mathbf{H}$. With the appropriate choice of the weight vectors, the effective input-output relation for the channel reduces to

$$z = \sqrt{\lambda_{max}} s + n, \quad (2.32)$$

where n is the zero mean CSCG noise with variance σ_n^2 and λ_{max} is the maximum eigenvalue of $\mathbf{H}^H \mathbf{H}$. The receiver weight vectors that maximize the SNR are the eigenvectors that correspond to maximum eigenvalue of $\mathbf{H}^H \mathbf{H}$. The average received SNR, $\mathcal{E}[\gamma]$, can be upper and lower bounded as follows [29], [31]

$$\rho \max\{M, N\} \leq \mathcal{E}[\gamma] \leq \rho MN, \quad (2.33)$$

where $\rho = E_s/\sigma_n^2$. Equation (2.33) implies that array gain when channel is known to transmitter is greater than or equal to the array gain when the channel is unknown. Both systems provide a *full* diversity gain that is equal to MN .

In Fig.2.6, the MIMO system ergodic capacities are plotted for different number of transmit and receive antennas with and without CSI knowledge at the transmitter. We observe that at high SNR values, spreading the available transmit power equally over all antennas is near optimal. On the other hand, for low SNR values, water-filling over channel directions based on the CSI knowledge provides a capacity increase when compared to the MIMO transmission with no CSI knowledge at the transmitter.

At the low SNR regime, the near optimal policy is to allocate power only to the strongest eigen-mode of the channel. This can be seen from the water-filling levels; when SNR is low, channel fluctuations will be sensed more deeply and the near optimal power allocation scheme is to direct all of the power on the strongest eigen-mode of the channel [28].

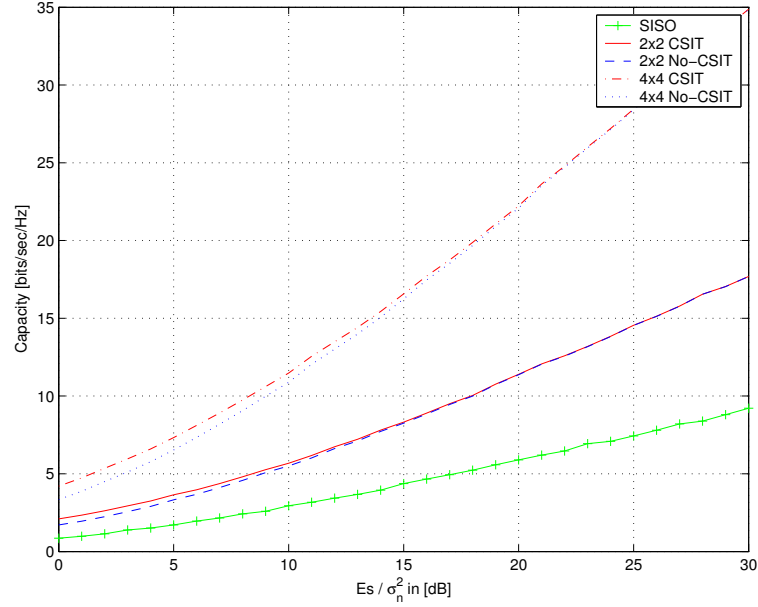


Figure 2.6: 2×2 and 4×4 MIMO system ergodic capacities with and without CSI at transmitter

Discussion on MIMO Beamforming and STBC

In the STBC scheme, the transmitter splits the available transmit power into equal parts among the antennas. This is because, there is no CSI knowledge available at the transmitter. The STBC scheme can not achieve the MIMO channel capacity since all available degrees of freedom cannot be utilized due to the lack of CSI knowledge at the transmitter. Specifically, the STBC scheme utilizes only a single degree of freedom since at the receiver we can partition all of the transmitted signal into a sub-channel in which there is no interference. This is identical to the system where a single symbol is transmitted in a symbol period in an AWGN channel with constant channel gain. So there is no utilization of added spatial degrees.

When beamforming is employed, all of the transmit power is directed to the best eigen-channel. At low SNR values the distribution of the transmit power will be more sensitive to channel variations, so we can gain more by exploiting the opportunistic power distribution. On the other hand, when the SNR values are high, the power distribution will not be as sensitive to the channel variations since the channel is generally good all over. Then, the system performance when transmit power is distributed equally among the transmit anten-

nas becomes near optimal. Also, in MIMO systems, beamforming does not use all of the degrees of freedom, truly but rather uses only one degree of freedom of the channel. This is because, beamforming is one dimensional (1-D) coding. Because of this, at intermediate and high SNR regimes beamforming can not achieve the true MIMO channel capacity.

2.2 Multiuser Diversity

Multiuser diversity is a form of diversity provided by the independent, time-varying channels observed by the different users in the wireless network. The diversity benefit is exploited by tracking the channel fluctuations of all users and scheduling transmissions to users only when their instantaneous channel quality is near the peak. Compared to the point-to-point setting, the multiuser settings offer more opportunities to exploit. In addition to the choice of *when* to transmit, there is now an additional choice of *which* user(s) to transmit from (in the uplink) or to transmit to (in the downlink) and the *amount* of power to allocate between the users [28]. These additional dimensions provide further performance gains not available to the point-to-point scenario. The diversity gain increases with the dynamic range of the fluctuations and is thus limited in environments with little scattering and/or slow fading [37]. The multiuser diversity is originally motivated by the information theoretic result of Knopp and Humblet in [16]. In the paper, the authors focus on the uplink of a single, isolated cell, with multiple users communicating to the base station via time-varying flat-fading channels. All of these channels are assumed to be perfectly and instantaneously tracked by the base station and this information is assumed to be fed back to the transmitters. To maximize the total information-theoretic capacity, the authors show that the optimal strategy is to schedule at any one time *only* the user with the best channel to transmit to the base station. Diversity gain arises from the fact that in a system with many users, whose channels vary independently, there is likely to be a user whose channel is near its peak at any one time. The scheme can also be thought of as a form of selection diversity.

The increase in the sum capacity comes from a *multiuser diversity effect*: when there are many users which fade independently, at any one time there is a high probability that one of the users will have a strong channel. By allowing only that user to transmit, the shared channel resource is used in the most efficient manner and the total system throughput is maximized. The larger the number of users, the more likely it is to have the best channel

experience an even higher peak, and thus the more the *multiuser diversity gain*.

The concept of multiuser diversity in [16] builds upon the work presented in [38]. Here, the optimal power control strategy is depicted along with adaptive modulation and coding whereby the users are allocated more power when their channels are good, and less when they are bad. When higher powers are allocated, users send more information by using higher order modulation. In this case, the channel model may be given as

$$r = \sum_{i=1}^K h_i x_i + n \quad (2.34)$$

where K is the number of users in the cell, h_i , and x_i are the channel gain and information for the i th user. We assume that the information sources x_i , are zero-mean, have unit energy and are mutually uncorrelated. The noise n is modeled as a zero mean CSCG random variable with variance σ_n^2 . If the h_i , are deterministic constants, this is simply a *Gaussian multiuser channel* whose capacity region is defined by the following set of equations

$$\begin{aligned} \forall S &\in \{1, 2, \dots, K\} \\ \sum_{i \in S} R_i &< \log \left(1 + \sum_{i \in S} \gamma_i \right) \end{aligned} \quad (2.35)$$

where R_i and $\gamma_i = \frac{h_i^2 E_s}{\sigma_n^2}$ are the information rate and received SNR for the i th user, respectively. In a fading environment the h_i and consequently the γ_i are modeled as random quantities. If we consider frequency-flat rayleigh fading, h_i is assumed to have a Rayleigh distribution, and in turn, the γ_i have the following exponential distribution

$$p(\gamma_i) = \begin{cases} \frac{1}{\gamma_i} \exp \left(-\frac{\gamma_i}{\gamma_{s_i}} \right), & \gamma_i > 0 \\ 0, & \text{otherwise} \end{cases} \quad (2.36)$$

where γ_{s_i} is the average received SNR for the i th user. Due to uncorrelated path losses between the users and the base station, these values will be different. A power control at users' sides can be done to ensure that the same received SNR, say γ_{s_i} is observed by each user user. Although this scheme is the perfect power control scheme over the Rayleigh fading channel, it assumes that the average transmit power can be infinitely large. Ignoring this practical limitation, the sum-of-rates capacity for this case is given by [16]

$$C_G = \log \left(1 + \sum_{i=1}^K \gamma_{s_i} \right) \quad (2.37)$$

which is simply the equation from the capacity region of a *Gaussian multiuser channel* (2.35) which contains the rates of all the users. For a system with *no power control*, the sum-of-rates capacity is given by

$$C_{npc} = \mathcal{E} \left[\log \left(1 + \sum_{i=1}^K \gamma_i \right) \right]_{\underline{\gamma}}. \quad (2.38)$$

This expression is simply an average of the sum-of-rates capacity for each possible realization of $\underline{\gamma} = [\gamma_1 \gamma_2 \dots \gamma_K]$. We remark that by Jensen's inequality $C_G \geq C_{npc}$. Furthermore, if we assure that $\gamma_{s_i} = \gamma_s$, $\forall i$, i.e. that we equalize the average received powers, and that the fading between users is independent, it is easily shown that $C_{npc} \rightarrow C_G$, when K becomes large. This is somewhat surprising, since it means that controlling the average power is sufficient from a sum-of-rates capacity standpoint, and that *perfect* power control will not yield a significant improvement [16].

Now, let us find a power control scheme under an average available power constraint law, expressed as

$$\mathcal{E} [S_i(\underline{\gamma})]_{\underline{\gamma}} = S \quad (2.39)$$

where $S_i(\underline{\gamma})$ is the transmit power for the i th user and S is the total average power constraint.

The problem is now to maximize

$$C_{pc} = \mathcal{E} \left[\log \left(1 + \sum_{i=1}^K \frac{S_i(\underline{\gamma}) \gamma_i}{S} \right) \right]_{\underline{\gamma}} \quad (2.40)$$

$$= \int \int \dots \int \log \left(1 + \sum_{i=1}^K \frac{S_i(\underline{\gamma}) \gamma_i}{S} \right) p(\underline{\gamma}) d\underline{\gamma}$$

subject to (2.39) and $S_i(\underline{\gamma}) \geq 0$, $\forall i$. Using the Lagrange multipliers, λ_i , corresponding to each constraint in (2.39), and using the convexity of the logarithm, we obtain the following power control law for user i [16]

$$\frac{S_i(\underline{\gamma})}{S} = \begin{cases} \frac{1}{\lambda_i} - \frac{1}{\gamma_i}, & \gamma_i > \lambda_i, \gamma_i > \frac{\lambda_i}{\lambda_j} \gamma_j, j \neq i \\ 0, & \text{otherwise.} \end{cases} \quad (2.41)$$

We see that a user is allocated more power when its channel is good and less when its channel is poor. This works in a completely opposite sense to the conventional power control and can

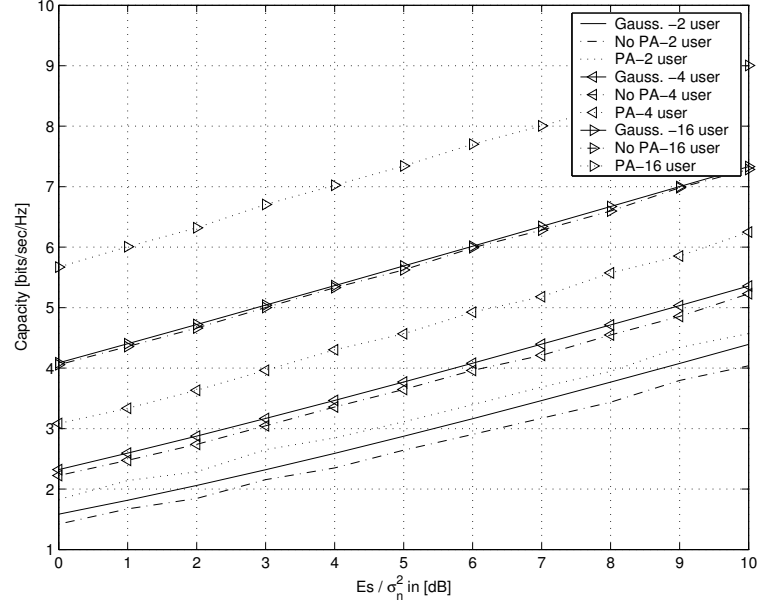


Figure 2.7: MU-SISO Gaussian and Rayleigh channel sum-of-rates for $K = \{2, 4, 16\}$ users with optimum Power Allocation (PA).

be interpreted as *water-filling in time*. The λ_i acts both as a cutoff SNR, below which a user may not transmit, and as a scaling factor used to determine the *strongest* user. Moreover, note that the peak power of the i th user is $1/\lambda_i$ [16]. This scenario can be seen as a multiple access scheme where time sharing is performed based on the instantaneous received power.

In Fig.2.7, we compare the average sum-of-rates for (2.37), (2.38) and (2.40) for the case of equal average received powers for $K = \{2, 4, 16\}$ users. We observe that as user numbers in the system increase, the multiuser gain increases, too.

In Fig.2.8, we plot the average sum-of-rates as a function of numbers of users for $E_s/\sigma_n^2 = 0$ dB where $E_{s_i} = E_s, \forall i \in \{1, 2, \dots, K\}$. As expected, we observe that allocation of all resources to only one user at a time provides significant gain over conventional uplink channels.

2.3 Cooperative Communications

The term *cooperative diversity* refers to the cooperative exploitation of the available spatial diversity in uncoordinated wireless networks. All users that are within the coverage area of the transmitted signal from a transmitter are able to detect the signal. In a wireless network,

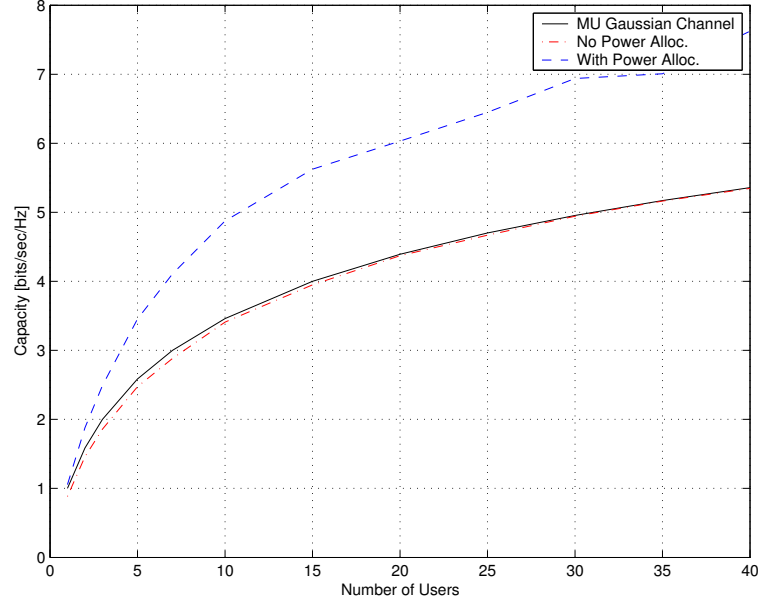


Figure 2.8: MU-SISO Gaussian and Rayleigh channel Sum-of-Rates for $E_{s_i} = E_s, \forall i \in \{1, 2, \dots, K\}$ and $E_s/\sigma_n^2 = 0$ dB with Optimum Power Allocation (PA).

this fact can be exploited by determining potential relay nodes to forward additional copies of the original source signal to the destination and thus create a virtual array of antennas. A simple such wireless scenario can be seen at Fig.2.9. As channel coefficients from the source and the relay to the destination nodes are independent, with proper processing and system resource usage, it is possible to achieve a second order diversity with this simple wireless network topology.

Cooperative communication can be divided into two categories depending on the relay processing. If the relay node only forwards its received signal from source node to destination node, such networks are called *Relay Networks* [17, 39]. If relay nodes also send their own data to the destination node with the assistance of the source node, such networks are called *Cooperative Networks* [24, 25].

The topic of cooperative communication is treated mainly from an information theoretic point of view in the literature and is generally limited to cases of one relay, for the sake of simplicity. Combiner design for receivers as well as performance results for cooperative diversity schemes using BPSK modulation can be found in [40]. Also, the influence of path loss and the relay's position on the performance of those protocols are shortly discussed in

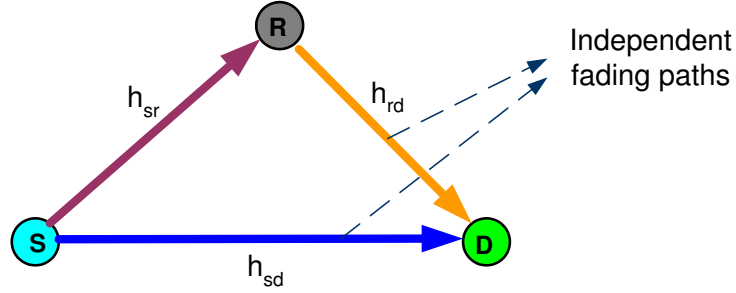


Figure 2.9: A relay network to exploit cooperative diversity with single source, single relay and single destination node

this work. Further information theoretic analysis can be found in [41]. Extensions to cases with a higher number of additional relays are treated in [42].

In this thesis, we focus on Relay Networks and main forwarding strategies for these networks. In this chapter we give some preliminary expressions for the considered forwarding strategies and compare results with point-to-point antenna diversity results in terms of ergodic capacity and outage probabilities (to observe the diversity order behaviors). Here, we only focus on the SISO single relay cooperation scheme to provide some basic understanding of these strategies. Chapters 3 and 4 provide novel results where multiple relays, all furnished with multiple antennas, are utilized.

2.3.1 System and Channel Model

Fig.2.9 provides the system model we use in this chapter to assess the performances of cooperative diversity utilizing various forwarding strategies. Additionally, the following system parameters are assumed. Every node in the system is identical. Source, relay and destination are all equipped with only a single antenna. The relay node only assists the source node transmission according to various relaying modes that are considered. The inability of the relay to simultaneously transmit and receive data on one frequency *orthogonality constraint* is taken into account. This can be done using either TDMA or FDMA. We also assume that the transmission period is divided into two consecutive time slots. In the first time slot, the source node broadcasts its data while relay and destination nodes listen for source transmission. In the second time slot, the source node stays silent and relay node assists the source transmission via a specific forwarding scheme. At the end of this period

the destination node combines the two received signals for proper decision. We assume that CSI is only available at the appropriate receivers. Orthogonal channels are assigned for the users in the system.

We assume that fading remains constant over one transmission block, i.e. *slow fading* and magnitudes of fading coefficients follow Rayleigh distribution. Fading is *frequency non-selective fading*. Clearly, neither spectral nor temporal diversity are available during the transmission of one source message block. This is a common approach to isolate the benefits arising from spatial diversity. However, one has to be aware of the fact that the performance gains through cooperative diversity protocols will be less substantial if diversity is available in the remaining two degrees of freedom (time and frequency).

2.3.2 Forwarding Strategies

In general, the relay node can take on one of various different approaches towards forwarding the data received from the source. It can either transmit a scaled version of its received signal subject to its transmit power constraint, referred to in the literature as *Amplify-and-Forward* (AF) transmission [1, 22, 39, 43, 44], or fully decode, re-encode and retransmit the received signal referred to in the literature as *Decode-and-Forward* (DF) transmission [1, 17, 39, 45, 46]. The relay node can also transmit a compressed version of its received signal according to a power constraint referred to as *Compress-and-Forward* (CF) transmission [17, 19, 20]. Last, but not least, the relay node can send an estimate of what it received from source node, referred to as *Estimate-and-Forward* (EF) transmission [47]. All approaches have their advantages and disadvantages. Here we show achievable rates via ergodic capacities and achievable diversity orders via outage probabilities for these relaying schemes.

Amplify-and-Forward Relaying

The AF relaying scheme provides cooperative diversity while minimizing the computational load at the relay node. In AF relaying, the received signal from source node is simply amplified according to a power constraint at the relay node. Two of the obvious hinderances of the AF relaying scheme are the storage limitations for large amounts of analog data required for relaying and required for combining at the destination due to the time division nature of the scheme and noise amplification at the relay node along with the received source

signal. For the relay channel shown in Fig.2.9, assuming i.i.d. channel coefficients and zero mean CSCG noise terms with variance σ_r^2 at the relay node and σ_n^2 at the destination node, the non-ergodic capacity of AF forwarding can be derived as

$$C_{AF} = \frac{1}{2} \log \left(1 + \frac{E_s |h_{sd}|^2}{\sigma_n^2} + \frac{E_s |h_{sd}|^2 E_r |h_{rd}|^2}{\sigma_n^2 (|h_{sr}|^2 E_s + \sigma_r^2) + E_r \sigma_r^2 |h_{rd}|^2} \right) \quad (2.42)$$

where E_s and E_r are the power constraints at the source and relay nodes, respectively. The $1/2$ factor in (2.42) is due to the TDMA nature of relay network transmission. Indeed in AF relaying, two consecutive time slots are used for a single data stream transmission. Ergodic capacity for AF mode can be evaluated by averaging (2.42) over all possible fading realizations.

The outage probability observed with AF relaying for a data rate of R [bit/sec/Hz] is given as

$$P_{out}(R) = P(C_{AF} < R). \quad (2.43)$$

Decode-and-Forward Relaying

DF type protocols have the advantage that only digital data has to be stored, but on the other hand they require the relay node to fully decode the signal stream which may impose a heavy computational load on the relay when the current prevailing FEC codes (e.g. convolutional, turbo, LDPC codes) are used. From network information theory, one realizes that the achievable information rate of DF relaying is determined by the max-flow min-cut theorem [27]. The cut set around the source forms a broadcast channel and the cut set around the destination forms a parallel channel (due to the orthogonality in time). The system capacity is the *minimum* value of the capacities of the two cut sets.

