NOVEL APPROACHES IN DEA-BASED MULTI-CRITERIA DECISION MAKING AND THEIR APPLICATIONS

(VERİ ZARFLAMA ANALİZİ TEMELLİ ÇOK ÖLÇÜTLÜ KARAR VERME YAKLAŞIMINDA YENİ MODEL ÖNERİLERİ VE UYGULAMALARI)

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TABLE OF CONTENTS

LIST OF SYMBOLS

ADMU	**:** Anti-ideal decision making unit
AVDMU	**:** Average decision making unit
CA	**:** Country attribute
CCR	**:** Charnes, Cooper, Rhodes
CI	**:** Composite indicator
CN	**:** Customer need
DEA	**:** Data envelopment analysis
DMU	**:** Decision making unit
EU	**:** European Union
FWA	**:** Fuzzy weighted average
GDP	**:** Gross domestic product
GNI	**:** Gross national income
HDI	**:** Human Development Index
HDR	**:** Human Development Report
HOQ	**:** House of quality
IDMU	**:** Ideal decision making unit
IT	**:** Information technology
LADMU	**:** Left ideal decision making unit
MCDM	**:** Multiple criteria decision making
MILP	**:** Mixed integer linear programming
MSCI	**:** Morgan Stanley Capital International
OECD	**:** Organization for Economic Cooperation and Development
OWA	**:** Ordered weighted average
PN	**:** Population need
QFD	**:** Quality function deployment
RIDMU	**:** Right ideal decision making unit
ROE	**:** Return on equity
TA	**:** Technical attribute
UNDP	**:** United Nations Development Program

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

Data envelopment analysis (DEA) is a mathematical programming-based decision making tool, which copes with decision problems that need to consider multiple inputs and multiple outputs to evaluate the relative efficiency of decision making units (DMUs) without a priori information about the weights of inputs and outputs. Albeit their use in performance assessment through yielding information whether evaluated DMUs are efficient or not, conventional DEA models also have several pitfalls. First, these models must be solved *n* times to compute the efficiency scores of all DMUs, where *n* is the number of DMUs to be evaluated. In addition, they do not utilize common attribute weights for performance assessment of DMUs enabling the evaluated DMUs to determine input and output weights in their own favor for maximizing their respective efficiency scores, which may yield results being far from practical. Furthermore, traditional DEA models assume that all DMUs with efficiency score of 1 are named as "efficient", while the DMUs with efficiency score less than 1 are identified as "inefficient". In other words, traditional DEA models are likely to fail to provide an adequate discriminating power for efficient DMUs with an efficiency score of 1.

In order to overcome these limitations, common weight DEA-based models have been proposed by a number of scholars. These models do not require subjective evaluation to determine weights for inputs and outputs while providing a common assessment for all DMUs. Thus, identifying input and output weights in favor of evaluated DMUs is limited that increases the discriminating power of the analysis. Hence, the use of common weights appears to be practical for computing the efficiency scores and obtaining a rank-order for all DMUs. Furthermore, common weight DEA-based models necessitate a smaller number of mathematical programs to be solved to determine efficiency scores compared with traditional DEA models. Thereby, common weight models also provide computational savings.

The objective of this thesis is to present common weight DEA-based decision making frameworks to provide common assessment with enhanced discriminating power and adequate weight dispersion. The developed methodologies in this thesis consider multiple inputs as well as multiple outputs, and proposed mathematical programming models are applicable when crisp and/or imprecise data are present.

The first proposed common weight DEA-based decision framework that includes multiple inputs and multiple outputs is a novel mathematical programming approach that improves the common weight MCDM model developed by Karsak and Ahiska (2007). The use of the proposed framework guarantees to identify the best performing DMU via solving one mixed integer linear programming model in addition to a single linear program for determining the set of minimax efficient DMUs. Moreover, it does not necessitate an arbitrarily determined parameter (k) value for improving the discriminating power and provides enhanced weight dispersion. The robustness of the developed model is illustrated by two case studies that aim to provide economic and financial performance assessment. The first study presents an evaluation of Morgan Stanley Capital International (MSCI) emerging markets, whereas the second case study ranks the Turkish deposit banks using the proposed methodology, while also providing a comparative evaluation with several other approaches addressed in earlier works.

The second proposed common weight DEA-based decision framework aims to incorporate imprecise data into the initially developed decision approach. The proposed approach assures to identify the best performing DMU via a two-stage mathematical programming framework based on α-cuts. Ordered weighted average (OWA) operator is utilized to obtain an aggregated efficiency score for each DMU, and then enable a unique rank-order of DMUs. Furthermore, the proposed decision framework provides enhanced weight dispersion for inputs and outputs. Three illustrative examples from the relevant literature are provided to demonstrate the robustness of the proposed methodology, and ranking results are compared with those of other approaches developed in earlier research papers. A case study, which focuses on identifying the most desirable country to work for an expatriate, is presented as well.

The third proposed DEA-based decision framework presents an integrated group decision making approach that ranks countries by taking into account ten attributes that are included in sustainable development goals set by United Nations Development Program (UNDP). House of quality (HOQ) is constructed to evaluate the relationships among the population needs and sustainable development goals as well as the inner dependence among these goals. In order to rank the countries and identify the best performer, a DEA-based mathematical programming approach, which uses the lower and upper bounds on weights of sustainable development indicators calculated by fuzzy weighted average (FWA) and the data derived from the HOQ, is employed. The robustness of the proposed approach is shown via a case study that aims to rank Latin American countries considering ten criteria that are included in sustainable development goals presented by UNDP.

ÖZET

Veri zarflama analizi (VZA), birden çok girdi ve birden çok çıktıyı dikkate alarak karar verme birimlerinin (KVBler) göreceli etkinliğinin değerlendirilmesini sağlayan matematiksel programlama temelli bir karar verme yaklaşımıdır. Geleneksel VZA modelleri, KVB'lerin etkin olup olmadığını belirleyen bir performans değerlendirmesi sunmasına rağmen bazı eksiklikler içermektedir. İlk olarak, değerlendirilecek *n* adet KVB söz konusu olduğunda, bu modeller tüm KVB'lerin etkinlik değerlerini hesaplamak için *n* kez çözülmelidir. Buna ek olarak geleneksel VZA modelleri ortak girdi ve çıktı ağırlıklarını kullanmayıp, bu durum KVB'lerin etkinlik değerlerini eniyilemek için girdi ve çıktı ağırlıklarını kendi lehlerine belirlemelerine olanak tanımaktadır. Ayrıca, etkinlik değeri 1'e eşit olan KVB "etkin", etkinlik değeri 1'den küçük olan KVB ise "etkin olmayan" olarak kabul edilmektedir. Böylelikle, geleneksel VZA modelleri etkinlik değeri 1 olan KVBler arasında bir etkinlik sıralaması yapamamaktadır.

Araştırmacılar, geleneksel VZA modellerinin eksikliklerinin giderilmesi için ortak ağırlıklı VZA temelli modeller önermiştir. Bu modeller, tüm KVB'ler için ortak bir değerlendirme sağlarken, girdi ve çıktıların ağırlıklarını belirlemek için öznel hesaplama gerektirmemektedir. Ayrıca, ortak ağırlıklı VZA temelli modeller analizin ayırt etme gücünü iyileştirmektedir.

Bu tezin amacı, geliştirilmiş ayırt etme gücü ve gerçekçi ağırlık dağılımı sağlayan ortak ağırlıklı VZA temelli karar verme modelleri önermektir. Bu tezde geliştirilen yaklaşımlar, karar problemlerinde birden çok girdinin yanı sıra birden çok çıktıyı da dikkate almaktadır. Bununla birlikte önerilen matematiksel programlama modelleri, kesin ve/veya kesin olmayan veri mevcut olduğunda uygulanabilmektedir.

Önerilen ilk ortak ağırlıklı VZA temelli karar verme yaklaşımında, Karsak ve Ahiska (2007) tarafından geliştirilen, birden çok girdiyi ve birden çok çıktıyı göz önünde

bulunduran ortak ağırlıklı çok ölçütlü karar verme (ÇÖKV) modelini iyileştiren yeni bir iki aşamalı matematiksel programlama prosedürü önerilmektedir. Önerilen yaklaşımda, tek bir doğrusal programlama modeli ve bir karma tam sayılı doğrusal programlama modeli çözülerek en etkin KVB'nin belirlenmesi sağlanmaktadır. Ayrıca, ayırt etme gücünün iyileştirilmesi için keyfi olarak belirlenen k parametresinin değerinin kullanımına gereksinim duyulmamakta ve gerçekçi ağırlık dağılımı sağlanmaktadır. Geliştirilen modelin uygulaması, ekonomik ve finansal performans değerlendirmesini amaçlayan iki vaka çalışması ile sağlanmaktadır. İlk çalışma, Morgan Stanley Capital International (MSCI) gelişmekte olan piyasaların bir değerlendirmesini sunarken, ikinci vaka çalışması ise Türk mevduat bankalarının göreceli performansını değerlendirmektedir. Ayrıca, önceki çalışmalarda ele alınan diğer birkaç yaklaşım ile karşılaştırmalı bir analiz sunulmaktadır.

Önerilen ikinci ortak ağırlıklı VZA temelli karar verme yaklaşımı, ilk sırada geliştirilen karar yaklaşımına kesin olmayan verinin dahil edilmesini amaçlamaktadır. Önerilen yaklaşım, α-kesmelerine dayalı iki aşamalı matematiksel programlama prosedürü aracılığı ile en etkin KBV'nin belirlenmesini sağlamaktadır. Sıralı ağırlıklı ortalama operatörü, her bir KVB için bütünleştirilmiş bir etkinlik değeri hesaplamakta ve son tahlilde KVBler arasında tek bir sıralama elde etmektedir. Ayrıca, önerilen karar yaklaşımında girdiler ve çıktılar için iyileştirilmiş ağırlık dağılımı sağlanmaktadır. Elde edilen sonuçlar daha önceki araştırma makalelerinde geliştirilen diğer yaklaşımların uygulanmasıyla elde edilenlerle karşılaştırılmaktadır. Ayrıca geliştirilen modelin uygulamasına yönelik olarak, yabancı bir ülkeye çalışmaya gidecek bir kişi için en uygun ülkeyi belirlemeye odaklanan bir vaka çalışması da verilmektedir.

Önerilen üçüncü VZA temelli karar analizi, sürdürülebilir kalkınma hedeflerinde yer alan özellikleri dikkate alarak ülkeleri sıralayan bütünleşik grup karar verme yaklaşımı sunmaktadır. Toplumun gereksinimleri ile sürdürülebilir kalkınma hedefleri arasındaki ilişkileri ve bu hedefler arasındaki bağımlılığı değerlendirmek için kalite evi kullanılmaktadır. Ülkeleri performans derecelerine göre sıralamak ve en iyi performans gösterenleri belirlemek için, bulanık ağırlıklı ortalama ve kalite evinden türetilen verileri kullanarak sürdürülebilir kalkınma göstergelerinin ağırlıklarının alt ve üst sınırlarını belirleyen VZA temelli bir matematiksel programlama yaklaşımı önerilmektedir.

Önerilen yaklaşımın uygulamasında, Birleşmiş Milletler Kalkınma Programı (UNDP) tarafından ortaya konan sürdürülebilir kalkınma hedefleri arasında yer alan on ölçütü dikkate alarak Latin Amerika ülkelerinin performans değerlendirmesini amaçlayan bir vaka çalışması sunulmaktadır.

1. INTRODUCTION

Data envelopment analysis (DEA) is a decision making tool that is based on mathematical programming and has been commonly utilized to deal with decision problems which require taking into account multiple inputs and outputs for evaluating the relative efficiency of decision making units (DMUs) without a priori knowledge concerning the importance of inputs and outputs (Karsak & Ahiska, 2007).

Albeit their use in performance assessment through yielding information whether evaluated DMUs are efficient or not, conventional DEA models also have several limitations. First of all, these models must be solved *n* times to obtain the efficiency scores for all DMUs, where *n* denotes the number of DMUs under evaluation. Second, traditional DEA models are likely to fail in discriminating among a group of DMUs since they consider all DMUs that obtain an efficiency score of 1 as efficient. In general, the flexibility of DMUs in determining the weights tends to result in more than one efficient DMU. Techniques such as cross-efficiency analysis (Doyle & Green, 1994) and weight restrictions (Wong & Beasley, 1990) were proposed to improve the discriminating power of DEA. Moreover, conventional DEA models do not provide common weights for evaluating DMUs and enable the DMUs to determine their own weights in order to maximize their efficiency scores, which is rather impractical. Hence, ranking DMUs with the efficiency scores acquired from different sets of weights leads to impossibility for common assessment (Sun et al., 2013).

In order to overcome the pitfalls mentioned above, common weight DEA-based models have been proposed by a number of scholars. These models do not require subjective evaluation to determine weights for inputs and outputs while providing a common assessment for all DMUs. Thus, the selection of input and output weights in favour of respective DMUs is limited that increases the discriminating power of the analysis (Karsak & Ahiska, 2005). Hence, the use of common weights appears to be practical for computing the efficiency scores and obtaining a rank-order for all DMUs (Jahanshahloo et al., 2010). Furthermore, common weight DEA-based models necessitate a smaller number of mathematical programs to be solved to determine efficiency scores compared with traditional DEA models. Thereby, common weight models also provide computational savings.

On the other hand, traditional DEA and common weight DEA-based models require crisp data for inputs and outputs. However, crisp data may become inadequate to handle qualitative issues that are commonly encountered in real-world cases. In order to include qualitative and imprecise data into the modeling framework, DEA models incorporating uncertain data should be developed. Vagueness, fuzziness, and uncertainty may be dealt with imprecise DEA models.

This thesis contributes to common weight DEA-based modeling literature incorporating multiple inputs and multiple outputs by proposing novel decision approaches in the presence of crisp as well as imprecise data. In this thesis, three common weight DEA-based decision approaches are proposed, and then illustrated by conducting several case studies in order to demonstrate the robustness of the developed methodologies.

The first proposed common weight DEA-based decision framework includes multiple inputs and multiple outputs by proposing a novel mathematical programming approach that improves the common weight MCDM model developed by Karsak and Ahiska (2007). The use of the proposed framework guarantees to identify the best performing DMU via solving one mixed integer linear programming model in addition to a single linear program for determining the set of minimax efficient DMUs. Moreover, it does not necessitate an arbitrarily determined parameter, k, value for improving the discriminating power and provides enhanced weight dispersion.

The robustness of the developed model, which provides a ranking with an improved discriminating power and enhanced weight dispersion, is illustrated by two case studies that aim to provide economic and financial performance assessment possessing an important role for efficient usage of available resources by taking into consideration market dynamics. The first study presents an evaluation of Morgan Stanley Capital International (MSCI) emerging markets, whereas the second case study ranks the Turkish deposit banks using the proposed methodology as well as providing a comparative evaluation with several other approaches addressed in earlier works. The results indicate that the proposed approach guarantees to identify the best performing DMU without including a discriminating parameter requiring an arbitrary step size value in model formulation while also achieving an improved weight dispersion for inputs and outputs.

The second proposed common weight DEA-based decision framework aims to enable to incorporate imprecise data into the initially developed decision approach. The proposed approach assures to identify the best performing DMU via a two-stage mathematical programming framework based on α-cuts. Ordered weighted average (OWA) operator is utilized to obtain an aggregated efficiency score for each DMU, and then enable a unique rank-order of DMUs. Furthermore, the proposed decision framework provides enhanced weight dispersion for inputs and outputs.

Three illustrative examples from the relevant literature are provided to demonstrate the robustness of the proposed methodology, and ranking results are compared with those of other approaches developed in earlier research papers. A case study, which focuses on identifying the most desirable country to work for an expatriate, is given as well. Crisp as well as fuzzy inputs and outputs are taken into consideration in the modelling framework. Resulting rankings indicate that the developed approach provides a ranking with improved discriminating power and enhanced weight dispersion with regard to inputs and outputs while also guaranteeing to determine a single best performing DMU.

The third proposed decision aid integrates quality function deployment (QFD) into the common weight DEA-based group decision making approach, and is applied for ranking Latin American countries by considering sustainable development goals. Sustainable development goals are adopted by United Nations Member States for promoting prosperity in all countries and protecting the planet. In order to achieve these goals, economic, social and environmental strategies should be constructed to provide progress in education, health, climate change, etc. (www.sustainabledevelopment.un.org).

The robustness of the proposed approach is shown via a case study that aims to rank Latin American countries considering 10 criteria that are included in sustainable development goals presented by United Nations Development Program (UNDP). Countries' ultimate aim is to improve these sustainable development indicators that guarantee a decent standard of living in terms of the characteristics of the population needs. Achieving these goals is mainly related to considering the relationships between population needs and sustainable development indicators along with the relationships among sustainable development criteria avoiding the assumption of impractical independence. As a result, forming a house of quality (HOQ), which enables the relations among population needs and sustainable development indicators as well as the inner dependence of sustainable development indicators to be considered, is crucial in identifying how successful each country feature is in meeting the needs determined from the population. The proposed decision-making procedure determines the characteristics that the countries should have to satisfy the population needs, and subsequently it aims to identify the pertinent sustainable development indicators. Quality function deployment (QFD) is a sound technique to obtain plausible outcomes that are thoroughly responsive and concentrated on the population needs. QFD guarantees that sustainable development indicators are in line with features required by the countries in order to achieve sustainable development goals.

In this thesis, we focus on the first of the four matrices in QFD, also known as the HOQ. Afterwards, a DEA-based mathematical programming model, which uses the weights of sustainable development indicators calculated by fuzzy weighted average (FWA) utilizing the data derived from the HOQ and the pertinent country ratings with respect to sustainable development indicators, is employed to determine the best performing Latin American country.

The remaining parts of this thesis are organized as follows. The following section reviews the literature with regard to common weight DEA models. In Section 3, basic concepts of DEA are summarized. The preliminaries of fuzzy sets are given in Section 4. Section 5 outlines the basic concepts of quality function deployment. Section 6 presents the developed decision-making approaches. The implementations of the proposed frameworks are provided in Section 7. Finally, concluding observations and future research directions are given in the last section.

2. LITERATURE REVIEW ON COMMON WEIGHT DEA-BASED MODELS

Several studies have been conducted in the recent past related to common weight DEA-based models. Some of these researches have focused on problems with single input and multiple outputs while others have addressed cases incorporating multiple inputs and multiple outputs. Apart from them, few researchers have employed common weight DEA-based models when data contain imprecision. Advantages and limitations of reviewed mathematical programming models are discussed in the respective tables.

2.1 Deterministic Models

In this subsection, common weight DEA-based models that are applicable when data are deterministic are reviewed. The advantages as well as the limitations of these models are discussed.

2.1.1 Models with Single Input and Multiple Outputs

Throughout the literature, scholars have proposed a number of common weight DEA-based mathematical programming models that are applicable when the data contain a single input and multiple outputs. In this subsection, the problems with single input and multiple outputs are reviewed.

2.1.1.1 Models that Use the Discriminating Parameter *k*

Several approaches make use of discriminating parameter k in order to obtain the single efficient DMU. Karsak and Ahiska (2005) aimed to identify the most efficient robot alternative by introducing a novel two-stage multi-criteria decision making methodology that can incorporate multiple outputs including qualitative factors represented as ordinal data, and a single input. The modeling framework is as follows:

$$\min \quad M$$

subject to (2.1)

$$\begin{aligned} M &\geq d_j, && \forall j, \\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{x_j} + d_j &= 1, && \forall j, \\ u_r &\geq \varepsilon, && \forall r, \\ d_j &\geq 0, && \forall j, \end{aligned}$$

where u_r is the weight assigned to output r, v_i is the weight assigned to input i, y_{rj} is the quantity of output r generated and x_{ij} is the amount of input i consumed by DMU j, respectively, d_j represents the deviation from efficiency (d_j=1-E_j, where E_j refers to the efficiency of DMU$_j$), ϵ is a small positive scalar, and M is the maximum deviation from efficiency. When the first stage yields more than one efficient DMU, Karsak and Ahiska (2005) proposes the following linear programming model.

$$\min M - k \sum_{j \in EF} d_j$$

subject to (2.2)

$$\begin{aligned} M &\geq d_j, && \forall j, \\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{x_j} + d_j &= 1, && \forall j, \\ u_r &\geq \varepsilon, && \forall r, \\ d_j &\geq 0, && \forall j, \end{aligned}$$

where $k \in [0,1]$ is the discriminating parameter, EF is the set of DMUs that are considered as efficient according to stage I. The advantages of the model are threefold. First, it has improved discriminating power. Second, the evaluation is based on both cardinal and ordinal outputs. Third, the developed methodology yields the best performing DMU by solving fewer linear programs compared with DEA-based approaches. However, the modeling framework possesses the following limitations. The developed methodology cannot be employed for problems with multiple inputs and multiple outputs. In stage II, the model requires a step size value for the discriminating parameter, k. Hatefi and Torabi (2010) employed the same formulation with Karsak and Ahiska (2005) in order to rank Asia Pacific Economic Countries according to sustainable energy index and human development index, and demonstrated that the model converges to a single efficient DMU by increasing the value of k from zero to one with a step size 0.1. Karsak and Ahiska (2008) proposed an approach for computing the values of discriminating parameter, k, via a bisection search algorithm as

Step 1: Let τ be the accuracy of tolerance, and c be the iteration counter. Let $k_c \in [k_c^L, k_c^R]$. Set c=1.
Step 2: Set $k_c^L = 0, \ \ k_c^R = 1$.
Step 3: Set $k_c = \frac{1}{2}(k_c^L + k_c^R)$.
Step 4: If $k_c - k_c^L < \tau$ then $k_c = k_c^L$.
Step 5: Solve $min\, M - k\sum_{j \in EF} d_j$ model (stage II of Karsak and Ahiska (2005)) with k=k_c to obtain $d_{j,c}$, which represents deviation from efficiency for DMU_j at iteration c, $\forall j$.
Step 6a: If $d_{j,c}$ >0 for $\forall j$, then $k_{c+1}^L = k_c^L$ and $k_{c+1}^R = k_c$. Set c=c+1. Go to Step 3.
Step 6b: If $\exists j$ such that $d_{j,c}$ =0, where the number of DMUs satisfying that condition is greater than 1 and $k_c = k_c^L$, then stop. The DMUs with $d_{j,c}$ =0 are efficient.
Step 6c: If $\exists j$ such that $d_{j,c}$ =0, where the number of DMUs satisfying that condition is greater than 1 and $k_c \neq k_c^L$, then $k_{c+1}^L = k_c$ and $k_{c+1}^R = k_c^R$. Set c=c+1. Go to Step 3.
Step 6d: If $\exists$ a unique j such that $d_{j,c}$ =0, then stop. The DMU with $d_{j,c}$ =0 is efficient.

The proposed approach has two main advantages. First, the paper introduces an improvement over the approach proposed by Karsak and Ahiska (2005) by making use of a bisection search algorithm. Second, the developed algorithm allows to calculate the values of discriminating parameter, k, in a robust manner rather than requiring the decision analyst to assign an arbitrary step size value.

Table 2.1: Advantages and Limitations of the Models that Use the Discriminating Parameter *k*

Author(s)	Year	Advantage(s)	Limitation(s)
Karsak and Ahiska	2005	• Improved discriminating power. • Based on both cardinal and ordinal outputs. • Yields the best performing DMU by solving fewer linear programs compared with DEA-based approaches.	• Cannot be employed for problems with multiple inputs and multiple outputs. • In stage II, the model requires an arbitrary step size value for the discriminating parameter, *k*.
Karsak and Ahiska	2008	• Introduces an improvement over the approach proposed by Karsak and Ahiska (2005) by making use of a bisection search algorithm. • The developed algorithm allows to calculate the values of discriminating parameter, k, in a robust manner rather than requiring the decision analyst to assign an arbitrary step size value.	• -
Hatefi and Torabi	2010	• The same formulation with Karsak and Ahiska (2005) is employed in order to rank Asia Pacific Economic Countries according to sustainable energy index and human development index, and demonstrated that the model converges to a single efficient DMU by increasing the value of *k* from zero to one with a step size 0.1.	

2.1.1.2 Models that Eliminate the Discriminating Parameter *k*

Several scholars aimed to eliminate the need of discriminating parameter k from the modelling framework. Amin and Emrouznejad (2007) aimed to propose a formulation that yields the maximum value of non-Archimedean infinitesimal, ϵ. The developed formulation is as

$$\varepsilon^* = \min\left\{\frac{x_j}{\phi_j} : j = 1, \dots, n\right\}, \qquad \text{where} \qquad \phi_j = \sum_{r \in EXO} y_{rj} + \sum_{r \in ORDO} l_{rj} \tag{2.3}$$

where *EXO* and *ORDO* are the sets of exact and ordinal outputs, respectively. Karsak and Ahiska (2005) solved a linear programming model to compute the maximum value of non-Archimedean infinitesimal, ϵ. However, the approach proposed by Amin and Emrouznejad (2007) does not require a linear programming model to obtain the maximum value of ϵ. Although an improvement over Karsak and Ahiska (2005) approach is proposed in terms of determining the maximum value of ϵ, a solution methodology developed in Karsak and Ahiska (2005) is required. In addition, the proposed approach cannot be generalized to multiple inputs and multiple outputs case.

Foroughi (2012) aimed to eliminate the discriminating parameter from the model proposed by Karsak and Ahiska (2005) while maintaining the discriminating power of the approach. The proposed model is as

$$Z_k = \min M - \sum_{j \in EF} d_j$$

subject to (2.4)

$$M - d_j \geq 0, \qquad \forall j,$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{x_j} + d_j = 1, \qquad \forall j,$$

$$d_k = 0, \qquad k \in EF,$$

$$u_r \geq \varepsilon, \qquad \forall r,$$

$$d_j \geq 0, \qquad \forall j,$$

where *EF* denotes the minimax efficient DMUs obtained by stage I of the procedure proposed by Karsak and Ahiska (2005). The model considers that the deviation of the evaluated DMU equals zero while maximizing the deviations of the other minimax DMUs. The model eliminates the discriminating parameter, hence it provides computational efficiency compared to the models that utilize step size value for discriminating parameter, *k*. However, it requires an additional mathematical programming model in order to discriminate the minimax efficient DMUs.

Toloo (2013) presented the following mixed integer programming approach that minimizes the maximum deviation from efficiency and applied it to 40 professional tennis players problem.

$$\min d_{max}$$

subject to (2.5)

$$
\begin{aligned}
& d_{\max} - d_j \geq 0, \quad \forall j, \\
& \sum_{r=1}^{s} u_r\, y_{rj} + d_j = 1 \qquad \forall j, \\
& \sum_{j=1}^{n} \theta_j = n - 1, \\
& d_j \leq \theta_j \qquad \forall j, \\
& \theta_j \leq M d_j \qquad \forall j, \\
& d_j \geq 0 \qquad \forall j, \\
& \theta_j \in \{0,1\} \qquad \forall j, \\
& u_r \geq \varepsilon,
\end{aligned}
$$

where *M* is a sufficiently large positive number, and θ_j is a binary variable. The model has an improved discriminating power, and does not require a discriminating parameter, *k*. However, it requires a penalty value, *M*, and an auxiliary binary variable θ_j. Furthermore, the author guarantees to identify the single best efficient DMU by adding a constraint with binary variables that restricts to obtain more than one efficient DMU. The model is applicable when the problem has a dummy input and multiple outputs.

Gan and Lee (2019) aimed to improve the mathematical programming model addressed in Toloo (2013), and proposed the following mixed integer linear programming (MILP) model.

$$\max(1 - d_{max}) - \varepsilon \sum_{j=1}^{n} (1 - \theta_j)$$

subject to (2.6)

$$\begin{aligned} & d_{\max} - d_j \geq 0, \quad \forall j, \\ & \sum_{r=1}^{s} u_r \, y_{rj} + d_j = 1 \qquad \forall j, \\ & d_j \leq \theta_j \qquad \forall j, \\ & \theta_j \leq M d_j \qquad \forall j, \\ & \theta_j \in \{0,1\} \qquad \forall j, \\ & u_r \geq \varepsilon \qquad \forall r. \end{aligned}$$

The authors mentioned that they used the same data set with Toloo (2013) in order to provide a comparative analysis, however, they utilized the data containing 20 tennis players in lieu of 40 tennis players problem. Hence, they did not utilize the same data set with Toloo (2013), thus the robustness of comparative analysis is ambiguous. Moreover, the approach addressed in Gan and Lee (2019) does not guarantee to obtain a single efficient DMU, hence an enhanced discriminating power is not provided.

Lately, Afsharian et al. (2019) developed a DEA-based model to incentivize the DMUs for optimizing the performance collectively when inefficient units do not demand the targets. The proposed approach provides clustering for different degrees of decentralization. The robustness of the proposed model, which is given below, is proved by conducting a case study in German banking sector.

$$\min \sum_{j=1}^{n} ER_j$$

subject to (2.7)

$$\sum_{r=1}^{s} u_{kr}\, y_{rj} \leq ER_j, \qquad \forall k, j,$$

$$\sum_{r=1}^{s} u_{kr}\, y_{rj} \geq \delta_j c_j, \qquad \forall k, j,$$

$$\sum_{k=1}^{K} \delta_{kj} = 1, \qquad \forall j,$$

$$\delta_{kj} \in \{0,1\}, \qquad \forall k, j,$$

$$ER_j \geq 0, \qquad \forall j,$$

$$u_{kr} \geq 0, \qquad \forall k, r,$$

where c_j denotes the aggregated cost to be minimized. The computational limitations of the proposed approach are twofold. The programming model requires a binary variable for enabling cost minimization. Moreover, an additional formulation is to be solved for obtaining efficiency scores of DMUs.

Table 2.2: Advantages and Limitations of the Models that Eliminate the Discriminating Parameter *k*

Author(s)	Year	Advantage(s)	Limitation(s)
Amin and Emrouznejad	2007	• It does not require a linear programming model to obtain the maximum value of ϵ.	• Although an improvement over Karsak and Ahiska (2005) approach is proposed in terms of determining the maximum value of ϵ, a solution methodology developed in Karsak and Ahiska (2005) is required. • Cannot be generalized to multiple inputs and multiple outputs case.
Foroughi	2012	• Eliminates the discriminating parameter from the model proposed by Karsak and Ahiska (2005) while maintaining the discriminating power of the approach. • Provides computational efficiency compared to the models that utilize step size value for discriminating parameter, *k*.	• Requires an additional mathematical programming model in order to discriminate the minimax efficient DMUs.
Toloo	2013	• Improved discriminating power without requiring a discriminating parameter, *k*.	• Requires a penalty value and an auxiliary binary variable. • Guarantees to identify the single best efficient DMU by transforming the model to a mixed integer linear program. • The model is applicable when the problem has a dummy input and multiple outputs.

Table 2.2: Advantages and Limitations of the Models that Eliminate the Discriminating Parameter *k* (cont.)

Gan and Lee	2019	• -	• Does not guarantee to obtain a single efficient DMU. • Cannot provide enhanced discriminating power.
Afsharian et al.	2019	• Provides clustering for different degrees of decentralization.	• Requires a binary variable for enabling cost minimization. • Requires an additional mathematical programming model that yields efficiency scores of DMUs.

2.1.1.3 Models that Use Composite Indicators

In the recent past, researchers made use of composite indicators (CIs) in common weight DEA-models. Peng et al. (2017) solved the same problem with Hatefi and Torabi (2010) by proposing two approaches that consider importance rankings of sub-indicators for constructing CIs. One of the developed methodologies aims to minimize the total deviation from the ideal point, whereas the other is to minimize the mean absolute deviation. The developed models are based on multiple objectives in order to optimize the performance of all DMUs. Likewise, Hatefi and Torabi (2018) provided a slack analysis methodology to improve CIs of human development and sustainable energy indices. Initially, they employed the following DEA based model that constructs the CI of DMU_j.

$$gI_j = \max \sum_{i=1}^{m} w_i I_{ij}$$

subject to (2.8)

$$\sum_{i=1}^{m} w_i I_{ij} \leq 1, \qquad \forall j,$$

$$w_i \geq \varepsilon, \qquad \forall i,$$

where I_{ij} denotes sub-indicator values w_i is the weight of I^{th} indicator. Formulation (2.8) fails to provide discriminating power, thus the following model is proposed in order to discriminate efficient DMUs.

$$\min \ M$$

subject to (2.9)

$$M + \sum_{i=1}^{m} w_i I_{ij} \geq 1, \qquad \forall j,$$

$$\sum_{i=1}^{m} w_i I_{ij} \leq 1, \qquad \forall j,$$

$$w_i \geq \varepsilon, \qquad \forall i.$$

Moreover, full ranking of DMUs can be provided by adding $-K\sum_{e\in EF}(1-\sum_{i=1}^{m} w_i I_{ie})$ or $+K\sum_{e\in EF}(1-\sum_{i=1}^{m} w_i I_{ie})$ to the objective function of Formulation (2.9), where K is a step size value. Additionally, the authors suggested a slack analysis by introducing the following model.

$$\max \varepsilon \sum_{i=1}^{n} \tau_i$$

subject to (2.10)

$$\sum_{k=1}^{K} v_k^{'} I_{kj} - \tau_i = \left(1/gI_j\right) I_{ij}, \qquad \forall i,$$

$$\sum_{k=1}^{K} v_k^{'} = 1,$$

$$v_k^{'} \geq 0, \qquad \forall k,$$

where v_k represents the dual variable of k^{th} constraint in Formulation (2.8). $\hat{E}_{ij}$ denotes the amount of improvement of the i^{th} sub-indicator of the j^{th} entity to increase its CI value. Finally, $\hat{E}_{ij}$ is computed as

$$\hat{I}_{ij} = \sum_{k=1}^{K} v_k^{*'} I_{kj} = \left(1/gI_j\right) I_{ij} + \tau_i^{*}, \qquad \forall i, \tag{2.11}$$

$$\hat{E}_{ij} = \hat{I}_{ij} - I_{ij}, \qquad \forall i. \tag{2.12}$$

This approach presents a minimax efficiency model in addition to a slack analysis for improving CIs of inefficient DMUs, however it possesses the following limitations. First, Formulation (2.9) requires a step size value k to be determined by a decision maker. Second, in order to provide slack analysis, Formulation (2.8) is required to be solved.

