Robust Optimization for Solving Stochastic Reverse Logistics Network Design Problem for Recycling Waste Batteries with an Application in Turkey

by

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Abstract

In recent decades, there has been an increasing interest in the mobile electronic devices that use batteries as their source of power. The growing demand for batteries creates environmental concerns for governments and companies because of their content of heavy metals that are usually significantly harmful to the environment. Waste batteries are known as hazardous waste and should therefore be appropriately disposed or recycled. Since finding suitable landfill areas is very hard—because people prefer to not live close to the landfill areas—recycling has become even more popular. In addition to the environmental benefits, recycling waste batteries also has economic advantages.

There are a number of mathematical models in the literature that address the design of recycling systems for different products. The design of a reverse logistics system for recycling waste batteries is expressed as a multi-period mixed integer linear programming (MILP) problem to address the collection, transportation, sorting, recycling, and landfill operations of the waste batteries. This model considers various types of collected batteries, several existing and potential facilities with different types and capacity options, various construction and operational costs, and revenue from selling recycled batteries at a secondary market. The objective of this MILP model is to maximize the profit—i.e., the total revenue minus the total cost.

The amount of used batteries that are returned by the end users is stochastic. Two-stage stochastic optimization method is used in the literature to address uncertainty in the model. In the first stage, the model makes strategic decisions—e.g., the capacity of the sorting and recycling facilities—while in the second stage, it makes the tactical decisions—e.g., inventory levels in warehouses. We extend the deterministic MILP model by using robust optimization approach to incorporate the uncertainty in the model. We consider four different scenarios for the amount of waste-batteries that are collected from the end users. We use GAMS (General Algebraic Modeling System) optimization package and the CPLEX solver to compare the results of these two approaches. We observe that the two-stage approach has

worse results than the robust optimization method with respect to the worst-case performance. We also conduct a sensitivity analysis to show the impact of the amount of collected waste-batteries on the situation of facilities in the model.

Özet

Son yıllarda, güç kaynağı olarak pil kullanan mobil elektronik cihazlara olan ilgi büyük bir artış göstermiştir. Piller için artan talep, pillerin içeriğindeki ağır metallerin genellikle çevreye önemli ölçüde zararının olması, hükümet ve şirketler için çevre kaygıları yaratmaktadır. Atık piller tehlikeli atık olarak kabul edilir ve bu nedenle pillerin uygun bir şekilde bertaraf edilmesi ya da geri dönüştürülmesi gerekir. İnsanlar depolama alanlarına yakın bölgelere yerleşmeyi tercih etmedikleri için uygun depolama alanları bulmak çok zorlaşmış ama geri dönüşüm alanları daha popüler hale gelmiştir. Geri dönüşmüş atık pillerin, çevresel avantajlarına ek olarak, ekonomik avantajları da vardır.

Literatürde farklı ürünler için geri dönüşüm sistemlerinin tasarımı ve matematiksel modelleri bulunmaktadır. Atık pillerin geri dönüşümü için bir tersine lojistik sistemi tasarlanır ve bu sistem, atık pillerin toplanması, taşınması, sıralaması, geri dönüşümü ve atık depolama işlemlerini içerir. Bu sistemi ele alan modeller, çok dönemli karma tam sayılı doğrusal programlama (MILP) problemi olarak ifade edilir. Bu modelde farklı tip ve kapasite seçenekleri, çeşitli inşaat ve işletme maliyetleri, geri dönüştürülmüş pillerin ikincil bir piyasada satış gelirleri, birkaç mevcut ve potansiyel tesis seçenekleri göz önünde bulundurur. Bu MILP modelinin amacı karı, yani toplam gelir eksi toplam maliyeti maksimize etmektir.

Son kullanıcılardan geri dönen kullanılmış pillerin miktarı olasılıksal ve değişkendir. Literatürde, modeldeki bu belirsizliği gidermek için iki aşamalı stokastik eniyileme yöntemi kullanılmaktadır. İlk aşamada, model, sıralama ve geri dönüşüm tesislerinin kapasitesi gibi stratejik kararlar alır. Depolardaki stok seviyeleri gibi taktik kararlar ise, ikinci aşamada alınır. Yaptığımız çalışmada, belirsizliği de göz önünde bulundurmak için robust eniyileme yaklaşımı kullanarak belirsizliği deterministik MILP modeline dahil ettik. Son kullanıcılardan toplanan atık pil miktarları için dört farklı senaryo göz önünde bulundurduk. Bu iki yaklaşımın sonuçlarını karşılaştırmak için GAMS (Genel Cebirsel Modelleme Sistemi) eniyileme paketi ve CPLEX çözücü kullandık. En kötü senaryoda, iki aşamalı yaklaşımın performans açısından robust eniyileme yönteminden daha kötü sonuçları olduğunu fark ettik. Bununla birlikte, toplanan atık pillerin miktarının tesislerin durumuna etkisini göstermek için bir duyarlılık analizi yapdık.

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Nomenclature

Sets

T set of time periods,

I set of waste battery collection nodes,

J_E set of pre-established battery sorting facilities,

J_N set of potential locations for sorting facilities, ($J_E \cup J_N = J$)

L_E set of landfill areas pre-established facilities,

L_N set of potential landfill areas, ($L_E \cup L_N = L$)

R set of potential locations for recycling facilities,

B set of battery-types,

M set of recycled material types,

K set of potential technological features,

C set of capacity extension modules,

Q set of capacity options for sorting facilities

Decision Variables

$x1_{ijt}$ amount of mixed batteries transported from the collection node $i \in I$ to the battery sorting facility $j \in J$ at time period $t \in T$,

$x2_{jbrt}$ amount of battery-type $b \in B$ transported from sorting facility $j \in J$ to recycling plant $r \in R$ at time period $t \in T$,

$x3_{jblt}$ amount of battery-type $b \in B$ transported from the sorting facility $j \in J$ to landfill area $l \in L$ at time period $t \in T$,

R_{brt} amount of battery-type $b \in B$, recycled at recycling facility $r \in R$ at time period $t \in T$,

Inv_{mrt} amount of recycled material inventory $m \in M$ at recycling facility $r \in R$ at time period $t \in T$,

$N1_{mrt}$	amount of recycled material $m \in M$ sold from recycling facility $r \in R$ at time period $t \in T$,
$N2_{rlt}$	amount of waste residue transported from recycling facility $r \in R$ to landfill area $l \in L$ at time period $t \in T$,
$z1_{jbt}$	amount of battery-type $b \in B$ processed at the sorting facility $j \in J$ at time period $t \in T$,
ys_{jqt}	1 if a sorting facility with capacity option $q \in Q$ is operational at potential location $j \in J$ at time period $t \in T$, 0 else
us_{jct}	1 if capacity expansion option $c \in C$ is added to sorting facility $j \in J$ at time period $t \in T$, 0 else
yr_{rt}	1 if battery recycling facility $r \in R$ is operational at time period $t \in T$, 0 else
ro_{rt}	1 if battery recycling facility $r \in R$ is built at time period $t \in T$, 0 else
yt_{krt}	1 if recycling technology $k \in K$ is operational at recycling facility $r \in R$ at time $t \in T$, 0 else
yl_{lt}	1 if landfill area $l \in L$ is operational at time period $t \in T$, 0 else
b_{brt}	1 if battery-type $b \in B$ can be processed at recycling facility $r \in R$ at time $t \in T$, 0 else

Parameters

a_{it}	amount of battery collected at source $i \in I$ during time period $t \in T$,
$d1_{ij}$	distance between collection node $i \in I$ and sorting facility $j \in J$,
$d2_{jr}$	distance between sorting facility $j \in J$ and recycling facility $r \in R$,
$d3_{jl}$	distance between sorting facility $j \in J$ and landfill area $l \in L$,
$d4_{rl}$	distance between recycling facility $r \in R$ and landfill area $l \in L$,
$UCT1_{ijt}$	unit variable cost of transportation between collection point $i \in I$ and sorting facility $j \in J$ at time period $t \in T$,
$UCT2_{jrt}$	unit variable cost of transportation between sorting facility $j \in J$ and recycling facility $r \in R$ at time period $t \in T$,

$UCT3_{jlt}$	unit variable cost of transportation between sorting facility $j \in J$ and landfill area $l \in L$ at time period $t \in T$,
$UCT4_{rlt}$	unit variable cost of transportation between recycling facility $r \in R$ and landfill area $l \in L$ at time period $t \in T$,
CCS_{jqt}	construction cost for sorting facility $j \in J$ at time period $t \in T$,
FCS_{jqt}	fixed cost for sorting operations at battery sorting facility $j \in J$ during time period $t \in T$,
$FCCS_{ct}$	capacity expansion cost of capacity expansion option $c \in C$ for sorting facilities at time period $t \in T$,
UCS_{jqt}	unit variable cost of sorting operations at sorting facility $j \in J$ at time period $t \in T$,
CCR_{rt}	construction cost for recycling facility $r \in R$ at time period $t \in T$,
FCT_{kt}	cost of installing technology $k \in K$ at time period $t \in T$,
FCR_{rt}	fixed cost of operating recycling facility $r \in R$ at time period $t \in T$,
UCR_{rbt}	unit variable cost of recycling operations for battery- type $b \in B$ at recycling facility $r \in R$ at time period $t \in T$,
CCL_{lt}	construction cost for landfill area $l \in L$ at time period $t \in T$,
FCL_{lt}	fixed cost of operating landfill area $l \in L$ at time period $t \in T$,
UCL_{lt}	unit variable cost of landfill operations at landfill area $l \in L$ at time period $t \in T$,
UCI_t	unit inventory cost for recycled materials at during time period $t \in T$,
CAP_s_q	capacity module q for sorting facilities,
CAP_r_r	capacity upper bound of recycling facility $r \in R$,
CAP_l_l	storage capacity for landfill area $l \in L$,
α_{ib}	share of battery-type $b \in B$ at collection node $i \in I$,
p_{bm}	percentage of material type $m \in M$ in battery-type $b \in B$,
$\mathrm{Re}v_{mt}$	sale price of recycled material $m \in M$ at time period $t \in T$,
tb_{kb}	1 if technology $k \in K$ is able to process battery-type $b \in B$, 0 else

c_c capacity expansion option $c \in C$,

$ParExistCap\_S_j$

capacity upper bound of sorting facility $j \in J$ in the beginning of planning horizon,

$ParExistCap\_L_l$

capacity upper bound of landfill area $l \in L$ in the beginning of planning horizon,

Definitions for the stochastic problem

Note. In the stochastic version of our problem, there is a finite set of scenarios *S* with index *s*.

a_{sit} Amount of waste batteries that are collected at region *i* during time period *t*, for scenario *s*,

P_s Probability of scenario *s*,

$x1_{sijt}$ Amount of mixed batteries transported from the collection node *i* to the battery sorting facility *j* at time period *t* in scenario *s*,

$x2_{sjbrt}$ Amount of battery- type b transported from sorting facility j to recycling plant *r*, at time period *t* in scenario *s*,

$x3_{sjbrt}$ Amount of battery transported from the sorting facility *j* to landfill area l at time period *t* in scenario *s*,

R_{sbrt} Amount of battery-type *b*, recycled at recycling facility *r*, at time period *t* in scenario *s*,

Inv_{smrt} Amount of recycled material inventory *m* at recycling facility *r*, at time period *t* in scenario *s*,

$N1_{smrt}$ Amount of recycled material *m* sold at time period *t* in scenario *s*,

$N2_{srlt}$ Amount of waste residue transported from recycling facility *r* to landfill area l at time period *t* in scenario *s*,

$z1_{sjbt}$ Amount of battery-type *b* processed at the sorting facility *j* at time period *t* in scenario *s*,

Chapter 1
Introduction

1.1. Problem description and motivation

During the last decade, a rapid growth of portable electronic devices increased the battery demand as a power resource of these devices. This increased demand was associated with various environmental risks due to hazardous heavy metals in the waste batteries. Therefore, people were strongly concerned about the hazardous materials that were contained in the waste batteries, and therefore were looking for an approach for the disposal of batteries at the end of their life-cycle (Shih, 2001).

Batteryuniversity.com claims that the significant improvements in battery technologies, its benefits and advantages made batteries a popular power. According to this website, the batteries' power satisfies all of today's needs for energy. However, one of the greatest environmental concerns is hazardous components of batteries such as mercury, lead, copper, zinc, cadmium, manganese, nickel and lithium (Lund, 2001). The batteries have significant harmful effects on the environment at the end of their life cycle. Bean (2003) states "Battery waste is an ever-growing problem for the environment worldwide".

As the battery technologies progress, the need for a waste battery management system increases. Waste battery management includes economic, social, environmental and technical aspects. The social aspect and its advantages can be explained as new job opportunities. Technical issues are associated with recycling technologies. Recycling all types of waste-batteries in one facility is not feasible, not only technically but also economically (Yender, 1998).

We design a reverse logistics system for the recycling of waste batteries and categorize it into collection, transportation, sorting, recycling and landfill phases. The consumers are the most important factor in the collection phase. If the battery is accessible in the electronic device and easily removable, then the collection step becomes more facile for the consumers. At the sorting phase, batteries are identified based on their chemical characteristics and they are separated to recyclable and non-recyclable batteries. The recyclable batteries are sent to recycling facilities and non-recyclable batteries are disposed. The most common method of disposal is using landfill. Receiving the collected batteries from the end users and dispatching them to other facilities, and then recycling the waste batteries require a large amount of expenditure. The need for a cost effective battery recycling process motivated us to design a mixed integer linear programming model that minimizes all related costs and maximizes revenue.

The problem of recycling waste batteries involves uncertainties in the amount of collected batteries. This issue makes the problem even more difficult. Uncertainty may occur in other parameters of the problem as well; however, we only focus on the variability of the amount of waste batteries that are collected by the end users. We consider four different scenarios for the amount of waste batteries that are collected from the customers. We incorporate the uncertainty in our model by using two-stage stochastic method and robust optimization approach. Our model maximizes revenue in all possible situations that are included in predefined scenario. Finally, we analyze and compare the results of the two-stage and robust methods.

1.2. Hazards of waste batteries

In this section, we explain the different types of batteries and the characteristics of each type to better understand the dimensions of the environmental effects that can be caused by an inappropriate disposal of waste batteries.

A battery is an electrochemical cell that can convert chemical energy to electrical energy. A battery generally consists of an anode, a cathode, an electrolyte, separators, and an external case. The cathode and anode are isolated by a separator. The separator is drenched with an electrolyte that forms a catalyst. It promotes the movement of ions between cathodes and anodes. Ions are atoms that loss or gain electrons. The transfer of ions from the cathode to the anode represents the charging of batteries and the reverse movement of ions represents the discharging. The external case is made of steel, polymeric materials or paperboard (Bernardes et al., 2004).

Batteries can be classified into primary and secondary types. The distinction between these two types of batteries is whether the battery is rechargeable or non-rechargeable. The combination of the electrolyte and electrode materials that are used in a battery specifies the chemistry of the battery. This combination creates a chemical reaction that makes electrical power. Electronic devices generally use batteries as their primary or secondary power source. Mobility and convenience have encouraged the electronic devices to use batteries as their source of power. Notebooks, laptop computers, music players and digital cameras are some of the popular portable electronic devices (Bean, 2003).

The chemistry of batteries makes some of them rechargeable and some of them non-rechargeable. Primary batteries are those that cannot be recharged; hence, they must be discarded after their charge is over. An alkaline battery is a primary type that contains manganese and zinc. The positive electrode is made of magnesium dioxide and the negative electrode is made of zinc. Commonly, this type of battery is used in watches, calculators, radios and remote controls. Secondary batteries are rechargeable and we can use them repeatedly. Nickel cadmium (NiCd) and Nickel metal hydride (NiMH) are secondary batteries which are commonly used in laptops and computers, mobile phones, digital cameras and house alarms. Many consumers dispose rechargeable batteries before the end of their useful cycle life. The wastes of both types of primary and secondary batteries create problems for the environment (Bean, 2003). In the following, we discuss some of the main categories of batteries and their properties.

1.2.1. Lead-acid batteries

The most common form of lead-acid batteries are used for starting, lighting, and ignition—the so-called SLI batteries. Since the power of lead-acid batteries is mainly used in the production of vehicles and equipment, the global demand for this kind of batteries increases while the population of vehicles increases. On the other hand, we gradually encounter with the lack of lead sources. Thus, recycling after the end of the lead-acid batteries' life cycle is extremely important (Ahmed, 1996). According to Battery Council International's (BCI) reports, lead-acid batteries are currently the most recycled material in the world. Lead-acid batteries are also used as an uninterruptable power supplies for electrical equipments such as computers, laptops et cetera (Yender, 1998).

There are two different methods for producing lead metal. The metal is either extracted directly from lead ore or recovered from scarp lead products. The second method is advantageous from the view of the environment. First, the recovery process needs less energy than extracting lead from ore. Second, recycling prevents products from becoming waste and returns them to usable products after the completion of their life cycle. Third, recycling is useful for future generations because it conserves natural resources (Tsoulfas et al., 2002).

