TÜRKİYE
FIRAT UNIVERSITY
THE GRADUADE SCHOOL OF NATURAL AND APPLIED SCIENCES
FACULTY OF SCIENCE

TUNING PID CONTROLLER DESIGN AND ANALYSIS OF AN INVERTED PENDULUM SYSTEM

Shivan Kareem ABDULLAH

Master's Thesis

Program: Machine Theory and Dynamics

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This Thesis, Which was prepared according to the thesis rule of the institute of the science and technology of Fırat university, was evaluated by the jury members who have signed the following signature and as a result of the defense made open to academic listeners

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Prof. Dr. Soner ÖZGEN
director of the institute

DECLARATION

I Declare That The Master Thesis Entitled "Tuning PID Controller Design And Analysis Of An Inverted Pendulum System", Is My Own Research And Prepared By Myself, And Hereby Certify That Unless Stated, All Work Contained Within This Thesis Is My Own Independent Research And It Is Being Submitted For The Degree Of Master Of Science (Machine Theory and Dynamics), At The Fırat University. It Has Not Been Submitted For The Award Of Any Other Degree At Any Institution.

Shivan Kareem ABDULLAH

ELAZIG,2020

PREFACE

Today, robotic systems have many popular uses. In addition to the intensive use of robotic systems, their control is also important in these areas. There are many methods of control for robotic systems where control is desired. These control systems; the aim of this course is to make the robots perform the desired work at the desired time in a way that minimizes the human labor and make them work in a regular and systematic way like a mind that knows what to do and does not ask questions. It is our responsibility to design and implement real-time control systems in accordance with the robotics era.

In this study, my dear professor, Prof.Dr. Servet SOYGÜDER I would like to thank you.

Shivan Kareem ABDULLAH

ELAZIG,2020

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ABSTRACT

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Shivan Kareem ABDULLAH

Master’s thesis

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Inverted Pendulum systems are one of the most important engineering systems used in the solution of many engineering problems and control analysis with the application of control techniques. Before real-time control, balancing obtained by running computer mathematical models of these systems has been very challenging and important for modern control theory. Studies on these systems shed light on the design and control algorithms of many systems ranging from automotive, aircraft, spatial vehicles, defense vehicles, missile launchers to human marches, load bearing robots and earthquake resistant building designs. Inverted-Pendulum is one of the most important basic systems used in control engineering. In this study, firstly, a mathematical model for the system was created. A mathematical model is a mathematical equation representing a process or the behavior of a system. The dynamic equations expressing these equations were obtained by using Newton’s Second Law of Motion. Inverted-Pendulum is a nonlinear system. The dynamics equations of system have been obtained by linearization of the obtained equations. After all these studies, the control analysis of the system was made. In order to make the control analysis has been found transfer function, characteristic equation, state-space model, poles, zeros and singular points of Inverted-Pendulum system. In control analysis of system, Root-Locus, Nyquist and Bode diagrams of system were obtained and discussed and were also investigated the controllability and observability of the system depending on the state variables. In addition to the PID (Proportional-Derivative-Derivative) and LQR (Linear Quadratic Regulator) controls made in the literature, the control of the system was also carried out successfully by designing the Tuning-PID control algorithm. Here, PID control coefficients (k_p, k_i, k_d) were determined by using the Ziegler-Nichols method. Finally, the performance of these control methods designed and the necessary control analyzes were performed and the results were obtained.

Keyword: Inverted pendulum, PID controller, LQR control, MATLAB Program, Root-Locus.

ÖZET

Bir Ters Sarkaç Sisteminin Ayarlamalı PID Denetleyici Tasarımı ve Analizi

Shivan Kareem ABDULLAH

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Ters Sarkaç sistemleri, kontrol tekniklerinin uygulanması ile birçok mühendislik probleminin ve kontrol analizinin çözümünde kullanılan en önemli mühendislik sistemlerinden biridir. Gerçek zamanlı kontrolden önce, bu sistemlerin bilgisayar ortamında matematiksel modellerinin benzetimi ile elde edilen dengeleme çalışmaları modern kontrol teorisi için çok zorlayıcı ve önemli olmuştur. Bu sistemler üzerinde yapılan çalışmalar, otomotiv, uçak, mekansal araçlar, savunma araçları, füze fırlatıcılarından insan yürüyüşlerine, yük taşıyan robotlara ve depreme dayanıklı bina tasarımlarına kadar birçok sistemin tasarım ve kontrol algoritmalarına ışık tutmuştur. Ters Sarkaç Sistemi, kontrol mühendisliğinde kullanılan en önemli temel sistemlerden biridir. Bu çalışmada ilk olarak sistem için bir matematiksel model oluşturulmuştur. Matematiksel bir model, bir süreci veya bir sistemin davranışını temsil eden matematiksel bir denklemdir. Bu denklemleri ifade eden dinamik denklemler Newton'un İkinci Hareket Yasası kullanılarak elde edilmiştir. Ters Sarkaç doğrusal olmayan bir sistemdir. Sistemin dinamik denklemleri, elde edilen denklemlerin doğrusallaştırılmasıyla elde edilmiştir. Tüm bu çalışmalardan sonra sistemin kontrol analizi yapılmıştır. Kontrol analizini yapabilmek için ters-Sarkaç sisteminin transfer fonksiyonu, karakteristik denklemi, durum-uzay modeli, kutuplar, sıfırlar ve tekil noktaları bulunmuştur. Sistemin kontrol analizinde, sistemin Root-Locus, Nyquist ve Bode diyagramları elde edildi ve tartışıldı ve ayrıca durum değişkenlerine bağlı olarak sistemin kontrol edilebilirliği ve gözlenebilirliği araştırıldı. Literatürde yapılan PID (Oransal-Türev-Türev) ve LQR (Lineer Kuadratik Regülatör) kontrollerine ek olarak, sistemin kontrolü aynı zamanda Tuning-PID kontrol algoritması tasarlanarak başarıyla gerçekleştirilmiştir. Burada PID kontrol katsayıları (k_p, k_i, k_d) Ziegler-Nichols yöntemi kullanılarak belirlenmiştir. Son olarak, tasarlanan bu kontrol yöntemlerinin performansı ve gerekli kontrol analizleri yapılmış ve sonuçlar elde edilmiştir.

Anahtar Kelimeler: Ters Sarkaç, PID Denetleyici, LQR Kontrol, MATLAB Program, Root-Locus

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LIST OF SYMBOLS

Symbols

(IP)	Inverted Pendulum
(PI)	Proportional Integral
(PID)	Proportional Integral Derivative Controller (PID) Controller
(PSO)	Particle Swarm Optimization
(FFOSMC)	Fuzzy Fractional Order Sliding Mode Controller
(SMC)	Sliding Mode Control
(LQR)	Linear Quadratic Regulator
(MMS)	Multi-purpose Module System
(SISO)	Single-Input, Single-Output
(F)	Force Applied to Cart
(M)	Mass of cart
(m)	Mass of pendulum
(l)	Length to Pendulum Center of mass
(N)	Reaction Force
(x)	Cart position coordinate
$(\boldsymbol{x'})$	Cart velocity
$(\boldsymbol{x''})$	Cart acceleration
(θ)	Pendulum angle from the vertical
$(\dot{\boldsymbol{\theta}})$	Pendulum angular velocity
$(\ddot{\boldsymbol{\theta}})$	Pendulum angular acceleration
r(s)	Reference signal
e(s)	Error signal
u(s)	Plant input
(u)	Input vector
(y)	Output vector
$\boldsymbol{e_{ss}}$	Study state error
$\boldsymbol{T_s}$	Settling time
$\boldsymbol{T_r}$	Rising time
L(s)	Open-loop transfer function
(b)	Coefficient of friction for cart
(I)	Mass Moment of Inertia of the pendulum
$(\boldsymbol{K_p})$	Proportional gain
$(\boldsymbol{K_i})$	Integral gain
$(\boldsymbol{K_d})$	Derivative gain
GUI	Graphic User Interface

1. INTRODUCTION

1.1. What Is Your Scientific Problem? Problem Definition?

The popularity of the inverted pendulum derives in part from the fact that it is unstable without control, that is, the pendulum will simply fall over if the cart isn't moved to balance it. In short, the problem in the inverted pendulum system is to ensure that the pendulum is balanced vertically to the ground. Also, the inverted pendulum system is a nonlinear system.

1.2. Why This Thesis Is Important? The Importance Of The Problem?

Inverted Pendulum systems are one of the most important engineering systems used in the solution of many engineering problems and control analysis with the application of control techniques. Studies on these systems shed light on the design and control algorithms of many systems ranging from automotive, aircraft, spatial vehicles, defense vehicles, missile launchers to human marches, load bearing robots and earthquake resistant building designs. Inverted-Pendulum is one of the most important basic systems used in control engineering.

1.3. What Is The Final Purpose Of The Thesis? Purpose (Objectives)

- Inverted pendulum systems represent dynamic structure of many physical systems such as a human walking, an airplane flight, take off of a rocket as well as many physical systems.
- To create and develop a mathematical modeling of system.
- To create transfer function of system to make control analysis.
- To design PID control algorithm to balance system.
- To design LQR control algorithm to balance system.
- To analysis the stability and control of pendulum during movement of the cart.

1.4. How Do You Assume The Relationship Between Which Variables Of The Problem? Hypotheses.

The inverted pendulum has two variables that are x and θ. These represent the linear position of the cart and the angular position of the pendulum, respectively. **x** is actively driven while θ is passively driven. We are trying to balance is variable θ depending on the variable x.

1.5. Simulink Tutorials

All simulation and analysis of system have been done by using MATLAB soft program. As known, MATLAB is very comprehensive to solve engineering problem and to analysis control of system. Also, MATLAB is both theoretically and practically.

1.6. Social Benefit Of This Problem?

Inverted Pendulum systems are one of the most important engineering systems used in the solution of many engineering problems and control analysis with the application of control techniques. Thus, the social life standards will be increased by solving these problems

2. LITERATURE REVIEW

Inverted pendulum systems (IPS) are significant engineering systems that are made use of in the solution of many engineering problems. The IPS has a non-linear, unstable structure. Because of systems, it attracts the attention of researchers intensively and there are many studies in the literature on the control of the IPS. Some of the studies are related to lifting the pendulum from the stable vertical position to an unstable peak. An important part of the studies in the literature is the stabilization of the IPS, which is the stabilization of the pendulum in an unstable vertical position. This thesis examined Inverted Pendulum Control, which is a textbook example of a system that is inherently unstable. It balances on the pendulum of the Mobile Inverted. To get a brief overview of the work going on in this area, a few papers were referred and a short survey was done for this purpose. Furthermore, the basic idea of the inverted mobile pendulum and 2 control strategies were discussion [2].

Automatic Control is a growing field of study in mechanical engineering, covering the control of proportional, integral and derivative (PID) and state-space. His nonlinear and erratic traits are the main reasons for his successful. This study started with a research description on the design and control of Inverted Pendulum system along with the methods of mathematical modeling. This project addressed the introduction and analysis in the MATLAB setting of two-dimensional inverted pendulum systems [2]. The research focused on researching the modeling process and developing various controllers for the inverted pendulum system. The first element that was derived was the nonlinear system's linear model, including transfer function and representation of state-space. The characteristic behavior of the system's phase and impulse responses was analyzed on the basis of such linear models: the open-loop characteristics, such as stable-state and transient response. [3].

A lot of research has already been done for the implementation of the Control Strategy in the Past on IPS. They had done a very small job in this study to plan and test Control Strategy with real-time experiments. $PID, PID + PI$ & LQR control for the IPS was introduced in this two-loop PID thesis and this control strategy provides a satisfactory response [4].

In particular, this paper addressed a quadrupled type MMS (MMS-G02), MMS-G02 modeling and simulation for stability control One of the mobile robots, the Multi-Purpose Mobile System (MMS) consists of combined modules and feature modules. MMS such as series-connected form and quadruped form can be transformed and suitable sensors can be added. MMS is therefore useful in an unknown situation as a rescue robot. The inverse cinematics were used to assess the location of each leg. Then simulation tests the leg trajectory of the MMS-G02. The results showed that without collision, the leg was properly moved. An inverted pendulum model for the MMS-G02 was proposed and a state feedback control was used to control the MMS-G02 to stay in balance. And simulation also measured the effect of friction force between the MMS-

G02 and the ground. The MMS-G02's overall lean value is 0.14 rad which is lower to the average angle without ground contact. T The results showed that the MMS-G02 should remain in balance and stay on both legs. The proposed calculation model, as the conclusion, was useful in achieving MMS-G02 regulation to adapt to a changing climate [5].

An inverted pendulum simulation was done and then three separate controllers (PID, LQR) were used to stabilize the pendulum. In adding two moving masses, the proposed system extends the traditional inverted pendulum. It is controllable the motion of two masses moving along the horizontal plane. Proportional, Integral, and Derivative (PID) & State Feedback Controllers computer simulation tests for the device were shown. The main focus was to extract the mathematical model and evaluate the performance of its system, then develop an LQR controller for better control. Index Terms-inverted pendulum, swing-up control, nonlinear control, gain formulae, pole positioning, PID controller, state feedback controller, linear quadratic controller; system performance [6].

One of several low-speed mobility systems is the Segway Human Transporter including (e.g., bikes, scooters, wheelchairs) operating on sidewalks, roadways, and other shared-use paths under certain circumstances. The aim of this research was to analyze the Segway's primary operating features. In this study, for a two-wheel balancing Segway robot, we designed and built a mechanically based system We introduced a Segway in this paper based on a gyro sensor, an accelerometer together with a microcontroller and used for mechanical and electrical hardware. The vehicle's dynamics are similar to an inverted pendulum's classic control problem, which means it is unstable and prone to tipping over. This is avoided by detecting the pitch angle and its time derivative by electronics regulating the engines to keep the vehicle balanced. This type of vehicle is worthy of interest because it contains a lot of technology that is important to an environmentally friendly and energy-efficient transport industry. This thesis developed a similar vehicle from scratch, integrating every step from a literature analysis to planning, design, construction of vehicles, and verification. The main goal was to create a vehicle capable of carrying a person weighing up to 70-80 kg and moving with varying speeds to some km distance. The rider controls were supposed to be natural movements; the only rider input needed to ride the vehicle should be leaning forward or backward in combination with tilting the handlebar sideways. It also takes into account the content used with the minimum possible expense. Segway Human Transporter's architecture is such that it provides the user with less room and comfort [7].

Razzaghi and Jalali were proposed two methods using PID controllers to control the car-reversing pendulum system. The first method is the control of the cart position under the pendulum angle and the second is the control of the pendulum angle under the car position. By comparing these two methods with each other, the second method has been shown to give better results [8] Shehu et al. proposed LQR, two PID and pole placement control methods for the control of an IPS and compared the simulation results of the system control they performed with these three methods. They obtained the best performance with the LQR controller in case of external disturbance or not [9]. The IPS are worked the first depending on the gravity by Åström and Furuta [10].

The positions (angular and horizontal) controlling of the cart have been was carry out by Yi et al. [11]. Wang was realized simulation studies using multiple PID control methods [12]. Prasad et al. carried out the control of the IPS with PID and LQR controllers and showed that PID + LQR control method gave better results than the PID control method [13]. Sánchez et al. have developed an IPS which can be controlled remotely over the internet for 24 hours [14]. Ghanbari and Farrokhi have introduced a new approach to biaxial inverse pendulum control. First, they divided the biaxial reverse pendulum system into two separate subsystems with the help of decentralized control theory. They then split each subsystem into two surfaces using a separation method to control the sliding-mode method. Sliding-mode controllers are also used to learn two fuzzy neural networks [15].

In the study of Jung et al., Neural networks distributed to the biaxial the IPS were applied. Each axis is controlled by two separate neural network controllers according to the separated control structure. Neural network controllers not only control the angle of the pendulum, but also the position of the carriage carrying the pendulum [16]. Huang et al. in their work with two-wheel the IPS have been controlled by the fuzzy control method [17]. Chiu et al. designed the AORCMA (adaptive output recurrent cerebellar-model-articulation-controller) controller with a simple calculation, good generalization and fast learning of the wheeled pendulum system [18].

Ertuğrul designed an IPS with a single degree of freedom and controlled this system. The study can guide researchers with the equipment used. A Siemens PLC is used for control and signal processing, a 400W servo motor of PANASONIC brand and a servo drive of MINAS A5 and two optical incremental encoders of 2500 pulse/revolution were used for measuring the motor position and the pendulum angle [19].

In a different study, Grămescu et al. designed a spatial the IPS with wheels and successfully controlled this system with LQR and PID method. It is important to note that the Arduino Mega 2560 microcontroller and the HC-05 Bluetooth module are used to communicate with a computer that is a controller. Researchers used two geared DC motors with a reduction ratio of 1/48 for movement, two 720 pulse/turn encoders to read the positions of these motors, and one Arduino-compatible MPU6050 electronic gyroscope to measure the angular acceleration of the system [20].

Kizir first created a physical experimental setup by placing a single IPS on a cart capable of linear motion in a single axis, and then controlled this system with the help of PID and fuzzy logic. The researcher using dSPACE DS1103 card, which can work in accordance with MATLAB / Simulink for signal communication, used a 400W power 3-phase Metronix servo motor and a motor driver of the same brand to give the necessary movement to the system. The researcher used a 1024 pulse / turn encoder to measure the cart's position and a 1000 pulse / turn encoder to measure the pendulum angles [21].

Lozano et al. have designed an energy-based controller to control two problems of the IPS. The pendulum has been explained by the simulation and experimental results about the equilibrium and equilibrium of the vehicle by controlling the vehicle position [22]. Ji, et al. presented the simulation results of the fuzzy logic controller developed by the IPS for problems of inverting, inverting and balancing the pendulum [23]. Magana and Holzapfel [15] image IPS fuzzy logic auditors [24]. Tang and Shoaee [7] compared classical control theory with fuzzy logic controllers based on simulation results on the IPS [25]. Nundrakwang et al. [9] used a PD position-controlled controller to raise the pendulum and state feedback methods to compensate [26].

Bugeja was used the hybrid approach by examining the feedback linearization technique and the state feedback methods for balancing the pendulum, taking into account the system energy for raising the pendulum [27]. Bettayeb et al. performed a real-time control of an IPS using the fractional grade PI-state feedback controller. The controller used is designed with pole placement. They compared the results of the fraction-grade PI-state feedback controller they proposed to the results of the integer-grade PI-state feedback controller. In addition, in the case of external disturbance and parameter change in the system, the durability of the controller is shown in real-time [28].

Mishra and Chandra proposed a fractional grade PID controller for robust control of an IPS. Using particle flock optimization, they designed one fraction-grade PID controller for pendulum angle and cart position. The performance of fraction-grade PID controllers is compared to conventional PID controllers. The results in case the proposed method is disturbing to demonstrate its durability are also given. In the study, the proposed method was not applied on a real system and only simulation study was performed [29]. In the study conducted by Hanwate and Hote a PID controller designed with stability boundary zones method was proposed for the control of an IPS. The results obtained in this study were compared with the results of PID + LQR designed with the pole placement method and the performance of the proposed method was shown to be better. In this study, the results are not real-time, but analyzes are based on simulation results [30].

