A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF ÇANKIRI KARATEKİN UNIVERSITY

UTILIZING THE LAPLACE RESIDUAL POWER SERIES APPROACH FOR THE FRACTIONAL KAWAHARA FORMULA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MATHEMATICAL SCIENCE

BY

ASEN HAMZAH KHALEEL KAHIYAH

ÇANKIRI

2024

UTILIZING THE LAPLACE RESIDUAL POWER SERIES APPROACH FOR THE FRACTIONAL KAWAHARA FORMULA

By Asen Hamzah Khaleel KAHIYAH

February 2024

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science

Advisor : Asst. Prof. Dr. Şerifenur CEBESOY ERDAL
Co-Advisor : Asst. Prof. Dr. Samer Raad YASEEN

Examining Committee Members:

Chairman : Prof. Dr. Faruk POLAT
Mathematics
Çankırı Karatekin University

Member : Assoc. Prof. Dr. Pembe İPEK AL
Mathematics
Karadeniz Technical University

Member : Asst. Prof. Dr. Şerifenur CEBESOY ERDAL
Mathematics
Çankırı Karatekin University

Approved for the Graduate School of Natural and Applied Sciences

Prof. Dr. Hamit ALYAR
Director of Graduate School

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Asen Hamzah Khaleel KAHIYAH

ABSTRACT

UTILIZING THE LAPLACE RESIDUAL POWER SERIES APPROACH FOR THE FRACTIONAL KAWAHARA FORMULA

Asen Hamzah Khaleel KAHIYAH

Master of Science in Mathematics

Advisor: Asst. Prof. Dr. Şerifenur CEBESOY ERDAL

Co-Advisor: Asst. Prof. Dr. Samer Raad YASEEN

Febuary 2024

By combining Laplace's residual power series technique with the concept of limits through finite energy and time compared to the residual power series technique, many nonlinear and fractional partial problems related to the Kawahara formula are conceptually solved in this thesis. This results in investigations and approximations in rapidly converging series shapes. The proposed strategy is tested on multiple subjects to determine whether it is practical, efficient, or clear using two tests and the graphs. Analysis of the collected data shows that the previously mentioned approach is suitable, accurate and simple to find solutions to non-linear technological problems.

2024, 51 pages

Keywords: Kawahara formula, Gamma function, Laplace residue energy series technique, Fractional differential equations

ÖZET

KESİRLİ KAWAHARA FORMÜLÜ İÇİN LAPLACE REZİDÜ KUVVET SERİSİ YAKLAŞIMININ KULLANILMASI

Asen Hamzah Khaleel KAHIYAH

Matematik, Yüksek Lisans

Tez Danışmanı: Dr. Öğr. Üyesi Şerifenur CEBESOY ERDAL

Eş Danışman: Dr. Öğr. Üyesi Samer Raad YASEEN

Şubat 2024

Bu tezde, Laplace rezidü kuvvet serisi tekniğini, rezidü kuvvet serisi tekniğine kıyasla sonlu enerji ve zaman yoluyla limitler kavramıyla birleştirerek, Kawahara formülü ile ilişkili birçok doğrusal olmayan ve kesirli kısmi problem kavramsal olarak çözülmüştür. Bu, hızla yakınsayan seri şekillerinde incelemeler ve yaklaşımlarla sonuçlanmıştır. Önerilen stratejinin pratik, verimli veya anlaşılır olup olmadığının belirlenmesi için iki test ve grafikler kullanılarak bu strateji birden fazla konu üzerinde test edilmiştir. Toplanan verilerin analizi, daha önce bahsedilen yaklaşımın doğrusal olmayan teknolojik problemlere çözüm bulmak için uygun, doğru ve basit olduğunu göstermektedir.

2024, 51 sayfa

Anahtar Kelimeler: Kawahara formülü, Gama fonksiyonu, Laplace rezidü enerji serisi tekniği, Kesirli diferensiyel denklemler

PREFACE AND ACKNOWLEDGEMENTS

I would like to thank my thesis advisor, Asst. Prof. Dr. Şerifenur CEBESOY ERDAL, for her patience, guidance and understanding.

Asen Hamzah Khaleel KAHIYAH
Çankırı-2024

CONTENTS

LIST OF SYMBOLS

$\mathcal{D}_t^{\alpha}\mathcal{M}(x,t)$	Caputo fractional derivative
$\mathcal{J}(x)$	Exponential ordering
$\Gamma(\mathcal{Z})$	Gamma function
$\mathcal{M}(\mathrm{x},t)$	Inverse Laplace transform
$\mathcal{L}Res(x,s)$	Laplace residual power series
$\mathcal{W}(x,s)$	Laplace transform
$\mathcal{D}_t^{\alpha}\mathcal{M}(x,t)$	The Riemann-Liouville fractional derivative
$J_t^{\alpha}\mathcal{M}(x,t)$	The Riemann-Liouville fractional integral
$\mathcal{L}Res_k(x,s)$	The kth-Laplace residual power series

LIST OF ABBREVIATIONS

ADS	Adomian strategy
DEs	Differential equations
FIVPs	Fractional initial value problems
FDEs	Fractional differential equations
FKDEs	Fractional Kawahara differential equations
FLE	Fractional laurient equation
FPDEs	Fractional partial differential equations
FPS	Fractional power series
FRPS	Fractional residiual power series
LRF	Laplace residual function
LRPS	Laplace residual power series
NPDEs	Nonlinear partial differential equations
q-HATM	q-homotopy assessment transformation technique
PDEs	Partial differential equations
RFPS	Residual fractional power series
RPS	Residual power series

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

In the domains of finance, biological science, software engineering, and various other disciplines, fractional derivatives are a reliable tool for precisely describing a variety of actual-life occurrences (Al-Smadi and Arqub 2019, Bira *et al.* 2018, Kilbas *et al.* 2006, Abu Arqub 2019, Alabedalhadi *et al.* 2020, Akdemir *et al.* 2020). This is primarily because, in cases where actual occurrences require upon the current time or a past temporal currently, fractional computation may be utilized to precisely model those scenarios (Al-Smadi and Gumah 2014, Al-Smadi *et al.* 2015, Baleanu *et al.* 2009, Kumar 2013). Because of this, a large number of scientists and technologists have focused on FDEs (Fractional Differential Equations) when creating responses to nonlinear issues and talking about nonlinear phenomena.

Additionally, FDE methods were studied by computations and predictions (Al-Smadi *et* taken an interest *al.* 2017, Al-Smadi 2018, Jleli *et al*. 2020, Momani *et al.* 2014). Many academics have in FIVPs (Fractional Initial Value Problems), an extension of conventional beginning value computations that are able to more precisely reflect certain practical aspects than typical DEs (Differential Equations). A great deal of attention has been paid to the concept that underlies the viability and dependability of reactions about FIVP applications (Abdeljawad *et al.* 2020, Al-Smadi *et al.* 2021a, Atluri and Shen 2002, Ateş and Yildirim 2009, Boon *et al.* 1988, Chambré 1952, Chawla 2002, Hasan *et al.* 2021, Luchko 2011, Sowa 2018, Talafha *et al.* 2020).

Studies on the IVP (Initial Value Problems), concept in particular is lacking, but, when it comes to talking about solutions. Many authors have showed dedication to solve these problems by using various hypotheses and computing methodologies because it is naturally difficult to evaluate them analytically. As a result, studies using numerical techniques have begun to emphasize this kind of structure (Rahman *et al.* 2021, Roul 2022).

FC (Fractional Calculus) has been studied throughout the 16th century. It is an extension of classical calculus that includes regular differentiation as well as integration for every degree. These days, FC is an excellent tool for preserving and exhibiting the genetic aspects of most events. Recently, a number of scholars have created a wide range of nonlinear problems in arithmetic and practical studies using the FC. Furthermore, non-linear quake motions, visual, flexibility, gas motion, dynamics, moving water, and concept are among actual-world issues for which NPDEs (Nonlinear partial differential equations) are routinely produced by mathematical models (Al-Smadi 2018, Agarwal *et al.* 2015, Baleanu *et al.* 2011, El-Ajou *et al.* 2015a, Miller and Ross 1993, Oldham and Spanier 1974).

Studying nonlinear physical effects with respect to their worldwide habits, including conduction, propagation, variation, and disintegration, while maintaining storage is the main goal of taking into account fractional-order differential equations (Diethelm and Ford 2002, Murad *et al.* 2014). It is evident that an approximate power-law approximation for the regional performance about non-differentiable variables may be obtained from a fractional derivative. Numerous numerical and analytical approaches, such as periodic solutions, tanh technique, homotopy perturbation technique, varitional technique and the q-homotopy assessment transformation technique (q-HATM) have been used in both science and math for investigating various kinds of FDEs (He and Abdou 2007, Abbasbandy 2010, Ganie *et al.* 2024, Yusufoglu 2007, Yusufoglu and Bekir 2007, Yusufoğlu and Bekir 2008, Ali *et al.* 2023, Hussain *et al.* 2022).

A direct mathematical approach to construct the correct moving wave results for nonlinear evolution problems is provided, utilizing the answers of a supplementary standard differential problem. The Kawahara and improved Kawahara formulas are examined in (Sirendaoreji 2004, Gepreel *et al.* 2019, Başhan 2021) using this methodology.

Further research on FDEs that have been conducted in a number of academic domains is referenced in (Atangana and Hammouch 2019, Al-Qurashi *et al.* 2021, Li *et al.* 2021, Rashid *et al.* 2019, Rashid *et al.* 2020 a, b, and c, Zhou *et al.* 2021). Furthermore, it may be difficult to pinpoint the precise solutions for a variety of FIVPs because of the numerical intricacy in fractional derivatives implicated. Techniques both analytic and computational were developed to investigate the outcomes of different kinds of nonlinear systems that contain FIVPs. Both numerical and analytical techniques were developed to investigate the remedies of different kinds of FIVP nonlinear systems. The Sumudu transformation technique, the homotopy disruption combined to Sumudu transformation technique, the homotopy perturbation technique, the Adomain categorized technique, the homotopy analysis turn technique, the Wavelet parameters interpolating technique, the variational repetition technique, the spectral colloquialism planning, the differential transforms technique, α-Sumudu transformation homotopy perturbation technique are a few examples (Momani *et al.* 2016, Al-Smadi *et al.* 2016, Atangana and Baleanu 2016, Ait Touchent *et al.* 2018, Gambo *et al.* 2018, Al-Smadi *et al.* 2020c, Al-Smadi *et al.* 2021b, Hasan *et al.* 2020, Kumar *et al.* 2021, Khader *et al.* 2021, Moazzam *et al.* 2023).

Since most FPDEs don't have sufficient mathematical solutions, researchers need to come up with an acceptable approach using these types of approximations responses. The research literature describes a number of ways for computations, some of which include the extended operational matrix procedure, Adomian strategy (ADS), fractional partial differentially transformation techniques (FDTT) (Garcia-Beristain and Remaki 2023, Saadatmandi and Dehghan 2010, Ertürk and Momani 2008), and others. There has been a lot of interest lately in the creation of useful matrices employing derivatives with fractional values. Operational matrices like these can be produced through the application of wavelet techniques, including structures like Legendre and Chebyshev wavelets, for instance. Utilizing such an operational matrices, a number of approximations algorithms for FDE computations were developed (Doha *et al.* 2013, Khan *et al.* 2020, Rehman and Khan 2011, Wang and Fan 2012). In order to solve FPDEs, the researchers developed a brand-new numerical technique depending upon the Wavelet in Yi and Chen (Yi and Chen 2012). The researchers' several new discoveries regarding the Jacobi parameters for operational matrices were made in 2014 (Khalil and Khan 2014). All operational matrices approaches are applied to the solution of FDE and FPDE issues. To the most effective of our understanding, their study has been carried out successfully, as has its utilization in the related PFDE technologies. Research on computing solutions of interrelated systems of fundamentals and PDEs having fractional ordering are scarce, therefore, as we can tell for additional details, see (Khalil and Khan 2015a, Khalil and Khan 2015b).

The RPS (Residual Power Series) technique is a numerical technique for computing fractional integro-differential formulas and other PDE shapes. It's an effective method since it provides solutions in a closed set of recognized formulas. Numerous FDEs, fuzzy FDEs, and fractions integral equation formulations have been solved via the application of the RFPS (Residual Fractional Power Series Technique). For instance, fractional relaxation–oscillation equation (Arora *et al.* 2023) and fractional Newell-Whitehead-Segel formulas (Saadeh *et al.* 2019). Value problems at the solitary boundaries (Komashynska *et al.* 2016) Massive Thirring formulae (Al-Smadi *et al.* 2020b, Wang *et al.* 2022), fractional Schrödinger methods combination (Al-Smadi *et al.* 2020a), certain formable transform decomposition formula (Saadeh *et al.* 2023), specific fuzzy FDE categories (Alaroud *et al.* 2018, Alaroud *et al.* 2019a and b, Moa'ath *et al.* 2022).

A powerful instrument to resolve a variety of intricate structures which arise in multiple areas of science is the transformation of Laplace approach. Nonlinear problems can be resolved more precisely and swiftly provided analytical techniques are used with Laplace transformation operators.

The primary goal of this study is to use the LRPS (Laplace Residual Power Series) approach, which was presented and demonstrated in (Eriqat *et al.* 2020), to analyze and approximate approaches for situations with nonlinear FIVP.

The LRPS technique combines the RPS and Laplace transformation procedures to produce fast FPS (Fractional Power Series) results that yield both approximate and exact results. The solutions to the new algebraic formulae are generated when the main problem is transformed to Laplace space. Using the Laplace opposite of outcomes, the main problem is finally resolved. By using the limit notion, the unknown components in a unique Laplace expanding can be recognized, contrast from the FRPS (Fractional Residual Power Series) technique, which mainly focuses on the partial derivative so may take some time to derive the different derivatives of fractions in phases of identifying the results. There are less accuracy- and time-intensive tiny computation requirements with the LRPS technique.

The following is how this study is structured: There is an overview in the first section. Section two goes over a number of crucial basic results on the transformation of Laplace using new types of FPS. The recommended procedure for obtaining outcomes from a fractional Kawahara differential formula is described in Section 3. Section 4 looks at the applicability and availability of an LRPS procedure by solving three instances of fractıonal Kawahara differential formula. The fifth section concludes with a few observations regarding the results and recommendation.

2. FRACTIONAL CALCULUS FOUNDATIONS

In this section, we cover a number of notions and concepts that are essential to complete our research. We will delve deeper through the details of a few unique functions, fractional calculation theories, Taylor series and notions connected to them, the notion of the Laplace transform and its fundamental idea, Laurent series along with associated notions, and others.

2.1 Function of Gamma

The Gamma Function $\Gamma(\mathcal{Z})$ is a the factorial function expanding that can be applied to any real or complex number. It appears to cover all values of $\mathcal{Z}$, with the possible exception of $\mathcal{Z} = 0, 1, 2, \dots,$ for which there are several possible descriptions.