1.2.2. Alkaline Manganese and Zinc-Carbon batteries

The alkaline manganese and zinc-carbon batteries are designed as non-rechargeable batteries that are thrown away after being discharged (Rayovac Corp.,1999). Both alkaline and zinc-carbon batteries are described as primary batteries with high volume, low cost and low energy. These kinds of batteries are mostly used in electronic devices such as calculators, watches, cameras recorders, and toys that need a little amount of power (Sayilgan et al., 2009). Mercury, manganese and zinc are heavy metals that are found in the residue of alkaline and zinc-carbon batteries (Bartolozi, 1990). These substances are potential threat to health (Kierkegaard, 2007) due to their huge amount of toxic waste in the environment (Li and Xi, 2005). According to the recent studies, although the amount of mercury in alkaline batteries decreases from 5% to 0.025% by weight, the environment is still menaced by the high volume of other heavy metals in these batteries. This issue forces countries to include alkaline-manganese in the list of hazardous waste batteries. Similarly, zinc-carbon batteries are recovered from mercury as a first step, and then they are transported to the recycling facilities so that their zinc content is extracted (Yender, 1998).

1.2.3. Nickel Cadmium and Nickel Metal Hydride batteries

The NiCd and NiMH batteries are designed as rechargeable batteries and they are used for multiple times in many electronic devices such as computers, laptops, and telephones. Cadmium content in these batteries is another toxic material that causes environmental problems. Although NiCd has low cost and long lifetime with high evacuation rate and it is popular among customers, its consumption has been decreased because of a high volume of cadmium in these batteries. NiMH is the new type of batteries that is less harmful to the environment comparing with NiCd. However, NiMH is more expensive and it has a lower lifetime. NiCd and NiMH are similar in charge and discharge traits (Tenorio and Espinosa, 2001).

Recycling NiCd and NiMH batteries has many advantages from the view of the environment. Moreover, it is significantly beneficial from economic view (Pietrelli et al. 2004). According to the stakeholders (EPBA, 1999; Sempels, 1999; CollectNiCad, 2001), increase in the amount of collected and recycled waste batteries—specially the portable NiCd and NiMH batteries—will have significant effects on economic development and may reduce the environmental hazards from these batteries.

1.2.4. Lithium-ion batteries

The lithium batteries are one of the most innovative technologies in the evolution of batteries (Contestabile et al., 1999). It is important to point out that Li and NiMH were generated to replace NiCd. The environmental problem due to the use of Cd is the main reason of this substitution (Tenorio and Espinosa, 2001). There are both types of non-rechargeable (primary) and rechargeable (secondary) lithium batteries in the market that are used in various electronic products (Contestabile et al., 1999). According to Lee and Rhee (2003), approximately 28% of the battery market in the world is provided by the primary and secondary lithium batteries.

The significant subject that NiCd, NiHM, and Li batteries compete is the heaviness in electrical devices (Tenorio and Espinosa, 2001). Lithium secondary batteries are very favorable in applications where weight is an important factor. Lithium rechargeable batteries are preferred due to their high ratio of energy to weight. They are mostly used by the electrical devices such as laptops and cellular phones, while high technological electronic cameras are energized by lithium primary batteries (Contestabile et al., 1999). In the future, the energy of electronic automobiles will be also provided by the secondary LIBs (Xu et al., 2008). Since the utilization of electronic appliances has become very popular, the amount of primary and secondary batteries are growing. Thus, correct disposal and recycling of batteries are becoming more highlighted (Contestabile et al., 1999). Lithium is not toxic but it has hazardous waste. Cobalt which is one of the most important components in lithium cobalt oxide (LiCoO2) is a good example of this waste (Lain, 2001).

1.3. Literature review

Researchers have defined reverse logistics in various ways. The initial definition of reverse logistics had given by Murphy and Poist (1989), Stock (1992) and Kopicki (1993) as a reverse flow of goods. Tibben-Lembke and Rogers (2002) described the reverse logistics by comparing the definition of forward and reverse logistics. Forward logistics is the part of the supply chain that organizes the flow and the storage of products, or process of services and data from the suppliers to the customers. However, reverse logistics is the flow of goods and services in the opposite direction for the purpose of several possible destinations such as returning to the vendor, selling as a new, remanufacturing, recycling and landfill. We observe that the reverse logistics has more tasks than the forward logistics.

Shih, L. H. (2001) used reverse logistics system planning for recycling electrical appliances and computers in the case of Taiwan. Kaya et al. (2014) considered the design of reverse logistics for disassembly, refurbishing, and production operation of modular productions such as mobile phones and computers. They considered various uncertainties in the system and observed that the uncertainties in the amount of return are much more important than the uncertainties in the customer demand. We use the reverse logistic system for the recycling of waste-batteries and we consider the variability in the amount of collected batteries from the end users.

According to the environmental protection and benefits, many research programs are planning to manage waste products by using recycling systems. Pati et al. (2008) designed a mixed integer goal programming model to improve the paper recycling management and used the model to solve a paper recycling problem in India. Barros et al. (1998) performed a case study to analyze the problem of sieved sand that is produced from the construction waste in Netherlands. The recycling of construction's waste is also studied in the case of Malaysia by Begum et al. (2010). Louwers et al. (1999) designed the allocation model for the recycling of waste carpet materials. For more studies on the recycling of waste materials, see Lam et al.

(2010), Lin et al. (2008), Kwon et al. (2004). Donmez and Turkay (2013) studied the recycling of waste batteries in the case of Turkey. Similar to Donmez and Turkay (2013), in this thesis, we research the recycling of waste-batteries in Turkey.

Two-stage stochastic approach and robust optimization model are very popular approaches to solve problems with uncertainties (Ben-Tal et al., 2009). Huang and Loucks (2000) formulated a two-stage stochastic model for managing water resources with uncertainties. Donmez and Turkay (2013) used the two stage optimization method to develop a MILP model for the recycling of waste-batteries in Turkey. For more studies that use two-stage stochastic programming, see Barbarosoglu and Arda (2004), Santoso et al. (2005), Wang et al. (2012). Realff et al. (2004) designed reverse production system for carpet recycling using robust-mixed-integer method. Kaya et al. (2014) studied the model for remanufacturing modular production. They extended their model by using two-stage stochastic and robust optimization methods. They compared the results of these two methods and observed that the two-stage approach has worse results than the robust optimization method with respect to the worst-case performance. For more studies on robust optimization, see Ben-Tal and Nemirovski (2002), Beyer and Sendhoff (2007), Bertsimas et al. (2011) and Bertsimas et al. (2004). In this thesis, we develop a mixed integer linear programming model using the robust optimization approach and compare the results of this model to the results of two-stage stochastic method that have been reported in the literature.

We model a reverse logistics system for the disposition of waste batteries in Turkey (chapter 2). We describe the two-stage and robust optimization approaches in Chapter 3 to analyze the impact of uncertainties in the system. We also provide a small example to illustrate these two approaches in Chapter 3. In chapter 4, we compare the results of the deterministic, stochastic and robust optimization methods. We provide a comparison between the results of the two-stage and robust optimization methods. A sensitivity analysis is also performed to see the effects of varying amount of collected batteries in the design of the network. This is followed by conclusions.

Chapter 2
Mathematical Model

2.1. Overview

This chapter presents the mixed integer linear programming (MILP) model by Donmez and Turkay 2013 which has been developed for the design of the waste battery management system using reverse logistics in Turkey. In Chapter 3, we use this deterministic MILP model and the robust optimization approach to formulate a MILP model that incorporates uncertainty in the amount of collected waste batteries. Despite forward logistics systems, the reverse logistics focuses on the backward flow of used products from customers to the recycling facilities. Facility location problems and determining system strategies are the main components of this problem. The nodes of the network correspond to collection points, sorting facilities, recycling facilities and landfill areas. Recycling facilities and landfill areas are the disposal options of the model.

2.2. Model explanation

The model involves multiple periods where all of the network design decisions are made at the beginning of the planning horizon. The data of collection nodes including the amount of waste-batteries that are collected and the locations of the nodes for the entire planning horizon is given. In this model, we ignore the costs associated with collecting the waste batteries. We assume that the capacities of the collection nodes are sufficiently large so that all of the waste batteries are collected. The collected waste batteries are sent to the sorting facilities where they are grouped into different classes. Each sorting facility has a limited capacity which we decide at the beginning of the planning horizon. In fact, our model selects the best capacity for each sorting facility from a set of possible options that are determined in advance. The capacity of an existing sorting facility can be expanded according to the capacity expansion decisions. We assume that a collection node does not keep inventory; hence, all of the collected waste batteries are directly sent to the sorting facilities.

The network consists of a number of collection points, existing and potential sorting facilities, recycling facilities, inventory holding warehouses and landfill areas. Candidate locations and capacities of potential facilities are given. All of the waste batteries that are collected are assumed to be recyclable in this model. We also assume that all of the recycled batteries can be sold in the market—the capacity of the market is infinite. The objective function of the model is to maximize the total profit which is the revenue that is gained from selling the recycled batteries minus the total cost of the network.

A reverse logistics network is designed where the product is waste-batteries. The end users—who are the individuals that are living in different cities—send their waste-batteries to the collection nodes. Mixed batteries that are sent from customers are transported to sorting facilities to be categorized into different groups. Then, some of the sorted batteries with specific chemical properties are directly transported to landfill areas and disposed. The rest are sent to the recycling facilities. The sorted batteries can be recyclable or non-recyclable. The non-recyclable sorted batteries are disposed at landfill areas while recyclable sorted batteries are recycled and sold in a secondary market and produce revenue for our network. All of the facilities in our network have limited capacities; therefore, the decision makers should pay attention to the recycled material inventory at recycling facilities and their market price.

2.3. Mathematical Formulation

Let I be the set of regions of a country as the waste-battery collection nodes. Let J_E denote the set of pre-established battery sorting facilities, J_N denote the set of candidate locations—potential sorting facilities. Index j is used for the sorting facilities; hence, $j \in J_E \cup J_N$. There are fixed and variable costs for sorting facilities. Additionally, waste batteries can be sorted by newly installed sorting facility. Newly installed sorting facilities are constructed in low, medium, and high capacity levels. There is also capacity expansion option for newly installed sorting facilities. There are pre-established landfill areas and potential landfill areas which are denoted by the sets L_E and L_N. Let l be the index for the landfill areas, $l \in L_E \bigcup L_N$. In the first year, J_E existing sorting facilities and L_E existing landfill areas are active. There are R potential recycling facilities that could be open at some time periods. The planning horizon consists of T time periods and there are M types of material that could be produced after recycling waste-batteries in recycling facilities (see Figure 1).

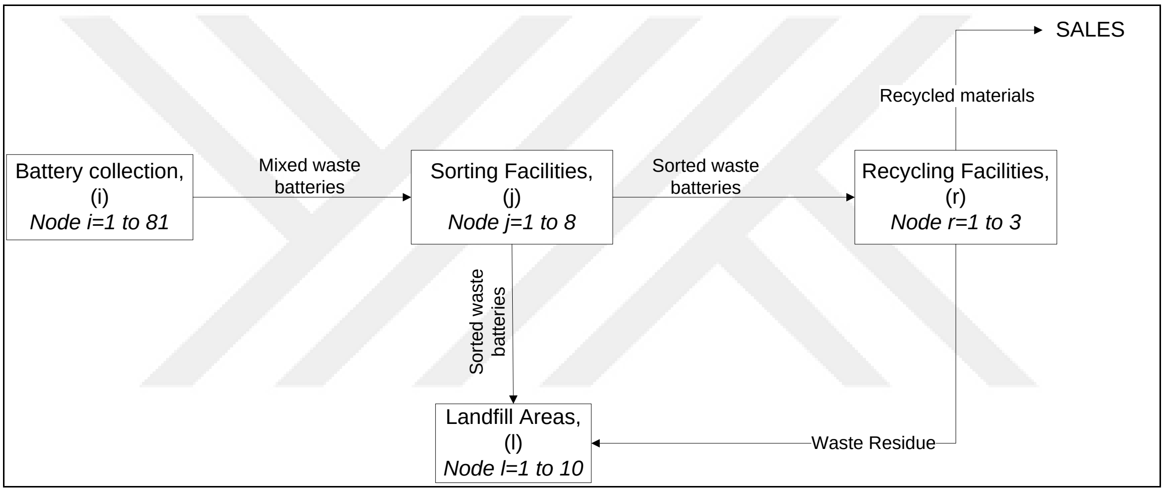

Figure 1: The reverse logistics of waste-battery recycling process

The model determines which set of potential facilities should be open in each time period, which battery types should be recycled in each time period, and which technological features should be installed in each recycling facility, how much the capacity of each facility should be expanded in each period, and the amount of waste batteries or residual batteries to be transported between facilities. Moreover, the model determines the amount of recycled materials that should be held as the inventory before being sold in the secondary market. In the remainder, we present and illustrate the mathematical model by Donmez and Turkay (2013).

2.3.1. Objective function

The total cost in the battery recycling network is the summation of transportation cost, sorting cost, recycling cost, landfill cost and inventory holding cost. On the other hand, the system gains its revenue from selling recycled materials to a secondary market. Therefore, the waste battery management system aims to achieve maximum profit; hence, the objective function is defined as the maximization of the net present value of the total revenue minus the total cost. Thus, the objective function is formulated as follows.

$$Maximize\ (Z) = \sum_{t \in T} \left((RM_t - TC_t - SC_t - RC_t - LC_t - IC_t) \times \frac{1}{(1+r)^t} \right) \quad (2\text{-}1)$$

where, r is a discount rate. To find the net present value of a cash flow, we multiply each monetary value at time t by the factor $1/(1+r)^t$—to obtain the equivalent monetary value at time 0—and sum the resulting values.

We denote the revenue of the system at time t by RM_t which is gained by selling the recycled materials. Let $N1_{mrt}$ denote the amount of recycled material $m \in M$ that is recycled in facility $r \in R$ at time period $t \in T$. The sale price of the recycled material $m \in M$ at time period $t \in T$ is denoted by $\mathrm{Re}v_{mt}$. Therefore, the revenue at time t from selling material m at facility r is $\mathrm{Re}v_{mt} \times N1_{mrt}$. Then, the total revenue at time t is given by the following equation.

$$RM_t = \sum_{m \in M} \sum_{r \in R} \mathrm{Re}v_{mt} \times N1_{mrt}, \ t \in T \quad (2\text{-}2)$$

Let TC_t represent the total transportation cost of the recycling network at time period t. Transportation is one of the main elements of the standard framework of the battery recycling system. The waste-batteries that are collected from the end-users in the collection nodes are transported to the sorting facilities. We define variables $x1_{ijt}$ as the amount of mixed batteries transported from the collection node $i \in I$ to the sorting facility $j \in J$ at time period $t \in T$, $x2_{jbrt}$ as the amount of battery type $b \in B$ transported from the sorting facility $j \in J$ to the recycling facility $r \in R$ at time period $t \in T$, and $x3_{jblt}$ as the amount of battery type $b \in B$ that is transported from the sorting facility $j \in J$ to the landfill area $l \in L$ at time period $t \in T$. The waste-batteries that are sent from the end-users have several chemistries and some of them cannot be recycled; hence, they are directly sent to the landfill areas for the disposal. The variable $N2_{rlt}$ is then defined as the amount of the remainder of the waste batteries that is transported from the recycling facilities to the landfill areas for the disposal. Let parameters $d1_{ij}$ denote the distance between the collection nodes and sorting facilities, $d2_{jr}$ denote the distance between sorting facilities and recycling facilities, $d3_{jl}$ denote the distance between the sorting facilities and landfill areas, and $d4_{rl}$ denote the distance between the recycling facilities and landfill areas. The transportation cost is formulated as follows.

$$TC_t = \sum_{i \in I}\sum_{j \in J} d1_{ij} \times x1_{ijt} \times UCT\ 1_{ijt} + \sum_{j \in J}\sum_{r \in R}\sum_{b \in B} d2_{jr} \times x2_{jbrt} \times UCT2_{jrt} + \sum_{j \in J}\sum_{l \in L}\sum_{b \in B} d3_{jl} \times x3_{jblt} \times UCT\ 3_{jlt} + \sum_{r \in R}\sum_{l \in L} d4_{rl} \times N2_{rlt} \times UCT\ 4_{rit} \tag{2-3}$$

In this equation, the unit variable cost of transportation between a pair of collection nodes, sorting facilities, recycling facilities and landfill areas are shown by $UCT\ 1_{ijt}$,$UCT\ 2_{jit}$, $UCT\ 3_{jlt}$, and $UCT\ 4_{rit}$, respectively.