Assuming the destination node has perfect knowledge of the channels, h_{sd} and h_{rd} (knowledge of h_{sr} is required only in the relay terminal) and assuming that the transmission rate is R , one has to ensure that

$$R \leq \log \left(1 + \frac{E_s |h_{sr}|^2}{\sigma_r^2} \right) \quad (2.44)$$

so that the relay node is able to decode the transmitted signal *correctly*. When this is the case, one can assume that the relay node produces an *error-free* estimate of the transmitted signal. In the second time slot, the cut set corresponds to Multiple-Access Channel (MAC)

set and for error-free decodability of the signal at the destination node the data rate should satisfy the inequality,

$$R \leq \log \left(1 + \frac{E_s |h_{sd}|^2}{\sigma_n^2} + \frac{E_r |h_{rd}|^2}{\sigma_n^2} \right). \quad (2.45)$$

Then, the capacity for DF relaying is given by the minimum of (2.44) and (2.45) which can be expressed as

$$C_{DF} = \frac{1}{2} \min \left\{ \log \left(1 + \frac{E_s |h_{sr}|^2}{\sigma_r^2} \right), \log \left(1 + \frac{E_s |h_{sd}|^2}{\sigma_n^2} + \frac{E_r |h_{rd}|^2}{\sigma_n^2} \right) \right\}. \quad (2.46)$$

where, once again the $1/2$ factor is due to the TDMA nature of relay network transmission.

Again, the ergodic capacity corresponding to DF forwarding is given by averaging (2.46) over all possible fading coefficients. The outage probability for a given R rate transmission is the same as in (2.43).

Adaptive Decode-and-Forward Relaying

This forwarding scheme is proposed in [45] where correct decoding at the relay node is not required. This relaxation provides the capacity not to be bounded by the $\mathcal{S} \rightarrow \mathcal{R}$ link. When the instantaneous channel information is available, the channels can be treated as Gaussian channels. Since the Gaussian broadcast channel is a degraded broadcast channel, the better channel can always also decode the information intended for the worse channel [45]. The capacity of the broadcast cut-set is the one with the better SNR and the capacity of the cut-set around the destination is the sum rate of the two parallel channels. Here the assumption is that at the relay node the same code book as used at the source node is used. Then, the corresponding adaptive DF mode capacity is expressed by

$$C_{AdDF} = \frac{1}{2} \min \left\{ \max \left\{ \log \left(1 + \frac{E_s |h_{sr}|^2}{\sigma_r^2} \right), \log \left(1 + \frac{E_s |h_{sd}|^2}{\sigma_n^2} \right) \right\}, \log \left(1 + \frac{E_s |h_{sd}|^2}{\sigma_n^2} + \frac{E_r |h_{rd}|^2}{\sigma_n^2} \right) \right\}. \quad (2.47)$$

With the adaptive DF scheme, the limitation on capacity of the conventional DF scheme is lifted since this mode does not require the relay node to decode all the time. If the $\mathcal{S} \rightarrow \mathcal{D}$ link is better than $\mathcal{S} \rightarrow \mathcal{R}$ link, then the system capacity will not be restricted by the $\mathcal{S} \rightarrow \mathcal{R}$ link. One can conclude that adaptive DF and conventional DF schemes will act different when relay node is far from the source node.

Estimate-and-Forward Relaying

The conventional DF scheme is limited by the fact that the relay needs to be able to correctly decode. This would result in a poor performance when the source-relay link is in a deep fade. Recently, a new transmission protocol called estimate and forward [47] has been proposed, and it relies on the principle that the relay uses a statistical estimate of the signal from the source instead of fully decoding. The achievable rate of this scheme is

$$C_{EF} = \frac{1}{2} \log \left(1 + \frac{E_s |h_{sd}|^2}{\sigma_n^2} + \frac{E_s E_r |h_{sr}|^2 |h_{rd}|^2}{\sigma_n^2 (\sigma_r^2 + E_s |h_{sr}|^2) + \sigma_r^2 (E_s |h_{sr}|^2 + E_r |h_{rd}|^2)} \right). \quad (2.48)$$

It is shown in [47] that when the relay is positioned near the source, then the DF protocol performs well, while the EF protocol has a high achievable rate for relays that are close to the destination.

Compress-and-Forward Relaying

Instead of using the DF scheme, an alternative approach is to let the relay use Wyner-Ziv lossy source coding [48] on the received signal to exploit the side information at the destination node. The relay node forwards a compressed version of its channel output to the destination node via error free channel encoding [17, Theorem 6], [19]. This information is used as side information at the destination. For the single relay case the capacity expression for compress and forward relaying can be given as

$$\begin{aligned} C_{CF} &= \frac{1}{2} \log \left(1 + \frac{E_s |h_{sd}|^2}{\sigma_n^2} + \frac{E_s |h_{sr}|^2}{\sigma_r^2 + \sigma_w^2} \right) \\ &= \frac{1}{2} \log \left(1 + \frac{E_s |h_{sd}|^2}{\sigma_n^2} + \frac{E_s E_r |h_{sr}|^2 |h_{rd}|^2 (E_s |h_{sd}|^2 + \sigma_n^2)}{\sigma_r^2 (\sigma_n^2 + E_s |h_{sd}|^2) (\sigma_n^2 + E_r |h_{rd}|^2) + \sigma_n^4 E_s |h_{sr}|^2} \right) \end{aligned} \quad (2.49)$$

where σ_w^2 is given by

$$\sigma_w^2 = \frac{\sigma_n^2 (\sigma_r^2 E_s |h_{sd}|^2 + \sigma_n^2 E_s |h_{sr}|^2 + \sigma_n^2 \sigma_r^2)}{E_r |h_{rd}|^2 (E_s |h_{sd}|^2 + \sigma_n^2)}. \quad (2.50)$$

The capacity derivations for the multiple relay network using the CF scheme can be found in [19].

In Fig.2.10, the ergodic capacities for all relay modes considered above are plotted for the case where each node is furnished with a single antenna. Results are given for $E_s/\sigma_n^2 =$

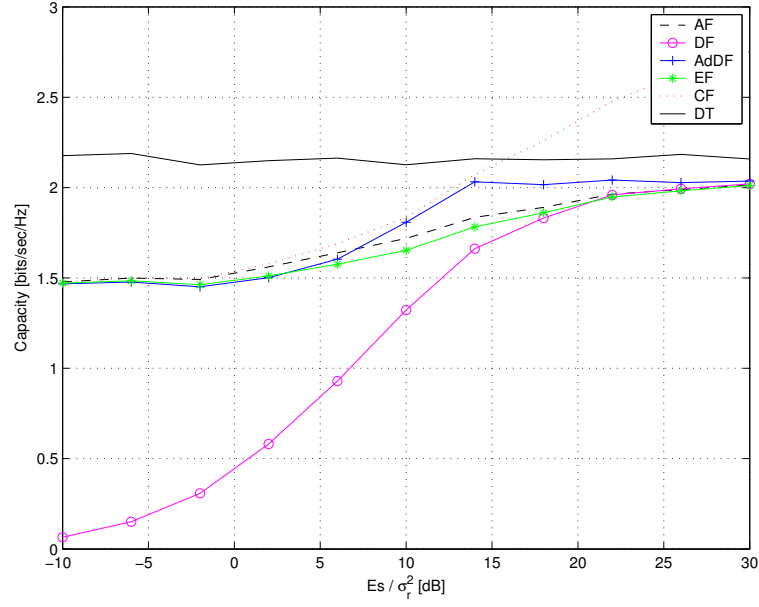


Figure 2.10: Ergodic capacities for single antenna of all relay modes considered in this section. $E_s/\sigma_n^2 = E_r/\sigma_n^2 = 10\text{dB}$. Results are given as a function of E_s/σ_r^2 .

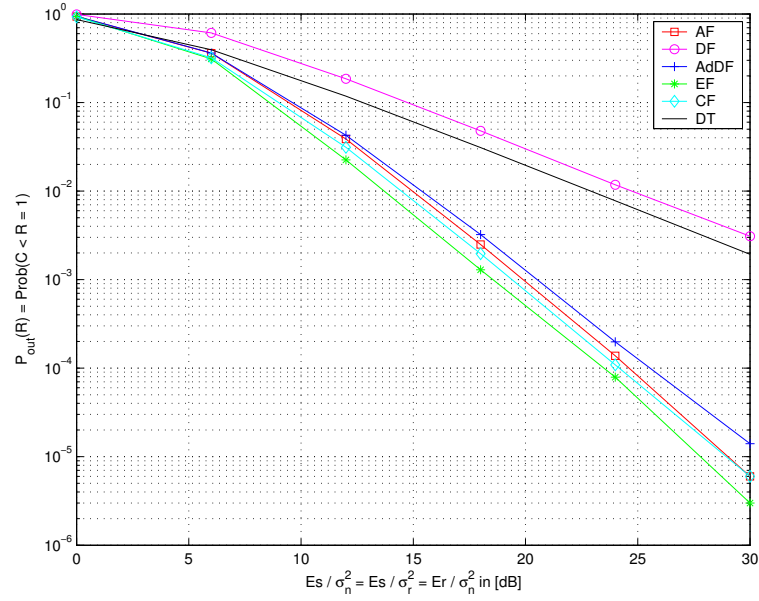


Figure 2.11: Single antenna relay modes' outage probabilities considered in this section for $R = 1$ [bits/sec/Hz]. Results are given as a function of E_s/σ_r^2 in [dB] where $E_s/\sigma_r^2 = E_s/\sigma_n^2 = E_r/\sigma_n^2$ is assumed.

$E_r/\sigma_n^2 = 10\text{dB}$. It can be seen that CF scheme outperforms all other modes. Furthermore, the DF scheme is the least capacity achieving mode amongst all. We observe that the AdDF scheme provides significant gains over the DF scheme especially when the $\mathcal{S} \rightarrow \mathcal{R}$ link is not good.

The outage probabilities corresponding to the relaying schemes considered in this section are plotted in Fig.2.11. Outage probabilities are calculated for the transmission rate $R = 1$ [bits/sec/Hz]. Results are given as a function of E_s/σ_r^2 in [dB] where the equality, $E_s/\sigma_r^2 = E_s/\sigma_n^2 = E_r/\sigma_n^2$ is assumed. It is observed that all relaying strategies except the DF scheme provide *second* order diversity. In [44], mathematical expressions for some other relaying modes can be found.

Chapter 3

MIMO MULTIPLE RELAY NETWORKS AND SOURCE BEAMFORMING

Over the last decade, cooperative diversity has captured significant attention in the research community due to its provision of spatial diversity gain in a *distributed* fashion. Simply stated, in cooperative communications, the source node cooperates with a partner (or multiple partners) to transmit its information to the destination node. This scheme provides a virtual transmit antenna array which results in a more reliable transmission towards the destination node compared to transmitting the stream alone [24, 44]. Cooperative diversity is also attractive because, like the antenna diversity, it requires no additional bandwidth. To increase the the channel capacity, several cooperative schemes involving relay nodes have been proposed in the literature [43, 44, 49]. Among them, the AF and DF schemes, overviewed in the previous chapter are two of the fundamental forwarding strategies. In the AF scheme, the relay node simply amplifies the received signal according to a power constraint and forwards amplified version of the signal to the destination node. There is no demodulation, decoding and re-encoding processes in this mode, but noise is also amplified with the signal. In the DF scheme on the other hand, the relay node fully decodes, re-encodes and retransmits the received signal. There is the possibility of decoding error propagation which results in a decrease in the overall system performance.

Up to now, these has been much work on this area in the literature. Distributed space-time code design and information-theoretic performance limits for single antenna fading relay channels (with a finite number of nodes) have recently been studied in [19, 39, 43, 44]. Capacity results for relay MIMO channels with a finite number of relays are derived in [19, 21, 50, 51].

As considered in Chapter 2, when beamforming is employed, all of the transmit power is directed to the direction characterized by the best eigen-channel. This provides gains in the system performance because, due to the randomness of the channel coefficients in a wireless

scenario, there is a higher possibility that the channel is good in one of the infinitely many directions [28]. Especially at the *low* SNR values, power distribution will be more sensitive to channel variations so that we can gain more by exploiting opportunistic power distribution. On the other hand, at high SNR values, the power distribution will not be as sensitive to channel variations (since the channel is good in general) so the system performance will degrade due to the lack of utilization of full degrees of freedom that is *array gain* achieved by beamforming will be so small compared to the loss induced by *spatial multiplexing*. In another word, in MIMO systems beamforming does not use all of the degrees of freedom, truly it uses only one degree of freedom of the channel. Beamforming is one dimensional (1-D) coding. Because of this, at intermediate and high SNR regimes beamforming can not achieve MIMO channel capacity.

Up to now, a significant body of work has been conducted on various aspects of beamforming for wireless relay channels. In [46] and [52], relay-destination beamforming algorithms are developed while the source node is assisted via multiple relays under a bandwidth constraint. Here, all of the communicating parts have single antennas. For the AF relaying scheme, ergodic and outage capacities of fading relay channels with single antennas is investigated in [39] and [45]. A general framework on the capacity of MIMO relay channel is proposed in [21]. In [22], the impact of multiple AF relays on the capacity of rank-deficient MIMO channels is studied when no CSI is available at the transmitter. The optimum weighting matrix at the relay node for maximizing the channel capacity is found for the AF scheme for MIMO relaying in [50].

In this chapter, we examine fading relay channels where the source, destination and multiple relay nodes are all equipped with multiple antennas. We study the ergodic capacities of multiple relay networks based on the two most prominent relaying strategies, namely *Amplify-and-Forward (AF)* and *Decode-and-Forward (DF)* schemes. We consider system performances based on CSI at transmitter sides, source node and relay nodes. We assume that all receivers in the system have their associated CSI available to them.

We consider two different scenarios corresponding to the availability of CSI at the transmitting nodes. In the first scenario, we find ergodic capacities for the relaying channel when no CSI is available at the transmitter nodes. In the second scenario, we examine the ergodic capacities for the relaying channel when perfect knowledge of CSI from all channels

is available at all transmitter node(s). For this case, we propose *simple but efficient* transmit beamforming algorithms at the source node for both AF and DF relaying strategies and derive ergodic capacity expressions. The impacts of source transmission via beamforming on the MIMO multi-relay channel capacity are investigated based on the inter-user channel conditions and relaying strategies. The proposed approaches are based on two basic concepts: beamforming and power allocation over multiple beams (waterfilling [27]). As conventional with beamforming, we direct our transmission power to the best eigen-channel. With waterfilling, we use the make use of channel knowledge for all relays and allocate more power on better channels and less power to worse channels. With these in mind, we distribute the total available transmit power in various channel directions: a portion for the best eigen-channel corresponding to channel between source and destination node, and the remaining distributed amongst the best eigen-channels corresponding to channels between source and each relay node. Due to the presence of multiple beamforms, the source coding has to be multi-dimensional coding. At this point one question comes to mind: How will we distribute the available power over these directions: uniformly or non-uniformly?

Before answering this question, we recall one fundamental concept from information theory: It is better to send more information on channels with better conditions. So, it is worth to allocate more power to the channel direction with a better condition. In this paper, with channel with better condition we refer to the more information conveying channel, i.e., high channel capacity. So, using the channel capacities between source-relay, source-destination and relay-destination, we propose a simple but efficient power allocation algorithm for AF and DF modes, similar in spirit to the conventional waterfilling idea.

For the AF scheme, we calculate the composite channel capacity for the source-relay(s)-destination direction and steer the transmit power according to the instantaneous channel capacities as well as source-destination instantaneous channel capacities.

For the case where the relaying strategy calls for the relays to decode the transmitted signal, we investigate two different variations of the conventional DF relaying scheme. In the first case, in the broadcast part only the best receiver is able to decode which we call *Best-Selection DF* (BS-DF) relaying mode. While in the second DF case, we assume that all relay nodes in the scenario are able to decode received signals which we call *All-Participate DF* (AP-DF) relaying mode. The results show that at low SNR proposed beamforming

Table 3.1: The TDMA based protocol considered in this thesis

Time Slots	Source node, $\mathcal{S}$	Relay nodes, $\mathcal{R}_k, k \in 1, 2, \dots, K$	Destination node, $\mathcal{D}$
Time Slot I	Broadcasts to $\mathcal{R}_k$ and $\mathcal{D}$	Listens for $\mathcal{S}$	Listens for $\mathcal{S}$
Time Slot II	Idle	Transmits to $\mathcal{D}$	Listens for $\mathcal{R}_k$

algorithms outperform than transmission with utilizing full degrees of freedom of the relay network. It is shown that BS-DF mode corresponds to an adaptive DF mode in the sense this mode turns to direct transmission when source-destination link is the best link compared to all other links. While in AP-DF mode, it is shown that when relay nodes are close to source node we have better performance than the previous case as all relays will make multiple-access part more reliable due to multi-relay diversity. Depending on relays' locations we compared all of the modes in terms of information theoretic view.

In the following section, we describe the system model as well as the relaying strategies investigated in this chapter. Then in Sec.3.2, we derive the ergodic capacity expressions for the system employing the AF scheme with and without CSI available at the transmitters. Next in Sec.3.3, we repeat these calculations for the DF scheme and finally, we provide numerical results in Sec.3.4.

3.1 System and Signal Model for MIMO Relay Channel

In this chapter, we consider scenarios that include a single source node and a single destination node with multiple partnering relay nodes in between. All nodes are assumed to be furnished with multiple antennas. The source node $\mathcal{S}$ transmits data to the destination node $\mathcal{D}$ with the assistance of relay nodes $\mathcal{R}_k, k \in \{1, 2, \dots, K\}$, where K is the number of relays capable of assisting the communication between the source and the destination. We consider, for realistic applications, that an orthogonality constraint for transmission and reception is present, in that, for all nodes, simultaneous transmission and reception is not possible. Thus, we divide the available channel into two orthogonal sub-channels in the time domain.

In this chapter, we use the protocol presented in [39] where no collision of packets at the

receiver is possible. Here, the source node communicates with relays and the destination node in the first time slot. In the second time slot, only the relay nodes communicate with the destination node. The operation steps for this protocol are summarized in Table 3.1. Since only a single data transmission is conducted over two consecutive time slots, the effective system throughput will be half when compared to direct transmission with the same effective channel transfer function.

The channels $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}$, for $k \in \{1, 2, \dots, K\}$, are all assumed to be Rayleigh block fading channels where the channels remain constant for the duration of the two consecutive time slots. The channels are assumed to be independent of each other and also in between the groups of two consecutive time slots realizations.

We define H_D , H_k and G_k as channel matrices for the $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}$ links, respectively. For each link, the channel coefficients are assumed to include the path-loss exponent; that is, the channel coefficient from m th transmit antenna to n th receiver antenna follows the general form of

$$h(n, m) = \alpha \phi(t), \quad (3.1)$$

where $\phi(t)$ is a complex Gaussian random variable with zero mean and unit variance and α is the path-loss exponent. We assume the square of path-loss is inversely proportional to a given power of the distance, i.e., $\alpha^2 = l^{-\delta}$. Here l is the distance between the transmitter and the receiver, and δ is the path-loss exponent that depends on the type of channel considered.

We define the number of antennas at the source node, relay nodes and destination node as M , $N_{\mathcal{R}_k}$ for $k \in \{1, 2, \dots, K\}$ and N , respectively. The noise terms at the receivers of the relay nodes and destination node are assumed to be i.i.d. zero-mean circularly symmetric complex Gaussian with $\sigma_{r_k}^2$ for $k \in \{1, 2, \dots, K\}$ and σ_n^2 , respectively, i.e., $\mathcal{CN}(0, \sigma_{r_k}^2)$ and $\mathcal{CN}(0, \sigma_n^2)$. For simplicity, we assume throughout the thesis that all relay nodes have the same noise variance, that is, $\sigma_{r_k}^2 = \sigma_r^2$.

The system model considered in this chapter is depicted in Fig.3.1. The total transmitted power is assumed to be independent of the number of antennas available for transmission. We assume that total available powers at the source and relay nodes are E_s and $E_{r_k}, \forall k$, respectively. In the first time slot, the source node is assumed to transmit the symbol vector $\mathbf{x}$, with $\mathbf{x} \in \mathbf{C}^{M \times 1}$, having the covariance matrix $\mathbf{Q}_s = \mathcal{E}[\mathbf{x}\mathbf{x}^H]$ and $E_s = \text{tr}(\mathbf{Q}_s)$. In

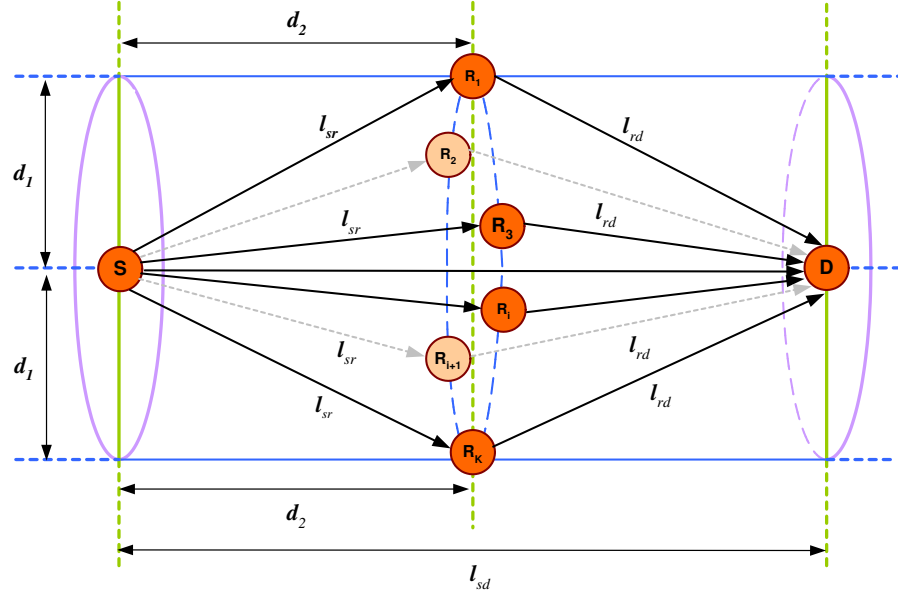


Figure 3.1: Relay location setup: 3D Parallel Case where all relay nodes are in same distance from both the source and the destination node.

the second time slot, the relay nodes are assumed to transmit symbol vectors $\mathbf{t}_k$, with $\mathbf{t}_k \in \mathbf{C}^{N_{\mathcal{R}_k} \times 1}$, having a covariance matrix $\mathbf{Q}_{t_k} = \mathcal{E} [\mathbf{t}_k \mathbf{t}_k^H]$ and $E_{r_k} = \text{tr}(\mathbf{Q}_{t_k})$.

