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ON TOPOLOGY OPTIMIZATION WITH APPLICATION



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**BY
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ON TOPOLOGY OPTIMIZATION WITH APPLICATION

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August 2022

We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science

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ABSTRACT

ON TOPOLOGY OPTIMIZATION WITH APPLICATION

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This thesis aims to give brief knowledge about topology optimization in the same context as optimizing geometric shapes topologically in the second chapter, a brief introduction to topological optimization, and some related matters. For chapter three, the new version of the firefly optimizer will provide the same mathematical analysis, changing the terms of the dynamic of attractiveness by releasing the standard measure of distance between flies by the Chebyshev distance, which is motivating for these purposes. While chapter four focused on the most appropriate application for topology optimization, the steel truss problem is a significant issue in structure design in engineering departments. We consider the 10-bar truss problem as a model. In the third and fourth chapters, we write the codes with Matlab and complete our simulations.

2022, 38 pages

Keywords: Topology optimization, Metaheuristic algorithms, Firefly optimization algorithm, Chebyshev distance, A 10-bar plane truss

ÖZET

UYGULAMAYA SAHİP TOPOLOJİ OPTİMİZASYONU ÜZERİNE

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Matematik, Yüksek Lisans

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Bu tez, ikinci bölümde geometrik şekilleri topolojik olarak optimize etmekle aynı bağlamda topoloji optimizasyonu hakkında kısa bilgi vermeyi, topolojik optimizasyona kısa bir giriş yapmayı ve ilgili bazı konuları vermeyi amaçlamaktadır. Üçüncü bölüm için, ateşböceği optimize edicinin yeni versiyonu, bu amaçlar için motive edici olan Chebyshev mesafesi ile sinekler arasındaki standart mesafe ölçüsünü serbest bırakarak çekiciliğin dinamiğinin şartlarını değiştirerek aynı matematiksel analizi sağlayacaktır. Dördüncü bölüm topoloji optimizasyonu için en uygun uygulamaya odaklanırken, çelik kafes problemi mühendislik bölümlerinde yapı tasarımında önemli bir konudur. 10 bar kafes problemini model olarak ele alıyoruz. Üçüncü ve dördüncü bölümlerde ise Matlab ile kodları yazıp simülasyonlarımızı tamamlıyoruz.

2022, 38 sayfa

Anahtar Kelimeler: Topoloji optimizasyonu, Metasezgisel algoritmalar, Ateşböceği optimizasyon algoritması, Chebyshev mesafesi, Kafes seviye 10 bar,

PREFACE AND ACKNOWLEDGEMENTS

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CONTENTS

ABSTRACT	i
ÖZET.....	ii
PREFACE AND ACKNOWLEDGEMENTS.....	iii
CONTENTS.....	iv
LIST OF SYMBOLS	vi
LIST OF ABBREVIATIONS	vii
LIST OF FIGURES	viii
LIST OF TABLES	ix
1. INTRODUCTION	1
2. PRELIMINARIES.....	3
2.1 General Topology	3
2.2 Optimization.....	6
2.3 General Optimization Problem	7
2.4 Topology Optimization.....	8
1.1 Problem Formulation in Topology Optimization	11
2.6 Solving the Topology Optimizing Problem	12
2.7 Metaheuristic Algorithms.....	13
3. CHEBYSHEV DISTANCE ADAPTIVE FIREFLY OPTIMIZATION ALGORITHM.....	15
3.1 Classical Firefly Algorithm(FA)	15
3.2 Adaptive Firefly Algorithm(MFA).....	16
3.3 Simulation Result.....	18
3.3.1 The first test problem is as follows.....	18
3.3.2 The second test problem.....	19
3.3.3 The third test problem.....	20
3.3.4 The fourth test problem	21
3.3.5 The fifth test problem.....	22
3.3.6 The sixth test problem	23
3.3.7 The seventh test problem	24
4. OPTIMIZATION OF TRUSS STRUCTURES VIA ALTERED FIREFLY ALGORITHMS.....	26

4.1 Sizing Optimal Design.....	26
4.2 Problem Formulation.....	26
4.3 Sizing Optimization Of Trusses	27
4.4 10 Bar Truss	28
4. RESULT AND DISCUSSION	30
5. CONCLUSIONS AND RECOMMENDATIONS	32
REFERENCES.....	33
CURRICULUM VITAE.....	38



LIST OF SYMBOLS

γ	The absorption coefficient
L_1	The chebyshev distance
A_i	The cross-section area of the truss element
l_i	The density of truss member
r	The distance function
ξ	The lenght of truss member
L_∞	The manhattan distance
X_i^t	The position of a firefly
α_t	The randomization parameter
β	The variance of attractiveness
ϵ_i^t	The vector of random values

LIST OF ABBREVIATIONS

FA	Standard firefly algorithm
FCM	Finite element numerical method
GA	The genetic algorithm
MFO	Modified firefly algorithm
PSO	The particle swarm optimization algorithm
s.t.	Subject to



LIST OF FIGURES

Figure 2.1	Structure optimization problem classifications	10
Figure 2.2	Typical topology optimization design steps.....	10
Figure 2.3	The topology optimization workflow(Gebisa and Lemu 2017).....	11
Figure 2.4	Classification-of-metaheuristic-algorithms (Ghaedi et al.2021).....	14
Figure 3.1	Conventional algorithm and Algorithm modified using a distance function.....	19
Figure 3.2	Conventional algorithm and Algorithm modified using a distance function.....	20
Figure 3.3	Conventional algorithm and Algorithm modified using a distance function.....	21
Figure 3.4	Conventional algorithm and Algorithm modified using a distance function.....	22
Figure 3.5	Conventional algorithm and Algorithm modified using a distance function.....	23
Figure 3.6	Conventional algorithm and Algorithm modified using a distance function.....	24
Figure 3.7	Conventional algorithm and Algorithm modified using a distance function.....	25
Figure 4.1	Ten bar truss model	29

LIST OF TABLES

Table 3.1 Include the collection of the experimental for Benchmark test functions comparing the classical and improved firefly optimization algorithms	25
Table 4.1 Design data for 10 Bar Truss and utilize firefly algorithm	29
Table 4.2 Optimal sizing variables for 10 bar truss in different population sizes	30
Table 4.3 Comparison between adaptive FA, classical FA, and GA in optimal sizing variables for 10 bar truss	31



1. INTRODUCTION

Topology optimization is a mathematical technique that spatially optimizes the distribution of material within a specified domain by satisfying previously set constraints and minimizing a predetermined cost function. Mechanical and civil engineers have utilized topology optimization to reduce the quantity of employed material and the strain energy of structures while preserving their mechanical strength (Hajirasouliha *et al.* 2011). This optimization technique comprises three main parts: design variables, cost functions, and restrictions. An engineer is constantly searching for solutions to do tasks with the least amount of money and computing time. These objectives include the creation of theaters, mechanical instruments, and electrical appliances. Currently, these objectives can be attained using two types of optimization techniques. The first kind consists of local optimizers, which utilize gradient information to search the solution space in close proximity to the initial starting point. In general, though, these strategies produce results more quickly. The variables and cost function of the generators for these approaches must be continuous if they are to perform local search tasks with greater precision than stochastic approaches. A solid starting point is required for the successful execution of these procedures (Kaveh and Khayatazad 2013). In contrast to mathematical methods, meta-heuristic optimization techniques are appropriate for global searches and eliminate the necessity for continuous cost functions and variables (Kaveh and Talatahari 2010). Metaheuristic algorithms are effective tools that are applied to a wide variety of engineering optimization issues. (Kaveh and Talatahari 2009, Kim and Park 2010, Reynolds and Ali 2008, Woo *et al.* 2011). among many others, apply these methods in computer engineering, mechanical engineering, electrical engineering, and civil engineering, respectively. There are numerous distinct algorithms within the family of meta-heuristic techniques. Genetic Method (Mirjalili 2019). Simulated Annealing algorithm (Kirkpatrick *et al.* 1983). Particle Swarm Optimizer (Eberhart and Kennedy 1995). Firefly Algorithm for multiple optimizations (Yang 2009) and Harmony Search are examples of such algorithms (Geem *et al.* 2001). These algorithms and other hybrid varieties were utilized to overcome the aforementioned issues. Among these issues is