Definition 2.1 (Andrews 1998) There exist multiple approaches to define $\Gamma(\mathcal{Z})$:

1. $\Gamma(\mathcal{Z}) = \int_0^\infty \zeta^{\mathcal{Z}-1} e^{-\zeta} d\zeta, \mathcal{Z} \in \mathbb{R}, \mathcal{Z} > 0.$
2. $\frac{1}{\Gamma(\mathcal{Z})} = \mathcal{Z} \lim_{n\to\infty} \left\{ n^{-\mathcal{Z}} \prod_{k=1}^{n} \left(1 + \frac{\mathcal{Z}}{k}\right) \right\}.$
3. $\Gamma(\mathcal{Z}) = \lim_{n\to\infty} \frac{n! n^{\mathcal{Z}}}{\mathcal{Z}(\mathcal{Z}+1)(\mathcal{Z}+2)\dots(\mathcal{Z}+n)}, \mathcal{Z} \in \mathbb{R}, \mathcal{Z} \neq 0, -1, -2, \dots$

Lemma 2.1 (Andrews 1998) Features of the Gamma Function include as follows:

1. $\Gamma(\mathcal{Z}) = (\mathcal{Z} - 1)!, \mathcal{Z} \in \mathbb{N}.$
2. $\Gamma(\mathcal{Z}) = \frac{\Gamma(\mathcal{Z}+1)}{\mathcal{Z}}, \mathcal{Z} < 0, \mathcal{Z} \neq -1, -2, -3, \dots .$
3. $\Gamma(\mathcal{Z} + 1) = \mathcal{Z}\Gamma(\mathcal{Z}), \mathcal{Z} > 0.$
4. $\Gamma\left(\frac{\mathcal{Z}+1}{2}\right) = \frac{\sqrt{\pi}}{2^{\mathcal{Z}}} (2\mathcal{Z} - 1)!, \mathcal{Z} \in \mathbb{N}.$
5. $\Gamma(\mathcal{Z})\Gamma(1 - \mathcal{Z}) = \frac{\pi}{\sin(\pi\mathcal{Z})}.$

2.2 Fractions Operations

The ideas of integrals and derivatives of fractionals in the context of Riemann-Liouville and Caputo are presented in this part, along with several of their distinguishing features.

Definition 2.2.1 (Podlubny 1999) The time-fractional derivatives of ranking $\alpha \in (k-1, k]$ for the multivariate function $\mathcal{M}(x,t)$ is provided as:

$$\mathcal{D}^{\alpha}\mathcal{M}(x,t) = \begin{cases} \dfrac{\partial^k}{\partial t^k} J_t^{k-\alpha}\mathcal{M}(x,t), & k-1 < \alpha < k, \\ \dfrac{\partial^k}{\partial t^k}\mathcal{M}(x,t) \quad , & \alpha = k. \end{cases}$$

Remark 2.2.2 (Andrews 1998) Suppose $\alpha > 0$ and $k, \gamma \in \mathbb{R}, t > 0$. Thus

1. $\mathcal{D}^{\alpha}k = \frac{k}{\Gamma(1-\beta)} t^{-\alpha}, \alpha \neq 1,2,3, \dots$

2. $\mathcal{D}^{\alpha}t^{\lambda} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma-\alpha+1)} t^{\lambda-\alpha}, \lambda > -1.$

Definition 2.2.3 (Podlubny 1999) The time-fractional integration of order $\alpha \in (k-1, k]$ for the multidimensional function $\mathcal{M}(x,t)$ is provided as

$$J_t^{\alpha}\mathcal{M}(x,t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\omega)^{\alpha-1}\mathcal{M}(x,t)d\omega.$$

Lemma 2.2.4 (Podlubny 1999) In the event where $\mathcal{M}(x,t)$ and $\mathcal{N}(x,t)$ are continuous functions, $\lambda, \zeta \in \mathbb{R}$, and $\kappa, \alpha > 0$ implies

1. $J_t^{\alpha}(\lambda\mathcal{M}(x,t) + \zeta\mathcal{N}(x,t)) = \lambda J_t^{\alpha}\mathcal{M}(x,t) + \zeta J_t^{\alpha}\mathcal{N}(x,t).$
2. $J_t^{\alpha}J_t^{\kappa}\mathcal{M}(x,t) = J_t^{\alpha+\kappa}\mathcal{M}(x,t) = J_t^{\kappa}J_t^{\alpha}\mathcal{M}(x,t)$.

Definition 2.2.5 (El-Ajou 2021) The Caputo time-fractional integration of order $\alpha \in (k-1, k]$ for the multivariate function $\mathcal{M}(x,t)$ is provided as

$$D^{\alpha}\mathcal{M}(x,t) = \begin{cases} J_t^{k-\alpha}\dfrac{\partial^k}{\partial t^k}\mathcal{M}(x,t), & k-1<\alpha<k, \\ \dfrac{\partial^k}{\partial t^k}\mathcal{M}(x,t), & \alpha = k. \end{cases}$$

Remark 2.2.6 (El-Ajou 2021) Suppose $\zeta, \lambda \in \mathbb{R}$ and $D^{\alpha}\mathcal{M}(x,t), D^{\alpha}\mathcal{N}(x,t)$ exist, then

1. $D_t^{\alpha}(\lambda\mathcal{M}(x,t) + \zeta\mathcal{N}(x,t)) = \lambda D_t^{\alpha}\mathcal{M}(x,t) + \zeta D_t^{\alpha}\mathcal{N}(x,t)$.
2. $D_t^{\beta} J_t^{\beta}\mathcal{M}(x,t) = \mathcal{M}(x,t)$.
3. $J_t^{\beta} D_t^{\beta}\mathcal{M}(x,t) = \mathcal{M}(x,t) - \sum_{n=0}^{k-1}\frac{\partial_t^n\mathcal{M}(x,t)}{n!}t^n$.

Remark 2.2.7 (El-Ajou 2021) Let $k-1<\alpha<k, k \in \mathbb{N}$ and $\mathcal{M}(x,t)$be a function such that $D_t^{\alpha}\mathcal{M}(x,t)$ exists. The following subsequent traits apply to the Caputo operator.

$$\lim_{\alpha\to k} D_t^{\alpha}\mathcal{M}(x,t) = \partial_t^k\mathcal{M}(x,t),$$

$$\lim_{\alpha\to k-1} D_t^{\alpha}\mathcal{M}(x,t) = \partial_t^{k-1}\mathcal{M}(x,t) - \partial_t^{k-1}\mathcal{M}(x,0).$$

2.3 Power Series with a Fraction and Laplace Transformation

This part goes into great length on the transformation of Laplace of fractions derivatives in the context of Caputo. Providing further theories and characteristics linked to our results, we will also explore some fundamental concepts to extend the interpretation of Caputo, aiming to generalize power series for fractional scenarios.

Definition 2.3.1 (Griffel 1991) The improper integral displayed as follows

$$\mathcal{W}(x,s)=\int_0^{\infty} e^{-st}\mathcal{M}(x,t)dt,$$

is known by the Laplace transformation of $\mathcal{M}(x,t)$ if it occurs as all s in a region $\mathcal{P}\subseteq\mathbb{R}$ and it can be expressed as $\mathcal{L}[\mathcal{M}(x,t)](x,s)$.

Definition 2.3.2 (Griffel 1991) In the area $I\times[c,d]$, the formula $\mathcal{M}(x,t)$ is piece-wise continuous if

(i) A limited number of sections, $c=t_0<t_1<\cdots<t_k=d$, can be formed from the range of elements $[c,d]$ provided that $\mathcal{M}$ is constantly within every sub-area $I\times(t_i,t_{i+1})$, for $i=0,1,2,\dots,k-1$.

(ii) The function $\mathcal{M}$ exhibits a jump discontinuity at (x,t_i), whenever

$$\left|\lim_{t\to t_i^+}\mathcal{M}(x,t_i)\right|<\infty, \text{and}\left|\lim_{t\to t_i^-}\mathcal{M}(x,t_i)\right|<\infty, i\in\{1,2,\dots,k-1\}.$$

A piecewise continuity of a function on $I\times[0,K], K>0$ is guaranteed if it is piecewise continuous on $I\times[0,\infty)$.

Definition 2.3.3 (Griffel 1991) The function $\mathcal{M}(x,t)$ is said to be of exponential order $\mathcal{J}(x)$ if two positive functions, $\mathscr{b}(x)$ and $\mathcal{J}(x)$ occur such that$|\ \mathcal{M}(x,t)\ |\leq \mathscr{b}(x)\ e^{\mathcal{J}(x)t}$ for sufficiently big t.

Definition 2.3.4 (Griffel 1991) The function $\mathcal{M}(x,t)$ can be obtained from the Laplace transformation $\mathcal{W}(x,s)$by using the inverse transformation of Laplace, as shown as follows:

$$\mathcal{M}(x,t)=\mathcal{L}^{-1}[\mathcal{W}(x,s)]=\int_{\mathcal{C}-i\infty}^{\mathcal{C}+i\infty} e^{st}\mathcal{W}(x,s)ds, \mathcal{C}=\mathrm{Re}(s)>\mathcal{C}_0,$$

where $\mathcal{C}_0$ is situated in the real closure plane that defines the Laplace integral's right part.

Lemma 2.3.5 (El-Ajou 2021) Presume that $\mathcal{M}(x,t)$ and $\mathcal{N}(x,t)$ are piece-wise continuous formulas of exponential ordering on $I \times [0,\infty)$. Moreover, let $\mathcal{W}(x,s) = L[\mathcal{M}(x,t)]$ and $\mathcal{H}(x,s) \ = L[\mathcal{N}(x,t)]$ hold, where $a, \mathfrak{b}$ and $\mathfrak{L}$ constant. Then, the following properties are satisfied:

1. $\mathcal{L}^{-1}[a\mathcal{W}(x,s) + \mathfrak{b}\mathcal{H}(x,s)\,] = a\mathcal{L}^{-1}[\mathcal{W}(x,s)] + \mathfrak{b}\mathcal{L}^{-1}[\mathcal{H}(x,s)] = a\mathcal{M}(x,t) + \mathfrak{b}\mathcal{N}(x,t)$.
2. $\mathcal{L}[a\mathcal{M}(x,t) + \mathfrak{b}\mathcal{N}(x,t)] = a\mathcal{L}[\mathcal{M}(x,t)] + \mathfrak{b}\mathcal{L}[\mathcal{N}(x,t)] = a\mathcal{W}(x,s) + \mathfrak{b}\mathcal{H}(x,s)$.
3. $\mathcal{L}[\mathcal{M}(\mathfrak{L}x, \mathfrak{L}t)] = \frac{1}{\mathfrak{L}}\mathcal{W}\left(\frac{x}{\mathfrak{L}}, \frac{s}{\mathfrak{L}}\right), \mathfrak{L} > 0$.
4. $\mathcal{L}[\partial_t^k \mathcal{M}(x,t)] = s^k\mathcal{W}(x,s) - \sum_n^{k-1} s^{\mathrm{k-n-1}}\, \partial_t^n \mathcal{M}(x,0)$.
5. $\lim\limits_{s\to\infty} s\mathcal{W}(x,s) = \mathcal{M}(x,0)$.

Remark 2.3.6 (El-Ajou 2021) Suppose $\mathcal{M}(x,t)$ be a piece-wise continuous function on $I \times [0,\infty)$ with exponential order, such that $\mathcal{W}(x,s) = \mathcal{L}[\mathcal{M}(x,t)]$. Afterwards, there is

1. $\mathcal{L}[D_t^\alpha \Omega(x,t)] = s^\beta \mathrm{H}(x,s) - \sum_i^{k-1} s^{\beta-\mathrm{i}-1}\, \partial_t^i \Omega(x,0)\,, \mathrm{k}-1 < \beta < \mathrm{k}$.
2. $\mathcal{L}[J_t^\alpha \Omega(x,t)] = s^{\beta-1}\mathrm{H}(x,s), \beta > 0$.
3. $\mathcal{L}[D_t^{k\alpha}\Omega(x,t)] = s^{k\beta}\Psi(x,s) - \sum_{\mathrm{n=0}}^{\mathrm{k-1}} s^{(\mathrm{k-n})\beta-1} D_t^{n\beta}\Omega(x,0), 0 < \beta < 1$,
   where $D_t^{k\alpha} = D_t^\alpha \cdot D_t^\alpha \ldots D_t^\alpha$ (k-times).

Definition 2.3.7 (El-Ajou *et al*. 2015b) A representation of a structure utilizing power series can be written as

$$\sum_{i=0}^{\infty} \omega_i(x)(t-t_0)^{i\alpha} = \varsigma_0 + \varsigma_1(t-t_0)^\alpha + \varsigma_2(t-t_0)^{2\alpha} + \varsigma_3(t-t_0)^{3\alpha} + \cdots,$$

where t is a variable, and $\omega_i(x)$ are functions corresponding to x that are called series components; however, $t \geq t_0$ and $0 \leq k-1 < \alpha \leq k$ are called FPS about t_0.

Proposition 2.3.8 (El-Ajou *et al.* 2015b) (Generalized Taylor's formulation). Let us assume that $\mathcal{M}(x,t)$ has an accompanying FPS formula at $t = t_0$.

$$\mathcal{M}(x,t) = \sum_{i=0}^{\infty} \omega_i(x)(t-t_0)^{i\alpha}, 0 \le k-1 < \alpha \le k, x \in I, t_0 \le t < t_0 + R. \quad (2.1)$$

In case $D_t^{i\Omega}\mathcal{M}(x,t)$, i $= 0,1,2,\dots$, are continuous on $I \times (t_0, t_0 + R)$, the coefficients $\omega_i(x)$ of Equation (2.1) can be determined by the following formula

$$\omega_i(x) = \frac{D_t^{i\Omega}\mathcal{M}(x,t)}{\Gamma(i\alpha+1)}, i = 0,1,2,\dots,$$

where $D_t^{k\alpha} = D_t^{\alpha} \cdot D_t^{\alpha} \dots D_t^{\alpha}$ (i-times).

Proposition 2.3.9 (El-Ajou 2021) Suppose $\mathcal{M}(x,t)$ be a piece-wise continuous function on the range $I \times [0,\infty)$ that is ordered exponential. Let us consider the following FLE for a given function $\mathcal{W}(x,s) = \mathcal{L}[\mathcal{M}(x,\mathrm{t})]$ is

$$\mathcal{W}(x,s) = \sum_{\mathrm{i}=0}^{\infty} \frac{\omega_i(x)}{s^{1+\mathrm{i}\alpha}}, 0 < \alpha \le 1, s > 0. \quad (2.2)$$

Then $\omega_i(x) = \left(D_t^{i\alpha}\mathcal{M}\right)(0)$.

Remark 2.3.10 (El-Ajou 2021) Proposition 2.3.8 of Equation (2.2) states that the inverse Laplace transformation for the FLE has a subsequent representation.

$$\mathcal{M}(x,t) = \sum_{\mathrm{i}=0}^{\infty} \frac{D_t^{i\alpha}\mathcal{M}(x,0)}{\Gamma(1+\mathrm{i}\alpha)} t^{i\alpha}, 0 < \alpha \le 1, t \ge 0. \quad (2.3)$$

3. CREATING THE LRPS FRACTIONAL KAWAHARA FORMULA

To understand or duplicate some of the benefits of these phenomena, the FKDEs (Fractional Kawahara Differential Equations) can rebuild a variety of actual scientific and technological events. Still, most FKDEs have been closed answers and are unable to determined mathematically. The current section aims to provide the basic notion of an LRPS approach used for solving certain systems for FKDEs analytically by defining the components of the FPS approximations solution. To make it easier, let's use the subsequent FKDE systems in perspective of the Caputo derivatives.

$$D_t^{\alpha} u_t(x,t) + \sigma u(x,t) u_x(x,t) + \eta u_{xxx}(x,t) - \gamma u_{xxxxx}(x,t) = 0, \tag{3.1}$$

where $0 < \alpha \leq 1$, $t \geq 0, x \in I \subseteq \mathbb{R}$ and the initial condition

$$u(x,0) = h(x). \tag{3.2}$$

First, we apply the Laplace transform to both sides of Equation (3.1) as

$$\mathcal{L}\{D_t^{\alpha} u_t(x,t)\} + \mathcal{L}\{\sigma u(x,t) u_x(x,t) + \eta u_{xxx}(x,t) - \gamma u_{xxxxx}(x,t)\} = 0. \tag{3.3}$$

Considering the circumstances, by using the formula $\mathcal{L}\{D_t^{\alpha} u(x,t)\} = s^{\alpha} U(x,s) - s^{\alpha-1} u(x,0)$ and Lemma 2.3.5, we may construct (3.3) as follows

$$s^{\alpha} U(x,s) - s^{\alpha-1}\, u(x,0) + \sigma \mathcal{L}\left\{\left(\mathcal{L}^{-1}\big(U_x(x,s)\big)\mathcal{L}^{-1} U(x,s)\right)\right\} + \eta U_{xxx}(x,s) - \gamma U_{xxxxx}(x,s)\,,\ s > 0. \tag{3.4}$$

By multiplying (3.4) by s^{α} and using the initial conditions of (3.4), the following shape of the equation can be obtained as follows:

$$U(x,s) = \frac{h(x)}{s} + \frac{\sigma}{s^{\alpha}} \mathcal{L}\left\{\left(\mathcal{L}^{-1}\big(U_x(x,s)\big)\mathcal{L}^{-1} U(x,s)\right)\right\} + \frac{\eta}{s^{\alpha}} U_{xxx}(x,s) - \frac{\gamma}{s^{\alpha}} U_{xxxxx}(x,s), s > 0, \tag{3.5}$$

where, $U(x,s) = \mathcal{L}\{u(x,t)\}$, $U_{xxx}(x,s) = \mathcal{L}\{u_{xxx}(x,t)\}$ and $U_{xxxxx}(x,s) = \mathcal{L}\{u_{xxxxx}(x,t)\}$.

