

Transition dynamics of single- and double Josephson junctions formed by spatially coupled soliton and surface plasmons

by

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**Transition dynamics of single- and double Josephson junctions formed by
spatially coupled soliton and surface plasmons**

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ABSTRACT

This thesis work investigates the crossing dynamics of photons between a spatially coupled copropagating soliton and a surface-plasmon, which constitute a type of photonic Josephson junction. By introducing modulations to the spatial coupling, the crossing dynamics exhibit features similar as well as different to that of nonlinear Landau-Zener or Rosen-Zener type transitions. The dependence of the coupling to the soliton amplitude provides an inherent dynamic, which may manifest distinct features in the transition characteristics. The dynamics of the system, which is formulated as a Josephson junction, is investigated by introducing fractional population imbalance and the relative phase variables. A full population conversion between the optical soliton and the surface plasmon is achieved. The governing equations of the system represents a set of nonlinear Schrödinger equations, and the eigenvalue analysis of these equations sheds light on the behaviour of the fixed points of the system, here-with the stability analysis will be investigated. Under a spatial periodic modulation, this type of Josephson junction may exhibit driven resonance states similar to Shapiro resonances. The stability of these resonances will be investigated in the presence of an external periodic field. The double Josephson junction is formed by coupling the soliton spatially to two surface-plasmons, which reside on either side of the soliton propagation axis, respectively. This three-state system may provide rich dynamics with features like collapse-revival and plasmon-plasmon coupling via a frozen soliton state. By introducing spatial modulations, further dynamical effects will be explored. Heuristic designs are analyzed for sensor applications by investigating realistic materials and associated parameters.

ÖZETÇE

Bu tez çalışmasında uzaysal olarak etkileşim içinde bulunan soliton ve yüzey plazmonları arasındaki foton geçişleri ve bu geçişlerin oluşturduğu Josephson eklemının dinamiği incelenmiştir. Bu geçişin dinamiği uzaysal etkileşime yapılan kipleme sayesinde lineer olmayan Landau-Zener ve Rosen-Zener geçişlerine benzer olduğu kadar bunlardan farklı özellikler de göstermektedir. Soliton ve plazmon arasındaki etkileşimin soliton genliğine olan bağımlılığı sağlanan geçişin özelliklerinde belirgin olgular ortaya çıkaran içsel bir dinamik sağlamaktadır. Fraksiyonel popülasyon oransızlığı ve bağlı faz terimlerini kullanarak sistemin dinamiği Josephson eklemi olarak ele alınabilir. Bu sayede optik soliton ve yüzey plazmonu arasında tam bir popülasyon transferi gerçekleşebilir. Sistemi yönlendiren esas denklemler doğrusal olmayan Schrödinger denklemi özellikleri göstermektedir ve Schrödinger denklemlerinin özdeğer analizi plazmonik sistemin sabit noktalarının bulunmasına ışık tutmaktadır. Böylelikle de sistemin denge analizi yapılabilmektedir. Uzaysal kipleme periyodik olarak tasarlandığında bu tarzda olan Josephson eklenleri Shapiro çınlamalarının benzeyen kararlı yapıda çınlama durumlarının ortaya çıkmasına neden olabilmektedir. Harici bir periyodik alanın varlığında bu tarz çınlama durumlarının denge analizi de yapılacaktır. Solitonun kendi yörüngesinin iki tarafında konumlanan yüzey plazmonlarıyla girdiği uzaysal etkileşim ikili Josephson eklemını oluşturur. Bu üç katmanlı sistem çöküş-canlanmış, donuk bir soliton aracılığı ile plazmon-plazmon etkileşimi gibi zengin dinamiği olan özel durumlara neden olabilir. Tekli Josephson eklenindeki gibi uzaysal kiplemelerin de eklenmesiyle bu kiplemenin sistem dinamiği üzerindeki etkileri incelenecektir. Ayrıca uygulanabilirliği olan gerçekçi materyallerin ve bunların oluşturduğu sistem parametrelerinin incelenmesiyle algılayıcı uygulamalarında kullanılmak üzere bulgusal tasarımların analizi yapılacaktır.

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Chapter 1

INTRODUCTION

One of the major problems of today's digital information technologies is to carry the signal between ends of a microprocessor, that is in centimeter scale. However, ultrafast transistors are in nanometer scale, and this causes problems regarding speed in signal processing, when copper wires are to be used. On the other hand, fiber optic cables are able to carry data much more efficient than electronic circuits, however combining the two counterparts on the same circuit have some significant constraints. The proposed solution is an element, that can carry optical signals in nanometer scale, surface plasmons [1]. The improvements and progresses in science and engineering exhibit renewed interest in surface plasmons, that allow metals to be designed in sub-wavelength scale. This property of surface plasmons has gotten attention from scientist of different disciplines. Manipulating the frequency spectrum of lasers, ultra-short pulse generation, signal processing, magneto-optic data storage, solar cells, and biosensors for health studies are main topics of surface plasmons, which are being used actively by researchers [2]. The inception of these studies is the transformation of light into surface plasmons, which enables scientist to suppress and channel light using sub-wavelength photonic structures. In this way, surface plasmons can be used as control parameters in circuits instead of light, which causes an enhancement in the non-linear phenomena. This new subdivision of photonics is called plasmonics, that offers to combine the advantage of carrying ultrafast signals and operating on small circuits at the same time. The mathematical formulation of surface plasmon propagation lies back to the Maxwell's equations, so that in order to investigate the properties of surface plasmons in nonlinear media, nonlinear Maxwell's equations for the TM

waves at the metal-dielectric interface should be analyzed. The roots of overcoming the diffraction limit by surface plasmon polaritons come from the slow wave properties of surface plasmons. The formation of surface plasmons by the interaction between surface charges and the electromagnetic field has two corollaries. First, the momentum of surface plasmon, $\hbar k_{\text{SP}}$, is greater than of a photon with the same frequency, $\hbar k_0$. The second corollary is that, the evanescent tail of the field, which is perpendicular to the interface decays exponentially throughout the direction perpendicular to the surface, which is caused by the confined and non-radiative nature of surface plasmons.

Anharmonic electromagnetic fields cause nonlinear effects in optical systems. This anharmonicity can be expanded as power series resulting in a mixture of incident fields and output fields, that oscillate at frequencies of a superposition of incident fields. Second harmonic generation and third harmonic generation are typical examples of these effects, that result in frequency conversion. Moreover, the third-order effects possess terms at the incident frequencies, which causes a change in the refractive index of the medium, which is called optical Kerr effect. The optical properties of Kerr effect enable to create bistability, furthermore diffraction caused by linear effects balances the bistability caused by nonlinear effects, producing a new type of optical wave, solitons.

An optical soliton is a stable optical wave, that propagates without an aberration. The Kerr effect caused by self-phase modulation creates a red shift at the evanescent tails of the wave, and a blue shift also occurs at the tails due to dispersion. The combination of these two effects result in a pulse, that maintains its shape while travelling. There are various application areas in which solitons can be used such as surface waves in shallow water, plasma waves, sound waves in ^3He , short optical pulses in fibers, and optical spatial solitons [3]. Although the physical systems behind these examples are quite different, they all obey the same rules, and they are a consequence of nonlinear partial differential equations. Between the various soliton kinds, optical solitons are a widespread research hotspot for the recent years, because of its applicability in laboratory environment and extensive control over the parameters.

The soliton concept was first discovered in hydrodynamics in the 19th century, and has been used extensively in various disciplines such as optics, condensed matter physics, fluid mechanics, particle physics, and even astrophysics. However, the most important contribution of solitons between these fields has been made in optical communications. Optical solitons have been studied under spatial and temporal domains, however spatial solitons arouse attention of researchers, because of their high-dimensional properties, whereas temporal solitons are basically one-dimensional elements. Moreover, the rich nonlinear properties of spatial solitons which have found application areas in photorefractive materials, liquid crystals, thermophoresis, and colloidal suspensions are important contributing factors. These various nonlinear effects enable scientist to study soliton phenomena more efficiently, since it was thought earlier that, solitons were exact solutions of the cubic nonlinear Schödinger equation. The earlier studies regarding nonlinear waves in metal-dielectric interfaces were taking into consideration only the transverse direction, however later studies show that, the temporal effects should also be taken into account, and so the metal-dielectric geometry should be studied with a slot width smaller than the wavelength, and the complicated structure of nonlinear localization requires the consideration of boundary conditions. A novel configuration has been proposed, where surface plasmon guided vertically on the metal surface and a soliton self-trapped by nonlinear Kerr effect combined to yield a plasmon-soliton coupling driven by a low-power continuous-wave optical source [4].

In this thesis work, we investigate the dynamical properties of optical solitons propagating on a Kerr type of nonlinearity and surface plasmons propagating on metal-dielectric interface. Matching of surface plasmon and soliton propagation constants gives rise to coupling of both waves forming a plasmonic Josephson junction. Nonlinear nature of the optical soliton causes the coupling between two channels to be a function of soliton amplitude, which makes the soliton amplitude a driving parameter of the system. The Josephson junction formulation can be set by introducing two variables, which have analogues on bosonic Josephson junctions: fractional pop-

ulation imbalance and relative phase difference. We then consider the interaction between two channels changing adiabatically, hence the soliton trajectory is modulated spatially such that, the distance between metal surface and Kerr nonlinearity becomes a hyperbolic function. A full population conversion between surface plasmon and optical soliton can be achieved under certain parameter values, indicating that metal/dielectric/Kerr interface can host a Rosen-Zener type of transition. Stability analysis of the system is also studied by inquiring the fixed points of the system. We next investigate the transition dynamics in a three-level system, in which the nonlinearity is sandwiched between metal/dielectric interfaces, which yields the interaction between two surface plasmons via optical soliton propagating between them. This multilayer interface gives rise to different phenomena such as complete population transfer, population trapping, and collapse and revival of the coupling function. The fixed points and eigenenergies of the system is also analyzed. The following step is the periodic modulation of the system in order to examine Shapiro resonances, which were originally studied in superconductors and Bose-Einstein condensates. Finally, a realistic approach into the materials has been made to limit the parameter values, and a few applications of surface plasmon-soliton coupling have been proposed.

Chapter 2

SINGLE JOSEPHSON JUNCTION IN A METAL-DIELECTRIC INTERFACE

2.1 *Theoretical Background*

2.1.1 *Surface Plasmons*

Surface plasmons (SPs, also called surface plasmon-polaritons) are coherent electron oscillations that propagate at the interface between two materials, where the permittivity changes sign across the interface. The most common candidate for SP excitation is a metal-dielectric interface. These electromagnetic surface waves originate from the coupling of the metal's electron plasma with photons. SPs are evanescently confined in the transverse direction, and they have the ability to propagate within small sized materials. These type of materials have the wavelength much smaller than the wavelength of the surface plasmon. Applying Maxwell's equations (Eq.'s 2.1 - 2.4) to the flat interface between a metal and a dielectric leads us to physical properties of SPPs. For a more detailed and clearer investigation of these properties, we should start with the derivation of the wave equation, that describes the guiding of electromagnetic waves, hence we start with the Maxwell's equations of macroscopic electromagnetism.

$$\nabla \cdot \mathbf{D} = \rho_{\text{ext}} \quad (2.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.2)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.3)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_{\text{ext}} + \frac{\partial \mathbf{D}}{\partial t}. \quad (2.4)$$

In the absence of external charge and current densities, Faraday's Law and Ampere's Law (2.3, 2.4) can be put together to generate

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\partial^2 \mathbf{D}}{\partial t^2} \quad (2.5)$$

Using identities $\nabla \times \nabla \times \mathbf{E} \equiv \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$ and $\nabla \cdot (\epsilon \mathbf{E}) \equiv \mathbf{E} \cdot \nabla \epsilon + \epsilon \nabla \cdot \mathbf{E}$, and also $\nabla \cdot \mathbf{D} = 0$, equation 2.5 can be rewritten as

$$\nabla \left(-\frac{1}{\epsilon} \mathbf{E} \cdot \nabla \epsilon \right) - \nabla^2 \mathbf{E} = -\mu_0 \epsilon_0 \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}. \quad (2.6)$$

Considering the negligible variation of the permittivity $\epsilon = \epsilon(r)$, Eq. (2.6) simplifies to the wave equation,

$$\nabla^2 \mathbf{E} - \frac{\epsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0. \quad (2.7)$$

Since the dielectric profile, ϵ , is constant in the medium, this equation has solutions in both metal and dielectric, and boundary conditions must be used to obtain these solutions. In order to describe the confined propagating waves with Eq.(2.7), we should rewrite this equation in a different form, the well-known Helmholtz equation. We should first consider, that there is a harmonic time dependence $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})e^{i\omega t}$ of the electric field. If we insert this function into Eq.(2.7), we obtain

$$\nabla^2 \mathbf{E} + k_0^2 \epsilon \mathbf{E} = 0, \quad (2.8)$$

where $k_0 = \frac{\omega}{c}$ is the wave vector of the propagating wave in vacuum.

The next step is introducing a specified geometry to apply the Helmholtz equation. We choose this geometry such that, ϵ depends only on one spatial coordinate, z-direction, and therefore ϵ does not have any dependence in the perpendicular, y-direction (see Fig. 2.1); this is chosen for simplicity, so that $\epsilon = \epsilon(x)$. Our electric field function can be written as $\mathbf{E}(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \mathbf{E}(\mathbf{x})e^{i\beta \mathbf{z}}$. β is defined as the complex parameter and can be written as $\beta = k_z$, which is the propagation constant of the traveling waves. Inserting this expression into Eq.(2.8) gives the wave equation in the

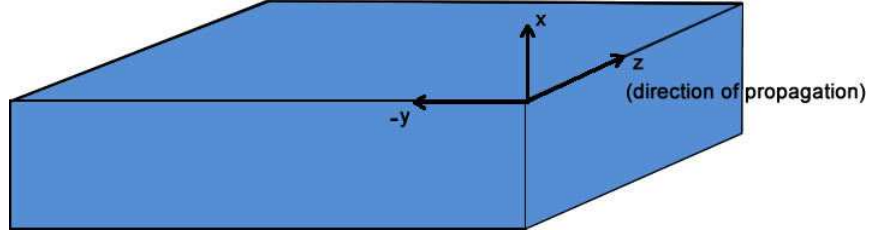


Figure 2.1: Illustration of a planar waveguide geometry. z - is the propagation direction of the waves.

expected form

$$\frac{\partial^2 \mathbf{E}(\mathbf{x})}{\partial x^2} + (k_0^2 \epsilon - \beta^2) \mathbf{E} = \mathbf{0}. \quad (2.9)$$

The magnetic field $\mathbf{H}$ can be obtained by following similar steps, which will not be shown here.

Equation (2.9) will be the key to analyze guided electromagnetic modes in waveguides. The next step now is to determine explicit expressions for the electric and magnetic fields. These expressions will help us determining the spatial field profile and dispersion of propagating waves. Two sets of solutions can be obtained from this type of systems: TM and TE modes. In TM (transverse magnetic) modes E_x , E_z , and H_y are nonzero and all other components are zero. In TE (transverse electric) modes H_x , H_z and E_y are nonzero and all other components are zero. For TM modes, the wave equation is

$$\frac{\partial^2 H_y}{\partial x^2} + (k_0^2 \epsilon - \beta^2) H_y = 0. \quad (2.10)$$

For TE modes, the corresponding wave equation is

$$\frac{\partial^2 E_y}{\partial x^2} + (k_0^2 \epsilon - \beta^2) E_y = 0. \quad (2.11)$$

A single, smooth interface (Fig. 2.2) composed of a two half-spaces on both sides, one being a dielectric non-absorbing material ($x > 0$) with positive permittivity ϵ_2 and the other being a conductive material ($x < 0$) described via a dielectric function $\epsilon_1(\omega)$ is the obvious candidate for a geometry, which sustains SPs. The metallic character

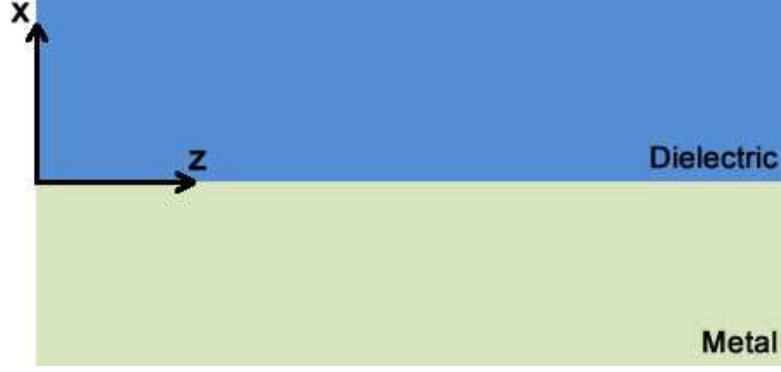


Figure 2.2: Geometry for SP propagation at a single interface between a metal and a dielectric.

requires $\text{Re}[\epsilon_1] < 0$. We should look for propagating wave solutions with evanescent decay in the perpendicular x-direction.

Let us first look at TM solutions.

$$H_y(x) = A_2 e^{i\beta z} e^{-k_2 x} \quad (2.12)$$

$$E_z(x) = iA_2 \frac{1}{\omega \epsilon_0 \epsilon_2} k_2 e^{i\beta z} e^{-k_2 x} \quad (2.13)$$

$$E_x(x) = -A_1 \frac{\beta}{\omega \epsilon_0 \epsilon_2} e^{i\beta z} e^{-k_2 x} \quad (2.14)$$

for $x > 0$, and

$$H_y(x) = A_1 e^{i\beta z} e^{k_1 x} \quad (2.15)$$

$$E_x(x) = -iA_1 \frac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta z} e^{k_1 x} \quad (2.16)$$

$$E_z(x) = -A_1 \frac{\beta}{\omega \epsilon_0 \epsilon_1} e^{i\beta z} e^{k_1 x} \quad (2.17)$$

for $x < 0$. Here $k_i \equiv k_{x,i}$ ($i = 1, 2$) is the wave vector, which is perpendicular to the propagation direction. Boundary conditions require the continuity of H_y and $\epsilon_i E_x$ at the interface ($A_1 = A_2$) and results in

$$\frac{k_2}{k_1} = -\frac{\epsilon_2}{\epsilon_1}. \quad (2.18)$$

The y - component of the magnetic field, H_y , should satisfy the wave equation 2.18. Plugging in the wave equation gives

$$k_1^2 = \beta^2 - k_0^2 \epsilon_1 \quad (2.19)$$

$$k_2^2 = \beta^2 - k_0^2 \epsilon_2. \quad (2.20)$$

Combination of these equations along with Eq. (2.18) yields the dispersion relation of SPs propagating at the interface between the two different media

$$\beta = k_0 \sqrt{\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2}}. \quad (2.21)$$

We arrived at this result by manipulating the TM solutions of the wave equation. However, no surface modes exist for TE polarization. Thus, surface plasmons only exist for TM polarization. We now investigate dispersion relation of SPs, which will reveal some important properties of SPs. As can be seen in Fig. 2.3, for small wave vectors (low frequencies), the SP propagation constant is close to k_0 at the light line. For large wave vectors, the free-electron dielectric function $\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$ can be plugged into the dispersion relation above. This shows that the frequency of the SPs approaches the characteristic surface plasmon frequency

$$\omega_{sp} = \frac{\omega_p}{\sqrt{1 + \epsilon_2}}, \quad (2.22)$$

If the damping of the conduction electron oscillation is neglected ($\text{Im}[\epsilon_1(\omega)] = 0$), the propagation constant β goes to infinity as the frequency approaches ω_{sp} . This would allow the group velocity goes to zero, $v_g \rightarrow 0$. Then this mode gains electrostatic character, which is known as the surface plasmon.

2.1.2 Optical Solitons

Optical spatial solitons are optical beams that travel without the effects of diffraction and show self-trapping properties with finite spatial cross section. The medium has to have a mechanism, that gives a self-focusing response to the light. The strong interaction between the medium and the electromagnetic wave causes the self-focusing.

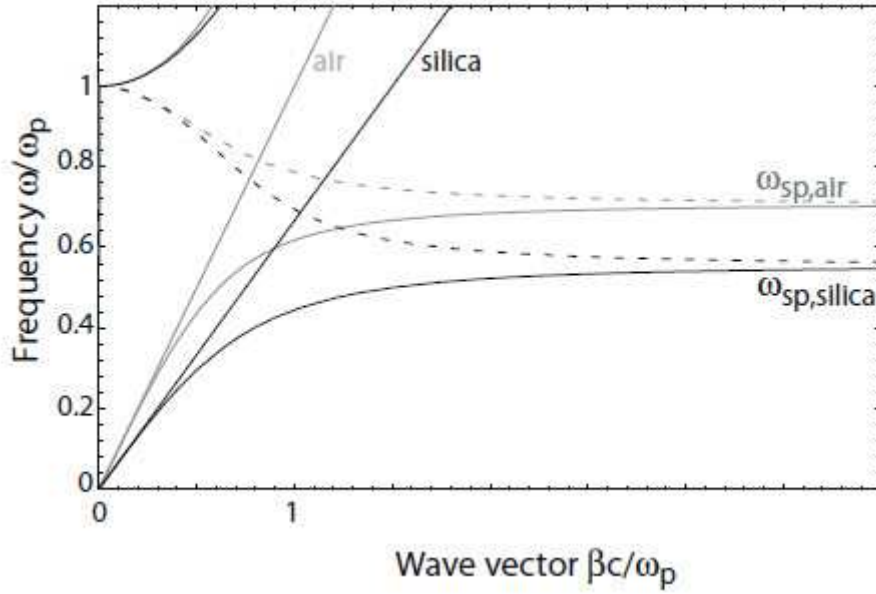


Figure 2.3: Dispersion relation of SPPs at the interface between a Drude metal with negligible collision frequency and air (gray curves) and silica (black curves) [5].

The wave modifies locally the medium and the medium in turn modifies the wave. Therefore, optical beams, whose fields intersect this particular region of the medium, are affected. This is a typical example of the interaction of solitons with one another, and with surface plasmons as well. We can now say that self trapping can be obtained as a result of a subsequent physical mechanisms.

In order to give insight about the SP-soliton interaction, some conceptual ideas should be presented first. Among these ideas the most fundamental ones are the definition of optical spatial solitons, structures that can host these type of solitons, and the generality of the interactions between solitons considering the extensively diverse physical origins of the self-trapping. Optical beams broaden with distance with the natural diffraction as they propagate, if they are very narrow in shape, which causes them to travel without changing the properties of a medium. If the width of the initial beam is thinner, then the beam diverges and therefore diffracts faster. However, in materials that possess nonlinear properties, the presence of light modifies the properties such as refractive index, absorption, or conversion to other

frequencies. The refractive index change forms an optical lens that increases the index in the beams center and leaves it unchanged in the beams tails. This induced lens focuses the beam, what is called self-focusing. Self-trapping can arise, when the self-focusing and the diffraction stabilize each other. The newly arising beam with a very narrow width is called an optical spatial soliton as shown in Fig. 2.4.

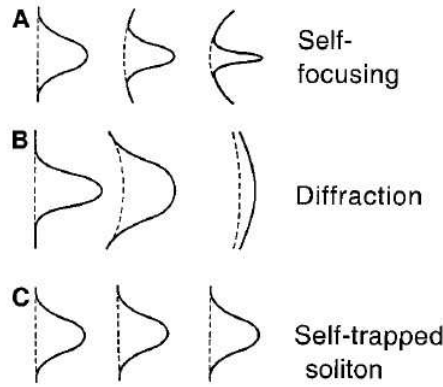


Figure 2.4: Illustration of different spatial beam profiles [7].

A simple convex lens model has to be explained in order to understand the existence of spatial solitons. As shown in the Fig. 2.5, an incident optical field is focused after getting through the lens, which introduces a x -dependent phase change which is a non-uniform function, $\phi(x) = k_0 n L(x)$, that causes the focusing effect [8], [9]. Here, $L(x)$ is the width of the lens. Since k_0 and n are constants, the width of the lens changes in each point with a shape similar to $\phi(x)$. This is another way of saying that, a focusing effect could be obtained by introducing a phase change of such a shape without a change of the width [10]. This method should be followed in order to apply this principle to optical solitons.

To create the same effect with a different approach, the value of the refractive index $n(x)$ should be changed, with the width L is kept fixed in each point. The modulation of the refractive index creates a focusing effect, that reduces the natural diffraction of the field. When the focusing effect stabilizes the diffraction of the field, a confined field can be observed within the medium. The existence of spatial solitons

are based on this principle. The Kerr effect creates a self-phase modulation, that alters the refractive index, which is a function of the intensity function, $\phi(x) = k_0 n(x)L$, [10].

$$n(x) = n_0 + n_2 I(x). \quad (2.23)$$

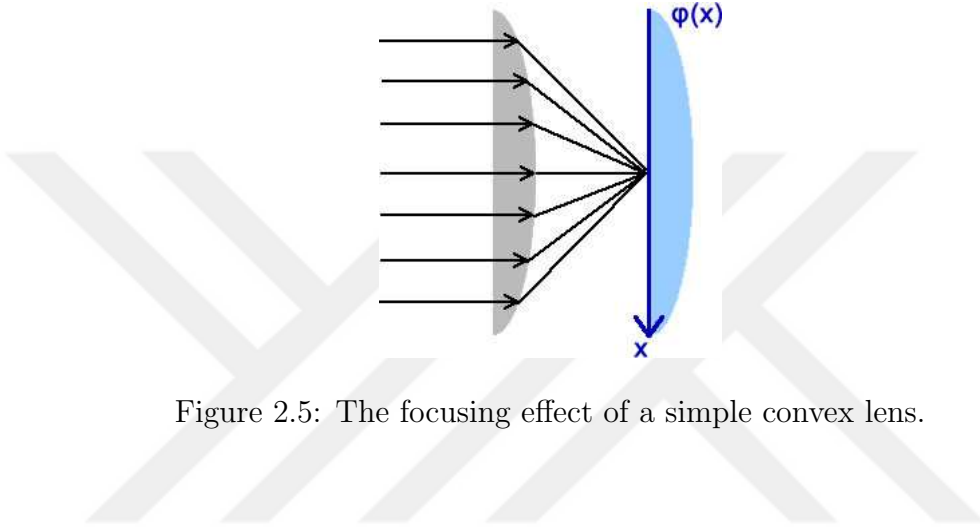


Figure 2.5: The focusing effect of a simple convex lens.

The propagating wave field develops a fiber-like guiding structure. Once this guiding is created, and if it is the mode of such a fiber at the same time, this indicates nonlinear effect, which causes focusing and linear effect, which causes diffraction are perfectly balanced, and the field does not experience a change in its shape. The condition of developing a nonlinear behavior is that n_2 must be positive, so that self-focusing can be observed.

The mathematical formulation of soliton waves is as follows. Kerr effect is the main reason for the existence of solitons, hence the refractive index is given by

$$n(I) = n + n_2 I \quad (2.24)$$

where $I = \frac{|E|^2}{2\eta}$ and $\eta = \eta_0/\eta$ and $\eta_0 = \sqrt{\mu_0/\epsilon_0} \approx 377 \Omega$. The field is propagating in the z direction with a phase constant $k_0 n$. Any dependence on y axis can be ignored, because the field is infinite in y direction, which can be expressed as

$$E(x, z, t) = A_m a(x, z) e^{i(k_0 n z - \omega t)} \quad (2.25)$$

where A_m is the maximum amplitude of the field and $a(x, z)$ is a dimensionless normalized function, that represents the shape of the electric field along x axis [11]. The next step is solving the Helmholtz equation $\nabla^2 E + k_0^2 n^2(I) E = 0$, and we assume that $a(x, z)$ changes slowly, while propagating, i.e. $|\frac{\partial^2 a(x, z)}{\partial z^2}| \ll |k_0 \frac{\partial a(x, z)}{\partial z}|$ and the following equation is obtained:

$$\frac{\partial^2 a}{\partial x^2} + i2k_0 n \frac{\partial a}{\partial z} + k_0^2 [n^2(I) - n^2] a = 0 \quad (2.26)$$

The effect of nonlinearity is always much smaller than the effect of linearity, so the following approximation can be considered valid:

$$[n^2(I) - n^2] = [n(I) - n][n(I) + n] = n_2 I (2n + n_2 I) \approx 2nn_2 I \quad (2.27)$$

then we express the intensity in terms of the electric field:

$$[n^2(I) - n^2] \approx 2nn_2 \frac{|A_m|^2 |a(x, z)|^2}{2\eta_0/n} = n^2 n_2 \frac{|A_m|^2 |a(x, z)|^2}{\eta_0} \quad (2.28)$$

n_2 is considered positive, because we assume that self-focusing is created by the nonlinearity. To clear this statement, n_2 will be written as $|n_2|$ from now on. Some new parameters should be introduced and substituted in the above equation.

First parameter, that should be introduced, is $\xi = \frac{x}{X_0}$, so the dependence on the x axis can be expressed with a dimensionless parameter; where X_0 is the characteristic length. Second parameter is $L_d = X_0^2 k_0 n$. Beyond the distance X_0 on the z axis, the diffraction caused by the linear effects becomes large enough, so they cannot be neglected. The third parameter is $\zeta = \frac{z}{L_d}$, a dimensionless variable, that describes the z -dependence. Next parameter is $L_{nl} = \frac{2\eta_0}{k_0 n |n_2| |A_m|^2}$, which depends upon the electric field intensity. Beyond the distance X_0 on the z axis, the self-focusing caused by the nonlinear effects becomes large enough, so they cannot be neglected. Final parameter is $N^2 = \frac{L_d}{L_{nl}}$. After the substitutions, Eq. 2.28 yields the nonlinear Schrödinger equation:

$$\frac{1}{2} \frac{\partial^2 a}{\partial \xi^2} + i \frac{\partial a}{\partial \zeta} + N^2 |a|^2 a = 0 \quad (2.29)$$

To interpret this equation in a better way, four different regimes should be considered separately.

For $N \ll 1$, the nonlinear part can be neglected.

For $N \gg 1$, the nonlinear effect will be dominant over diffraction.

For $N \approx 1$, the two effects balance each other and the equation can be solved.