truss optimization. One of the most important forms of structures in civil and mechanical engineering are trusses. In amusement parks, these buildings are frequently used as roofs, transmission towers, and supports for other buildings like roller coasters. Truss optimization typically falls into one of three broad categories: size, form, or topology. Size optimization is within the first category. In this category, the cross-sectional areas serve as design variables, and the objective of optimization is to minimize weight or cost. The geometry of the truss is examined in the second category, shape optimization. The cost or weight of the truss is the goal function in this category of truss optimization, and the nodal coordinates of a particular truss are used as design variables. For instance, studies on the seismic design of truss-like structures have been conducted by (Hajirasouliha *et al.* 2011). It should be noted that there are several constraints, such as node displacements and member stresses, that must be met in truss optimization, regardless of size or shape. The connectivity of the structure is optimized in the third category, topology optimization. The thesis layout is organized and divided into five chapters. The following are the general contents of the thesis: The chapter two gives a brief overview of topological optimization in the field of applications called structural engineering optimization. Chapter three introduces the modified firefly algorithm and compares the results with the standard firefly algorithm. In chapter four, the firefly algorithm and its adaptive version are applied to truss structures for sizing optimization, and the results are compared with other literature. Moreover, in the end, some conclusions and future work are provided in chapter five.

2. PRELIMINARIES

2.1 General Topology

The science of topology extends its roots to the era of Greek civilization, where the Greeks studied the concept of continuity. Nevertheless, the science of topology did not appear in its current state until the beginning of this century when Frisch published his thesis in 1906, which dealt with the distance of the coupling (function) and the relationship between it and the concept of continuity. The two worlds, Riesz and Hausdorff, later pointed out that there is no need for this coupling, and continuity can be studied without reference to distance coupling. Thus, the so-called general topology emerged. Mathematicians associate the emergence of topology as a distinct field of mathematics with the 1895 publication of *Analysis Situs* by the Frenchman Henri Poincaré. However, many topological ideas had found their way into mathematics during the previous century and a half. The Latin phrase *analysis situs* may be translated as "analysis of position" and is similar to the phrase *geometria situs*, meaning "geometry of position," used in 1735 by the Swiss mathematician Leonhard Euler to describe his solution to the Königsberg bridge problem. Euler's work on this problem is also cited as the beginning of graph theory, the study of networks of vertices connected by edges, which shares many ideas with topology. Topology is a mathematical system derived from a Greek word (*topos* meaning place or structures and *logos* meaning study or science). It is the study of changing sets that do not change the nature of their contents. That is, studying geometric shapes differently from geometry science, topology discusses volumes, areas, and angles, representing the fundamental issues in our geometry study. In geometry, rectangles, squares, and other shapes set laws for spaces, volumes, angles, Etc. In topology, we ignore all these things that do not care about areas and angles. A square and a rectangle are the same things. Also, parallelograms and trapezoids. All quadrilaterals are topologically the same. Moreover, if we look further by ignoring the angle, the proverb becomes like a square, and a pentagon is like a hexagon. All of these shapes represent one shape in the structure. Does topology say there is no difference at all between geometric shapes? Of course not, but the things we consider in topology differ from geometry. In our surveys, topology is concerned with

the shape's expansion and continuity; it believes that the shape remains the same no matter how much it is stretched without rupture. For example, we have a ball; no matter how much it compresses and changes its length, it will stay the same topological shape, so baseball is the same as soccer and tennis. Also, the Möbius tape cannot match the square, but the two topologies are the same shape. The further application would create the truss problem. In the following branches, we see the altered shape with the same structure after optimizing the same materials feature, precisely what the topology demonstrates.

Definition 2.1. Let $\mathbb{X}$ be a nonempty set and $d: \mathbb{X} \times \mathbb{X} \rightarrow \mathbb{R}$ be a function. The function $d: \mathbb{X} \times \mathbb{X} \rightarrow \mathbb{R}$ considered metric (distance function) if the following conditions are met:

- A. $d(x, y) \geq 0$ for all $x, y \in \mathbb{X}$.
- B. $d(x, y) = 0$ if and only if $x = y$.
- C. $d(x, y) = d(y, x)$ for all $x, y \in \mathbb{X}$.
- D. $d(x, y) \leq d(x, w) + d(w, y)$ for all $x, y, w \in \mathbb{X}$.

Additionally, $(\mathbb{X}, d)$ is metric space (Clarkson 2006).

The following examples give some types of essential metric spaces.

Examples 2.2. i) The usual metric is defined on $\mathbb{R}$ as $d(x, y) = |x - y|$ for all $x, y \in \mathbb{R}$.

ii) The discrete metric define on any non-empty set X as

$$d(x, y) = \begin{cases} 0 & x = y \\ 1 & x \neq y \end{cases}.$$

iii) The Euclidean metric is defined on $\mathbb{R}^2$ by

$$d((x_1, x_2), (y_1, y_2)) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}.$$

Some functions of distance given by the following definitions.

iv) Let $\mathbb{X}$ be a set of real valued continuous functions defined on $[x, y]$ and $d: \mathbb{X} \times \mathbb{X} \rightarrow \mathbb{R}$ be a function defined by

$$d(f, g) = \max_{t \in [x, y]} |f(t) - g(t)|$$

for all $f, g \in \mathbb{X}$. In this case, $(\mathbb{X}, d)$ is a metric space. This metric is called as uniform metric.

Definition 2.3. Let $(\mathbb{X}, d)$ be a metric space, the set

$$B(x_0, r) = \{x \in \mathbb{X}: d(x, x_0) < r\}$$

is open ball in X with radius $r > 0$ and centre x_0 , and the set

$$\bar{B}(r, x_0) = \{x \in \mathbb{X}: d(x, x_0) \leq r\}$$

is closed ball in X with radius $r > 0$ and centre x_0 .

Definition 2.4. Let $\mathbb{X}$ be a nonempty set. A set τ containing a subset of $\mathbb{X}$ is said a topology on $\mathbb{X}$ if (Ash 2009):

- A. If both $\mathbb{X}$ and ϕ are open.
- B. The intersection of any two sets in τ is open.
- C. The union of any (finite or infinite) number of sets in τ is open.

$(\mathbb{X}, \tau)$ is referred to as a topological space.

The following examples give some types of essential topology spaces.

Examples 2.5. a) The metric topology generated by the usual metric on any subset of $\mathbb{R}^n$ will be called the usual topology.

b) Let X be any set and τ be the collection of all subsets of X . Then τ is clearly a topology for X it is called the discrete topology.

c) Let X be any set and let $\tau = \{\emptyset, X\}$. Then τ is a topology for X , called the trivial (indiscrete) topology for X .

2.2 Optimization

For any improvement, workers should make decisions. The goal of all these decisions is either to reduce the effort or increase the interest. We can express the voltage or interest as a function of the design variables. Hence, the optimization finds the conditions that give the maximum or minimum of the function. If the point x^* corresponds to the minimum function value $f(x)$, then the same point corresponds to the maximum value of the function $-f(x)$. The way to all optimization problems efficiently is to optimize. Then developed several methods known as mathematical programming techniques, a branch of operational research.

Optimization is an essential tool in the science of resolution and analysis of the physical system to take advantage of this tool. First, we must define the goal and the quantitative measure of the system's performance under study. The target can be the profit, time, potential energy, or any quantity represented by a single number that depends on the system's specific properties, called unknown variables that improve the goal. The variables may be constrained or unconstrained, so the interest cannot be negative.

An optimization strategy consists of three primary elements:

- A function to optimize,
- A possible solution set from which to select a value for a variable,

- Furthermore, an optimization procedure either minimizes or maximizes the variable's value.