When using the LRPS method, the series of Laplace result of (3.5) has the following form:

$$U(x,s) = \sum_{n=0}^{\infty} \frac{h_i(x)}{s^{1+n\alpha}}, s > 0. \tag{3.6}$$

Proposition 2.3.8 allows us to rewrite Equation (3.6) as follows:

$$U(x,s) = \frac{h(x)}{s} + \sum_{n=1}^{\infty} \frac{h_i(x)}{s^{1+n\alpha}}, s > 0. \tag{3.7}$$

The following equation represents the kth-truncated sequence of $U(x,s)$ in (3.7):

$$U_k(x,s) = \frac{h(x)}{s} + \sum_{n=1}^{k} \frac{h_i(x)}{s^{1+n\alpha}}, s > 0. \tag{3.8}$$

The first approximation is recognized as follows:

$$U_1(x,s) = \frac{h(x)}{s} + \frac{h_1(x)}{s^{1+\alpha}}, s > 0. \tag{3.9}$$

To determine the quantity of the unidentified parameter h(x) in (3.9), it's necessary to establish the primary LRPS methods, including the LRF of (3.5).

$$\begin{aligned} \mathcal{L}Res(x,s) = U_1(x,s) - \frac{h(x)}{s} - \frac{\sigma}{s^{\alpha}} \mathcal{L}\left\{\left(\mathcal{L}^{-1}\big(U_x(x,s)\big)\mathcal{L}^{-1}U(x,s)\right)\right\} \\ - \frac{\eta}{s^{\alpha}} U_{xxx}(x,s) + \frac{\gamma}{s^{\alpha}} U_{xxxxx}(x,s), s > 0, \end{aligned} \tag{3.10}$$

as well as the kth-LRF of (3.10) as:

$$\mathcal{L}Res_k(x,s) = U_k(x,s) - \frac{h(x)}{s} - \frac{\sigma}{s^\alpha}\mathcal{L}\left\{\left(\mathcal{L}^{-1}\left(U_{x(k)}(x,s)\right)\mathcal{L}^{-1}U_k(x,s)\right)\right\} - \frac{\eta}{s^\alpha}U_{xxx(k)}(x,s) + \frac{\gamma}{s^\alpha}U_{xxxxx(k)}(x,s), s > 0, \tag{3.11}$$

for $s > 0$ and $k = 0,1,2,3,\ldots$, it is evident that $\lim_{k\to\infty}\mathcal{L}\mathrm{Res}_k(x,s) = \mathcal{L}\mathrm{Res}(x,s)$, $\mathcal{L}\mathrm{Res}_\mathrm{k}(x,s) = 0$ for $\mathcal{L}\mathrm{Res}_\mathrm{k}(x,s) = 0$. As a consequence, $\lim_{s\to\infty}\left(s^k\mathcal{L}\mathrm{Res}(x,s)\right) = 0$. It was also created, according to (Eriqat *et al*. 2020). Additionally,

$$\lim_{s\to\infty}\left(s^{k+1}\mathcal{L}\mathrm{Res}(x,s)\right) = \lim_{s\to\infty}\left(s^{k+1}\mathcal{L}\mathrm{Res}_k(x,s)\right) = 0, k = 1,2,3,\ldots \tag{3.12}$$

kth-LRF's subsequent extended shape is produced by substituting the extensions found in steps required (3.8) and (3.11):

$$\mathcal{L}Res_1(x,s) = \frac{h_1(x)}{s^{\alpha+1}} - \frac{\sigma}{s^\alpha}\mathcal{L}\left\{\left(\mathcal{L}^{-1}\left(\frac{h'(x)}{s} + \frac{{h_1}'(x)}{s^{\alpha+1}}\right)\mathcal{L}^{-1}\left(\frac{h_1(x)}{s^{\alpha+1}} + \frac{h(x)}{s}\right)\right)\right\} - \frac{\eta}{s^\alpha}\left(\frac{h^{(3)}(x)}{s} + \frac{{h_1}^{(3)}(x)}{s^{\alpha+1}}\right) + \frac{\gamma}{s^\alpha}\left(\frac{h^{(5)}(x)}{s} + \frac{{h_1}^{(5)}(x)}{s^{\alpha+1}}\right), s > 0. \tag{3.13}$$

As a result, the initial approximation of (3.13) was as follows:

$$\begin{aligned}\mathcal{L}Res_1(x,s) &= \gamma h^{(5)}(x)s^{-\alpha-1} - \eta h^{(3)}(x)s^{-\alpha-1} - \sigma h\,(x)s^{-\alpha-1}h'(x) \\ &- \sigma h_1(x)s^{-2\alpha-1}h'(x) + \gamma {h_1}^{(5)}(x)s^{-2\alpha-1} - \eta {h_1}^{(3)}(x)s^{-2\alpha-1} \\ &- \sigma h\,(x)s^{-2\alpha-1}{h_1}'(x) - \frac{\sigma\Gamma\,(2\alpha+1)h_1(x)s^{-3\alpha-1}{h_1}'(x)}{\Gamma\,(\alpha+1)^2} \\ &+ h_1(x)s^{-\alpha-1}, s > 0.\end{aligned} \tag{3.14}$$

Multiplying the two sides of Equation (3.14) by $s^{1+\alpha}$ yields

$$
\begin{aligned}
& s^{1+\alpha}\mathcal{L}Res_1(x,s) \\
&\qquad = \gamma h^{(5)}(x) - \eta h^{(3)}(x) - \sigma\, h(x)\, h'(x) - \sigma h_1(x) s^{-\alpha} h'(x) \\
&\qquad + \gamma {h_1}^{(5)}(x) s^{-\alpha} - \eta {h_1}^{(3)}(x) s^{-\alpha} - \sigma\, h(x)\, s^{-\alpha} {h_1}'(x) \\
&\qquad - \frac{\sigma\, \Gamma(2\alpha+1)\, h_1(x) s^{-2\alpha} {h_1}'(x)}{\Gamma(\alpha+1)^2} + h_1(x), \qquad s > 0. \qquad (3.15)
\end{aligned}
$$

Following that, we can rapidly determine the outcome of $h_1(x)$ by calculating the equation and applying the limits as s approachs to ∞ from each side of (3.15), as well as the source (Eriqat *et al.* 2020):

$$
h_1(x) = \eta h^{(3)}(x) + \sigma\, h(x)\, h'(x) - \gamma h^{(5)}(x). \qquad (3.16)
$$

Similarly, substitute the second-truncated of (3.8) with (3.11) to find the value of $h_2(x)$. These yield

$$
h_2(x) = \frac{h(x)}{s} + \frac{h_1(x)}{s^{1+\alpha}} + \frac{h_2(x)}{s^{1+2\alpha}}
$$

into the second to get the corresponding:

$$
\begin{aligned}
\mathcal{L}Res_2(x,s) &= \frac{h_2(x)}{s^{2\alpha+1}} \qquad (3.17) \\
& - \frac{\sigma}{s^{\alpha}} \mathcal{L}\left\{ \left(\mathcal{L}^{-1}\left(\frac{h'(x)}{s} + \frac{{h_1}'(x)}{s^{\alpha+1}} + \frac{{h_2}'(x)}{s^{2\alpha+1}} \right) \mathcal{L}^{-1}\left(\frac{h_1(x)}{s^{\alpha+1}} + \frac{h(x)}{s} \right.\right.\right. \\
& \left.\left.\left. + \frac{h_2(x)}{s^{1+2\alpha}} \right) \right) \right\} - \frac{\eta}{s^{\alpha}} \left(\frac{h^{(3)}(x)}{s} + \frac{{h_1}^{(3)}(x)}{s^{\alpha+1}} + \frac{{h_2}^{(3)}(x)}{s^{2\alpha+1}} \right) \\
& + \frac{\gamma}{s^{\alpha}} \left(\frac{h^{(5)}(x)}{s} + \frac{{h_1}^{(5)}(x)}{s^{\alpha+1}} + \frac{{h_2}^{(5)}(x)}{s^{2\alpha+1}} \right), s > 0.
\end{aligned}
$$

As a result, the second approximations of (3.17) was as follows:

$$
\begin{aligned}
\mathcal{L}Res_2(x,s) = {} & \gamma h^{(5)}(x)s^{-\alpha-1} - \eta h^{(3)}(x)s^{-\alpha-1} - \sigma h\,(x)s^{-\alpha-1}h'(x) \\
& - \sigma h_1(x)s^{-2\alpha-1}h'(x) - \sigma h_2(x)s^{-3\alpha-1}h'(x) \\
& + \gamma {h_1}^{(5)}(x)s^{-2\alpha-1} + \gamma {h_2}^{(5)}(x)s^{-3\alpha-1} - \eta {h_1}^{(3)}(x)s^{-2\alpha-1} \\
& - \eta {h_2}^{(3)}(x)s^{-3\alpha-1} - \sigma h\,(x)s^{-2\alpha-1}{h_1}'(x) \\
& - \sigma h\,(x)s^{-3\alpha-1}{h_2}'(x) - \frac{\sigma\Gamma\,(2\alpha+1)h_1(x)s^{-3\alpha-1}{h_1}'(x)}{\Gamma\,(\alpha+1)^2} \\
& - \frac{\sigma\Gamma\,(3\alpha+1)h_2(x)s^{-4\alpha-1}{h_1}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(2\alpha+1)} \\
& - \frac{\sigma\Gamma\,(3\alpha+1)h_1(x)s^{-4\alpha-1}{h_2}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(2\alpha+1)} \\
& - \frac{\sigma\Gamma\,(4\alpha+1)h_2(x)s^{-5\alpha-1}{h_2}'(x)}{\Gamma\,(2\alpha+1)^2} + h_2(x)s^{-2\alpha-1}
\end{aligned}
\tag{3.18}
$$

Multiplying both sides of above equation by $s^{2\alpha+1}$ yields

$$
\begin{aligned}
& s^{1+2\alpha}\mathcal{L}Res_2(x,s) \\
& \quad = -\sigma h_1(x)h'(x) - \sigma h_2(x)s^{-\alpha}h'(x) + \gamma {h_1}^{(5)}(x) \\
& \quad + \gamma {h_2}^{(5)}(x)s^{-\alpha} - \eta {h_1}^{(3)}(x) - \eta {h_2}^{(3)}(x)s^{-\alpha} - \sigma h\,(x){h_1}'(x) \\
& \quad - \sigma h\,(x)s^{-\alpha}{h_2}'(x) - \frac{\sigma\Gamma\,(2\alpha+1)h_1(x)s^{-\alpha}{h_1}'(x)}{\Gamma\,(\alpha+1)^2} \\
& \quad - \frac{\sigma\Gamma\,(3\alpha+1)h_2(x)s^{-2\alpha}{h_1}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(2\alpha+1)} \\
& \quad - \frac{\sigma\Gamma\,(3\alpha+1)h_1(x)s^{-2\alpha}{h_2}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(2\alpha+1)} \\
& \quad - \frac{\sigma\Gamma\,(4\alpha+1)h_2(x)s^{-3\alpha}{h_2}'(x)}{\Gamma\,(2\alpha+1)^2} + h_2(x), s > 0.
\end{aligned}
\tag{3.19}
$$

Additionally, we can rapidly determine the result by calculating this equation and applying the limits as s approach to ∞ from each side of (3.19), we have

$$
h_2(x) \ = \sigma h_1(x)h'(x) + \eta {h_1}^{(3)}(x) + \sigma\, h(x)\, {h_1}'(x) - \gamma {h_1}^{(5)}(x). \tag{3.20}
$$

Furthermore, substitute the second-truncated of (3.8) in (3.11) to find the value of $h_3(x)$. Then, we write

$$h_3(x) = \frac{h(x)}{s} + \frac{h_1(x)}{s^{1+\alpha}} + \frac{h_2(x)}{s^{1+2\alpha}} + \frac{h_3(x)}{s^{1+3\alpha}},$$

into 3nd to get the accompanying:

$$\begin{aligned} \mathcal{L}Res_3(x,s) = \frac{h_3(x)}{s^{3\alpha+1}} & \\ - \frac{\sigma}{s^{\alpha}}\mathcal{L}\Bigg\{\Bigg(\mathcal{L}^{-1}\Bigg(\frac{h'(x)}{s} + \frac{{h_1}'(x)}{s^{\alpha+1}} + \frac{{h_2}'(x)}{s^{2\alpha+1}} & \\ + \frac{{h_3}'(x)}{s^{3\alpha+1}}\Bigg)\mathcal{L}^{-1}\Bigg(\frac{h_1(x)}{s^{\alpha+1}} + \frac{h(x)}{s} + \frac{h_2(x)}{s^{1+2\alpha}} + \frac{h_3(x)}{s^{1+3\alpha}}\Bigg)\Bigg)\Bigg\} & \\ - \frac{\eta}{s^{\alpha}}\Bigg(\frac{h^{(3)}(x)}{s} + \frac{{h_1}^{(3)}(x)}{s^{\alpha+1}} + \frac{{h_2}^{(3)}(x)}{s^{2\alpha+1}} + \frac{{h_3}^{(3)}(x)}{s^{3\alpha+1}}\Bigg) & \\ + \frac{\gamma}{s^{\alpha}}\Bigg(\frac{h^{(5)}(x)}{s} + \frac{{h_1}^{(5)}(x)}{s^{\alpha+1}} + \frac{{h_2}^{(5)}(x)}{s^{2\alpha+1}} + \frac{{h_3}^{(5)}(x)}{s^{3\alpha+1}}\Bigg). & \end{aligned} \tag{3.21}$$