The variable SC_t represents the total sorting cost at time *t*, which is obtained using equation (2-4). The parameter UCS_{jqt} demonstrates the unit variable cost of sorting operations at sorting facilities and CCS_{jqt} shows the fixed cost of opening the sorting facility *j* with capacity $q \in Q$ at time *t*. Define the binary variable ys_{jqt} that is 1 if the sorting facility is active at location $j \in J$ with capacity *q* at time *t*, and 0 otherwise. The fixed cost of keeping the sorting facility *j* open with capacity $q \in Q$ at time *t* is denoted by FCS_{jqt}. In our model, we consider a capacity expansion option for the sorting facilities. Define a binary variable us_{jct} which takes a value of 1 if the capacity expansion option *c* is added to the sorting facility *j* at time *t*, and 0 otherwise. If we expand the capacity of an open sorting facility, we incur a fixed cost of $FCCS_{ct}$. Therefore, the total sorting cost of network is formulated as follows.

$$SC_t = \sum_{i\in I}\sum_{j\in J}\sum_{q\in Q} UCS_{jqt} \times x1_{ijt} \times ys_{jqt} + \sum_{j\in J}\sum_{q\in Q} FCS_{jqt} \times ys_{jqt}$$
$$\sum_{j\in J_N}\sum_{q\in Q} CCS_{jqt} \times \left(ys_{jqt} - ys_{jq,t-1}\right) + \sum_{j\in J}\sum_{c\in C} FCCS_{ct} \times us_{jct} \tag{2-4}$$

Note that in equation (2-4), $x1_{ijt} \times ys_{jqt}$ is nonlinear because of the multiplication of two variables. This nonlinearity is eliminated by defining a new variable $TS_{jqt} \geq 0$ and adding the following constrains to the model.

$$TS_{jqt} \leq ys_{jqt} \times \mathrm{K}, \quad \forall j \in J, \forall q \in Q, \forall t \in T \tag{2-5}$$

$$TS_{jqt} \geq \sum_{i\in I} x1_{ijt} - \left(1 - ys_{jqt}\right) \times \mathrm{K}, \quad \forall j \in J, \forall q \in Q, \forall t \in T \tag{2-6}$$

$$TS_{jqt} \leq \sum_{i\in I} x1_{ijt} + \left(1 - ys_{jqt}\right) \times \mathrm{K}, \quad \forall j \in J, \forall q \in Q, \forall t \in T \tag{2-7}$$

where, K is a sufficiently large number. We must have $TS_{jqt} = x1_{ijt} \times ys_{jqt}$. Based on equation (2-5), TS_{jqt} will be less than or equal to 0 if ys_{jqt} is 0. As we mentioned before, TS_{jqt} is also greater than or equal to zero; hence, TS_{jqt} will be equal to 0 if ys_{jqt} is 0. However, if ys_{jqt} becomes 1, based on equation (2-6), TS_{jqt} will be greater than or equal to $x1_{ijt}$ and according to equation (2-7) TS_{jqt} will be less than or equal to $x1_{ijt}$. Therefore, TS_{jqt} will be equal to $x1_{ijt}$ if ys_{jqt} becomes 1. Therefore, equations (2-5) to (2-7) correctly formulate $TS_{jqt} = x1_{ijt} \times ys_{jqt}$ using linear inequalities.

The variable RC_t is the total recycling cost of the network at time period t. In a reverse logistics network, reusable products such as waste-batteries are returned from the customers to the collection nodes and then sent to recycling facilities to be converted into new products. Recycling includes the treatment, recovery and reprocessing of waste-batteries and using them in the production of new goods. Clearly, because of the various activities that are used and the technologies that are employed in the recycling process, this is the most expensive process in the model. We will analyze these concepts and will show comparisons between the different cost components in the experimental part of this thesis.

The recycling cost at time *t* is formulated as in equation (2-7). In this equation, CCR_{rt} is the construction cost for recycling facility $r \in R$ at time period $t \in T$. Fixed cost and unit variable cost of operations in the recycling facilities are FCR_{rt} and UCR_{brt}, respectively. The cost of installing technology $k \in K$ is denoted by FCT_{kt} and included in equation (2-7) for the recycling facilities. The amount of battery type *b* that is recycled at recycling facility *r* at time period *t* is denoted by R_{brt}.

$$RC_t = \sum_{r \in R} CCR_{rt} \times ro_{rt} + \sum_{r \in R} FCR_{rt} \times yr_{rt} + \sum_{b \in B} \sum_{r \in R} UCR_{brt} \times R_{brt} + \sum_{k \in K} \sum_{r \in R} FCT_{kt} \times \left(yt_{krt} - yt_{kr,t-1} \right) \tag{2-7}$$

Equation (2-7) includes three sets of binary variables. If recycling facility *r* is built at time period *t*, ro_{rt} becomes 1, and otherwise 0. If recycling facility *r* is operational at time period *t*, then the binary variable yr_{rt} takes a value of 1, and otherwise it becomes 0. If recycling technology *k* is used at facility *r* at time *t*, then yt_{krt} becomes 1, and otherwise 0. The statement ($yt_{krt} - yt_{kr,t-1}$) represents that we will pay fix cost of technology *k* once in the entire horizon.

After the completion of the recycling process, two possibilities can happen for the batteries: they are either transferred to the landfill areas for disposal or sold in a secondary market as recycled products. Since waste-batteries are considered as hazardous waste, landfill areas for their disposal become a major issue with a potential to have significant effects on the environment. In this model, the unit variable cost of the disposal operations at the landfill area $l \in L$ is denoted by UCL_{lt}. The construction cost for landfill area *l* at time period *t* is shown by CCL_{lt}. Define the variable LC_t as the total landfill cost of the recycling network at time *t*. The formulation of this cost is as follows.

$$LC_t = \sum_{j \in J} \sum_{b \in B} \sum_{l \in L} UCL_{lt} \times x3_{jblt} + \sum_{r \in R} \sum_{l \in L} UCL_{lt} \times N2_{rlt} + \sum_{l \in L_N} CCL_{lt} \left(yl_{lt} - yl_{l,t-1} \right) \tag{2-8}$$

In equation (2-8), if landfill area *l* is active at time *t*, then the binary variable yl_{lt} becomes 1, and otherwise it becomes 0. The statement $(yl_{lt} - yl_{l,t-1})$ shows that we pay the construction cost of the landfill area *l* once in the entire horizon.

Inventory management is one of the important parts of the recycling network. The objective of the inventory management is to hold the recycled batteries and subsequently sell them in the secondary markets. Inventory costs are associated with the amount of waste-batteries that are returned from the customers to the collection nodes. The total inventory holding cost is defined as the variable IC_t and formulated as given in equation (2-9). In this equation, UCI_{mt} is the unit inventory holding cost of the recycled materials during time period $t \in T$. The amount of material $m \in M$ that is produced from recycling operations at facility *r* and time *t* is denoted by Inv_{mrt}.

$$IC_t = \sum_m \sum_r UCI_t \times Inv_{mrt} \tag{2-9}$$

2.3.2. Mass Balance constraints

Recall that $x1_{ijt}$ refers to the amount of mixed batteries that are transported from the collection nodes to the sorting facilities. Moreover, a_{it} is the parameter that shows the amount of batteries which are collected at the collection node $i \in I$, and time *t*. Summation of $x1_{ijt}$ overall *j* 's must equal to a_{it}. The flow balance constraints are formulated as given in equation (2-10).

$$\sum_{j \in J} x1_{ijt} - a_{it} = 0, \quad \forall i \in I, \forall t \in T \tag{2-10}$$

The model also includes a flow balance constraint for each battery type. In equation (2-11), $x1_{ijt}$ refers to the amount of mixed batteries that are transported from collection node *i* to sorting facility *j*. The percentage of the waste-battery type *b* that are collected collection node *i is* denoted by α_{ib}. Moreover, $z1_{jbt}$ is the amount of battery-type *b* that is processed at sorting facility *j* at time *t*.

$$\sum_{i \in I} x1_{ijt} \times \alpha_{ib} = z1_{jbt}, \quad \forall j \in J, \forall b \in B, \forall t \in T \tag{2-11}$$

The amount of battery-type $b \in B$ that is processed at sorting facility *j* at time *t*, $z1_{jbt}$, must be equal to the total amount of battery-type *b* that is transported from sorting facility *j* to the recycling plants, $x2_{jbrt}$, and landfill areas, $x3_{jblt}$. This constraint is formulated in equation (2-12). In other words, this equation guarantees that all of the waste batteries that are sorted are either sent to the recycling facilities or landfill areas.

$$z1_{jbt} = \sum_{r \in R} x2_{jbrt} + \sum_{l \in L} x3_{jblt}, \quad \forall j \in J, \forall b \in B, \forall t \in T \tag{2-12}$$

Because of technological restrictions, only a proportion of the waste batteries can be recycled; the rest that are non-recyclable are sent to the disposal areas. Equation (2-13) represents this constraint where p_{bm} is the percentage of material *m* in battery type $b \in B$ that can be recycled.

$$\sum_{b \in B} R_{brt} \times \left(1 - \sum_{m \in M} p_{bm}\right) = \sum_{l \in L} N2_{rlt}, \quad \forall r \in R, \forall t \in T \tag{2-13}$$

2.3.3. Inventory balance constraints

In traditional supply chain systems, the primary reason for holding inventory is to meet the customer demand, taking the advantage of ordering in large quantities, and reducing the probability of stock-outs. In reverse logistics network design systems, inventory is also hold due to the scarcities of raw materials or an expected rise in the price of the material that are produced in the recycling process in the secondary market.

Equation (2-14) represents the inventory balance constraints for the recycled material at each recycling facility. In this equation, p_{bm} denotes the percentage of the recyclable material *m* from the battery type $b \in B$, R_{brt} denotes the amount of battery-type $b \in B$ that is recycled at recycling facility $r \in R$ at time period $t \in T$, and $N1_{mrt}$ denotes the amount of recycled material $m \in M$ sold from recycling facility $r \in R$ at time period $t \in T$.

$$Inv_{mrt} = Inv_{mr,t-1} + \sum_{b \in B} p_{bm} \times R_{brt} - N1_{mrt}, \quad \forall m \in M, \forall r \in R, \forall t \in T \tag{2-14}$$

The amount of battery-type $b \in B$ that is recycled at each recycling facility $r \in R$ at time period $t \in T$ must be equal to the total amount of the waste battery-type $b \in B$ that is transported from sorting facility $j \in J$ to recycling facility $r \in R$ at time period $t \in T$. This is formulated in equation (2-15).

$$R_{brt} = \sum_{j \in J} x2_{jbrt}, \quad \forall b \in B, \forall r \in R, \forall t \in T \tag{2-15}$$

2.3.4. Capacity constraints

There is a given set of capacity options for each sorting facility that is denoted by *Q*. Constraint (2-16) ensures that for each sorting facility, only one capacity option can be chosen at time period $t \in T$.

$$\sum_{q \in Q} ys_{jqt} \le 1, \quad \forall j \in J, \forall t \in T \tag{2-16}$$

The model considers capacity expansion options for sorting facilities. Therefore, the total capacity for sorting facility *j* includes its initial capacity CAP_s_q and the expanded capacity c_c. This is guaranteed by the following equation.

$$\sum_{i \in I} x1_{ijt} \le \sum_{q \in Q} CAP\_s_q \times ys_{jqt} + \sum_{c \in C} c_c \times us_{jct'}, \quad \forall j \in J, t' \le t \tag{2-17}$$

Equations (2-18) and (2-19) enforce the capacity expansion constraints for the sorting facilities. Equation (2-18) shows that at most one extra capacity can be installed in sorting facility $j \in J$ at each time period $t \in T$. Equation (2-19) ensures that at most three extra capacities can be installed in each sorting facility $j \in J$ in the entire planning horizon of *T* years.

$$\sum_{c \in C} us_{jct} \le 1, \quad \forall j \in J, \forall t \in T \tag{2-18}$$

$$\sum_{c\in C}\sum_{t\in T} us_{jct} \le 3, \quad \forall j \in J \qquad (2\text{-}19)$$

Equation (2-20) restricts the amount of waste-battery type $b \in B$ which will be recycled at recycling facility $r \in R$. In this equation, CAP_r_{br} denotes the upper bound of the capacity for the recycling facility $r \in R$ at time period $t \in T$. The binary variable yr_{rt} in this constraint takes a value of 1 if the battery recycling facility $r \in R$ is operational at time period $t \in T$, and 0 otherwise.

$$R_{brt} \le CAP\_r_{br} \times yr_{rt}, \quad \forall b \in B, \forall r \in R, \forall t \in T \qquad (2\text{-}20)$$

The sorting facility $j \in J$ is constructed with a specific capacity $q \in Q$ in the network. Constraint (2-21) means that if the sorting facility $j \in J$ with capacity $q \in Q$ is open at time *t*, then it must be open until the end of the time horizon. The same condition is included for recycling facilities $r \in R$, technologies of the recycling facilities $k \in K$, and landfill areas $l \in L$ (see equations (2-22), (2-23) and (2-24)).

$$ys_{jqt} \le ys_{jq,t+1}, \quad \forall j \in J, \forall q \in Q, t < T \qquad (2\text{-}21)$$

$$yr_{rt} \le yr_{r,t+1}, \quad \forall r \in R, t < T \qquad (2\text{-}22)$$

$$yt_{krt} \le yt_{kr,t+1}, \quad \forall k \in K, \forall r \in R, t < T \qquad (2\text{-}23)$$

$$yl_{lt} \le yl_{l,t+1}, \quad \forall l \in L, t < T \qquad (2\text{-}24)$$

The total amount of waste-batteries that is transported from the recycling facilities to the landfill areas, $x3_{jblt}$, plus the total amount of residual waste-batteries that is transported from the recycling facilities to the landfill areas, $N2_{rlt}$, must be less than or equal to some given capacities. This is ensured by constraints (2-25), (2-26), and (2-27). Based on equation (2-25), if landfill area $l \in L$ is operational at time period $t \in T$, the sum of $x3_{jblt}$ and $N2_{rlt}$ can be nonzero, and otherwise these two variables become zero. According to constraint (2-26), sum of $x3_{jblt}$ and $N2_{rlt}$ must be less than or equal to the capacity of the landfill area, CAP_l_l

. Moreover, there is a capacity limitation for the pre-established landfill areas that is denoted by $parExistCap\_L_l$ (see equation (2-27)).

$$\sum_{j\in J}\sum_{b\in B} x3_{jblt} + \sum_{r\in R} N2_{rlt} \le y_{lt} \times M, \quad \forall l \in L, \forall t \in T \tag{2-25}$$

$$\sum_{j\in J}\sum_{b\in B}\sum_{t\in T} x3_{jblt} + \sum_{r\in R}\sum_{t\in T} N2_{rlt} \le CAP\_l_l, \quad \forall l \in L_N \tag{2-26}$$

$$\sum_{j\in J}\sum_{b\in B}\sum_{t\in T} x3_{jblt} + \sum_{r\in R}\sum_{t\in T} N2_{rlt} \le parExistCap\_L_l, \quad \forall l \in L_E \tag{2-27}$$

In equation (2-26), CAP_l_l represents the storage capacity for landfill area $l \in L$. Equation (2-27) ensures the capacity upper bound on the landfill area at the beginning of the planning horizon.

Constraints (2-29) and (2-30) enforce the existence of sorting facilities J_E and landfill areas L_E at the beginning of the planning horizon. In other words, we start the recycling network such that sorting facilities J_E and landfill areas L_E are open with specific capacities at the start of the time.

$$ys_{jqt} = 1, \quad \forall j \in J_E, q = 1, t = 1 \tag{2-29}$$

$$yl_{lt} = 1, \quad \forall l \in L_E, t = 1 \tag{2-30}$$

As we previously discussed, the model incorporates a multi-period scenario and it considers a lead time for constructing the recycling facilities. Based on constraint (2-31), the construction lead time is one year.

$$ro_{r,t-1} = yr_{rt} - yr_{r,t-1}, \quad \forall r \in R, \forall t \in T \tag{2-31}$$

For example, if a decision is made to construct a recycling facility at the beginning of period 3, the new facility starts up at the beginning of period 4. In this model, based on constraint (2-32), additional capacity can be added for sorting facility $j \in J$ while the sorting facility $j \in J$ is operational. Moreover, technological features can be installed while the recycling facility $r \in R$ is active. This issue is modeled in equation (2-33).

$$us_{jct} \leq \sum_{q \in Q} ys_{jqt}, \quad \forall j \in J, \forall c \in C, \forall t \in T \tag{2-32}$$

$$yt_{krt} \leq yr_{rt}, \quad k \in K, r \in R, t \in T \tag{2-33}$$

$$R_{brt} \leq b_{brt} \times M, \quad \forall b \in B, \forall r \in R, \forall t \in T \tag{2-34}$$

In constraint (2-34), b_{brt} is a binary variable which takes a value of 1 if battery-type $b \in B$ can be processed at recycling facility r. This constraint provides information about the characteristics and location of the waste batteries that are processed in recycling facility $r \in R$ at time t.