These systems, which have become highly complex by combining five-rod mechanisms and spatial IPS, have been controlled in a similar manner to the third type of spatial IPS. The nonlinear dynamic model of the system is obtained and the model is linearized in a similar way and the problem is reduced to a less complicated structure and the controller is designed [31-32]. In the third case, the robotic arms of series configuration are used to compensate for a spatial pendulum. Although there are too many nonlinear terms in the dynamic equations in these systems, although the control process seems to be difficult, successful controller designs have been made with the help of linearization of the equations. In this type of study, first of all, the dynamic equations are linearized at the unstable top of the spatial IPS and a relatively easy problem is obtained and then the controller design is made on this problem [33-34]. The IPS were controlled in real-time with fuzzy logic controllers by Abut and Soyguder [35]. Kizir and et al. were realized an IPS in real time using fuzzy logic control method [36]. Ahmadi et al. were utilized the PID type fuzzy logic controller in their studies and were viewed with various parameters [37].

Magdy et al. were obtained modeling and the PID controller is used to control the IPS. The PID coefficients are tuned by using Gravitational Search Algorithm (GSA) and Genetic Algorithm (GA) [38]. The article optimal control of nonlinear IPS utilization PID controller & LQR were realized by authors [39]. Kajita et al. biped walking stabilization based on linear IPS tracking was implementation [40]. Wang et al. the fractional PID controller method have realized for the IPS [41]. IPS has examined under Dry Friction Influence on Cart Pendulum Dynamics [42]. In this study, a geometrical approach has been used to analyze the influence of bearing friction on cart-pendulum dynamics [43]. The article was aimed at a nonlinear optimal control method for the model of the wheeled inverted pendulum (WIP). The authors, an optimal (H-infinity) feedback controller was improved [44]. The self-balancing robot was utilized as the fuzzy PD control method [45]. The model predictive control (MPC) method has used for a two-WIP mobile robot [46].

The real IPS has used an adaptive swing-up sliding mode controller method. Also, in the proposed control algorithm has used the Lyapunov stability method and the Culture-Bees algorithm [47]. In an IPS, Kharola and Patil have proposed three different PID control methods for keeping the pendulum in the vertical position and positioning the cart in the desired position [48]. For a IPS, only the local stability of the system was investigated using the combined sliding mode control method [49]. Gautam has used ADALINE artificial neural network with LQR optimal control for the IPS [50]. Control of the IPS under a limited car length was performed by Chatterjee et al. [51]. In another works, an interval type-2 fuzzy PD controller was made for a nonlinear IPS [52].

Control design, mathematical modeling, optimal control, swing-up control, position control, intelligent transfer control of double IPS have been realized [53]. In the other study, hybrid control for global stabilization of the IPS was an investigation [54]. The article authors are used hybrid sliding mode controller method on fire algorithm [55]. Wadi et al. have studied the dynamic analysis of a pendulum. They developed analytical and Adams models for a balanced position of the pendulum. In this study, the moment of inertia of the pendulum and the viscous and can friction of the plant be considered by the authors [56]. This study was carried out two steps, the first one is controlled by the position of the system using an energy function and the second one consists of the Lyapunov function for an event-based state feedback function [57].

Neuro-tabu fuzzy controller to stabilize an IPS was proposed by Khwan-on [58]. In authors study, learning to control the IPS using neural networks was aimed [59]. Huang et al. were realized of an inverted pendulum using grey prediction model [60]. FPGA-based of an IPS were controlled with fuzzy sliding mode method [61]. In this study, controllers with an IPS holistic optimization approach are proposed. Holistic optimization is accomplished by a simplified Ant Colony Optimization method with the restricted Nelder-Mead algorithm (ACO-NM). A simplified Ant Colony Optimization was implemented with the Nelder-Mead algorithm (ACO-NM) which is one of the optimization techniques. These optimization methods are aimed to keep the cost function at the minimum value by finding the optimum values of Q and R parameters in the Linear Quadratic Regulator (LQR) method which is one of the optimal control techniques. The study was carried out simulation and experimentally and the results were compared. Furthermore, these optimization techniques have been compared and examined with other techniques [62]. In the study by Masrom et al., A three-link IPS fuzzy logic type 2 research was conducted [63].

Taur et al. were developed a fuzzy logic model for the system and were designed the FLC for the system [64]. Roose et al. were proposed a parallel distributed compensation (PDC) controller based on a fuzzy model for the IPS and were simulated it[65]. Yadav et al. used PID and three different fuzzy controls for an IPS system and compared the results [66]. Irfan and et. al. tried to control the system with advanced sliding-mode control techniques. They also modeled the system and carried out studies in the simulation environment [67].

A new fuzzy inference model has been proposed for balancing the parallel type double IPS and this model is based on single entry rule models [68]. Muskinja and Tovarnik have controlled the IP system with fuzzy logic controllers in real time [69]. Chen and Liang Chen proposed a new robust adaptive control architecture based on the fuzzy model for the operation of an IPS mechanical system [70]. An IPS is the study made by Yüce and Tan the system is controlled in real-time with phase regressor and phase advance compensators designed by pole placement method. The control approach used was compared with the PID controller and both the simulation and real-time application results outperformed the PID controller [68] Mahmoodabadi and Khoobroo proposed an optimal adaptive hybrid controller for a fourth order IPS. To improve the performance of the controller, there is a combination of approximate feedback linearization and floating mode control approaches. The performance of the controller has been compared with several studies in the literature [69].

Larcombe was obtained the dynamic motion equations of the multi-part IPS and formed the mathematical model of the system [Larcombe, 1991]. It is aimed to control the angle position variable of an IPS. The problem is solved by using the linearization method with feedback in a different way. Prediction based control and LPV model control were performed. A compensating gain timing controller is proposed and the variable parameter of the system is shown as speed. In another study, the methods for stabilization were investigated based on geometric techniques and information was given. This paper put forwards evolutionary approaches for the IPS controller design, including various optimization methods. The aim is to balance the pendulum. Hirata et al. Examined the control of a rotational pendulum system with unknown parameters with a variable, self-adjusting, adaptive control system. In terms of control methods applied IPS can be classified as follows: linear methods such as PID and state feedback, parameter determination using genetic algorithms, artificial neural networks, energy-based, fuzzy logic, feedback linearization, sliding mode control and so on. nonlinear methods such as [72-81].

3. MATERIAL AND METHOD

3.1. Mathematical Model Of The System

The system in this sample occurs an inverted pendulum mounted to an active propulsion car cart. This inverted pendulum system is a sample usually finding in command system handbooks and literature. the interesting aspect of this mechanism is that it is unstable while uncontrolled, this is, the pendulum is going to simply fall over if the cart isn't moved to balance it. Also, the system is non-linear. The objective of the control system is to balance the inverted pendulum by applying a force to the cart that the pendulum is attached to. In that condition, as shown in Figure 3.1, we will examine a two-dimensional problem of the pendulum at 90 degrees to the ground plane. The control input of this system is the force f that moves the cart horizontally while the outputs of this system are the angular position of the pendulum θ and the horizontal position of the cart x.

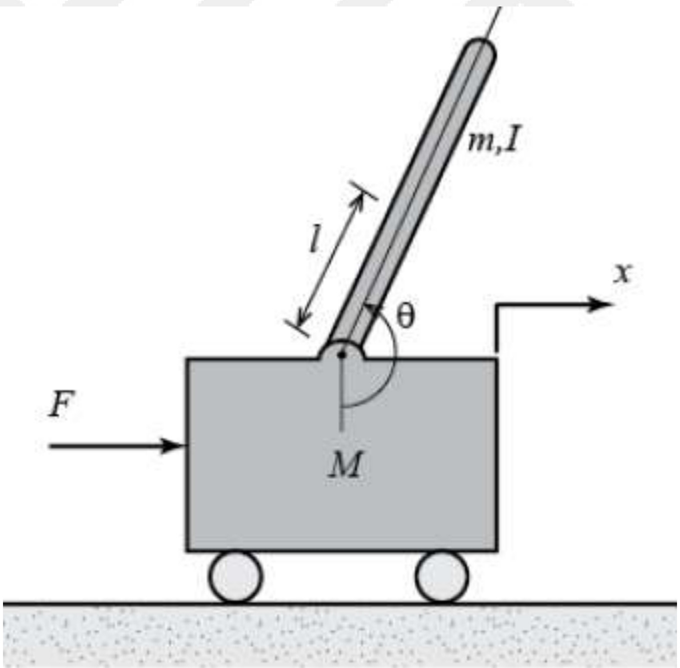

Figure 3.1. Free Body Diagram (FBD) of Inverted Pendulum [1]

Kinematic and kinetic values of the system are given in the Table 3.1. as shown below.

Table 0.1. Kinematic and kinetic values of the system

Unit	Property	Value
(M)	Mass of the cart	0.5 kg
(m)	Mass of the pendulum	0.2 kg
(b)	Coefficient of friction for cart	0.1 N.s/m
(l)	Length to pendulum center of the mass	0.3
(I)	Mass moment of inertia of the pendulum	0.006 kg.m^2
(F)	Force applied to the cart	N
(x)	Cart position coordinate	cm

The free-body diagrams (FBD) of the two links of the inverted pendulum system are shown below.

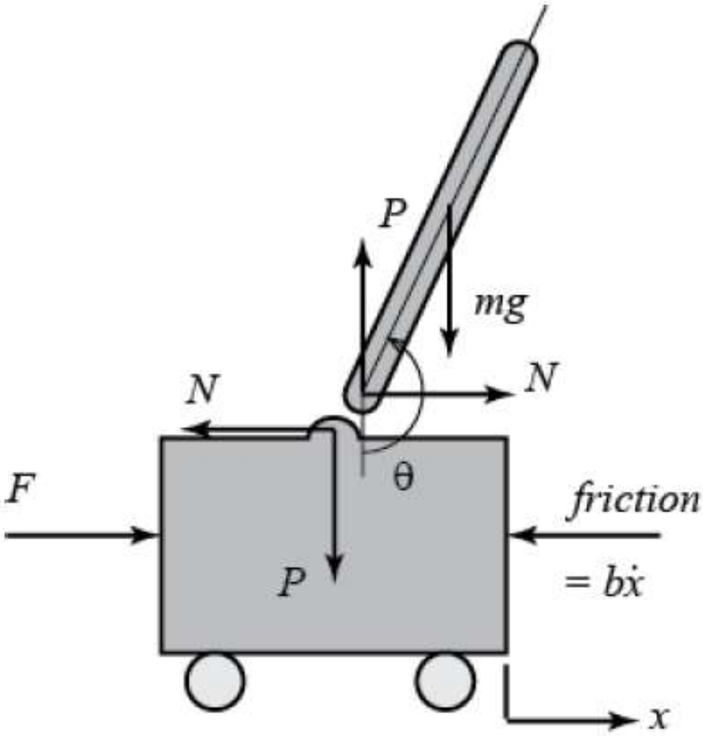

Figure 3.2. Free Body Diagram (FBD) of Inverted Pendulum [1]

In The Fig.3.2. According To The Free-Body Diagram (FBD) Of The System, We Get The Following Equation Of Motion By Using Newton's Second Law.

$$M\ddot{X} + b\dot{x} + N = F \tag{3.1}$$

We get the following expression for the reaction force N.

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \tag{3.2}$$

If Equation 2 Is Replaced By Equation 1, The First Main Equation 3.3 Of The System Is Obtained.

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F \tag{3.3}$$

if the vertical forces of the pendulum are summed to obtain the second main motion Equation of the system, the mathematical equation of the system becomes as follows

$$P\sin\theta+N\cos\theta-mg\sin\theta=ml\ddot{\theta}+m\ddot{x}\cos\theta \tag{3.4}$$

If total the moment is taken around the center of gravity of the pendulum to get Equ.5.

$$-Pl\sin\theta-Nl\cos\theta=I\ddot{\theta} \tag{3.5}$$

If the equation is rearranged again, the second main mathematical equation of the system is obtained.

$$(I+ml^2)\ddot{\theta}+mgl\sin\theta=\ -ml\ddot{x}\cos\theta \tag{3.6}$$

In This Study, Since We Do Control Analysis And Design For A Linear System, We Should Linearize The Nonlinear Mathematical Model Of The System. Especially, Ours Desire Linearize The Equations Around The Perpendicularly Upward Equilibrium Position, Θ = Π. Also, We Are Assuming That The System Stays Inside A Little Neighborhood Of This Equilibrium, That Is, This Equilibrium Angle Should Be Lesser Than 20° From The Vertical Upward Position. This Aim Is This Study's Hypothesis. Let Φ Represent The Deflection Of The Pendulum's Position From Equilibrium That Is Θ = Π+ Φ.

If We Linearize The System For Very Small Angles Of Φ, We Obtain The Linear Differential Equations Of The System:

$$\cos\theta\approx-1\ and\ \cos(\pi+\phi)\ \approx-1 \tag{3.7}$$

$$\sin\theta\approx-\phi\ and\ \sin(\pi+\ \phi)\ \approx-\phi \tag{3.8}$$

$$\dot{\theta}^2\ \approx 0\ and\ \dot{\phi}^2\ \approx 0 \tag{3.9}$$

Note that u represents the input F.

$$(\ I+ml^2)\ddot{\phi}\ -\ mgl\phi=ml\ddot{x} \tag{3.10}$$

$$(M+m)\ddot{x}+b\dot{x}-ml\ddot{\phi}=u \tag{3.11}$$

3.1.1. Transfer Function Of System

We set the transfer function of the linearized system differential equations by taking the Laplace transform of the system differential equations to assumed zero initial condition. Transfer function of system are as shown in Equ.3.12 and 3.13.

$$(I + ml^2)\phi(s)s^2 - mgl\phi(s) = mlX(s)s^2 \tag{3.12}$$

$$(M + m)X(s)s^2 + bX(s)s - ml\phi(s)s^2 = U(s) \tag{3.13}$$

If eliminate $X(s)$ with the Equ.12, we obtain the first transfer function for the output $\phi(s)$ and input of $U(s)$,

$$X(s) = \left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right] \phi(s) \tag{3.14}$$

After That, If Eq. 3.14 Is Substituted Into Eq. 3.13, We Obtain Below Eq. 3.15.

$$(M - m)\left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)s^2 + b\left[\frac{I + ml^2}{ml} - \frac{g}{s^2}\right]\phi(s)s - ml\phi(s)s^2 = U(s) \tag{3.15}$$

Again rearranging, the transfer function is then obtained the following.

$$\frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}\, s^2}{s^4 + \frac{b(I + ml^2)}{q}\, s^3 - \frac{(M + m)mgl}{q}\, s^2 - \frac{bmgl}{q}\, s} \tag{3.16}$$

Where

$$q = [\,(M + m)(I + ml^2) - (ml)^2] \tag{3.17}$$

As shown, both a pole and a zero in the transfer function is reduced from the system to obtain a new transfer function.

$$P_{pend}(s) = \frac{X(s)}{U(s)} = \frac{\phi(s)}{U(s)}$$

$$= \frac{\frac{ml}{q}\, s}{s^3 + \frac{b(I + ml^2)}{q}\, s^2 - \frac{(M + m)mgl}{q}\, s - \frac{bmgl}{q}} \quad \left[\frac{rad}{N}\right] \tag{3.18}$$

Also, the transfer function about the cart position X(s) can be derived in the same manner to come at that under.

$$P_{cart}(s) = \frac{X(s)}{U(s)}$$

$$= \frac{\frac{(I+ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \qquad \left[\frac{m}{N}\right] \tag{3.19}$$

The transfer functions obtained manually above can also be obtained with the MATLAB program. Figure 3.3 shows MATLAB command program.

```
M = 0.5;
m = 0.2;
b = 0.1;
I = 0.006;
g = 9.8;
l = 0.3;
q = (M+m)*(I+m*l^2)-(m*l)^2;
s = tf('s');
P_cart = (((I+m*l^2)/q)*s^2 - (m*g*l/q))/(s^4 + (b*(I + m*l^2))*s^3/q - ((M + m)*m*g*l)*s^2/q - b*m*g*l*s/q);
P_pend = (m*l*s/q)/(s^3 + (b*(I + m*l^2))*s^2/q - ((M + m)*m*g*l)*s/q - b*m*g*l/q);
sys_tf = [P_cart ; P_pend];
inputs = {'u'};
outputs = {'x'; 'phi'};
set(sys_tf,'InputName',inputs)
set(sys_tf,'OutputName',outputs)
sys_tf
```

Figure 3.3. MATLAB command program The obtained transfer functions in MATLAB program

```
sys_tf =
 From input "u" to output...
                 4.182e-06 s^2 - 0.0001025
  x:  -----------------------------------------------------------
      2.3e-06 s^4 + 4.182e-07 s^3 - 7.172e-05 s^2 - 1.025e-05 s
                       1.045e-05 s
  phi:  -----------------------------------------------------
       2.3e-06 s^3 + 4.182e-07 s^2 - 7.172e-05 s - 1.025e-05
```

Figure 3.4. MATLAB command program The obtained Transfer Function

3.1.2. State-Space Model Of System

The equations of motion of the above linearized system are converted to first order non-complex algebraic equations by assigning new state variables to first order differential equations. Then can be shown in state-space form;

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Ml^2} & \frac{m^2gl^2}{I+(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u \tag{3.20}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u \tag{3.21}$$

The state-space matrix obtained manually above can also be obtained with the MATLAB program. Figure 3.5 shows the MATLAB command program.

```
M = 0.5;
m = 0.2;
b = 0.1;
I = 0.006;
g = 9.8;
l = 0.3;
q = (M+m)*(I+m*l^2)-(m*l)^2;
s = tf('s');
P_cart = (((I+m*l^2)/q)*s^2 - (m*g*l/q))/(s^4 + (b*(I + m*l^2))*s^3/q - ((M + m)*m*g*l)*s^2/q - b*m*g*l*s/q);
P_pend = (m*l*s/q)/(s^3 + (b*(I + m*l^2))*s^2/q - ((M + m)*m*g*l)*s/q - b*m*g*l/q);
sys_tf = [P_cart ; P_pend];
inputs = {'u'};
outputs = {'x'; 'phi'};
set(sys_tf,'InputName',inputs)
set(sys_tf,'OutputName',outputs)
sys_tf
```

Figure 3.5. MATLAB Command Program The Obtained State-Space Matrix In MATLAB

```
A =
                      x     x_dot       phi   phi_dot
  x                   0         1         0         0
  x_dot               0   -0.1818     2.673         0
  phi                 0         0         0         1
  phi_dot             0   -0.4545     31.18         0

B =
                u
  x             0
  x_dot     1.818
  phi           0
  phi_dot   4.545

C =
              x     x_dot       phi   phi_dot
  x           1         0         0         0
  phi         0         0         1         0

D =
        u
  x     0
  phi   0

Continuous-time state-space model.
```

