DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

COMPARISON OF SINGLE AND MODIFIED EXPONENTIAL SMOOTHING METHODS IN THE PRESENCE OF A STRUCTURAL BREAK

by

İrem EFE

February, 2019
İZMİR

COMPARISON OF SINGLE AND MODIFIED EXPONENTIAL SMOOTHING METHODS IN THE PRESENCE OF A STRUCTURAL BREAK

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We have read the thesis entitled **"COMPARISON OF SINGLE AND MODIFIED EXPONENTIAL SMOOTHING METHODS IN THE PRESENCE OF A STRUCTURAL BREAK"** completed by **İREM EFE** under supervision of **PROF. DR. ESİN FİRUZAN** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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COMPARISON OF SINGLE AND MODIFIED EXPONENTIAL SMOOTHING METHODS IN THE PRESENCE OF A STRUCTURAL BREAK

ABSTRACT

The essential aim of the time series modelling is applied for the forecasting as well as the examination of correlation. One of the most widely used methods in the literature is exponential smoothing (ES) methods. It is a method preferred by many researchers because of its easy application, calculation efficiency, high accuracy and automatic prediction.

Such as policy changes, financial crises, natural disasters in the data production processes of the series, permanent structural changes can change affect model parameters as well as analysis results. Having no consideration of such affects, leads parameter estimator to be biased, tests tend to be useless in terms of power and incorrect modelling arise. The main purpose of this study is to compare the predictive performances of the newly developed Modified Exponential Smoothing (MSES) (2016) methods with the simple exponential smoothing (SES) when there are structural breaks in the series. Determining the initial value and misspecification in the selection of the optimum smoothing parameter, as a disadvantage, adversely affect the estimation results. The MSES method gives more weight to the current observations on the series, so that the predictions that are calculated give better performance than the classical method. The MSES method against structural break has not been examined yet. In this study, received from the Central Bank of the Republic of Turkey and traded on the Istanbul Gold Exchange "weighted average price of gold (TL/kg)" data are used. This data set with different break points compares the forecast performance of MSES and SES methods.

Keywords: Simple exponential smoothing, modified simple exponential smoothing, structural break and forecast

YAPISAL KIRILMALAR OLDUĞUNDA UYARLANMIŞ VE BASİT ÜSTEL DÜZELTME YÖNTEMLERİNİN KARŞILAŞTIRILMASI

ÖZ

Zaman serisi modellemesinin amacı, korelasyonun incelenmesinin yanı sıra öngörümleme yapmaktır. Literatürde en yaygın kullanılan yöntemlerden biri üstel düzeltme yöntemleridir. Kolay uygulanması, hesaplama etkinliği, doğruluk oranın yüksek olması ve otomatik tahmin işleminin olmasından dolayı birçok araştırmacının tercih ettiği bir yöntemdir.

Veri üretim süreçlerinde, politika değişikliği, finansal krizler, doğal afetler gibi birçok nedenden dolayı kalıcı yapısal değişimler meydana gelebilmektedir. Bu değişimler model parametrelerini değiştirebildiği gibi analiz sonuçlarına da etki etmektedirler. Bu etkilerin dikkate alınmaması parametre tahminlerinin yanlı olmasına, testlerin güçlerinin düşmesine, yanlış modelleme yapılmasına sebep olabilmektedir. Bu çalışmadaki temel amaç, seride yapısal kırılmalar olduğunda basit üstel düzeltme (BÜD) ile yeni geliştirilmiş olan Modifiye Üstel Düzeltme (MBÜD) (2016) yöntemlerinin tahminleme performanslarını karşılaştırmaktır. Dezavantaj olarak, ilk değer (initial value) belirleme ve optimum düzeltme parametresinin seçiminde hatalar yapılması tahminleme sonuçlarını olumsuz etkilemektedir. MSES yöntemi serideki güncel gözlemlere daha fazla ağırlık vererek hesapladığı tahminlerin performansı klasik yönteme göre daha iyi sonuçlar vermesini sağlamaktadır. MSES yönteminin yapısal kırılma karşısında etkinliği henüz incelenmemiştir. Bu çalışmada Türkiye Cumhuriyet Merkez Bankasından alınan ve İstanbul Altın Borsasında işlem gören "altın ağırlıklı ortalama fiyat (TL/kg)" verileri kullanılmıştır. Farklı kırılma noktalarına sahip bu veri seti ile MBÜD ve BÜD yöntemlerinin öngörümleme performansları karşılaştırılmıştır.

Anahtar Kelimeler: Basit üstel düzeltme, modifiye edilmiş basit üstel düzeltme, yapısal kırılma ve öngörümleme

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CHAPTER ONE
INTRODUCTION

The time series is the chronological sequence of observations. The time series analysis is carried out in order to accurately predict the future values of the variables (Bowerman & O'Connel, 1993). In many areas such as industry, economy, science and politics, in the process of making decisions, it is very important to make forecasting in an accurate way. The number of hospitals to open in a city, production of a company in the following month based on the sales figures, the determination of the flight route of a pilot according to the weather conditions are some examples time series analysis is used. In order to carry out forecasting, a forecasting system is needed. In general, such a system consists of two main stages. In the first phase of the forecasting system, valid data and appropriate theory are used to build a forecasting model, data are applied and finally the model's sufficiency is tested. If the model is insufficient, the process is repeated until the appropriate model is found. In the second stage, that is in the prediction stage, the final model is used to obtain forecasts. New data is applied to test if the forecasting model is stable. As described in Figure 1.1, if the model is stable, errors are calculated and the possible improvement areas in the model are determined. If the model is not stable, the model is built again and this process continues until the appropriate model is selected.

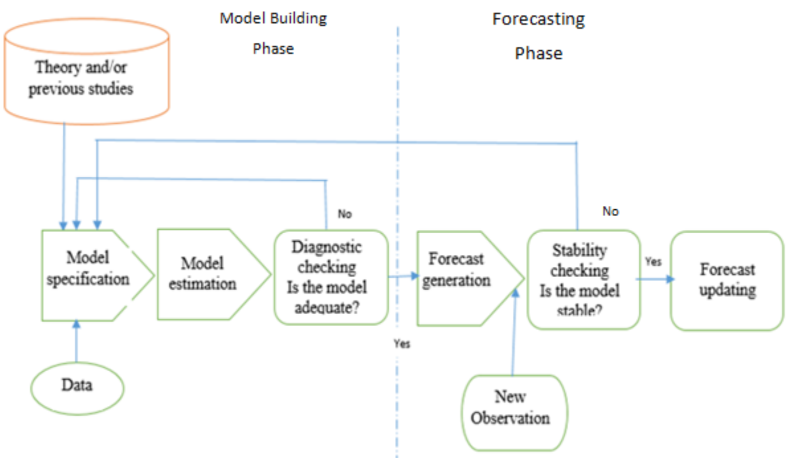

Figure 1.1 Conceptual framework of a forecasting system (Abraham & Ledolter, 1983)

While choosing the appropriate forecasting model or method, it is important to pay attention to which degree of accuracy is needed, what the lead time is, how high a cost can be tolerated for forecasting, whether a less accurate but simpler or a more accurate but more complex model should be preferred, which data would be appropriate, along with other criteria.

The data obtained over time vary randomly and smoothed methods are used to reduce or eliminate this variability in the data. In the 1940s, methods such as time series regression, moving average, in 1970s methods such as decomposition, exponential smoothing, Box-Jenkins methods were developed and started to be used. Afterwards, many different methods have been developed in line with the needs.

In time series regression, the dependent variable is associated with the function of time and gives better results when the parameters defining the time series to be estimated are stationary over time.

The moving average has a simple structure. It is obtained by taking the average of the observation values for a period of '*k*'. When the new observation value is obtained, this model adds the new value to its average account, while reducing the oldest value included in the previous average. Thus, the average of the latest *k* period's observation values of the last period is taken as the forecasting value (Brown, 1962). If the *k* value is chosen small, the method gives a faster response to the change in the series, as more weight is given to the new observations. If a large *k* value is chosen, the method cannot give a fast response to the change in the series, as equal weight is given to the new and old observations. In this case, this method may yield to incorrect results. The moving average method helps to show the tendency of the series in the long term, but cannot capture the change in the variables. This is the disadvantages of this method.

Decomposition models are used to predict time series having trend and seasonality components. These models do not rely on a theoretical basis, that is they are intuitive approaches. However, if the parameters defining the time series do not change over time, it is stated that the decomposition method is determined to be more useful and the estimates obtained are stated as more advantageous in the estimation of point estimatiors. The purpose of the decomposition method is to seperate time series into factors such as trend, seasonal, cyclic and irregular (Bowerman & O'Connel, 1993).

Exponential smoothing (ES) method, is has a similar methodology to the moving average. However, the weights are not given equally to all observation in the ES method. Here, the current observation is weighted the most and the most distant observation is weighted the oldest. This weighting process is carried out using one or more smoothing coefficient which determines the weight of each observation. ES has been widely used for many years due to its ease of calculation, accuracy, applicability and robustness. However, the determination of the initial value and the smoothing coefficient by the researcher has been challenge for this method. It can be said that this method is not based on any theoretical background, and it is rather an intuitive method that produces sufficient foresight in some applications.

The Integrated Autoregressive Moving Average Model (ARIMA), also known as the Box-Jenkins method, is used to predict non-stationary time series. This method consists of four stages. In the first stage, a tentative model is identified. In the second stage, the parameters of the model are estimated using historical data. In the third stage, the model's accuracy is checked. Finally, if the model is adequate, future time series values are forecasted.

Holt & Brown worked on ES in the 1950s and it is developed by Brown in 1959. In his article, Brown mentioned how to determine the value of initial value and correction constant, and how important they are for analysis (Brown, 1961). In his study in 1960, Winter carried out forecasting using seasonal data and reported that ES performed better than traditional methods. Makridakis et al., (1982), compared the different forecasting methods using the 1001 data set, and examined different factors affecting the precision of forecasting. It has been observed that the ES performed surprisingly better in forecasting compared to the sophisticated methods. As a result of these comparisons, it is argued that, if the user can select the method based on the data type, the size of the data set and the time domain, combining the forecasts of many methods rather than using one single method will improve the overall forecasting sufficiency. Gardner (1985) developed the technology guidelines for the implementation of the exponential smoothing methodology. He could find answers to the questions such as: Which parameters should be used? How to start the modelling? What are the useful features of these models? Gardner argued that ES has both simple and powerful approaches to forecasting. In addition, he carried out an empirical study on measuring the effect of the selection of initial value and the loss function on the sufficiency of the forecast. Makridakis & Hibon (1991) carried out the comparison in this analysis using the M competition 1001 data sets and 3 different initial values.

Time series are assumed to be constructed from latent components such as trend and seasonal effects. Due to the structural changes in the series, these components should be adapted over time. When the components are combined with operators such as addition and multiplication, 24 different variations emerge. Hyndman et al., (2002), and Taylor (2003) had updated Holt's report of 1957, then they published a study in 2004. In this study, these variations were examined and a systematic improvement of the forecast expressions was provided. In an empirical analysis, Goodwin (2010) estimated short term demand prediction of a company.

Hyndman et al., (2002) provided a framework for exponential smoothing based on an earlier work of Ord, Kohler and Synder (1997). This framework combined the stochastic models to different forms of exponential smoothing, and provided the maximum likelihood estimates of the smoothing parameters. This framework also used the Akaike's Information Criterion for the choice of method (Billah et al., 2006).

ES is very popular, widely used, easy and understandable. In addition to these advantages, however, the weakness of ES is the lack of a statistical framework that produces both confindence intervals and point estimators. With the state-space approach, not only the intuitive nature of ES in measurement and state equations is preserved, but also prediction intervals, probability estimates, procedures for model selection, and much more are provided. However, although this approach has been subject to many articles, newspapers and journals, there has not been a systematic explanation and development of the ideas (Hyndman et al., 2008).

Proper selection of smoothing coefficient and initial values plays an important role in ES, because assigning more weight to current observations without giving up remaining observations provides a rather realistic forecast for the user. In other words, the performance of the ES is mainly dependent on the most recent observations and the smoothing constant reflecting the relative weight assigned to the initial value. If the smoothing constant in the range [0,1] is close to 1, the values of the last observations directly affect the estimation of the series, which can lead to excessive spikes in the estimation series. If the smoothing constant is close to 0, it gives more weight to the old observations. Then, no significant difference occurs between the initial data and the latest data in the forecast series. These two cases are not desirable in terms of reliable estimates. As mentioned in the paragraphs above, a lot of research has been done on ES for many years and new methods have been developed. However, the estimators have not consensus on how to select the initial value and smoothing coefficient, and there is no consistent rule in the literature regarding this situation (Yapar, 2016).

To address the disadvantages of the simple exponential smoothing method, Yapar (2016) developed a modified simple exponential smoothing (MSES) method, as an alternative method. The purpose of this method is to develop a parametric procedure to obtain weights. In this way, the selection of the smoothing factor is not left to the user preference. Also there is no initial problem, as the initial value is chosen as the oldest observation or the mean of all the observations in the series. When the traditional method and the improved method are compared, it is emphasised that MSES method has smaller errors and thus has better predictions, less number of iterations and thus a more significant weighting scheme, a younger average age and smaller variance and thus a more flexible model. This study is carried out with a stable data set. However, as MSES is more adaptable than ES, it is indicated that MSES would perform better even if there exists a level-shift in the series. In one of the studies carried out by Yapar et al., in 2017, Holt's linear trend method is modified and thus the result of initialization is eliminated and the optimization process is simplified. In another study conducted in the same year, the performances of MSES and SES models are compared.

In this thesis, it is investigated whether MSES performs better than SES in a series containing structural break. If there is a structural break in the series, the mean and / or variance can be distorted, so that a stationary series can be observed as is it has a non-stationary structure. Therefore, it becomes very difficult to obtain forecast in such a series. Biased and inconsistent estimates can be obtained. Therefore, it becomes very difficult to obtain a correct forecast in such a series.

In this study, as a contribution to the literature, what results from MSES can be obtained will be discussed when there is a structural break in the time series. In the second part of the study, time series components (trend, seasonal, cyclic and irregular) and the sudden change in series, that is structural break is mentioned. In the third part, the exponential smoothing method, and in the fourth part of the modified exponential smoothing method is carried out. In the fifth chapter, Monte-Carlo simulation carried out in the R 3.2. program to compare the predictive performances of MSES and SES. In the sixth chapter, structural break was added to the YAF-2 data set, which is widely used in the literature, and the new YAF-2 data set. In order to observe the effectiveness of method with real-world data, application is performed on the "gold weighted average price (TL/kg)" data, provided by the Central Bank of the Republic of Turkey (CBRT) and traded in the Istanbul Gold Exchange (IGE). In the conclusion, the results of all analyzes are discussed.

CHAPTER TWO

TIME SERIES COMPONENTS AND STRUCTURAL BREAKING

2.1 Time Series Components

The key to analyzing the time series is to understand the pattern of the data observed over time. This pattern consists of several different components. Time series analysis decomposes each component from each other and measures the extent to which each component affects the observed data (Gerbing, 2016). The changes arising from the differences in the direction and intensity of various reasons such as economic, social, psychological etc. on time series observation values can be named as trend, seasonal, cyclic and irregular components. These changes are generally referred to as time series components (Yüzer et al., 2003).

2.1.1 Trend Component

Trend component is the steady state that occurs after the rise and fall periods of time series occuring in a long period of time. This component explains the long-term effects that cause the general tendency of the time-dependent variable. Figure 2.1. shows the types of trend component.

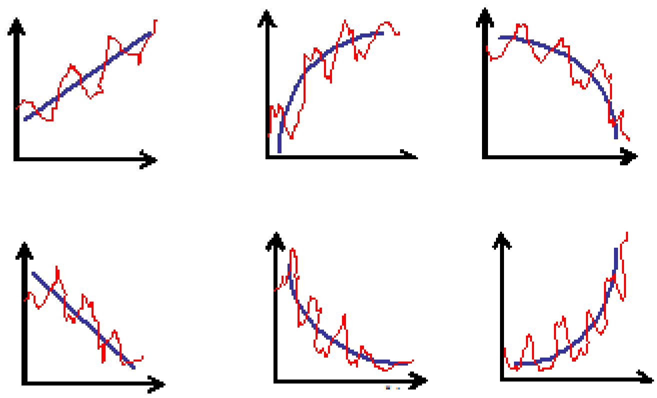

Figure 2.1 Trend components graphic (Yazilimagiris, 2017)

2.1.2 Seasonal Component

Seasonal component is the systematic behavior in the time series. It explains the changes in the time series observation values, in terms of increases and decreases at the same time points of successive years, seasons, quarters, months or days. Seasonal components are shown in detail in Figure 2.2.

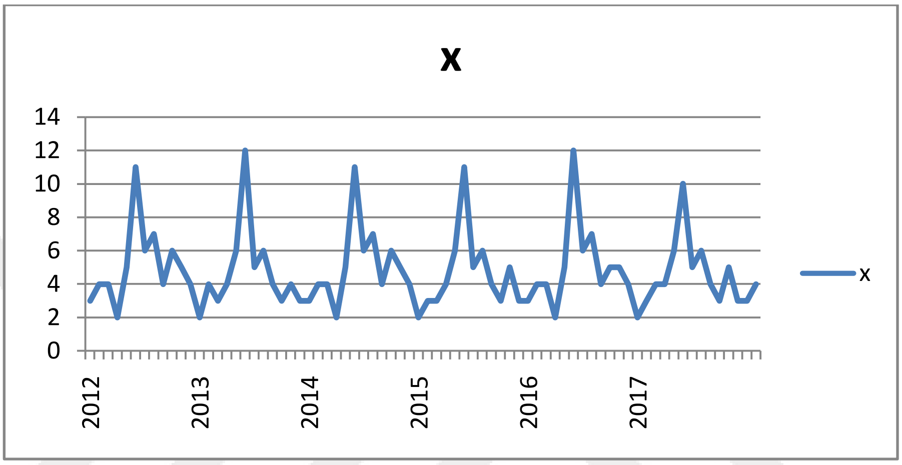

Figure 2.2 Seasonal component graphic

2.1.3 Cyclical Component

The fluctuations that occur due to the increase or decrease around the long-term trend curve are called cyclical component. These changes are not about seasonal changes, but they rather that involve periods of prosperity or stagnation of the economy or a sector. Unlike the seasonal component, the periods in the cyclical component are irregular, but not periodic. Cyclical component is described in Figure 2.3.

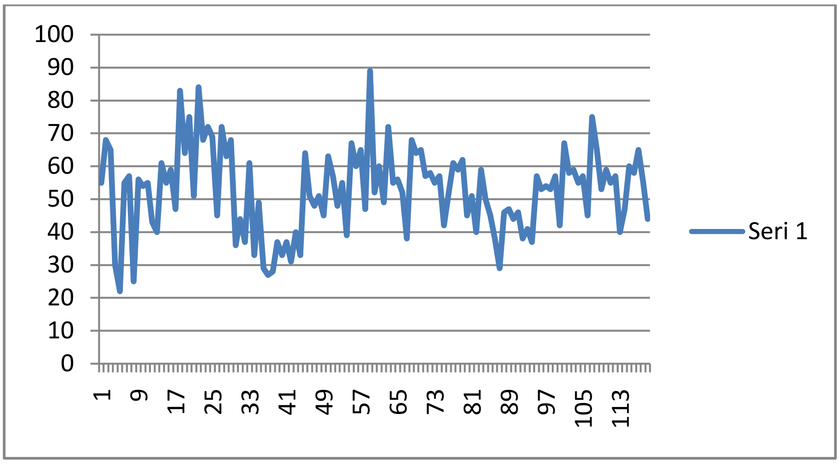

Figure 2.3 Time series with cyclical component

2.1.4 Irregular Component

An irregular component is a change through the effect of unexpected events on time series, and none of the other components can be shown as the cause of these changes. Earthquake, war, strike, changes in the policies of the competitors are examples of irregular component, and these lead to occurrence of irregular component. Irregular component is the component that explains the remaining effect after the decomposition of the cyclical, seasonal and trend components. It is shown in detail in Figure 2.4.