2.3.5. Non-negativity and Integrality constraints

All of the decision variables for determining the amounts of waste batteries in different parts of the network are non-negative (see equation (2-35)). The variables that are used for capacity expansion, facility location and installing technological features are binary (see equation (2-36)).

$$x1_{ijt}, x2_{jbrt}, x3_{jblt}, R_{brt}, Inv_{mrt}, N1_{mrt}, N2_{rlt}, z1_{jbt} \geq 0 \tag{2-35}$$

$$ys_{jqt}, yr_{rt}, ro_{rt}, yl_{lt}, yt_{krt}, us_{jct}, b_{brt} \in \{0,1\} \tag{2-36}$$

Chapter 3
Model Extension

3.1. Overview

The mathematical model that was presented in chapter 2 considers a multi-period supply chain that collects waste batteries from the end-users, sorts them and then either recycles or disposes them. The amount of collected batteries is not deterministic which introduce uncertainty into the problem. This uncertainty has significant effects on the design of the supply chain. We extend the model that was presented in chapter 2 to incorporate the uncertainty in the amount of collected waste-batteries from different regions. We assume that a finite number of independent scenarios are known and each scenario happen with a given probability. We also assume that the probability of each scenario is known at the beginning of the planning horizon. We first discuss the formulation using the two-stage stochastic programming approach by Donmez and Turkay (2013). We then present our new model using the robust optimization method. Finally, we provide a small example to illustrate the two models.

3.2. Two-stage stochastic programming approach

One of the most popular frameworks for planning under uncertainty is the two-stage stochastic programming approach with a fixed recourse (Birge and Louveaux, 1997). The two-stage stochastic programming is a method that can deal with uncertainty in any parameter of the model. Two-stage stochastic programming problems refer to situations where decisions are made in two stages and the observation of some random events takes place in between (Fabian and Szoke, 2006). The first-stage decisions are strategic-level decisions (such as the capacity of sorting and recycling facilities, and the number and location of facilities) and the second-stage decisions are tactical-level decisions (such as the amount of transported materials and inventory levels). The first decision must be made when the outcome of the random event is not known (Fabian and Szoke, 2006). In other words, the first stage decisions should be implemented before the actual demand or supply quantities are known. Then the second-stage decisions are made (Kaya et al., 2014). In the literature, the uncertainty is incorporated in the model by defining different scenarios (see for example Kaya et al., 2014, Donmez and Turkay 2013). Let *S* be the set of possible scenarios for the amount of collected waste-batteries and s be the specific scenario in the set S—i.e., index for scenarios. The classical two-stage stochastic program is presented as follows (see Birge and Louveaux (1997) for a detailed explanation).

$$\begin{aligned} &\min_{y \in R^n} c^T y + E[Q(y,\varepsilon)] \\ &s.t. \quad Ay = b, \; y \geq 0 \end{aligned} \tag{3-1}$$

where, $Q(y,\varepsilon)$ is the optimal value of the second-stage problem. The associated cost of the first-stage decision variable is denoted by the vector c (see equation (3-1). Here, $Q(y,\varepsilon)$ is the optimal value of the following minimization problem.

$$\begin{aligned} &Q(y,\varepsilon) = \min_{x \in R^m} q^T x \\ &s.t. \quad Ty + Wx = h, \quad x \geq 0 \end{aligned} \tag{3-2}$$

where vector y represents the strategic-level decisions (first-stage). The variable that are used in the tactical-level decisions (second-stage) is denoted by the vector x. The vector ε in equation (3-2) includes the parameters of the second-stage problem which are *T*, *W*, *h*, and *q*. Since we only consider the discrete distributions, a vector ε has a finite number of scenarios; hence, we can use the notation $\varepsilon_s(q_s, h_s, T_s, W_s)$ which is specific to each scenario. We are also given probability values for each scenario p_s, $s = 1,...,S$. Therefore, considering a finite number of scenarios and the given probability values, we can obtain the expected value using the following equation.

$$E[Q(y,\varepsilon)] = \sum_{s=1}^{S} p_s Q(y,\varepsilon_s) \tag{3-3}$$

Hence, using equation (3-3), the minimization problem given in equation (3-2) can be written as follows.

$$\begin{aligned} &\min_{x_1,x_2,\ldots,x_S} \sum_{s=1}^{S} p_s\, q_s^T x_s \\ &s.t. \quad T_s y + W_s x_s = h_s \\ &\qquad\;\; x_s \geq 0, \quad \forall s \end{aligned} \tag{3-4}$$

Combining equations (3-3) and (3-4), we can formulate the following linear program (see equation (3-5)). This linear program is in fact the deterministic equivalent program of the stochastic problem that we previously discussed. This approach was originally proposed by Dantzig (1955).

$$\begin{aligned} &\min_{y,x_a,\ldots,x_S} c^T y + \sum_{s=1}^{S} p_s q_s^T x_s \\ &s.t. \quad T_s y + W_s x_s = h_s, \quad \forall s \\ &\qquad\;\; Ay = b, \\ &\qquad\;\; y \geq 0, x_s \geq 0, \quad \forall s \end{aligned} \tag{3-5}$$

In the remainder, we show the steps that must be performed to formulate the stochastic version of the waste-battery recycling network using a two-stage stochastic programming and scenario-based approach. The more detailed explanation is given by Donmez and Turkay (2013). The waste-battery recycling problem can be decomposed into two stages including strategic-level (first-stage) and tactical-level (second-stage) decisions. The strategic-level decisions should be made first. Then the demand quantities are determined. Finally, the second-stage decisions are made based on the results obtained in the first stage and the known values of the demand. The model includes the following sets of first-stage decision variables. The number, locations and capacities of the operational sorting facilities, ys_{jqt}, the additional capacity options for the sorting facilities, us_{jct}, the number and locations of the operational recycling facilities, yr_{rt}, the construction time of recycling facilities, ro_{rt}, the number and locations of the operational landfill areas, yl_{lt}, the number and locations of the operational recycling technologies, yt_{krt}, and the number of battery type that can be processed at recycling facilities, b_{brt}.

Recall that a finite number of scenarios with given probability values are used to handle the uncertainty in the amount of the collected waste-batteries. In the case of Turkey (to be discussed in Chapter 4), there are only four possible scenarios. Thus, the two-stage stochastic program by using a scenario-based approach is as follows (Donmez and Turkay, 2013, also see Kaya et al, 2014).

$$\begin{aligned} &\min_{y \in R^n} fy + E_s[Q(y,s)] \\ &s.t. \quad Ay = b \\ &y \in z^+ \end{aligned} \tag{3-6}$$

Where

$$Q(y,s) = \min\,(cx) \tag{3-7}$$

$$W_1 x = b(s),\ \forall s \in S$$

$$W_2 x = d$$

$$W_3 x \le Ty$$

$$x \ge 0$$

Equation (3-6) is the first-stage problem in the two-stage stochastic optimization approach. The decision variables of the first-stage, y, are observed at this stage (their values are determined). The total cost of the optimization is given by the expectation of $Q(y,s)$ over s plus fy, where $Q(y,s)$ is the minimum cost solution for a particular scenario s and f is the vector of the costs associated with the first-stage decision variables. The second-stage objective function is shown by equation (3-7) while its minimum cost is denoted by $Q(y,s)$ for each scenario s. Note that x is the decision variable of the second-stage. After solving the two-stage optimization problem, we find the values of the variables x_s for each scenario.

In the two-stage stochastic optimization approach, we assign probabilities to each scenario and solve the problem for each scenario. Since the solution of each scenario is not an optimal solution for the general problem, we reformulate the problem and solve it as a single-stage optimization problem.

$$\begin{aligned}
&\min_{x \in R^n} \; fy + E_s[cx_s] \\
&s.t. \quad Ay = b \\
&\mathrm{W_1 x_s = b(s), \forall s \in S} \\
&\mathrm{W_2 x_s = d, \forall s \in S} \qquad (3\text{-}8) \\
&\mathrm{W_3 x_s \le Ty, \forall s \in S} \\
&y \in Z^+ \\
&\mathrm{x_s \ge 0, \forall s \in S}
\end{aligned}$$

Donmez and Turkay (2013) and Kaya et al. (2014) used model (3-8) to solve the stochastic version of their problem. For the detailed model, we refer the reader to these papers. In the remainder, we discuss the robust optimization approach that we use in this paper to formulate the stochastic version of the waste-battery recycling network. In Chapter 4, we will compare the result of the robust optimization approach with the result of the model given by Donmez and Turkay (2013).

3.3. Robust optimization approach

We aim in this thesis to address the network design problem for waste batteries considering uncertainty in the input data—quantities of waste batteries which are collected from the end-users. Similar to the two-stage stochastic approach that we previously discussed, we have strategic level decisions—e.g., the capacity of sorting and recycling facilities, and the number and location of facilities—and tactical level decisions—e.g., the amount of transported materials, and inventory levels. The uncertainty in the input data forces the designers to search for a network design that is "good" (nearly optimal) for a variety of future operating scenarios. This is referred to as the "robustness" property of the network designs (Gutierrez et al., 1996).

We define the long-term and short-term decisions similar to the two-stage stochastic programming approach in previous section. We first determine the long-term capacity decisions and after observing capacity levels, we determine the short-term operational decisions. The main objective of the robust optimization approach is to find a solution to minimize the maximum difference between the optimal values of scenarios and their values collectively. The general model for the robust optimization approach is given in the following.

$$Minimize\ \delta \tag{3-9}$$

s.t. $$\delta \geq R_s - Q_s^*\ ,\ \forall s \in S \tag{3-10}$$

$$\begin{aligned} & Ay = b \\ & W_1 x_s = b(s), \forall s \in S \\ & W_2 x_s = d, \forall s \in S \\ & W_3 x_s \leq Ty, \forall s \in S \\ & y \in Z^+ \\ & x_s \geq 0, \forall s \in S \end{aligned} \tag{3-11}$$

Where, Q_S^* and R_s are the costs of the optimal solution and the robust solution for each scenario *s*, respectively. In the first step of the robust optimization approach, we solve the model by using data of each scenario and find the optimal solution for each scenario which we denote by Q_S^* . In the second step, we solve the problem given in equations (3-9) to (3-11) and find the value of the variables $R_{s,}\ \ s \in S$. We also obtain the minimum value of the objective function, δ.

In the remainder, we present the detailed formulation of the waste-battery recycling network design problem using the robust optimization approach. This model is similar to the model which is given in Chapter 2. Therefore, for the explanation of the model, the reader is referred to chapter 2 where we thoroughly illustrate the model, its parameters and variables, and the purpose of each constraint.

$$Minimize \quad \delta \tag{3-12}$$

$$\delta \geq R_s - Q_S^*\ ,\ \forall s \in S \tag{3-13}$$

$$R_s = \sum_{t \in T} \left(\left(RM_{st} - TC_{st} - SC_{st} - RC_{st} - LC_{st} - IC_{st} \right) \times \frac{1}{(1+r)^t} \right), \quad \forall s \in S \tag{3-14}$$

$$RM_{st} = \sum_{m \in M} \sum_{r \in R} \mathrm{Re}\, v_{mt} \times N1_{smrt}, \quad \forall s \in S, \forall t \in T \tag{3-15}$$

$$\begin{aligned} TC_{st} = & \sum_{i \in I} \sum_{j \in J} d1_{ij} \times x1_{sijt} \times UCT\ 1_{ij} + \sum_{j \in J} \sum_{r \in R} \sum_{b \in B} d\,2_{jr} \times x\,2_{sjbrt} \times UCT\ 2_{jr} + \\ & \sum_{j \in J} \sum_{l \in L} \sum_{b \in B} d\,3_{jl} \times x3_{sjblt} \times UCT\ 3_{jl} + \sum_{r \in R} \sum_{l \in L} d4_{rl} \times N2_{srlt} \times UCT\ 4_{rl}, \quad \forall s \in S, \forall t \in T \end{aligned} \tag{3-16}$$

$$SC_{st} = \sum_{j\in J}\sum_{q\in Q} USC_{jqt} \times TS_{sjqt} + \sum_{j\in J}\sum_{q\in Q} FCS_{jqt} \times ys_{jqt} + \sum_{j\in J_N}\sum_{q\in Q} CCS_{jqt} \times \left(ys_{jqt} - ys_{jq,t-1}\right) + \sum_{j\in J_N}\sum_{c\in C} FCCS_{ct} \times us_{jct}, \quad \forall s\in S, \forall t\in T \tag{3-17}$$

$$RC_{st} = \sum_{r\in R} CCR_{rt} \times ro_{rt} + \sum_{r\in R} FCR_{rt} \times yr_{rt} + \sum_{b\in B}\sum_{r\in R} UCR_{brt} \times R_{sbrt} + \sum_{k\in K}\sum_{r\in R} FCT_{kt} \times \left(yt_{krt} - yt_{kr,t-1}\right), \quad \forall s\in S, \forall t\in T \tag{3-18}$$

$$LC_{st} = \sum_{l\in L_N} CCL_{lt}\left(yl_{lt} - yl_{l,t-1}\right) + \sum_{j\in J}\sum_{b\in \mathbf{B}}\sum_{l\in L} UCL_{lt} \times x3_{sjblt} + \sum_{r\in R}\sum_{l\in L} UCL_{lt} \times N2_{srlt}, \quad \forall s\in S, \forall t\in T \tag{3-19}$$

$$IC_{st} = \sum_{m}\sum_{r} UCI_{t} \times Inv_{smrt}, \quad \forall s\in S, \forall t\in T \tag{3-20}$$

The constraints of the mathematical model are given as follows (see Chapter 2 for detailed illustration).

$$\sum_{j\in J} x1_{sijt} - a_{sit} = 0, \quad \forall s\in S, \forall i\in I, \forall t\in T \tag{3-21}$$

$$\sum_{i\in I} x1_{sijt} \times \alpha_{ib} = z1_{sjbt}, \quad \forall s\in S, \forall j\in J, \forall b\in B, \forall t\in T \tag{3-22}$$

$$z1_{sjbt} = \sum_{r\in R} x2_{sjbrt} + \sum_{l\in L} x3_{sjblt}, \quad \forall s\in S, \forall j\in J, \forall b\in B, \forall t\in T \tag{3-23}$$

$$\sum_{b\in B} R_{sbrt} \times \left(1 - \sum_{m\in M} p_{bm}\right) = \sum_{l\in L} N2_{srlt}, \quad \forall s\in S, \forall r\in R, \forall t\in T \tag{3-24}$$

$$Inv_{smrt} = Inv_{smr,t-1} + \sum_{b\in B} p_{bm} \times R_{sbrt} - N1_{smrt}, \quad \forall s\in S, \forall m\in M, \forall r\in R, \forall t\in T \tag{3-25}$$

$$R_{sbrt} = \sum_{j\in J} x2_{sjbrt}, \quad \forall s\in S, \forall b\in B, \forall r\in R, \forall t\in T \tag{3-26}$$

$$\sum_{q\in Q} ys_{jqt} \le 1, \quad \forall j\in J, \forall t\in T \tag{3-27}$$

$$\sum_{i \in I} x1_{sijt} \leq \sum_{q \in Q} CAP\_s_q \times ys_{jqt} + \sum_{c \in C} c_c \times us_{jct'}, \quad \forall s \in S, \forall j \in J, t' \leq t \tag{3-28}$$

$$\sum_{c \in C} us_{jct} \leq 1, \quad \forall j \in J, \forall t \in T \tag{3-29}$$

$$\sum_{c \in C} \sum_{t \in T} us_{jct} \leq 3, \quad \forall j \in J \tag{3-30}$$

$$\sum_{b \in B} R_{sbrt} \leq CAP\_r_r \times yr_{rt}, \quad \forall s \in S, \forall r \in R, \forall t \in T \tag{3-31}$$

$$ys_{jqt} \leq ys_{jq,t+1}, \quad \forall j \in J, \forall q \in Q, t < T \tag{3-32}$$

$$yr_{rt} \leq yr_{r,t+1}, \quad \forall r \in R, t < T \tag{3-33}$$

$$yt_{krt} \leq yt_{kr,t+1}, \quad \forall k \in K, \forall r \in R, t < T \tag{3-34}$$

$$yl_{rt} \leq yl_{r,t+1}, \quad 1 \leq t < T \tag{3-35}$$

$$\sum_{j \in J} \sum_{b \in B} x3_{sjblt} + \sum_{r \in R} N2_{srlt} \leq y_{lt} \times M, \quad \forall s \in S, \forall l \in L, \forall t \in T \tag{3-36}$$

$$\sum_{j \in J} \sum_{b \in B} \sum_{t \in T} x3_{sjblt} + \sum_{r \in R} \sum_{t \in T} N2_{srlt} \leq CAP\_l_l, \quad \forall s \in S, \forall l \in L_N \tag{3-38}$$

$$\sum_{j \in J} \sum_{b \in B} \sum_{t \in T} x3_{sjblt} + \sum_{r \in R} \sum_{t \in T} N2_{srlt} \leq parExistCap\_L_l, \quad \forall s \in S, \forall l \in L_E \tag{3-39}$$

$$ys_{jqt} = 1, \quad \forall j \in J_E, q = 1, t = 1 \tag{3-40}$$

$$yl_{lt} = 1, \quad \forall l \in L_E, t = 1 \tag{3-41}$$

$$ro_{r,t-1} = yr_{rt} - yr_{r,t-1}, \quad \forall r \in R, \forall t \in T \tag{3-42}$$

$$us_{jct} \leq \sum_{q \in Q} ys_{jqt}, \quad \forall j \in J, \forall c \in C, \forall t \in T \tag{3-43}$$

$$yt_{krt} \le yr_{rt}, \quad k \in K, r \in R, t \in T \tag{3-44}$$

$$R_{sbrt} \le b_{brt} \times M, \quad \forall s \in S, \forall b \in B, \forall r \in R, \forall t \in T \tag{3-45}$$

$$b_{brt} \le \sum_{k|tb_{kb}=1} yt_{krt}, \quad \forall b \in B, \forall r \in R, \forall t \in T \tag{3-46}$$

$$TS_{sjqt} \ge \sum_{i \in I} x1_{sijt} - \left(1 - ys_{jqt}\right) \times M, \quad \forall s \in S, \forall j \in J, \forall q \in Q, \forall t \in T \tag{3-47}$$

$$TS_{sjqt} \le \sum_{i \in I} x1_{sijt} + \left(1 - ys_{jqt}\right) \times M, \quad \forall s \in S, \forall j \in J, \forall q \in Q, \forall t \in T \tag{3-48}$$

$$x1_{ijt}, x2_{jbrt}, x3_{jblt}, R_{brt}, Inv\_m_{mrt}, Inv\_b_{brt}, N1_{mrt}, N2_{rlt}, z1_{jbt} \ge 0 \tag{3-49}$$

$$ys_{jqt}, yr_{rt}, ro_{rt}, yl_{lt}, yt_{krt}, us_{jct}, b_{brt} \in \{0,1\} \tag{3-50}$$

3.4. Example

In this section, we use a small example to explain the two-stage and robust optimization methods that are illustrated in previous sections. This example helps the reader to better understand the concepts of the two methods for handling uncertainties. The extension of this example to the general model of this thesis will be given afterwards.