Figure 3.6. MATLAB command program And Results

3.2. System Analysis Of Inverted Pendulum

3.2.1. Open-Loop Impulse Response:

The open-loop step answer of the IPS was composed and we set up a new m file and composed the system model. After that if the design requirements for the pendulum and MATLAB program shown in Figure 3.7 are done then we obtain Figure 3.8.

```
M = 0.5;
m = 0.2;
b = 0.1;
I = 0.006;
g = 9.8;
l = 0.3;
q = (M+m)*(I+m*l^2)-(m*l)^2;
s = tf('s');
P_cart = (((I+m*l^2)/q)*s^2 - (m*g*l/q))/(s^4 + (b*(I + m*l^2))*s^3/q - ((M + m)*m*g*l)*s^2/q - b*m*g*l*s/q);
P_pend = (m*l*s/q)/(s^3 + (b*(I + m*l^2))*s^2/q - ((M + m)*m*g*l)*s/q - b*m*g*l/q);
sys_tf = [P_cart ; P_pend];
inputs = {'u'};
outputs = {'x'; 'phi'};
set(sys_tf,'InputName',inputs)
set(sys_tf,'OutputName',outputs)
   t=0:0.01:1;
impulse(sys_tf,t);
title('Open-Loop Impulse Response');
```

Figure 3.7. MATLAB command program and results

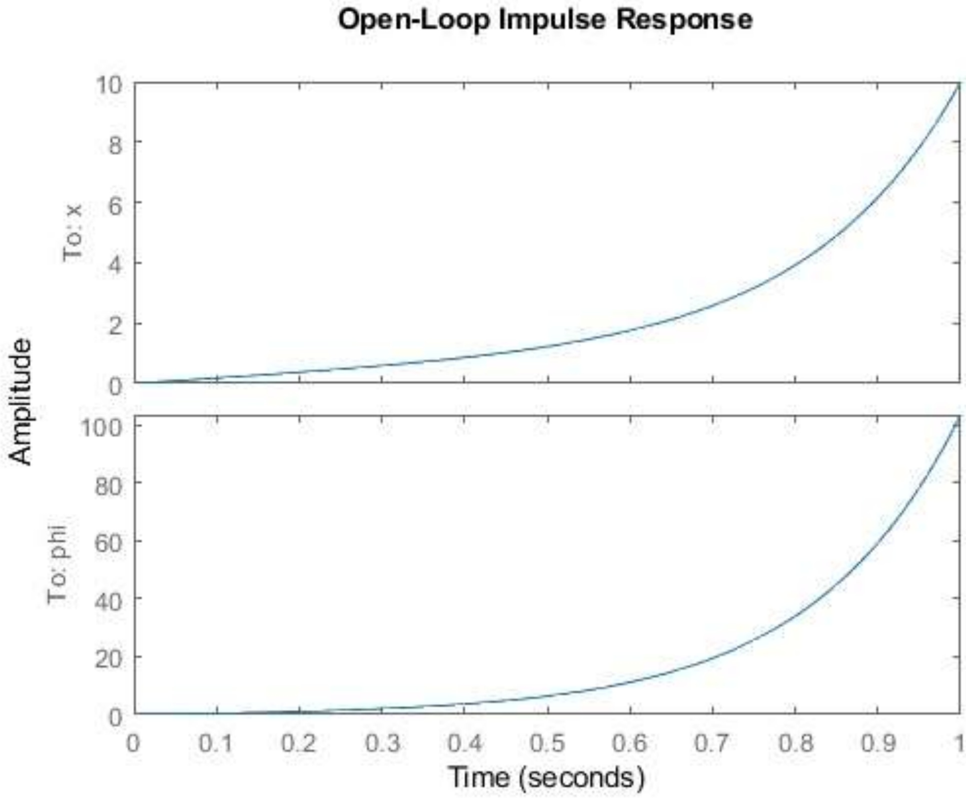

Figure 3.8. Open-loop impulse response

Since the IPS has one input and two outputs, it has two transfer functions. Also, we can find the poles and zeros of the IPS utilizing the MATLAB function.

```
[zeros poles] = zpkdata(P_pend,'v')

zeros =

     0


poles =

    5.5651
   -5.6041
   -0.1428


[zeros poles] = zpkdata(P_cart,'v')

    zeros =

        4.9497
       -4.9497


    poles =

             0
        5.5651
       -5.6041
       -0.1428
```

Figure 3.9. MATLAB command program And Results

In The Figure 3.9. As shown, the poles for both transfer functions are equal. The pole at 5.5651 shows that the system is unstable because the pole has positive real part. This means that the pole is in the right half of the complex s-plane.

3.2.2. Open Loops Step Response:

The open-loop step answer of the IPS was created and we set up a new m file and composed the system model. After that if the design requirements for the pendulum and MATLAB program shown in Figure 3.10 are done then we obtain Figure 3.11. Also let's not forget, the system has a pole with a positive real part its response to a step input will increase unlimitedly. Thus, Newton step input is going to be used.

```
t = 0:0.05:10;
u = ones(size(t));
[y,t] = lsim(sys_tf,u,t);
plot(t,y)
title('Open-Loop Step Response')
axis([0 3 0 50])
legend('x','phi')
```

Figure 3.10. MATLAB command program and graph

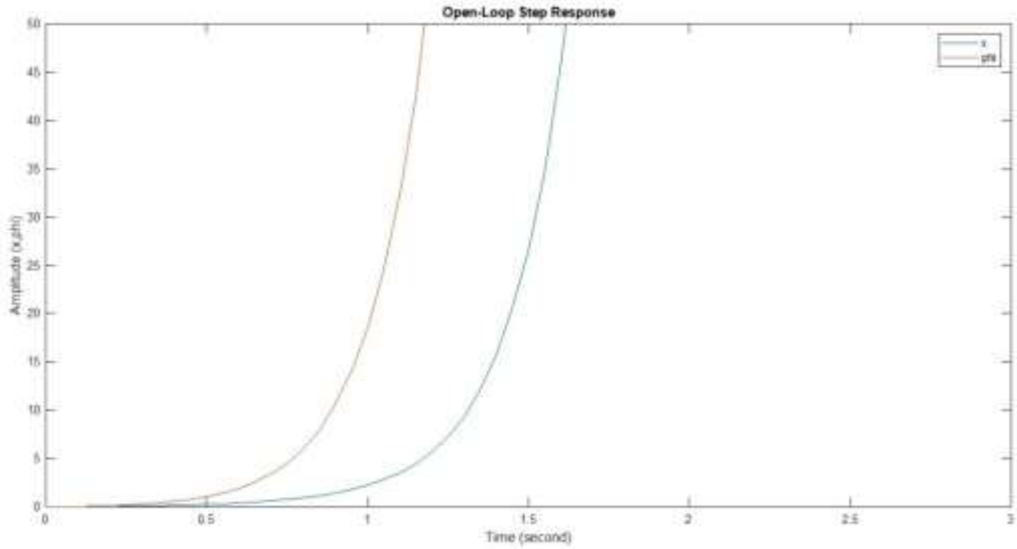

Figure 3.11. Open-Loop Step Response

We will also figure out some significant characteristics of the answer about the system by using the limoniform command as shown in the Figure 3.12. below.

```
step_info = lsiminfo(y,t);
cart_info = step_info(1)
pend_info = step_info(2)

cart_info =

  struct with fields:

    SettlingTime: 9.9959
             Min: 0
         MinTime: 0
             Max: 8.7918e+21
         MaxTime: 10


pend_info =

  struct with fields:

    SettlingTime: 9.9959
             Min: 0
         MinTime: 0
             Max: 1.0520e+23
         MaxTime: 10
```

Figure 3.12. MATLAB command program And Results

As shown, the results provide our expectancy that the system's response to a step input is unstable. This means that we be required designed to develop the answer of the system for some control. We especially try to create PID control, root-locus, frequency-response, and state-space. Finally, we show what happens to the cart's position and pendulum’s position on the system.

3.3. PID Controller Design Of Inverted Pendulum

We carried out PID control algorithm to control the system pendulum's angle without regard for the cart's position. Our control system is a single-input, single-output plant as shown by the following transfer function.

$$p_{pend}(s) = \frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \left[\frac{rad}{N}\right] \quad (3.22)$$

Where

$$q = (M+m)(I+ml^2) - (ml)^2 \quad (3.23)$$

Here the most important, the designed PID-controller try to hold the pendulum vertically upward when the cart is applied to a 1-Nsec impulse. Control bloc diagram for this balancing problem is given in Figure 3.13. Here *F* is disturbance effect while *u* is output of control system.

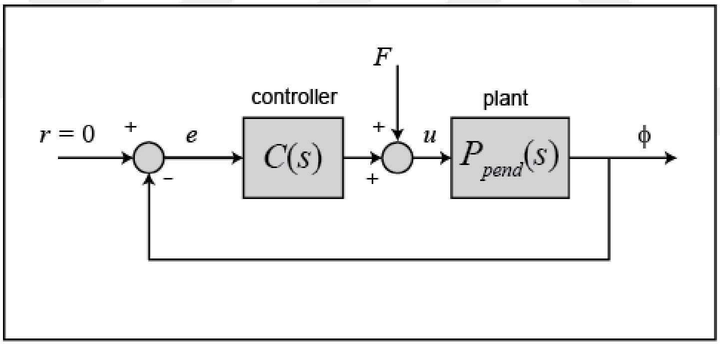

Figure 3.13. Control Bloc Diagram of with An Impulse Disturbance [1]

The transfer function T(s) of system for the closed-loop system as shown below;

$$T(s) = \frac{\phi(s)}{F(s)} = \frac{P_{pend}(s)}{1 + C(s)P_{pend}(s)} \quad (3.24)$$

To design and create PID controller, first we need to determine our system inside MATLAB. That is, we need transfer function of system. As shown in Figure 3.14, it is obtaining by using MATLAB. After that we can create PID controller m-file in MATLAB as shown in Figure 3.15.

```
M = 0.5;
m = 0.2;
b = 0.1;
I = 0.006;
g = 9.8;
l = 0.3;
q = (M+m)*(I+m*l^2)-(m*l)^2;
s = tf('s');
P_pend = (m*l*s/q)/(s^3 + (b*(I + m*l^2))*s^2/q - ((M + m)*m*g*l)*s/q - b*m*g*l/q)

P_pend =

                      1.045e-05 s
  -----------------------------------------------------
  2.3e-06 s^3 + 4.182e-07 s^2 - 7.172e-05 s - 1.025e-05

Continuous-time transfer function.
```

Figure 3.14. MATLAB command program of System transfer function

```
Kp = 1;
Ki = 1;
Kd = 1;
C = pid(Kp,Ki,Kd);
T = feedback(P_pend,C);
 t=0:0.01:10;
impulse(T,t)
title({'Response of Pendulum Position to an Impulse Disturbance';'under PID Control: Kp = 1, Ki = 1, Kd = 1'})
```

Figure 3.15. MATLAB command program for PID Controller

First, we provide the response of the closed-loop system with applying an impulse disturbance. PID control coefficient have been got K_p=1, K_d=1, K_i=1. PID control's response plot is shown in Figure 3.16. If we use these coefficients, system will be unstable as shown in Figure 3.17. Thus, we must determine new coefficient to balance the system. This work can be done by trial error. Therefore, we tried to determine them. Then they are found as K_p= 100, K_d = 20, K_i = 1. Its m-file program and response are given in Figure 3.17 and 3.18, respectively. System become stable now as shown in Figure 3.18. Finally, if you use these coefficients such as K_p = 100, K_d = 1, K_i = 1, then we will have Figure 3.19. and Figure 3.20.

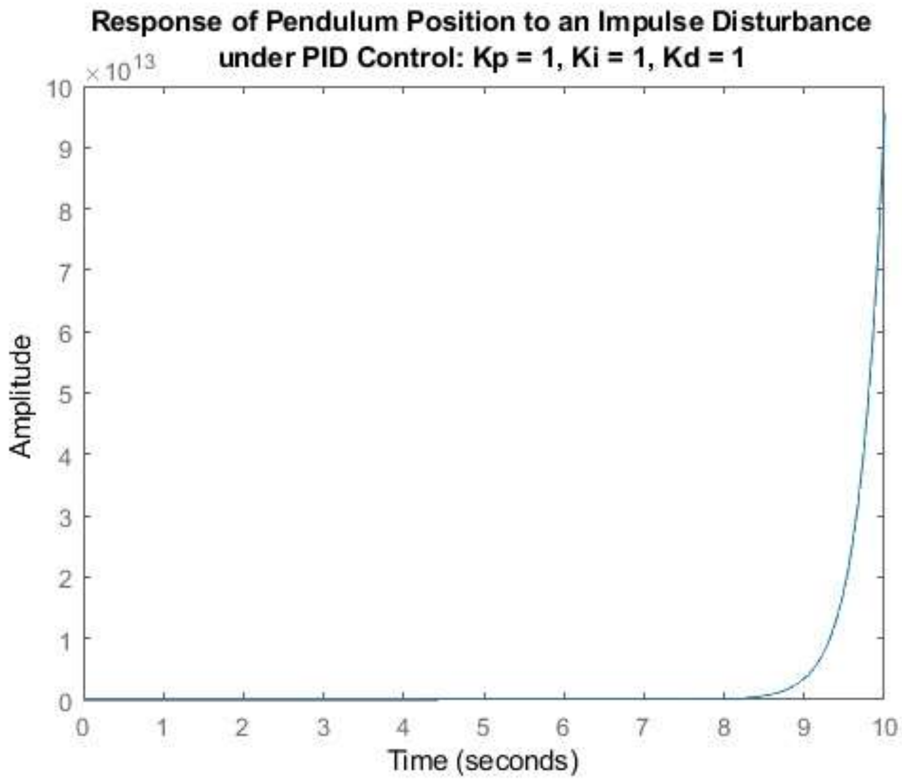

Figure 3.16. PID control Response of Pendulum to An Impulse Disturbance Under PID control: $K_p = 1, K_i = 1, K_d = 1$.

```
Kp = 100;
Ki = 1;
Kd = 20;
C = pid(Kp,Ki,Kd);
T = feedback(P_pend,C);
t=0:0.01:10;
impulse(T,t)
axis([0, 2.5, -0.2, 0.2]);
title({'Response of Pendulum Position to an Impulse Disturbance';'under PID Control: Kp = 100, Ki = 1, Kd = 20'})
```

Figure 3.17. MATLAB command program for PID Controller

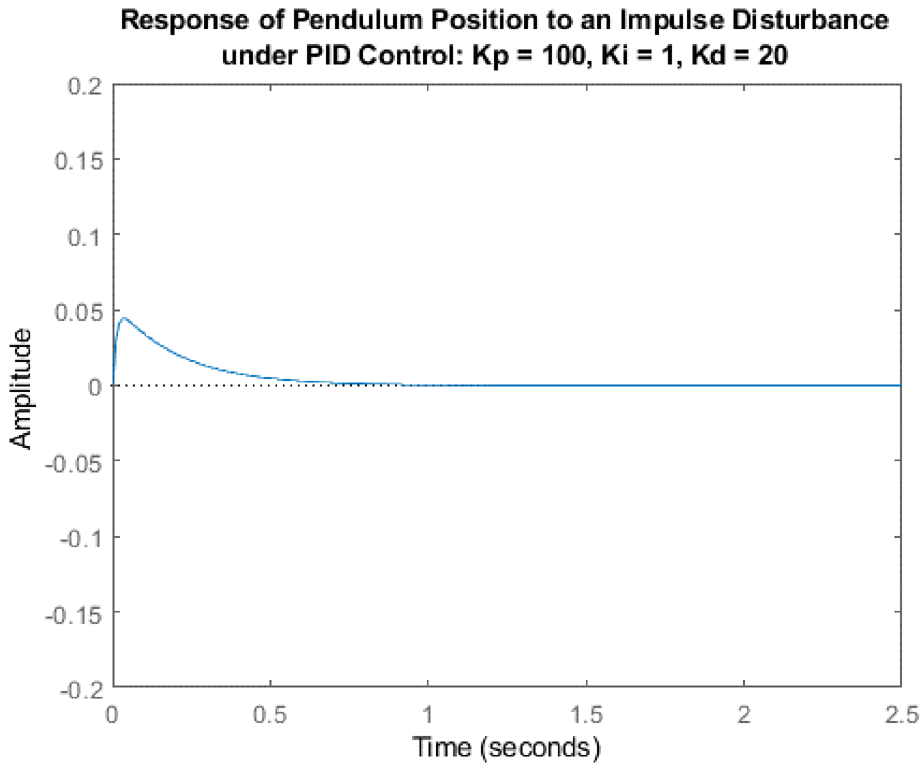

Figure 3.18. PID control Response of Pendulum to An Impulse Disturbance Under PID control: $K_p = 100, K_i = 1, K_d = 20$.

```
Kp = 100;
Ki = 1;
Kd = 1;
C = pid(Kp,Ki,Kd);
T = feedback(P_pend,C);
t=0:0.01:10;
impulse(T,t)
axis([0, 2.5, -0.2, 0.2])
title({'Response of Pendulum Position to an Impulse Disturbance';'under PID Control: Kp = 100, Ki = 1, Kd = 1'})
```

Figure 3.19. MATLAB command program for PID Controller

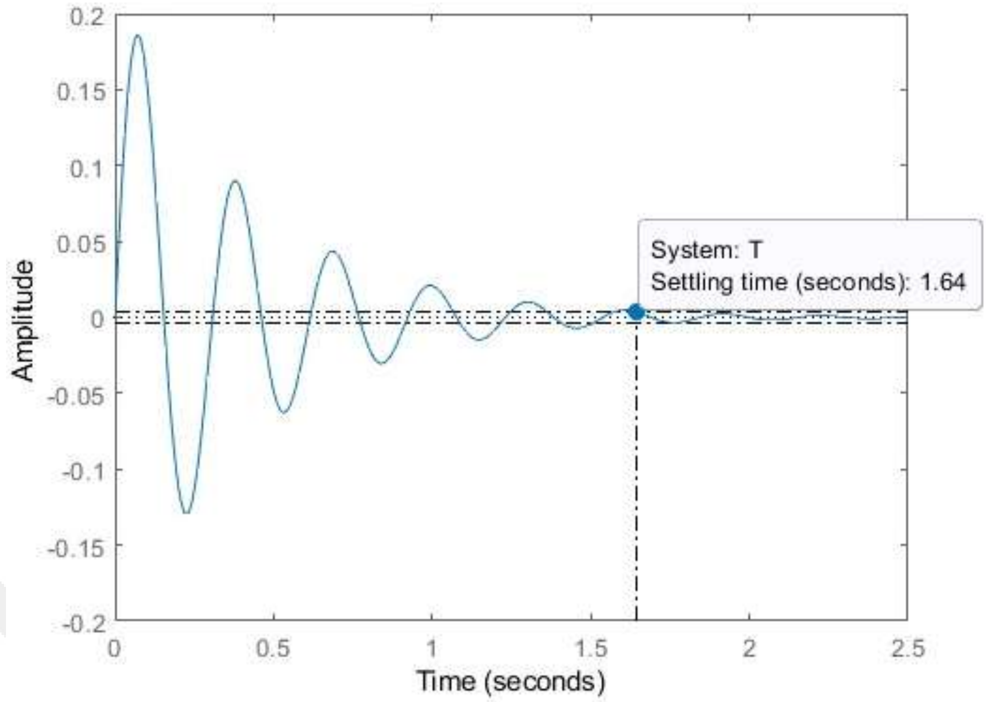

Figure 3.20. Response of Pendulum Position to An Impulse Disturbance Under PID control: $K_p = 100, K_i = 1, K_d = 1.$

Now we will examine the position of the cart. The control block diagram for the system has been given below. System block diagram is shown in the following Figure 3.21.