3.2 AF with Multiple Antennas

In this section we want to determine the capacity of multiple antenna equipped relay networks operating on AF mode with the specified transmission protocol for both single and multiple relay cases. As explained in Section 3.1 we assume that source, relays and destination terminals have M , $N_{\mathcal{R}_k}$ and N antennas, respectively. In AF mode, relay nodes simply scale received signals without any demodulation and decoding/reencoding with respect to transmit power limitation of $\text{tr}(\mathbf{Q}_{t_k}) = E_{r_k}$, where $\mathbf{Q}_{t_k}$ is covariance matrix of the scaled signal vector of the k th relay node. In next subsections, we will determine the capacity of AF mode both with and without transmit beamforming at Source node.

3.2.1 Transmission Without Beamforming

In this section, we will consider MIMO multi-relay channel capacity for no channel side information at transmitter sides but at receiver sides. According to the specified protocol,

at first time slot source node broadcasts a signal vector and at second time slot all of the relay nodes access to the destination node. Here we assume that synchronous transmission at relay nodes. With the specified transmission protocol, at first time slot received signal vectors at the destination and the relay nodes will be, respectively

$$\mathbf{y}_1 = \mathbf{H}_D \mathbf{x} + \mathbf{n}_1, \quad (3.2)$$

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_{r_k} \quad k \in \{1, 2, \dots, K\}.$$

where $\mathbf{H}_D$ and $\mathbf{H}_k$ are the channel matrices from $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$, respectively. The received signal vectors at relay nodes are scaled to construct a transmit signal vectors of power $E_{r_k}, \forall k$. The received signal vector $\mathbf{r}_k$ is multiplied by the gain matrix $\mathbf{B}_k$ prior to retransmission, as in [22]

$$\mathbf{t}_k = \mathbf{B}_k \mathbf{r}_k \quad \forall k. \quad (3.3)$$

Covariance matrices of the received signal vectors $\mathbf{r}_k$ at each relay node have the following general structure

$$\mathbf{Q}_{r_k} = \mathcal{E} [\mathbf{r}_k \mathbf{r}_k^H] = \mathbf{H}_k \mathbf{Q}_s \mathbf{H}_k^H + \sigma_r^2 \mathbf{I}_{N_{\mathcal{R}_k}} \quad \forall k. \quad (3.4)$$

To meet our transmit power constraint at the relay node, the gain matrix, $\mathbf{B}_k$, should be of the form

$$\mathbf{B}_k = \sqrt{\frac{E_{r_k}}{N_{\mathcal{R}_k}}} \mathbf{Q}_{r_k}^{-1/2} \quad k \in \{1, 2, \dots, K\}. \quad (3.5)$$

At the second time slot, assuming synchronous reception, the received signal at destination node will be

$$\begin{aligned} \mathbf{y}_2 &= \sum_{k=1}^K \mathbf{G}_k \mathbf{t}_k + \mathbf{n}_2 = \sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{r}_k + \mathbf{n}_2 \\ &= \sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{H}_k \mathbf{x} + \sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{n}_{r_k} + \mathbf{n}_2 \\ &= \sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{H}_k \mathbf{x} + \mathbf{n}_{eq} \\ &= \mathbf{H}_2 \mathbf{x} + \mathbf{n}_{eq}. \end{aligned} \quad (3.6)$$

where

$$\mathbf{H}_2 = \sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{H}_k, \quad (3.7)$$

and

$$\mathbf{n}_{eq} = \sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{n}_{r_k} + \mathbf{n}_2. \quad (3.8)$$

Due to the gain matrices $\mathbf{B}_k$ and the channel matrices $\mathbf{G}_k$ the equivalent noise at destination node is not white. Following the same procedure at [22], the covariance matrix of $\mathbf{n}_{eq}$ is

$$\begin{aligned}\mathbf{Q}_{n_{eq}} &= \mathcal{E} [\mathbf{n}_{eq} \mathbf{n}_{eq}^H] \\ &= \sigma_n^2 \left(\sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{B}_k^H \mathbf{G}_k^H \frac{\sigma_r^2}{\sigma_n^2} + \mathbf{I}_N \right) \\ &= \sigma_n^2 \Lambda.\end{aligned}\tag{3.9}$$

the matrix Λ is Hermitian matrix and it is unitarily diagonalizable, such that $\Lambda = \mathbf{V} \mathbf{D} \mathbf{V}^H$, where the unitary matrix¹ $\mathbf{V}$ contains the eigenvalues of Λ and the diagonal matrix $\mathbf{D}$ contains corresponding eigenvalues. The whitening matrix

$$\mathbf{W} = \mathbf{D}^{-1/2} \mathbf{V}^H \tag{3.10}$$

decorrelates the noise vector $\mathbf{n}_{eq}$, because

$$\mathcal{E} [(\mathbf{W} \mathbf{n}_{eq}) (\mathbf{W} \mathbf{n}_{eq})^H] = \mathbf{W} \Lambda \mathbf{W}^H = \sigma_n^2 \mathbf{I}_N. \tag{3.11}$$

If we multiply received vector $\mathbf{y}_2$ with $\mathbf{W}$ then we get an equivalent received signal vector with white noise $\mathbf{w} = \mathbf{W} \mathbf{n}_{eq}$, i.e, $\mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_N)$. Then at second time slot, we have following equivalent received signal vector

$$\begin{aligned}\tilde{\mathbf{y}}_2 &= \mathbf{W} \mathbf{y}_2 \\ &= \mathbf{W} \left(\sum_{k=1}^K \mathbf{G}_k \mathbf{B}_k \mathbf{G}_k \mathbf{x} + \mathbf{n}_{eq} \right) \\ &= \mathbf{W} \mathbf{H}_2 \mathbf{x} + \mathbf{w}.\end{aligned}\tag{3.12}$$

The signal received at first time slot is given in 3.2. If we take in account both time slots and write down the received signals in compound form we get

$$\underbrace{\begin{bmatrix} \mathbf{y}_1 \\ \tilde{\mathbf{y}}_2 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \mathbf{H}_D \\ \mathbf{W} \mathbf{H}_2 \end{bmatrix}}_{\mathbf{H}} \mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{n}_1 \\ \mathbf{w} \end{bmatrix}}_{\mathbf{n}}. \tag{3.13}$$

¹If a matrix $\mathbf{A} \in \mathbf{C}^{n \times n}$ is unitary than it satisfies $\mathbf{A}^H \mathbf{A} = \mathbf{A} \mathbf{A}^H = \mathbf{I}_n$

Equation (3.13) represents a $2N \times M$ MIMO channel with additive white Gaussian noise. The generalized ergodic capacity of AF mode multi-relay channels for the case that channel coefficients are unknown to the source node but perfectly known to the destination node is given by [13]

$$\begin{aligned} C_{NCSI} &= \max_{\text{tr}(\mathbf{Q}_s)=E_s} \frac{1}{2} \mathcal{E}_{\mathbf{H}} \left[\log \det \left(\mathbf{I}_{2N} + \frac{1}{\sigma_n^2} \mathbf{H} \mathbf{Q}_s \mathbf{H}^H \right) \right] \\ &= \frac{1}{2} \mathcal{E}_{\mathbf{H}} \left[\log \det \left(\mathbf{I}_{2N} + \frac{E_s}{M \sigma_n^2} \mathbf{H} \mathbf{H}^H \right) \right]. \end{aligned} \quad (3.14)$$

Here the factor $1/2$ is introduced to account for the fact that two consecutive time slots are used for each symbol vector.

3.2.2 Transmission With Beamforming

In the previous section, we find the capacity of MIMO relay channels without any power adaptation at the source node. As specified in [13], optimum power allocation in this case is equal power distribution over transmit antennas, i.e., $\mathbf{Q}_s = E_s/M \mathbf{I}_M$. In this section, we propose a simple but efficient power adaptation policy over beams corresponding to maximum eigen-modes of channels which outperforms than the previous cases with the cost of feedback knowledge. First we will look at single relay case and then we generalize our power allocation approach to multi-relay case.

We assume that source node have instantaneous channel knowledge of $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$ for $k \in \{1, 2, \dots, K\}$. Here we define the equivalent channels $\mathcal{S} \rightarrow \mathcal{R}_k \rightarrow \mathcal{D}$ as $\mathbf{H}_{SkD}$ and the equivalent channel capacities as C_{SkD} for all k values.

According to the channels knowledge, or equivalently the individual channel capacities, we do beamforming on the directions of the eigenvectors corresponding to maximum eigen-values of $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$ links. Our proposed beamforming algorithm is similar to conventional *water-filling* on the aspect of distributing transmit power among all of the channel directions: one for $\mathcal{S} \rightarrow \mathcal{D}$ channel and the rest $\mathcal{S} \rightarrow \mathcal{R}_k$ channels.

The amount of power allocated to each direction is specified by the instant channel capacities of C_{SD} and all C_{SkD} , $k = 1, 2, \dots, K$, where C_{SD} and C_{SkD} are the channel capacities corresponding to $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k \rightarrow \mathcal{D}$, respectively. On the information theoretic aspect, allocating more power on the channel direction which is capable of convey-

ing more information results in capacity increase compared to transmission with no power allocation.

We define $\mathbf{w}_{sd}$ and $\mathbf{w}_{sk}$, $k \in \{1, 2, \dots, K\}$, as the eigenvectors corresponding to maximum eigenvalues of matrices $\mathbf{H}_D^H \mathbf{H}_D$ and $\mathbf{H}_k^H \mathbf{H}_k$, respectively. Note that all matrices are Hermitian matrices, so they are unitarily diagonalizable. Thus, the eigenvectors have the following property

$$\|\mathbf{w}_{sd}\|^2 = \|\mathbf{w}_{sk}\|^2 = 1 \quad \forall k. \quad (3.15)$$

Next, We define our generalized transmit covariance matrix as follow

$$\mathbf{Q}_s = \begin{bmatrix} \mathbf{w}_{sd} & \mathbf{w}_{s1} & \cdots & \mathbf{w}_{sK} \end{bmatrix} \begin{bmatrix} \alpha_1 E_s & & & \\ & \alpha_2 E_s & & \\ & & \ddots & \\ & & & \alpha_{K+1} E_s \end{bmatrix} \begin{bmatrix} \mathbf{w}_{sd}^H \\ \mathbf{w}_{s1}^H \\ \vdots \\ \mathbf{w}_{sK}^H \end{bmatrix} \quad (3.16)$$

$$= \alpha_1 E_s \mathbf{w}_{sd} \mathbf{w}_{sd}^H + \sum_{k=1}^K \alpha_{k+1} E_s \mathbf{w}_{sk} \mathbf{w}_{sk}^H.$$

where

$$\sum_{k=1}^{K+1} \alpha_k = 1. \quad (3.17)$$

Note that under this construction

$$\begin{aligned} \text{tr}(\mathbf{Q}_s) &= \text{tr}(\alpha_1 E_s \mathbf{w}_{sd} \mathbf{w}_{sd}^H + \sum_{k=1}^K \alpha_{k+1} E_s \mathbf{w}_{sk} \mathbf{w}_{sk}^H) \\ &= \alpha_1 E_s \text{tr}(\mathbf{w}_{sd} \mathbf{w}_{sd}^H) + \sum_{k=1}^K \alpha_{k+1} E_s \text{tr}(\mathbf{w}_{sk} \mathbf{w}_{sk}^H) \\ &= \alpha_1 E_s + \sum_{k=1}^K \alpha_{k+1} E_s \\ &= E_s \end{aligned} \quad (3.18)$$

so with the above settings we meet the power constraint of E_s at the source node. Next, we will introduce power allocation strategies over multiple beams and give ultimate capacity expressions.

Now, we will give a general multi-relay beamforming algorithm. We will give capacity expression for multi-relay assisted communication systems with the availability of beamforming at source node via the directions of best eigen-modes of channels between $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$. Eigenvectors corresponding to these channels are described in Eq.(3.15).

Our power adaptation policy is to determine the α_k values, $k = 1, 2, \dots, K$. To determine α_k values, we first probe the channel capacities with initializing α_k values to $\alpha_k = 1/(K + 1) \quad \forall k$, i.e., equal power allocated to each channel direction. Then after probing the instantaneous channel capacities, C_{SD} and C_{SkD} , we adjust our α_k values as follow

$$\alpha_1 = \frac{C_{SD}}{C_{SD} + \sum_{k=1}^K C_{SkD}} \quad (3.19)$$

which corresponds to $\mathcal{S} \rightarrow \mathcal{D}$ channel direction and

$$\alpha_{k+1} = \frac{C_{SkD}}{C_{SD} + \sum_{j=1}^K C_{SjD}}, \quad k = 1, 2, \dots, K. \quad (3.20)$$

which corresponds to $\mathcal{S} \rightarrow \mathcal{R}_k$ channel directions. We follow the same procedure in each turn of two consecutive time slots. After determining the α_k values, we update the transmit covariance matrix given in Eq.(3.16) for K relays and find corresponding ergodic capacity expression similar to Eq.(3.14) as follow

$$C_{beam} = \frac{1}{2} \mathcal{E}_{\mathbf{H}} \left[\log \det \left(\mathbf{I}_{2N} + \frac{1}{\sigma_n^2} \mathbf{H} \mathbf{Q}_s \mathbf{H}^H \right) \right]. \quad (3.21)$$

where $\mathbf{H}$ is defined as in Eq.(3.13).

3.3 DF with Multiple Antennas

In this section we want to determine the capacity of multiple antenna equipped single and multiple relay networks operating on DF mode with the specified transmission protocol. In DF mode, signal vector received by relay node at the first time slot is demodulated and decoded before transmission with respect to transmit power limitations. We assumed *Unconstraint Coding* [51] at relay nodes and we consider two different relay participation schemes:

- *perfect* decoding only at the best capacity achieving relay node: *Best-Selection DF*(BS-DF) relaying mode, and
- *perfect* decodability at all of the relay nodes in the scenario: *All-Participate DF*(AP-DF) relaying mode.

In BS-DF mode, we will use the opportunity of multiple relay nodes by enabling the best capacity achieving relay to decode perfectly and participate in the transmission in the

second time slot. The other relay nodes as not being able to decode will be silent in the second time slot. We will indicate the best capacity achieving relay node with the index l , which means l th relay node has the best channel among all of the relays and it will be the only relay that will be active in the second time slot.

In AP-DF mode, we will assume that all of the participating relay nodes can decode source transmissions perfectly. As all have the exact transmitted signals, all of them will help source transmission in the second time slot with each re-encode with different code books.

From network information theory, one realizes that the achievable information rate of the DF mode is determined by the *max-flow min-cut* of the system for both cases. The cut set around the source forms a broadcast channel and the cut set around the destination forms a parallel channel (due to orthogonality in time) [45]. The system capacity is the minimum value of the two cut sets' capacity.

In the next subsections, following the above expressions, we will detail the ergodic capacities of BS-DF and AP-DF modes both with and without transmit beamforming at source node.

3.3.1 Transmission Without Beamforming

In this section, we will give capacity expressions for relay network operating on DF mode with no CSI at transmitting nodes. After we give general signal models for both DF modes in the following subsections we will derive capacity expressions for multi-relay networks. Operating on the specified transmission protocol, the signal vectors received by the destination and relay nodes at the first time slot are

$$\mathbf{y}_1 = \mathbf{H}_D \mathbf{x}_s + \mathbf{n}_1, \quad (3.22)$$

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{x}_s + \mathbf{n}_{r_k} \quad k = 1, 2, \dots, K.$$

Next, we will derive general capacity expression for both cases of DF mode when there is no CSI at transmitting nodes. Then it is easy to find capacity for any number of relays. Before proceeding, we define the instantaneous channel capacities of the links $\mathcal{S} \rightarrow \mathcal{D}$,

$\mathcal{S} \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}$ as

$$\begin{aligned} C_{SD} &= \log \det(\mathbf{I}_N + \frac{1}{\sigma_n^2} \mathbf{H}_D \mathbf{Q}_s \mathbf{H}_D^H), \\ C_{Sk} &= \log \det(\mathbf{I}_{N_{\mathcal{R}_k}} + \frac{1}{\sigma_r^2} \mathbf{H}_k \mathbf{Q}_s \mathbf{H}_k^H) \quad \text{for } k = 1, 2, \dots, K, \\ C_{kD} &= \log \det(\mathbf{I}_N + \frac{1}{\sigma_n^2} \mathbf{G}_k \mathbf{Q}_{t_k} \mathbf{G}_k^H) \quad \text{for } k = 1, 2, \dots, K, \end{aligned} \quad (3.23)$$

where $\mathbf{Q}_s$ and $\mathbf{Q}_{t_k}$ are transmit covariance matrices of source and relay nodes, respectively. For following, we assume that re-encoded, if possible, signals at relay nodes are $\mathbf{t}_k \forall k$. In general, achievable rates for DF mode relay networks can be derived from min-cut max-flow theorem [27]

$$C_{BC} = \max_{p(\mathbf{x}_s, \mathbf{t}_1, \dots, \mathbf{t}_K)} \mathbb{I}(\mathbf{x}_s; \mathbf{y}, \mathbf{r}_1, \dots, \mathbf{r}_K | \mathbf{t}_1, \dots, \mathbf{t}_K) \quad (3.24)$$

and the capacity of the cut-set around the destination is given by

$$C_{MAC} = \max_{p(\mathbf{x}_s, \mathbf{t}_1, \dots, \mathbf{t}_K)} I(\mathbf{x}_s, \mathbf{t}_1, \dots, \mathbf{t}_K; \mathbf{y}) \quad (3.25)$$

and the achievable rate is given by

$$C = \max_{p(\mathbf{x}_s, \mathbf{t}_1, \dots, \mathbf{t}_K)} \min \{C_{BC}, C_{MAC}\} \quad (3.26)$$

where expectation over all channels gives ergodic capacity.

BS-DF mode

In BS-DF mode, we assume that in broadcast part of communication only the best received node is able to decode among all of the nodes. This can be interpreted as if $\mathcal{S} \rightarrow \mathcal{D}$ is the best quality link than the transmission will convert to direct transmission; on the other hand the best relay node will assist the source transmission in the second time slot.

Among all of the relay nodes, selection of the best relay is done as follows:

$$l = \arg \max_{k \in \{1, 2, \dots, K\}} C_{Sk}. \quad (3.27)$$

where l is the index of relay that has the best channel capacity. The received signal vector at l th relay node in the first time slot are demodulated, decoded and re-encoded, and at the second time slot the re-encoded signal vector is transmitted according to the power

constraint of E_{r_l} for $l \in \{1, 2, \dots, K\}$ where K is the number of relay nodes. Then the received signal vector at the destination node in the second time slot will be

$$\mathbf{y}_2 = \mathbf{G}_l \mathbf{t}_l + \mathbf{n}_2. \quad (3.28)$$

Then the overall channel input-output relationship can be written as [51]

$$\underbrace{\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \mathbf{H}_D & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_l \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{x}_s \\ \mathbf{t}_l \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix}}_{\mathbf{n}} \quad (3.29)$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}.$$

where $\mathbf{x}_s \in \mathbf{C}^{M \times 1}$ is the signal vector transmitted from source node with covariance matrix satisfying the constraint $\text{tr}(\mathbf{Q}_s) = \text{tr}(\mathcal{E}[\mathbf{x}_s \mathbf{x}_s^H]) = E_s$, $\mathbf{t}_l \in \mathbf{C}^{N_{\mathcal{R}_l} \times 1}$ is the re-encoded signal vector with $\text{tr}(\mathbf{Q}_{t_l}) = \text{tr}(\mathcal{E}[\mathbf{t}_l \mathbf{t}_l^H]) = E_{r_l}$ for l th relay node where $l \in \{1, 2, \dots, K\}$, and, $\mathbf{n}_1$ and $\mathbf{n}_2$ are zero-mean circularly symmetric complex Gaussian noises with a common covariance matrix of $\mathbf{Q}_{n_d} = \sigma_n^2 \mathbf{I}_N$. We can write the covariance matrix of overall transmitted signal vectors, $\mathbf{x}$ as follow

$$\mathbf{Q} = \mathcal{E}[\mathbf{x} \mathbf{x}^H] = \begin{bmatrix} \mathbf{Q}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{r_l} \end{bmatrix}. \quad (3.30)$$

Assuming instantaneous channel information is available, the channels can be treated as Gaussian channels. Since Gaussian broadcast channel is degraded broadcast channel [27], the better channel can always decode the information intended for the worse channels. Following the *max-flow min-cut* theory, the capacity of broadcast cut set is the one with better SNR

$$C_{BC,1} = \max \{C_{SD}, C_{Sl}\} \quad (3.31)$$

and the capacity of the cut-set around the destination is given by

$$\begin{aligned}
C_{MAC,1} &= \log \det \left(\mathbf{I}_{2N} + \frac{1}{\sigma_n^2} \mathbf{H} \mathbf{Q} \mathbf{H}^H \right) \\
&= \log \det \left(\begin{bmatrix} \mathbf{I}_N + \frac{1}{\sigma_n^2} \mathbf{H}_D \mathbf{Q}_s \mathbf{H}_D^H & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_N + \frac{1}{\sigma_n^2} \mathbf{G}_l \mathbf{Q}_{t_l} \mathbf{G}_l^H \end{bmatrix} \right) \\
&= \log \det \left(\mathbf{I}_N + \frac{1}{\sigma_n^2} \mathbf{H}_D \mathbf{Q}_s \mathbf{H}_D^H \right) + \log \det \left(\mathbf{I}_N + \frac{1}{\sigma_n^2} \mathbf{G}_l \mathbf{Q}_{t_l} \mathbf{G}_l^H \right).
\end{aligned} \tag{3.32}$$

Then the system's ergodic capacity can be found as follow

$$C_{DF,1}^{NC SI} = \max_{\substack{tr(\mathbf{Q}_s) = E_s \\ tr(\mathbf{Q}_{t_l}) = E_{r_l}}} \frac{1}{2} \min \{ \mathcal{E} [C_{BC,1}], \mathcal{E} [C_{MAC,1}] \}. \tag{3.33}$$

where expectations are over the channels $\mathbf{H}_D$, $\mathbf{H}_k$ and $\mathbf{G}_k$ for $k = 1, 2, \dots, K$. And the factor of $1/2$ is due to transmission of a single signal vector over two consecutive time slots. As there is no channel knowledge at transmitter sides, from [13] the capacity maximizing transmit covariance matrices are $\mathbf{Q}_s = E_s/M \mathbf{I}_M$ and $\mathbf{Q}_{t_l} = E_{r_l}/N_{\mathcal{R}_l} \mathbf{I}_{N_{\mathcal{R}_l}}$.