The example that we use in this section is in fact a simplified version of the complete model that we aim to solve in this thesis. The mathematical model of the example is given as follows.

$$Min \quad \sum_{i \in I} \sum_{j \in J} d_{ij} \times x_{ij} \times UCT_{ij} + \sum_{j \in J} \mathrm{F_j} \times y_j \tag{3-51}$$

$$\sum_{j \in J} x_{ij} = a_i, \quad \forall i \in I \tag{3-52}$$

$$\sum_{i \in I} x_{ij} \le K_j \times y_j, \quad \forall j \in J \tag{3-53}$$

$$x_{ij} \ge 0, \; y_j \in \{0,1\} \tag{3-54}$$

where, i is the index for collection nodes and j is the index for sorting facilities. In this model, we define parameter a_i as the amount of waste-batteries that are collected at collection node i. We define variable x_{ij} as the amount of waste-batteries that are transported from collection node i to sorting facility j. We have a binary variable y_j that takes a value of 1 if the sorting facility i is built and otherwise, it becomes 0.

Our example is shown in Figure 2. We consider three collection nodes and three sorting facilities (we assume i= 1, 2, 3 and j=1, 2, 3). The rectangles represent collection nodes of our example and the circles represent the sorting facilities. As we show in Figure 2, we have transportation from each collection node to every sorting facility.

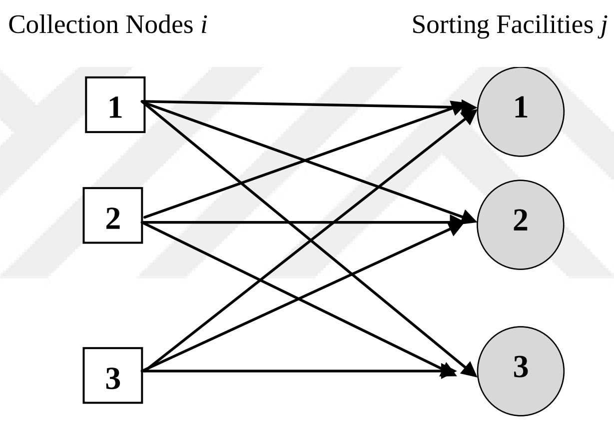

Figure 2: schema of our sample network

The distances between the collection nodes $i \in I$ and sorting facilities $j \in J$ are shown by d_{ij} (see Table 1). The parameter UCT_{ij} represents the cost of transportation for a unit of waste-battery that is transported from the collection node $i \in I$ to the sorting facility $j \in J$ (see Table 2). We define three different scenarios for the amount of waste-batteries that are collected from end-users at the collection nodes (see Table 3). The capacity of sorting facilities, K_j, and the fix-cost of constructing sorting facilities, F_j, are shown in Table 4.

Table 1: Distances between the collection nodes and sorting facilities

C/S	1	2	3
1	30	10	10
2	20	30	10
3	10	50	20

Table 2: unit cost of transportation of waste battery from *i* to *j*

C/S	1	2	3
1	0.1	0.5	0.2
2	0.2	0.1	0.1
3	0.4	0.1	0.1

Table 3: The amount of collection of waste-batteries for each scenario

Collection Node	Scenario		
	1	2	3
1	30	40	50
2	20	30	40
3	10	20	30

Table 4: The value of fix and construction cost for sorting facilities

Sorting facilities *i*	F_j \$	K_j per unit
1	60	50
2	70	60
3	120	70

In the remainder, we will apply the two methods—two-stage and robust—to solve this example. We will demonstrate the mechanism of the two approaches, compare the results and discuss their advantages and disadvantages.

3.4.1. Two-stage approach for solving the example

We use the application of the two-stage method for solving the example that we presented in the previous section. We begin with considering all possible situations for the three potential sorting facilities. There are eight different situations for (y_1, y_2, y_3) that represent which ones of the sorting facilities are open. We find the minimum transportation cost for each of these eight situations by using the value of each scenario. Then, we add fixed costs of construction. The value of F_j depends on which sorting facilities are open. Finally, we find the expected value of each situation by using equal probabilities for scenarios.

Table 5, 6 and 7 show all of the operations that we perform in the two-stage approach. The results of solving our example using the two-stage are also shown. Table 5 shows how we calculate transportation costs of sending material from the collection node $i \in I$ to the sorting facility $j \in J$. In Table 6, we describe the eight situations for the sorting facilities. We also explain how these situations are effective for the transportation of waste-batteries from the collection node $i \in I$ to the sorting facility $j \in J$. We show the mathematics for finding the optimal solution in Table 7.

Table 5: calculating the transportation costs of sending material from $i \in I$ to $j \in J$.

$d_{11} \times x_{11} \times UCT_{11} =$ $30 \times x_{11} \times 0.1 = 3x_{11}$	$d_{21} \times x_{21} \times UCT_{21} =$ $20 \times x_{21} \times 0.2 = 4x_{21}$	$d_{31} \times x_{31} \times UCT_{31} =$ $10 \times x_{31} \times 0.4 = 4x_{31}$
$d_{12} \times x_{12} \times UCT_{12} =$ $10 \times x_{12} \times 0.5 = 5x_{12}$	$d_{22} \times x_{22} \times UCT_{22} =$ $30 \times x_{22} \times 0.1 = 3x_{22}$	$d_{32} \times x_{32} \times UCT_{32} =$ $50 \times x_{32} \times 0.1 = 5x_{32}$
$d_{13} \times x_{13} \times UCT_{13} =$ $10 \times x_{13} \times 0.2 = 2x_{13}$	$d_{23} \times x_{23} \times UCT_{23} =$ $10 \times x_{23} \times 0.1 = 1x_{23}$	$d_{33} \times x_{33} \times UCT_{33} =$ $20 \times x_{33} \times 0.1 = 2x_{33}$

Table 6: the variety situation for sorting facilities in sample problem

(y_1, y_2, y_3)	Description	$(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33})$
(0,0,0)	We don't open any of the sorting facilities	(0,0,0,0,0,0,0,0,0)
(1,0,0)	The first sorting facility is open but its capacity is not satisfying the model for each the scenarios.	(1,0,0,1,0,0,1,0,0)
(0,1,0)	The second sorting facility is open but its capacity is satisfying the model only for the first scenario.	(0,1,0,0,1,0,0,1,0)
(0,0,1)	The third sorting facility is open but its capacity is satisfying the model only for the first scenario.	(0,0,1,0,0,1,0,0,1)
(1,1,0)	The first and second sorting facilities are open but the summation of their capacities are not satisfying S_3.	(1,1,0,1,1,0,1,1,0)
(1,0,1)	The first and third sorting facilities are open.	(1,0,1,1,0,1,1,0,1)
(0,1,1)	The second and third sorting facilities are open.	(0,1,1,0,1,1,0,1,1)
(1,1,1)	All of the sorting facilities are open.	(1,1,1,1,1,1,1,1,1)

Table 7: The calculation of the objective function for each design option

Min $3x_{11}+5x_{12}+2x_{13}+4x_{21}+3x_{22}+1x_{23}+4x_{31}+5x_{32}+2x_{33}+(F_j \times y_j + E[Q(x,s)])=$?					
Design Options	(y_1, y_2, y_3)	S_1 (30,20,10)	S_2 (40,30,20)	S_3 (50,40,30)	$F_j \times y_j + E[Q(x,s)]$
1	(0,0,0)	∞[1]	∞	∞	∞
2	(1,0,0) F_1=60$	∞	∞	∞	∞
3	(0,1,0) F_2=70$	$30\times5x_{12}+20\times3x_{22}+10\times5x_{32}=$ $30\times5+20\times3+10\times5=260$ 260+70=330$	∞	∞	∞
4	(0,0,1) F_3=120$	$30\times2+20\times1+10\times2=100$ 100+120=220$	∞	∞	∞
5	(1,1,0) F_1+F_2=130$	$30\times3+20\times3+10\times4=190$ 190+130=320$	430$	∞	∞
6	(1,0,1) F_1+F_3=180$	$30\times2+20\times1+10\times2=100$ 100+180=280$	350$	430$	(280 + 350 + 430)/3 = 353$
7	(0,1,1) F_2+F_3=190$	$30\times2+20\times1+10\times2=100$ 100+190=290	380$	500$	(290 + 380 + 500)/3= 390$
8	(1,1,1) $F_1+F_2+F_3$=250$	$30\times2+20\times1+10\times2=100$ 100+250=350	420$	500$	(350 + 420 + 500)/3 =423.33 $

[1] This situation violates the capacity constrains in our model; hence, the problem is infeasible.

Consider Table 7. Since we are looking for the minimum cost solution, the optimal decision is the design option 6 where we open sorting facilities 2 and 3. The optimal value is 353$. Note that the above analyses were performed for the case where we assume equal probabilities for different scenarios.

3.4.2. GAMS codes for the two-stage stochastic programming approach for solving the example

We provide our GAMS code for the two-stage stochastic approach for the example that we previously presented. We also provide a detailed explanation of our code. This code is suitable for solving two-stage optimization problems if the number of design options is small. We use this code here because it makes it easy to understand the mechanism that is followed in the two-stage optimization approach. For the cases where the number of design options is large, one could use the code that has been developed by Donmez and Turkay (2013).

We define the set of collection nodes by *i*=1,2,3 meaning that we consider three collection nodes. We also define the set of sorting facilities by *j*=1,2,3 that represent the three sorting facilities in the example problem. We consider three scenarios for the amount of the waste-batteries that are collected in the collection nodes. We denote the set of scenarios by *S*. The GAMS codes for defining sets are as follows.

```
** Defining sets **
set i     /1*3/;
set j     /1*3/;
set s     /1*3/;
```

We define variables *Q* as the optimal value. We define a set of positive variables *x(i,j)* that shows the amount of the transported waste-batteries from the collection nod *i* to the sorting facility *j*. We also define a binary variable *y(j)* that is 1 if the sorting facility *j* is open, and 0 otherwise. The codes for defining variables are as follows.

```
** Defining variables **
variable Q;
positive variables x(i,j);
binary variables y(j);
```

We are given the fixed construction cost for the sorting facilities which we denote in our codes by ``fcost(j)''. We denote the capacities of the sorting facilities by ``capacity(j)'', and the transportation cost of one unit waste-battery from the collection node *i* to the sorting facility *j* by ``tcost(i,j)''. These parameters are defined in our GAMS codes as shown bellow. The parameters ``fcost(j)'' and ``capacity(j)'' are one dimensional, but ``tcost(i,j)'' is a two dimensional parameter; hence, we use a table to define ``tcost(i,j)''.

```
** Defining one-dim parameters **
parameter fcost(j)/
1 60
2 70
3 120/;
parameter cappacity(j)/
1 50
2 60
3 70/;
** Defining two-dim parameters **
table tcost(i,j)
     1   2   3
1    3   5   2
2    4   3   1
3    4   5   2;
```

In the remainder, we explain the way we formulate the equations of our instance in GAM. We have an objective function and two sets of constraints. The objective function that we label as ``EQ1'' is the summation of the fixed cost and transportation cost which we aim to minimize and it is equal to the objective function value *Q*. Equation ``EQ2'' ensures that the total amount of the waste-batteries at sorting facilities is equal to the total amount of the collected batteries—i.e., all of the collected batteries will be sorted.

```
** Defining equations **
Equation
EQ1
EQ2
EQ3;
EQ1.. Q =e=
       sum(j,fcost(j)*y(j)) + sum((i,j),x(i,j)*tcost(i,j));
EQ2(i).. sum(j,x(i,j))=e=supply(i);
EQ3(j).. sum(i,x(i,j))=l=cappacity(j)*y(j);
model mymodel /all/;
```

Recall that there are 8 possible design options. We define the set ``ysSet'' to account for these design options. The table ``QsSet(ysSet,s)'' contains the objective values of each design option for each scenario.

```
** Defining design options and the corresponding objective
        values **
set ysSet/1*8/;
parameter QsSet(ysSet,s) ;
```

For the convenience, we define the table ``YFX(j,ysSet)'' with 0-1 entries that show whether or not a particular sorting facility is open in the corresponding design option. For example, in the first design option (ysSet=1) all of the three facilities are closed.

```
** Defining YFX (j,ysSet); 1 if facility j is open in the
        design option ysSet **
table YFX (j,ysSet)
      1  2  3  4  5  6  7  8
1     0  1  0  0  1  1  0  1
2     0  0  1  0  1  0  1  1
3     0  0  0  1  0  1  1  1;
```

We define table ``SUP(i,s)'' to show the amount of waste-batteries that are collected at node *i* in scenario *s*. We also have parameter ``supply(i)'' which takes its value from ``SUP(i,s)'' at each iteration based on scenario *s*. The way that we use these parameters will be seen bellow and discussed in more detail.

```
** Defining supplies for different scenarios **
table SUP(i,s)
   1    2    3
1 30   40   50
2 20   30   40
3 10   20   30     ;
parameter supply(i);
```

We solve the problem in a loop for each scenario. At each iteration, we initialize the parameter ``supply(i)'' and then evaluate the objective value for all of the eight possible design options. The results are saved in table ``QsSet(ysSet,s)''.

```
loop(s,
supply(i)= SUP(i,s);
LOOP(ysSet,
         y.fx(j) =YFX(j,ysSet);
```

```
Q.l = 10000000000;
solve twoStageModel using MIP minimizing Q;
QsSet(ysSet,s)=Q.l; ); );
```

Now, we need to calculate the expected value of the table ``QsSet(ysSet,s)'' with respect to scenarios. We define a parameter for the probabilities of scenarios labeled as ``pvalues(s)''. It is assumed that the probabilities of scenarios are 0.3, 0.4, and 0.3, respectively. We define parameter ``expectedvalues(ysSet)'' to save the expected values for each design option. In order to calculate these expected values, we multiply values with their probabilities and add them up. Finally, we need to select the minimum of the ``expectedvalues(ysSet)''. This will identify the best design option.

```
parameter pvalues(s) /
1   .3
2   .4
3   .3/;
parameter expectedvalues(ysSet);
expectedvalues(ysSet)=sum(s,pvalues(s)*QsSet(ysSet,s)) ;
scalar best;
best = smin(ysSet,expectedvalues(ysSet));
parameter Qoptimal(s);
Qoptimal(s)= smin(ysSet,QsSet(ysSet,s));
display  QsSet,expectedvalues,best,Qoptimal;
```

Finally, we ask the GAMS to display the optimal solution for each scenario according to the best design option. The parameter ``Qoptimal'' will be also used in solving this example by the robust optimization approach in the next section.

3.4.3. Robust optimization for solving the example

We use the robust optimization approach to solve the small example that we previously presented. Recall that the main objective of the robust optimization is to find a solution that minimizes the maximum difference between the optimal solution for each scenario and the robust solution for each scenario. In the first step of the robust optimization approach, we solve all specific scenarios one by one and find the optimal solution for each scenario (we denote them by Q_S^*). For instance, for the example problem that we previously presented, one could use Table 7 to determine the values of Q_S^*. These values are given in Table 8.