This combination causes to solve two mathematical programming models, hence the computational efficiency decreases. Furthermore, the reason of this combination is questionable.

Yang et al. (2018) assessed the economic situation of emerging markets by introducing a common weight decision approach constructing CIs. In the first step, the output oriented stratification DEA is employed to partition evaluated DMUs as higher (*hD*), lower (*lD*), and overlapped (*oD*) DMUs. In the second step, the output-oriented super efficiency DEA model is utilized for classifying the DMUs that have the same efficiency level with virtual DMU. Finally, the authors proposed the following programming model to rank DMUs using CIs.

$$D = \min \sum_{j=1}^{p} \delta_j + \sum_{j=1}^{k} \phi_j$$

subject to (2.13)

$$\sum_{i=1}^{m} w_i \, I_{ij}^{h} - \delta_j = 1, \qquad \forall j,$$

$$\sum_{i=1}^{m} w_i \, I_{ij}^{l} - \phi_j = 1, \qquad \forall j,$$

$$\sum_{i=1}^{m} w_i \, \bar{I}_i = 1, \qquad \forall j,$$

$$w_i \geq \varepsilon > 0, \qquad \forall i,$$

where I_{ij}^{h} and I_{ij}^{l} represent the indicator value of hD_i and lD_i, respectively. $\bar{I}_i$ denotes the indicator value of a virtual DMU. The developed approach provides full ranking of DMUs, however it has two limitations. First, the output oriented stratification DEA model and the output-oriented super efficiency model are required to solve for employing Formulation (2.13), thus the requirement of solving three mathematical programming models decreases computational efficiency of the modeling framework. Second, the approach combines the output-oriented stratification DEA model and the output-oriented super efficiency model that do not yield common set of weights with the developed model that generates common weights for indicators. Hence, inconsistent models are provided. Sayed et al. (2018) provided a CI of human development index of countries by combining goal programming and data envelopment analysis obtaining common set of weights. Initially, they proposed the following linear benefit-of-the-doubt model.

$$CI_j^* = \max \sum_{r=1}^{s} u_{rj} y_{rj}$$

subject to (2.14)

$$\sum_{r=1}^{s} u_{rj} y_{rj}, \qquad \forall j,$$

$$u_{rj} \geq 0, \qquad \forall r.$$

Using the optimal solution of the model above, the authors developed the following model that yields the optimal value for non-Archimedean infinitesimal.

$$\max \varepsilon$$

subject to (2.15)

$$\sum_{r=1}^{s} u_{rj}^* y_{rj} \leq 1, \qquad \forall j,$$

$$u_{rj}^* \geq \varepsilon, \qquad \forall r.$$

The authors defined two goals as follows:

$$CI_j \leq 1, \qquad \forall j, \tag{2.16}$$

$$\left| CI_j^* - CI_j \right| \leq \varepsilon, \qquad \forall j, \tag{2.17}$$

In order to calculate the final CI values and determine a common set of weights, the following linear programming model is developed.

$$\min\left(\sum_{j=1}^{n} sp1_j + \sum_{j=1}^{n} sp2_j\right)$$

subject to (2.18)

$$\sum_{r=1}^{s} u_r y_{rj} + sn1_j - sp1_j = 1, \qquad \forall j,$$

$$\left|\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{r=1}^{s} u_r y_{rj}\right| + sn2_j - sp2_j = 0.0000001, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r y_{rj} \geq 0,$$

$$\sum_{r=1}^{s} u_r^* y_{rj} \geq 0,$$

$$sn1_j, sp1_j, sn2_j, sp2_j \geq 0,$$

$$u_r \geq \varepsilon, \qquad \forall r,$$

where $sn1_j$ and $sp1_j$ denote the negative and positive deviation variables associated with the first goal, $sn2_j$ and $sp2_j$ represent the negative and positive deviation variables associated with the second goal. The modeling framework provides consistent and stable CI rankings and considers two goals, however it possesses following shortcoming. The proposed approach requires three mathematical programming models to be solved, whereas reducing these steps to a single one will increase computational efficiency of the model.

Peiro-Palomino and Picazo-Tadeo (2018) introduced a decision approach that provides a CI of well-being for countries taking into consideration 10 different well-being domains. Initially they developed the following model for ranking the DMUs.

$$\max w\, b_{c'} = \sum_{d=1}^{10} \eta_{dc'} wbd_{dc'}$$

subject to (2.19)

$$\sum_{d=1}^{10} \eta_{dc'} wbd_{dc} \leq 1, \qquad c = 1,2,\ldots,38,$$
$$\eta_{dc'} \geq 0, \qquad d = 1,2,\ldots,10,$$

where 10 is the number of domains, 38 is the number of countries under evaluation, $wbd_{dc'}$ denotes the observed score for well-being domain d in country c', and $\eta_{dc'}$ represents the weight appointed to the domain d. Since the discriminating power of Formulation (2.19) remains inadequate, the authors proposed the following linear programming model.

$$\min t \frac{1}{38} \sum_{c=1}^{38} m_c + (1-t)z$$

subject to (2.20)

$$\sum_{d=1}^{10} \eta_d wbd_{dc} + m_c = wb_c^*, \qquad c = 1,2,\ldots,38,$$
$$(m_c - z) \leq 0, \qquad c = 1,2,\ldots,38,$$
$$m_c \geq 0, \qquad c = 1,2,\ldots,38,$$
$$\eta_d \geq \varepsilon, \qquad d = 1,2,\ldots,10,$$
$$z \geq 0,$$

where m_c denotes the deviation between the score of country c, namely wb_c^* obtained from Formulation (2.19), and the score of country c derived from Formulation (2.20). z is a non-negative parameter to be forecasted, and t is a parameter which enables assigning different importance to the first and second components in the objective function of Formulation (2.20). The same modeling framework is employed by Castillo-Gimenez (2019) to evaluate 27 European Union countries according to waste management performance. Formulation (2.20) provides full ranking of DMUs, however the optimal value of the objective function changes according to the values assigned to the parameter t. In order to overcome this limitation, Gomez-Vega and Picazo-Tadeo (2019) equaled t value to 1 and employed the model for ranking world tourist destinations.

More recently, Omrani et al. (2019a) introduced DEA-based approach for ranking provinces in Iran according to their development degrees using 131 CIs. The authors initially divide these indicators into 14 classes, and then the following model is solved for each class.

$$\min \sum_{j=1}^{n} \left(d_j^- + d_j^+\right)$$

subject to (2.21)

$$\sum_{r=1}^{s} u_r y_{rj} - \theta^{CRS} \leq 0, \ \ \forall j,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \theta^{CRS} + d_j^- - d_j^+ = 0, \ \ \forall j,$$

$$\sum_{r=1}^{s} u_r = 1,$$

$$u_r \geq \varepsilon,$$

where θ^{CRS} represents the optimal value of the objective function of DEA-CCR model where all DMUs use only one input equal to 1 to produce s outputs. The developed approach provides classifying the indicators, however it has following shortcomings. First, CCR model does not yield common set of weights while the proposed model aims to provide common assessment using common weight for the indicators. Hence, inconsistent models are developed. Second, weight dispersion obtained from the abovementioned model is poor. Third, full ranking of DMUs is not provided for each class, and thus an additional decision approach apart from DEA is required. Therefore, the proposed model does not have an adequate discriminating power. Likewise, Omrani et al. (2019b) assessed the environmental efficiency of provinces of Iran by introducing an integrated common weight decision approach using CIs. Initially, the authors made use of input-oriented DEA-BCC model, and then they utilized the obtained weights in order to construct cross-efficiency matrix. Afterwards, the following common weight linear programming model is developed in order to identify the weights assigned to the indicators. Finally, efficiency scores of DMUs are obtained via a formulation and resulting ranking is achieved.

$$\min p$$

subject to (2.22)

$$wE_j' + S_j^+ - S_j^- = \varphi_j, \ \ \forall j,$$
$$w_1 + \cdots + w_n = 1,$$
$$S_j^+ \leq p, \ \ S_j^- \leq p,$$
$$w_i \geq 0, \ \ \forall i,$$
$$S_j^+ \geq 0, \ \ S_j^- \geq 0, \ \ \forall j,$$

where $E^{'}$ denotes the normalized cross-efficiency matrix, w_d^* is the optimal weights of indicators obtained from the abovementioned model. Hence, the final cross-efficiency DEA scores of DMUs are computed as

$$E_j^{DEA} = \sum_{d=1}^{n} w_d^* E_{dj}, \quad \forall j. \tag{2.23}$$

The developed approach provides an enhanced discriminating power and full ranking of DMUs. However, it requires three mathematical programming models. The first and the second models do not yield common set of weights for the indicators while the third one obtains common weights in order to provide common assessment. The logic of this combination is argumentative, inconsistent models are developed. Moreover, solving three mathematical programming models does not provide computation efficiency, since throughout the literature there are several approaches that provides full ranking of DMUs via solving fewer models.

Lately, Sun et al. (2020) developed two-stage common weight DEA-based methodology by employing CIs for evaluating environmental sustainability of South Asia. Initially, they proposed the linear mathematical programming model as

$$\min M$$

subject to (2.24)

$$M - d_j \geq 0,$$

$$\sum_{i=1}^{m} w_i I_{ij} + d_j = 1, \ \ \forall j,$$

$$w_i \geq \varepsilon, \ \ \forall i,$$

$$d_j \geq 0, \ \ \forall j,$$

where I_{ij} denotes sub-indicator values, w_i is the weight of I^{th} indicator. In the case of obtaining more than one efficient DMUs, the following model is presented as the second stage of the decision approach.

$$\min M - k \sum_{e \in EF} d_e$$

subject to (2.25)

$$M - d_j \geq 0,$$

$$\sum_{i=1}^{m} w_i I_{ij} + d_j = 1, \ \ \forall j,$$

$$w_i \geq \varepsilon, \ \ \forall i,$$

$$d_j \geq 0, \ \ \forall j.$$

where *EF* represent the set of efficient DMUs obtained from Formulation (2.24), *k* denotes the discriminating parameter. The advantages of Sun et al. (2020) approach are twofold. First, it provides efficiency in computation and improved discriminating power compared with several models in the literature. However, the methodology has the following shortcomings. First, the model requires a decision analyst to determine the value of *k* subjectively. Second, the final evaluation in their approach may yield lower efficiency scores for some minimax efficient DMUs compared with other DMUs that are considered to be inefficient according to the initial stage of the developed framework.

Finally, it provides no further insight than presenting the input-oriented version of Karsak and Ahiska (2005).

Table 2.3: Advantages and Limitations of the Models that Use Composite Indicators

Author(s)	Year	Advantage(s)	Limitation(s)
Peng et al.	2017	• Considers importance rankings of sub-indicators for constructing CIs. • Proposes two methodologies and provide comparative analysis between them. • Is based on multiple objective programming approach in order to optimize the performance of all DMUs.	• -
Hatefi and Torabi	2018	• Provides a slack analysis for improving CIs of inefficient DMUs • Full ranking of DMUs	• Requires a step size value to be determined by a decision maker. • Requires computational efforts.
Yang et al.	2018	• Provides full ranking of DMUs.	• The output oriented stratification DEA model and the output-oriented super efficiency model are required to solve for employing the final formulation that yields common set of weights. • Combines the output-oriented stratification DEA model and the output-oriented super efficiency model that do not yield common set of weights with the developed model that generates common weights for indicators. • Inconsistent models are provided. • Requirement of three mathematical programming models decreases computational efficiency of the modeling framework.

Table 2.3: Advantages and Limitations of the Models that Use Composite Indicators (cont.)

Sayed et al.	2018	• Provides consistent and stable CI rankings. • Considers two goals.	• Requires three mathematical programming models to be solved
Peiro-Palomino and Picazo-Tadeo; Castillo-Gimenez	2018; 2019	• Provides full ranking of DMUs.	• Requires a parameter to be determined by a decision analyst. • Optimal value of the objective function changes according to the values assigned to the parameter *t*.
Gomez-Vega and Picazo-Tadeo	2019	• Provides full ranking of DMUs. • Improves the approach employed by Peiro-Palomino and Picazo-Tadeo (2018) and Castillo-Gimenez (2019) by eliminating the parameter to be determined by a decision analyst.	• Supposes that the parameter *t* equals to 1.
Omrani et al.	2019a	• Provides classifying the indicators.	• Integrates DEA-CCR model into the developed common weight DEA-based framework. • DEA-CCR model does not yield common set of weights while the proposed model aims to provide common assessment using common weight for the indicators. • Inconsistent models are developed. • Weight dispersion obtained from the abovementioned model is poor. • Full ranking of DMUs is not provided for each class, and thus an additional decision approach apart from DEA is required.

Table 2.3: Advantages and Limitations of the Models that Use Composite Indicators (cont.)

Omrani et al.	2019b	• Provides an enhanced discriminating power and full ranking of DMUs by making use of cross-efficiency-based analysis.	• Requires three mathematical programming models. • Combines common weight model with the models that do not yield common set of weights. • Inconsistent models are developed.
Sun et al.	2020	• Provides efficiency in computation and improved discriminating power compared with several models in the literature.	• Requires a decision analyst to determine the value of *k* subjectively. • May yield lower efficiency scores for some minimax efficient DMUs compared with other DMUs that are considered to be inefficient according to the initial stage of the developed framework. • Provides no further insight than presenting the input-oriented version of Karsak and Ahiska (2005).

2.1.2 Models with Multiple Inputs and Multiple Outputs

Throughout the literature, researchers have developed a number of common weight DEA-based mathematical programming models that are applicable when the data contain multiple inputs and multiple outputs. In this subsection, the problems with multiple inputs and multiple outputs are reviewed.

2.1.2.1 Distance-Based Models

Scholars made use of the concept of deviation from efficiency as a distance measure. Jahanshahloo et al. (2010) proposed two mathematical programming models. First formulation aimed to minimize the sum of deviations from efficiency and obtain the common set of weights. Second formulation defined a special DMU and compares all the other DMUs with it and ranked them. First formulation is as follows:

$$\varDelta^* = \min \sum_{j=1}^{n} \varDelta_j$$

subject to (2.26)

$$\sum_{r=1}^{s} u_r \bar{y}_r - \sum_{i=1}^{m} v_i \bar{x}_i = 0,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \varDelta_j = 0, \qquad \forall j,$$

$$\varDelta_j \geq 0, \qquad \forall j,$$

$$u_r \geq \varepsilon > 0, \qquad \forall r,$$

$$v_i \geq \varepsilon > 0, \qquad \forall i,$$

where $\varDelta_j$ is the deviation from efficiency, $\bar{x}_i = \min\{x_{ij} | j = 1, \dots, n\}$, i=1,2,…,m and $\bar{y}_r = \max\{y_{rj} | j = 1, \dots, n\}$, r=1,2,…,s. Second, Jahanshahloo et al. (2010) developed the following formulation that provides comparison of the DMUs with respect to a special unit.

$$\Delta^* = \min \sum_{j=1}^{n} \Delta_j$$

subject to (2.27)

$$\sum_{r=1}^{s} y_{rj}\, u_r - \sum_{i=1}^{m} x_{ij}\, v_i + \Delta_j = 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} y_{rl}\, u_r - \sum_{i=1}^{m} x_{il}\, v_i = 0,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r \geq \varepsilon > 0, \qquad \forall r,$$

$$v_i \geq \varepsilon > 0, \qquad \forall i,$$

$$\Delta_j \geq 0, \qquad \forall j,$$

where DMU$_l$=(X_l,Y_l) denotes the special DMU. The second model enables to compare the DMUs with respect to a special unit with the requirement of defining the special DMU. Toloo (2012) extended a linear programming model and obtained the single best efficient DMU by providing the following mixed integer linear programming model for cases with multiple inputs and outputs.

$$z^* = \min d_{max}$$

subject to (2.28)

$$d_{\max} - d_j \geq 0, \quad \forall j,$$

$$\sum_{i=1}^{m} v_i\, x_{ij} \leq 1 \qquad \forall j,$$

$$\sum_{r=1}^{n} u_r\, y_{rj} - u_0 - \sum_{i=1}^{m} v_i\, x_{ij} + d_j = 0 \qquad \forall j,$$

$$\sum_{j=1}^{n} \theta_j = n - 1$$

$$d_j \leq M\theta_j \qquad \forall j,$$
$$\theta_j \leq N d_j \qquad \forall j,$$
$$d_j \geq 0 \qquad \forall j,$$
$$\theta_j \in \{0,1\} \qquad \forall j,$$
$$v_{i,} u_r \geq \varepsilon^* \qquad \forall i, r,$$

where $\varepsilon^* = \frac{1}{\max\left\{\sum_{j=1}^{m} x_{ij}, \quad \forall j\right\}}$, M and N are large numbers, $d_{\max}$ is the maximum deviation from efficiency, and u_o is a free variable. The model guarantees to identify the single efficient DMU by solving one mixed integer linear programming model. However, the author guarantees to identify the single best efficient DMU by adding constraints that restrict to obtain more than one efficient DMU. In addition, introducing the binary variable θ_j, and the penalty values M and N are required to solve the model.

Carrillo and Jorge (2016) proposed the following DEA-based common-weight model, which minimizes the total Manhattan (rectangular) distances of each DMU to an ideal point.

$$\min \sum_{j=1}^{n} \bar{D}_j$$

subject to (2.29)

$$\bar{D}_j - \sum_{i=1}^{m} x_{ij} v_i + m \geq 0, \qquad \forall j,$$
$$\bar{D}_j - M + \sum_{r=1}^{s} y_{rj} u_r \geq 0, \qquad \forall j,$$
$$m - \sum_{i=1}^{m} x_{ij} v_i \leq 0, \qquad \forall j,$$
$$M - \sum_{r=1}^{s} y_{rj} u_r \geq 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} y_{rj}u_r - \sum_{i=1}^{m} x_{ij}v_i \leq 0, \qquad \forall j,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

$$m, M, \bar{D}_j \geq 0, \qquad \forall j.$$

where $\bar{D}_j$ refers to the distance of DMU$_j$ to an ideal point. The computational experiments are performed with randomly generated data sets with acceptable CPU times.

Kritikos (2017) aimed to develop a distance based approach by identifying the ideal (IDMU), the anti-ideal (ADMU), the right ideal (RIDMU), the left ideal (LADMU), the average DMU (AVDMU), and constructing the right relative closeness and the left relative closeness indexes that are used to determine the common set of weights. Initially, the author determined the efficiencies of IDMU, ADMU, RIDMU, LADMU, and AVDMU by developing the following formulations.

$$\max \theta_{IDMU} = \sum_{r=1}^{s} u_r y_r^{max}$$

subject to (2.30)

$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \leq 0, \qquad \forall j,$$

$$\sum_{i=1}^{m} v_i\, x_i^{min} = 1,$$

$$u_r, v_i \geq 0, \qquad \forall r, i,$$

where $y_r^{max} = \max_j\{y_{rj}\}$ and $x_i^{min} = \min_j\{x_{ij}\}$.

$$\max \theta_{ADMU} = \sum_{r=1}^{s} u_r y_r^{min}$$

subject to (2.31)

$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \leq 0, \qquad \forall j,$$

$$\sum_{i=1}^{m} v_i\, x_i^{max} = 1,$$

$$u_r, v_i \geq 0, \qquad \forall r, i,$$

where y_r^{min} represents $\min_j\{y_{rj}\}$ and x_i^{max} represents $\max_j\{x_{ij}\}$.

$$\min L\, RC_{AVDMU} + RRC_{AVDMU}$$

subject to (2.32)

$$\sum_{r=1}^{s} u_r\left(y_r^{max} - y_r^{min}\right) + \sum_{i=1}^{m} v_i\left(x_i^{max} - x_i^{min}\right) = 1,$$

$$\sum_{r=1}^{s} u_r\left(y_r^{avg} - y_r^{min}\right) + \sum_{i=1}^{m} v_i\left(x_i^{avg} - x_i^{min}\right) + LRC_{AVDMU} = 1,$$

$$\sum_{r=1}^{s} u_r\left(y_r^{max} - y_r^{avg}\right) + \sum_{i=1}^{m} v_i\left(x_i^{max} - x_i^{avg}\right) + RRC_{AVDMU} = 1,$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq \theta_{IDMU}^{*}, \qquad \forall j,$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \geq \theta_{ADMU}^{*}, \qquad \forall j,$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad \forall j,$$

$$\frac{\sum_{r=1}^{s} u_r y_{rj} + t_j}{\sum_{i=1}^{m} v_i x_{ij}} = 1, \qquad \forall j,$$

$$u_r, v_i \geq 0, \qquad \forall r, i.$$

$$t_j \geq 0, \qquad \forall j,$$

$$LRC_{AVDMU}, RRC_{AVDMU} \geq 0,$$

where $y_r^{max} = \max_j\{y_{rj}\}$, $x_i^{min} = \min_j\{x_{ij}\}$, $x_i^{avg} = \sum_{j=1}^{n} x_{ij}/n$, $y_r^{avg} = \sum_{j=1}^{n} y_{rj}/n$,

$t_j = \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj}$, LRC_{AVDMU} and RRC_{AVDMU} represent left relative closeness of average DMU and right relative closeness of average DMU, respectively. Finally, efficiency score of DMU_j is computed as

$$\theta_j^* = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}}, \qquad \forall j. \tag{2.33}$$

The model requires computing $y_r^{max} = \max_j \{y_{rj}\}$, $y_r^{min} = \min_j \{y_{rj}\}$, $x_i^{max} = \max_j \{x_{ij}\}$ and $x_i^{min} = \min_j \{x_{ij}\}$. Limitations of the proposed approach can be listed as follows. The developed approach requires at least three mathematical programming models to be solved for identifying the best performer. Each model may yield more than one efficient DMU, hence the discriminating power of the proposed approach is questionable.

Lately, Toloo et al. (2019) developed a MILP model, which eliminates the free variable u_0 and large positive number M from the model addressed in Toloo (2012) while all the other variables and the constraints remain the same.

Table 2.4: Advantages and Limitations of Distance-Based Models

Author(s)	Year	Advantage(s)	Limitation(s)
Jahanshahloo et al.	2010	• Enables to compare the DMUs with respect to a special unit with the requirement of defining the special DMU.	• -
Toloo	2012	• Guarantees to identify the single efficient DMU by solving one mixed integer linear program.	• Guarantees to identify the single best efficient DMU by adding constraints that restrict to obtain more than one efficient DMU. • A binary variable and two penalty values are required to solve the model.
Carrillo and Jorge	2016	• Minimizes the total Manhattan (rectangular) distances of each DMU to an ideal point.	• The computational experiments are performed with randomly generated data sets.
Kritikos	2017	• Considers ideal and anti-ideal points.	• Requires at least three mathematical programming models to be solved for identifying the best performer. • May yield more than one efficient DMU. • Poor discriminating power.
Toloo et al.	2019	• Guarantees to identify the single efficient DMU by solving one mixed integer linear program. • Eliminates the binary variable and two penalty values from the model addressed in Toloo (2012).	• Guarantees to identify the single best efficient DMU by adding constraints that restrict to obtain more than one efficient DMU.

2.1.2.2 Two-Stage Models

Throughout the literature, researchers have also developed two-stage common weight DEA-based decision models. Karsak and Ahiska (2007) proposed a two-stage approach that minimizes the maximum deviation from efficiency. The first stage of the developed approach is as follows:

$$\min M$$

subject to (2.34)

$$\begin{aligned}
M - d_j &\geq 0, && \forall j,\\
\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + d_j &= 0, && \forall j,\\
\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i &= 1,\\
u_r, v_i, d_j &\geq 0, && \forall r, i, j.
\end{aligned}$$

For the case of obtaining more than one efficient DMU according to stage I, Karsak and Ahiska (2007) suggested the following common-weight model for identifying the best DMU.

$$\min M - k \sum_{j \in EF} d_j$$

subject to (2.35)

$$\begin{aligned}
M - d_j &\geq 0, && \forall j,\\
\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + d_j &= 0, && \forall j,\\
\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i &= 1,
\end{aligned}$$

$$u_r, v_i, d_j \geq 0, \qquad \forall r, i, j.$$

The advantages of the proposed methodology are threefold. First, it aims to extend the work by Karsak and Ahiska (2005) by integrating multiple inputs into the decision framework. Second, it provides computational savings compared with conventional DEA models. Third, the model has an improved discriminating power compared with conventional DEA. On the other hand, the methodology has several limitations. First, the model requires a decision analyst to determine the value of k subjectively. Second, the final evaluation in their approach may yield lower efficiency scores for some minimax efficient DMUs compared with other DMUs that are considered to be inefficient according to stage I. Thus, the proposed model may not provide consistent results as for the rankings of minimax efficient DMUs considering two minimax efficiency models provided in the study.

Likewise, Liu and Peng (2008) proposed two-stage common weight DEA-based modelling framework and demonstrated the robustness of their approach through simulated data. At the initial stage, their model minimizes the sum of deviations from efficiency while the objective function of the second stage minimizes the sum of output weights and maximizes the sum of input weights. The initial stage is as follows:

$$\Delta^* = \min \sum_{j=1}^{n} \Delta_j$$

subject to (2.36)

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \Delta_j = 0, \qquad \forall j,$$

$$u_r \geq \varepsilon > 0, \qquad \forall r,$$

$$v_i \geq \varepsilon > 0, \qquad \forall i,$$

$$\Delta_j \geq 0, \qquad \forall j,$$

where Δ_j is the deviation from efficiency. Second stage is formulated as follows:

$$\min \sum_{r=1}^{s} u_r - \sum_{i=1}^{m} v_i$$

subject to (2.37)

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + \Delta_j = 0, \qquad \forall j,$$
$$\sum_{j=1}^{n} \Delta_j = \Delta^*,$$
$$u_r \geq \varepsilon > 0, \qquad \forall r,$$
$$v_i \geq \varepsilon > 0, \qquad \forall i,$$
$$\Delta_j \geq 0, \qquad \forall j,$$

The developed approach requires two mathematical programming models that do not guarantee full ranking of DMUs. Moreover, in some cases, all the deviations, and input and output weights equal to 0. Thus, their models fail to provide a full ranking of DMUs and prove to be impractical.

Sun et al. (2013) suggested two model formulations, one of which considers the virtual ideal unit as the reference object and the other takes into account the virtual anti-ideal unit as the reference object. Initially, Sun et al. (2013) developed the following linear programming model.

$$\min \sum_{j=1}^{n} d_j$$

subject to (2.38)

$$\sum_{i=1}^{m} v_i\, x_{ij} - d_j = \sum_{r=1}^{s} u_r y_{rj}, \qquad \forall j,$$

$$\sum_{i=1}^{m} v_i\, x_{min} = 1,$$
$$\sum_{r=1}^{s} u_r\, y_{max} = 1,$$
$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$
$$d_j \geq 0, \qquad \forall j,$$

where $x_{\min} = \min\{x_{ij} | j = 1, \ldots, n\}$, i=1,2,…,m and $y_{\max} = \max\{y_{rj} | j = 1, \ldots, n\}$, r=1,2,…,s. Sun et al. (2013) stated that different software may yield different optimal weights. In order to improve the usefulness of this formulation, they proposed the following non-linear programming model.

$$\max \sum_{i=1}^{m} v_i^2 + \sum_{r=1}^{s} u_r^2$$

subject to (2.39)

$$\sum_{j=1}^{n} \left(\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \right) = D^*,$$
$$\sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, y_{rj} \geq 0, \quad \forall j,$$
$$\sum_{i=1}^{m} v_i\, x_{min} = 1,$$
$$\sum_{r=1}^{s} u_r\, y_{max} = 1,$$
$$u_r, v_i \geq \varepsilon, \qquad \forall r, i.$$

where D^* is the optimal objective function value of the first modeling framework. This model provides full ranking of DMUs via a single mathematical programming model. However, it yields poor weight dispersion for inputs and outputs, and the implementation of the objective function is ambiguous. Alternatively, Sun et al. (2013) employed the following linear programming model that results in super efficiency scores for DMUs.

$$\min \sum_{j=1}^{n} \left(\sum_{i=1}^{m} v_i x_{\max} - \sum_{i=1}^{m} v_i x_{ij} \right) + \sum_{j=1}^{n} \left(\sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} u_r y_{\min} \right)$$

subject to (2.40)

$$\sum_{i=1}^{m} v_i \, x_{ij} - \sum_{r=1}^{s} u_r \, y_{rj} \leq 0, \quad \forall j,$$

$$\sum_{i=1}^{m} v_i \, x_{max} = 1,$$

$$\sum_{r=1}^{s} u_r \, y_{min} = 1,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i.$$

The developed model provides full ranking of DMUs via a single mathematical programming model, whereas it yields poor weight dispersion for inputs and outputs. Likewise, Mavi et al. (2019) developed a two-stage network DEA model that includes desirable and undesirable factors namely inputs, intermediate products, and final outputs. In order to obtain efficiency scores of DMUs, they proposed the following goal programming model, and applied it for identifying the most eco-efficient and eco-innovative Organization for Economic Cooperation and Development (OECD) country.

$$\min \sum_{j=1}^{n} \alpha_j^+ + \alpha_j^- + \beta_j^+ + \beta_j^-$$

subject to (2.41)

$$\sum_{f=1}^{l} p_f^D z_{fj}^D + \sum_{i=k+1}^{m} v_i^{UD} x_{ij}^{UD} - E_j^{1*} \left(\sum_{i=1}^{k} v_i^D x_{ij}^D + \sum_{f=l+1}^{h} p_f^{UD} z_{fj}^{UD} \right) - \alpha_j^+ + \alpha_j^- = 0,$$

$$\sum_{r=1}^{t} u_r^D y_{rj}^D + \sum_{f=l+1}^{h} q_f^{UD} w_{fj}^{UD} - E_j^{2*}\left(\sum_{f=1}^{l} q_f^D w_{fj}^D + \sum_{r=l+1}^{s} u_r^{UD} y_{rj}^{UD}\right) - \beta_j^+ + \beta_j^- = 0,$$

$$\sum_{f=1}^{l} p_f^D z_{fj}^D - \sum_{f=l+1}^{h} p_f^{UD} z_{fj}^{UD} - \sum_{i=1}^{k} v_i^D x_{ij}^D + \sum_{i=k+1}^{m} v_i^{UD} x_{ij}^{UD} \le 0,$$

$$\sum_{r=1}^{l} u_r^D y_{rj}^D - \sum_{r=l+1}^{s} u_r^{UD} y_{rj}^{UD} - \sum_{f=1}^{l} q_f^D w_{fj}^D + \sum_{f=l+1}^{h} q_f^{UD} w_{fj}^{UD} \le 0,$$

$$\sum_{r=1}^{l} u_r^D + \sum_{r=l+1}^{s} u_r^{UD} + \sum_{i=1}^{k} v_i^D + \sum_{i=k+1}^{m} v_i^{UD} = 1,$$

$$u_r^D, u_r^{UD}, q_f^D, q_f^{UD}, p_f^D, p_f^{UD}, v_i^D, v_i^{UD} \ge \varepsilon,$$

$$\alpha_j^+, \alpha_j^-, \beta_j^+, \beta_j^- \ge 0,$$

where D and UD represent the set of desirable and undesirable factors, respectively. p and q represent weights assigned to intermediate outputs and intermediate inputs, respectively. W_j denotes the amount of intermediate product Z_j utilized as inputs for stage 2. E_j^{1*} and E_j^{2*} represent the optimal value of first and second stages efficiencies, respectively. These values are obtained by solving two additional programming models addressed in Mavi et al. (2019).

To identify the overall efficiency for each DMU, the following formulation is proposed.

$$E_j^{*(csw)} = \Big(\textstyle\sum_{r=1}^{l} u_r^{*D} y_{rj}^D - \sum_{f=1}^{l} q_f^{*D} w_{fj}^D + \sum_{f=l+1}^{h} q_f^{*UD} w_{fj}^{UD} + \sum_{f=1}^{l} p_f^{*D} z_{fj}^D - \sum_{f=l+1}^{h} p_f^{*UD} z_{fj}^{UD} + \sum_{i=k+1}^{m} v_i^{*UD} x_{ij}^{UD}\Big) / \Big(\sum_{i=1}^{k} v_i^{*D} x_{ij}^D + \sum_{r=l+1}^{s} u_r^{*UD} y_{rj}^{UD}\Big) \qquad (2.42)$$

The proposed methodology provides full ranking of DMUs. However, it possesses the following limitations. First, a mathematical programming model, which does not yield common weights for inputs and outputs, is required to solve the models that yield E_j^{1*} and E_j^{2*} values. Hence, inconsistent models regarding weighting schemes are developed.

Second, the modelling framework is applicable just when the problem contains desirable and undesirable factors. Third, four mathematical programming models in addition to the formulation above are required to be solved for obtaining efficiency scores of DMUs, which impair computational efficiency.

Table 2.5: Advantages and Limitations of Two-Stage Models

Author(s)	Year	Advantage(s)	Limitation(s)
Karsak and Ahiska	2007	• Extends the work by Karsak and Ahiska (2005) by integrating multiple inputs into the decision framework. • Provides computational savings compared with conventional DEA models. • Improved discriminating power compared with conventional DEA.	• Requires a decision analyst to determine the value of *k* subjectively. • May not provide consistent results as for the rankings of minimax efficient DMUs considering two-stage approach provided in the study.
Liu and Peng	2008	• -	• Does not guarantee full ranking of DMUs. • Fail to provide a ranking of DMUs for some cases.
Sun et al.	2013	• Provides full ranking of DMUs.	• Yields poor weight dispersion for inputs and outputs. • The implementation of the objective function is ambiguous.
Mavi et al.	2019	• Provides full ranking of DMUs.	• Combines a common weight DEA-based model with the models that do not yield common set of weights for inputs and outputs. • Inconsistent models in terms of weighting schemes are developed. • Is applicable just when the problem contains desirable and undesirable factors. • Requires four mathematical programming models to rank DMUs.