For $N = 1$, the solution of the equation is the fundamental soliton [10]:

$$a(\xi, \zeta) = \text{sech}(\xi)e^{i\zeta/2} \quad (2.30)$$

Although the solution is a function of z , the shape of the field will be stationary while it propagates, because the z dependence is in the phase term. The illustration of the soliton can be shown in Fig. 2.6.

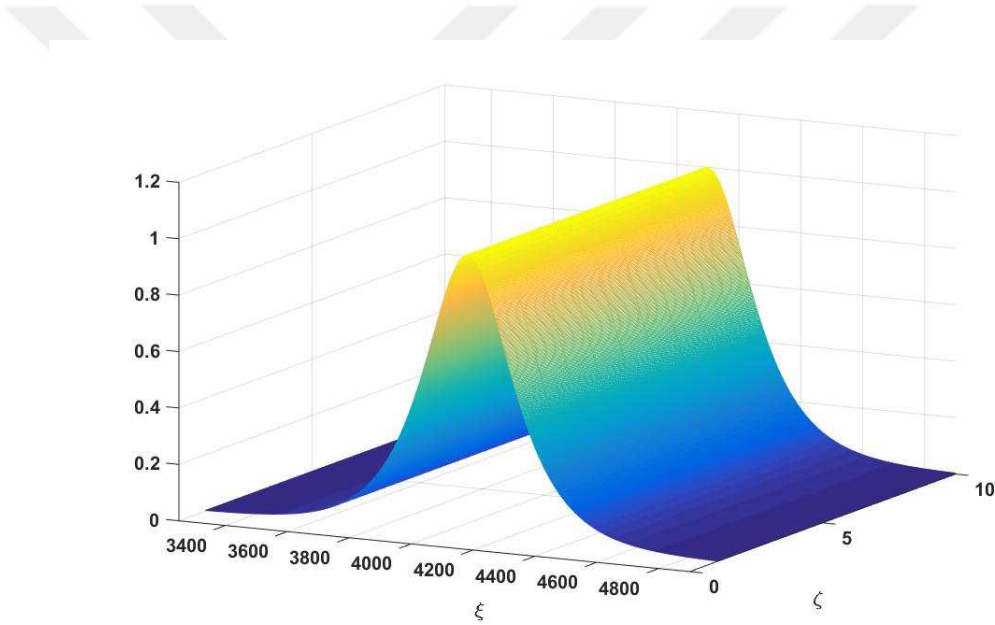


Figure 2.6: The shape of the soliton, while propagating with $N=1$.

As far as soliton solutions are concerned, N must be an integer, and is called the order of the soliton. For the solitons with higher orders, closed form expressions does not exist. These higher order solitons are all periodic, which have different periods. Their shape can be identified after generation:

$$a(\xi, \zeta = 0) = N \text{sech}(\xi). \quad (2.31)$$

The mathematical formulation of the optical waveguide introduced by the propagating soliton is not only in theory, but also a realistic model. The optical soliton

wave could be utilized to interact with other waves at different frequencies, which would otherwise be impossible in linear media.

2.1.3 Josephson Junction

The weak coupling of two macroscopic quantum objects features the difference between classical and quantum mechanical dynamics. This fact was predicted by Brian D Josephson in 1962. The system is described as a thin insulator sandwiched between two superconductors without an applied external voltage. He predicted that a DC current can flow through these layers. This is called the dc-Josephson effect. Moreover, an external voltage will result in a rapidly oscillating current; this effect is known as the ac-Josephson effect. In general, a Josephson junction can be constructed by a three layer system, one nonsuperconducting layer between two superconducting layers as shown in the figure.

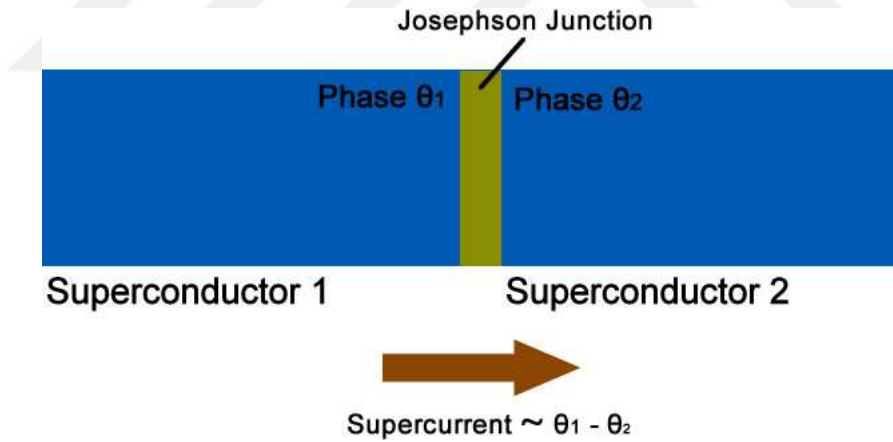


Figure 2.7: The basic geometry of a Josephson junction formed between two superconductors is illustrated.

In a Josephson junction, the middle nonsuperconducting section should be narrow. If the barrier is made of an insulator, it has to be on the order of 30 angstroms thick or less. If the barrier is made of another kind of metal, its thickness should be in units of micron. A supercurrent can flow across the insulator; so that electron pairs encounter no resistance, while crossing the barrier from one side to the other. However

when the threshold current is exceeded, an AC voltage evolves across the junction. This lowers the junction's threshold current, generates even more normal current to flow and causes a larger AC voltage close to 500 GHz per mV across the junction. The voltage would be zero, until the current through the junction is less than the threshold current. Once the current reaches the threshold current, the voltage would be nonzero and oscillates in time. Detecting and measuring the change between two states is the key to the many applications for Josephson junctions.

The Josephson effects has been used in various applications such as voltage standards of the Shaphiro effect, ultrasensitive magnetic field sensors, and superconducting quantum interference devices (SQUIDS). Furthermore fundamental questions on quantum physics have been studied both theoretically and experimentally with Josephson junctions in various configurations such as ultra small junctions and long junction arrays. In this thesis study, the discussion of bosonic Josephson junctions (BJJ), generated by confining a single Bose-Einstein condensate (BEC) in a double-well potential plays an important role, since we draw an analogy between surface plasmon-optical soliton coupling and bosonic Josephson junctions.

The macroscopic quantum phase difference has been exhibited by setting up a double-well trap for confined Bose-Einstein condensates, which are divided by a strong laser sheet, that builds an impenetrable barrier between the traps. Turning off the barrier, the two released condensates overlapped, producing a strong two-slit atomic interference pattern, which is a sign for macroscopic phase coherence. The method for observing the phase differences between two trapped BEC is turning off the barrier gradually, in other words the intensity of the laser sheet should be lowered to enable atomic tunneling through the barrier, and Josephson-like current-phase effects manifest themselves. The structure is shown in Fig. 2.8, [12].

The importance of detection of the phase difference reveals itself, when monitoring the population of the trapped condansates with phase-contrast microscopy. In this method, the geometry of the wells and the barrier can be adjusted by the location and the strength of the laser sheet that divides the trap. The analogy between bosonic

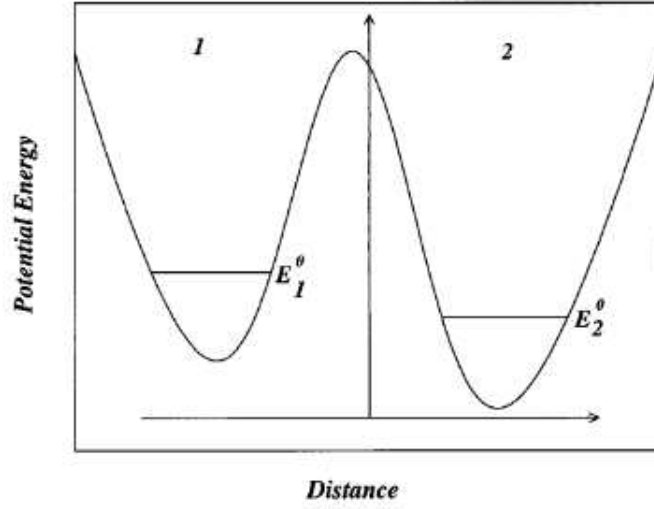


Figure 2.8: The BEC double-well trap. E_1^0 and E_2^0 are the zero-point energy of each condensate [12].

Josephson junction (BJJ) and superconducting Josephson junction (SJJ) requires some similarities between system components. An external direct current voltage in SJJ could be replaced by the chemical potential between the BECs, which is a function of the initial energy bias, furthermore capacitive SJJ charging energy could be replaced by the initial population imbalance. These replacements between two systems along with the adjusting capability of traps and the nonlinear barrier draw a fulfilling analogy between BJJ and SJJ.

The similarities between Bosonic Josephson junction and superconducting Josephson junctions should be mentioned as well as the differences between them. The comparison includes some obvious results. In both models the motivation is to obtain a tunneling between two states and to find the alternating current. However, getting this alternating current requires different conditions for both cases. For example, BJJ requires no potential difference for producing ac, although zero potential difference produces direct current in SJJ. Another example is that, the ac frequency in BJJ depends on the barrier transmissivity, on the other hand the frequency depends on the external voltage in SJJ.

Experimental realization of Bosonic Josephson junctions has been compared with the theoretical results [13]. A Bose-Einstein condensate is trapped in a double-well, which are separated by a potential barrier. The coupling is constructed weakly so that, the particle can tunnel through the barrier. The population imbalance between the two wells, which is a function of the energy bias shows an agreement with the theoretical expectation. The population imbalance and the relative phase difference were found to be the key elements, that control the dynamics of the self-trapping regime with the agreement to the solution of the Gross-Pitaevskii equation.

Bosonic Josephson junctions can find many applications within the field of BECs and nanophysics, since the rich dynamics of Bose-Einstein condensates in double and triple-well traps reveals some particular properties such as the macroscopic quantum self trapping and adiabatic transition between the wells, in which BECs are trapped [14], [15]. These examples encounter some problems regarding the nonlinearity created by the interaction of BECs, and few new approaches have been proposed to overcome these difficulties by introducing a modified nonlinear Landau-Zener and Rosen-Zener models [16], [17].

2.1.4 Landau-Zener Transition

Landau-Zener [LZ] model [20], [21] is an extensively used two-state model in quantum physics, however its applied fields extend over a large area such as ac current driven Josephson junctions, atomic collision studies, BEC in optical lattices, QED circuits, multiple level crossings, and three-level systems. There are many reasons for this wide usage of Landau-Zener model. One of the reasons is that, this model portrays an interaction between a double-well quantum system and an external field near the resonance. Another reason is the realistic presumption of the detuning, which is a time-dependent function, however the coupling is constant. And the final reason is the accurate definition of transition probability, that this model gives. All of these reasons considered, Landau-Zener model is a promising and satisfying model, that can be adapted to many different systems in both applied and theoretical physics.

Despite the wide application fields of Landau-Zener model, the extension of the model to the nonlinear case urged itself, when the transition probability in the adiabatic limit has been started to be investigated. As a result of this investigation, it has been realized, that strong nonlinearity breaks the adiabaticity to enable a tunneling between two states. This breakdown of adiabaticity is caused by the emergence of extra fixed points and the collision of these fixed points, which was thoroughly explained by analysing the energy levels and quantum eigenstates of the system [22], [23], [24].

The promising applicability of Landau-Zener model to the two state systems encourages researchers to examine the three-level systems [25]. Though the triple-well model shares some dynamic properties with its two-level counterpart such as the breakdown of adiabaticity and nonzero transition probability, it has some distinguished specifications like self-trapping of populations in a single well, topological change of the system near crossing points, and sensitivity to the initial system parameters, which results in chaos. An elaborated analysis has been done on this topic both numerically and analytically [26], and tunneling dynamics of the nonlinear three-level system has been exhibited.

2.1.5 Rosen-Zener Transition

The the Rosen-Zener [RZ] model [27] was introduced to investigate the double Stern-Gerlach experiments regarding the hyperfine Zeeman levels of energy under a rotating magnetic field. It was proposed at the same times as the Landau-Zener model with the motivation of encompassing a different system structure. The Landau-Zener model describes the transition properties of two avoided-crossing levels, in which the coupling is constant and the energy gap between levels are slowly changing. On the contrary, the Rosen-Zener model describes the transition probability between two levels, in which the energy difference is fixed and the coupling is a time-dependent function. The structure of this coupling function defines the driving properties of the system.

One of the advantages of this model is to provide both numerical and analytical solutions to two- or three- level systems such as exchange of ion population in nonres-

onant ion-atom collision, excitations induced by lasers, nuclear magnetic resonance techniques, and quantum computation.

In a recent study [32], the Rosen-Zener model has been enlarged to take the nonlinearity into account, and the effects of this nonlinearity have been investigated thoroughly. Bose-Einstein condensates have been used as a subject in this investigation because of their rich nonlinear properties, which was caused by the interaction between condensates. These nonlinear properties enable to observe various significant phenomena such as macroscopic quantum self-trapping, superfluidity, and Landau-Zener transition in a nonlinear perspective suggesting the benefits of analysing the nonlinear Rosen-Zener transition.

Here a nonlinear Schrödinger equation has been solved numerically to obtain the transition probabilities between traps.

$$i \frac{\partial}{\partial t} \begin{pmatrix} a \\ b \end{pmatrix} = H(t) \begin{pmatrix} a \\ b \end{pmatrix}, \quad (2.32)$$

where the Hamiltonian is given by

$$H(t) = \begin{pmatrix} \frac{\gamma}{2} + \frac{c}{2}(|b|^2 - |a|^2) & \frac{v}{2} \\ \frac{v}{2} & -\frac{\gamma}{2} - \frac{c}{2}(|b|^2 - |a|^2) \end{pmatrix}. \quad (2.33)$$

Here, a and b are the probability amplitudes of the condensates, γ is energy difference between traps, and c is the nonlinearity parameter indicating the interaction between condensates. The coupling parameter, v , is given as a time-dependent function

$$v = \begin{cases} 0, & t < 0, t > T, \\ v_0 \sin^2(\frac{\pi t}{T}), & 0 < t < T, \end{cases} \quad (2.34)$$

where T is the external period of the system.

The probability of one of the condensates, which resides in the other trap after the coupling loses its effect on them is considered as the transition probability. The effect of system parameters to the transition probability has been investigated, the results for the nonlinearity are presented in Fig. 2.9.

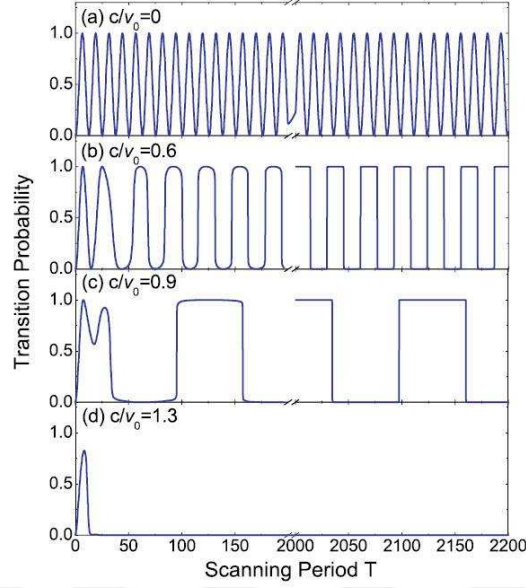


Figure 2.9: Transition probability versus external period under different nonlinearity strengths [32].

2.2 Surface Plasmon-Spatial Soliton Coupling

2.2.1 Model Description

To design a system, that has both optical soliton and surface plasmon components, that are copropagating along the Kerr-dielectric and metal-dielectric interfaces respectively, we adopt the following model in [35].

The system shown in Fig. 2.10 is formed by two interfaces that host optical wave formation: a metal/linear-dielectric interface (surface plasmon formation) and a linear-dielectric/nonlinear-dielectric (Kerr) interface (spatial soliton formation). The distance between these two interfaces, in other words the distance from the surface plasmon propagation axis to the soliton center axis is d . The interaction between a surface plasmon guided wave at the metal/dielectric interface, that propagates along the z direction and a spatial soliton in the nonlinear medium propagating along the same direction will be analyzed. It is assumed that, the nonlinearity is self-focusing, and the nonlinear behavior of the soliton is able to provide a propagation constant β_s .

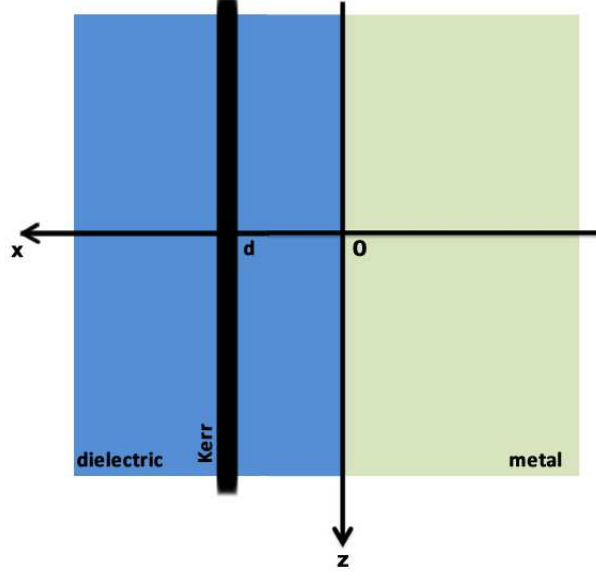


Figure 2.10: Metal-dielectric-nonlinearity interfaces for plasmon-soliton propagation.

that grows with the soliton peak amplitude. On the other hand, the surface plasmon is formed by two evanescent wave tails both in the metal and in the dielectric with propagation constant β_p . It is expected that, under appropriate conditions controlled by the soliton power, it is possible to achieve the matching of the propagation constants of plasmon and soliton. This phase-matching condition, $\beta_s = \beta_p$, should give rise to a mechanism of nonlinear resonant transfer of energy from soliton to surface plasmon.

The total wave field of the system is represented by the following ansatz, which indicates the superposition of surface plasmon and soliton fields,

$$\Psi(x, z) = c_p(z)\psi_p(x) + c_s(z)\psi_s(x, |c_s|). \quad (2.35)$$

We emphasize that, the transverse profile of the soliton, $\psi_s = \text{sech} \left[k\sqrt{\gamma/2}|c_s|(x-d) \right]$, depends on its amplitude $|c_s|$ which is the driving parameter of the system, whereas the transverse profile of the plasmon, $\psi_p = \exp(-x\sqrt{k_p^2 - k^2})$ at $x > 0$, represents two exponents decaying away from the metal surface with normalizations $\psi_p(0) = 1$

and $\psi_s(d) = 1$. We assume that, the soliton amplitude varies slowly enough to use quasistationary adiabatic approximation. Here γ is the nonlinearity parameter of the medium.

Our motivation is to construct an effective plasmon-soliton interaction, and in the first order approximation, we obtain the nonlinear coupled oscillator equations from Ref. [35]:

$$\ddot{c}_p + \beta_p^2 c_p = q(|c_s|)c_s, \quad \ddot{c}_s + \beta_s^2(|c_s|)c_s = q(|c_s|)c_p. \quad (2.36)$$

where the derivatives are with respect to the propagation parameter z and $q(|c_s|)$ is the coupling function which is the overlap of the tails of the plasmon and soliton waves in the area between metal and dielectric. The first equation implies that, the right-hand side represents an external source, which excites surface plasmons at the metal-dielectric interface, and due to weak-coupling approximation it is equal to the soliton field at the metal surface, $q(|c_s|) \sim \psi_s|_{x=0} \simeq \exp(-d\sqrt{\gamma/2}|c_s|)$. We need to remind that, in the zero-order approximation, surface plasmon and soliton amplitudes are independent, because coupling between them is assumed to be zero. However, in the first-order approximation, they become linearly coupled due to the spatial overlapping of the wave fields of surface plasmon and soliton.

These coupled oscillator equations are similar to the system of two weakly coupled nonlinear waveguides. However, there are significant differences between two systems. In the plasmon - soliton coupled system, only one subsystem is nonlinear, while the other is linear. Furthermore, coupling between surface plasmon and soliton is not constant, but soliton dependent.

The coupled oscillator equations can be reduced to first order differential equations by making the substitution $c_{p,s} = C_{p,s}e^{iz}$ and by adapting the slowly varying amplitude approximation, hence we arrive,

$$-i \begin{pmatrix} C_p \\ C_s \end{pmatrix}' = \begin{pmatrix} \nu_p & -q(|C_s|)/2 \\ -q(|C_s|)/2 & \nu_s(|C_s|) \end{pmatrix} \begin{pmatrix} C_p \\ C_s \end{pmatrix}, \quad (2.37)$$

where $C_{p,s}$ are the z -dependent plasmon and soliton amplitudes, which satisfy the nonlinear Schrödinger equation with a nondiagonal Hamiltonian. $\nu_p \ll 1$ and $\nu_s =$

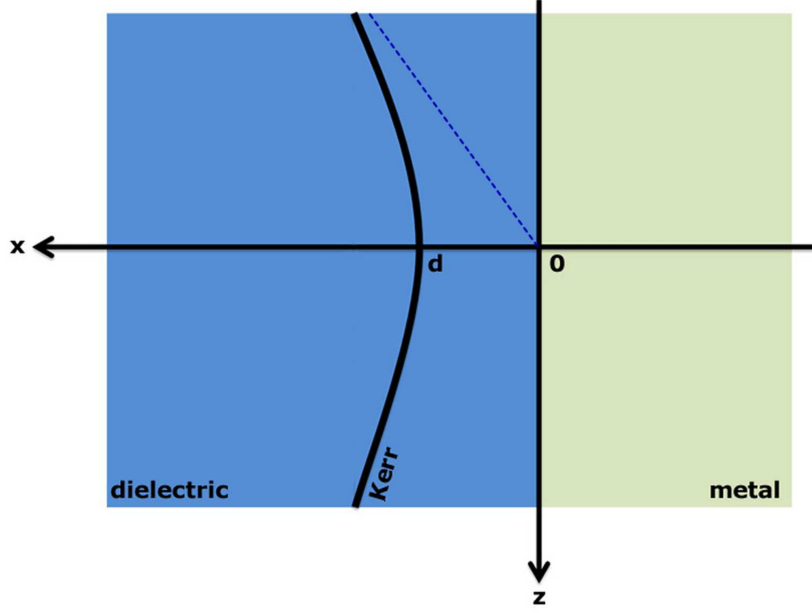


Figure 2.11: Hyperbolic two-level system.

$\gamma|C_s|^2/4 \ll 1$ are small deviations of β_p and β_s such that $\nu_p \equiv \beta_p - 1 \ll 1$ and $\nu_s \equiv \beta_s - 1 \ll 1$. The second inequality points the weakness of nonlinearity.

2.2.2 Hyperbolic Modulation to the Model

One of the objectives of this study is to obtain a Landau-Zener type of transition, so we should configure our system such that, surface plasmon and soliton are uncoupled first, then, when they enter an effective interaction area, coupling between them manifests itself via the evanescent tails of the waves. After they leave the effective interaction area, coupling vanishes, and we again have a system of two uncoupled waveguides. The geometry of this effective interaction area should be arranged such that, the resonant transfer of energy between soliton and surface plasmon can be observed at maximum efficiency. The optimum candidate for such a geometry is the one, in which the distance between metal surface and nonlinearity is a hyperbolic function of z' .

In this modulated geometry the coupling function $q(|C_s|)$ can be represented as:

$$q(|C_s|) \simeq |C_s| \exp(-\sqrt{d^2 + z^2 \alpha^2} \sqrt{\gamma/2} |C_s|), \quad (2.38)$$

where α is the parameter of the hyperbolic trajectory, (i.e., fiber), followed by the soliton (Fig. 2.11). The dashed lines in Fig. 2.11 represent the asymptotic line and θ_c is the angle between the asymptotic line and the z axis. α is defined as the tangent of this critical angle, i.e. $\alpha = \tan \theta_c$. Here, $\sqrt{d^2 + z^2 \alpha^2}$ is the distance between the metal surface and the fiber defined by the general hyperbola equation:

$$\frac{x^2}{d^2} - \frac{z^2}{(d/\alpha)^2} = 1 \quad (2.39)$$

Since the weak-coupling approximation fails at small soliton amplitudes, $|C_s|$ must be inserted before the exponential term to hold the approximation [38].

To draw an analogy to the Josephson junction dynamics, we introduce 'fractional population imbalance' and 'relative phase difference', respectively

$$Z = \frac{|C_s|^2 - |C_p|^2}{|C_s|^2 + |C_p|^2} \quad (2.40)$$

$$\phi = \phi_s - \phi_p \quad (2.41)$$

by applying the substitution $C_{s,p} = |C_{s,p}| e^{i\phi_{s,p}}$ [12], [36], [37]. Using this substitution and the fact that $|C_s|^2 + |C_p|^2 = N$ (N is a normalized constant for isolated system and equal to 1), Eq. (2.3) becomes

$$\dot{Z} = -q(Z) \sqrt{1 - Z^2} \sin \phi, \quad (2.42)$$

$$\dot{\phi} = \Lambda Z + \Delta E + \frac{q(Z) Z \cos \phi}{\sqrt{1 - Z^2}}. \quad (2.43)$$

Here the parameters Λ is the nonlinearity (i.e., the soliton strength) with $\Lambda = \gamma/8$, and ΔE parametrizes the asymmetry between the soliton and surface plasmon states occupied by the photons with $\Delta E = \Lambda - \nu_p$. We can now express the coupling parameter as a function of Z

$$q(Z) \simeq \sqrt{\frac{1+Z}{2}} e^{-\sqrt{d^2 + z^2 \alpha^2} \sqrt{2\Lambda(1+Z)}}. \quad (2.44)$$

The range of the parameters in our system is set by the insight of physical circumstances. Λ is investigated between 0.01 and 0.1, because the nonlinearity should be weak by definition $\nu_s = \gamma|C_s|^2/4 \ll 1$, ΔE is investigated between -0.04 and 0.04, because $\nu_p \ll 1$, and the range of Λ values allow only this range for ΔE .

In a recent study [39], a different analytical formulation of the coupling function is stated, revealing that, the equation system is not symmetric in the coupling function, hence two separate coupling functions are defined q and $\bar{q}$, implying that, a strong soliton drives a weak surface plasmon at a rate q , whereas a strong surface plasmon drives a weak soliton at a much smaller rate $\bar{q} \sim |C_s|$ and $\bar{q}/q \sim 10^{-3}$ [40]. By taking the new considerations into account, it is expected that the new dynamic equations should be different from Eq. (2.42) and Eq. (2.43). However, the equations cannot be written in Z and ϕ formation, because the presence of two separate coupling functions breaks the isolation of the system and the equality $|C_s|^2 + |C_p|^2 = N$ is no longer obeyed. The reason for this violation is that, the change in total surface plasmon and soliton amplitudes is different from zero, since $q \neq \bar{q}$ in $\dot{N} = (\bar{q} - q)C_s C_p \sin\phi$.

In the following sections, we will investigate the system, which is symmetric in coupling function ($q \simeq \bar{q}$) described by Eq. (2.42) - (2.43).

2.2.3 Analogy to Landau-Zener [LZ] and Rosen-Zener [RZ] Transitions

The original Landau-Zener model [20],[21] contains a constant coupling coefficient and a level separation which changes adiabatically with time. The optical analog of the problem is discussed in [28]. In a previous study [29], [30], coupling coefficient has been modified such that, the coupling is effective only for a finite duration and then the transition dynamics have been investigated. The analog of the level separation in the plasmon-soliton system is the asymmetry between surface plasmon and soliton states (ΔE), which is a constant coefficient. However the time dependent variable in this system is the coupling function. In [35], a 100% conversion of energy from soliton to plasmon channel could not be achieved, because the adiabaticity of the system fails due to the nonlinear effects and the system undergoes diabatic evolution, which

causes the energy to be stored in the soliton channel. The reason for this failure is that, the coupling parameter is a function of soliton amplitude, which changes swiftly. However, with the hyperbolic modification the velocity of the coupling function can be slowed down, hence almost a full conversion of energy from soliton to plasmon channel can be obtained. In the light of these results, it can be said that, the hyperbolic plasmon-soliton system might be a proper host to Landau-Zener tunneling. In order to understand the concept of adiabatic transitions which result in Landau-Zener tunneling, the following system should be analyzed.

The so-called nonlinear Landau-Zener model consists of two levels, in which the energy levels depends on populations of the condensates trapped in the wells. The model can be identified by the following Hamiltonian:

$$H(\gamma) = \begin{pmatrix} \frac{\gamma}{2} + \frac{C}{2}(|b|^2 - |a|^2) & \frac{V}{2} \\ \frac{V}{2} & -\frac{\gamma}{2} - \frac{C}{2}(|b|^2 - |a|^2) \end{pmatrix} \quad (2.45)$$

where population amplitudes are represented as a and b , respectively. V is the coupling constant, γ is the energy difference, and C is the nonlinear parameter. In BEC trapped wells, the nonlinearity identifies, how much the level energies depend on the populations. Total population and total probability is held constant, $|a|^2 + |b|^2 = 1$. The eigenvalues of this Hamiltonian for two different nonlinearity values are given in Fig. 2.12. For the weak nonlinearity, $C = 0.1$, there are two eigenvalues and the system does not experience any unexpected dynamical behaviour. However, for the strong nonlinearity, $C = 0.4$, two more eigenvalues appear, resulting a loop at the top of the lower level. When a quantum state moves in the lower level until the right end of the loop it has to jump to either upper or lower levels, since there is no further way to move. This jump between states points out that, nonlinear Landau-Zener tunneling is nonzero in the adiabatic limit.