2.3 General Optimization Problem

Optimization methods will be used to solve a problem when there is a need to formulate a problem using mathematical terms (Resende and Pardalos 2008). However, many traditional methods exist (Bonnans *et al.* 2006, Resende and Pardalos 2008). Some solving techniques depend on the derivatives of the objective function. These can be solved by metaheuristic algorithms that run randomly. The following describes a typical minimization problem (Yang 2011).

$$\begin{aligned}
 & \max f(x) \\
 & x \in \mathcal{S} \subseteq \mathbb{R}^n \\
 \text{Subject to} \quad & \\
 & C_j(x) \leq 0 \quad j \in I \\
 & C_j(x) = 0 \quad j \in E
 \end{aligned} \tag{2.1}$$

Here I and E are disjoint sets of integers that indices for equality and inequality constraints, respectively. we thus have m_I inequality constraints indexed in I and m_E equality constraints indexed in E . The number $m_I + m_E$ of functions $C_j: \mathbb{R}^n \rightarrow \mathbb{R}$ is called constraints $j = 1, 2, \dots, m_I + m_E$ (Bonnans et al. 2006). A solution to this problem is a component of the set $\mathcal{S}$ that has the highest value of $f(x)$ achieved among all elements in $\mathcal{S}$.

Consider the following problem as an example:

$$\min f = (x_1 - 2)^2 - (x_1 - 1)^2$$

$$s.t. \quad \begin{aligned} x_1^2 - x_2 &\leq 0 \\ x_1 + x_2 &\leq 2 \end{aligned}$$

We can write this problem in the form above by defining

$$f(x) = (x_1 - 2)^2 - (x_1 - 1)^2, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$C(x) = \begin{bmatrix} C_1(x) \\ C_2(x) \end{bmatrix} = \begin{bmatrix} -x_1^2 + x_2 \\ -x_1 - x_2 + 2 \end{bmatrix}, I = \{1,2\}, E = \emptyset$$

Mathematically, x^* is a solution if and only if $x^* \in S$ and

$$f(x^*) \leq f(x), \text{ for all } x \in S. \quad (2.2)$$

All solutions met to find such x^* . We learned about the optimization problem because the built Algorithm was tested on it.

2.4 Topology Optimization

Bends and Kikuchi (Bendsøe and Kikuchi 1988) say that topology optimization research has grown and is now one of the most active research areas in structural and multidisciplinary optimization. Topology optimization was developed as an advanced structural design methodology to generate innovative, lightweight, and high-performance configurations that are difficult to obtain with conventional ideas. Any optimization technique can be used as a tool of optimization in topology optimization. An objective function has to be formulated, which has to be optimized, maximized, or minimized. Then, using the optimization techniques, several probabilities are produced, and the result that suits best is chosen. Some examples of topological optimization are shown in (Figure 2.1), (Figuer2.2) and (Figuer2.3). In recent decades, topology

optimization has been applied to discrete structures such as trusses (Pedersen 1970, Ringertz 1988, Sankaranarayanan *et al.* 1993) to discontinuous structures such as plates (Cheng and Olhoff 1981) or membranes (Bendsøe and Kikuchi 1988). Objective functions are most often compliance or weight. Still, topology optimization has also been performed with other performance criteria such as dynamic behavior (Díaz and Kikuchi 1992) (Huiskes and Hollister 1993). looked at several ways that medical engineering has used or been influenced by topology optimization. Topology optimization of structures is a rapidly growing field of particular interest to the automotive and aerospace industries. Topology optimization, as opposed to shape optimization, allows the introduction of holes or cavities in structures, which usually results in significant savings in weight or improvement of structural behavior such as stiffness, strength, or dynamic response. Topology optimization has received revived interest since the introduction of the "homogenization approach to topology optimization" (Bendsøe and Kikuchi 1988). Still, it originated in minimum-weight structures (Michell 1904). Topology optimization is a mathematical method that optimizes material layout within a given design space for a given set of loads, boundary conditions, and constraints to obtain optimal mechanical performance of the design concept. The optimization process moves things around the domain in a planned way to minimize or maximize a specific goal. The problem of topology optimization can be divided into three categories: structure optimization, which attempts to find the optimal layout of the elements connecting the joints in the structure; shape optimization, which seeks to find the optimal coordinates of the joints in the design space; and volume optimization, which attempts to find the optimal cross-sectional regions for members in the structure. A- Size optimization, b- Shape optimization, c- Topology optimization. The combination of volume optimization and topology of the structure is to find the optimal size of the elements and to add or remove some elements from the structure, as shown in(Figure 2.1) below.

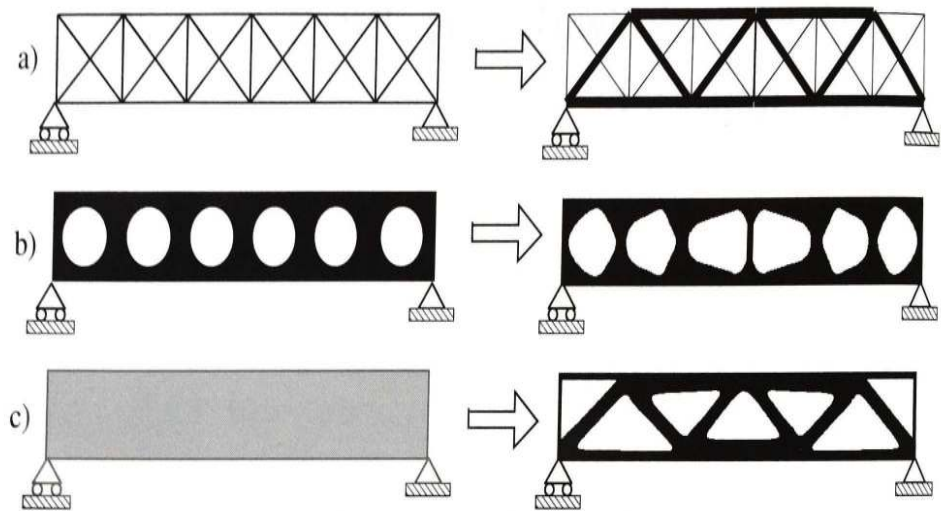


Figure 2.1 Structure optimization problem classifications (Tyflopoulos *et al.* 2018)



Figure 2.2 Typical topology optimization design steps (Gebisa and Lemu 2017)

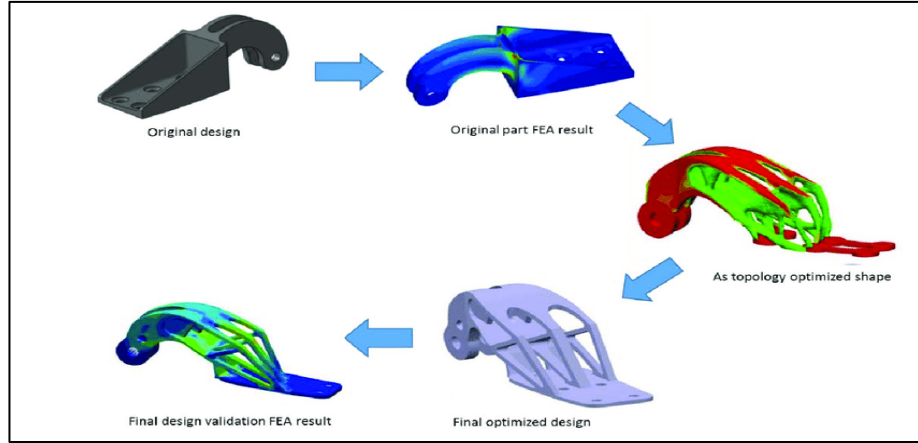


Figure 2.3 The topology optimization workflow (Gebisa and Lemu 2017)