2.1.2.3 Multi-Period Models

Hajiagha et al. (2018) proposed a mathematical programming model to determine common set of weights in multi-period DEA.

$$\max \frac{1}{M^+}\left[\frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s} u_r\bar{y}_{rj} - \frac{1}{\phi_u - 1}\sum_{j=1}^{n}\sum_{i=1}^{m} v_i\bar{x}_{ij}\right] + \frac{1}{V^-}\left[V^- - \frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s}(u_r)^2\delta_{rj}^2 - \frac{1}{(\phi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m}(v_i)^2\sigma_{ij}^2\right]$$

subject to (2.43)

$$0 \le \frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s} u_r\bar{y}_{rj} - \frac{1}{\phi_u - 1}\sum_{j=1}^{n}\sum_{i=1}^{m} v_i\bar{x}_{ij} \le M^+,$$

$$\frac{1}{(1-\theta_l)^2}\sum_{j=1}^{n}\sum_{r=1}^{s}(u_r)^2\delta_{rj}^2 - \frac{1}{(\phi_u-1)^2}\sum_{j=1}^{n}\sum_{i=1}^{m}(v_i)^2\sigma_{ij}^2 \ge 0,$$

$$\sum_{r=1}^{s} u_r\left(\bar{y}_{rj} + k\delta_{rj}\right) \le 1, \qquad \forall j,$$

$$\sum_{i=1}^{m} v_i\left(\bar{x}_{ij} - k\sigma_{ij}\right) \ge 1, \qquad \forall j,$$

$$u_r, v_i \ge 0, \qquad \forall r, i,$$

where t, t=1,2,…,T , represent time period, $\bar{x}_{ij} = \sum_{t=1}^{T} x_{ij}^t/T$, $\bar{y}_{rj} = \sum_{t=1}^{T} y_{rj}^t/T$, $\sigma_{ij}^2 = \sum_{t=1}^{T}\left(x_{ij}^t - \bar{x}_{ij}\right)^2/(T-1)$, $\delta_{rj}^2 = \sum_{t=1}^{T}\left(y_{rj}^t - \bar{y}_{rj}\right)/(T-1)$, $\theta_l$and $\phi_u$denote lower and upper bound threshold determined by decision maker. Initially, the following models are solved in order to obtain $M^+$and V^-values.

$$M^+ = \max \frac{1}{1-\theta_l}\sum_{j=1}^{n}\sum_{r=1}^{s} u_r\bar{y}_{rj} - \frac{1}{\phi_u - 1}\sum_{j=1}^{n}\sum_{i=1}^{m} v_i\bar{x}_{ij}$$

subject to (2.44)

$$u_r, v_i \ge 0, \qquad \forall r, i.$$

and

$$V^{-} = \min \frac{1}{(1-\theta_l)^2} \sum_{j=1}^{n} \sum_{r=1}^{s} (u_r)^2 \delta_{rj}^2 - \frac{1}{(\phi_u - 1)^2} \sum_{j=1}^{n} \sum_{i=1}^{m} (v_i)^2 \sigma_{ij}^2$$

subject to (2.45)

$$u_r, v_i \geq 0, \qquad \forall r, i.$$

Finally, the mean relative efficiency of a DMU_0 and its upper bound efficiency are calculated employing the following formulations, respectively.

$$E_0 = \frac{\sum_{r=1}^{s} u_r \bar{y}_{r0}}{\sum_{i=1}^{m} v_i \bar{x}_{i0}} \tag{2.46}$$

$$\bar{E}_0 = \frac{\sum_{r=1}^{s} u_r (\bar{y}_{r0} + k\delta_{r0})}{\sum_{i=1}^{m} v_i (\bar{x}_{i0} - k\sigma_{i0})} \tag{2.47}$$

The developed approach provides full ranking of DMUs over different periods. The model requires computing $\bar{x}_{ij} = \sum_{t=1}^{T} x_{ij}^t / T$, $\bar{y}_{rj} = \sum_{t=1}^{T} y_{rj}^t / T$, $\sigma_{ij}^2 = \sum_{t=1}^{T} \left(x_{ij}^t - \bar{x}_{ij}\right)^2 / (T-1)$, $\delta_{rj}^2 = \sum_{t=1}^{T} \left(y_{rj}^t - \bar{y}_{rj}\right) / (T-1)$. The values $\theta_l$and $\phi_u$are needed to be determined by a decision maker. Furthermore, it requires three mathematical programming models to be solved for identifying the best performing unit, hence its computational efficiency is inadequate. Mavi and Mavi (2019) aimed to improve the abovementioned methodology, and proposed multi-period common weight DEA approach for analyzing environmental efficiency of OECD countries using Malmquist productivity index during 2012-2015. First, they developed the following mathematical programming model that minimizes the sum of deviations from efficiency.

$$\min Z = \sum_{j=1}^{n} \Delta_j$$

subject to (2.48)

$$\sum_{r=1}^{s} u_r y_{rj}^t - \sum_{i=1}^{m} v_i x_{ij}^t + \Delta_j = 0, \quad \forall j,$$
$$\sum_{i=1}^{m} v_i x_{i(\min)}^{t+1} = 1,$$
$$\sum_{r=1}^{s} u_r y_{r(\max)}^{t+1} = 1,$$
$$v_i, u_r, \Delta_j \geq 0, \quad \forall i, r.$$

Second, they proposed the following non-linear mathematical programming model in order to obtain common set of weights.

$$\max \sum_{r=1}^{s} u_r^2 + \sum_{i=1}^{m} v_i^2$$

subject to (2.49)

$$\sum_{j=1}^{n} \left(\sum_{r=1}^{s} u_r y_{rj}^t - \sum_{i=1}^{m} v_i x_{ij}^t \right) = Z^*,$$
$$\sum_{r=1}^{s} u_r y_{rj}^{t+1} - \sum_{i=1}^{m} v_i x_{ij}^{t+1} \leq 0, \quad \forall j,$$
$$\sum_{i=1}^{m} v_i x_{i(\min)}^{t+1} = 1,$$
$$\sum_{r=1}^{s} u_r y_{r(\max)}^{t+1} = 1,$$
$$v_i, u_r \geq 0, \quad \forall i, r.$$

where Z^* is the optimal value of the objective function of the first stage of the developed method. Finally, Malmquist productivity index for each DMU is calculated and full ranking is provided. With regard to the model addressed in Mavi et al. (2019), they improved computational efficiency and integrated multi-period data into the modelling framework. However, the implementation of the objective function is ambiguous. Lately, Li et al. (2020a) proposed a DEA-based common weight multi-period production system for ranking Chinese commercial banks, and achieved full ranking of DMUs. However, non-linear properties of the objective function may cause computational difficulties. Moreover, a mixed binary integer programming model is required to be solved.

Table 2.6: Advantages and Limitations of Multi-Period Models

Author(s)	Year	Advantage(s)	Limitation(s)
Hajiagha et al.	2018	• Provides full ranking of DMUs over different periods.	• Needs two parameters to be determined by a decision maker. • Requires three mathematical programming models to be solved for identifying the best performing unit.
Mavi and Mavi	2019	• Has an improved computational efficiency compared with the approach addressed in Hajiagha et al. (2018). • Provides full ranking of DMUs over different periods.	• The implementation of the objective function is ambiguous.
Li et al.	2020a	• Provides full ranking of DMUs.	• Non-linear properties of the objective function may cause computational difficulties.

2.1.2.4 Input / Output Reduction Models

Over the last decade, scholars employed common weight DEA-based models that aim to reduce inputs or outputs for certain reasons. Amirteimoori and Emrouznejad (2011) aimed to determine the highest possible reduction in inputs and the lowest possible deterioration in outputs by proposing the following mathematical programming model.

$$\min \psi$$

subject to (2.50)

$$\begin{aligned}
&\psi \geq \lambda_p, \\
&\psi \geq \left(\hat{\varphi}_{ip} - \breve{\varphi}_{ip}\right), \qquad i \in I_1, \qquad \forall p, \\
&\psi \geq \left(\hat{\theta}_{rp} - \breve{\theta}_{rp}\right), \qquad r \in O_1, \qquad \forall p, \\
&0 \leq \left(\sum_{r=1}^{s} u_r y_{rp} - \sum_{r \in O_1} \bar{f}_{rp}\right) - \left(\sum_{i=1}^{m} v_i x_{ip} - \sum_{i \in I_1} \bar{c}_{ip}\right) * e_p^* \leq \lambda_p, \qquad \forall p, \\
&\breve{\theta}_{rp} \leq \bar{f}_{rp} - u_r\left(\mu_{rp} F_r\right) \leq \hat{\theta}_{rp}, \qquad r \in O_1, \qquad \forall p, \\
&\breve{\varphi}_{ip} \leq \bar{c}_{ip} - v_i\left(\rho_{ip} C_i\right) \leq \hat{\varphi}_{ip}, \qquad i \in I_1, \qquad \forall p, \\
&\sum_{p=1}^{n} \bar{c}_{ip} = v_i C_i, \qquad i \in I_1, \qquad \forall p, \\
&\sum_{p=1}^{n} \bar{f}_{rp} = u_r F_r, \qquad r \in O_1, \qquad \forall p, \\
&\bar{c}_{ip} \leq v_i x_{ip}, \qquad i \in I_1, \qquad \forall p, \\
&\bar{f}_{rp} \leq u_r y_{rp}, \qquad r \in O_1, \qquad \forall p, \\
&u_r, v_i, \bar{c}_{ip}, \bar{f}_{rp} \geq 0, \qquad \forall i, r, p,
\end{aligned}$$

where the total i^{th} and r^{th} reduced inputs and outputs are stated $C_i: i \in I_1$ and $F_r: r \in O_1$, the total i^{th} and r^{th} reduced input/output of DMU$_j$ are denoted as c_{ip} and f_{rp} with $\sum_{p=1}^{n} c_{ip} = C_i$and $\sum_{p=1}^{n} f_{rp} \leq F_r$, respectively. We assume $\bar{f}_{rp} = u_r f_{rp}$ and $\bar{c}_{ip} = v_i c_{ip}$. In the model, e_p^* is CCR efficiency of DMU$_j$, $\lambda_j, \breve{\varphi}_{ij}, \breve{\theta}_{rj} \hat{\varphi}_{ij}, \hat{\theta}_{rj}$denote the goal achievement variable, lower and upper goal achievement variables for i^{th} reduced input and r^{th} reduced output. At a rational sight, an equitable deterioration is to reduce $c_{ij} = \rho_{ij}C_i$ and $f_{rj} = \mu_{rj}F_r$. $\rho_{ip} = \frac{x_{ip}}{\sum_{j=1}^{n} x_{ij}}$ and $\mu_{rp} = \frac{y_{rp}}{\sum_{j=1}^{n} y_{rj}}$ for each p=1,2,…,n with $\sum_{p=1}^{n} \mu_{rp} = 1$ and $\sum_{p=1}^{n} \rho_{ip} = 1$.

Banks may have to reduce their inputs and outputs due to some budget constraints or financial regulations enforced by the government. In such situations, the proposed model enables to reduce inputs and outputs, and calculate efficiency by using reduced data. On the other hand, the developed approach requires initially employing CCR model that has to be solved n times, where n is the number of DMUs, and then the mathematical programming model above. Hence, it lacks computational efficiency. In addition, the amount of reduction of inputs and outputs has to be determined to solve the model, which is applicable only when inputs and outputs have to be reduced. Amirteimoori and Emrouznejad (2012) proposed an approach to input/output reduction problem that may occur in organizations, and assess the impact of information technology (IT) on bank performance by solving the same problem with Amirteimoori and Emrouznejad (2011). The developed formulation is as

$$\min \psi = \beta_1 \sum_{p=1}^{n} T_p^{+} + \beta_2 \sum_{p=1}^{n} \left(S_p^{+} + S_p^{-}\right)$$

$$+ \beta_3 \sum_{p=1}^{n} \sum_{r \in O_1} \left(\theta_{rp}^{+} + \theta_{rp}^{-}\right) + \beta_4 \sum_{j=p}^{n} \sum_{i \in I_1} \left(\phi_{ip}^{+} + \phi_{ip}^{-}\right)$$

subject to (2.51)

$$\sum_{r=1}^{s} u_r y_{rp} - \sum_{r \in O_1} \bar{f}_{rp} - e_p^{*} \left(\sum_{i=1}^{m} v_i x_{ip} - \sum_{i \in I_1} \bar{c}_{ip} \right) = T_p^{+} - T_p^{-}, \qquad \forall p,$$

$$\sum_{r=1}^{s} u_r y_{rp} - \sum_{r \in O_1} \bar{f}_{rp} - \sum_{i=1}^{m} v_i x_{ip} + \sum_{i \in I_1} \bar{c}_{ip} = S_p^{+} - S_p^{-}, \qquad \forall p,$$

$$\bar{f}_{rp} - \mu_{rp} u_r F_r = \theta_{rp}^{+} - \theta_{rp}^{-}, \qquad r \in O_1, \qquad \forall p,$$

$$\bar{c}_{ip} - \rho_{ip} v_i C_i = \phi_{ip}^{+} - \phi_{ip}^{-}, \qquad i \in I_1, \qquad \forall p,$$

$$\sum_{p=1}^{n} e_p^* \leq \sum_{p=1}^{n}\left(\sum_{r=1}^{s} u_r y_{rp} + \sum_{r\in O_1} \bar{f}_{rp}\right),$$

$$\sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} \leq 0, \qquad p \in EF,$$

$$\sum_{p=1}^{n} \bar{c}_{ip} = v_i C_i, \qquad i \in I_1, \qquad \forall p,$$

$$\sum_{j=1}^{n} \bar{f}_{rp} = u_r F_r, \qquad r \in O_1, \qquad \forall p,$$

$$\bar{c}_{ip} \leq v_i x_{ip}, \qquad i \in I_1, \qquad \forall p,$$

$$\bar{f}_{rp} \leq u_r y_{rp}, \qquad r \in O_1, \qquad \forall p,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall i, r,$$

$$\bar{f}_{rp}, \bar{c}_{ip} \geq 0, \qquad \forall i, r, p,$$

where $T_p^+, T_p^-, S_p^+, S_p^-, \theta_{rj}^+, \theta_{rj}^-, \phi_{ij}^+, \phi_{ij}^-$ are deviational variables that state the deviations above and below the goals, $\beta_1, \beta_2, \beta_3, \beta_4$ denote the importance weight of the respective objectives with $\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1$. CCR efficiency, the amount of reduction of inputs and outputs, the importance weight of the objectives $\beta_1, \beta_2, \beta_3$ and β_4 are required to solve the proposed model. The advantages of this model are twofold. First, the developed model enables input or output reduction when some system constraints force to reduce them. Second, DEA-based mathematical program is formulated as a weighted goal programming model. However, the developed approach requires initially employing CCR model that has to be solved *n* times, where *n* is the number of DMUs, and then the mathematical programming model given above. Hence, it lacks computational efficiency. Moreover, the model is applicable only when inputs and outputs have to be reduced. Hatami-Marbini et al. (2015) determined input and output reduction required for each DMU in order to increase the efficiency of all the DMUs, and evaluates the impact of IT on bank performance by solving the same problem with Amirteimoori and Emrouznejad (2011). The modelling framework is as follows:

$$\min \sum_{j=1}^{n} S_j$$

subject to (2.52)

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + S_j = 1, \qquad \forall j,$$

$$u_r, v_i, S_j \geq \varepsilon, \qquad \forall r, i, j.$$

Let the optimal solution of the programming model above be $\left(u_r^*, v_i^*, S_j^*\right)$, the efficiency score of DMU$_j$ is computed as follows:

$$\theta_j^* = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} = 1 - \frac{S_j^*}{\sum_{i=1}^{m} v_i^* \, x_{ij}}, \qquad \forall j. \tag{2.53}$$

In order to determine the amount of input and output reduction, the following mathematical programming model is developed.

$$\min \zeta = \sum_{j=1}^{n} \sum_{i \in I_1} \left(\alpha_{ij}^- + \alpha_{ij}^+\right) + \sum_{j=1}^{n} \sum_{r \in O_1} \left(\beta_{rj}^- + \beta_{rj}^+\right)$$

subject to (2.54)

$$\left(\sum_{r=1}^{s} u_r y_{rj} - \sum_{r \in O_1} p_{rj}\right) - \theta_j^* \left(\sum_{i=1}^{m} v_i x_{ij} - \sum_{i \in I_1} c_{ij}\right) \geq 0, \qquad \forall j,$$

$$\left(\sum_{r=1}^{s} u_r y_{rj} - \sum_{r \in O_1} p_{rj}\right) - \left(\sum_{i=1}^{m} v_i x_{ij} - \sum_{i \in I_1} c_{ij}\right) \geq 0, \qquad \forall j,$$

$$c_{ij} + \alpha_{ij}^- - \alpha_{ij}^+ = v_i\left(\rho_{ij} C_i\right), \qquad i \in I_1, \qquad \forall j,$$

$$p_{rj} + \beta_{rj}^- - \beta_{rj}^+ = u_r\left(\mu_{rj} P_r\right), \qquad r \in O_1, \qquad \forall j,$$

$$c_{ij} \leq v_i x_{ij}, \qquad i \in I_1, \qquad \forall j,$$

$$p_{rj} \leq u_r y_{rj}, \qquad r \in O_1, \qquad \forall j,$$

$$\sum_{j=1}^{n} c_{ij} = v_i C_i, \qquad i \in I_1,$$

$$\sum_{j=1}^{n} p_{rj} = u_r P_r, \qquad r \in O_1,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

$$c_{ij}, p_{rj}, \alpha_{ij}^{-}, \alpha_{ij}^{+}, \beta_{rj}^{-}, \beta_{rj}^{+} \geq 0, \qquad \forall r, i, j,$$

where $\alpha_{ij}^{-}, \beta_{ij}^{-}, \alpha_{ij}^{+}, \beta_{ij}^{-}$denote negative and positive deviations from reductions with $c_{ij}^{*} = \rho_{ij} C_i$ and $p_{rj}^{*} = \mu_{rj} P_r$, θ_j^{*} refer to the efficiency score of DMU$_j$ obtained from the first stage of the developed approach. The financial amount of input and output reduction, C_i and P_r, and the efficiency of DMU$_j$ obtained from the first stage are required for solving the model. The proposed approach solves a single model, whereas the formulation developed by Amirteimoori and Emrouznejad (2011) requires additional CCR models to be solved for each DMU. The model allows input or output reduction when some financial constraints force to reduce them. On the other hand, the model is applicable only when inputs and outputs have to be reduced.

Table 2.7: Advantages and Limitations of Input/Output Reduction Models

Author(s)	Year	Advantage(s)	Limitation(s)
Amirteimoori and Emrouznejad	2011, 2012	• DEA-based mathematical program is formulated as a weighted goal programming model.	• Requires initially employing CCR. • Lacks computational efficiency. • Applicable only when inputs and outputs have to be reduced.
Hatami-Marbini et al.	2015	• Improves the approach addressed in Amirteimoori and Emrouznejad (2011) by solving the problem by a single mathematical programming model.	• Applicable only when inputs and outputs have to be reduced.

2.1.2.5 Cost Allocation, Resource Allocation, Target Setting Allocation Models

Over the past few years, researchers employed common weight DEA-based modelling frameworks for solving allocation problems. Li et al. (2017) proposed DEA-based models that aim to allocate multiple input resources and determine multiple output targets for the DMUs by considering common weights and efficiency invariance principles. The main topic is to develop two plans that are achieved at the same time by minimizing the distance between them. The case study, which is provided in order to demonstrate the application of the developed framework in addition to a numerical illustration taken from previous literature, is conducted in an urban bus company located in China. The linearized model is as

$$d^* = \min \sum_{j=1}^{n} \left[\sum_{f=1}^{g} \left(a_{fj} + b_{fj} \right) + \sum_{p=1}^{q} \left(\alpha_{pj} + \beta_{pj} \right) \right]$$

subject to (2.55)

$$\sum_{r=1}^{s} u_r y_{rj} + \sum_{p=1}^{q} \tau_{pj}^{cw} = e_j^* \left(\sum_{i=1}^{m} v_i x_{ij} + \sum_{f=1}^{g} \gamma_{fj}^{cw} \right), \qquad \forall j,$$

$$\sum_{j=1}^{n} \tau_{pj}^{cw} = u_{s+p} T_{p,} \qquad \forall p,$$

$$\sum_{j=1}^{n} \gamma_{fj}^{cw} = v_{m+f} R_{f,} \qquad \forall f,$$

$$\sum_{j \in E} \lambda_j^{k*} \gamma_{fj}^{ei} \leq \theta_j^* \gamma_{fk}^{ei}, \qquad \forall f, k \in N,$$

$$\sum_{j \in E} \lambda_j^{k*} \tau_{pj}^{ei} \geq \tau_{pk}^{ei}, \qquad \forall p, k \in N,$$

$$\sum_{j=1}^{n} \tau_{pj}^{ei} = u_{s+p} T_{p,} \qquad \forall p,$$

$$\sum_{j=1}^{n} \gamma_{fj}^{ei} = v_{m+f} R_{f,} \qquad \forall f,$$

$$\gamma_{fj}^{cw} - \gamma_{fj}^{ei} = a_{fj} - b_{fj,} \qquad \forall f, j,$$
$$\tau_{pj}^{cw} - \tau_{pj}^{ei} = \alpha_{pj} - \beta_{pj,} \qquad \forall p, j,$$
$$\tau_{pj}^{cw}, \tau_{pj}^{ei} \geq 0, \qquad \forall p, f, j,$$
$$\gamma_{fj}^{cw}, \gamma_{fj}^{ei} \geq 0, \qquad \forall p, f, j,$$
$$u_r, u_{s+p}, v_i, v_{m+f} \geq \varepsilon, \qquad \forall r, p, i, f,$$
$$a_{fj}, b_{fj}, \alpha_{pj}, \beta_{pj} \geq 0, \qquad \forall f, p, j,$$

where $a_{fj}, b_{fj}, \alpha_{pj}, \beta_{pj}$ are deviation variables, R_f and T_p represent the set of input resources to be allocated and output targets to be achieved, respectively. γ_{fj}^{cw} and τ_{pj}^{cw} denote deviational variables for common weight principle whereas γ_{fj}^{ei} and τ_{pj}^{ei} refer to the deviations for efficiency invariance principle. e_j^* is CCR efficiency, θ_j^* is the objective function value of the envelopment form of CCR model, λ_j^{k*} are intensity variables obtained from the envelopment model, in other words the dual form of CCR. The proposed model in this paper is based on the constant return to scale, however the model may yield infeasible solutions in some circumstances if the variable return to scale is encountered. Likewise, Jahanshahloo et al. (2017) proposed a DEA-based method for fixed allocation based on efficiency invariance by providing a case study that is conducted in Iranian banking sector, and provided full ranking of DMUs. They initially introduced a DEA model that does not yield common set of weights for inputs and outputs, and must be solved n times, where n is the number of DMUs. Afterwards, three additional common weight models are employed in order to reach the final allocation plan. Thus, n+3 programming models are to be solved. In addition, inconsistent models are developed.

Li et al. (2018) aimed to solve fix cost allocation problem in banking sector by introducing DEA-game cross-efficiency model that yields common set of weights for inputs and outputs. The following linearized goal programming model is presented.

$$\min \rho$$

subject to (2.56)

$$\sum_{r=1}^{s} \pi_r y_{rj} + \phi_j^+ - \phi_j^- = \left({\sum_{d=1}^{n} \hat{E}_{dj}} \big/ {n} + \varphi_j \right) \left(\sum_{i=1}^{m} \tau_i x_{ij} + r_j \right), \qquad \forall j,$$

$$\sum_{r=1}^{s} \pi_r + \sum_{i=1}^{m} \tau_i + \tau_{m+1} = 1,$$

$$\sum_{j=1}^{n} r_j = \tau_{m+1} R,$$

$$\phi_j^+, \phi_j^- \le \rho, \qquad \forall j,$$

$$\phi_j^+, \phi_j^-, r_j, \pi_r, \tau_i, \tau_{m+1} \ge 0, \qquad \forall i, r, j,$$

where $\sum_{d=1}^{n} \hat{E}_{dj}/n + \varphi_j$ represents the original average cross-efficiency, $\varphi_j$is coalition surplus, ϕ_j^+andϕ_j^-denote non-negative deviational variables, R refers to the fixed cost to be fully allocated. The allocation plan is provided by taking into account the interaction among the DMUs via cross-efficiency analysis-based goal programming model, however constructing cross-efficiency matrix requires computational efforts.

Likewise, Wu et al. (2018) introduced a common weight DEA approach for resource allocation and target setting for highway transportation systems in 30 provincial-level regions in China. The developed approach has two advantages. First, it guarantees minimum acceptable satisfaction degree for all DMUs. Second, the allocation plan is completed via solving one single linear programming model, hence the approach has an improved computational efficiency compared with the other cost allocation problems in the literature. Likewise, Li et al. (2019) aimed to solve cost allocation problem in banking sector by introducing DEA-based two-stage network framework. The modeling framework requires two free variables for being able to solve the model.

Ghazi and Lotfi (2019) proposed the following linearized model in order to solve budget allocation problem in Iranian has industry.

$$\max \sum_{j=1}^{n} \left(\sum_{r=1}^{s} u_r y_{rj} - \bar{b}_j - \sum_{i=1}^{m} v_i x_{ij} \right)$$

subject to (2.57)

$$\sum_{r=1}^{s} u_r y_{rj} - \bar{b}_j - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad \forall j,$$

$$\sum_{j=1}^{n} \left(v_1 x_{1j} + \bar{b}_j \right) = v_1 B,$$

$$v_1 L_j \leq \bar{b}_j \leq v_1 U_j, \qquad \forall j,$$

$$v_1 x_{1j} + \bar{b}_j \geq v_1 A_j, \qquad \forall j,$$

$$\bar{b}_j \qquad free, \qquad \forall j,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i.$$

The proposed model aims to allocate the total budget, however it requires free variable $\bar{b}_j$, which causes to decrease computational efficiency of the model, however this approach is more efficient in computation than Li et al. (2019). Similarly, Li et al. (2020b) aimed to improve their earlier approach by proposing the following goal programming model in order to allocate the fixed cost, and illustrated the developed methodology based on the efficiency evaluation of 27 bank branches.

$$\max \rho$$

subject to (2.58)

$$\frac{\left(R_j^{max} + R_j^{min} - 2R_j \right)}{\left(2\left(R_j^{max} - R_j^{min} \right) \right)} \geq \rho, \quad \forall j,$$

$$R_j = \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0, \quad \forall j,$$

$$\sum_{i=1}^{m} v_i - v_{m+1} = -1,$$

$$\sum_{j=1}^{n} R_j = R,$$

$$R_j^{min} \leq R_j \leq R_j^{max}, \quad \forall j,$$

$$v_{m+1}, u_r, v_i, R_j \geq 0, \quad \forall r, i, j.$$

where R is the fixed cost, v_{m+1} is the relative importance of allocated cost R_j. The developed model possesses non-linear properties as well as requires a free variable u_0. Moreover, only a single objective is considered.

On the other hand, An et al. (2019) combined game theory with network DEA to solve cost allocation problem by proposing a three-stage framework. The authors demonstrated the applicability of the developed system providing a proof, however, they did not incorporate real data into numerical illustration, utilized only an illustrative example to verify the proposed model. In a similar manner, Lin and Chen (2020) proposed a common weight DEA approach for fixed cost allocation. The developed approach can be applied under variable return to scale conditions and zero input data, hence wide application range is provided. However, the final cost allocation is provided after solving 4 mathematical programming models and the approach is not applied to a real-life example.

Table 2.8: Advantages and Limitations of Cost Allocation, Resource Allocation, Target Setting Allocation Models

Author(s)	Year	Advantage(s)	Limitation(s)
Li et al.	2017	• -	• The developed model is based on the constant return to scale, however the model may yield infeasible solutions in some circumstances if the variable return to scale is encountered.
Jahanshahloo et al.	2017	• Provides full ranking of DMUs.	• Introduces a DEA model that does not yield common set of weights for inputs and outputs, and must be solved *n* times, where n is the number of DMUs. • Three additional common weight models are employed in order to reach the final allocation plan. • Inconsistent models are developed.
Li et al.	2018	• Provides a fixed cost allocation by taking into account the interaction among the DMUs via cross-efficiency analysis-based goal programming model.	• Requires computational efforts for constructing cross-efficiency matrix. • Inconsistent models are developed.
Wu et al.	2018	• Guarantees minimum acceptable satisfaction degree for all DMUs. • The allocation plan is completed via solving one single linear programming model. • Has an improved computational efficiency compared with the other cost allocation problems in the literature.	• -
Li et al.	2019	• Introduces DEA-based two-stage network framework.	• Requires two free variables.

Table 2.8: Advantages and Limitations of Cost Allocation, Resource Allocation, Target Setting Allocation Models (cont.)

Ghazi and Lotfi	2019	• Allocates the total budget. • Improves the models addressed in Li et al. (2019) in terms of computational efficiency.	• Requires a free variable
An et al.	2019	• Combines game theory with network DEA.	• Does not provide a real-life example.
Lin and Chen	2020	• Can be applied under variable return to scale conditions and zero input data. • Provides wide application range.	• Requires solving 4 mathematical programming models. • Does not provide a real-life example.
Li et al.	2020b	• Improves their earlier study by proposing the following goal programming model in order to allocate the fixed cost.	• Considers only a single objective.

2.1.2.6 Other Models

Carrillo and Jorge (2017) introduced the following linearized mathematical programming model in order to evaluate the efficiency of DMUs.

$$
\begin{aligned}
&\min \sum_{j=1}^{n} \big(X_j(v) - m\big) + \big(M - Y_j(u)\big) \\
&\text{subject to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.59)\\
&Y_j(u) - X_j(v) \leq 0, \qquad \forall j, \\
&M - Y_j(u) \geq 0, \qquad \forall j, \\
&m - X_j(v) \leq 0, \qquad \forall j, \\
&u_r, v_i \geq \varepsilon, \qquad \forall r, i,
\end{aligned}
$$

where $X_j(v) = \sum_{i=1}^{m} v_i x_{ij}$, $Y_j(u) = \sum_{r=1}^{s} u_r y_{rj}$. The model provides full ranking of DMUs while resulting in poor weight dispersion. Wang et al. (2017) ranked 14 international passenger airlines in order to illustrate the performance and credibility of the developed DEA-based methodology that employs weight restriction in common assessment for the inputs and outputs. Initially, the authors proposed the following mathematical programming models, which restrict virtual weights.

$$
\begin{aligned}
&\max \theta_j = \sum_{r=1}^{s} u_r y_{rj} \\
&\text{subject to} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (2.60)\\
&\sum_{i=1}^{m} v_i x_{ij} = 1, \\
&\sum_{r=1}^{s} u_r \, y_{rj} - \sum_{i=1}^{m} v_i \, x_{ij} \leq 0, \qquad \forall j, \\
&v_i x_{ij} \geq \alpha, \qquad \forall i, \\
&u_r y_{rj} \geq \beta \theta_{low}, \qquad \forall r.
\end{aligned}
$$

To obtain common set of weights for inputs and outputs, the authors proposed the following model.

$$\max \sum_{r=1}^{s}\left(u_r \sum_{j=1}^{n} y_{rj}\right) - \sum_{i=1}^{m}\left(v_i \sum_{j=1}^{n} x_{ij}\right)$$

subject to (2.61)

$$\sum_{i=1}^{m} v_i x_{ij} = 1,$$

$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \leq 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r\, y_{rj} - \theta_j^* \sum_{i=1}^{m} v_i\, x_{ij} = 0,$$

$$v_i x_{ij} \geq \alpha, \qquad \forall i,$$

$$u_r y_{rj} \geq \beta \theta_{low}, \qquad \forall r.$$

where θ_j^* denotes the value of objective function of the Formulation (2.60) that refers to the optimistic self-evaluation efficiency of DMU$_j$. α, β, and θ_{low} represent the constants that are determined by the decision makers. To obtain a set of common weights for the inputs and outputs, the weights derived from the above models are rescaled as

$$v_i' = v_i . \left\|X_j\right\| . \cos \alpha_j\,, \qquad \forall i, \tag{2.62}$$

$$u_r' = u_r . \left\|X_j\right\| . \cos \alpha_j\,, \qquad \forall r, \tag{2.63}$$

where $\left\|X_j\right\|$ refers to the module of input vector $X_j = \left(x_{1j}, x_{2j}, \dots, x_{mj}\right)$ of DMU$_j$, α_j is the intersection angle between input vector X_j and the weight vector $v_j = (v_1, v_2, \dots, v_m)$. Afterwards, common set of weights for inputs and outputs are obtained from the following equations, respectively.

$$\bar{v}_i^* = \sum_{i=1}^{n} c_j v_i' \Big/ \sum_{i=1}^{n} c_j\,, \qquad \forall i, \tag{2.64}$$

$$\bar{u}_r^* = \left. \sum_{i=1}^{n} c_j u_r^{'} \middle/ \sum_{i=1}^{n} c_j \right., \qquad \forall r. \tag{2.65}$$

where c_j denotes the consensus degree identified solving an equation. Finally, the efficiency scores that provides the ranking of all the DMUs are determined employing the formulation below.

$$A_k^* = \left. \sum_{r=1}^{s} \bar{u}_r^* y_{rj} \middle/ \sum_{i=1}^{m} \bar{v}_i^* x_{ij} \right. \tag{2.66}$$

The developed approach requires solving two mathematical programming models, two rescaling equations, calculating the consensus degree to obtain common weights for inputs and outputs. Thus, computational efficiency of the proposed decision approach remains inadequate when comparing to the numerous other models provided in the literature. Moreover, the model requires a decision analyst to determine the values of α, β, and θ_{low}, subjectively.