Further analysis on this system can be made by investigating the fixed points of the Hamiltonian [31]. First, Eq.2.45 can be converted into a classical Hamiltonian:

$$H(s, \theta, \gamma) = \frac{C}{2}s^2 + \gamma s - V\sqrt{1-s^2}\cos\theta. \quad (2.46)$$

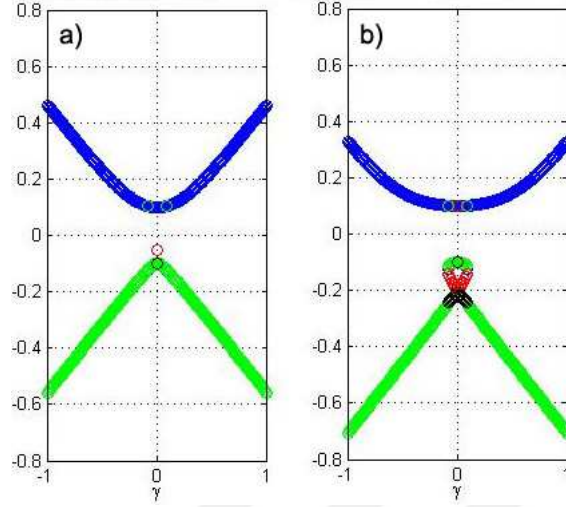


Figure 2.12: Eigenenergies of the Hamiltonian (Eq. 2.45) [31]. a) $c = 0.1$ and $v = 0.2$, b) $c = 0.4$ and $v = 0.2$.

Here s and θ can be treated as conjugate variables and obey the canonical Hamilton equations:

$$\dot{s} = -\frac{\partial H}{\partial \theta}, \quad (2.47)$$

and

$$\dot{\theta} = \frac{\partial H}{\partial s}. \quad (2.48)$$

Fixed points of the system can be found by setting $\dot{s}$ and $\dot{\theta}$ equal to zero:

$$\theta = 0, \text{ or } \pi, \quad (2.49)$$

$$\gamma + Cs + \frac{Vs}{\sqrt{1-s^2}} \cos \theta = 0. \quad (2.50)$$

The eigenenergies of the system can be found by substituting the fixed points into the classical Hamiltonian. This shows a direct correlation between figures 2.12 and 2.13. The loop structure in Fig. 2.12-b is caused by the emergence of two additional fixed points in Fig. 2.13-b which is caused by the strong nonlinearity. The problems with the Landau-Zener model can be overcome by introducing another tunneling model, which is the Rosen-Zener model. The $\dot{Z}$ and $\dot{\phi}$ equations allow us to investigate

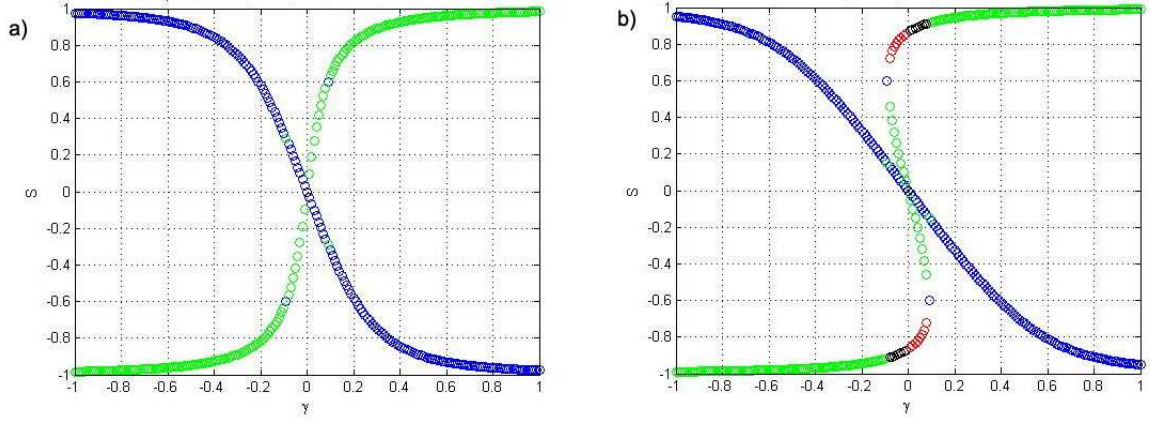


Figure 2.13: Fixed points associated with the eigenenergies in Fig. 2.12 [31]. a) $c/v = 0.5$, b) $c/v = 2$.

the dynamics of Rosen-Zener transition in a nonlinear two-level system, where the asymmetry between the surface plasmon and soliton states, ΔE , is fixed, and the coupling between two channels, $q(Z)$, is a time dependent exponential function. This is a much more closer analogy than the LZ model as far as the mathematical modeling is concerned. A similar study has been done with Bose Einstein condensates [32] by constructing the effective classical Hamiltonian, that describes the dynamic properties of nonlinear quantum Rosen-Zener system. Although our system cannot be cast into a classical hamiltonian, because the driving parameter is part of the coupling function, Eq. (2.42) and Eq. (2.43) are fully capable of describing the dynamical properties of plasmon-soliton interaction.

2.3 Numerical Analysis

In this thesis study, our main objective is to obtain a full population transfer between soliton and surface plasmon channels. With the hyperbolic modification of the coupling function we implemented in the previous section, we observe that such a transition occur in the neighbourhood of the minimum distance between the metal surface and the nonlinearity. Since the distance is increasing hyperbolically, outside of this effective interaction area the coupling between soliton and surface plasmon is

negligible. As shown in the Fig. 2.14(a) at certain parameter values a nearly ideal transition between soliton and surface plasmon amplitudes is observed.

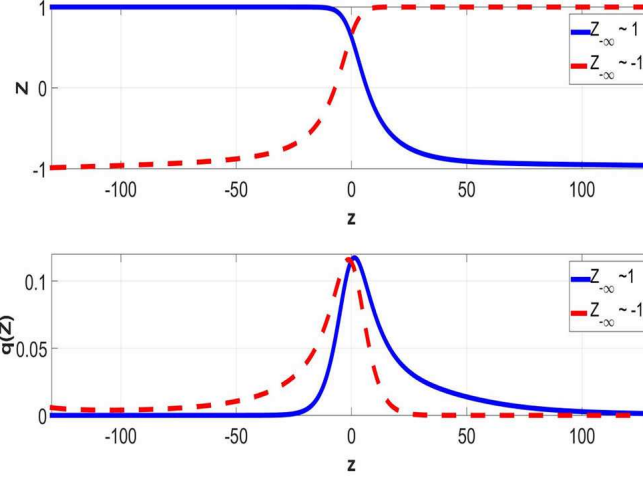


Figure 2.14: (a) Solid line shows the transition from soliton to surface plasmon amplitude, dashed lines show the reverse transition from surface plasmon to soliton amplitude. $\alpha = 0.63$, $d = 5.7$, $\Lambda = 0.04$, and $\Delta E = 0.02$. (b) Solid line shows the plot of coupling function ' $q(Z)$ ' vs. z of the first transition, and the dashed lines show the plot of ' $q(Z)$ ' vs. z of the reverse transition.

When the system starts with $Z_{-\infty} \sim 1$, and it approaches -1 implying that, the optical soliton propagating inside the fiber in the absence of surface plasmon enters the effective interaction region, where the population change occurs, it transfers almost all of its energy to the surface plasmon emerging on the metal surface, and the surface plasmon continues propagating in the absence of optical soliton. Since the dissipation in the system is negligible, and the system is symmetric in coupling function ($q \simeq \bar{q}$), total soliton and plasmon amplitudes are conserved as expected from $\dot{N} = 0$. This population transfer is reversible and symmetrical. In the case of $Z_{-\infty} \sim -1$, it approaches 1 after the transition. The initial Z values can not be taken exactly 1 or -1, because that would make the denominator term zero in Eq. (2.43). Beyond this reason, there is another consideration, why Z should not be chosen -1. It implies that the initial soliton amplitude is zero, which makes the coupling function $q(Z) = 0$ and

in that case, no transition would occur.

Figure 2.14(b) shows the plot of $q(Z)$ vs. z for two different cases, and the difference can be seen clearly between two coupling parameter plots. In the first case, where the soliton starts to propagate in the absence of surface plasmon, the rate of change of $q(Z)$ with respect to z is greater than the second case, where the surface plasmon starts to propagate in the absence of soliton. After the transition, the rate of change of $q(Z)$ with respect to z is less than the second case, as expected. This is caused by the direct dependency of coupling function 'q' to the soliton amplitude $|C_s|$. An asymmetrized generalization of the Rosen-Zener model with a similar coupling function has been investigated in a previous study [41]. Although this full population transfer is our desired result, some further analysis should be followed, since there are six parameters ($\alpha, d, \Lambda, \Delta E, Z_0$, and ϕ_0), that effects the outcome. First we discuss the effects of d and α .

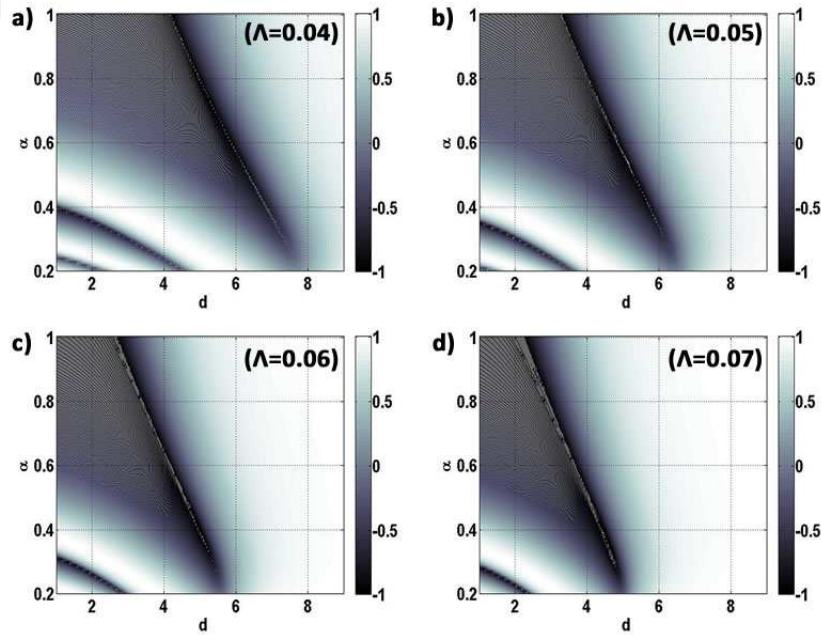


Figure 2.15: Contour plot of final Z values with respect to α and d for different Λ values. Every position on the d - α plane corresponds to steady-state 'Z' values after each transition. The maximum energy transfer from soliton to surface plasmon is observed, when $d = 5.7$, and $\alpha = 0.63$ for panel (a). Here ΔE is taken 0.02.

The panels in Fig. 2.15 show the final values of Z as a function of d and α for four different Λ values. From these results, we try to determine the optimum system parameters, which yield maximum transition from soliton to surface plasmon. Here ΔE is kept constant at 0.02. The darker territories correspond to strong transitions. A long, thin and dark slit can be seen on each panel, which indicates the optimum parameters for a successful transition. The transition gets weaker when the position on the d - α plane moves away from the slit, and after some point no transition occurs at all (wide and light area). The reason for this lack of transition is the reverse proportionality of d and α to the $q(Z)$ itself. In order to keep the coupling parameter above some threshold value, which would cause a successful transition, we should investigate the area where the Z value is close to -1. It can be seen in panel (a) that, the optimum d and α values are 5.7 and 0.63, respectively.

The meaning of α being small is crucial, because Eq. (2.37) is derived from 1-D Maxwell's equations, and in our system we assumed that, the hyperbolicity in the fiber is small enough to keep the paraxial approximation valid. For the sake of the validity of our calculations, we do not allow α be greater than 0.7, so our modification to the coupling parameter does not cause the system to lose its physical meaning. In Fig. 2.16, the error percentage - α comparison can be seen. The paraxial

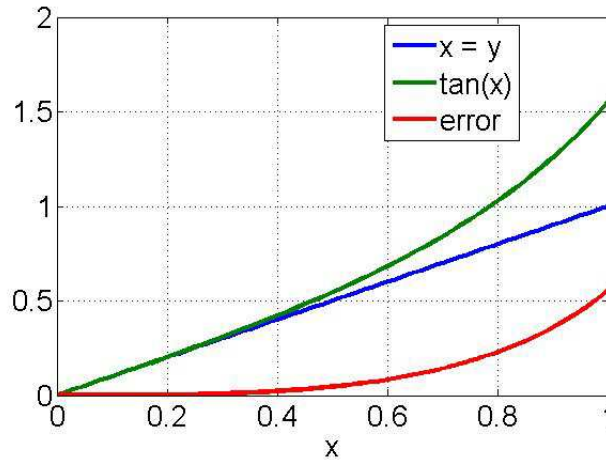


Figure 2.16: Error percentage of the α values. Red line shows the error.

approximation suggests: $\tan(\theta) \approx \theta$. α is identified as the tangent of the angle between z axis and the asymptotic line, called θ_c . Hence,

$$\tan\theta = \alpha \approx \theta_c. \quad (2.51)$$

So the red line in Fig. 2.16 represents the deviation from the approximation, and it is defined as the difference between the green and blue lines.

The panels in Fig. 2.17 reveal the role of nonlinearity and asymmetry in plasmon-soliton coupling. The dark slit moves left in the former, as Λ increases pointing out that, strong nonlinearity requires closer distance, in order to obtain a successful transition. In the latter, the slit expands, when ΔE decreases, which gives us a wider area of transitions. This result comes from the definition of ΔE :

$$\Delta E = \Lambda - \nu_p = \frac{\nu_s}{2|C_s|^2} - \nu_p \quad (2.52)$$

ΔE can be defined as the asymmetry between soliton and plasmon wave vectors. As ΔE increases, soliton channel takes over the dominant role of the coupling.

Figure 2.18 shows the wavefunction profiles of surface plasmon and optical soliton. It is shown that, soliton initially propagating on the curved fiber transfers almost all of its energy to the surface plasmon. The exponent of the surface plasmon profile on the metal ($x < 0$) decays away from the metal-dielectric interface quicker than the exponent on the dielectric, because of the small skin depth of the metal δ , which is taken 0.1. As discussed earlier, the decay in the soliton amplitude is greater than the increase in surface plasmon amplitude, which is caused by the ' $|C_s|$ ' dependency of the coupling parameter.

Figure 2.18 also shows the wavefunction profiles of surface plasmon and optical soliton in the case that, surface plasmon propagates initially in the absence of soliton, and transfers almost all of its energy to soliton. The increase in the soliton amplitude is greater than the decrease in the surface plasmon amplitude as expected. The tail of the newly forming soliton wave is seen to be on the metal surface, where the distance between the metal and the fiber is at its minimum (around $z = 0$), which may differ

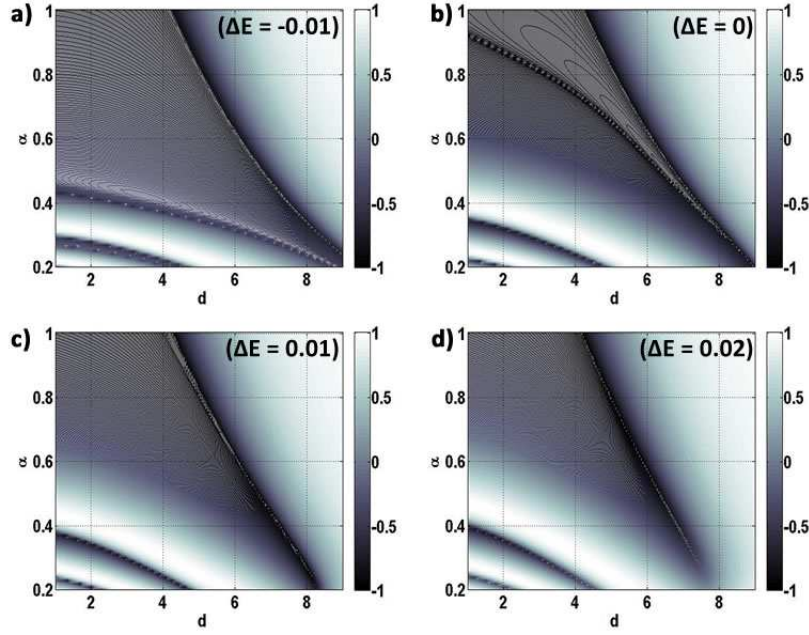


Figure 2.17: Contour plot of final Z values with respect to α and d for different ΔE values. Every position on the d - α plane corresponds to steady-state ' Z ' values after each transition. The maximum energy transfer from soliton to surface plasmon is observed, when $d = 5.7$, and $\alpha = 0.63$ for panel (d). Here Λ is taken 0.04 respectively.

by the choice of the d value. In both cases, the total energy of the surface plasmon and soliton is normalized to 1.

The interaction between surface plasmon and optical soliton with a hyperbolic distance between them gives rise to other significant results. Changing the system geometry and the strength of the nonlinearity allow us to manipulate the energy transfer between surface plasmon and soliton. Fig. 2.19 shows some of the significant transitions under different α and Λ values.

We can divide the energy of the soliton propagating in the absence of the surface plasmon equally between them, and similarly, when we have both soliton and surface plasmon propagating initially, we can manipulate the total energy to form either surface plasmon or optical soliton only. In the case, where α is 0.25, we have subsequent transitions between soliton and surface plasmon. First, soliton transfers almost all of its energy to surface plasmon, then surface plasmon transfers all the energy back

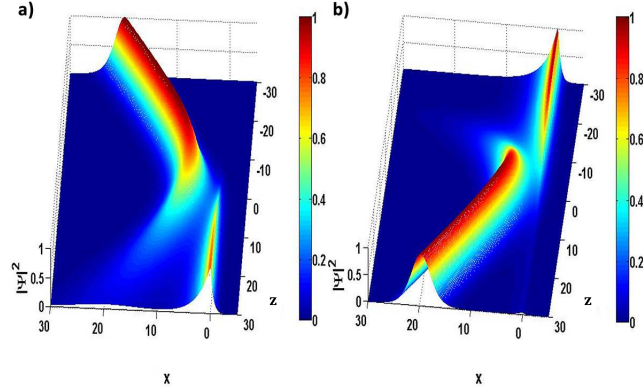


Figure 2.18: (a) Propagation of surface plasmon and optical soliton wavefunctions. Optical soliton is propagating in the absence of surface plasmon initially and after the transition it transfers almost all of its energy to surface plasmon on the metal surface. Here α is 0.63, Λ is 0.04, ΔE is 0.02, and d is 5.7. (b) Propagation of surface plasmon and optical soliton wavefunctions. Surface plasmon is propagating in the absence of optical soliton initially and after the transition it transfers almost all of its energy to optical soliton on the fiber. The same parameter values are used.

to soliton. These subsequent transitions show similar properties of localized surface plasmons.

Localized surface plasmons are non-propagating excitations of the conduction electrons of sub-wavelength metallic nanostructures coupled to the electromagnetic field [5]. These excitations emerge inherently from metallic nanoparticles of dimensions below 100 nm in an oscillating electromagnetic field. Applying an effective restoring force by the curved surface of the particle on the driven electrons generates localized surface plasmon resonance, which leads to a field enhancement in the near-field zone outside the particle. In our system, the small area around the minimum distance from the Kerr nonlinearity to the metal surface can be considered as the nanoparticle mentioned above. Since the nonlinearity in the dielectric is structured as a curved path characterized by the parameter α , and this area is in nanometer scale, our system can be counted as a proper host to localized surface plasmons. The wavefunction profiles of surface plasmon and soliton of these four cases can be seen in Fig. 2.20.

The distance between nonlinearity and the metal surface d can be arranged as

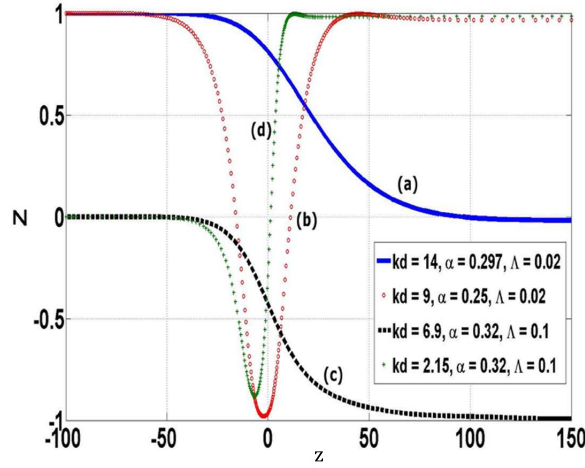


Figure 2.19: Significant transitions between surface plasmon and soliton under different d , α and λ values. (a) shows the case that soliton is propagating initially in the absence of surface plasmon, and its energy is divided equally between soliton and surface plasmon. (b) shows the case that soliton transfers almost all of its energy to surface plasmon and then surface plasmon transfers its energy back to soliton. (c) and (d) show the case that in the presence of equal surface plasmon and soliton, the energy can be manipulated to be transferred to surface plasmon or soliton, respectively.

desired in order to obtain different significant results as shown in the figure. The change of d could result in outranging the system from 100 nm scale. In this case, the quasi-static approximation breaks down due to retardation effects, however localized surface plasmon resonance can still be obtained with damping according to the Mie Theory [5], which will not be discussed here.

2.4 Stability Analysis

2.4.1 Effect of relative phase difference

Up to this point, the relative phase difference ' ϕ ' is chosen zero in our calculations, indicating that, soliton and surface plasmon are propagating in phase. However, our system allows the outcome of the transition to be different and significant, when soliton and surface plasmon are travelling out of phase. Our previous examples of transitions are immune to effects of phase change, because the absence of surface

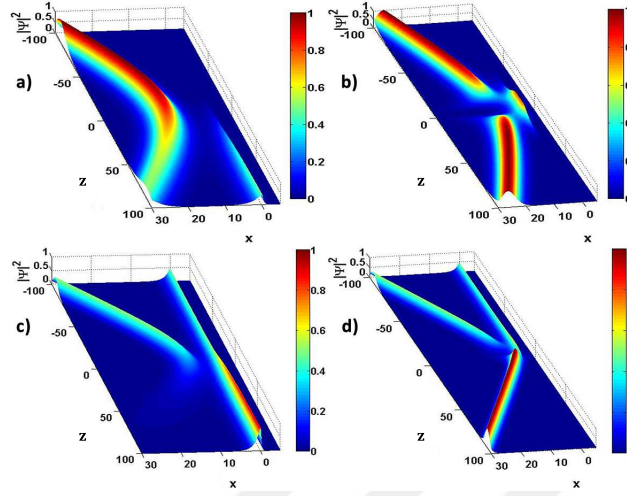


Figure 2.20: The wavefunction profiles of surface plasmon and optical soliton of the same four cases.

plasmon or soliton means that, the relative phase between them is irrelevant. However, when initial Z value is different from 1 or -1 , we observe that the system is highly sensitive to initial relative phase value. Fig. 2.21 shows the effect of initial relative phase to different initial Z values.

The sensibility of the system to the initial relative phase difference can be used to manipulate the system to give desired outcomes. Starting with equal surface plasmon and soliton amplitudes, it can be arranged to transfer all the energy of the system either to surface plasmon or soliton only. Figure 2.22 shows manipulation of energy to one direction by changing only the initial relative phase difference.

The effect of relative phase difference can be best perceived once the phase-space analysis of the full population transfer is discussed [18], [19]. In Fig. 2.23, a comprehensive analysis of the evolution of the population imbalance and the phase difference is shown. In the bosonic Josephson junction models the phase-space of the system changes adiabatically, since they change the nonlinearity parameter with constant velocity. This repeated change in the system results in a repeated deformation in the phase-space as expected. However the nonlinearity parameter is determined by the material used as the dielectric in the plasmonic Josephson junction system, therefore,

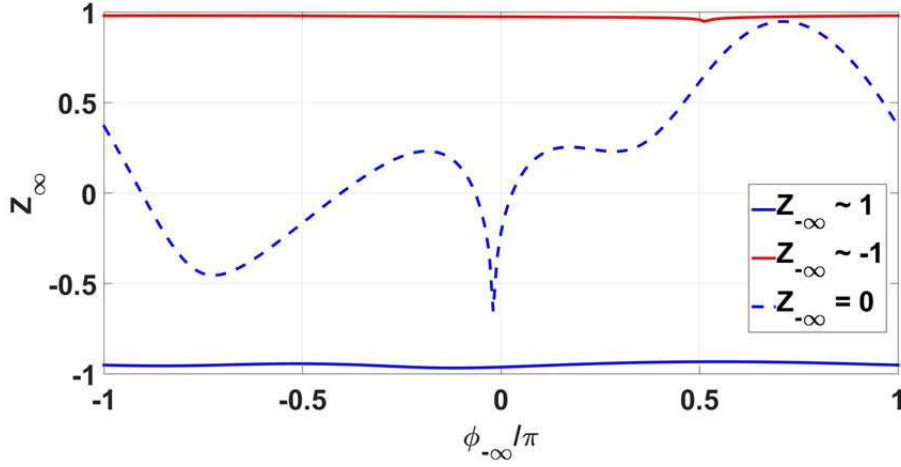


Figure 2.21: The effect of different initial phase differences. Solid red line shows the final Z values for every single different ϕ_0 value when the system starts with $Z_{-\infty} \sim -1$. Solid blue line shows the final Z values for every single different ϕ_0 value when the system starts with $Z_{-\infty} \sim 1$. Dashed lines show the final Z values for every single different ϕ_0 value when the system starts with $Z = 0$. Here the full transition parameter values are used; $d = 5.7$, $\alpha = 0.63$, $\Lambda = 0.04$, and $\Delta E = 0.02$.

it cannot be changed externally along the propagation. In this case, the phase-space of the system would not undergo a deformation, ergo the movement of the population imbalance would be restricted by the current phase-space trajectories indicating that, a Landau-Zener transition would not manifest itself. In order to overcome this problem we modified our system spatially, which was the main motivation of this work. When the distance between metal surface and the Kerr nonlinearity decreases adiabatically to the minimum distance d and then, starts to increase, in other words, when z changes, the phase-space trajectories also change, and the expected deformations manifest themselves under the control of adiabatic parameter α . On every trajectory the transition probability can be determined explicitly. In Fig. 2.23 some of the significant trajectories are shown. Comparing this figure with Fig. 2.14 concurrently would give a good insight to the interpretation of these trajectories.

Panel (a) in Fig. 2.23 corresponds to the initial state of the system, $z = -100$. The red dot represents the system with $Z = 1$ and $\phi = 0$, which means the whole energy is in the soliton channel, and the phase difference between soliton and surface plasmon

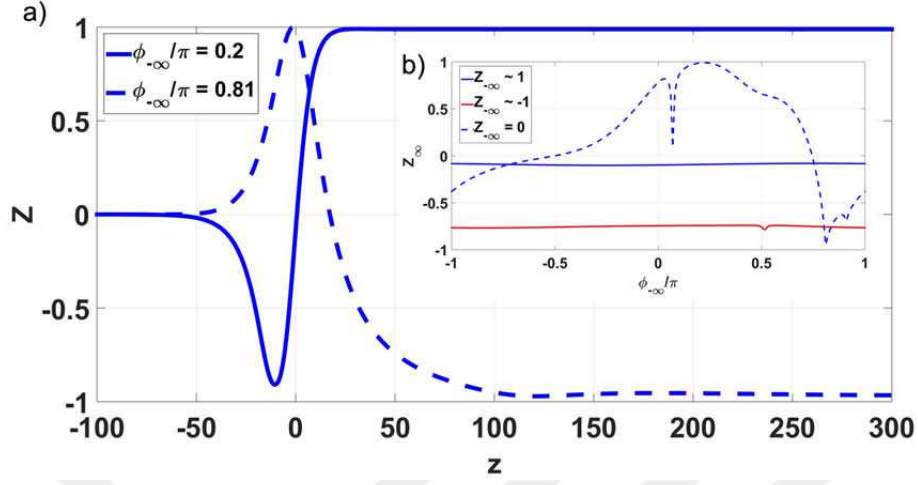


Figure 2.22: (a) Solid line shows the case when $\phi_0 = 0.2$, in which all the energy is transferred to soliton. Dashed lines show the case when $\phi_0 = 0.81$, in which almost all the energy is transferred to surface plasmon. Here $d = 5.6$, $\alpha = 0.4$, $\Lambda = 0.04$, and $\Delta E = 0.02$. (b) Solid red line shows the final Z values when the system starts with $Z_{-\infty} \sim -1$. Solid blue line shows the final Z values when the system starts with $Z_{-\infty} \sim 1$. Dashed lines show the final Z values when the system starts with $Z = 0$.

is zero. When the soliton propagates until $z = -30$ the trajectory is still unaffected by the geometry of the nonlinearity, so the interaction is still negligible. When the soliton enters the effective interaction region, $z = -13$, a fixed point emerges at $Z = 1$, $\Phi = \pi$ which causes a closed trajectory around itself, and surface plasmon-soliton coupling falls in to this trajectory and change its course. When z increases, this fixed point starts to move downwards along with its trajectories. Surface plasmon-soliton coupling has no choice but to follow these trajectories, and ends up trapped by this fixed point at $Z = -1$, $\phi = \pi$, which corresponds to transfer of energy to plasmon channel completely.

A similar transition can be observed when the system starts with surface plasmon only. The green dot starts at $Z = -1$, $\phi = 0$ and follows its current trajectory. The fixed point around $Z = -0.5$ affects its direction, and leads it to a stationary-state at $Z = 1$. The change in the relative phase can also be clearly seen in Fig. 2.24. Panel (a) shows the phase differences of the green dot with different initial phase difference

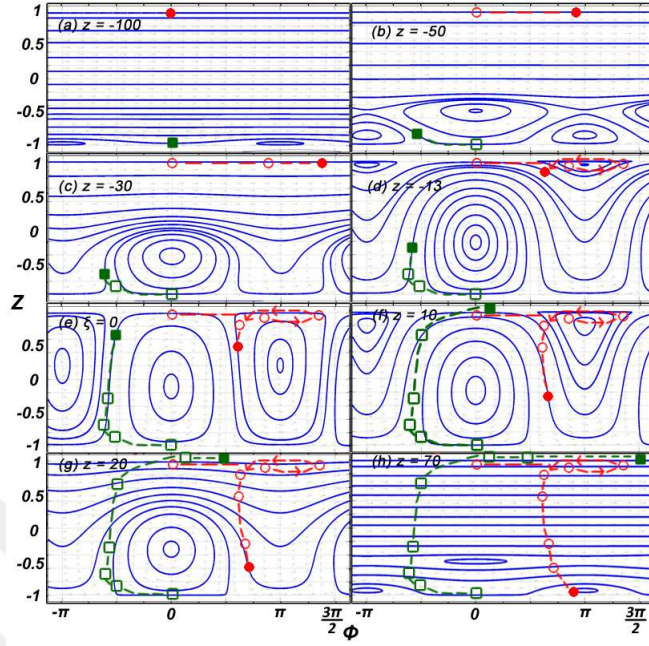


Figure 2.23: The evolution of the phase-space trajectories of the system as z changes adiabatically. The red dots indicate the motion of the population imbalance in phase-space when $Z_{-\infty} \sim 1$. The green dot indicates the motion when $Z_{-\infty} \sim -1$. Here the parameters are the same as in Fig. 2.14.

values. Depending on the choice of the initial value, there are two different trajectories, one of them tends to move left in the phase-space, and the other tends to move right. Whichever the choice might be, the phase difference increases constantly at the stationary state, indicating the coupling moves constantly right on the uppermost trajectory, since there isn't any fixed points emerging at the top. Although the phase difference increases, the population imbalance reaches its final value, $Z = 1$.

