Pre-disaster Relief Item Inventory Planning Under Cooperation of Response Agencies

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,

and that any and all revisions required by the final

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To my family

ABSTRACT

Frequent disasters with devastating consequences have been experienced worldwide within the last decades. In the preparedness phase of disaster management, relief inventory pre-positioning plays a vital role to achieve successful delivery of relief aid. In this study, we analyze two cooperation scenarios between two relief agencies which aim to optimize their pre-disaster stocking decisions by means of a newsvendor-based quantitative model. In the first scenario, both of the organizations agree to share their residual inventory and also decide their stocking levels together to optimize their joint efforts. In the second cooperation scenario, they agree to share their residual inventory, but they give their stocking decisions independently and try to optimize ”returns” from their own aid efforts. We obtain the unique optimal inventory levels under Centralized Model (CM) and the unique Nash Equilibrium under Decentralized Model (DM) analytically. We also conduct extensive numerical analyses to get insights about the importance of cooperative efforts by introducing the term ”price of anarchy” into the humanitarian operations literature stream. Moreover, relief items may not reach the other organization intact or the arrival time of the aid may be late. Therefore, the organization that faces stock-out after demand realization can not use all the items which come from the other organization. As another novel contribution, we introduce the ratio of usable relief items as a ”reliability factor” and the existence of this factor changes the analytical findings completely. We believe that our study leads to useful insights for humanitarian organizations which may ignore the importance of horizontal cooperation and doubt the resulting opportunities.

ÖZETÇE

Son yıllarda dünya çapında yıkıcı sonuçlara neden olan büyük felaketler yaşanmıştır. Afet yönetiminin hazırlık aşamasında, yardım malzemesi envanterinin ön pozisyonlandırılması afet sonrası yardımların başarılı bir şekilde yerine ulaştırılması için hayati bir rol oynamaktadır. Bu çalışmada, afet öncesi stok kararlarını gazete bayi (newsvendor-based) bazlı bir kantitatif model ile optimize etmeyi amaçlayan iki yardım kurumu arasındaki iki işbirliği senaryosunu analiz ediyoruz. İlk işbirliği senaryosunda, her iki kuruluş da afet sonrası kalan envanterlerini paylaşmayı kabul ediyor ve ortak çabalarını optimize etmek için stok seviyelerine birlikte karar veriyor. İkinci işbirliği senaryosunda, afet sonrası kalan envanterlerini paylaşmayı kabul ediyorlar, ancak stok kararlarını bağımsız olarak veriyorlar ve kendi yardım çabalarını optimize etmeye çalışıyorlar. Merkezi model (CM) altında benzersiz optimum envanter seviyelerini ve merkezi olmayan model (DM) altında benzersiz Nash Dengesini analitik olarak elde ediyoruz. Ayrıca, anarşi fiyatı (price of anarchy) terimini insani yardım literatürüne ekleyerek işbirlikçi çabaların önemi hakkında bilgi edinmek için kapsamlı sayısal analizler yapıyoruz. Öte yandan, yardım öğeleri diğer kuruluşlara eksiksiz olarak ulaşamayabilir veya yardımın varış zamanı gecikebilir. Bu nedenle, talep gerçekleştikten sonra stok yetersizliği ile karşı karşıya kalan kuruluş, diğer kuruluştan gelen tüm öğeleri kullanamayabilir. Bir diğer yeni katkı olarak, kullanılabilir yardım malzemelerinin oranını "güvenilirlik faktörü" olarak tanımlıyoruz ve bu faktörün varlığı analitik bulguları tamamen değiştiriyor. Çalışmamızın, yatay işbirliğinin önemini göz ardı eden ya da olası fırsatlardan şüphe edebilecek insani yardım kuruluşları için yararlı içgörülere yol açtığına inanıyoruz.

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NOMENCLATURE

Chapter 1

INTRODUCTION

Earthquakes, volcanic activities, floods, storms, cyclones, and tsunamis are examples of natural disasters that cause an unignorable number of deaths, injuries, displacements and direct economic losses almost every hour all around the world [18]. Natural disasters possess an incredible range of magnitudes and impacts where major ones cause widespread destruction and affect large numbers of people across an extensive geographic area, such as the Japan, Haiti, Indonesian earthquakes, and the Hurricane Katrina. According to a recent report [39], natural disasters killed 1.3 million people and left around 4.4 billion affected people in need of emergency assistance during the last twenty years. The report divides natural disasters into two types: climate-related disasters and geophysical disasters. Furthermore, it remarks that the majority of fatalities were due to geophysical events such as tsunamis and earthquakes. Direct economic losses due to natural disasters almost tripled in the last twenty years and amounted to almost $3 trillion. Moreover, man-made disasters such as nuclear accidents, terrorist attacks, and civil wars increase the number of victims and the need for relief aid. For example, the Syrian Civil War and the ISIS terrorist group has caused the world′s largest humanitarian crisis in the 21st century. More than 200,000 people were killed, and more than 6 million people were displaced because of this conflict [16]. All these growing statistics put forward significant and increasing needs for effective humanitarian management and especially humanitarian supply chain management, which has started to receive the attention it deserves by OR/MS scholars within the last couple of decades. ([42], [41], and [6])

According to the Comprehensive Emergency Management concept introduced in

1978, disaster management can be grouped under four programmatic phases: mitigation, preparedness, response, and recovery [1]. Mitigation is defined as activities that eliminate or lessen the probability of occurrence or the effect of a disaster situation. Preparedness activities prepare societies for an emergency before it occurs. The response includes the actions taken immediately before, during and after a disaster occurs to save lives, and the social, economic and political structures of the communities. Providing relief to people affected by large-scale emergencies like natural disasters is also the main objective of disaster response in humanitarian relief [4]. Recovery refers to both short-term and long-term activities to return life to normal. Disaster response is a complicated phase of the disaster management cycle due to the uncertainty of the time, magnitude and probable effects of a disaster. Distributing relief aid is one of the critical response activities along with intelligence gathering, needs assessment, debris clearance, temporary shelter and medical post location, casualty transportation, and evacuation.

