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M.Sc. in Physics Engineering

SACIDE SAFADI

REPUBLIC OF TÜRKİYE

GAZİANTEP UNIVERSITY

GRADUATE SCHOOL OF NATURAL & APPLIED SCIENCES

VARIATION OF MULTIPLICATION FACTOR WITH ENRICHMENT FOR AN ENRICHED-URANIUM REACTOR

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BY

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Sacide SAFADI

ABSTRACT

VARIATION OF MULTIPLICATION FACTOR WITH ENRICHMENT FOR AN ENRICHED-URANIUM REACTOR

SAFADI, Sacide
M.Sc. in Physics Engineering
Supervisor: Prof. Dr. Okan OZER
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The criticality of a nuclear power reactor is one of the most important factors to be considered in any nuclear world. Multiplication factor, k, sometimes called as survival factor f, is the crucial parameter for achieving the criticality. There are many factors affecting the criticality and in the calculation of k: The thermal fission factor (η), the resonance escape probability (p), and the thermal utilization factor (f) are some of them. Implicitly, the Moderator (graphite) to Fuel ratio (NMF) and the uranium-235 enrichment percent are what all these factors depend on. In order to determine the effects of these various parameters and to illustrate the relationship between these implicit factors and their impact on nuclear reactor criticality a computational method is applied, and the uranium-235 enrichment from 0.7 percent (uranium percent in nature) to 5.0 percent by weight (the maximum allowable in LEU) are examined to calculate the multiplication factor with the corresponding size and volume of the system under investigation. The minimum enrichment for a range of NMF as well as the minimum NMF ratio for a range of enrichment has been considered for the criticality as an optimization problem. Results are presented in Tables and Graphs, and the optimum conditions for the criticality have been discussed.

Key Words: Diffusion and Moderation, Diffusion Equation, Fission, Monte Carlo Method, Critical Size,

ÖZET

ZENGINLEŞTIRILMIŞ BIR URANYUM REAKTÖRÜ IÇIN ZENGINLEŞTIRME ILE ÇARPMA FAKTÖRÜNÜN DEĞIŞIMI

SAFADI, Sacide
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Danışman: Prof. Dr. Okan OZER
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59 sayfa

Bir nükleer güç reaktörünün kritikliği, herhangi bir nükleer dünyada dikkate alınması gereken en önemli faktörlerden biridir. Bazen hayatta kalma faktörü f olarak da adlandırılan çarpım faktörü k, kritikliğe ulaşmak için çok önemli bir parametredir. Kritikliği ve k'nin hesaplanmasını etkileyen birçok faktör vardır: Termal fisyon faktörü (η), rezonans kaçış olasılığı (p) ve termal kullanım faktörü (*f*) bunlardan bazılarıdır. Örtük olarak, Moderatör (karbon)/Yakıt (uranyum) oranı (NMF) ve uranyum-235 zenginleştirme yüzdesi tüm bu faktörlerin bağlı olduğu parametrelerdir. Bu çeşitli parametrelerin etkilerini belirlemek ve bu örtülü faktörler ile bunların nükleer reaktör kritikliği üzerindeki etkileri arasındaki ilişkiyi göstermek için bir hesaplama yöntemi uygulandı ve uranyum 235 zenginleştirmesi (doğadaki uranyum yüzdesi) yüzde 0,7'den (LEU'da izin verilen maksimum) yüzde 5,0'a çıkarıldı, ve incelenen sistemin karşılık gelen boyutu ve hacmi ile çarpım faktörü incelendi. Bir optimizasyon problemi olarak kritiklik açısından, bir dizi NMF için minimum zenginleştirmenin yanı sıra bir dizi zenginleştirme için minimum NMF oranı dikkate alınmıştır. Sonuçlar Tablo ve Grafiklerde sunulmuş ve kritiklik için optimum koşullar tartışılmıştır.

Anahtar Kelimeler: Difüzyon ve Moderasyon, Difüzyon Denklemi, Fisyon, Monte Carlo Yöntemi, Kritik Boyut.

"Dedicated to Salaheddin Safadi"

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To my wonderful **family**, whose love and constant support have been my anchor, I owe my heartfelt gratitude. Their belief in me has been the foundation of my perseverance, and their unending support has carried me through every challenge.

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LIST OF SYMBOLS

b	Barns
σ	Sigma
Σ	Sigma
∇	Nable
ϕ	Phi
η	Eta
ε	Epsilon
τ	Tau
ξ	Zeta
π	Pi

LIST OFABBREVIATIONS

PWR	Pressurized Water Reactor
PHWR	Pressurized Heavy Water Reactor
BWR	Boiling Water Reactor
LWGR	Light Water Graphite Reactors
GCR	Gas-Cooled Reactors
IAEA	International Atomic Energy Agency
MCM	Monte Carlo Method
N_M/N_F	Moderator-to-Fuel Ratio

CHAPTER 1
INTRODUCTION

1.1 Introduction

Throughout history, energy sources have played a crucial role in human progress, with fossil fuels dominating for many years. However, as we look to the future, alternative energy sources are becoming increasingly important. Nuclear energy, a significant contender in this shift, is primarily used to generate electricity. In the global effort to combat climate change, reducing carbon dioxide emissions is paramount. Nuclear power plants offer a promising solution, as they can produce substantial amounts of energy without emitting carbon dioxide (CO2)[1].

Additionally, advancements in nuclear technology are making it a safer and more efficient option. As the world seeks sustainable energy solutions, understanding the mechanics and benefits of nuclear power is essential.

1.2 Number of reactors

The global number of power reactors has been steadily increasing since the 1950s, which accounts for 10% of the electrical energy generated in the world [2]. The Figure 1.1 illustrates the changes in the number of power reactors from 2003 to 2022 [3].

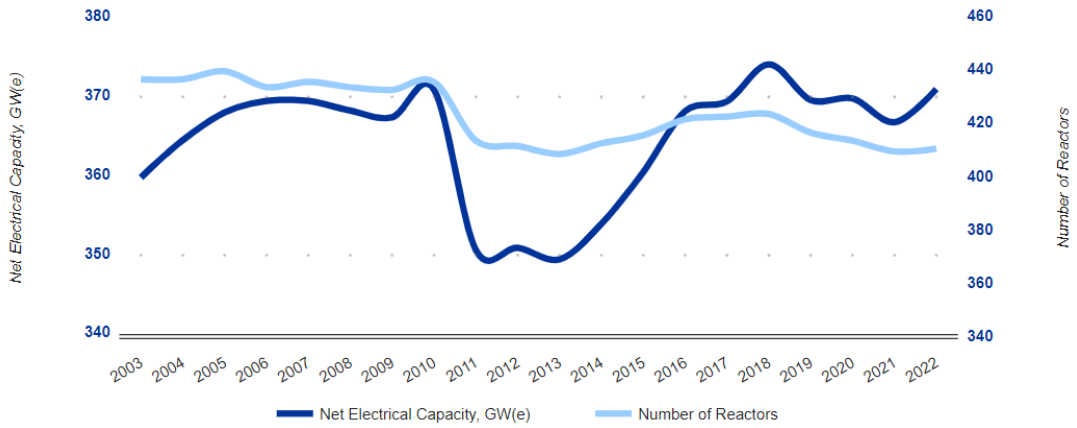

Figure 1.1 Nuclear power capacity trend. [3]

Currently, nuclear power plants are operational in 31 countries (see Figure 1.2). In several of these countries, they contribute significantly to the overall electrical demand[3].

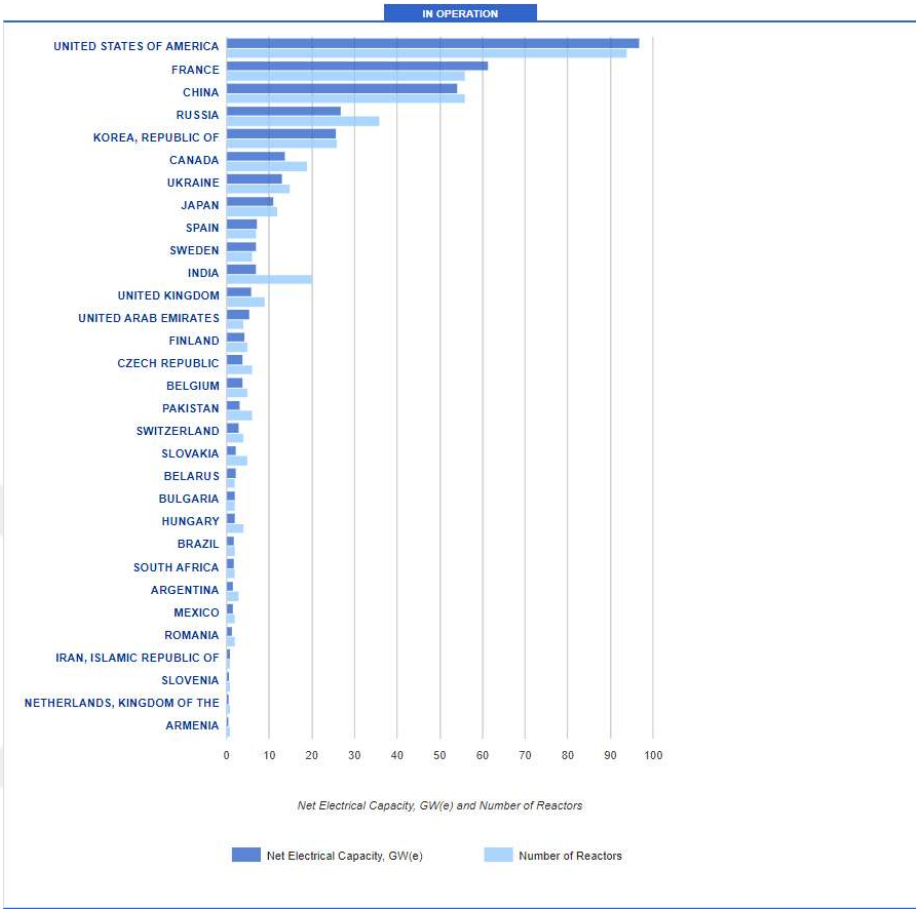

Figure 1.2 Worldwide nuclear power plants in operation.[3]

Energy released from nuclear processes is being harnessed for commercial use in nuclear reactors [4]. As of 2024, there are approximately 416 operational reactors worldwide[3], The majority of nuclear electricity production occurs in the United States, France, China, and Russia.

The most prevalent type of commercial nuclear reactor globally is the Pressurized Water Reactor (PWR)[2], with 307 operating units, which account for 73.8% of all operating nuclear reactors and have an electric power generation capacity of 294.08 GW(e). Following PWRs are Pressurized Heavy Water Reactors (PHWR), with 47 units representing 11.3% of the total and a capacity of 24.75 GW(e). Boiling Water Reactors (BWR) are next, with 41 units making up 9.9% of the total and a capacity of 34.07 GW(e). Other reactor designs comprise the remaining 5% of the operating

nuclear reactors, capable of generating 12.72 GW(e), including designs such as Light Water Graphite Reactors (LWGR) and Gas-Cooled Reactors (GCR).

According to the International Atomic Energy Agency (IAEA)[3], there are currently 59 reactors under construction worldwide, with a total net electrical capacity of 61.63 GW(e). Among these, four reactors are being built in Mersin, Türkiye, all of which are Pressurized Water Reactors (PWR). Each of these reactors has an electrical power generation capacity of 1.114 GW(e), resulting in a combined total of 4.46 GW(e).

According to another attribute, the reactors responsible for producing electricity are either moderated and cooled by light water (H2O) or heavy water (D2O) [5]. Both types of reactors can use water or gas for cooling and can operate with natural uranium-based fuel assemblies.

1.3 Types of reactors

Among all the elements in the universe, only four isotopes of uranium and plutonium (Uranium-233, Uranium-235, Uranium-238, and Plutonium-239) are available in sufficient quantities to generate nuclear energy on a commercial scale. Uranium is relatively rare compared to other elements in the universe, and there is much less of it on Earth today due to radioactive decay than there was several billion years ago [2].

There are five primary types of nuclear power plants commonly found around the world. These commercial nuclear reactors can be broadly categorized as follows [3]:

1. **Pressurized Light-Water Reactors (PWRs):** in these reactors, the coolant does not boil, and both the moderator and coolant are ordinary water. PWRs were developed in the 1950s.
2. **Pressurized Heavy-Water Reactors (PHWRs):** These reactors use heavy water (deuterium dioxide, D_2O) as the moderator and coolant. PHWRs were developed in Canada in the 1960s.
3. **Boiling Water Reactors (BWRs):** BWRs use light water as both the moderator and coolant. They were developed in the 1950s, similar to PWRs.
4. **Light-Water Cooled, Graphite Moderated Reactors (LWGRs):** These reactors use light water as the coolant and graphite as the moderator. LWGRs were developed in the Soviet Union.

5. **Gas Cooled, Graphite Moderated Reactors (GCRs):** This type of reactor uses graphite as a moderator and gas as a coolant. GCRs were developed in the UK after World War II.

Over the past years, numerous nuclear reactor designs have been developed worldwide. These reactors can be classified based on their configuration and objectives:

1. **Fast Reactors:** These reactors sustain the fission chain reaction using fast neutrons with energies typically ranging from 1-10 MeV and velocities around 50,000 km/s.
2. **Thermal Reactors:** These reactors operate using thermal neutrons, which have a speed of approximately 2200 m/s and an energy of 0.0253 eV.

Most commercial nuclear power plants use enriched uranium to sustain a nuclear chain reaction with thermal neutron-based reactions. The concentration of Uranium-235 in their fuel rods (the enrichment level) typically ranges from about 2.5% to 5.0%. However, some reactors, such as those used in military applications, use enrichment levels as high as 95%. The Canadian pressurized heavy water reactor, known as the CANDU reactor, does not require enriched uranium at all.