Panel (c) in Fig. 2.24 shows a similar phase difference pattern for the red dot. Instead of staying on a stationary state, in which the phase difference increases with constant velocity, this coupling is trapped at a fixed point, and the change in the relative phase is quite small. Panel (b) shows the behaviour of the relative phase, when the system starts from $Z = 0$. As can be seen from the figure, the final result is more sensitive to the initial choice of ϕ , because population of one of the channels is not zero and coupling is more flexible.

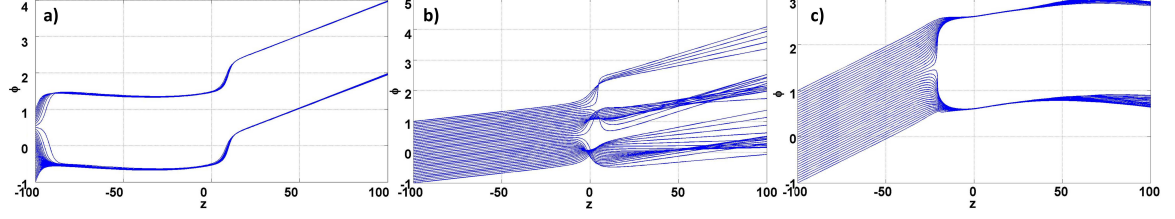


Figure 2.24: Initial relative phase difference vs. z graph. (a) $Z = -1$, (b) $Z = 0$, (c) $Z = 1$.

These two transitions are a clear presentation of the jump from one state to another in the metal-dielectric-Kerr system, which leads us to make a comparison with the Landau-Zener tunneling [20],[21].

The phase-space analysis of the system with parameters in Fig. 2.22 is also insightful. The route of the population imbalance in phase-space is shown in Fig. 2.25, such that different initial relative phase differences lead the system to two different steady-state solutions.

2.4.2 Fixed Point Analysis

The trajectory of the system in the phase-space is radically effected by the fixed points of the system, as seen in the figures 2.23 and 2.25. Since the distance between metal surface and the nonlinearity in the dielectric is not constant, but a function of α , it can be said that, the geometry of the system is changing slowly. This geometry change causes a continuous and regular deformation in the phase-space of the system. This is another way of saying that, the fixed points can move, disappear, and/or reappear in the effective interaction region between metal and nonlinearity. The evolution of fixed points of the system plays crucial role in the surface plasmon - optical soliton interaction, therefore attention should be given to the analysis of fixed points.

Fixed points of the system can be found numerically by setting the main dynamic equations equal to zero:

$$\dot{Z} = -q(Z)\sqrt{1 - Z^2}\sin\phi = 0, \quad (2.53)$$

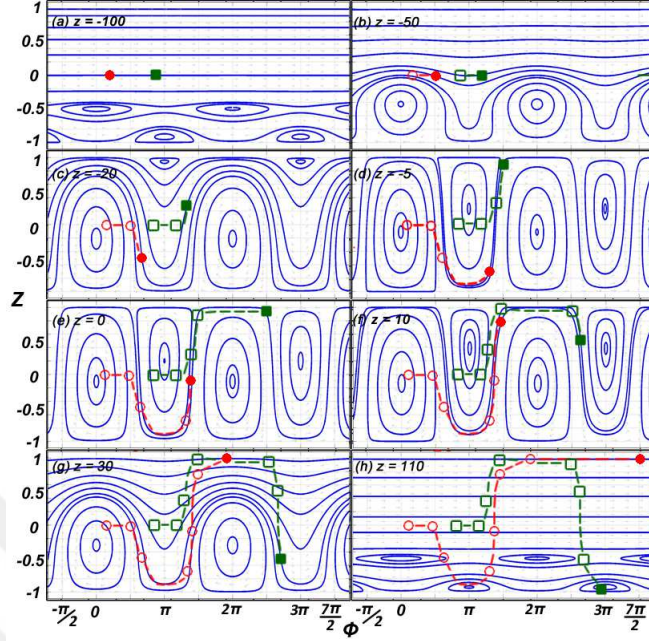


Figure 2.25: The evolution of the phase-space trajectories of the system with equal initial amplitude of surface plasmon and optical soliton as z changes adiabatically. The red dot indicate the motion of the population imbalance in phase-space with $\phi_0 = 0.2$, and green squares indicate the motion of the population imbalance in phase-space with $\phi_0 = 0.81$. The population imbalance changes dramatically from 0 to 1 (red dot) or from 0 to -1 (green dot). Here the parameters are $d = 5.6$, $\alpha = 0.4$, $\Lambda = 0.04$, and $\Delta E = 0.02$.

$$\dot{\phi} = \Lambda Z + \Delta E + \frac{q(Z)Z}{\sqrt{1-Z^2}} \cos \phi = 0. \quad (2.54)$$

It is easier to solve the first equation above as $\sin \phi = 0$ would give the solution. $q(Z)$ shouldn't be zero, otherwise no interaction would occur between surface plasmon and soliton, and Z cannot be 1 or -1, which would make the denominator in the second equation zero. Hence, we should plug in $\phi = 0$, and $\phi = \pi$ solutions in the second equation, and solve for ' Z ' solutions. Since $q(Z)$ is z dependent, for every coordinate on the z -axis, there may be fixed Z points for $\phi = 0$ and $\phi = \pi$, separately. The graph of fixed points with respect to z coordinate can be seen in Fig. 2.26. These fixed points are retrieved from the system which corresponds to the transition in Fig. 2.14.

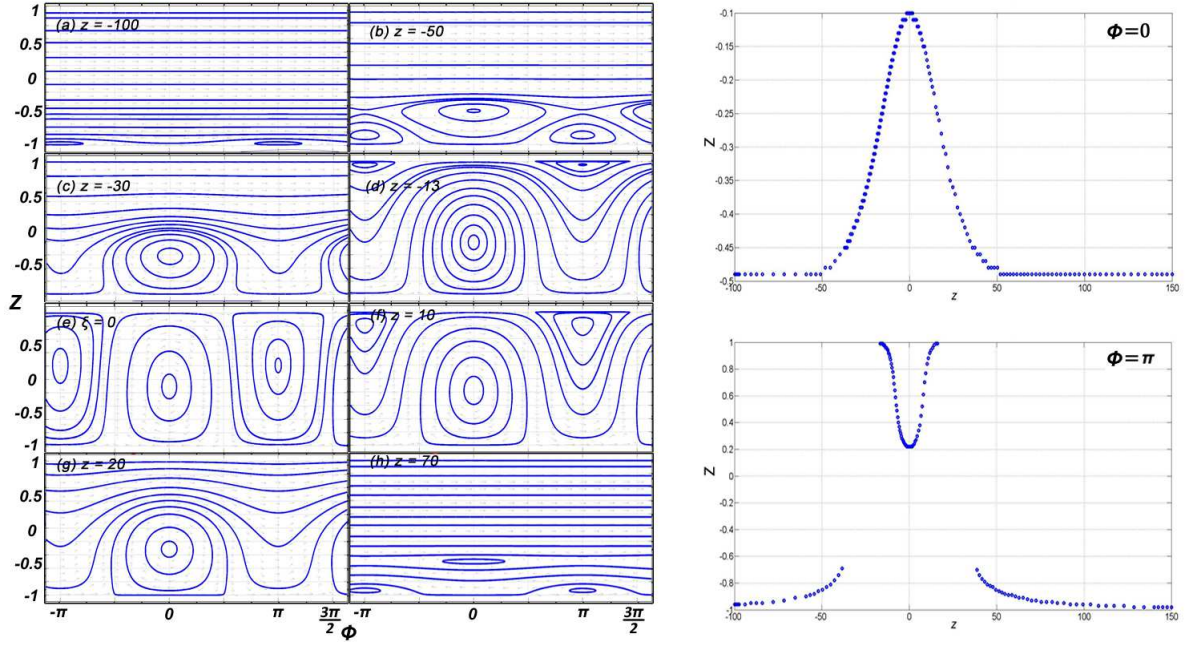


Figure 2.26: Evolution of fixed points for $\phi = 0$ and $\phi = \pi$ can be clearly seen in the phase-space diagram on the left. Corresponding fixed point vs. z graphs are on the right. Here, $d = 5.7$, $\alpha = 0.63$, $\Lambda = 0.04$, and $\Delta E = 0.02$ are used.

The evolution of fixed points can be seen clearly in Fig. 2.26. The first fixed point, ($\phi = 0$), moves between $Z \simeq -0.5$ and $Z \simeq -0.1$. The second fixed point, ($\phi = \pi$) however shows a different behaviour. It first starts at $Z \simeq -1$, then around $z = -20$ it disappears, then around $z = -10$ it reappears at $Z \simeq 1$. After $z = 0$, the movement repeats itself symmetrically. As a matter of fact, this symmetrical evolution of fixed points is observed for every combination of parameters in the hyperbolic system, because the mathematical modeling of this geometry points that, the distance between metal surface and the nonlinearity depends on z^2 , so being on the negative side or positive side of the z -axis does not make a difference. Another correlation of fixed points and phase-space is given in Fig. 2.27. These fixed points are retrieved from the system which corresponds to Fig. 2.22.

This constraint is the main reason, for why this system cannot host an adiabatic transition. In the previous studies, the authors showed that, a fixed point can start

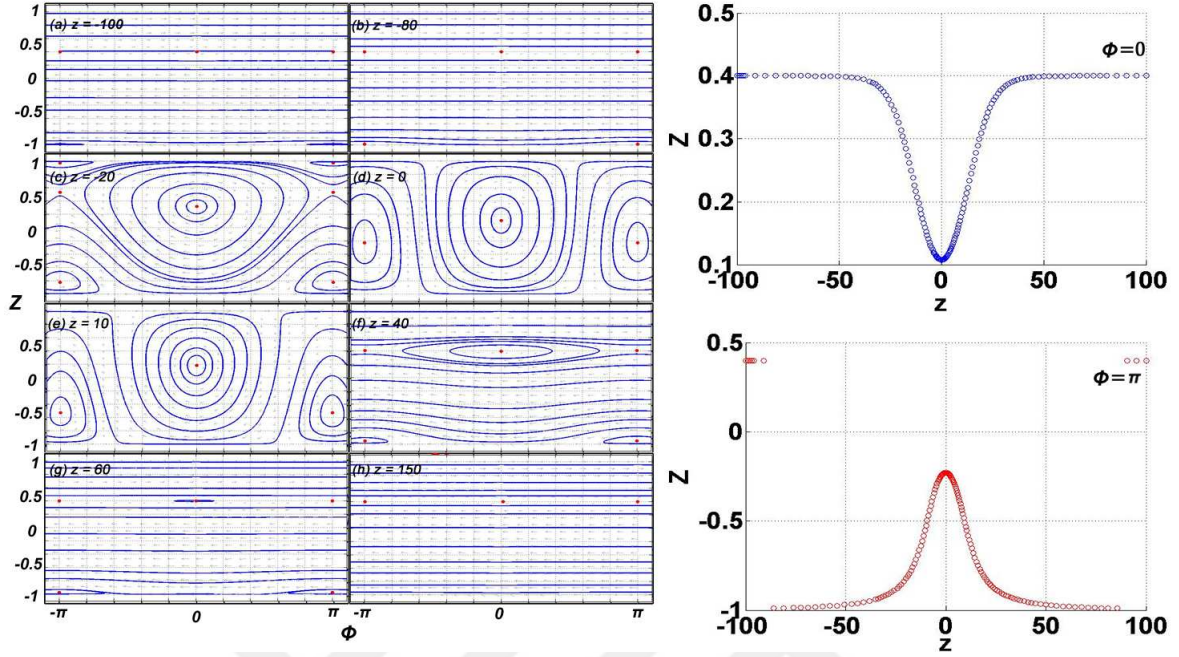


Figure 2.27: Evolution of fixed points for $\phi = 0$ and $\phi = \pi$ can be clearly seen in the phase-space diagram on the left. Corresponding fixed point vs. z graphs are on the right. Here, $d = 2.15$, $\alpha = 0.32$, $\Lambda = 0.1$, and $\Delta E = -0.04$ are used.

its journey from one end of the phase-space, and evolve such that at the end the fixed point reaches the other end of the phase-space. In the meantime BECs trapped in the fixed point would move with it, so a slow external change in the Hamiltonian would result in a 100% transition without a character change in eigenvalues. This is called adiabatic transition. In those studies the adiabatic parameter in the system is the energy gap between traps, however in the plasmonic system the adiabatic parameter is taken as the distance between surface plasmon and soliton. Since the distance cannot be negative, fully adiabatic transition cannot be acquired in the hyperbolically modulated Josephson junction.

To understand the properties of adiabatic transition, eigenenergies of the system should be analyzed. Eq. (2.37) can be considered as nonlinear Schrödinger equation

and written as :

$$-i\frac{d}{dt}\begin{pmatrix} C_p \\ C_s \end{pmatrix} = H(z)\begin{pmatrix} C_p \\ C_s \end{pmatrix}, \quad (2.55)$$

where the Hamiltonian is a function of distance between surface plasmon and soliton:

$$H(z) = \begin{pmatrix} \nu_p & -q(|C_s|)/2 \\ -q(|C_s|)/2 & \nu_s(|C_s|) \end{pmatrix}. \quad (2.56)$$

There are two conventional ways of determining the eigenvalues of this Hamiltonian [34]. The first way is to construct the classical Hamiltonian for the conjugate variables Z and ϕ . These variables satisfy the Hamilton's dynamic equations: $\dot{Z} = -\frac{\partial H}{\partial \phi}$, $\dot{\phi} = \frac{\partial H}{\partial Z}$. This way canonical equations (2.42) and (2.43) can be derived and the fixed points can be found by setting equations ??eq:2.42) and (2.43) equal to zero. Then the results should be substituted into the classical Hamiltonian to obtain the eigenenergies. These eigenenergies can be considered as the energy levels of the system. However this method cannot be applicable to the plasmonic system, because in the systems, which previously used this method Hamiltonian has the physical meaning of being total energy of the system. Unfortunately, total energy of the plasmonic system cannot be defined by the classical Hamiltonian, since C_s and C_p are merely the amplitudes of surface plasmon and soliton. The total energy of this system can be calculated by considering the frequencies of the waves and the momentum, that they carry, while propagating. Moreover, the plasmonic system cannot be cast into classical Hamiltonian analytically, therefore this method is proved to be useless for the plasmonic two-level system.

The second method is to present a quartic equation by substituting $-i\frac{d}{dt}$ in Eq. (2.43) with ϵ , which is called the eigenenergy of the Hamiltonian.

$$\epsilon\begin{pmatrix} C_p \\ C_s \end{pmatrix} = H(z)\begin{pmatrix} C_p \\ C_s \end{pmatrix}, \quad (2.57)$$

After some elaboration the above equation yields the following quartic equation:

$$-\frac{\nu_p^2}{2} + \frac{\epsilon\nu_p}{2} + \left(\frac{\nu_p - \epsilon}{2}\right)\sqrt{\Delta} - \frac{2\epsilon\nu_p + \sqrt{\Delta}}{2\gamma}\exp\left[-2\sqrt{d^2 + \alpha^2 z^2}\sqrt{2\epsilon - \nu_p + \sqrt{\Delta}}\right] = 0$$

$$(2.58)$$

where $\Delta = \nu_p^2 + 4\epsilon^2 - 4\epsilon\nu_p - \gamma\epsilon + \nu_p\gamma$. Although the analytical solution of this equation cannot be presented, a numerical approach might be offered, however this approach would be time inefficient due to the complexity of the equation, so another method is presented here.

We start with the Eq.2.57, but instead of constructing it into the quartic equation, we divert it into a relation between ϵ and Z :

$$\epsilon = \frac{\Lambda(1+Z)^2 - \nu_p(1-Z)}{2Z} \quad (2.59)$$

Afterwards, we find the fixed points by equating the equations (2.42) and (2.43) equal to zero as previously explained and practiced. Then we plug in the fixed points that we have found into the Eq. (2.59) as Z . The resulting values of ϵ are considered as the eigenenergies of the plasmonic two-level system. The eigenenergies of the system with parameters used in Fig. 2.26 and the associated fixed points are given in Fig. 2.28.

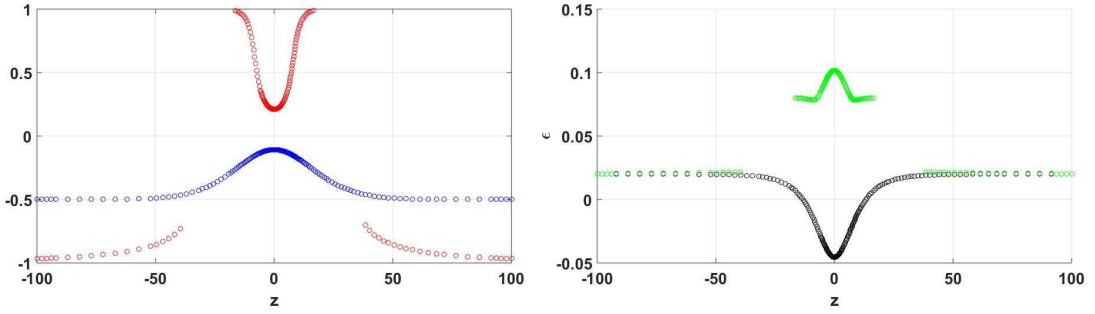


Figure 2.28: Eigenenergies and the corresponding fixed points.

In order to understand, how the system behaves under different parameters, Fig. 2.29 could give insight into the effect of ΔE to the fixed points of the system and the associated eigenvalues. Panels a)-c) shows the fixed points of the system with parameters $d = 2.15$, $\alpha = 0.32$, and $\Lambda = 0.1$, whereas panels d)-f) shows the eigenenergies of the corresponding fixed points. It can be seen that, the choice of ΔE might create three different regimes. When ΔE is negative, the first fixed point ($\phi = 0$)

always takes positive values. The second fixed point, however, starts its evolution on the positive side, when the system is decoupled, then jumps into the negative side, and follows the path shown in panel a). When ΔE is positive, the behaviour of fixed points is contrary. Not only the first fixed point is on the negative side, its trajectory is upside down as well. Although the change in the ΔE is symmetric, the flip of the trajectories are not. This is caused by the soliton dominancy in the governing equations of the system. An interesting behaviour manifests itself, when $\Delta E = 0$. The first fixed point becomes zero at all points, and the second fixed point follows a trajectory from -1 to 0 , and then to -1 again. At the effective transition area, both fixed points coincide. In panels d)-f) associated eigenenergies of these fixed points can be seen. Effect of ΔE to these eigenenergies is similar to the effect to fixed points.

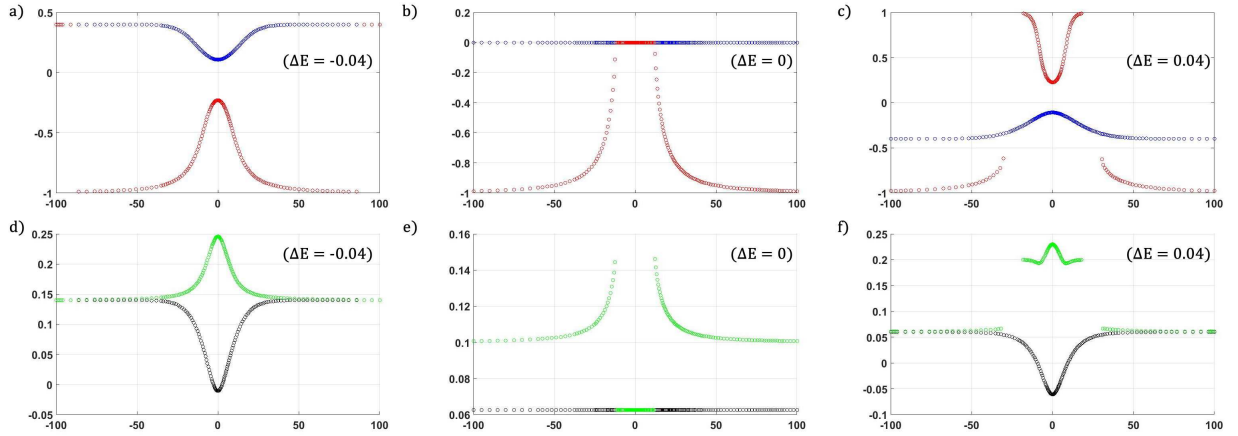


Figure 2.29: Effect of ΔE . Upper panels are fixed points, lower panels are eigenenergies. Black and green dots in the lower panels correspond to blue and red dots in the upper panels, respectively. Here, $d = 2.15$, $\alpha = 0.32$, $\Lambda = 0.1$.

Figure 2.30 shows the behaviour of fixed points, when the nonlinearity changes. To understand the effect of nonlinearity is quite important, since it is directly related to the soliton propagation, $\nu_s = 2\Lambda|C_s|^2$. Unlike ΔE , nonlinearity does not change the pattern of fixed point trajectories, but slowly reduces the area, in which the second fixed point reappears at $Z = 1$. Weak nonlinearity, see Fig. 2.30 panel d), causes a more symmetric eigenenergy pattern, whereas the strong nonlinear regime, see panel

f), causes an irregular pattern, in which the evolution along z -axis follows the green dots, then around $z = 20$ there is no path to follow but to jump to upper or lower states. This behaviour, however, is not concluded with an adiabatic transfer of energy, because the system is symmetric with respect to $z = 0$, so when the evolution along z -axis reaches another dead end, it jumps one of the lower states, which coincides eventually at the value that is exactly equal its initial value at $z = -100$.

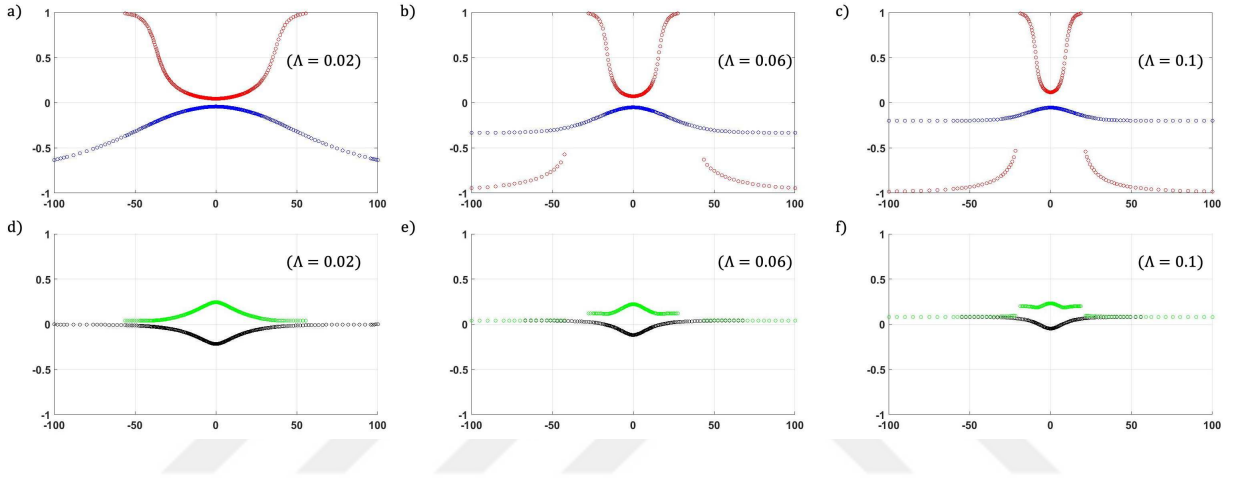


Figure 2.30: Effect of Λ . Upper panels are fixed points, lower panels are eigenenergies. Black and green dots in the lower panels correspond to blue and red dots in the upper panels, respectively. Here, $d = 2.15$, $\alpha = 0.32$, $\Delta E = 0.02$.

In order to gain more perception about the effect of Λ and ΔE , another set of parameters is presented in the following figures. Fig. 2.31 and 2.32 show the fixed point and eigenenergy plots of the system in Fig. 2.19 d), where the system starts with equal soliton and surface plasmon amplitude, and after the transition all of the population is canalized to the soliton channel. The parameters are $d = 14$, $\alpha = 0.297$, $\Lambda = 0.02$, and $\Delta E = 0.03$. Also note that, the parameters used in figures 2.29 and 2.30 are the same as the system in Fig. 2.19-a, where the initial soliton amplitude is divided equally between soliton and surface plasmon channels after the transition.

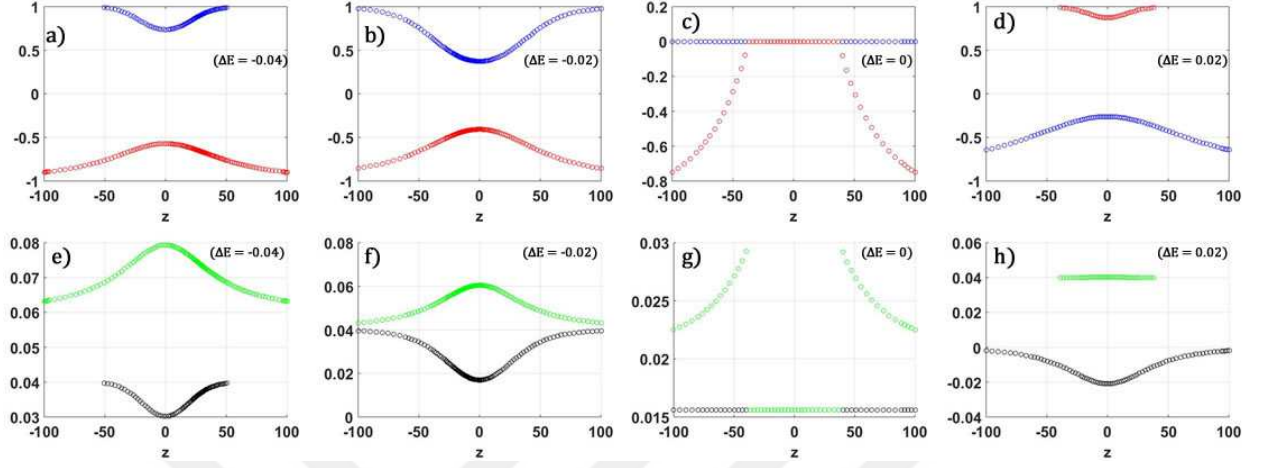


Figure 2.31: Effect of ΔE . Upper panels are fixed points, lower panels are eigenenergies. Black and green dots in the lower panels correspond to blue and red dots in the upper panels, respectively. Here, $d = 14, \alpha = 0.297, \Lambda = 0.02$.

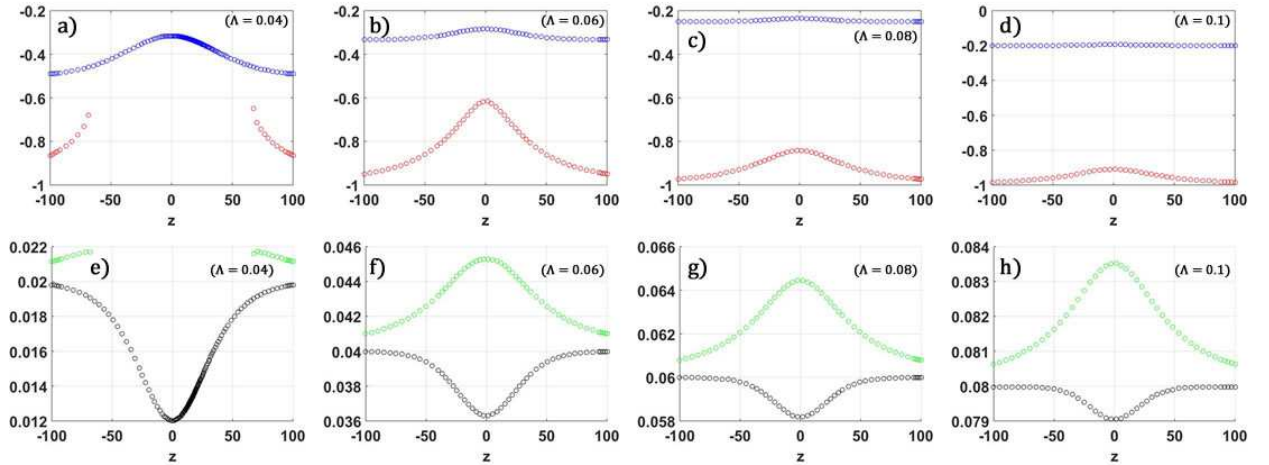


Figure 2.32: Effect of Λ . Upper panels are fixed points, lower panels are eigenenergies. Black and green dots in the lower panels correspond to blue and red dots in the upper panels, respectively. Here, $d = 14, \alpha = 0.297, \Delta E = 0.03$.

Chapter 3

DOUBLE JOSEPHSON JUNCTION IN A METAL-DIELECTRIC-METAL INTERFACE

3.1 Theoretical Background

3.1.1 Surface Plasmons in Multilayers

Surface plasmon excitation on a metal-dielectric interface has been discussed throughly in Chapter 2. The key property about this subject is that, when the tail of the evanescent waves encounters the nonlinearity spatial solitons arise, and these two waves obey coupled oscillator equations. When the tail of the soliton wave encounters the metal surface, surface plasmons arise as well. This mechanism can also occur on a three level system, metal-dielectric-metal interface, so that, surface plasmons on both sides can interact indirectly via optical soliton. The geometrical condition for coupled modes to occur from the interactions of these optical waves is that, the distance between neighboring interfaces should be comparable to the evanescent tails of the waves. To investigate the general features of coupled surface plasmon-soliton modes, the system in Fig. 3.1 should be analyzed [5], [6].