As stated in the previous paragraph, preparedness is one of the four phases of disaster management. In this phase, relief inventory pre-positioning plays a vital role to achieve successful delivery of relief aid. From a perspective of a humanitarian organization, while keeping cash is easier, it carries the risk of not being able to respond to the disaster fast enough, and it forces the organization to stock relief items to use them in case of need. Since having a large number of stockpile costs too much, finding the proper level of inventory is always one of the main challenges for humanitarian aid organizations. Furthermore, although inventory management for commercial supply chains is well-established, the mathematical models and insights introduced for commercial supply chains may not reflect the unique characteristics of disaster settings. Therefore, it can be concluded that inventory models which focus on disaster management have been relatively limited in number and scope.

There are countless counterparts of humanitarian relief efforts such as international relief organizations, governments, local and regional humanitarian organizations, individual benefactors and some for-profit corporations. All of these organizations have their own aims and agendas and they are generally conflicting with each other. For example, international relief agencies may seek to help a maximum number of victims, on the other hand, some non-for-profit organizations may seek to get maximum media attention by acting selectively when deciding to respond to which major disaster as international outside support. Including conflicting objectives, many elements such as insufficient relief materials and chaotic post-disaster environment make coordination between actors difficult. Moreover, relief organizations often fail to make the effort, or simply find it too difficult to collaborate [19]. Coordination between humanitarian agencies can significantly increase the efficiency of aid efforts and many recent studies acknowledge this fact ([32], [37], and [41]). Coordination between humanitarian organizations can take place in supply acquisition, pre-positioning, fundraising, and transportation. Although the importance of cooperation between relief agencies has started to receive attention, there are only a few quantitative analyses which focus on cooperative relief pre-positioning efforts and our work intends to fill this research gap. One of the reasons for the gap in the literature is that the chaotic nature of the post-disaster environment is one of the main barriers to effective coordination. Since the inventory decisions are made before the sudden-onset disasters and there is no time limitation for pre-positioning, creating efficient coordination mechanisms in inventory management between humanitarian organizations is promising from an academic perspective and viable in practice.

In this study, we analyze two cooperation scenarios between two relief agencies which aim to optimize their stocking decisions by means of a newsvendor-based quantitative model. In the first scenario, both of the organizations agree to share their residual inventory and also decide their stocking levels together to optimize their joint efforts. In the second cooperation scenario, they agree to share their residual inventory, but they give their stocking decisions independently and try to optimize "returns" from their own aid efforts. We obtain the optimal inventory levels analytically under both cooperation scenarios and conduct extensive numerical analyses to get insights about the importance of cooperative efforts by introducing the term "price of anarchy" into the humanitarian operations literature stream. Moreover, relief items may not reach the other organization intact or the arrival time of the aid may be late. Therefore, the organization that faces stock-out after demand realization can not use all the items which come from the other organization. As another novel contribution, we introduce the ratio of usable relief items as a "reliability factor" and the existence of this factor changes the analytical findings completely. We believe that our study leads to useful insights for humanitarian organizations which may ignore the importance of horizontal cooperation and doubt the resulting opportunities.

Chapter 2

LITERATURE REVIEW

In this chapter, we first review studies that propose inventory management models in pre-disaster planning. Then, we briefly review the relevant literature on the game-theoretic newsvendor models in supply chain management, focusing on the horizontal competition and cooperation models.

2.1 Inventory management in disaster management

Frequent disasters with devastating consequences have been experienced worldwide within the last decades. The annual average number of natural disasters between 2003 and 2014 was 384 and about 200 million people were affected from these disasters worldwide every year [21]. For instance, in the Wenchuan earthquake (8.0 Ms) more than 3.3 million tents were demanded, while in the Yaan earthquake (7.0 Ms) about 60 thousand tents were in need for the affected people [11]. The lives and well-being of thousands of people affected by a natural or man-made disaster depend on the humanitarian aid they will receive in a short time. It is not surprising that the effectiveness and efficiency of humanitarian logistics operations rely on the successful management of relief inventory [6]. The importance of pre-disaster stocking decisions for relief items has been emphasized frequently in disaster management literature. The literature of pre-positioning of the relief items can be divided into two streams according to how they incorporate the uncertainty of disaster. They are the pre-positioning models with stochastic programming and the inventory models with newsvendor approach. Balcik et al. [6] and Sabbaghtorkan et al. [31] are extensive literature review papers which focused on inventory management in humanitarian supply chains and these studies were discussed in detail.

2.1.1 Pre-positioning models with stochastic programming

Beamon and Kotleba [7] developed one of the earliest inventory models in humanitarian management. They claimed that humanitarian emergency operations can continue for many years and obtained optimal order quantities by assuming a continuous demand approximation. As an extension of [7], The authors introduced and compared three different inventory management strategies: nave, heuristic and mathematical models as conducted to the complex emergency in Sudan, Africa [8]. Taskin and Lodree [36] explored a multi-period stochastic inventory problem for profit-driven firms that are facing difficulties to estimate surge demand and to optimize stocking decisions in hurricane disasters. They assumed that the inventory manager knows the demand distribution of the pre-hurricane season. However, a Markov chain model was suggested to estimate hurricane landfall count rates which are used to define the demand distribution of the hurricane season. In real-world applications, the suggested stochastic programming approach becomes ineffective because of the computation challenges and they implemented scenario reduction techniques to find the optimal set of scenarios to represent the underlying distribution.