As a result, the second approximations of (3.21) was as follows:

$$
\begin{aligned}
\mathcal{L}Res_3(x,s) = {} & \gamma h^{(5)}(x)s^{-\alpha-1} - \eta h^{(3)}(x)s^{-\alpha-1} - \sigma h\,(x)s^{-\alpha-1}h'(x) \\
& - \sigma h_1(x)s^{-2\alpha-1}h'(x) - \sigma h_2(x)s^{-3\alpha-1}h'(x) \\
& - \sigma h_3(x)s^{-4\alpha-1}h'(x) + \gamma {h_1}^{(5)}(x)s^{-2\alpha-1} + \gamma {h_2}^{(5)}(x)s^{-3\alpha-1} \\
& + \gamma {h_3}^{(5)}(x)s^{-4\alpha-1} - \eta {h_1}^{(3)}(x)s^{-2\alpha-1} - \eta {h_2}^{(3)}(x)s^{-3\alpha-1} \\
& - \eta {h_3}^{(3)}(x)s^{-4\alpha-1} - \sigma h\,(x)s^{-2\alpha-1}{h_1}'(x) \\
& - \sigma h\,(x)s^{-3\alpha-1}{h_2}'(x) - \sigma h\,(x)s^{-4\alpha-1}{h_3}'(x) \\
& - \frac{\sigma\Gamma\,(2\alpha+1)h_1(x)s^{-3\alpha-1}{h_1}'(x)}{\Gamma\,(\alpha+1)^2} \\
& - \frac{\sigma\Gamma\,(3\alpha+1)h_2(x)s^{-4\alpha-1}{h_1}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(2\alpha+1)} \\
& - \frac{\sigma\Gamma\,(3\alpha+1)h_1(x)s^{-4\alpha-1}{h_2}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(2\alpha+1)} \\
& - \frac{\sigma\Gamma\,(4\alpha+1)h_2(x)s^{-5\alpha-1}{h_2}'(x)}{\Gamma\,(2\alpha+1)^2} \\
& - \frac{\sigma\Gamma\,(4\alpha+1)h_3(x)s^{-5\alpha-1}{h_1}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(3\alpha+1)} \\
& - \frac{\sigma\Gamma\,(4\alpha+1)h_1(x)s^{-5\alpha-1}{h_3}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(3\alpha+1)} \\
& - \frac{\sigma\Gamma\,(5\alpha+1)h_3(x)s^{-6\alpha-1}{h_2}'(x)}{\Gamma\,(2\alpha+1)\Gamma\,(3\alpha+1)} \\
& - \frac{\sigma\Gamma\,(5\alpha+1)h_2(x)s^{-6\alpha-1}{h_3}'(x)}{\Gamma\,(2\alpha+1)\Gamma\,(3\alpha+1)} \\
& - \frac{\sigma\Gamma\,(6\alpha+1)h_3(x)s^{-7\alpha-1}{h_3}'(x)}{\Gamma\,(3\alpha+1)^2} + h_3(x)s^{-3\alpha-1}
\end{aligned} \tag{3.22}
$$

where s is greater than 0. Multiplying each of the parts of (3.22) by $s^{1+3\alpha}$ yields:

$$
\begin{aligned}
&s^{1+3\alpha}\mathcal{L}Res_3(x,s)\\
&= -\sigma h_2(x)h'(x) - \sigma h_3(x)s^{-\alpha}h'(x) + \gamma {h_2}^{(5)}(x)\\
&+ \gamma {h_3}^{(5)}(x)s^{-\alpha} - \eta {h_2}^{(3)}(x) - \eta {h_3}^{(3)}(x)s^{-\alpha} - \sigma h\,(x){h_2}'(x)\\
&- \sigma h\,(x)s^{-\alpha}{h_3}'(x) - \frac{\sigma\Gamma\,(2\alpha+1)h_1(x){h_1}'(x)}{\Gamma\,(\alpha+1)^2}\\
&- \frac{\sigma\Gamma\,(3\alpha+1)h_2(x)s^{-\alpha}{h_1}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(2\alpha+1)}\\
&- \frac{\sigma\Gamma\,(3\alpha+1)h_1(x)s^{-\alpha}{h_2}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(2\alpha+1)}\\
&- \frac{\sigma\Gamma\,(4\alpha+1)h_2(x)s^{-2\alpha}{h_2}'(x)}{\Gamma\,(2\alpha+1)^2}\\
&- \frac{\sigma\Gamma\,(4\alpha+1)h_3(x)s^{-2\alpha}{h_1}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(3\alpha+1)}\\
&- \frac{\sigma\Gamma\,(4\alpha+1)h_1(x)s^{-2\alpha}{h_3}'(x)}{\Gamma\,(\alpha+1)\Gamma\,(3\alpha+1)}\\
&- \frac{\sigma\Gamma\,(5\alpha+1)h_3(x)s^{-3\alpha}{h_2}'(x)}{\Gamma\,(2\alpha+1)\Gamma\,(3\alpha+1)}\\
&- \frac{\sigma\Gamma\,(5\alpha+1)h_2(x)s^{-3\alpha}{h_3}'(x)}{\Gamma\,(2\alpha+1)\Gamma\,(3\alpha+1)}\\
&- \frac{\sigma\Gamma\,(6\alpha+1)h_3(x)s^{-4\alpha}{h_3}'(x)}{\Gamma\,(3\alpha+1)^2}\\
&+ h_3(x)
\end{aligned}
\tag{3.23}
$$

Additionally, we can rapidly determine the outcome of by calculating this equation and applying the limits as s approach to ∞ from each side of (3.23), we have

$$
\begin{aligned}
h_3(x) &= \sigma h_2(x)h'(x) + \eta {h_2}^{(3)}(x) + \sigma\, h(x)\, {h_2}'(x)\\
&+ \frac{\sigma\Gamma\,(2\alpha+1)h_1(x){h_1}'(x)}{\Gamma\,(\alpha+1)^2} - \gamma {h_2}^{(5)}(x).
\end{aligned}
\tag{3.24}
$$

Thus, the outcome of (3.4), (3.16), (3.20) and (3.24) in infinite series structure is:

$$U(x,s) = \frac{h(x)}{s} + \frac{(\eta h^{(3)}(x) + \sigma\, h(x)\, h'(x) - \gamma h^{(5)}(x))}{s^{1+\alpha}} + \frac{\left(\sigma h_1(x) h'(x) + \eta {h_1}^{(3)}(x) + \sigma h\,(x) {h_1}'(x) - \gamma {h_1}^{(5)}(x)\right)}{s^{1+2\alpha}} + \frac{\left(\sigma h_2(x) h'(x) + \eta {h_2}^{(3)}(x) + \sigma\, h(x)\, {h_2}'(x) + \frac{\sigma \Gamma\,(2\alpha+1) h_1(x) {h_1}'(x)}{\Gamma\,(\alpha+1)^2} - \gamma {h_2}^{(5)}(x)\right)}{s^{1+3\alpha}} + \cdots \quad . \tag{3.25}$$

Using Equation (2.3) and the inverse Laplace transformation for (3.25) in infinite series structure yields the LRPS result for Equations (3.1) and (3.2) as follows

$$u(x,t) = \frac{h(x)}{s} + \frac{(\eta h^{(3)}(x) + \sigma\, h(x)\, h'(x) - \gamma h^{(5)}(x))}{\Gamma(1+\alpha)} t^{\alpha} + \frac{\left(\sigma h_1(x) h'(x) + \eta {h_1}^{(3)}(x) + \sigma h\,(x) {h_1}'(x) - \gamma {h_1}^{(5)}(x)\right)}{\Gamma(1+2\alpha)} t^{2\alpha} + \frac{\left(\sigma h_2(x) h'(x) + \eta {h_2}^{(3)}(x) + \sigma\, h(x)\, {h_2}'(x) + \frac{\sigma \Gamma\,(2\alpha+1) h_1(x) {h_1}'(x)}{\Gamma\,(\alpha+1)^2} - \gamma {h_2}^{(5)}(x)\right)}{\Gamma(1+3\alpha)} t^{3\alpha} + \cdots.$$

4. RESULTS AND DISCUSSION

The current section looks at two FKDEs situations to show the usefulness and implementation of the proposed method.

Application 1: Consider the following formula of FKDEs:

$$D_t^{\alpha} u_t(x,t) + u(x,t)u_x(x,t) + 5u_{xxx}(x,t) - 2u_{xxxxx}(x,t) = 0, \quad (4.1)$$

$$0 < \alpha \leq 1, \mathrm{t} \geq 0, x \in I \subseteq \mathbb{R}.$$

and the original situation

$$u(x,0) = \sin x \quad (4.2)$$

First, convert the result to Laplace space using the transformation of Laplace to Equation (4.1) by doing the following

$$\mathcal{L}\{D_t^{\alpha} u_t(x,t)\} + \mathcal{L}\{u(x,t)u_x(x,t) + 5u_{xxx}(x,t) - 2u_{xxxxx}(x,t)\} = 0. \quad (4.3)$$

Considering the circumstances and using the formula $\mathcal{L}\{D_t^{\alpha} u(x,t)\} = s^{\alpha} U(x,s) - s^{\alpha-1} u(x,0)$, where $\sigma = 1, \eta = 5, \gamma = 2$, we may construct (4.3) as follows by Lemma 2.3.5

$$\begin{aligned} s^{\alpha} U(x,s) - s^{\alpha-1}\, u(x,0) + \mathcal{L}\left\{\left(\mathcal{L}^{-1}\big(U_x(x,s)\big)\mathcal{L}^{-1} U(x,s)\right)\right\} + 5U_{xxx}(x,s) \\ - 2U_{xxxxx}(x,s)\,, \qquad s > 0. \end{aligned}$$

(4.4)

By multiplying (4.4) by s^{α} and using the initial conditions of (4.4), we get the new equation as follows

$$U(x,s) = \frac{\sin x}{s} + \frac{1}{s^\alpha}\mathcal{L}\left\{\left(\mathcal{L}^{-1}\left(U_x(x,s)\right)\mathcal{L}^{-1}U(x,s)\right)\right\} + \frac{5}{s^\alpha}U_{xxx}(x,s) - \frac{2}{s^\alpha}U_{xxxxx}(x,s), s > 0, \tag{4.5}$$

where $U(x,s) = \mathcal{L}\{u(x,t)\}$, $U_{xxx}(x,s) = \mathcal{L}\{u_{xxx}(x,t)\}$ and $U_{xxxxx}(x,s) = \mathcal{L}\{u_{xxxxx}(x,t)\}$.

When using the LRPS method, the series of Laplace result of (4.5) has the following form:

$$U(x,s) = \sum_{n=0}^{\infty} \frac{h_i(x)}{s^{1+n\alpha}}, s > 0. \tag{4.6}$$

Proposition 2.3.8 allows us to rewrite Equation (4.6) as follows:

$$U(x,s) = \frac{\sin x}{s} + \sum_{n=1}^{\infty} \frac{h_i(x)}{s^{1+n\alpha}}, s > 0. \tag{4.7}$$

The following represents the kth-truncated sequence of $U(x,s)$ in (4.7):

$$U_k(x,s) = \frac{\sin x}{s} + \sum_{n=1}^{k} \frac{h_i(x)}{s^{1+n\alpha}} \text{ }, s > 0. \tag{4.8}$$

The first approximation is recognized as follows:

$$U_1(x,s) = \frac{\sin x}{s} + \frac{h_1(x)}{s^{1+\alpha}} \text{ }, s > 0. \tag{4.9}$$

To determine the quantity of the unidentified parameter $h(x)$ in (4.9), it's necessary to establish the primary LRPS methods, including the LRF of (4.5).

$$\mathcal{L}Res(x,s) = U_1(x,s) - \frac{\sin x}{s} - \frac{1}{s^\alpha}\mathcal{L}\left\{\left(\mathcal{L}^{-1}\left(U_x(x,s)\right)\mathcal{L}^{-1}U(x,s)\right)\right\} - \frac{5}{s^\alpha}U_{xxx}(x,s) + \frac{2}{s^\alpha}U_{xxxxx}(x,s), s > 0, \tag{4.10}$$

as well as the kth-LRF of (4.10) as:

$$\mathcal{L}Res_k(x,s) = U_k(x,s) - \frac{\sin x}{s} - \frac{1}{s^\alpha}\mathcal{L}\left\{\left(\mathcal{L}^{-1}\left(U_{x(k)}(x,s)\right)\mathcal{L}^{-1}U_k(x,s)\right)\right\} - \frac{5}{s^\alpha}U_{xxx(k)}(x,s) + \frac{2}{s^\alpha}U_{xxxxx(k)}(x,s), s > 0. \tag{4.11}$$

Putting the k-th Laplace consulting method of (4.4) into the k-th LRPS of (4.3) yields the values $h_n(x)$ in the expanded (4.4). Subsequently, by computing the resulting formulas $\lim_{s\to\infty} s^{k\alpha+1}\mathcal{L}Res_k\ (U_1\ (x,s)) = 0$ and $\lim_{s\to\infty} s^{k\alpha+1}\mathcal{L}\mathrm{Res}^k\ (U_2(x,s)) = 0$ for $k = 1,2,3,\cdots$, we find a summary of (4.5) three parameters are displayed:

$$\begin{aligned}
h_1(x) &= -5\cos x + \sin x\cos x - 2\cos x = -7\cos x + \sin x\cos x \\
h_2(x) &= -2\cos x + \frac{3}{4}(-47\sin x + \sin 3x - 36\cos\ 2x) \\
h_3(x) &= \frac{1}{4}((7\sin x + \cos 2x)(-8\sin x + 729\cos 3x) + 3(2304\sin x - 47\cos x)(\sin 2x \\
&\quad - 14\cos x)\frac{1}{\Gamma\ (\alpha+1)} + 5(-8\sin x + 864\sin 2x - 141\cos x \\
&\quad + 81\cos 3x) + \sin x(8(\sin x - 27\sin 2x) + 141\cos x - 9\cos 3x) \\
&\quad - 2\cos x(8\cos x + 3(-47\sin x + \sin 3x - 36\cos 2x)))
\end{aligned} \tag{4.12}$$

Consequently, the sequence of Laplace outcomes to (4.3) as well as (4.12) will be as follows:

$$U(x,s)=\frac{sinx}{s}+\frac{(-5\cos x+\sin x\cos x-2cosx=-7\cos x+\sin x\cos x)}{s^{1+\alpha}}$$
$$+\frac{\left(-2\cos x+\frac{3}{4}(-47\sin x+\sin 3x-36\cos\ 2x)\right)}{s^{1+2\alpha}}$$
$$+\left(\frac{1}{4}((7\sin x+\cos 2x)(-8\sin x+729\cos 3x+3(2304\sin x\right.$$
$$-47)\cos x)(\sin 2x-14\cos x)\frac{1}{\Gamma\ (\alpha+1)}+5(-8\sin x+864\sin 2x$$
$$-141\cos x+81\cos 3x)+8\sin x(\sin x-27\sin 2x)+141\cos x-9\cos\ 3x)$$
$$\left.-2\cos x(8\cos x+3(-47\sin x+\sin 3x-36\cos 2x)))\right)\frac{1}{s^{1+3\alpha}}+\cdots\quad. \tag{4.13}$$

Finally, through using the inverse of the Laplace transformation to an enlarged series (4.7), we can determine the LFPS outcomes of (4.13).

$$u(x,t)=\sin x+\frac{(-7\cos x+\sin x\cos x)\,t^{\alpha}}{\Gamma(1+\alpha)}$$
$$+\frac{\left(-2\cos x+\frac{3}{4}(-47\sin x+\sin 3x-36\cos 2x)\right)t^{2\alpha}}{\Gamma(1+2\alpha)}$$
$$+\left(\left(\frac{1}{4}((7\sin x+\cos 2x)(-8\sin x+729\cos 3x+3(2304\sin x\right.\right.$$
$$-47)\cos x)(\sin 2x-14\cos x)\frac{1}{\Gamma(\alpha+1)}+5(-8\sin x+864\sin 2x \tag{4.14}$$
$$-141\cos x+81\cos 3x)+\sin x(8(\sin x-27\sin 2x)+141\cos x$$
$$-9\cos 3x)-2\cos x(8\cos x+3(-47\sin x+\sin 3x$$
$$\left.\left.-36\cos 2x)))\right)\right)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)}+\cdots$$

Upon obtaining $\alpha=1$ in (4.14), the precise outcomes are outlined below:

$$u(x,t)=\frac{1}{24}t^3((7\sin x+\cos 2x)(-8\sin x+729\cos 3x+3(2304\sin x-47)\cos x)(\sin 2x-14\cos x)+5(-8\sin x+864\sin 2x-141\cos x+81\cos 3x)+\sin x(8(\sin x-27\sin 2x)+141\cos x-9\cos 3x)-2\cos x(8\cos x+3(-47\sin x+\sin 3x-36\cos 2x)))+\frac{1}{2}t^2(\frac{3}{4}(-47\sin x+\sin 3x-36\cos 2x)-2\cos x)+t(\sin x\cos x-7\cos x)+\sin x+\cdots \tag{4.15}$$

A brief description of the numerical results involving the generated conclusions of (4.1), (4.2), and (4.9) is given in Table 4.1 and Table 4.2. Moreover, the real error value $\left|u_i(x,t)-u_i^3(x,t)\right|$ is determined for t_i. Additionally, Figures 4.1 through 4.6 illustrate how precisely the outcomes of the LFPS behave in comparison to different parameters of $\alpha=1, 0.90, 0.80, 0.75, 0.60$ and α =0.50. The visualizations show that the precise and approximating solutions match each other. The success of our method is illustrated using only five objects from the LFPS approximations findings. Naturally, the outcome that follows is superior.