Rezaeiani and Foroughi (2018) developed a DEA-based approach to rank efficient DMUs based on the share of each efficient unit in reference frontier of inefficient ones. Initially, they employed the simple additive model for identifying the efficient DMUs as

$$\max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$$

subject to (2.67)

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{i0,} \qquad \forall i,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0,} \qquad \forall r,$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$s_i^-, s_r^+ \geq 0, \qquad \forall i, r,$$
$$\lambda_j \geq 0, \qquad \forall j.$$

DMU$_0$ is efficient if and only if $s_i^{-*} = s_i^{+*} = 0$. The set of efficient and inefficient DMUs are denoted by E and IE, respectively. The efficient DMUs are ranked regarding their reference frontier share by employing the following model.

$$\lambda_{p0} = \max \lambda_0$$

subject to (2.68)

$$\begin{gathered}
\sum_{j \in E} \lambda_j x_{ij} \leq x_{ip}, \qquad \forall i, \\
\sum_{j \in E} \lambda_j y_{rj} \leq y_{rp}, \qquad \forall r, \\
\sum_{j \in E} \lambda_j = 1 \\
\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + u_0 = d_j. \qquad j \in E, \\
\lambda_j \leq t_j, \qquad j \in E, \\
d_j \leq M(1 - t_j), \qquad j \in E, \\
v_i, u_r \geq 1, \qquad \forall i, r, \\
u_0 \ \text{free}, \\
\lambda_j, d_j \geq 0, \qquad j \in E, \\
t_j \in \{0,1\}.
\end{gathered}$$

where M is a sufficiently large positive number, u_0 is a free variable. The limitations of the developed approach are twofold. First, introducing the binary variable t_j is required to solve the model. Second, the reason of combining the simple additive model and Formulation (2.68) is questionable. In addition, the cause of considering input and output weights greater than or equal to 1 is not discussed.

Zhu et al. (2018) calculated the attractiveness and progress of DMUs for different levels of efficiency frontier by combining the context dependent DEA with Dinkelbach algorithm, which yields common weights for inputs and outputs. In the first step, output-oriented CCR model is solved to determine the levels of efficiency frontier. In the second step, maximum efficiency is obtained employing the following formulation.

$$\min \gamma$$

subject to (2.69)

$$uy_{l_j} - (\theta_{l_j} - \gamma)vx_{l_j} \geq 0, \qquad l_j \in E^l,$$
$$uy_{l_j} - vx_{l_j} \leq 0, \qquad l_j \in E^l,$$
$$u_r, v_i, \gamma \geq 0, \qquad \forall r, i, j,$$

where θ_{l_j} is the maximum efficiency value, which is required for the next step, E^l denotes l^{th} efficiency frontier. In the third step, the following model is developed for determining common set of weights.

$$\min s$$

subject to (2.70)

$$(\theta_{l_j} - \gamma)vx_{l_j} - uy_{l_j} \leq s, \qquad l_j \in E^l,$$
$$uy_{l_j} - vx_{l_j} \leq 0, \qquad l_j \in E^l,$$
$$u_r, v_i, \gamma \geq 0, \qquad \forall r, i, j.$$

Finally, the attractiveness and progress scores are computed through the ratio of the total weighted outputs to the total weighted inputs by defining different evaluation contexts for attractiveness and progress values. However, the combination of the output-oriented CCR model, which does not generate common set of weights, with two common weight models leads inconsistency. Omrani et al. (2020) introduced a goal programming-based approach for evaluating Iranian electricity distribution companies and provided full ranking of DMUs. For that reason, DEA-CCR model in addition to a DEA-based model that does not yield common set of weights for inputs and outputs are required to be solved for employing the goal programming model that obtains common weights. DEA-CCR model and the other DEA-based model are solved *n* times, where *n* is the number of DMUs. The modelling framework needs 2*n*+1 mathematical programming models in order to obtain common set of weights, which impairs computational efficiency. Moreover, combining the common weight approach with the models that do not yield common weights for inputs and outputs causes inconsistency. Furthermore, a decision analyst is required to determine two preference vectors that are integrated into the modelling framework.

Recently, Shirdel and Ramezani-Tarkhorani (2020) proposed the following linear programming model in order to improve inadequate weight dispersion of the approach addressed in Liu and Peng (2008).

$$\min \sum_{j=1}^{n}\sum_{i=1}^{m} v_i x_{ij} - 2\sum_{j=1}^{n}\sum_{r=1}^{s} u_r y_{rj}$$

subject to (2.71)

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r, v_i \geq 0.$$

The abovementioned model is applied to the same dataset containing 4 inputs and 2 outputs addressed in Liu and Peng (2008). Formulation (2.71) takes into account 2 inputs and 2 outputs while the approach of Liu and Peng (2008) equals the weight of second output to 0.000004 and all the other weights to ϵ. Thus, the approach addressed in Shirdel and Ramezani-Tarkhorani (2020) provides enhanced weight dispersion, but it still ignores two inputs according to the weighting scheme.

Table 2.9: Advantages and Limitations of Other Models

Author(s)	Year	Advantage(s)	Limitation(s)
Carrillo and Jorge	2017	• Provides full ranking of DMUs.	• Results in poor weight dispersion for inputs and outputs.
Wang et al.	2017	• Employs weight restriction in common assessment for the inputs and outputs.	• Requires solving two mathematical programming models, two rescaling equations, in addition to calculating the consensus degree to obtain common weights for inputs and outputs. • Requires three parameters to be determined by a decision maker.
Rezaeiani and Foroughi	2018	• Rank efficient DMUs based on the share of each efficient unit in reference frontier of inefficient ones.	• Requires a binary variable to solve the model. • The reason of combining the simple additive model and DEA model is questionable. • Does not discuss the reason of considering input and output weights greater than or equal to 1.
Zhu et al.	2018	• Calculates the attractiveness and progress of DMUs for different levels of efficiency frontier.	• Combines DEA-CCR model with common weight DEA-based model. • Inconsistent models in terms of weighting scheme are developed.

Table 2.9: Advantages and Limitations of Other Models (cont.)

Omrani et al.	2020	• Introduces a goal programming-based approach.	• Requires 2n+1 models to obtain common set of weights, where n is the number of DMUs. • Combines DEA-CCR model with common weight DEA-based model. • Inconsistent models are developed. • Requires a decision analyst to determine two preference vectors.
Shirdel and Ramezani-Tarkhorani	2020	• Improves inadequate weight dispersion of the model addressed in Liu and Peng (2008).	• Provides enhanced weight dispersion, but it still ignores several inputs.

2.2 Non-Deterministic Models

In this subsection, non-deterministic common weight DEA-based models are reviewed. If the inputs and outputs are evaluated through different weights for each DMU in the presence of imprecision, the same factor will be accorded differing weights, and thus it may be unacceptable. Therefore, obtaining common set of weights for the inputs and outputs is very important (Saati & Memariani, 2005). For that reason, Saati and Memariani (2005) determined common set of weights for different α-cut values by defining bounds on inputs and outputs, and obtained efficiency values of DMUs as fuzzy numbers.

Over the last few years, researchers have focused on solving the problems that contain imprecise data, i.e. interval data and fuzzy data, by proposing non-deterministic common weight DEA-based models. Hadi-Vencheh et al. (2014) proposed a common weight DEA-based approach for solving fixed cost allocation problem in fuzzy network systems. Initially, a linearized mathematical programming model is solved to obtain fuzzy efficiency scores for all DMUs. Afterwards, a goal programming model is presented to allocate fixed cost. The developed approach enables to add new sub-DMUs of each DMU. However, the robustness of the proposed approach is not illustrated by providing a real-life example. Shirdel et al. (2016) developed a DEA-based model that evaluates DMUs with interval data. The modeling framework is as

$$\min \sum_{j=1}^{n} d_j$$

subject to (2.72)

$$u_r y_{rj}^{u} - v_i x_{ij}^{l} \leq 0, \qquad \forall j,$$
$$u_r y_{rj}^{l} - v_i x_{ij}^{u} + d_j = 0, \qquad \forall j,$$
$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$
$$d_j \geq 0, \qquad \forall j,$$
$$u_r, v_i \geq \varepsilon.$$

where y_{rj}^{u} and y_{rj}^{l}denote the upper bound and the lower bound of y_{rj}, x_{ij}^{u} and x_{ij}^{l}refer to the upper bound and the lower bound of x_{ij}, respectively.

The worst possible efficiency of DMU*j* is as

$$E_j^l = \frac{\sum_{r=1}^{s} u_r^* y_{rj}^l}{\sum_{i=1}^{m} v_i^* x_{ij}^u} \tag{2.73}$$

The best possible efficiency of DMU*j* is as

$$E_j^u = \frac{\sum_{r=1}^{s} u_r^* y_{rj}^u}{\sum_{i=1}^{m} v_i^* x_{ij}^l} \tag{2.74}$$

The average of efficiency for DMU*j* is as follows:

$$\bar{E}_j = \frac{E_j^l + E_j^u}{2} \tag{2.75}$$

The approach provides full ranking of DMUs while the average of the lower and upper bounds of interval efficiency is the main criterion for ranking DMUs. Additionally, the proposed methodology requires calculating the worst, best, and average efficiency for all the DMUs. The model is applicable when the problem has interval data. Salahi et al. (2016) proposed two DEA-based formulations that can be employed in the presence of interval data. First, the following model is introduced to obtain the efficiency score of each DMU.

$$\max \sum_{r=1}^{s} \bar{y}_{r_0} u_r$$

subject to (2.76)

$$\sum_{r=1}^{s} \underline{y}_{rj} u_r - \sum_{i=1}^{m} \bar{x}_{ij} v_i \leq 0, \qquad \forall j,$$

$$\sum_{i=1}^{m} \underline{x}_{i_0} v_i \leq 1,$$

$$\sum_{i=1}^{m} \bar{x}_{i_0} v_i \geq 1,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i.$$

where $y_{rj} \in [\underline{y}_{rj}, \bar{y}_{rj}]$ and $x_{ij} \in [\underline{x}_{ij}, \bar{x}_{ij}]$ denote the interval output and interval input data, respectively.

Second, the following common weight DEA-based model is proposed in order to obtain common set of weights.

$$\min \sum_{j=1}^{n} \left(\theta_j \sum_{i=1}^{m} \bar{x}_{ij} v_i - \sum_{r=1}^{s} \underline{y}_{rj} u_r \right)$$

subject to (2.77)

$$\sum_{r=1}^{s} \bar{y}_{rj} u_r - \sum_{i=1}^{m} \underline{x}_{ij} v_i \leq 0, \qquad \forall j,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i.$$

where θ_j refers to the efficiency value of the DMU$_j$ obtained from the first stage of the developed approach.

Finally, efficiency scores of DMUs based on common weights are calculated by

$$\theta_j^{'} = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} \qquad (2.78)$$

The efficiency value of the DMU$_j$ (θ_j) is required to solve the proposed methodology, which is applicable for the problems with interval data. On the other hand, the developed approach possesses several limitations. First, first stage is required to be solved for *n* times, where *n* is the number of DMUs, and then, second stage is solved for obtaining common set of weights of the inputs and the outputs. Subsequently, efficiency score for each DMU is computed. Hence, their approach solves *n*+1 models to identify the best performing DMU. Throughout the literature, there are several approaches (Toloo, 2012; Sun et al., 2013; Toloo, 2013) that yield a single efficient DMU via one common weight DEA-based mathematical programing model. Thus, the approach of Salahi et al. (2016) lacks computational efficiency compared with these models. Second, first stage of the developed approach computes different set of weights for each DMU whereas second stage yields common set of weights. Thus, inconsistent models are developed. Third, the data set is in fact not interval data. The other authors that utilize the same data set consider them as crisp numbers (Kao & Yang, 1992; Kao & Hung, 2005; Wu et al., 2016). Salahi et al. (2016) set the negative and positive deviations from the data to 1. Likewise, Aghayi et al. (2016) proposed the following programming model that is applicable when the problem contains interval data, and conducted a case study in the banking industry.

$$\min \sum_{j=1}^{n} \rho_j$$

subject to (2.79)

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + \rho_j = 0, \ \ \forall j,$$

$$y_{rj}^{U} - y_{rj} - z_{rj}^{y} \gamma_j^{y} - p_{rj} \geq 0, \ \ \forall r, j,$$

$$y_{rj}^{L} - y_{rj} \leq 0,$$

$$x_{ij}^{L} - x_{ij} + z_{ij}^{x} \gamma_j^{x} + q_{ij} \leq 0, \ \ \forall i, j,$$

$$x_{ij}^{U} - x_{ij} \geq 0, \ \ \forall i, j,$$

$$z_j^{y} = \sum_{r=1}^{s} z_{rj}^{y}, \ \ \forall j,$$

$$z_j^x = \sum_{i=1}^{m} z_{ij}^x, \ \ \forall j,$$

$$z_j^y + p_{rj} \geq y_{rj}^U - y_{rj}^L, \ \ \forall r, j,$$

$$z_j^x + q_{ij} \geq x_{ij}^U - x_{ij}^L, \ \ \forall i, j,$$

$$x_{ij}, y_{rj}, z_{ij}^x, z_{rj}^y, z_j^x, z_j^y, q_{ij}, p_{rj}, \rho_j \geq 0, \ \ \forall i, r, j,$$

$$v_i, u_r \geq \varepsilon, \ \ \forall i, r.$$

where $z_j^x, z_j^y, q_{ij}, p_{rj}$ are the variables utilized to assign fixed values to the inputs and outputs in their interval, γ_j^x and γ_j^y are adjusting parameters, ρ_j represents the deviation from efficiency, and L and U denote the lower and upper bounds of the interval of the inputs and outputs, respectively. The developed model provides full ranking of DMUs, however, it necessitates the parameters to be determined by a decision maker. Moreover, x_{ij} and y_{rj}, which represent the data for respective inputs and outputs, are considered as variables and included to non-negativity constraints. Hu et al. (2017) proposed a distance based approach that reduces multiple objective fuzzy DEA model into an auxiliary bi-objective fuzzy optimization problem in order to identify the compromise solution of the fuzzy DEA model. The developed framework, which is applicable when crisp, fuzzy, and ordinal data are incorporated into decision approach, yields common weights for inputs and outputs, thus a common evaluation is provided. The approach possesses the following limitations. Initially, the distance parameter is required to be determined by a decision maker, subjectively. The positive ideal solution, negative ideal solution, distance to the positive ideal solution, distance to the negative ideal solution, and degree of satisfactory level are obtained by solving one mathematical programming model for each of them. Hence, at least five formulations are solved to obtain the efficiency scores of DMUs, computational efficiency of the proposed decision framework remains inadequate.

Wen et al. (2018) proposed the following modified minimax efficiency model based on common weight DEA taking into account interval or imprecise data for project selection problem.

$$\min M$$

subject to (2.80)

$$M - d_j \geq 0, \qquad \forall j,$$

$$\sum_{r=1}^{s}\left[u_r y_{rj}^L + p_{rj}\left(y_{rj}^U - y_{rj}^L\right)\right] - \sum_{i=1}^{m}\left[v_i x_{ij}^L + q_{ij}\left(x_{ij}^U - x_{ij}^L\right)\right] + d_j = 0, \qquad \forall j,$$

$$\sum_{j=1}^{n}\delta_j = n - 1,$$

$$\delta_j - d_j\beta_j = 0, \qquad \forall j,$$

$$p_{rj} - u_r \leq 0, \qquad \forall r, j,$$

$$q_{ij} - v_i \leq 0, \qquad \forall i, j,$$

$$\varepsilon \leq u_r \leq 1, \qquad \forall r,$$

$$\varepsilon \leq v_i \leq 1, \qquad \forall i,$$

$$p_{rj} \geq 0, \qquad \forall r, j,$$

$$q_{ij} \geq 0, \qquad \forall i, j,$$

$$\delta_j = \{0,1\}, \qquad \forall j,$$

$$\beta_j \geq 1,$$

where $y_{rj} \in [y_{rj}^L, y_{rj}^U]$ and $x_{ij} \in [x_{ij}^L, x_{ij}^U]$. The model guarantees to identify the single efficient DMU by solving one mixed integer linear programming model. However, the author guarantees to obtain the single best performer by adding constraints that restrict to obtain more than one efficient DMU. In addition, introducing the binary variable δ_j is required to solve the model. Hatami-Marbini and Saati (2018) presented a common weight DEA-based approach for two-stage structures that enable efficiency evaluation in fuzzy environment.

Let $\tilde{x}_{ij} = \left(x_{ij}^m, x_{ij}^\alpha, x_{ij}^\beta\right)$, $\tilde{y}_{rj} = \left(y_{rj}^m, y_{rj}^\alpha, y_{rj}^\beta\right)$, and $\tilde{z}_{dj} = \left(z_{dj}^m, z_{dj}^\alpha, z_{dj}^\beta\right)$, the authors developed the following programming model.

$$\max u_r$$

subject to (2.81)

$$\sum_{i=1}^{m} \bar{x}_{ij} \leq 1, \qquad \forall j,$$
$$\sum_{d=1}^{p} \bar{z}_{dj} - \sum_{i=1}^{m} \bar{x}_{ij} \leq 0, \qquad \forall j,$$
$$\sum_{r=1}^{s} \bar{y}_{rj} - \sum_{d=1}^{p} \bar{z}_{dj} \leq 0, \qquad \forall j,$$
$$u_r\left(y_{rj}^m - (1-\gamma)y_{rj}^\alpha\right) \leq \bar{y}_{rj} \leq u_r\left(y_{rj}^m + (1-\gamma)y_{rj}^\beta\right), \qquad \forall r, j,$$
$$v_i\left(x_{ij}^m - (1-\gamma)x_{ij}^\alpha\right) \leq \bar{x}_{ij} \leq v_i\left(x_{ij}^m + (1-\gamma)x_{ij}^\beta\right), \qquad \forall i, j,$$
$$w_d\left(z_{dj}^m - (1-\gamma)z_{dj}^\alpha\right) \leq \bar{z}_{dj} \leq w_d\left(z_{dj}^m + (1-\gamma)z_{dj}^\beta\right), \qquad \forall d, j,$$
$$\bar{y}_{rj}, \bar{x}_{ij}, \bar{z}_{dj}, u_r, v_i, w_d \geq 0, \qquad \forall r, i, d, j,$$

where z_{dj} are called intermediate products that are the outputs of first stage and the inputs of second stage, w_d denotes the weight of intermediate product, $\gamma \in [0,1]$is a parameter. The upper bound on input weights and intermediate measure weights are calculated changing the objective function to $\max v_i$ and $\max w_d$, respectively.

$$\max Q$$

subject to (2.82)

$$\sum_{d=1}^{p} \bar{z}_{dj} - \sum_{i=1}^{m} \bar{x}_{ij} \leq 0, \qquad \forall j,$$
$$\sum_{r=1}^{s} \bar{y}_{rj} - \sum_{d=1}^{p} \bar{z}_{dj} \leq 0, \qquad \forall j,$$
$$u_r\left(y_{rj}^m - (1-\gamma)y_{rj}^\alpha\right) \leq \bar{y}_{rj} \leq u_r\left(y_{rj}^m + (1-\gamma)y_{rj}^\beta\right), \qquad \forall r, j,$$
$$v_i\left(x_{ij}^m - (1-\gamma)x_{ij}^\alpha\right) \leq \bar{x}_{ij} \leq v_i\left(x_{ij}^m + (1-\gamma)x_{ij}^\beta\right), \qquad \forall i, j,$$
$$w_d\left(z_{dj}^m - (1-\gamma)z_{dj}^\alpha\right) \leq \bar{z}_{dj} \leq w_d\left(z_{dj}^m + (1-\gamma)z_{dj}^\beta\right), \qquad \forall d, j,$$
$$Qu_r^u \leq u_r \leq (1-Q)u_r^u, \qquad \forall r,$$
$$Qv_i^u \leq v_i \leq (1-Q)v_i^u, \qquad \forall i,$$
$$Qw_d^u \leq w_d \leq (1-Q)w_d^u, \qquad \forall d,$$

$$\bar{y}_{rj}, \bar{x}_{ij}, \bar{z}_{dj} \geq 0, \qquad \forall r, i, d, j.$$

The upper bounds of system and process efficiencies of DMU_0 are defined as

$$(E_0)_\gamma^U = \sum_{r=1}^{s} u_r^* (y_{r0})_\gamma^U \Bigg/ \sum_{i=1}^{m} v_i^* (x_{i0})_\gamma^L \tag{2.83}$$

$$(E_0^1)_\gamma^U = \sum_{d=1}^{p} w_d^* \bar{z}_{d0} \Bigg/ \sum_{i=1}^{m} v_i^* (x_{i0})_\gamma^L \tag{2.84}$$

$$(E_0^2)_\gamma^U = \sum_{r=1}^{s} u_r^* (y_{r0})_\gamma^U \Bigg/ \sum_{d=1}^{p} w_d^* \bar{z}_{d0} \tag{2.85}$$

The lower bounds of system and process efficiencies of DMU_0 are defined as the similar way. The shortcomings of the modeling framework are summarized as follows. The developed approach determines upper bounds on input and output weights, thus, three mathematical programming models are required to solve. Moreover, Formulations (2.81-2.82) are solved for each γ -level. Additionally, Formulations (2.83-2.85) are employed to obtain the upper bounds of the system and process efficiencies. Likewise, the lower bounds of the system and process efficiencies are computed. Hence, the proposed approach lacks computational efficiency. In addition, dispersion of input and output weights is not powerful. Recently, Tavana et al. (2019) developed the following mathematical programming model that computes the efficiency scores of the DMUs in a multi-period dynamic fuzzy environment based on a common weight methodology.

$$\max a$$

subject to (2.86)

$$\sum_{r=1}^{s} u_r^t y_{rj}^t - \sum_{i=1}^{m} v_i^t x_{ij}^t \leq 0, \qquad \forall j, t,$$

$$\sum_{r=1}^{s} u_r^t y_{rj}^t - \sum_{i=1}^{m} \mu_i^t x_{ij}^t \leq 0, \qquad \forall j, t,$$

$$\mu_i^t \leq v_i^t, \qquad \forall i, t,$$

$$v_i^t \geq \varepsilon, \qquad \forall i, t,$$

$$u_r^t \geq \varepsilon, \qquad \forall r, t,$$

$$0 \leq a \leq 1,$$

where $\mu_i^t = a.v_i^t$, and T is the number of periods (t=1,2,…,T). The developed model is applicable in dynamic environment, and obtains common set of weights and then efficiency score of DMUs for each time of period. However, the model does not yield powerful weight dispersion. The model equals the weights of approximately half of the inputs and outputs to non-Archimedean for each period of the oil refineries evaluation problem.

Table 2.10: Advantages and Limitations of Non-Deterministic Models

Author(s)	Year	Advantage(s)	Limitation(s)
Hadi-Vencheh et al.	2014	• Solves fixed cost allocation problem in fuzzy network systems. • Presents a goal programming model to allocate fixed cost. • Enables to add new sub-DMUs of each DMU.	• The robustness of the proposed approach is not illustrated by providing a real-life example.
Shirdel et al.	2016	• Provides full ranking of DMUs. • Applicable for the problems with interval data.	• Requires calculating the worst, best, and average efficiency for all the DMUs.
Salahi et al.	2016	• Applicable for the problems with interval data.	• Solves *n*+1 models to identify the best performing DMU, *n* is the number of DMUs. • Lacks computational efficiency. • Inconsistent models, where first stage of the developed approach computes different set of weights for each DMU whereas second stage yields common set of weights, are developed.
Aghayi et al.	2016	• Provides full ranking of DMUs.	• Requires the parameters to be determined by a decision maker. • Considers the data for inputs and outputs as variables and included to non-negativity constraints.

Table 2.10: Advantages and Limitations of Non-Deterministic Models (cont.)

Hu et al.	2017	• Applicable for the problems with crisp, fuzzy, and ordinal data.	• Requires the distance parameter to be determined by a decision maker, subjectively. • Requires solving the positive ideal solution, negative ideal solution, distance to the positive ideal solution, distance to the negative ideal solution, and degree of satisfactory level by solving one mathematical programming model for each of them.
Wen et al.	2018	• Guarantees to identify the single efficient DMU by solving one mixed integer linear programming model.	• Guarantees to obtain the single best performer by adding constraints that restrict to obtain more than one efficient DMU. • Requires a binary variable.
Hatami-Marbini and Saati	2018	• Enables efficiency evaluation in fuzzy environment.	• Requires three mathematical programming models to solve. • Dispersion of input and output weights is not powerful.
Tavana et al.	2019	• Applicable in multi-period dynamic fuzzy environment. • Obtains efficiency score of DMUs for each time of period.	• Does not yield powerful weight dispersion. • Equals the weights of approximately half of the inputs and outputs to non-Archimedean for each period.

3. DATA ENVELOPMENT ANALYSIS

The original data envelopment analysis (DEA) model, also named as the CCR (Charnes, Cooper, & Rhodes) model, proposed by Charnes et al. (1978), computes the relative efficiency of a decision making unit (DMU) by maximizing the ratio of its total weighted outputs to its total weighted inputs subject to the condition that the output to input ratio of every DMU be less than or equal to unity. The conventional DEA formulation can be represented as follows:

$$\max E_{j_0} = \frac{\sum_{r=1}^{s} u_r y_{rj_0}}{\sum_{i=1}^{m} v_i x_{ij_0}}$$

subject to (3.1)

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad \forall j,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

where E_{j_0} is the efficiency score of the evaluated DMU, u_r is the weight assigned to output r, v_i is the weight assigned to input i, y_{rj} is the quantity of output r generated and x_{ij} is the amount of input i consumed by DMU j, respectively, and ϵ is a small positive scalar.

Formulation (3.1) possesses non-linear and non-convex properties, however, it can be converted into a linear programming model via a transformation (Charnes & Cooper, 1962). The linear programming model for computing the relative efficiency of a DMU is as

$$\max E_{j_0} = \sum_{r=1}^{s} u_r y_{rj_0}$$

subject to (3.2)

$$\sum_{i=1}^{m} v_i \, x_{ij_0} = 1$$

$$\sum_{r=1}^{s} u_r \, y_{rj} - \sum_{i=1}^{m} v_i \, x_{ij} \leq 0, \qquad \forall j,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i.$$

The efficiency scores of DMUs are calculated by solving Formulation (3.2) n times, where n is the number of DMUs. Formulation (3.2) classifies DMUs with efficiency score of 1 as efficient while DMUs with lower efficiency scores are considered as inefficient. In order to eliminate the unrealistic weight dispersion and improve the poor discriminating power of DEA, a number of approaches have been proposed (Karsak & Ahiska, 2005). To avoid the unrealistic weight dispersion and improve the poor discriminating power of DEA, Wong and Beasley (1990) employed weight restrictions which enforce some frontiers on weights. The proposed model is as

$$\max e_{j0}$$

subject to (3.3)

$$e_{j_0} = \frac{\left(\sum_{r=1}^{s} u_r y_{rj}\right)}{\left(\sum_{i=1}^{m} v_i x_{ij}\right)}, \quad \forall j,$$

$$e_{j_0} \leq 1, \quad \forall j,$$

$$a_r \leq {\left(u_k y_{kj}\right)} \Big/ {\left(\sum_{r=1}^{s} u_r y_{rj}\right)} \leq b_r,$$

$$a_i \leq {\left(v_k x_{ij}\right)} \Big/ {\left(\sum_{i=1}^{m} v_i x_{ij}\right)} \leq b_i,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

where e_{j_0} denotes the relative efficiency of DMU$_0$, a_r and a_i represent the lower bounds of the proportion of total outputs and inputs, b_r and b_i are the upper bounds of the proportion of total outputs and inputs, respectively.

Another widely used mathematical technique to improve the discriminating power of DEA is cross-efficiency analysis (Sexton et al., 1986). The aggressive cross-efficiency model is as (Doyle & Green, 1994)

$$\min \sum_{r=1}^{s} u_r \left(\sum_{\substack{j=1 \\ j\neq 0}}^{n} y_{rj} \right)$$

subject to (3.4)

$$\sum_{i=1}^{m} v_i \left(\sum_{\substack{j=1 \\ j\neq 0}}^{n} x_{ij} \right) = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj_0} - E_{kk}^{*} \sum_{i=1}^{m} v_i\, x_{ij_0} = 0,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \leq 0, \qquad \forall j, \qquad j \neq 0,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

where $E_{kk}^{*}$is the CCR efficiency of DMU$_0$ derived from the CCR model. The aggressive cross-efficiency model leads to choosing input and output weights, which yield the maximum efficiency for a DMU under evaluation, while minimizing the other DMUs' cross-efficiencies. Alternatively, the benevolent cross-efficiency formulation aims to obtain maximum efficiency for a DMU to be evaluated, while maximizing the other DMUs' cross-efficiencies (Doyle & Green, 1994). Hence, the benevolent formulation for cross-efficiency analysis is obtained by changing the objective function of Formulation (3.4) from min to max, and is presented as

$$\max \sum_{r=1}^{s} u_r \left(\sum_{\substack{j=1 \\ j \neq 0}}^{n} y_{rj} \right)$$

subject to (3.5)

$$\sum_{i=1}^{m} v_i \left(\sum_{\substack{j=1 \\ j \neq 0}}^{n} x_{ij} \right) = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj_0} - E_{kk}^{*} \sum_{i=1}^{m} v_i \, x_{ij_0} = 0,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i \, x_{ij} \leq 0, \qquad \forall j, \qquad j \neq 0,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i.$$

Alternatively, minimax and minsum efficiency measures do not give favorable consideration to the evaluated DMU unlike the conventional DEA model. Minimax efficiency minimizes maximum deviation from efficiency (Li & Reeves, 1999). The minimax efficiency model can be represented as follows:

$$\min M$$

subject to (3.6)

$$\sum_{i=1}^{m} v_i \, x_{ij_0} = 1,$$

$$\sum_{r=1}^{s} u_r \, y_{rj} - \sum_{i=1}^{m} v_i \, x_{ij} + d_j = 0, \qquad \forall j,$$

$$M \geq d_j, \qquad \forall j,$$
$$u_r, v_i, d_j \geq 0, \qquad \forall r, i, j,$$

where d_j is the deviation from efficiency for DMU$_j$, (i.e. d_j = 1 – E_j when E_j is the efficiency score of DMU$_j$), and M is the maximum deviation from efficiency.

Likewise, minsum efficiency is to minimize the total deviation from efficiency (Li & Reeves, 1999). The resulting model is as

$$\min \sum_{j=1}^{n} d_j$$

subject to (3.7)

$$\sum_{i=1}^{m} v_i\, x_{ij_0} = 1,$$
$$\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + d_j = 0, \qquad \forall j,$$
$$u_r, v_i, d_j \geq 0, \qquad \forall r, i, j.$$

In addition, super efficiency, which excludes the constraint that the optimal weights for the evaluated DMU does not imply an efficiency exceeding unity, is presented for ranking DEA-efficient DMUs as (Andersen & Petersen, 1993)

$$\max \sum_{r=1}^{s} u_r y_{rk}$$

subject to (3.8)

$$\sum_{i=1}^{m} v_i x_{ik} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0,\ j = 1,2, \dots, n; j \neq k,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

Super efficiency measure has an improved discriminating power through enabling super-efficient units with efficiency scores greater than one; however, it possesses several limitations (Adler et al., 2002). First, super efficiency models provide full ranking of DMUs, while enabling each DMU to be evaluated according to different weights. Second, super efficiency approach can assign specialized DMUs an excessively high efficiency score. Moreover, super efficiency model may yield an infeasible solution, where full ranking of DMUs cannot be provided.

4. PRELIMINARIES OF FUZZY SETS

Fuzzy set theory, which was initially introduced by Zadeh (1965), aims to cope with the decision frameworks that contain vagueness and imprecision. A fuzzy set $\tilde{A}$ can be represented by a membership function $\mu_{\tilde{A}}(x)$, which assigns each element x in the universe of discourse X a real number in the interval [0,1]. The membership degree of the element is determined with the concept represented by the fuzzy set. In the following paragraph, some basic definitions of fuzzy sets are briefly reviewed (Chen, 2001).

Definition 4.1 A fuzzy set $\tilde{A}$ is convex if and only if for all x_1 and $x_2 \in X$:

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right), \quad \lambda \in [0,1] \tag{4.1}$$

Definition 4.2 A fuzzy set $\tilde{A}$ is called a normal fuzzy set implying

$$\exists\, x_i \in X, \mu_{\tilde{A}}(x_i) = 1 \tag{4.2}$$

Definition 4.3 α-cut is represented as

$$\tilde{A}_\alpha = \{x_i : \mu_{\tilde{A}}(x_i) \geq \alpha, x_i \in X\} \text{ where } \alpha \in [0,1] \tag{4.3}$$

$\tilde{A}_\alpha$ is a limited nonempty bounded interval in X and it can be defined by $\tilde{A}_\alpha = \left[(\tilde{A})_\alpha^L, (\tilde{A})_\alpha^U\right]$, where $(\tilde{A})_\alpha^L$ and $(\tilde{A})_\alpha^U$ are the lower and upper bounds of the closed interval, respectively.

Definition 4.4 A triangular fuzzy number $\tilde{A}$ can be represented by (a_1, a_2, a_3). The membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{x - a_3}{a_2 - a_3}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \tag{4.4}$$

Triangular fuzzy numbers are suitable to quantify uncertain data about the decision problems (Dursun & Karsak, 2010). The main reason for utilizing triangular fuzzy numbers can be denoted as their intuitive and computational efficient representation (Karsak, 2002).