Table 8: The optimal solution for each scenario of sample problem

Q_1^*	Q_2^*	Q_3^*
220	350	430

For each design option and for each scenario, we find the differences between the robust objective value and the objective value when considering the optimal solution for that scenario. The result of this calculation is given in Table 9. Since some of the results for the first five design options in table 7 are infeasible, they are omitted.

Table 9: the robust solution cost minus the optimal cost for each scenario

Design options	$R_1 - Q_1^*$	$R_2 - Q_2^*$	$R_3 - Q_3^*$
6	280−220=60	350−350= 0	430−430= 0
7	290−220=70	380−350=30	500−430=80
8	350−220=130	420−350=70	500−430= 80

We are looking for the minimum of the maximum deviation of the optimal solutions (Q_s^*) and robust solutions (R_s). As it is shown in Table 9, the best design option is 6 with differences 60, 0, and 0, respectively. Other design options have larger maximum differences. For example, the differences for option 7 are 70, 30, and 80; hence, the maximum difference for the design option 7 is 80.

Though they employ difference mechanism, the two-stage method and the robust optimization approach both find the same design option for this small example. However, we note that this may not be true for all problems—especially large real world instances. We will compare the results of these two approaches and show the advantages and disadvantages of one over the other in the next chapter.

3.4.4. GAMS codes for the robust optimization approach for solving the example

We present the GAMS codes that we use to solve the small example that we discussed before. Although this code has been written for the small example that we presented above, its extension to the general model is straightforward. Most part of this code is similar to the code that we used for the two-stage approach. We add the variable ``R(s)'' as the cost of robust solution for each scenario. We also define the variable ``delta'' that we aim to minimize. This variable is the difference between the cost of the robust solution ``R(s)'' and the cost of the optimal solution ``Qoptimal''. Comparing to the code that we used for the two-stage approach, we add a new dimension of scenarios to the positive variables ``x(i,j,s)'', the transportation costs ``tcost(i,j,s)'', and the capacity parameters ``cappacity(j,s)''.

```
set i/1*3/;
set j/1*3/;
set s/1*3/;
variables delta;
variables R(s);
positive variables x(i,j,s);
binary variables y(j);
parameter fcost(j)/
1 60
2 70
3 120/;
table supply(i,s)
   1    2    3
1 30   40   50
2 20   30   40
3 10   20   30     ;
table cappacity(j,s)
   1    2    3
1 50   50   50
2 60   60   60
3 70   70   70     ;
table tcost(i,j,s)
       1   2   3
1.1    3   3   3
1.2    5   5   5
1.3    2   2   2
2.1    4   4   4
2.2    3   3   3
2.3    1   1   1
3.1    4   4   4
3.2    5   5   5
3.3    2   2   2;
```

We define a parameter ``Copt(s)'' with values that we found in our previous code for the parameter ``Qoptimal'' in pervious code. Hence, after solving the two-stage method, we use the values that we find for ``Qoptimal'' as the values of ``Copt(s)''—the same values as in table 8.

```
parameter Copt(s)/
1 220
2 350
3 430/;
```

In the remainder, we present the codes for the equations. Despite the two-stage method, we have four equations in the robust optimization method. The variable ``delta'' is the objective value. According to ``EQ1'', delta must be greater than or equal to the difference between the cost of the robust solution and the cost of the optimal solution for all scenarios. Equation ``EQ2'' represents that the summation of the fix cost of the sorting facilities and the transportation cost of waste-batteries must equal to ``R(s)''. Equations ``EQ3'' and ``EQ4'' are similar to what we had in the two-stage method—except for the addition of the new dimension *s*.

```
Equation
EQ1
EQ2
EQ3
EQ4;
EQ1(s).. delta =g= R(s)-Copt(s);
   EQ2(s).. R(s)=e=
sum(j,fcost(j)*y(j))+sum((i,j),x(i,j,s)*tcost(i,j,s));
EQ3(i,s)..sum(j,x(i,j,s))=e=supply(i,s);
EQ4(j,s).. sum(i,x(i,j,s))=l=cappacity(j,s)*y(j);
```

We define the model as ``robustModel'' which includes all of the equations that we defined above and ask the solver to solve it while minimizing ``delta''. Finally, we display the optimal value, ``delta”, and the robust solutions, ``R''.

```
model robustModel /all/;
solve robustModel using MIP minimizing delta;
display delta.l,R.l;
```

In the next chapter, we apply our codes to a real world problem which is the design of reverse logistics network for waste batteries with an Application in Turkey. We will present a comparative analysis of the results that are obtained by using the two approaches that we discussed in this chapter.

Chapter 4
Numerical Results

4.1. Overview

In this chapter, we solve the reverse logistics network design problem for recycling waste-batteries in the case of Turkey. We first explain the problem in detail and then use the two-stage stochastic approach and the robust optimization method to solve the problem. We provide a detailed discussion of the results of each method and a comparison of the solutions that are found when applying these methods.

Our problem is formulated as a mixed-integer linear programming problem (see Chapter 2). We use a GAMS code which is similar to, and in fact the extension of what we discussed in Chapter 3. This GAMS code is available as a supplementary file to this thesis. We solve the problem using GAMS 24.1.3 and CPLEX 12.5.1.0 on a personal computer with the 32-bit operating system, 2.50 GHz CPU, and 4.00 GB of RAM.

4.2. Waste-battery recycling in Turkey

We describe the problem of recycling waste-batteries in the case of Turkey. Donmez and Turkay (2013) used the same benchmark problem to test their mathematical model. The data for this case is given to us by TAP (Portable Battery Manufacturers and Importers Association) that is the only institution in Turkey that is authorized by the Ministry of Environment and Urban Planning for the activities according to recycling of waste-battery. In this case, we assume that the centers of the 81 provinces in Turkey are the collection nodes of our network (see Figure 3). The planning horizon is assumed to be 10 years.

Figure 3: The 81 provinces of Turkey as the waste-battery collection nodes of our reverse logistics network

There are varying amount of waste-batteries with different combination of materials. We consider the battery types Alkaline Manganese, Zinc-carbon, Portable NiCd, Portable NiMH, and Portable Li-based which are the most used batteries in Turkey. Each battery may include specific combination of materials. For example, battery type ``portable NiCD" contains 0.06 percent Zinc, 0.083 percent manganese, and 29-40 percent Iron. A waste battery may include some of the following materials: Alkaline, Zinc-carbon, NiCd, NiMH, and Li-ion. The weight percentages of these materials for each battery type are given in Table 10.

Table 10: Material composition of different batteries in % by weight

Battery Type	Mercury	Lead	Zinc	Manganese	Iron
Alkaline Manganese	0.0013	0.04-2	35	28	28
Zinc-carbon	0.0005	0.15-2	35	18	21
Portable NiCd			0.06	0.083	29-40
Portable NiMH				0.81-3	20-25
Portable Li-based				10-15	4.7-25

The revenues of our model is obtained from the sale of recycled materials in a single secondary market which is considered to have unlimited capacity—i.e., we assume that the market demand is unlimited. Our model includes two pre-established sorting facilities in

Ankara and Kocaeli. In addition, we consider the following 6 regions from the 81 provinces to be the potential places for constructing sorting facilities: Corum, Diyarbakir, Erzurum, Izmir, Kirklareli, and Mersin. Hence, we can have at most 8 sorting facilities in Turkey. We assume that there are two existing landfill areas in Istanbul and Izmir. Moreover, we consider 8 candidate regions for constructing new landfill areas in Ankara, Corum, Diyarbakir, Erzurum, Bingol, Kirklareli, Kocaeli, and Mersin. Therefore, in total our network can have 10 landfill areas. We assume no existing recycling facilities; however, new recycling facilities can be built in Ankara, Izmir, and/or Kocaeli.

The potential and pre-established sorting facilities, landfill areas, and recycling facilities are shown in Figure 4. We mark the existing sorting facilities by red squares and the potential sorting facilities by blue squares. The existing landfill areas are depicted using green circles and the potential landfill areas are shown by yellow circles. The candidate locations for recycling facilities are marked by purple triangles in Figure 4. In addition to the assumptions that we previously discussed, we consider the following features for the case of Turkey:

1. Different types of the facilities are located in same provinces, because of the transportation cost.
2. We consider a small number of facilities in eastern and south eastern of Anatoli. This assumption is due to insufficient waste battery collection in these regions.
3. There are three levels of capacity for each potential sorting facility: 200,000,400,000 and 500,000 kg.
4. The capacity levels of the two existing sorting facilities—in Ankara and Kocaeli—are 200,000 and 500,000 kg.
5. The cost of constructing a landfill area is proportional to the population density of the corresponding region.
6. The unit transportation cost between the nodes of the network for the first year is assumed to be 0.0037TL/km-kg.

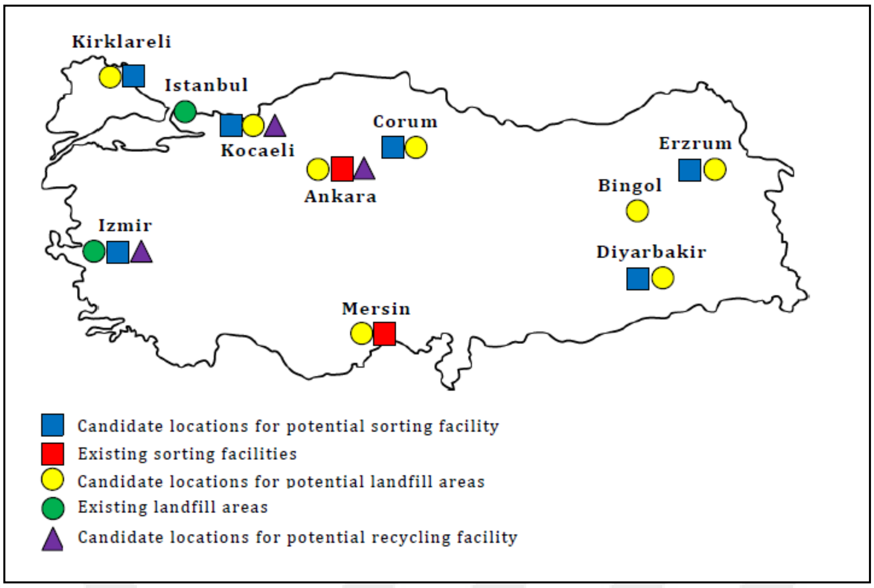

Figure 4: Candidate and existing locations for sorting facilities, recycling facilities, and landfill areas

In the remainder, we consider different scenarios and solve using the two methods discussed in chapter 3. We present a detailed discussion on the results that we obtain by applying each method.

4.3. Scenario 1: Base Case

The first part of our experimental work is to solve our model without considering scenarios and probabilities which we refer to as the base case. In this situation, the problem instance—for the case of Turkey—includes 126,234 non-zero elements, 4,413 single equations, 14,211 single variables, and 876 discrete variables.

According to the base case, the sorting facilities in Ankara, Kocaeli, and Mersin with 200,000 capacities and Izmir with 500,000 capacities are built in year 1 and they are operational throughout the planning horizon. According to the solution of the base case, we must add the option of 80,000 extra capacities to the sorting facility in Ankara in the first year. In addition, we must add 125,000 extra capacities to the sorting facility in Kocaeliin the first three years.

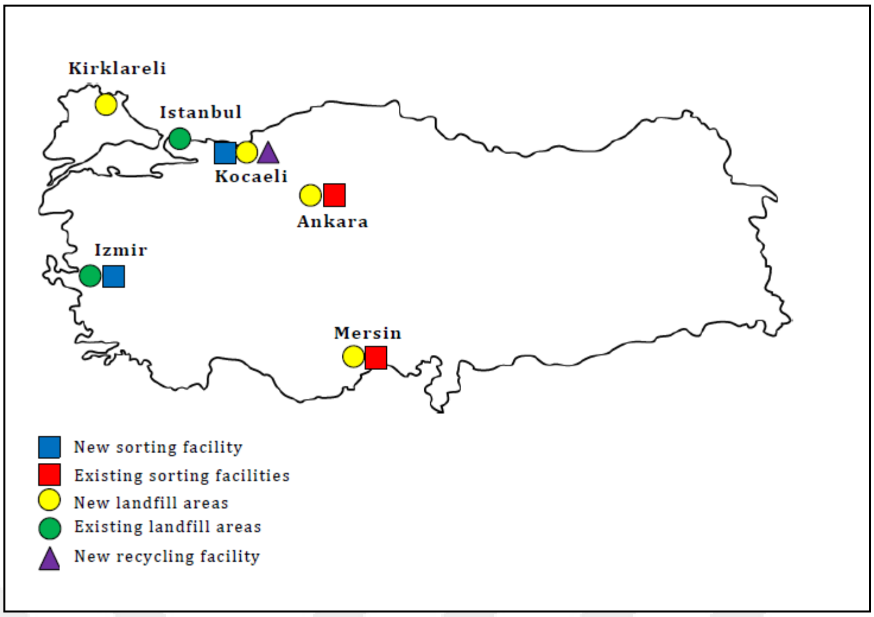

Figure 5: The solution of the base case: new and existing sorting facilities, recycling facilities, and landfill areas

The solution of the base case suggests that there must be only one recycling facility in Kocaeli. This sorting facility can serve the entire country and there is no need to open any other sorting facility in other cities. The landfill areas in Ankara, Istanbul, Izmir, Kocaeli, Kirklareli, and Mersin must be operational for the whole 10 years—the entire planning horizon. These results are also represented graphically in Figure 5.

The total cost for transportation, sorting, recycling, landfill and inventory facilities and the total revenue from the sale of the recycled materials during the 10 years are shown in Table 11. The values are given in dollars. The largest cost is the recycling cost and the smallest cost is the sorting cost. These costs are $21,299,670 and $639,683.2, respectively.

Table 11: Total costs and revenue for the base case

	Base Case
Total Revenue	40,089,730
Transportation cost	10,869,500
Sorting cost	639,683.2
Recycling cost	21,299,670
Landfill cost	1,097,337.7
Inventory cost	80,961.64

The total revenue of the base case—for the optimal solution—is $40,089,730 which is fortunately bigger than the sum of all cost items. One could calculate the profit by subtracting the sum of all costs from the total revenue. This results in a total profit of $345,426.8 for the base case.

4.4. Two-stage stochastic optimization approach

In this section, we use the two-stage optimization approach to solve the reverse logistics network design problem for the case of Turkey (see also Donmez and Turkay, 2013). Recall that in the stochastic version of our problem, we assume that the amount of waste-batteries that are collected in different collection points is uncertain. We consider four scenarios for the amount of the collected batteries to handle the uncertainty in our model. The actual data for the amount of waste-batteries that are collected in 2011 is given. The four scenarios are created for the entire planning horizon—10 years—using the data from year 2011. We describe these scenarios as follows (the same approach is used by I.Donmez and M.Turkay, 2013):

- Scenario 1 is the base case—that we discussed in the previous section of this chapter. In this scenario, the collection amounts are the same as the amounts that are collected in 2011.
- Scenario 2: we assume that the waste-batteries that are collected in a region is proportional to the population of that region.
- Scenario 3: the average amounts of the waste-batteries that are collected per inhabitant for each state are calculated based on the data of 2011. It is assumed that if the amount of the collected batteries per inhabitant is below the average, it rises to the average amount; if the amount of the collected batteries per inhabitant is above the average, it reaches the highest amount of collection per inhabitant.
- Scenario 4: A similar approach to scenario 3 is used, but considering the quartile of the data. In this case, we assume that the inhabitants increase their collection amount up to the next break point.

As we do not have the accurate probability of each scenario happening, we consider a uniform distribution; hence, each scenario occurs with a probability of 0.25. We solve our mixed-integer linear programming model using the above scenarios and considering equal probability values for each scenario. Based on the statistics of the two-stage stochastic model, there are 492,162 non-zero elements, 14,133 single equations, 53,961 single variables, and 876 discrete variables.

According to the solution of the two-stage stochastic model, we must construct sorting facilities in Ankara, Kocaeli, Diyarbakir and Mersin each with a capacity of 200,000, and another sorting facility in Izmir with a capacity of 500,000. The sorting facility that is constructed in Izmir must start its work at the beginning of the fifth year. We must also use an extra capacity of 80,000 in Ankara in the first year and an extra capacity of 125,000 in Kocaeli in the first, second and third years. The solution also suggests that we must build a recycling facility at Kocaeli in the first year. Moreover, all of the landfill areas except for the one in Bingol must be operational in the entire planning horizon. These results are graphically shown in Figure 6.