AP-DF Mode

In AP-DF mode, we assume that in broadcast part of communication all relay nodes are able to decode perfectly. With this case transmission data rate will be upper bounded by the worst broadcast channel since a transmission rate higher than the worst channel capacity will make this node unreliable. On the other hand, we expect that when $\mathcal{S} \rightarrow \mathcal{R}_k$ links are strong compared to direct transmission using multiple relay will increase the system capacity compared to single relay channel due to the effect of *multi-relay diversity*.

In this case, at the second time slot received signals at the destination node will as follow

$$\mathbf{y}_2 = \sum_{k=1}^K \mathbf{G}_k \mathbf{t}_k + \mathbf{n}_2. \tag{3.34}$$

Then the overall channel input-output relationship can be written as [51]

$$\underbrace{\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \mathbf{H}_D & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_1 & \cdots & \mathbf{G}_K \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{x}_s \\ \mathbf{t}_1 \\ \vdots \\ \mathbf{t}_K \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix}}_{\mathbf{n}} \quad (3.35)$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}.$$

The covariance matrix of all transmitted signal vectors, $\mathbf{x} = [\mathbf{x}_s^T, \mathbf{t}_1^T, \dots, \mathbf{t}_K^T]^T$, assuming all the transmitted signals are independent, is given as follows

$$\mathbf{Q} = \mathcal{E} [\mathbf{x}\mathbf{x}^H] = \begin{bmatrix} \mathbf{Q}_s & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_{t_1} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{Q}_{t_K} \end{bmatrix}. \quad (3.36)$$

Following the *max-flow min-cut* theory, the capacity of broadcast cut-set is the minimum of all of the relay nodes

$$C_{BC,1} = \min \{C_{S1}, \dots, C_{SK}\} \quad (3.37)$$

and the capacity of the cut-set around the destination is given by

$$\begin{aligned} C_{MAC,2} &= \log \det \left(\mathbf{I}_{2N} + \frac{1}{\sigma_n^2} \mathbf{H}\mathbf{Q}\mathbf{H}^H \right) \\ &= \log \det \left(\begin{bmatrix} \mathbf{I}_N + \frac{1}{\sigma_n^2} \mathbf{H}_D \mathbf{Q}_s \mathbf{H}_D^H & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_N + \frac{1}{\sigma_n^2} \sum_{k=1}^K \mathbf{G}_k \mathbf{Q}_{t_k} \mathbf{G}_k^H \end{bmatrix} \right) \\ &= \log \det \left(\mathbf{I}_N + \frac{1}{\sigma_n^2} \mathbf{H}_D \mathbf{Q}_s \mathbf{H}_D^H \right) + \log \det \left(\mathbf{I}_N + \frac{1}{\sigma_n^2} \sum_{k=1}^K \mathbf{G}_k \mathbf{Q}_{t_k} \mathbf{G}_k^H \right). \end{aligned} \quad (3.38)$$

Then the system's ergodic capacity can be found as follow

$$C_{DF,2}^{NCSI} = \max_{\substack{tr(\mathbf{Q}_s) = E_s \\ tr(\mathbf{Q}_{t_k}) = E_{r_k}, \forall k}} \frac{1}{2} \min \{ \mathcal{E} [C_{BC,2}], \mathcal{E} [C_{MAC,2}] \}. \quad (3.39)$$

As there is no channel knowledge at transmitter sides, from [13] the capacity maximizing transmit covariance matrices are $\mathbf{Q}_s = \frac{E_s}{M} \mathbf{I}_M$ and $\mathbf{Q}_{t_k} = \frac{E_{r_k}}{N_{\mathcal{R}_k}} \mathbf{I}_{N_{\mathcal{R}_k}}$ where $k \in \{1, 2, \dots, K\}$.

Next, we will propose a power allocation algorithm on multiple beams that take the advantage of channel knowledge at transmitting nodes, i.e. source and relay nodes.

3.3.2 Transmission With Beamforming

In the previous section for MIMO relay channels operating in DF mode, we showed that capacity is maximized with equally power distribution over transmit antennas of the source and relay nodes. As in AF mode, in this section we propose a simple but efficient beam power allocation policy which outperforms than the previous case with the cost of feedback knowledge. We assume that source node has instantaneous channel knowledge of $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$, $k \in \{1, 2, \dots, K\}$. We assume that relays do not have forward channel side information such that they distribute total power over transmit antennas equally, $\mathbf{Q}_{t_k} = \frac{E_{r_k}}{N_{\mathcal{R}_k}} \mathbf{I}_{N_{\mathcal{R}_k}}$.

According to the channel knowledge, or equivalently the individual channel capacities, we do beamforming on the directions of the eigenvectors corresponding to maximum eigenvalues of $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$ channels, $k \in \{1, 2, \dots, K\}$, at the source node. For the source node, our proposed beamforming algorithm is similar to conventional *water-filling* on the aspect of distributing transmit power between channel directions.

The amount of power allocated to each direction is specified by the instantaneous channel capacities C_{SD} , C_{Sk} and C_{kD} which correspond to $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}$, respectively. On the information theoretic aspect, allocating more power on the channel which is capable of conveying more information results in capacity increase compared to disregarding power allocation based on channel knowledge.

We define $\mathbf{w}_{sd}$ and $\mathbf{w}_{sk}$ as the eigenvectors corresponding to maximum eigenvalues of matrices $\mathbf{H}_D^H \mathbf{H}_D$ and $\mathbf{H}_k^H \mathbf{H}_k$, respectively. Note that all matrices are Hermitian matrices, so they are unitarily diagonalizable. Thus, the eigenvectors have the following property

$$\|\mathbf{w}_{sd}\|^2 = \|\mathbf{w}_{sk}\|^2 = 1, \quad k \in \{1, 2, \dots, K\}. \quad (3.40)$$

After the definitions, now we will look at power allocation schemes for both cases of DF mode.

BS-DF Mode

In this mode, we do beamforming on the directions of the eigenvectors corresponding to maximum eigenvalues of $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_l$ channels at the source node where l th relay is the best capacity achieving relay. For the source node, our proposed beamforming algorithm is similar to conventional *water-filling* on the aspect of distributing transmit power between channel directions: one for $\mathcal{S} \rightarrow \mathcal{D}$ channel and the remaining for $\mathcal{S} \rightarrow \mathcal{R}_l$ channel. Power allocation will be done as defined above, and the resulting transmit covariance matrix will be

$$\mathbf{Q}_s = \begin{bmatrix} \mathbf{w}_{sd} & \mathbf{w}_{sl} \end{bmatrix} \begin{bmatrix} \alpha_1 E_s & 0 \\ 0 & \alpha_2 E_s \end{bmatrix} \begin{bmatrix} \mathbf{w}_{sd}^H \\ \mathbf{w}_{sl}^H \end{bmatrix}. \quad (3.41)$$

Our power adaptation policy is to determine the α_k values, $k = 1, 2$. To determine α_k values, we first probe the channel capacities with initializing α_k values to $\alpha_k = 1/2 \quad \forall k$, i.e., equal power allocated to each channel direction. Then after probing the instantaneous channel capacities C_{SD} , C_{Sl} and C_{lD} , we adjust our α_k values as follow

$$\alpha_1 = \frac{C_{SD}}{C_{SD} + \min \{C_{Sl}, C_{lD}\}} \quad (3.42)$$

which corresponds to $\mathcal{S} \rightarrow \mathcal{D}$ channel direction and

$$\alpha_2 = \frac{\min \{C_{Sl}, C_{lD}\}}{C_{SD} + \min \{C_{Sl}, C_{lD}\}} \quad (3.43)$$

which corresponds to $\mathcal{S} \rightarrow \mathcal{R}_l$ channel direction. We follow the same procedure in each turn of two consecutive time slots. After determining the α_k values, we update the transmit covariance matrix, $\mathbf{Q}_s$, given in Eq.(3.41) and find corresponding ergodic capacity expression with Eq.(3.33) using the updated source transmit covariance matrix.

AP-DF Mode

In this mode, as we will use all of the relays in the scenario, we will have $K + 1$ channel directions each corresponds to different receiver nodes. with this in mind, the transmit covariance matrix will be the same as in AF mode transmission given by Eq.(3.16). As in the first DF mode case, to determine α_k values, where $k \in \{1, 2, \dots, K + 1\}$, we use the

following power allocation policy

$$\alpha_1 = \frac{C_{SD}}{C_{SD} + \sum_{j=1}^K \min \{C_{Sj}, C_{jD}\}} \quad (3.44)$$

which corresponds to $\mathcal{S} \rightarrow \mathcal{D}$ channel direction and

$$\alpha_{k+1} = \frac{\min \{C_{Sk}, C_{kD}\}}{C_{SD} + \sum_{j=1}^K \min \{C_{Sj}, C_{jD}\}}, \quad i = 1, 2, \dots, K \quad (3.45)$$

which corresponds to $\mathcal{S} \rightarrow \mathcal{R}_k$ channel direction. Again, after determining the α_k values, we update the transmit covariance matrix, $\mathbf{Q}_s$, given in Eq.(3.16) and find corresponding ergodic capacity expression with Eq.(3.39) using the updated source transmit covariance matrix.

3.4 Simulation Scenarios and Numerical Results

We conclude our discussion of the performance limits of the individual transmission protocol under AF and DF (both BS and AP cases) modes with numerical results. In this setup, we studied the capacity as function of geometry where all relay nodes are in the same distance to both the source and the destination node, assumed relay nodes are on a line over the surface of a cylinder. This scenario is depicted in Fig.3.1 which is the same scenario as in [45] where only single relay was considered. We consider the system for Rayleigh block fading channels using power law air propagation model with path loss exponent $\delta = 4$, that is channel coefficients do not change during a block period but change independently from a block to the next. As shown in Fig.3.1, in parallel case the relay nodes can be at various places on the surface of the cylinder. The horizontal dashed line is paralleled to the link between the source and the destination with a distance of d_1 . The distance between the source and the destination node is normalized to unity.

To see the performance of given relaying modes, we first show in Fig.3.2 ergodic capacities as function of E_s/σ_r^2 while setting E_r/σ_n^2 and E_s/σ_n^2 to -10dB and $l_{SD} = 1$, $l_{SR} = 1$ and $l_{RD} = 1$ where l_{SD} , l_{SR} and l_{RD} are distances between $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}$ as shown in Fig.3.1, respectively.

For further analysis of the given relaying modes, we select system parameters as $d_1 = 0.5$, $l_{SD} = 1$, $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10\text{dB}$ for all receiving nodes for our following

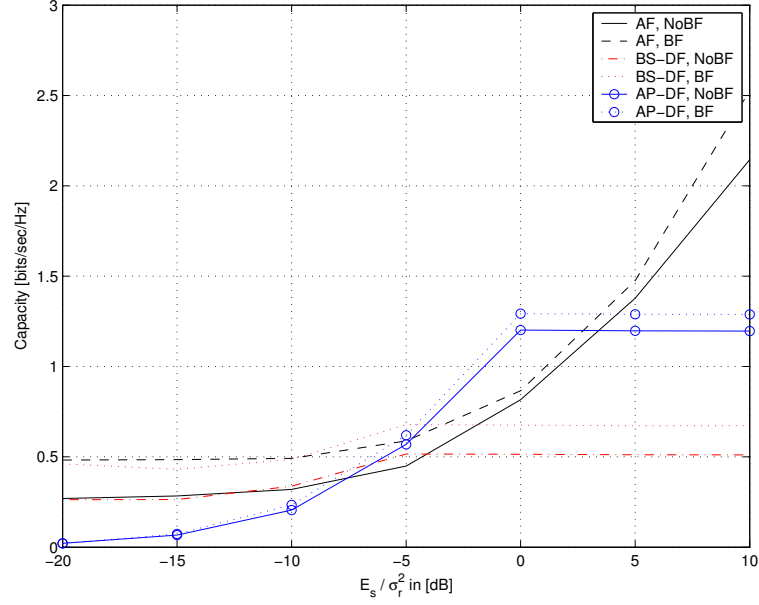


Figure 3.2: Ergodic capacities for 4 Relays and $M = N = N_{\mathcal{R}_k} = 4$ for AF, BS-DF and AP-DF relaying modes. $E_s/\sigma_n^2 = E_r/\sigma_n^2 = -10\text{dB}$. Results are given as a function of E_s/σ_r^2 .

simulation setups. All channel coefficients assumed to include path-loss components as specified in Section 3.1.

For all nodes having $M = N_{\mathcal{R}_k} = N = 4$ antennas for $k \in \{1, 2, \dots, K\}$, simulation results of AF, BS-DF and AP-DF relaying modes with 4 relay nodes are shown concurrently in Fig.3.3.

To see the effect of number of relays in the communication scenario, we give capacity expressions for AF, BS-DF and AP-DF modes in Fig.3.4, Fig.3.5 and Fig.3.6, respectively, for all nodes with $M = N_{\mathcal{R}_k} = N = 4$ antennas and $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10[\text{dB}]$ for $K = \{1, 2, 3, 4\}$ relays and $E_r = E_{r_k}, \forall k$.

For AF mode, the increase in capacity for beamforming compared to non-beamforming transmission get smaller with increasing number of relays. For BS-DF mode, with the specified settings beamforming has better performance compared to non-beamform transmission. But, when all participating relays are near to the source node there is no relative increase in capacity with relay numbers due to the protocol we used. After a certain point (relays close to the destination), we get gain with increasing relay numbers; we can regard this as multi-relay diversity gain. For AP-DF mode, it is seen that the increase in relay nodes

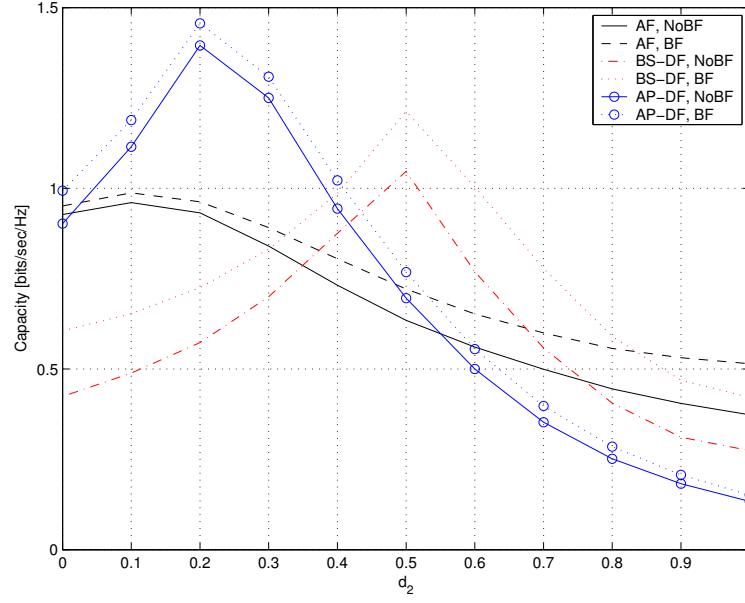


Figure 3.3: Ergodic capacities for 4 Relays and $M = N = N_{\mathcal{R}_k} = 4$ for AF, BS-DF and AP-DF relaying modes. $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10\text{dB}$. Results are given for path-loss exponent $\delta = 4$.

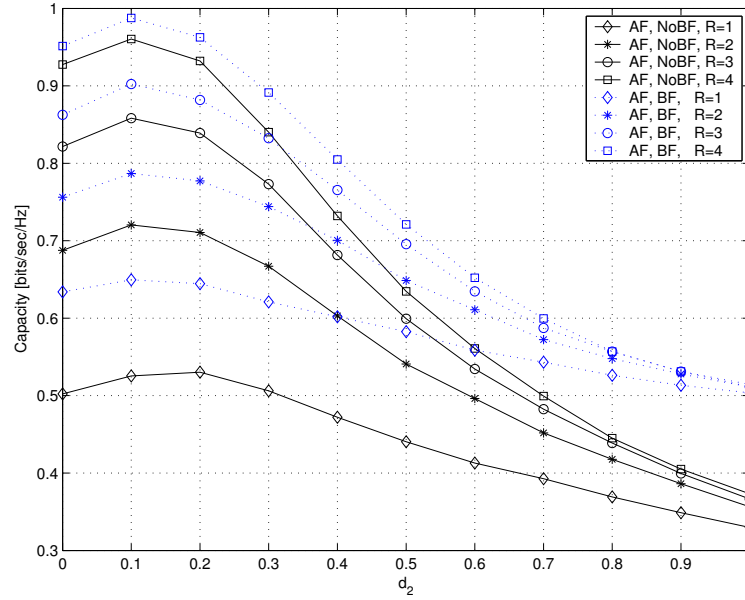


Figure 3.4: Ergodic capacities for $K = \{1, 2, 3, 4\}$ relays and $M = N = N_{\mathcal{R}_k} = 4$ for AF relaying mode. $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10\text{dB}$. Results are given for path-loss exponent $\delta = 4$.

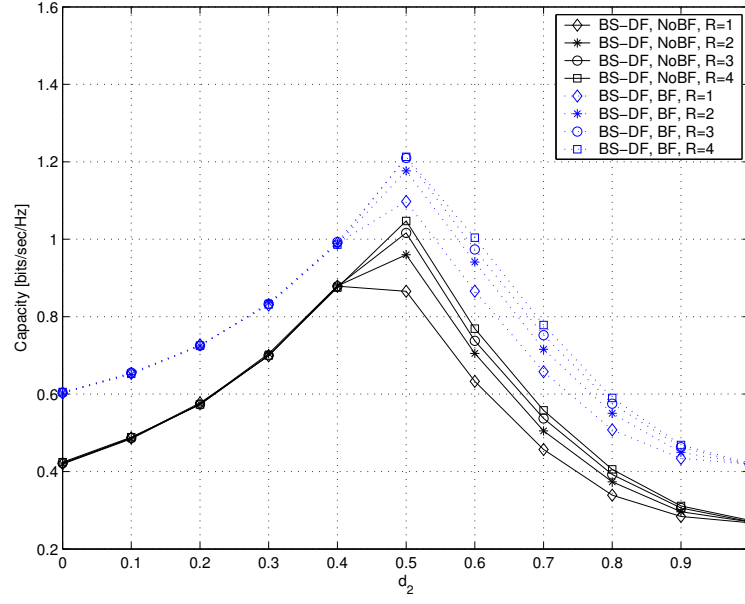


Figure 3.5: Ergodic capacities for $K = \{1, 2, 3, 4\}$ relays and $M = N = N_{\mathcal{R}_k} = 4$ for BS-DF relaying mode. $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10$ dB. Results are given for path-loss exponent $\delta = 4$.

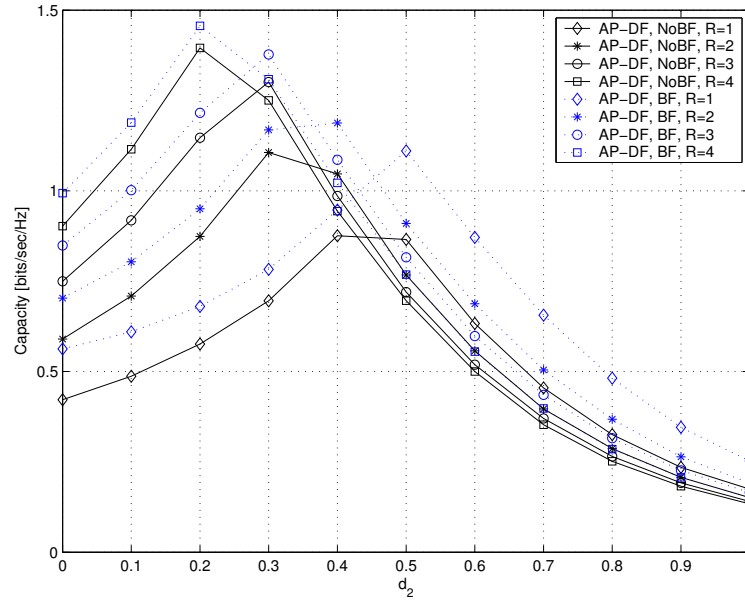


Figure 3.6: Ergodic capacities for $K = \{1, 2, 3, 4\}$ relays and $M = N = N_{\mathcal{R}_k} = 4$ for AP-DF relaying mode. $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10$ dB. Results are given for path-loss exponent $\delta = 4$.