Definition 4.5 If $\tilde{A}$ is a fuzzy number and $a_1^{\alpha} > 0$ for $\alpha \in [0,1]$, then $\tilde{A}$ is called as a positive fuzzy number.

Let $\tilde{A}$ and $\tilde{B}$ denote two positive fuzzy numbers and *k* a positive real number. The α-cuts of two fuzzy numbers are $\tilde{A}_{\alpha} = \left[\left(\tilde{A}\right)_{\alpha}^{L}, \left(\tilde{A}\right)_{\alpha}^{U}\right]$, and $\tilde{B}_{\alpha} = \left[\left(\tilde{B}\right)_{\alpha}^{L}, \left(\tilde{B}\right)_{\alpha}^{U}\right]$, respectively, where $\alpha \in [0,1]$.

Basic arithmetic operations of positive fuzzy numbers can be stated as

$$\left(\tilde{A}(+)\tilde{B}\right)_{\alpha} = [(A)_{\alpha}^{L} + (B)_{\alpha}^{L}, (A)_{\alpha}^{U} + (B)_{\alpha}^{U}] \tag{4.5}$$

$$\left(\tilde{A}(-)\tilde{B}\right)_{\alpha} = [(A)_{\alpha}^{L} - (B)_{\alpha}^{U}, (A)_{\alpha}^{U} - (B)_{\alpha}^{L}] \tag{4.6}$$

$$\left(\tilde{A}(*)\tilde{B}\right)_{\alpha} = [(A)_{\alpha}^{L} * (B)_{\alpha}^{L}, (A)_{\alpha}^{U} * (B)_{\alpha}^{U}] \tag{4.7}$$

$$\left(\tilde{A}(\div)\tilde{B}\right)_\alpha = \left[\frac{(A)_\alpha^L}{(B)_\alpha^U}, \frac{(A)_\alpha^U}{(B)_\alpha^L}\right] \tag{4.8}$$

$$\left(\tilde{A}_\alpha\right)^{-1} = \left[\frac{1}{(A)_\alpha^U}, \frac{1}{(A)_\alpha^L}\right] \tag{4.9}$$

$$\left(\tilde{A}(*)k\right)_\alpha = [(A)_\alpha^L * k, (B)_\alpha^U * k] \tag{4.10}$$

$$\left(\tilde{A}(\div)k\right)_\alpha = \left[\frac{(A)_\alpha^L}{k}, \frac{(A)_\alpha^U}{k}\right] \tag{4.11}$$

Definition 4.6 If $\tilde{A}$ is a fuzzy number and $(A)_\alpha^L > 0$, $(A)_\alpha^U \leq 1$ for $\alpha \in [0,1]$, then $\tilde{A}$ is named a normalized positive fuzzy number (Chen, 2001).

Definition 4.7 A linguistic term is represented as a term whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic term provides approximate characterization of the problems that are too complex or impossible to be described in quantitative variables (Zadeh, 1975).

5. QUALITY FUNCTION DEPLOYMENT

In this section, quality function deployment (QFD) is summarized, and the use of fuzzy weighted average in calculating the priorities of the technical attributes in house of quality is delineated.

5.1 Basic Concepts

QFD is a pivotal product development framework that is used in converting customer needs (CNs) into technical attributes (TAs) for developing services and products (Carnevalli & Miguel 2008). QFD aims to deliver value by considering the customer requirements, and to deploy this information through whole development steps (Karsak, 2004). It leads to decrease risk and uncertainty in product design process, hence it is to enhance customer satisfaction degree, business processes levels, competition capability, and profitability.

QFD enables company to provide resource allocation and skill coordination based on CNs, and hence, helps reduce manufacturing costs and cycle time. Further, it provides making decisions for development and change in designing stage while minimizing the failures during the whole process (Karsak et al., 2003). In general, QFD needs four matrices that each are associated with a stage of the product development cycle. These stages are named product planning, part deployment, process planning, and production/operation planning matrices, respectively. The four phases of QFD are shown in Figure 5.1.

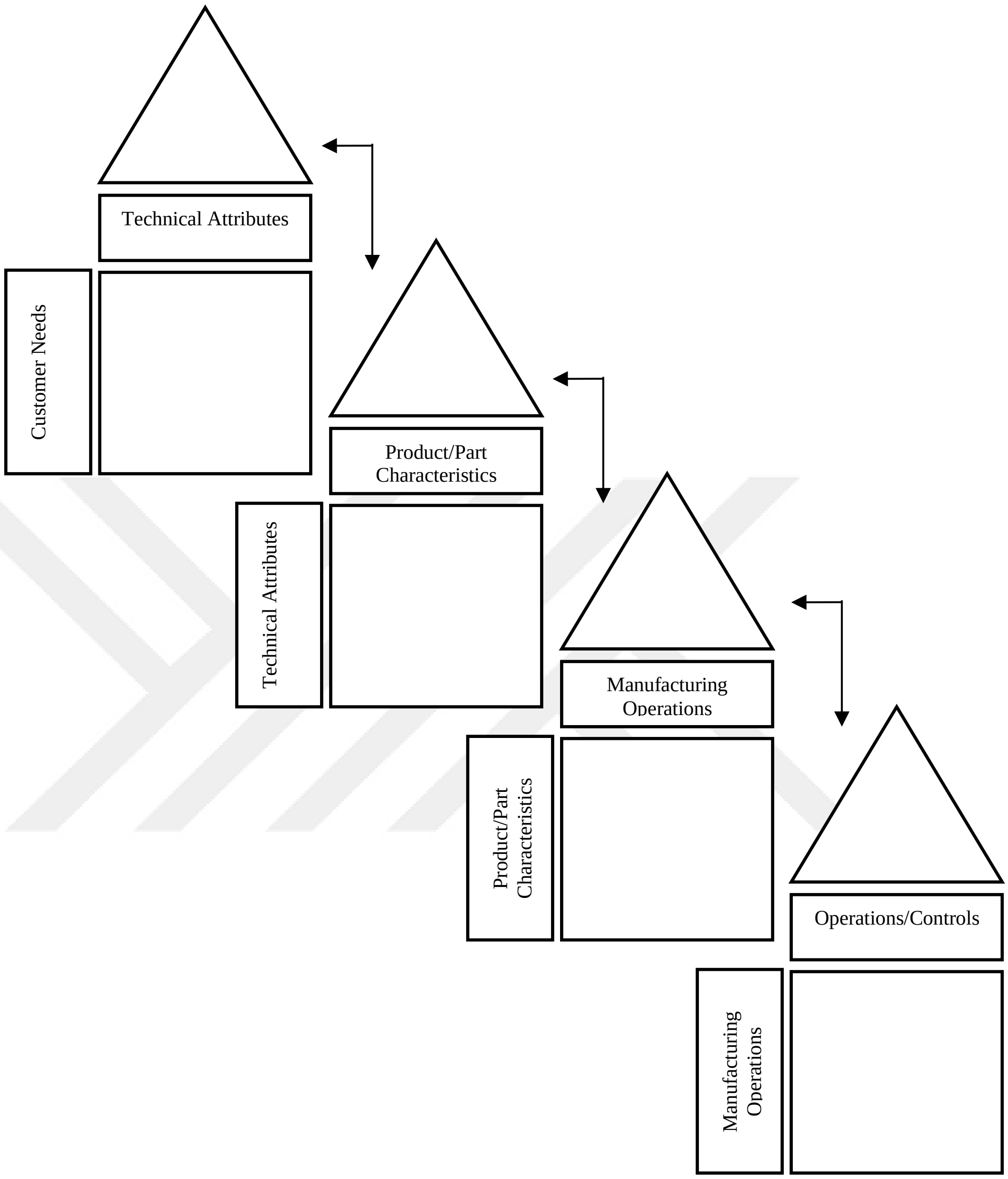


Figure 5.1: The four phases of QFD process (Shillito, 1994)

The product planning matrix provides the translation of CNs into TAs; the part deployment matrix presents the translation of the important TAs into product properties; the process planning matrix provides the translation of the important product properties into production processes; the production/operation planning matrix presents the translation of the important production processes into daily controls and operations (Shillito, 1994).

5.2 House of Quality

The quality chart, which is created by Toyota Auto Body, was initially named as "house of quality (HOQ)" at a Japanese Society for Quality Control research presentation conference by Sawada in 1979 because of its shape (Akao, 1997).

According to Hauser and Clausing (1998), the HOQ is a conceptual map that provides functional planning and communications. Relationships between CNs and TAs and among the TAs are defined by answering a specific question corresponding to each cell in HOQ. It consists of seven elements as given in Figure 5.2.

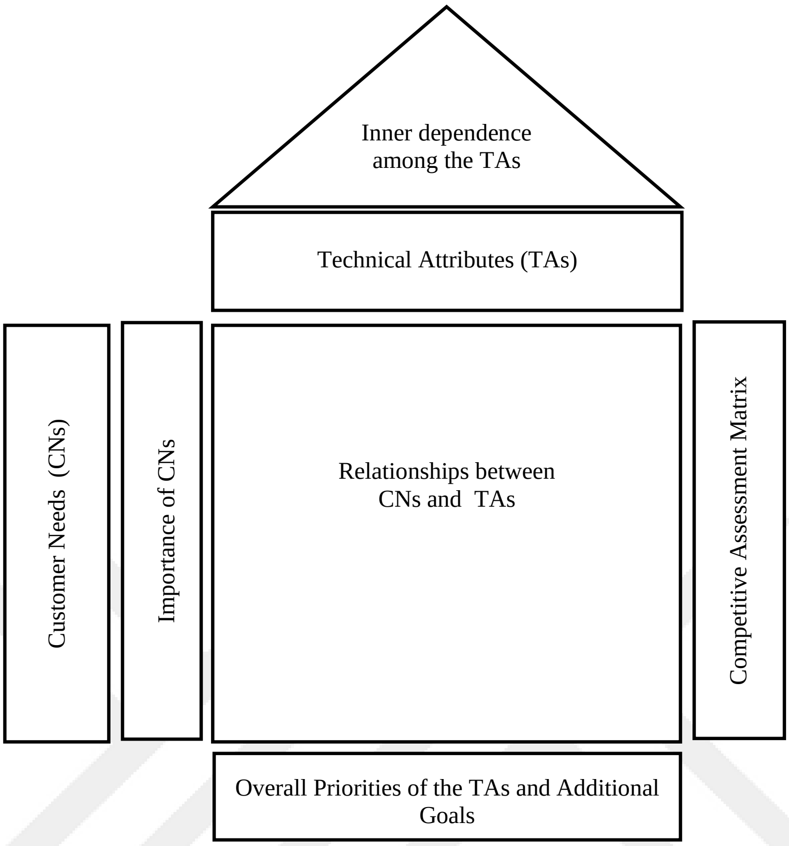


Figure 5.2: The house of quality (Shillito, 1994)

The seven elements of the HOQ summarized in Figure 5.2 can be briefly described as

1. Customer needs (CNs), which are also expressed as customer requirements, provides a guideline of attributes that the product should obtain.
2. Technical attributes (TAs), which design technical needs of the product, determines how well the company meets the customer requirements. Weighting customer needs is a crucial decision process since the providers should focus on the most important needs while disregarding less important ones because of the limited resources.
3. Importance of customer needs can be identified employing a Likert scale, linguistic terms, or a decision aid approach.
4. Relationships between the CNs and TAs are represented in the relation matrix that shows how technical attributes influence customer needs.
5. Inner dependences among the TAs, which are provided in the HOQ's roof matrix, measure the relationships between the pair of TAs.

6. Competitive assessment matrix provides a comparative evaluation between the performance of the case company and its competitors.
7. Prioritized technical descriptors and targets determine the most required TAs in order to meet CNs.

5.3 Fuzzy Weighted Average

The calculation of the priorities of the TAs falls into the category of fuzzy weighted average (FWA) when the importance weights of CNs, the relations between CNs and TAs, and the inner dependencies among TAs are expressed with fuzzy numbers in QFD framework (Liu, 2005). Throughout the literature, there are several methodologies introduced for computing FWA (Dong & Wong, 1987; Liou & Wang, 1992; Lee & Park, 1997; Kao & Liu, 2001; Wang & Chin, 2011). These methodologies provide ratings of the importance of TAs in fuzzy environments accurately through α-level sets.

The normalization of the relationships between CNs and TAs is required; otherwise, the importance of TAs cannot be correctly rated (Wasserman, 1993). Using fuzzy arithmetic for performing fuzzy normalization or computing FWA is not appropriate since fuzzy arithmetic operations increase the fuzziness of normalized fuzzy relationships and FWA (Wang & Chin, 2011).

Apart from these advantages, the methods using FWA technique rectify the problem of loss of information that may occur when including imprecise data and subjective assessment for computing the lower and upper bounds of the weights of the TAs. Hence, these methods yield more realistic overall desirability levels. Moreover, improved discriminatory characteristics among fuzzy numbers are provided by employing these methods since they utilize a fuzzy number ranking technique based on area measurement.

Wang and Chin (2011) proposed a pair of nonlinear programming models and two equivalent pairs of linear programming models for determining the α-cut of $\tilde{Y}_p$. Their method produces normalized fuzzy importance ratings for TAs. The method can be summarized as follows:

Let $\tilde{W}_p$ represent the fuzzy relative weight of CN_p, $\tilde{X}_{pc}$ denote the fuzzy relationship measure between TA_c and CN p, and $\tilde{\rho}_{kc}$ represent the degree of dependence of the kth TA on the c^{th} TA. Denote $\left[\left(W_p\right)_\alpha^L, \left(W_p\right)_\alpha^U\right]$, $\left[\left(X_{pc}\right)_\alpha^L, \left(X_{pc}\right)_\alpha^U\right]$, $[(\rho_{kc})_\alpha^L, (\rho_{kc})_\alpha^U]$ be the α-level sets of the fuzzy relative weight, fuzzy relationships, and fuzzy correlations, respectively. Compute the normalized fuzzy relationships employing the following formulation.

$$\tilde{X}_{pc}' = \frac{\sum_{k=1}^{b} \tilde{X}_{pk}\tilde{\rho}_{kc}}{\sum_{\substack{l=1 \\ l\neq c}}^{b} \sum_{k=1}^{b} \tilde{X}_{pk}\tilde{\rho}_{kl} + \sum_{k=1}^{b} \tilde{X}_{pk}\tilde{\rho}_{kc}}, \quad p = 1,2,\dots,q; \quad c = 1,2,\dots,b. \tag{5.1}$$

Formulation (5.1) can be characterized using α-level sets by the following pair of non-linear programming models for each p and c.

$$\left(\tilde{X}_{pc}'\right)_\alpha^L = \min \frac{\sum_{k=1}^{b} X_{pk}(\rho_{kc})_\alpha^L}{\sum_{\substack{l=1 \\ l\neq c}}^{b} \sum_{k=1}^{b} X_{pk}(\rho_{kl})_\alpha^U + \sum_{k=1}^{b} X_{pk}(\rho_{kc})_\alpha^L}$$

subject to (5.2)

$$\left(X_{pk}\right)_\alpha^L \leq X_{pk} \leq \left(X_{pk}\right)_\alpha^U, k = 1,2,\dots,b.$$

$$\left(\tilde{X}_{pc}'\right)_\alpha^U = \max \frac{\sum_{k=1}^{b} X_{pk}(\rho_{kc})_\alpha^U}{\sum_{\substack{l=1 \\ l\neq c}}^{b} \sum_{k=1}^{b} X_{pk}(\rho_{kl})_\alpha^L + \sum_{k=1}^{b} X_{pk}(\rho_{kc})_\alpha^U}$$

subject to (5.3)

$$\left(X_{pk}\right)_\alpha^L \leq X_{pk} \leq \left(X_{pk}\right)_\alpha^U, k = 1,2,\dots,b.$$

By letting

$$t = \frac{1}{\sum_{\substack{l=1 \\ l\neq c}}^{b} \sum_{k=1}^{b} X_{pk} (\rho_{kl})_\alpha^U + \sum_{k=1}^{b} X_{pk} (\rho_{kc})_\alpha^L} \text{ and } z_{pk} = tX_{pk}, \quad k = 1,2,\dots,b \tag{5.4}$$

$$\delta = \frac{1}{\sum_{\substack{l=1 \\ l\neq c}}^{b} \sum_{k=1}^{b} X_{pk} (\rho_{kl})_\alpha^L + \sum_{k=1}^{b} X_{pk} (\rho_{kc})_\alpha^U} \text{ and } u_{pk} = \delta X_{pk}, k = 1,2,\dots,b. \tag{5.5}$$

Formulations (5.4) and (5.5) can be transformed into the following linear programming models:

$$\left(\tilde{X}'_{pc}\right)_\alpha^L = \min \sum_{k=1}^{b} z_{pk} (\rho_{kc})_\alpha^L$$

subject to (5.6)

$$\sum_{k=1}^{b} z_{pk} \left((\rho_{kc})_\alpha^L + \sum_{\substack{l=1 \\ l\neq c}}^{b} (\rho_{kl})_\alpha^U \right) = 1,$$

$$\left(X_{pk}\right)_\alpha^L t \leq z_{pk} \leq \left(X_{pk}\right)_\alpha^U t, \quad k = 1,2,\dots,b,$$

$$t > 0.$$

$$\left(\tilde{X}'_{pc}\right)_\alpha^U = \max \sum_{k=1}^{b} \varphi_{pk} (\rho_{kc})_\alpha^U$$

subject to (5.7)

$$\sum_{k=1}^{b} \varphi_{pk} \left((\rho_{kc})_\alpha^U + \sum_{\substack{l=1 \\ l\neq c}}^{b} (\rho_{kl})_\alpha^L \right) = 1$$

$$\left(X_{pk}\right)_{\alpha}^{L}\delta \le \varphi_{pk} \le \left(X_{pk}\right)_{\alpha}^{U}\delta, \quad k = 1,2,\ldots,b$$
$$\delta > 0,$$

where t, δ, z_{pk}, and $\varphi_{pk}$are decision variables. By solving this pair of linear programming models for each α-level, the normalized fuzzy relationship matrix $\tilde{X}' = \left(\tilde{X}'_{pc}\right)_{q\times b}$can be obtained. Once the normalized fuzzy relationships are generated, the FWA of the normalized fuzzy relationship can be formulated as

$$\tilde{\Theta}_c = \sum_{p=1}^{q} \widetilde{W}_p \tilde{X}'_{pc} \Big/ \sum_{p=1}^{q} \widetilde{W}_p\,, \quad c = 1,2,\ldots,b \tag{5.8}$$

and the lower and the upper bounds of the α-cut of $\tilde{\Theta}_c$ can be solved as

$$(\tilde{\Theta}_c)_{\alpha}^{L} = \min \sum_{p=1}^{q} w_p \left(X'_{pc}\right)_{\alpha}^{L} \Big/ \sum_{p=1}^{q} w_p$$

subject to (5.9)

$$\left(W_p\right)_{\alpha}^{L} \le w_p \le \left(W_p\right)_{\alpha}^{U}, \quad p = 1,2,\ldots,q.$$

$$(\tilde{\Theta}_c)_{\alpha}^{U} = \max \sum_{p=1}^{q} w_p \left(X'_{pc}\right)_{\alpha}^{U} \Big/ \sum_{p=1}^{q} w_p$$

subject to (5.10)

$$\left(W_p\right)_{\alpha}^{L} \le w_p \le \left(W_p\right)_{\alpha}^{U}, \quad p = 1,2,\ldots,q.$$

Following the variable substitution of Charnes and Cooper (1962), by letting $\lambda^{-1} = \sum_{p=1}^{q} w_p$ and $\eta_p = \lambda w_p$, Formulations (5.9) and (5.10) can be transformed to the following linear programs, which are utilized to identify the lower and upper bounds for $\alpha = 0$ as

$$\beta_c = \min \sum_{p=1}^{q} \eta_p \left(X'_{pc}\right)_\alpha^L$$

subject to (5.11)

$$\lambda \left(W_p\right)_\alpha^L \leq \eta_p \leq \lambda \left(W_p\right)_\alpha^U, \quad \mathrm{p} = 1,2,\ldots,q$$

$$\sum_{p=1}^{q} \eta_p = 1$$

$$\lambda, \eta_p \geq 0, \qquad p = 1,2,\ldots,q.$$

$$\gamma_c = \max \sum_{p=1}^{q} \eta_p \left(X'_{pc}\right)_\alpha^U$$

subject to (5.12)

$$\lambda \left(W_p\right)_\alpha^L \leq \eta_p \leq \lambda \left(W_p\right)_\alpha^U, \quad \mathrm{p} = 1,2,\ldots,q$$

$$\sum_{p=1}^{q} \eta_p = 1$$

$$\lambda, \eta_p \geq 0, \qquad p = 1,2,\ldots,q.$$

The α-cuts of $\tilde{\Theta}_c$ is the crisp interval $[(\Theta_c)_\alpha^L, (\Theta_c)_\alpha^U]$ obtained from Formulations (5.11) and (5.12). Computing for differing α values, the membership function $\mu_{\tilde{\Theta}_c}$ can be formed.

6. PROPOSED COMMON WEIGHT DEA-BASED DECISION MAKING APPROACHES

This section outlines the common weight data envelopment analysis (DEA)-based decision making procedures. Conventional DEA models do not provide common weights for evaluating decision making units (DMUs) and enable the DMUs to determine their own weights in order to maximize their efficiency scores, which is rather impractical. Hence, ranking DMUs with the efficiency scores acquired from different sets of weights leads to impossibility for common assessment (Sun et al., 2013).

On the other hand, common weight DEA models do not require subjective evaluation to determine weights for inputs and outputs while providing a common assessment for all DMUs. Thus, the selection of input and output weights in favour of respective DMUs is limited that increases the discriminating power of the analysis (Karsak & Ahiska, 2005). Hence, the use of common weights appears to be practical for computing the efficiency scores for all DMUs (Jahanshahloo et al., 2010). Furthermore, common weight DEA-based models necessitate a smaller number of mathematical programs to be solved to determine efficiency scores compared with traditional DEA models. Thereby, common weight models also provide computational savings. On the other hand, traditional DEA and common weight DEA-based models require crisp inputs and outputs. However, crisp data may become inadequate to handle qualitative issues that are commonly incorporated in real world cases. In order to include qualitative data into the modeling framework, DEA models incorporating uncertain data should be proposed. Vagueness, fuzziness, and uncertainty may be handled with imprecise DEA models.

In this section of the thesis, three common weight DEA-based approaches are outlined in order to identify the best performing unit. The first approach incorporates multiple inputs and multiple outputs by proposing a novel mathematical programming approach that improves the common weight multiple criteria decision making (MCDM) model developed by Karsak and Ahiska (2007). The second approach enables to include imprecise data into the first decision making framework. The third approach provides group decision making procedure by integrating quality function deployment (QFD) and fuzzy weighted average (FWA) into the common weight DEA-based decision approach.

6.1 Common Weight DEA-Based Methodology

In this subsection, the first common weight DEA-based methodology proposed in this thesis is explained.

6.1.1 Background

Due to aforementioned limitations of conventional DEA models such as poor discriminating power and impractical assessment of alternatives lack of common weights, researchers intended to extend the evaluation/selection process of alternatives using common weight models. Karsak and Ahiska (2007) developed a common weight multi-criteria decision making (MCDM) framework for decision problems in the presence of multiple inputs and multiple outputs that is based on minimizing maximum deviation from efficiency as well as equating the sum of criteria weights to 1 as in other typical MCDM models.

The minimax efficiency model proposed in Karsak and Ahiska (2007) for assessing alternatives with multiple inputs and multiple outputs is as follows:

$$\min M$$

subject to (6.1)

$$\begin{aligned} & M - d_j \geq 0, && \forall j, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i \, x_{ij} + d_j = 0, && \forall j, \\ & \sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1, && \\ & u_r, v_i, d_j \geq 0, && \forall r, i, j, \end{aligned}$$

where d_j is the deviation from the efficiency score of DMU j, (i.e. $d_j = 1 - E_j$, E_j is the efficiency score of DMU$_j$), and M denotes the maximum deviation from efficiency.

Formulation (6.1) aims at minimizing the maximum deviation from efficiency and results in computation savings by calculating efficiency scores of all DMUs with a single model. Furthermore, this formulation enables the assessment of relative efficiency of all DMUs with common weights unlike traditional DEA models in which each DMU is evaluated by different set of weights.

For the case where more than one efficient DMU is obtained by solving Formulation (6.1), Karsak and Ahiska (2007) proposed the subsequent common weight model to identify the most efficient DMU.

$$\min M - k \sum_{j \in EF} d_j$$

subject to (6.2)

$$M - d_j \geq 0, \qquad \forall j,$$

$$M - \sum_{j \in EF} d_j \geq 0,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r, v_i, d_j \geq 0, \qquad \forall r, i, j,$$

where $k \in (0,1]$ is a discriminating parameter which has to be determined by the analyst, and EF indicates the set of minimax efficient DMUs that are identified via Formulation (6.1).

The methodology presented by Karsak and Ahiska (2007) enables to rank efficient DMUs with superior computational efficiency compared with traditional DEA models. However, a shortcoming of this methodology is that a decision analyst is required to determine the value of discriminating parameter *k.* Another limitation of the model is that Formulation (6.2) may result in lower final efficiency scores for some minimax efficient DMUs than several other DMUs that are found to be inefficient with respect to Formulation (6.1).

More recently, Sun et al. (2013) employed two mathematical programming models to rank the common weight efficiency scores. First, they proposed the linear programming model given below.

$$\min \sum_{j=1}^{n} d_j$$

subject to (6.3)

$$\sum_{i=1}^{m} v_i\, x_{ij} - d_j = \sum_{r=1}^{s} u_r y_{rj}, \qquad \forall j,$$

$$\sum_{i=1}^{m} v_i\, x_{min} = 1,$$

$$\sum_{r=1}^{s} u_r\, y_{max} = 1,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

$$d_j \geq 0, \qquad \forall j,$$

where $x_{\min} = \{x_{ij} | j = 1, \dots, n\}$ and $y_{max} = \{y_{rj} | j = 1, \dots, n\}$. Sun et al. (2013) demonstrated that the optimal weights computed by Formulation (6.3) may not be unique and different software may result in disparate optimal weights. Hence, they developed the following nonlinear programming model in order to improve the convenience of this approach.

$$\max \sum_{i=1}^{m} v_i^2 + \sum_{r=1}^{s} u_r^2$$

subject to (6.4)

$$\sum_{j=1}^{n}\left(\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj}\right) = D^*,$$

$$\sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, y_{rj} \geq 0, \quad \forall j,$$

$$\sum_{i=1}^{m} v_i\, x_{min} = 1,$$

$$\sum_{r=1}^{s} u_r\, y_{max} = 1,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

where D^* represents the optimal objective function value of Formulation (6.3). As an alternative to their model mentioned before, Sun et al. (2013) proposed the following model.

$$\min \sum_{j=1}^{n}\left(\sum_{i=1}^{m} v_i x_{max} - \sum_{i=1}^{m} v_i\, x_{ij}\right) + \sum_{j=1}^{n}\left(\sum_{r=1}^{s} u_r\, y_{rj} - \sum_{r=1}^{s} u_r\, y_{min}\right)$$

subject to (6.5)

$$\sum_{i=1}^{m} v_i\, x_{ij} - \sum_{r=1}^{s} u_r\, y_{rj} \leq 0, \quad \forall j,$$

$$\sum_{i=1}^{m} v_i\, x_{max} = 1,$$

$$\sum_{r=1}^{s} u_r\, y_{min} = 1,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i.$$

Although these models enable full ranking of DMUs and thus improve the discriminatory characteristics of DEA, the weight dispersion for inputs and outputs is relatively poor.

6.1.2 Proposed Models

The common weight DEA-based approach developed in this thesis aims to improve the modeling framework in Karsak and Ahiska (2007). The proposed approach incorporates a non-Archimedean infinitesimal ϵ to Formulation (6.1), and the resulting common weight model is as follows:

$$\min \theta$$

subject to (6.6)

$$\theta - d_j \geq 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

$$d_j \geq 0, \qquad \forall j,$$

where $\theta = \max_j d_j$ and ϵ is a small positive scalar. Then, in lieu of Formulation (6.2), an improved model is proposed by eliminating discriminating parameter k. The developed model for multiple inputs and multiple outputs is as

$$\min \theta$$

subject to (6.7)

$$\theta - d_j \geq 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$d_j + M z_j \geq \varepsilon, \qquad j \in EF,$$

$$\sum_{j \in EF} z_j = 1,$$

$$z_j \in \{0,1\}, \qquad j \in EF,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

$$d_j \geq 0, \qquad \forall j,$$

where *M* denotes a very large positive number, and z_j is a binary variable. The proposed model assures to have a single efficient DMU as demonstrated below, and yields improved weight dispersion for inputs and outputs.

Theorem. Formulation (6.7) yields a single efficient DMU.

Proof. Let DMU_k and DMU_l be decision making units found to be efficient ($k,l \in EF$) by the common set of optimal weights u_r^* and v_i^*. This will lead to $u_r^* y_{rk}^* - v_i^* x_{ik}^* = 0$ and $u_r^* y_{rl}^* - v_i^* x_{il}^* = 0$ which result in $d_k^* = d_l^* = 0$, and thus, $z_k^* = z_l^* = 1$. However, this is not possible according to constraint $\sum_{j \in EF} z_j = 1$. Consequently, DMU_k and DMU_l cannot be efficient at the same time. □

6.2 Proposed Imprecise Common Weight DEA-Based Decision Framework

The imprecise common weight DEA-based approach developed in this thesis aims to identify the best performer when fuzzy data exist in the modeling scheme. The preliminaries of the modeling framework are given below (Karsak, 2008).

Define $\tilde{x}_{ij} = (x_{ija}, x_{ijb}, x_{ijc})$ where $x_{ija} \leq x_{ijb} \leq x_{ijc}$ for the fuzzy input *i* used by DMU_j and $\tilde{y}_{rj} = (y_{rja}, y_{rjb}, y_{rjc})$ where $y_{rja} \leq y_{rjb} \leq y_{rjc}$ for the fuzzy output *r* produced by DMU_j. $(x_{ij})_\alpha^L$ and $(x_{ij})_\alpha^U$ represent lower and upper bounds of the α-cut of the membership function of $\tilde{x}_{ij}$, and $(y_{rj})_\alpha^L$ and $(y_{rj})_\alpha^U$ denote lower and upper bounds of the α-cut of the membership function $\tilde{y}_{rj}$, respectively. Define $w_i = v_i \alpha_i$, where $\alpha_i \in [0,1]$ and $0 \leq w_i \leq v_i$. Likewise, let $\mu_r = u_r \alpha_r$, where $\alpha_r \in [0,1]$ and $0 \leq \mu_r \leq u_r$.

$(x_{ij})_\alpha^L$,$(x_{ij})_\alpha^U$,$(y_{rj})_\alpha^L$, and $(y_{rj})_\alpha^U$ are as

$$\sum_{i=1}^{m} v_i\left(x_{ij}\right)_{\alpha}^{L} = \sum_{i=1}^{m} v_i x_{ija} + w_i\left(x_{ijb} - x_{ija}\right) \tag{6.8}$$

$$\sum_{i=1}^{m} v_i\left(x_{ij}\right)_{\alpha}^{U} = \sum_{i=1}^{m} v_i x_{ijc} - w_i\left(x_{ijc} - x_{ijb}\right) \tag{6.9}$$

$$\sum_{r=1}^{s} u_r\left(y_{rj}\right)_{\alpha}^{L} = \sum_{r=1}^{s} u_r y_{rja} + \mu_r\left(y_{rjb} - y_{rja}\right) \tag{6.10}$$

$$\sum_{r=1}^{s} u_r\left(y_{rj}\right)_{\alpha}^{U} = \sum_{r=1}^{s} u_r y_{rjc} - \mu_r\left(y_{rjc} - y_{rjb}\right) \tag{6.11}$$

The first stage of the proposed imprecise common weight approach is as follows:

$$\min \quad \theta$$

subject to (6.12)

$$\theta - d_j \geq 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r y_{rjc} - \mu_r\left(y_{rjc} - y_{rjb}\right) - \sum_{i=1}^{m} v_i x_{ija} + w_i\left(x_{ijb} - x_{ija}\right) + d_j = 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

where d_j is the deviation from the efficiency score of DMU j, (i.e. $d_j = 1 - E_j$, for E_j being the efficiency score of DMU$_j$), $\theta = \max_j d_j$, and ϵ is a small positive scalar.

For the case where more than one efficient DMU is obtained by solving Formulation (6.12), common weight model given below is proposed to identify the best performing DMU.

$$\min \ \theta$$

subject to (6.13)

$$\theta - d_j \geq 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r y_{rjc} - \mu_r\left(y_{rjc} - y_{rjb}\right) - \sum_{i=1}^{m} v_i x_{ija} + w_i\left(x_{ijb} - x_{ija}\right) + d_j = 0, \qquad \forall j,$$

$$\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1,$$

$$d_j + z_j M \geq \varepsilon, \qquad j \in EF,$$

$$\sum_{j \in EF} z_j = 1,$$

$$z_j \in \{0,1\}, \qquad j \in EF,$$

$$u_r, v_i \geq \varepsilon, \qquad \forall r, i,$$

where *M* denotes a very large positive number, z_j is a binary variable, and *EF* indicates the set of minimax efficient DMUs that are identified via Formulation (6.12). The proposed model sets forth a common evaluation for all DMUs by generating a common set of weights for inputs and outputs. It ensures to have a single efficient DMU as shown by the proof below. It also yields enhanced weight dispersion for inputs and outputs. Furthermore, the resulting rankings are consistent with the differentiation between efficient and inefficient DMUs in Formulation (6.12).