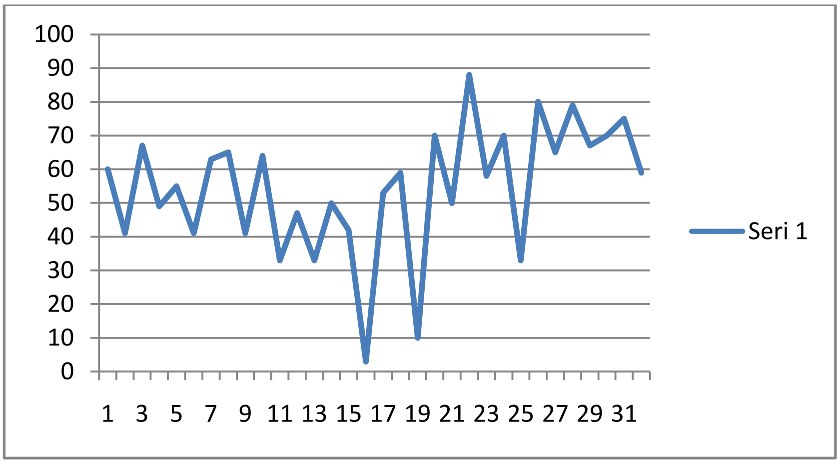

Figure 2.4 The pattern of irregular component

2.2 Structural Break

The structural breaks which cause the interruptions of the series long and/or termed changes in their trend components are expressed as the outlier observations. Structural changes may occur in the data generating process of time series due to policy changes, financial crisis and natural disasters. These changes in the series, without any exact definition, are generally called as the change in the model parameter (Çoban & Firuzan, 2016). In such cases, it is difficult to obtain accurate forecasts, because in these circumstances estimation result would be biased and inconsistent.

Last decades, researchers developed new methods take into account of structural breaks in time series analysis. Primarily, Nelson & Plosser (1982), demonstrated a new methods as a unit root test using statistical techniques developed by Dickey & Fuller (1979, 1981), because current shocks have a permanent effect on the long-run level of most macro-economic and financial series.

Knowledge of the break point enables the inclusion of these shocks into the model as dummy variables. Such inclusion of the break into the model as a dummy variable does not express the models which are built for the variable that represent the series, but it is used to just remove the effects of the shocks in the series.

There are a lot of structural breaks types in the time series models. For instance Perron (1989) and Zivot & Andrews (1992) examined for unit root analysis on three different models. Most widely used model can be defined as break in level shift model equation (2.1), break in trend equation (2.2) and break in regime shift equation (2.3). As it can be seen in Figure 2.5, 2.6 and 2.7. Model A is constructed by taking a structural change in the level (intercept) of the series into consideration; while Model B is a structural change in the trend of the series; and Model C is taking into consideration the structural changes both in the regime of the series. Models can be expressed by figures as below:

Model A: $Y_t = \mu_1 + \mu_2\varphi_\tau + \alpha' Y_{t-1} + e_t \qquad t = 1,2,\cdots,n \qquad (2.1)$

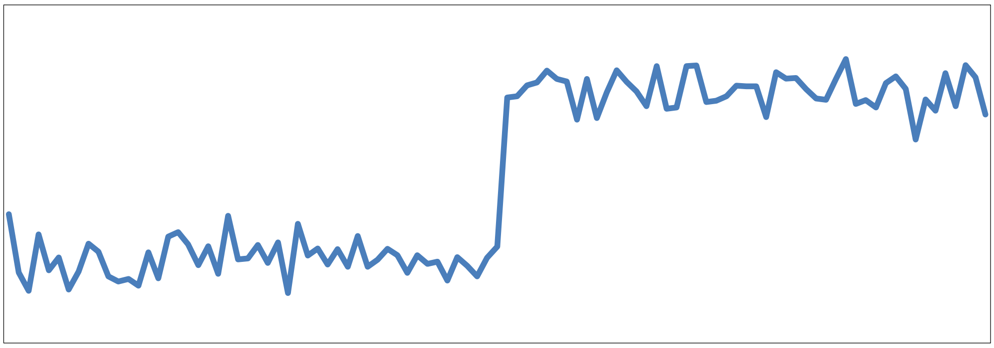

Figure 2.5 Break in level shift model

Model B: $Y_t = \mu_1 + \alpha_1' Y_{t-1} + \varphi_\tau Y_{t-1} + e_t \qquad t = 1,2,\cdots,n \qquad (2.2)$

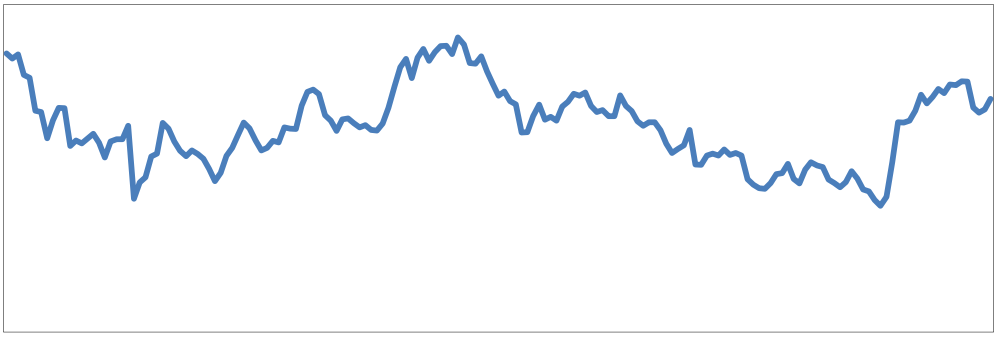

Figure 2.6 Break in trend shift model

Model C: $Y_t = \mu_1 + \mu_2\varphi_\tau + \alpha_1' Y_{t-1} + \varphi_\tau Y_{t-1} + e_t \qquad t = 1,2,\cdots,n \qquad (2.3)$

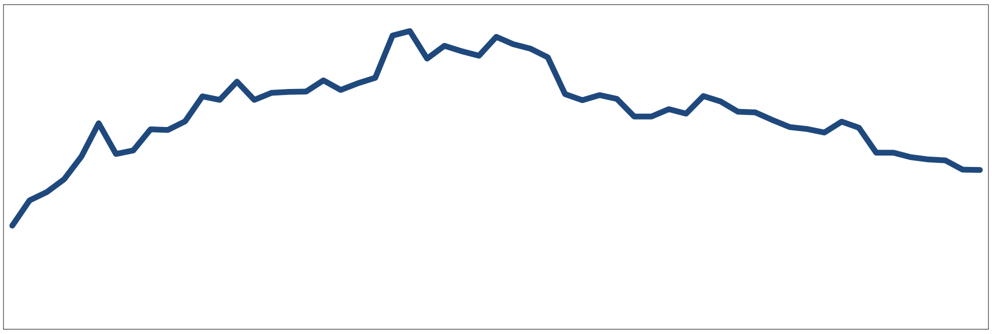

Figure 2.7 Break in regime shift model

Let T_B be $1 < T_B < n$ and indicate the location of break, the dummy variable is defined as below:

$$\varphi_\tau = \begin{cases} 1, & t > [T_B n] \\ 0, & t < [T_B n] \end{cases}$$

When there is structural break in the series, the type I error increases, in which case the correct hypothesis can be rejected. Model parameters do not remain the same throughout this process, and varying parameters give some indications about structural breaks. Many statistical methods have been developed to test the presence of these changes in time series. The most common of these is the “chow test” proposed by Gregery Chow (1960). The most important feature of the Chow test is the need for information before the structural breaks enter the model. This test has not been used in this study, since no information was previously obtained about the structural breaks in the series used in this study. The Chow test assumes that the variances of the errors are homogeneous. Toyoda (1974) presented a study based on the least square estimation of this assumption, where there is no assumption of information. Jayatissa (1977) and Watt (1979) later described some methods for extending the assumption of homogeneous variance. Dong (2004) took into account the changing variance case and determined the structural break in the series.

In the time series analysis, the researcher has a preliminary knowledge of the point of structural break, which is not possible in some stochastic processes. Brown, Durbin, and Evans (1975) developed recursive residuals, CUSUM and CUSUMSQR tests that did not require prior knowledge. In this thesis, a brief information will be given about the CUSUM test, which is applied to determine the structural break points in the series.

2.2.1 Recursive Residuals Test

Primarily, the regression equation 2.4,

$$y_t = x_t^{'}\beta_t + e_t \tag{2.4}$$

is estimated by least squares method using a small observation set of n_1 of size in the sample for that the uses recursive residuals test. The number of observations (n_1) to be taken at the beginning should be at least k number of parameters ($n_1 \geq k$). Then repeat the least squares estimates by increasing the data period. Thus, *n-k+1* number of coefficient estimates are obtained sequentially. The test hypothesis is shown below:

H_0 = there is no structural break
H_a = there is a structural break

Assuming that the H_0 hypothesis is not rejected, let b_r be the least-squares estimate of β based on the first r observations, $b_r = (X_r^{'}X_r)^{-1}X_r^{'}Y_r$ where the matrix $X_r^{'}X_r$ is assumed to be non-singular in equation 2.4 (Brown et al., 1975).

The last estimate of b_r at each step is used to forecast the next value of the dependent variable. One step beyond, this forecast is called "recursive residuals" in the estimation error. The step-by-step forecast error is obtained from relevant forecast, this is called as the "recursive residual".

Thus, the error of forecast in period r^{th} is obtained as follows;

$$v_r = y_r - x_r^{'}\, b_{r-1} \qquad\qquad r = k+1, \cdots, n \tag{2.5}$$

The standardized recursive residuals w_r, which are the standardized error estimator of the estimated y_r by y_1 ,...., y_{r-1}, can also be calculated as follows:

$$w_r = \frac{y_r - x_r' b_{r-1}}{\sqrt{\left(1 + x_r' \left(X_{r-1}' X_{r-1}\right)^{-1} x_r\right)}} \quad r = k+1, \cdots, n \tag{2.6}$$

It is possible to use the time graph of recursive residuals to decide whether there is structural break in the process. If the values of v_r exceed the upper and lower bounds calculated by formula $E(v_r) \pm 2\sqrt{Var(v_r)}$, there is a structural break (Güler, 2014).

2.2.2 CUSUM Test

One of the tests used to determine whether there is a structural break in the regression coefficients is CUSUM Test. This test statistics are based on Cumulative summations of standardized recurring residuals. If the β_t in equation 2.4 is constant until $t = t_0$ and then difference in the mean of standardized recursive residuals (w_r) is zero until t_0, then it is different from zero. Using the formulas in equation 2.6 and 2.7 found below, when it is investigated the locations where the means of w_r are different from zero throughout the series, it can be determined the points at which structural breaks occur.

$$Cusum_r = \sum_{t=k+1}^{r} \frac{w_t}{\hat{\sigma}} \quad . \tag{2.7}$$

where $r = k+1, \cdots, n$.

$$\hat{\sigma}^2 = \frac{1}{n-k} \sum_{t=k+1}^{n} (w_t - \bar{w})^2 \tag{2.8}$$

$$\bar{w} = \frac{\sum_{t=1}^{n} w_t}{n-k} \tag{2.9}$$

Time graphs are drawn with the calculated CUSUM values. Then the lower $(\pm a\sqrt{n-k})$ and upper $(\pm 3a\sqrt{n-k})$ confidence intervals are determined to α significance level. Where a is a parameter determined by the selected level of significance (Brown et al., 1975). Some of them are like the ones below:

$$\alpha = 0.01 \;\Rightarrow\; a = 1.143$$
$$\alpha = 0.05 \;\Rightarrow\; a = 0.948$$
$$\alpha = 0.10 \;\Rightarrow\; a = 0.850$$

The test hypothesis is established as follows:

H_0 = there is no structural break
H_a = there is structural break

If the Cusum values on the Figure 2.4 intersect any of the lower and upper limits, the hypothesis “no structural break” is rejected, so there is a structural break. But if the CUSUM values remain between the boundaries, the hypothesis “no structural break” cannot be rejected.

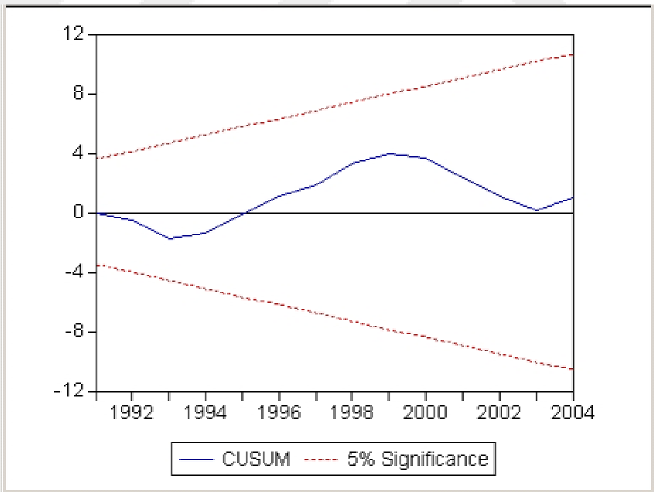

Figure 2.8 Graph of CUSUM test (Dokuz Eylül Üniversitesi, 2018)

2.2.3 The CUSUM of Squares Test

In the CUSUM of Squares Test, the hypotheses are determined as in the CUSUM test, but the test statistics are calculated using the squares of the standardized residuals in this test.

$$Cusum_r^2 = \frac{\sum_{t=k+1}^{r} w_t^2}{\sum_{t=k+1}^{n} w_t^2} \tag{2.10}$$

where $r = k + 1, \cdots$.

$$E(Cusum_r^2) \cong \frac{r-k}{n-k} \tag{2.11}$$

If $r = k$, then $E(Cusum_r^2) = 0$, if $r = n$ then $E(Cusum_r^2) = 1$.

In this test, the confidence interval is obtained by $E(Cusum^2) \pm c_0$. The tables in which the value of c_0 are given in Brown et al., (1975).

If the obtained test statistics values result in any interception between these two boundary, it is decided that there is a structural break, if the obtained test statistics values a result stay between these two boundary, it is decided that there is no structural break. As it can be seen in Figure 2.5.

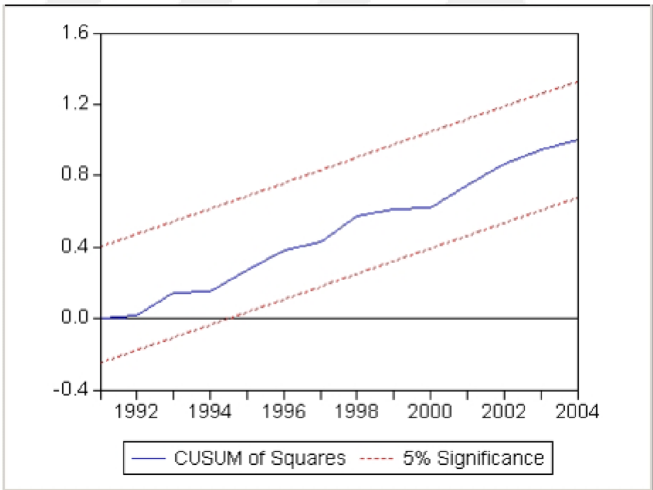

Figure 2.9 Graph of CUSUMSQR test (Dokuz Eylül Üniversitesi, 2018)

CHAPTER THREE
EXPONENTIAL SMOOTHING

3.1 Exponential Smoothing Method (ES)

Exponential smoothing is a forecasting method that weights the observed time series values exponentially. More recent observations are weighted more than more past observations. The exponential weighting is constituted by using one or more smoothing constant, which determine how much weight is given to each observation. Exponential smoothing has been found to be most effective when the parameters describing the time series may be changing slowly over time (Bowerman & O'Connel, 1993).

ES models assume that the time series have up to three underlying data components: level, trend and seasonality. Estimates, for the final values of these components are used to construct the forecast. In Figure 3.1 and as can be seen, an ES model can include of one of five types of trend (none, additive, damped additive, multiplicative, or damped multiplicative) and one of three types of seasonality (none additive, or multiplicative). Thus, there are 15 different ES models, the best known of the which are simple exponential smoothing (SES), Holt's linear trend model (HLT), Holt-Winters's additive model (HWA), Holt-Winter's multiplicative model (HWM), additive damped trend model (ADT), multiplicative damped trend model (MDT), Holt-Winter's damped model (HWD) and exponential trend model (ET) (Yapar, 2016) and as it can be seen in Table 3.1. These methods will be mentioned in the next section.

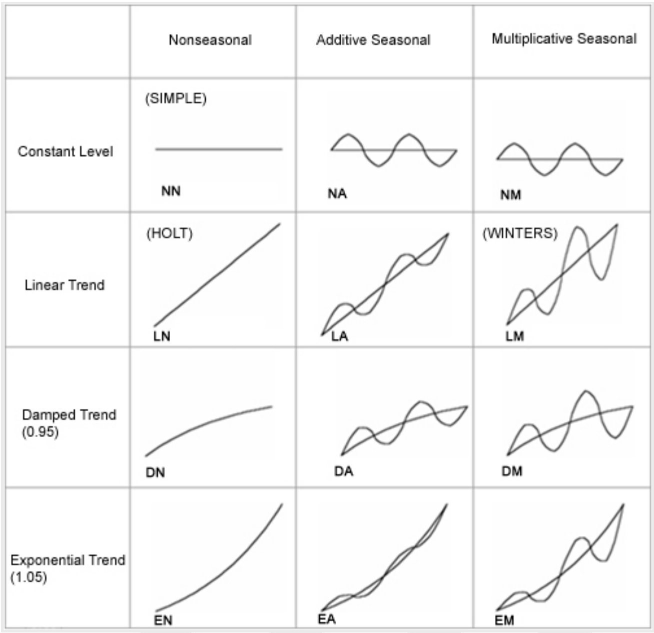

Figure 3.1 Types of exponential smoothing models (Gardner, 1985)

Table 3.1 Classification of exponential smoothing model

TREND COMPONENT	SEASONAL COMPONENTS		
	N (None)	A (Additive)	M (Multiplicative)
N (None)	**SES**		
A (Additive)	**HLT**	**HWA**	**MHW**
Ad (Additive damped)	**ADT**		**HWD**
M (Multiplicative)	**ET**		
Md (Multiplicative damped)	**MDT**		

3.1.1 Simple Exponential Smoothing Method (SES)

In Brown (1959) published, the SES method is a classical and widely-known approach where is used with the smoothing constant $\alpha \in [0,1]$ (Selamlar 2017). If it has been observed data up to and including time *t*, and the next value of the time series is forecasted x_t, the forecast is denoted by $S_t(x)$ (Hyndman et al., 2008). When the observation x_t becomes available, the forecast error is found to be $x_t - S_{t-1}(x)$. The forecast for the next period is,

$$S_t(x) = S_{t-1}(x) + \alpha\big(x_t - S_{t-1}(x)\big) \tag{3.1}$$

$$S_t(x) = S_{t-1}(x) + \alpha x_t - \alpha S_{t-1}(x) \tag{3.2}$$

$$S_t(x) = \alpha x_t + (1-\alpha) S_{t-1}(x) \tag{3.3}$$

The forecast $S_t(x)$ in equation (3.3) is based on weighting the most recent observation X_t with a weight value α, and weighting the most recent forecast $S_{t-1}(x)$ with a weight of $(1-\alpha)$. Thus, it can be interpreted as a weighted average of the most recent forecast and the most recent observation (Hyndman et al., 2008).

$$S_t(x) = \alpha \sum_{k=0}^{t-1} (1-\alpha)^k X_{t-k} + (1-\alpha)^t S_0(x) \tag{3.4}$$

where $\alpha \epsilon [0,1]$.

$S_0(x)$ in equation (3.4) represents the initial value. The SES always requires a previous value of the smoothing function. When the process is started, there must be some value that can be used as the previous value $S_{t-1}(x)$.

$S_t(x)$ in equation (3.4) represents a weighted moving average of all past observations with the weights decreasing exponentially; hence, the name “exponential smoothing“ (Brown, 1962). It can be seen that for large α, the recent observations get more weight.

Weights assigned by the SES are non-negative and sum to unity. If α is small, more weight is given to observations from the more distant past. If α is large, more weight is given to the more recent observations. In the SES process, the weight given data t periods ago is $\alpha(1-\alpha)^t$ (Selamlar, 2017). For instance, if smoothing constant is equal to 0.2 then the weight associated with the last observation is equal to 0.2 and the weights assigned to previous observations are 0.16, 0.128, 0.1024, 0.0819, and so on. These weights appear to decline exponentially when connected by a smooth curve. More weights given to most recent observations and weights decrease geometrically with age (Çapar, 2009).