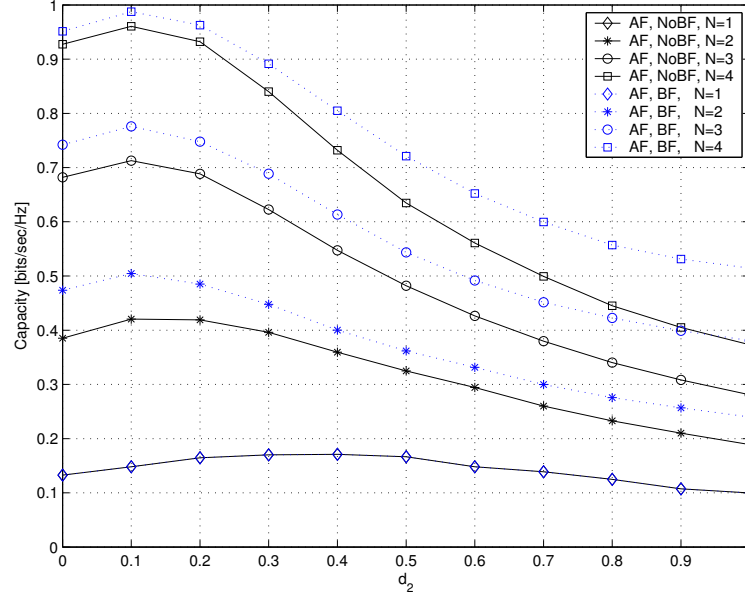


Figure 3.7: Ergodic capacities for $K = 4$ relays and $M = N = N_{\mathcal{R}_k} = \{1, 2, 3, 4\}$ for AF relaying mode. $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10\text{dB}$. Results are given for path-loss exponent $\delta = 4$.

results in an increase in capacity when relays are located closer to source node. On the other hand, when relay nodes are get closer to destination node, increase in relay number results in a decrease in capacity. The reason is that since AP-DF mode assumes all relays to be participated the transmission going apart from source node will make it mode likely that one of the relay be in deep fading which will bound our transmission rate.

Again for a detailed study of beamforming effect, we considered the effect of antenna size of participating nodes with the assumption that all nodes have the same antenna number, $M = N_{\mathcal{R}_k} = N = \{1, 2, 3, 4\}$. In Fig.3.7, Fig.3.8 and Fig.3.9, ergodic capacity expressions for AF, BS-DF and AP-DF relaying modes are given, respectively. In these figures, 4 relays participate communication and all other settings are the same.

In AF relaying mode, increasing antenna number results in a decrease in relative capacity gain. Increasing nodes' antenna size results in an increased degree of freedom in the system as we utilize only a limited degrees in beamforming the increase in capacity will be diminished with increased antenna number. While for BS-DF and AP-DF relaying modes, the increase in beamforming capacities compared to non-beamforming case increase as antenna number increase.

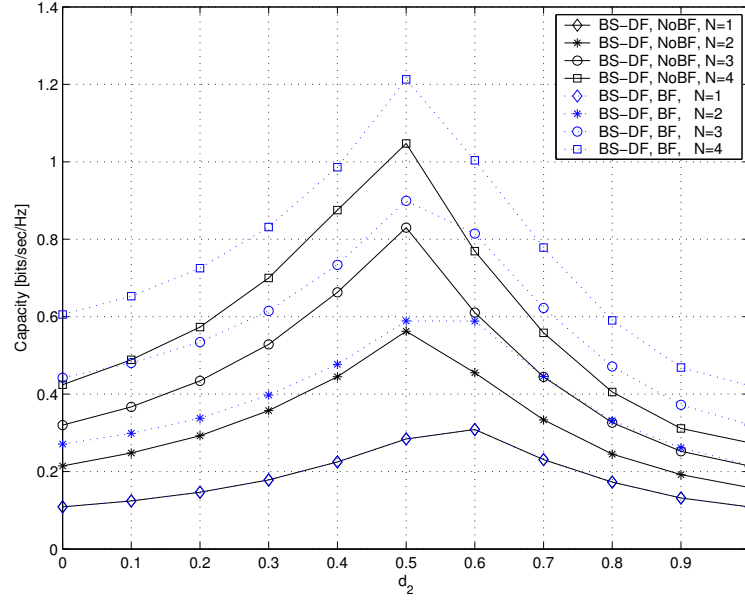


Figure 3.8: Ergodic capacities for $K = 4$ relays and $M = N = N_{\mathcal{R}_k} = \{1, 2, 3, 4\}$ for BS-DF relaying mode. $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10\text{dB}$. Results are given for path-loss exponent $\delta = 4$.

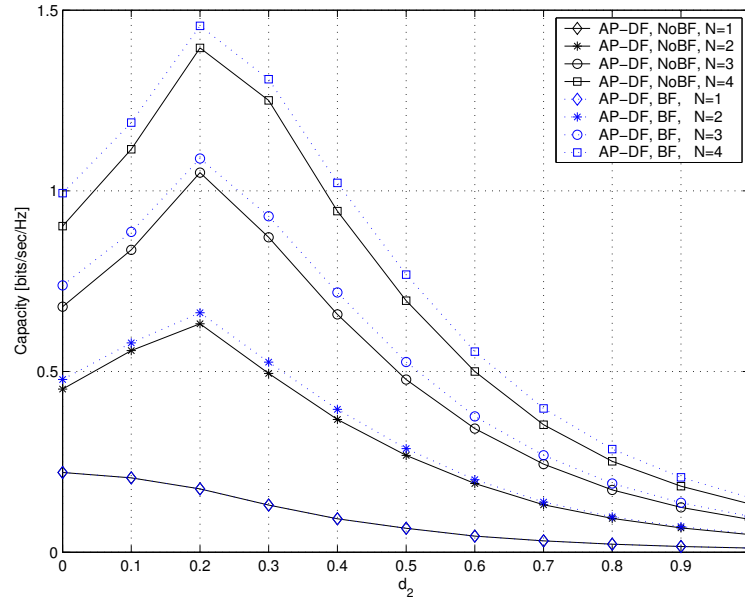


Figure 3.9: Ergodic capacities for $K = 4$ relays and $M = N = N_{\mathcal{R}_k} = \{1, 2, 3, 4\}$ for AP-DF relaying mode. $E_s/\sigma_n^2 = E_s/\sigma_r^2 = E_r/\sigma_n^2 = -10\text{dB}$. Results are given for path-loss exponent $\delta = 4$.

Chapter 4

MULTI-USER BEAMFORMING FOR MIMO MULTIPLE RELAY NETWORKS

In Chapter 2 and Chapter 3 it is pointed out that for point to point wireless communication systems, using multiple antennas at the transmit and/or receiver side introduces additional *degrees of freedom*. It has been shown that this property can be used to have increased *diversity order* and *link capacity* through *space-time coding* and *spatial multiplexing*, respectively.

In this chapter, we consider a MIMO wireless relay network scheme where a designated source and destination node communication assisted (for non-line-of-sight case) or relayed (for 2-hop case) by multiple relays. We consider the coherent AF mode transmission proposed in [23], where each relay node has been assigned to one of the multiplexed data streams and source and destination nodes assumed to have the same antenna numbers and each transmit-receiver antenna was paired virtually. Additionally, we assume that source node has full CSI, i.e., $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$ channel coefficients. To mitigate the interference seen by non-assigned relay nodes of an individual data stream, we propose a multi-user source node beamforming method by which system capacity is increased. We propose to use a beamforming method based on maximizing Sum of Signal-to-Leakage Ratios (SSLR) for all data streams. With this method, sum of signal strengths at the set of relay nodes that are assigned to corresponding data stream will be maximized and corresponding interference terms at other relay nodes that are not in the assigned set will be diminished. So the propagation of interference terms is diminished as well. This method does not have any constraints on transmit antenna number as other interference cancelation methods which makes this multi-user beamforming method more efficient than other beamforming methods aimed to fully cancel co-channel interference at receiving nodes.

In this chapter, we use a multi-user beamforming method where source node uses all of its spatial degrees of freedom by assigning different disjoint set of receiver(s) to each

data stream. Each data stream is multiplied by a corresponding beamforming vector to make Signal-to-Interference-and-Noise Ratios (SINR) at all receiver nodes maximized. It has been shown in [53] that multi-user MIMO downlink system capacity can be improved by a beamforming algorithm which is based on maximizing Signal-to-Leakage Ratio (SLR) for each multiplexed data where each data stream is intended for different receivers. Due to the objective function used in this method, unlike other perfect co-channel interference canceling methods, there is no constraint on transmit antenna numbers which makes this approach a convenient for wireless networks.

We analyze effects of multi-user beamforming at source node when the system members have multiple antennas for the proposed AF transmission mode given in [23] which pairs each antennas of source and destination nodes for individual communications and assigns each data stream to a set of relay nodes and those relay nodes coherently assist the transmission of assigned data stream and the remainder data streams assumed to be interference. It has been shown that this AF mode, asymptotically in relay number, turns point-to-point MIMO link with multiplexing gain $N/2$ and distributed array gain of K where N is source node antenna number and K is relay number.

In this chapter, we proposed a multi-user beamforming scheme builded upon the AF mode proposed in [23]. Considering the AF mode structure, we selected our problem statement similar to the one in [53]. We selected our beamforming vectors so as to maximize, for each data stream, the ratio of sum of one data stream's power at intended receivers to sum of that stream's power at non-intended receivers. With this, we will not only mitigate the interference seen by each relay node but also prevent the propagation of interference terms to destination node. We give the capacity expressions in a compact form and for different scenarios give some numerical results. It is seen that with multi-user beamforming the AF mode benefit from *transmitter array gain*. We showed capacity expressions as a function of relay numbers for fixed antenna numbers to see the effect of increase in relay numbers. It is seen that increase in relay number result in *array gains* due to each data stream is relayed over increase set of relays which result in an increase in average SINR at the destination node.

In the next section, we describe the system model and the protocol description used in whole paper. We introduced multi-user beamforming algorithm in Section- 4.2 and

capacity upper bound of AF mode transmission in Section- 4.3. Then in Section-4.4, we derive ergodic capacity expressions for AF mode with and without CSI at transmitters and the AF relaying mode in Section- 4.4.1. We give numerical results in Section- 4.4.2 and conclude with Section- 4.5.

4.1 Protocol Description and System Model

In this chapter, we consider the same system model and relaying protocol given in Chapter 3. The channel coefficients between $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}$ links, for $k = 1, 2, \dots, K$, are Rayleigh block fading channels where the channels remain constant for the duration of two consecutive time slots. The channels are assumed to be independent each other and between two consecutive time slots realizations.

In terms of channel knowledge at participating nodes, the system performance is investigated in Section- 4.4 in two cases:

- Perfect forward and backward CSI at each corresponding relay nodes and full CSI at destination node, and
- Additional to previous case, source node have perfect CSI which means it has perfect $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$ channel coefficients.

The second case enables us to have the opportunity of multi-user beamforming for mitigating interference terms at each relay node.

We use $\mathbf{H}_D$, $\mathbf{H}_k$ and $\mathbf{G}_k$ notations for channel matrices for $\mathcal{S} \rightarrow \mathcal{D}$, $\mathcal{S} \rightarrow \mathcal{R}_k$ and $\mathcal{R}_k \rightarrow \mathcal{D}$ links, respectively. Throughout this chapter we will assume for simplicity that the source and destination nodes have the same number of antennas and denoted by N , while each relay node has M , $\forall k$, antennas. Signal models, input-output relations are derived for general case. Noise terms at the front end of each antenna of relay nodes and destination node are assumed to be i.i.d RVs with $\mathcal{CN}(0, \sigma_{r_k}^2)$, $\forall k$ and $\mathcal{CN}(0, \sigma_n^2)$, respectively.

The system model considered in this chapter is depicted in Fig.4.1. With or without CSI at transmitters, the total transmitted power is independent of transmit antenna numbers and we assume that total transmitted powers for source node and relay nodes are E_s and E_{r_k} for $k \in \{1, 2, \dots, K\}$, respectively. At first time slot, source node transmits symbol

vector $\mathbf{x}$, where $\mathbf{x} \in \mathbf{C}^{N \times 1}$, which has the covariance matrix $\mathbf{Q}_s = \mathcal{E}[\mathbf{x}\mathbf{x}^H]$ and $E_s = \text{tr}(\mathbf{Q}_s)$. At the second time slot, relay nodes transmit symbol vectors $\mathbf{t}_k$, where $\mathbf{t}_k \in \mathbf{C}^{M \times 1}$, with covariance matrix $\mathbf{Q}_{r_k} = \mathcal{E}[\mathbf{t}_k\mathbf{t}_k^H]$ and $E_{r_k} = \text{tr}(\mathbf{Q}_{r_k})$ for $k \in \{1, 2, \dots, K\}$.

4.1.1 Signal Models at Destination and Relay Nodes

In this chapter, we consider the same wireless network scenario as in [23] which consists of $K + 2$ nodes, with a designated source-destination node pair and K relay nodes located randomly and independently in a fixed area. Opposed to [23], we also assume direct-link (DL) between source and destination node is present. We follow the same notations as in [23]. We assumed perfect synchronization between nodes during transmissions and receptions. At the first time slot, the input-output relations for $\mathcal{S} \rightarrow \mathcal{D}$ and $\mathcal{S} \rightarrow \mathcal{R}_k$ links are given by, respectively,

$$\mathbf{y}_1 = \mathbf{H}_D \mathbf{x} + \mathbf{n}_{d,1}, \quad (4.1)$$

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k \quad k \in \{1, 2, \dots, K\}.$$

where $\mathbf{y}_1$ and $\mathbf{r}_k$ denotes $N \times 1$ and $M \times 1$ received vector signals, $\mathbf{H}_D = [\mathbf{h}_{d,1} \ \mathbf{h}_{d,2} \ \dots \ \mathbf{h}_{d,N}]^T$ denotes $N \times N$ matrix for $\mathcal{S} \rightarrow \mathcal{D}$ link. $\mathbf{H}_k = [\mathbf{h}_{k,1} \ \mathbf{h}_{k,2} \ \dots \ \mathbf{h}_{k,N}]$ is the $M \times N$ random channel matrix corresponding to $\mathcal{S} \rightarrow \mathcal{R}_k$ link, independent across relay nodes and consisting of i.i.d. $\mathcal{CN}(0, 1)$ entries, $\mathbf{x} = [x_1^T \ x_2^T \ \dots \ x_N^T]^T$ is the zero mean $N \times 1$ circularly symmetric complex Gaussian transmit signal vector satisfying $\mathcal{E}[\mathbf{x}\mathbf{x}^H] = (E_s/N)\mathbf{I}_N$, and $\mathbf{n}_{d,i}$, $\mathbf{n}_k$ are $N \times 1$ and $M \times 1$ spatio-temporally white circularly symmetric complex Gaussian noise vector sequences with the covariance matrices $\mathcal{E}[\mathbf{n}_{d,i}\mathbf{n}_{d,i}^H] = \sigma_n^2 \mathbf{I}_N$ for $i = \{1, 2\}$, $\mathcal{E}[\mathbf{n}_k\mathbf{n}_k^H] = \sigma_{r_k}^2 \mathbf{I}_M$ for $k \in \{1, 2, \dots, K\}$.

At the second time slot, due to the specified protocol in Chapter 3, all of the relay nodes transmit and source node stays in idle mode. Each relay node processes its received vector signal $\mathbf{r}_k$ to produce the $M \times 1$ vector signal $\mathbf{t}_k$, which is then transmitted to the destination node over the second time slot. The $N \times 1$ vector signal received at the destination node is consequently given by

$$\mathbf{y}_2 = \sum_{k=1}^K \mathbf{G}_k \mathbf{t}_k + \mathbf{n}_{d,2} \quad (4.2)$$

where $\mathbf{G}_k = [\mathbf{g}_{k,1} \ \mathbf{g}_{k,2} \ \dots \ \mathbf{g}_{k,N}]^T$ is the corresponding $N \times M$ channel matrix for $\mathcal{R}_k \rightarrow \mathcal{D}$

link with i.i.d. $\mathcal{CN}(0, 1)$ entries, independent across k . The transmit signal vectors $\mathbf{t}_k$ will in general depend both on the forward and backward channels, $\mathbf{G}_k$ and $\mathbf{H}_k$, respectively, and are chosen to satisfy the average power constraint $\text{tr}(\mathcal{E}[\mathbf{t}_k \mathbf{t}_k^H]) \leq E_{r_k}$ for $k \in \{1, 2, \dots, K\}$.

4.1.2 A New AF Relaying Mode for Wireless Networks

In this chapter, we investigate the AF mode transmission that has been proposed in [23] with an additional assumption of DL between $\mathcal{S}$ and $\mathcal{D}$ nodes. Apart from other proposed AF modes for MIMO multi-relay wireless channels, i.e. [22], in this mode each of the antenna stream at source node is paired with a receiver antenna at destination node. Assisting relay nodes are partitioned to each of data stream. The set of relay nodes assigned to i th data stream is denoted by $\mathcal{X}_i$, and we denote the cardinality of set $\mathcal{X}_i$ by $|\mathcal{X}_i|$. During the first time slot, source node broadcasts data streams and both relay nodes and destination node listen for source transmission. In the second time slot, source node turns to idle mode and relay nodes multiply received signal with appropriate match filters. Destination node performs independent decoding at each antenna, i.e., the receiver side of a transmit-receiver pair will assume other participating signal terms as interference and perform decoding with this structure. After the second time slot, destination node combines its received signal by MRC method and implement independent decoding for each antenna part.

For relay nodes, we assume they have their both forward CSI and backward CSI. During the first time slot, according to their assigned set they multiply their received signals with proper vector (match filtering) and after scaling to appropriate power level they direct their signal by beamforming to intended antenna at destination node. The system model considered in this paper is shown in Fig.4.1.

In the following section, we give a proper source beamforming model which is aimed to maximize SSLR at each set of relay nodes for each data streams.

4.2 Multi-User Beamforming For Maximizing Sum of Signal-To-Leakage Ratio

In single user point-to-point communications, beamforming is done by sending a single data to the channel direction which is the eigenvector of MIMO channel corresponding to maximum eigenvalue of that channel. While in multi-user MIMO wireless systems, for each user transmitter has multiple data streams these streams are transmitted by a proper

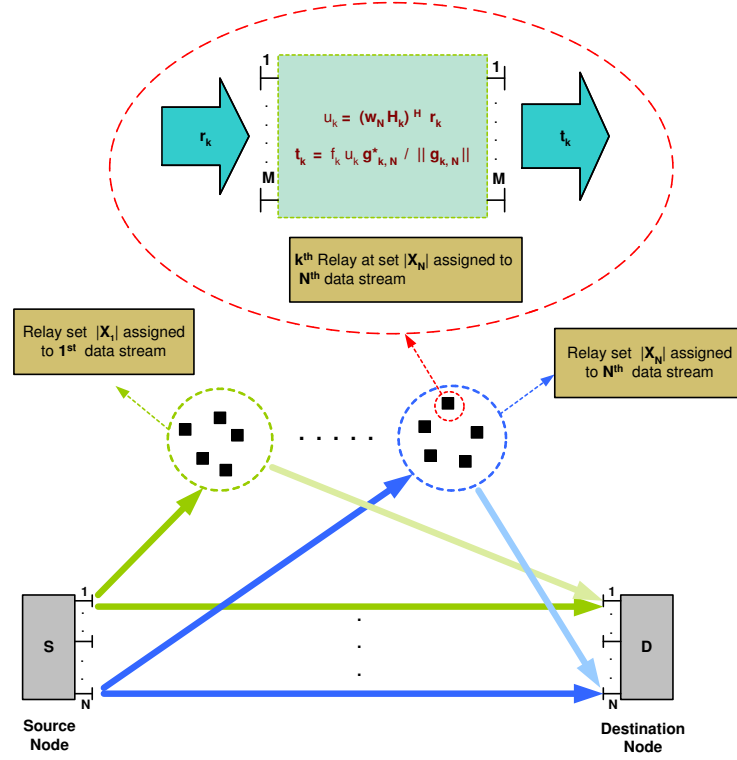


Figure 4.1: MIMO multiple relay scenario for the proposed AF mode transmission where each relay node set is assigned to only one data stream for assistance.

beamforming vector multiplication. The main purpose in multi-user beamforming is to minimize interference terms at each user.

For the AF mode we analyze, as each relay node set has an intended data stream for themselves, implementing source node beamforming would lead to system performance improvement due to interference cancelation at relay node and would mitigate the propagation of those interference terms to destination node which at the end will improve the system performance.

As performance metric we use a similar metric as in [53] where beamforming vectors are assessed to maximize SLR which results in minimizing interference terms seen by each of non-intended users for each data stream.

The transmit signal model at source node can be expressed as

$$\begin{aligned} \mathbf{x} &= \sum_{n=1}^N \mathbf{w}_n s_n \\ &= [\mathbf{w}_1 \mathbf{w}_2 \cdots \mathbf{w}_N] \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_N \end{bmatrix} = \mathbf{W} \mathbf{s} \end{aligned} \quad (4.3)$$

where s_n denotes the transmitted data intended for the relay set $\mathcal{X}_n$ and $\mathbf{w}_n$ represents $N \times 1$ beamforming vector where they satisfy the following constraints:

$$\sum_{n=1}^N \mathcal{E}[|s_n|^2] = \sum_{n=1}^N E_n = E_s \quad (4.4)$$

and

$$\|\mathbf{w}_n\|^2 = 1 \quad (4.5)$$

for $n \in \{1, 2, \dots, N\}$.