Theorem. Formulation (6.13) yields a single efficient DMU.

Proof. Let DMU_k and DMU_l be decision making units found to be efficient ($k,l \in EF$) by the common set of optimal weights u_r^* and v_i^*. This will lead to $u_r^* y_{rk}^* - v_i^* x_{ik}^* = 0$ and $u_r^* y_{rl}^* - v_i^* x_{il}^* = 0$ which result in $d_k^* = d_l^* = 0$, and thus, $z_k^* = z_l^* = 1$. However, this is not possible according to constraint $\sum_{j \in EF} z_j = 1$. Consequently, DMU$_k$ and DMU$_l$ cannot be efficient at the same time. □

The proposed approach guarantees full ranking of DMUs according to their efficiency scores for every α value, however a single rank-order is demanded for a more robust and consistent assessment of DMUs. The efficiency scores are aggregated using ordered weighted averaging (OWA) operator introduced by Yager (1998), since it can apply different aggregation procedures by changing the order weights. The OWA operator is a common representation of the three basic aggregation operators, i.e. max, min, and the arithmetic mean. The OWA operator combines the information while assigning weights to the values regarding their ordered position unlike the arithmetic mean (Karsak & Dursun, 2015). This operator is generally employed to aggregate data by taking into consideration a nonlinear weighted sum of its arguments. The OWA operator can be stated as the inner product of two vectors: the weighting vector, and the arguments to be aggregated (Yager, 1998). The OWA operator provides a unified framework for decision making under uncertainty, in which different decision criteria named as maximax, maximin, equally likely (Laplace) and Hurwicz criteria are represented by different OWA operator weights (Karsak & Dursun, 2015). A significant goal in the application of the OWA operator for decision making is identifying the weights, which can be generated as follows:

Let $E = \{e_1, e_2, \dots, e_k\}$ be a set of values to be aggregated, OWA operator F is defined as

$$F(e_1, e_2, \dots, e_k) = \boldsymbol{w} f^T = \sum_{t=1}^{k} w_t f_t, \tag{6.14}$$

where $\boldsymbol{w} = \{w_1, w_2, \dots, w_k\}$ is a weighting vector, such that $w_t \in [0,1]$ and $\sum_t w_t = 1$ and f is the associated ordered value vector, where $f_t \in f$ is the t^{th} largest value in E.

OWA aggregation generalizes the basic fuzzy operators that are used in fuzzy set theory: min and max. It is worth highlighting that if most of the weight in an OWA vector is related to weights that are at the top of the vector, then the aggregation will cause to favor higher valued arguments in the aggregation. However, if most of the weight is connected to weights at the bottom of **w**, the aggregation will favor smaller valued arguments. Max is the most optimistic OWA operator, while min is the most pessimistic. The usual average is in the middle of this dimension (Yager & Kelman, 1996; Yager, 1998).

The weights of the OWA operator are calculated employing fuzzy linguistic quantifiers, which for a non-decreasing relative quantifier Q, are given by

$$w_t = Q(t/h) - Q\big((t-1)/h\big), \;\; i = 1, \ldots, h, \tag{6.15}$$

The non-decreasing relative quantifier, Q, is defined as (Herrera et al., 2000)

$$Q(g) = \begin{cases} 0 & , g < e, \\ \frac{g-e}{f-e} & , e \le g \le f, \\ 1 & , g > f, \end{cases} \tag{6.16}$$

with $e, f, g \in [0,1]$, and $Q(g)$ refer to the degree to which the proportion g is compatible with the meaning of the quantifier it represents. Some non-decreasing relative quantifiers are represented by terms ‘most’, ‘at least half’, and ‘as many as possible’, with parameters (e, f) are $(0.3,0.8)$, $(0,0.5)$, and $(0.5,1)$, respectively.

6.3 Proposed QFD-Integrated Common Weight DEA-Based Decision Framework

The modeling framework developed in this work aims to identify the best performing unit proposing a two-stage common weight DEA-based decision approach, which uses lower and upper bounds on input and output weights that are determined employing the Formulations (5.11) and (5.12). The first stage of the resulting common weight approach is as follows:

$$\min \sum_{j=1}^{n} d_j$$

subject to (6.17)

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + d_j = 0, \qquad \forall j,$$

$$v_i \geq \beta_i, \qquad \forall i,$$

$$v_i \leq \gamma_i, \qquad \forall i,$$

$$u_r \geq \beta_{r,} \qquad \forall r,$$

$$u_r \leq \gamma_{r,} \qquad \forall r,$$

$$d_j \geq 0, \qquad \forall j.$$

where d_j represents the deviation from efficiency. β_r and β_i denote the lower bounds of outputs and inputs, whereas γ_r and γ_i represent the upper bounds outputs and inputs obtained from the Formulations (5.11) and (5.12), respectively. In case of obtaining more than one efficient DMU according to the Formulation (6.17), the following model, which guarantees to obtain a single efficient unit, is developed.

$$\min \sum_{j=1}^{n} d_j$$

subject to (6.18)

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} + d_j = 0, \qquad \forall j,$$

$$d_j + M z_j \geq \varepsilon, \qquad j \in EF,$$

$$\sum_{j \in EF} z_j = 1,$$

$$z_j \in \{0,1\}, \qquad j \in EF,$$

$$v_i \geq \beta_i, \qquad \forall i,$$

$$v_i \leq \gamma_i, \qquad \forall i,$$

$$u_r \geq \beta_{r,} \qquad \forall r,$$

$$u_r \leq \gamma_{r,} \qquad \forall r,$$

$$d_j \geq 0, \qquad \forall j,$$

where M refers to a very large positive number, z_j is a binary variable, and ϵ is non-Archimedean infinitesimal.

The proposed model guarantees to yield a single efficient DMU as shown below, and the resulting rankings are consistent with the differentiation between efficient and inefficient DMUs in Formulation (6.17).

Theorem. Formulation (6.18) yields a single efficient DMU.

Proof. Let DMU_μ and DMU_ϑ be the efficient DMUs ($\mu, \vartheta \in EF$) with the common set of optimal weights u_r^* and v_i^*. This will prompt $u_r^* y_{r\mu}^* - v_i^* x_{i\mu}^* = 0$ and $u_r^* y_{r\vartheta}^* - v_i^* x_{i\vartheta}^* = 0$ which result in $d_\mu^* = d_\vartheta^* = 0$, and thus, $z_\mu^* = z_\vartheta^* = 1$. However, this is not possible according to constraint $\sum_{j \in EF} z_j = 1$. Consequently, DMU_μ and DMU_ϑ cannot be efficient at the same time. □

The detailed stepwise representation of the proposed integrated common weight DEA-based decision framework is given below.

Step 1. Form an experts' committee of Z decision-makers ($\varsigma = 1,2,\ldots,Z$). Determine the population needs (PNs) in order to meet countries' needs, and the country attributes (CAs).

Step 2. Create the decision matrices for each expert that refer to importance weights of PNs, the fuzzy evaluation to identify the relationship scores between the pair of PNs-CAs, and the inner dependencies among CAs.

Step 3. Let the fuzzy score assign to the importance weight of p^{th} PN (p=1,2,…,q) for ζth expert, relationship score between p^{th} PN and c^{th} CA (c=1,2,…,b) for ζth expert, and the degree of inner dependence of k^{th} CA on c^{th} CA for ζth expert be $\widetilde{W}_{p\zeta} = (W_{p\zeta 1}, W_{p\zeta 2}, W_{p\zeta 3})$, $\tilde{X}_{pc\zeta} = (X_{pc\zeta 1}, X_{pc\zeta 2}, X_{pc\zeta 3})$ and $\tilde{\rho}_{kc\zeta} = (\rho_{kc\zeta 1}, \rho_{kc\zeta 2}, \rho_{kc\zeta 3})$, respectively. Calculate the aggregated importance weight of p^{th} PN ($\widetilde{W}_p$), aggregated fuzzy evaluation of the relationship scores between p^{th} PN and c^{th} CA ($\tilde{X}_{pc}$), and aggregated degree of inner dependence of the k^{th} CA on c^{th} CA ($\tilde{\rho}_{kc}$) as follows:

$$\widetilde{W}_p = \sum_{\zeta=1}^{Z} \Omega_\zeta \widetilde{W}_{p\zeta} \tag{6.19}$$

$$\tilde{X}_{pc} = \sum_{\zeta=1}^{Z} \Omega_\zeta \tilde{X}_{pc\zeta} \tag{6.20}$$

$$\tilde{\rho}_{kc} = \sum_{\zeta=1}^{Z} \Omega_\zeta \tilde{\rho}_{kc\zeta} \tag{6.21}$$

where $\Omega_\zeta \in [0,1]$ is the weight of ζth expert with $\sum_{\zeta=1}^{Z} \Omega_\zeta = 1$.

Step 4. Compute the lower and upper bounds of the input and output weights assigned to each CA by employing Formulations (5.11) and (5.12).

Step 5. Provide related data that represent the rating of each country regarding each CA.

Step 6. Create the DEA-based models for country evaluation and selection. The factors to be minimized are considered is inputs, while the attributes that are to be maximized are viewed as outputs.

Step 7. Calculate the deviation scores for the countries by using a DEA model with including weight restrictions. Rank the countries with regard to their deviation scores in decreasing order, and determine the best performing the country with a deviation score of 0.

7. NUMERICAL ILLUSTRATIONS AND CASE STUDIES

In order to illustrate the application of the proposed common weight DEA-based decision making methodologies, four numerical illustrations are provided. The first and the second illustrations that are given to demonstrate the robustness of the proposed common weight DEA-based methodology provide economic and financial assessment by ranking Morgan Stanley Capital International (MSCI) emerging markets and banks operating in Turkish banking sector by taking into consideration economic and financial indicators, respectively. The third one illustrates the application of the proposed imprecise decision making procedure for determining the best performing Latin American country for an expatriate who is assigned to working abroad for a time period. Moreover, three numerical examples taken from earlier research studies are illustrated to show the robustness of the proposed imprecise decision making approach. The fourth illustration provides the application of the proposed QFD-integrated group decision aid for ranking Latin American countries considering sustainable development goals set by United Nations Development Program (UNDP).

The detailed stepwise representation of the proposed decision making algorithms are given in the following subsections.

7.1 Economic and Financial Performance Assessment Using the Proposed Decision Approach

This subsection provides performance evaluation of MSCI emerging markets and banks operating in Turkish banking sector by taking into consideration economic and financial indicators. Some of these indicators are to be minimized whereas others should be maximized to increase the efficiency of decision making units (DMUs). For that reason, the indicators that are to be minimized and maximized are considered as inputs and outputs, respectively. The logic of the categorization of indicators is in conformance with the principles of DEA, and thus, it is appropriate to develop a DEA-based decision framework for economic and financial assessment of MSCI emerging markets and banks operating in Turkish banking sector. DEA was previously used as a decision aid in economic and financial performance assessment. In this section, several studies that employ DEA to provide economic and financial performance assessment of countries as well as banks are reviewed.

First, research works that implement DEA models for analyzing performance of countries considering economic and financial indicators are reviewed. Lozano-Vivas and Pastor (2006) employed DEA to evaluate financial performance of 15 OECD countries over time period by demonstrating the correlation between banking productivity and macroeconomic efficiency. Giambona and Vassallo (2014) proposed a DEA-based approach to evaluate 27 European Union (EU) countries in terms of their social inclusion, which denotes a key component of EU sustainable development strategy. Hsu and Lee (2014) employed DEA to evaluate the performance of public spending and observe how productivity has changed over time for 18 OECD countries from 1995 to 2002. Degl'Innocenti et al. (2017) used a two-stage DEA model for examining the bank productivity for 28 EU countries during global financial crisis. Feng et al. (2017) employed DEA for evaluating 41 regions by considering green development performance index that provides ecologic and economic improvement. More recently, Chodakowska and Nazarko (2017) developed a DEA-based decision approach for evaluating European countries by taking into account financial and environmental factors.

A number of research papers focus on employing DEA for evaluating economic and financial performance of banks. Sathye (2003) used the conventional DEA model to compute the efficiency scores of Indian banks by classifying them under three subclasses. Sherman and Zhu (2006) determined the efficiency of bank branches by developing an integrated approach that incorporates quality criterion into DEA efficiency. Grigoroudis et al. (2013) proposed a multi-stage DEA network model to compute the efficiency scores of bank branches, and utilized a set of performance factors combining business success indicators, customer satisfaction and personnel assessment. Titko et al. (2014) applied input-oriented DEA model under variable return to scale assumption to compute efficiency of Latvian banks, and solved the introduced model with fourteen different input and output combinations. Puri and Yadav (2016) developed a DEA approach with uncertain data and undesirable outputs to evaluate cost efficiency and revenue efficiency of Indian banks as well as bank groups for the periods 2011-2012 and 2012-2013. Ray (2016) aimed to identify the optimum number of bank branches in Indian banking sector for efficient allocation of the available resources. For that reason, a DEA-based mixed integer linear programming model, which aims at minimizing cost subject to the number of bank branches of the market region, is developed.

This subsection shows the implementation of the proposed decision making approach via case studies conducted in MSCI emerging markets and Turkish banking sector, and provides a comparative evaluation with the models developed by Karsak and Ahiska (2007) and Sun et al. (2013). The stepwise representation of the proposed common weight DEA-based methodology is depicted in Figure 7.1.

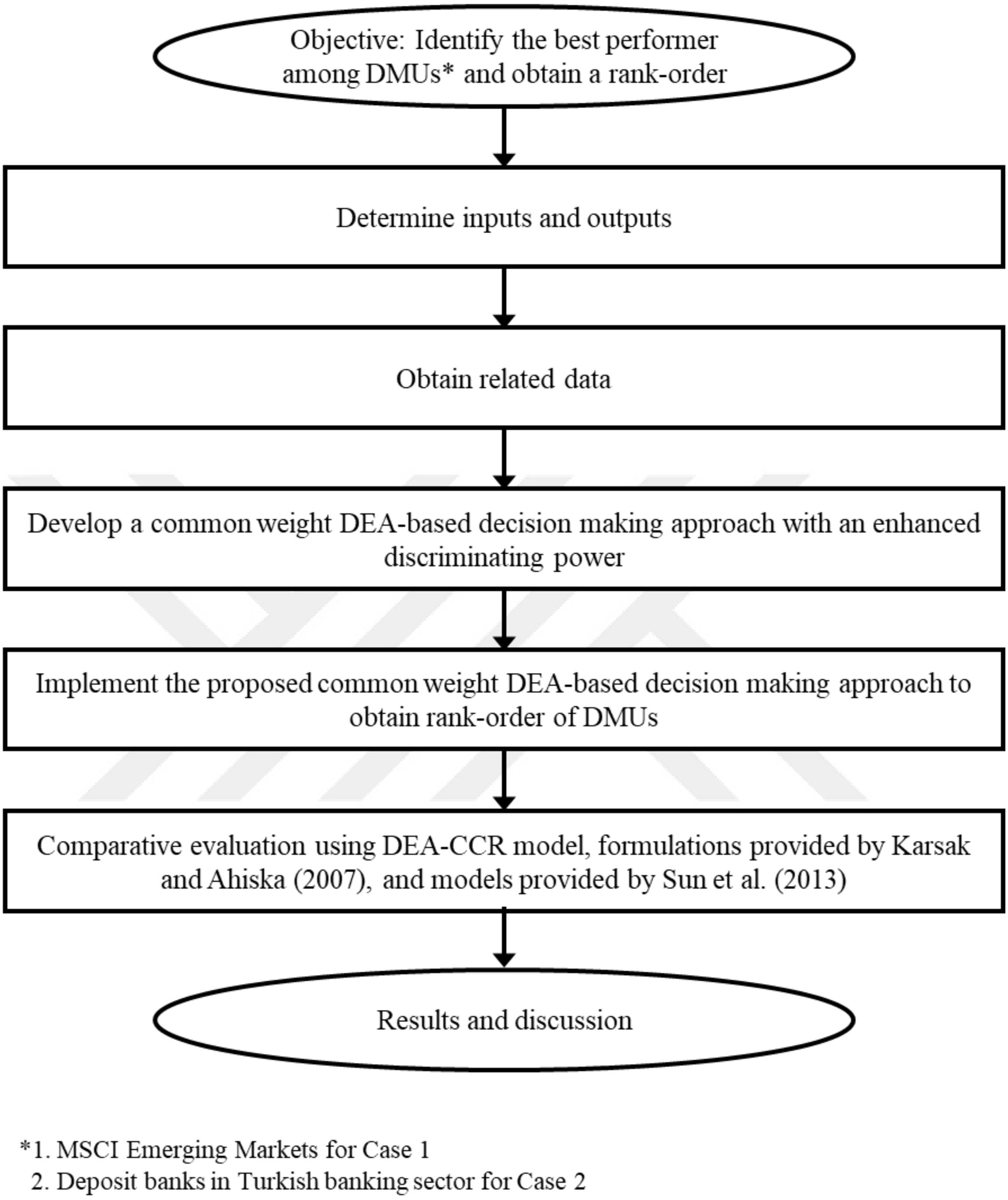


Figure 7.1: Representation of the proposed common weight DEA-based methodology

7.1.1 Performance Evaluation of MSCI Emerging Markets

MSCI was established in 1998 to provide investment decision aid tools worldwide. MSCI offers Market Cap Indexes that are divided into four different indexes as “MSCI World

Index”, “MSCI Emerging Markets Index”, “MSCI Frontier Markets Index”, and “MSCI Standalone Market Indexes” (www.msci.com).

This subsection illustrates the application of the proposed decision aid using MSCI Emerging Markets data, and compares the results with those of earlier models developed by Karsak and Ahiska (2007) and Sun et al. (2013). In order to identify the best performer among the countries in MSCI Emerging Markets Index, 23 countries are evaluated according to two inputs, namely “inflation” and “government debt to gross domestic product (GDP)”, and four outputs, namely “GDP growth rate”, “gross savings to GDP”, “current account balance (% of GDP)”, and “market capitalization of listed domestic companies (% of GDP)”. Raw data for related inputs and outputs for the year 2015 are given in Table 7.1. Since the raw data includes negative as well as positive values, input data are normalized as $\left[1-\frac{x_i^*-x_{ij}}{\sum_{i=1}^{m}(x_i^*-x_{ij})}\right]$, where $x_i^* = \max_j x_{ij}$ for $\forall i$, while output data are normalized using $\left[1-\frac{y_r^*-y_{rj}}{\sum_{r=1}^{s}(y_r^*-y_{rj})}\right]$, where $y_r^* = \max_j y_{rj}$ for $\forall r$ (Jahan et al., 2016).

Table 7.1: Input and output data for MSCI Markets
(http://data.worldbank.org; https://tradingeconomics.com)

DMU (j)	Country	Input1	Input2	Output1	Output2	Output3	Output4
1	Brazil	9	65.45	-3.8	14	-3.3	27.2
2	Chile	4.3	17.4	2.3	21	-2	79.1
3	China	1.4	42.6	6.9	48	3	74
4	Colombia	5	50.7	3.1	18	-6.5	29.4
5	Czech Republic	0.3	40.3	4.5	27	0.9	17.39
6	Egypt	10.4	85	4.2	11	-5.1	16.7
7	Greece	-1.7	177.4	-0.2	10	0.1	21.6
8	Hungary	-0.1	74.7	3.1	25	3.2	14.5
9	India	4.9	69.6	7.9	32	-1.1	72.6
10	Indonesia	6.4	26.9	4.8	32	-2	41
11	Korea	0.7	37.8	2.6	36	7.7	89.4
12	Malaysia	2.1	54.5	5	28	3	129.3
13	Mexico	2.7	43.2	2.5	22	-2.9	35.2
14	Pakistan	2.5	63.2	4.7	23	-0.6	15.25
15	Peru	3.6	23	3.3	19	-4.9	29.9
16	Philippines	1.4	45.05	5.9	44	2.5	81.7
17	Poland	-1	51.1	3.9	20	-0.6	28.9
18	Qatar	1.9	34.9	3.6	47	8.4	86.6
19	Russia	15.5	15.9	-2.8	27	5.1	28.8
20	South Africa	4.6	49.3	1.3	16	-4.3	234
21	Thailand	-0.9	43.9	2.8	30	8.1	88.3
22	Turkey	7.7	27.5	4	25	-4.5	26.3
23	United Arab Emirates	4.1	18.1	3.8	29	5.3	52.9

Overall ranking results according to efficiency scores are shown in Table 7.2. To provide a comparative evaluation of models, ϵ is taken to be 0.000001 as in Sun et al. (2013). The DEA-CCR model yields five efficient countries that are DMU_3, DMU_{12}, DMU_{18}, DMU_{20} and DMU_{21}, whereas other DMUs are inefficient. On the other hand, DMU_{18}, DMU_{20} and DMU_{21} are considered as minimax efficient as regards Formulation (6.1). Subsequently, three additional linear programs are solved with a step size of 0.1 until k equals to 0.3 in Formulation (6.2), and DMU_{20} is identified as the best performing country. Although DMU_{18} and DMU_{21} obtain efficiency score of 1 according to Formulation (6.1), with respect to Formulation (6.2) they are rank-ordered after DMU_{12} which is a minimax inefficient alternative. Alternatively, DMU_3 and DMU_{20} are the most efficient countries with regard to Formulation (6.4) and Formulation (6.5), respectively, developed by Sun et al. (2013). Instead, Formulation (6.7) of the developed methodology identifies DMU_{18} as the best performing country. DMU_{20} and DMU_{21}, which are the minimax efficient countries as well regarding Formulation (6.6), follow DMU_{18} in the efficiency rankings of Formulation (6.7).

Table 7.2: Rankings with respect to efficiency scores of countries

DMU (j)	DEA-CCR	Formulation (6.1) developed by Karsak and Ahiska (2007)	Formulation (6.2) developed by Karsak and Ahiska (2007)	Formulation (6.4) developed by Sun et al. (2013)	Formulation (6.5) developed by Sun et al. (2013)	Proposed formulation (6.6)	Proposed formulation (6.7)
1	23	21	20	22	19	21	21
2	10	9	8	13	2	9	9
3	1	6	4	1	9	6	6
4	21	19	17	14	17	19	19
5	14	10	12	6	16	10	10
6	22	21	20	17	22	21	21
7	18	21	20	23	23	21	21
8	17	14	16	19	21	14	14
9	7	10	10	2	14	10	10
10	11	13	14	4	11	13	13
11	6	4	4	15	5	4	4
12	1	5	2	8	3	5	5
13	20	14	14	18	15	14	14
14	19	17	17	10	20	17	17
15	16	14	12	9	12	14	14
16	9	6	4	3	8	6	6
17	12	10	10	12	18	10	10
18	1	1	4	11	4	1	1
19	13	20	20	21	10	20	20
20	1	1	1	20	1	1	2
21	1	1	3	16	7	1	2
22	15	18	17	7	13	18	18
23	8	8	9	5	6	8	8

The developed approach that enables to identify the best performing country obviously enhances the discriminatory characteristics of DEA-CCR model yielding five efficient countries. In comparison to models in Karsak and Ahiska (2007), the ranking inconsistency encountered in the outcomes of Formulation (6.2) among the DMUs found to be efficient according to Formulation (6.1) is avoided in the proposed approach. Moreover, in contrast with Formulation (6.2), a discriminating parameter is not required in the proposed model.

Although presenting a new common weight modelling perspective, the models addressed in Sun et al. (2013) are inapt to provide robust weight distribution for assessing MSCI emerging markets as given in Table 7.3. Formulation (6.5) developed by Sun et al. (2013) considers a single input ($v_2 > \epsilon$) and a single output ($u_4 > \epsilon$), while Formulation (6.4) considers one output ($u_1 > \epsilon$) and two inputs (v_1, $v_2 > \epsilon$). One shall also note that v_1 equals to 0.000008 that is quite close to ϵ. All the other weights are determined to be equal to non-Archimedean infinitesimal ϵ. On the other hand, Formulation (6.7) proposed in this thesis provides improved weight dispersion with all inputs and outputs possessing weights significantly greater than ϵ as depicted in Table 7.3.

Table 7.3: Comparative evaluation of input and output weights

Weight	**Formulation (6.2) developed by Karsak and Ahiska (2007)**	**Formulation (6.4) developed by Sun et al. (2013)**	**Formulation (6.5) developed by Sun et al. (2013)**	**Proposed formulation (6.7)**
v_1	0.233098	0.000008	0.000001	0.21042
v_2	0.27573	1.058481	0.999999	0.294784
u_1	0.056867	0.999997	0.000001	0.065238
u_2	0	0.000001	0.000001	0.023153
u_3	0.00454	0.000001	0.000001	0.079532
u_4	0.429765	0.000001	1.057122	0.326874

7.1.2 Performance Evaluation of Banks in Turkish Banking Sector

Banking is an important component of the financial framework that mediates the transfer of resources in the economy. The banking sector is a major field that affects the whole economy as well as being a crucial element of the Turkish financial framework. The relative share of the banking sector in the financial system may vary in each country depending on the level of economic and social development. The relative share of the banking sector in the financial system generally decreases in parallel to social and economic development.

After the financial crisis in 2001, the banking sector recovery continued until 2003 in Turkey. The number of branches decreased due to the bankruptcies in the industry. After 2004, number of branches started to increase rapidly although the number of banks continued to decrease between 2004 and 2010. Due to the upsurge in total number of branches, the number of personnel has also increased. Moreover, the banks have managed to increase regularly the number of ATM and POS machines along with the number of credit cards (www.tbb.org.tr).

The increasing level of competition in the banking industry has led the banks to focus on provided services and efficient allocation of available resources (Grigoroudis et al., 2013). Turkish financial services developed dramatically during the period 2002-2012 when Turkish banks obtained return on equity (ROE) ratios above 20% every year. Although the global economic crisis contracted the gross domestic product (GDP) by 4.8% in 2009, a drastic rebound of 9.2% growth in GDP followed in 2010. By the end of 2012, Turkish banking system was no longer an underdeveloped sector, while also being a client-focused market. The average GDP growth rate for Turkey decreased to 3.1% during 2013-2016 period, whereas average ROEs of banks decreased to about 12% (Elhadef et al., 2016). In Turkish banking sector, loans to total assets ratio is high compared with a number of other countries while non-deposit funds are substantially based on liabilities (www.tbb.org.tr). As of 2016, there are 34 deposit banks, 13 investment banks and 6 participation banks in Turkey.

This subsection illustrates the implementation of the proposed approach through a case study conducted in Turkish banking sector based on real data, and provides a comparative evaluation with the models developed by Karsak and Ahiska (2007) and Sun et al. (2013). The case study, which aims to identify the most efficient bank, evaluates 18 deposit banks selected considering the size of total assets according to three inputs as "total non-deposit funds", "deposits" and "number of branches", and three outputs namely "total loans", "total income from interest" and "total financial assets", respectively. Raw data for related inputs and outputs for the year 2015 are depicted in Table 7.4. Normalization for data is performed via a linear normalization scheme. Input data are normalized as x_{ij}/x_i^*, where $x_i^* = \max_j x_{ij}$ for $\forall i$, whereas output data are normalized as y_{rj}/y_r^*, where $y_r^* = \max_j y_{rj}$ for $\forall r$ (Karsak & Ahiska, 2007).

Table 7.4: Input and output data for deposit banks (www.tbb.org.tr)

DMU (*j*)	Bank	Input1 (millions of TRY)	Input2 (millions of TRY)	Input3	Output1 (millions of TRY)	Output2 (millions of TRY)	Output3 (millions of TRY)
1	Turkiye Cumhuriyeti Ziraat Bankasi	73847.159	186469.435	1812	186813	22050.495	64871.349
2	Turkiye Is Bankasi	68258.775	153802.426	1377	177934	19200.361	46044.289
3	Turkiye Garanti Bankasi	62704.727	140899.332	980	159140	17420.007	44805.077
4	Akbank	57808.513	138942.497	902	141763	15247.388	55524.719
5	Yapi ve Kredi Bankasi	45478.233	126908.893	1000	148779	15292.461	31862.498
6	Turkiye Halk Bankasi	39540.87	122145.965	949	126745	13656.908	28155.131
7	Turkiye Vakiflar Bankasi	41852.025	109922.534	920	123781	13630.05	25337.165
8	Finans Bank	14615.651	48565.837	642	57226	7597.377	14721.072
9	Denizbank	19142.426	46587.577	692	51349	6804.782	12882.837
10	Turk Ekonomi Bankasi	14250.187	44395.86	532	53213	6219.447	5226.274
11	ING Bank	16688.666	23648.977	298	35205	3726.152	5078.701
12	Odea Bank	3445.322	25333.496	55	21807	2352.473	1587.352
13	HSBC Bank	6142.302	19056.359	284	20491	2402.378	2318.259
14	Sekerbank	5422.869	14867.633	301	16726	2283.308	3151.962
15	Alternatifbank	4588.029	6288.12	59	9345	1071.06	843.604
16	Fibabanka	2033.009	7460.485	67	8615	891.475	728.302
17	Anadolubank	1789.426	7322.809	106	6815	959.21	1513.841
18	Burgan Bank	2122.614	6695.608	56	8186	846.777	774.839

Overall ranking outcomes with regard to efficiency scores are reported in Table 7.5. In order to enable a comparison of the results, ϵ is taken to be 0.000001 as in Sun et al. (2013). The DEA-CCR model results in nine efficient banks that are DMU_3, DMU_4, DMU_5, DMU_8, DMU_{11}, DMU_{12}, DMU_{15}, DMU_{16} and DMU_{18} while the other DMUs are considered as inefficient. Besides, DMU_4, DMU_8 and DMU_{11} are minimax efficient DMUs according to Formulation (6.1). Afterwards, seven additional linear programs are solved with a step size of 0.1 until k equals to 0.7 in Formulation (6.2), and DMU_8 is found to be the best performing bank. However, DMU_4 and DMU_{11} with efficiency scores of 1 according to Formulation (6.1) rank below a number of minimax inefficient alternatives with respect to Formulation (6.2) that results in an inconsistency between the outcomes of Formulations (6.1) and (6.2). On the other hand, DMU_{11} and DMU_{15} are the best alternatives as regards Formulation (6.4) and Formulation (6.5), respectively, addressed in Sun et al. (2013). Alternatively, DMU_8 is the best performing bank according to Formulation (6.7) of the proposed methodology. DMU_4 and DMU_{11}, which are also the minimax efficient banks with respect to Formulation (6.6), follow DMU_8 in the efficiency rankings of Formulation (6.7).

Table 7.5: Rankings with respect to efficiency scores of banks (ϵ = 0.000001)

DMU (*j*)	DEA-CCR	Formulation (6.1) developed by Karsak and Ahiska (2007)	Formulation (6.2) developed by Karsak and Ahiska (2007)	Formulation (6.4) developed by Sun et al. (2013)	Formulation (6.5) developed by Sun et al. (2013)	Proposed formulation (6.6)	Proposed formulation (6.7)
1	15	17	16	16	15	17	17
2	11	11	16	7	10	11	11
3	1	7	14	9	12	7	7
4	1	1	11	15	17	1	2
5	1	9	13	6	13	9	9
6	16	17	16	14	16	17	17
7	14	16	15	10	11	16	16
8	1	1	1	5	3	1	1
9	17	13	9	12	5	13	13
10	10	14	12	4	6	14	14
11	1	1	8	1	2	1	2
12	1	15	9	18	18	15	15
13	18	12	7	13	9	12	12
14	13	9	6	11	4	9	9
15	1	4	5	2	1	4	4
16	1	6	4	8	14	6	6
17	12	7	2	17	7	7	7
18	1	5	2	3	8	5	5

The proposed approach provides enhanced discriminating power by enabling to determine the best performing bank while DEA-CCR model results in nine efficient banks. Contrary to the models developed in Karsak and Ahiska (2007), Formulation (6.7) provides consistent rankings regarding the sets of efficient and inefficient banks obtained from Formulation (6.6). Furthermore, the discriminating parameter, which is required in Karsak and Ahiska (2007) despite its previously discussed drawbacks, is not used in the proposed framework.

The formulations employed in Sun et al. (2013) do not provide sound weight distribution compared to Formulation (6.7) for evaluating the banks as depicted in Table 7.6. Formulation (6.4) proposed by Sun et al. (2013) considers a single input ($v_2 > \epsilon$) and an output ($u_1 > \epsilon$), whereas Formulation (6.5) takes into account one input ($v_2 > \epsilon$) and one output ($u_2 > \epsilon$) to rank the alternatives. All the other weights are determined to be equal to ϵ. Contrarily, Formulation (6.7) proposed in this thesis results in enhanced dispersion of weights assigned to inputs and outputs as shown in Table 7.6. These diversity in weights result in different efficiency scores for assessing alternatives, and thus, obtained rankings are obviously not the same for three approaches. The unrealistic weighting schemes, which result in a single input and a single output to be considered while ignoring other inputs as well as outputs in efficiency computations, distort the robustness of the related models.

Table 7.6: Comparative evaluation of input and output weights

Weight	Formulation (6.2) developed by Karsak and Ahiska (2007)	Formulation (6.4) developed by Sun et al. (2013)	Formulation (6.5) developed by Sun et al. (2013)	Proposed formulation (6.7)
v_1	0.179158	0.000001	0.000001	0.063572
v_2	0.350691	29.65423	0.999998	0.416027
v_3	0	0.000001	0.000001	0.039816
u_1	0.253216	0.999998	0.000001	0.327263
u_2	0	0.000001	26.071539	0.000001
u_3	0.216935	0.000001	0.000001	0.153321

7.2 Application of the Proposed Imprecise Decision Approach

In this subsection of the thesis, three illustrative examples taken from the literature as well as conducted case study are provided. Initially, data are normalized employing a linear normalization scheme in order to avoid problems related to scale differences. Input data are normalized as ${\tilde{x}_{ij}}/{x^*_{ijc}}$, where $x^*_{ijc} = \max_j x_{ijc}$ for $\forall i$, while output data are normalized as ${\tilde{y}_{rj}}/{y^*_{rjc}}$, where $y^*_{rjc} = \max_j y_{rjc}$ for $\forall r$. Afterwards, Formulation (6.12) is applied to each data set. In case of Formulation (6.12) yields multiple efficient DMUs, Formulation (6.13) is employed to identify the best performer, and provide full ranking of DMUs for each α-cut value. By using the linguistic quantifier “most” and Equations (6.15) and (6.16), the OWA weights for 11 α-cut values are calculated as **w** = (0,0,0,0.127273,0.181818,0.181818,0.181818,0.181818,0.145455,0,0). Afterwards, aggregated efficiencies of DMUs are determined employing Equation (6.14), and then the DMUs are ranked regarding their aggregated efficiency scores in descending order.