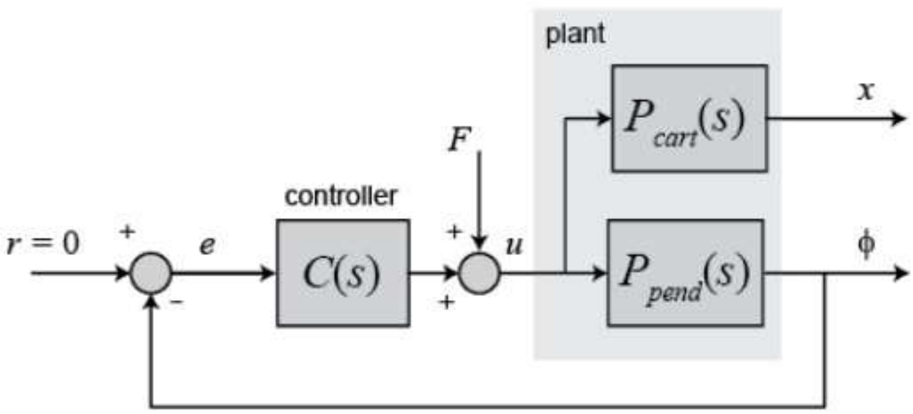

Figure 3.21. System block diagram for cart position [1]

The block C(s) represents the controller designed to balance the vertical pendulum. Thus, the closed-loop transfer function $T_2(s)$ is shown below from an input force applied to the cart to a cart position output.

$$T_2(s) = \frac{X(s)}{U(s)} = \frac{P_{cart}(s)}{1 + P_{cart}(s)C(s)} \tag{3.25}$$

The transfer function for P cart(s) is defined as follows,

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{\frac{(I+ml^2)s^2 - gml}{q}}{s^4 + \frac{b(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{bmgl}{q}s} \quad \left[\frac{m}{N}\right] \tag{3.26}$$

Where,

$$q = [(M+m)(I+ml^2) - (ml)^2] \tag{3.27}$$

We create transfer function of the chart in MATLAB in Figure 3.23. Also, we generate the location of cart response to the same impulse disturbance. Response of cart Position to an Impulse Disturbance is given Figure 3.23.

```
P_cart = (((I+m*l^2)/q)*s^2 - (m*g*l/q))/(s^4 + (b*(I + m*l^2))*s^3/q - ((M + m)*m*g*l)*s^2/q - b*m*g*l*s/q);
T2 = feedback(1,P_pend*C)*P_cart;
t = 0:0.01:5;
impulse(T2, t);
title({'Response of Cart Position to an Impulse Disturbance';'under PID Control: Kp = 100, Ki = 1, Kd = 20'})
```

Figure 3.22. MATLAB command program for Cart Position with PID Controller

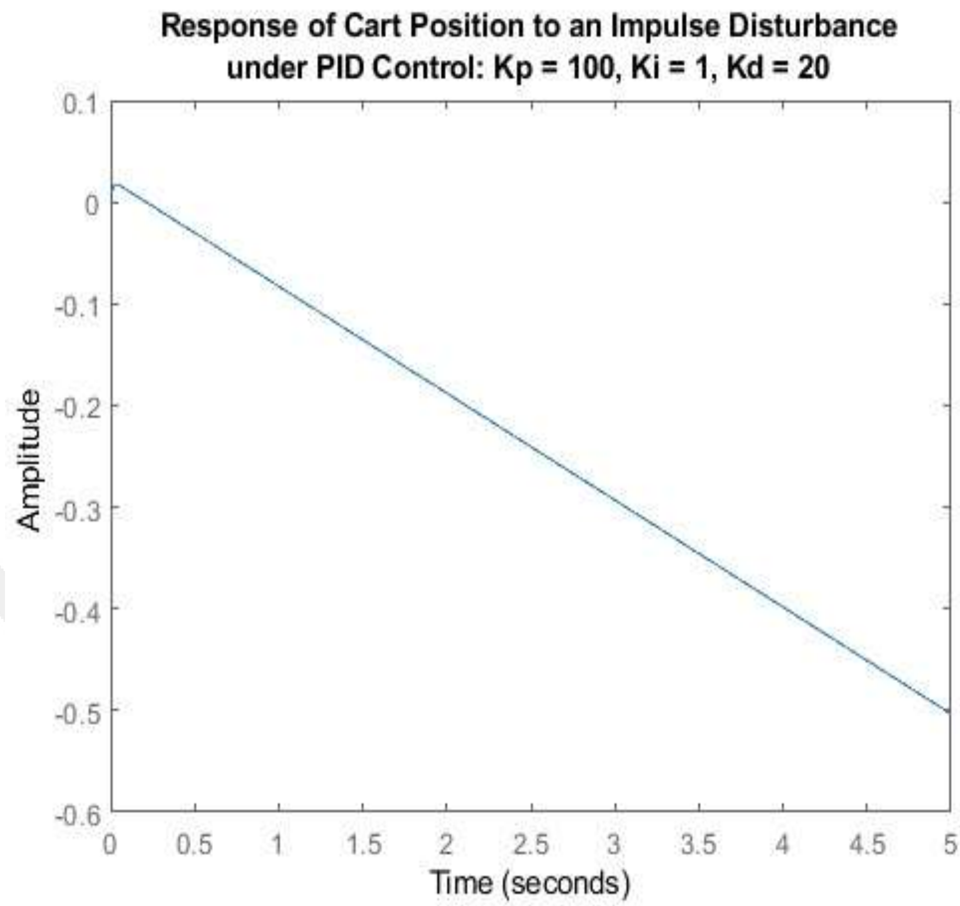

Figure 3.23. Response of cart Position to An Impulse Disturbance Under PID control: $K_p = 100, K_i = 1, K_d = 20$.

In The Figure 3.23, the cart is traveling constant velocity in the negative way. Thus, while the PID controller stabilizes the angle of the pendulum, real device would not be possible to apply.

3.4. Root Locus Design Of System

We carried out root-locus models under the designed control algorithms to control the system pendulum's angle without regard for the cart's position. As known that root locus shows us where the roots and zeros of the system are located on the complex plane. That is, it shows root curves to us. Again here, control system is a single-input, single-output plant as shown by the following transfer function.

$$\mathrm{p_{pend}}(\mathrm{s}) = \frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q} s}{s^3 + \frac{b(I+ml^2)}{q} s^2 - \frac{(M+m)mgl}{q} s - \frac{bmgl}{q}} \quad \left[\frac{rad}{N}\right] \tag{3.28}$$

Where,

$$q = (M+m)(I+ml^2) - (ml)^2 \tag{3.29}$$

Here the most important, the designed controller try to hold the pendulum vertically upward when the cart is applied to a 1-Nsec impulse. Control bloc diagram for this balancing problem is given in Figure 3.24. Here *F* is disturbance effect while *u* is output of control system.

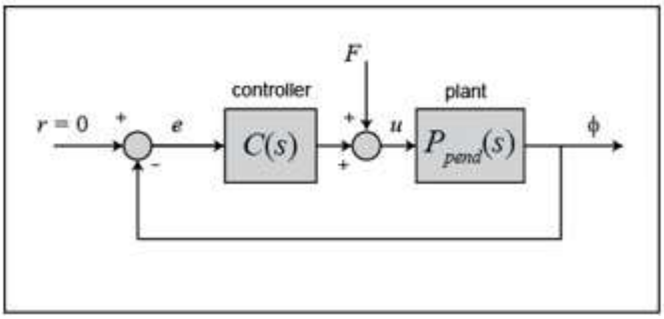

Figure 3.24. Control Bloc Diagram for a disturbance of impulses [1]

The transfer function T(s) of system for the closed-loop system as shown below;

$$T(s) = \frac{\phi(s)}{F(s)} = \frac{P_{pend}(s)}{1 + C(s)P_{pend}(s)} \tag{3.30}$$

First, to design and create proportional (P) controller, first we need to determine our system inside MATLAB. That is, we need transfer function of system. As shown in Figure 3.25, it is obtaining by using MATLAB. After that we can create proportional (P) controller m-file in MATLAB as shown in Figure 3.25. Also, proportional (P) controller's response is given in Figure 3.26.

```
M = 0.5;
m = 0.2;
b = 0.1;
I = 0.006;
g = 9.8;
l = 0.3;
q = (M+m)*(I+m*l^2)-(m*l)^2;
s = tf('s');
P_pend = (m*l*s/q)/(s^3 + (b*(I + m*l^2))*s^2/q - ((M + m)*m*g*l)*s/q - b*m*g*l/q)
[zeros poles] = zpkdata(P_pend,'v');rlocus(P_pend)
title('Root Locus of Plant (under Proportional Control)')

P_pend =

                      1.045e-05 s
  -----------------------------------------------------
  2.3e-06 s^3 + 4.182e-07 s^2 - 7.172e-05 s - 1.025e-05

Continuous-time transfer function.
```

Figure 3.25. MATLAB command program

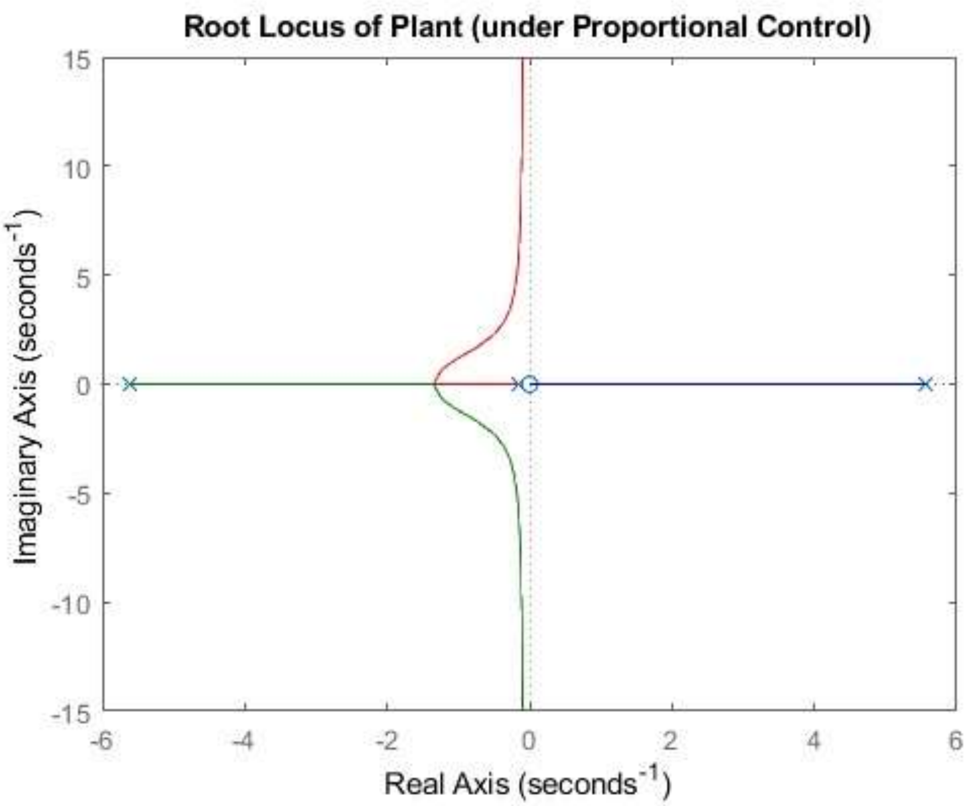

Figure 3.26. Root Locus Model Under Proportional Control (P)

Second, to design and create integral (I) controller, first we need to determine our system inside MATLAB. That is, we need transfer function of system. As shown in Figure 3.26, it is obtaining by using MATLAB. After that we can create integral (I) controller m-file in MATLAB as shown in Figure 3.27. Also, integral (I) controller's response is given in Figure 3.28.

```
C = 1/s;
rlocus(C*P_pend)
title('Root Locus with Integral Control')
```

Figure 3.27. MATLAB command program

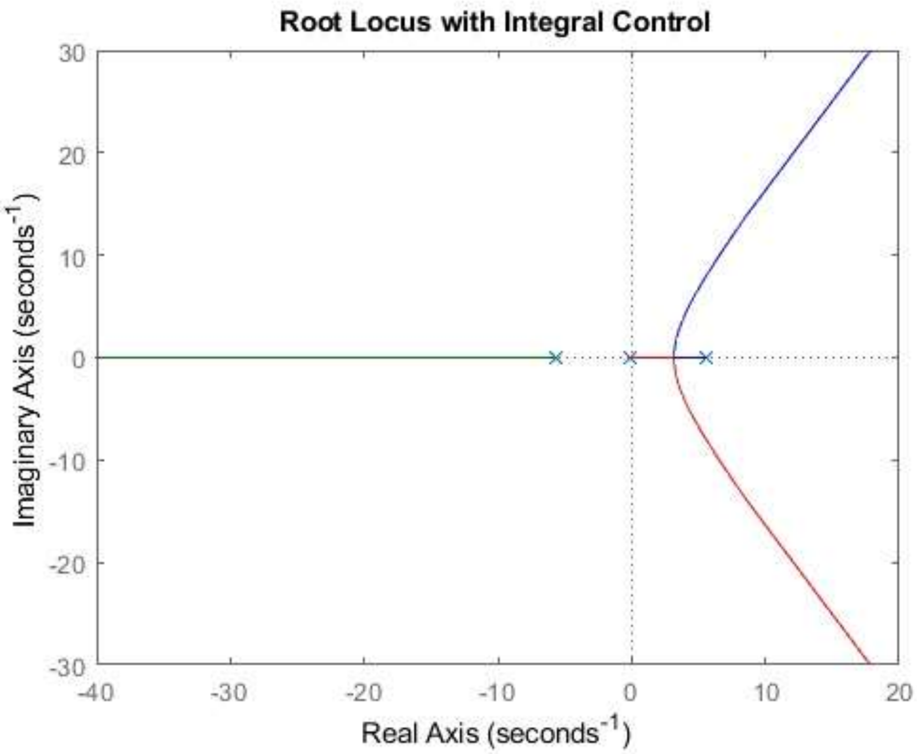

Figure 3.28. Root Locus Modal Under Integral Control (I)

Third, to design and create proportional-integral-derivative (PID) controller, first we need to determine our system inside MATLAB. That is, we need transfer function of system. As shown in Figure 3.25, it is obtaining by using MATLAB. After that we can create PID controller m-file in MATLAB as shown in Figure 3.29. Also, PID controller's response is given in Figure 3.30.

```
z = [-3 -4];
p = 0;
k = 1;
C = zpk(z,p,k);
rlocus(C*P_pend)
title('Root Locus with PID Controller')
```

Figure 3.29. MATLAB command program

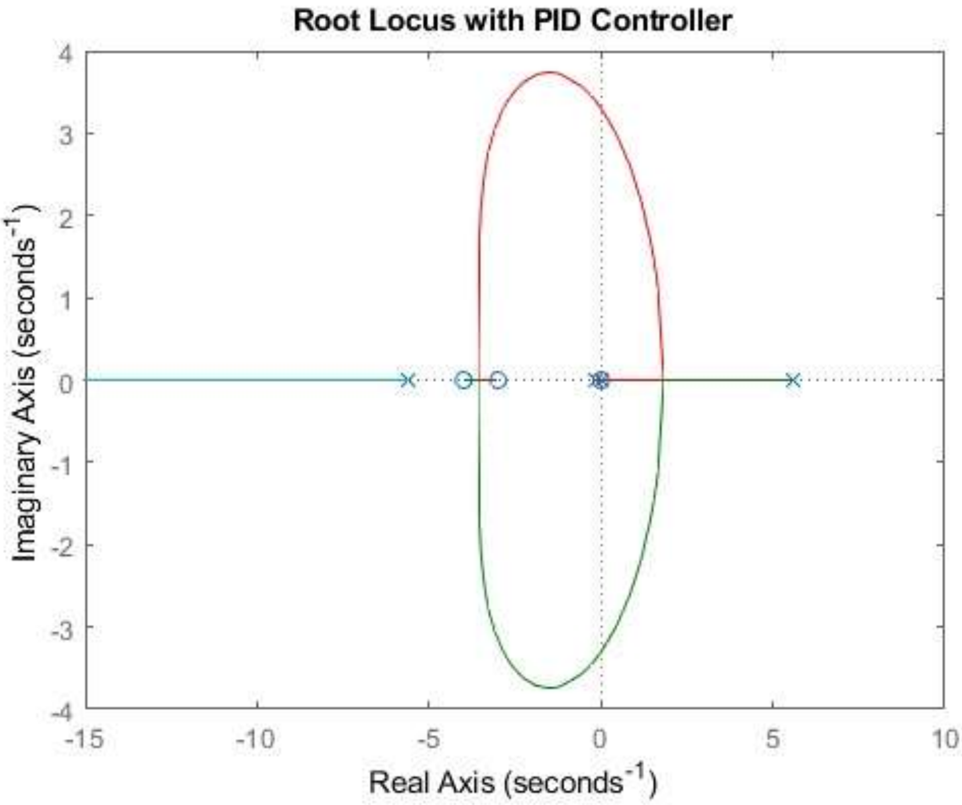

Figure 3.30. Root Locus Modal with PID Control

3.5. Frequency Domain Methods For Controller Design Of System

We carried out a frequency domain response design method under the designed control algorithms to control the system pendulum's angle without regard for the cart's position. As known that frequency response design gives important information about system balancing to us. Again here, control system is a single-input, single-output plant as shown by the following transfer function.

$$p_{pend}(s) = \frac{\phi(s)}{U(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{b(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{bmgl}{q}} \quad \left[\frac{rad}{N}\right] \tag{3.31}$$

Where,

$$q = (M+m)(I+ml^2) - (ml)^2 \tag{3.32}$$

Here again the most important, the designed controller try to hold the pendulum vertically upward when the cart is applied to a 1-Nsec impulse. Control bloc diagram for this balancing problem is given in Figure 3.31. Here *F* is disturbance effect while *u* is output of control system.