Rawls and Turnquist [30] formulated stochastic mixed integer program which minimizes the expected total cost of relief efforts for hurricanes. Their model considers the possibility that the stocked relief items may be destroyed, and some parts of the transportation network may be damaged during the hurricane. Since two-stage stochastic programming approach performs poorly at large-scale problems, they introduced a heuristic algorithm which is called as Lagrangian L-shaped method (LLSM) and showed that their heuristic algorithm consistently finds solutions with minimal optimality gaps. They, finally, presented a realistic case study which is focusing the disaster threats in the Gulf Coast of the United States. Duran et al. [15] developed a two-stage stochastic programming model to minimize the expected average response time to sudden-onset disasters. They collaborated with CARE to improve CAREs overall relief aid emergency performance. They obtained the historical data for relief aid from the International Disaster Database to estimate the demand for a given period. They identified best candidate locations for pre-positioning warehouses by cooperating with CARE. They run their models with many instances and concluded that the best strategy for CARE is the implementation of gradual network expansion. Their model showed that the potential benefits of each potential warehouses. Thanks to their work and by cooperating with other relief agencies, CARE established its first warehouses in Dubai. Rabbani et al. [28] developed two mathematical models to optimize the pre-disaster and post-disaster phases of humanitarian operations. They, first, introduced a two-stage stochastic programming model which incorporates the perishability of relief items. In this model, they optimized the warehouse locations, stocking decisions for each warehouse, and replenishment schedules for perishable relief items. Secondly, they proposed a continuous review (Q,r) model to optimize post-disaster inventory policy which incorporates uncertainty by using the fuzzy programming and obtained a sufficient condition for optimal r^* and Q^*. As a unique contribution to the literature, their post-disaster inventory model considers backorder and lost demand simultaneously.

2.1.2 Disaster inventory management with newsvendor-based models

Lodree and Taskin [35] proposed one of the earliest newsvendor-variant models in the humanitarian inventory management literature that accounts for demand uncertainty and disaster risk of a given period. The concept of an insurance premium and the pay-off framework were introduced to quantify the risks and benefit associated with the proactive disruption planning. The authors conducted numerical analysis and sensitivity analysis to compare proposed methods and to gain insights that can guide the relief agencies. Campbell and Jones [10] employed a newsvendor-variant model to determine where to stock relief items and how much to stock relief items at a location. In this setting, if supplies are located closer to the disaster, the delivery time of the items can be shorter but the items may be in a riskier location if the disaster occurs. By considering these two objectives, they found optimal stocking quantity and location from a discrete set of choices.

Purchase of relief supplies immediately after a disaster (instant purchasing) and pre-purchasing relief items are two general methods which are used by humanitarian organizations and they have both their own disadvantages. Purchasing price is high in the instant purchasing approach because of the supply scarcity and pre-purchasing contains a loss because of the demand uncertainty and inventory cost. Wang et al. [40] integrated pre-purchasing relief items with option contract into humanitarian inventory management by considering the system as a relief supply chain to solve these problems. They showed that the proposed mechanism is superior to the two above mentioned methods in terms of the total expected profit. They further showed that the proposed model can coordinate a humanitarian organization and its supplier, and both of them can make higher profits by adopting an option contract model.

Chen, Liang, and Yao [11] developed a generalized two-stage delivery process model to optimize relief material's delivery after a disaster by incorporating social donation and emergency spot purchasing. They introduced a newsvendor-based analytical model that aims to minimize the total costs when all demands incurred by a sudden-onset disaster must be satisfied. This paper also suggests a risk-sharing mechanism, in which a humanitarian organization and its supplier jointly stockpile relief items. They showed that implementing this scheme could increase the joint storage quantity and decrease the total costs simultaneously. As an extension of [11], Chen, Liang, and Yao [12] proposed a newsvendor-based inventory model with more than one relief material which aims to minimize the total expected cost of a single humanitarian organization. The probability of occurrence of a disaster, the demand after a sudden-onset disaster and the uncertainty of the quantity of social donation are considered in their proposed model. They showed the implicit conditions that quantify the optimal stocking decisions when there is no budget constraint. They developed a two-binary iterative approach to solve the model with a budget constraint. The authors further proposed a risk-sharing mechanism between the humanitarian organization and its suppliers which can decrease the expected total cost after a disaster like their previous study [11].

Studies that investigate the cooperation of agencies explicitly in their model are rare. Eftekhar et al. [17] empirically examined the effect of operational performance and media exposure on their individual and institutional fund-raising. They further applied these empirical findings in a stylized mathematical model to characterize the preferred humanitarian coordination mechanisms with respect to the main objective and the funding source of a relief agency. Eftekhar et al. [17] is one of the few empirical papers which studied the intersection of media exposure, social donation and their effects on the strategy of policymakers and provided insights for decision makers on designing effective mechanisms that incentivize coordination between relief agencies. Almost every study in the literature focus on the optimizing the operations of relief agencies under disaster risk. On the other hand, Adida et al. [2] proposed an inventory model to minimize the cost of the group of hospitals which store medical supplies for the possible terrorist attacks or natural disasters. In their model, a group of hospital faces a newsvendor problem and they agree to share the corresponding cost which is resulting from unmet demand. Under this setting, they proved the existence of the Nash Equilibrium and provided insights that can give direction to the public health policy by conducting extensive numerical analysis. Coskun et al. [13] introduced a mathematical model which considers two humanitarian organizations that pre-purchase and stock the same type of relief supplies at different locations. They also prone to individual sudden-onset disaster risks and have an agreement to help each other in case of need after demand realizations. The authors incorporated the disaster risk and the demand uncertainty by using Newsvendor model. They computed a Nash equilibrium solution numerically and showed that relief agencies can decrease their expected cost by helping each other. They further applied their proposed model to the case of Istanbul and they found optimal stocking quantities to Istanbul under various cooperation and earthquake risk scenarios.

Despite the fact that the importance of the cooperation between relief agencies has been addressed frequently ([5], [32], [41]), almost every study in the literature incorporated with single agency inventory models. As a novel contribution, we consider more than a single agency and demonstrate the importance of their cooperation by introducing the investigation of the price of anarchy (PoA) to the humanitarian operations literature. Also, we are able to find analytically tractable centralized and decentralized optimal quantities. Moreover, we conduct numerical analyses to show the effects of the parameters on the PoA which denotes the inefficiency of the system and is calculated the ratio of rewards between coordinated system and decentralized system.