1.4 Nuclear Reaction

A nuclear reaction refers to a process in which the nucleus of an atom undergoes a change, resulting in the formation of one or more new nuclei, along with the release or absorption of energy. These reactions involve the interaction of atomic nuclei, either with each other or with other particles, and can result in the emission of energy in various forms, such as gamma rays, neutrons, or charged particles. The number of protons and neutrons in the nucleus can change during these reactions, leading to the transformation of one element into another. These reactions involve the forces acting within the nucleus, including the strong nuclear force and the electrostatic force.

There are two main types of nuclear reactions: nuclear fission and nuclear fusion. For this study nuclear fission is discussed.

1.4.1 Nuclear Fission

Nuclear fission is a process in which a heavy nucleus, like uranium or plutonium, undergoes a split into two or more smaller nuclei, accompanied by the release of a

substantial amount of energy. For further understanding of the process, Figure 1.3 illustrate the sequence of events involving the reaction with U-235 as follows:

1. In Stage A, a neutron approaches the U-235 nucleus.
2. In Stage B, the U-236 nucleus is formed and exists in an excited state. In some cases, excess energy may be emitted as a gamma ray,
3. but more frequently, the energy leads to distortions of the nucleus, creating a dumbbell shape, as seen in Stage C.

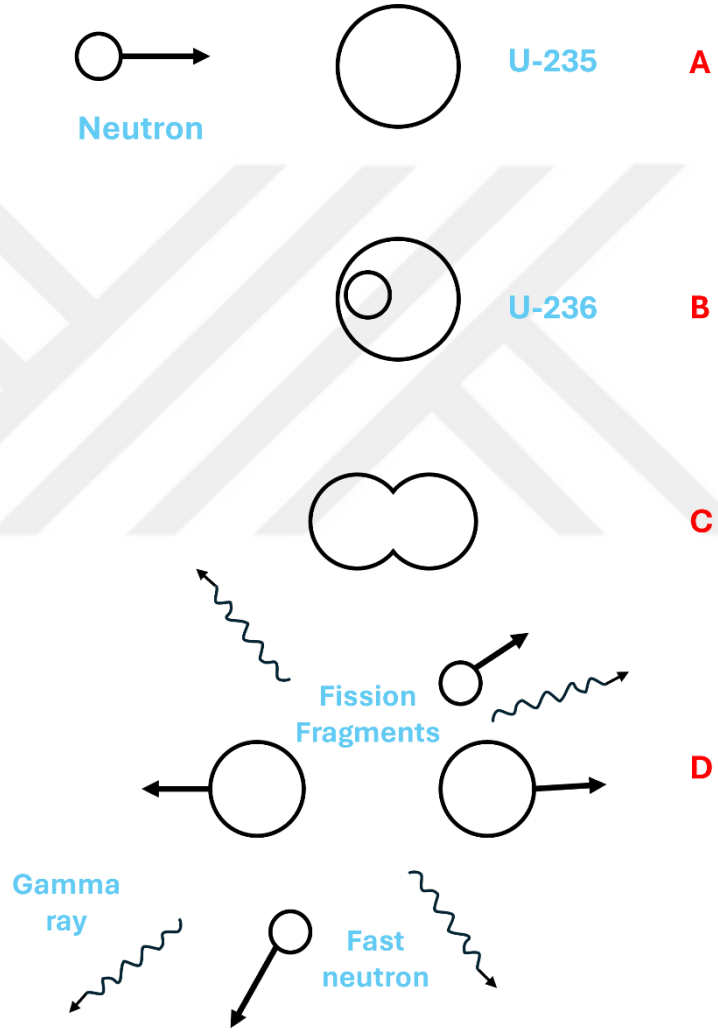

Figure 1.3 Fission process.

4. The nucleus parts oscillate, resembling the motion of a liquid drop. Due to the prevalence of electrostatic repulsion over nuclear attraction, the two parts can separate, as depicted in Stage D. The fission of U-236 occurs rapidly following the absorption of a free neutron by the U-235 nucleus. On average, this entire process

takes approximately 10^{-12} seconds. These separated parts, known as fission fragments, carry the majority of the released energy. They rapidly move apart, carrying around 166 MeV of kinetic energy out of the total of approximately 200 MeV released throughout the entire process. This energy results from the conversion of a small amount of nuclear mass into energy, as expressed by Einstein's equation, E=mc^2. As the fragments separate, they shed atomic electrons, and the resulting high-speed ions gradually lose energy through interactions with the atoms and molecules of the surrounding medium.

In this process, a U-236 nucleus can undergo fragmentation in over 60 recognized channels or ways. Fission of the uranium-235 isotope can be made to give up new cerium, tellurium and iodine isotopes. Some of Uranium fission reactions are represented in Table 1.1:

Table 1.1 Uranium fission reaction

$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{56}Ba + {}^{90}_{36}Kr + 2{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{100}_{38}Sr + {}^{134}_{54}Xe + 2{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{142}_{56}Ba + {}^{91}_{36}Kr + 3{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{132}_{50}Sn + {}^{101}_{42}Mo + 3{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{54}Xe + {}^{90}_{38}Sr + 2{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{92}_{37}Rb + {}^{140}_{55}Cs + 4{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{95}_{42}Mo + {}^{139}_{57}La + 2{}^{1}_{0}n$
$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{93}_{37}Rb + {}^{141}_{55}Cs + 2{}^{1}_{0}n$

From the Table 1.1 it is noticed that:

1. The quantity of emitted neutrons is not consistently uniform; it follows a statistical probability distribution ranging from one to four. The most fission channels typically release an average of approximately two neutrons per fission, whereas alternative fission channels emit an average of around three neutrons.
2. Also, for a nuclear chain reaction to be sustained, at least two neutrons must be generated for each neutron absorbed. This requirement arises from the fact that approximately 20% of the absorbed neutrons do not lead to the fission of the U-236 nucleus[2].

This reaction, induced by bombarding the nucleus with a neutron, is the foundation of nuclear power generation. Nuclear fission provides a high energy density, enabling the production of significant electricity without emitting large amounts of greenhouse gases. However, it generates radioactive waste, necessitating careful management to ensure proper disposal and prevent environmental contamination.

1.4.2 Nuclear Cross Section

Cross sections can be defined for any type of process involving a collision between two nuclear or subatomic particles, such as the scattering of photons, the scattering of electrons, and the absorption of a neutron by an atomic nucleus[2]. Essentially, a nuclear cross section measures the probability that an incoming particle will interact with the nucleus (or another comparably sized object like an electron), resulting in a specific nuclear reaction or set of reactions.

In other words, it is the probability that a particle will interact per unit surface area of the target. This area defines the unit for cross-section in barns (b), where 1 b=10^{-24} cm^2. A larger cross-sectional area indicates a higher probability of a reaction occurring, while a smaller cross-sectional area indicates a lower probability.

A nuclear cross section is usually represented in the literature by the Greek symbol σ (sigma). The number of particles that are scattered and the number of particles that are absorbed can vary significantly depending on the following factors:

1. The energy of the incoming particle
2. The type of particle
3. The material from which the target is made. As shown in the figure below

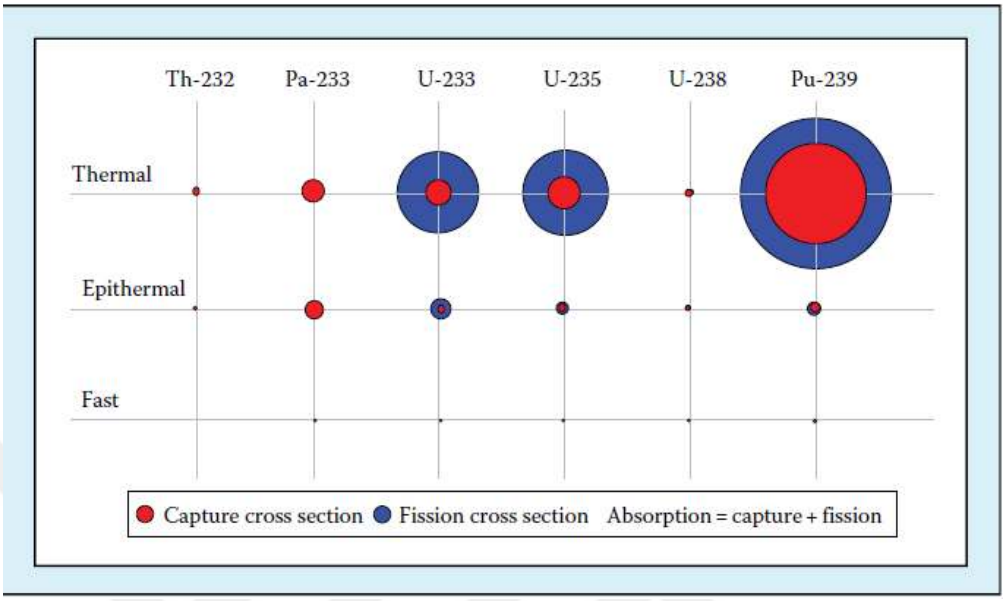

Figure 1.4 Reaction probabilities at different neutron energies.[2]

Figure 1.3 represents the Reaction probabilities for some common nuclear materials at different neutron energies.

There are four basic types of nuclear cross sections:

1. Scattering Cross Section (σ_s): Represents the probability of scattering.
2. Absorption Cross Section (σ_a): Represents the probability of absorption.
3. Fission Cross Section (σ_f): Represents the probability of causing fission.
4. Capture Cross Section (σ_c): Represents the probability of being captured without causing fission.

At low or thermal energies (E<1 eV), the absorption cross sections are usually the largest. A total nuclear cross section σ_t can be defined to represent the sum of the probabilities that a neutron will be scattered or absorbed:

$$\boldsymbol{\sigma_t = \sigma_s + \sigma_a = \sigma_s + \sigma_c + \sigma_f} \qquad (1.1)$$

Additionally, a total absorption cross section Σ_a can be defined to represent the probability of either being captured without causing a nucleus to fission (σ_c) or being captured and causing a nucleus to fission (σ_f):

$$\boldsymbol{\Sigma_a = \sigma_c + \sigma_f} \tag{2.1}$$

Nuclear cross sections are fundamental in understanding and quantifying the interactions between subatomic particles and atomic nuclei.

1.4.3 Fissionable Material

A nuclide capable of undergoing fission after capturing either high-energy (fast) neutrons or low-energy thermal (slow) neutrons is known as a fissionable material. While this term was once synonymous with fissile material, fissionable materials include those, such as uranium-238, that can be fissioned only with high-energy neutrons. Consequently, fissile materials (e.g., uranium-235) are a subset of fissionable materials[6].

When discussing nuclear reactors, it is essential to distinguish between two classes of fissionable materials:[7]

1. **Fissile Material**: A nuclide that is capable of undergoing fission after capturing low-energy thermal (slow) neutrons. The three primary fissile materials are uranium-233, uranium-235, and plutonium-239.
2. **Fertile Material:** A material, which is not itself fissile (fissionable by thermal neutrons), that can be converted into a fissile material by irradiation in a reactor. The two basic fertile materials: uranium-238 and thorium-232. When these fertile materials capture neutrons, they are converted into fissile plutonium-239 and uranium-233, respectively.

The following equations illustrate the production of fissile materials from fertile materials[7].

$$^{238}_{92}U + {}^{1}_{0}n \rightarrow {}^{239}_{92}U \rightarrow {}^{239}_{93}Np \rightarrow {}^{239}_{94}Pu$$

$$^{232}_{90}Th + {}^{1}_{0}n \rightarrow {}^{233}_{90}Th \rightarrow {}^{233}_{91}Pa \rightarrow {}^{233}_{92}U$$

Uranium-235 is the only naturally occurring fissile material. Fissile materials can be produced by including the parent fertile material in a reactor core. Since most power reactors are fueled by natural or partially enriched uranium, there is a plentiful supply of uranium-238 in the reactor for conversion to plutonium. By utilizing the conversion process of fertile materials to produce fissile materials, reactors can maintain a sustainable and efficient fuel cycle.

The aim of this study is twofold: first, to review the solution of the diffusion equation in two basic geometries, and second, to examine the variation of the multiplication factor with enrichment for an enriched uranium reactor and its effect on volume using Monte Carlo methods (MCM). Numerical results and graphs will be presented for a hypothetical uranium-graphite assembly. It is anticipated that this study will provide researchers with valuable insights into the effects of the moderator-to-fuel ratio and variations in enrichment on the multiplication factor and minimum volume.

In Chapter 2, we review the solution of the one-group steady-state diffusion equation and present the equations for the buckling parameters and minimum volume for spherical and cylindrical reactors. This chapter describes the relationship between fuel and moderator materials within a reactor and the dimensions of the reactor's shape. In Chapter 3, we detail the parameters and equations used in the Monte Carlo Method (MCM) to calculate the multiplication factor, as well as the radius of the spherical reactor and the radius and height of the cylindrical reactor. We also explore how these parameters are influenced by the enrichment factor and the moderator-to-fuel ratios. Our results are presented in tables and figures for each geometry, and it is evident that they align perfectly with the references. Chapter 4 provides a conclusion.

CHAPTER 2
DIFFUSION THEORY

2.1 Introduction

Diffusion theory is an important concept in the field of nuclear reactor physics. It provides a mathematical framework for analyzing and understanding the behavior of neutrons within a nuclear reactor.

At its core, diffusion theory is based on the principle that the motion of neutrons in a reactor can be approximated as a diffusive process. This approximation is valid when the mean free path of neutrons is much smaller than the size of the reactor core. In other words, it assumes that neutrons are constantly scattering off atomic nuclei and effectively undergo random walks.