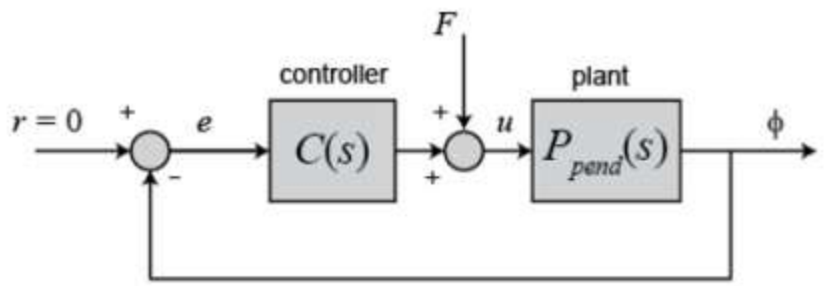

Figure 3.31. Control Bloc Diagram for a disturbance of impulses [1]

The transfer function T(s) of system for the closed-loop system as shown below;

$$T(s) = \frac{\phi(s)}{F(s)} = \frac{P_{pend}(s)}{1 + C(s)P_{pend}(s)} \tag{3.33}$$

First, to design and create frequency domain response design, first we need to determine our system inside MATLAB. That is, we need transfer function of system. As shown in Figure 3.32, it is obtaining by using MATLAB.

```
M = 0.5;
m = 0.2;
b = 0.1;
I = 0.006;
g = 9.8;
l = 0.3;
q = (M+m)*(I+m*l^2)-(m*l)^2; s = tf('s');
P_pend = (m*l*s/q)/(s^3 + (b*(I + m*l^2))*s^2/q - ((M + m)*m*g*l)*s/q - b*m*g*l/q);
[zeros poles] = zpkdata(P_pend,'v')
s = tf('s');
P_pend = (m*l*s/q)/(s^3 + (b*(I + m*l^2))*s^2/q - ((M + m)*m*g*l)*s/q - b*m*g*l/q);
```

Figure 3.32 . MATLAB command program

Inverted pendulum system is unstable without control. We can indicate this by using the Matlab command which is zpkdata. Zpkdata command finds the zeros and poles of the transfer function of system. If we put the code in the MATLAB then it finds them as shown below in the Figure 3.33.

```
[zeros poles] = zpkdata(P_pend,'v')

zeros =

     0


poles =

    5.5651
   -5.6041
   -0.1428
```

Figure 3.33. MATLAB command code

Now we will examine the position of the cart. The control block diagram for the system has been given below. System block diagram is shown in the following Figure 3.34.

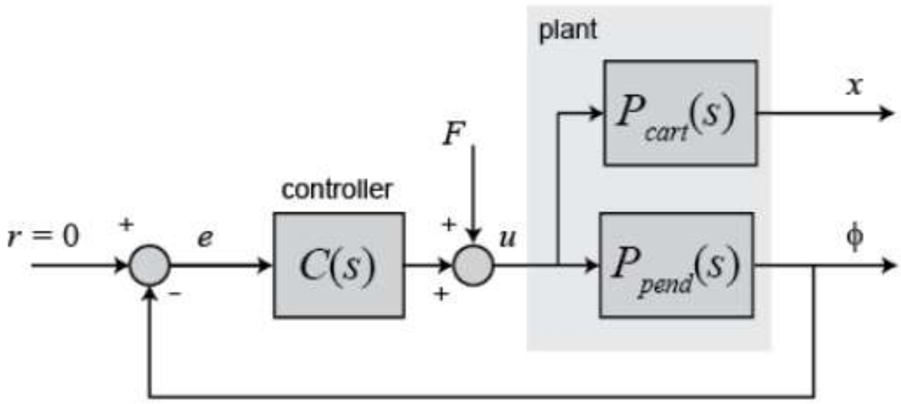

Figure 3.34. The inverted pendulum block diagram [1]

The block C(s) represents the controller designed to balance the vertical pendulum. Thus, the closed-loop transfer function $T_2(s)$ is shown below from an input force applied to the cart to a cart position output.

$$T_2(s) = \frac{X(s)}{F(s)} = \frac{P_{cart}(s)}{1 + P_{pend}(s)C(s)} \tag{3.34}$$

The transfer function for P cart(s) is defined as follows,

$$P_{cart}(s) = \frac{X(s)}{U(s)} = \frac{(I + ml^2)s^2 - gml}{s^4 + \frac{b(I + ml^2)}{q}s^3 - \frac{(M + m)mgl}{q}s^2 - \frac{bmgl}{q}s} \quad \left[\frac{m}{N}\right] \tag{3.35}$$

Where,

$$q = [(M + m)(I + ml^2) - (ml)^2] \tag{3.36}$$

We create transfer function of the chart in MATLAB in Figure 3.4. Also, we generate the location of cart response to the same impulse disturbance. Response of cart Position to an Impulse Disturbance is given Figure 3.36.

```
% fit past
P_cart = (((I+m*l^2)/q)*s^2 - (m*g*l/q))/(s^4 + (b*(I + m*l^2))*s^3/q - ((M + m)*m*g*l)*s^2/q - b*m*g*l*s/q);
T2 = feedback(1,P_pend*C)*P_cart;
T2 = minreal(T2);
t = 0:0.01:10;
impulse(T2, t), grid
title({'Response of Cart Position to an Impulse Disturbance';'under Closed-loop Control'});
```

Figure 3.35. MATLAB command program

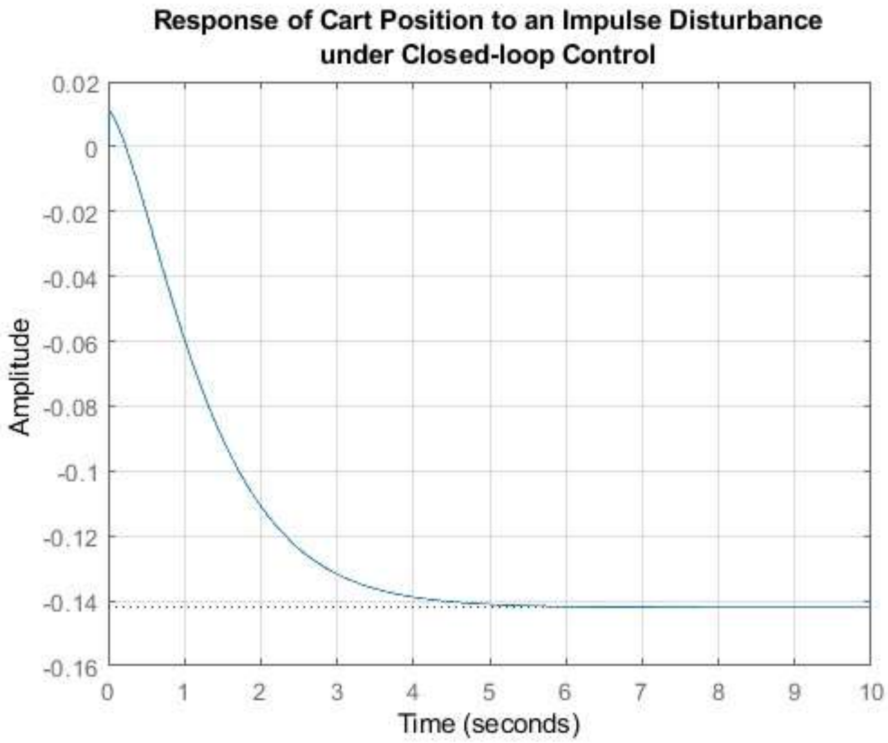

Figure 3.36. Response od cart position to an impulse Disturbance under closed loop system

3.5.1. Closed-Loop Response Without Compensation

We now study the response of the closed-loop system without compensation after that we start to design controller and control analysis. we plot many graphics such as Root-Locus, Bode, Nyquist, and impulse with done analysis. This analysis has been done by using MATLAB command control system. Bode diagram of the system is shown in Figure 3.37. Also, Architecture of Control System has been given in Figure 3.38. Especially, when studying with nonminimum-phase systems it is preferable to analyze relative stability using the Nyquist plot of the open-loop transfer function. In generally, the Nyquist plot is also used for higher-order systems. This is because the Bode plot shows frequencies that are 360 degrees apart as being different when in fact they are the same. Since the Nyquist plot is a polar-type plot, Nyquist response of the system is shown in Figure 3.39.

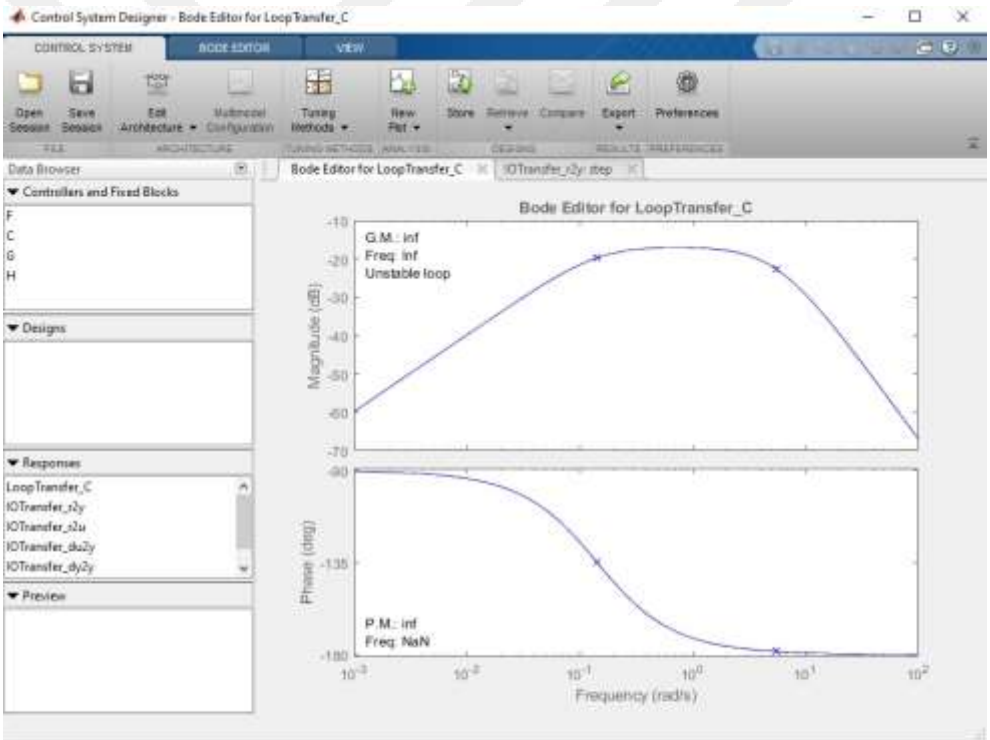

Figure 3.37. Bode diagram of system ('bode',P_pend)

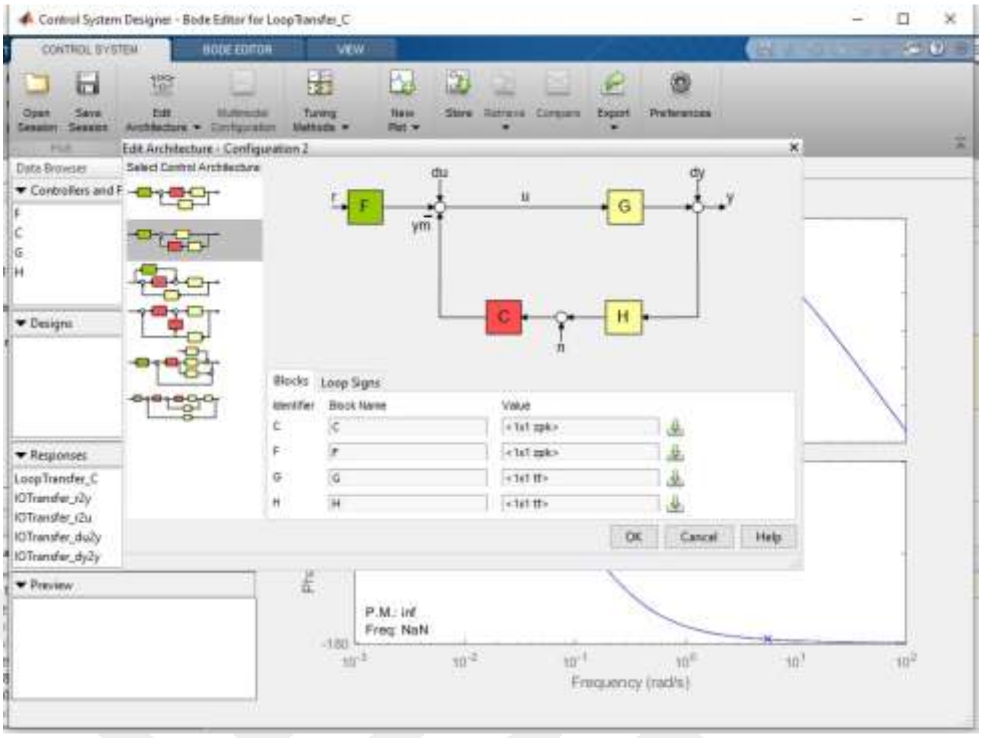

Figure 3.38. Architecture of Control System

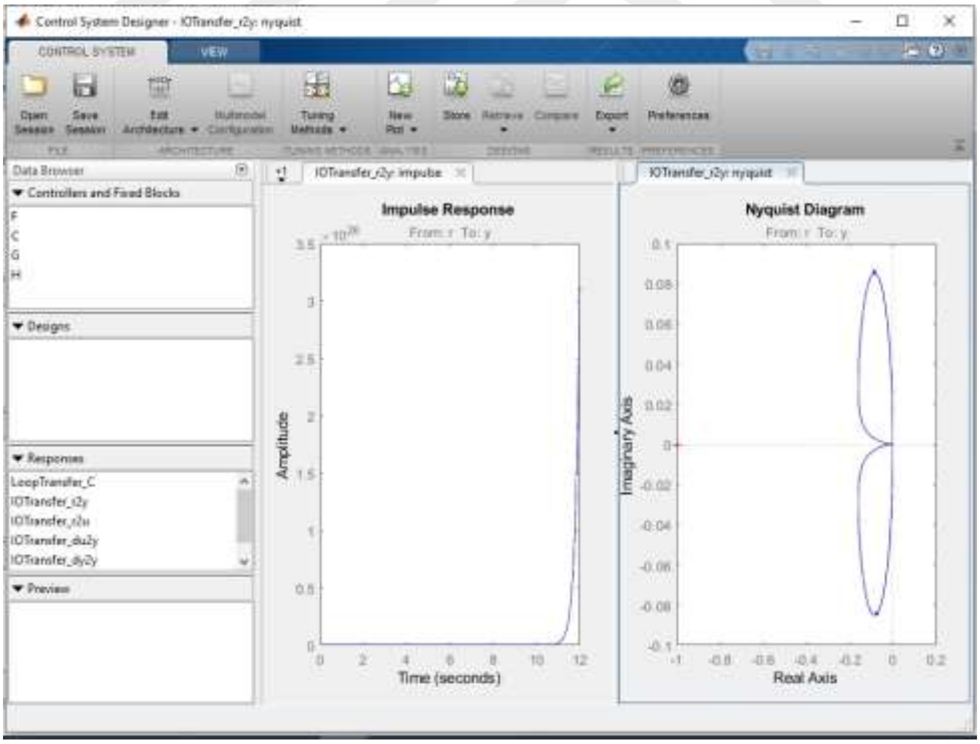

Figure 3.39. Control System Designer new plot Impulse, and Nyquist response.

3.5.2. Closed-Loop Response With Compensation

As shown above, the closed-loop system of system is unstable without compensation. Thus, we need to use the designed controller to balance the system. First, we will add an integrator to cancel the zero at the origin. Bode plot that is generated is shown in Figure 3.40.

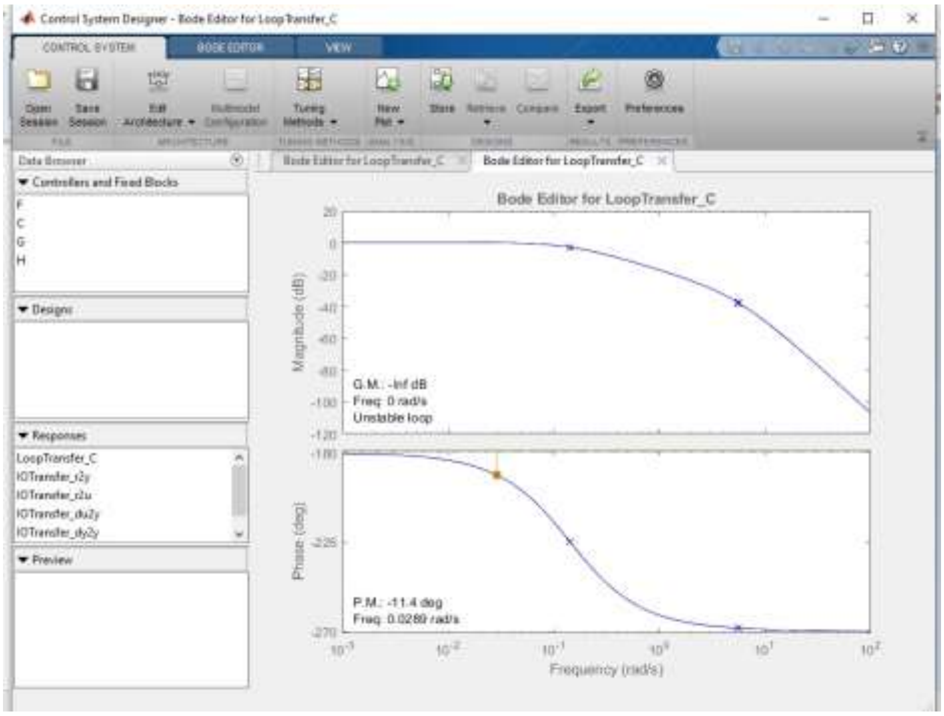

Figure 3.40. Control System Designer, Bode plot that is generated.

The closed-loop system is still unstable eventhough with the addition of this integrator. We can work to better understand the instability by looking more closely at the Nyquist plot. We can generate this analysis Nyquist plot in the same manner we did. Nyquist plot of system is given in Figure 3.41. If you add a zero on system, automatically the Bode and Nyquist plots will change. According to the added a zero, Nyquist plot has been shown in Figure 3.42.