1.1 Problem Formulation in Topology Optimization

Optimization of topology can be viewed as a material distribution problem. The objective is to discover the optimal material distribution $\rho(x)$ that minimizes the objective function F . This function is governed by m constraints $\widehat{G}_i$, the first of which is typically a volume constraint. This is the mathematical form of the topology optimization issue (Groen 2015):

$$\begin{aligned}
 \min_{\rho} : \widehat{F}(\rho) &= F(\rho, u(\rho)) = \int_{\Omega} f(\rho, u(\rho)) dV \\
 \text{s. t. : } \widehat{G}_1(\rho) &= \int_{\Omega} \rho(x) dV - V_{mar} \leq 0 \\
 \widehat{G}_i(\rho) &= G_i(\rho, u(\rho)) \leq 0, & i = 2, \dots, m \\
 \rho(x) &= 0 \text{ or } 1. & \forall x \in \Omega
 \end{aligned}$$

where f is a local function, for instance, the strain energy density for minimal compliance issues? The material distribution is either 0 or 1, and the FCM is used to solve for u in a separate step. This optimization problem is referred to as a nested topology optimization problem since the total potential energy is stationary at each optimization stage. Other ways exist in which the displacement and optimal material distribution are determined simultaneously. Nonetheless, this sort of optimization will not be examined in this study. It is important to remember that the volume limitation is

not always a constraint in optimization issues. Consequently, minimization of volume might also be an objective. However, in the majority of academic optimization formulations, a topology that satisfies this volume constraint must be identified as optimal. There are a number of solutions to the aforementioned optimization problem, which can be grouped into two primary categories. The density approach to deriving geometry employs a nodal or element-wise density distribution. The second method optimizes the form of the design domain. In contrast to conventional form optimization, this method must permit the introduction of new holes inside the material domain. Several Tendencies in Topological Optimization

- Evolutionary topology optimization.
- Large-scale topology optimization.
- Topology optimization via the phase field method.
- Optimization of topology using a continuous density distribution.
- Optimization of topology using the level-set method.

2.6 Solving the Topology Optimizing Problem

They start with a rough estimate of the variable $f(x)$ and iterate through a series of better estimates (known as 'iterates') until they reach a solution, hopefully. This method is used to differentiate between algorithms. Specific algorithms keep track of information gathered in previous iterations, while others depend solely on local data collected during the current iteration (Nocedal and Wright 2006). The value of the function $f(x)$ and the first and second derivatives are used in most strategies.

Definition 2.6. (Iteration methods) Choose $x^0 \in \mathbb{R}^n$. Iterate step k : Given $x^k \in \mathbb{R}^n$, find a new point x^{k+1} with $f(x^{k+1}) < f(x^k)$. We hope that: $x^k \rightarrow \dot{x}$ with $\dot{x}$ is a local minimizer (Rao 2019).

2.7 Metaheuristic Algorithms

Swarm intelligence, or metaheuristic algorithms, are stochastic optimization algorithms that run iteratively to improve the quality of the solution. Instead of traditional solving methods, most of these algorithms are inspired by nature (Talbi 2009). It is characterized as the collective behavior of natural or artificial decentralized, self-organizing systems. Intelligent algorithms can be inspired by evolutionary phenomena or the collective behavior of creatures, or they may mimic physical rules and basic concepts related to humans. As follows:

- First: Swarm Intelligence Techniques (SIT) such as Ant Colony Optimization (ACO), Ant Colony Optimization (Kaveh and Talatahari 2009), Particle Swarm Optimization (PSO) (Kaveh and Talatahari 2009), and Firefly Optimization Algorithm (Yang 2009).
- Second: Evolutionary Techniques (ET) such as Genetic Algorithms (GA) (Mirjalili 2019), Differential Evolution (DE) (Storn and Price 1997), Biogeography-based Optimization Algorithm (BBO) (Simon 2008), Evolution Strategy (ES).
- Third: Physics-based Techniques (PT) such as Gravitational Search Algorithm (GSA) (Rashedi *et al.* 2009), Colliding Bodies Optimization (CBO) (Kaveh and Mahdavi 2014) and Black Hole (BH) (Hatamlou 2013).
- Fourth: Human-related Techniques (HT) such as the League Championship Algorithm (LCA) (Kashan 2014), Mine Blast Algorithm (MBA) (Sadollah *et al.* 2013), and Teaching-learning-Based Improvement Optimization (TLBO) (Vakharia *et al.* 2011).

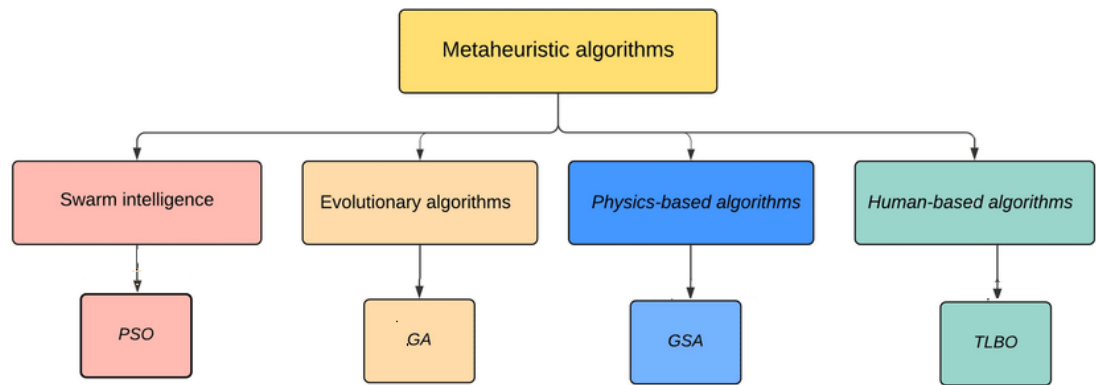


Figure 2.4 Classification-of-metaheuristic-algorithms modified from (Ghaedi *et al.* 2021)

3. CHEBYSHEV DISTANCE ADAPTIVE FIREFLY OPTIMIZATION ALGORITHM

In this section, we address one of the meta-heuristic optimization algorithms, the fireflies algorithm, and modify this Algorithm. Some optimization issues are applied, and finally, the results are compared between the standard Algorithm and the modified Algorithm.

3.1 Classical Firefly Algorithm(FA)

In 2007 and 2008, based on the flashing patterns and behavior at Cambridge University. Xin-She Yang developed the Firefly Algorithm (FA) (Yang 2009). In essence, FA employs the following algorithms in pursuit of the following three ideal rules:

- a) Since fireflies are only attracted to other fireflies, it does not matter what sex they are.
- b) As distance increases, the attractiveness decreases in proportion to the brightness. If no other firefly is brighter than a given firefly, that firefly will move at random. Therefore, the less brilliant of two flashing fireflies will travel toward the more brilliant.
- c) The objective function's landscape determines a firefly's brightness.

As a firefly's attraction is based on how bright its light is to other fireflies, the variation of attractiveness β concerning distance r is specified by

$$\beta = \beta_0 e^{-\gamma r^2}, \quad (3.1)$$

where β_0 is the level of attraction at $r = 0$ and the light intensity change with distance r for a given material with a fixed light absorption coefficient γ in the range (0,1). The movement of a firefly i is determined by its attraction to another (brighter) firefly j .

$$x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2} (x_j^t - x_i^t) + \alpha_t \epsilon_i^t, \quad (3.2)$$

x_i represents the position of a firefly in the iteration t , $\beta_0 e^{-\gamma r_{ij}^2} (x_j^t - x_i^t)$ describes the attraction between fireflies j and i , and ϵ_i^t is a vector of random values with a randomization parameter specified by α_t .

The second term is due to the attraction, and the third term represents randomization, with α_t defining the randomization parameters and ϵ_i^t being a vector of random values chosen at time t from the uniform distribution. If $\beta_0 = 0$, it becomes a straightforward random walk. Alternatively, if $\gamma = 0$, it simplifies to a particle swarm optimization variation (Yang and Karamanoglu 2013). Moreover, the random ϵ_i^t can be easily extended to other distributions of silky Levy flights (Yang and Karamanoglu 2013). This value represents the initial randomness scaling factor.

$$\alpha_{t+1} = \alpha_t \delta^t, \quad (3.3)$$

Where δ is between 0 and 1, experimentally determined values for α , β , and δ were utilized in this study.