The results generated via the present strategy are the same as those produced by the approach of Adomian decomposition (Li and Pang 2020) and Atangana-Baleanu derivative operator technique (Dhaigude and Bhadgaonkar 2021).

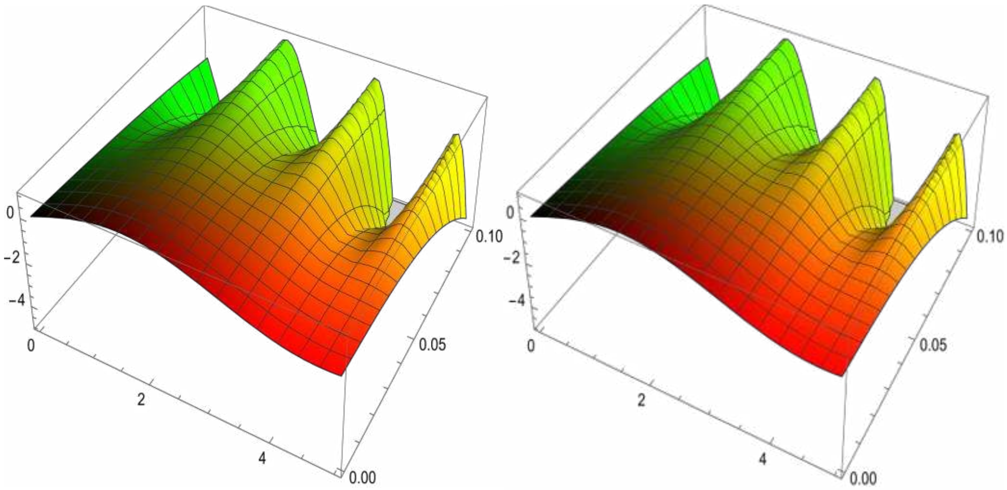
Table 4.1 Comparing digitally the exact value for $u(x,t)$ and the 3rd estimate for $u(x,t)$ at $\alpha = 1$.

$\boldsymbol{x}$	$\boldsymbol{t}$	$\boldsymbol{u(x,t)}$ − Approximation	$\boldsymbol{u(x,t)}$ − exact	Abs. Err.
1	0.01	0.803807	0.803807	2.1684×10^{-19}
2.5		0.64264	0.64264	4.33681×10^{-19}
5		-0.982315	-0.982315	4.33681×10^{-19}
1	0.04	0.473995	0.473995	3.46945×10^{-18}
2.5		0.411563	0.411563	7.63278×10^{-17}
5		-1.23664	-1.23664	1.38778×10^{-17}
1	0.07	-0.611919	-0.611919	1.11022×10^{-16}
2.5		-1.09508	-1.09508	0
5		-2.22498	-2.22498	2.77556×10^{-17}
1	0.1	-3.00792	-3.00792	1.38778×10^{-17}
2.5		-4.8161	-4.8161	1.02696×10^{-15}
5		-4.53588	-4.53588	5.55112×10^{-17}

Table 4.2 Comparing digitally the exact value for $u(x,t)$ and the 3rd estimate for $u(x,t)$ at $\alpha = 1$ and $\alpha = 0.90$

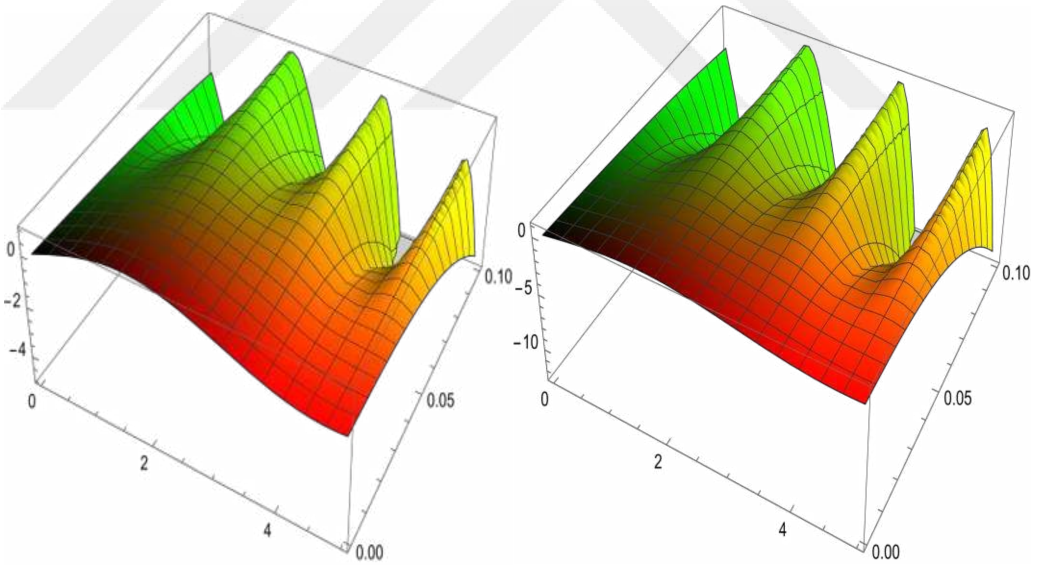
$\boldsymbol{x}$	$\boldsymbol{t}$	$\boldsymbol{u(x,t)}$ − Approximation	$\boldsymbol{u(x,t)}$ −exact	Abs. Err.
1	0.01	0.763338	0.803807	0.0404689
2.5		0.644547	0.64264	0.00190664
5		-1.00929	-0.982315	0.0269714
1	0.04	-0.245803	0.473995	0.661674
2.5		-0.611008	0.411563	1.02257
5		-1.89832	-1.23664	0.661674
1	0.07	0.139997	-0.611919	2.86074
2.5		-3.47266	-1.09508	4.54704
5		-5.02641	-2.22498	2.80143
1	0.1	-9.99828	-3.00792	6.99037
2.5		-16.2591	-4.8161	11.443
5		-11.5498	-4.53588	7.01393

The performance of the LFPS findings with different parameters of $\alpha = 1, 0.90, 0.80, 0.75, 0.60$ and 0.50 compared to the precise findings is now displayed in Figures 4.1 through 4.6.



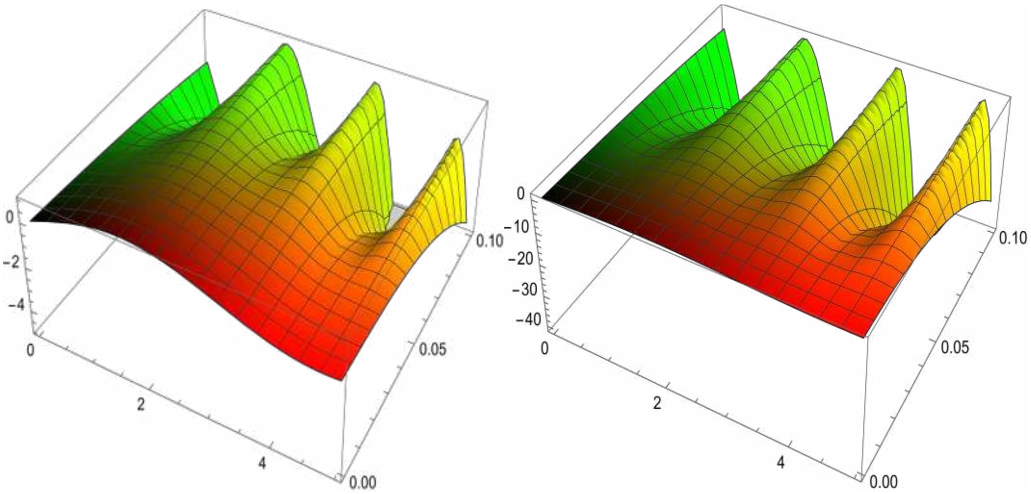
(a) exact result at $\alpha = 1$ (b) approximat result at $\alpha = 1$

Figure 4.1 The 3[rd] approximations of $\boldsymbol{u(x,t)}$ appears graphically with the exact outcome at values $\boldsymbol{\alpha = 1}$ particularly.



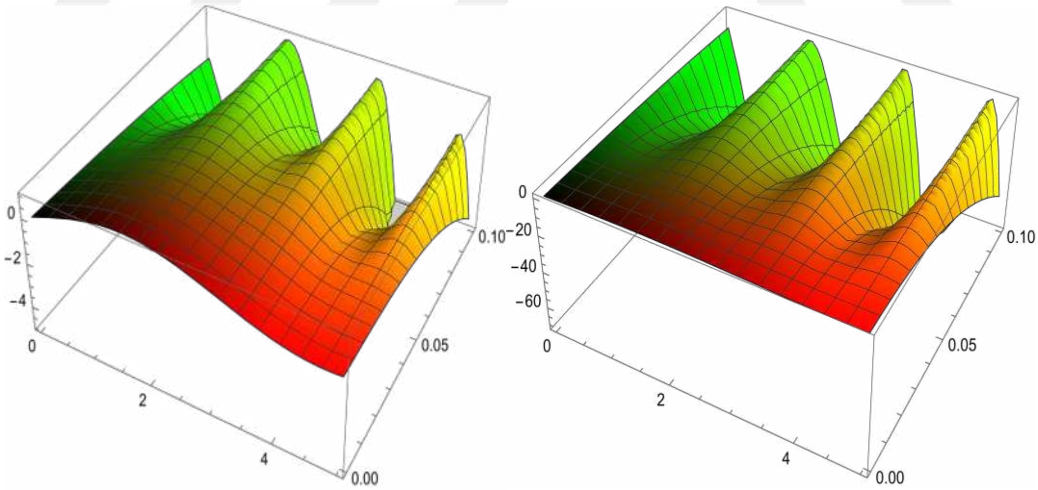
(a) exact result at $\alpha = 1$ (b) approximat result at $\alpha = 0.90$

Figure 4.2 The 3[rd] approximations of $\boldsymbol{u(x,t)}$ appears graphically with the exact outcome at values $\boldsymbol{\alpha = 1 \text{ and } 0.90}$ particularly.



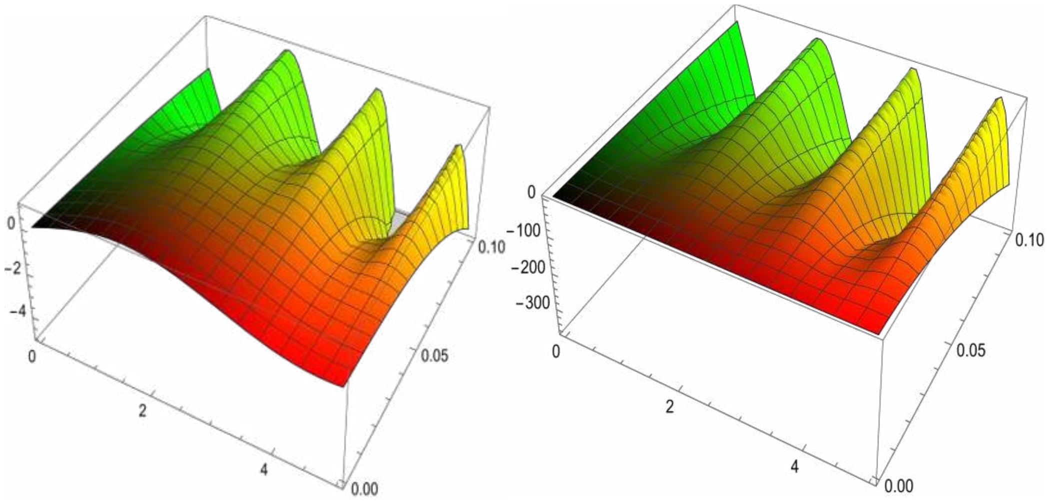
$(\mathbf{a})$ **exact result at** $\boldsymbol{\alpha = 1}$ $(\mathbf{b})$ **approximat result at** $\boldsymbol{\alpha = 0.80}$

Figure 4.3 The 3rd approximations of $\boldsymbol{u(x,t)}$ appears graphically with the exact outcome at values $\boldsymbol{\alpha = 1}$ **and** $\mathbf{0.80}$ particularly.



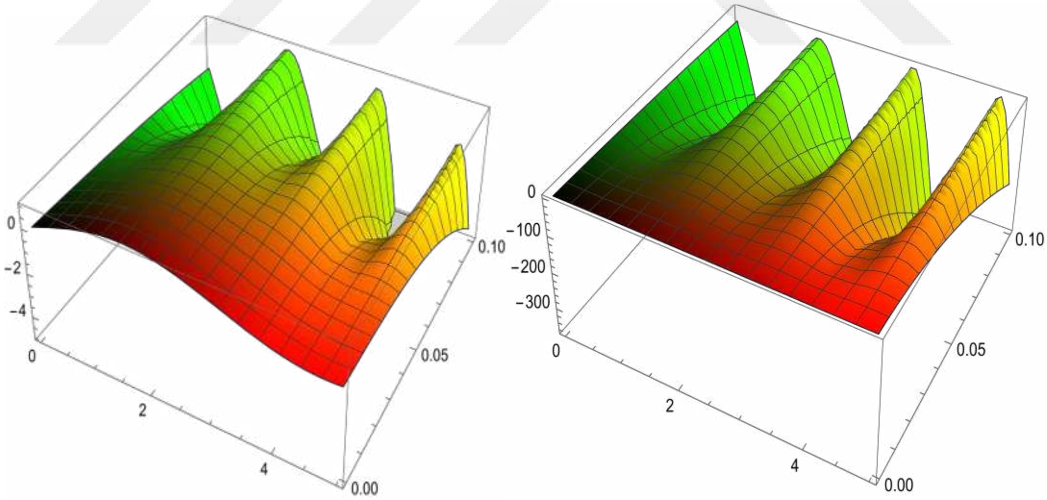
$(\mathbf{a})$ **exact result at** $\boldsymbol{\alpha = 1}$ $(\mathbf{b})$ **approximat result at** $\boldsymbol{\alpha = 0.75}$

Figure 4.4 The 3rd approximations of $\boldsymbol{u(x,t)}$ appears graphically with the exact outcome at values $\boldsymbol{\alpha = 1}$ **and** $\mathbf{0.75}$ particularly.



$(\mathbf{a})$ **exact result at** $\boldsymbol{\alpha = 1}$ $(\mathbf{b})$ **approximat result at** $\boldsymbol{\alpha = 0.60}$

Figure 4.5 The 3rd approximations of $\boldsymbol{u(x,t)}$ appears graphically with the exact outcome at values $\boldsymbol{\alpha = 1}$ **and** $\mathbf{0.60}$ particularly.