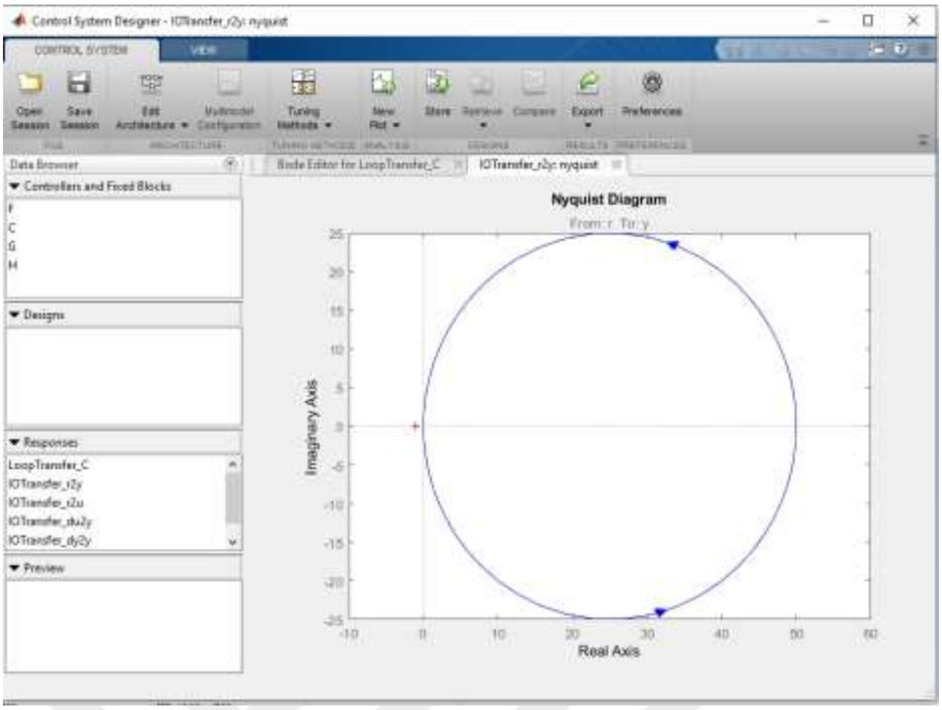

Figure 3.41. Control System Designer New Nyquist.

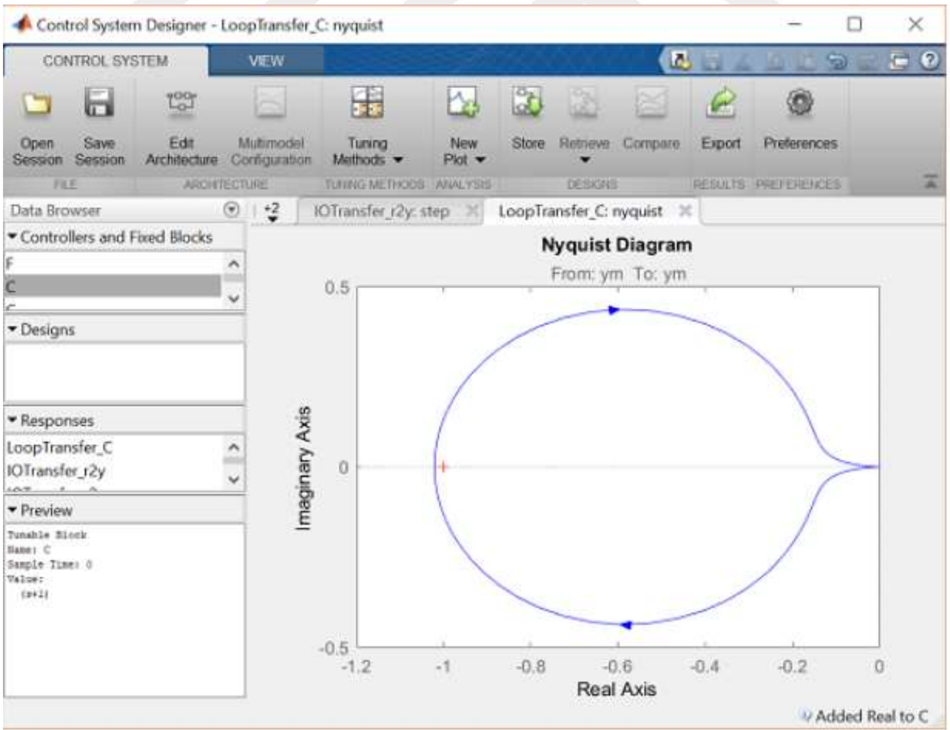

Figure 3.42. Control System Designer New Nyquist additional zero.

As may seem from Figure 3.41, this change has been not provide enough phase. The encirclement around -1 is still clockwise. Thus, We try to add a second zero at -1 in the same manner. New Nyquist plot has been shown in Figure 3.43.

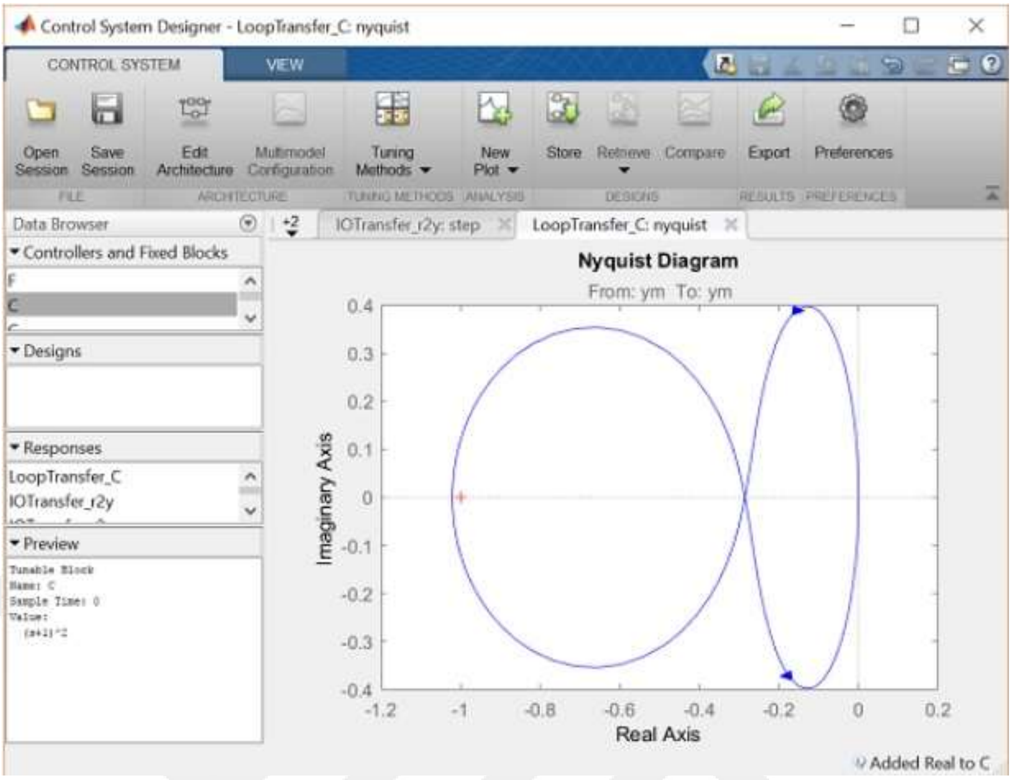

Figure 3.43. Control System Designer New Nyquist Additional Second Zero.

3.6. Control With Linear Quadratic Regulation (LQR) Of System

Linear quadratic ragulator (LQR) control, which is used in many industrial fields as well as in educational control laboratories, is an optimal control theory with state space feedback. Optimal control attempts to minimize a specified cost function.

This controller design successfully achieves stability and noise rejection in multi-input multi-output systems and in theory determines the most appropriate input parameters, ie optimal control input parameters. State matrices and performance index of the system are used in determining the parameters. The performance index is based on the selection of Q and R matrices, known as weight matrices.

The LQR control method is performed by leaving the gain values found in the controller by using the state space equation and the Q and R matrices known as weight matrices. The controllability matrix of the system takes the form shown Equ. 37.

$$C = [B\ AB\ A^2B\ \dots\ A^{n-1}B] \tag{37}$$

Matrix C of controllability is 4x4 diomention, thus, the rank of matrix must be 4. The rank and controllability of the system can be To determined by using Matlab command which is shown in Figure 3.44 below:

```
M = 0.5;
m = 0.2;
b = 0.1;
I = 0.006;
g = 9.8;
l = 0.3;
p = I*(M+m)+M*m*l^2; %denominator for the A and B matrices
A = [0 1 0 0; 0 -(I+m*l^2)*b/p (m^2*g*l^2)/p 0; 0 0 0 1; 0 -(m*l*b)/p m*g*l*(M+m)/p 0];
B = [ 0; (I+m*l^2)/p; 0; m*l/p];
C = [1 0 0 0; 0 0 1 0];
D = [0; 0];
states = {'x' 'x_dot' 'phi' 'phi_dot'};
inputs = {'u'}; outputs = {'x'; 'phi'};
sys_ss = ss(A,B,C,D,'statename',states,'inputname',inputs,'outputname',outputs);
poles = eig(A);co = ctrb(sys_ss);
controllability = rank(co)

controllability =

     4
```

Figure 3.44. The rank and controllability of the system

In particular, the linear quadratic regulatör control approach is used to evaluate contol gain matrix K. The MATLAB function lqr allows you to choose two parameters, R matrix and Q matrix, to balance the relative importance of the control effort (u) and error, respectively. Assum that $R = 1$, and $Q = C\,[\,C]C$ is to be the simplest case. Figure 3.45 shows how to find Q matrix by using Matlab.

```
Q = C'*C

Q =

     1     0     0     0
     0     0     0     0
     0     0     1     0
     0     0     0     0
```

Figure 3.45. Q Matrix of the system

Our aim is to find the control gain matrix K based on the q and r matrices. As mentioned above, control gain matrix K can be found by using MATLAB program. The designed m-file program is shown in Figure 3.46.

```
Q = C'*C;
R = 1;
K = lqr(A,B,Q,R)
Ac = [(A-B*K)];
Bc = [B];
Cc = [C];
Dc = [D];
states = {'x' 'x_dot' 'phi' 'phi_dot'};
inputs = {'r'};
outputs = {'x'; 'phi'};
sys_cl = ss(Ac,Bc,Cc,Dc,'statename',states,'inputname',inputs,'outputname',outputs);
t = 0:0.01:5;
r =0.2*ones(size(t));
[y,t,x]=lsim(sys_cl,r,t);
[AX,H1,H2] = plotyy(t,y(:,1),t,y(:,2),'plot');
set(get(AX(1),'Ylabel'),'String','cart position (m)')
set(get(AX(2),'Ylabel'),'String','pendulum angle (radians)')
title('Step Response with LQR Control')

K =

   -1.0000   -1.6567   18.6854    3.4594
```

Figure 3.46. MATLAB command program

The chart is shown the position of the carriage and the angle of the pendulum. These are shown in red and blue colors respectively. Figure 3.47 is not enough results in the graph shown. The pendulum and cart's overshoot come out fine, but their setting times requirement improvement and also the cart's rise time needs to be decreased. Then we should again find a new Q matrix and K matrix depending on Q, as shown in Figure 3.48. Also the ploted graphic with new control gain K matrix has been given in Figure 3.49.

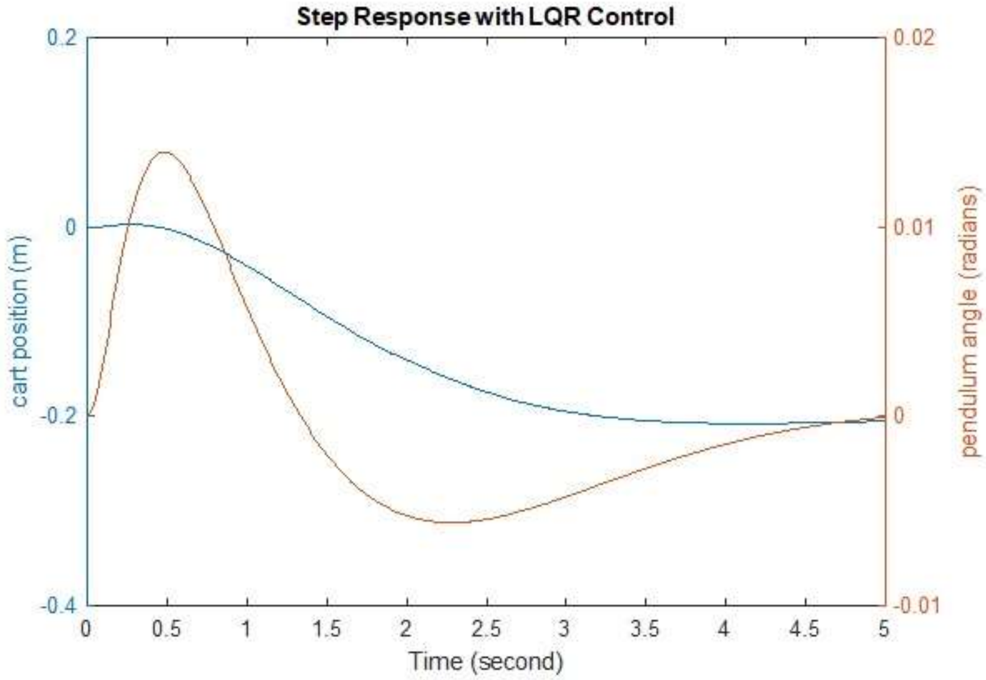

Figure 3.47. Step response with LQR control

```
Q = C'*C;
Q(1,1) = 5000;
Q(3,3) = 100
R = 1;
K = lqr(A,B,Q,R)
Ac = [(A-B*K)];
Bc = [B];
Cc = [C];
Dc = [D];
states = {'x' 'x_dot' 'phi' 'phi_dot'};
inputs = {'r'};
outputs = {'x'; 'phi'};
sys_cl = ss(Ac,Bc,Cc,Dc,'statename',states,'inputname',inputs,'outputname',outputs);
t = 0:0.01:5;
r =0.2*ones(size(t));
[y,t,x]=lsim(sys_cl,r,t);
[AX,H1,H2] = plotyy(t,y(:,1),t,y(:,2),'plot');
set(get(AX(1),'Ylabel'),'String','cart position (m)')
set(get(AX(2),'Ylabel'),'String','pendulum angle (radians)')
title('Step Response with LQR Control')
```

Figure 3.48. MATLAB command program and results And continues the results as shown in the Figure 3.49. and 3.50. below.

```
Q =

        5000           0           0           0
           0           0           0           0
           0           0         100           0
           0           0           0           0


K =

  -70.7107  -37.8345  105.5298   20.9238
```

Figure 3.49. MATLAB command program and results

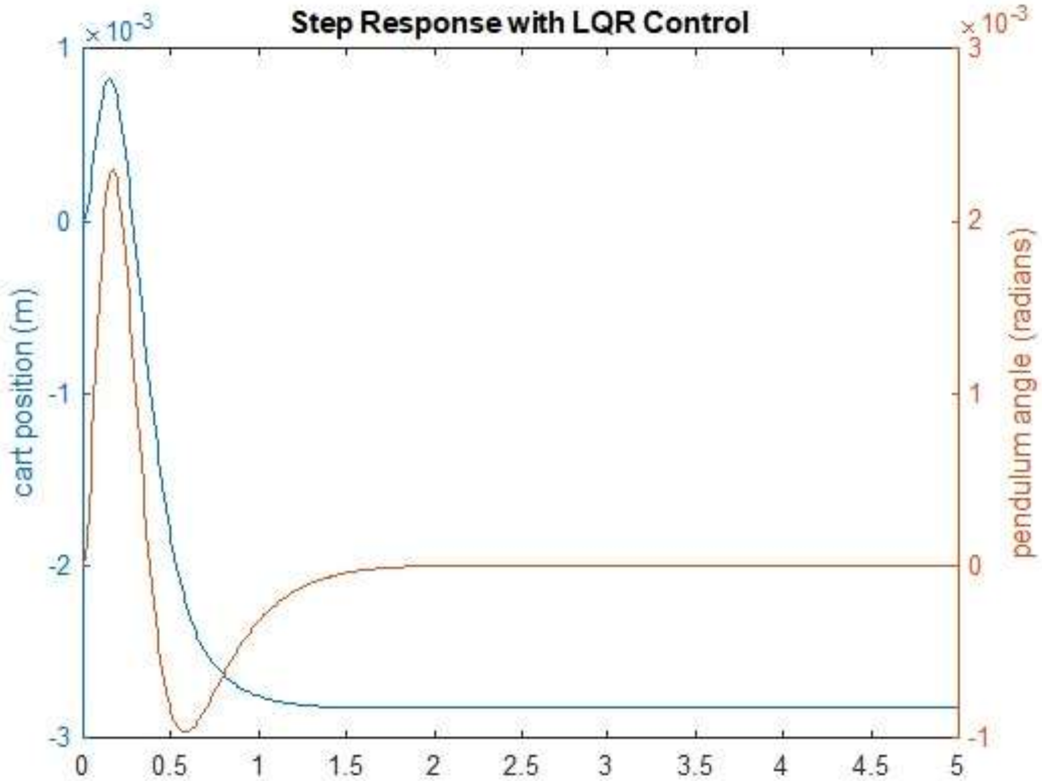

Figure 3.50. Step response with LQR control

3.7. Controllability And Observability

In this step is to determine the controllability and the observability of the system. The controllability and the observability of the system is depending on matrix C that is formed with matrix A and B of system. Matrix C of controllability is 4x4 diomention, thus, the rank of matrix must be 4. The controllability and observability of the system can be to determined by using Matlab command which is shown in Figure 3.50 below:

$$C = [B \quad AB \quad A^2B \quad . \; . \; . \quad A^{n-1}B] \tag{3.38}$$

$$O = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{3.39}$$

If the number of state variables in the system is 4 then the rank of both matrices must be 4, otherwise we cannot obtain any information about whether or not the system is the controllability and the observability.

```
M = 0.5;
m = 0.2;
b = 0.1;
I = 0.006;
g = 9.8;
l = 0.3;
p = I*(M+m)+M*m*l^2; %denominator for the A and B matrices
A = [0 1 0 0; 0 -(I+m*l^2)*b/p (m^2*g*l^2)/p 0; 0 0 0 1; 0 -(m*l*b)/p m*g*l*(M+m)/p 0];
B = [ 0; (I+m*l^2)/p; 0; m*l/p];
C = [1 0 0 0; 0 0 1 0]; D = [0; 0];
states = {'x' 'x_dot' 'phi' 'phi_dot'};
inputs = {'u'};
outputs = {'x'; 'phi'};
sys_ss = ss(A,B,C,D,'statename',states,'inputname',inputs,'outputname',outputs);
Ts = 1/100;
sys_d = c2d(sys_ss,Ts,'zoh');
co = ctrb(sys_d);
ob = obsv(sys_d);
controllability = rank(co)
observability = rank(ob)

controllability =

     4

observability =

     4
```

Figure 3.51. MATLAB m-file command program

3.8. Control Design With Pole Placement Of System

The control system of a system can be designed by making use of root curves. This is called Pole Placement in the literature. The poles are the root of the equation in the denominator of a closed-loop transfer function, that is, the characteristic equation. The behavior of a polarized system can be determined. The block diagram of a full-state feedback control system is shown in Figure 3.51. To control system by using pole placement, we need to determine matrix K that represents control gain of controller. As mentioned before, matrix K depend on Matrix Q and R. These depend on state-space methods, as well.