2.2 Game theoretical newsvendor models in inventory management

Game theoretic analysis in inventory management is extensive in terms of both theory and applications. Parlar [26] is one of the earliest studies which combined game theory and inventory management. He modeled an inventory management problem with two newsvendors who make ordering decisions for two substitutable products. It is assumed that players know the demand probability density functions of both products, substitution rates between the products and the other parameter values. In this context, each player's decision affects the other player's pay-off and Parlar used game theory to prove the existence and uniqueness of the Nash equilibrium. Netessine and Rudi [24] considered an extension of Parlar's [26] study with have N products and N retailers. They found analytically tractable solutions that are useful when comparing the centralized and competitive inventory management under substitution with multi-newsvendor.

Anupundi et al. [3] developed a general scheme for the analysis of decentralized distribution systems. They introduced both cooperative and coopetitive frameworks in this problem and developed the conditions for the existence of the core and a pure strategy Nash equilibrium, respectively. They also showed that there exists an allocation mechanism that achieves the centralized solution. Rudi et al. [29] is one of the closest studies to ours. They considered a two-location inventory model with two sellers and the possibility of transshipment of the surplus inventory between the sellers. In case a seller is stocked out, it can buy from the surplus stock of another seller. They analyzed both centralized and decentralized cases and found coordinating transshipment prices to maximize total profit. On the other hand, Hu et al. [23] showed that there does not always exist a unique pair of coordinating transshipment prices in the same problem. They found the necessary conditions to have coordinating prices. Finally, they analyzed the effects of some parameters on the magnitude of coordinating transshipment prices.

Slikker et al. [34] introduced a problem with N retailers which face a newsvendor problem. They analyzed this problem by defining a cooperative game and showed that these general newsvendor games have non-empty cores. This means that every subset of retailers has an incentive to form a coalition. As a continuation of this study, Ozen et al. [25] examined an inventory problem that consists of N newsvendors and M warehouses. They carried out a game-theoretical analysis of a distribution structure with warehouses, in which newsvendors can cooperate to increase their expected profit by reallocating their orders after demand realizations and from coordinating their orders accordingly.

During our modeling and analysis, we benefit from the supply chain literature frequently. Our model can be interpreted as an extension of Rudi et al.'s [29] study. On the other hand, it is clear that humanitarian operations differ from the classical supply chain management and the classical definitions of some parameters are not valid in humanitarian operations such as the price for a product and the penalty cost. We need to define some of the parameters to understand the behavior and the objective of the relief organizations. The main motivation of humanitarian organizations is to protect lives and improve the health condition of affected people as well as facilitating their dignity. In this respect, we introduce a reward parameter instead of the price which can be interpreted as the monetary value of helping one person affected from the disaster. Furthermore, it is known that humanitarian organizations can increase their donations by being active and contributing to relief activities properly. Therefore the reward can be interpreted as the total estimated gain by helping a person. On the other hand, the penalty cost in humanitarian operations can be viewed as the cost of not being able to help the casualties, and it is likely to be high compared to the reward. However, we must note that estimating the reward and the penalty cost is not trivial and it depends on the perspective of each humanitarian organizations. There are some recent works which acknowledge this social cost of not being able to help the casualties and give some methods to estimate the penalty cost ([2], [31], and [22]). Also, there are no transshipment prices in humanitarian operations and the agency which helps the other agency's region can get a reward from this effort. Therefore, the reward function is different than the one modeled in supply chain management studies.

Sometimes, relief items do not reach the other organization intact or the arrival time of the aid is late. Therefore, the organization who faces stock-out after demand realization can not use all the items which come from the other organization. As another novel contribution, we introduced the ratio of usable relief items as a reliability factor and the existence of this factor changes the analysis completely.

Chapter 3

MATHEMATICAL MODEL WITH A DETERMINISTIC RELIABILITY FACTOR

The fundamental framework of this model relies on Rudi et al. [29]. In this chapter, even if we introduce the reliability factor, we followed the same procedure to obtain the theoretical findings with Rudi et al. [29]. Therefore findings in this chapter do not completely belong to us and Rudi et al. [29] is the first study who analyzed the two location inventory model. We consider two nonprofit organizations at distinct regions, indexed *i,j=1,2*. Relief agency i purchases Q_i units of one aid item, such as a shelter unit or a kit of essentials, from one or more suppliers before the disaster at a fixed unit cost $c_i > 0$. (Note that c_i and c_j can be different.) When deciding the number of items to be kept in inventory, the relief agencies do not know the level of demand X_i, but it is assumed that the joint distribution over demand realizations ($X_i \times X_j$) is known to everyone. The probability distribution of demand is assumed to be continuous and differentiable for analytical convenience.

Relief agency i obtains a reward $r_i > c_i$, which is estimated in monetary units, for every unit which is used for help, and if it cannot meet the entire demand, it is penalized $p_i \geq 0$ monetary units for every unit of unmet demand. Here, p_i can be viewed as the cost of not helping the casualties, and it is likely to be high compared to r_i. However, we do not make any assumptions on its comparative value. Also, unused inventory has salvage value $s_i < c_i$. Relief agency can sell unused relief items after demand realizations. We consider a time horizon such as a year and the order is given at the beginning. That is, in this model, relief agencies place their orders simultaneously. After that, demand realizations take place. Both agencies can observe the order quantity of the other agency and the demand realizations of the other region.

If there is a surplus inventory of one relief agency, this agency may help the other agency by sending its units from surplus inventory. The other agency would appeal for help request only if its demand realization exceeds its inventory level.