$(\mathbf{a})$ **exact result at** $\boldsymbol{\alpha = 1}$ $(\mathbf{b})$ **approximat result at** $\boldsymbol{\alpha = 0.50}$

Figure 4.6 The 3rd approximations of $\boldsymbol{u(x,t)}$ appears graphically with the exact outcome at values $\boldsymbol{\alpha = 1}$ **and** $\mathbf{0.50}$ particularly.

Application 2 : Consider the following formula of FKDEs:

$$D_t^{\alpha} u_t(x,t) + 2u(x,t)u_x(x,t) + u_{xxx}(x,t) = 0, \tag{4.16}$$

$$0 < \alpha \leq 1, \mathrm{t} \geq 0, x \in I \subseteq \mathbb{R}.$$

and the original situation

$$u(x,0) = e^x. \tag{4.17}$$

First, convert the result to Laplace space using the transformation of Laplace to Equation (4.16) by doing the following:

$$\mathcal{L}\{D_t^{\alpha} u_t(x,t)\} + \mathcal{L}\{2u(x,t)u_x(x,t) + u_{xxx}(x,t)\} = 0. \tag{4.18}$$

Considering the circumstances Additionally, by using the formula $\mathcal{L}\{D_t^{\alpha} u(x,t)\} = s^{\alpha} U(x,s) - s^{\alpha-1} u(x,0)$, and $\sigma = 2, \eta = 1, \gamma = 0$. We may construct (4.18), as following using Lemma 2.3.5:

$$s^{\alpha} U(x,s) - s^{\alpha-1}\, u(x,0) + 2\mathcal{L}\left\{\left(\mathcal{L}^{-1}\big(U_x(x,s)\big)\mathcal{L}^{-1} U(x,s)\right)\right\} + U_{xxx}(x,s)\,,\ \ s > 0. \tag{4.19}$$

By multiplying (4.19) by s^{α} and using the initial conditions of (4.19), the following shape of the equation is obtained as follows:

$$U(x,s) = \frac{\mathrm{e}^x}{s} + \frac{2}{s^{\alpha}}\mathcal{L}\left\{\left(\mathcal{L}^{-1}\big(U_x(x,s)\big)\mathcal{L}^{-1} U(x,s)\right)\right\} + \frac{1}{s^{\alpha}} U_{xxx}(x,s), s > 0, \tag{4.20}$$

where $U(x,s) = \mathcal{L}\{u(x,t)\}$ and $U_{xxx}(x,s) = \mathcal{L}\{u_{xxx}(x,t)\}$.

When using the LRPS method, the series of Laplace result of (4.20) has the following form:

$$U(x,s)=\sum_{n=0}^{\infty}\frac{h_i(x)}{s^{1+n\alpha}},s>0. \tag{4.21}$$

Proposition 2.3.8 allows us to rewrite Equation (4.21) as follows:

$$U(x,s)=\frac{\mathrm{e}^x}{s}+\sum_{n=1}^{\infty}\frac{h_i(x)}{s^{1+n\alpha}},s>0. \tag{4.22}$$

The following represents the kth-truncated sequence of $U(x,s)$ in (4.22):

$$U_k(x,s)=\frac{\mathrm{e}^x}{s}+\sum_{n=1}^{k}\frac{h_i(x)}{s^{1+n\alpha}}\ ,s>0. \tag{4.23}$$

The first approximation is recognized as follows:

$$U_1(x,s)=\frac{\mathrm{e}^x}{s}+\frac{h_1(x)}{s^{1+\alpha}}\ ,s>0. \tag{4.24}$$

To determine the quantity of the unidentified parameter $h(x)$ in (4.24), it's necessary to establish the primary LRPS methods, including the LRF of (4.20).

$$\begin{aligned}\mathcal{L}Res(x,s)=U_1(x,s)-\frac{\mathrm{e}^x}{s}-\frac{1}{s^\alpha}\mathcal{L}\left\{\left(\mathcal{L}^{-1}\big(U_x(x,s)\big)\mathcal{L}^{-1}U(x,s)\right)\right\}\\-\frac{5}{s^\alpha}U_{xxx}(x,s),s>0,\end{aligned} \tag{4.25}$$

as well as the kth-LRF of (4.25) as:

$$\mathcal{L}Res_k(x,s) = U_k(x,s) - \frac{\mathrm{e}^x}{s} - \frac{2}{s^\alpha}\mathcal{L}\left\{\left(\mathcal{L}^{-1}\left(U_{x(k)}(x,s)\right)\mathcal{L}^{-1}U_k(x,s)\right)\right\} - \frac{1}{s^\alpha}U_{xxx(k)}(x,s), s > 0. \tag{4.26}$$

Putting the k-th Laplace consulting method of (4.19) into the k-th LRPS of (4.18) yields the values $h_n(x)$ in the expanded (4.19). Subsequently, by computing the resulting formulas $\lim_{s\to\infty} s^{k\alpha+1}\mathcal{L}Res_k\,(U_1\,(x,s)) = 0$ and $\lim_{s\to\infty} s^{k\alpha+1}\mathcal{L}\mathrm{Res}^k\,(U_2(x,s)) = 0$ for $k = 1,2,3,\cdots$, we find a summary of (4.20) three parameters are displayed:

$$\begin{gathered} h_1(x) = \mathrm{e}^x + 2\mathrm{e}^{2x} \\ h_2(x) = e^x(1 + 19e^x + 10e^{2x}) \\ h_3(x) = e^x(1 + 156e^x + 384e^{2x} + 80e^{3x} + \frac{2e^x(6e^x + 8e^{2x} + 1)\Gamma\,(2\alpha + 1)}{\Gamma\,(\alpha + 1)^2}) \end{gathered} \tag{4.27}$$

Consequently, the sequence of Laplace outcomes to (4.18) as well as (4.27) will be as follows:

$$\begin{aligned} U(x,s) = \frac{e^x}{s} + \frac{(e^x(1+e^x))}{s^{1+\alpha}} + \frac{(e^x(1 + 19e^x + 10e^{2x}))}{s^{1+2\alpha}} \\ + \Bigg(e^x(1 + 156e^x + 384e^{2x} + 80e^{3x} \\ + \frac{2e^x(6e^x + 8e^{2x} + 1)\Gamma\,(2\alpha + 1)}{\Gamma\,(\alpha + 1)^2})\Bigg)\frac{1}{s^{1+3\alpha}} + \cdots \quad . \end{aligned} \tag{4.28}$$

Finally, through using the inverse of the Laplace transformation to an enlarged series (4.22), we can determine the LFPS outcomes of (4.28).

$$u(x,t) = e^x + \frac{(e^x(1+e^x))t^\alpha}{\Gamma(1+\alpha)} + \frac{(e^x(1+19e^x+10e^{2x}))t^{2\alpha}}{\Gamma(1+2\alpha)} + \left(\left(e^x(1+156e^x+384e^{2x}+80e^{3x} + \frac{2e^x(6e^x+8e^{2x}+1)\Gamma\,(2\alpha+1)}{\Gamma\,(\alpha+1)^2}\right)\right)\frac{t^{3\alpha}}{\Gamma(1+3\alpha)} + \cdots \tag{4.29}$$

Upon obtaining $\alpha = 1$ in (4.29), the precise outcomes are outlined below:

$$u(x,t) = 4(e^x + 2e^{2x})(e^x + 4e^{2x}) + e^x(1+19e^x+10e^{2x}) + 2e^{2x}(1+19e^x + 10e^{2x}) + 3e^x(19e^x+20e^{2x}) + 3e^x(19e^x+40e^{2x}) + e^x(19e^x + 80e^{2x}) + 2e^x(e^x(1+19e^x+10e^{2x}) + e^x(19e^x+20e^{2x})) + \cdots \tag{4.30}$$

A brief description of the numerical results involving the generated conclusions of (4.16), (4.17), and (4.25) is given in Table 4.3 and Table 4.4. Moreover, the real error value $\left|u_i(x,t) - u_i^3(x,t)\right|$ is determined for t_i. Additionally, Figures 4.7 through 4.12 illustrate how precisely the outcomes of the LFPS behave in comparison to different parameters of $\alpha = 1$ and α =0.90. The visualizations show that the precise and approximating solutions match each other. The success of our method is illustrated using only five objects from the LFPS approximations findings. Naturally, the outcome that follows is superior.

The results generated via the present strategy are the same as those produced by the approach of Adomian decomposition (Li and Pang 2020) and Atangana-Baleanu derivative operator technique (Dhaigude and Bhadgaonkar 2021).

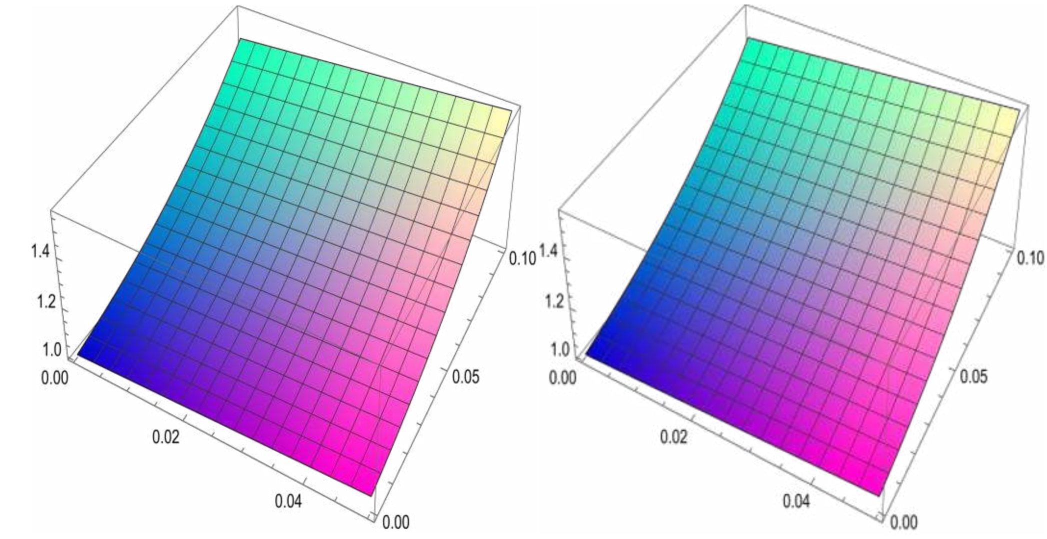
Table 4.3 Comparing numerically the 5th estimation for $u(x,t)$ at $\alpha = 1$ and the precise value for $u(x,t)$

x	t	$u(x,t)$ − Approximation	$u(x,t)$ − exact	Abs. Err.
0.01	0.01	1.032	1.032	0
0.03		1.05311	1.05311	0
0.05		1.07465	1.07465	0
0.01	0.04	1.1233	1.1233	0
0.03		1.1478	1.1478	0
0.05		1.17288	1.17288	0
0.01	0.07	1.26747	1.26747	0
0.03		1.29819	1.29819	0
0.05		1.3298	1.3298	0
0.01	0.1	1.48344	1.48344	0
0.03		1.52436	1.52436	5.55112×10^{-17}
0.05		1.56671	1.56671	1.11022×10^{-16}

Table 4.4 Comparing numerically the 5th estimation for $u(x,t)$ at $\alpha = 0.90$ and the precise value for $u(x,t)$

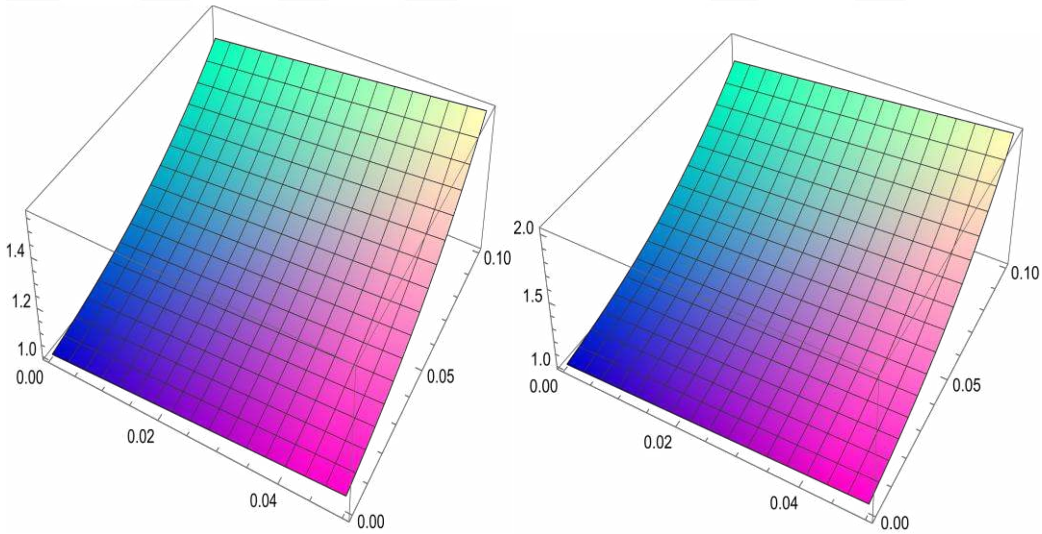
x	t	$u(x,t)$ − approximation	$u(x,t)$ −exact	Abs. Err.
0.01	0.01	1.04877	1.032	0.0167655
0.03		1.07045	1.05311	0.0173451
0.05		1.0926	1.07465	0.0179476
0.01	0.04	1.21035	1.1233	0.087055
0.03		1.23864	1.1478	0.0908459
0.05		1.2677	1.17288	0.0948252
0.01	0.07	1.48255	1.26747	0.215085
0.03		1.52372	1.29819	0.225532
0.05		1.56634	1.3298	0.236546
0.01	0.1	1.89871	1.48344	0.415265
0.03		1.961	1.52436	0.436647
0.05		2.02594	1.56671	0.459239

Now, Figure 4.7 – Figure 4.12 show the behavior of LFPS results with various constants of $\alpha = 1, 0.90, 0.80, 0.75, 0.60$ and 0.50 versus the exact results.



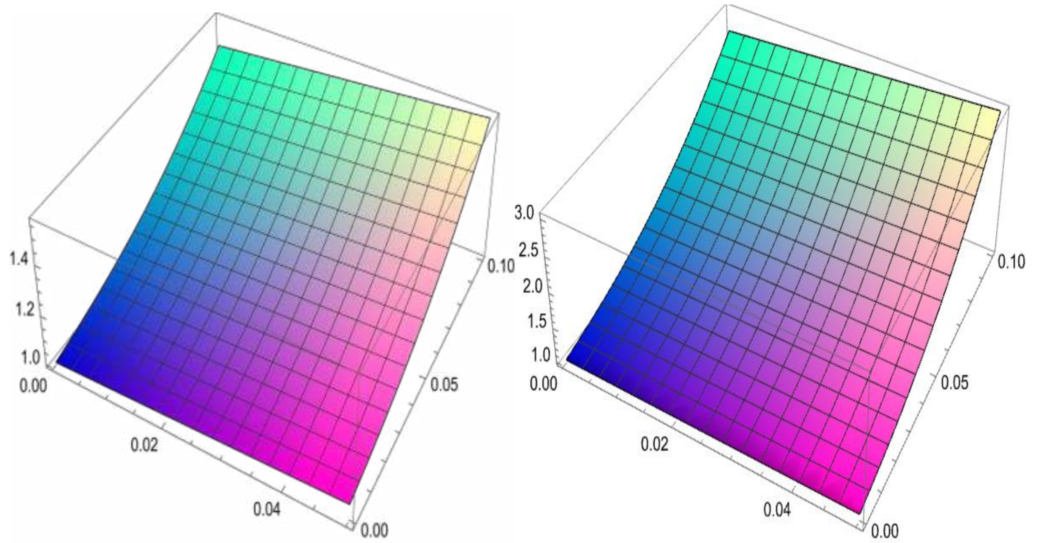
(a) exact result at $\alpha = 1$ (b) approximat result at $\alpha = 1$

Figure 4.7 The 3rd estimations of $\boldsymbol{u(x,t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 1}$.