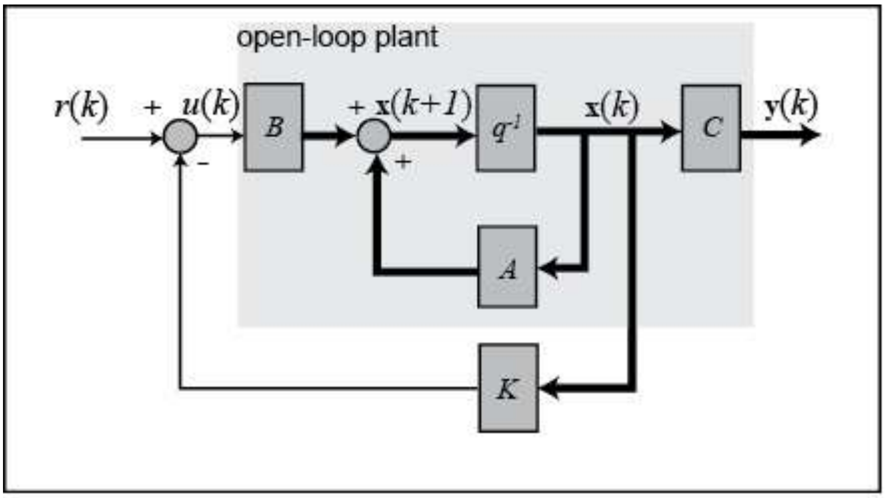

Figure 3.52. The block diagram of a full-state feedback control system [1]

If you agree State-Space Methods for controller design, then the Linear Quadratic Regulator (LQR) method is used to determine the control gain K. To be simple, we initially choose the performance index matrix equal to 1, and the state-cost matrix R equal to $C'C$. The relative weightings of these two matrices will then be tuned by trial and error. The state-cost matrix Q is given in Equ.40, below. Notice that state-space matrices A, B, C, and D can be found by using MATLAB command c2d, as shown in Figure 3.52.

$$\boldsymbol{Q} = \acute{C}\,C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.40}$$

```
A = sys_d.a;
B = sys_d.b;
C = sys_d.c;
D = sys_d.d;
Q = C'*C
R = 1;
[K] = dlqr(A,B,Q,R)
Ac = [(A-B*K)];
Bc = [B];
Cc = [C];
Dc = [D];
states = {'x' 'x_dot' 'phi' 'phi_dot'};
inputs = {'r'};
outputs = {'x'; 'phi'};
sys_cl = ss(Ac,Bc,Cc,Dc,Ts,'statename',states,'inputname',inputs,'outputname',outputs);
t = 0:0.01:5;
r =0.2*ones(size(t));
[y,t,x]=lsim(sys_cl,r,t);
[AX,H1,H2] = plotyy(t,y(:,1),t,y(:,2),'plot');
set(get(AX(1),'Ylabel'),'String','cart position (m)')
set(get(AX(2),'Ylabel'),'String','pendulum angle (radians)')
title('Step Response with Digital LQR Control')
```

```
Q =

     1     0     0     0
     0     0     0     0
     0     0     1     0
     0     0     0     0


K =

   -0.9384   -1.5656   18.0351    3.3368
```

Figure 3.53. MATLAB m-file command program

The chart is shown the position of the carriage and the angle of the pendulum. These are shown in red and blue colors respectively. Figure 3.53 is not enough results in the graph shown. Then we should again find a new Q matrix and K matrix depending on Q, as shown in Figure 3.52. Also the ploted graphic with new control gain K matrix has been given in Figure 3.54.

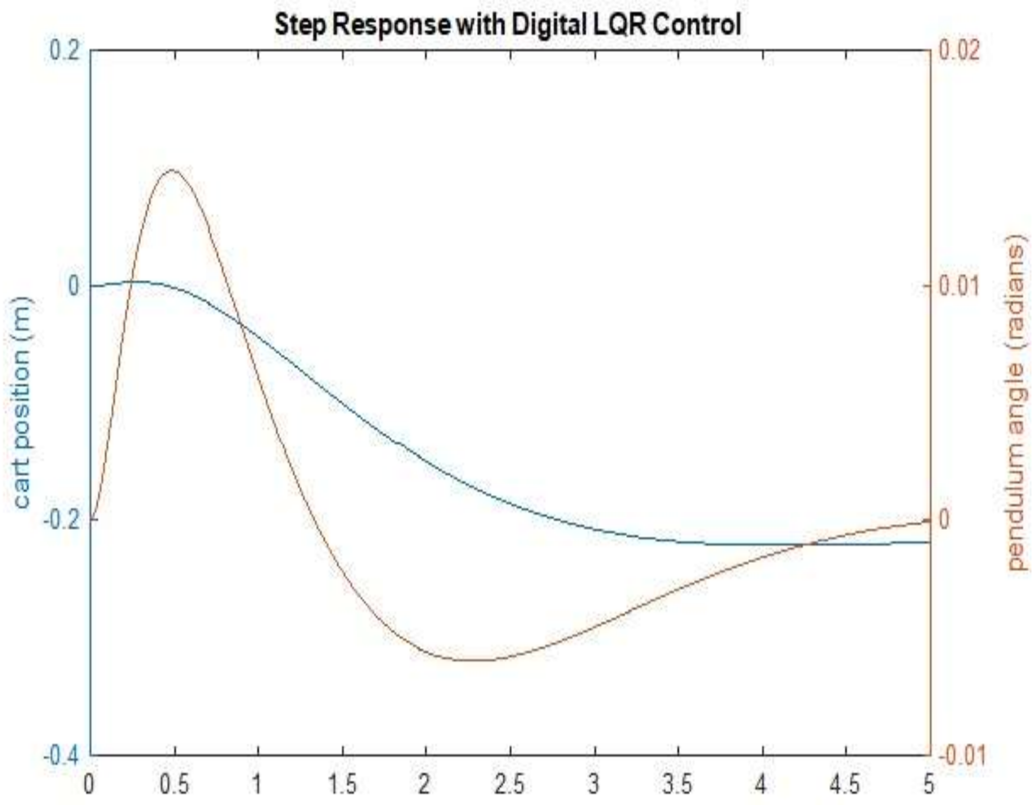

Figure 3.54. Step Response with LQR control

```
A = sys_d.a;
B = sys_d.b;
C = sys_d.c;
D = sys_d.d;
Q = C'*C;
Q(1,1) = 5000;
Q(3,3) = 100
R = 1;
[K] = dlqr(A,B,Q,R)
Ac = [(A-B*K)];
Bc = [B];
Cc = [C];
Dc = [D];
states = {'x' 'x_dot' 'phi' 'phi_dot'};
inputs = {'r'};
outputs = {'x'; 'phi'};
sys_cl = ss(Ac,Bc,Cc,Dc,Ts,'statename',states,'inputname',inputs,'outputname',outputs);
t = 0:0.01:5;
r =0.2*ones(size(t));
[y,t,x]=lsim(sys_cl,r,t);
[AX,H1,H2] = plotyy(t,y(:,1),t,y(:,2),'plot');
set(get(AX(1),'Ylabel'),'String','cart position (m)')
set(get(AX(2),'Ylabel'),'String','pendulum angle (radians)')
title('Step Response with Digital LQR Control')
Q =

        5000           0           0           0
           0           0           0           0
           0           0         100           0
           0           0           0           0


K =

  -61.9933  -33.5040   95.0597   18.8300
```

Figure 3.55. MATLAB m-file Command Program

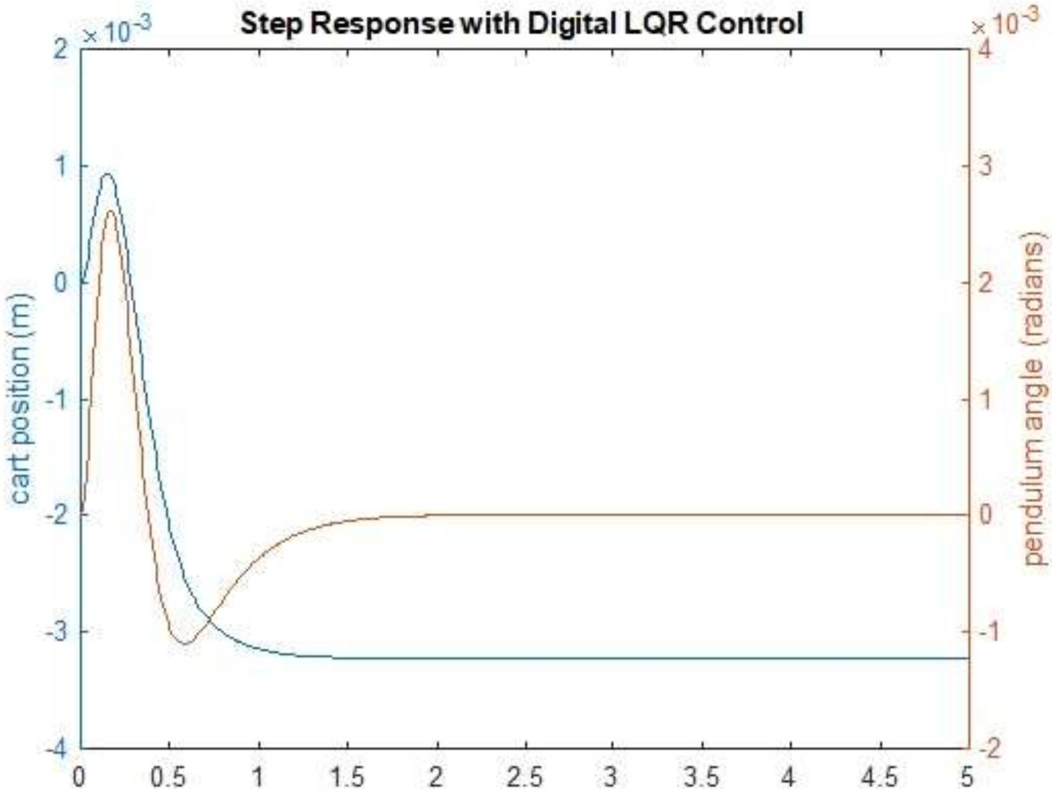

Figure 3.56. Step Response with LQR control

4. RESULT AND DISCUSSION

PID control method is widely used to control many engineering systems and also too cheaper than other control methods. PID represents proportional, integral, and derivative, respectively. Basic of PID control depends on the mathematical structure of PID control as shown in Equation 4.1. There are three coefficients of PID control that are k_p, k_i, and k_d, respectively. These coefficients can be determined both trial error and by using some optimization method. In this study, we found them by a using tuning PID method which is Ziegler-Nichols method [34].

$$u(t) = K_p e(t) + K_i \int_0^{\tau} e(t)dt + K_d \frac{d}{dt} e(t) \tag{4.1}$$

Table 4.1. shows a linear moving car with the physical parameters of an inverted pendulum. Table 4.2. shows the parameters of control obtained using the process Ziegler-Nichols. P_{cr} represents any duration of output oscillation of the system while K_{cr} represent the max. gain of oscillation of system.

Table 4.1. Physical Parameters of Inverted pendulum system

Description	Symbol	Value	Unit
Mass of the cart	M	0.5	kg
Mass of the pendulum	M	0.2	Kg
Mass moment of Inertia	I	0.006	$kg.m^2$
Length of pendulum the center of the mass	L	0.3	M
Coefficient of friction of the cart	B	0.1	N/m/sec
Force applied due to the gravity	G	9.81	m/sec^2

Table 4.2. Control parameters obtained by the Ziegler-Nichols method

Control Types	k_p	k_i	k_d
P	0.5*k_{cr}	∞	0
PI	0.4*k_{cr}	0.8*P_{cr}	0
PID	0.3*k_{cr}	0.5*P_{cr}	0.125*P_{cr}

Model of the inverted pendulum system was created by using the MATLAB Simulink toolbox program as digital. Figure 4.1 shows the feedback block diagram of the PID controller.

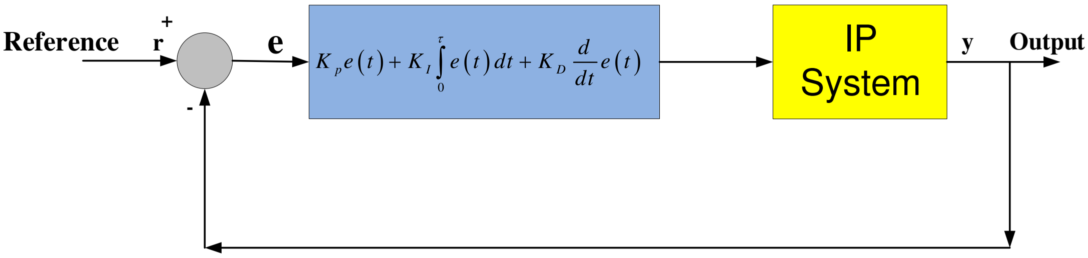

Figure 4.1. PID controller block diagram

In this study, we use PID control algorithm to balance at 90 degrees the angle of pendulum of the inverted pendulum system, which consist of two parts that, one of them is the cart while the second part is the pendulum. The performances of system balancing have been shown in Figure 4.2 and 4.3.

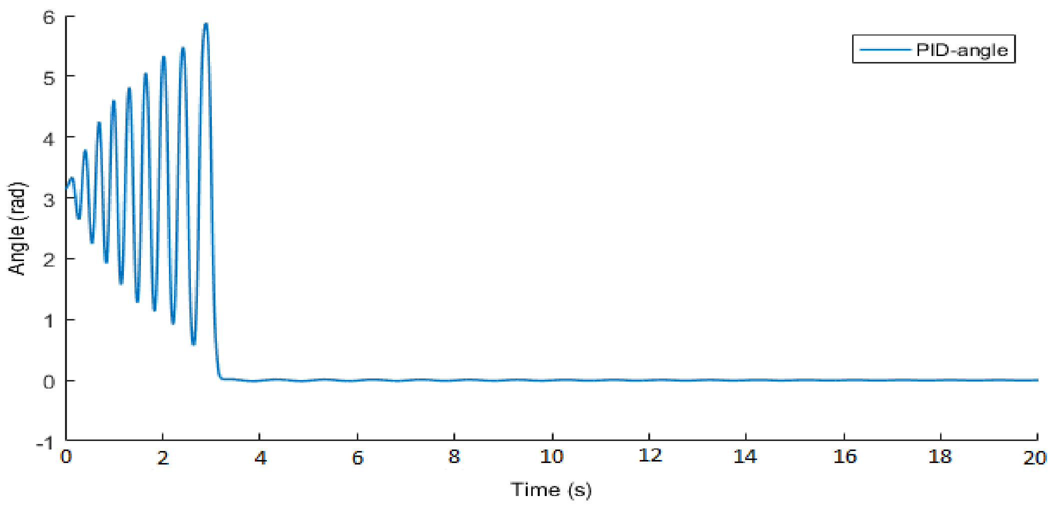

Figure 4.2. Pendulum angle obtained by the PID control method (θ , rad)

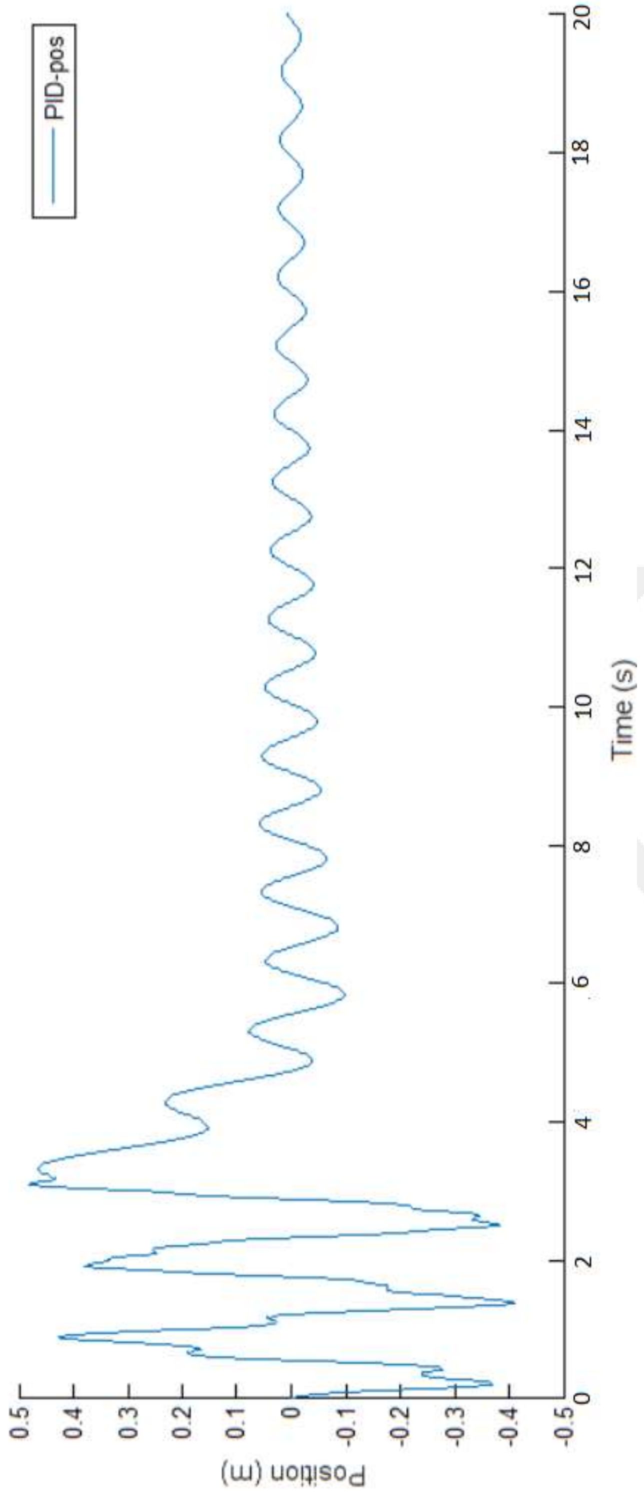

Figure 4.3. Position of cart position obtained through the PID control method (x, m)

5. CONCLUSION

Dynamic equations for the control of inverted pendulum system were obtained by Newton's Second Law of motion. Inverted pendulum system is a non-linear system with 2 degrees of freedom. The pendulum-bearing cart is actively driven while the pendulum is passively driven. The obtained dynamic equations were linearized for the purpose of analysis and design. After obtaining the differential equations of the system, the transfer function of the system was created to perform the control analysis of the system. The equation that determines the roots of the transfer function is called the characteristic equation and is used to determine the stability analysis of the system. In addition, the state-space model, which is the basis of optimal control theory, has been formed from the model of inverted pendulum. Observability and controllability conditions were examined for the control analysis of the inverted pendulum system using the generated state space model. Poles (poles), zeros (zeros) and singular points of the system were found after the transfer function and characteristic equation obtained in order to perform the control analysis of the inverted pendulum system in an appropriate manner. The control of the inverted pendulum system has been realized by using the developed PID and linear quadratic regulator (LQR) control methods. Also control analysis like Closed-Loop Response, Open-Loop Response, Root Locus Design, Nyquist and Bode diagrams of system have been done. Simulations of the system were performed in MATLAB package program. The analyzes and performances of the control algorithms designed were examined and examined with the obtained graphs. PID and Linear Quadratic Regulator (LQR) controls designed and simulated in MATLAB have been found to be very successful in stabilizing the inverted pendulum system. In addition to the PID (Proportional-Derivative-Derivative) and LQR (Linear Quadratic Regulator) controls made in the literature, the control of the system was also carried out successfully by designing the Tuning-PID control algorithm. Here, PID control coefficients (k_p, k_i, k_d) were determined by using the Ziegler-Nichols method. Finally, the performance of these control methods designed and the necessary control analyzes were performed and the results were obtained.