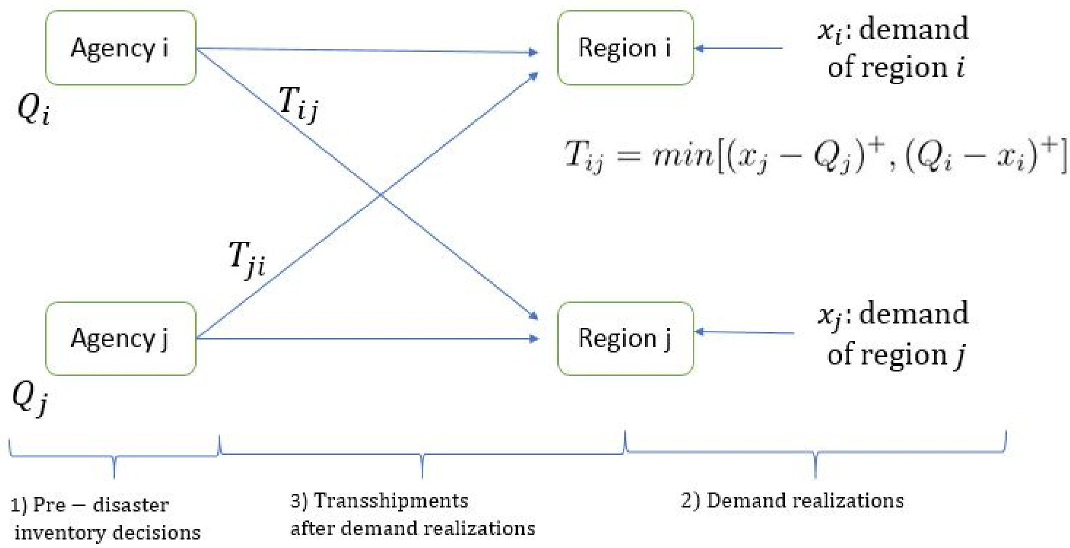


Figure 3.1: The representation of the mathematical model

When relief agency i sends aid to the other agency, it is charged τ_{ij} for every unit of transshipment. In other words, we assume that cost τ_{ij} is incurred by agency i. We further assume that relief agency i gets a reward r_{ij} per unit item which is used in location j. This reward represents the benefit/utility from helping the agency in need. There is also a reliability factor when there is a transshipment between relief agencies i and j. We assume that ρ_{ij} portion of the transshipped items can be used by relief agency j, when agency i helps the casualties at location j. This may be because, for instance, all of the sent units may not arrive on time due to difficulties in transportation or relief aids may be damaged because of the chaotic environment of the post-disaster. To the best of our knowledge, this is the first study which considers above-defined reliability factor. Since the usable amount depends on the transshipment amount, we determine to use the multiplicative model.

Let us define x^+=$max\{x,0\}$. Then, the transshipment from relief agency i to relief agency j, T_{ij}, is given by the smaller of the two values, the excess demand $(X_j - Q_j)^+$ of relief agency j and the surplus $(Q_i - X_i)^+$ at relief agency i. That is,

$$T_{ij} = min[(x_j - Q_j)^+, (Q_i - X_i)^+]$$

Given the transshipments, let us define

$$R_i = min(X_i, Q_i)$$

as the amount of help at location of agency i.

Left-over inventory amount at location i is

$$L_i = (Q_i - X_i - T_{ij})^+$$

and unmet demand is

$$S_i = (X_i - Q_i - \rho_{ji}T_{ji})^+$$

Note that all these expressions depend on Q_i, Q_j, X_i and X_j. Then, the expected reward for relief agency i is given by

$$\pi_i(Q_i, Q_j) = E[r_iR_i + (r_{ij}\rho_{ij} - \tau_{ij})T_{ij} + s_iL_i - p_iS_i] - c_iQ_i$$

where,

Q_i : stocking quantity of agency i

X_i : demand for relief items at location i. x_i is positive random variable.

T_{ij} : transshipment amount from agency i to agency j after demand realizations

$f(x_i, x_j)$: joint probability density function of X_i and X_j

r_i : reward value per item for agency i

s_i : salvage value per item for agency i

c_i : cost per item for agency i

p_i : penalty cost per item for agency i

r_{ij} : reward value for sending aid from agency i to agency j

ρ_{ij} : deterministic portion of the transshipped items can be used by relief agency j

τ_{ij} : transshipment cost per item which transshipped from agency i to agency j

To obtain the open form of the reward functions, let us define the all possible cases and corresponding rewards that agencies may face.

Case 1: $Q_i > X_i$ and $Q_j > X_j$

(Both stocking quantities are more than demand quantities.)

$$\pi_i(Q_i, Q_j) = r_i X_i - c_i Q_i + s_i(Q_i - X_i)$$

Case 2: $Q_i > X_i$, $Q_j < X_j$, and $Q_i + Q_j < X_i + X_j$

(Stocking quantity for agency i is more than demand quantity in region i, stocking quantity for agency j is less than demand quantity in region j, and the total stocking quantity is less than total demand quantity.)

$$\pi_i(Q_i, Q_j) = r_i X_i - c_i Q_i + (\rho_{ij} r_{ij} - \tau_{ij})(Q_i - X_i)$$

Case 3: $Q_i > X_i$, $Q_j < X_j$, and $Q_i + Q_j > X_i + X_j$

(Stocking quantity for agency i is more than demand quantity in region i, stocking quantity for agency j is less than demand quantity in region j, and the total stocking quantity is more than total demand quantity.)

$$\pi_i(Q_i, Q_j) = r_i X_i - c_i Q_i + (\rho_{ij} r_{ij} - \tau_{ij}) + s_i(Q_i + Q_j - X_i - X_j)$$

Case 4: $Q_i < X_i$, $Q_j > X_j$, and $Q_i + Q_j < X_i + X_j$

(Stocking quantity for agency i is less than demand quantity in region i, stocking quantity for agency j is more than demand quantity in region j, and the total stocking quantity is less than total demand quantity.)

$$\pi_i(Q_i, Q_j) = r_iQ_i - c_iQ_i - p_i(X_i - Q_i - \rho_{ji}(Q_j - X_j))$$

Case 5: $Q_i < X_i$, $Q_j > X_j$, and $Q_i + Q_j > X_i + X_j$

(Stocking quantity for agency i is less than demand quantity in region i, stocking quantity for agency j is more than demand quantity in region j, and the total stocking quantity is more than total demand quantity.)

$$\pi_i(Q_i, Q_j) = r_iQ_i - c_iQ_i - p_i(1 - \rho_{ij})(x_i - Q_i)$$

Case 6: $Q_i < X_i$ and $Q_j < X_j$

(Both stocking quantities are less than demand quantites.)

$$\pi_i(Q_i, Q_j) = r_iQ_i - c_iQ_i - p_i(X_i - Q_i)$$

We consider two cooperation scenarios between two relief agencies. In the first scenario, both of the organizations agree to share their residual inventory and also decide their stocking levels together to optimize their joint efforts. In the second cooperation scenario, they agree to share their residual inventory, but they give their stocking decisions independently and try to optimize "returns" from their own aid efforts. First cooperation scenario is called as "Centralized System" and second cooperation scenario is called as "Decentralized System" in the rest of the thesis.