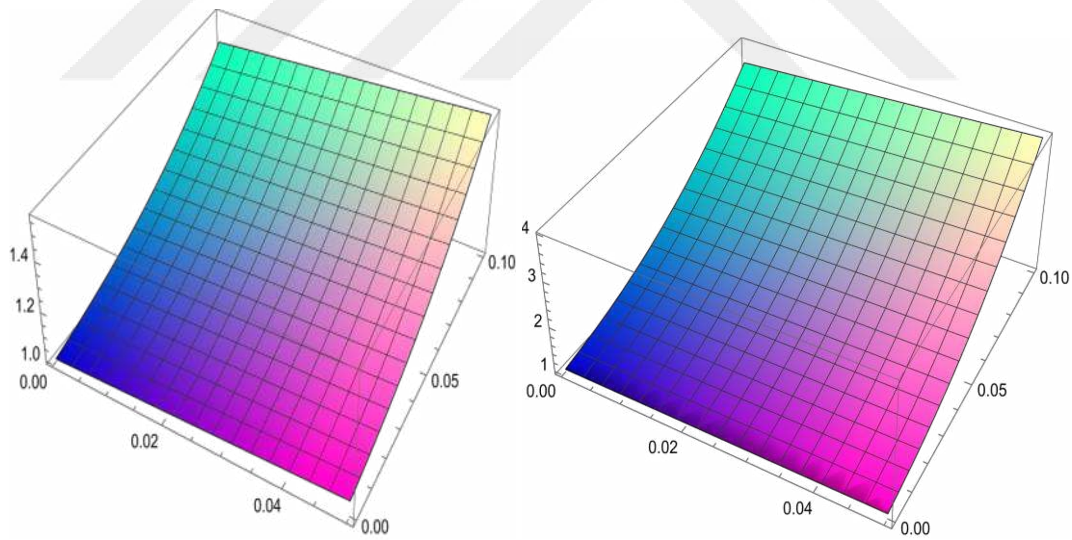
(a) exact result at $\alpha = 1$ (b) approximat result at $\alpha = 0.90$

Figure 4.8 The 3rd estimations of $\boldsymbol{u(x,t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 1 \text{ and } 0.90}$.



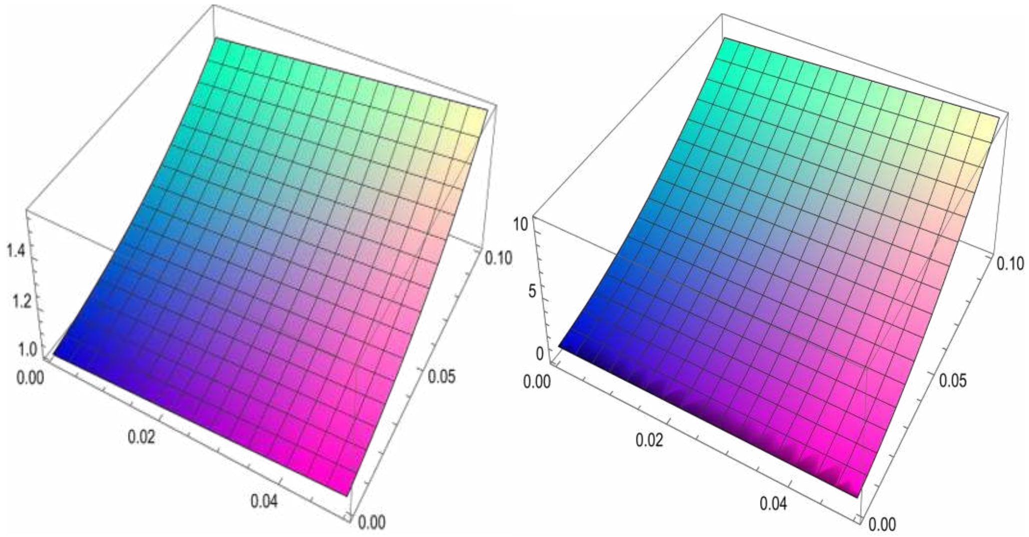
$(\mathbf{a})$ $\mathbf{exact\ result\ at}\ \boldsymbol{\alpha = 1}$ $(\mathbf{b})$ $\mathbf{approximat\ result\ at}\ \boldsymbol{\alpha = 0.80}$

Figure 4.9 The 5th estimations of $\boldsymbol{u(x,t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 1}$ **and** $\boldsymbol{0.80}$.



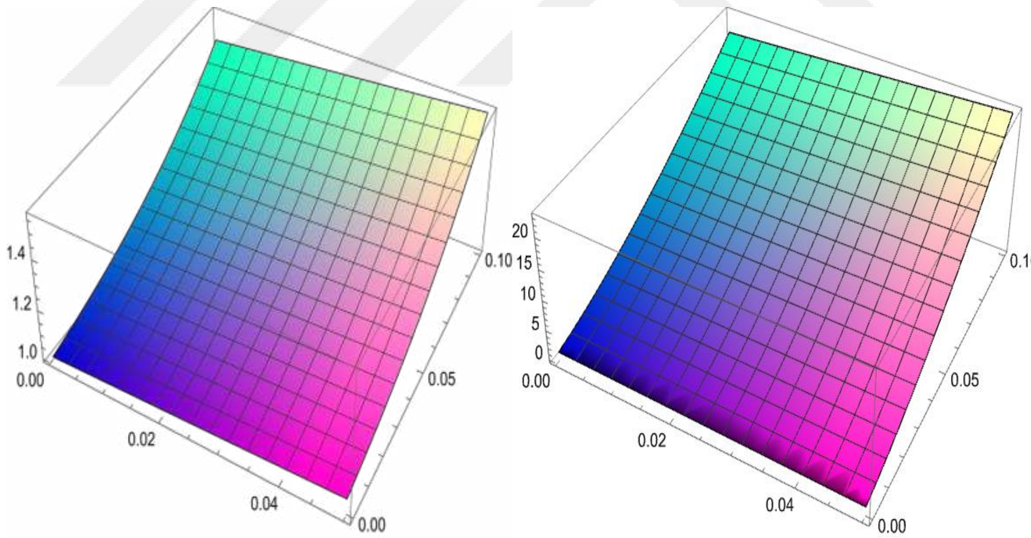
$(\mathbf{a})$ $\mathbf{exact\ result\ at}\ \boldsymbol{\alpha = 1}$ $(\mathbf{b})$ $\mathbf{approximat\ result\ at}\ \boldsymbol{\alpha = 0.75}$

Figure 4.10 The 3[rd] estimations of $\boldsymbol{u(x,t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 1}$ **and** $\boldsymbol{0.75}$



$(\mathbf{a})$ **exact result at** $\boldsymbol{\alpha = 1}$ $(\mathbf{b})$ **approximat result at** $\boldsymbol{\alpha = 0.60}$

Figure 4.11 The 3rd estimations of $\boldsymbol{u(x,t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 1}$ **and** $\mathbf{0.60}$



$(\mathbf{a})$ **exact result at** $\boldsymbol{\alpha = 1}$ $(\mathbf{b})$ **approximat result at** $\boldsymbol{\alpha = 0.50}$

Figure 4.12 The 3rd estimations of $\boldsymbol{u(x,t)}$ surfaces graphing, as well as the precise result for value $\boldsymbol{\alpha = 1}$ **and** $\mathbf{0.50}$

5. CONCLUSIONS AND RECOMMENDATION

5.1 Conclusions

The LRPS strategy is considered as an analytical tool to solve certain types of FPDEs almost exactly. This process is based on constructing expanded series using the Laplace transform, where the concept of the limit at infinity is used to determine the coefficients of the series. When it comes to overcoming FPDEs, the LRPS strategy has various benefits. Initially of all, given appropriate initial circumstances, it offers precise outcomes for particular kinds of FPDEs. It also enables the production of approximations solutions by varying the amount of series parts utilized. The trade-off involving accurate and processing complication is made possible by the versatility in the number of parts utilized. A numerical and visual study is carried out to evaluate the LRPS methodology's efficacy by examining how the produced results behave under various scenarios. This analysis aids in assessing the precision and dependability of the approximations produced by the LRPS technique. Whenever α (the fractions derivative's ordering) has been set to 1, the numerical findings show that the approximations produced through the LRPS technique match very well with exact solutions. Additionally, the approximations derived utilizing the LRPS technique are often near to each other. According to this result, the LRPS technique is a good and efficient way to get approximation and analytical approaches for a variety of real-world issues including fractional formulas in various fields of science. All things considered, the LRPS approach provides a simple analytical approach that can yield precise answers for some kinds of FPDEs as well as the capacity to produce approximations by adjusting the number of series constituents. It is an invaluable tool for solving real-world issues with fractional formulas because of its efficiency and adaptability.

5.2 Recommendation

Following provide a few suggestions and possible future paths for the LRPS technique and its use for resolving FPDEs according to the data presented in the advancement:

1- Validation and Comparisons: It's essential to verify the LRPS methodology's efficacy by contrasting its outcomes with those of other analytically and numeric FPDE problem-solving methods already in use. This can assist in evaluating its precision, effectiveness, and suitability for a variety of FPDEs, such solving both linear and nonlinear formulas.

2- Expansion to several FPDE Kinds: The LRPS technique can be investigated and expanded to address several FPDE kinds, including fractional heat formulas, fractional diffusion formulas, and fractional wave formulas. Examining the LRPS methodology's efficacy for these formulas can provide light on its possible drawbacks as well as its adaptability.

3- Parameter Optimizing: The limitation idea at infinity is used in the LRPS technique to determine the elements of an expanding series. In order to increase the precision and convergent of the results, future work can concentrate on improving these factors. This may entail creating optimization plans or procedures especially to suit the LRPS approach.

4- Numerical Execution: Although the LRPS strategy is mostly based on analysis, it can be useful to investigate its numerically application. In real-world situations, the development of numerical methods founded on the LRPS approach can yield effective mathematical instruments to solve FPDEs.

5- Applications to Real-World Issues: The applicability and efficacy of the LRPS approach can be shown by extending it to real-world issues in a range of scientific fields. This can involve addressing particular research topics or real-world problems by solving FPDEs that come up in science, technology, biological sciences, financing, and other disciplines.

6- Sensitive Analysis: By examining how parameters, beginning circumstances, and boundaries affect the LRPS approaches, sensitivity research can help us comprehend this approach and pinpoint its strengths and weaknesses in various contexts.

7- Error Evaluation: Thoroughly analyzing the LRPS results' errors can reveal information about the methodology's efficiency and convergent characteristics. To empirically evaluate the error features, this may include matching the LRPS results with established accurate remedies or benchmarking issues.

8- Development of Software: Scientists and those working in the discipline of FPDEs can find the LRPS technique easier to use by creating intuitive programs or computational resources designed for using it.

REFERENCES

Abbasbandy, S. 2010. Homotopy analysis method for the Kawahara equation. Nonlinear Analysis: Real World Applications, 11(1): 307-312.

Abdeljawad, T., Amin, R., Shah, K., Al-Mdallal, Q. and Jarad, F. 2020. Efficient sustainable algorithm for numerical solutions of systems of fractional order differential equations by Haar wavelet collocation method. Alexandria Engineering Journal, 59(4): 2391-2400.

Abu Arqub, O. 2019. Computational algorithm for solving singular Fredholm time-fractional partial integrodifferential equations with error estimates. Journal of Applied Mathematics and Computing, 59(1): 227-243.

Agarwal, R. P., Lupulescu, V., O'Regan, D. and ur Rahman, G. 2015. Fractional calculus and fractional differential equations in nonreflexive Banach spaces. Communications in Nonlinear Science and Numerical Simulation, 20(1): 59-73.

Ait Touchent, K., Hammouch, Z., Mekkaoui, T. and Belgacem, F. B. 2018. Implementation and convergence analysis of homotopy perturbation coupled with Sumudu transform to construct solutions of local-fractional PDEs. Fractal and Fractional, 2(3): 22.

Akdemir, A. O., Hemen, D. and Abdon, A. 2020. Fractional order analysis: theory, methods and applications. John Wiley & Sons, 336 Page, USA.

Alabedalhadi, M., Al-Smadi, M., Al-Omari, S., Baleanu, D. and Momani, S. 2020. Structure of optical solution for nonliear resonant space-time Schrödinger equation in conformable sense with full nonlinearity term. Physica Scripta, 95(10): 105215.

Alaroud, M., Al-Smadi, M., Ahmad, R. R. and Salma Din, U. K. 2018. Computational optimization of residual power series algorithm for certain classes of fuzzy fractional differential equations. International Journal of Differential Equations, 2018.

Alaroud, M., Al-Smadi, M., Rozita Ahmad, R. and Salma Din, U. K. 2019a. An analytical numerical method for solving fuzzy fractional Volterra integro-differential equations. Symmetry, 11(2): 205.

Alaroud, M., Ahmad, R. R. and Salma Din, U. K. 2019b. An efficient analytical-numerical technique for handling model of fuzzy differential equations of fractional-order. Filomat, 33(2): 617-632.

Ali, K. K., Maneea, M. and Mohamed, M. S. 2023. Solving nonlinear fractional models in superconductivity using the q-Homotopy analysis transform method. Journal of Mathematics, 1-23.

Al-Qurashi, M., Rashid, S., Karaca, Y., Hammouch, Z., Baleanu, D. and Chu, Y. M. 2021. Achieving more precise bounds based on double and triple integral as proposed by generalized proportional fractional operators in the Hilfer sense. Fractals, 29(05): 2140027.

Al-Smadi, M. H. and Gumah, G. N. 2014. On the homotopy analysis method for fractional SEIR epidemic model. Research Journal of Applied Sciences, Engineering and Technology, 7(18): 3809-3820.

Al-Smadi, M., Freihat, A., Arqub, O. A. and Shawagfeh, N. 2015. A Novel Multistep Generalized Differential Transform Method for Solving Fractional-order. Lü Chaotic and Hyperchaotic Systems. Journal of Computational Analysis & Applications, 19(1): 713-724.

Al-Smadi, M., Freihat, A., Hammad, M. M. A., Momani, S. and Arqub, O. A. 2016. Analytical approximations of partial differential equations of fractional order with multistep approach. Journal of Computational and Theoretical Nanoscience, 13(11): 7793-7801.

Al-Smadi, M., Freihat, A., Khalil, H., Momani, S. and Ali Khan, R. 2017. Numerical multistep approach for solving fractional partial differential equations. International Journal of Computational Methods, 14(03): 1750029.

Al-Smadi, M. 2018. Simplified iterative reproducing kernel method for handling time-fractional BVPs with error estimation. Ain Shams Engineering Journal, 9(4): 2517-2525.

Al-Smadi, M. and Arqub, O. A. 2019. Computational algorithm for solving fredholm time-fractional partial integrodifferential equations of dirichlet functions type with error estimates. Applied Mathematics and Computation, 342: 280-294.

Al-Smadi, M., Arqub, O. A.and Momani, S. 2020a. Numerical computations of coupled fractional resonant Schrödinger equations arising in quantum mechanics under conformable fractional derivative sense. Physica Scripta, 95(7): 075218.

Al-Smadi, M., Arqub, O. A. and Hadid, S. 2020b. Approximate solutions of nonlinear fractional Kundu-Eckhaus and coupled fractional massive Thirring equations emerging in quantum field theory using conformable residual power series method. Physica Scripta, 95(10): 105205.

Al-Smadi, M., Arqub, O. A. and Hadid, S. 2020c. An attractive analytical technique for coupled system of fractional partial differential equations in shallow water waves with conformable derivative. Communications in Theoretical Physics, 72(8): 085001.