We will use $\mathcal{S} \rightarrow \mathcal{D}_l$ notation for link from source node to destination's l th antenna. With this in mind, we will now give our multi-user beamforming method according to the AF mode we are going to use. As mentioned above, in the first time slot l th data stream is intended to be received by both a set of relay nodes and at l th antenna of destination node. Our statement is to maximize the ratio of sum of the power at intended users and the interference power at other sides for each data stream. We call this statement as SSLR. So the power of l th data stream at the assigned set of relays, $\mathcal{X}_l$, and at l th antenna of destination node is given by

$$\sum_{k, \mathcal{R}_k \in \mathcal{X}_l} \|\mathbf{H}_k \mathbf{w}_l\|^2 + |\mathbf{h}_{d,l}^T \mathbf{w}_l|^2 \quad (4.6)$$

and the interference powers seen by non intended receivers can be expressed as follows

$$\sum_{k, \mathcal{R}_k \notin \mathcal{X}_l} \|\mathbf{H}_k \mathbf{w}_l\|^2 + \sum_{i=1, i \neq l}^N |\mathbf{h}_{d,i}^T \mathbf{w}_l|^2. \quad (4.7)$$

Our SSLR expression for l th data stream is given by ratio of (4.6) and (4.7) which is

$$SSLR_l = \frac{\sum_{k, \mathcal{R}_k \in \mathcal{X}_l} \|\mathbf{H}_k \mathbf{w}_l\|^2 + |\mathbf{h}_{d,l}^T \mathbf{w}_l|^2}{\sum_{k, \mathcal{R}_k \notin \mathcal{X}_l} \|\mathbf{H}_k \mathbf{w}_l\|^2 + \sum_{i=1, i \neq l}^N |\mathbf{h}_{d,i}^T \mathbf{w}_l|^2}. \quad (4.8)$$

As in [53], our problem statement is to design $\mathbf{w}_l$, $l = \{1, 2, \dots, N\}$ such that the SSLR is maximized for every data stream:

$$\mathbf{w}_l^{max} = \arg \max_{\mathbf{w}_l \in \mathbf{C}^{N \times 1}} \{SSLR_l\} \quad (4.9)$$

subject to $\|\mathbf{w}_l\|^2 = 1$, $l = \{1, 2, \dots, N\}$.

We can rewrite the expression in (4.8) as follows:

$$SSLR_l = \frac{\mathbf{w}_l^H \bar{\mathbf{H}}_l^H \bar{\mathbf{H}}_l \mathbf{w}_l}{\mathbf{w}_l^H \tilde{\mathbf{H}}_l^H \tilde{\mathbf{H}}_l \mathbf{w}_l} \quad (4.10)$$

where $\bar{\mathbf{H}}_l$ and $\tilde{\mathbf{H}}_l$ are given by

$$\bar{\mathbf{H}}_l = \left[\mathbf{H}_{\mathbf{u}_l(1)}^H \mathbf{H}_{\mathbf{u}_l(2)}^H \cdots \mathbf{H}_{\mathbf{u}_l(|\mathcal{X}_l|)}^H (\mathbf{h}_{d,l}^T)^H \right]^H \quad (4.11)$$

and

$$\tilde{\mathbf{H}}_l = \left[\mathbf{H}_{\mathbf{v}_l(1)}^H \mathbf{H}_{\mathbf{v}_l(2)}^H \cdots \mathbf{H}_{\mathbf{v}_l(K-|\mathcal{X}_l|)}^H (\mathbf{h}_{d,1}^T)^H \cdots (\mathbf{h}_{d,l-1}^T)^H (\mathbf{h}_{d,l+1}^T)^H \cdots (\mathbf{h}_{d,N}^T)^H \right]^H \quad (4.12)$$

where $\mathbf{u}_l$ is $|\mathcal{X}_l| \times 1$ size array that holds the relay indexes that are assigned to l th data stream and $\mathbf{v}_l$ is $(K - |\mathcal{X}_l|) \times 1$ size array that holds indexes of relays that are the compliment of the set $\mathcal{X}_l$.

It is easy to see that (4.10) has the form of generalizes eigenvalue problem¹ which have the following form of inequality:

$$\frac{\mathbf{w}_l^H \bar{\mathbf{H}}_l^H \bar{\mathbf{H}}_l \mathbf{w}_l}{\mathbf{w}_l^H \tilde{\mathbf{H}}_l^H \tilde{\mathbf{H}}_l \mathbf{w}_l} \leq \lambda_{max} \left(\bar{\mathbf{H}}_l^H \bar{\mathbf{H}}_l, \tilde{\mathbf{H}}_l^H \tilde{\mathbf{H}}_l \right) \quad (4.13)$$

where λ_{max} is the largest generalized eigenvalue of the matrix pair $\bar{\mathbf{H}}_l^H \bar{\mathbf{H}}_l$ and $\tilde{\mathbf{H}}_l^H \tilde{\mathbf{H}}_l$. Equality occurs if $\mathbf{w}_l$ is proportional to a generalized eigenvector that corresponds to the largest generalized eigenvalue.

4.3 Capacity Upper-Bound on MIMO Relay Networks

As described in Section-4.1, we will look for ergodic capacity of coherent MIMO relay networks when DL between $\mathcal{S}$ and $\mathcal{D}$ node is present. In this section, we will derive upper bound on the capacity of proposed network. Applying the cut-set theorem in [27, Theorem

¹Generalized eigenvalue problem is given by $\mathbf{Ax} = \lambda \mathbf{Bx} \Rightarrow (\mathbf{A} - \lambda \mathbf{B})\mathbf{x} = \mathbf{0}$

14.10.1] and using the broadcast cut, the capacity of the MIMO relay network is upper bounded by

$$C_u = \frac{1}{2} \mathcal{E}_{\{\mathbf{H}_D, \{\mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K\}} [I(\mathbf{x}; \mathbf{r}_1, \dots, \mathbf{r}_K, \mathbf{y} | \mathbf{t}_1, \dots, \mathbf{t}_K)] \quad (4.14)$$

where the factor $1/2$ results from the fact that data is transmitted over two time slots. Applying the chain rule for mutual information [27] we obtain

$$I(\mathbf{x}; \mathbf{r}_1, \dots, \mathbf{r}_K, \mathbf{y} | \mathbf{t}_1, \dots, \mathbf{t}_K) = I(\mathbf{x}; \mathbf{r}_1, \dots, \mathbf{r}_K | \mathbf{t}_1, \dots, \mathbf{t}_K) + I(\mathbf{x}; \mathbf{y} | \mathbf{r}_1, \dots, \mathbf{r}_K, \mathbf{t}_1, \dots, \mathbf{t}_K) \quad (4.15)$$

where $\mathbf{y} = [\mathbf{y}_1^T \ \mathbf{y}_2^T]^T$ is the collection of received signals for first and second time slots. Conditioning on the $\mathbf{t}_k$ in the first term on the right-hand-side (RHS) can be dropped since neither $\mathbf{x}$ nor the $\mathbf{r}_k$ depend on the $\mathbf{t}_k$.

Conditioning on the $\mathbf{t}_k$ and assuming perfect knowledge of $\mathbf{H}_D$ and $\mathbf{G}_k$ channels at the receiver results in the second term on the RHS of (4.15) being equal to $I(\mathbf{x}; \mathbf{y}_1)$ due to both of them are independent of $\mathbf{t}_k$ and $\mathbf{r}_k$. We can therefore summarize our result as

$$C_u = \frac{1}{2} \{ \mathcal{E}_{\{\mathbf{H}_k\}_{k=1}^K} [I(\mathbf{x}; \mathbf{r}_1, \dots, \mathbf{r}_K)] + \mathcal{E}_{\mathbf{H}_D} [I(\mathbf{x}; \mathbf{y}_1)] \} \quad (4.16)$$

Recalling that $\mathbf{x}$ was assumed circularly symmetric complex Gaussian with uniform power distribution over antennas that is $\mathcal{E}[\mathbf{x}\mathbf{x}^H] = (E_s/N) \mathbf{I}_N$ and all of relay noise terms have the same covariance of $\mathcal{E}[\mathbf{n}_k\mathbf{n}_k^H] = \sigma_r^2 \mathbf{I}_M$, for $k = 1, 2, \dots, K$, we get [13]

$$C_u = \frac{1}{2} \left\{ \mathcal{E}_{\{\mathbf{H}_k\}_{k=1}^K} \left[\log \left(\det \left(\mathbf{I}_N + \frac{E_s}{N\sigma_r^2} \sum_{k=1}^K \mathbf{H}_k^H \mathbf{H}_k \right) \right) \right] \right. \\ \left. + \mathcal{E}_{\mathbf{H}_D} \left[\log \left(\det \left(\mathbf{I}_N + \frac{E_s}{N\sigma_n^2} \mathbf{H}_D^H \mathbf{H}_D \right) \right) \right] \right\} \quad (4.17)$$

and with Jensens inequality [27] the capacity expression is upper bounded as follow:

$$C_u \leq \frac{N}{2} \log \left(1 + \frac{KME_s}{N\sigma_r^2} \right) + \frac{N}{2} \log \left(1 + \frac{E_s}{\sigma_n^2} \right) = C_{upper} \quad (4.18)$$

which concludes upper bound of the coherent MIMO relay network when no CSI is available at transmitting node.

In the next section, we will derive signal models for the proposed AF mode transmission and resulting capacity expressions for both non-beamforming and beamforming cases at source node.

4.4 Coherent AF Mode Multi-Relay Network Capacity

In this section, we will give signal models for the AF mode proposed in [23] which does not require any cooperation between nodes and employs single antenna decoding at the destination node where decoding is done independently antenna by antenna. It has been shown in [23] that the capacity achieved by this mode transmission asymptotically in K approaches the cut-set upper bound.

4.4.1 The AF Relaying Protocol and Its Capacity

In the proposed transmission mode each relay nodes assigned to one out of the N transmit-receiver antenna pairs [23]. We assume that the l th transmit antenna and l th receiver antenna constitute the l th pair. The relay nodes are partitioned into groups with each group being responsible for one of the N source-destination antenna pairs which is depicted in Fig.4.1. As explained, the set of relay nodes assigned to l th transmit-receive antenna pair will be denoted by $\mathcal{X}_l$.

In the following expressions, we will use the general expression for transmitted signal given in (4.3). For no CSI case, beamforming matrix will be $\mathbf{W} = \mathbf{I}_N$, while with CSI availability at source node $\mathbf{W}$ is evaluated as explained in Section-4.2. So capacity expressions for both cases is the same except beamforming matrix entries. We will now find the general capacity expressions for both cases.

At first time slot, the received signal expressions at destination node and k th relay node given in (4.1) can be rewritten as

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{H}_D \sum_{i=1}^N \mathbf{w}_i s_i + \mathbf{n}_{d,1}, \\ \mathbf{r}_k &= \mathbf{H}_k \sum_{i=1}^N \mathbf{w}_i s_i + \mathbf{n}_k \\ &= \mathbf{H}_k \mathbf{w}_l s_l + \mathbf{H}_k \sum_{i=1, i \neq l}^N \mathbf{w}_i s_i + \mathbf{n}_k, \quad k \in \{1, 2, \dots, K\}, \end{aligned} \tag{4.19}$$

respectively.

We follow the same steps in [23]. Assuming the k th relay node is assigned to the l th transmit-receive antenna pair. Upon reception of $\mathbf{r}_k$, the relay performs matched-filtering

with respect to the assigned backward channel to obtain

$$u_k = \mathbf{w}_l^H \mathbf{H}_k^H \mathbf{r}_k = \|\mathbf{H}_k \mathbf{w}_l\|^2 s_l + (\mathbf{H}_k \mathbf{w}_l)^H \mathbf{H}_k \sum_{i=1, i \neq l}^N \mathbf{w}_i s_i + (\mathbf{H}_k \mathbf{w}_l)^H \mathbf{n}_k \quad (4.20)$$

for $k = \{1, 2, \dots, K\}$. As described in Section- 4.1, $\mathcal{E}[|s_i|^2] = E_i$, and according to relay transmit power constraint, we scale received signal by

$$f_k = \sqrt{\frac{E_{r_k}}{\|\mathbf{H}_k \mathbf{w}_l\|^4 E_l + \sum_{i=1, i \neq l}^N |(\mathbf{H}_k \mathbf{w}_l)^H \mathbf{H}_k \mathbf{w}_i|^2 E_i + \|(\mathbf{H}_k \mathbf{w}_l)^H\|^2 \sigma_{r_k}^2}} \quad (4.21)$$

and assuming forward channel knowledge at the relay we multiply the scaled signal with a unit-norm beamforming vector corresponding to $\mathcal{R}_k \rightarrow \mathcal{D}_l$ link and get the k th relay's transmitted signal as

$$\mathbf{t}_k = f_k u_k \frac{\mathbf{g}_{k,l}^*}{\|\mathbf{g}_{k,l}\|} \quad (4.22)$$

Now, the signal forms received at i th receiver antenna of the destination node at the first and second time slots is given by, respectively,

$$y_{1,i} = \mathbf{h}_{d,i}^T \mathbf{w}_i s_i + \mathbf{h}_{d,i}^T \sum_{j=1, j \neq i}^N \mathbf{w}_j s_j + n_{1,i} \quad (4.23)$$

$$y_{2,i} = \sum_{n=1}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_n} \mathbf{g}_{k,i}^T \mathbf{t}_k \right) + n_{2,i}, \quad i = \{1, 2, \dots, K\}.$$

For second time slot, by using (4.20), (4.21) and (4.22) the received signal $y_{2,i}$ can be rewritten as

$$\begin{aligned} y_{2,i} &= \sum_{k, \mathcal{R}_k \in \mathcal{X}_i} \mathbf{g}_{k,i}^T \mathbf{t}_k + \sum_{n=1, n \neq i}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_n} \mathbf{g}_{k,i}^T \mathbf{t}_k \right) + n_{2,i} \\ &= \sum_{k, \mathcal{R}_k \in \mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k u_k + \sum_{n=1, n \neq i}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_n} \frac{\mathbf{g}_{k,i}^T \mathbf{g}_{k,n}^*}{\|\mathbf{g}_{k,n}\|} f_k u_k \right) + n_{2,i} \\ &= \sum_{k, \mathcal{R}_k \in \mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k \left(\|\mathbf{H}_k \mathbf{w}_i\|^2 s_i + (\mathbf{H}_k \mathbf{w}_i)^H \mathbf{H}_k \sum_{j=1, j \neq i}^N \mathbf{w}_j s_j + (\mathbf{H}_k \mathbf{w}_i)^H \mathbf{n}_k \right) \\ &\quad + \sum_{n=1, n \neq i}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_n} \frac{\mathbf{g}_{k,i}^T \mathbf{g}_{k,n}^*}{\|\mathbf{g}_{k,n}\|} f_k \left(\|\mathbf{H}_k \mathbf{w}_n\|^2 s_n + (\mathbf{H}_k \mathbf{w}_n)^H \mathbf{H}_k \sum_{j=1, j \neq n}^N \mathbf{w}_j s_j \right. \right. \\ &\quad \left. \left. + (\mathbf{H}_k \mathbf{w}_n)^H \mathbf{n}_k \right) \right) + n_{2,i} \end{aligned} \quad (4.24)$$

We observe that $y_{2,i}$ consists of a contribution from the desired data stream s_i , interference signal terms from the data streams s_n with $n \neq i$, forwarded noise from the relay terminals and receiver noise. To be able to express capacity, we should show the signal, interference, and noise contributions to $y_{2,i}$ in detail. After some manipulation on the signal model at (4.24), we get the following compact form

$$\begin{aligned}
y_{2,i} = & \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k \|\mathbf{H}_k \mathbf{w}_i\|^2 + \sum_{n=1, n \neq i}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_n} \frac{\mathbf{g}_{k,i}^T \mathbf{g}_{k,n}^*}{\|\mathbf{g}_{k,n}\|} f_k (\mathbf{H}_k \mathbf{w}_n)^H \mathbf{H}_k \mathbf{w}_i \right) \right) s_i \\
& + \sum_{j=1, j \neq i}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k (\mathbf{H}_k \mathbf{w}_i)^H \mathbf{H}_k \mathbf{w}_j + \sum_{k, \mathcal{R}_k \in \mathcal{X}_j} \frac{\mathbf{g}_{k,i}^T \mathbf{g}_{k,j}^*}{\|\mathbf{g}_{k,j}\|} f_k \|\mathbf{H}_k \mathbf{w}_j\|^2 \right. \\
& \left. + \sum_{m=1, m \neq \{i,j\}}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_m} \frac{\mathbf{g}_{k,i}^T \mathbf{g}_{k,m}^*}{\|\mathbf{g}_{k,m}\|} f_k (\mathbf{H}_k \mathbf{w}_m)^H \mathbf{H}_k \mathbf{w}_j \right) \right) s_j + N_{2,i}
\end{aligned} \tag{4.25}$$

which can be expressed as

$$y_{2,i} = h_i^{sig} s_i + \sum_{j=1, j \neq i}^N h_{i,j}^{int} s_j + N_{2,i} \tag{4.26}$$

where h_i^{sig} denotes the effective scalar channel gain for the data stream transmitted from the i th source terminal antenna, $h_{i,j}^{int}$ stands for the effective channel seen by the interference from s_j to s_i and $N_{2,i}$ denotes the effective noise term. The effective noise term is given by following equation

$$N_{2,i} = \sum_{k, \mathcal{R}_k \in \mathcal{X}_i} \|\mathbf{g}_{k,i}\| f_k (\mathbf{H}_k \mathbf{w}_i)^H \mathbf{n}_k + \sum_{n=1, n \neq i}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_n} \frac{\mathbf{g}_{k,i}^T \mathbf{g}_{k,n}^*}{\|\mathbf{g}_{k,n}\|} f_k (\mathbf{H}_k \mathbf{w}_n)^H \mathbf{n}_k \right) + n_{2,i}. \tag{4.27}$$

We assume that noise vector have the same covariance matrices $\mathcal{E}[\mathbf{n}_k \mathbf{n}_k^H] = \sigma_r^2 \mathbf{I}_M$ for $k = \{1, 2, \dots, K\}$, and received signals at each antenna of destination node at first and second time slot are combined with MRC which result in summation of SINRs at each time slot. SINRs at first and second time slots are given by

$$SINR_{1,i} = \frac{|\mathbf{h}_{d,i}^T \mathbf{w}_i|^2 E_i}{\sum_{j=1, j \neq i}^N |\mathbf{h}_{d,i} \mathbf{w}_j|^2 E_j + \sigma_n^2} \tag{4.28}$$

$$SINR_{2,i} = \frac{|h_i^{sig}|^2 E_i}{\sum_{j=1, j \neq i}^N |h_{i,j}^{int}|^2 E_j + \sigma_r^2 \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_i} a_{k,i} + \sum_{n=1, n \neq i}^N \left(\sum_{k, \mathcal{R}_k \in \mathcal{X}_n} b_{k,i,n} \right) \right) + \sigma_n^2} \quad (4.29)$$

where $a_{k,i}$ and $b_{k,i,n}$ are given by

$$a_{k,i} = f_k^2 \|\mathbf{g}_{k,i}\|^2 \|\mathbf{H}_k \mathbf{w}_i\|^2, \quad b_{k,i,n} = \frac{f_k^2 \|\mathbf{H}_k \mathbf{w}_n\|^2 |\mathbf{g}_{k,i}^T \mathbf{g}_{k,n}^*|^2}{\|\mathbf{g}_{k,n}\|^2}. \quad (4.30)$$

With (4.28) and (4.28) in mind, we can express the capacity of the proposed AF mode as follow

$$C = \sum_{i=1}^N \mathcal{E}_{\{\mathbf{H}_D, \{\mathbf{H}_k, \mathbf{G}_k\}_{k=1}^K\}} [I_i] \quad (4.31)$$

where mutual information for i th data stream is given by

$$I_i = \frac{1}{2} \log(1 + SINR_{1,i} + SINR_{2,i}), \quad i = 1, 2, \dots, N. \quad (4.32)$$

Here, we used the fact that perfect channel knowledge at destination node which results in the noise plus interference contributions in (4.26) being circularly symmetric complex Gaussian.

After expressing the capacity in compact form for the proposed AF mode transmission, it is now easy to see that for both scenarios we interested the only difference will be the beamforming vectors, $\mathbf{w}_i$ for $i = \{1, 2, \dots, N\}$ or $\mathbf{W}$ matrix given in (4.3) which will be $\mathbf{I}_N$ for no CSI case at source node and capacity expressions boils down the expression given in [23].

4.4.2 Numerical Results

To give the performance improvement supplied by source beamforming with increasing relay number, we provide a numerical result for $N = 2$, $M = 1$ and $\sigma_r^2 = \sigma_n^2$ and $E_s/\sigma_n^2 = E_{r_k}/\sigma_n^2 = 10\text{dB}$ for $k = \{1, 2, \dots, K\}$. For simplicity we assign each data stream the same number of relays, which mean K is a multiple of N and $|\mathcal{X}_1| = |\mathcal{X}_1| = \dots = |\mathcal{X}_N| = K/N$. Fig. 4.2 shows the upper bound on coherent network capacity given in (4.18) and the capacity achieved by the matched-filtering based relaying protocol described in Section-4.4.1 for both beamforming and non-beamforming cases. With increasing the relay number in both cases capacity goes up. And beamforming is always higher than non-beamform

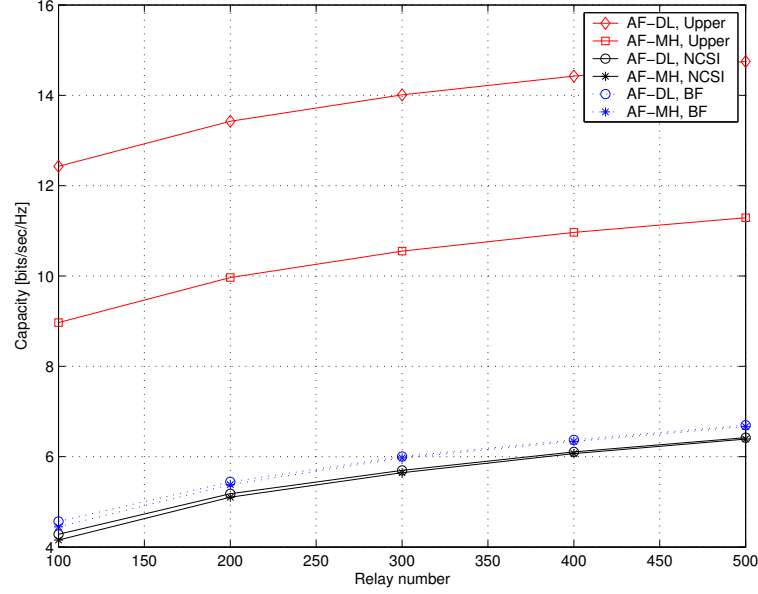


Figure 4.2: Achievable capacities for the proposed AF mode with Direct-Link(DL) and Multi-Hopping(MH) cases. $N = 2$, $M = 1$ and $E_s/\sigma_n^2 = E_s/\sigma_r^2 = 10\text{dB}$ for $\sigma_n^2 = \sigma_r^2$.

case with an amount of ten percent. It is also intuitive that increasing relay nodes' antenna number will make this difference higher due to additional array gain achieved by increased antenna size at relay nodes.