We have the following assumptions to avoid trivial solutions throughout the thesis.

- The assumption, $c_i < c_j + \tau_{ji}$, ensures that it is not beneficial to buy another location.

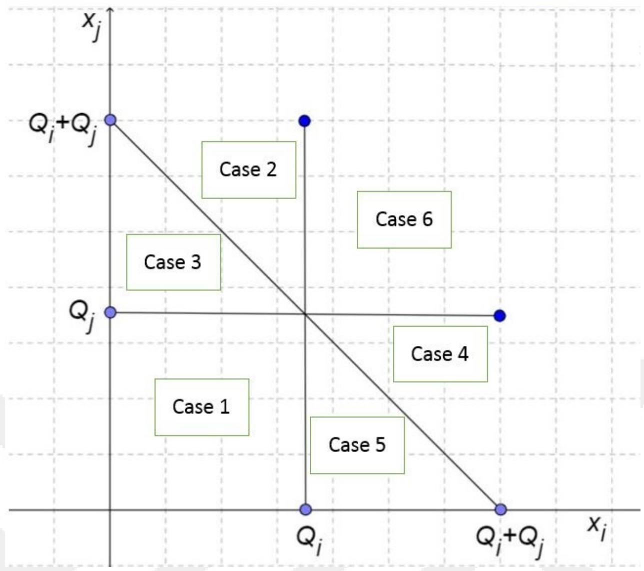


Figure 3.2: The representation of the cases

- The assumption, $s_i - \tau_{ij} < s_j$, ensures that it is not beneficial to salvage another location.
- The assumption, $\rho_{ij}r_{ij} + p_i - \tau_{ij} < r_j + p_j$ ensures that it is not beneficial to send relief item to another location without having surplus inventory.
- The assumption, $\rho_{ij}r_{ij} > s_i + \tau_{ij}$ ensures that it is beneficial to send relief item to another location. Otherwise, there is no meaning of cooperation. Since the reward value is very high compared to the salvage value and the transportation cost in the disaster context, this assumption is also realistic and is not restrictive.

3.1 The Centralized System

First, we consider the case in which inventory decisions for each relief agency are centrally coordinated to maximize the joint reward. An umbrella organization chooses inventory levels (Q_1, Q_2) to maximize the sum of the rewards across the relief agencies. Note that, in this cooperation scenario, they also agree to share their residual inventory. Then, total expected rewards for the two agencies, denoted as π^c, is

$$\pi^c(Q_1,Q_2) = E[r_1R_1 + r_2R_2 + (r_{21}\rho_{21} - \tau_{21})T_{21} + (r_{12}\rho_{12} - \tau_{12})T_{12} + s_1L_1 + s_2L_2 - p_1S_1 - p_2S_2] - c_1Q_1 - c_2Q_2 \tag{3.1}$$

Also, we can write the open form of $\pi^c(Q_i,Q_j)$ by using above defined cases and corresponding reward values.

$$\begin{aligned}
\pi^c(Q_i,Q_j) = & \int_0^{Q_i}\int_0^{Q_j}[r_ix_i - c_iQ_i + s_i(Q_i - x_i)] \\
& +[r_jx_j - c_jQ_j + s_j(Q_j - x_j)]f(x_i,x_j)dx_idx_j \\
+ & \int_0^{Q_i}\int_{Q_j+(Q_i-x_i)}^{\infty}[r_ix_i - c_iQ_i + (\rho_{ij}r_i - \tau_{ij})(Q_i - x_i)] \\
& +[r_jQ_j - c_jQ_j - p_j(x_j - Q_j - \rho_{ij}(Q_i - x_i))]f(x_i,x_j)dx_idx_j \\
+ & \int_0^{Q_i}\int_{Q_j}^{Q_j+(Q_i-x_i)}[r_ix_i - c_iQ_i + (\rho_{ij}r_i - \tau_{ij})(x_j - Q_j) + s_i(Q_i + Q_j - x_i - x_j)] \\
& +[r_jQ_j - c_jQ_j - p_j(1-\rho_{ij})(x_j - Q_j)]f(x_i,x_j)dx_idx_j \\
+ & \int_{Q_i+(Q_j-x_j)}^{\infty}\int_0^{Q_j}[r_iQ_i - c_iQ_i - p_i(x_i - Q_i - \rho_{ji}(Q_j - x_j))] \\
& +[r_jx_j - c_jQ_j + (\rho_{ji}r_j - \tau_{ji})(Q_j - x_j)]f(x_i,x_j)dx_idx_j \\
+ & \int_{Q_i}^{Q_i+(Q_j-x_j)}\int_0^{Q_j}[r_iQ_i - c_iQ_i - p_i(1-\rho_{ji})(x_i - Q_i)] \\
& +[r_jx_j - c_jQ_j(\rho_{ji}r_j - \tau_{ji})(x_i - Q_i) + s_j(Q_j + Q_i - x_j - x_i)]f(x_i,x_j)dx_idx_j \\
+ & \int_{Q_i}^{\infty}\int_{Q_j}^{\infty}[r_iQ_i - c_iQ_i - p_i(x_i - Q_i)] \\
& +[r_jQ_j - c_jQ_j - p_j(x_j - Q_j)]f(x_i,x_j)dx_idx_j
\end{aligned} \tag{3.2}$$