REFERENCE

[1] Vijayanand Kurdekar1, Samarth Borkar2. Sep - Oct. 2013. "INVERTED PENDULUM CONTROL: A BRIEF OVERVIEW." International Journal Of Modern Engineering Research (IJMER) Pp-2924-2927 ISSN: 2249-6645.

[2] Roshan Kumar, Ravi Bhushan Singh, Jayanta Das. Oct-2013. "Modeling And Simulation Of Inverted Pendulum System Using Matlab: Overview." International Journal of Mechanical and Production Engineering, ISSN: 2320-2092,.

[3] Dotihal, Rohan Rajiv. May 2017. "Study Of Inverted Pendulum And."

[4] Chakraborty, Sudipta. Date: 03-05-2014 Sudipta Chakraborty. "An Experimental study for Stabilization of Inverted Pendulum."

[5] Shinya Tachino*1, Takuya Sugiura*2, Tasuku Hatakeyama*3, Masaru Urushibata*4 and Yoshihisa Uchida*5. Pilsen, Czech Republic, August 31 – September 3, 2014. "Modeling and Simulation of Quadruped Form Multi-purpose Module System MMS for Stability Control." The 3rd International Conference on Design Engineering and Science, ICDES 2014 104-109.

[6] Mr.Pankaj Kumar, Mr. KunalChakraborty,Mr. RabiRanjan Mukherjee,Mr.SuvobratraMukherjee. January 2013. "Modelling and Controller Design of Inverted Pendulum." International Journal of Advanced Research in Computer Engineering & Technology (IJARCET) ISSN: 2278 – 1323.

[7] Transporter, Segway – The Human . April 2015. "Mayank Sharma,Rahul Sharma." IJIRST – International Journal for Innovative Research in Science & Technology 128-132.

[8] Razzaghi, K., Jalali, A. A. 2012. A new approach on stabilization control of an inverted pendulum, using PID controller. Advanced Materials Research, 403: 4674–4680.

[9] Shehu, M., Ahmad, M. R., Shehu, A., Alhassan, A. 2015. LQR, double-PID and pole placement stabilization and tracking control of single link inverted pendulum. 2015 IEEE International Conference on Control System, Computing and Engineering (ICCSCE), 27-29 Kasım 2015, Penang. S, 218–223.

[10] Åström, K.J. and Furuta, K. (2000), "Swinging up a pendulum by energy control", Automatica, Vol. 36 No. 2, pp. 287-295.

[11] Yi, J., Yubazaki, N. and Hirota, K. (2001), "Upswing and stabilization control of inverted pendulum system based on the SIRMs dynamically connected fuzzy inference model", Fuzzy Sets and Systems, Vol. 122 No. 1, pp. 139-152.

[12] Wang, J.J. (2011), "Simulation studies of inverted pendulum based on PID controllers", Simulation Modelling Practice and Theory, Vol. 19No. 1, pp. 440-449.

[13] Prasad, L. B., Tyagi, B., Gupta, H. O. 2014. Optimal control of nonlinear inverted pendulum system using PID controller and LQR: performance analysis without and with disturbance input. International Journal of Automation and Computing, 11 (6): 661–670.

[14] Sánchez, J., Dormido, S., Pastor, R., Morilla F., "A Java/Matlab-Based Environment for Remote Control System Laboratories: Illustrated With an Inverted Pendulum", IEEE Transactions on Education, Cilt 47, Sayı 3, Sayfa 321-329, (2004).

[15] Ghanbari, A., Farrokhi, M., "Decentralized Neuro-Fuzzy Controller Design Using Decoupled Sliding-Mode Structure for Two-Dimensional Inverted Pendulum", Engineering of Intelligent Systems International Conference, (2006).

[16] Jung, S., Cho, H., Hsia, T. C., "Neural Network Control for Position Tracking of a Two-Axis Inverted Pendulum System: Experimental Studies", IEEE Transactıons on Neural Networks, Cilt 18, Sayı 4, Sayfa 1042-1048, (2007).

[17] Huang, C., Wang, W., Chiu, C. , “Design and Implementation of Fuzzy Control on a Two-Wheel Inverted Pendulum”, IEEE Transactions on Industrıal Electronics, Cilt 58, Sayı 7, Sayfa 2988-3001, (2011).

[18] Chiu, C., Peng, Y., Lin, Y., “Intelligent backstepping control for wheeled inverted pendulum”, Expert Systems with Applications, Sayı 38, Sayfa 3364–3371, (2011).

[19] Ertuğrul, İ., 2015. Bir Ters Sarkaç Sisteminde Kullanilan Genel Kontrol Algoritmalarinin Karşilaştirilmasi, (yüksek lisans tezi). Marmara Üniversitesi, Fen Bilimleri Enstitüsü, İstanbul.

[20] Grămescu, B., Niţu, C., Su, N., Phuc, P., Borzea, C. I., 2015. PID control for two-wheeled inverted pendulum (WIP) system. The Romanian Review Precision Mechanics, Optics & Mechatronics, 2015 (48): 246–252.

[21] Kizir, S., 2008. Doğrusal Olmayan Ters Sarkaç Sisteminin Tasarımı ve Kontrolü (yüksek lisans tezi). Kocaeli Üniversitesi, Fen Bilimleri Enstitüsü, İzmit, Kocaeli.

[22] Lozano R., Fantoni I., Block D.J., “Stabilization of the inverted pendulum around its homoclinic orbit”, Systems and Control Letters, Volume 40, Number 3, 5 July 2000 , pp. 197-204(8) (2000).

[23] Ji, C.W., Lei, F. and Kin, L.K. (1997), “Fuzzy logic controller for an inverted pendulum system”, IEEE International Conference on Intelligent Processing Systems, ICIPS’97, Vol. 1, pp. 185-189.

[24] Magana, M.E. and Holzapfel, F. (1998), “Fuzzy-logic control of an inverted pendulum with vision feedback”, IEEE Transactions on Education, Vol. 41No. 2, pp. 165-170.

[25] Tang, Y., Zhou, D. and Jiang, W. (2016), “A new fuzzyevidential controller for stabilization of the planar inverted pendulum system”, PloS One, Vol. 11 No. 8, p. e0160416.

[26] Nundrakwang, S., Benjanarasuth, T., Ngamwiwit, J., Komine, N. (2005). Hybrid PD-servo state feedback control algorithm for swing up inverted pendulum system. 690-693.

[27] Bugeja, M., “Non-Linear Swing-Up and Stabilizing Control of an Inverted Pendulum System”, EUROCON Ljubljana, Slovenia (2003).

[28] Bettayeb, M., Boussalem, C., Mansouri, R., Al-Saggaf, U. M. 2014. Stabilization of an inverted pendulum-cart system by fractional PI-state feedback. ISA Transactions, 53 (2): 508–516.

[29] Mishra, S. K., Chandra, D. 2014. Stabilization and tracking control of inverted pendulum using fractional order PID controllers. Journal of Engineering, 2014: 1–9.

[30] Hanwate, S. D., Hote, Y. V. 2014. Design of PID controller for inverted pendulum using stability boundary locus. 2014 Annual IEEE India Conference (INDICON), 11-13 Aralık 2014, Pune. S, 1–6.

[31] Ishii, C. ., Nishitani, Y., Hashimoto, H., 2009. Modelling and robust stabilisation of a closed-link 2-dof inverted pendulum with gain scheduled control. International Journal of Modelling, Identification and Control, 6 (4): 320–332.

[32] Soto, I., Campa, R., 2015. Modelling and control of a spherical inverted pendulum on a five-bar mechanism. International Journal of Advanced Robotic Systems, 12(7): 95.

[33] Albouy, X., Praly, L., 2000. On the use of dynamic invariants and forwarding for swinging up a spherical inverted pendulum. Proceedings of the 39th IEEE Conference on Decision and Control. Aralık, Sydney. 1667–1672.

[34] Chung, C. Y., 2002. Balancing of an inverted pendulum with a kinematically redundant robot. International Journal of General Systems, 31 (1): 1–15.

[35] Abut,T., Soyguder, S., (2019) "Real-time control and application with self-tuning PID-type fuzzy adaptive controller of an inverted pendulum", Industrial Robot: the international journal of robotics research and application, Vol. 46 Issue: 1, pp.159-170,

[36] Kizir, S., Bingul, Z. and Oysu, C. (2010), “Fuzzy control of a real time inverted pendulum system”, Journal of Intelligent & Fuzzy Systems, Vol. 21 Nos 1/2, pp. 121-133.

[37] Ahmadi, A., Abdul Rahim, H. and Abdul Rahim, R. (2014), “Optimization of a self-tuning PID type fuzzy controller and a PID controller for an inverted pendulum”, Journal of Intelligent&Fuzzy Systems, Vol. 26No. 4, pp. 1987-1999.

[38] Magdy, M., El Marhomy, A., & Attia, M. A. (2019). Modeling of inverted pendulum system with gravitational search algorithm optimized controller. Ain Shams Engineering Journal, 10(1), 129-149.

[39] Prasad Lal Bahadur, Tyagi Barjeev, Gupta Hari Om. Modelling & simulation for optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR. IEEE Comput Soc 2012;978. pp. 7695-4730.

[40] Kajita, S., Morisawa, M., Miura, K., Nakaoka, S. I., Harada, K., Kaneko, K.& Yokoi, K. (2010, October). Biped walking stabilization based on linear inverted pendulum tracking. In 2010 IEEE/RSJ International Conference on Intelligent Robots and Systems (pp. 4489-4496). IEEE.

[41] Wang Chunyang, Yin Gaofeng, Liu Canglong, Fu Weicheng. Design and simulation of inverted pendulum system based on the fractional PID controller. IEEE 11th conference on industrial electronics and applications (ICIEA), China, 2016. pp. 4673-8644.

[42] Guida Domenico, Nilvetti Fabio, Pappalardo Carmine Maria. Dry friction influence on cart pendulum dynamics. Int J Mech 2009;3(2).

[43] Guida, D., Nilvetti, F., & Pappalardo, C. M. (2010). Dry friction of bearings on dynamics and control of an inverted pendulum. Journal of Achievements in Materials and Manufacturing Engineering, 38(1), 80-94.

[44] Rigatos, G., Busawon, K., Pomares, J., & Abbaszadeh, M. (2019). Nonlinear Optimal Control for the Wheeled Inverted Pendulum System. Robotica, 1-19.

[45] Wu J , Zhang W , Wang S . ‘A two-wheeled self-balancing robot with the fuzzy PD control method’. Math Prob Eng 2012;2012:1–13.

[46] Bature AA , Buyamin S , Ahmad MN , Muhammad M , Muhammad AA . ‘Identification and model predictive position control of two wheeled inverted pendulum mobile robot’. Jurnal Teknologi 2015;73(6).

[47] Al-Araji, Ahmed S. "An adaptive swing-up sliding mode controller design for a real inverted pendulum system based on Culture-Bees algorithm." European Journal of Control 45 (2019): 45-56.

[48] A. Kharola, P. Patil, PID control of two-stage inverted pendulum. Proceedings of the IEEE International Conference on Computational Intelligence and Com- puting Research (ICCIC), doi. 10.1109/ICCIC.2016.7919521 .

[49] M-N. Park , D. Chwa , Swing-up and stabilization control of inverted-pendulum systems via coupled sliding-mode control method, IEEE Trans. Indust. Electron. 56 (2009) 3541–3555 .

[50] Gautam, Pooja. "Optimal control of Inverted Pendulum system using ADALINE artificial neural network with LQR." 2016 International Conference on Recent Advances and Innovations in Engineering (ICRAIE). IEEE, 2016.

[51] D. Chatterjee, A. Patra, H. K. Joglekar, “Swing-up and Stabilization of a cart-pendulum system under restricted cart length”, Systems and Control Letters, vol. 47, pp. 355-364, July 2002.

[52] El-Nagar, A. M., El-Bardini, M., & El-Rabaie, N. M. (2014). Intelligent control for nonlinear inverted pendulum based on interval type-2 fuzzy PD controller. Alexandria Engineering Journal, 53(1), 23-32.

[53] LiW, Ding H, Cheng K. (2012) An investigation on the design and performance assessment of double-PID and LQR controllers for the inverted pendulum. In: Proceedings of 2012 UKACC international conference on control, pp 190–196.

[54] Zhao , M.W. Spong , Hybrid control for global stabilization of the cart pendulum system, Automatica 37 (2001) 1941–1951 .

[55] Su, T. J., Wang, S. M., Li, T. Y., Shih, S. T., & Hoang, V. M. (2016). Design of hybrid sliding mode controller based on fireworks algorithm for nonlinear inverted pendulum systems. Advances in Mechanical Engineering, 9(1), 1687814016684273.

[56] A. Wadi, J.-H. Lee, and L. Romdhane, "Dynamic Analysis of the Tilted Furuta Pendulum," MATEC Web of Conferences, 2017.

[57] Durand, S., Castellanos, J. F. G., Marchand, N., & Sanchez, W. F. G. (2013). Event-based control of the inverted pendulum: Swing up and stabilization. Journal of Control Engineering and Applied Informatics, 15(3), 96-104

[58] Khwan-on S., Kulworawanichpong T., Srikaew A., Sujitjorn S., "Neuro-tabu fuzzy controller to stabilize an inverted pendulum system", IEEE Region 10 conference, Vol. 4, (2004), 562.

[59] Anderson C. W., "Learning to control an inverted pendulum using neural network," IEEE Control Syst. Mag., Vol. 9, no 3, (1989), 31.

[60] Huang S. J., Huang C. L., "Control of an inverted pendulum using grey prediction model", IEEE Transactions on industry applications, Vol. 36, Issue 2, (2000), 452.

[61] Li J. H., Li T. H. S., Tsai M..C., Lin S. H., "Design and implementation of dynamic weighted fuzzy sliding-mode controller for an FPGA-based inverted pendulum car", Proceedings of the international conference on advanced intelligent mechatronics, vol. 1, (2003).

[62] Blondin, M. J., & Pardalos, P. M. (2019). A holistic optimization approach for inverted cart-pendulum control tuning. Soft Computing, 1-17.

[63] Masrom, M. F., A Ghani, N. M., & Tokhi, M. O. (2019). Particle swarm optimization and spiral dynamic algorithm-based interval type-2 fuzzy logic control of triple-link inverted pendulum system: A comparative assessment. Journal of Low Frequency Noise, Vibration and Active Control, 1461348419873780.

[64] Taur, J. S., Tao, C. W., Hsieh, T. W., Tsai, C. L. (2008). Design of a fuzzy controller with fuzzy swing-up and parallel distributed pole assignment schemes for an inverted pendulum and cart system. IEEE Transactions on Control Systems Technology, 16(6), 1277-1288.

[65] Roose, A. I., Yahya, S., Al-Rizzo, H. (2017). Fuzzy-logic control of an inverted pendulum on a cart. Computers & Electrical Engineering, 61, 31-47.

[66] Yadav, A. K., Gaur, P., Mittal, A. P., Anzar, M. (2011). Comparative analysis of various control techniques for inverted pendulum. In India International Conference on Power Electronics 2010 (IICPE2010) (pp. 1-6). IEEE.

[67] Irfan, S., Mehmood, A., Razzaq, M. T., Iqbal, J. (2018). Advanced sliding mode control techniques for inverted pendulum: Modelling and simulation. Engineering science and technology, an international journal, 21(4), 753-759.

[68] Yoshida, K. 1999. Swing-up control of an inverted pendulum by energy-based methods. American Control Conference, 2-4 Haziran 1999, San Diego. Cilt VI. S, 4045–4047.

[69] Muskinja, N. and Tovornik, B. (2006), "Swinging up and stabilization of a real inverted pendulum", IEEE Transactions on Industrial Electronics, Vol. 53 No. 2, pp. 631-639.

[70] Chen, C. S., & Chen, W. L. (1998). Robust adaptive sliding-mode control using fuzzy modeling for an inverted-pendulum system. IEEE Transactions on Industrial Electronics, 45(2), 297-306.

[71] Yüce, A., Tan, N., & Doğruer, T. (2018, October). Root-Locus Analysis of Fractional Order Transfer Functions Using LabVIEW: An Interactive Application. In 2018 6th International Conference on Control Engineering & Information Technology (CEIT) (pp. 1-6). IEEE.

[72] Mahmoodabadi, M. J., & Khoobroo Haghbayan, H. (2019). An optimal adaptive hybrid controller for a fourth-order under-actuated nonlinear inverted pendulum system. Transactions of the Institute of Measurement and Control, 0142331219868589.

[73] Larcombe, P. J. (1991, December). On the control of a two-dimensional multi-link inverted pendulum: co-ordinate system suitability for dynamic formulation. In [1991] Proceedings of the 30th IEEE Conference on Decision and Control (pp. 136-141). IEEE.

[74] Özbey, A., & Uzal, E. Ters Sarkacin Nonlineer Kontrolü.

[75] Kajiwara, H., Apkarian, P., & Gahinet, P. (1999). LPV techniques for control of an inverted pendulum. IEEE Control Systems Magazine, 19(1), 44-54.

[76] Ratiroch-Anant, P., Anabuki, M., & Hirata, H. (2004, November). Self-tuning control for rotational inverted pendulum by eigenvalue approach. In Proceedings of the IEEE Region 10 Conference: Analog and Digital Techniques in Electrical Engineering (TENCON'04) (Vol. 500, pp. 542-545).

[77] Rabah M, Rohan A, Kim SH (2018) Comparison of position control of a gyroscopic inverted pendulum using PID, fuzzy logic and fuzzy PID controllers. Int J Fuzzy Logic Intell Syst 18(2):103–110.

[78] Wu Q., Sepehri N., He, S., "On control of a base-excited inverted pendulum using neural networks", Journal of the Franklin Institute, Sayı 337, Sayfa 267-286, (2000).

[79] Maity, S., Luecke, G. R. (2019). Stabilization and Optimization of Design Parameters for Control of Inverted Pendulum. Journal of Dynamic Systems, Measurement, and Control, 141(8), 081007.

[80] Morasso, P., Cherif, A., Zenzeri, J. (2019). Quiet standing: The Single Inverted Pendulum model is not so bad after all. PloS one, 14(3), e0213870.

[81] Ma, J., Coogler, K., Suh, M. (2019), Inquiry-based learning: Development of an introductory manufacturing processes course based on a mobile inverted pendulum robot. International Journal of Mechanical Engineering Education, 0306419019844257.

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