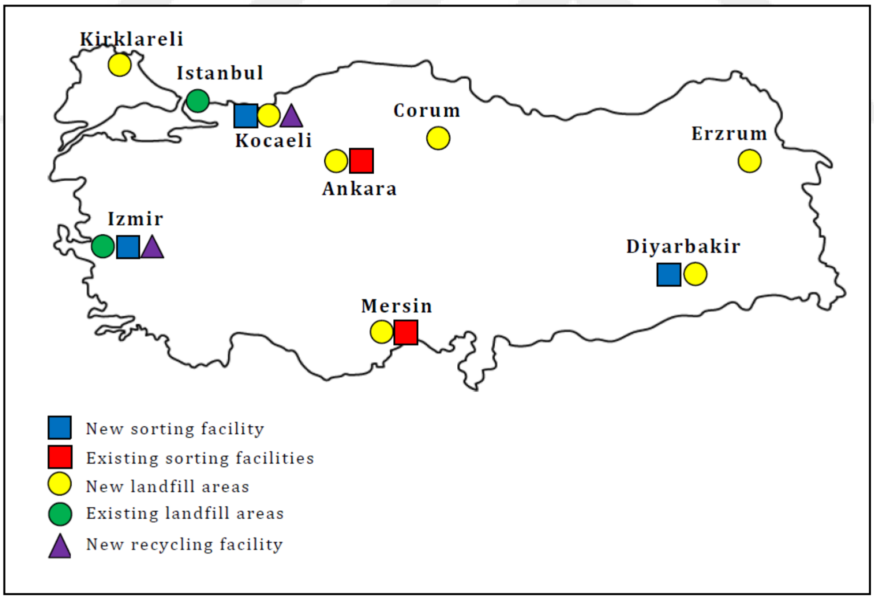

Figure 6: The solution of the two-stage method: the new and existing sorting facilities, recycling facilities, and landfill areas

As we mentioned before the revenue of our reverse logistics network is obtained from the sale of the recycled materials in the secondary market. Table 12 shows all of the revenues we will gain in the 10 years of the planning horizon. It is seen that no sale occurs in the first year; hence, the revenue is 0 for 2013. The system gains revenue from the second year and the revenue increases in the next years. There is a jump in the amount of revenue in year 2018 but the revenue decreases afterwards. Generally—ignoring the jump in 2018—the revenue increases slowly throughout the planning horizon.

Table 12: The annual revenues that are gained according to the solution of the two-stage stochastic approach

Year	Revenue
2013	0
2014	2528900.00
2015	2902700.00
2016	3302800.00
2017	3751900.00
2018	11564000.00
2019	6802600.00
2020	7715700.00
2021	8662200.00
2022	9713400.00

The results in Table 12 are also depicted in Figure 7 to provide a graphical illustration of the trend. The increases in the amount of revenue over the entire planning horizon as well as the jump in year 2018 are shown in Figure 7.

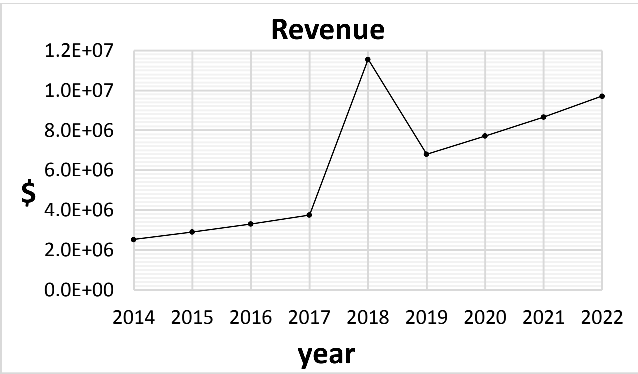

Figure 7: The revenue per year during the 10 years of the planning horizon according to the solution of the two-stage approach

It is also very interesting to see the total revenue for each scenario. This result is shown in Table 13. It is seen that the lowest revenue is for the second scenario and the highest revenue is for the last scenario. Although the second scenario is more probable than the other three scenarios—because in scenario 2 the amount of collected waste batteries is proportional to the population of a region which is very credible—its total revenue is the lowest. This suggests that if the government promotes the collection of batteries such that this scenario happens, the revenue will be the highest.

Table 13: The total revenue for each scenario

Scenario	Revenue
1	39953480
2	38175880
3	41606310
4	43028650

The detailed values of cost components for each year are shown in Table 14. Each value in this table is the expected value of the scenarios. Similarly, Table 15 shows the total cost components for each scenario. These values are found by summing the corresponding cost values over all 10 years. This table will be used for comparing the results of the two-stage stochastic approach and the robust optimization method in the next section.

Table 14: The cost components per year for the entire planning horizon according to the solution of the two-stage approach

Years	Transportation cost	Sorting cost	Recycling cost	Landfill cost	Inventory cost
2013	806300.00	888570.00	5000000.00	680720.00	0
2014	1024000.00	551720.00	1596300.00	53977.29	7452.964
2015	1169800.00	621790.00	1583900.00	61274.20	16530.44
2016	1045300.00	649275.00	1813600.00	70813.94	27356.07
2017	1574700.00	1292090.00	2081900.00	81198.45	40245.24
2018	1388300.00	822080.00	2382700.00	93313.22	7452.964
2019	1766800.00	925340.00	2720000.00	107450.00	0
2020	1898700.00	1030610.00	3108700.00	123270.00	0
2021	2826100.00	1157420.00	3510300.00	144070.00	0
2022	3361400.00	1289020.00	3967600.00	167840.00	0

Table 15: The total cost for each scenario (two-stage stochastic approach)

Years	Transportation cost	Sorting cost	Recycling cost	Landfill cost	Inventory cost
1	10616390.0	6670057.95	1686668.26	1194249.19	80450.33
2	13387170.0	7189608.51	19632870.0	1437227.02	64495.93
3	12413980.0	6983353.79	21596220.0	1306184.57	75358.49
4	12634660.0	6902664.16	22408680.0	1251957.81	80010.36

Figure 9 shows the total cost and cost components for the 10 years of the planning horizon according to the solution of the two-stage approach. Based on figure 9, it is clearly seen that the recycling process is the most expensive operation and the landfill is the least one in the network for each period. There is a big difference between the total cost in the first year and the other years. This is because of the construction cost that must be paid for building the sorting and recycling facilities in the first year. There is a slow increase in the total cost during the 10 years of the planning horizon. The reason for this increasing trend is the increase in the amount of the collected waste-batteries. Therefore, since the collection amount increases, then the total cost and cost components also increase accordingly. Additionally, the solution suggests constructing a new sorting facility in Izmir in 2017 which is the main reason of the jump in this year.

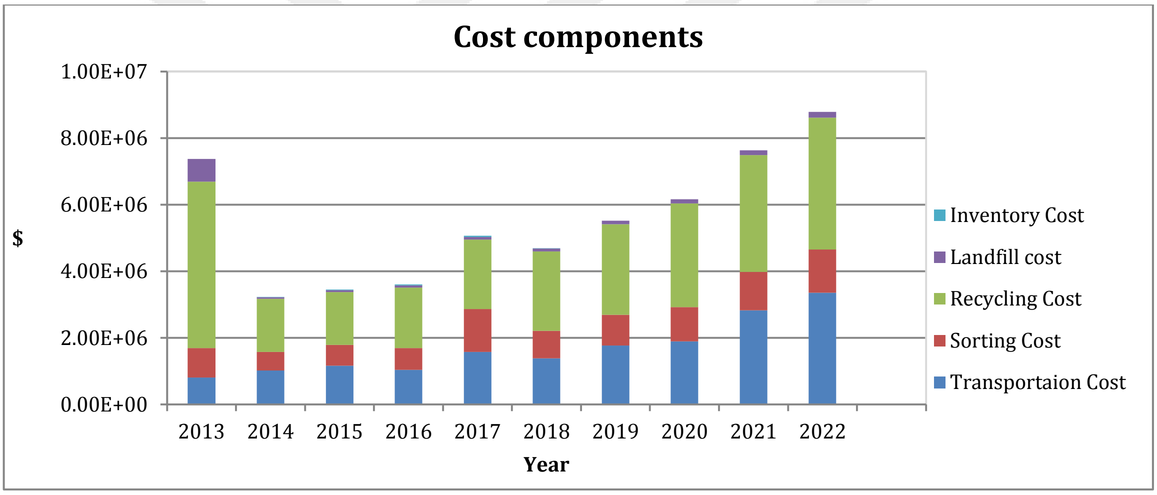

Figure 8: The total cost and cost components for the 10 years of the planning horizon according to the solution of the two-stage approach

4.5. Robust Optimization Approach

In this section, we present the results of solving the reverse logistics network design problem for recycling waste-batteries in the case of Turkey using the robust optimization approach. Based on the model statistics of the robust optimization approach, there are 495,728 non-zero elements, 14,380 single equations, 54,205 single variables, and 876 discrete variables. In the robust optimization method, we do not use probability values for scenarios. Solving the problem in GAMS suggests that sorting facilities in Ankara, Erzurum, Kocaeli and Mersin should be built in the first year each with a capacity of 200,000. These facilities will stay operation throughout the planning horizon. The solution also suggests that a sorting facility in Diyarbakir must be constructed in 2017 with a capacity of 200,000. In addition, a sorting facility must be opened in Izmir in the second year with a capacity of 500,000. We must also add an extra capacity of 80,000 to the sorting facility in Ankara in the first year, and an additional capacity of 125,000 to the sorting facility in Kocaeli in years 2013, 2014 and 2016. There is only one recycling facility that is constructed in Kocaeli in the first year and is operational throughout the planning horizon.

According to the solution, we use the existing landfill areas in Istanbul and Izmir. Additionally, we must build new landfill areas in Ankara, Erzurum, Kirklareli, and Mersin in the first year. The landfill areas in Kocaeli and Diyarbakir are constructed in 2014 and 2016, respectively, and are operation until the end of year 10. The result of solving the problem using the robust optimization approach is graphically shown in Figure 9.

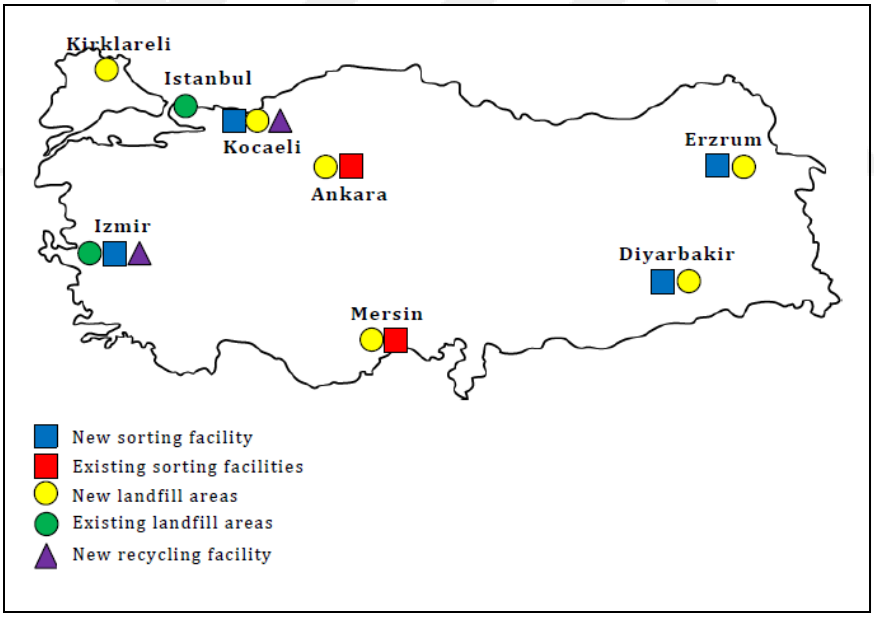

Figure 9: The solution of the robust optimization method: the new and existing sorting facilities, recycling facilities, and landfill areas

Table 16 shows the amount of revenue for each year and for each scenario that is found by using the robust optimization method. The revenue in the first year is zero for all of the scenarios. The system gains revenue in the second year and onwards and the revenue increases over time. The highest revenue is gained in year 2022 for the scenario 4 which is $10,408,980. There is a jump in the revenues of the year 2018 for scenarios 2, 3, and 4;

however, for scenario 1, a jump in revenue occurs in 2016—but the magnitude of this jump is less than the increases in year 2018 for other scenarios.

Table 16: Revenue per year for each scenario during the 10 years according to the solution of the robust optimization approach

Years	Scenario 1	Scenario 2	Scenario 3	Scenario 4
2013	0	0	0	0
2014	2606471	2351588	2506115	2599125
2015	2969484	2671689	2878772	2973605
2016	6893214	3063860	3225540	3397054
2017	5542257	3469733	3750031	3873749
2018	5946678	9812485	11394180	12223310
2019	6617357	5803431	6802246	7144601
2020	7277656	6918646	7775507	8169511
2021	7216050	7183206	9540017	9235328
2022	8780742	8873083	9572964	10408980

In Table 17, we show the total revenue for each scenario based on the solution of the robust optimization approach. Each value is in fact the summation of the corresponding values in Table 16 over the 10 years in the planning horizon. For example, the total revenue for scenario 1 is $39,051,150 which can be found by summing the values that are given in the second column of Table 16. The lowest revenue is for scenario 2 and the highest revenue is for scenario 4.

Table 17: The total revenue for each scenario according to the solution of the robust optimization approach

Scenario	RM_s
1	39051150.00
2	35887230.00
3	40977880.00
4	42827740.00

Figure 11 graphically illustrates the revenues that are found for each scenario and for each year. We can compare the revenues of different scenarios using this figure. As it is seen, the revenue is increasing in all scenarios over the time horizon. But there is an extreme growth in

2016 for the first scenario because of the increase in the amount of recycled battery—that will be sold—in that year. The revenue for the first scenario shows a different pattern compared with the other scenarios. The reason for this is that we have only a sale of recycled battery type Cupper for the first scenario.

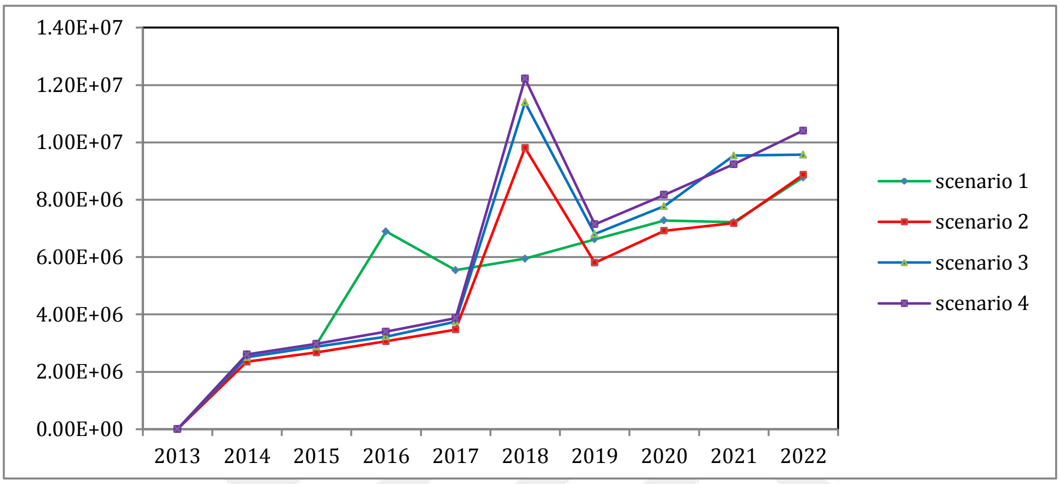

Figure 10: Revenue per year for each scenario during the 10 years according to the solution of the robust optimization approach

There is a large growth in revenues for scenarios 2, 3 and 4 comparing to the first scenario in 2018 (see Figure 11). This is because of the amount of recycled waste-batteries of the type of Cadmium, Nickle, Cobalt, and steel which are approximately the same for scenarios 2, 3 and 4. But for scenario 1, the amount of these recycled waste-batteries is very smaller than for the other scenarios. Moreover, there are different amount of waste-battery types Zinc, Manganese and Iron between the first scenario and the other scenarios but because of their low price, they are not so effective in increasing or decreasing the revenue (see Table 18).

Table 18: The amount of recycled material that is sold from recycling facilities according to the solution of the robust optimization approach

Battery-type	scenario 1	scenario 2	scenario 3	scenario 4
Zinc	121539.8	91954.92	107309.2	123597.3
Manganese	121539.8	91954.92	107309.2	123597.3
Iron	121539.8	91954.92	107309.2	123597.3
Cupper	70268.47	0	0	0
Cadmium	5711.158	5771.194	5771.191	5771.194

Nickel	24748.35	25008.51	25008.5	25008.51
Cobalt	6472.645	6540.687	6540.684	6540.687
Steel	28555.79	28855.97	28855.96	28855.97

Table 19 shows the transportation, sorting, recycling, inventory and landfill costs for each scenario. In the robust optimization approach, we observe that the highest and the least costs belong to recycling and inventory holding, respectively. The cost of transportation in scenario 4 is greater than the cost of transportation in scenario3 and both of them are greater than the cost of transportation in the second scenario. The cost of transportation for the first scenario is the lowest. The same ordering happens for the total inventory holding cost, but not for the other cost components (see Table 19).