Again to see the performance of both cases, we also give some numerical results where relay number is fixed and capacity expressions are given as a function of E_s/σ_r^2 with $N = 2$ and $M = 1$ and $E_s/\sigma_n^2 = E_{r_k}/\sigma_n^2 = 10\text{dB}$, $\forall k$. With these settings Fig. 4.3 and Fig. 4.4 show the corresponding results for both cases of, with and without CSI at source node cases, the proposed AF mode transmission for relay numbers $K \in \{4, 8\}$, respectively. It can be seen that increasing relay number results in an increase with higher rate for beamforming case. We also give capacity results in Fig. 4.5 and Fig. 4.6 for the same settings expect source and destination node antenna number which is set to $N = 4$. With increased number of transmit antenna number, it can easily be seen that resulting capacity terms get smaller. We can say that for the proposed AF relaying mode instead of increasing source and destination node antenna number, increasing number of assisting relay numbers gives better performance. This can be interpreted as follows : with increasing transmit antenna number, while $\mathcal{S} \rightarrow \mathcal{D}$ link degrees of freedom increases, interference terms seen by

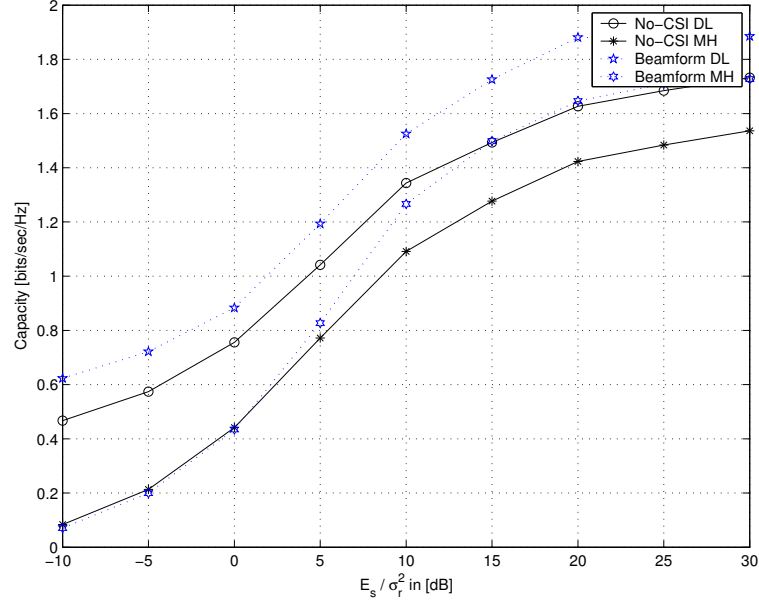


Figure 4.3: Achievable capacities for the proposed AF mode with Direct-Link(DL) and Multi-Hopping(MH) cases. $N = 2$, $M = 1$ and $E_s/\sigma_n^2 = E_r/\sigma_n^2 = 10\text{dB}$ for $K = 4$ relays in the scenario.

relay nodes and also by destination node deteriorates the system performance more and as a result the effective capacity get lower.

4.5 Conclusion Remarks

The coherent AF relaying mode that we consider here has been shown that it asymptotically (in K) turns the network into a point-to-point MIMO link with *multiplexing gain* $N/2$ and a *distributed per-stream array gain* K . Besides realizing multiplexing gain and *distributed array gain*, the proposed relaying mode also orthogonalizes the effective MIMO channel between source and destination nodes and hence multi-stream interference is completely eliminated without cooperation between the relay nodes [23].

In this proposed mode, there is no need for sophisticated *joint decoding* schemes such as successive cancelation or sphere decoding at destination node which makes this transmission mode attractive when compared to other modes which use *joint processing* between the receive antennas to achieving a multiplexing gain of N . Due to coherent combining of each data stream at their intended destination node antennas, the received power of the desired

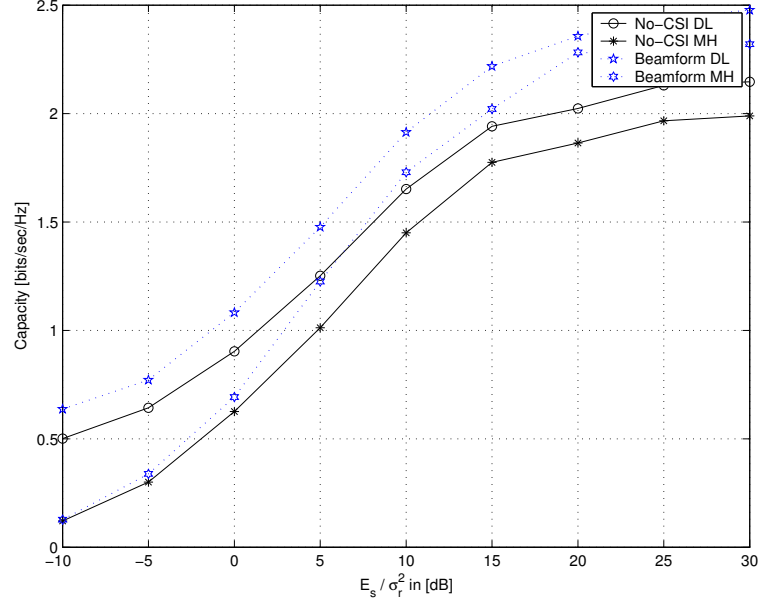


Figure 4.4: Achievable capacities for the proposed AF mode with Direct-Link(DL) and Multi-Hopping(MH) cases. $N = 2$, $M = 1$ and $E_s/\sigma_n^2 = E_r/\sigma_n^2 = 10\text{dB}$ for $K = 8$ relays in the scenario.

signal is increased by a factor of K over the noise and interference terms which can be interpreted as *distributed diversity gain*.

In this paper, we showed with the usage of multi-user beamforming that a *transmit array gain* is also achievable with the proposed AF mode. The multi-user beamforming we consider here is to maximize SSLR of each data stream. Considering the structure of the AF mode, with this beamforming mechanism the interference seen by each set of relays that are not assigned to a data stream is mitigated and consequently the propagation of interference terms are also mitigated. This makes our proposition to supply additional array gain with the knowledge of CSI for $\mathcal{S} \rightarrow \mathcal{R}_k, \forall k$ and $\mathcal{S} \rightarrow \mathcal{D}$, no need of the knowledge of $\mathcal{R}_k \rightarrow \mathcal{D}$ links.

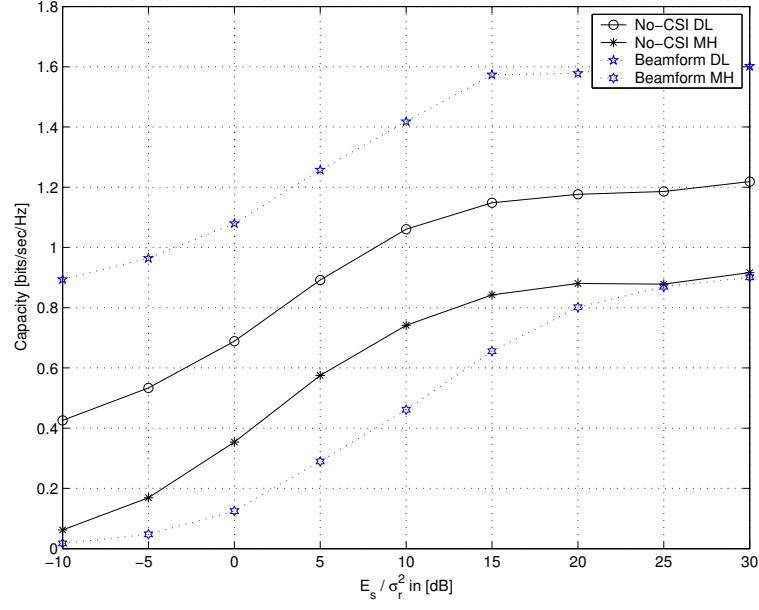


Figure 4.5: Achievable capacities for the proposed AF mode with Direct-Link(DL) and Multi-Hopping(MH) cases. $N = 4$, $M = 1$ and $E_s/\sigma_n^2 = E_r/\sigma_n^2 = 10\text{dB}$ for $K = 4$ relays in the scenario.

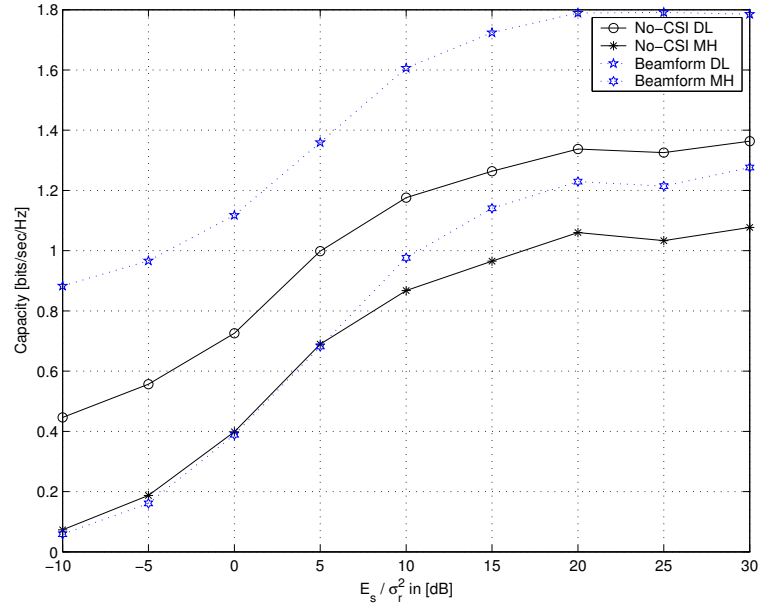


Figure 4.6: Achievable capacities for the proposed AF mode with Direct-Link(DL) and Multi-Hopping(MH) cases. $N = 4$, $M = 1$ and $E_s/\sigma_n^2 = E_r/\sigma_n^2 = 10\text{dB}$ for $K = 8$ relays in the scenario.

Chapter 5

COOPERATIVE COMMUNICATIONS

Up to now, in Chapter 3 and Chapter 4 we considered wireless relay networks which consist of only single information source node. We assumed that relay nodes can not transmit and receive at the same time which is the case of realistic radio hardware constraint. In the previous chapters due to the specified transmission protocol, we further assumed that source and relay node do not transmit at the same time.

In this chapter, we include wireless relay networks with multiple information source node. In contrast to wireless relay networks in this scenarios users cooperate with each other to convey information through a destination node (user or base station). In cooperative communication, transmission through a destination node is done by sharing system resources (time, bandwidth and power) and cooperating to transmit each others data. This type of cooperation among user provides additional benefits over direct transmission or no cooperation case. This type of systems is called *User Cooperative Diversity* or *Cooperative Communication* and first examined in [24] and [25].

Cooperative diversity is an emerging diversity technique in wireless networks which allows single-antenna mobiles to reap some of the benefits of MIMO systems. In [24], [25] and [1] it is showed that cooperating mobiles in a wireless network provide not only higher data rates, but also decreases sensitivity to channel variations (increasing diversity order). In [1] cooperation among source nodes is investigated and achievable rate regions for different coding schemes is derived and it is shown that cooperation among users provides significant gains over non-cooperation case with CSI assumption at source nodes. Besides, in [1], it is showed that with no CSI at source nodes cooperation does not provide any gain over non-cooperation in terms of the maximum sum-rate. In the all studies specified above, full-duplex transmission is assumed.

To exploit spatial diversity in wireless environment various cooperative coding schemes have been proposed in [2, 26, 54, 55]. The increasing studies for designing effective channel

codes came with the concept of *Coded Cooperation* first proposed by Hunter and Nosratinia in [2, 49, 55–57]. Coded cooperation works by sending different portions of each user’s code word via two independent fading paths. The basic idea is that each user tries to transmit incremental redundancy for its partner. Whenever that is not possible, the users automatically revert to non-cooperative mode. The key to the efficiency of coded cooperation is that all this is managed automatically through code design, with no feedback between the users [55]. Hunter and Nosratinia [2] proposed the use of rate compatible punctured convolution (RCPC) codes for the partnering mobiles. In [54] channel coding for single antenna case and in [26] space-time coding for multiple antenna case are considered and it was shown that these codes provide full cooperative diversity. In [49] turbo codes are applied to the coded cooperation framework.

In this chapter, we will briefly give signal models and system performance in terms of achievable sum-of-rates for cooperative communication. While we consider the signal models proposed in [1], the ones proposed in [24] and [25] are similar in terms of utilization of cooperation. After understanding the benefits of cooperative communications, we will give a recently proposed implementation of cooperative communication which is called *coded cooperation*. For better illustration of benefits of cooperative communication, we will give some results for different scenarios.

In this chapter, first Section-5.1 we are going to briefly describe the system model and give the conclusions derived in [24] and [1]. Then, in Section-5.2 we will again briefly mention about an implementation of user cooperation via coded cooperation. And finally, we will show some numerical results for various scenarios for coded cooperation in Section-5.2.2.

5.1 Cooperative Diversity

Cooperative communication refers to processing of broadcasted information, which is targeted for a pre-assigned destination node, at the surrounding nodes and retransmission towards the destination to create spatial diversity, thereby to obtain higher throughput and reliability [1, 24]. In cooperative communication, apart from relay channels, each of the partners is responsible for transmitting not only their own information, but also the information of their partners, which they receive and detect. With these in mind, cooperative systems can be thought as a virtual antenna array, where each antenna in the array corresponds

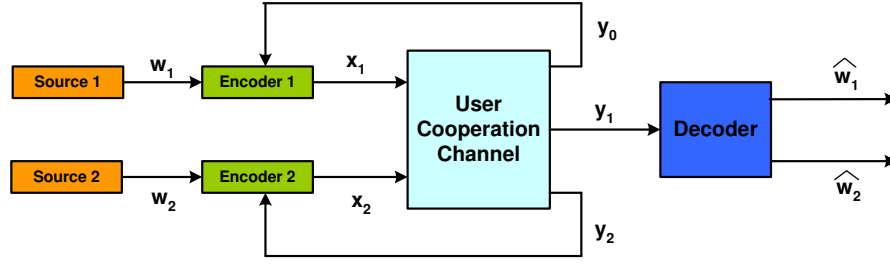


Figure 5.1: Block diagram for a multiple-access channel with cooperative diversity (This figure is taken from [1]).

to one of the partners. The partners can overhear each others transmissions though the wireless medium, process this information and retransmit to collaborate [24].

In this section, we consider the multiple-access channels with cooperation proposed by Laneman [1, Chapter 4] to present the benefits of cooperation in terms of sum-of-rate metric. This system is similar to the cooperation model proposed by Sendonaris *et. al* [24,25]. The system model for two cooperating users is given in Fig. 5.1 while it is easy to generalize this model to multi-user schemes. Next, we follow the same procedures as in [1].

Two independent source messages w_1 and w_2 are encoded into codewords x_1 and x_2 that can also depend causally on the channel outputs y_1 and y_2 , respectively. The decoder receives the channel output y_0 and estimates the source messages as $\hat{w}_1$ and $\hat{w}_2$, respectively. Generalized channel model can be given as follow

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{0,1} & h_{0,2} \\ h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ n_2 \end{bmatrix}. \quad (5.1)$$

It is worth to note that in this section we assume that the encoders can simultaneously transmit and receive, i.e, full-duplex transmission case. In the signal model given in (5.1), we further assume that each encoder has perfect echo-cancelation ability which results in channel outputs y_1 and y_2 are not effected by x_1 and x_2 , respectively. Here, $h_{j,i}$ represents the effects of attenuation and multi-path fading between input i and output j , and n_j is modeled as mutually independent, zero-mean, circularly-symmetric complex Gaussian white noise process, each with variance σ_j^2 . In this section, $h_{j,i}$ are modeled as stationary and ergodic random processes known to at least the decoder and possibly the encoders as

in [1]. The transmit signals' powers are constrained by P_i for $i = 1, 2$. For the following derivations, we define SNRs as follow

$$\gamma_{j,i} = \frac{|h_{j,i}|^2 P_i}{\sigma_j^2}. \quad (5.2)$$

Encoding and decoding definitions for two user memoryless Gaussian multiple-access channel with cooperation are given in [1] with details. It is worth to say that each encoder is a causal function of the encoder's message w_i and channel output y_i , $i = 1, 2$. Outer bounds and inner bounds on capacity region are derived in [1] for Gaussian channel case. The results show that cooperation among users provide significant gains over non-cooperation case in terms of sum rate. Assuming transmit signals have the following covariance matrix

$$\mathbf{Q}_{x_1, x_2} = \begin{bmatrix} P_1 & \rho\sqrt{P_1 P_2} \\ \rho^*\sqrt{P_1 P_2} & P_2 \end{bmatrix} \quad (5.3)$$

where ρ is the correlation coefficient between x_1 and x_2 , the set of achievable rates for Gaussian multiple-access channel with cooperation is given by

$$\begin{aligned} R_1 &\leq \log(1 + (1 - |\rho|^2) [\gamma_{0,1} + \gamma_{2,1}]) \\ R_2 &\leq \log(1 + (1 - |\rho|^2) [\gamma_{0,2} + \gamma_{1,2}]) \\ R_1 + R_2 &\leq \log(1 + \gamma_{0,1} + \gamma_{0,2} + 2|\rho|\sqrt{\gamma_{0,1}\gamma_{0,2}}) \end{aligned} \quad (5.4)$$

where R_1 and R_2 are transmission rates for user 1 and user 2, respectively. It is worth to specify that when one of the users does not have information to convey through destination node, i.e., relay channel case, then the capacity upper bound for this case is given by

$$\begin{aligned} R_1 &= \max_{0 \leq |\rho| \leq 1} \min \{ \log(1 + (1 - |\rho|^2) [\gamma_{0,1} + \gamma_{2,1}]), \log(1 + \gamma_{0,1} + \gamma_{0,2} + 2|\rho|\sqrt{\gamma_{0,1}\gamma_{0,2}}) \} \\ R_2 &= \max_{0 \leq |\rho| \leq 1} \min \{ \log(1 + (1 - |\rho|^2) [\gamma_{0,2} + \gamma_{1,2}]), \log(1 + \gamma_{0,1} + \gamma_{0,2} + 2|\rho|\sqrt{\gamma_{0,1}\gamma_{0,2}}) \} \end{aligned} \quad (5.5)$$

where R_1 corresponds to the case when user 2 acts as relay node and R_2 corresponds to the case when user 1 acts as relay node.

For non-cooperative transmission (conventional Gaussian MAC with independent transmit signals), the set of achievable rate pair (R_1, R_2) is given by [27]

$$\begin{aligned} R_1 &\leq \log(1 + \gamma_{0,1}) \\ R_2 &\leq \log(1 + \gamma_{0,2}) \\ R_1 + R_2 &\leq \log(1 + \gamma_{0,1} + \gamma_{0,2}). \end{aligned} \quad (5.6)$$

As specified above, for memoryless Gaussian multiple-access channel with cooperative diversity provides higher sum rates compared to non-cooperative case. For fading coefficients $h_{j,i}$ being i.i.d. random processes in time and mutually independent one another, in [1] it has been shown that cooperation does not provide higher sum rates compared to non-cooperative transmission when transmitters do not have CSI. While CSI is available at both transmitters and receivers, it is shown that cooperation among users provides higher sum rate compared to non-cooperation case. For detailed analysis, the reader is encouraged to look at [1].

5.2 Coded Cooperations

Recently, a new user cooperative communication methodology, called *coded cooperation*, in which cooperative signaling is integrated with channel coding was proposed by Hunter and Nostarinia [55]. This method has two key characteristics. First, cooperation occurs through partitioning a user's code word such that part of the code word is transmitted by the user itself, while the remainder is transmitted by the partner through partial or complete decoding. In previous methods cooperation occurs via repetition, which may not be the best use of available bandwidth. Second, we employ error detection at the partner to avoid error propagation. Many of the previous methods either admit forwarding of erroneous estimates of the partner's symbols, or include propagation of the partner's noise. Error propagation diminishes the performance, particularly when the channel between partners is poor [49,55]. In coded cooperation method, cooperative signaling is based on channel coding. In this method, each user tries to transmit incremental redundancy for its partner. Whenever that is not possible, the users automatically revert back to a non-cooperative mode. The system model can be seen in Fig.5.2. The key to the efficiency of coded cooperation is that all this is managed automatically through code design and there is no need for feedback between users.

In this section, we are briefly going to introduce the signaling model of coded cooperation and give some analytical results under different cooperation levels and channel conditions. We are going to assess the diversity performance of this scheme compared to non-cooperation case. More detailed system performance derivations of coded cooperation signaling in terms of bit error rate can be found in [2, 49, 55, 56].