The stepwise representation of the proposed imprecise common weight DEA-based decision framework is depicted in Figure 7.2.

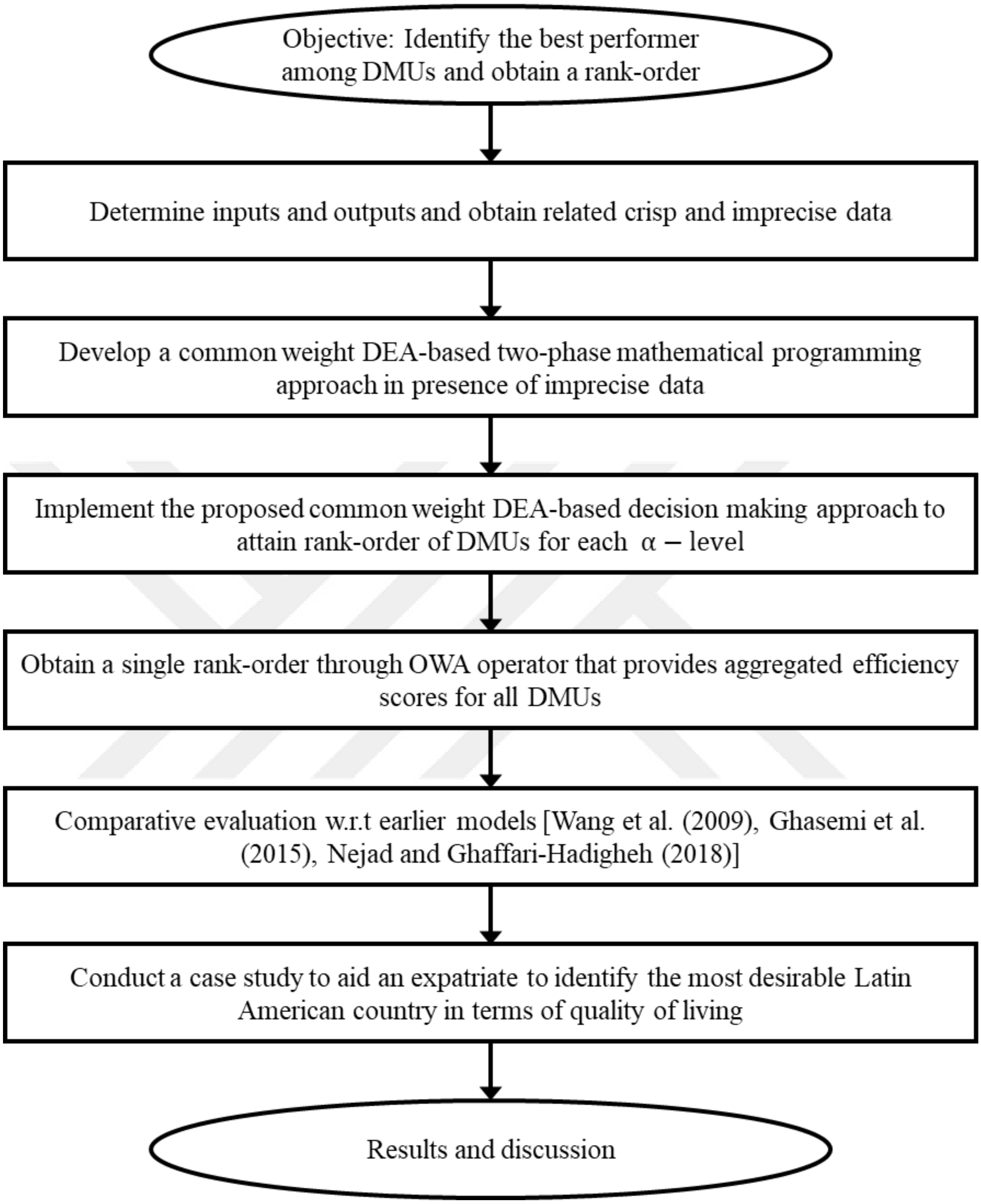


Figure 7.2: Representation of the proposed imprecise common weight DEA-based decision framework

7.2.1 Illustrative Examples

This subsection includes three illustrative examples which have been reported in earlier research papers.

The first illustrative example, which was initially given in Nejad and Ghaffari-Hadigheh (2018), evaluates 10 DMUs with respect to two inputs (I1 and I2) and two outputs (O1 and O2). Data regarding the problem are denoted in Table 7.7.

Table 7.7: Fuzzy data related to example 1 (Nejad & Ghaffari-Hadigheh, 2018)

DMU	I1	I2	O1	O2
1	(58.5, 95, 97)	(85.8, 135, 137)	(62, 67, 126)	(153, 159, 289.8)
2	(52, 83, 90)	(97.5, 154, 157)	(71, 75, 142.2)	(189, 191, 351)
3	(39, 69, 70)	(91, 145, 150)	(58, 59, 108)	(200, 204, 374.4)
4	(45.5, 78, 90)	(58.5, 99, 107)	(91, 93, 171)	(175, 178, 324)
5	(52.65, 85, 86)	(78, 121, 122)	(80, 84, 153)	(180, 183, 333)
6	(37.05, 59, 64)	(71.5, 112, 114)	(61, 65, 126)	(141, 149, 273.6)
7	(57.85, 90, 92)	(130, 201, 205)	(51, 58, 120.6)	(190, 200, 369)
8	(39, 67, 70)	(65, 109, 110)	(72, 74, 140.4)	(188, 189, 343.8)
9	(58.5, 96, 100)	(74.1, 119, 122)	(48, 49, 95.4)	(170, 174, 315)
10	(65, 101, 110)	(84.5, 131, 132)	(86, 91, 171)	(150, 151, 279)

Nejad and Ghaffari-Hadigheh (2018) set ϵ to 0.001, and solved their proposed model for four different α values that are 0.6, 0.7, 0.8, and 0.9; and provided four different rank-orders as represented in Table 7.8. They ignored smaller α values in order to improve discriminating power of their approach; however, identifying the best performing DMU is not possible for any of α values. Besides, one can conclude that their model yields varying rankings for different α values. For instance, DMU_1 ranks ninth for α=0.6 and α=0.7, however the same DMU is ranked as fifth for α=0.8. Moreover, for α=0.9, their model results in infeasible solution for DMU_1, DMU_2, DMU_5, DMU_7, DMU_9, and DMU_{10}. In addition, the model yields different set of weights for inputs and outputs, thus common assessment for all DMUs cannot be provided.

Table 7.8: Ranking results given in Nejad & Ghaffari-Hadigheh (2018)

DMU	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$
1	9	9	5	Infeasible
2	5	5	6	Infeasible
3	2	2	2	2
4	1	1	1	1
5	4	4	4	Infeasible
6	3	3	3	3
7	8	7	8	Infeasible
8	1	1	1	1
9	6	6	7	Infeasible
10	7	8	9	Infeasible

On the other hand, overall ranking results of the model proposed in this work are reported in Table 7.9. As in the model proposed by Nejad and Ghaffari-Hadigheh (2018), ϵ is set as 0.001 to provide a robust comparison. Akin to Nejad and Ghaffari-Hadigheh (2018), DMU_8 is determined as the best performer. Unlike their model, Formulation (6.12) provides full ranking of DMUs for each of the α values, and identifies the best performer as DMU_8 for every value of α. Since Formulation (6.12) yields the single best performer, Formulation (6.13) is not implemented. As shown in Table 7.9, Formulation (6.12) generates very similar ranking results for different values of α, and provides a common evaluation for DMUs by using a common set of weights for inputs and outputs while infeasibility problem does not occur. Although the first ranking DMU is the same for all α-levels, the remaining rankings for the DMUs change with respect to α values. Hence, a single rank-order that is given in Table 7.10 is attained by employing OWA operator, which aggregates the ranking results for different values of α.

Table 7.9: Efficiency scores obtained from Formulation (6.12) for example 1

DMU	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
1	0.852084	0.850601	0.849675	0.849407	0.849931	0.851022	0.852708	0.85502	0.857992	0.861665	0.866121
2	0.913068	0.913307	0.913776	0.914385	0.914974	0.91588	0.917124	0.918727	0.920712	0.925089	0.928037
3	0.968068	0.967183	0.966541	0.966179	0.966152	0.966316	0.966668	0.96721	0.967946	0.966109	0.967525
4	0.986888	0.985999	0.985239	0.984528	0.983753	0.983157	0.982738	0.982498	0.982439	0.986399	0.986711
5	0.925813	0.926713	0.92782	0.929128	0.93062	0.932329	0.934261	0.936427	0.938837	0.943407	0.946446
6	0.964578	0.965046	0.965578	0.965985	0.965986	0.966174	0.96655	0.967115	0.967871	0.972242	0.973366
7	0.849269	0.848927	0.849009	0.849407	0.849931	0.851022	0.852708	0.85502	0.857992	0.861665	0.866121
8	1	1	1	1	1	1	1	1	1	1	1
9	0.849269	0.848927	0.849009	0.850012	0.852654	0.855505	0.858603	0.861987	0.865687	0.861665	0.866121
10	0.849269	0.848927	0.849009	0.849407	0.849931	0.851022	0.852708	0.85502	0.857992	0.867812	0.872196

Table 7.10: Aggregated efficiency and ranking results for example 1

DMU	Aggregated efficiency	Rank
1	0.851795	8
2	0.915726	6
3	0.966672	3
4	0.984194	2
5	0.931653	5
6	0.966207	4
7	0.85109	9
8	1	1
9	0.854428	7
10	0.85109	9

The model proposed by Nejad and Ghaffari-Hadigheh (2018) yields different set of weights for inputs and outputs for each DMU, which renders a common assessment impractical. On the other hand, Formulation (6.12) proposed in this thesis provides more meaningful weight dispersion with all inputs and outputs possessing weights significantly greater than ϵ as depicted in Table 7.11. Thereby, rankings of the proposed model and the approach developed by Nejad and Ghaffari-Hadigheh (2018) differ as expected.

Table 7.11: Input and output weights for example 1

Weight	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
v_1	0.553216	0.526023	0.500269	0.474034	0.445029	0.418106	0.392876	0.369032	0.346323	0.340129	0.318611
v_2	0.163359	0.167312	0.169175	0.171003	0.175329	0.176758	0.175633	0.172217	0.166707	0.139574	0.130579
u_1	0.126761	0.130852	0.135224	0.138572	0.138681	0.13948	0.141011	0.143343	0.146585	0.193546	0.202049
u_2	0.156664	0.175813	0.195332	0.216391	0.240961	0.265656	0.29048	0.315408	0.340385	0.326751	0.348761

The second illustrative example is taken from Wang et al. (2009) for which data are given in Table 7.12. The example evaluates eight manufacturing enterprises with "manufacturing cost (I1)" and "number of employees (I2)" employed as inputs, and "gross output value (O1)" and "product quality (O2)" as outputs, respectively. The manufacturing cost and gross output value are considered as fuzzy data due to imprecision, and product quality is evaluated by customers using a fuzzy linguistic term set whose membership functions are represented by triangular fuzzy numbers.

Table 7.12: Data related to example 2 (Wang et al., 2009)

DMU	I1	I2	O1	O2
1	(2120, 2170, 2210)	1870	(14500, 14790, 14860)	(3.1, 4.1, 4.9)
2	(1420, 1460, 1500)	1340	(12470, 12720, 12790)	(1.2, 2.1, 3.0)
3	(2510, 2570, 2610)	2360	(17900, 18260, 18400)	(3.3, 4.3, 5.0)
4	(2300, 2350, 2400)	2020	(14970, 15270, 15400)	(2.7, 3.7, 4.6)
5	(1480, 1520, 1560)	1550	(13980, 14260, 14330)	(1.0, 1.8, 2.7)
6	(1990, 2030, 2100)	1760	(14030, 14310, 14400)	(1.6, 2.6, 3.6)
7	(2200, 2260, 2300)	1980	(16540, 16870, 17000)	(2.4, 3.4, 4.4)
8	(2400, 2460, 2520)	2250	(17600, 17960, 18100)	(2.6, 3.6, 4.6)

The modeling framework proposed by Wang et al. (2009) requires 6*n* linear programs, where *n* is the number of DMUs, for determining the best possible fuzzy efficiency values for all DMUs in addition to an analytical approach based on degree of preference to provide full ranking. As a result, rankings that are given in Table 7.13 are obtained. According to the rank-order, DMU_2 is determined as the best performer followed by DMU_5 and DMU_1, respectively.

Table 7.13: Ranking results given in Wang et al. (2009)

DMU	Rank
1	3
2	1
3	7
4	8
5	2
6	5
7	4
8	6

Table 7.14 and Table 7.15 outline the ranking results of example 2 obtained by Formulation (6.12) and Formulation (6.13), respectively, with $\epsilon = 0$ as in Wang et al.

(2009). According to Formulation (6.12) of the proposed methodology, DMU_1 and DMU_2 are minimax efficient units for all α-cut values. Since Formulation (6.12) results in two minimax efficient DMUs, Formulation (6.13) is solved to identify the single best performer. As depicted in Table 7.15, DMU_1 is the best performing alternative, and it is followed by DMU_2 for all α-levels. Though the first two DMUs are identical for all α-levels, the resulting rankings for other DMUs vary with respect to α values. Therefore, an aggregation operator, which provides a single rank-order for all DMUs, is required to evaluate the alternatives in a robust manner. For that reason, OWA operator is utilized to aggregate the outcomes for α-levels and yield a final rank-order for all DMUs as in Table 7.16.

The proposed methodology guarantees to obtain the best performing DMU by solving a mixed integer linear programming model in addition to a linear programming model for each α-level. Moreover, ranking results of the proposed methodology show a higher level of consistency with respect to different α values.

Table 7.14: Efficiency scores obtained from Formulation (6.12) for example 2

DMU	α = 0	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5	α = 0.6	α = 0.7	α = 0.8	α = 0.9	α = 1
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1
3	0.945824	0.948306	0.950588	0.952692	0.954636	0.956436	0.958106	0.959658	0.961102	0.962449	0.963706
4	0.950393	0.951373	0.952264	0.953076	0.953818	0.954496	0.955117	0.955686	0.956208	0.956686	0.957126
5	0.969967	0.970494	0.970977	0.971421	0.97183	0.972208	0.972558	0.972882	0.973183	0.973462	0.973722
6	0.941946	0.943275	0.944493	0.945613	0.946645	0.947597	0.948477	0.949292	0.950048	0.950749	0.951402
7	0.968234	0.969694	0.971043	0.97229	0.973448	0.974524	0.975527	0.976463	0.977338	0.978159	0.978929
8	0.941946	0.943275	0.944493	0.945613	0.946645	0.947597	0.948477	0.949292	0.950048	0.950749	0.951402

Table 7.15: Efficiency scores obtained from Formulation (6.13) for example 2

DMU	α = 0	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5	α = 0.6	α = 0.7	α = 0.8	α = 0.9	α = 1
1	1	1	1	1	1	1	1	1	1	1	1
2	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999
3	0.945823	0.948305	0.950587	0.952691	0.954636	0.956436	0.958106	0.959657	0.961102	0.962448	0.963705
4	0.950393	0.951372	0.952263	0.953076	0.953817	0.954496	0.955117	0.955686	0.956207	0.956686	0.957125
5	0.969966	0.970492	0.970975	0.971419	0.971829	0.972207	0.972557	0.972881	0.973181	0.97346	0.97372
6	0.941945	0.943274	0.944492	0.945612	0.946644	0.947596	0.948476	0.949291	0.950047	0.950748	0.951401
7	0.968233	0.969693	0.971041	0.972289	0.973447	0.974523	0.975526	0.976462	0.977338	0.978158	0.978928
8	0.941945	0.943274	0.944492	0.945612	0.946644	0.947596	0.948476	0.949291	0.950047	0.950748	0.951401

Table 7.16: Aggregated efficiency and ranking results for example 2

DMU	Aggregated efficiency	Rank
1	1	1
2	0.999999	2
3	0.95529	5
4	0.954054	6
5	0.971965	4
6	0.946986	7
7	0.973844	3
8	0.946986	7

Rankings for Wang et al. (2009) and the proposed approach differ since the proposed decision model yields common weights for inputs and outputs unlike the approach proposed by Wang et al. (2009). The proposed modeling framework enables common performance evaluation, thus it is more practical and plausible than the approach in Wang et al. (2009). Input and output weights for each α-level that are depicted in Table 7.17, obtained by employing Formulation (6.13) convey sound dispersion.

Table 7.17: Input and output weights for example 2

Weight	**α = 0**	**α = 0.1**	**α = 0.2**	**α = 0.3**	**α = 0.4**	**α = 0.5**	**α = 0.6**	**α = 0.7**	**α = 0.8**	**α = 0.9**	**α = 1**
v_1	0.343629	0.314278	0.287096	0.26185	0.238342	0.216399	0.195872	0.176626	0.158548	0.141535	0.125495
v_2	0.190042	0.21509	0.238317	0.259919	0.280061	0.298889	0.316529	0.333092	0.348675	0.363364	0.377236
u_1	0.158369	0.174643	0.18969	0.20364	0.216607	0.228687	0.239965	0.250518	0.260409	0.269697	0.278434
u_2	0.30796	0.295989	0.284897	0.274591	0.26499	0.256025	0.247634	0.239764	0.232368	0.225404	0.218835

The third example, which was presented in Ghasemi et al. (2015), illustrates energy dependency among European Union countries except Bulgaria, Luxembourg, Malta, and Romania. The two inputs and three outputs data set for 23 European Union countries is given in Table 7.18. “Allocated carbon allowances (I1)” and “gross inland energy consumption (I2)” are used as inputs, while “electricity generated from renewable sources (O1)”, “verified emissions (O2)”, and “share of renewable energy in gross final energy consumption (O3)” are identified as outputs.

Table 7.18: Data related to example 3 (Ghasemi et al., 2015)

DMU	I1	I2	O1	O2	O3
1	(3.853, 3.859, 4.088)	(4.105, 4.130, 4.143)	(59.038, 61.363, 64.980)	(0.3043, 0.3088, 0.3426)	29.7
2	(5.482, 5.570, 5.931)	(5.501, 5.567, 5.719)	(3.960, 4.359, 5.391)	(0.5091, 0.5231, 0.5885)	4.6
3	(5.129, 5.168, 5.931)	(2.503, 2.544, 2.615)	(0.080, 0.105, 0.241)	(0.0530, 0.0540, 0.0555)	4.6
4	(9.118, 9.143, 9.958)	(4.384, 4.445, 4.545)	(5.278, 5.400, 6.535)	(0.7843, 0.8141, 0.8609)	8.5
5	(4.127, 5.369, 5.892)	(3.575, 3.742, 3.828)	(24.109, 26.276, 26.757)	(0.2551, 0.2890, 0.2967)	9.9
6	(11.869, 12.645, 17.731)	(4.195, 4.263, 4.361)	(2.744, 2.770, 5.642)	(0.1054, 0.1282, 0.1510)	22.8
7	(8.043, 8.074, 9.179)	(6.531, 6.934, 7.151)	(25.214, 26.613, 30.189)	(0.3793, 0.3940, 0.4073)	30.3
8	(2.339, 2.355, 2.595)	(4.396, 4.450, 4.468)	(12.655, 13.210, 13.641)	(1.2194, 1.2219, 1.3136)	12.3
9	(5.663, 5.681, 6.609)	(4.148, 4.173, 4.201)	(12.144, 14.079, 15.187)	(4.6013, 4.6655, 4.9795)	9.8
10	(6.153, 6.167, 6.650)	(2.807, 2.812, 2.821)	(6.221, 9.788, 12.606)	(0.6710, 0.6905, 0.7343)	8.2
11	(2.867, 2.872, 3.226)	(2.698, 2.709, 2.716)	(4.447, 5.026, 6.174)	(0.2480, 0.2552, 0.2895)	7.7
12	(4.510, 4.562, 4.636)	(3.664, 3.671, 3.719)	(10.202, 10.817, 12.493)	(0.1981, 0.2014, 0.2238)	5.0
13	(3.377, 3.528, 3.618)	(3.082, 3.114, 3.162)	(15.417, 16.020, 19.090)	(2.1412, 2.1485, 2.4016)	8.9
14	(1.646, 1.649, 1.985)	(1.967, 2.018, 2.063)	(40.230, 41.122, 46.793)	(0.0269, 0.0276, 0.0293)	34.3
15	(2.495, 3.088, 3.667)	(2.594, 2.622, 2.635)	(3.890, 4.590, 5.196)	(0.0574, 0.0610, 0.0633)	17.0
16	(5.102, 5.122, 5.552)	(4.949, 5.065, 5.164)	(7.060, 7.440, 8.455)	(0.7804, 0.8028, 0.8203)	4.1
17	(5.981, 5.982, 6.661)	(2.449, 2.534, 2.564)	(3.632, 4.112, 5.195)	(2.0363, 2.0363, 2.1277)	8.9
18	(3.326, 3.334, 3.766)	(2.349, 2.469, 2.544)	(28.999, 29.543, 34.383)	(0.2931, 0.3063, 0.3181)	24.5
19	(5.691, 5.694, 5.904)	(3.399, 3.424, 3.485)	(16.570, 16.609, 18.308)	(0.2352, 0.2425, 0.2689)	10.3
20	(4.101, 4.266, 4.285)	(3.693, 3.697, 3.836)	(26.492, 28.110, 33.534)	(0.0856, 0.0870, 0.0927)	16.9
21	(3.548, 3.683, 3.805)	(3.230, 3.260, 3.356)	(19.523, 20.841, 24.492)	(1.6183, 1.6667, 1.8543)	13.3
22	(2.418, 2.421, 2.594)	(5.417, 5.544, 5.597)	(49.794, 52.610, 53.601)	(0.1852, 0.1912, 0.2095)	47.3
23	(3.453, 3.467, 3.501)	(3.671, 3.724, 3.764)	(5.063, 5.356, 6.250)	(2.4735, 2.5119, 2.7460)	2.9

As can be observed from Table 7.19, three models proposed by Ghasemi et al. (2015), which are named as “CCR form”, “BCC form”, and “FDH form”, provide three different rankings of DMUs. The authors set ϵ as 0. According to CCR form, DMU_{14} is the best performer followed by DMU_9 and DMU_{22}, respectively. BCC form determines DMU_9 as the most efficient unit followed by DMU_{22} and DMU_{14}, respectively. Alternatively, with respect to FDH form, DMU_1 is identified as the best performing alternative followed by DMU_{22} and DMU_{14}, respectively.

Table 7.19: Ranking results given in Ghasemi et al. (2015)

DMU	CCR form	BCC form	FDH form
1	6	4	1
2	23	23	21
3	22	12	15
4	19	21	23
5	12	16	16
6	14	17	22
7	15	15	12
8	7	7	8
9	2	1	4
10	16	14	11
11	18	11	13
12	21	20	18
13	5	8	7
14	1	3	3
15	11	13	14
16	20	22	20
17	8	5	6
18	10	10	10
19	17	19	19
20	13	18	17
21	9	9	5
22	3	2	2
23	4	6	9

Each of three methodologies proposed by Ghasemi et al. (2015) require solving $2n$ linear programs, n indicating the number of DMUs. As depicted in Table 7.20, by setting ϵ equal to 0 as in Ghasemi et al. (2015), Formulation (6.12) yields two minimax efficient DMUs that are DMU_9 and DMU_{14}.

Table 7.20: Efficiency scores obtained from Formulation (6.12) for example 3

DMU	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
1	0.871217	0.87149	0.871763	0.872036	0.872309	0.872583	0.872857	0.873132	0.873407	0.873682	0.873958
2	0.714521	0.714548	0.714578	0.71461	0.714646	0.714684	0.714725	0.714769	0.714817	0.714868	0.714922
3	0.823788	0.823925	0.824065	0.824209	0.824356	0.824507	0.824661	0.824819	0.824981	0.825147	0.825316
4	0.727695	0.728012	0.728335	0.728663	0.728997	0.729336	0.729681	0.730031	0.730387	0.730749	0.731116
5	0.823214	0.820887	0.818572	0.816269	0.813979	0.811703	0.809439	0.80719	0.804954	0.802732	0.800525
6	0.701571	0.700626	0.699691	0.698766	0.697852	0.696948	0.696054	0.695172	0.6943	0.693439	0.692589
7	0.701571	0.700626	0.699691	0.698766	0.697852	0.696948	0.696054	0.695172	0.6943	0.693439	0.692589
8	0.874901	0.874949	0.874998	0.875048	0.875098	0.875149	0.8752	0.875253	0.875306	0.87536	0.875415
9	1	1	1	1	1	1	1	1	1	1	1
10	0.838193	0.83853	0.83887	0.839213	0.839558	0.839906	0.840257	0.84061	0.840966	0.841325	0.841687
11	0.874651	0.874816	0.874981	0.875146	0.875312	0.875478	0.875645	0.875812	0.87598	0.876149	0.876318
12	0.793576	0.793855	0.794136	0.794418	0.794703	0.79499	0.79528	0.795571	0.795865	0.796162	0.796461
13	0.956059	0.955354	0.954645	0.953932	0.953216	0.952497	0.951774	0.951048	0.950318	0.949584	0.948847
14	1	1	1	1	1	1	1	1	1	1	1
15	0.904451	0.903646	0.902846	0.90205	0.901259	0.900473	0.899692	0.898916	0.898145	0.89738	0.89662
16	0.754328	0.754481	0.75464	0.754804	0.754975	0.755151	0.755333	0.755522	0.755717	0.755919	0.756127
17	0.926473	0.926539	0.926608	0.926683	0.926763	0.926848	0.926937	0.927032	0.927131	0.927236	0.927345
18	0.937749	0.937541	0.937337	0.937135	0.936937	0.936741	0.936548	0.936358	0.93617	0.935986	0.935805
19	0.804575	0.804862	0.805151	0.805442	0.805735	0.806031	0.806329	0.80663	0.806933	0.807239	0.807547
20	0.83129	0.831437	0.831586	0.831738	0.831891	0.832048	0.832207	0.832368	0.832533	0.8327	0.83287
21	0.934456	0.933999	0.93354	0.933079	0.932616	0.932151	0.931685	0.931216	0.930746	0.930273	0.929799
22	0.889112	0.889071	0.889031	0.888992	0.888953	0.888916	0.888879	0.888844	0.888809	0.888776	0.888744
23	0.926715	0.926372	0.926027	0.92568	0.925332	0.924982	0.92463	0.924276	0.92392	0.923563	0.923203

Since Formulation (6.12) results in two efficient DMUs, Formulation (6.13) is solved to identify the single best performing unit. According to the results depicted in Table 7.21, DMU_9 ranks first, and it is followed by DMU_{14} according to the efficiency scores of DMUs. Albeit the first and second DMUs are the same for all α-levels, the rankings for other DMUs differ as α changes. Thus, a rank-order based on aggregated efficiency values is obtained via OWA operator as shown in Table 7.22.

Table 7.21: Efficiency scores obtained from Formulation (6.13) for example 3

DMU	α = 0	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5	α = 0.6	α = 0.7	α = 0.8	α = 0.9	α = 1
1	0.871216	0.871489	0.871762	0.872035	0.872308	0.872582	0.872856	0.873131	0.873406	0.873681	0.873957
2	0.714521	0.714548	0.714578	0.71461	0.714645	0.714684	0.714725	0.714769	0.714816	0.714867	0.714921
3	0.823788	0.823925	0.824065	0.824209	0.824356	0.824507	0.824661	0.824819	0.824981	0.825146	0.825316
4	0.727694	0.728012	0.728335	0.728663	0.728996	0.729336	0.72968	0.730031	0.730386	0.730748	0.731116
5	0.823214	0.820886	0.818571	0.816269	0.813979	0.811702	0.809439	0.807189	0.804954	0.802732	0.800525
6	0.70157	0.700625	0.69969	0.698765	0.697851	0.696947	0.696053	0.695171	0.694299	0.693438	0.692588
7	0.70157	0.700625	0.69969	0.698765	0.697851	0.696947	0.696053	0.695171	0.694299	0.693438	0.692588
8	0.8749	0.874949	0.874998	0.875048	0.875098	0.875149	0.8752	0.875252	0.875305	0.875359	0.875414
9	1	1	1	1	1	1	1	1	1	1	1
10	0.838193	0.83853	0.83887	0.839212	0.839558	0.839906	0.840256	0.84061	0.840966	0.841325	0.841686
11	0.874651	0.874815	0.87498	0.875146	0.875312	0.875478	0.875645	0.875812	0.87598	0.876149	0.876318
12	0.793576	0.793855	0.794135	0.794418	0.794703	0.79499	0.795279	0.795571	0.795865	0.796161	0.79646
13	0.956059	0.955353	0.954644	0.953932	0.953216	0.952497	0.951774	0.951047	0.950317	0.949584	0.948847
14	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999
15	0.90445	0.903645	0.902845	0.902049	0.901258	0.900472	0.899691	0.898915	0.898145	0.897379	0.89662
16	0.754328	0.754481	0.75464	0.754804	0.754974	0.755151	0.755333	0.755522	0.755717	0.755918	0.756127
17	0.926473	0.926538	0.926608	0.926683	0.926763	0.926847	0.926937	0.927031	0.927131	0.927235	0.927345
18	0.937748	0.937541	0.937336	0.937135	0.936936	0.93674	0.936547	0.936357	0.93617	0.935985	0.935804
19	0.804575	0.804861	0.80515	0.805441	0.805735	0.806031	0.806329	0.806629	0.806933	0.807238	0.807547
20	0.83129	0.831437	0.831586	0.831737	0.831891	0.832047	0.832206	0.832368	0.832532	0.832699	0.832869
21	0.934456	0.933998	0.933539	0.933079	0.932616	0.932151	0.931684	0.931216	0.930746	0.930273	0.929799
22	0.889111	0.88907	0.88903	0.88899	0.888952	0.888914	0.888878	0.888842	0.888808	0.888775	0.888743
23	0.926715	0.926372	0.926027	0.92568	0.925332	0.924982	0.92463	0.924276	0.92392	0.923563	0.923203

The proposed methodology enables a common assessment for all DMUs by generating a common set of weights for inputs and outputs. Consequently, the proposed approach, which provides reasonable weight dispersion, is more practical compared to the approach employed by Ghasemi et al. (2015). Since the weight assignments are provided in different forms in the proposed model and Ghasemi et al. (2015), ranking results also differ.

Table 7.22: Aggregated efficiency and ranking results for example 3

DMU	Aggregated efficiency	Rank
1	0.872433	12
2	0.714666	21
3	0.824429	15
4	0.729157	20
5	0.810484	16
6	0.696472	22
7	0.696472	22
8	0.875122	11
9	1	1
10	0.839719	13
11	0.875388	10
12	0.794836	18
13	0.952098	3
14	0.999999	2
15	0.900052	8
16	0.755062	19
17	0.926807	6
18	0.936638	4
19	0.805872	17
20	0.831965	14
21	0.931894	5
22	0.888896	9
23	0.924788	7

7.2.2 Performance Assessment of Countries

This section illustrates the application of the proposed imprecise decision making procedure for determining the country with the highest quality of living for an expatriate who is assigned to working in Latin America for a time period. In order to identify the best performer among the Latin American countries in terms of quality of living, 22 countries are evaluated regarding three inputs, namely “security risk (I1)”, " CO2 emissions from energy (tCO2/capita) (I2)”, and “climate change vulnerability (0-1) (I3)” and two outputs, namely “gross national income (GNI) per capita ($) (O1)”, and “world press freedom index (O2)”. Raw data for related inputs and outputs for the year 2018 are given in Table 7.23. Since security risk and world press freedom index are represented by linguistic variables, triangular fuzzy numbers are employed to express them numerically according to the membership functions given in Figure 7.3. The data is normalized using a linear normalization scheme.