3.1.2 Holt's Linear Trend Method (HLT)

Holt, added to the SES method in 1957 to make forecasting of the time series with trend (Holt, 2004). This new method has two smoothing parameters, α and β, as presented in equations (3.5) and (3.6).

$$S_t(x) = \alpha x_t + (1-\alpha)\big(S_{t-1}(x) + T_{t-1}(x)\big) \qquad (3.5)$$

$$T_t(x) = \beta\big(S_t(x) - S_{t-1}(x)\big) + (1-\beta)T_{t-1}(x) \qquad (3.6)$$

In equations (3.5) and (3.6) $S_t(x)$ denotes an estimate of the level of the series at time t and $T_t(x)$ denotes an estimate of the slope of the series at time t (Hyndman, 2018). When HLT was compared with SES, HLT stated that the errors were always better because the method was optimized on an additional parameter.

3.1.3 Exponential Trend Method (ET)

Exponential trend method which is another variation of Holt's linear trend method can be achieved through multiplying instead of adding level and trend as equations are illustrated both in (3.7) and (3.8).

$$S_t(x) = \alpha x_t + (1-\alpha)\left(\big(S_{t-1}(x) \times T_{t-1}(x)\big)\right) \qquad (3.7)$$

$$T_t(x) = \beta\big(S_t(x)/S_{t-1}(x)\big) + (1-\beta)T_{t-1}(x) \qquad (3.8)$$

3.1.4 Additive Damped Trend Method (ADT)

Gardner Jr (1985) proposed that the trends need to be "damped" to be more conservative for longer forecast horizons (Hyndman, 2018). The formulation of this method as follows:

$$S_t(x) = \alpha x_t + (1-\alpha)\big(S_{t-1}(x) + \phi T_{t-1}(x)\big) \tag{3.9}$$

$$T_t(x) = \beta\big(S_t(x) - S_{t-1}(x)\big) + (1-\beta)\phi T_{t-1}(x) \tag{3.10}$$

Damped parameter is ϕ, $0 < \phi < 1$.

3.1.5 Multiplicative Damped Trend Method (MDT)

Taylor (2003) suggested a damping parameter to the exponential trend method resulting to a multiplicative damped trend method (Selamlar, 2017):

$$S_t(x) = \alpha x_t + (1-\alpha)\left(\left(S_{t-1}(x) \times T_{t-1}^{\phi}(x)\right)\right) \tag{3.11}$$

$$T_t(x) = \beta\big(S_t(x)/S_{t-1}(x)\big) + (1-\beta)T_{t-1}^{\phi}(x) \tag{3.12}$$

The smoothing constant and initial values for any SES method and HLTM can be estimated by minimizing the sum of the squared errors. Deciding smoothing constant an initial value is very important for any type SES model and HLT. Despite the successful research on this subject, forecasters were unable to have a consensus on how to select smoothing constant and initial value. Determining the initial value and making mistakes in the selection of the optimum smoothing parameter affect the estimation results adversely. Different methods are applied to solve these problems. The MSES method (Yapar, 2016) and MHES (Yapar et al., 2017) have been developed to deal with these problems.

CHAPTER FOUR
MODIFIED EXPONENTIAL SMOOTHING

4.1 Modified Simple Exponential Smoothing Method (MSES)

The MSES method, which has developed in the recent years, calculates the most appropriate smoothing constant by giving more weight to the current observations in the series so that the calculated estimates perform better than the classical method.

The MSES method is similar to the SES method, but in this method smoothing parameter is not determined by the user as in the constant SES method. The smoothing parameters are modified so that when obtaining a smoothed value at a specific time point the weights among the observations are distributed taking into account how many observations can contribute to the value being smoothed.

“Therefore the smoothing parameter for this method is a function of t unlike exponential smoothing in regardless of the location of smoothed value on the time line, the observation receive weights only depending on their distances from the value being smoothed” (Selamlar, 2017, p. 48).

When it does not have trend and seasonal, model reduces to simple model. For the series X_t, $t = 1, \cdots, n$, can be written as:

$$S_t(x) = \left(\frac{m}{t}\right) X_t + \left(\frac{t-m}{t}\right) S_{t-1}(x) \tag{4.1}$$

for $t > m$

where $m = 0,1, \cdots, n$.

Recognize that MSES is similar to the SES, but the smoothing parameters are now dependent on the number of observations.

When the equation (4.1) is applied recursively to all successive observations in the series, the smoothed value at time *n* obtained by MSES can be re-written as:

$$S_t(x) = \sum_{k=0}^{t-(m+1)} \frac{\binom{t-k-1}{m-1}}{\binom{t}{m}} x_{t-k} + \frac{1}{\binom{t}{m}} S_m(x) \tag{4.2}$$

Proof (4.2):

$$S_t(x) = \left(\frac{m}{t}\right)x_t + \left(\frac{t-m}{t}\right)S_{t-1}(x)$$

$$S_{t-1}(x) = \left(\frac{m}{t-1}\right)x_t + \left(\frac{t-m-1}{t-1}\right)S_{t-2}(x)$$

$$S_t(x) = \left(\frac{m}{t}\right)x_t + \left(\frac{t-m}{t}\right)\left[\left(\frac{m}{t-1}\right)x_t + \left(\frac{t-m-1}{t-1}\right)S_{t-2}(x)\right]$$

$$S_t(x) = \left(\frac{m}{t}\right)x_t + \left(\frac{m}{t}\right)\left(\frac{t-m}{t-1}\right)x_{t-1} + \left(\frac{t-m}{t}\right)\left(\frac{t-m-1}{t-1}\right)S_{t-2}(x)$$

$$S_{t-2}(x) = \left(\frac{m}{t-2}\right)x_{t-2} + \left(\frac{t-m-2}{t-2}\right)S_{t-3}(x)$$

$$S_t(x) = \left(\frac{m}{t}\right)x_t + \left(\frac{m}{t}\right)\left(\frac{t-m}{t-1}\right)x_{t-1} + \left(\frac{t-m}{t}\right)\left(\frac{t-m-1}{t-1}\right)\left[\left(\frac{m}{t-2}\right)x_{t-2} + \left(\frac{t-m-2}{t-2}\right)S_{t-3}(x)\right]$$

$$S_t(x) = \left(\frac{m}{t}\right)x_t + \left(\frac{m}{t}\right)\left(\frac{t-m}{t-1}\right)x_{t-1} + \left(\frac{t-m}{t}\right)\left(\frac{t-m-1}{t-1}\right)\left(\frac{m}{t-2}\right)x_{t-2} + \left(\frac{t-m}{t}\right)\left(\frac{t-m-1}{t-1}\right)\left(\frac{t-m-2}{t-2}\right)S_{t-3}(x)$$

$$\vdots$$

$$S_t(x) = \left(\frac{m}{t}\right)x_t + \left(\frac{m}{t}\right)\left(\frac{n-m}{t-1}\right)x_{t-1} + \cdots + \frac{\binom{m}{m-1}}{\binom{t}{m}}x_{m+1} + \left(\frac{t-m}{t}\right)\left(\frac{t-m-1}{t-1}\right)\cdots\left(\frac{1}{m+1}\right)S_m$$

where $S_m(x)$ is the initial value for the MSES which can simply be the m^{th} observation or the average of the oldest m observations. If the initial value is $S_m(x)$, average of the *m* observation;

$$S_m(x) = \frac{X_1+X_2+\cdots+X_m}{m} \quad (4.3)$$

The weight for observation x_{t-k} is computed as follows:

$$w_{t-k} = \left(\frac{m}{t}\right)\frac{(t-m)!(t-k-1)!}{(t-k-1)!(t-1)!} \quad (4.4)$$

where $k = 0,1,\ldots,t-(m+1)$.

The weights of the MSES as given in equation (4.2) can be considered as the probabilities from a Negative Hyper-Geometric distribution with parameters *(t,m,1)* and a random variable *x* (Bailey, 1935).

4.2 Modified Holt's Linear Trend Method (MHES)

As shown in equation (3.5) and (3.6) in the previous section, HES method estimates expansion, T_t; by smoothing sequential differences, $(S_t(x) - S_{t-1}(x))$ of the local level, $S_t(x)$ and has two smoothing parameters, α and β, $0 < \alpha, \beta < 1$. There are two smoothing parameters to estimate and initial values for both the level and trend must be supplied (Yapar et al., 2017). The modified form of Holt's linear trend method is as follows:

$$S_t(x) = \left(\frac{p}{t}\right)x_t + \left(\frac{t-p}{t}\right)\left(S_{t-1}(x) + T_{t-1}(x)\right) \quad (4.5)$$

$$T_t(x) = \left(\frac{q}{t}\right)\left(S_{t-1}(x) - S_t(x)\right) + \left(\frac{t-q}{t}\right)T_{t-1}(x) \quad n \geq p \geq q \geq 0 \quad (4.6)$$

$$\hat{x}_t(h) = S_t(x) + hT_t(x) \quad (4.7)$$

for $p \in \{1,\ldots,n\}$ and $q \in \{1,\ldots,n\}$.

4.3 Average Age

The average age (AA) is the age of each piece of data used in the average, weighted as the data of that age would be weighted (Brown, 1962). Average age is used for measuring a smoothing method's ability to utilize fresh data (Yapar, 2016). The formulated state is represented by:

$$AA = n - \sum_{t=1}^{n} t w_t \qquad (4.8)$$

The assumption of these weights are:

1) $w_t \in [0,1] \qquad t = 1,....,n$
2) $\sum_{t=1}^{n} w_t = 1$
3) $w_1 \leq w_2 \leq .... \leq w_n$

In the SES method, AA and variance are obtained as shown in equations (4.9) and (4.10);

$$AA = \frac{1-\alpha}{\alpha} \qquad (4.9)$$

$$Var = \frac{\alpha}{2-\alpha} \qquad (4.10)$$

In the MSES method, AA and variance are obtained as shown in equations (4.11), (4.12) and (4.13);

$$AA = \frac{n-m}{m+1} \qquad (4.11)$$

$$Var = \sum_{t-1}^{n} w_t^2 = \left(\frac{m}{n}\right)^2 \left[1 + \sum_{t=0}^{n-m-1} \prod_{j=0}^{i} \left(\frac{n-m-j}{n-1-j}\right)^2\right] \qquad (4.12)$$

$$Var = \left(\frac{m}{n}\right)^2 {}_3F_2\left(\left(1, m-n, m-n\right), \left(1-n, 1-n\right), 1\right) \quad (4.13)$$

In the next section, the SES and MSES methods will be compared and the metrics mentioned above will decide which method is better.

CHAPTER FIVE
COMPARISON OF SES AND MSES WITH MONTE-CARLO SIMULATION

5.1 Comparison Criteria's

5.1.1 Mean Absolute Error (MAE)

MAE is estimated measure the average size of errors without considering direction of variables. That is, the average of the absolute values of the differences between forecast and observation.

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t| \tag{5.1}$$

where $\hat{y}_t$ is estimator of the observation values for time *t*.

5.1.2 Mean Absolute Percent Error (MAPE)

The MAPE measures the size of the error in percentage terms. It is calculated as shown in the formula below:

$$MAPE = \left(\frac{1}{n}\sum_{t=1}^{n}\frac{|y_t - \hat{y}_t|}{y}\right) \times 100 \tag{5.2}$$

The MAPE is a percentage, so it can be easily compared it between series, easily understood and interpreted. However, MAPE has the significant disadvantage that it produces infinite or undefined values for zero or close-to-zero actual values (Kim & Kim, 2016).

5.1.3 Mean Absolute Deviation (MAD)

MAD calculate the average distance between each data value and the mean.

$$MAD = \frac{1}{n}\sum_{t=1}^{n}|y_t - \bar{y}_t| \tag{5.3}$$

Many methods that provide an understanding of the differences between data sets are based on the use of standard deviations. However, the calculation and interpretation of these methods, including squaring and square-rooting, are more difficult than MAD. This method is that gives similar results in almost all cases with simple mean based on absolute deviation.

5.2 Monte-Carlo Simulation

In this study, Monte-Carlo simulations were carried out in R 3.2 program to compare the performances of SES and MSES methods when structural break occurred. Each data series that did not have seasonality and trend component was generated with 20, 50, 200 sample size and was repeated 1.000 times and.

Since it was thought that the magnitude of breaks in the series could affect the power of the model, the performance of the model were investigated with 1,5 and 10 breaks' magnitudes (M). Similarly, since it was also thought that the breaks' occurring in different location (l) of the series affect the power of the model, the breaks were applied in the first quarter (0.25n), the second quarter (0.50n) and in the third quarter (0.75n) (Firuzan & Çoban, 2016).

Since both methods must have equal smoothing coefficients to compare, R 3.2. program, using MSES method, in this study was calculated alpha value by divide the value of m that get the smallest error by sample width for using in the SES method. It was obtained the MAE (Mean Absolute Error) values both methods and was computed mean and variance of these values. These values are shown in Figure 5.1, 5.2, 5.3, 5.4, 5.5, 5.6 and Table 5.1, 5.2, 5.3.

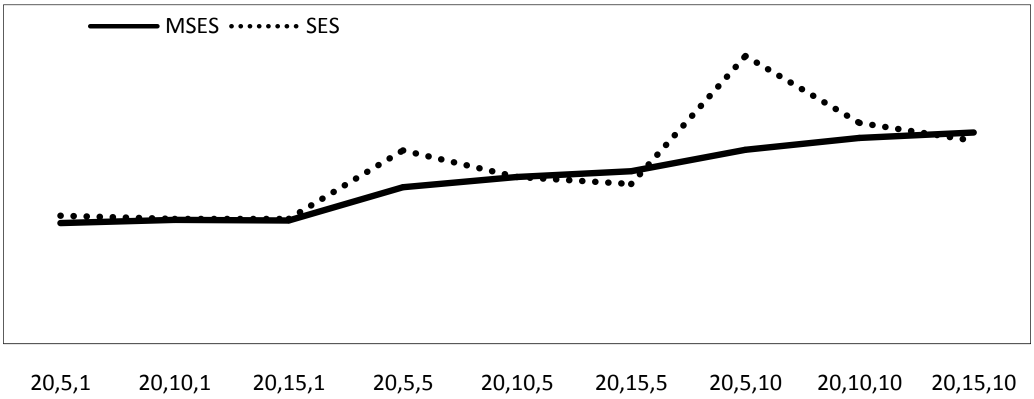

Figure 5.1 Comparison of the mean of the errors of MSES and SES methods for *n, l, M*

In Figure 5.1, n represents the sample size, *l* is used for location symbol and the last one is M which is abroreviated for breaking magnitude.

When it is investigated data set with a sample size of 20, it is seen that when the breaking magnitude is 1, the error averages are similar in both methods. As the magnitude of the break increases, the average of the errors increases. When the magnitude of breaking has increased to 5 and 10, while the MSES gives better results in the first quarter of the series, SES is better in the last quarter of the series.

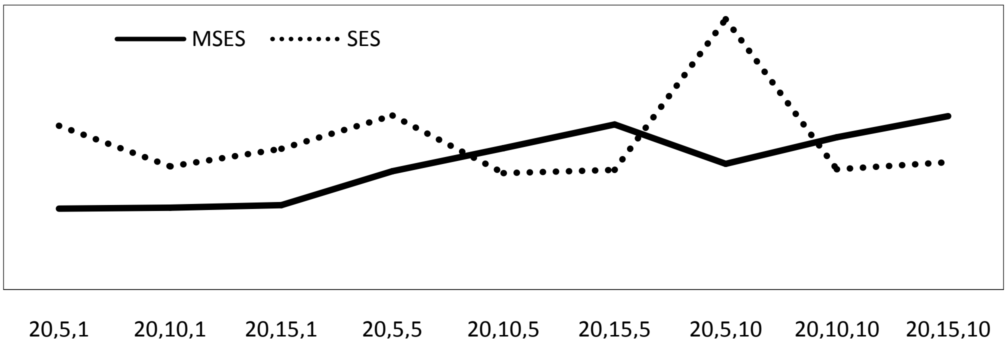

Figure 5.2 Comparison of the variance of the errors of MSES and SES methods sample size 20 (*n, l, M)*

As can be easily seen on the Figure 5.2, the variance of the MSES method increases as the magnitude of the break increases by 20 sample size. In the SES method, the variances of errors' in all breaking magnitudes show sudden jumps. When it is compare the two methods, it is generally seen that the errors' mean and variance of the MSES are smaller than SES. If break is at the first quadrand of series, MSES gets better; if break is at the last quadrand of series, SES is better.

Table 5.1 Mean and variance comparison of MSES and SES methods for sample size 20 under 1000 repetition

n, l, M	$\mathbf{MSES_{error\ mean}}$	$\mathbf{SES_{error\ mean}}$	$\mathbf{MSES_{error\ var}}$	$\mathbf{SES_{error\ var}}$
20, 5, 1	0.8907	0.9431	0.0228	0.0461
20, 10, 1	0.9132	0.9202	0.0231	0.0347
20, 15, 1	0.9096	0.9212	0.0238	0.0396
20, 5, 5	1.1544	1.4265	0.0333	0.0490
20, 10, 5	1.2292	1.2300	0.0398	0.0328
20, 15, 5	1.2727	1.1775	0.0465	0.0337
20, 5, 10	1.4311	2.1207	0.0354	0.0761
20, 10, 10	1.5168	1.6276	0.0429	0.0339
20, 15, 10	1.5587	1.4960	0.0488	0.0358

When it is researched the data set with a sample size of 50, it can be seen that there is a difference between the average of the errors of the MSES and the average of the errors of the SES methods when the magnitude of the break is 1. As with 20 sample size, as the magnitude of break increases, the average of the errors in both methods increases. It is observed that the error of the MSES method is smaller when the break is at the beginning, and the error of the SES method is smaller when the break is at the end of the series.

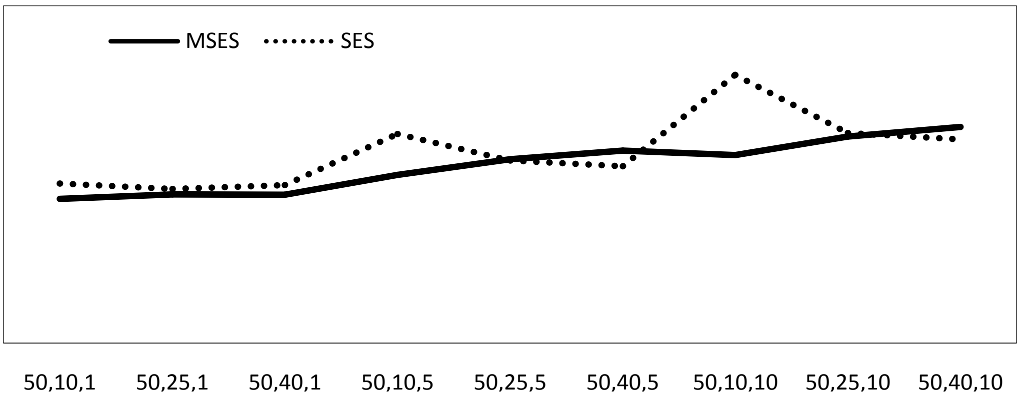

Figure 5.3 Comparison of the mean of the errors of MSES and SES methods at 50 sample size (*n, l, M*)

It is observed in Figure 5.3 that the error' mean of the MSES method is smaller when the break is at the beginning, and the error' mean of the SES method is smaller when the break is at the end.

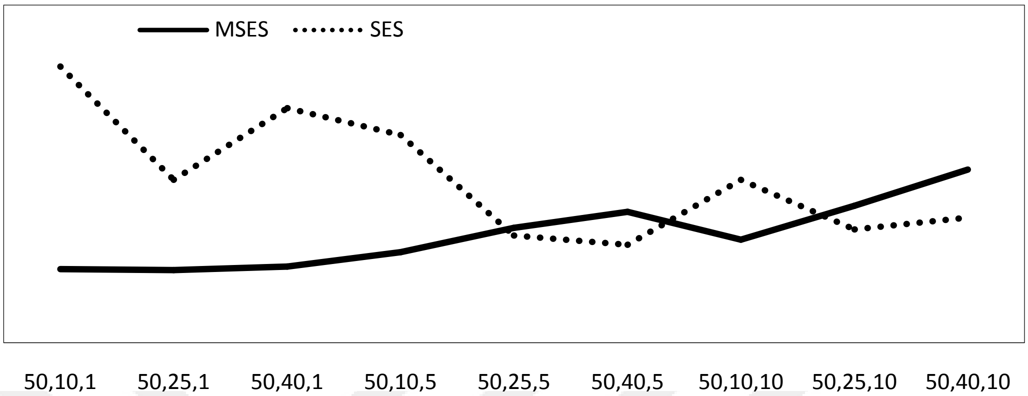

Figure 5.4 Comparison of the variance of the errors of MSES and SES methods at 50 sample size (*n*, *l*, *M*)

If it is compared the variances of the errors for both methods at 50 sample size, when the magnitude of the breaks are 5 and 10, and at the same time in the middle and last quadrant of the series, the MSES method can give worse results. When it is investigated in general, it is seen that the errors of the MSES method are smaller than the SES method. The following Figure 5.4 and Table 5.2 summarize the results.