Surface plasmons occur only in TM modes, so the field components of TM modes will be discussed. The electric and magnetic fields for $z > a$ region are

$$H_y(x) = Ae^{i\beta z}e^{-k_3x}, \quad (3.1)$$

$$E_z(x) = iA\frac{1}{\omega\epsilon_0\epsilon_3}k_3e^{i\beta z}e^{-k_3x}, \quad (3.2)$$

$$E_x = -A\frac{\beta}{\omega\epsilon_0\epsilon_3}e^{i\beta z}e^{-k_3x}, \quad (3.3)$$

for $x < -a$ region

$$H_y(x) = Be^{i\beta z}e^{k_2x}, \quad (3.4)$$

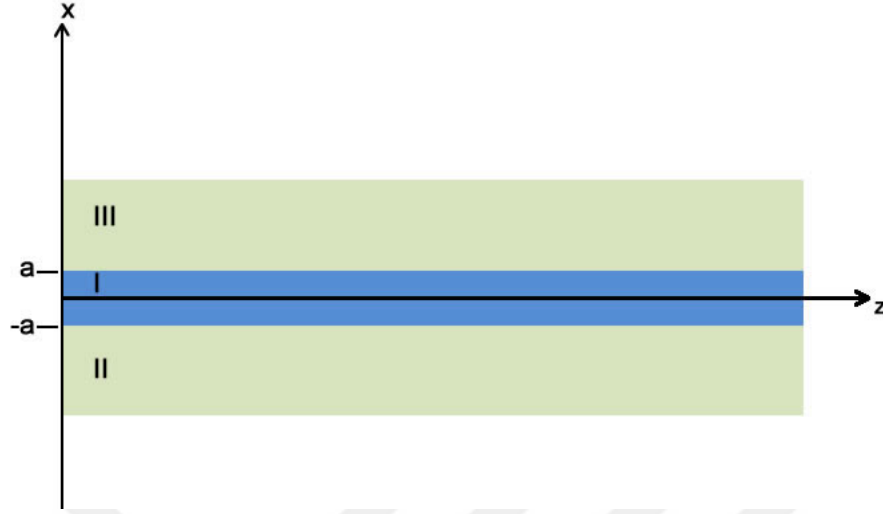


Figure 3.1: Illustration of a three-level system consisting of a middle layer I trapped between two large half spaces II and III.

$$E_z(x) = -iB \frac{1}{\omega \epsilon_0 \epsilon_2} k_2 e^{i\beta z} e^{k_2 x}, \quad (3.5)$$

$$E_x = -B \frac{\beta}{\omega \epsilon_0 \epsilon_2} e^{i\beta z} e^{k_2 x}. \quad (3.6)$$

In the center region $-a < x < a$, the bottom and top modes couple, yielding

$$H_y = C e^{i\beta z} e^{k_1 x} + D e^{i\beta z} e^{-k_1 x}, \quad (3.7)$$

$$E_z = -iC \frac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta z} e^{k_1 x} + iD \frac{1}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta z} e^{-k_1 x}, \quad (3.8)$$

$$E_x = C \frac{\beta}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta z} e^{k_1 x} + D \frac{\beta}{\omega \epsilon_0 \epsilon_1} k_1 e^{i\beta z} e^{-k_1 x}. \quad (3.9)$$

The boundary condition requires continuity of H_y and E_z at $x = a$, which leads to

$$A e^{-k_3 a} = C e^{k_1 a} + D e^{-k_1 a}, \quad (3.10)$$

$$\frac{A}{\epsilon_3} k_3 e^{-k_3 a} = -\frac{C}{\epsilon_1} k_1 e^{k_1 a} + \frac{D}{\epsilon_1} k_1 e^{-k_1 a}, \quad (3.11)$$

and at $x = -a$

$$B e^{-k_2 a} = C e^{-k_1 a} + D e^{k_1 a}, \quad (3.12)$$

$$\frac{B}{\epsilon_2} k_2 e^{-k_2 a} = -\frac{C}{\epsilon_1} k_1 e^{-k_1 a} + \frac{D}{\epsilon_1} k_1 e^{k_1 a}. \quad (3.13)$$

We are left with a linear system, that consist of four coupled equations. The solution of these coupled equations comes with H_y fulfilling the wave equation in the three specific regions, via

$$k_i^2 = \beta^2 - k_0^2 \epsilon_i \quad (3.14)$$

for $i = 1, 2, 3$. The desired result of this analytical calculation can be achieved by solving this system of linear equations, which is an implicit expression for the dispersion relation relating β and ω via

$$e^{-4k_1 a} = \frac{k_1/\epsilon_1 + k_2/\epsilon_2}{k_1/\epsilon_1 - k_2/\epsilon_2} \frac{k_1/\epsilon_1 + k_3/\epsilon_3}{k_1/\epsilon_1 - k_3/\epsilon_3}. \quad (3.15)$$

In the special case of $\epsilon_2 = \epsilon_3$ and $k_2 = k_3$, the dispersion relation can be divided into a couple of equations

$$\tanh(k_1 a) = -\frac{k_2 \epsilon_1}{k_1 \epsilon_2}, \quad (3.16)$$

$$\tanh(k_1 a) = -\frac{k_1 \epsilon_2}{k_2 \epsilon_1}. \quad (3.17)$$

These equations can be applied to metal/dielectric/Kerr/dielectric/metal structures to investigate the surface plasmon-soliton-surface plasmon interaction, which forms a double Josephson junction.

3.1.2 Bose-Einstein Condensates in Triple-Well Traps

BECs in double-well structures have been discussed previously. The unique properties of BEC systems could earn advantages over superconducting Josephson junctions such as the possibility to investigate the nonlinear effects in double-well, or the control of particle number as it is impracticable in nonlinear waveguide systems. With these advantages many physical phenomena can be investigated such as Landau-Zener tunneling and macroscopic self-trapping. These phenomena could also be observed in triple-well systems, which have two structural types: chain-shaped and ring-shaped potentials. In the former, the wells are positioned alongside so the first and the third well does not interact each other directly [53]. In the latter, the wells are positioned

on a triangle, so that all wells have a direct interaction. The ring-shaped model has been investigated in detail [54], the fixed points and the associated eigenstates have been found, and the phase-space of the system have been discussed through Poincare maps to determine chaos in the system.

3.2 Surface Plasmon-Spatial Soliton Coupling in Multilayer Parallel Systems

3.2.1 Model Description

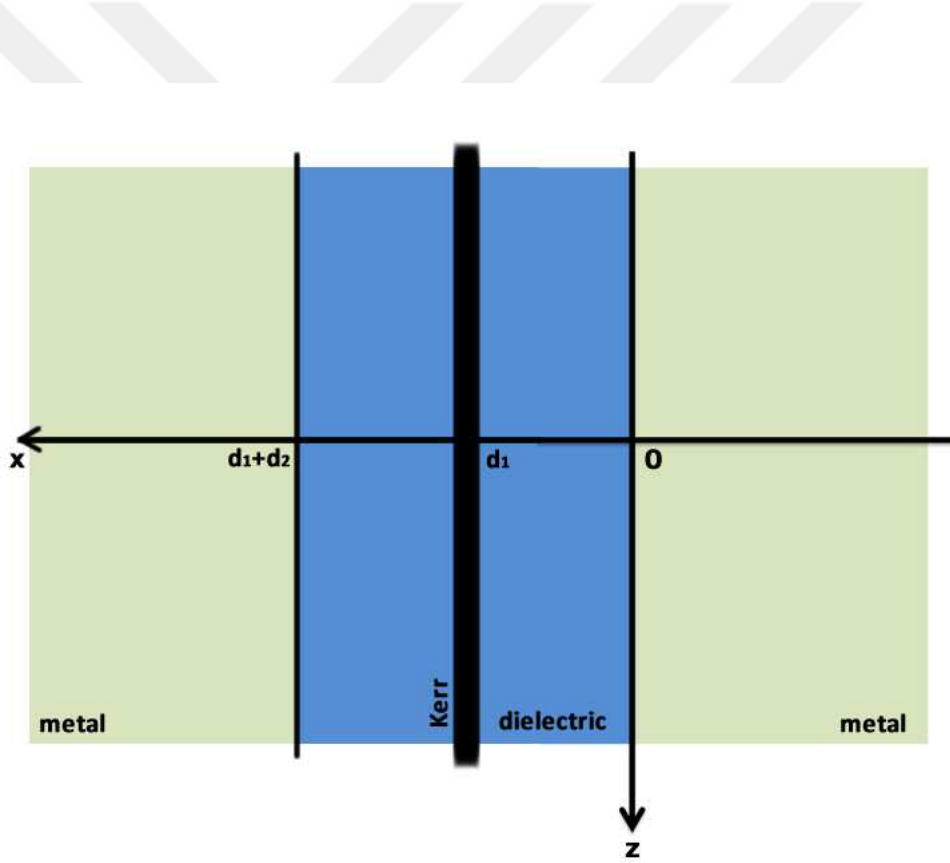


Figure 3.2: Two surface plasmons and a spatial soliton in a metal/dielectric/Kerr/dielectric/metal multilayer.

Single Josephson junction model between surface plasmon and soliton can be extended to a three-level system by adding another metal-dielectric interface to the

other side of the nonlinearity, which forms a double Josephson junction. In this way, the soliton in the middle can act like a bridge between two surface plasmons, resulting in rich dynamical results. The distance from the first metal to the nonlinearity in the dielectric is held constant at d_1 , hence $\alpha_1 = 0$. The distance from the second metal to the nonlinearity is taken as both a parallel and a hyperbolic function of α_2 , and the minimum distance between the two layers is d_2 , as shown in Fig. 3.2. Both cases will be investigated in detail.

In this three-level system, two different coupling functions are defined corresponding two different surface plasmon-spatial soliton interactions:

$$q_1(|C_s|) = |C_s|e^{-d_1\sqrt{\gamma/2}|C_s|}, \quad (3.18)$$

$$q_2(|C_s|) = |C_s|e^{-d_2\sqrt{\gamma/2}|C_s|}. \quad (3.19)$$

The coupled oscillator equations, Eq. 2.36, previously used in two-layer system need to be adapted to three-layer system. The new equations are

$$c_{p1}'' + \beta_{p1}^2 c_{p1} = q_1(|c_s|)c_s \quad (3.20)$$

$$c_{p2}'' + \beta_{p2}^2 c_{p2} = q_2(|c_s|)c_s \quad (3.21)$$

$$c_s'' + \beta_s^2 c_s = q_1(|c_s|)c_{p1} + q_2(|c_s|)c_{p2} \quad (3.22)$$

Making the substitutions $c_{p1} = C_{p1}e^{iz}$, $c_{p2} = C_{p2}e^{iz}$, and $c_s = C_se^{iz}$, the equations yields the following nonlinear Schrödinger equation:

$$-i \begin{pmatrix} \dot{C}_{p1} \\ \dot{C}_s \\ \dot{C}_{p2} \end{pmatrix} = \begin{pmatrix} \nu_{p1} & -\frac{q_1}{2} & 0 \\ \frac{-q_1}{2} & \nu_s & -\frac{q_2}{2} \\ 0 & -\frac{q_2}{2} & \nu_{p2} \end{pmatrix} \begin{pmatrix} C_{p1} \\ C_s \\ C_{p2} \end{pmatrix} \quad (3.23)$$

After the following substitutions is implemented,

$$C_{p1} = |C_{p1}|e^{i\phi_{p1}} \quad (3.24)$$

$$C_{p2} = |C_{p2}|e^{i\phi_{p2}} \quad (3.25)$$

$$C_s = |C_s|e^{i\phi_s} \quad (3.26)$$

the governing equations yield

$$\dot{C}_{p1} = \frac{q_1}{2}|C_s|\sin(\phi_s - \phi_{p1}), \quad (3.27)$$

$$\dot{C}_{p2} = \frac{q_2}{2}|C_s|\sin(\phi_s - \phi_{p2}), \quad (3.28)$$

$$\dot{C}_s = \frac{q_1}{2}|C_{p1}|\sin(\phi_{p1} - \phi_s) + \frac{q_2}{2}|C_{p2}|\sin(\phi_{p2} - \phi_s), \quad (3.29)$$

$$\dot{\phi}_{p1} = \Lambda - \Delta E_1 - \frac{q_1}{2} \frac{|C_s|}{|C_{p1}|} \cos(\phi_s - \phi_{p1}), \quad (3.30)$$

$$\dot{\phi}_{p2} = \Lambda - \Delta E_2 - \frac{q_2}{2} \frac{|C_s|}{|C_{p2}|} \cos(\phi_s - \phi_{p2}), \quad (3.31)$$

$$\dot{\phi}_s = 2\Lambda|C_s|^2 - \frac{q_1}{2} \frac{|C_{p1}|}{|C_s|} \cos(\phi_{p1} - \phi_s) - \frac{q_2}{2} \frac{|C_{p2}|}{|C_s|} \cos(\phi_{p2} - \phi_s). \quad (3.32)$$

Since solving these equations analytically may seem impossible, numerical approach is needed. An ODE solver function in MATLAB -ode45- has been used to solve these differential equations. With no dissipation in the system, subsequent transitions will occur between three channels. Depending on the parameter choice different proportions of energy transfer will be observed. Some of them are shown in Fig. 3.3 and Fig. 3.4.

3.2.2 Population Trapping

In the metal/dielectric/metal interface without a spatial modification, a single transition between surface plasmon and soliton amplitudes cannot be achieved, since the distance is unchanged, and there is no energy dissipation, so a stationary state after the transition could not be reached. However, the system can be configured into a significant state, in which the soliton amplitude oscillations can be suppressed into a stationary soliton propagation, so that the soliton channel acts like a bridge between two surface plasmon channels. The resonant transfer of energy occurs between surface plasmons via the soliton. This effect can be obtained for a variety of system parameters as well as the initial surface plasmon and the soliton amplitudes. This phenomenon is called population trapping, which is based on a quantum theory of light propagation in two optical waveguides [43],[44]. A common continuum of

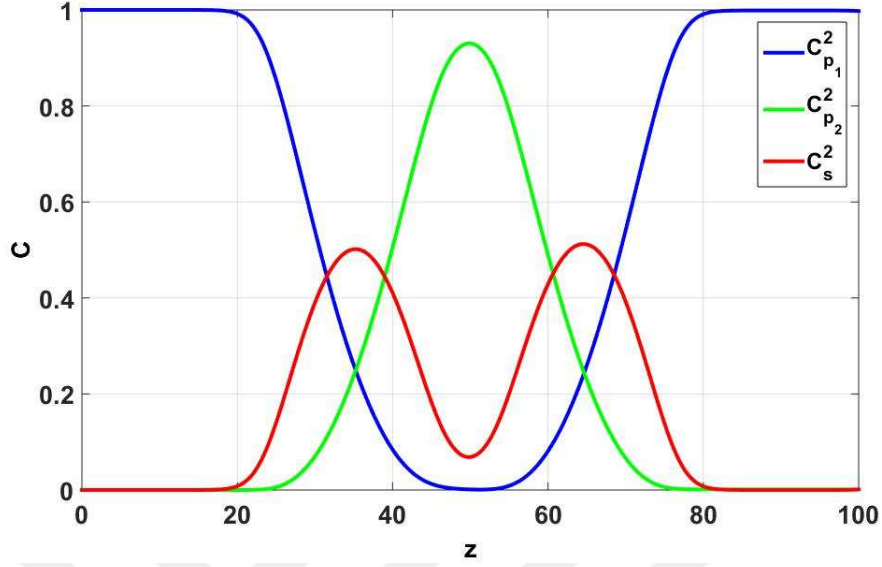


Figure 3.3: Dynamics of plasmon-soliton-plasmon interaction. Parameters are $d_1 = 6$, $d_2 = 6$, $\Lambda = 0.04$, $\Delta E_1 = 0.03$, $\Delta E_2 = -0.01$, $Ap_0 \simeq 1$, $As \simeq 0$

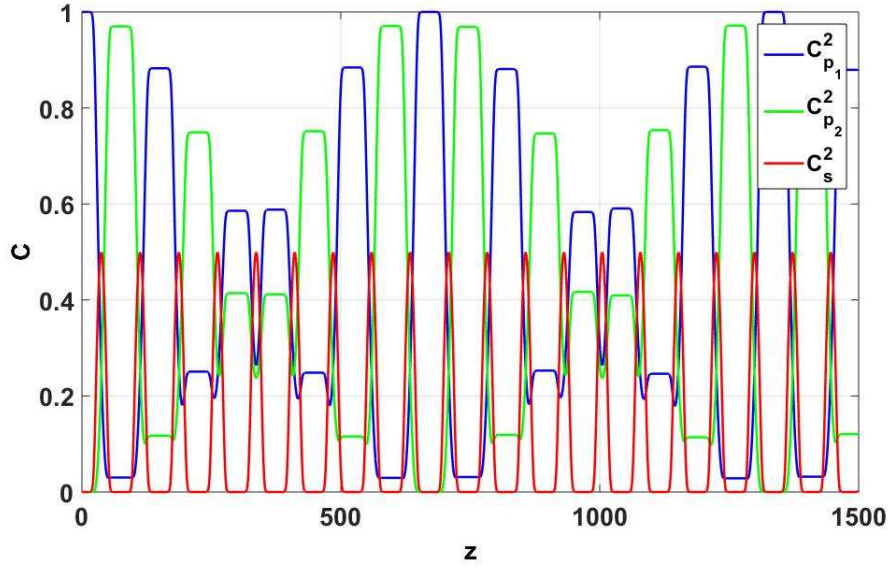


Figure 3.4: Dynamics of plasmon-soliton-plasmon interaction. Parameters are $d_1 = 6$, $d_2 = 6$, $\Lambda = 0.05$, $\Delta E_1 = 0.03$, $\Delta E_2 = 0.03$, $Ap_0 \simeq 1$, $As \simeq 0$

modes coupled to both optical channels enables the tunneling between them [45]. For classical light waves, coupled-mode equation analysis indicates that, the emergence

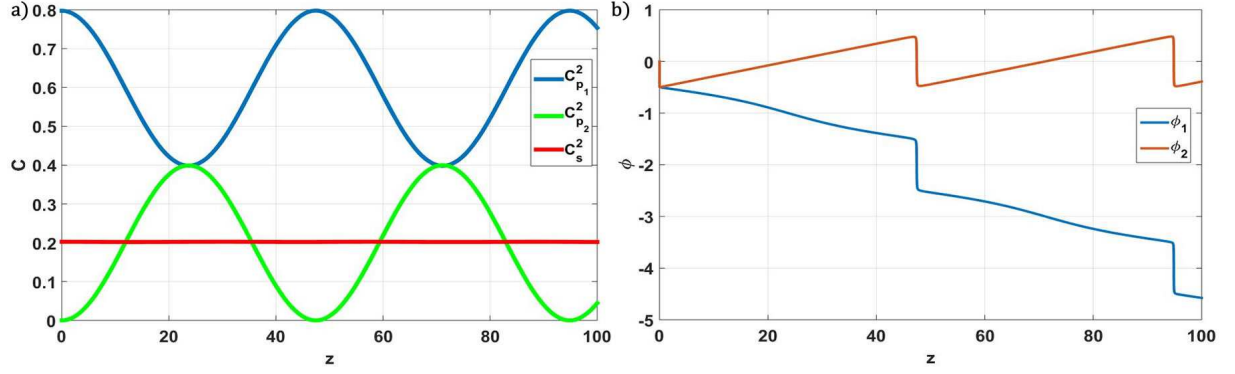


Figure 3.5: Dynamics of amplitudes of soliton and surface plasmons (a) and relative phase differences (b). Parameters are $d_1 = 8, d_2 = 4, \Lambda = 0.06, \Delta E_1 = -0.02, \Delta E_2 = -0.02, A_{p_{10}} \sim 0.9, A_{s_0} \sim 0.45$.

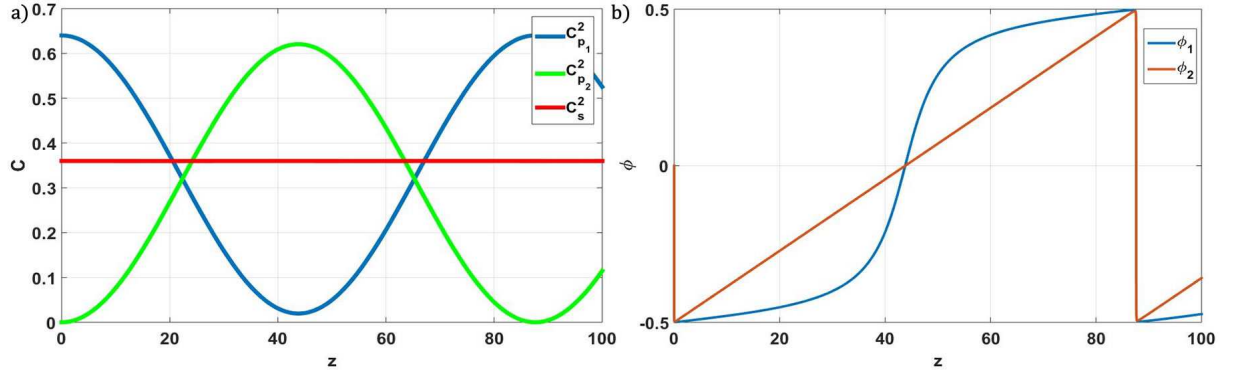


Figure 3.6: Dynamics of amplitudes of soliton and surface plasmons (a) and relative phase differences (b). Parameters are $d_1 = 5.7, d_2 = 6.3, \Lambda = 0.06, \Delta E_1 = 0.02, \Delta E_2 = 0.02, A_{p_{10}} \sim 0.8, A_{s_0} \sim 0.6$.

of a trapped state placed in the continuum is caused by Fano interference between different light leakage channels. Results show that, plasmonic structures originally designed to mimic the quantum mechanical phenomena may exhibit themselves as proper analogues of their quantum counterparts.

Figure 3.5 shows the first example of population trapping in non-dissipative parallel metal-dielectric-metal interface. As mentioned before, amplitudes of surface plasmons exchange their energies repetitively via the soliton bridge. In this configuration the soliton amplitude is arranged so that surface plasmon in the second metal sur-

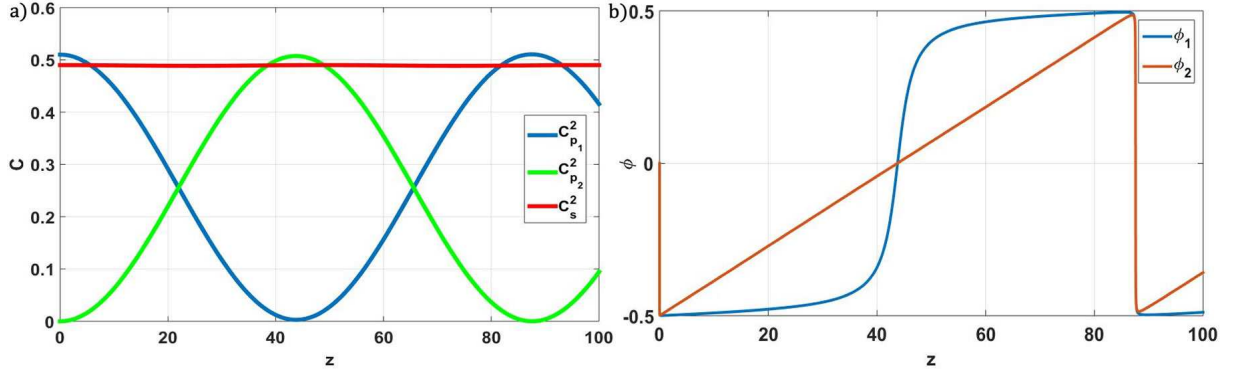


Figure 3.7: Dynamics of amplitudes of soliton and surface plasmons (a) and relative phase differences (b). Parameters are $d_1 = 5.9, d_2 = 6.1, \Lambda = 0.07, \Delta E_1 = -0.03, \Delta E_2 = -0.03, A_{p_{10}} \sim 0.7, A_{s_0} \sim 0.7$.

face is not excited initially, and the first surface plasmon can share half of its energy at most. This makes the nodes and the anti-nodes of the oscillatory waves full and half populated states respectively. The anti-nodes are the points where a phase-shift occurs as shown in panel b.

Figures 3.6 and 3.7 show the trapping of soliton population for completely different set of parameters indicating that, this phenomenon is not bounded to a particular system property. The relative phase differences in these two examples have the same jumping points, whereas the behaviours of the phase evolution are different. The only constraint to this behaviour is that, the asymmetry between the soliton and both surface plasmons, ΔE_1 and ΔE_2 , must be equal. An analytical approach might be insightful for explaining this constraint. In order to have a constant soliton amplitude, the derivative of C_s must equal to zero.

$$\dot{C}_s = \frac{q_1}{2}|C_{p_1}|\sin(\phi_{p_1} - \phi_s) + \frac{q_2}{2}|C_{p_2}|\sin(\phi_{p_2} - \phi_s) = 0. \quad (3.33)$$

There are a few ways to ensure this. Since coupling functions, q_1 and q_2 , and the amplitudes, $|C_{p_1}|$ and $|C_{p_2}|$, are nonzero, we must have both sinus functions equal to zero, which yields:

$$\phi_1 = \phi_2 = 0, \text{ or } \pi. \quad (3.34)$$

Unfortunately, we know from the phase difference plots of the trapped systems, that ϕ_1 and ϕ_2 are not equal to 0 or π at all times. This means that, the only way to have $\dot{C}_s$ equal to zero, is to obey the following condition:

$$-q_1|C_{p1}|\sin(\phi_1) = q_2|C_{p2}|\sin(\phi_2) \quad (3.35)$$

It must be noted that, in the trapped soliton system coupling functions, q_1 and q_2 , are constants, since they are only soliton amplitude dependent. If we look into the dynamic equations of phases and considering the above condition, it can be understood that, the only different term in the two equations are ΔE_1 and ΔE_2 and why they must be equal in order to have a trapped soliton amplitude.

$$\dot{\phi}_1 = 2\Lambda|C_s|^2 - \frac{q_1}{2} \frac{|C_{p1}|}{|C_s|} \cos(\phi_{p1} - \phi_s) - \frac{q_2}{2} \frac{|C_{p2}|}{|C_s|} \cos(\phi_{p2} - \phi_s) - \Lambda - \Delta E_1 - \frac{q_1}{2} \frac{|C_s|}{|C_{p1}|} \cos(\phi_s - \phi_{p1}), \quad (3.36)$$

$$\dot{\phi}_2 = 2\Lambda|C_s|^2 - \frac{q_1}{2} \frac{|C_{p1}|}{|C_s|} \cos(\phi_{p1} - \phi_s) - \frac{q_2}{2} \frac{|C_{p2}|}{|C_s|} \cos(\phi_{p2} - \phi_s) - \Lambda - \Delta E_2 - \frac{q_2}{2} \frac{|C_s|}{|C_{p2}|} \cos(\phi_s - \phi_{p2}). \quad (3.37)$$

3.2.3 Classical Collapse and Revival of Coupling Function

The investigation of two quantum states, that enable macroscopic nonlinear tunneling between themselves reveals that, the populations of one of the states might reduce to zero from time to time. However, the mean-field approximation is violated, when one of the populations become negligible, since the following is one of the assumptions of mean-field theory [42]: Large number of particles can be treated as a single averaged particle, which reduces a many-body problem to a one-body problem. This difficulty can be seen in systems of two-level atoms, that mimic the properties of a Josephson junction in which the amplitudes of the states oscillate between each other [47]. Throughout the years, many different systems have been used as a host for quantum collapse-revival phenomenon such as a single-mode resonant field propagating in a Kerr-like medium [48], an effective giant spin model constructed from

a coupled two-mode Bose-Einstein condensate, that possesses adiabatic and cyclic time-varying Raman coupling [49], Bose-Einstein condensates trapped in a triple-well [50], or quantum bouncing ball on a truncated lattice [51]. In [47], a two-dimensional optical lattice filled with BEC is taken as an example of a two-level system with equal energy, which allows population oscillations, and this system is realized experimentally to examine its dynamic behaviours. The quantization of the motion yields collapse and revival of these oscillations. The evolution of BEC atoms and quantum collapses can be seen in Fig. 3.8.

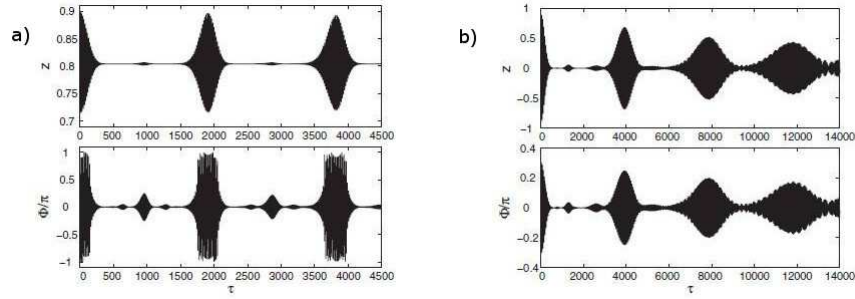


Figure 3.8: Quantum collapse and revivals can be seen in BEC atoms [47]. In a) population imbalance (Z) of $N = 350$ BEC atoms can be seen. Most of the population is trapped in of the states. The relative phase of both states are equal. In b) $N = 500$ BEC atoms can be seen. Both states are equally populated during the collapse. The relative phases are equal.

Some regimes of collapses, such as equal population or disbalanced population oscillations can be useful to predict the tunneling properties, since they don't violate the mean-field approximation. So, we shouldn't look for a collapse in the population of one of the states, but the collapse of the oscillations, in other words, the coupling function. However, the quantization of the motion in the above study still restricts us from obtaining collapse of coupling in the plasmonic Josephson junction, therefore a classical approach should be in order.

A classical approach has been proposed in [52], where the analysis of wave packets in weakly anharmonic potentials is given. According to this study, when the wave packets propagate along classical trajectories in harmonic potential, they spread along

the motion, and regain their shape. Introducing a small anharmonicity to the system leads to a reversible quantum dephasing of wave packets, which is called revival. The amplitude of the oscillation disappears between these revivals, which is called collapse. A realization of this study has been experimented with coherent A_{1g} phonons in Bismuth. The dynamics of the phonon wave packet in a two-atom system is shown in Fig. 3.9.