When we calculate the first order partial derivatives of the above equation, we obtain the expected value of a marginal unit of inventory at location i as

$$
\begin{aligned}
\frac{\partial \pi^c}{\partial Q_i} &= s_i Pr(x_i < Q_i, x_j < Q_j) \\
&+ \big(-\tau_{ij} + \rho_{ij}(r_{ij} + p_j)\big) Pr(x_i < Q_i, x_j > Q_j + Q_i - x_i) \\
&+ \big(s_i\big) Pr(x_i < Q_i, Q_j < x_j < Q_j + Q_i - x_i) \\
&+ (r_i + p_i) Pr(x_i > Q_i + Q_j - x_j, x_j < Q_j) \\
&+ (r_i - r_{ji}\rho_{ji} + \tau_{ji} + s_j + p_i(1 - \rho_{ji})) Pr(Q_i < x_i < Q_i + Q_j - x_j, x_j < Q_j) \\
&+ (r_i + p_i) Pr(x_i > Q_i, x_j > Q_j) - c_i
\end{aligned}
\tag{3.3}
$$

When we collect the terms, we obtain

$$
\begin{aligned}
\frac{\partial \pi^c}{\partial Q_i} &= (r_i + p_i)(1 - Pr(x_i < Q_i - Pr(Q_i < x_i < Q_i + Q_j - x_j)) \\
&+ (r_i - r_{ji}\rho_{ji} + \tau_{ji} + s_j + p_i(1 - \rho_{ji})) Pr(Q_i < x_i < Q_i + Q_j - x_j) \\
&+ (-\tau_{ij} + \rho_{ij}(r_{ij} + p_j)) Pr(Q_i + Q_j - x_j < x_i < Q_i) \\
&+ s_i (Pr(x_i < Q_i - Pr(Q_i + Q_j - x_j < x_i < Q_i)) - c_i
\end{aligned}
\tag{3.4}
$$

To simplfy the notation, for given inventory levels (Q_1, Q_2), we define events and associated probablities as shown in Table 1.

Table 3.1: Cases and associated probability functions

Case	Description	Probability
E_i^1	$X_i < Q_i$	$A_i(Q_i)$
E_i^2	$Q_i + Q_j - X_j < X_i < Q_i$	$B_i(Q_i, Q_j)$
E_i^3	$Q_i < X_i < Q_i + Q_j - X_j$	$C_i(Q_i, Q_j)$

If we rearrange equation 3.4, we obtain first order conditions which characterize the optimal order quantity as follows;

$$A_i(Q_i) - B_i(Q_i,Q_j)\left(\frac{-\tau_{ij}+\rho_{ij}(r_{ij}+p_j)-s_i}{r_i+p_i-s_i}\right) \qquad (3.5)$$
$$+C_i(Q_i,Q_j)\left(\frac{\rho_{ji}(p_i+r_{ji})-s_j-\tau_{ji}}{r_i+p_i-s_i}\right) = \frac{r_i+p_i-c_i}{r_i+p_i-s_i}.$$

Proposition 1 *There exists a unique optimal solution, (Q_1^c, Q_2^c) and first order conditions are sufficient for optimality.*

Proof.

To prove the uniqueness of the optimal solution for centralized model, it is sufficient to show that the concavity of centralized reward function. To show that we can obtain the hessian matrix of the reward function by taking the derivatives.

$$\begin{aligned}\frac{\partial\pi^c}{\partial Q_i\partial Q_j} &= (s_i+\tau_{ij}-\rho_{ij}(r_{ij}+p_j))\int_0^{Q_i} f(x_i,Q_i+Q_j-x_i)dx_i \\ &+ (s_j+\tau_{ji}-\rho_{ji}(r_{ji}+p_i))\int_{Q_i}^{Q_i+Q_j} f(x_i,Q_i+Q_j-x_i)dx_i \qquad (3.6)\end{aligned}$$

$$\begin{aligned}\frac{\partial\pi^c}{\partial Q_i^2} &= (s_i-(r_i+p_i)+\rho_{ji}(r_{ji}+p_i)-s_j-\tau_{ji})\int_0^{Q_j} f(Q_i,x_j)dx_j \\ &+ (-r_i-p_i+\rho_{ij}(r_i+p_j))\int_{Q_j}^{\infty} f(Q_i,x_j)dx_j + \frac{\partial\pi^c}{\partial Q_i\partial Q_j}\end{aligned} \qquad (3.7)$$

$$\begin{aligned}\frac{\partial\pi^c}{\partial Q_j^2} &= (s_j-(r_j+p_j)+\rho_{ij}(r_{ij}+p_j)-s_i-\tau_{ij})\int_0^{Q_i} f(x_i,Q_j)dx_i \\ &+ (-r_j-p_j+\rho_{ji}(r_j+p_i))\int_{Q_i}^{\infty} f(x_i,Q_j)dx_i + \frac{\partial\pi^c}{\partial Q_i\partial Q_j}\end{aligned} \qquad (3.8)$$

Note that $\rho_{ij}(r_{ij}+p_j) \geq s_i+\tau_{ji}$. Additionaly, since $s_i+\tau_{ji} \geq s_j$ and $\rho_{ij}r_{ij}+p_i-\tau_{ij} < r_j+p_j$ all three functions are negative. Also, $\frac{\partial\pi^c}{\partial Q_i^2} \leq \frac{\partial\pi^c}{\partial Q_i\partial Q_j}$ and $\frac{\partial\pi^c}{\partial Q_j^2} \leq \frac{\partial\pi^c}{\partial Q_i\partial Q_j}$ which means that

$$\frac{\partial \pi^c}{\partial Q_i^2}\frac{\partial \pi^c}{\partial Q_j^2} - (\frac{\partial \pi^c}{\partial Q_i \partial Q_j})^2 \geq 0. \tag{3.9}$$

Therefore, π^c *is concave in* (Q_i, Q_j).