Table 5.2 Mean and Variance Comparison of MSES and SES Methods for Sample Size 50 under 1000 repetition

n, l, M	$\mathbf{MSES_{error\ mean}}$	$\mathbf{SES_{error\ mean}}$	$\mathbf{MSES_{error\ var}}$	$\mathbf{SES_{error\ var}}$
50, 10, 1	0.8550	0.9467	0.0087	0.0327
50, 25, 1	0.8806	0.9140	0.0086	0.0193
50, 40, 1	0.8803	0.9356	0.0090	0.0278
50, 10, 5	0.9976	1.2388	0.0107	0.0246
50, 25, 5	1.0891	1.0829	0.0136	0.0127
50, 40, 5	1.1415	1.0472	0.0155	0.0116
50, 10, 10	1.1152	1.5898	0.0122	0.0193
50, 25, 10	1.2244	1.2461	0.0162	0.0134
50, 40, 10	1.2818	1.2077	0.0205	0.0148

When it is researched the data set with a sample size of 200, it can be seen that there is a difference between the average and variance of the errors of the MSES, and the average and variance of the errors of the SES methods when the magnitude of the break is 1,5 and 10. As with 20 sample size, as the magnitude of break increases, the average of the errors in both methods increases. The following Figure 5.5, 5.6 and Table 5.3 summarize the situation.

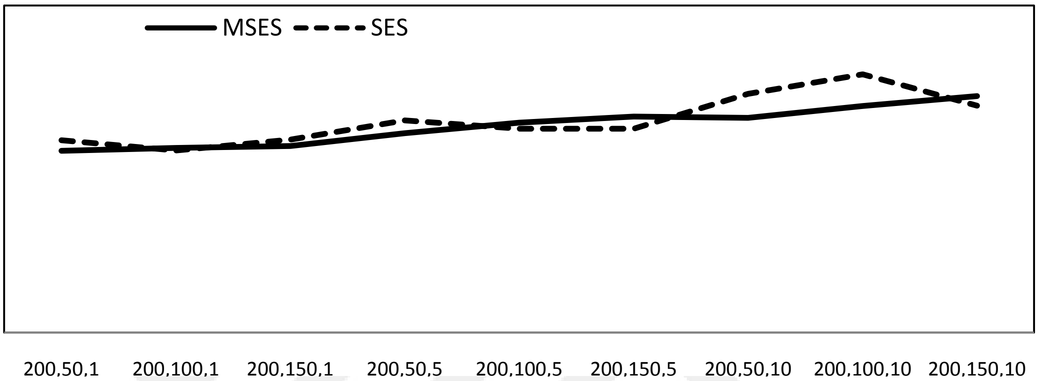

Figure 5.5 Comparison of the mean of the errors of MSES and SES methods at 200 sample size (*n, l, M)*

It is observed that the mean of the error of the MSES method is almost same when the break is at the beginning. The mean of the error of the SES method is smaller when the break is at the middle of series and magnitude of the break is 5. In case of the magnitude of the break increases, the average of MSES 'errors is better.

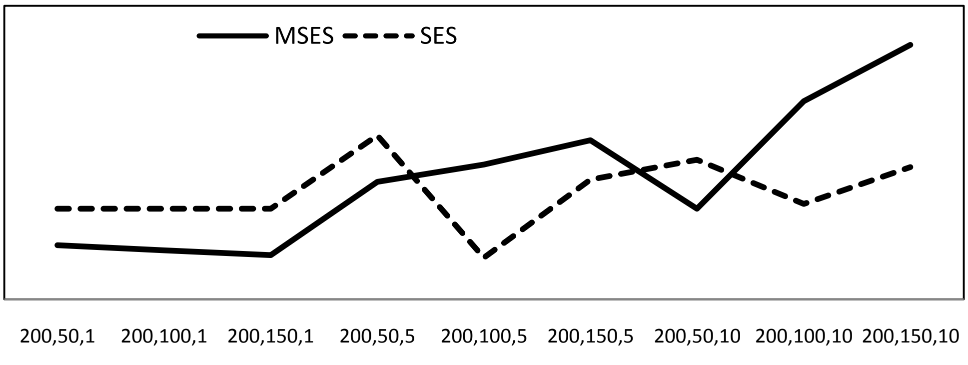

Figure 5.6 Comparison of the variance of the errors of MSES and SES methods at 200 sample size (n, l, M)

If it is compared the variances of the errors of both methods at 200 sample size, when the magnitude of the break is 1 the MSES method can give better results, but when the magnitude of the break are 5 and 10 the MSES method can give worse results.

Table 5.3 Mean and variance comparison of MSES and SES methods for sample size 200 under 1000 repetition

n, l, M	$\mathbf{MSES_{error\ mean}}$	$\mathbf{SES_{error\ mean}}$	$\mathbf{MSES_{error\ var}}$	$\mathbf{SES_{error\ var}}$
200, 50, 1	0.8336	0.8831	0.0022	0.0037
200, 100, 1	0.8475	0.8356	0.0020	0.0037
200, 150, 1	0.8555	0.8856	0.0018	0.0037
200, 50, 5	0.9139	0.9737	0.0048	0.0067
200, 100, 5	0.9622	0.9356	0.0055	0.0017
200, 150, 5	0.9900	0.9354	0.0065	0.0049
200, 50, 10	0.9846	1.0960	0.0037	0.0057
200, 100, 10	1.0392	1.1848	0.0081	0.0039
200, 150, 10	1.0846	1.0410	0.0104	0.0054

Compared to MSES and SES methods, three sample sizes are combined and presented in Figures 5.7 and 5.8.

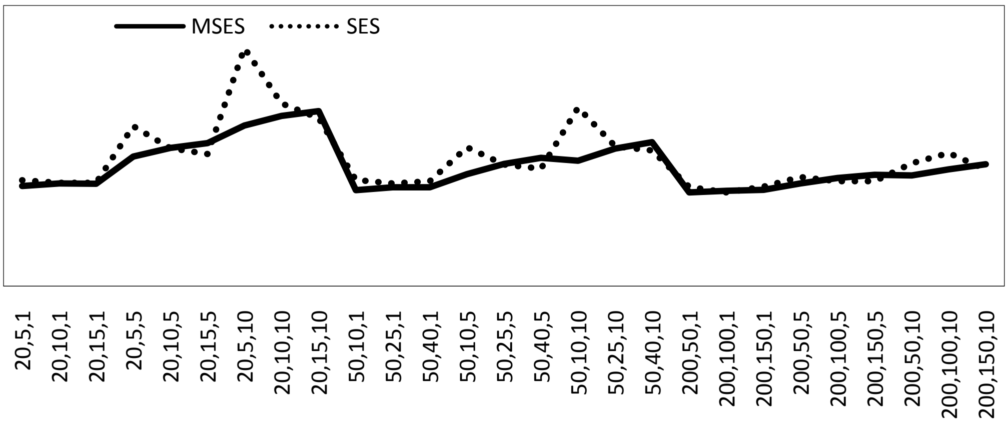

Figure 5.7 Comparison of the mean of the errors of MSES and SES methods for all sample size (*n, l, M)*

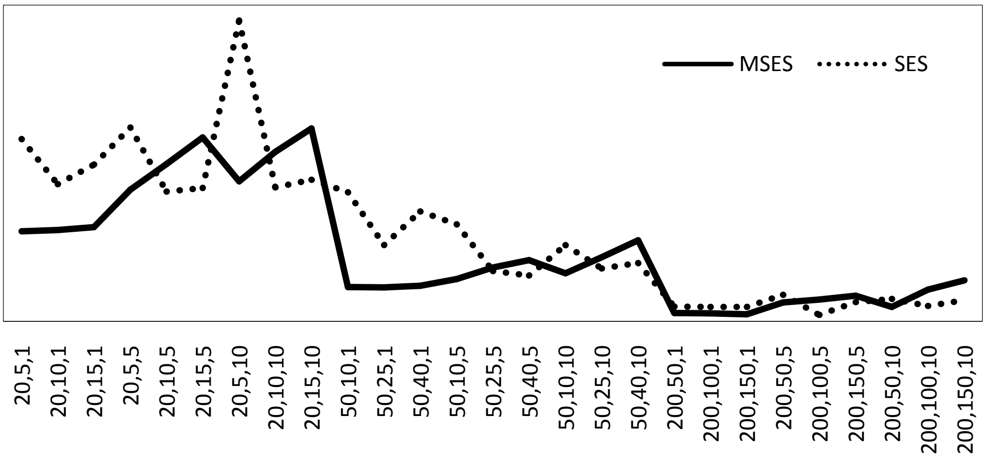

Figure 5.8 Comparison of the variance of the errors of MSES and SES methods all for all sample size (*n, l, M)*

The mean absolute error values under the different sample size, different break magnitude and different break locations of the MSES and the SES methods are shown in the tables and figures. When the mean absolute errors change under the breaks is examined, it is observed that it decreases for both of the methods while the sample size increases and it decreases while the break magnitude increases. Considering both of the methods, while almost the same results are obtained when the break magnitude is low (*M=1*), the MSES method's error appears to be lower than the SES method's while the magnitude of the break increases. Regardless of this superiority of the MSES method, when the magnitude of the break is medium and high, the location of the break is in the middle and at the end, the error of the MSES method is higher than the SES method's. It can be said that due to the MSES method's concentrating on the recent observations, the effect of the break type affects the model estimation negatively.

Mean Absolute Error's mean and variance are affected by the sample's size, break's magnitude and location. In fact, the breaks in the data set would affect the model estimation negatively. Especially, the potential breaks being in the end of the series increases the error of estimation of the method. Possible breaks' magnitude and locations should be taken into consideration in the use of the MSES method.

CHAPTER SIX

APPLICATION

In this study, it is mention about the SES method that has been widely used for many years and the MSES method which has been developed to make up for deficiencies of this method. By comparing these two methods with applications, it will be decided which one is better than the other.

In the first application, it used data set “YAF2” from the 1001 series of the M-competition data (Makridakis et al., 1982). The data set measured yearly, non-seasonal, consists of 22 observations. In SES method, the comparison of different α levels according to YAF2 series is shown in the Figure 6.1. below.

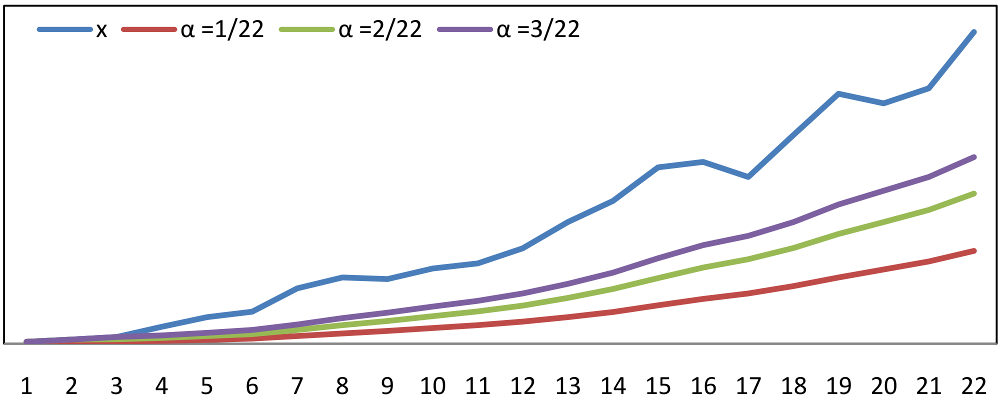

Figure 6.1 The SES method comparison of different α levels

According to the MSES method, we compare the smoothed values at different *m* levels which mentioned in Chapter 4. The results could be seen in follewing Figure 6.2.

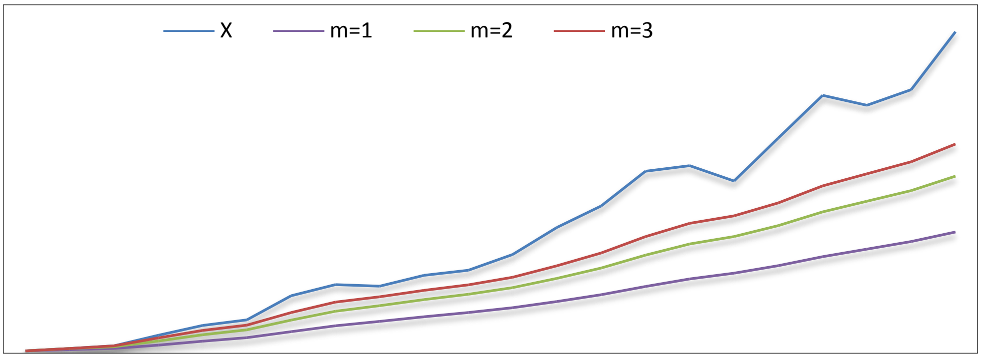

Figure 6.2 The MSES method comparison of smoothed value in different *m* levels

The Table 6.1 below also shows the results of both methods using YAF2 data set.

Table 6.1 Comparing of SES and MSES with YAF2 data set

		S(x)					
		***m*=1 and *α* =1/22**		***m*=2 and *α* =2/22**		***m*=3 and *α* =3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.00	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.30	10,152.90	25,950.00	14,781.80
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.10	38,814.00	19,228.40
6	57,006.00	26,416.00	9,540.28	39,892.70	17,489.70	47,910.00	24,379.90
7	98,563.00	36,722.70	13,586.80	56,655.60	24,860.00	69,618.40	34,495.80
8	117,759.00	46,852.30	18,321.90	71,931.50	33,305.40	87,671.10	45,849.80
9	115,097.00	54,435.00	22,720.70	81,523.80	40,741.00	96,813.10	55,292.60
10	133,759.00	62,367.40	27,767.90	91,970.80	49,197.20	107,897.00	65,992.60
11	142,485.00	69,650.80	32,982.30	101,155.00	57,677.90	117,330.00	76,423.40
12	169,611.00	77,980.80	39,192.70	112,565.00	67,853.60	130,400.00	89,130.80
13	216,229.00	88,615.30	47,239.80	128,513.00	81,342.30	150,207.00	106,462.00
14	253,227.00	100,373.00	56,602.90	146,329.00	96,968.20	172,283.00	126,476.00
15	313,096.00	114,555.00	68,261.70	168,565.00	116,616.00	200,445.00	151,924.00
16	322,681.00	127,563.00	79,826.20	187,829.00	135,349.00	223,364.00	175,209.00
17	296,245.00	137,485.00	89,663.40	200,584.00	149,976.00	236,226.00	191,714.00
18	370,333.00	150,421.00	102,421.00	219,445.00	170,009.00	258,577.00	216,071.00
19	443,826.00	165,864.00	117,940.00	243,064.00	194,901.00	287,827.00	247,129.00
20	426,751.00	178,908.00	131,976.00	261,433.00	215,978.00	308,665.00	271,623.00
21	453,627.00	191,990.00	146,597.00	279,737.00	237,583.00	329,374.00	296,441.00
22	553,400.00	208,418.00	165,088.00	304,615.00	266,293.00	359,923.00	331,481.00

The following Figures 6.3, 6.4 and 6.5 illustrade these results in better way. In these figures the smoothed values are compared in both methods at the same level of smoothing constant.

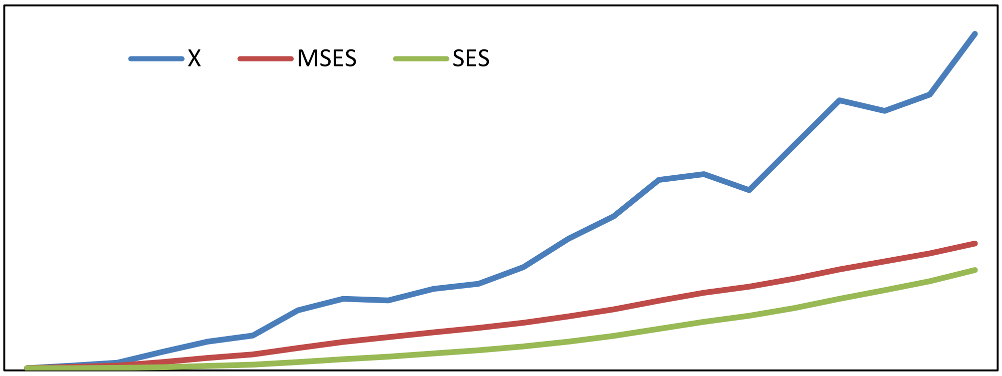

Figure 6.3 The smoothed values comparison of both methods for m = 1

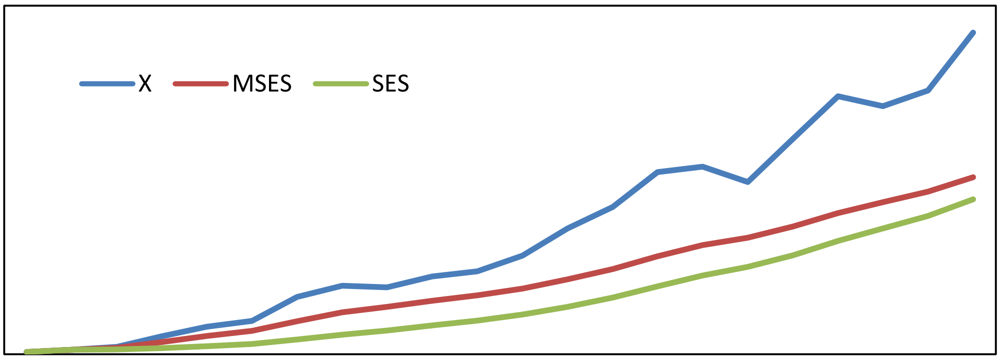

Figure 6.4 The smoothed values comparison of both methods for m = 2

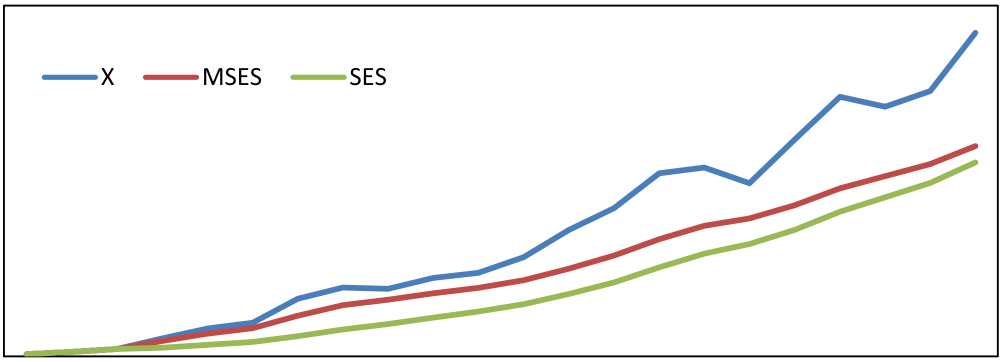

Figure 6.5 The Smoothed values comparison of both methods for m = 3

According to the results shown in the Table 6.2, the errors of the MSES method are smaller than the SES method.

Table 6.2 The MAPE values of both methods

MAPE					
***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m*=3 and *α* =3/22**	
MSES	**SES**	**MSES**	**SES**	**MSES**	**SES**
0.5274	0.71945	0.32919	0.54029	0.22034	0.41866

When it is compared different smoothing levels (either m or α), it is seen that as the smoothing level increases, the smoothed value approaches the actual value and it can be say that the MSES method is better than the SES method in the dataset where there is no structural break.