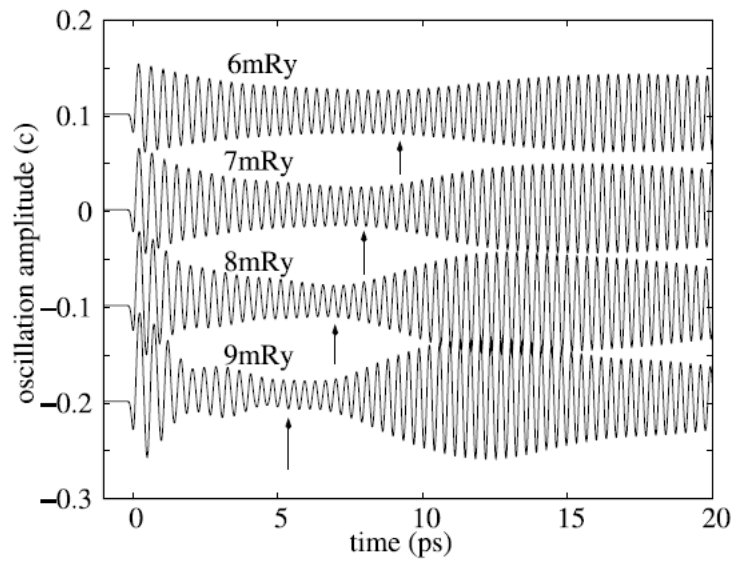


Figure 3.9: Oscillatory part of the z coordinate for different absorbed energies n_0 . Arrows indicate the amplitude collapse of the oscillations [52].

This study shows that, the system size is the key factor, which affects the behaviour of the excited phonons, that approach the classical behavior swiftly. This behaviour indicates that, quantum effects are not included in the revival of the oscillations, which is a substantial discovery, because collapse and revival phenomenon was originally considered a pure quantum effect with no classical analogy [46]. The physical interpretation of this effect was given as a linear combination of stationary states of a system. These linear combinations are determined as the coherent oscillations, that show collapse and revival properties. Furthermore an important natural specification of a stationary state is the number of quanta as a result of being a

quantum mechanical phenomenon.

The changeover from quantum systems without a classical counterpart to quantum optical systems, and finally to opto-photonic systems with classical effects urge us to move to the next step: surface plasmon-soliton Josephson junctions. Unlike the quantum collapse-revival phenomenon, the plasmonic system does not lose its total energy, however it is the interaction between these three states, ergo, the coupling function, that collapses as shown in figures 3.10 - 3.18.

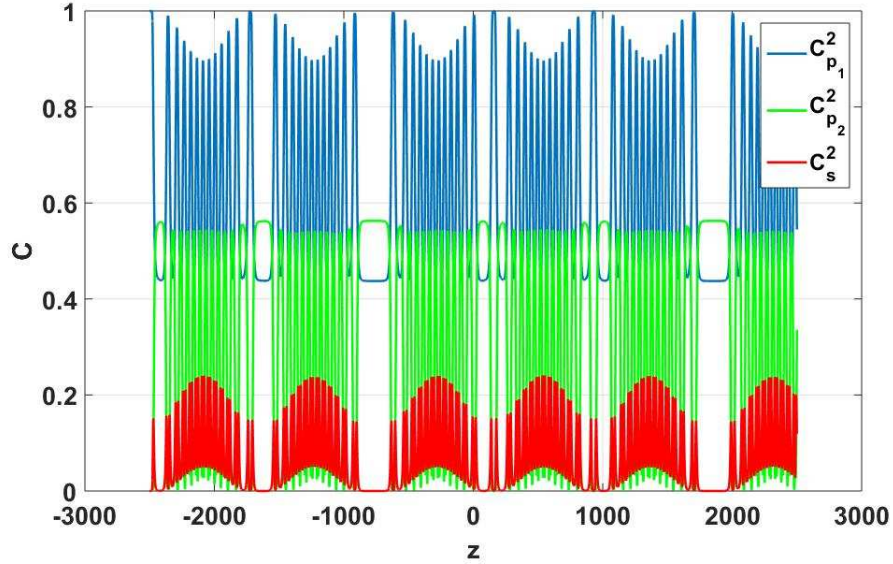


Figure 3.10: Dynamics of amplitudes of soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 10.1, d_2 = 1.9, \Lambda = 0.03, \Delta E_1 = 0.01, \Delta E_2 = 0.01, A_{p_{10}} \sim 1, A_{s_0} \sim 0$

Fig. 3.10 shows an obvious collapse-revival behaviour. The oscillations of amplitudes between two surface plasmons and soliton channels manifest periodic patterns, however the non-oscillating parts, i.e., the length of the collapse time, differ from each other. In order to obtain this behaviour the following set of parameters are used: $d_1 = 10.1, d_2 = 1.9, \Lambda = 0.03, \Delta E_1 = 0.01$, and $\Delta E_2 = 0.01$. Initial surface plasmon amplitude in the first metal surface carries the whole energy of the system, $A_{p_{10}} \sim 1$, therefore $A_{s_0} \sim 0$, and $A_{p_{20}} \sim 0$. The outcome of the CR behaviour differs

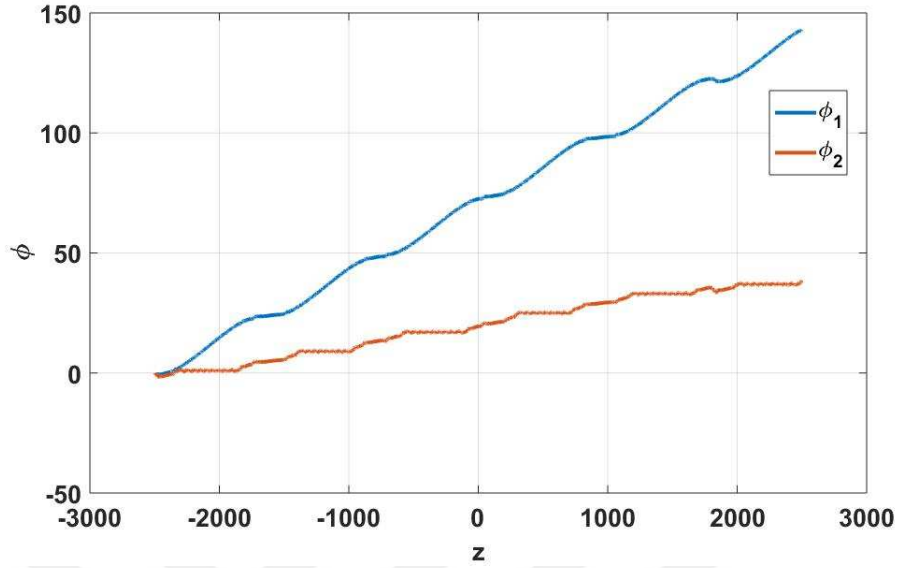


Figure 3.11: Dynamics of relative phase differences between soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 10.1, d_2 = 1.9, \Lambda = 0.03, \Delta E_1 = 0.01, \Delta E_2 = 0.01, Ap_{10} \sim 1, As_0 \sim 0$

from the population trapping, that has been discussed in the previous section. In the population trapping phenomenon, oscillations of the soliton amplitude reduces to zero, stabilizing the soliton population permanently, and in this period of time the amplitudes of surface plasmons continue oscillating. However in the CR phenomenon, not only the oscillations of the soliton amplitude, but the soliton population collapses to zero temporarily, and in this temporal interval of time oscillations of surface plasmons collapses to zero as well.

The relative phase differences between surface plasmons and the soliton should also be discussed. Although the effect of relative phases to the double Josephson junctions will be explained in the following sections, a brief discourse on relative phase differences in the CR-systems might be insightful. Fig. 3.11 shows the phases of the system, that was explained in Fig. 3.10. As can be understand from the figure, the relative phases between channels also manifest a non-oscillating behaviour during the collapse time. The fact that, both populations and phases of the optical channels

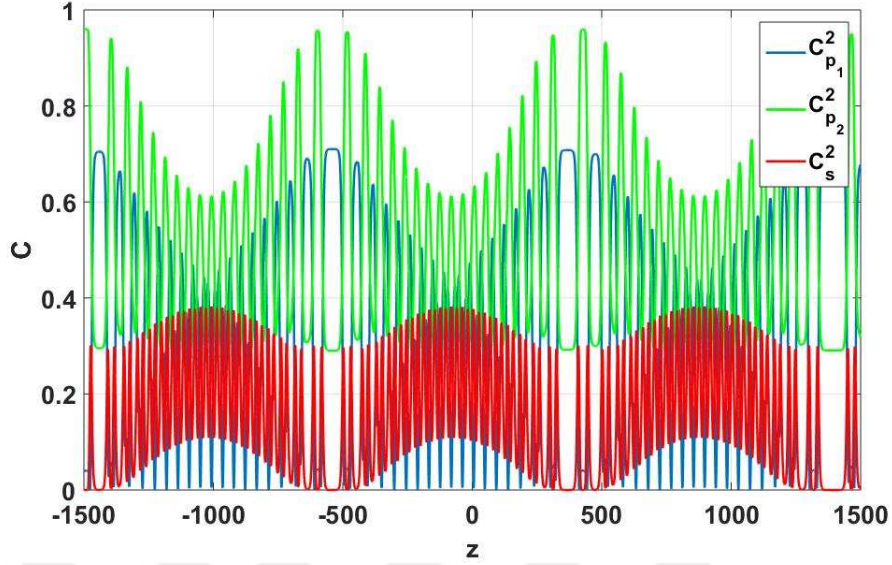


Figure 3.12: Dynamics of amplitudes of soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 3, d_2 = 9, \Lambda = 0.03, \Delta E_1 = 0, \Delta E_2 = 0, A_{p_{1_0}} \sim 0.2, A_{s_0} \sim 0$

do not change their current state, indicates that, the system may fall into one of the fixed points.

Figure 3.12 also shows a CR behaviour, but with a more regular pattern. The collapse time between periodic islands seems closer to each other than Fig. 3.10. Both the period of those islands, $z \sim 1000$, and the shape of the envelopes are similar, although system parameters are quite different. This is a sign that CR behaviour is not bound to specific set of parameters, it can be observed in different geometry (d, α) and material properties ($\Lambda, \Delta E$). The relative phase difference plot can be seen in Fig. 3.13.

We have defined collapse-revival phenomenon in plasmonic Josephson junction as the collapse of the coupling between surface plasmons and the soliton. More to that, interesting patterns of system dynamics, that does not obey the CR behaviour might be worth mentioning. In Fig. 3.14 and 3.15, the oscillations of soliton draws a regular pattern so that, the oscillations of surface plasmons follows a sequential pattern, which creates a sinusoidal envelope. This sinusoidal envelope can also be

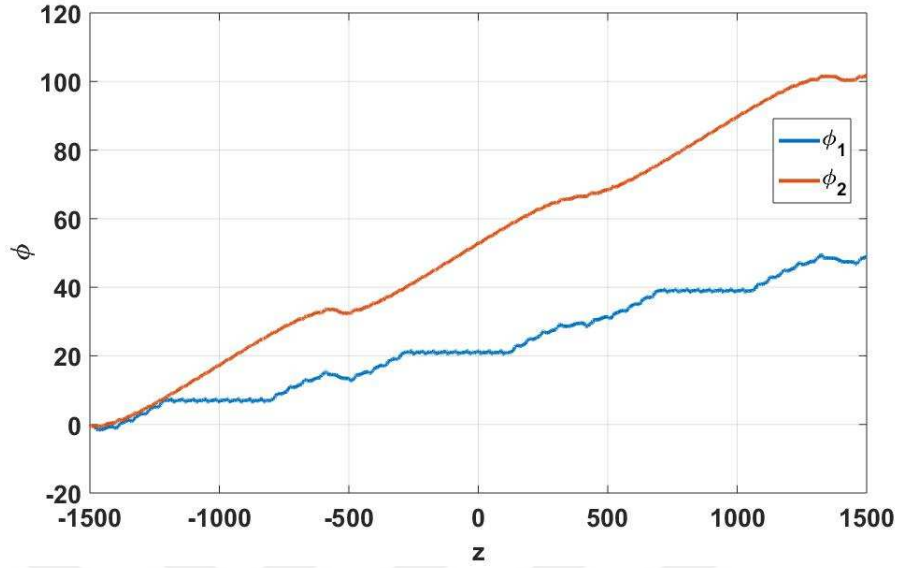


Figure 3.13: Dynamics of relative phase differences between soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 3, d_2 = 9, \Lambda = 0.03, \Delta E_1 = 0, \Delta E_2 = 0, Ap_{10} \sim 0.2, As_0 \sim 0$

seen in Fig. 3.16 and 3.17 with strong soliton oscillations, and a very steep collapse and revival behaviour. And finally, Fig. 3.18 and 3.19 shows an irregular type of CR oscillations.

3.3 Surface Plasmon-Spatial Soliton Coupling in Hyperbolically Modulated Multilayer Systems

The rich dynamic properties of single Josephson junction formed by surface plasmon and spatial soliton previously discussed in Chapter 2 can also be investigated in double Josephson junctions with one of the metal surfaces being spatially modulated. In this system, nonlinearity is kept straight, indicating a parallel junction on one side and a hyperbolic junction on the other side as can be seen in Fig. 3.20.

Dynamic evolution of the system can be described with a similar set of equations as the parallel system.

$$\dot{C}_{p1} = \frac{q_1}{2} |C_s| \sin(\phi_s - \phi_{p1}), \quad (3.38)$$

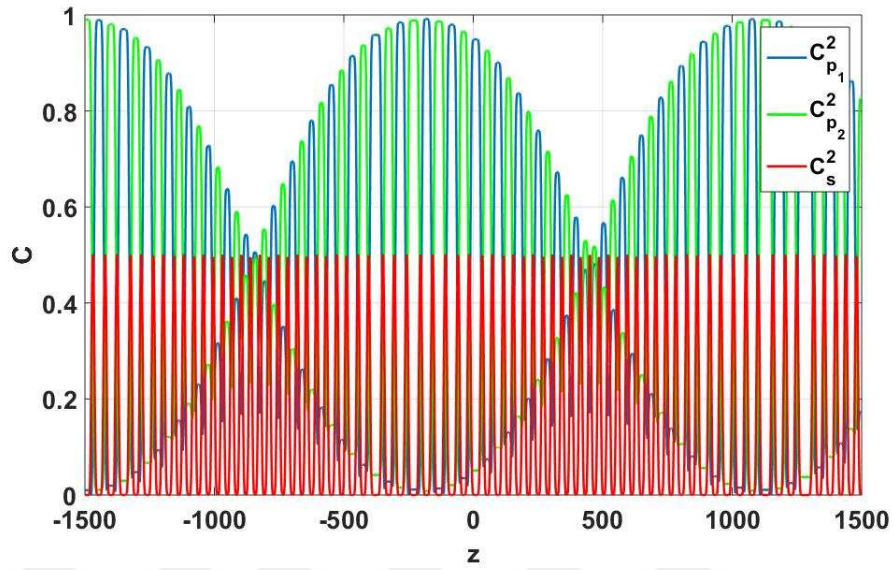


Figure 3.14: Dynamics of amplitudes of soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 5.5, d_2 = 6.5, \Lambda = 0.03, \Delta E_1 = 0, \Delta E_2 = 0, Ap_{1_0} \sim 0.1, As_0 \sim 0$

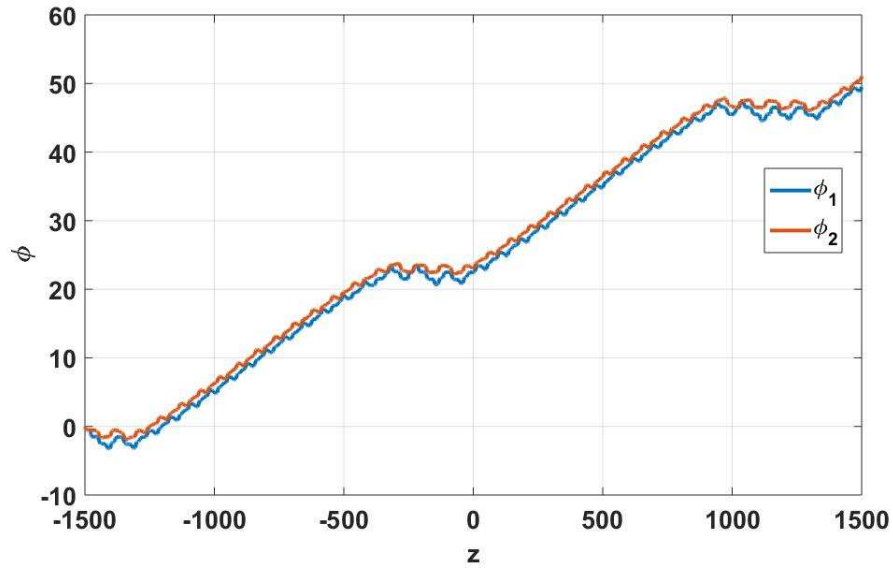


Figure 3.15: Dynamics of relative phase differences between soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 5.5, d_2 = 6.5, \Lambda = 0.03, \Delta E_1 = 0, \Delta E_2 = 0, Ap_{1_0} \sim 0.1, As_0 \sim 0$

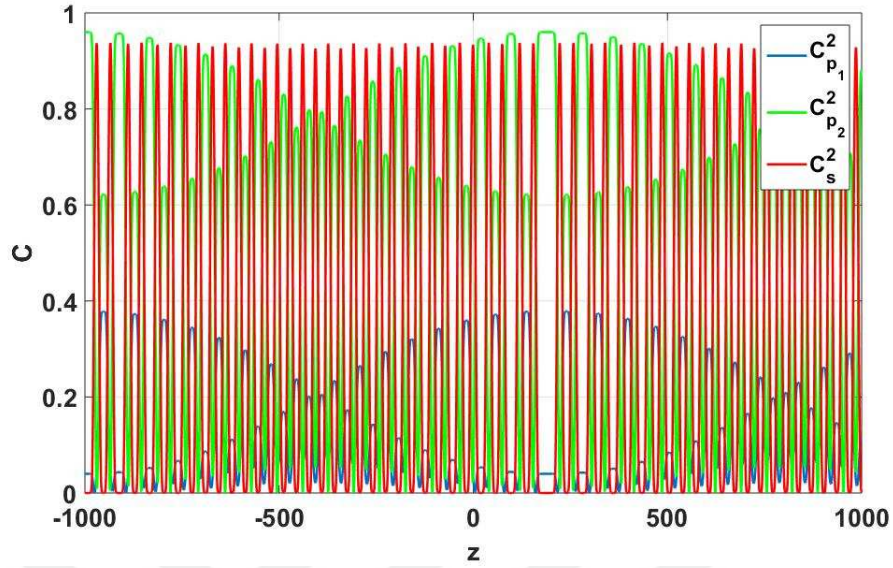


Figure 3.16: Dynamics of amplitudes of soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 7.5, d_2 = 4.5, \Lambda = 0.03, \Delta E_1 = -0.02, \Delta E_2 = -0.02, Ap_{10} \sim 0.2, As_0 \sim 0$

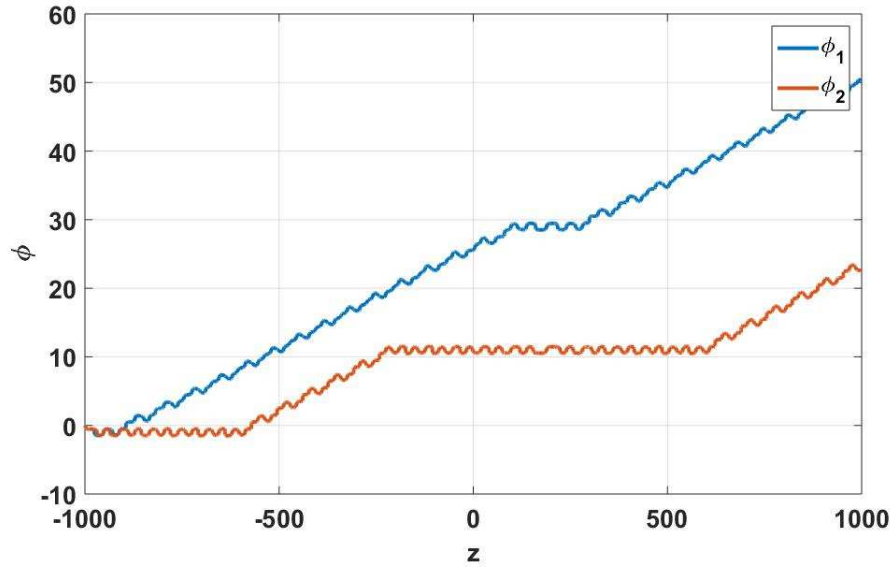


Figure 3.17: Dynamics of relative phase differences between soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 7.5, d_2 = 4.5, \Lambda = 0.03, \Delta E_1 = -0.02, \Delta E_2 = -0.02, Ap_{10} \sim 0.2, As_0 \sim 0$

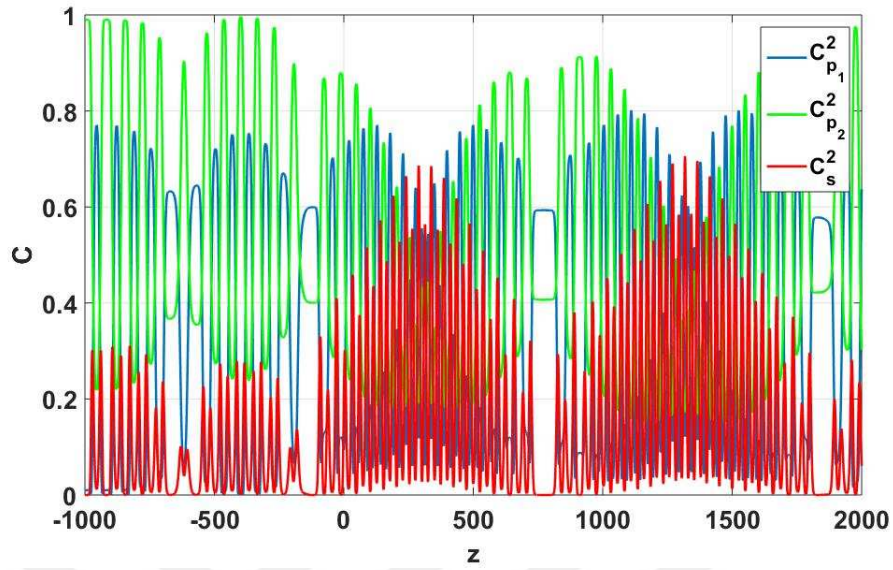


Figure 3.18: Dynamics of amplitudes of soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 4, d_2 = 8, \Lambda = 0.03, \Delta E_1 = -0.02, \Delta E_2 = 0, Ap_{1_0} \sim 0.1, As_0 \sim 0$

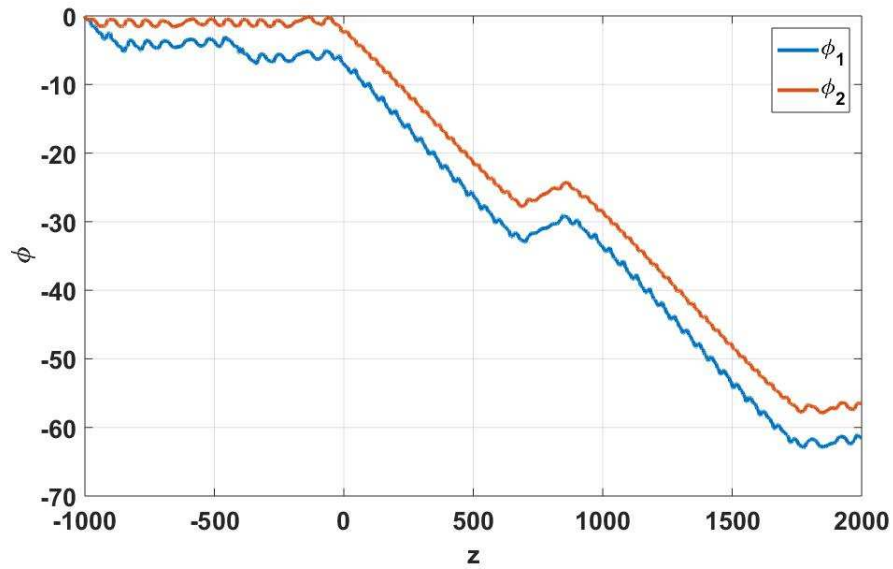


Figure 3.19: Dynamics of relative phase differences between soliton and surface plasmons in a collapse and revival case. Parameters are $d_1 = 4, d_2 = 8, \Lambda = 0.03, \Delta E_1 = -0.02, \Delta E_2 = 0, Ap_{1_0} \sim 0.1, As_0 \sim 0$

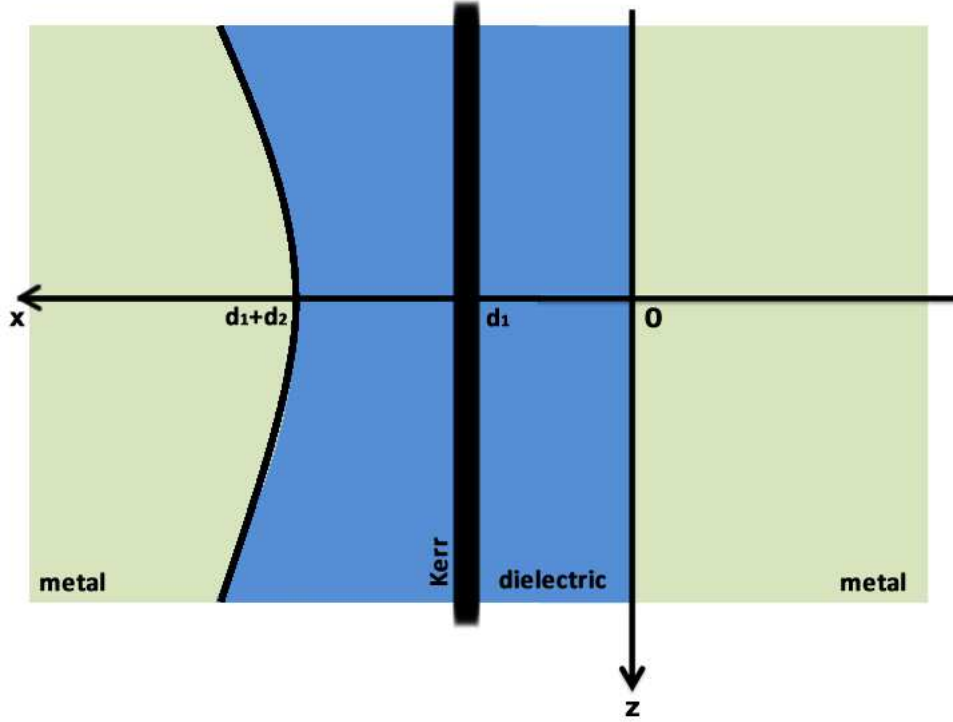


Figure 3.20: Two surface plasmons and a spatial soliton in a metal/dielectric/Kerr/dielectric/metal multilayer.

$$\dot{C}_{p2} = \frac{q_2}{2} |C_s| \sin(\phi_s - \phi_{p2}), \quad (3.39)$$

$$\dot{C}_s = \frac{q_1}{2} |C_{p1}| \sin(\phi_{p1} - \phi_s) + \frac{q_2}{2} |C_{p2}| \sin(\phi_{p2} - \phi_s), \quad (3.40)$$

$$\dot{\phi}_{p1} = \Lambda - \Delta E_1 - \frac{q_1}{2} \frac{|C_s|}{|C_{p1}|} \cos(\phi_s - \phi_{p1}), \quad (3.41)$$

$$\dot{\phi}_{p2} = \Lambda - \Delta E_2 - \frac{q_2}{2} \frac{|C_s|}{|C_{p2}|} \cos(\phi_s - \phi_{p2}), \quad (3.42)$$

$$\dot{\phi}_s = 2\Lambda |C_s|^2 - \frac{q_1}{2} \frac{|C_{p1}|}{|C_s|} \cos(\phi_{p1} - \phi_s) - \frac{q_2}{2} \frac{|C_{p2}|}{|C_s|} \cos(\phi_{p2} - \phi_s). \quad (3.43)$$

with a change in the coupling function

$$q_1(|C_s|) = |C_s| e^{-d_1 \sqrt{\gamma/2} |C_s|}, \quad (3.44)$$

$$q_2(|C_s|) = |C_s| e^{-\sqrt{d_2^2 + \alpha^2 z^2} \sqrt{\gamma/2} |C_s|}. \quad (3.45)$$

The main motivation of constructing a single surface plasmon-soliton Josephson junction was to investigate the tunneling dynamics, and to obtain a full population transfer between two optical channels. In order to establish a slowly changing adiabatic parameter, we modulated the geometry of the system spatially, and this modulation paid off with a successful resonant transfer of energy from surface plasmon to spatial soliton and visa versa. This achievement sets a good example for the double Josephson junction as well, and searching a full transition of energy from the surface plasmon channel on the unmodified metal surface to the surface plasmon channel on the hyperbolically modified metal surface is the main goal of this section.

When the system starts with surface plasmon propagating on the unmodified metal surface only, the system can be considered a single Josephson junction initially, because the distance between nonlinearity and the modified metal surface is far enough to neglect the second metal surface. Naturally an oscillation between surface plasmon and soliton arises, however, when the waves enter the effective interaction area near $z = 0$, the second surface plasmon is excited. If the system parameters are chosen correctly, it can be arranged that, the second surface plasmon steals all the energy of the first surface plasmon, and keeps propagating on its own hyperbolic trajectory leaving the upper single junction with zero energy. In this scenario, surface plasmons do not directly interact with each other because of the distance between metal surfaces, however, they transfer energy between them via the soliton channel, since one of the evanescent tails of the soliton intersects with the first surface plasmon, and the other tail intersects with the second surface plasmon. This interaction is shown in Fig. 3.21 with parameters $d_1 = 8.21$, $d_2 = 3.79$, $\alpha = 0.6$, $\lambda = 0.07$, $\Delta E_1 = 0.02$, and $\Delta E_2 = 0.04$.