In diffusion theory, the neutron flux is described by the diffusion equation, which is a partial differential equation that relates the neutron flux distribution to various parameters such as the neutron diffusion coefficient, the absorption cross section, and the fission cross section. The diffusion equation is derived from the transport equation through simplifications and assumptions, making it more computationally tractable.

Diffusion theory also allows for the analysis of different types of reactors, such as reactors with different geometries, fuel compositions, and neutron moderators. By solving the diffusion equation under specific conditions, one can obtain valuable insights into reactor behavior, such as the flux distribution, fuel utilization, and the impact of control rods. It can also be used to calculate important parameters such as the power distribution, neutron flux, and criticality of the reactor. This information is crucial for reactor design, safety analysis, and optimization of reactor performance.

2.2 Diffusion Equation for Critical Reactor

The diffusion equation describes the spatial distribution of neutron flux within a nuclear reactor. It considers both the neutrons that are being produced by fission reactions and the neutrons that are undergoing absorption or scattering interactions. In chain reaction, neutrons produced when atomic nuclei undergo fission trigger

additional fission reactions in other fissile or fissionable nuclei. The neutrons generated from these subsequent fission reactions, in turn, initiate even more fission reactions in additional fissile or fissionable nuclei, creating a self-sustaining cascade of reactions.

In a critical reactor, the neutron flux remains constant over time, meaning that the rate of neutron production is balanced by the rate of neutron loss [8]. So, the rule for reactor to remain stable and critical is [2]:

The Condition for a Critical Reactor

Neutron production rate (P) ≈ Neutron destruction rate (D)

In other words, the difference between these two numbers determines how a nuclear reactor behaves.

The chain reaction can be quantitatively characterized by a parameter called the multiplication factor, denoted as 'k' The multiplication factor, k, is defined as the ratio of the number of fissions (or fission neutrons) in one generation to the number of fissions (or fission neutrons) in the previous generation [9].

$$k = \frac{number\ of\ fissions\ in\ one\ generation\ (n+1)}{number\ of\ fissions\ \ in\ the\ previous\ generation\ (n)} \qquad (2.1)$$

Controlling the value of the multiplication factor is essential to maintaining reactor stability and preventing the risk of accidents or uncontrolled reactions. k should always be maintained at or close to 1 to ensure reactor stability and control. If 'k' were to exceed 1, it signifies that the number of neutron fissions in the current generation surpasses the number of neutron fissions in the previous generation. This indicates that the reactor has entered a supercritical state, posing a risk of a nuclear explosion. If 'k' is less than 1, it signifies that the number of neutron fissions in the current generation is less than the number of neutron fissions in the previous generation, indicating a subcritical state. Leading to a reactor shutdown or a decrease in reactor power.

In maintaining a reactor at a critical state, it is imperative to achieve equilibrium in regulating the neutron population within the volume. This equilibrium involves

balancing the production rate of neutrons within the reactor, the absorption rate of neutrons in nuclear reactions, and the leakage rate of neutrons escaping from the reactor's surface [10]. Consequently, to ensure a k-value of one, the sum of the absorption rate of neutrons and the escaping rates must equal the production rate of neutrons.

$$k_{\mathrm{c}} = \frac{\text{neutrons productions}}{\text{neutrons absorbtions} + \text{neutrons escaping (leakage)}} = 1 \qquad (2.2)$$

The dynamics of neutron population within a reactor's volume (V) are influenced by various factors. Neutrons are constantly produced, absorbed, and may also escape (leak) from the volume. These processes are inherently dynamic and occur randomly over time. Understanding the rate of change of neutrons within this volume is crucial for predicting and controlling reactor behavior.

One fundamental equation that helps describe this dynamic process is the equation of continuity [9], [11],[12]. This equation provides a formal representation of how neutrons are produced, absorbed, and escape within the reactor volume. By analyzing the terms in this equation, one can gain insights into the factors influencing the neutron population and, consequently, the overall reactivity of the reactor.

$$\left[\begin{matrix}\text{Rate of change in number} \\ \text{of neutrons in V}\end{matrix}\right] = \left[\begin{matrix}\text{Production rate} \\ \text{of neutrons in V}\end{matrix}\right] - \left[\begin{matrix}\text{Absorbtion rate} \\ \text{of neutrons in V}\end{matrix}\right] - \left[\begin{matrix}\text{Leakage rate} \\ \text{of neutrons in V}\end{matrix}\right] \qquad (2.3)$$

where:

$$\text{Rate of change in number of neutrons in V} = \int_{\mathrm{V}} \frac{\partial \mathrm{n}}{\partial \mathrm{t}}\, \mathrm{dv}$$

$$\text{Production rate of neutrons in V} = \int_{\mathrm{v}} \mathrm{S}_n\, \mathrm{dv}$$

$$\text{Absorbtion rate of nutrons in V} = \int_{\mathrm{v}} \Sigma_{\mathrm{a}} \phi\, \mathrm{dv}$$

To explain the leakage rate, Fick's law becomes a valuable tool. Fick's law is rooted in the concept of solute diffusion, as shown in Figure 1.1 where a substance moves from a region of higher concentration to one of lower concentration within a solution. In the

context of reactor volume, envisioning the medium as a solution and the neutrons as the solute material is a reasonable approximation.

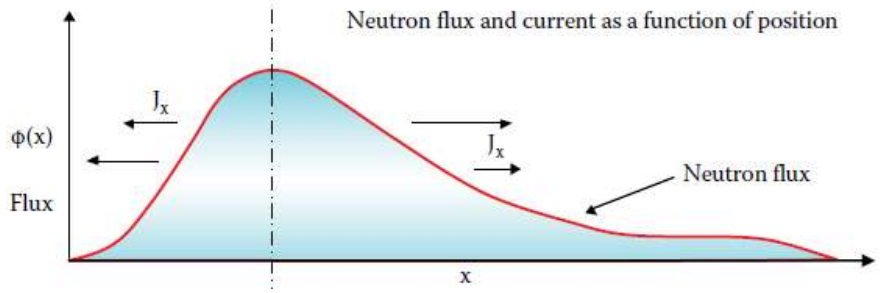

Figure 2.1 Neutron flux and current density as function of position[2].

According to Fick's law, the current density vector of the solute within a solution is defined by a specific relationship

$$\vec{J} = -D\,\vec{\nabla}\phi$$

where $\vec{J}$ repersents the net number of neutrons passing through unit area per unit time. specifically in a direction perpendicular to the x-axis. It is observed that $\vec{J}$ has the same unit as flux $(neutrons/cm^2 - sec)$ and it is a vector quantity. D is defined as the Diffusion coefficient has unit of *cm*. $\vec{\nabla}$ represents the gradient operator applied to three spatial variables.

Therefore, the neutron leakage rate within the volume can be expressed as:

$$\text{Neutron Leakage rate in V} = \int_A \vec{J} \cdot d\vec{A}$$

By employing the divergence theorem, the surface integral can be converted into a volume integral as follows:

$$\text{Leakage rate of neutrons in V} = \int_A \vec{J} \cdot dA = \int_V \text{div}\,\vec{J}\ dV$$

The equation of continuity can now be derived, and get

$$\int_V \frac{\partial n}{\partial t}\, dv = \int_v S_n\, dv - \int_v \Sigma_a \phi\, dv - \int_V \text{div}\,\vec{J}\ dV$$

The integrals we've discussed are all over the same volume, meaning their contents must be equal. This equality should hold for any volume, so when we sum the integrands on the right, they must equal the integrand on the left, thus:[9]

$$\frac{\partial n}{\partial t} = S_n - \Sigma_a \phi - \text{div}\,\vec{J} \qquad (2.4)$$

The Eq. (2.4) represents the general form of the equation of continuity (which is also known as *steady-state equation of continuity*). When the neutron density is independent of time, the equation simplifies to:

$$S_n - \Sigma_a \phi - \operatorname{div} \vec{J} = 0 \tag{2.5}$$

From Fick's law, it is known.

$$\vec{J} = -D \vec{\nabla} \phi$$

and

$$\operatorname{div} \vec{J} = -D \nabla^2 \phi$$

where the flux for the one-group energy of neutrons is

$$\phi = nv$$

Where v is the neutron speed.

Therefore, from the Eq. (2.4)

$$D \nabla^2 \phi - \Sigma_a \phi + S_n = 0 \tag{2.6}$$

The Eq. (2.6) is called the *steady-state diffusion equation*.

In a reactor operating at measurable power, the neutrons emitted from fission events. To determine s, consider Σ_f as the macroscopic fission cross-section for the fuel. Assuming ν neutrons are produced per fission; the source can be expressed as:

$$S_n = \nu \Sigma_f \phi$$

If the fission source doesn't equate to the combined leakage and absorption terms, the right-hand side of Eq. (2.6), is non-zero. To restore balance, the source term is multiplied by a constant $1 / k$, where k is an unknown constant. If the source is insufficient (too small), k is less than 1. Conversely, if it is excessive (too large), k is greater than 1. Thus:

$$D \nabla^2 \phi - \Sigma_a \phi + \frac{1}{k} \nu \Sigma_f \phi = 0 \tag{2.7}$$

rearranging the previous equation, one could obtain

$$B^2 = \frac{1}{D}\left(\frac{1}{k} \Sigma_f - \Sigma_a\right)$$

After that Eq. (2.7) may written in the form:

$$\nabla^2\phi = -B^2\phi \tag{2.8}$$

where B^2 is defined as the *geometric buckling*.

Substituting Eq. (2.8) into Eq. (2.7) for $\nabla^2\phi$, to get

$$-D\,B^2\phi - \Sigma_a\phi + \frac{1}{k}v\,\Sigma_f\,\phi = 0 \tag{2.9}$$

Eq. (2.9) is called the *one group reactor equation*.

Within the reactor volume, neutrons are not solely absorbed by the fuel but also by environmental elements like the moderator. Hence, the source term in the one-group equation should be expressed in its more comprehensive form. If Σ_{aF} represents the macroscopic absorption cross-section for the fuel element in the one-group energy equation, and η denotes the average number of fission neutrons emitted per neutron absorbed by the fuel, then the source term can be written as:

$$S_n = \eta\,\Sigma_{aF}\,\phi \tag{2.10}$$

It could be written also as:

$$S_n = \eta\,\Sigma_a\frac{\Sigma_{aF}}{\Sigma_a}\,\phi = \eta\,f\,\Sigma_a\,\phi \tag{2.11}$$

Where the term Σ_a represents the summation of cross sections for the mixture of fuel and moderator elements, while Σ_{aF} specifically denotes the absorption cross section of the fuel itself.

Hence f is called as the *fuel utilization coefficient* of the system. It is written as

$$f = \frac{\Sigma_{aF}}{\Sigma_a} \tag{2.12}$$

The utilization coefficient for thermal neutrons is commonly known as the thermal utilization factor. Once neutrons undergo thermalization, they continue to diffuse in the reactor while being scattered. Some of these neutrons are absorbed by the moderator and structural components. Thermal utilization factor is the number of these neutrons who's absorbed back into the fuel. It is written as

$$f = \frac{\sigma_{a(F)}}{\sigma_{a(F)}+\sigma_{a(M)}} = \frac{1}{1+\frac{N_M \sigma_{a(M)}}{N_F\, \sigma_{a(F)}}} \tag{2.13}$$

Where N_F is the number of fuel atoms in the mixture,

N_M is the number of moderator atoms in the mixture,

N_M/N_F is what it called moderator to fuel ratio,

$\sigma_{a(F)}$ is the microscopic absorption cross section of fuel mixture,

$\sigma_{a(M)}$ is the microscopic absorption cross section of moderator.

To determine the multiplication factor for an infinite reactor, an infinite reactor is considered. In this configuration, there is no neutron escape. Instead, all neutrons are eventually absorbed, either within the fuel or the coolant.

As all born neutrons must eventually be absorbed within the system, this quantity is equivalent to $\Sigma_a\, \phi$, representing the number of neutrons absorbed per cubic centimeter per second throughout the entire system. Among these absorbed neutrons, a fraction $f\Sigma_a\, \phi$ is absorbed in the fuel, releasing $\eta f\Sigma_a\, \phi$ fission neutrons in the subsequent generation.

Since the multiplication factor is defined as the ratio of the number of fissions in one generation to that in the preceding generation. Given that, in an infinite medium, these quantities are connected to the absorption rate, it can be concluded that:

$$k_\infty = \frac{\eta\, f\, \Sigma_a\, \phi}{\Sigma_a\, \phi} = \eta f \tag{2.14}$$

This result holds true only for an infinite reactor volume.

Where η represents the number of fast neutrons generated per fission event. Each time a thermal neutron is absorbed in the fuel, a certain number of fast neutrons, denoted by η, are produced. This value is less than the average of 2.42 neutrons produced in a fission reaction because not all absorbed neutrons in the fuel lead to fission events.

Considering pure uranium U-235 for more detailed explanation η is written as:

$$\eta = \nu \, x \, [\frac{\sigma_f}{\sigma_f + \sigma_c}] = 2.06 \tag{2.15}$$

For natural uranium mixture (U-235 and U-238) is it written as:

$$\eta = \nu \, x \, [\frac{0.72\sigma_f(235)}{0.72\sigma_a(235) + 99.28\sigma_c(238)}] = 1.328 \tag{2.16}$$

where ν = 2.42 is the average number of neutrons produced in fission reaction,

σ_f represent neutron fission cross section,

σ_c represent neutron capture cross section,

σ_a represent neutron absorption cross section.

The Eq. (2.15) holds true only for an infinite reactor volume.

In aim to analyze the finite reactor volume. **four factor formula: $k_\infty = \varepsilon \, p \, \eta \, f$** should be considered. It was developed to estimate the value of the infinite multiplication factor in reactor cores where neutron leakage is negligible. Furthermore, it is entirely independent of the size and shape of the reactor, offering a description of the inherent multiplication ability of both fuel and moderator.