Table 19: The cost components for each scenario according to the solution of the robust optimization method

scenario	*TCs*	*SCs*	*RCs*	*LCs*	*ICs*
1	10119520.00	6923760.04	20749600.00	1303289.62	23904.35
2	11886100.00	7467618.70	18320710.00	1602329.62	59716.61
3	12156830.00	7242251.87	21236170.00	1407740.12	72722.11
4	12344520.00	7118323.79	22293290.00	1337183.00	80630.08

Figure 12 graphically illustrates the values that are given in Table 19. We can see the differences between the total costs of scenarios according to the solution of the robust optimization method. The largest cost is for the recycling, then transportation, then sorting, then landfill, and then inventory holding which has the smallest amount of total cost for all scenarios—the inventory holding cost is negligible comparing to the other cost components.

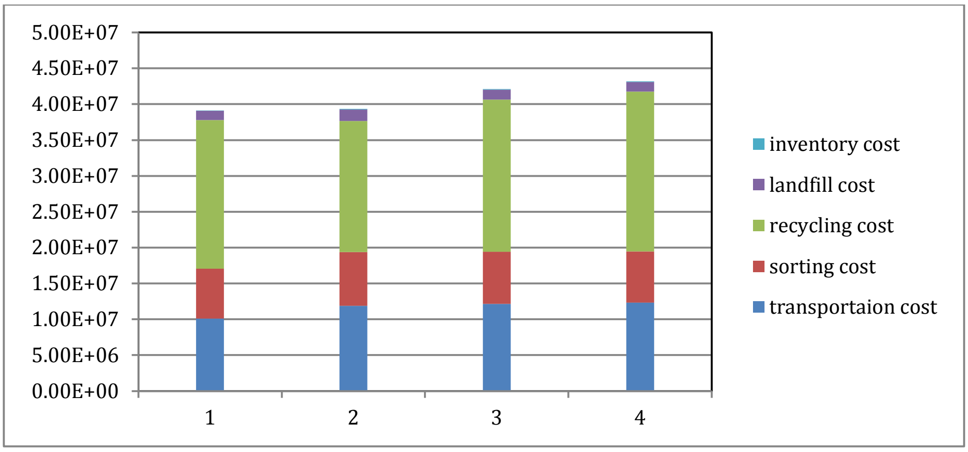

Figure 11: The rate of the variety costs for each scenario (robust optimization method)

In the remainder, we compare the results that we have obtained for the two-stage method and the robust optimization approach. We also perform a sensitivity analysis to show the effects of varying amount of collected batteries on the location and capacities if the sorting facilities and landfill areas. The inventory holding and recycling facilities are not very sensitive to the small changes in the amount of collected waste-batteries.

4.6. Comparison of the models

We analyze and compare the results of the two-stage method and the robust optimization approach that we have obtained from solving the reverse logistics network design problem for recycling waste-batteries in the case of Turkey.

The comparison between scenarios is summarized in Table 20. The first column presents the optimal values of the profit for each scenario when they are solved separately. The second and third columns provide the total profit of the two-stage stochastic approach and the robust optimization method, respectively. We observe that the average value of the total revenues with stochastic optimization approach is 31% lower than the average value of the optimal profit while the average value of the total profit with the robust optimization approach is approximately 50% lower.

Table 20: The comparison of final results of the two-stage and robust optimization approaches for each scenario

Scenario	Optimal Profit	Total Profit two-stage	Total Profit Robust
1	345426.88	170533.10	-68921.11
2	-3034900.00	-3535480.00	-3449250.00
3	-723481.42	-768786.74	-1137830.00
4	68134.57	-249324.49	-346213.43
Expected value	-836204.9925	-1095764.533	-1250553.635
Max. loss		-500580	-414350

Considering the worst case scenarios, we see that the maximum difference between the revenues and their optimal values can be as high as 500580 when the two-stage stochastic approach is used; however, the maximum difference for the robust optimization approach is 414350—approximately 17% less than for the two-stage method. Hence, the result of the robust optimization method is better as it involves a lower efficiency loss in the worst case, while the two-stage stochastic approach performs better on average.

4.7. Comparison of scenarios

We also analyze the importance of the amount of collected waste-batteries and its impact on the number of facilities that must be built in the network. Our results suggests that the recycling and inventory holding facilities remain the same in all situations and both of them will be constructed in Kocaeli throughout the planning horizon; hence, the decision on the recycling and inventory holding facilities is not very sensitive to the amount of the collected waste-batteries. However, in the following we show that the sorting facilities and landfill areas are quite sensitive to varying amount of collected waste-batteries batteries.

We previously analyzed the results of the two-stage stochastic approach and robust optimization method and compared the locations of facilities in their results. In this section, we focus on each scenario separately—for each scenario, one could think of a case where that scenario happens with a probability of 1. We show the impact of changing the probability values on the situation of facilities in our model. In the remainder, we first analyze the sorting facilities with different capacity options and the possibility of adding extra capacities. Then we will focus on the landfill areas.

4.7.1. The sorting facilities

Table 21 presents the amount of collected waste-batteries for each scenario, the number of sorting facilities for each scenario and for the two methods that we use in this thesis, the capacities of the sorting facilities and the extra capacities that must be added.

Table 21: The data of sorting facilities and their capacity qualities

Scenario	The amount of waste-battery collected	Optimum Sorting facilities	Capacity options			Extra capacity options		
			200,000	300,000	500,000	50,000	80,000	125,000
1	8531663.900	4	3	0	1	1	0	3
2	9213919.700	6	5	0	1	0	0	3
3	9213920.300	5	4	0	1	0	0	3
4	9213920.800	4	2	0	2	0	0	3
Two.stage	--	5	4	0	1	0	1	3
Robust	--	6	5	0	1	0	1	3

According to the results of our model, for scenario 1, we have four sorting facilities where three of them have 200,000 capacities (Ankara, Kocaeli, and Mersin) and one of them has a capacity of 500,000 (Izmir). We also have an extra capacity of 50,000 in the first year for Ankara and an extra capacity of 125000 in the first three years for Kocaeli (see Figure 12).

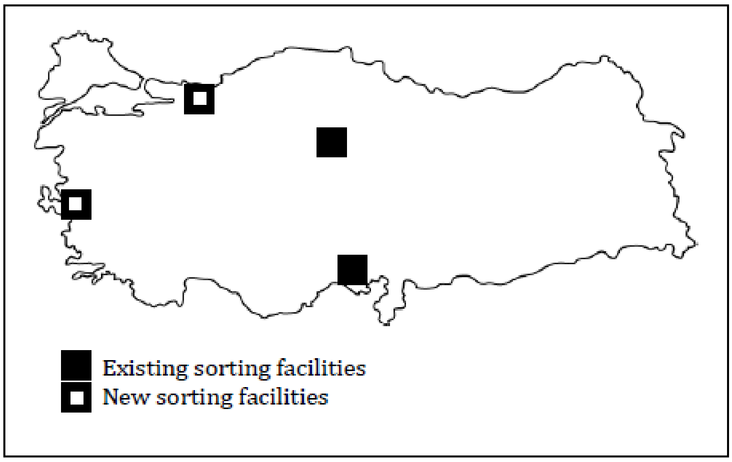

Figure 12: The new and existing sorting facilities for scenario 1

The amount of collected waste-battery is increased in the second scenario compared with the first scenario; therefore, we expect to see changes in the number and locations of the sorting facilities. In scenario 2, we must construct sorting facilities in Ankara, Erzurum, Corum, Mersin and Kocaeli with 200,000 capacities. We must also build a large sorting facility in Diyarbakir with a capacity of 500,000. The sorting facility in Kocaeli must increase its capacity by 125,000 in the first three years (see Figure 13).

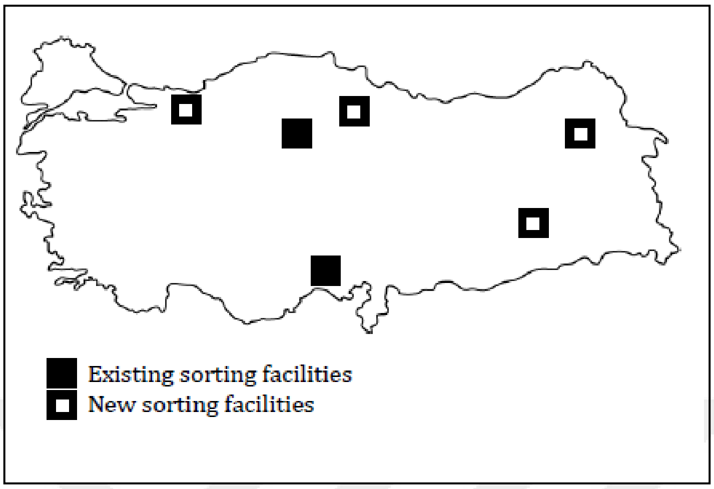

Figure 13: The new and existing sorting facilities for scenario 2

In scenario 3, although we see an increase in the amount of collection, there is a decrease in the number of sorting facilities comparing with the results of the second scenario. The distribution of the collection is changed; the collection amount decreases in the East of Turkey and increases in the West of Turkey. According to the results that we have obtained for scenario 3, we have a sorting facility in Izmir with a capacity of 500,000—instead of Diyarbakir. We must build sorting facilities in Ankara, Erzurum, Mersin and Kocaeli with 200,000 capacities. The sorting facility in Kocaeli has 125,000 extra capacities for the first three years (Figure 15).

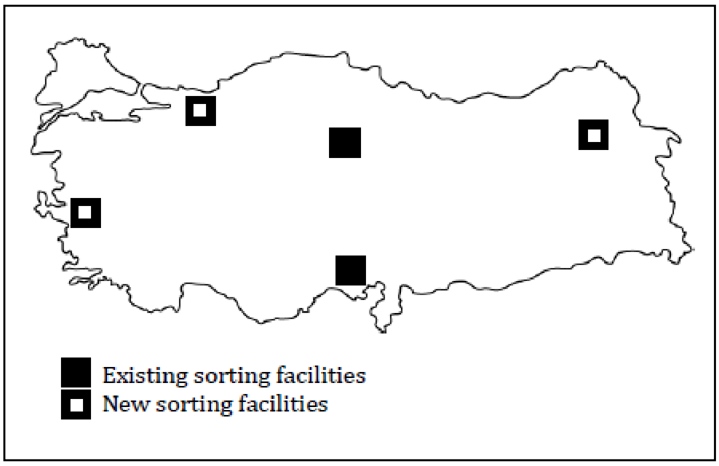

Figure 14: The new and existing sorting facilities for scenario 3

In scenario 4, we observe that the amount of the waste-battery collection is more than scenario 3; however, this increase does not require opening extra facilities. In this scenario, Corum and Izmir facilities are constructed with 500,000 capacities and the other two facilities—in Ankara and Kocaeli—have 200,000 capacities. Kocaeli has 125000 extra capacities for the first three years (see Figure 16).

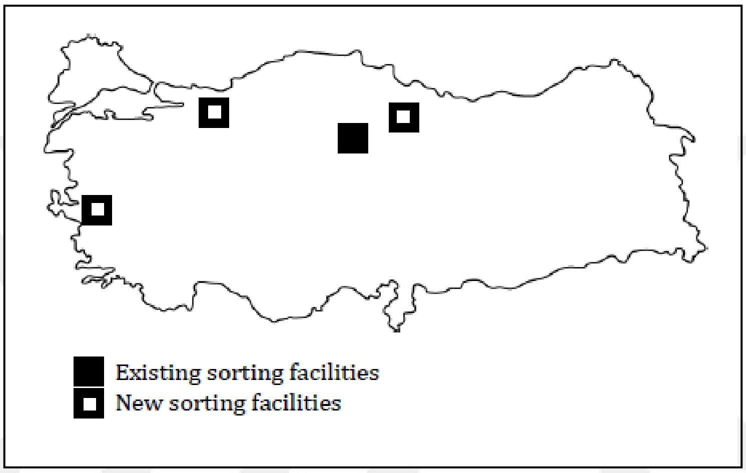

Figure 15: The new and existing sorting facilities for scenario 4

Thus, we analyzed the sensitivity of our results to different scenarios. In the remainder, we provide a detailed analysis of the impact of the amount of collected batteries (or different scenarios) on the locations of the landfill areas.

4.7.2. The landfill areas

In this section, we discuss the effects of changing the amount of collected waste-batteries on the land fill areas. We solve the problem of reverse logistics network design for recycling waste-batteries in the case of Turkey for each scenario separately with a probability of 1. The results for the locations of the landfill areas are shown in Table 22 and depicted in Figure 16. It is seen that except for the existing landfill areas in Istanbul and Izmir, each scenario has different locations for the landfill areas. Ankara and Kocaeli are the common landfill locations for all scenarios. Diyarbakir and Erzurum are active only for scenario 2. Mersin is open in all scenarios except for scenario 4 and Kirklareli is open in all scenarios except for scenario 2. The distinction between scenario 3 and the other scenarios is that the landfill area in Kilrklareli starts its work at the begging of the third year in scenario 3. Moreover, the landfill area in Corum is constructed in scenario 2 and scenario 4.

Table 22: The location of landfill areas in each scenario (Y: Yes, N: No)

Scenario	Ankara	Istanbul	Izmir	Diyarbakir	Erzurum	Kocaeli	Kirklareli	Mersin	Corum
1	Y	Y	Y	N	N	Y	Y	Y	N
2	Y	Y	Y	Y	Y	Y	N	Y	Y
3	Y	Y	Y	N	N	Y	Y	Y	N
4	Y	Y	Y	N	N	Y	Y	N	Y

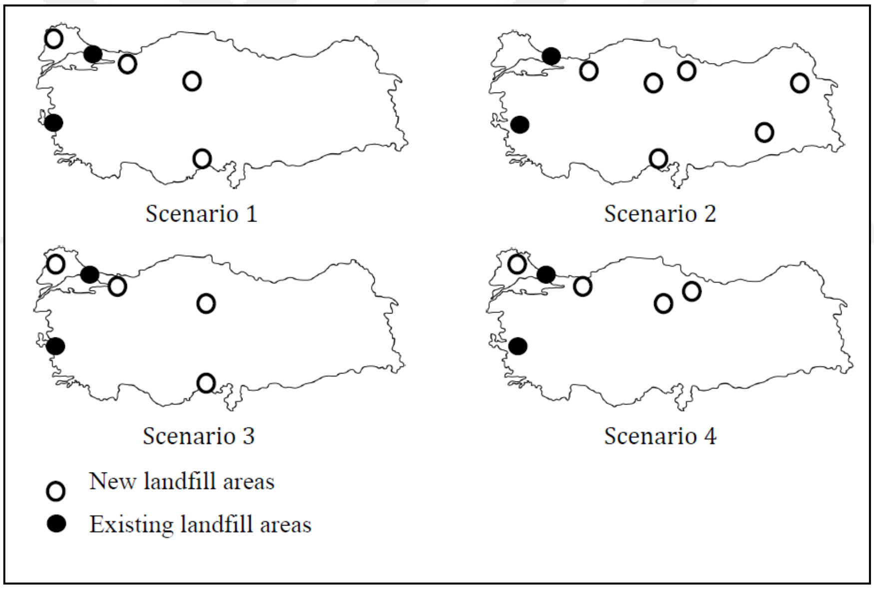

Figure 16: The new and existing landfill areas for each scenario

Chapter 5
Conclusions

5.1. Conclusions

In this thesis, we develop a multi period mixed integer programming model (MILP) for recycling waste batteries in a reverse logistics system using the robust optimization approach. Our model considers uncertainty in the amount of collected waste batteries in different region. We consider four different scenarios to handle the uncertainty in the amount of the collected waste batteries. Our model involves strategic and tactical decisions. In the first stage, the model determines the capacity of the sorting and recycling facilities and the number of facilities. After observing the actual value of the collected batteries, in the second stage, the model finds the location of the facilities and inventory levels. These are known as the second stage decisions.

We analyze numerical results of both approaches and observe an average of 31% efficiency loss in revenue when we are using two-stage stochastic optimization method. However, the efficiency loss of revenue increases to 50% when we use robust optimization method. On the other hand, we consider the worst case scenario and observe that the robust optimization method performs better than the two-stage stochastic method. The maximum loss of revenue for the robust optimization is less than 17% of the loss for two-stage stochastic approach.

5.2. Recommendations for future research

The model can be extended by considering cost for collection activities and different options for collection nodes. Moreover, we do not apply export agencies in this model due to the lack of data. It can be included in the model as a third option for sending sorted waste batteries in the network. The relation between landfill and recycling facilities would be explained better by using export agencies option.

This problem contains various uncertainties while we only focus on the most important one in this thesis which is the uncertainty in the amount of collected waste batteries in different regions. However, other parameters of the model can be stochastic too—e.g., cost parameters, market demand for recycled materials, and price. As a future work, one could consider uncertainty in other parameters of the model.

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