If we summarize the advantages of the MSES method over the SES method in the series without structural break,

- There is no initial problem since the MSES method selects the average of m^{th} observation and the first observation.
- Since the errors of the MSES method are smaller than the SES, the predictions are better.
- The MSES also provides more meaningful weighting schemes when the number of iterations is small.
- The smoothing coefficient is calculated according to certain parameters, not according to user preference.
- Since the variance of the new model is smaller, the model is more flexible.
- The MSES achieves more precious predictions because it assigns more weight to recent observations than to the SES.

It was investigated the MSES and the SES methods in the inexistence of structural breaks. So, how does the MSES and SES methods give results when there is a structural break in the series? To investigate this situation in the YAF2 data set, in point 5, 10, 15 we added 10,000, 50,000 and 100,000 magnitude of breaks. The Table 6.3 shows the data we have generated.

Table 6.3 Original values increased values with location *l* and magnitude *M*

		***M*=10,000**			***M*=50,000**			***M*=100,000**		
t	***x***	***l*=5**	***l*=10**	***l*=15**	***l*=5**	***l*=10**	***l*=15**	***l*=5**	***l*=10**	***l*=15**
1	3,600	3,600	3,600	3,600	3,600	3,600	3,600	3,600	3,600	3,600
2	7,700	7,700	7,700	7,700	7,700	7,700	7,700	7,700	7,700	7,700
3	12,300	12,300	12,300	12,300	12,300	12,300	12,300	12,300	12,300	12,300
4	30,500	30,500	30,500	30,500	30,500	30,500	30,500	30,500	30,500	30,500
5	47,390	47,390	47,390	47,390	47,390	47,390	47,390	47,390	47,390	47,390
6	57,006	67,006	57,006	57,006	107,006	57,006	57,006	157,006	57,006	57,006
7	98,563	108,563	98,563	98,563	148,563	98,563	98,563	198,563	98,563	98,563
8	117,759	127,759	117,759	117,759	167,759	117,759	117,759	217,759	117,759	117,759
9	115,097	125,097	115,097	115,097	165,097	115,097	115,097	215,097	115,097	115,097
10	133,759	143,759	133,759	133,759	183,759	133,759	133,759	233,759	133,759	133,759
11	142,485	152,485	152,485	142,485	192,485	192,485	142,485	242,485	242,485	142,485
12	169,611	179,611	179,611	169,611	219,611	219,611	169,611	269,611	269,611	169,611
13	216,229	226,229	226,229	216,229	266,229	266,229	216,229	316,229	316,229	216,229
14	253,227	263,227	263,227	253,227	303,227	303,227	253,227	353,227	353,227	253,227
15	313,096	323,096	323,096	313,096	363,096	363,096	313,096	413,096	413,096	313,096
16	322,681	332,681	332,681	332,681	372,681	372,681	372,681	422,681	422,681	422,681
17	296,245	306,245	306,245	306,245	346,245	346,245	346,245	396,245	396,245	396,245
18	370,333	380,333	380,333	380,333	420,333	420,333	420,333	470,333	470,333	470,333
19	443,826	453,826	453,826	453,826	493,826	493,826	493,826	543,826	543,826	543,826
20	426,751	436,751	436,751	436,751	476,751	476,751	476,751	526,751	526,751	526,751
21	453,627	463,627	463,627	463,627	503,627	503,627	503,627	553,627	553,627	553,627
22	553,400	563,400	563,400	563,400	603,400	603,400	603,400	653,400	653,400	653,400

When the structural break is in the 5'th point of the series and at the 10,000 of magnitude, smoothed values are calculated with both of the methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.6, m = 2 and α = 2/22 in the Figure 6.7, m = 3 and α = 3/22 in the Figure 6.8; and in the Table 6.4.

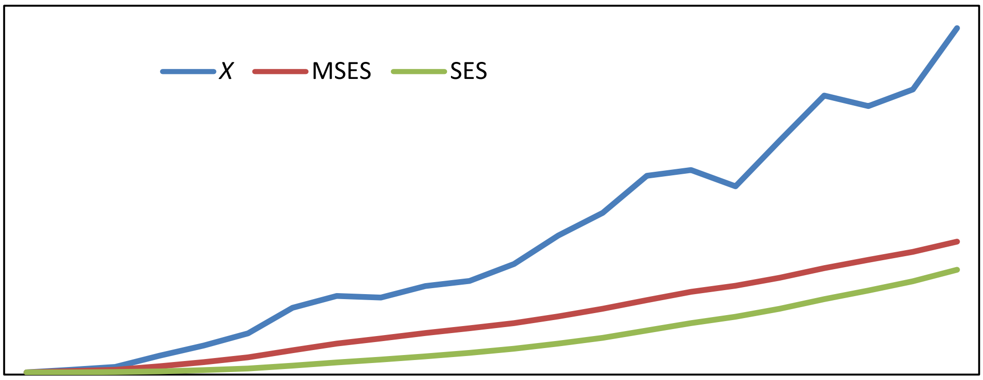

Figure 6.6 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 5$, $M = 10{,}000$

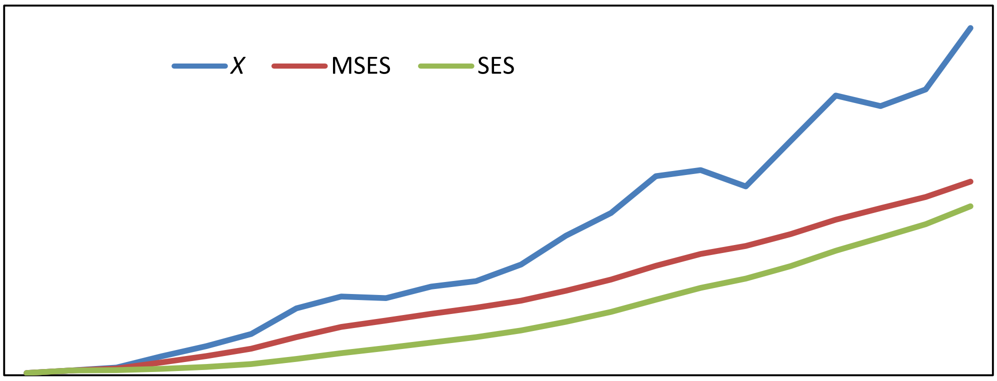

Figure 6.7 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 5$, $M = 10{,}000$

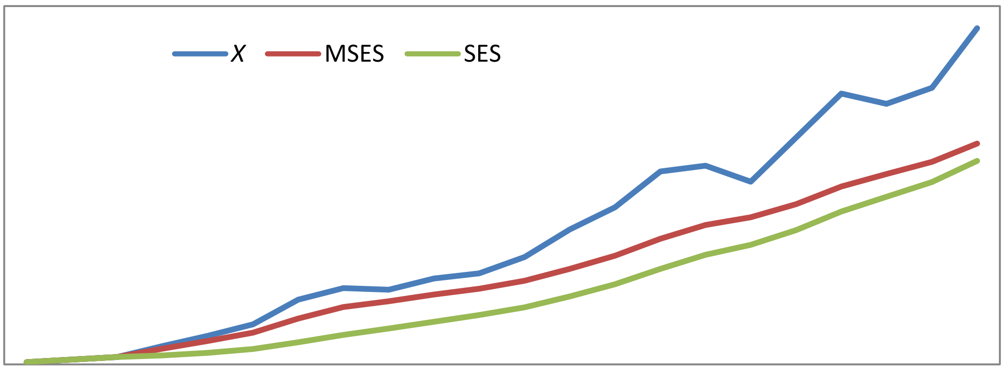

Figure 6.8 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 5$, $M = 10{,}000$

Table 6.4 Smoothed values for, $l = 5$, $M = 10{,}000$

		S(X)					
		m = 1 and α = 1/22		**m = 2 and α = 2/22**		**m = 3 and α = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.37	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.67	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.33	10,152.89	25,950.00	14,781.82
5	47,390.00	20,298.00	7,280.13	31,336.00	13,538.08	38,814.00	19,228.39
6	67,006.00	28,082.67	9,994.83	43,226.00	18,398.80	52,910.00	25,743.52
7	108,563.00	39,579.86	14,475.2	61,893.71	26,595.55	76,761.29	37,037.08
8	127,759.00	50,602.25	19,624.47	78,360.04	35,792.23	95,885.43	49,408.25
9	125,097.00	58,879.44	24,418.67	88,746.03	43,910.84	105,622.60	59,729.45
10	143,759.00	67,367.40	29,843.23	99,748.62	52,987.95	117,063.50	71,188.02
11	152,485.00	75,105.36	35,417.86	109,337.10	62,033.13	126,723.90	82,273.97
12	179,611.00	83,814.17	41,972.09	121,049.40	72,722.03	139,945.70	95,547.20
13	226,229.00	94,769.15	50,347.41	137,230.90	86,677.21	159,857.20	113,367.40
14	263,227.00	106,801.90	60,023.75	155,230.30	102,727.20	182,007.90	133,802.80
15	323,096.00	121,221.50	71,981.58	177,612.40	122,760.70	210,225.50	159,615.50
16	332,681.00	134,437.70	83,831.55	196,996.00	141,844.40	233,185.90	183,215.40
17	306,245.00	144,544.00	93,941.26	209,848.80	156,789.90	246,078.70	199,992.10
18	380,333.00	157,643.40	106,959.10	228,791.50	177,112.00	268,454.40	224,584.10
19	453,826.00	173,231.90	122,725.70	252,479.30	202,267.80	297,723.60	255,844.30
20	436,751.00	186,407.90	136,999.60	270,906.50	223,584.50	318,577.70	280,513.40
21	463,627.00	199,608.80	151,846.30	289,260.80	245,406.50	339,299.00	305,483.50
22	563,400.00	216,144.80	170,553.30	314,182.60	274,315.00	369,858.30	340,653.90

When the structural break is in the 10’th point of the series and at the 10,000 of magnitude, smoothed values are calculated with both of the methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.9, m = 2 and α = 2/22 in the Figure 6.10, m = 3 and α = 3/22 in the Figure 6.11; and in the Table 6.5.

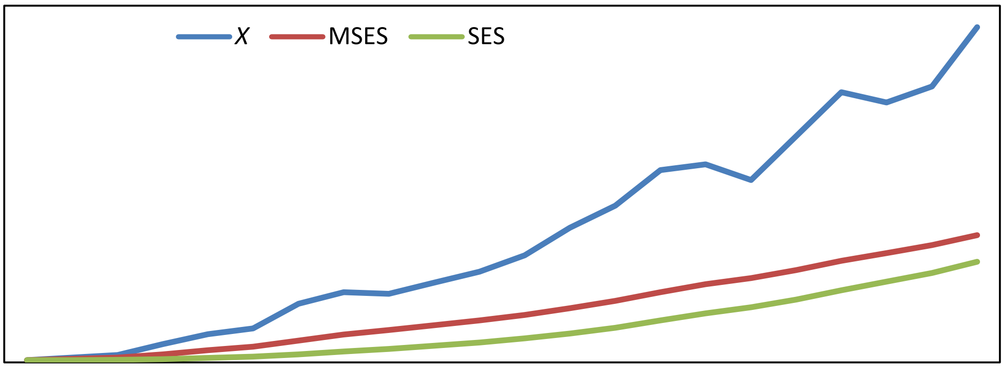

Figure 6.9 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 10$, $M = 10{,}000$

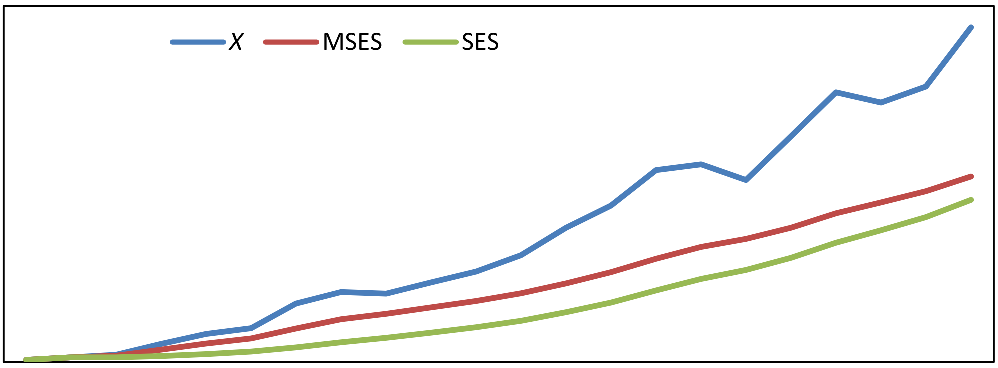

Figure 6.10 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 10$, $M = 10{,}000$

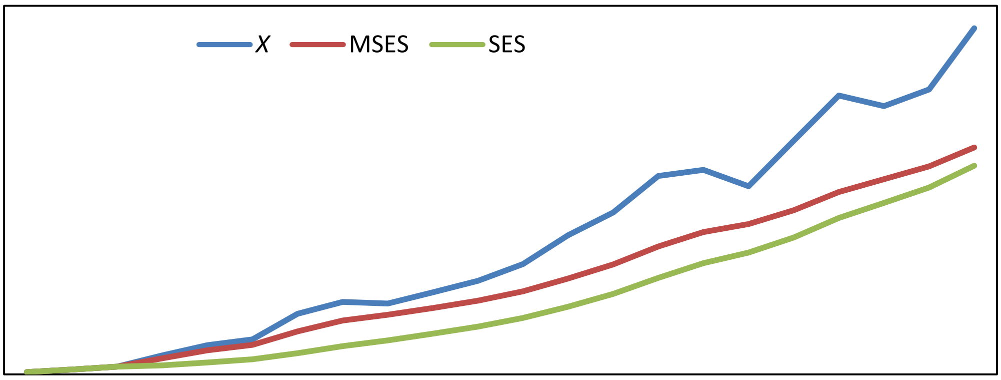

Figure 6.11 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 10$, $M = 10{,}000$

Table 6.5 Smoothed values for, $l = 10$ and $M = 10{,}000$

		S(X)					
		***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m* = 3 and *α* = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.67	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.33	10,152.89	25,950.00	14,781.82
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.08	38,814.00	19,228.39
6	57,006.00	26,416.00	9,540.29	39,892.67	17,489.71	47,910.00	24,379.88
7	98,563.00	36,722.71	13,586.77	56,655.62	24,860.01	69,618.43	34,495.76
8	117,759.00	46,852.25	18,321.87	71,931.46	33,305.37	87,671.14	45,849.84
9	115,097.00	54,435.00	22,720.74	81,523.81	40,740.98	96,813.10	55,292.63
10	133,759.00	62,367.40	27,767.94	91,970.84	49,197.16	107,896.90	65,992.59
11	152,485.00	70,559.91	33,436.89	102,973.40	58,586.96	120,057.30	77,787.01
12	179,611.00	79,647.50	40,081.17	115,746.30	69,589.15	134,945.70	91,672.10
13	226,229.00	90,923.00	48,542.44	132,743.70	83,829.14	156,011.10	110,020.80
14	263,227.00	103,230.40	58,300.83	151,384.20	100,138.00	178,985.90	130,912.50
15	323,096.00	117,888.10	70,336.97	174,279.10	120,406.90	207,807.90	157,119.40
16	332,681.00	131,312.70	82,261.70	194,079.30	139,704.60	231,221.60	181,059.60
17	306,245.00	141,602.80	92,442.76	207,275.30	154,844.60	244,461.10	198,130.30
18	380,333.00	154,865.60	105,528.70	226,503.90	175,343.60	267,106.40	222,976.10
19	453,826.00	170,600.40	121,360.40	250,432.50	200,660.10	296,588.40	254,455.70
20	436,751.00	183,907.90	135,696.30	269,064.40	222,123.00	317,612.80	279,314.10
21	463,627.00	197,227.90	150,602.30	287,594.20	244,077.90	338,472.00	304,447.70
22	563,400.00	213,872.00	169,365.80	312,667.40	273,107.20	369,144.00	339,759.40

When the structural break is in the 15’th point of the series and at the 10,000 of magnitude, smoothed values are calculated with both of methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.12, m = 2 and α = 2/22 in the Figure 6.13, m = 3 and α = 3/22 in the Figure 6.14; and in the Table 6.6.

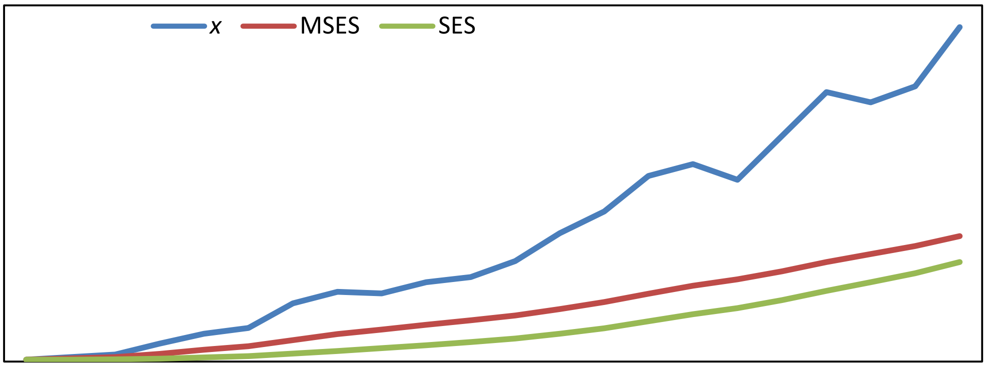

Figure 6.12 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 15$, $M = 10{,}000$

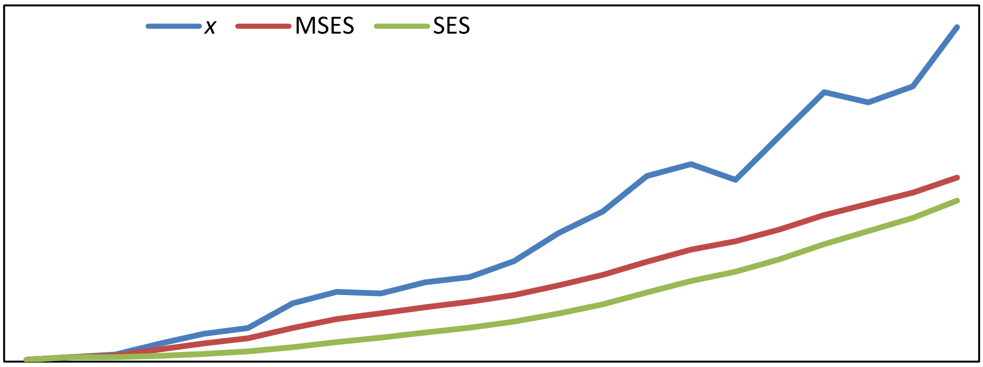

Figure 6.13 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 15$, $M = 10{,}000$

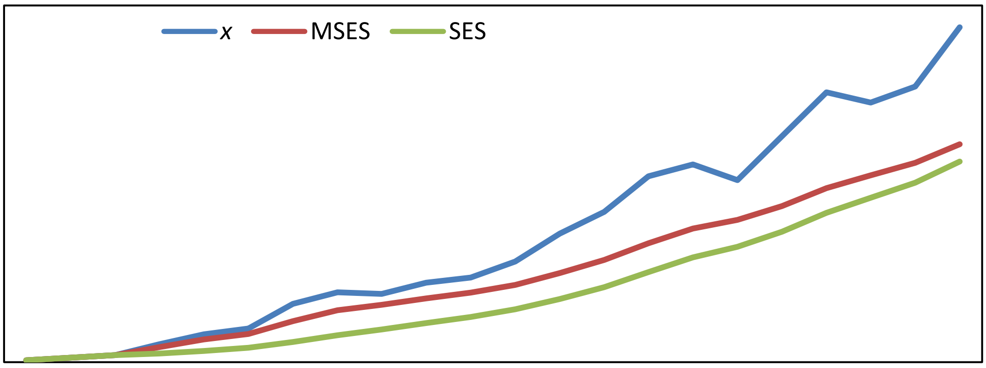

Figure 6.14 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 15$, $M = 10{,}000$

Table 6.6 Smoothed values for, $l = 15$ and $M = 10{,}000$

		S(X)					
		***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m* = 3 and *α* = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.67	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.33	10,152.89	25,950.00	14,781.82
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.08	38,814.00	19,228.39
6	57,006.00	26,416.00	9,540.29	39,892.67	17,489.71	47,910.00	24,379.88
7	98,563.00	36,722.71	13,586.77	56,655.62	24,860.01	69,618.43	34,495.76
8	117,759.00	46,852.25	18,321.87	71,931.46	33,305.37	87,671.14	45,849.84
9	115,097.00	54,435.00	22,720.74	81,523.81	40,740.98	96,813.10	55,292.63
10	133,759.00	62,367.40	27,767.94	91,970.84	49,197.16	107,896.90	65,992.59
11	142,485.00	69,650.82	32,982.35	101,155.20	57,677.87	117,330.00	76,423.38
12	169,611.00	77,980.83	39,192.74	112,564.50	67,853.61	130,400.20	89,130.78
13	216,229.00	88,615.31	47,239.84	128,512.90	81,342.28	150,206.90	106,462.40
14	253,227.00	100,373.30	56,602.90	146,329.20	96,968.17	172,282.60	126,475.70
15	313,096.00	114,554.80	68,261.67	168,564.80	116,616.20	200,445.30	151,923.90
16	332,681.00	128,187.70	80,280.73	189,079.30	136,258.40	225,239.50	176,572.60
17	306,245.00	138,661.60	90,551.84	202,863.50	151,711.70	239,534.60	194,255.20
18	380,333.00	152,087.80	103,723.70	222,582.30	172,495.50	263,001.00	219,629.50
19	453,826.00	167,968.80	119,637.40	246,923.80	198,071.00	293,131.20	251,565.40
20	436,751.00	181,407.90	134,051.70	265,906.50	219,769.20	314,674.20	276,817.90
21	463,627.00	194,846.90	149,032.40	284,737.00	241,938.10	335,953.20	302,291.90
22	563,400.00	211,599.30	167,867.30	310,070.00	271,161.90	366,968.70	337,897.60

When the structural break is in the 5'th point of the series and at the 50,000 of magnitude, smoothed values are calculated with both of methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.15, m = 2 and α = 2/22 in the Figure 6.16, m = 3 and α = 3/22 in the Figure 6.17; and in the Table 6.7.