Besides the full transition objective, different significant results can be obtained with different choice of parameters. Dynamics of population trapping can be observed in this system as well. Furthermore, the final amplitudes of first surface plasmon and soliton could be arranged to be equal indicating population imbalance between first

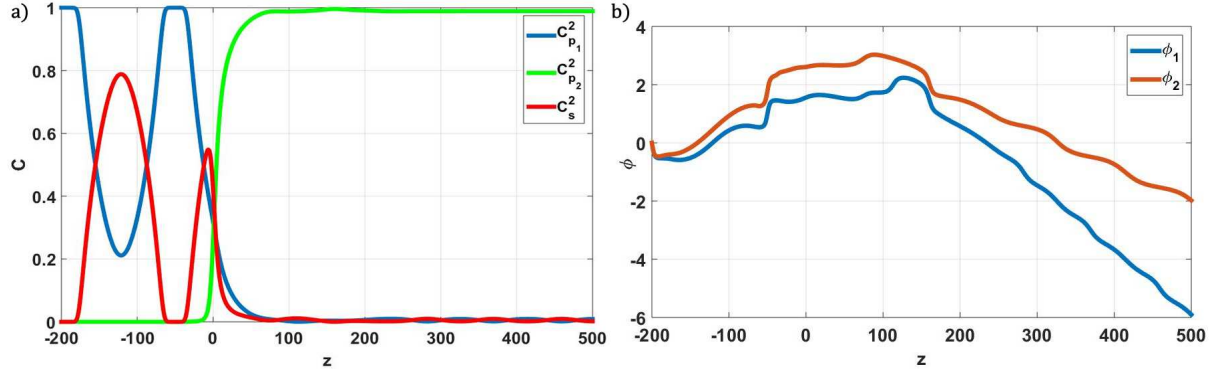


Figure 3.21: A %100 transfer of energy between two surface plasmon channels. Parameters are $d_1 = 8.21$, $d_2 = 3.79$, $\alpha = 0.6$, $\lambda = 0.07$, $\Delta E_1 = 0.02$, and $\Delta E_2 = 0.04$.

surface plasmon and soliton in the middle becomes zero, as shown in Fig. 3.22.

$$Z_1 = \frac{|C_s|^2 - |C_{p1}|^2}{|C_s|^2 + |C_{p1}|^2} \sim 0. \quad (3.46)$$

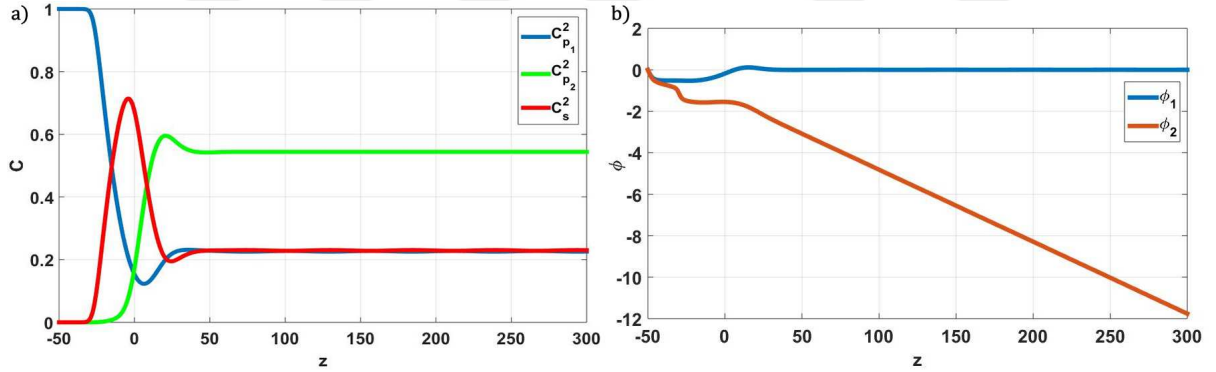


Figure 3.22: Population trapping could be observed with $Z_1 \sim 0$. The parameters are $d_1 = 6.5$, $d_2 = 5.5$, $\alpha = 0.51$, $\lambda = 0.062$, $\Delta E_1 = 0.033$, and $\Delta E_2 = -0.025$.

Population trapping of surface plasmon and spatial soliton could result in interesting outcomes. Although the system parameters are the decisive elements of the transfer ratio, the initial amplitudes are as important as them for manipulating the junction strength. When the initial value of the soliton is different than zero, full transition is no longer sought, but a population trapping could still be achieved. This type of transition could create a mechanism, which stabilizes an oscillating single Josephson

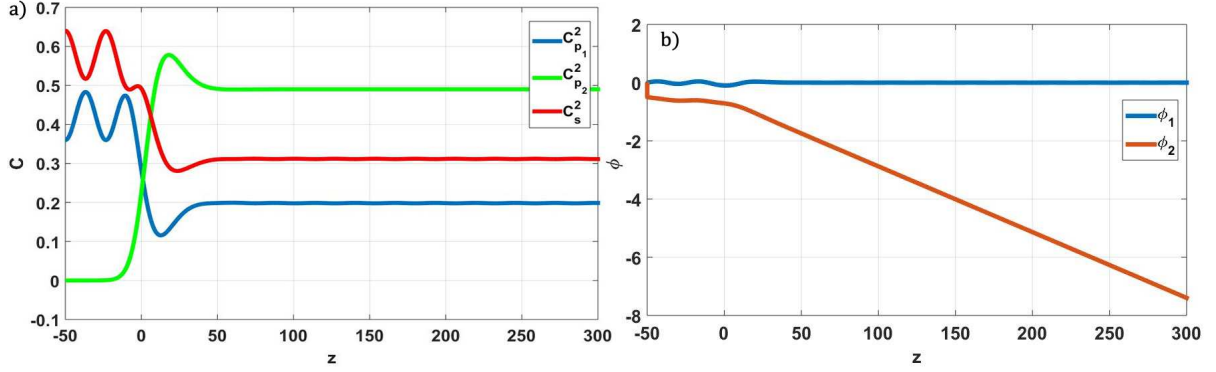


Figure 3.23: A population trapping could be achieved when the initial soliton amplitude is different than zero. The parameters are $d_1 = 4.7$, $d_2 = 7.3$, $\alpha = 0.6$, $\lambda = 0.03$, $\Delta E_1 = -0.04$, $\Delta E_2 = 0.03$, $A_{p1} = 0.6$, $A_s = 0.8$.

junction with the correct usage of second metal surface as shown in Fig. 3.23. This mechanism will be discussed in detail in the subsequent chapter.

3.4 Stability Analysis

Dynamical analysis of the metal-dielectric-metal structure gives various outcomes, which are presented in the previous sections. Although transfer of energy between surface plasmon and soliton channels is known to be possible by manipulating system parameters such as nonlinearity, plasmon-soliton asymmetry, plasmon-soliton distance, and hyperbolicity parameter, complete understanding of the behavior of the system needs further analysis of fixed points and eigenvalues.

Similar analysis of eigenstates has been done previously [33], where the nonlinear Hamiltonian has been cast into classical Hamiltonian, then using the conjugate equations, the dynamic equations of the system have been obtained. Furthermore, the relation between classical Hamiltonian and eigenvalues has been given in order to find the eigenenergies of the system. These eigenenergies reveal the properties of adiabatic transition of eigenstates and also the adiabatic breakdown due to strong nonlinearity. Landau-Zener tunneling has been investigated and nonlinear stimulated Raman adiabatic passage (STIRAP) has been observed, which allows a coherent population

transfer between quantum states and even a full population transfer [55],[56].

Another detailed analysis of tunneling dynamics of nonlinear Josephson oscillation in triple-well systems [57] includes self-trapping of BECs, the control of transition by relative phase difference, and chaotic motion of phase-space of the system, which was demonstrated by Poincare section method [58],[59] in a reduced two-dimensional phase plane.

The eigenvalue and fixed point analysis on triple-well BEC systems sheds light upon plasmonic double Josephson junction model. The fixed points and the associated eigenenergies give further insight on full population transfer between two surface plasmon states. The relation between energy levels and the evolution of fixed points of a nonlinear Landau-Zener model was comprehensively discussed in both two- and three-level systems [60]. This relation can be obtained by replacing ϵ for $-i\frac{\partial}{\partial t}$ in the evolution equations.

$$\epsilon \begin{pmatrix} C_{p1} \\ C_s \\ C_{p2} \end{pmatrix} = \begin{pmatrix} \nu_{p1} & -\frac{q_1}{2} & 0 \\ -\frac{q_1}{2} & \nu_s & -\frac{q_2}{2} \\ 0 & -\frac{q_2}{2} & \nu_{p2} \end{pmatrix} \begin{pmatrix} C_{p1} \\ C_s \\ C_{p2} \end{pmatrix} \quad (3.47)$$

Unravelling the matrices yields the following set of equations

$$\epsilon C_{p1} = \nu_{p1} C_{p1} - \frac{q_1}{2} C_s, \quad (3.48)$$

$$\epsilon C_{p2} = \nu_{p2} C_{p2} - \frac{q_2}{2} C_s, \quad (3.49)$$

$$\epsilon C_s = \nu_s C_s - \frac{q_1}{2} C_{p1} - \frac{q_2}{2} C_{p2}. \quad (3.50)$$

Multiplying the first row by C_{p1} , the second row by C_{p2} , and the third row by C_s would give

$$\epsilon |C_{p1}|^2 = \nu_{p1} |C_{p1}|^2 - \frac{q_1}{2} C_s C_{p1}^*, \quad (3.51)$$

$$\epsilon |C_{p2}|^2 = \nu_{p2} |C_{p2}|^2 - \frac{q_2}{2} C_s C_{p2}^*, \quad (3.52)$$

$$\epsilon |C_s|^2 = \nu_s |C_s|^2 - \frac{q_1}{2} C_{p1} C_s^* - \frac{q_2}{2} C_{p2} C_s^*. \quad (3.53)$$

After implemeting the substitution for relative phase $C_{p/s} = |C_{p/s}|e^{i\phi_{p/s}}$ and $\phi_1 = \phi_s - \phi_{p_1}$ and $\phi_2 = \phi_s - \phi_{p_2}$, we have

$$\epsilon|C_{p_1}|^2 = \nu_{p_1}|C_{p_1}|^2 - \frac{q_1}{2}|C_s||C_{p_1}|e^{i\phi_1}, \quad (3.54)$$

$$\epsilon|C_{p_2}|^2 = \nu_{p_2}|C_{p_2}|^2 - \frac{q_2}{2}|C_s||C_{p_2}|e^{i\phi_2}, \quad (3.55)$$

$$\epsilon|C_s|^2 = \frac{\gamma}{4}|C_s|^4 - \frac{q_1}{2}|C_s||C_{p_1}|e^{-i\phi_1} - \frac{q_2}{2}|C_s||C_{p_2}|e^{-i\phi_2}. \quad (3.56)$$

Anyone of these three equations can be used as a relation between ϵ and population amplitudes since $|C_s|^2 + |C_{p_1}|^2 + |C_{p_2}|^2 = 1$. In general, the following equation can be used as a key substitution between energy levels and evolution of fixed points:

$$\epsilon = \frac{\frac{\gamma}{4}|C_s|^4 - \nu_{p_1}|C_{p_1}|^2 - \nu_{p_2}|C_{p_2}|^2}{|C_s|^2 - |C_{p_1}|^2 - |C_{p_2}|^2}. \quad (3.57)$$

Here ϵ is the eigenvalue of the nonlinear Hamiltonian, and it can be considered as the eigenenergy of the corresponding quantum state. In previous studies the eigenenergies could be found by inserting the fixed points of the system into the classical Hamiltonian, which corresponds by definition the energy of the system. However it was mentioned that, plasmonic system cannot be cast into classical Hamiltonian, so we should first obtain the fixed points by equating the dynamical equations to zero, and we substitute the obtained fixed points into Eq. 3.57 instead of a classical Hamiltonian. We start with

$$\dot{C}_{p_1} = \frac{q_1}{2}|C_s|\sin(\phi_1) = 0, \quad (3.58)$$

$$\dot{C}_{p_2} = \frac{q_2}{2}|C_s|\sin(\phi_2) = 0, \quad (3.59)$$

$$\dot{C}_s = \frac{q_1}{2}|C_{p_1}|\sin(-\phi_1) + \frac{q_2}{2}|C_{p_2}|\sin(-\phi_2) = 0, \quad (3.60)$$

$$\dot{\phi}_1 = 2\Lambda|C_s|^2 - \frac{q_1}{2}\frac{|C_{p_1}|}{|C_s|}\cos(-\phi_1) - \frac{q_2}{2}\frac{|C_{p_2}|}{|C_s|}\cos(-\phi_2) - \Lambda - \Delta E_1 - \frac{q_1}{2}\frac{|C_s|}{|C_{p_1}|}\cos(\phi_1) = 0, \quad (3.61)$$

$$\dot{\phi}_2 = 2\Lambda|C_s|^2 - \frac{q_1}{2}\frac{|C_{p_1}|}{|C_s|}\cos(-\phi_1) - \frac{q_2}{2}\frac{|C_{p_2}|}{|C_s|}\cos(-\phi_2) - \Lambda - \Delta E_2 - \frac{q_2}{2}\frac{|C_s|}{|C_{p_2}|}\cos(\phi_2) = 0.$$

(3.62)

since $\phi_1 = \phi_s - \phi_{p1}$ and $\phi_2 = \phi_s - \phi_{p2}$. The first three equations give solutions for only $\phi_{1/2} = 0$ and $\phi_{1/2} = \pi$, which creates four cases:

Case 1: $\phi_1 = 0$ and $\phi_2 = 0$

Case 2: $\phi_1 = 0$ and $\phi_2 = \pi$

Case 3: $\phi_1 = \pi$ and $\phi_2 = 0$

Case 4: $\phi_1 = \pi$ and $\phi_2 = \pi$

The numerical results of stability analysis of the interactions, which was discussed in the previous section are presented here. The fixed points and the associated eigenenergies of Fig. 3.21 is shown in Fig. 3.24. As can be seen from the panels in

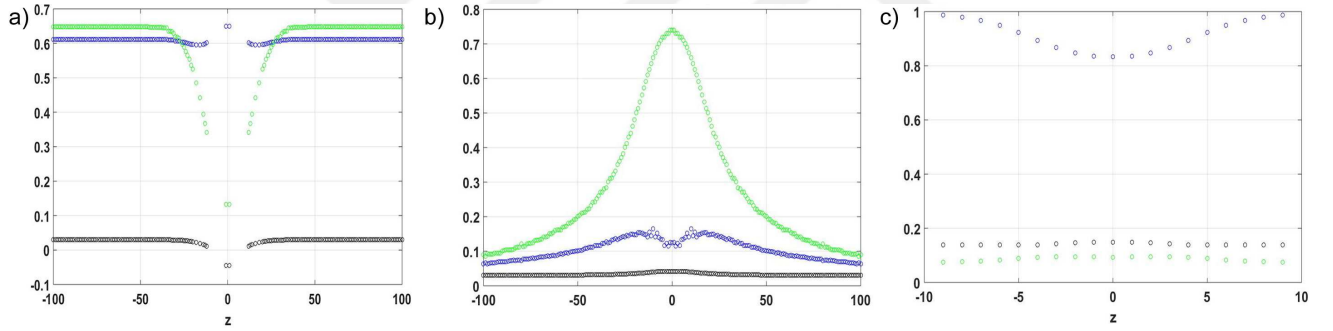


Figure 3.24: Fixed points (blue circles are A_s and green circles are A_{p1} points) and corresponding eigenenergies of the system with parameters $d_1 = 8.21$, $d_2 = 3.79$, $\alpha = 0.6$, $\lambda = 0.07$, $\Delta E_1 = 0.02$, and $\Delta E_2 = 0.04$ for cases $\phi_1 = 0$, $\phi_2 = 0$ (a), $\phi_1 = 0$, $\phi_2 = \pi$ (b), and $\phi_1 = \pi$, $\phi_2 = \pi$ (c).

Fig. 3.24, different ϕ_1 and ϕ_2 cases give completely different results, and furthermore note that $\phi_1 = \pi$, $\phi_2 = 0$ case possess no fixed points, and therefore no eigenvalues. The evolution of fixed points in three panels are quite different. In panel (a) fixed points disappear near $z = 0$ and immediately reappear. In panel (b) no such behaviour can be observed. However in panel (c), fixed points only emerge between a very narrow interval near the effective interaction region.

The evolution of fixed points and corresponding eigenenergies of the system in

Fig. 3.22 can be seen in Fig. 3.25. No fixed points emerge for the case $\phi_1 = \pi$, and $\phi_2 = 0$.

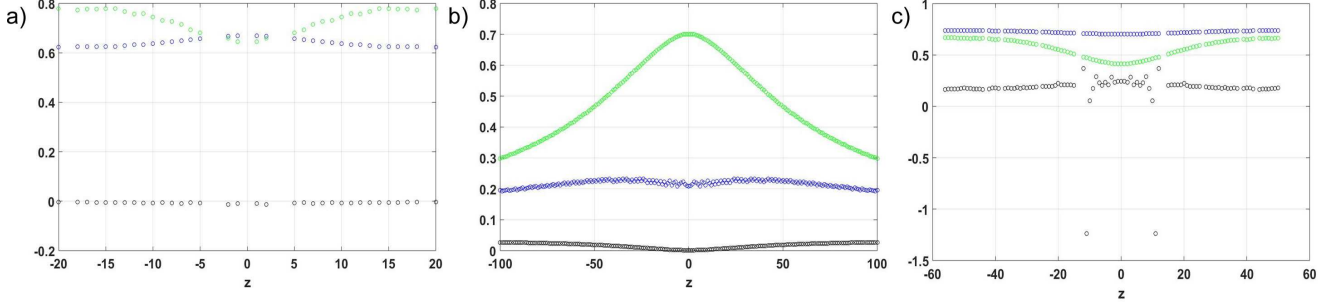


Figure 3.25: Fixed points (blue circles are A_s and green circles are A_{p1} points) and corresponding eigenenergies of the system with parameters $d_1 = 6.5$, $d_2 = 5.5$, $\alpha = 0.51$, $\lambda = 0.062$, $\Delta E_1 = 0.033$, and $\Delta E_2 = -0.025$ for cases $\phi_1 = 0$, $\phi_2 = 0$ (a), $\phi_1 = \pi$, $\phi_2 = 0$ (b), and $\phi_1 = \pi$, $\phi_2 = \pi$ (c).

The evolution of fixed points and corresponding eigenenergies of the system in Fig. 3.23 can be seen in Fig. 3.26. No fixed points emerge for the case $\phi_1 = \pi$, and $\phi_2 = 0$.

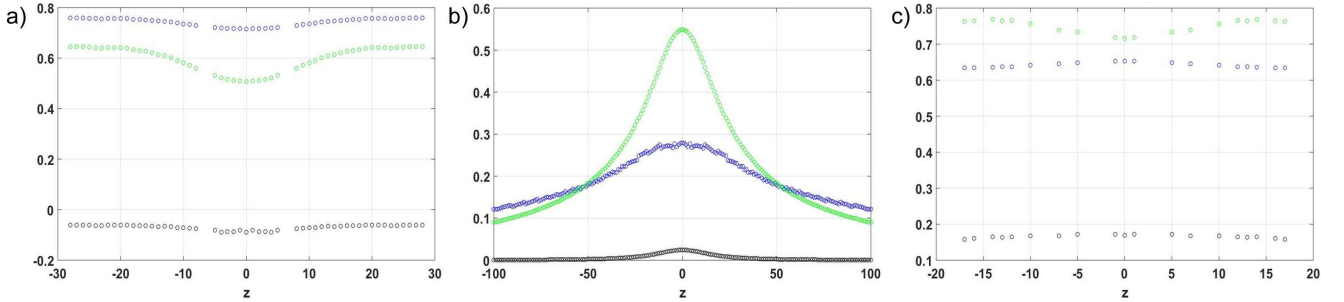


Figure 3.26: Fixed points (blue circles are A_s and green circles are A_{p1} points) and corresponding eigenenergies of the system with parameters $d_1 = 4.7$, $d_2 = 7.3$, $\alpha = 0.6$, $\lambda = 0.03$, $\Delta E_1 = -0.04$, $\Delta E_2 = 0.03$ for cases $\phi_1 = 0$, $\phi_2 = 0$ (a), $\phi_1 = \pi$, $\phi_2 = 0$ (b), and $\phi_1 = \pi$, $\phi_2 = \pi$ (c).

Chapter 4

SHAPIRO RESONANCES

4.1 *Theoretical Background*

The applications of Bose-Einstein condensation in atomic gases have encouraged researchers to investigate the macroscopic behaviour of coherent quantum states. This helped people to study superfluidity in different systems other than superconductors and helium liquids. In this context, the Josephson effect between BECs is the most remarkable and fundamental example to witness superfluidity in a macroscopic scale. This effect was also observed in weak interactions between superfluid ^3He baths. These examples of the Josephson effect were in progress for a long time, though its relation to the atomic condensates for the superconductivity and superfluidity studies is still a hot topic. The suitability of BECs to the superfluidity applications comes from the richness and the tunability of condensate setups.

One of the specific phenomena regarding the Josephson effect in macroscopic BECs is Shapiro resonance, which can be observed when the external frequency of the field balances the internal frequency of the system. The idea behind the formation of Shapiro resonance is similar as the formation of spatial solitons, the self-focusing effect caused by the nonlinearity balances the diffraction caused by the linear effects. In bosonic Josephson junctions, the direct current of the junction creates resonant values, that is the shape of a spike. Experiments of bosonic Josephson junctions show that, half-integer Shapiro resonances can be obtained under realistic parameters.

The experiments regarding Shapiro resonances have been made with two weakly coupled superconductors which are exposed to a voltage difference V and a signal $V_m \sin(ft)$. This voltage difference can be described as the resultant voltage of dc components and the corresponding signal can be identified as periodic. There is

a condition between these two components that enables an interval of nonzero dc currents:

$$V = \frac{\hbar}{2e}kf. \quad (4.1)$$

Here $2e$ is the Cooper-pair charge, $\hbar$ is Planck's constant, and k is an integer. When this condition is met, the constants in the right-hand side, $2e/\hbar$, can be measured with certainty and precision. This is one of the most important uses of Shapiro resonances, so that it describes a standard unit of voltage since 1972. Beyond this measurement, there are novel studies currently done on Shapiro effect.

In [61], [62], the authors discussed to arrange the BEC setup in a way to meet the resonance condition in order to obtain macroscopic population trapping with the presence of dissipation. However, it is not yet fully understood, how to drive the system with controlled dissipation. The known methods are the incoherent exchange of atoms, emergence of quasi-particles caused by the condensate, that fluctuates, or atoms that are lost incidently. The effects of both the atom loss or the incoherent particle exchange are similar, only if loss of the atoms are not large enough. Both methods enable Ohmic dissipation to the system. An important point is the balance between Josephson current and the dissipative current, which causes the Shapiro resonances. This Josephson current can be obtained by the application of an external magnetic field or dipole forces.

The mathematical formulation of this model can be done by considering the formulation of a driven nonrigid pendulum. The equations of motion of such a pendulum are given as

$$\dot{\phi} = \frac{z}{\sqrt{1-z^2}}\cos\phi + \Lambda z + \epsilon\cos(\Omega t) = \Delta\mu/\hbar, \quad (4.2)$$

$$\dot{z} = -\sqrt{1-z^2}\sin\phi - \gamma\dot{\phi}. \quad (4.3)$$

where ϕ and z are relative phase difference and population imbalance, respectively. Λ is identified as the effective interaction constant, $\cos(\Omega t)$ is the time-dependent harmonic control parameter, and $\Delta\mu$ is the chemical potential difference. These equations of motion are derived from the classical Hamiltonian of the system as canonically

conjugate coordinates:

$$H(z, \phi, \Lambda) = -\sqrt{1-z^2}\cos\phi + \frac{1}{2}\Lambda z^2 + \epsilon z\cos(\Omega t), \quad (4.4)$$

and

$$\dot{\phi} = -\frac{\partial H}{\partial z}, \quad (4.5)$$

$$\dot{z} = \frac{\partial H}{\partial \phi}. \quad (4.6)$$

The interaction term, Λ , forces the system to undergo Bloch oscillations, which is caused as the ac Josephson reaction to the stationary behaviour of $\Delta\mu$. The center of these oscillations is the average population imbalance, $\bar{z}$, which is called macroscopic quantum self-trapping. Although this behaviour is called self-trapping, the oscillations are not stable, but slowly decay towards the equilibrium points in the phase space. The objective is to trap these oscillations in an emerging fixed point, so that decaying towards the equilibrium can be prevented as shown in Fig. 4.1. The

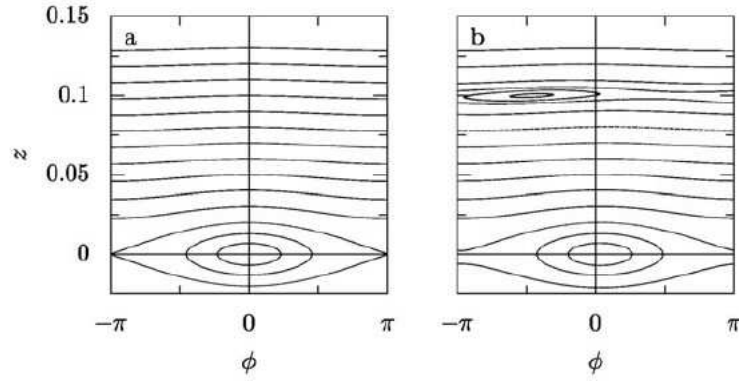


Figure 4.1: Phase space illustration of two different pendulums. (a) is $\epsilon = 0$, and (b) is $\epsilon = 100$. The interaction is $\Lambda = 10^4$, and the driving frequency is $\Omega = 1000$.

conditions for such a behaviour are

$$\Lambda \bar{z} = k\Omega, \quad (4.7)$$

and

$$d\bar{z}/dt = 0. \quad (4.8)$$

When the system parameters are met with above conditions two possible outcomes can be observed. The self-trapping can decay towards zero, until it leaves the macroscopic quantum self-trapping regime, and then it shows damped plasma oscillations. In the second case, the population imbalance can be trapped in a fixed point, which can be seen in Fig. 4.1-b. This type of behaviour is called Shapiro resonance which is shown in detail in Fig. 4.2.

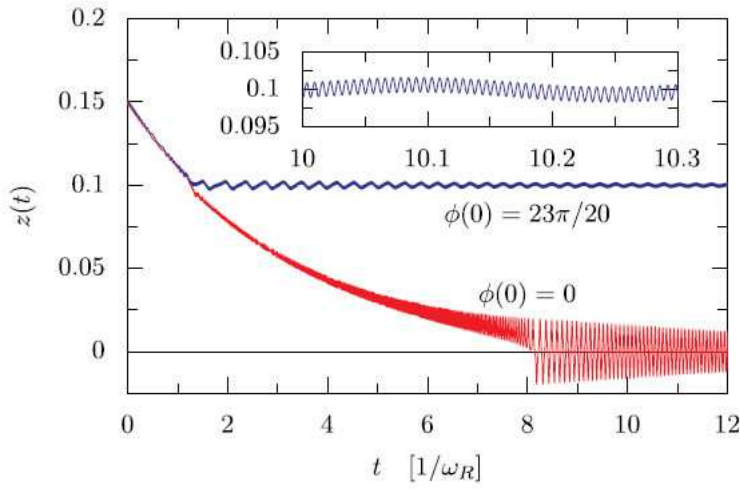


Figure 4.2: Population imbalance vs. time when $\Lambda = 10^4$ and $\gamma = 3 \times 10^{-5}$ with $\epsilon = 100$, $\Omega = 1000$. Shapiro resonance can be observed when time-averaged population imbalance is trapped around $\bar{z} = 0.1$.

Shapiro resonance effect can also be observed in dissipationless weakly coupled Bose-Einstein condensates [37]. When the system is exposed to an external periodic field, resonance condition might be satisfied. When the frequency of this external field becomes equal to the internal frequency of the system, a sharp shift can be seen in the average population imbalance. The periodic component should be added to energy bias such that $\Delta E \rightarrow \Delta E(1 + \delta_0 \cos \omega_0 t)$, which plays the role of a dc voltage in superconducting Josephson junctions. The sinusoidal variation can be implemented on the asymmetry between traps by adjusting the position of laser barrier without changing its intensity. The average population imbalance creates resonant spikes when the period from the energy bias $1/\Delta E$ commensurates the period from the intrinsic

oscillations $1/\omega_0$. The numerical solution of the time averaging dc value, $\langle \dot{z}(t) \rangle$ is plotted as a function of $\Delta E/\omega_0$ in Fig. 4.3.

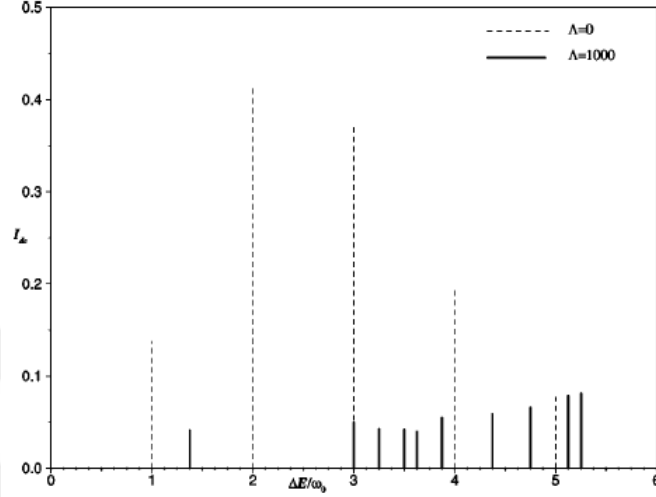


Figure 4.3: Shapiro resonance effects in weakly coupled Bose-Einstein condensates. I_{dc} is the intensity value which is proportional to average population imbalance. Λ is the nonlinearity parameter [37].