3.2 Adaptive Firefly Algorithm(MFA)

The FA, according to Yang (Yang 2009). has so many advantages. It can also locate the global finest and the local optimal in a timely and efficient manner. Another benefit of FA is that separate fireflies can function independently, making it ideal for parallel processing. It is better than the genetic algorithm (GA) and the particle swarm optimization algorithm(PSO) because the fireflies congregate closely around each other (Miguel and Miguel 2012). The interactions between the various sub-regions can be expected to be minimal in parallel execution. However, it makes mistakes in finding the proper distance to attract and talk itself toward the best point, so our contributions to the change were analytical. We changed the distance function and built the Algorithm using the Chebyshev distance function. Although Firefly Algorithm(FA) is as simple as it is

valid, there is still a lack of a reasonable explanation as to why the Algorithm does not work well. We have shown that the second step in the recursive scheme eq (3.2) depends on taking the Euclidean distance between two flies (two points) in a multi-dimensional space. The distance function represented by the parameter r plays a prominent role in controlling this limit. We suggested, among other things, using Chebyshev distance instead. We noticed that the Chebyshev distance formula is more precise in terms of displacement than the Euclidean distance formula (short in the distance length). In the firefly algorithm, the closer fireflies are, the more robust the attraction, and vice versa. The greater the attraction between fireflies, the more quickly the goal achieves its objective and the fewer repeats required. An empirical test of our proposals showed that they led to improved performance. The Chebyshev distance is the distance between two vectors or two points, x and y , as follows (Coghetto 2016).

$$D_{chebyshev}(x, y) = \max(|x_i - y_i|). \quad (3.4)$$

The Euclidean distance used in standard FA is the measurement of the hypotenuse of the resulting right triangle, and the Chebyshev distance is going to be the length of one of the sides of the triangle. The Chebyshev distance maximum metric, also known as the L infinity metric, is defined in a vector space in which the distance between two vectors corresponds to their most incredible difference along any coordinate dimension. In common, this metric is sometimes referred to as the L infinity metric. In common parlance, the number of moves necessary for a king to move from one square on a chessboard to another is equal to the Chebyshev distance between the centers of the squares if the squares have a side length of one, it is also known as the "chessboard distance." This is because the minimum number of moves required for a king to move from one square on a chessboard to another equals the Chebyshev distance between the centers of the squares. As a result of rotation and scaling, the Chebyshev distance can be considered analogous to the plane's Manhattan distance. This similarity between L_1 and L_∞ metrics does not extend to higher dimensions; a sphere formed using the Chebyshev distance as a metric is equivalent to a cube with each face parallel to one of the coordinate axes. An octahedron was constructed as a result of this investigation using Manhattan distance. This consists of two different polyhedrons. The square is the only

self-dual polytope that contains cubes as its components. Because it accurately measures the amount of time needed to move an object from one grid to another using an overhead crane, the Chebyshev distance is sometimes used in the logistics of warehouses. It is an extremely long distance between the two locations and the point that is the closest to them.

3.3 Simulation Result

Seven test examples were chosen to assess the performance of the modified Firefly algorithm(MFA) and compare it to the version of the original Firefly technique(FA). The evaluation functions comprise well-known random, multimodal, continuous, and discontinuous tasks. The modified and conventional methods utilize the same collection of randomly generated solutions for each test problem.

3.3.1 The first test problem is as follows

This problem's objective function is :

$$\begin{aligned} \min f_1 &= (\cos x_1 \cos x_2) e^{-(x_1-\pi)^2-(x_2-\pi)^2} \\ \text{s.t} \quad & -5 \leq x_1, x_2 \leq 5, \end{aligned} \tag{3.5}$$

After running the program, it was determined that the modified MFA algorithm performs better than the classical Algorithm in terms of iterations, as the results in the standard Algorithm were (-0.00896) at iteration (17), while they were (-0.00896) at iteration (4) in the modified Algorithm (out of a total of 20). repeatedly, as depicted in (Figure 3.1).

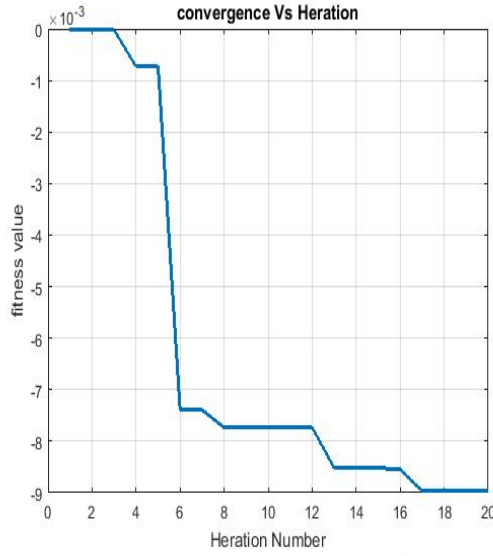


Figure 3.1.a Conventional algorithm

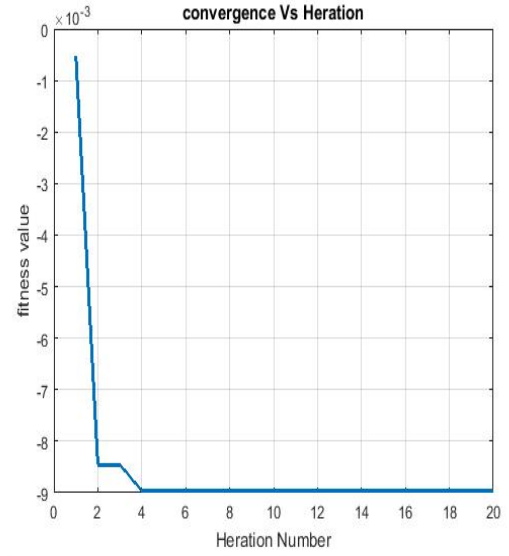


Figure 3.1.b Algorithm modified using Chebyshev distance function

3.3.2 The second test problem

The following is an explanation of the objective function :

$$\begin{aligned} \min f_2 = & -(20 + (x_1^2 - 10 \cos 2\pi x_1) + (x_2^2 - 10 \cos 2\pi x_2)), \\ \text{s.t} \quad & -4 \leq x_1, x_2 \leq 4 \end{aligned} \quad (3.6)$$

As illustrated in (Figure 3.2). the traditional Algorithm produced (-63) at iteration (48) while the modified approach produced (-63) at iteration (44) out of a total of (50) iterations.

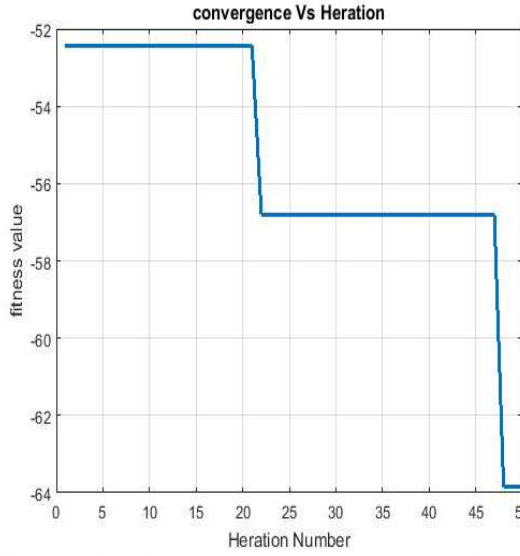


Figure 3.2.a Conventional algorithm

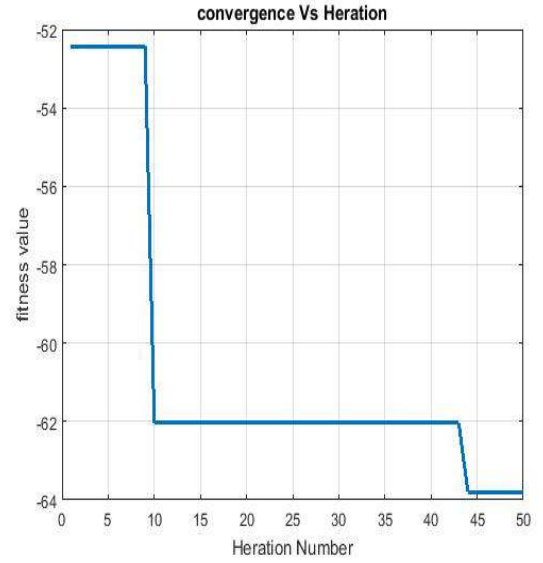


Figure 3.1.b Algorithm modified using Chebyshev distance function

3.3.3 The third test problem

The following explains the objective function:

$$\begin{aligned} \min f_3 &= 280 - \left(\frac{x_1^4 - 16x_1^2 + 5x_1}{2} + \frac{x_2^4 - 16x_2^2 + 5x_2}{2} \right) \\ s. t \quad & -6 \leq x_1, x_2 \leq 6 \end{aligned} \quad (3.7)$$

After running the software, the following results were displayed on the screen: In the standard Algorithm, the value was (-1160) at iteration (5), but in the modified Algorithm, the value was the same at iteration (3), as seen in (Figure 3.3).