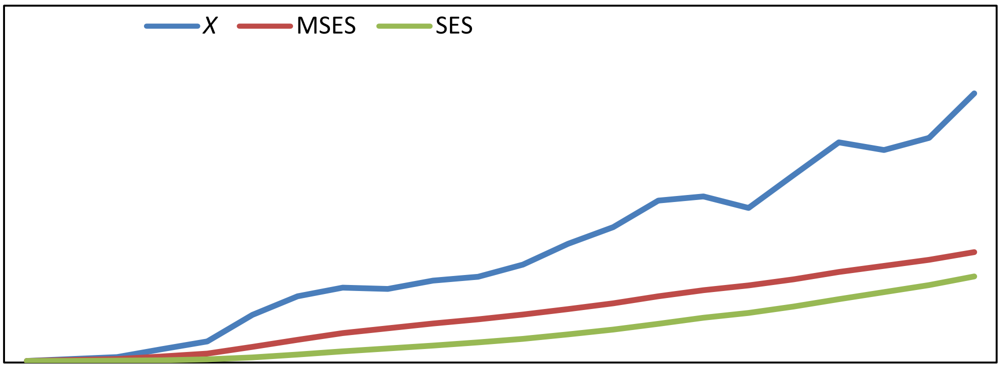

Figure 6.15 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 5$, $M = 50{,}000$

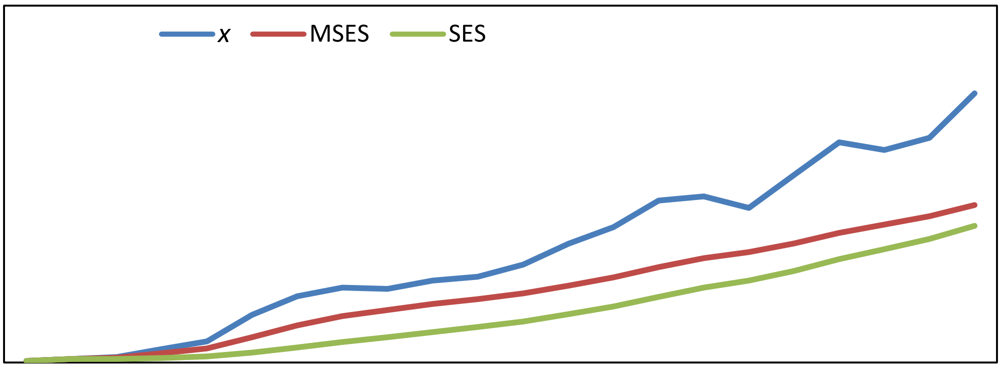

Figure 6.16 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 5$, $M = 50{,}000$

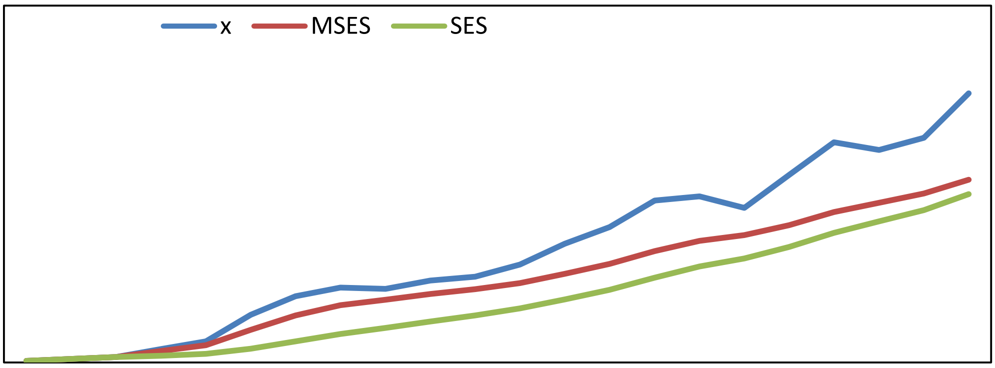

Figure 6.17 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 5$, $M = 50{,}000$

Table 6.7 Smoothed values for, $l = 5$ and $M = 50{,}000$

		S(X)					
		***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m* = 3 and *α* = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.67	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.33	10,152.89	25,950.00	14,781.82
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.08	38,814.00	19,228.39
6	107,006.00	34,749.33	11,813.01	56,559.33	22,035.17	72,910.00	31,198.06
7	148,563.00	51,008.43	18,028.92	82,846.10	33,537.70	105,332.70	47,202.37
8	167,759.00	65,602.25	24,834.83	104,074.30	45,739.63	128,742.60	63,641.91
9	165,097.00	76,657.22	31,210.39	117,634.90	565,90.30	140,860.70	77,476.70
10	183,759.00	87,367.40	38,144.41	130,859.70	68,151.09	153,730.20	91,969.74
11	192,485.00	96,923.55	45,159.90	142,064.30	79,454.18	164,299.70	105,676.40
12	219,611.00	107,147.50	53,089.49	154,988.80	92,195.71	178,127.50	121,212.90
13	266,229.00	119,384.50	62,777.65	172,102.70	108,016.90	198,458.60	140,987.80
14	303,227.00	132,516.10	73,707.17	190,834.70	125,763.30	220,909.00	163,111.40
15	363,096.00	147,888.10	86,861.20	213,802.90	147,339.00	249,346.40	190,382.00
16	372,681.00	161,937.70	99,853.01	233,662.60	167,824.60	272,471.60	215,240.90
17	346,245.00	172,779.30	111,052.60	246,907.60	184,044.70	285,490.50	233,105.10
18	420,333.00	186,532.30	125,110.80	266,177.10	205,525.40	307,964.20	258,636.20
19	493,826.00	202,705.60	141,870.60	290,140.20	231,734.60	337,310.80	290,707.50
20	476,751.00	216,407.90	157,092.50	308,801.20	254,008.80	358,226.80	316,077.10
21	503,627.00	230,085.00	172,844.00	327,356.10	276,701.30	378,998.30	341,652.10
22	603,400.00	247,053.90	192,414.80	352,451.00	306,401.20	409,598.50	377,345.00

When the structural break is in the 10'th point of the series and at the 50,000 of magnitude, smoothed values are calculated with both of methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.18, m = 2 and α = 2/22 in the Figure 6.19, m = 3 and α=3/22 in the Figure 6.20; and in the Table 6.8.

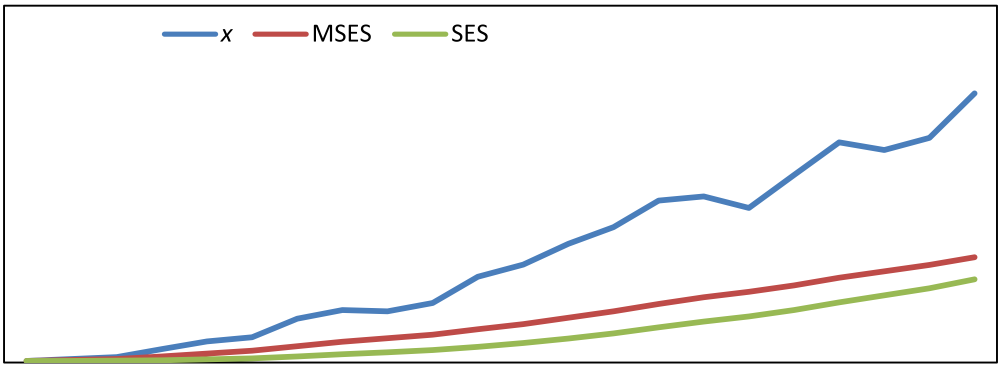

Figure 6.18 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 10$, $M = 50{,}000$

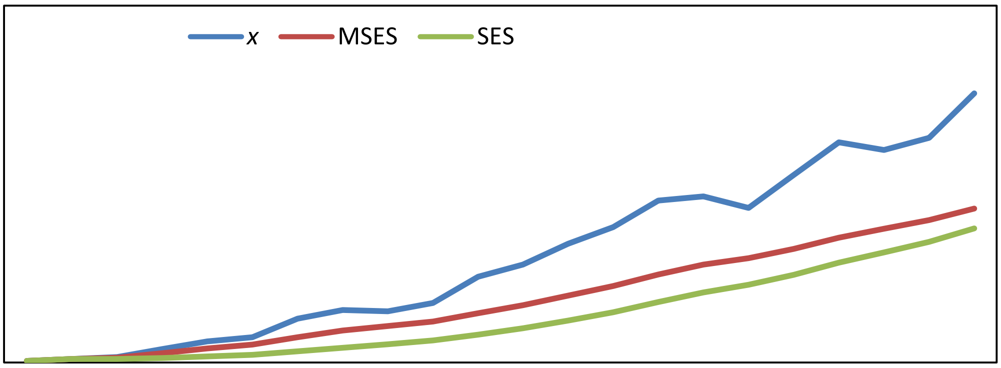

Figure 6.19 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 10$, $M = 50{,}000$

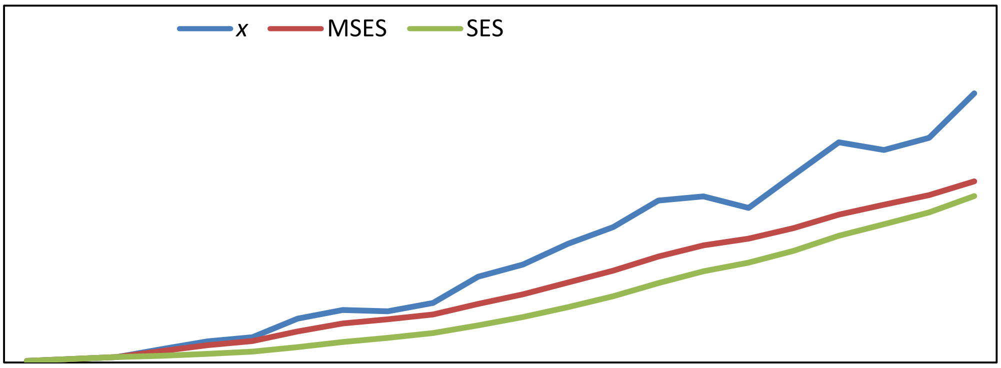

Figure 6.20 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 10$, $M = 50{,}000$

Table 6.8 Smoothed values for, $l = 10$ and $M = 50{,}000$

		S(X)					
		***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m* = 3 and *α* = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.67	8,118,18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.33	10,152.89	25,950.00	14,781.82
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.08	38,814.00	19,228.39
6	57,006.00	26,416.00	9,540.29	39,892.67	17,489.71	47,910.00	24,379.88
7	98,563.00	36,722.71	13,586.77	56,655.62	24,860.01	69,618.43	34,495.76
8	117,759.00	46,852.25	18,321.87	71,931.46	33,305.37	87,671.14	45,849.84
9	115,097.00	54,435.00	22,720.74	81,523.81	40,740.98	96,813.10	55,292.63
10	133,759.00	62,367.40	27,767.94	91,970.84	49,197.16	107,896.90	65,992.59
11	192,485.00	74,196.27	35,255.08	110,246.10	62,223.33	130,966.40	83,241.56
12	219,611.00	86,314.17	43,634.89	128,473.60	765,31.30	153,127.50	101,837.40
13	266,229.00	100,153.80	53,752.80	149,666.80	93,776.54	179,227.90	124,254.40
14	303,227.00	114,659.00	65,092.54	171,603.90	112,817.50	205,799.10	148,659.80
15	363,096.00	131,221.50	78,638.15	197,136.20	135,570.10	237,258.50	177,901.10
16	372,681.00	146,312.70	92,003.74	219,079.30	157,125.60	262,650.20	204,462.00
17	346,245.00	158,073.40	103,560.20	234,040.00	174,318.30	277,402.20	223,796.00
18	420,333.00	172,643.40	117,958.90	254,739.20	196,683.30	301,224.00	250,596.50
19	493,826.00	189,547.70	135,043.80	279,906.20	223,696.20	331,634.90	283,764.20
20	476,751.00	203,907.90	150,575.90	299,590.70	246,701.20	353,402.30	310,080.60
21	503,627.00	218,180.20	166,623.70	319,022.70	270,058.10	374,863.00	336,473.30
22	603,400.00	235,690.20	186,477.20	344,875.20	300,361.90	406,027.10	372,872.40

When the structural break is in the 15'th point of the series and at the 50,000 of magnitude, smoothed values are calculated with both of methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.21, m = 2 and α = 2/22 in the Figure 6.22, m = 3 and α = 3/22 in the Figure 6.23; and in the Table 6.9.

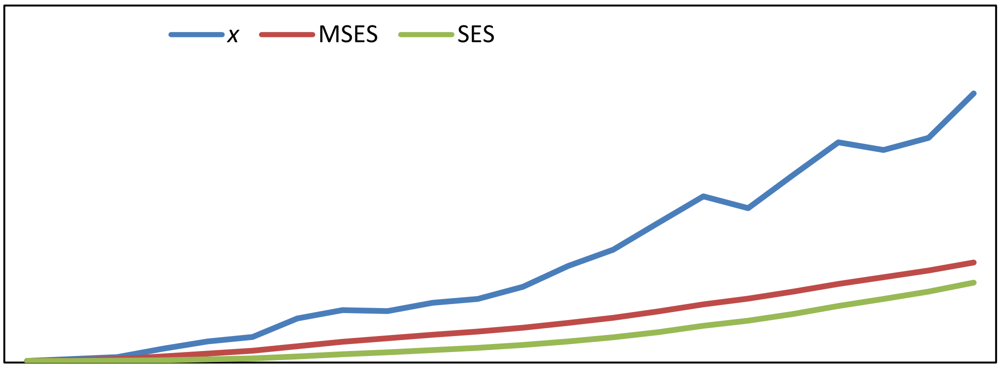

Figure 6.21 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 15$, $M = 50{,}000$

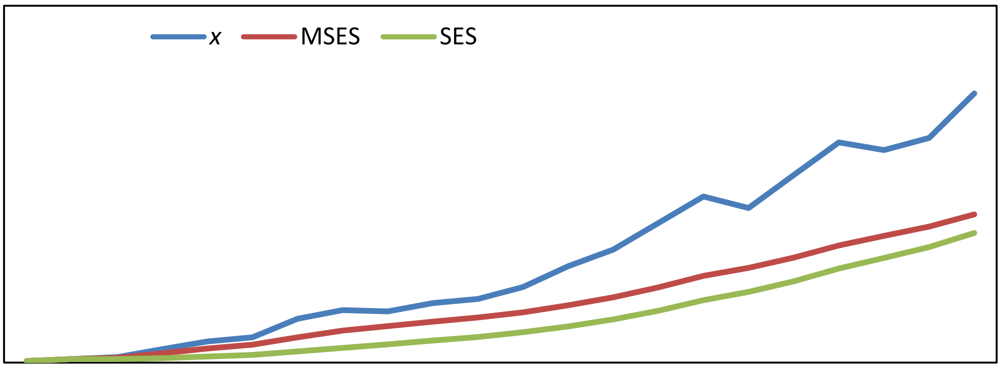

Figure 6.22 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 15$, $M = 50{,}000$

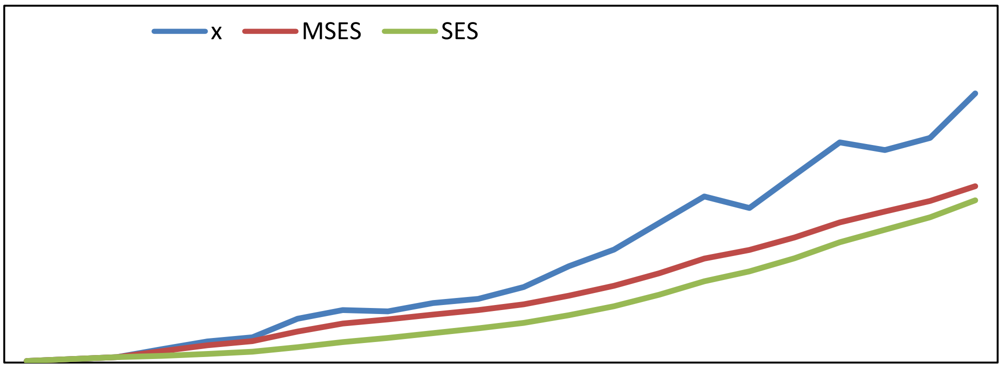

Figure 6.23 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 15$, $M = 50{,}000$

Table 6.9 Smoothed values for, $l = 15$ and $M = 50{,}000$

		S(X)					
		***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m* = 3 and *α* = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.70	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.30	10,152.90	25,950.00	14,781.80
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.10	38,814.00	19,228.40
6	57,006.00	26,416.00	9,540.28	39,892.70	17,489.70	47,910.00	24,379.90
7	98,563.00	36,722.70	13,586.80	56,655.60	24,860.00	69,618.40	34,495.80
8	117,759.00	46,852.30	18,321.90	71,931.50	33,305.40	87,671.10	45,849.80
9	115,097.00	54,435.00	22,720.70	81,523.80	40,741.00	96,813.10	55,292.60
10	133,759.00	62,367.40	27,767.90	91,970.80	49,197.20	107,897.00	65,992.60
11	142,485.00	69,650.80	32,982.30	101,155.00	57,677.90	117,330.00	76,423.40
12	169,611.00	77,980.80	39,192.70	112,565.00	67,853.60	130,400.00	89,130.80
13	216,229.00	88,615.30	47,239.80	128,513.00	81,342.30	150,207.00	106,462.00
14	253,227.00	100,373.00	56,602.90	146,329.00	96,968.20	172,283.00	126,476.00
15	313,096.00	114,555.00	68,261.70	168,565.00	116,616.00	200,445.00	151,924.00
16	372,681.00	130,688.00	82,098.90	194,079.00	139,895.00	232,739.00	182,027.00
17	346,245.00	143,368.00	94,105.60	211,981.00	158,654.00	252,770.00	204,421.00
18	420,333.00	158,755.00	108,934.00	235,131.00	182,443.00	280,697.00	233,863.00
19	493,826.00	176,390.00	126,429.00	262,362.00	210,750.00	314,349.00	269,313.00
20	476,751.00	191,408.00	142,353.00	283,801.00	234,932.00	338,709.00	297,600.00
21	503,627.00	206,275.00	158,774.00	304,737.00	259,359.00	362,269.00	325,694.00
22	603,400.00	224,327.00	178,985.00	331,888.00	290,636.00	395,150.00	363,563.00

When the structural break is in the 5'th point of the series and at the 100,000 of magnitude, smoothed values are calculated with both of methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.24, m = 2 and α = 2/22 in the Figure 6.25, m = 3 and α = 3/22 in the Figure 6.26; and in the Table 6.10.