From this stage one should consider two more terms to achieve the complete understanding of the four-factor formula the fast fission factor and the resonance escape probability.

Fast fission factor represented as ' ε '. It accounts for the fact that some fast neutrons induce fission before they slow down. This increases the neutron population, known as the fast fission factor. In this study, it will assume it to be 1, indicating that neutrons interact with the moderator and lose energy before causing fission.

In case of Uranium fuel mixture ' p ' represents the Resonance escape probability. It is the probability that a neutron will avoid resonance capture by uranium-238. It depends on the rate at which neutrons are slowed down and on the number of uranium-238 atoms, denoted as N_{238}, relative to moderator atoms.

$$p = \exp\left[-\frac{2.73}{\langle\xi\rangle}\left\{\frac{N_{238}}{\Sigma_s}\right\}^{0.514}\right] \tag{2.17}$$

where Σ_s is total macroscopic scattering cross section.

ξ is the logarithmic energy decrement for a single substance.

It is important to note that the four factor formula is a simplified representation of reactor physics, and actual reactor behavior is influenced by various other factors and phenomena. Nonetheless, the formula provides a useful starting point to analyze and understand reactor criticality and stability.

After analyzing the multiplication factor, substituting it in Eq. (2.11) gives:

$$\mathrm{S}_n = \frac{k_\infty}{p}\,\Sigma_\mathrm{a}\,\phi_T \tag{2.18}$$

Replacing Eq. (2.14) and Eq. (2.8) into Eq. (2.7) one gets

$$-\,\mathrm{D}\,\mathrm{B}^2\phi - \,\Sigma_1\phi + \frac{k_\infty}{p}\Sigma_\mathrm{a}\phi_T = 0 \tag{2.19}$$

Considering the resonance absorption, only $p\,\Sigma_1\,\phi_1$ neutrons per cubic centimeter per second of the fast group manage to enter the thermal group. Consequently, the thermal source term becomes:

$$s_T = p\,\Sigma_1\,\phi \tag{2.20}$$

Replacing Eq. (2.16) into Eq. (2.15) gives

$$-\,\mathrm{D}\,\mathrm{B}^2\phi - \,\Sigma_\mathrm{a}\phi + p\,\Sigma_1\,\phi = 0 \tag{2.21}$$

Using linear algebraic equations ones get

$$\frac{k_\infty}{(1+B^2L_T^2)(1+B^2\tau_T)} = 1 \tag{2.22}$$

If the reactor system is in critical condition, one defines the parameter, called as *diffusion area*, by

$$L_T^2 = \frac{\mathrm{D}}{\Sigma_\mathrm{a}}$$

And the parameter called *neutron age* as:

$$\tau_T = \frac{\mathrm{D}}{\Sigma_1}$$

Rearrange the Eq. (2.22) ones get:

$$\frac{k_\infty}{(1+B^2M_T^2)} = 1 \tag{2.23}$$

Where *thermal migration area* is

$$M_T^2 = L_T^2 + \tau_T$$

Then Eq. (2.21) can now be written as

$$-\mathrm{B}^2\phi + \left(\frac{k_\infty - 1}{M_T^2}\right)\phi = 0 \tag{2.24}$$

In this context, one addresses the material properties of the reactor system, including factors such as the type of fuel and moderator. Eq. (2.24) introduces a revised definition of the buckling parameter for a critical reactor, encapsulating these material characteristics.

$$\mathrm{B}^2 = \frac{k_\infty - 1}{M_T^2} = \mathrm{B}_g^2 = \mathrm{B}_m^2 \tag{2.25}$$

Here B_g is referred to as the geometrical buckling, while and B_m is termed the material buckling.

2.2.1 Theoretical Calculation of Multiplication Factor

In this study, a hypothesis system was built to theoretically calculate the multiplication factor. Using computer code, the following system specifications were incorporated to determine η, p, f and k_∞

- Type of reactor: **Infinite Thermal reactor**
- Fuel material: **Uranium mixture (U-235 and U-238)**
- Moderator: **Graphite**
- Enrichment: **Varying enrichment percentages between 0.72% to 5%**
- Moderator to Fuel ratio: **Varying between 100 to 1000**

Noting that in this study, neutrons are considered as thermal neutrons with a speed of 2200 m/s, a temperature of 293.6 K, and an energy of 0.0253 eV. Consequently, all related parameters, including cross sections, absorption coefficients, etc., are assumed to be in the thermal context.

To calculate the k_∞ for the system, additional data is required. The uranium data necessary for the calculation is provided in Table 2.1.

Table 2.1 Uranium data for calculation

Nucleus	**Density**	$\boldsymbol{\sigma_f}$	$\boldsymbol{\sigma_c}$	$\boldsymbol{\sigma_a}$	$\boldsymbol{\nu}$
U-235	18.7	579	101	680	2.42
U-238	18.9	-	2.72	2.72	-
U-nat	18.9	4.17	3.43	7.60	-

Consider the fact that neutrons primarily undergo successive scattering collisions with nuclei to slow down. This process is most efficiently achieved when the scattering nuclei are light. Three commonly utilized materials for this purpose include water, heavy water, and graphite, with graphite being acknowledged as the ideal moderator[8]. However relevant properties for graphite, which is used in this system, are listed in Table 2.2.

Table 2.2 Graphite moderator properties

Material	**M.Wt**	**Density (g/cm^3)**	σ_s	σ_a	ξ	**SPD (m^{-1})**
Graphite	12.01	1.6	4.7	0.0045	0.158	6.0

To enhance the design, the code incorporates equations Eq. (2.13), Eq. (2.15), Eq. (2.16), and Eq. (2.17) for calculating k_∞. considering a specified U-235 enrichment of 0.72% and 1.6% which gives from Eq. (2.16) the values for η =1.328 and η = 1.653 respectively. Table 2.3 presents the results obtained for varying moderator to fuel ratios (N_M/N_F) between 100 and 1000.

Table 2.3 Assembly of Uranium-Graphite with enrichment of 0.72% and 1.6%

	U-235: 0.72 %, η = 1.328			**U-235: 1.6 %, η = 1.654**		
N_M/N_F	f	p	k_∞	f	P	k_∞
100	0.94407	0.48579	0.60908	0.96787	0.48740	0.78014
200	0.89407	0.60179	0.71455	0.93774	0.60318	0.93540
300	0.84910	0.66171	0.74618	0.90943	0.66295	0.99706
400	0.80844	0.70016	0.75173	0.88278	0.70130	1.02383
500	0.77149	0.72763	0.74552	0.85765	0.72868	1.03352
600	0.73777	0.74856	0.73344	0.83391	0.74955	1.03368
700	0.70688	0.76521	0.71836	0.81145	0.76614	1.02811
800	0.67847	0.77888	0.70181	0.79016	0.77977	1.01895
900	0.65225	0.79038	0.68465	0.76997	0.79122	1.00750
1000	0.62799	0.80022	0.66739	0.75078	0.80103	0.99456

The data obtained from the optimized system demonstrates compatibility with J.S. Lilley's reference, accurately matching two decimal places.

Table 2.4 shows Lilley's results which are in agreement with Table 1.3 representing our ones in this study.

Table 2.4 J.S. Lilley's, Uranium-Graphite assembly with enrichment of 0.72%

	U-235: 0.72 %, η = 1.328			**U-235: 1.6 %, η = 1.654**		
N_M/N_F	***F***	***P***	k_∞	***f***	***P***	k_∞
100	0.944	0.480	0.602	0.968	0.482	0.771
200	0.894	0.599	0.712	0.938	0.601	0.931
300	0.849	0.660	0.744	0.909	0.661	0.995
400	0.808	0.699	0.751	0.883	0.700	1.022
500	0.771	0.727	0.745	0.858	0.728	1.032
600	0.738	0.748	0.733	0.834	0.749	1.033
700	0.707	0.765	0.718	0.811	0.766	1.027
800	0.678	0.778	0.701	0.790	0.779	1.018
900	0.652	0.790	0.684	0.770	0.791	1.007
1000	0.628	0.800	0.667	0.751	0.801	0.994

This alignment provides reliable support for the subsequent calculations of buckling and geometric parameters in the reactor study. It is understood from the table above that, for natural uranium, the reactor remains subcritical within the N_M/N_F ratios ranging between 100 and 1000. Increasing the U-235 content will lead to achieving criticality at lower N_M/N_F values.

2.3 Calculation of Geometric Buckling for Some Reactor Shapes

Geometric buckling refers to the spatial distribution of the neutron flux in a nuclear reactor due to its geometry. It is a measure of how the neutron flux changes as a function of position within the reactor. Geometric buckling is an important parameter in reactor design and safety analysis. It helps to characterize the behavior of the neutron flux as it interacts with the reactor materials and influences the overall reactor performance. By understanding and analyzing the geometric buckling, engineers and physicists can optimize the design and operation of nuclear reactors, in this study detailed explanation about spherical and finite- cylindrical reactor is represented.

2.3.1 Spherical Reactor

To construct the reactor equation, first considering a spherical reactor of radius R. where the flux is a function of r only. As illustrated in Figure 2.2.

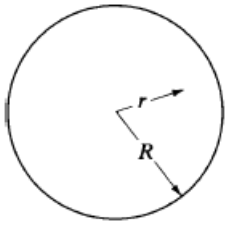

Figure 2.2 The Spherical Reactor

The reactor equation driven from the Laplacian spherical coordinate is now.

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d\phi}{dr} + B^2\phi = 0 \tag{2.26}$$

The flux is required to meet the boundary condition $\phi(R) = 0$ and maintain to stay finite throughout the entire interior of the reactor. Here we assume that the extrapolation distance is too small. Therefore, d is omitted.

The general solution for Eq. (2.26) is found to be

$$\phi = A\frac{\sin B\,R}{r} + C\frac{\cos B\,R}{r} \tag{2.27}$$

Here A and C are constants. When r reaches zero, the second term becomes infinite. Since the shape is finite, the constant C must be set equal to zero. Here Eq. (2.27) become.

$$\phi = A\frac{\sin B\,R}{r} \tag{2.28}$$

Applying the boundary condition at $\phi(R) = 0$, the ones could get the geometrical buckling as

$$B^2 = (\frac{\pi}{R})^2 \tag{2.29}$$

The buckling is function of R only.

The critical radius is then given by

$$R = \frac{\pi}{B} \tag{2.30}$$

And the critical volume V

$$V = \frac{4\pi}{3}R^3 = \frac{4\pi^4}{3B^3} = \frac{130}{B^3} \tag{2.31}$$

2.3.1.1 Theoretical Calculation

The calculation procedure for the multiplication factor, as demonstrated earlier in this study, is closely tied to material properties. In this section, the same calculations as in Section 1.2.1 will be performed. However, additional focus will be given to determining the critical volume of the sphere and assessing geometric buckling.

For natural uranium enrichment of 0.72%, as specified in Table 1.3, the system is demonstrated to be subcritical within the range of 100-1000 N_M/N_F. According to Eq. (2.25), if $k_\infty$is smaller than unity, it will lead to k_∞-1 yielding negative values and no real roots for B^2, resulting in the inability to calculate the radius (R) or volume (V) of this system.

For 1.6% enriched uranium with a N_M/N_F ratio of 600, we can use Eq. (2.25), Eq. (2.29), Eq. (2.30), and Eq. (2.31) to calculate the multiplication factor, buckling, radius, and volume, respectively. The results are represented in Table 2.5:

Table 2.5 Calculation of 1.6% enriched uranium with a N_M/N_F ratio of 600

	U-235: 1.6 %, η = 1.654			
N_M/N_F	**k_∞**	**B (cm^2)**	**R (m)**	**V (m^3)**
600	1.03	$4.1x10^{-5}$	4.86	482.6

The results align with J.S. Lilley's findings, which indicate a radius of approximately 4.92 m for the same enrichment value and moderator-to-fuel ratio. This strong correlation supports further estimations in the study with different enrichments or geometric shapes. Optimizing the same system for enrichment of 1.5 % and 2.5 % with the same varying moderator to fuel ratio. The results are given in Table 2.6.

Table 2.6 Assembly of Uranium-Graphite with enrichment of 1.5% and 2.5%

	U-235: 1.5 %, η = 1.63191			U-235: 2.5 %, η = 1.78249		
N_M/N_F	**k_∞**	**R (m)**	**V (m^3)**	**k_∞**	**R (m)**	**V (m^3)**
100	0.76826	Nan	Nan	0.85223	nan	nan
200	0.91981	Nan	Nan	1.03054	3.95	259.11
300	0.97903	Nan	Nan	1.1079	2.21	45.77
400	1.00394	13.17	9575.12	1.14709	1.99	33.02
500	1.01214	7.87	2043.69	1.16719	1.94	30.77
600	1.01106	8.59	2658.53	1.17627	1.96	31.67
700	1.00444	14.05	11618.96	1.17847	2.01	34.26
800	0.9944	Nan	nan	1.17612	2.08	38.15
900	0.9822	Nan	nan	1.17066	2.17	43.32
1000	0.96863	Nan	nan	1.16302	2.28	49.91

In Table 2.6, the data illustrate two main findings. Firstly, considering the multiplication factor (Blue font color in Table 2.6) for both 1.5% and 2.5% enriched uranium, criticality is achieved at lower N_M/N_F values for higher enrichment levels: 400 for 1.5% and 200 for 2.5% enriched uranium, respectively.

Secondly, examining the minimum critical volume (Red font color in Table 2.6) for an enrichment level of 1.5%, the critical volume is 2043.69 m³, attained at an N_M/N_F ratio of 500, resulting in a k_∞ of 1.01214 and a corresponding radius of 7.87 m. For an enrichment level of 2.5%, the critical volume condition is 30.77 m³, also at an N_M/N_F ratio of 500, yielding a $k_\infty$of 1.16719 and a corresponding radius of 1.94 m. Comparing the two systems, as the enrichment increases, the corresponding critical volume values decrease.