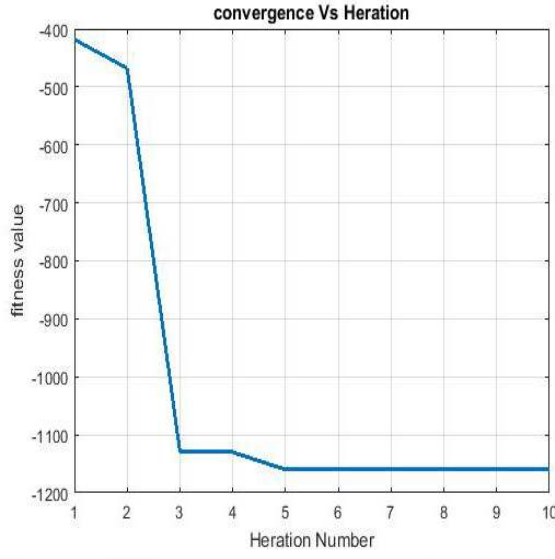


Figure 3.3.a Conventional algorithm

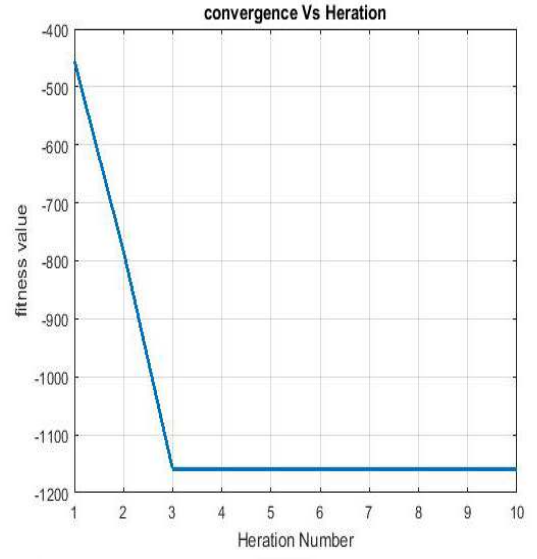


Figure 3.3.b Algorithm modified using chebyshev distance function

3.3.4 The fourth test problem

The following is an explanation of the objective function :

$$\begin{aligned} \min f_4 &= (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2, \\ \text{s.t} \quad &-5 \leq x_1, x_2 \leq 5 \end{aligned} \quad (3.8)$$

Figure 3.4 shows that the optimal value of the tested function (0) was found at iteration (11) for the usual method but only at iteration (6) for the Proposed Algorithm:

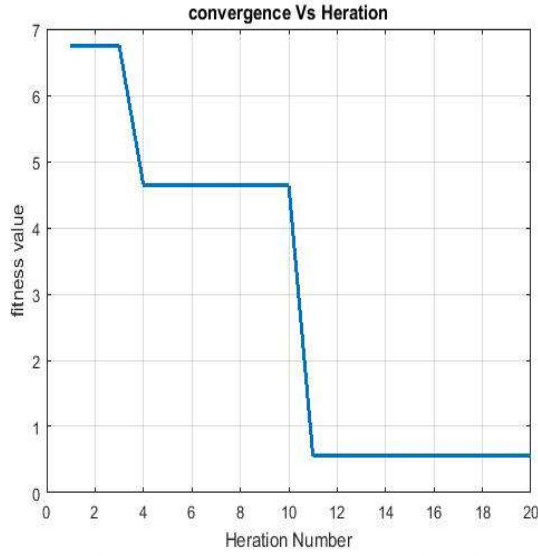


Figure 3.4.a Conventional algorithm

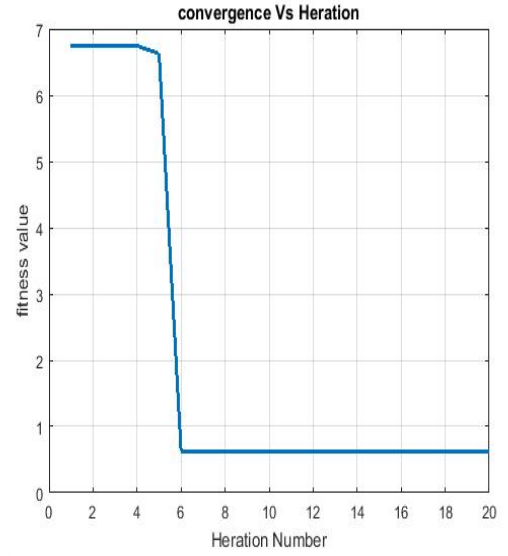


Figure 3.4.b Algorithm modified using chebyshev distance function

3.3.5 The fifth test problem

The following is an explanation of the objective function :

$$\begin{aligned} \min f_5 = & -(x_2 + 47) \sin \sqrt{\left| \frac{x_1}{2} + (x_2 + 47) \right|} - x \sin \sqrt{|x_1 - (x_2 + 47)|}, \\ \text{s.t} \quad & -500 \leq x_1, x_2 \leq 500 \end{aligned} \quad (3.9)$$

The test result was encouraging for the fifth function as well, where the results were: (-895) at iteration (15) in the standard Algorithm and the modified Algorithm by iteration (14), as shown in (Figure 3.5).

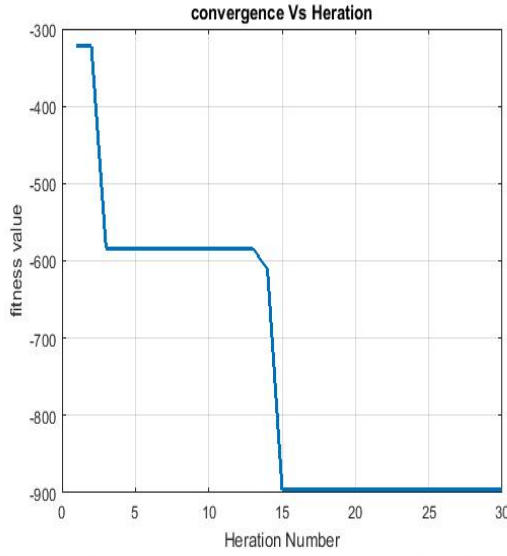


Figure 3.5.a Conventional algorithm

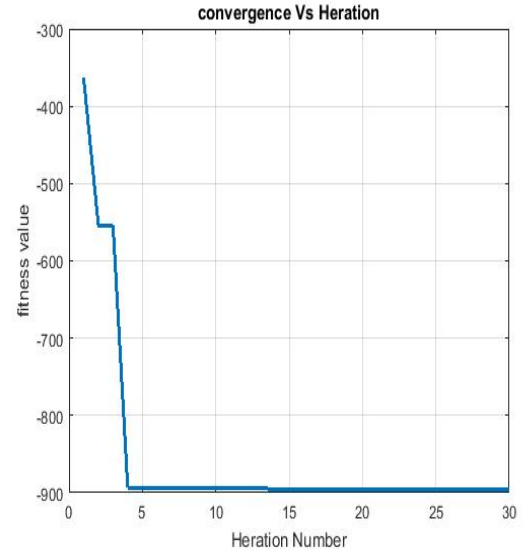


Figure 3.5.b Algorithm modified using chebyshev distance function

3.3.6 The sixth test problem

The following is an explanation of the objective function :

$$\begin{aligned} \min f_6 &= 100\sqrt{|x_2 - 0.01x_1^2|} + 0.01|x_1 + 10| \\ \text{s. t} \quad &-5 \leq x_1, x_2 \leq 5 \end{aligned} \quad (3.9)$$

From a total of (25) iterations, it was found that the standard Algorithm reached its peak state, which is the value (1) at iteration (15). In contrast, the updated Algorithm reached its peak state at iteration (10) and the same value as announced in (Figure 3.6).