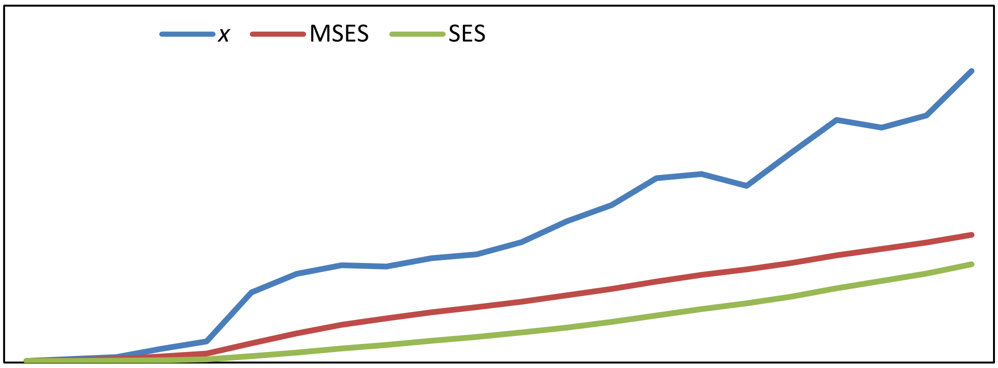

Figure 6.24 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 5$, $M = 100{,}000$

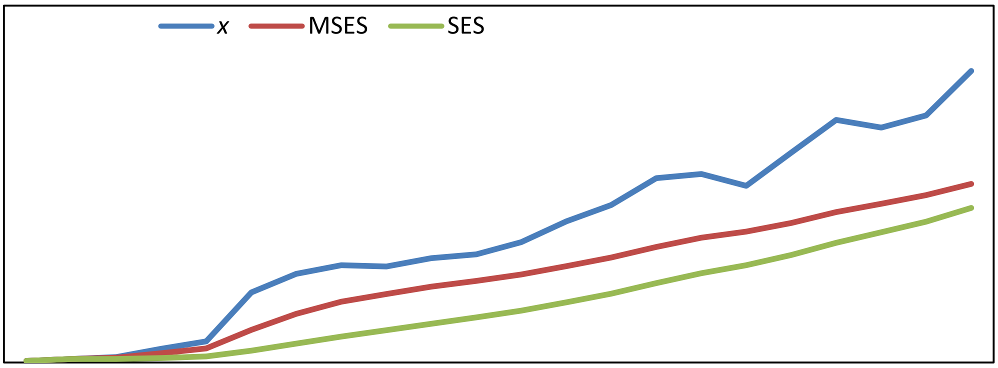

Figure 6.25 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 5$, $M = 100{,}000$

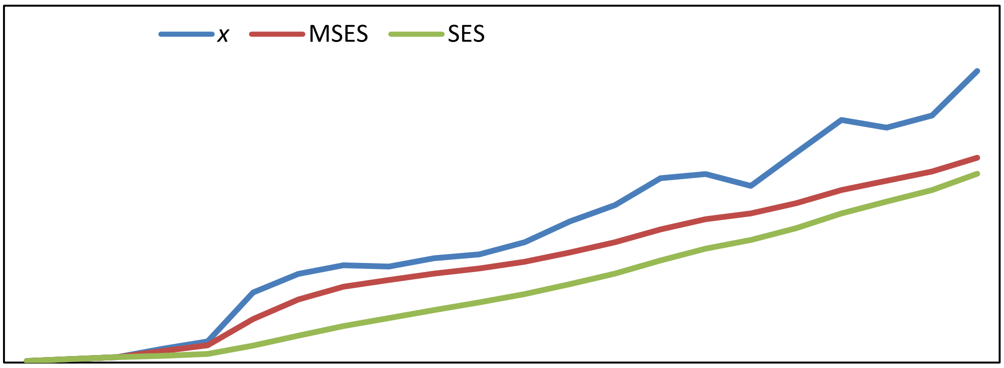

Figure 6.26 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 5$, $M = 100{,}000$

Table 6.10 Smoothed values for, $l = 5$ and $M = 100{,}000$

		S(X)					
		***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m* = 3 and *α* = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.67	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.33	10,152.89	25,950.00	14,781.82
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.08	38,814.00	19,228.39
6	157,006.00	43,082.67	14,085.74	73,226.00	26,580.62	97,910.00	38,016.24
7	198,563.00	65,294.14	22,471.07	109,036.60	42,215.38	141,047.00	59,908.98
8	217,759.00	84,352.25	31,347.79	136,217.20	58,173.89	169,814.00	81,433.99
9	215,097.00	98,879.44	39,700.03	153,746.00	72,439.63	184,908.30	99,660.76
10	233,759.00	112,367.40	48,520.89	169,748.60	87,105.03	199,563.50	117,946.90
11	242,485.00	124,196.30	57,337.44	182,973.40	101,230.50	211,269.40	134,929.40
12	269,611.00	136,314.20	66,986.24	197,413.00	116,537.80	225,854.80	153,295.00
13	316,229.00	150,153.80	78,315.46	215,692.40	134,691.50	246,710.40	175,513.30
14	353,227.00	164,659.00	90,811.44	235,340.20	154,558.40	269,535.40	199,747.00
15	413,096.00	181,221.50	10,5460.70	259,041.00	178,061.80	298,247.50	228,840.00
16	422,681.00	196,312.70	119,879.80	279,496.00	200,299.90	321,578.80	255,272.90
17	396,245.00	208,073.40	132,441.90	293,231.20	218,113.10	334,755.20	274,496.40
18	470,333.00	222,643.40	147,800.60	312,909.10	241,042.20	357,351.50	301,201.40
19	543,826.00	239,547.70	165,801.70	337,216.20	268,568.00	386,794.80	334,286.50
20	526,751.00	253,907.90	182,208.50	356,169.70	292,039.20	407,788.20	360,531.70
21	553,627.00	268,180.20	199,091.20	374,975.10	315,819.90	428,622.40	386,862.90
22	653,400.00	285,690.20	219,741.60	400,286.50	346,509.00	459,273.90	423,208.80

When the structural break is in the 10'th point of the series and at the 100,000 of magnitude, smoothed values are calculated with both of methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.27, m = 2 and α = 2/22 in the Figure 6.28, m = 3 and α = 3/22 in the Figure 6.29; and in the Table 6.11.

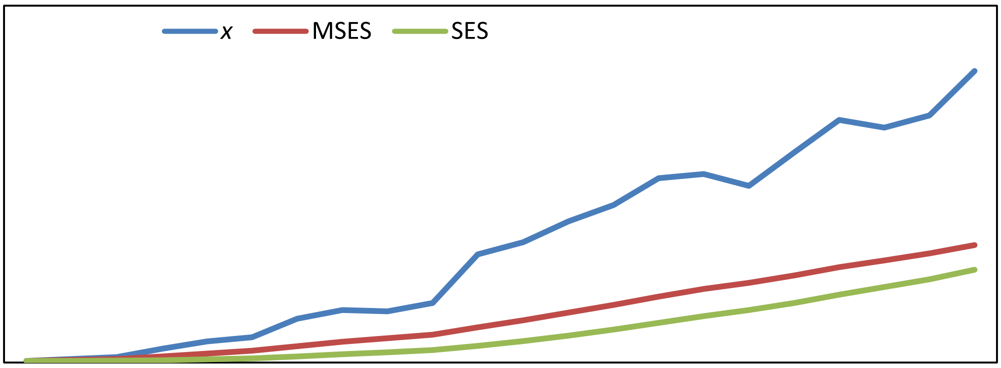

Figure 6.27 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 10$, $M = 100{,}000$

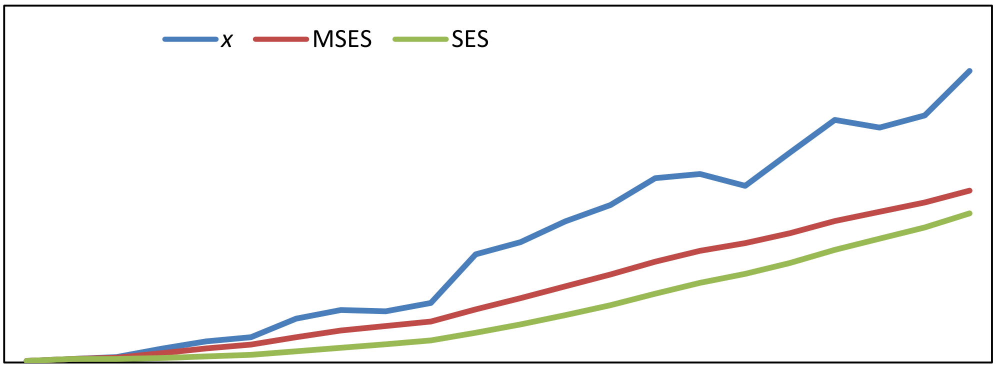

Figure 6.28 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 10$, $M = 100{,}000$

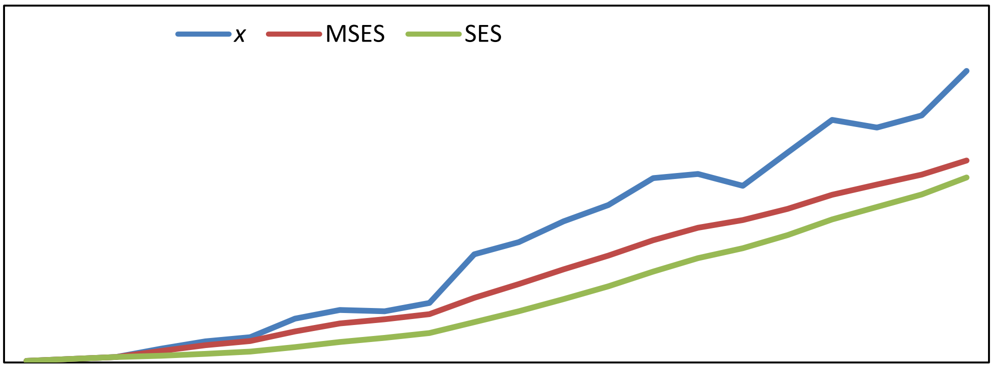

Figure 6.29 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 10$, $M = 100{,}000$

Table 6.11 Smoothed values for, $l = 10$ and $M = 100{,}000$

		S(X)					
		***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m* = 3 and *α* = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.70	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.30	10,152.90	25,950.00	14,781.80
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.10	38,814.00	19,228.40
6	57,006.00	26,416.00	9,540.28	39,892.70	17,489.70	47,910.00	24,379.90
7	98,563.00	36,722.70	13,586.80	56,655.60	24,860.00	69,618.40	34,495.80
8	117,759.00	46,852.30	18,321.90	71,931.50	33,305.40	87,671.10	45,849.80
9	115,097.00	54,435.00	22,720.70	81,523.80	40,741.00	96,813.10	55,292.60
10	133,759.00	62,367.40	27,767.90	91,970.80	49,197.20	107,897.00	65,992.60
11	242,485.00	78,741.70	37,527.80	119,337.00	66,768.80	144,603.00	90,059.70
12	269,611.00	94,647.50	48,077.00	144,383.00	85,209.00	175,855.00	114,544.00
13	316,229.00	111,692.00	60,265.80	170,821.00	106,211.00	208,249.00	142,047.00
14	353,227.00	128,945.00	73,582.20	196,879.00	128,667.00	239,316.00	170,844.00
15	413,096.00	147,888.00	89,014.60	225,708.00	154,524.00	274,072.00	203,878.00
16	422,681.00	165,063.00	104,181.00	250,329.00	178,902.00	301,936.00	233,715.00
17	396,245.00	178,662.00	117,457.00	267,496.00	198,660.00	318,579.00	255,878.00
18	470,333.00	194,866.00	133,497.00	290,033.00	223,358.00	343,871.00	285,122.00
19	543,826.00	213,232.00	152,148.00	316,748.00	252,491.00	375,443.00	320,400.00
20	526,751.00	228,908.00	169,175.00	337,749.00	277,424.00	398,139.00	348,539.00
21	553,627.00	244,371.00	186,651.00	358,308.00	302,533.00	420,352.00	376,505.00
22	653,400.00	262,963.00	207,866.00	385,135.00	334,430.00	452,131.00	414,264.00

When the structural break is in the 15'th point of the series and at the 100,000 of magnitude, smoothed values are calculated with both of methods. The results obtained are shown when the smoothing coefficients are m = 1 and α = 1/22 in the Figure 6.30, m = 2 and α = 2/22 in the Figure 6.31, m = 3 and α = 3/22 in the Figure 6.32; and in the Table 6.12.

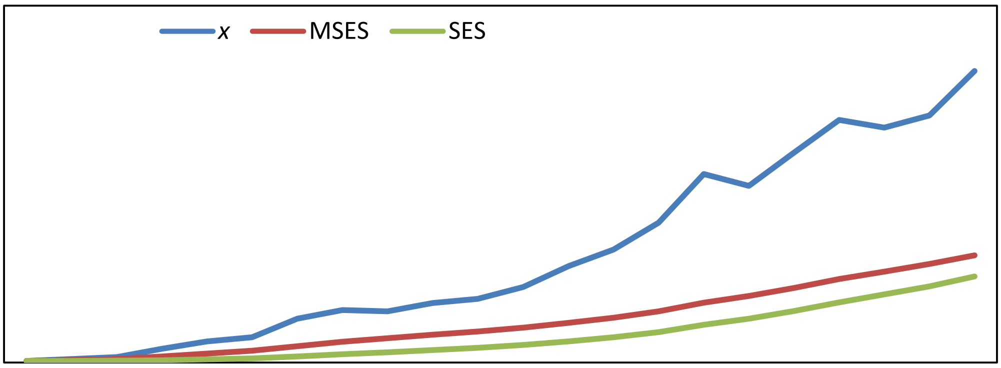

Figure 6.30 Smoothed values of MSES and SES for $m = 1$, $\alpha = 1/22$, $l = 15$, $M = 100{,}000$

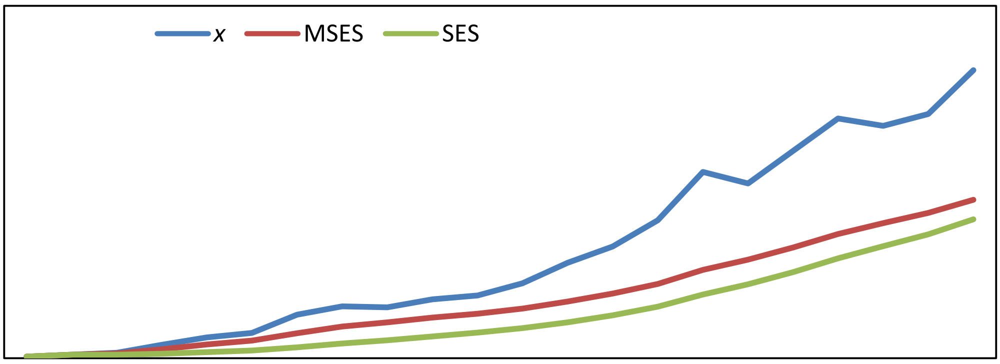

Figure 6.31 Smoothed values of MSES and SES for $m = 2$, $\alpha = 2/22$, $l = 15$, $M = 100{,}000$

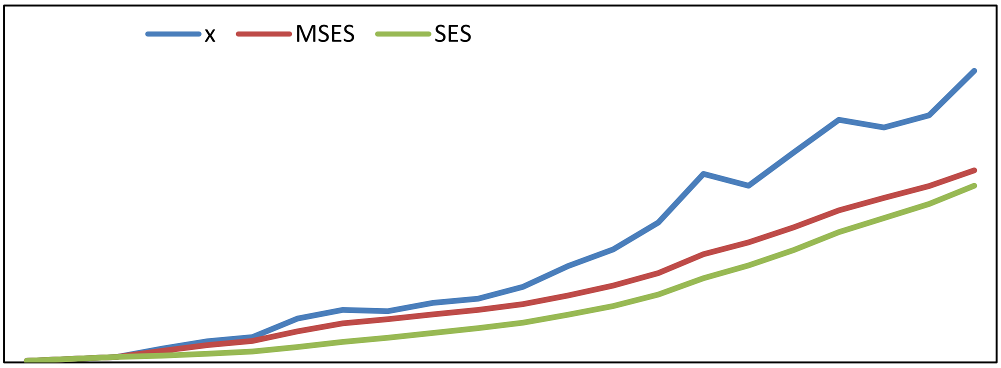

Figure 6.32 Smoothed values of MSES and SES for $m = 3$, $\alpha = 3/22$, $l = 15$, $M = 100{,}000$

Table 6.12 Smoothed values for, $l = 15$ and $M = 100{,}000$

		S(X)					
		***m* = 1 and *α* = 1/22**		***m* = 2 and *α* = 2/22**		***m* = 3 and *α* = 3/22**	
t	***x***	**MSES**	**SES**	**MSES**	**SES**	**MSES**	**SES**
1	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00	3,600.00
2	7,700.00	5,650.00	3,786.36	7,700.00	7,700.00	7,700.00	7,700.00
3	12,300.00	7,866.67	4,173.35	10,766.70	8,118.18	12,300.00	12,300.00
4	30,500.00	13,525.00	5,370.01	20,633.30	10,152.90	25,950.00	14,781.80
5	47,390.00	20,298.00	7,280.01	31,336.00	13,538.10	38,814.00	19,228.40
6	57,006.00	26,416.00	9,540.28	39,892.70	17,489.70	47,910.00	24,379.90
7	98,563.00	36,722.70	13,586.80	56,655.60	24,860.00	69,618.40	34,495.80
8	117,759.00	46,852.30	18,321.90	71,931.50	33,305.40	87,671.10	45,849.80
9	115,097.00	54,435.00	22,720.70	81,523.80	40,741.00	96,813.10	55,292.60
10	133,759.00	62,367.40	27,767.90	91,970.80	49,197.20	107,897.00	65,992.60
11	142,485.00	69,650.80	32,982.30	101,155.00	57,677.90	117,330.00	76,423.40
12	169,611.00	77,980.80	39,192.70	112,565.00	67,853.60	130,400.00	89,130.80
13	216,229.00	88,615.30	47,239.80	128,513.00	81,342.30	150,207.00	106,462.00
14	253,227.00	100,373.00	56,602.90	146,329.00	96,968.20	172,283.00	126,476.00
15	313,096.00	114,555.00	68,261.70	168,565.00	116,616.00	200,445.00	151,924.00
16	422,681.00	133,813.00	84,371.60	200,329.00	144,440.00	242,114.00	188,845.00
17	396,245.00	149,250.00	98,547.70	223,378.00	167,332.00	269,314.00	217,127.00
18	470,333.00	167,088.00	115,447.00	250,818.00	194,877.00	302,817.00	251,655.00
19	543,826.00	186,916.00	134,919.00	281,661.00	226,600.00	340,871.00	291,497.00
20	526,751.00	203,908.00	152,729.00	306,170.00	253,886.00	368,753.00	323,577.00
21	553,627.00	220,561.00	170,952.00	329,737.00	281,135.00	395,164.00	354,947.00
22	653,400.00	240,236.00	192,881.00	359,161.00	314,978.00	430,378.00	395,645.00

When it is analyzed the YAF 2 data set to which has structural break in level shift regardless of, location and magnitude of break, the *m* increases, it can be seen that the forecasted values approach the actual values as Thus MSES method is better than SES method. Table 6.13 shows the MAPE values of MSES and SES method and also Figure 6.33 illustrates the can parison of MAPE values.