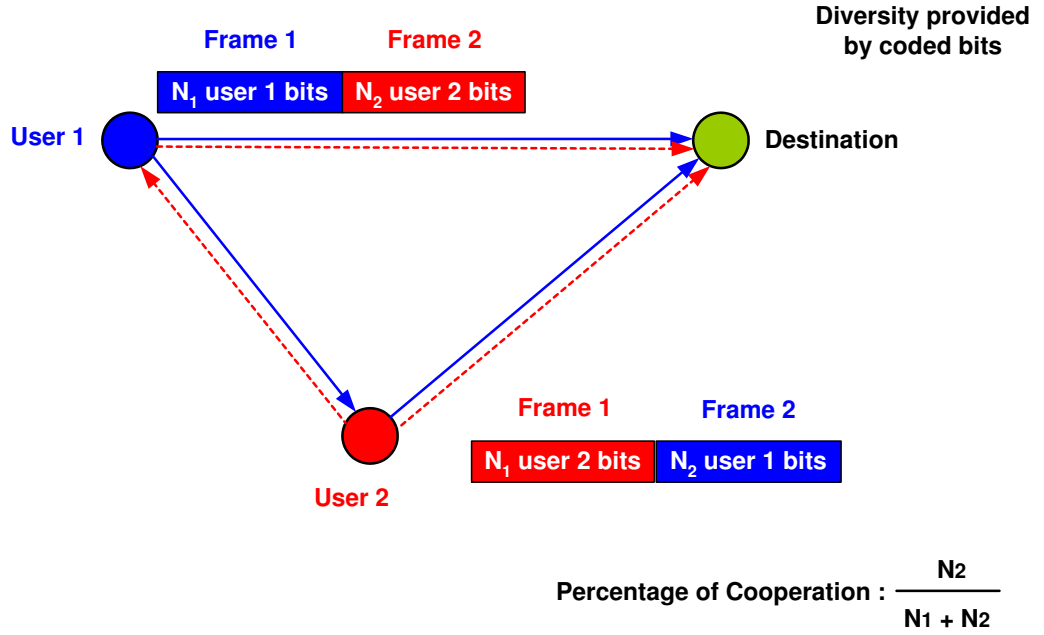


Figure 5.2: Coded Cooperation system model. This figure is taken from [2].

5.2.1 System Model for Coded Cooperation

For the purposes of exposition, we consider two users both transmitting to a single destination. It is in the context of this multiuser communication that cooperation takes place. The channels between users (inter-user channels) and from each user to the destination (uplink channels) are mutually independent and subject to at Rayleigh fading. We consider frequency flat fading in order to isolate the benefits of spatial diversity provided by coded cooperation, however the technique easily extends to systems that experience frequency selective fading.

The users transmit on orthogonal channels (e.g., TDMA, CDMA, or FDMA), which allows the destination, and other users in the cooperative case, to separately detect each user. However, the basic idea and operation of our technique does not depend on the specifics of the channel access protocol. In our analysis and simulations, we assume that the receivers maintain CSI and employ coherent detection, so that we need only consider the magnitudes of the fading coefficients in the system model and analysis that follow. To simplify the presentation, we assume BPSK modulation, for which the baseband-equivalent

discrete-time signal transmitted by user $i \in \{1, 2\}$ and received by user $j \in \{0, 1, 2\}$ ($j \neq i$, and $j = 0$ denotes the destination node) is given by [55]

$$r_{i,j}(n) = \alpha_{i,j}(n)\sqrt{E_{b,i}}b_i(n) + n_j \quad (5.7)$$

where $E_{b,i}$ is the transmitted energy per bit for user i , $b_i(n) \in \{-1, +1\}$ is the BPSK modulated code bit at time n , $\alpha_{i,j}(n)$ is the fading coefficient magnitude between users i and j , and $n_j(n)$ accounts for noise and other additive interference at the receiver. We model $\alpha_{i,j}(n)$ as independent samples of a Rayleigh distributed random variable. For slow (quasi-static) fading, the fading coefficients remain constant $\alpha_{i,j}(n) = \alpha_{i,j}$ over the transmission of each source block, while for fast fading, they are i.i.d. for each transmitted symbol. The noise term $n_j(n)$ is modeled as independent, zero-mean additive white Gaussian noise with variance N_j (i.e., samples of a band-pass white noise process with two-sided power spectral density $N_j/2$) [55].

We assume that mean-square value of Rayleigh fading channel is equal to unity and consequently we define average received SNRs as follow

$$\Gamma_{i,j} = \mathcal{E}_{\alpha_{i,j}} \left[\frac{\alpha_{i,j}^2(n)E_{b,i}}{N_j} \right] = \frac{E_{b,i}}{N_j} \quad (5.8)$$

We consider cases in which the average received SNR for the two uplink channels, $\Gamma_{1,0}$ and $\Gamma_{2,0}$, are equal (statistically similar channels) and unequal (statistically dissimilar channels). We also assume that inter-user channel is reciprocal, $\alpha_{i,j}(n) = \alpha_{j,i}(n)$ for slow fading case. This assumption is valid under the scheme of TDMA manner communications. For fast fading, inter-user channel coefficients are assumed to be independent to each other.

The users segment their source data into blocks which are augmented with a *Cyclic Redundancy Check*(CRC) code, for a total of K bits per source block (including the CRC bits). Each block is then encoded with a forward error-correcting code, so that, for an overall rate R code, we have $N = K/R$ total code bits per block. We emphasize that since most current and future wireless systems already employ CRC codes, this does not represent additional overhead required by coded cooperation.

The two users cooperate by dividing the transmission of their N -bit code words into two successive time segments, or frames. In the first frame, each user transmits a rate $R_1 \geq R$ code word with $N_1 = K/R_1$ bits. This itself is a valid code word which can be decoded to obtain the original information. Each user also receives and decodes the partner's

transmission. If the user successfully decodes the partner's rate R_1 code word, determined by checking the CRC bits, the user computes and transmits N_2 additional parity bits for the partner's data in the second frame, where $N_1 + N_2 = N$. These additional parity bits are selected such that they can be combined with the first frame code word to produce a more powerful rate R code word. If the user does not successfully decode the partner, N_2 additional parity bits for the user's own data are transmitted. Each user always transmits a total of N bits per source block over the two frames, and the users only transmit in their own multiple access channels [56].

As in [2], the level of cooperation is defined as N_2/N , the percentage of the total bits per each source block that the user transmits for its partner. A smaller percentage implies a more powerful code for the first frame and increased probability that a user successfully decodes the partner. However, this also means a smaller N_2 , thus reducing gain from diversity [2].

To obtain the performance results, we employ a simple but very effective implementation using *Rate-Compatible Punctured Convolutional*(RCPC) codes [58]. In this implementation, the overall rate R code is selected from a given RCPC code family (e.g., the mother code). The code word for the first frame is obtained by applying the puncturing matrix corresponding to rate R_1 , and the additional parity bits transmitted in the second frame are those punctured from the first frame. Fig.5.3 illustrates a user's implementation of coded cooperation using RCPC codes. The users act independently in the second frame, with no knowledge of whether their own first frame was correctly decoded. Upon this knowledge there are four possible situations for the second frame period. We will assume that destination node has the knowledge of decoding correctness in the second frame.

5.2.2 Numerical Results

In order to evaluate the performance of coded cooperation, we employ the implementation using RCPC codes. Specially, we use the family of RCPC codes with memory $M = 4$, puncturing period $P = 8$, and rate $1/4$ mother code given by Hagenauer [58]. For slow fading, we choose overall code rate $R = 1/4$, while for fast fading we use $R = 2/5$. In all cases, the source block size is $K = 128$ bits. For the simulations, we use a 16-bit CRC code with generator polynomial given by coefficients 15935 (hexadecimal notation). For our analysis we assume perfect error detection. Our results indicate that suboptimal error

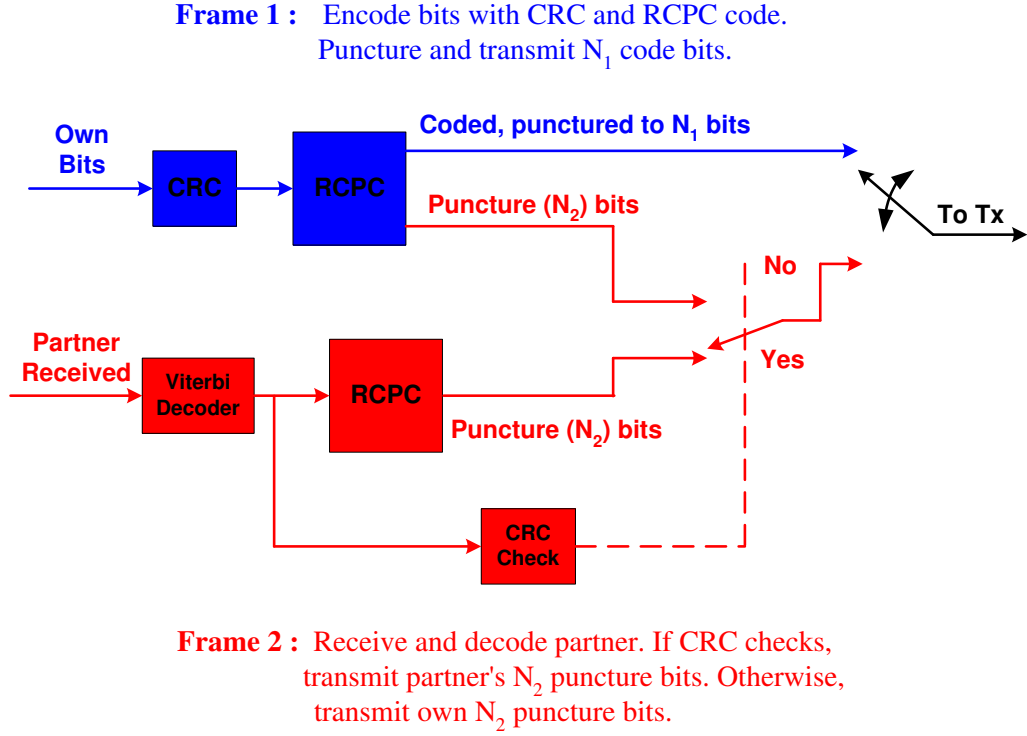


Figure 5.3: Implementation of Coded Cooperation by using RCPC coding. This figure is taken from [2].

detection of the CRC code has a negligible effect on the overall performance. Since all comparisons are between systems with equal information rate K bits per source block, and equal code rate R , we plot the BER versus the channel $SNR = E_{b,i}/N_j$, where $E_{b,i} = RE_{bit}$.

In Fig.5.4 and Fig.5.5, the BERs are shown for slow fading with reciprocal inter-user channels of various qualities. The users have statistically similar uplink channels ($\Gamma_{1,0} = \Gamma_{2,0}$), and the levels of cooperation are %25 and %50. We see that coded cooperation with a perfect inter-user channel performs virtually identically to a comparable two-antenna transmit diversity system. In this system, we use as an outer code the same rate 1/4 RCPC code that we use for coded cooperation, concatenated with the STBC proposed by Alamouti [5]. Since the Alamouti code is known to provide full diversity order two, this result confirms that coded cooperation does achieve full diversity for the case that both cooperating users perfectly decode each others information and send partner's data in the second time frame. The improvement over no cooperation remains dramatic for good inter-

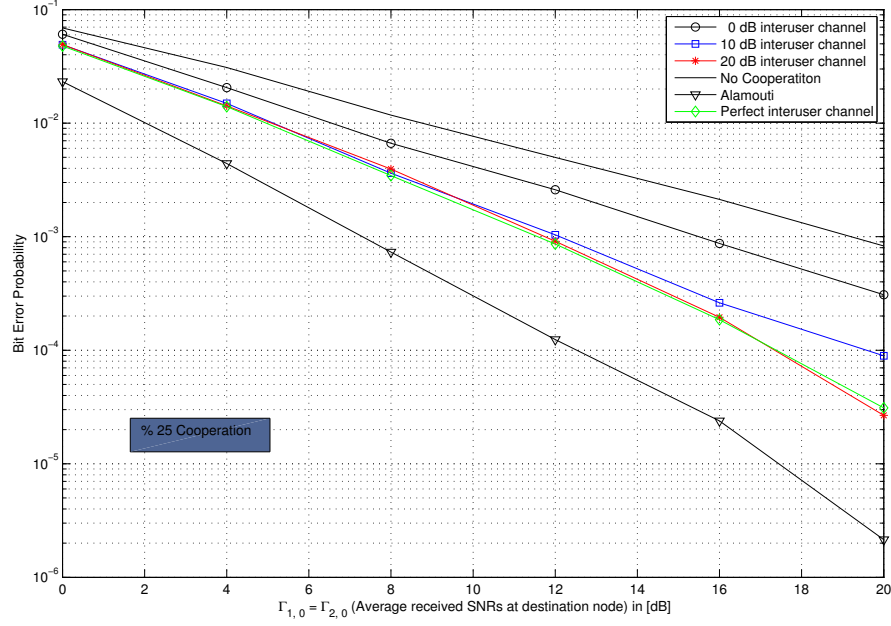


Figure 5.4: Performance in slow Rayleigh fading with %25 cooperation, equal uplink SNRs ($\Gamma_{1,0} = \Gamma_{2,0}$), and reciprocal inter-user channels ($\Gamma_{1,2} = \Gamma_{2,1}$).

user channels.

Fig.5.6 shows simulation results for coded cooperation when the destination has two receive antennas. We consider slow Rayleigh fading, equal average SNR uplink channels, 50% cooperation, and reciprocal inter-user channels with various average SNR. We see that again coded cooperation provides substantial gains.

In this chapter, we investigated cooperative communication where each user in the system has both its own and its partners' information to broadcast through a destination node in contrast to relay channel model where only a single source has information to send to destination node while other the surrounding nodes only assists in communication. First we briefly gave some remarks from [1] and [24, 25] for better understanding the cooperation concept. Then to see implementation point of view of this communication scheme we presented a new cooperation concept: coded cooperation.

We looked at system model for coded cooperation where there are one source, one relay and one destination. Coded cooperation provides both users to utilize system resources more

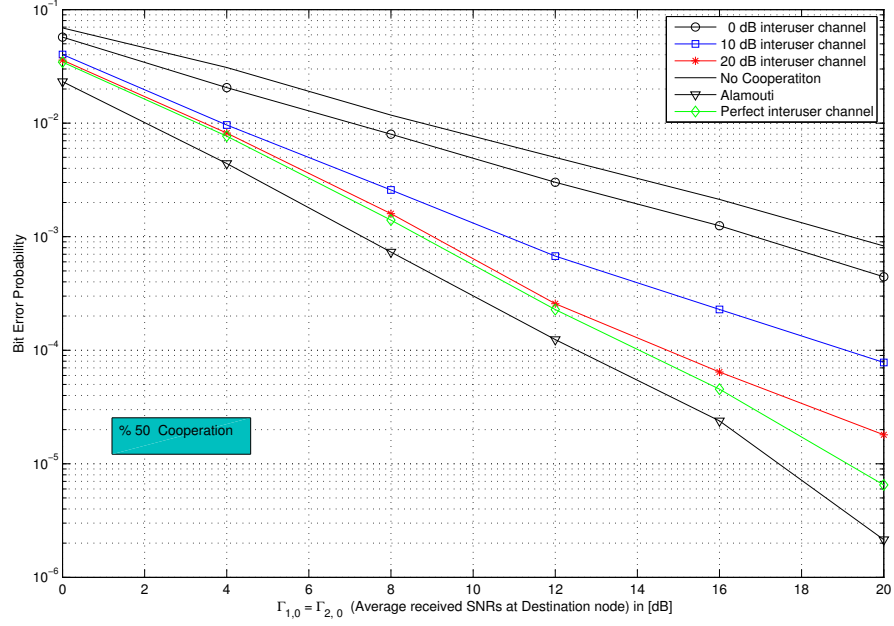


Figure 5.5: Performance in slow Rayleigh fading with %50 cooperation, equal uplink SNRs ($\Gamma_{1,0} = \Gamma_{2,0}$), and reciprocal inter-user channels ($\Gamma_{1,2} = \Gamma_{2,1}$).

efficiently. This method does not require channel feedback and there is no error propagation over relay node due to error detection code utilization. It is shown that coded cooperation provides full diversity when inter-user channel is perfect, less when it is worsen. But in terms of system resources there is no extra resource usage in this method. We give analytical results of different cooperation levels and channel conditions for coded cooperation. It is shown that coded cooperation when inter-user channel is perfect provides full diversity of two. On the other hand, when inter-user channel get worsen the diversity gain decreases. We also compare the performance results for multiple-antenna usage at destination node. It is seen that coded cooperation provides less diversity gains when other form of diversity techniques are used.

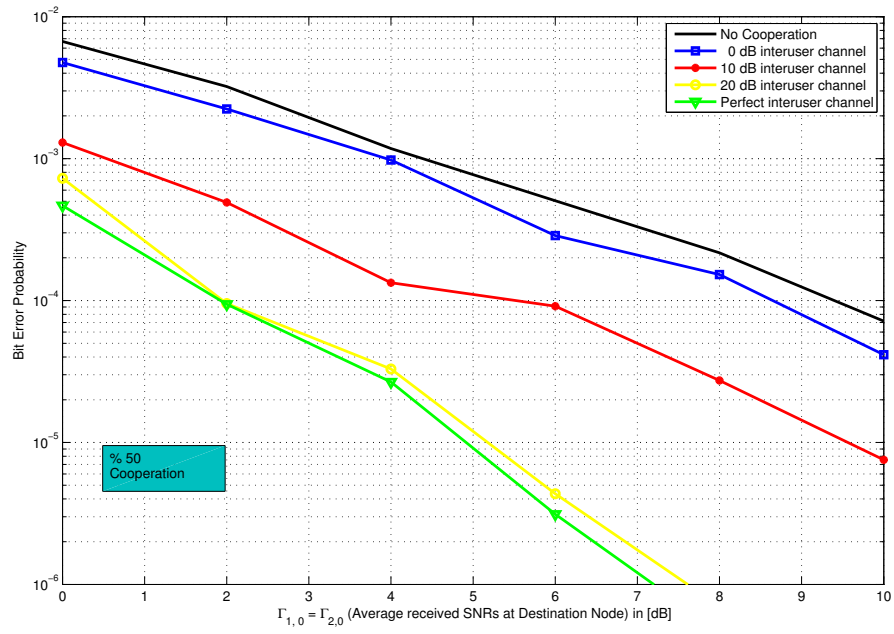


Figure 5.6: Performance in slow Rayleigh fading with %50 cooperation, equal uplink SNRs ($\Gamma_{1,0} = \Gamma_{2,0}$), and reciprocal inter-user channels ($\Gamma_{1,2} = \Gamma_{2,1}$) where destination node has 2 antennas.

Chapter 6

CONCLUSIONS

Throughout this thesis, spatial diversity is our main focus point and we investigate the performance such diversity schemes provide in wireless channels. We show that spatial diversity can be exploited via geographically separated antennas and/or users. After considering antenna diversity and multiuser diversity separately, we investigate the performance of systems that employ both schemes simultaneously. In general, this is the main novelty of this thesis.

In Chapter 2, we provide the necessary literature survey and provide definitions for the performance measures, i.e., ergodic capacity to measure the multiplexing gain and outage probability to measure the diversity gain. We investigate the potential benefits of all spatial diversity techniques over direct transmission with single transmit and receiver antennas. We compare relative benefits of transmit diversity and transmit beamforming. We show that transmit beamforming provides additional array gain over point-to-point communication performance.

To see the potential benefits of transmit beamforming at wireless relay channel with multiple relaying nodes, each furnished with multiple antennas, in Chapter 3, we propose new signaling models for source node beamforming. We study the ergodic capacity of AF, best-selection DF (BS-DF) and all-participating DF (AP-DF) relaying modes operating on a TDMA based cooperation protocol. We assume that CSI from all channels are available at all relevant transmitters. For each considered protocol, we derive ergodic capacities for general MIMO systems both with and without CSI available at the transmitters for single as well as multiple relay nodes. We propose a simple beamforming method where the total transmit power is allocated over channel directions which are maximum eigenvalues corresponding to the eigenvectors of H_{SD} , H_{Sk} for the AF mode and H_{SD} , H_{Sk} for the DF relaying modes, $k = 1, 2, \dots, K$ where K is the number of relays in the system. With channel side information available at transmitters, we propose a simple power allocation

algorithm over the multiple beams which is similar to the conventional waterfilling.

In Chapter 4, we show that with the use of multi-user beamforming a *transmit array gain* is also achievable with the AF relaying scheme proposed by [23]. We assume that source node has full channel side information (CSI). To mitigate the interference seen by non-assigned relay nodes of an individual data stream, we propose a source beamforming method by which the goal is to maximize the system capacity. For this purpose, we propose a beamforming method based on maximizing sum of signal-to-leakage ratios (SSLR) for all data streams. With this method, the sum of signal strengths at the set of the relay node assigned to that stream will be maximized and the interferences to other relay nodes will be suppressed. Thus the propagation of interference terms is suppressed as well. Unlike other interference cancellation methods, the proposed method does not have any constraints on the number of transmit antennas. The proposed scheme supplies an additional array gain with the knowledge of CSI for $\mathcal{S} \rightarrow \mathcal{R}_k, \forall k$ and $\mathcal{S} \rightarrow \mathcal{D}$, and there is no need for the knowledge of $\mathcal{R}_k \rightarrow \mathcal{D}$ links.

We briefly examine cooperative diversity in Chapter 5 so as to accomplish the goal of this thesis which is an exhaustive study on the information theoretic behavior of all spatial diversity schemes. We observe that cooperation among mobile users provides significant gains over non-cooperative case in terms of diversity and sum rate.

6.1 Future Work

Extensions of this work are possible in the following directions:

- The concept of relay nodes with multiple antennas in relay networks can be generalized to cooperative networks in general. This would pave the way for efficient, wireless multiple antenna peer-to-peer networks. Various cooperation protocols could be established based on different performance goals, such as minimization of the download time, spectral efficiency, minimization of interference, etc.
- The concept of multi-user beamforming applied to the AF relaying scheme could be extended to other relaying strategies. Furthermore, one should investigate whether it is really advantageous to have mutually exclusive relay groups for each of the transmit antennas when the relays themselves are equipped with multiple antennas.

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