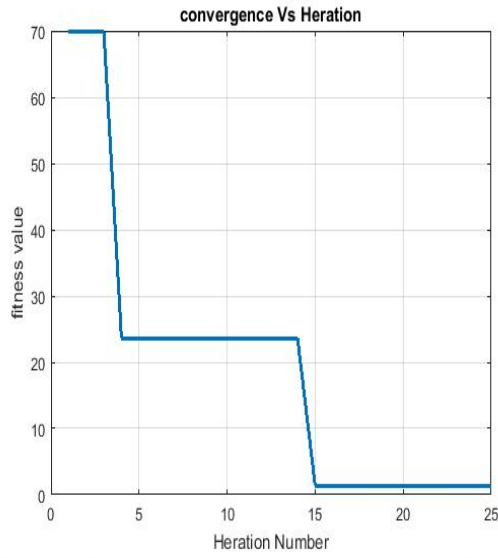


Figure 3.6.a Conventional algorithm

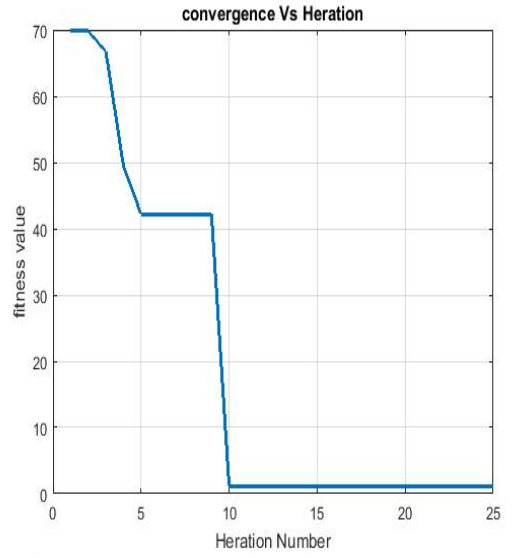


Figure 3.6.b Algorithm modified using chebyshev distance function

3.3.7 The seventh test problem

The following is an explanation of the objective function :

$$\min f_7 = (1.5 - x_1 + x_1 x_2)^2 + (2.25 - x_1 + x_1 x_2^2)^2 + (2.625 - x_1 + x_1 x_2^3)^2, \quad (2.10)$$

$$\text{s. t} \quad -5 \leq x_1, x_2 \leq 5$$

In (Figure 3.7). it is clear that the standard Algorithm reached the optimal value (0) at iteration 15. However, the modified Algorithm reached the optimal value (0) at iteration (4) out of a total of iterations (15).

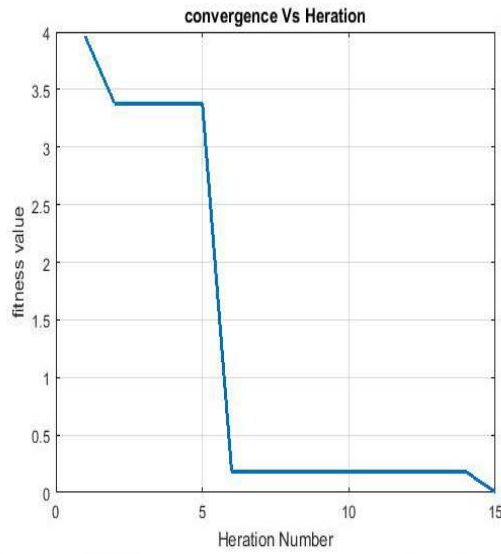


Figure 3.7.a Conventional algorithm

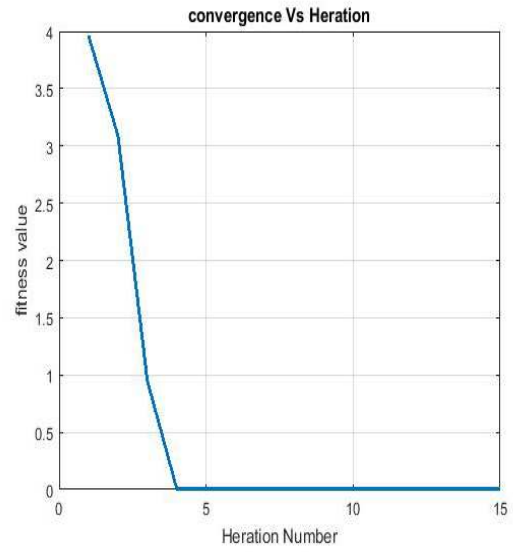


Figure 3.7.b Algorithm modified using chebyshev distance function

Table 3.1 Include the collection of the experimental for Benchmark test functions comparing the Classical and improved Firefly Optimization algorithms

Fun. Num.	Name functions	The number of iterations Standard FA	The number of iterations Modified MFA	MAX iteration	Global Minimum	Search Domain
1	Esom function (Gebisa and Lemu 2017)	17	4	20	-0.00896	$-5 \leq x_1, x_2 \leq 5$
2	De Jong (Molga and S 2005)	48	44	50	-63	$-4 \leq x_1, x_2 \leq 4$
3	Styblinski (Ittipong et al. 2008)	5	3	10	-1160	$-6 \leq x_1, x_2 \leq 6$
4	Himmelblau's (Back 1996)	11	6	20	0	$-5 \leq x_1, x_2 \leq 5$
5	Egg holder (Vanaret 2020)	15	14	30	-895	$-500 \leq x_1, x_2 \leq 500$
6	Bukin (Binh 1999)	15	10	25	1	$-5 \leq x_1, x_2 \leq 5$
7	Beale (Simionescu and Beale 2002)	15	4	15	0	$-5 \leq x_1, x_2 \leq 5$

4. OPTIMIZATION OF TRUSS STRUCTURES VIA ALTERED FIREFLY ALGORITHMS

In this section, we improve the 10-bar truss size, intending to reduce the weight by employing the new fireflies algorithm as mentioned in the third section, and we compare the results with some literature of the same class, namely the original fireflies algorithm and the gene algorithm, putting them in a detailed table.

4.1 Sizing Optimal Design

In optimal sizing design problems, the design variables are the finite element model's size parameters, such as the cross-sectional areas. The assumption is that they are available as discrete values. The objective of this challenge is to discover the optimal cross-sections for each piece in order to minimize material and construction costs.

4.2 Problem Formulation

The purpose is to determine the ideal cross-sectional area of each member for the particular stress condition in order to minimize the truss's weight.

$$f(x) = \sum_{i=1}^m \xi A_i l_i \quad (4.1)$$

The constraint equation $g(x)$ is written as

$$d_i/d_{max} - 1 \leq 0 \quad (4.2)$$

$$\frac{\zeta_i}{\zeta_{max}} - 1 \leq 0 \quad (4.3)$$

$$\frac{\zeta_i}{\zeta_{cri}} - 1 \leq 0 \quad (4.4)$$

A violation coefficient $\mathcal{M}$ is computed in the following manner.

If $h(x) \geq 0$, then $\mathcal{M}_i = h(x)$ And for, $g(x) \leq 0$, then $\mathcal{M}_i = 0$

$$\mathcal{M} = \sum_{i=1}^I \mathcal{M}_i \quad (4.5)$$

The modified objective function is written incorporating constraint violation as

$$\varphi(x) = f(x)(1 + K\mathcal{M}) \quad (4.6)$$

where k is judiciously selected depending on the influence of a violated individual in the next generation.

The fitness of the individual population is given by

$$F(x) = 1/\varphi(x) \quad (4.7)$$

Based on the fitness function, each population is given a count to create a mating pool for the next generators

$$Count = F_i^x / F'(x)$$

Where $F'(x)$ is the average fitness in a generation.

4.3 Sizing Optimization Of Trusses

An example of a 10-bar truss is presented to illustrate the operation of the Firefly algorithm's method for two-dimensional trusses. The objective of the problems is to minimize the weight subjected to a constraint.