Figure 2.3 and Figure 2.4 illustrate the relationship between the multiplication factor and the N_M/N_F ratio. In Figure 2.3, where enrichment is 1.5%, criticality is achieved at an N_M/N_F ratio near 400. In Figure 2.4, with an enrichment of 2.5%, criticality is achieved at a lower N_M/N_F ratio of approximately 200. In both graphs, k_∞reaches its maximum and then decreases.

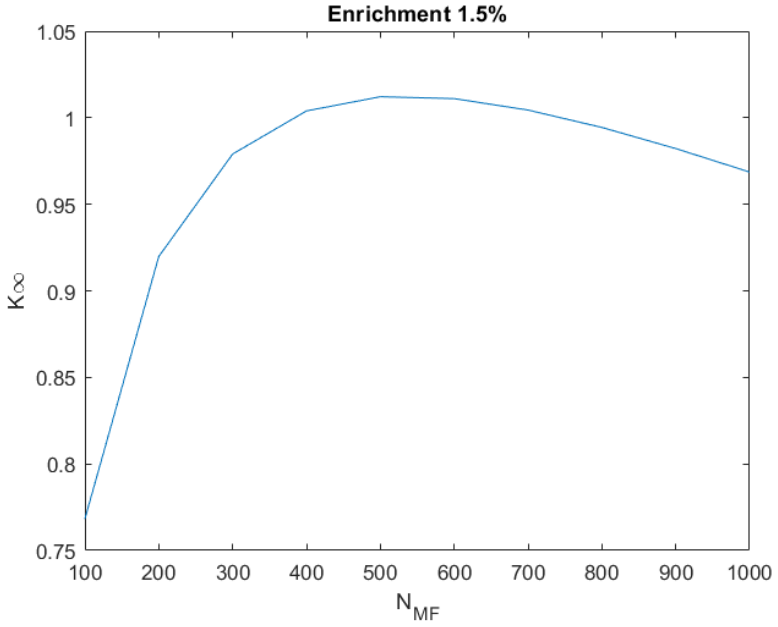

Figure 2.3 k_∞VS. N_{MF} with enrichment of 1.5%.

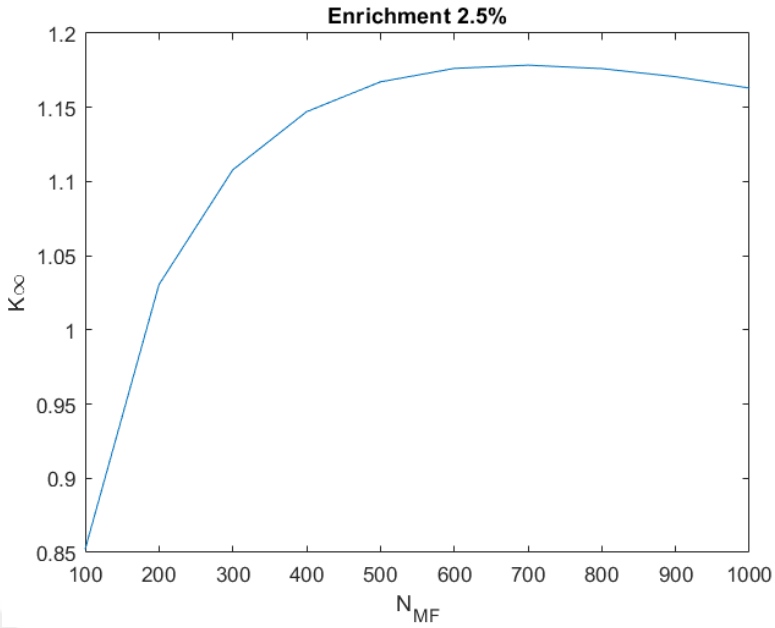

Figure 2.4 k_∞ VS. N_{MF} with enrichment of 2.5%.

Further calculations have been performed on the same system with a 5% enrichment level to estimate the corresponding k_∞ and volume. The results are presented in Table 2.7.

From Table 2.7, the multiplication factor is close to unity within the examined range of 100-1000 N_M/N_F, with a step of 100, and is yielded at an N_M/N_F ratio of 200 (Blue font color in Table 2.7). This emphasizes the observations made in Table 2.6, That higher enrichment % values achieve criticality at lower levels.

Considering the critical minimum volume is 8.31 m^3, this is attained at an N_M/N_F ratio of 600. This condition corresponds to a k_∞ of 1.34367 and a radius of 1.25 m. (Red font color in Table 2.7)

This is consistent with the observations in Table 2.6, where the minimum critical volume is not achieved at the critical multiplication factor.

These findings From Table 2.6 and Table 2.7 highlight the relationship between enrichment levels and critical volumes, demonstrating that higher enrichment levels lead to significantly smaller critical volumes.

Table 2.7 Assembly of Uranium-Graphite with enrichment of 5%

U-235: 5 %, η = 1.91502			
N_M/N_F	**k_∞**	**R (m)**	**V (m^3)**
100	0.93403	nan	nan
200	1.13765	1.75	22.80
300	1.23345	1.39	11.44
400	1.2882	1.29	9.16
500	1.32208	1.26	8.46
600	1.34367	1.25	8.31
700	1.35734	1.26	8.45
800	1.36557	1.27	8.75
900	1.36993	1.29	9.17
1000	1.37142	1.32	9.69

From Table 2.6 and Table 2.7, the volume column, indicated as 'V (m^3)', shows a U-shaped distribution where the volume decreases to a minimum value and then starts to increase.

This follows an opposite pattern to the distribution of the multiplication factor, which increases to a maximum value and then decreases. Notably, for 5% enriched uranium, the decrease in the multiplication factor is not shown in the data (Table 2.7) due to the limitation of N_M/N_F values between 100 and 1000. The decrease starts at an N_M/N_F ratio of 1100, where k_∞ equals 1.37074.

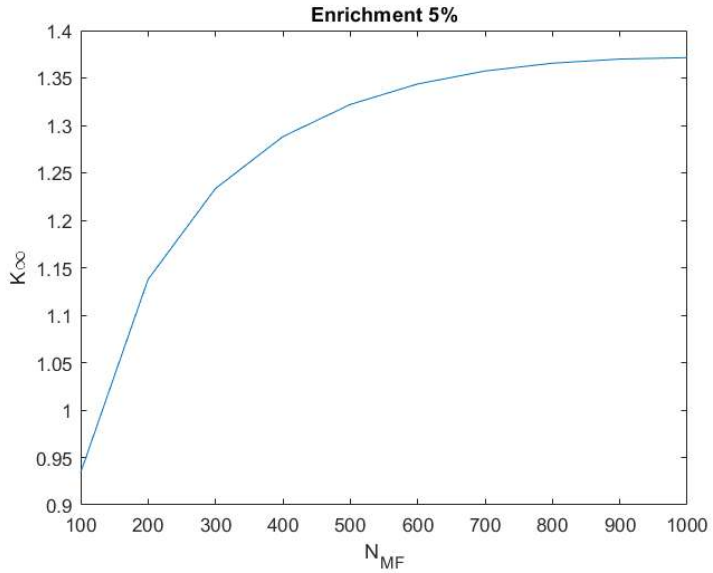

Figure 2.5 k_∞ VS. N_{MF} with enrichment of 5 %.

In Figure 2.5 where enrichment is 5%, criticality is achieved at an N_M/N_F ratio near 125. This emphasizes the idea that increasing the enrichment decreases the N_M/N_F ratio required to achieve criticality. Figure 2.6 shows a comparison between reactors with enrichments of 1.5%, 2.5%, and 5% for the same N_M/N_F ratio range. It clearly illustrates the relationship between enrichment and N_M/N_F ratio and their effect on the multiplication factor.

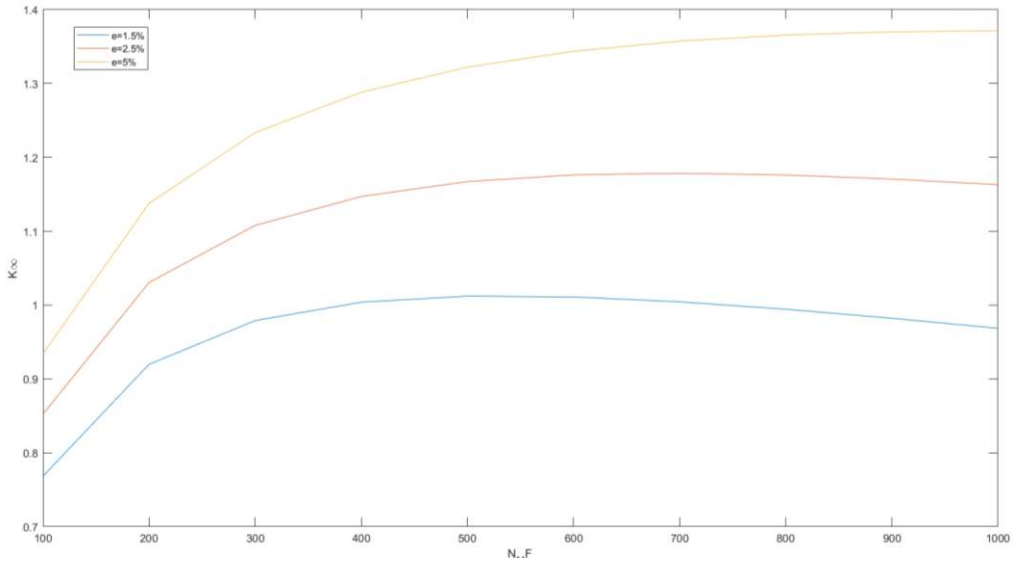

Figure 2.6 Comparison between 1.5%, 2.5% and 5% enrichments.

2.3.2 Finite - Cylindrical Reactor

To construct the reactor equation, first considering a finite cylindrical reactor of radius R and height H. where the flux is a function of r and z. As illustrated in Figure 2.7.

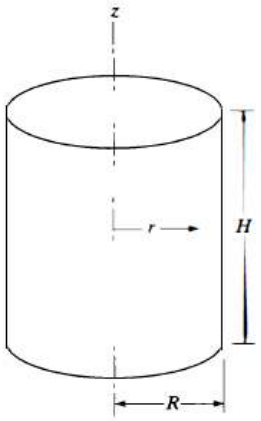

Figure 2.7 The finite cylindrical reactor.

The reactor equation driven from the Laplacian spherical coordinate is now.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right)+\frac{d^2\phi}{dz^2}+B^2\phi=0 \tag{2.32}$$

In addition to satisfying this equation, ϕ must also satisfy to the standard boundary conditions, which include $\phi(R, z) = 0$ and $\phi(r, H/2) = 0$. The solution is derived by the assumption of separation of variables. Where extrapolation distance (d) was omitted.

$$\phi(r, z) = R(r)\, Z(z) \tag{2.33}$$

One could obtain

$$\frac{1}{R}\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+\frac{1}{Z}\frac{d^2Z}{dz^2}=B^2 \tag{2.34}$$

Subsequently, the solution is obtained

$$\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}=B_r^2R$$
$$\frac{d^2Z}{dz^2}=B_z^2Z \tag{2.35}$$

where the total buckling B^2 is

$$B^2 = B_r^2 + B_z^2 \tag{2.36}$$

The flux function within the reactor volume can be determined by solving Eq. (2.35) subject to the boundary condition that each function must vanish at the boundaries.

$$B_r^2 = \left(\frac{2.405}{R}\right)^2$$
$$B_z^2 = \left(\frac{\pi}{H}\right)^2 \tag{2.37}$$

For each axis, the final result for the geometric buckling is provided as

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 \tag{2.38}$$

In aim of finding the minimum critical volume for the cylinder. First minimizing its volume to $V = \pi R^2 H$ by taking its derivative and assign it to zero. Ones then get

$$B^2 = \frac{3\pi^2}{H^2} \tag{2.39}$$

And the minimum critical volume V is

$$V_{min} = \frac{148}{B^3} \tag{2.40}$$

2.3.2.1 Theoretical Calculation

Repeating the calculations for a cylindrical shape, as was done for a spherical reactor, while considering that the cylindrical reactor is defined by two parameters: height and radius. We assumed h values in a specific range between 1 and 300 m. From there, one could calculate the corresponding R and search for the minimum volume that satisfies the Eq. (2.40).

To avoid repetition and present the data efficiently, we will focus on the minimum critical volume for specific multiplication factors near unity for each uranium enrichment level.

- **1.5% Enrichment:** Based on Table 2.6, k_∞ approaches unity between 400 and 700 N_M/N_F. Therefore, we will analyze the minimum critical volume for k_∞ values of 400, 500, 600, and 700.
- **2.5% Enrichment:** Table 2.6 indicates k_∞ reaches unity at 200 N_M/N_F. We will solely investigate the minimum critical volume at this k_∞ value.
- **5% Enrichment:** As shown in Table 2.7, criticality occurs between 100 and 200 N_M/N_F. Figure 2.5 reveals k_∞ reaching unity near 125 N_M/N_F. We will examine the minimum critical volume within this range to identify the k_∞ value closest to one and its corresponding minimum volume.

Tables 2.8 through 2.11 present the minimum critical volume for 1.5% enriched uranium at different N_M/N_F ratios:

- N_M/N_F = 400 (Table 2.8): V = 13491.8 m³ (H/R = 1.85)
- N_M/N_F = 500 (Table 2.9): V = 2937.901 m³ (H/R = 1.84)
- N_M/N_F = 600 (Table 2.10): V = 3881.943 m³ (H/R = 1.85)
- N_M/N_F = 700 (Table 2.11): V = 17179.31 m³ (H/R = 1.84)

Noting that in all four cases, the minimum critical volume is achieved at an H/R ratio around 1.8, which aligns with the equation for critical minimum volume in cylindrical geometry.