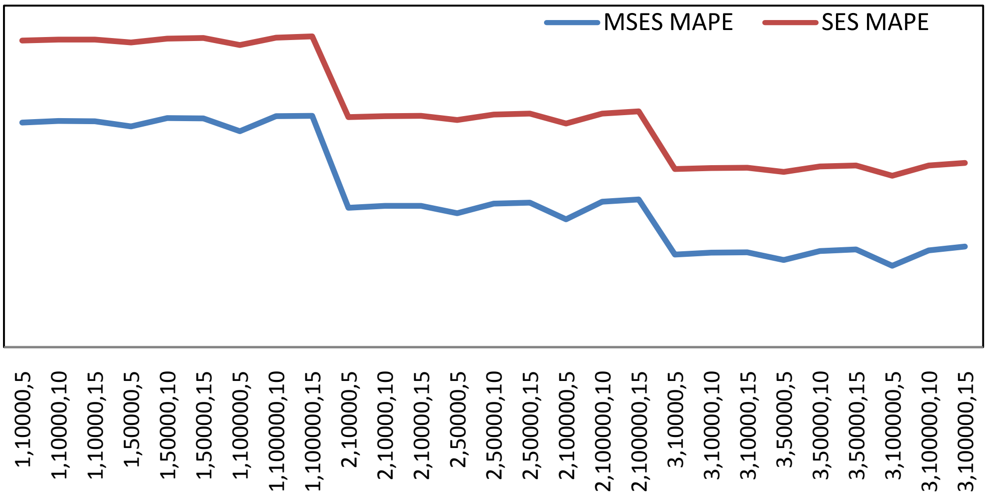

Figure 6.33 MAPE values of MSES and SES methods (*n, l, M)*

Table 6.13 MAPE values of MSES and SES methods

m	*M*	*l*	MSES $_{MAPE}$	SES $_{MAPE}$
1	10,000	5	0.52561628651182	0.718489059177856
		10	0.529565378674772	0.720324904948803
		15	0.529176880973823	0.720511218454235
	50,000	5	0.516537723323662	0.713782713176149
		10	0.536018929123304	0.722850071110798
		15	0.535397317673585	0.724235202632128
	100,000	5	0.505426067952795	0.708024606119696
		10	0.541072603753853	0.72468559959018
		15	0.541628029375497	0.727960133059277
2	10,000	5	0.326406282264917	0.539054835571483
		10	0.331166343318315	0.541544562115934
		15	0.331271596925365	0.541822188635443
	50,000	5	0.313844082273621	0.5324782778501
		10	0.336670521649373	0.545042601153874
		15	0.338515318398693	0.547161036866767
	100,000	5	0.299544841665511	0.524221210338938
		10	0.340442703199855	0.547429998188029
		15	0.345727380166183	0.552476516932086
3	10,000	5	0.217331112582323	0.417548228001375
		10	0.221731007739892	0.420042452973993
		15	0.222296038305481	0.42034003391351
	50,000	5	0.204685284784819	0.410768122959835
		10	0.225248063311727	0.42379404603884
		15	0.229093489903714	0.426177803513881
	100,000	5	0.190950211016458	0.401895411261687
		10	0.227094780852371	0.426197899795799
		15	0.235817375612892	0.431962067362489

In general, when the MAPE values are examined, it is seen that the MAPE values decrease as the smoothing coefficients (*m*) increases. When two methods are compared, the MAPE values of the MSES method are smaller than the MAPE values of the SES method, that is, the MSES method gives better results than SES method.

It have been discoursed about the SES and the MSES in this work up to now and studied the both models in the YAF2 data set.

In this section of our study, "gold-weighted average price" received from the electronic data system of the Central Bank of the Republic of Turkey is compared to both methods. This data is traded on the Istanbul gold market.

The comparison is separated in to three part. The first data set is 300 samples wide between 22nd September 2016 and 30th November 2017, and the break is in the 75’th observation, in other words the first quarter of the series. The break location is tested using the CUSUM test in the R 3.2 program. The test results are given in the Table 6.14, in the Figure 6.34, in the Figure 6.35 and in the Figure 6.36.

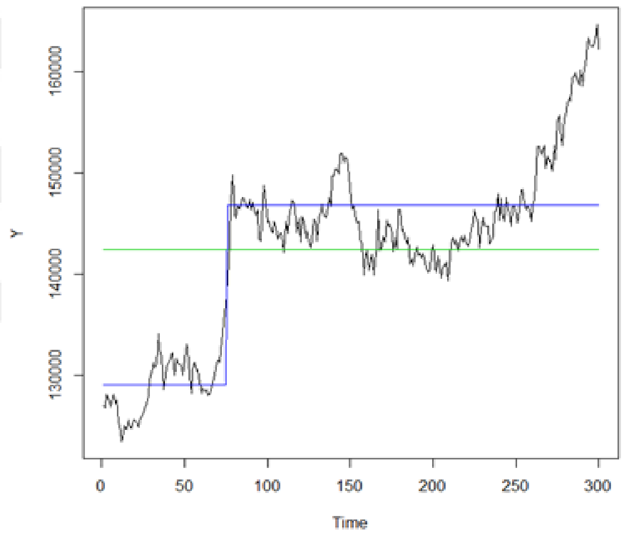

Figure 6.34 Time series plot when the break is in the first quarter

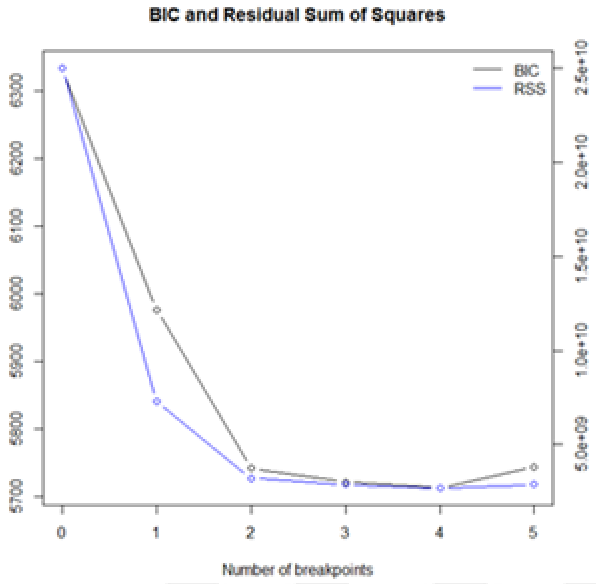

Figure 6.35 BIC and residual sum of squares when the break is in the first quarter

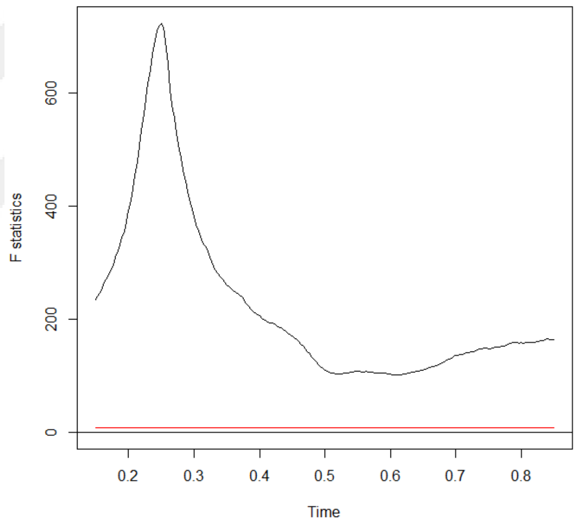

Figure 6.36 F statistics values when the break is in the first quarter

Table 6.14 First data set CUSUM test result

```
         Optimal 2-segment partition:

Call:
breakpoints.Fstats(obj = testkır)

Breakpoints at observation number:
75

Corresponding to breakdates:
0.2466667


        supF test

data:  testkır
sup.F = 722.86, p-value < 2.2e-16
```

According to the result of the MSES analysis with the R.3.2 program, for the first part of series, the value of *m* with the lowest error is 293. That means, the most suitable m value is 293. The MAE values of SES and MSES calculated according to this value and the forecast values calculated by the both methods of the last observation are given in the Table 6.15.

Table 6.15 According to both methods MAE and forecast values for first series

	MAE	**FORECAST *x* = 300**
MSES	1,070.567	162,275.5478
SES	1,066.375	162,275.4413
***x* = 300**		162,219.3

When it is looked at the MAE and the forecast results, the values of the both methods are close to each other when the break is in the first quarter of the series.

The second data set is 450 samples wide between 11th February 2016 and 30th November 2017, and the break is in the 223th observation, in the middle of the series. The break point was tested using the CUSUM test. The test results are given in the Table 6.16, in the Figure 6.37, in the Figure 6.38 and in the Figure 6.39.

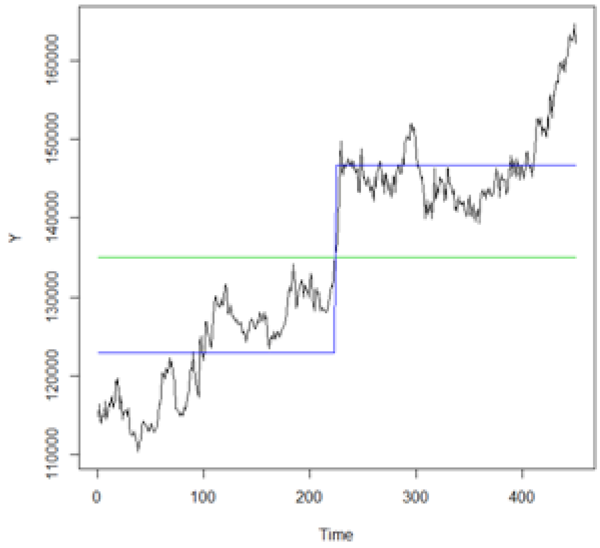

Figure 6.37 Time series plot when the break is in the middle

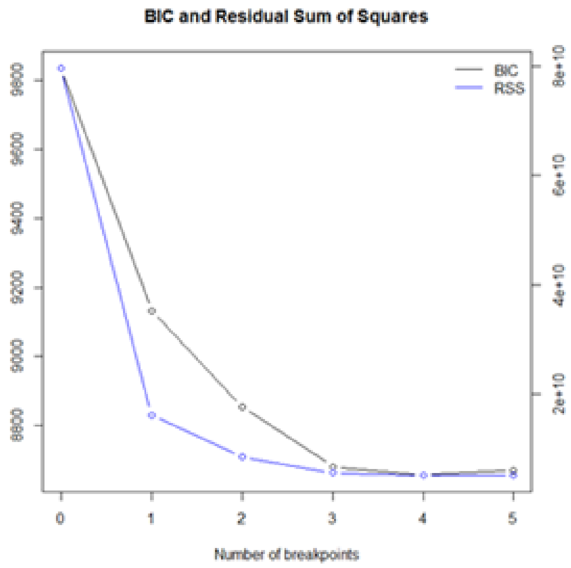

Figure 6.38 BIC and the residual sum of squares when the break is in the middle

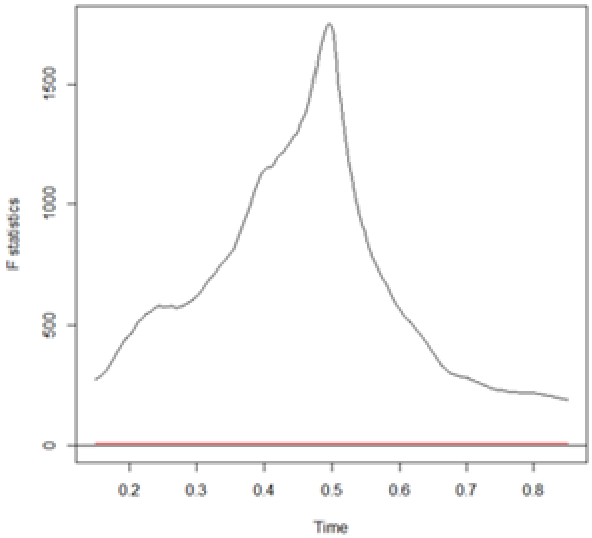

Figure 6.39 F statistics values when the break is in the middle

Table 6.16 Second data set CUSUM test result

```
         Optimal 2-segment partition:

Call:
breakpoints.Fstats(obj = testkır)

Breakpoints at observation number:
223

Corresponding to breakdates:
0.4933333


        supF test

data:  testkır
sup.F = 1753.5, p-value < 2.2e-16
```

When it is looked at the result of the MSES analysis, in the second series, the value of *m* with the lowest error is 245. So, the best fit *m* value is 245. The MAE values of SES and MSES calculated according to this value and the forecast values calculated by both methods of last observation are given in the Table 6.17.

Table 6.17 According to both methods MAE and forecast values for second series

	MAE	FORECAST $x = 450$
MSES	997.1656	162,974.734
SES	1,080.965	162,973.193
$x = 450$		162,219.33

When it is examined the results of the Table 6.17 above, it is understood that the MSES gives better results when it is in the middle of the break series.

The third data set is 600 samples wide between 17th April 2014 and 21th September 2016, and the break is in the 447'th observation, so in the third quarter of the series. The break point was tested using the CUSUM test. Test results are given in the Table 6.18, in the Figure 6.40, in the Figure 6.41 and in the Figure 6.42.

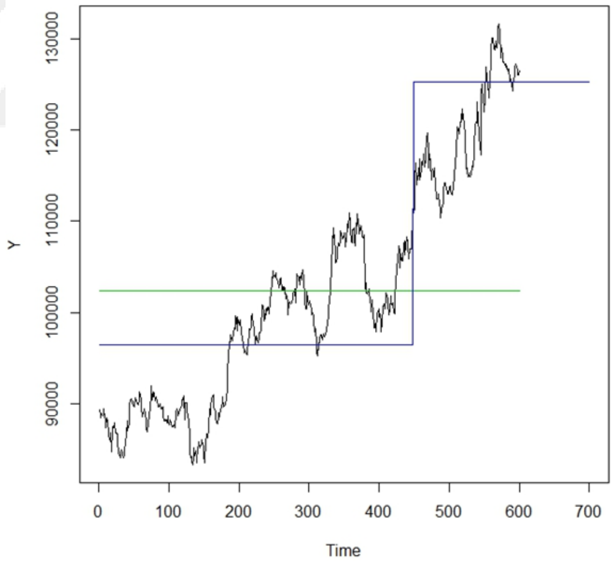

Figure 6.40 Time series plot when the break is in the third quarter

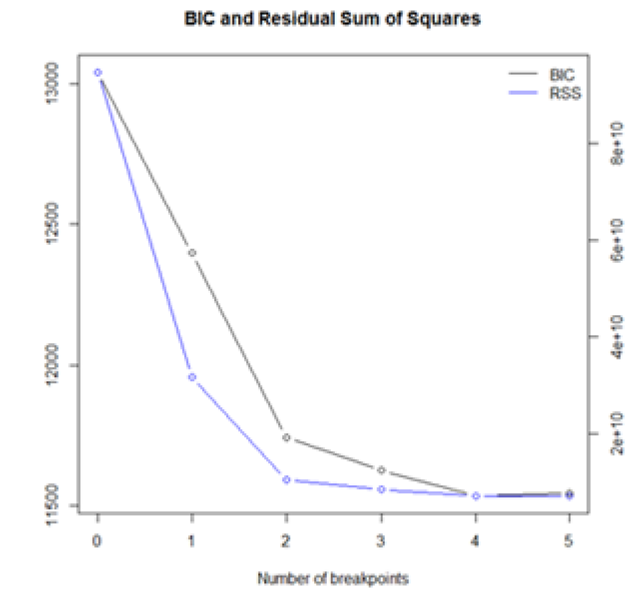

Figure 6.41 BIC and residual sum of squares when the break is in the third quarter

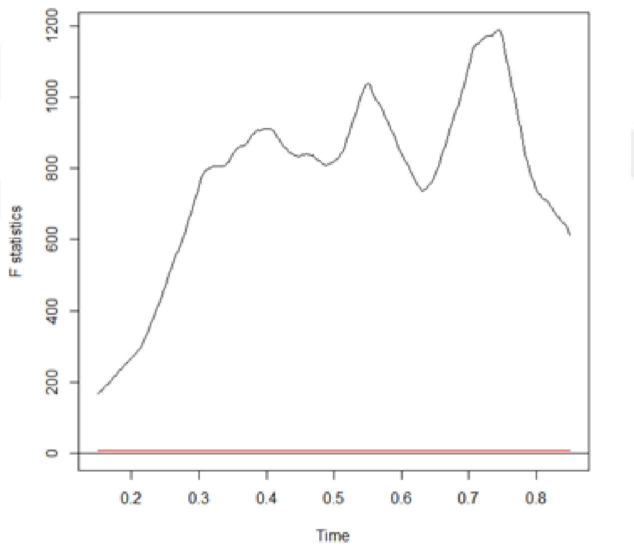

Figure 6.42 F statistics values when the break is in the third quarter

Table 6.18 For the third data set CUSUM test result

```
	 Optimal 2-segment partition: 

Call:
breakpoints.Fstats(obj = testkır)

Breakpoints at observation number:
447 

Corresponding to breakdates:
0.7433333 


	supF test

data:  testkır
sup.F = 1189, p-value < 2.2e-16
```

According to the result of the MSES analysis, in the third series, the value of *m* with the lowest error is 597. So, the most suitable *m* value is 597. The MAE values of SES and MSES calculated according to this value and the forecast values calculated by the both methods of the last observation are given in the Table 6.19.

Table 6.19 According to both methods MAE and forecast values for first series

	MAE	**FORECAST x = 600**
MSES	723.313	126,466.45
SES	722.2466	126,466.54
x= 600		126,468.6

When it is investigated the Table 6.19, it is seen that the MAE values obtained by both methods are very close to each other.

When the SES and MSES analysis results of the three data sets are examined, it is seen that the results of both methods are very close to each other when the break is at the beginning and end of the series, but the MSES method gives better results when the break is in the middle of the series.

CHAPTER SEVEN
CONCLUSION

In this study, it is expressed that the predictions made by time series analysis which shed light on many areas such as industry, economy, science and policy, helps to give right decisions. Various methods have been mentioned using literature researches.

For time series analysis, it is mentioned firstly that the components of the series should be examined (trend, seasonality, cyclicity or random). It has been explained that it should be determined by some tests whether there are sudden breaks in the series due to reason of earthquake, economic crisis and war etc.

After examining the structure of the series, it is told which forecasting methods will be used. In this study; the MSES method developed by Yapar (2016) was compared with the SES method, which is easy, understandable and gives more weight to the last observations. In the first analysis, the time series where the break point is located at the beginning, middle and at the end of the series, with sample size 20,50 and 200 and with a break magnitude of 1, 5 and 10 were obtained by using Monte- Carlo simulation technique. As a result of the analysis made with this time series;

1) Regardless of the sample size and location of the break, the MSES and SES methods give similar results when the breaking magnitude is 1.

2) In the case of three breaks, the MSES method is more accurate than the SES method if the break occurs in the beginning of the series.

3) MSES and SES methods give very close results, if break is at the end of the series at all break magnitude and sample size.

As a second analysis, in the YAF2 data set from the 1001 series of M. Competition; breakings magnitude of 10,000, 50,000 and 100,000 were added to the 5th, 10th and 15th observations. As a result of the analysis with the new data set;

1) In both methods, MAPE values decreased as the sample size increased and MAPE values increased when the breaking was at the end of the series;

2) In all sample size, breaking magnitude and breaking points; it is seen that the MAPE values of the MSES method are lower than the SES method that is the MSES method gives more accurate results.

Finally, in order to see the effectiveness of the methods on the real data, both methods were compared by using the weight-average price of gold values that were traded in the IAB. The first data set consists of 300 observations between 22nd September 2016 and 30th November 2017. According to the analysis obtained by program R. 3.2, it is seen that the breaks occurred in the first quarter of the series, and the MSES and SES method gave very close results. The second data set consists of 450 observations between 11th February 2016 and 30th November 2017. In this series, it was determined that the break was in the 223rd observation. The MSES method had a lower MAE value than the SES method that is, if the breaking is in the middle of the series; it is understood that the MSES method is better than the SES method. The last data set consists of 600 samples between 17th April 2014 and 21th September 2016, and the break in the series is in the 447th observations, that is; in the last quarter of the series. In this data set, very similar MAE values were calculated in both methods as in the first data set.

Generally, when all the analyzes in this study are interpreted, the choice of the initial value and smoothing constant (*m*) in the MSES method is not left to the user preference. On the contrary to the SES method; the initial value and smoothing constant is determined by parametric methods in MSES method. Even though the SES method is known to be easily computable even at large sample sizes; MSES is an easier method than SES due to its lower number of iterations in its calculations. As the value of m moves away from the area of the break, the alpha value is given more weight. SES methods works better than MSES method when alpha value increases. For efficient results of the MSES method, the value of m should be close to the break point.. Since MSES gives more weight to the recent data, the average age of data of MSES are lower compared to that of the SES method. This situation shows that MSES works with more recent data. As a result, even if both methods have similar results, the MSES method is preferred since it is easy and parametric.

In this study, the MSES and SES methods were compared by using data sets which have structural breaks and did not include trend and seasonality, thus contributed to the literature. Similar studies have been carried out in the data sets which include other time series components by Yapar et al.

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