Al-Smadi, M., Abu Arqub, O. and Gaith, M. 2021a. Numerical simulation of telegraph and Cattaneo fractional-type models using adaptive reproducing kernel framework. Mathematical Methods in the Applied Sciences, 44(10): 8472-8489.

Al-Smadi, M., Arqub, O. A. and Zeidan, D. 2021b. Fuzzy fractional differential equations under the Mittag-Leffler kernel differential operator of the ABC approach: Theorems and applications. Chaos, Solitons & Fractals, 146, 110891.

Andrews, L. C. 1998. Special functions of mathematics for engineers, Vol. 49. Spie Press, 512 page, New York.

Arora, G., Pant, R., Emadifar, H. and Khademi, M. 2023. Numerical solution of fractional relaxation–oscillation equation by using residual power series method. Alexandria Engineering Journal, 73: 249-257.

Atangana, A. and Baleanu, D. 2016. New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model. arXiv preprint arXiv:1602.03408.

Atangana, A. and Hammouch, Z. 2019. Fractional calculus with power law: The cradle of our ancestors. The European Physical Journal Plus, 134(9): 429.

Ateş, İ., and Yildirim, A. 2009. Application of variational iteration method to fractional initial-value problems. International Journal of Nonlinear Sciences and Numerical Simulation, 10(7): 877-884.

Atluri, S. N. and Shen, S. 2002. The meshless method. Encino: Tech Science Press,700 page, USA.

Baleanu, D., Golmankhaneh, A. K., Golmankhaneh, A. K. and Baleanu, M. C. 2009. Fractional electromagnetic equations using fractional forms. International Journal of Theoretical Physics, 48(11): 3114-3123.

Baleanu, D., Machado, J. A. T. and Luo, A. C. (Eds.). 2011. Fractional dynamics and control. Springer Science & Business Media, 310 page, London.

Başhan, A. 2021. Bell-shaped soliton solutions and travelling wave solutions of the fifth-order nonlinear modified Kawahara equation. International Journal of Nonlinear Sciences and Numerical Simulation, 22(6): 781-795.

Bira, B., Raja Sekhar, T. and Zeidan, D. 2018. Exact solutions for some time-fractional evolution equations using Lie group theory. Mathematical Methods in the Applied Sciences, 41(16): 6717-6725.

Boon, J. P., Yip, S., Burgers, J. M., Van de Hulst, H. C., Chandrasekhar, S., Chen, F. F. and Smit, B. 1988. An introduction to fluid mechanics. Cambridge, UK: Cambridge University Press, 371 page, London.

Chambré, P. L. 1952. On the solution of the Poisson-Boltzmann equation with application to the theory of thermal explosions. The Journal of Chemical Physics, 20(11):1795-1797.

Chawla, M. M. 2002. Generalized newmark schemes for singular second order initial-value problems. International Journal of Pure and Applied Mathematics, 1(3): 261-274.

Dhaigude, D. B. and Bhadgaonkar, V. N. 2021. A novel approach for fractional Kawahara and modified Kawahara equations using Atangana-Baleanu derivative operator. J. Math. Comput. Sci., 11(3): 2792-2813.

Diethelm, K. and Ford, N. J. 2002. Analysis of fractional differential equations. Journal of Mathematical Analysis and Applications, 265(2), 229-248.

Doha, E. H., Bhrawy, A. H., Baleanu, D. and Ezz-Eldien, S. S. 2013. On shifted Jacobi spectral approximations for solving fractional differential equations. Applied Mathematics and Computation, 219(15): 8042-8056.

El-Ajou, A., Arqub, O. A. and Momani, S. 2015a. Approximate analytical solution of the nonlinear fractional KdV–Burgers equation: a new iterative algorithm. Journal of Computational Physics, 293: 81-95.

El-Ajou, A., Arqub, O. A. and Al-Smadi, M. 2015b. A general form of the generalized Taylor's formula with some applications. Applied Mathematics and Computation, 256: 851-859.

El-Ajou, A. 2021. Adapting the Laplace transform to create solitary solutions for the nonlinear time-fractional dispersive PDEs via a new approach. The European Physical Journal Plus, 136(2): 1-22.

Eriqat, T., El-Ajou, A., Moa'ath, N. O., Al-Zhour, Z. and Momani, S. 2020. A new attractive analytic approach for solutions of linear and nonlinear neutral fractional pantograph equations. Chaos, Solitons & Fractals, 138: 109957.

Ertürk, V. S. and Momani, S. 2008. Solving systems of fractional differential equations using differential transform method. Journal of Computational and Applied Mathematics, 215(1): 142-151.

Gambo, Y. Y., Ameen, R., Jarad, F. and Abdeljawad, T. 2018. Existence and uniqueness of solutions to fractional differential equations in the frame of generalized Caputo fractional derivatives. Advances in Difference Equations, 2018(1): 1-13.

Ganie, A. H., Mofarreh, F. and Khan, A. 2024. A novel analysis of the time-fractional nonlinear dispersive K (m, n, 1) equations using the homotopy perturbation transform method and Yang transform decomposition method. AIMS Mathematics, 9(1): 1877-1898.

Garcia-Beristain, I. and Remaki, L. 2023. Time-adaptive Adomian decomposition-based numerical scheme for Euler equations. Numerical Methods for Partial Differential Equations, 39(1): 329-355.

Gepreel, K. A., Nofal, T. A. and Al-Asmari, A. A. 2019. Abundant travelling wave solutions for nonlinear Kawahara partial differential equation using extended trial equation method. International Journal of Computer Mathematics, 96(7): 1357-1376.

Griffel, D. H. 1991. Fourier series, transforms and boundary value problems , by JR Hanna and JH Rowland. Pp 354.£ 43· 65. 1990. ISBN 0-471-61983-3 (Wiley). The Mathematical Gazette, 75(473): 389-390.

Hasan, S., El-Ajou, A., Hadid, S., Al-Smadi, M. and Momani, S. 2020. Atangana-Baleanu fractional framework of reproducing kernel technique in solving fractional population dynamics system. Chaos, Solitons & Fractals, 133: 109624.

Hasan, S., Al-Smadi, M., El-Ajou, A., Momani, S., Hadid, S. and Al-Zhour, Z. 2021. Numerical approach in the Hilbert space to solve a fuzzy Atangana-Baleanu fractional hybrid system. Chaos, Solitons & Fractals, 143: 110506.

He, J. H. and Abdou, M. A. 2007. New periodic solutions for nonlinear evolution equations using Exp-function method. Chaos, Solitons & Fractals, 34(5): 1421-1429.

Hussain, S., Shah, A., Ullah, A. and Haq, F. 2022. The q-homotopy analysis method for a solution of the Cahn–Hilliard equation in the presence of advection and reaction terms. Journal of Taibah University for Science, 16(1): 813-819.

Jleli, M., Kumar, S., Kumar, R. and Samet, B. 2020. Analytical approach for time fractional wave equations in the sense of Yang-Abdel-Aty-Cattani via the homotopy perturbation transform method. Alexandria Engineering Journal, 59(5): 2859-2863.

Khader, M. M., Saad, K. M., Hammouch, Z. and Baleanu, D. 2021. A spectral collocation method for solving fractional KdV and KdV-Burgers equations with non-singular kernel derivatives. Applied Numerical Mathematics, 161: 137-146.

Khalil, H. and Khan, R. A. 2014. A new method based on Legendre polynomials for solutions of the fractional two-dimensional heat conduction equation. Computers & Mathematics with Applications, 67(10): 1938-1953.

Khalil, H. and Khan, R. A. 2015a. The use of Jacobi polynomials in the numerical solution of coupled system of fractional differential equations. International Journal of Computer Mathematics, 92(7): 1452-1472.

Khalil, H. and Khan, R. A. 2015b. Numerical scheme for solution of coupled system of initial value fractional order Fredholm integro differential equations with smooth solutions. Journal of Mathematical Extension, 9: 39-58.

Khan, N. A., Ahmad, S., Razzaq, O. A. and Ayaz, M. 2020. Rational approximation with cuckoo search algorithm for multifarious Painlevé type differential equations. Ain Shams Engineering Journal, 11(1): 179-190.

Kilbas, A. A., Srivastava, H. M. and Trujillo, J. J. 2006 . Theory and applications of fractional differential equations ,Vol. 204. Elsevier, 550 page, London.

Komashynska, I., Al-Smadi, M., Arqub, O. A. and Momani, S. 2016. An efficient analytical method for solving singular initial value problems of nonlinear systems. Appl. Math. Inf. Sci, 10(2): 647-656.

Kumar, S. 2013. A new fractional modeling arising in engineering sciences and its analytical approximate solution. Alexandria Engineering Journal, 52(4): 813-819.

Kumar, S., Kumar, A., Samet, B. and Dutta, H. 2021. A study on fractional host–parasitoid population dynamical model to describe insect species. Numerical Methods for Partial Differential Equations, 37(2): 1673-1692.

Li, W. and Pang, Y. 2020. Application of Adomian decomposition method to nonlinear systems. Advances in Difference Equations, 1-17.

Li, Y. M., Rashid, S., Hammouch, Z., Baleanu, D. and Chu, Y. M. 2021. New Newton's type estimates pertaining to local fractional integral via generalized p-convexity with applications. Fractals, 29(05): 2140018.

Luchko, Y. 2011. Initial-boundary-value problems for the generalized multi-term time-fractional diffusion equation. Journal of Mathematical Analysis and Applications, 374(2): 538-548.

Miller, K. S. and Ross, B. 1993. An introduction to the fractional calculus and fractional differential equations. Wiley, 366 page, London.

Moa'ath, N. O., El-Ajou, A., Al-Zhour, Z., Eriqat, T. and Al-Smadi, M. 2022. A new approach to solving fuzzy quadratic Riccati differential equations. International Journal of Fuzzy Logic and Intelligent Systems, 22(1): 23-47.

Moazzam, A., Shokat, A. and Kuffi, E. A. 2023. α-Sumudu Transformation Homotopy Perturbation Technique on Fractional Gas Dynamical Equation. Ibn AL-Haitham Journal For Pure and Applied Sciences, 36(1): 325-332.

Momani, S., Freihat, A. and Al-Smadi, M. 2014. Analytical study of fractional-order multiple chaotic FitzHugh-Nagumo neurons model using multistep generalized differential transform method. In Abstract and Applied Analysis, 2014:1-10, Hindawi.

Momani, S., Arqub, O. A., Freihat, A. and Al-Smadi, M. 2016. Analytical approximations for Fokker-Planck equations of fractional order in multistep schemes. Applied and computational mathematics, 15(3): 319-330.

Murad, S. A., Melhum, A. I. and Omar, L. A. 2014. Numerical Solution of Kawahara Equation Using Neural Network. Science Journal of University of Zakho, 2(1): 196-203.

Oldham, K. and Spanier, J. 1974. The fractional calculus theory and applications of differentiation and integration to arbitrary order. Vol III, 322 page, Elsevier.

Podlubny, I. 1999. Fractional differential equations. Mathematics in science and engineering, 198: 41-119.

Rahman, H., Hasan, M. K., Ali, A. and Alam, M. S. 2021. Implicit methods for numerical solution of singular initial value problems. Applied Mathematics and Nonlinear Sciences, 6(1): 1-8.

Rashid, S., Jarad, F., Noor, M. A., Kalsoom, H. and Chu, Y. M. 2019. Inequalities by means of generalized proportional fractional integral operators with respect to another function. Mathematics, 7(12): 1225.

Rashid, S., Jarad, F. and Chu, Y. M. 2020 a. A note on reverse Minkowski inequality via generalized proportional fractional integral operator with respect to another function. Mathematical Problems in Engineering, 2020:1-12.

Rashid, S., Kalsoom, H., Hammouch, Z., Ashraf, R., Baleanu, D. and Chu, Y. M. 2020 b. New multi-parametrized estimates having pth-order differentiability in fractional calculus for predominating $\hbar$-convex functions in Hilbert space. Symmetry, 12(2): 222.

Rashid, S., Hammouch, Z., Baleanu, D. and Chu, Y. M. 2020 c. New generalizations in the sense of the weighted non-singular fractional integral operator. Fractals, 28(08): 2040003.

Rehman, M. and Khan, R. A. 2011. The Legendre wavelet method for solving fractional differential equations. Communications in Nonlinear Science and Numerical Simulation, 16(11): 4163-4173.

Roul, P. 2022. A novel approach for solving nonlinear singular boundary value problems arising in various physical models. Journal of Mathematical Chemistry, 60(8): 1584-1609.

Saadeh, R., Alaroud, M., Al-Smadi, M., Ahmad, R. R. and Salma Din, U. K. 2019. Application of fractional residual power series algorithm to solve Newell–Whitehead–Segel equation of fractional order. Symmetry, 11(12): 1431.

Saadeh, R., Qazza, A., Burqan, A. and Al-Omari, S. 2023. On Time Fractional Partial Differential Equations and Their Solution by Certain Formable Transform Decomposition Method. CMES-Computer Modeling in Engineering & Sciences, 136(3): 3121-3139.

Saadatmandi, A. and Dehghan, M. 2010. A new operational matrix for solving fractional-order differential equations. Computers & mathematics with applications, 59(3): 1326-1336.

Sirendaoreji, N. 2004. New exact travelling wave solutions for the Kawahara and modified Kawahara equations. Chaos, Solitons and Fractals, 19(1): 147-150.

Sowa, M. 2018. Application of SubIval in solving initial value problems with fractional derivatives. Applied Mathematics and Computation, 319: 86-103.

Talafha, A. G., Alqaraleh, S. M., Al-Smadi, M., Hadid, S. and Momani, S. 2020. Analytic solutions for a modified fractional three wave interaction equations with conformable derivative by unified method. Alexandria Engineering Journal, 59(5): 3731-3739.

Wang, Y. and Fan, Q. 2012. The second kind Chebyshev wavelet method for solving fractional differential equations. Applied Mathematics and Computation, 218(17): 8592-8601..052.

Wang, Y., Veeresha, P., Prakasha, D. G., Baskonus, H. M. and Gao, W. 2022. Regarding deeper properties of the fractional order Kundu-Eckhaus equation and massive thirring model. Computer Modeling in Engineering & Sciences, 133 (3): 697-717.

Yi, M. and Chen, Y. 2012. Haar wavelet operational matrix method for solving fractional partial differential equations. Computer Modeling in Engineering & Sciences(CMES), 88(3): 229-243.

Yusufoglu, E. 2007. Homotopy perturbation method for solving a nonlinear system of second order boundary value problems. International Journal of Nonlinear Sciences and Numerical Simulation, 8(3): 353-358.

Yusufoglu, E. and Bekir, A. 2007. The variational iteration method for solitary patterns solutions of gBBM equation. Physics Letters A, 367(6): 461-464.

Yusufoğlu, E. and Bekir, A. 2008. Symbolic computation and new families of exact travelling solutions for the Kawahara and modified Kawahara equations. Computers & Mathematics with Applications, 55(6): 1113-1121.

Zhou, S. S., Rashid, S., Parveen, S., Akdemir, A. O. and Hammouch, Z. 2021. New computations for extended weighted functionals within the Hilfer generalized proportional fractional integral operators. AIMS Mathematics, 6(5): 4507-4525.

CURRICULUM VITAE

Personal Information

Name and Surname : Asen Hamzah Khaleel KAHIYAH

Education

MSc	Çankırı Karatekin University Graduate School of Natural and Applied Sciences Department of Mathematics	2019-2024
Undergraduate	Tikrit University Faculty Mathematical and Computer Sciences Department of Mathematics	2014-2015

Work Experience

Year	Institution	Position
2021-Present	College of Education for Human Sciences	Documents section