Table 2.8 Uranium-Graphite assembly with enrichment 1.5% and N_M/N_F =400

U-235: 1.5 %, η = 1.63191, N_M/N_F =400, k_∞ = 1.00394			
R (m)	**H(m)**	**V (m^3)**	**H/R**
13.5037	23.6	13519.81	1.74766
13.4720	23.7	13513.52	1.75919
13.4410	23.8	13508.09	1.77069
13.4106	23.9	13503.47	1.78217
13.3807	24	13499.66	1.79362
13.3514	24.1	13496.62	1.80504
13.3227	24.2	13494.34	1.81644
13.2945	24.3	13492.79	1.82782
13.2668	24.4	13491.95	1.83917
13.2396	24.5	13491.8	1.8505
13.2129	24.6	13492.32	1.86181
13.1867	24.7	13493.49	1.87309
13.1610	24.8	13495.29	1.88435
13.1357	24.9	13497.71	1.89559
13.1109	25	13500.73	1.9068
13.0865	25.1	13504.34	1.918
13.0625	25.2	13508.51	1.92918
13.0390	25.3	13513.24	1.94033

Table 2.9 Uranium-Graphite assembly with enrichment 1.5% and N_M/N_F =500

U-235: 1.5 %, η = 1.63191, N_M/N_F =500, k_∞ = 1.01214			
R (m)	**H(m)**	**V (m^3)**	**H/R**
8.22437	13.9	2953.727	1.6901
8.18962	14	2949.893	1.70948
8.15604	14.1	2946.645	1.72878
8.12355	14.2	2943.953	1.748
8.09212	14.3	2941.789	1.76715
8.06170	14.4	2940.125	1.78622
8.03223	14.5	2938.938	1.80523
8.00367	14.6	2938.204	1.82416
7.97599	14.7	2937.901	1.84303
7.94915	14.8	2938.009	1.86183
7.92310	14.9	2938.51	1.88058
7.89782	15	2939.385	1.89926
7.87328	15.1	2940.617	1.91788
7.84944	15.2	2942.192	1.93644
7.82627	15.3	2944.093	1.95495
7.80375	15.4	2946.307	1.97341
7.78185	15.5	2948.821	1.99181
7.76056	15.6	2951.622	2.01016

Table 2.10 Uranium-Graphite assembly with enrichment 1.5% and N_M/N_F =600

U-235: 1.5 %, η = 1.63191, N_M/N_F=600, k_∞ = 1.01106			
R (m)	**H(m)**	**V (m^3)**	**H/R**
8.97398	15.4	3896.202	1.71607
8.94073	15.5	3892.493	1.73364
8.90847	15.6	3889.387	1.75114
8.87716	15.7	3886.854	1.76858
8.84677	15.8	3884.87	1.78596
8.81724	15.9	3883.408	1.80328
8.78856	16	3882.445	1.82055
8.76067	16.1	3881.961	1.83776
8.73356	16.2	3881.934	1.85491
8.70719	16.3	3882.345	1.87201
8.68153	16.4	3883.175	1.88907
8.65656	16.5	3884.407	1.90607
8.63224	16.6	3886.026	1.92302
8.60856	16.7	3888.014	1.93993
8.58549	16.8	3890.359	1.95679
8.56301	16.9	3893.045	1.97361
8.54109	17	3896.061	1.99038
8.51972	17.1	3899.392	2.00711

Table 2.11 Uranium-Graphite assembly with enrichment 1.5% and N_M/N_F=700

U-235: 1.5 %, η = 1.63191, N_M/N_F=700, k_∞ = 1.00444			
R (m)	**H(m)**	**V (m^3)**	**H/R**
14.5982	25.7	17206.15	1.76049
14.5672	25.8	17199.83	1.7711
14.5368	25.9	17194.41	1.78168
14.5068	26	17189.86	1.79225
14.4775	26.1	17186.15	1.8028
14.4486	26.2	17183.26	1.81332
14.4202	26.3	17181.17	1.82382
14.3923	26.4	17179.86	1.8343
14.3649	26.5	17179.31	1.84476
14.3380	26.6	17179.49	1.85521
14.3115	26.7	17180.39	1.86563
14.2854	26.8	17181.99	1.87603
14.2598	26.9	17184.28	1.88642
14.2346	27	17187.22	1.89678
14.2098	27.1	17190.82	1.90713
14.1854	27.2	17195.05	1.91746
14.1614	27.3	17199.89	1.92777
14.1377	27.4	17205.34	1.93807

For a 1.5% enrichment, criticality is attained at N_M/N_F values of 400, 500, 600, and 700. Figure 2.8 and Figure 2.9 illustrate the relationship among the three primary parameters: height, radius, and volume.

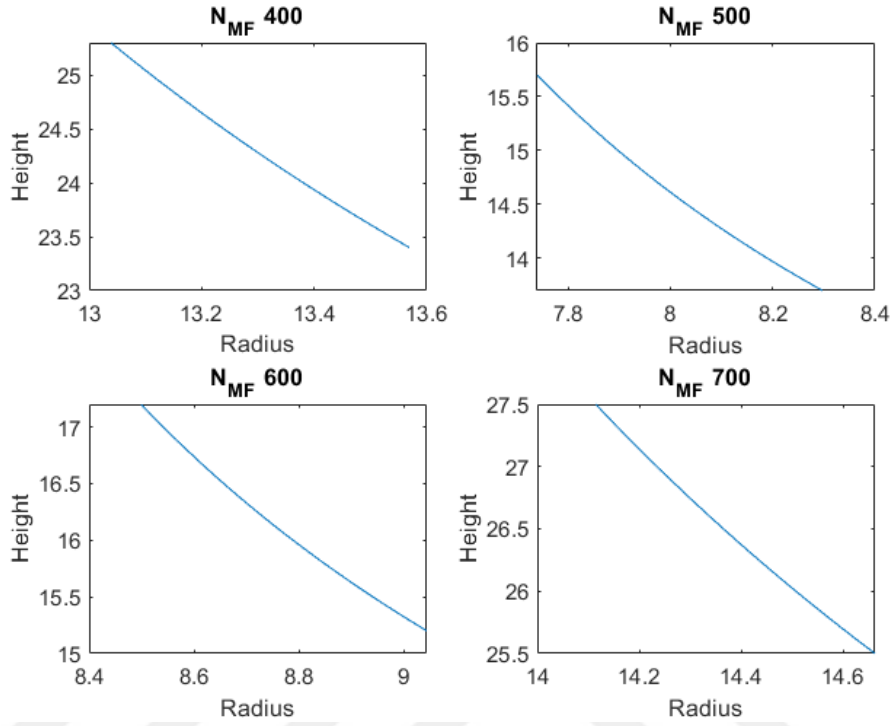

Figure 2.8 Relation between radius and height for enrichment value of 1.5 %.

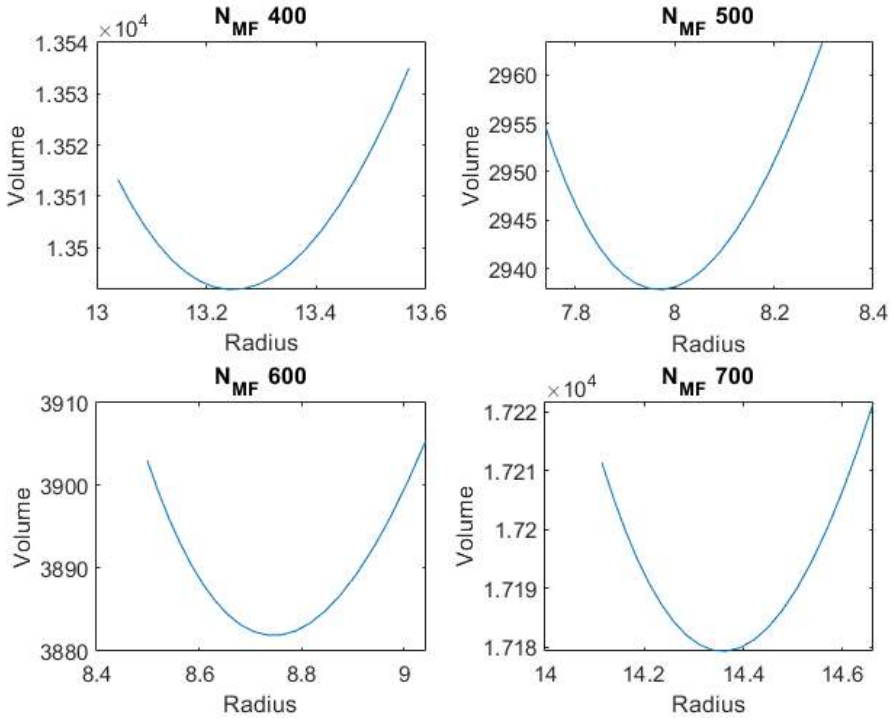

Figure 2.9 Relation between radius and volume for enrichment value of 1.5 %

In Figure 2.9, the relationship between volume and radius for an enrichment value of 1.5% is depicted. At certain radius values, the reactor volume reaches its minimum critical volume.

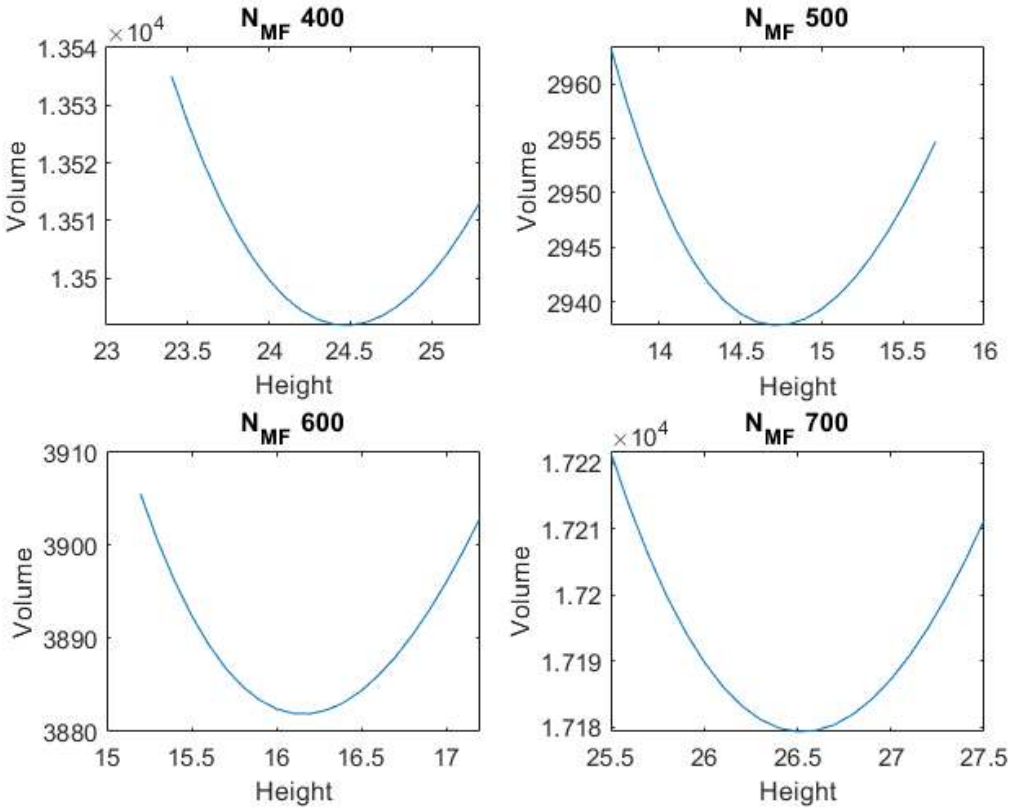

Figure 2.10 Relation between height and volume for enrichment value of 1.5 %

Figure 2.10 illustrates the correlation between volume and height for an enrichment value of 1.5%. At specific height values, the reactor volume attains its minimum critical volume.

After studying the system for 1.5% enriched uranium, the system will be examined for 2.5% enrichment. For this enrichment level, k_∞ is closest to unity only when N_M/N_F equal 200.

Table 2.12 provides a deeper understanding of the relationship between the three parameters. The minimum volume is achieved at an H/R ratio of 1.8442.

Table 2.12 Uranium-Graphite assembly with enrichment 2.5% and N_M/N_F =200

U-235: 2.5 %, η = 1.78249, N_M/N_F=200, k_∞ = 1.03054			
R (m)	**H(m)**	**V (m^3)**	**H/R**
4.14113	6.3	339.413	1.52132
4.09459	6.4	337.0946	1.56304
4.05162	6.5	335.2132	1.6043
4.01182	6.6	333.7166	1.64514
3.97487	6.7	332.5608	1.68559
3.94047	6.8	331.7082	1.72568
3.90838	6.9	331.1266	1.76543
3.87838	7	330.7883	1.80487
3.85028	7.1	330.6691	1.84402
3.82390	7.2	330.7481	1.88289
3.79911	7.3	331.0069	1.9215
3.77576	7.4	331.4294	1.95987
3.75374	7.5	332.0014	1.99801
3.73294	7.6	332.7102	2.03593
3.71327	7.7	333.5447	2.07364
3.69464	7.8	334.4948	2.11117
3.67697	7.9	335.5515	2.1485
3.66020	8	336.7068	2.18567

For 2.5 % enrichment, criticality is achieved at N_M/N_F =200, k_∞ = 1.03054. Figure 2.11 shows the relationship between radius and height. As observed previously, increasing the radius leads to a decrease in height to achieve the minimum volume.