Shapiro effect analogies can also be observed in several different systems without any dissipation mechanisms [63], [64]. Based on these studies plasmonic analogy of Shapiro-like resonance can be made by introducing a sinusoidal modulation to the trajectory of the optical soliton, which will be discussed in the following section.

4.2 Periodic Modulation to the Model

Shapiro-like resonance, which was observed in Bosonic Josephson Junctions can also be observed in plasmonic Josephson junctions with the proper modifications to the model. The resonance can only be manifested under an external sinusoidal modification, which matches the internal frequency of the system. Although implementing a dissipation mechanism is a useful method, modulating the distance between surface plasmon and soliton waveguides sinusoidally to create an external ac-effect is more favorable to our system. Hence the distance between metal surface and nonlinearity

in the dielectric medium becomes

$$x = d + A \sin \frac{2\pi z}{a}. \quad (4.9)$$

Here a is the wavelength of the sinusoidal waveguide, and A is the amplitude. The model is illustrated in Fig. 4.4.

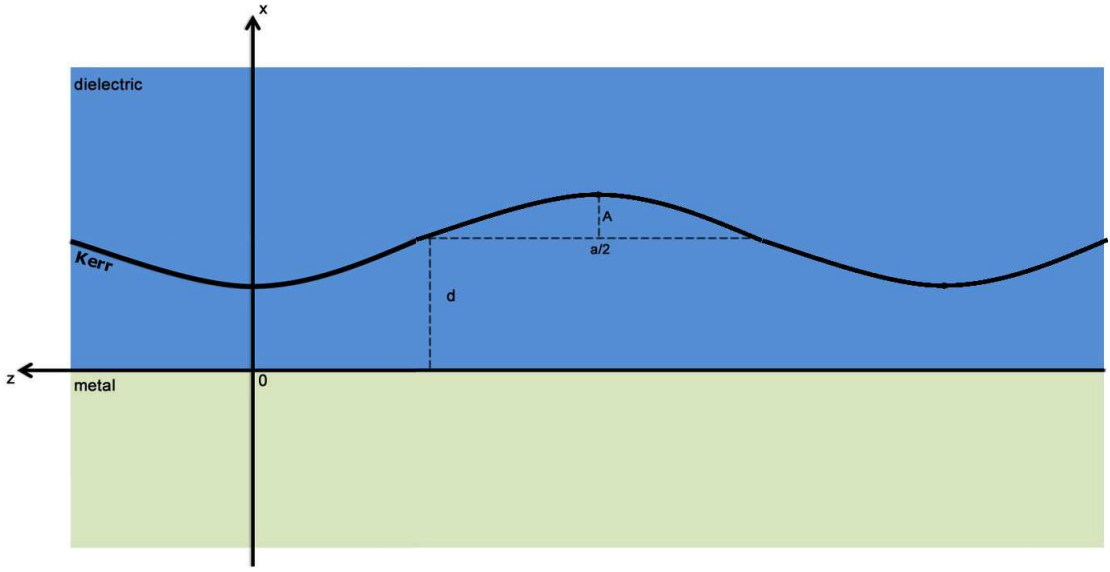


Figure 4.4: Illustration of the sinusoidally modulated single Josephson junction.

The dynamic equations of the system should be adapted to the change of geometry. Therefore the coupling function becomes

$$q(Z) = \sqrt{\frac{1+Z}{2}} \exp \left[- \left(d + A \sin \left(\frac{2\pi z}{a} \right) \right) \sqrt{2\Lambda(1+Z)} \right] \quad (4.10)$$

For the sake of the paraxial approximation, that has been discussed in the previous chapters, the wavelength and the amplitude of the sinusoidal soliton trajectory are investigated between $10 < a < 90$ and $0.2 < A < 1$, respectively. In this range of parameters, one can observe resonant spikes in the average population imbalance when the external periodic field -distance between soliton and surface plasmon- is modulated slowly. The periodic modification of the soliton trajectory does not allow the soliton

and plasmon channels decouple like they do in the hyperbolic modulated system, therefore transfer of energy between two channels continues without distortion. The time-averaged value of these subsequent oscillations doesn't vary very much unless a resonant state is achieved. These resonance features indicate, that Shapiro resonances can be observed in a plasmonic two-level system formed by optical soliton and surface plasmon. Two different examples of Shapiro resonance can be seen in the following figures.

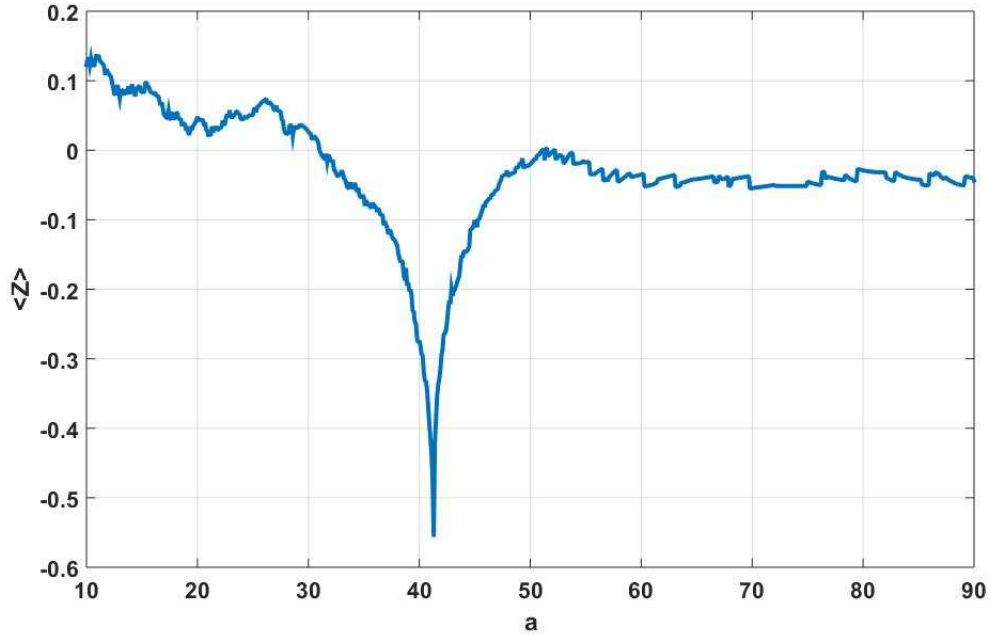


Figure 4.5: Shapiro resonance in plasmonic Josephson junction. A drop in the magnitude of average population imbalance can be observed, when the external modulation parameter changes between 10 and 90. System parameters are $d = 5$, $\Lambda = 0.08$, $\Delta E = 0$, $A = 0.8$, $Z_{-\infty} = 0.9049$.

Figure 4.5 shows an instant drop in the average population imbalance. The system parameters used in this configuration are $d = 3$, $\Lambda = 0.07$, $\Delta E = -0.04$, and $A = 0.6$ with an initial population imbalance $Z_{-\infty} = 0.9049$ and relative phase $\phi_{-\infty} = 0$.

Figure 4.6 also shows two instant rises in the average population imbalance. The system parameters are $d = 3$, $\Lambda = 0.07$, $\Delta E = -0.04$, $A = 0.6$ with $Z_{-\infty} = 0.9049$.

and $\phi_\infty = 0$.

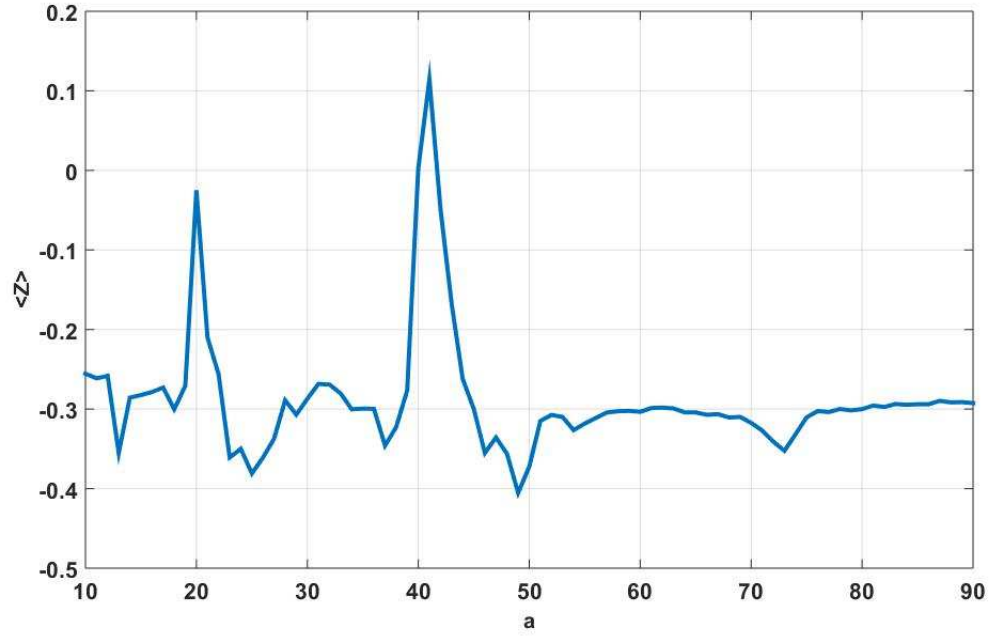


Figure 4.6: Shapiro resonance in plasmonic Josephson junction. Two different spikes in the magnitude of average population imbalance can be observed, when the external modulation parameter changes between 10 and 90. System parameters are $d = 3$, $\Lambda = 0.07$, $\Delta E = -0.04$, $A = 0.6$, $Z_{-\infty} = 0.9049$.

Chapter 5

APPLICATIONS

5.1 *Surface Plasmon-Soliton Coupling in Realistic Structures*

The first realization of surface plasmons and optical solitons have been done separately more than three decades ago. These experiments contain surface waves on metal-dielectric or dielectric-dielectric interfaces which host self-focused solutions, that connect soliton and surface plasmon properties. However, experimental results regarding surface plasmon-soliton coupling have not been presented, mainly because of the high nonlinear refractive index requirement of these proposed models. In other words, it can be said that, the peak power is too high for conventional materials used in optical structures. In a recent study [65], low peak power solutions, that support soliton propagation with a plasmonic counterpart has been proposed. The proposed structure is made of conventional materials like silica and gold films coated into chalcogenide glass. Some of these materials have already been produced for non-plasmonic experiments, and the peak power that has been observed in these structures is 2 GW/cm^2 [66],[67]. The plasmonic usage of these materials gave close results such as 1.07 GW/cm^2 , which is not enough for required soliton solutions, however this is a promising result considering twice this value will be enough.

The conversion of realistic values into our system parameters has been explained [35]. The permittivity of the dielectric material (chalcogenide glass) is $\epsilon_0 \sim 5 - 10$, the vacuum wavelength is $\lambda_0 \sim 5 - 10 \mu\text{m}$, which is in mid-infrared regime. Within this range, the propagation constants of plasmon and soliton becomes $\nu_p = \nu_s \sim 10^{-3}$. Parameters, that identify evanescent tails of the plasmon and soliton becomes $\kappa_p \sim \kappa_s \sim 0.1 \mu\text{m}^{-1}$, which yields the propagation distance of the soliton from the metal interface as $d \sim 20 - 40 \mu\text{m}$, that results in coupling coefficient of $q_{\text{res}} \sim 10^{-1} - 10^{-2}$.

In a different study [68], the dispersion relation of a four layer model (metal/linear dielectric/nonlinear dielectric/linear dielectric) is investigated and the existence of low-power plasmon-soliton solutions are determined. The structure can be seen in Fig. 5.1 and has the following parameters: $\tilde{\epsilon}_{l,1} = 2.4707^2 - 10^{-5}i$, $n_2^{(1)} = 10^{-17}\text{m}^2/\text{W}$ (chalcogenide glass), $\tilde{\epsilon}_2 = 1.443^2 - 10^{-5}i$ (silica), $\tilde{\epsilon}_3 = -96 - 10i$ (gold), $\tilde{\epsilon}_4 = 2.4707^2 - 10^{-5}i$, $L = 15\text{ nm}$, $d = 40\text{ nm}$, and $\lambda = 1.55\text{ }\mu\text{m}$. Here ϵ is defined as the sum of linear and nonlinear parts of the permittivity function of the medium, $\epsilon_j(x) = \epsilon_{j,1}(x) + \epsilon_{nl}(x)$. Nonlinear part can be defined as $\epsilon_0 c \epsilon_{l,1} n_2^{(1)} |E(x)|^2$.

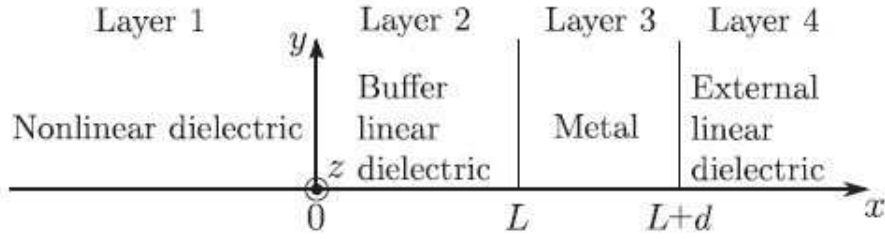


Figure 5.1: Geometry of the four-layer configuration.

Although there are some experiments regarding surface plasmon and soliton coupling as mentioned above, these optical elements have been used separately for many applications such as magnetic field measurement by using soliton stability [69], multicolor solitons for biosensors [70], a refractive index sensor made of surface solitons [71], and surface plasmon resonance biosensors [72]. However the resonant condition of surface plasmon and optical soliton was never used for sensor applications due to difficulties in obtaining the realistic parameters mentioned above.

5.2 Proposed Instruments Regarding Surface Plasmon-Soliton Coupling

In this section several outcomes of the coupling mechanism of surface plasmon and soliton regarding both single and double Josephson junction formulation will be presented. As mentioned in the previous chapters, a self-focusing optical soliton originated from a light source can propagate on a nonlinear waveguide, and if the tail of

this soliton reaches the neighboring curved metal-dielectric interface a co-propagating surface plasmon can be excited. Depending on the system parameters, i.e., the distance between metal surface and the nonlinear waveguide, the curvature of the metal, the strength of the nonlinearity or propagation constants of the waves, the transfer of energy between soliton and surface plasmon can be adjusted, even a 100% population transfer can be achieved. This phenomenon can lead a heuristic design of a refractive index sensor, which can be seen in Fig. 5.2.

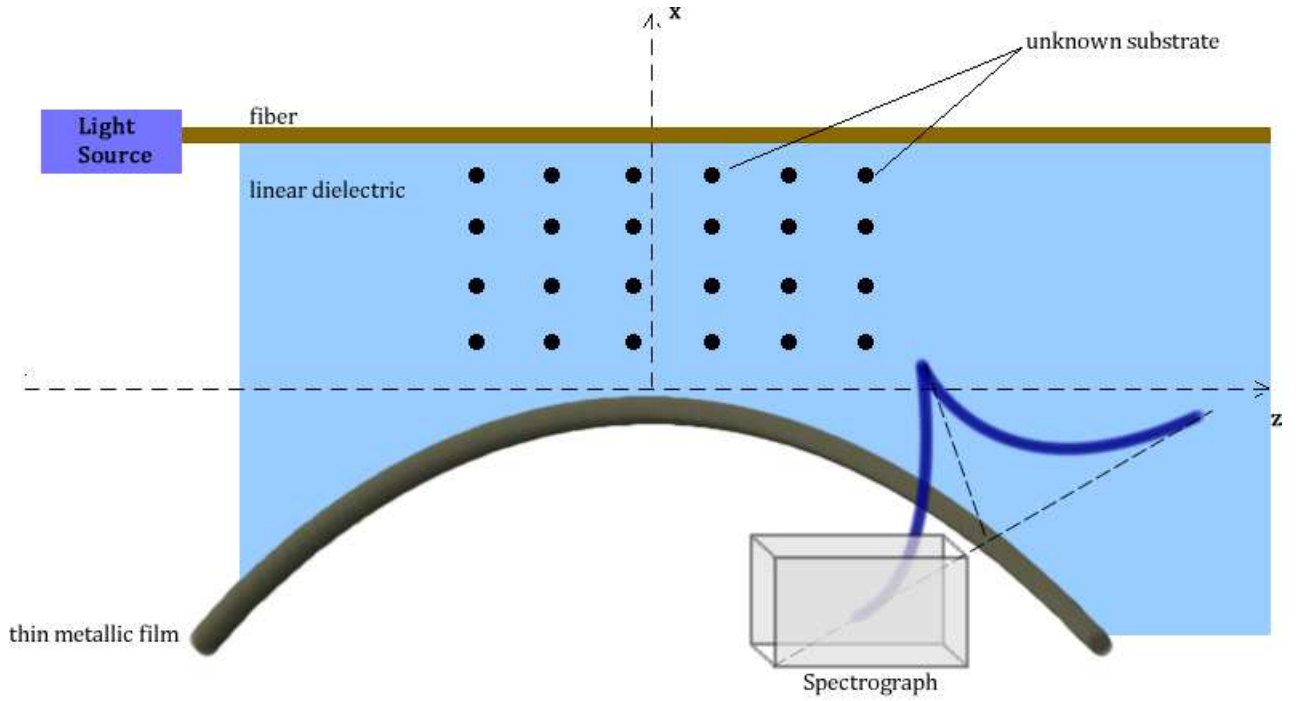


Figure 5.2: Refractive index sensor based on single Josephson junction formulation of surface plasmon-optical soliton coupling.

If an unknown material is placed uniformly onto the region between the nonlinear waveguide and the metal surface, this substrate alters the permittivity of the dielectric material, ϵ_d . The permittivity of the nonlinear material is given as $\epsilon_n = \epsilon_d(1 + 8\Lambda|\Psi|^2)$ [35]. Since it depends on both ϵ_d and soliton and plasmon amplitudes, $|C_p|$ and $|C_s|$, the transition probability of the interaction with the substrates would result

in determining the altered permittivity. The final surface plasmon amplitude can be measured by a spectrograph from the evanescent tail on the metal side, and this would give the transition probability, since the total amplitudes are known. This measurement should be done both before and after the unknown material is placed, so the change in the dielectric permittivity would be verified. In order to detect the dielectric permittivity, the system should be arranged to yield a full population transfer from soliton to surface plasmon initially. The detection of the dielectric permittivity would initiate the calculation of the refractive index by the following relation

$$n = \sqrt{\frac{\epsilon_d}{\epsilon_0}} \quad (5.1)$$

For the sake of applicability, the metallic film should be responsive to hyperbolic modulation, α , and the nonlinearity parameter of the Kerr medium, Λ , should also be adaptable to change, so that the full transition can be obtained under different circumstances.

Another application of surface plasmon-soliton coupling is made of double Josephson junction with one of the metal films is bent with a hyperbolic surface as shown in Fig. 5.3.

This mechanism is inspired by Fig. 3.23, in which the populations of two copropagating surface plasmon and soliton waves are oscillating repeatedly until the soliton wave starts to fall into the effects of lower metallic film. With the proper parametric configurations these oscillations can be suppressed and surface plasmon and soliton on the upper waveguides can propagate without any interaction while the distance between the waveguides allows them to engage. This stabilizer mechanism can find many application areas in sensor development purposes.

The spectrograph takes information from the evanescent tail of the lower surface plasmon about the amplitude of the surface plasmon and naturally other waves. The lever on the left side of the lower metallic film, which takes information from the spectrograph by the feedback mechanism, can be used to align the distance between the nonlinear fiber and the lower metal surface vertically, so that the proper adjust-

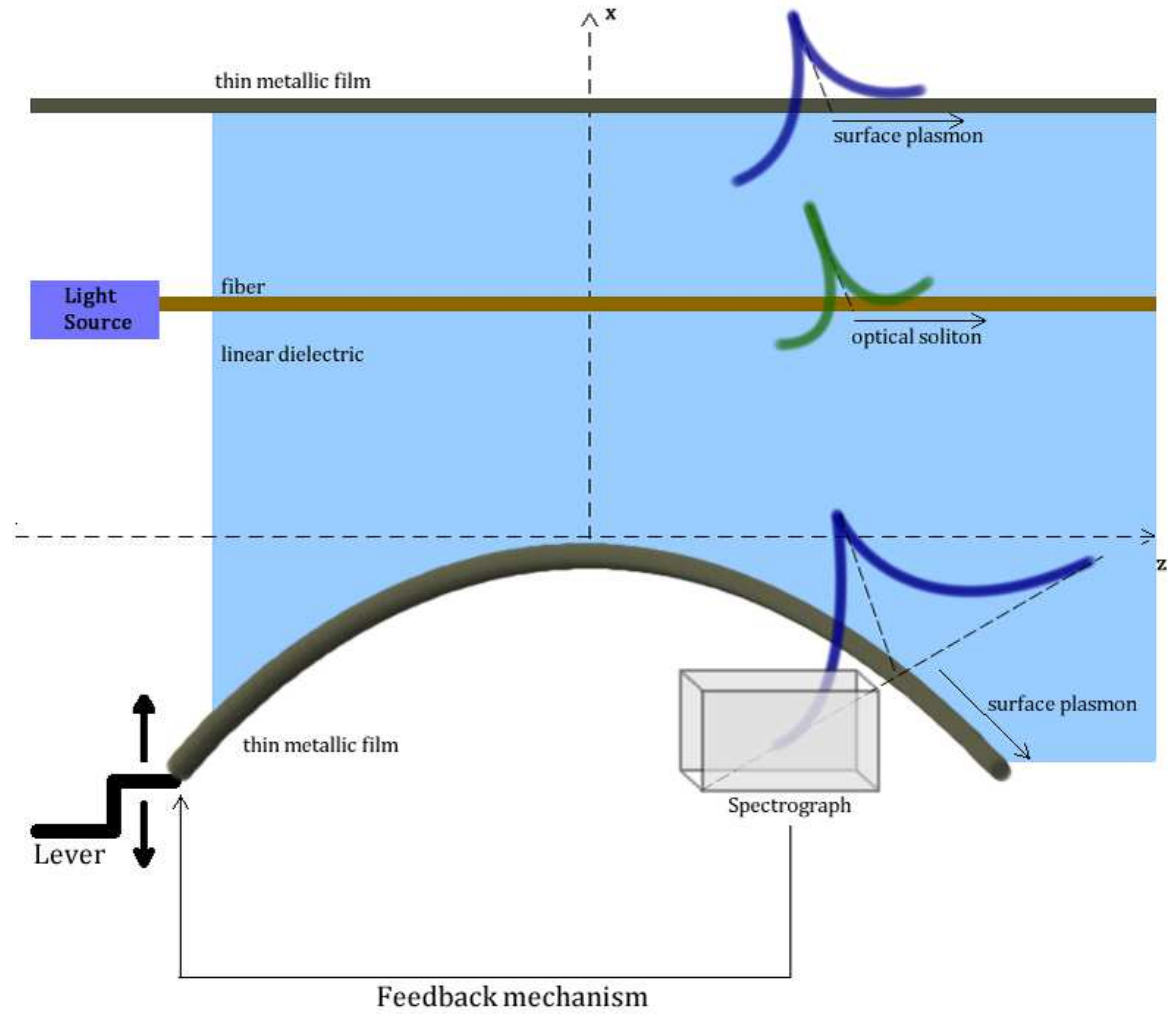


Figure 5.3: A coupling modulator made of metal-dielectric-nonlinear dielectric-dielectric-metal multilayers.

ment for the stabilizing of oscillations can be ensured. The effect of the adjustment of distance can be seen in Fig. 5.4.

By using the same parameters as Fig. 3.23, the behaviour of the plasmon-soliton coupling after the interaction with the second metal surface can be analyzed. No surface plasmon is propagating on the lower metal surface initially indicating that, the upper coupling mechanism is only effected by the presence of a thin metallic film. This effect shifts the amplitudes of the oscillations by stealing some portion of energy. This energy loss of the upper mechanism could force the populations to be

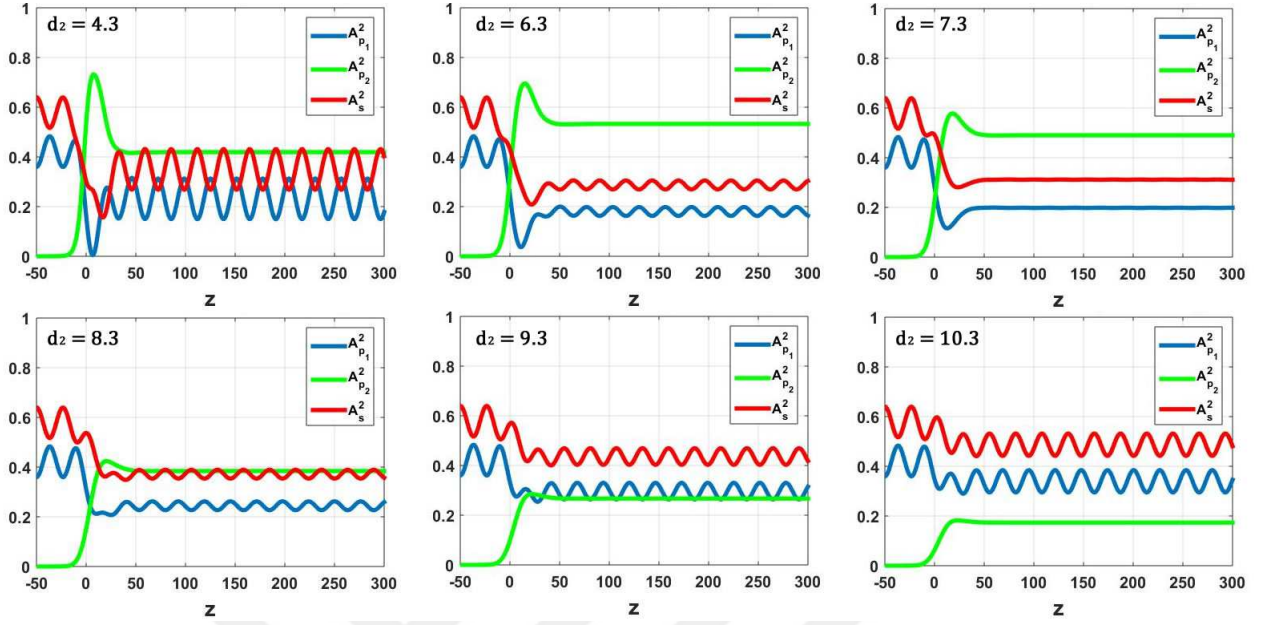


Figure 5.4: The effect of d_2 . The other parameters are $d_1 = 4.7$, $\alpha = 0.6$, $\lambda = 0.03$, $\Delta E_1 = -0.04$, $\Delta E_2 = 0.03$, $A_{p1} = 0.6$, $A_s = 0.8$.

trapped in their current states. The reason for the usage of a lever mechanism to change the distance instead of other system parameters such as nonlinearity, Λ , or hyperbolicity, α , is that the difficulties of changing these parameters, while the system is running. They can be change before the system starts by modulating the nonlinear material, or the curvature of the metal, however moving the metal film is much easier and allows the feedback mechanism to work faster. This proposed instrument is a heuristic model with no experimental counterparts, since it is still early to develop such a mechanism with the current experimental findings of optical solitons, but it is a promising research area with many potential applications.

Chapter 6

CONCLUSION

In this thesis work, dynamical properties of surface plasmon-spatial soliton coupling, which forms an analog of bosonic Josephson junction is investigated in both two and three layer systems. Following the 80-year old theoretical quantum phenomena Landau-Zener and Rosen-Zener transitions, tunneling dynamics between two decoupled optical channels are switched on by coupling the system for a brief period of time with a spatial modulation to the soliton trajectory. In the proposed system the surface plasmon is propagating on metal-dielectric interface and the optical soliton is propagating on Kerr type of nonlinearity located at an appropriate distance from the metal surface such that, tails of both waves intersect. This intersection procreates a coupling function, that determines the time-dependent coupling between channels defined in the Rosen-Zener transition. By introducing a hyperbolical modulation to the nonlinearity, a full population transfer between soliton and surface plasmon is observed.

The plasmonic Josephson junction configuration gives rise to rich dynamical properties apart from a full transition. Different combinations of parameters allow the system to be manipulated into significant transitions such as splitting the population into two equal parts or visa versa. Stability analysis of the system is also studied by exploring the dynamics of the fixed points in the phase-space. The mathematical setup of the system represents a nonlinear Schrödinger equation, and the correlation between the eigenvalues of this equation and the fixed points in the phase-space is demonstrated.

The dynamics of surface plasmon-soliton interaction extends beyond the two-level system to form double Josephson junction. The nonlinearity is sandwiched between

two metal surfaces and linear dielectric buffers. Since this configuration is not decoupled at the beginning and at the end, Landau-Zener analogy does not apply, however interesting outcomes can be attained such as trapping of soliton amplitude, while transfer is continuing between surface plasmons, or the classical demonstration of collapse and revival of the coupling function for a brief period of time, which is originally a quantum mechanical effect. While these two distinguishable physical phenomena are manifested in parallel three layer system, different outcomes can be observed in the spatially modulated three layer system. When a hyperbolical trajectory is constructed for one of the surface plasmons, population trapping for not only soliton but for all three channels can be obtained, which also gives inspiration to a coupling modulator proposed as an realistic application.

The hyperbolical trajectory of the surface plasmon causes an adiabatic change in the distance between metal surface and the nonlinearity under the paraxial approximation. This adiabatic change disturbs the phase-space of the system and ergo the stability of the fixed points, whose analysis has been made by investigating the trajectory of the fixed points and the corresponding eigenenergies, which are the eigenvalues of the nonlinear Schrödinger equation.

The spatial modulation, that has been implemented on the soliton trajectory has also been constructed periodically to demonstrate a phenomenon called Shapiro resonances, that can be observed, when an external periodic field is introduced to the system. The resonance condition is satisfied, when the soliton wave follows a sinusoidal trajectory, and the results are given.

Finally the realistic materials and parameters, that can host a surface plasmon-soliton Josephson junction are introduced. In the light of these values, two instruments, that can be used in sensor applications are proposed. A two-layer surface plasmon-soliton coupling is designed as a heuristic refractive index sensor and a three-layer surface plasmon-soliton-surface plasmon coupling is designed as a stabilizer mechanism. The physical properties behind both instruments come from the results of the spatially modulated geometries.

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