1. Stress Constraint
2. Displacement constraint
3. Stress and Displacement constraint

There are ten design variables for the 10-bar truss, and the overall string length is forty feet. In these issues, it is assumed that each design variable can accept any 16 values from the higher and lower bounds' lists. Consequently, the length of the substring representing a design variable is set to four characters. The total length of the string is equal to the substring length multiplied by the design variable. The generational histories of populations with 10, 20, 40, 70, and 100 members are calculated. The following parameters of the firefly and modified-firefly algorithms are utilized to solve the full problem.

The Probability of randomization (α) = 0.5

The Probability of attractiveness (β) = 0.2

The Column absorption (γ) = 1.0

In sizing optimization of trusses, members' cross-sectional areas are considered design variables, and the coordinates of nodes and connectivity among various members are considered fixed. The cross-sectional areas called sizing variables in the truss are assumed to be discrete and are chosen from a list of discrete variables which are commercially available. The member cross-section is restricted to taking certain pre-specified discrete values in sizing optimization problems.

4.4 10 Bar Truss

The ten bar truss is optimally designed for the truss. (Figure 4.1) depicts a 10-bar truss with its dimensions and load. In this report, the governing dimensions of the 10-bar truss, analysis and design data, and loading specifications are provided (Table 4.1 and

Table 4.2). The maximum and minimum areas are 21612.85 mm² and 1045.16 mm², respectively. A force of 445.374 kN is applied vertically downward to the unsupported bottom joints. The maximum permissible displacement is 50.8 mm, and the maximum allowable stress is 172.25 MPa. Fig. 4.1 depicts the generation history of the ten-bar truss for stress, stress, and displacement constraint.

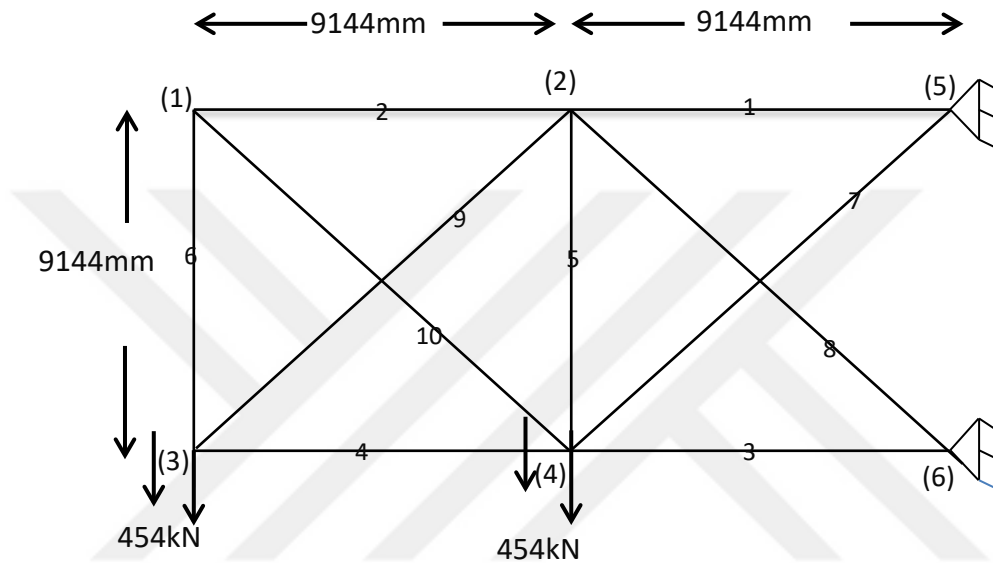


Figure 4.1 Ten-bar truss model

Table 4.1 Design data for 10 bar truss and utilize firefly algorithm

S.NO	Parameter	Date
1-	Density	$2.77 \times 10^{-5} \text{ N/mm}^3$
2-	Young's Modulus	$6.87 \times 10^4 \text{ N/mm}^2$
3-	Stress	172.5 N/mm^2
4-	Deflection	50.8mm
5-	randomization (α)	0.5
6-	attractiveness (β)	0.2
7-	absorption (γ)	1.0

4. RESULT AND DISCUSSION

In this item, the ten-bar truss problem shown in the above figure is modeled and solved using the improved firefly algorithm. The results are presented in (Table 4.2) when the population size varies between 10 and 100. In (Table 4.3), we compare the altered firefly algorithm for solving the problem with that of the traditional shepherd methods and genetic algorithms at a sample size of 10.

Table 4.2 Optimal sizing variables for 10 bar truss in different population sizes

Areas in mm ²	Population Size					Average
	10	20	40	70	100	
A1	32.397	33.009	26.982	29.587	29.786	30.3522
A2	0.100	0.100	0.100	0.185	0.100	0.117
A3	25.391	24.479	27.798	28.706	23.256	25.926
A4	14.336	15.737	18.436	16.264	17.549	16.4644
A5	0.100	0.100	0.100	0.100	0.100	0.1
A6	0.393	0.703	0.567	0.101	0.618	0.4764
A7	18.986	21.061	19.447	20.832	20.557	20.1766
A8	7.868	7.476	6.833	8.220	7.451	7.5696
A9	0.100	0.100	0.100	0.102	0.100	0.1004
A10	21.786	18.943	22.181	18.834	21.098	20.5684
Optimum Weight in N	5.099e+03	5.091e+03	5.135e+03	5.141e+03	5.076e+03	5.11E+03

Weight optimization of steel trusses using firefly algorithm

Table 4.3 Comparison between adaptive MFA, classical FA, and GA in optimal sizing variables for 10 bar truss

Areas in mm ²	Population Size 10		
	Genetic Algorithm (Mirjalili 2019)	Classical Firefly Algorithm (Yang and Slowik 2020)	Modified Firefly Algorithm
A1	30.765	28.767	30.712
A2	4.254	0.105	0.100
A3	23.808	24.543	22.401
A4	13.250	15.493	15.506
A5	8.570	0.100	0.100
A6	4.215	0.101	0.381
A7	16.665	21.066	20.922
A8	16.903	9.248	8.363
A9	7.386	0.100	0.100
A10	20.695	21.180	21.485
Optimum Weight in N	6.193+03	5.114e+03	5.081e+03

There is a presentation of a two-dimensional truss as well as generation histories for these populations for a two-dimensional truss. The best sizing factors for ten-bar trusses exposed to the restrictions of stress, stress, and displacement are provided in (Tables 4.2 and 4.3). provides a comparison of the discrete values of the areas and weights for distinct populations with a size of 10 by comparing measurements acquired under different limitations. The parameters for the AFO and FO algorithms were identical, as indicated at the conclusion of (Table 4.1). The probability of cross-over (P_c) is 0.80, the probability of mutation (P_m) is 0.001, and column convergence is 0.10.

5. CONCLUSIONS AND RECOMMENDATIONS

An investigation about topological optimization, which has recently received much attention, mentions some of its most important uses as an effective tool in many applications of structural engineering optimization and some physical problems. Swarm intelligent algorithms were highlighted from the nature-inspired metaheuristic algorithms, namely the firefly algorithm, and then developed using the Chebyshev distance function instead of the Euclidean distance function by writing the code in Matlab language and testing the two methods for seven mathematical optimization functions. The results were good, and the results for the method made are in the third section. The new advanced method of FA algorithms has been applied to one of the essential truss issues, a 10-bar truss, to obtain the optimal weight against the ideal weight for each bar separately, in detail through the tables included in Chapter 4. Now, we present some recommendations.

1. The work applies to higher capacity truss structures such as 36-, 52-, and 72-bar trusses.
2. The FA algorithm can also be modified or combined with other algorithms of the same type.
3. Topological optimization can be applied to different fields and engineering structures, such as airplanes, bridges, building roofs, car bodies, and many more.
4. Neural network training to solve topological optimization problems.
5. Attempting to find the optimal shape and measurements for some basic materials in an engineering structure in some applications by minimizing or maximizing these parameters using an intelligent algorithm rather than traditional algorithms that require additional conditions to be provided in the cost function.

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