Table 7.23: Data related to countries

Country	DMU	I1	I2	I3	O1	O2
Argentina	1	L	4.7	0.046	19820	M
Bolivia	2	M	1.9	0.114	7670	L
Brazil	3	M	2.6	0.058	15820	M
Chile	4	L	4.7	0.037	24190	M
Colombia	5	M	1.8	0.061	14490	L
Costa Rica	6	M	1.6	0.058	16670	VH
Cuba	7	VL	3.0	0.093	6679	VL
Dominican Republic	8	M	2.1	0.051	16960	M
Ecuador	9	L	2.8	0.070	11410	M
El Salvador	10	VH	1.0	0.062	7850	M
Guatemala	11	M	1.2	0.092	8310	L
Haiti	12	L	0.3	0.086	1870	M
Honduras	13	H	1.1	0.095	4780	L
Jamaica	14	M	2.6	0.099	8930	VH
Mexico	15	M	3.9	0.042	19360	L
Nicaragua	16	M	0.8	0.097	5390	L
Panama	17	M	2.3	0.047	23510	M
Paraguay	18	L	0.9	0.131	13180	M
Peru	19	M	2.0	0.115	13180	M
Trinidad and Tobago	20	M	34.2	0.041	32060	H
Uruguay	21	L	2.0	0.061	21900	H
Venezuela, RB	22	H	6.0	0.045	17900	L

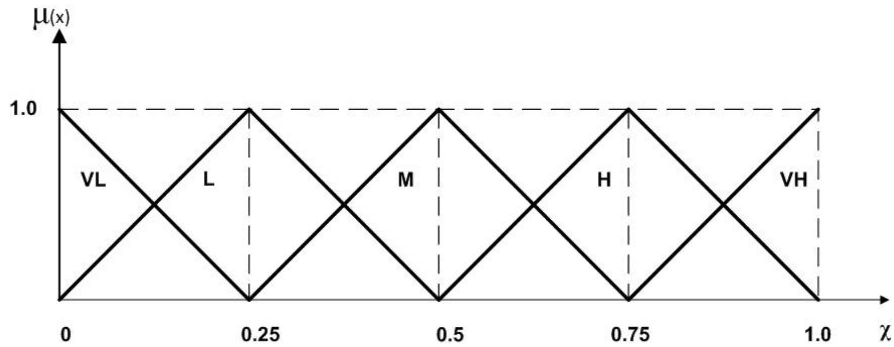


Figure 7.3: A linguistic term set where VL: (0, 0, 0.25), L: (0, 0.25, 0.5), M: (0.25, 0.5, 0.75), H: (0.5, 0.75, 1), VH: (0.75, 1, 1).

Table 7.24 and Table 7.25 provide the ranking results obtained by Formulation (6.12) and Formulation (6.13), respectively, with ϵ=0.000001. According to Formulation (6.12) of the proposed decision making procedure, Chile and Uruguay are minimax efficient countries for all α-cut values. Since Formulation (6.12) yields two minimax efficient DMUs, Formulation (6.13) is solved to identify the single best performing country. As outlined in Table 7.25, Chile is the best performer, followed by Uruguay for all α-levels.

Table 7.24: Efficiency scores obtained from Formulation (6.12) for countries

DMU	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
1	0.974018	0.972732	0.971522	0.970380	0.969301	0.968280	0.967313	0.966394	0.965521	0.964691	0.963900
2	0.730423	0.730906	0.731360	0.731789	0.732194	0.732577	0.732940	0.733285	0.733613	0.733924	0.734221
3	0.907446	0.907495	0.907542	0.907586	0.907627	0.907666	0.907703	0.907709	0.907772	0.907804	0.907834
4	1	1	1	1	1	1	1	1	1	1	1
5	0.868159	0.868656	0.869124	0.869566	0.869983	0.870378	0.870752	0.871107	0.871444	0.871765	0.872071
6	0.954272	0.957820	0.961246	0.964560	0.967771	0.970888	0.973918	0.976868	0.979743	0.982549	0.985292
7	0.786518	0.789094	0.791435	0.793562	0.795491	0.797239	0.798819	0.800245	0.801529	0.802680	0.803707
8	0.930085	0.930302	0.930507	0.930700	0.930882	0.931054	0.931218	0.931373	0.931521	0.931661	0.931795
9	0.925354	0.922235	0.919297	0.916526	0.913908	0.911430	0.909082	0.906853	0.904735	0.902719	0.900799
10	0.800775	0.802095	0.803338	0.804510	0.805618	0.806666	0.807660	0.808603	0.809499	0.810352	0.811165
11	0.792049	0.791765	0.791498	0.791246	0.791008	0.790783	0.790569	0.790366	0.790174	0.789991	0.789816
12	0.899999	0.894255	0.888847	0.883745	0.878925	0.874362	0.870039	0.865935	0.862035	0.858324	0.854788
13	0.730423	0.730906	0.731360	0.731789	0.732194	0.732577	0.732940	0.733285	0.733613	0.733924	0.734221
14	0.83887	0.841309	0.843690	0.846019	0.848299	0.850535	0.852730	0.854887	0.857010	0.859100	0.861159
15	0.900824	0.901986	0.903080	0.904112	0.905087	0.906010	0.906884	0.907714	0.908503	0.909254	0.909969
16	0.780455	0.779314	0.778240	0.777227	0.776270	0.775363	0.774505	0.773690	0.772915	0.772178	0.771476
17	0.94319	0.945783	0.948224	0.950528	0.952704	0.954764	0.956716	0.958569	0.960330	0.962006	0.963602
18	0.792636	0.792624	0.793555	0.794433	0.795262	0.796047	0.796791	0.797498	0.798169	0.798807	0.799416
19	0.769433	0.771447	0.773343	0.775132	0.776822	0.778422	0.779938	0.781377	0.782744	0.784045	0.785285
20	0.730423	0.730906	0.731360	0.731789	0.732194	0.732577	0.732940	0.733285	0.733613	0.733924	0.734221
21	1	1	1	1	1	1	1	1	1	1	1
22	0.821662	0.824099	0.826393	0.828557	0.830603	0.832538	0.834372	0.836113	0.837768	0.839342	0.840842

Table 7.25: Efficiency scores obtained from Formulation (6.13) for countries

DMU	$\alpha = 0$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 1$
1	0.974017	0.972732	0.971522	0.970380	0.969301	0.968280	0.967312	0.966394	0.965521	0.964691	0.963899
2	0.730423	0.730905	0.731359	0.731788	0.732193	0.732576	0.732940	0.733284	0.733612	0.733924	0.734221
3	0.907446	0.907495	0.907541	0.907585	0.907626	0.907666	0.907703	0.907708	0.907771	0.907803	0.907834
4	1	1	1	1	1	1	1	1	1	1	1
5	0.868159	0.868656	0.869124	0.869566	0.869983	0.870378	0.870752	0.871107	0.871444	0.871765	0.872071
6	0.954271	0.957819	0.961244	0.964558	0.967769	0.970886	0.973916	0.976866	0.979741	0.982547	0.985290
7	0.786517	0.789094	0.791435	0.793562	0.795491	0.797239	0.798819	0.800245	0.801529	0.802680	0.803707
8	0.930084	0.930302	0.930506	0.930699	0.930881	0.931054	0.931217	0.931373	0.931520	0.931661	0.931794
9	0.925353	0.922234	0.919296	0.916525	0.913907	0.911429	0.909081	0.906852	0.904733	0.902718	0.900798
10	0.800774	0.802094	0.803337	0.804509	0.805617	0.806666	0.807659	0.808602	0.809498	0.810351	0.811164
11	0.792048	0.791765	0.791497	0.791246	0.791007	0.790782	0.790569	0.790366	0.790173	0.789990	0.789816
12	0.899996	0.894253	0.888845	0.883743	0.878923	0.874361	0.870037	0.865933	0.862033	0.858322	0.854787
13	0.730423	0.730905	0.731359	0.731788	0.732193	0.732576	0.732940	0.733284	0.733612	0.733924	0.734221
14	0.838868	0.841306	0.843688	0.846016	0.848297	0.850533	0.852728	0.854885	0.857007	0.859097	0.861157
15	0.900825	0.901987	0.903081	0.904113	0.905088	0.906010	0.906885	0.907715	0.908504	0.909254	0.909970
16	0.780454	0.779314	0.778239	0.777226	0.776269	0.775363	0.774504	0.773689	0.772914	0.772177	0.771475
17	0.943190	0.945783	0.948225	0.950528	0.952705	0.954764	0.956717	0.958569	0.960330	0.962006	0.963602
18	0.792635	0.792623	0.793554	0.794432	0.795261	0.796046	0.796790	0.797496	0.798167	0.798806	0.799414
19	0.769433	0.771446	0.773343	0.775131	0.776822	0.778421	0.779937	0.781376	0.782743	0.784044	0.785284
20	0.730423	0.730905	0.731359	0.731788	0.732193	0.732576	0.732940	0.733284	0.733612	0.733924	0.734221
21	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999	0.999999
22	0.821663	0.824099	0.826394	0.828558	0.830603	0.832539	0.834373	0.836114	0.837769	0.839343	0.840843

Though the first two DMUs are identical for all α-levels, the ranking results for other DMUs vary regarding α values. Thus, OWA operator is used to aggregate the outcomes for α-levels and result in a final rank-order for all DMUs as in Table 7.26.

Table 7.26: Aggregated efficiency scores and ranks for countries

DMU	Aggregated efficiency	Rank
1	0.968903	4
2	0.732342	20
3	0.907642	8
4	1	1
5	0.870137	11
6	0.969078	3
7	0.796078	15
8	0.930949	6
9	0.912941	7
10	0.806026	14
11	0.79092	17
12	0.877144	10
13	0.732342	20
14	0.849263	12
15	0.905448	9
16	0.775916	19
17	0.953508	5
18	0.795567	16
19	0.777445	18
20	0.732342	20
21	0.999999	2
22	0.831358	13

The weights assigned to inputs and output for each α-level obtained by employing Formulation (6.13) are depicted in Table 7.27. The developed common weight modeling framework provides quite powerful weight dispersion that renders it practical and plausible.

Table 7.27: Input and output weights for country selection

Weight	α **= 0**	α **= 0.1**	α **= 0.2**	α **= 0.3**	α **= 0.4**	α **= 0.5**	α **= 0.6**	α **= 0.7**	α **= 0.8**	α **= 0.9**	α **= 1**
v_1	0.209081	0.201055	0.193498	0.186368	0.179632	0.173257	0.167214	0.161480	0.156030	0.150844	0.145903
v_2	0.291058	0.295876	0.300414	0.304694	0.308739	0.312566	0.316194	0.319637	0.322909	0.326022	0.328989
v_3	0.324537	0.317803	0.311460	0.305477	0.299824	0.294474	0.289403	0.284591	0.280017	0.275665	0.271519
u_1	0.023329	0.036092	0.048110	0.059448	0.070160	0.080298	0.089907	0.099026	0.107693	0.115940	0.123797
u_2	0.151995	0.149174	0.146518	0.144013	0.141645	0.139405	0.137282	0.135266	0.133351	0.131529	0.129792

7.3 Performance Assessment of Countries Using the Proposed QFD-Integrated Fuzzy Group Decision Making Approach

United Nations Development Program (UNDP) aims to end poverty, minimize inequalities and exclusion by working in more than 170 countries. Countries develop policies, institutional capabilities, partnering abilities, and leadership skills with the support provided by UNDP in order to achieve sustainable development (www.undp.org). Prior to the first Human Development Report (HDR), released by the UNDP in 1990, the only measure for representing development had been gross domestic product (GDP). Then, the UNDP introduced Human Development Index (HDI) for ranking countries according to their human-related factors (www.hdr.undp.org; www.unstats.un.org).

The human development approach aims to allocate the people at the core of the activities related to economy, politics, and society (Vierstraete, 2012). The HDI refers to a measure of average accomplishment in three aspects of human development: being knowledgeable, a long and healthy life, and having an adequate standard of living. Initially calculated as an arithmetic average of these three socio-economic indicators representing three principal dimensions of human development, the HDI is subsequently computed as the geometric mean of normalized indices for each of the three dimensions.

In 2015, "2030 Agenda for Sustainable Development" was introduced and its 17 sustainable development goals, entitled "no poverty", "zero hunger", "good health and well-being", "quality education", "gender equality", "clean water and sanitation", "affordable and clean energy", "decent work and economic growth", "industry, innovation and infrastructure", "reduced inequalities", "sustainable cities and communities", "responsible consumption and production", "climate action", "life below water", "life on land", "peace, justice and strong institutions", and "partnerships for the goals", were identified. The agenda copes with poverty, inequality and exclusion, while developing knowledge, skills and technology (Mariano et al., 2015). The main objective of these sustainable goals, also named as Global Goals, is to improve the countries in terms of economy, social skills, and environment (www.un.org).

Sustainable development goals are adopted by United Nations Member States for promoting prosperity in all countries, and protecting the planet. For achieving these goals, economic, social and environmental strategies should be constructed to provide progress in education, health, climate change, etc. (www.sustainabledevelopment.un.org).

This subsection illustrates the application of the integrated fuzzy multiple criteria decision making (MCDM) procedure for ranking Latin American countries combining quality function deployment (QFD) and common weight data envelopment analysis (DEA). The developed approach enables to include the interactions among country evaluation criteria via forming a house of quality (HOQ). The lower and upper bounds on country evaluation criteria are determined by utilizing fuzzy weighted average (FWA) technique. A common-weight DEA-based modeling framework, which uses the weights of country attributes calculated by FWA with the data from HOQ, is employed for identifying country ratings with respect to related country attributes. The stepwise representation of the proposed HOQ-integrated DEA-based decision framework is depicted in Figure 7.4.

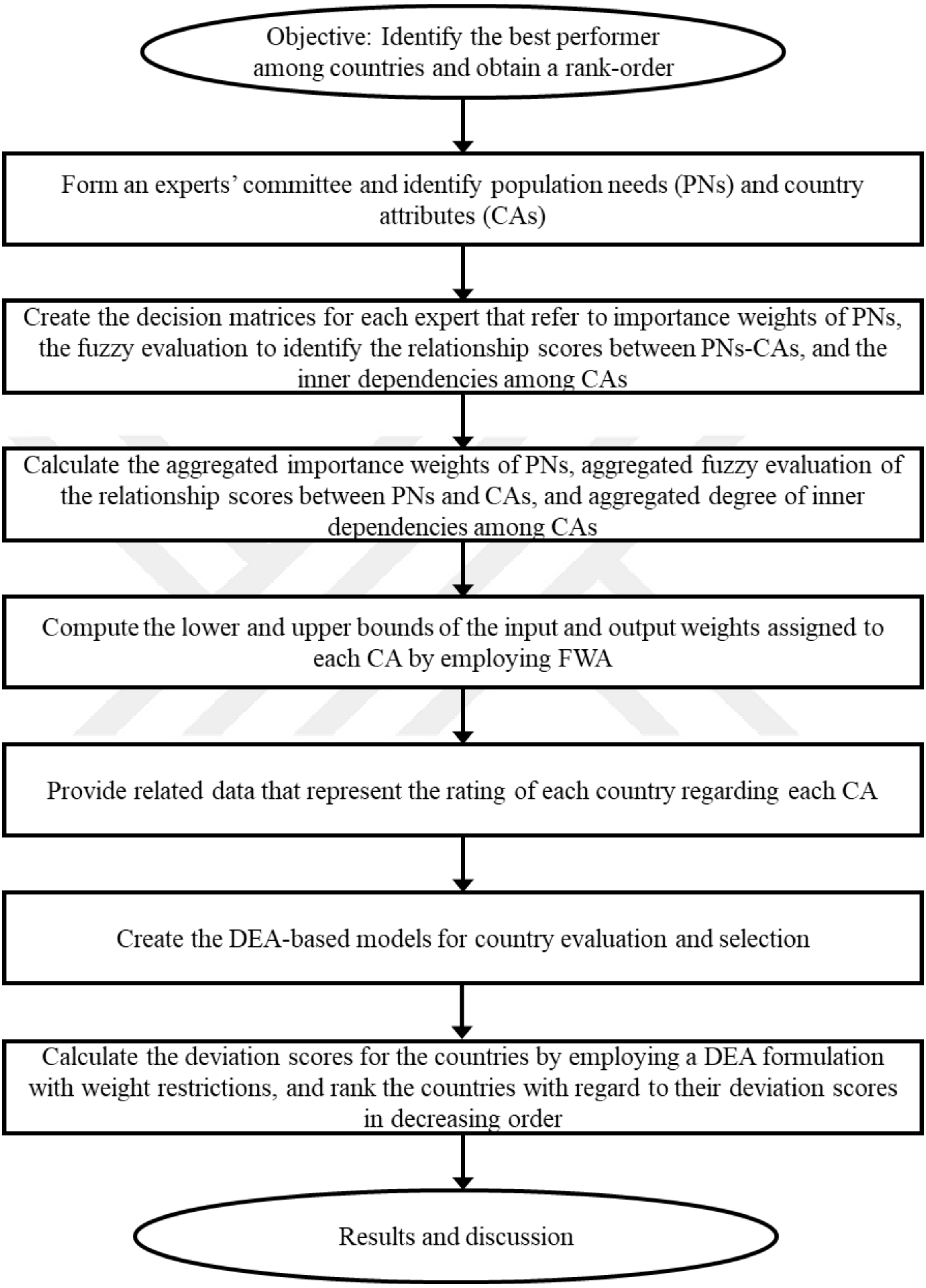


Figure 7.4: Representation of the proposed HOQ-integrated DEA-based decision framework

In the conducted case study, 19 Latin American countries considering 10 factors that are included in 17 sustainable development goals are ranked. As a result of interviews with a group of experts composed of academics and related practitioners, five goals that the population aims to achieve from sustainable development are determined as PNs, which are named as “good health (PN1)”, “education & work (PN2)”, “equality (PN3)”, “economy & wealth (PN4)”, and “environmental concern (PN5)”.

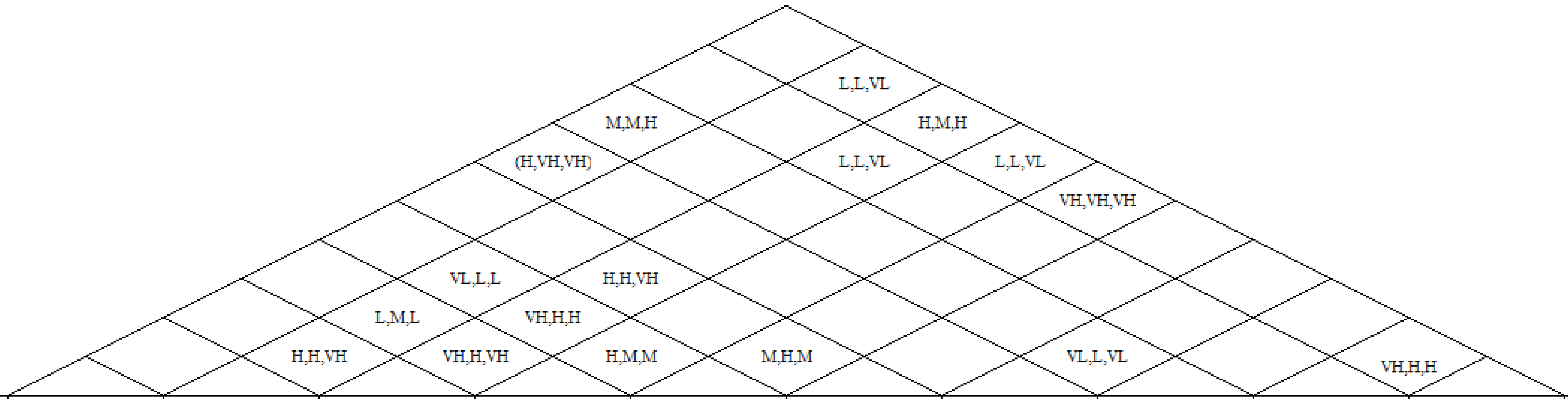
Ten attributes are considered as five inputs and five outputs with regard to DEA terminology. Inputs are “under 5 mortality (per 1000 live births) (CA1)”, “adolescent fertility (births per 1000) (CA2)”, “unemployment rate (%) (CA5)”, “CO2 emissions from energy (tCO2/capita) (CA8)” and “Gini index (CA10)”, while “expected years of schooling (CA3)”, “women in national parliaments (%) (CA4)”, “internet use (%) (CA6)”, “wastewater treated (%) (CA7)” and “corruption perception index (0-100) (CA9)” are treated as outputs.

A committee with three decision-makers (DM_1, DM_2, DM_3) conducted the evaluation utilizing linguistic terms namely “very low (VL)”, “low (L)”, “medium (M)”, “high (H)”, and “very high (VH)”. Each linguistic variable is depicted with triangular fuzzy numbers that are associated with the membership functions given in Figure 7.3.

The data for country evaluation is given in the HOQ depicted in Figure 7.5, and raw data of the countries for the respective inputs and outputs for the year 2018 are provided in Table 7.28.

Table 7.28: Ratings of Latin American countries with respect to CAs (www.unstats.un.org)

DMU (*j*)	**Country**	**Input1**	**Input2**	**Input3**	**Input4**	**Input5**	**Output1**	**Output2**	**Output3**	**Output4**	**Output5**
1	Argentina	11.1	63.027	8.738	4.747	48.920	9.8	38.9	70.969	7.670	39
2	Bolivia	36.9	69.032	3.111	1.932	45.750	8.2	53.1	39.698	3.509	33
3	Brazil	15.1	62.677	12.882	2.594	53.925	7.8	10.7	60.873	17.500	37
4	Chile	8.3	46.581	7.220	4.687	53.303	9.9	15.8	66.010	87.510	67
5	Colombia	15.3	49.497	8.960	1.760	55.310	7.6	18.7	58.136	12.243	37
6	Costa Rica	8.8	54.580	8.533	1.631	50.668	8.7	35.1	66.029	3.891	59
7	Dominican Republic	30.7	96.102	5.486	2.070	51.356	7.7	26.8	61.328	40.421	29
8	El Salvador	15.0	70.270	4.516	1.001	49.181	6.5	32.1	28.997	1.038	33
9	Guatemala	28.5	72.458	2.735	1.151	55.813	6.3	12.7	34.509	8.000	28
10	Haiti	67.0	38.213	13.988	0.271	40.890	5.2	2.5	12.233	0.000	22
11	Honduras	18.7	72.183	4.462	1.075	53.536	6.2	25.8	30.000	1.780	29
12	Jamaica	15.3	54.356	12.448	2.593	45.500	9.6	17.5	45.000	12.000	44
13	Mexico	14.6	61.405	3.320	3.866	57.829	8.6	42.6	59.540	45.632	29
14	Nicaragua	19.7	86.874	4.422	0.809	46.523	6.5	45.7	24.572	0.000	26
15	Panama	16.4	82.835	5.562	2.254	54.000	9.9	18.3	54.000	13.320	37
16	Paraguay	19.9	56.603	5.797	0.870	47.947	8.1	13.8	51.350	0.735	29
17	Peru	15.3	48.401	3.675	1.993	48.150	9.0	27.7	45.462	34.646	37
18	Uruguay	9.2	55.378	8.131	1.973	45.204	8.6	20.2	66.400	2.250	70
19	Venezuela. RB	16.3	85.834	8.061	6.026	53.789	9.4	22.2	60.000	16.960	18



CAs / PNs	CA_1	CA_2	CA_3	CA_4	CA_5	CA_6	CA_7	CA_8	CA_9	CA_{10}	Importance of PNs
PN_1	VH,H,VH	VL,L,L					H,M,VH	M,M,H			**H,VH,H**
PN_2		M,H,H	VH,H,VH	L,VL,VL	VH,H,VH	H,M,VH			L,L,VL	VL,VL,VL	**VH,H,VH**
PN_3			L,M,L	H,VH,VH		L,M,L			M,M,L	VH,M,H	**H,M,H**
PN_4	M,L,M	L,VL,VL	H,M,H		H,VH,VH				L,M,M	H,H,VH	**VH,VH,H**
PN_5							VH,H,VH	VH,VH,VH			**H,M,M**

Figure 7.5: HOQ for county selection problem

The aggregated relative importance degrees of PNs, aggregated relations between PNs and CAs, and aggregated level of dependencies among CAs are obtained by utilizing Equations (6.19)-(6.21). Each expert is assigned equal weight as $\Omega_1 = \Omega_2 = \Omega_3$. The lower and upper bounds on the weights of inputs and outputs are computed solving Formulations (5.11) and (5.12) as in Tables 7.29 and 7.30, respectively.

Table 7.29: Lower and upper bounds of the weights of inputs

CA_1	β_1	0.0589
	γ_1	0.2680
CA_2	β_2	0.0136
	γ_2	0.1800
CA_3	β_3	0.0492
	γ_3	0.2359
CA_4	β_4	0.0340
	γ_4	0.2360
CA_5	β_5	0.0394
	γ_5	0.2361

Table 7.30: Lower and upper bounds of the weights of outputs

CA_1	β_1	0.0570
	γ_1	0.2734
CA_2	β_2	0.0246
	γ_2	0.1969
CA_3	β_3	0.0200
	γ_3	0.1629
CA_4	β_4	0.0435
	γ_4	0.2473
CA_5	β_5	0.0095
	γ_5	0.1581

In order to employ DEA-based decision-making procedure for identifying the best performing country, data are initially normalized via linear normalization scheme. Normalization for input data using x_{ij}/x_i^*, where $x_i^* = \max_j x_{ij}$ for $\forall i$, while output data are normalized as y_{rj}/y_r^*, where $y_r^* = \max_j y_{rj}$ for $\forall r$ (Karsak & Ahiska, 2007).

Using normalized data as well as lower and upper bounds on input and output weights, the deviation scores and then resulting rankings of the countries are calculated using

Formulations (6.17) and (6.18) as given in Table 7.31. Moreover, input and output weights obtained from Formulation (6.18) are illustrated in Table 7.32.

Table 7.31: Ranking results of the countries (ϵ = 0.000001)

DMU (j)	Proposed Formulation (6.17)	Deviation scores obtained from the proposed model (6.18)	Proposed Formulation (6.18)
1	10	0.032	10
2	9	0.030	9
3	16	0.059	16
4	1	0.000001	2
5	11	0.036	11
6	1	0.000001	2
7	7	0.022	7
8	13	0.040	12
9	17	0.060	17
10	19	0.133	19
11	15	0.055	15
12	14	0.048	14
13	5	0.017	5
14	12	0.040	12
15	8	0.026	8
16	6	0.020	6
17	1	0	1
18	4	0.002	4
19	18	0.083	18

Table 7.32: Input and output weights

Weight	Formulation (6.18)
v_1	0.0589
v_2	0.0136
v_3	0.0492
v_4	0.0843
v_5	0.0884
u_1	0.0570
u_2	0.0246
u_3	0.0746
u_4	0.0435
u_5	0.0095

Chile, Costa Rica, and Peru are efficient countries with respect to Formulation (6.17). To obtain the most efficient country, Formulation (6.18), which guarantees to obtain the best performer, is employed. Finally, Peru ranks first, followed by Chile and Costa Rica. According to the ranking results, it is also noted that Formulation (6.18) provides consistent rankings with respect to the sets of inefficient and efficient countries obtained from Formulation (6.17).

Further, rankings obtained from HDI 2019 and the proposed Formulation (6.18) are compared via Spearman's rank correlation. Spearman's rank correlation coefficient is computed as 0.624. For n = 19 and α = 0.05, $r_s = 0.624 > r_{s,0.05} = 0.391$, and thus, we reject H_0, stating there is no positive correlation between the rankings due to the proposed formulation and the HDI, and conclude that the rankings obtained by Formulation (6.18) are positively correlated with rankings according to HDI values. One shall note that the differences among the rankings possibly result from the fact that the HDI measures performance achievement of countries with only three dimensions whereas the proposed approach provides a more in-depth analysis considering additional aspects for evaluation as well as accounting for interactions among them.

8. CONCLUSIONS

Data envelopment analysis (DEA) is a mathematical programming-based decision making tool, which copes with decision problems that need to consider multiple inputs and multiple outputs to evaluate the relative efficiency of decision making units (DMUs) without a priori information about the weights of inputs and outputs (Karsak & Ahiska, 2007).

Conventional DEA models yield performance assessment with information whether evaluated DMUs are efficient or not, whereas they possess several shortcomings in terms of making evaluation/selection decisions. First, these models must be solved *n* times to compute the efficiency scores of all DMUs, where *n* is the number of DMUs to be evaluated. In addition, they do not utilize common attribute weights for performance evaluation of DMUs enabling the DMUs to determine input and output weights in their own favor for maximizing their respective efficiency scores, which may yield results being far from practical. Furthermore, traditional DEA models assume that all DMUs with efficiency score of 1 are named as "efficient", while the DMUs with efficiency score less than 1 are identified as "inefficient". In other words, traditional DEA models are likely to fail to provide an adequate discriminating power since all efficient DMUs obtain an efficiency score of 1 (Karsak & Ahiska, 2005).

On the other hand, common weight DEA-based models do not require subjective evaluation to determine weights for inputs and outputs while providing a common assessment for all DMUs. Thus, the selection of input and output weights in favor of respective DMUs is limited that increases the discriminating power of the analysis (Karsak & Ahiska, 2005). In this thesis, common weight DEA-based decision making frameworks are presented to provide common assessment with enhanced discriminating power and adequate weight dispersion. The developed methodologies in this thesis consider multiple inputs as well as multiple outputs in the decision frameworks, and proposed mathematical programming models are applicable when crisp and/or imprecise data are present.

The first methodology introduces an improved common weight multiple criteria decision making (MCDM) approach, which can be successfully employed to identify the best performing DMU in the presence of multiple inputs and multiple outputs. An earlier two-stage efficiency model proposed by Karsak and Ahiska (2007) with an objective given in the second stage as $\min \mathrm{M} - k \sum_{j \in EF} d_j$ for determining the most efficient unit has several limitations. First of all, their approach requires a step size value for *k* that is chosen arbitrarily, and further, it may not provide consistent results as for the rankings of minimax efficient DMUs considering two minimax efficiency models provided in their study. The model is solved by increasing the value of *k* by an arbitrary step size until a single efficient DMU is obtained.

The contributions of the proposed approach to the DEA literature can be summarized as follows: First, the use of the proposed framework guarantees to identify the best performing DMU via solving one mixed integer linear programming model in addition to a single linear program for determining the set of minimax efficient DMUs. Second, it does not necessitate an arbitrarily determined parameter (*k*) value for improving the discriminating power. Third, it provides enhanced weight dispersion for inputs and outputs. Fourth, two real-world applications concerning economic and financial performance evaluation are given to show the robustness of the proposed decision framework. Finally, a comparison in terms of ranking and weight dispersion of the approaches developed in earlier research papers is provided to set forth the merits of the proposed methodology.

Two case studies, aiming to identify the best DMU in terms of economic and financial performance, are conducted to illustrate the implementation of the proposed decision making approach. The ranking results are compared with those of the models provided in Karsak and Ahiska (2007) and Sun et al. (2013). Two minimax efficiency models employed in Karsak and Ahiska (2007) do not provide consistent results in the rankings of minimax efficient DMUs while the proposed methodology reveals consistent rankings.

The models developed by Sun et al. (2013) fall short of providing practical weight dispersion for inputs and outputs, whereas the proposed methodology yields improved weight distribution compared with both of their models.

The second framework improves the initially proposed model by providing a common weight DEA-based decision making approach, which can be successfully employed to identify the best performing DMU in the presence of multiple inputs and multiple outputs incorporating imprecise data. The proposed two-stage approach, which determines the best performing DMU via solving one mixed integer linear programming model and a linear program for each α-level, yields consistent results. A single rank-order is obtained through ordered weighted average (OWA) operator that is employed for aggregating the efficiency scores regarding α-levels for each DMU.

Three numerical examples, aiming to identify the best performing DMU in the presence of imprecise data, are provided in order to illustrate the robustness of the proposed decision approach. The ranking results are compared with those of the models addressed in Nejad and Ghaffari-Hadigheh (2018), Wang et al. (2009), and Ghasemi et al. (2015). The models in these studies enable DMUs to generate input and output weights for maximizing their efficiency values, which is rather impractical in a decision making setting. On the contrary, the approach proposed in this thesis yields common set of weights for inputs and outputs, thus common performance assessment is enabled. The model proposed by Nejad and Ghaffari-Hadigheh (2018) suffers from infeasibility for some α-cut values whereas the approach developed in this work yields feasible solutions for all α-cut values. On the other hand, the decision approach introduced by Wang et al. (2009) requires 6n linear programs to be solved to obtain the best possible fuzzy efficiency values for all the DMUs in addition to an analytical approach based on degree of preference for full ranking of performers, which demonstrates a relatively increased computational burden. Moreover, a case study, which aims to identify the best performing Latin American country regarding quality of living for an expatriate, is conducted.

The developed decision-making framework provides a number of contributions to the literature on common weight DEA-based models. First, the proposed two-stage approach guarantees to obtain the best performing DMU by solving one mixed integer linear programming model in addition to a linear program for each α-cut value. Second, the proposed methodology results in common weights for inputs and outputs, hence a common assessment is provided. Third, consistent ranking results between the first and the second stages of the proposed approach are provided. Fourth, OWA operator is utilized to aggregate the efficiencies of DMUs that are obtained according to each α-level, and thus a unique rank-order for all DMUs is provided. Finally, the proposed framework yields enhanced weight dispersion for inputs and outputs compared with earlier models.

The third approach developed in this thesis presents an integrated group decision making approach that ranks countries by taking into account ten attributes that are included in sustainable development goals. House of quality (HOQ) is constructed to evaluate the relationships among the population needs (PNs) and sustainable development goals as well as the inner dependence among these goals. In order to rank the countries and identify the best performer, a DEA-based mathematical programming approach, which uses the lower and upper bounds on weights of sustainable development indicators calculated by fuzzy weighted average (FWA) and the data derived from the HOQ, is employed.

The proposed decision making procedure provides the following contributions to the related literature: First, the developed decision-making procedure enables to include vague and imprecise data into the analysis utilizing linguistic terms. Second, the proposed approach takes into account the effects of relationships between the PNs and country evaluation attributes, and the inner dependencies among country attributes (CAs). Third, fuzzy weighted average (FWA) technique, which deals with the problem of loss of information that may occur when including imprecise data, is employed to compute the lower and upper bounds of the weights of country evaluation attributes. Fourth, common weight DEA-based modelling framework assures to single out the best performer by solving a mixed integer linear programming model together with a linear program. Hence, developed common assessment provides improved discriminatory characteristics. Finally, weight restrictions in DEA-based methodology provide plausible weights for CAs.

It is worth highlighting that the decision models presented here are not restricted to any specific application area and could be implemented in various disciplines without any difficulty. One shall also note that the decision approaches proposed in here for evaluating alternatives (decision making units) can be easily programmed. Future research will focus on applying the decision frameworks presented in here to real-world group decision making problems in diverse disciplines that can be represented in DEA structure. A user interface can also be developed for users who are novice in mathematical programming.

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