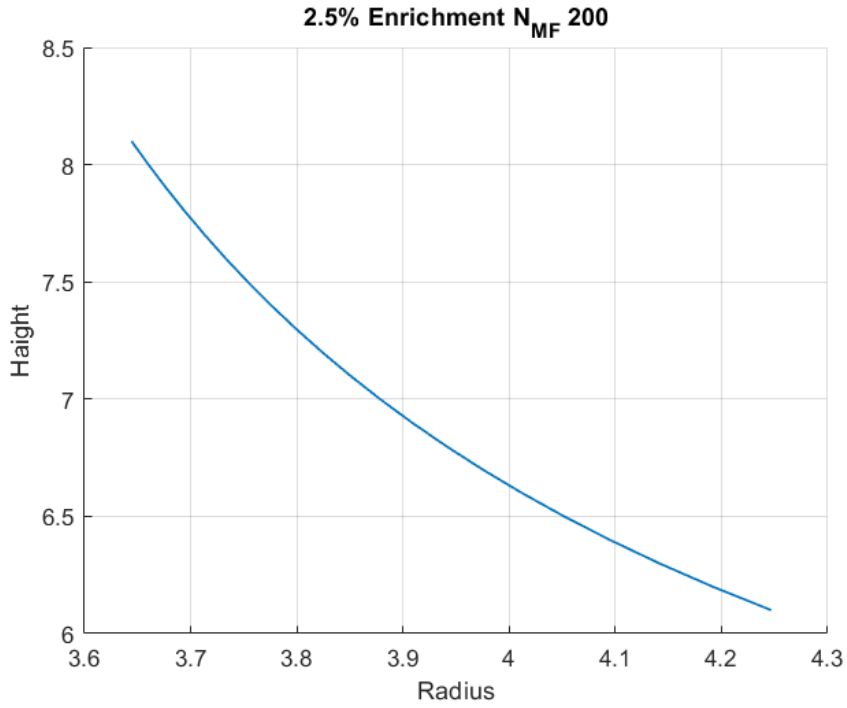

Figure 2.11 Relation between radius and height for enrichment value of 2.5 %

Figures 2.12 and 2.13 illustrate the effects of radius and height on volume, respectively. Figure 2.12 demonstrates how varying the radius influences the volume, while Figure 2.13 shows the impact of height changes on volume. As shown previously, the volume reaches its minimum at specific values of radius and height. These figures help to clearly understand the interplay between these parameters in achieving the minimum critical volume.

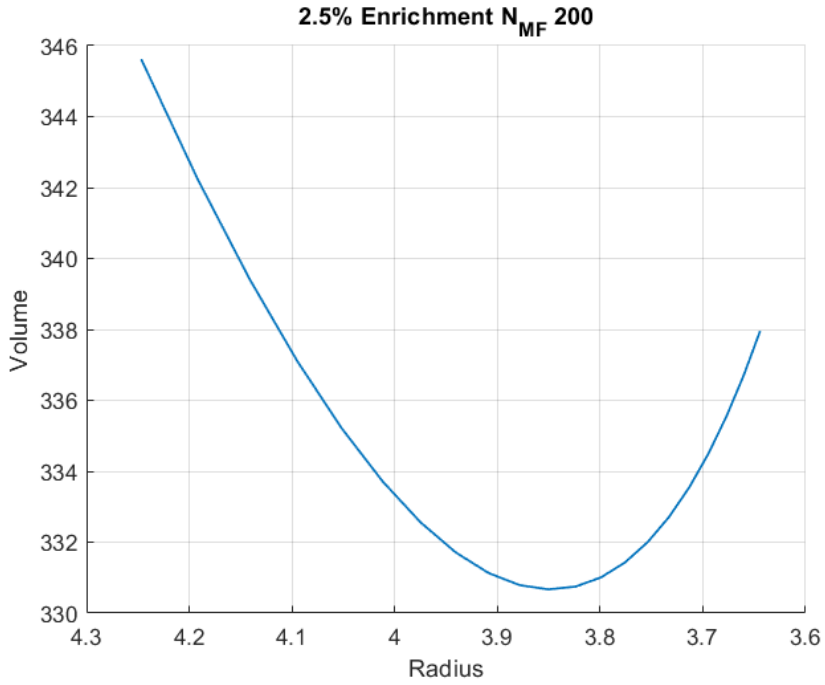

Figure 2.12 Relation between radius and volume for enrichment value of 2.5 %

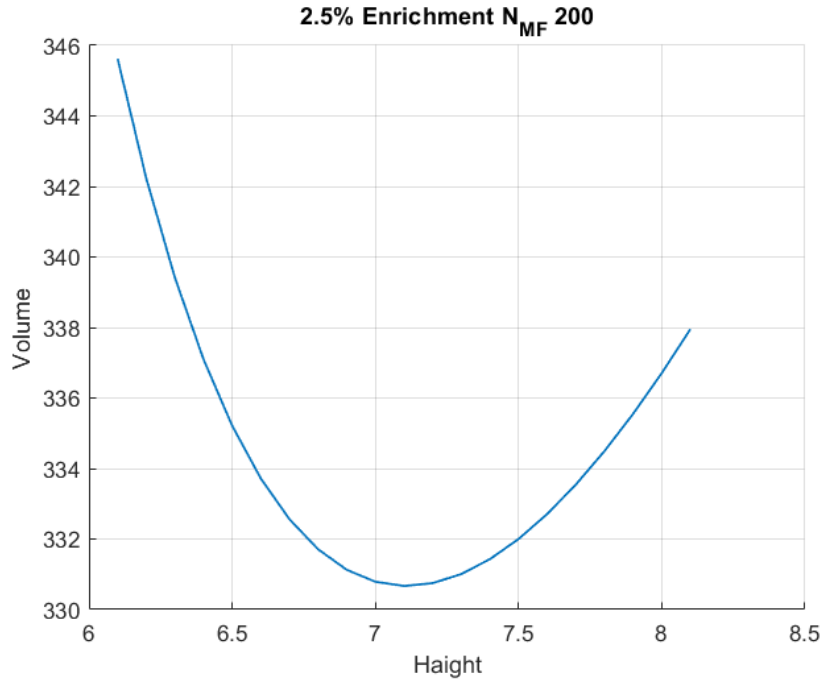

Figure 2.13 Relation between height and volume for enrichment value of 2.5 %

It is observed that the minimum volume is attained around the H/R ratio of 1.8 for both 1.5% and 2.5% enrichment values. To further investigate this relationship, additional calculations were conducted with a 5% enrichment. An additional calculation was carried out to validate this relationship, focusing on a 5% enrichment.

In the context of the sphere reactor for 5% enrichment, criticality is achieved at N_M/N_F values between 100 and 200. At this point, the calculations will be conducted within this N_M/N_F range, with a step number set to 5. The corresponding data is presented in Table 2.13.

Table 2.13 Assembly of Uranium-Graphite with enrichment of 5%

U-235: 5 %, η = 1.91502					
N_M/N_F	**k_∞**	**R (m)**	**H(m)**	**V (m^3)**	**H/R**
125	1.00407	9.476964	17.5	4937.719	1.84658
130	1.01599	4.778161	8.9	638.3537	1.86264
135	1.02734	3.669105	6.8	287.5935	1.85331
140	1.03817	3.105096	5.8	175.6823	1.8679
145	1.0485	2.776684	5.1	123.5301	1.83672
150	1.05838	2.526149	4.7	94.22488	1.86054
155	1.06783	2.341519	4.4	75.78758	1.87912
160	1.07688	2.215794	4.1	63.24012	1.85035
165	1.08557	2.104177	3.9	54.24742	1.85346

The range where criticality is achieved at N_M/N_F ratio between 125 and 165. Where it could be noticed that for enrichment of 5 % increasing N_M/N_F ratio will decrease corresponding volume, radius, and height. The minimum volume is at N_M/N_F ratio 165, is 54.24742.

Figure 2.14 shows the relation between the three main parameters and N_M/N_F ratio.

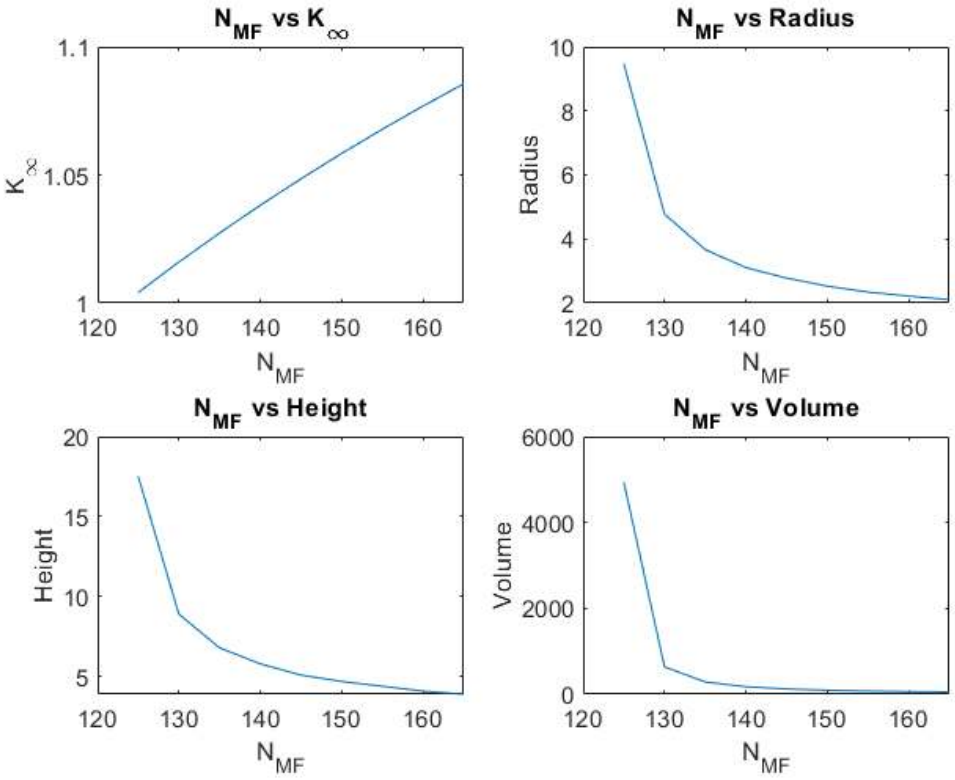

Figure 2.14 Effect of N_M/N_F on k_∞, radius, height, and volume.

Whereas shown in previous figure for 1.5% and 2.5% enriched uranium. The effect of N_M/N_F ratio in multiplication factor, radius, height and volume.

CHAPTER 3
MONTE CARLO METHOD FOR THE CRITICAL SIZE

3.1 Introduction

Monte Carlo methods are a wide range class of computational algorithms relying on repetition of random samples to obtain numerical results. The underlying main concept is to use arbitrary randomness to solve such problems that might be deterministic in principle[13–18]. The name of the method comes from a place the Monte Carlo Casino in Monaco, where the essential developer of the method was inspired by the physicist Stanislaw Ulam because of his uncle's habits in gambling [19].

Monte Carlo Method (MCM) is fundamentally used in three distinct classes of problems: numerical integration, optimization and generating draws from a function of probability distribution. These solutions can also be used to model the phenomena with significant uncertainty in inputs as an example of calculating the risk analysis in any failure of a nuclear power plant. MCM is frequently applied by using computer simulations and it can effectively provide approximate solutions to such problems that are otherwise intractable in theoretical analysis or too complex to analyze mathematically. The simulation model is not only resistant to mathematical equations, it is resistant to trials and does not produce optimum results of the model, but alternative solutions are presented with the help of simulation solutions and a solution with optimum quality and close to it is selected [20].

In this Chapter, we explain the parameters, equations and their presentations used in the MCM method for the calculation of multiplication factor; radius of the spherical reactor; the radius and height of the cylindrical reactor and their relations with the enrichment factor and the moderator to fuel ratios. Numerical results are presented for sphere and cylinder in their own sections.

3.2 The MCM Code Parameters

We basically present here the main parameters for the calculation of multiplication factor for a nuclear reactor of radius r (for spherical reactor) and h (for finite-

cylindrical reactor) consisting of mixture of graphite and uranium with atomic ratio $ANMF=N_M/N_F$ where N_M is moderators' and N_F is fuel elements' atoms [21, 22]. The uranium in the mixture is also enriched by a factor from the composition of natural uranium which has composition rate as $FRAC35=U_{235}:U_{238}=1:138$ (that corresponds to *0.71942%* for U_{235} in natural uranium) and *FRAC35*=1-FRAC38. It is defined as The MCM method follows the path of prompt neutrons in the reactor volume. The numerical values used in the code are also given in the previous Chapter and not repeated here. Since the moderator is graphite, it is well known that the neutrons will have thermal energy after *114* scattering by moderator atoms and then these neutrons may be either just absorbed by U_{235} or they cause fission by interaction with U_{235}.

We assume that the graphite has an approximate density of $2200kg/m^3$. Since the neutrons will be scattered in the volume, the equation for the calculation of scattering probability per unit length for graphite, *sigsc*, is

$$sigsc = 4.8 \times concc \times barn \tag{3.1}$$

where barn is the unit of cross section area as a constant of $10^{-28}\ cm^2$ and we set the concentration of graphite as $concc \approx 10^{29} atoms/cm^3$. For all other scatterings one can be written as

$$\Sigma_s = n_M\sigma_{s,M} + n_{235}\,\sigma_{s,235} + n_{238}\sigma_{s,238}$$
$$sigst = sigsc + (10.0 \times conc35 + 8.3 \times conc38) \times barn \tag{3.2}$$

in which

$$conc35 = concc \times ANMF \times FRAC35$$
$$conc38 = concc \times ANMF \times FRAC38 \tag{3.3}$$

And considering all absorption processes in reactor volume one writes the total interaction probability for any interaction per unit path length is

$$\Sigma_T = n_M\sigma_{s,M} + n_{235}\,\sigma_{s,235} + n_{238}\sigma_{s,238} + n_{235}\,\sigma_{a,235} + n_{238}\sigma_{a,238} + n_{235}\sigma_{f,235} \tag{3.4}$$

Because the moderator is graphite the fixed number of scattering events must be taken into account for the slowing-down of neutrons from its birth to thermal energy point. Since the high energetic neutrons will be scattered by moderator, one can use the momentum and energy conservation equations in the calculation of energy lost per scattering process. Using the mean reduction factor calculation for the energy per any scattering process for the graphite is given as [23]

$$R_{energy} = \left(\frac{A}{1+A}\right)^2 = \left(\frac{12}{13}\right)^2 = \frac{144}{169} \tag{3.5}$$

For the number of such factors per scattering the reduce in energy of neutron, for example, from 2 MeV down to thermal level of 0.025 eV we require a certain number of collisions, *n*, such as

$$\frac{0.025}{2 \times 10^6} = \left(\frac{144}{169}\right)^n \tag{3.6}$$

Thus, one gets n=114 the number of scatterings for neutron in graphite moderator.

Since there are basically two types of interactions then the probabilities p1 and p2 can take place and the random choice for which occurs in the volume can be made by generating r from interval of 0 to 1. If r is less than p1/(p1+p2), the first reaction takes place, otherwise the second one. Thus, a sequence of such probabilities will be taken into account for the neutron's interactions [24]. Here is the summary of these sequences:

- If the neutron is just absorbed without fission reaction, the next step for Monte Carlo trial is initiated.
- If there is scattering of neutron, then a new random direction is generated and the procedure is followed as in the previous step. If it is the moderator scattering the neutron then one is added to the counter which was set equal to zero in initial case. If the counter reaches at number 114 the neutron is assumed to be thermalized and absorbed by ^{235}U and activated. Once it is the thermalized neutron it stays in that state for all other steps.
- If it is the fission reaction, then a number 2.42 is added to the counter which is recorded as fission-generated neutrons and the next Monte Carlo trial is triggered.

After all these steps Monte Carlo sequence is completed, and the multiplication factor is determined by using its definition as the ratio of number of generated neutrons to that of all trials.

3.2.1 The MCM for Spherical Reactor

Using the same procedure in the previous section, the outputs are given in the table below. In all MCM calculations, total trial neutron number is set to $n=10000$. Table 3.1 shows the minimum volume (V) of spherical reactor by the variation of the uranium-graphite mixture (N_M/N_F) for enriched-uranium (in %) with respect to the Multiplication Factor (k_∞). The radius of sphere is R.

Table 3.1 Assembly of Uranium-Graphite with enrichment of 1.5% and 2.5%

	U-235: 1.5 %, η = 1.63191			**U-235: 2.5 %, η = 1.78249**		
N_M/N_F	**k_∞**	**R (m)**	**V (m^3)**	**k_∞**	**R (m)**	**V (m^3)**
100	Nan	Nan	Nan	Nan	Nan	Nan
200	Nan	Nan	Nan	1,03	3,95	259,11
300	Nan	Nan	Nan	1,11	2,22	45,77
400	1.00	13.15	9520.2	1,15	1,99	33,02
500	1.01	7.87	2040.7	1,17	1,94	30,78
600	1.01	8.60	2663.9	1,18	1,96	31,68
700	1.03	14.04	11586.9	1,18	2,01	34,26
800	Nan	Nan	Nan	1,18	2,09	38,15
900	Nan	Nan	Nan	1,17	2,18	43,32
1000	Nan	Nan	Nan	1,16	2,28	49,92

In Table 3.1, the minimum volume is obtained for N_M/N_F=500 if the enrichment is 1.5%. When the enrichment is raised to 2.5% then the minimum critical volume is

obtained for N_M/N_F =600. As it is seen in the results above that the multiplication factor is not available for a range of N_M/N_F because k_∞ is smaller than 1.0 in the MCM. The volume changes a curve with a deepest value (from higher to lower and then lower to higher) as the radius changes for values of N_M/N_F from 400 to 700 for 1.5% enrichment and for N_M/N_F from 200 to 1000 for 2.5% enrichment. The results are in agreement with these of analytical ones. In Table 3.2, the results by MCM are presented for 5% enriched uranium for a range of N_M/N_F. It is observed that the minimum volume (that means size) is obtained for N_M/N_F=600 at which the radius of sphere is the minimum one. Again, one can see the behavior of the volume change with respect to N_M/N_F is like a curve with minimum point. Table 3.2 shows the minimum volume (V) of spherical reactor by the variation of the uranium-graphite ratio (N_M/N_F) for enriched-uranium (%) with respect to the Multiplication Factor (k_∞). The radius of sphere is R.

Table 3.2 Assembly of Uranium-Graphite with enrichment of 5%

U-235: 5 %, η = 1.91502			
N_M/N_F	**k_∞**	**R (m)**	**V (m^3)**
100	0,93	nan	nan
200	1,14	1,76	22,81
300	1,23	1,40	11,45
400	1,29	1,30	9,17
500	1,32	1,26	8,46
600	1,34	1,25	8,32
700	1,36	1,26	8,45
800	1,37	1,28	8,75
900	1,37	1,30	9,17
1000	1,37	1,32	9,69

3.2.2 The MCM for Finite - Cylindrical Reactor

Using the same procedure in the previous section, the outputs are given in the table below. In all MCM calculations, total trial neutron number is set to *n=10000*.

Table 3.3 shows the minimum volume (*V*) of cylindrical reactor by the variation of the uranium-graphite ratio (N_M/N_F) for enriched-uranium (%) with respect to the Multiplication Factor that satisfies k_∞>1.0. The radius of cylinder is *R* and the height is *H*.

Table 3.3 Assembly of Uranium-Graphite with enrichment of 1.5% and 2.5%

	U-235: 1.5 %, η = 1.63191			**U-235: 2.5 %, η = 1.78249**		
N_M/N_F	**H (m)**	**R (m)**	**V (m^3)**	**H (m)**	**R (m)**	**V (m^3)**
100	Nan	Nan	Nan	Nan	Nan	Nan
200	Nan	Nan	Nan	6,85	3,71	295,84
300	Nan	Nan	Nan	3,84	2,08	52,26
400	22,82	12,35	10932,411	3,45	1,87	37,70
500	13,64	7,38	2333,35	3,37	1,82	35,14
600	14,89	8,06	3035,39	3,40	1,84	36,17
700	24,34	13,17	13266,01	3,49	1,89	39,12
800	Nan	Nan	Nan	3,62	1,96	43,56
900	Nan	Nan	Nan	3,77	2,04	49,46
1000	Nan	Nan	Nan	3,96	2,14	56,99

In Table 3.3, the minimum volume is obtained again for the moderator-to-fuel ratio of N_M/N_F=500 if the enrichment is 1.5%. If the enrichment is increased to 2.5% then the minimum critical volume is obtained for N_M/N_F =500. As it is seen in the results above that the multiplication factor is not available for a range of N_M/N_F because k_∞ is again smaller than 1.0 in the MCM. The results are in agreement with those of analytical ones.

In Table 3.4, the results by MCM are presented for 5% enriched uranium for a range of N_M/N_F. It is shown here that the minimum volume (that means size) is obtained for N_M/N_F=600. Again, one can observe that the change in volume of the cylinder with respect to the change in moderator-to-fuel ratio behaves like a curve with a deepest value around N_M/N_F=600. One can see the behavior of the volume here changes with respect to N_M/N_F but not like a curve with minimum point. The H/R ratio is also shown in the last column where one observes it satisfies the analytical solution for H/R=1.84.

Range of N_M/N_F=125 to N_M/N_F=160 because the multiplication factor is about 1.0 for these values. If one increases the N_M/N_F ratio, it is observed that the multiplication factor is also increased and its value becomes much greater than 1.0, meaning of supercritical reactor. Therefore, it is shown here again the change of volume of the cylinder with respect to the change in moderator-to-fuel ratio. The H/R ratio is also observed that it satisfies the analytical solution for H/R=1.84. Table 3.4 shows the minimum volume (V) of cylindrical reactor with respect to the Multiplication Factor that satisfies k_∞>1.0. The radius of cylinder is R and the height is H.

Table 3.4 Assembly of Uranium-Graphite with enrichment of 5% and k_∞>1.0.

U-235: 5 %, η = 1.91502			
N_M/N_F	**R (m)**	**H(m)**	**V (m^3)**
100	Nan	Nan	Nan
200	1,65	3,05	26,04
300	1,31	2,42	13,07
400	1,22	2,25	10,47
500	1,19	2,19	9,66
600	1,18	2,18	9,50
700	1,18	2,19	9,65
800	1,20	2,21	9,99
900	1,22	2,25	10,47

Here a question arises about the optimization of the system: If one looks for the minimum size (that is the volume) of the system, then it only needs to focus on the volume term and the corresponding value of moderator-to-fuel ratio. However, it is also necessary to control the multiplication factor, k_∞>1.0. If there is no any limitation on k_∞ value for higher than 1.10, then the volume term is determined as what Table 3.4 does. On the other hand, the limitation on k_∞ value will change the procedure: If one sets up the limitation on k_∞, as $k_\infty \approx 1.0$, then one needs to determine the volume between N_M/N_F=100 and N_M/N_F=200 because k_∞<1.0 in this range. We present our results in Table 3.5. It is seen that the volume decreases gradually as the multiplication factor increases. When $k_\infty \approx 1.0$, then the volume reaches its maximum point and then it starts to decrease as k_∞ value increases. There is direct relation between moderator-to-fuel ratio and k_∞ value as shown in Eq. (2.5).

Table 3.5 shows the volume (V) of cylindrical reactor by the variation of the uranium-graphite mixture (N_M/N_F) for enriched-uranium (in %) with respect to the Multiplication Factor that satisfies $k_\infty \approx 1.0$. The radius of cylinder is R, and the height is H.

Table 3.5 Assembly of Uranium-Graphite with enrichment of 5% $k_\infty \approx 1.0$.

U-235: 5 %, η = 1.91502				
N_M/N_F	**k_∞**	**R (m)**	**H(m)**	**V (m^3)**
125	1,00	9,33	17,24	4712,23
130	1,02	4,71	8,71	608,19
135	1,03	3,61	6,67	273,57
140	1,04	3,06	5,66	166,84
145	1,05	2,72	5,03	117,14
150	1,06	2,49	4,59	89,21

CHAPTER 4
CONCLUSION

In this thesis, we run a computer program to examine the effect of enrichment and the variation of the moderator-to-fuel ratio in a uranium- graphite assembly with two geometric shapes. Additionally, an MCM code is used to study and simulate the fission chain reaction for uranium fissile elements using the Monte Carlo Method (MCM). The critical minimum volume for spherical and cylindrical shapes of fissile material is first determined by solving the diffusion equation for the geometric and material buckling parameters. Then, the main parameters for calculating the multiplication factor and minimum volume across a range of moderator-to-fuel ratios and different shapes are discussed throughout the chapter.

In this study, we write and run our code for two different geometries. In the case of spherical reactor, we find that as uranium enrichment increases, the critical volume required for achieving criticality decreases. The multiplication factor increases with the N_M/N_F ratio up to a certain point before decreasing. This relationship is evident in the U-shaped distribution of volume. Higher uranium enrichment levels allow the system to achieve criticality at lower N_M/N_F ratios. For example, criticality is reached at an N_M/N_F ratio of approximately 400 for 1.5% enrichment and around 200 for 2.5% enrichment. At 5% enrichment, the system achieves criticality at even lower N_M/N_F ratios, this demonstrates that increasing the uranium enrichment significantly reduces the required volume and N_M/N_F ratio to achieve criticality.

In the case of cylindrical reactor, Increased uranium enrichment reduces the critical volume. For example, 1.5% enrichment requires 2043.69 m^3, whereas 2.5% enrichment needs only 30.77 m^3 at an N_M/N_F ratio of 500. The multiplication factor first increases and then decreases with the NM/NF ratio, showing a U-shaped trend in volume distribution. These findings outline the relationship between uranium enrichment levels, NM/NF ratios, and critical reactor volumes, demonstrating that higher enrichment leads to lower critical volumes and more efficient reactor designs.

We obtain the critical size and volume of the spherical and cylindrical bare reactors and show that there is very good agreement among our results with analytical results. One can observe that there is no direct effect of moderator-to-fuel ration in the determination of criticality of the system. However, the enrichment of uranium plays a critical role in the determination of the critical size and volume. Our results show that there is optimization problem between the determination of critical size of the system under investigation and determination and choice of the multiplication factor: Since k_∞>1.0 is required for any reactor system, then what should be the highest value for k_∞ plays a key role in the determination of critical size and volume. This is the main question here.

For future study, one can study water, heavy water or Beryllium as moderator considering the macroscopic cross-sections and other the material properties such as weight percent of materials. Criticality of systems with different types of reflectors can also be studied by extending the equations throughout this study.

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CURRICULUM VITAE

Sacide Safadi

Education:

August 25 2022 – Present

Gaziantep University, Gaziantep

- Master of Science in Nuclear Physics Engineering

September 2019 – June 2022

Gaziantep University, Gaziantep

- Bachelor of Science in Physics Engineering (GPA: 3.41/4.0)

Department first place

August 2018 – June 2022

Gaziantep University, Gaziantep

- Bachelor of Science in Optic and Acoustic Engineering (GPA: 3.53/4.0)

Department first place

Computer Skills:

- MATLAB
- C++
- AutoCAD
- DiaLUX
- Python (basics)
- Adobe Photoshop

- SketchUp
- Microsoft Office
- LaTeX
- Familiar with HTML, CSS, and JavaScript
- ZEMAX

Languages:

- Arabic: Native speaker
- English: C1 advanced
- Turkish: B1 upper-intermediate