■

3.2 The Decentralized System

In the decentralized system, each agency decides its inventory level to maximize its own reward and they agree to share their residual inventory after demand realizations. Then, the expected reward at each decentralized organization i is denoted as π_i^d.

$$\pi_i^d(Q_i, Q_j) = E[r_i R_i + (r_{ij}\rho_{ij} - \tau_{ij})T_{ij} + s_i L_i - p_i S_i] - c_i Q_i$$

Open form of $\pi_i^d(Q_i, Q_j)$ is as follows:

$$\begin{aligned}
\pi_i^d(Q_i, Q_j) &= \int_0^{Q_i}\int_0^{Q_j}[r_i x_i - c_i Q_i + s_i(Q_i - x_i)]f(x_i, x_j)dx_i dx_j \\
+ &\int_0^{Q_i}\int_{Q_j+(Q_i-x_i)}^{\infty}[r_i x_i - c_i Q_i + (\rho_{ij} r_i - \tau_{ij})(Q_i - x_i)]f(x_i, x_j)dx_i dx_j \\
+ &\int_0^{Q_i}\int_{Q_j}^{Q_j+(Q_i-x_i)}[r_i x_i - c_i Q_i(\rho_{ij} r_i - \tau_{ij})(x_j - Q_j) + s_i(Q_i + Q_j - x_i - x_j)]f(x_i, x_j)dx_i dx_j \\
+ &\int_{Q_i+(Q_j-x_j)}^{\infty}\int_0^{Q_j}[r_i Q_i - c_i Q_i - p_i(x_i - Q_i - \rho_{ji}(Q_j - x_j))]f(x_i, x_j)dx_i dx_j \\
+ &\int_{Q_i}^{Q_i+(Q_j-x_j)}\int_0^{Q_j}[r_i Q_i - c_i Q_i - p_i(1 - \rho_{ji})(x_i - Q_i)]f(x_i, x_j)dx_i dx_j \\
+ &\int_{Q_i}^{\infty}\int_{Q_j}^{\infty}[r_i Q_i - c_i Q_i - p_i(x_i - Q_i)]f(x_i, x_j)dx_i dx_j
\end{aligned} \tag{3.10}$$

When we calculate the derivatives of the above equation, we obtain the expected value of a marginal unit of inventory at location i as

$$
\begin{aligned}
\frac{\partial \pi_i^d}{\partial Q_i} &= s_i Pr(x_i < Q_i, x_j < Q_j) \\
&+ \big(-\tau_{ij} + \rho_{ij} r_{ij} \big) Pr(x_i < Q_i, x_j > Q_j + Q_i - x_i) \\
&+ \big(s_i \big) Pr(x_i < Q_i, Q_j < x_j < Q_j + Q_i - x_i) \\
&+ (r_i + p_i) Pr(x_i > Q_i + Q_j - x_j, x_j < Q_j) \\
&+ (r_i + p_i(1 - \rho_{ji})) Pr(Q_i < x_i < Q_i + Q_j - x_j, x_j < Q_j) \\
&+ (r_i + p_i) Pr(x_i > Q_i, x_j > Q_j) - c_i
\end{aligned} \tag{3.11}
$$

When we collect the terms, we obtain

$$
\begin{aligned}
\frac{\partial \pi_i^d}{\partial Q_i} &= (r_i + p_i)(1 - Pr(x_i < Q_i) - Pr(Q_i < x_i < Q_i + Q_j - x_j)) \\
&+ (r_i + p_i(1 - \rho_{ji})) Pr(Q_i < x_i < Q_i + Q_j - x_j) \\
&+ (-\tau_{ij} + \rho_{ij} r_{ij}) Pr(Q_i + Q_j - x_j < x_i < Q_i) \\
&+ s_i (Pr(x_i < Q_i - Pr(Q_i + Q_j - x_j < x_i < Q_i)) - c_i
\end{aligned} \tag{3.12}
$$

If we rearrange equation 3.12, we obtain Nash Equilibrium which is given by (Q_1^d, Q_2^d) such that equation 3.13) holds for *i,j=1,2*.

$$
\begin{aligned}
& A_i(Q_i) - B_i(Q_i, Q_j) \left(\frac{\rho_{ij} r_{ij} - s_i - \tau_{ij}}{r_i + p_i - s_i} \right) \\
& + C_i(Q_i, Q_j) \left(\frac{\rho_{ji} p_i}{r_i + p_i - s_i} \right) = \frac{r_i + p_i - c_i}{r_i + p_i - s_i}
\end{aligned} \tag{3.13}
$$

Proposition 2 *There exists a unique Nash equilibrium, (Q_1^d, Q_2^d).*

Proof.

It is sufficient to show that the best response functions are monotonic, and the absolute value of the slope is less than 1 (see Fudenberg and Tirole [20]) to prove the existence and the uniqueness of the Nash Equilibrium.

Let us define the following marginal probabilities:

$$\beta_{ij}^1 = Pr(x_i < Q_i) f_{x_i+x_j|x_i<Q_i}(Q_i + Q_j)$$

$$\beta_{ij}^2 = Pr(x_i + x_j > Q_i + Q_j) f_{x_i|x_i+x_j>Q_i+Q_j}(Q_i)$$

$$\gamma_{ij}^1 = Pr(x_i > Q_i) f_{x_i+x_j|x_i>Q_i}(Q_i + Q_j)$$

$$\gamma_{ij}^2 = Pr(x_i + x_j < Q_i + Q_j) f_{x_i|x_i+x_j<Q_i+Q_j}(Q_i)$$

and

$$\alpha_i = f_{x_i}(Q_i).$$

By taking implicit differentiation of Equation 3.13 and rearranging results, we obtain the best response function as follows:

$$\frac{\partial Q_i}{\partial Q_j} = -\frac{(\rho_{ij} r_{ij} - s_i - \tau_{ij})\beta_{ij}^1 + (\rho_{ji} p_i)\gamma_{ij}^1}{(r_i + p_i - s_i)\alpha_i + (\rho_{ij} r_{ij} - s_i - \tau_{ij})(\beta_{ij}^1 - \beta_{ij}^2) + (\rho_{ji} p_i)(\gamma_{ij}^1 - \gamma_{ij}^2)} \tag{3.14}$$

Since $(\rho_{ij} r_{ij} - s_i - \tau_{ij}) > 0$, $(\rho_{ji} p_i) > 0$, $\alpha_i > \beta_{ij}^2$ *,and* $\alpha_i > \gamma_{ij}^2$*, the slope of the best response curve is negative and bigger than -1. Thus, the proof follows.*

■

In this chapter, we consider two cooperation scenarios, namely Centralized System and Decentralized System. First, we define our mathematical model and cooperation scenarios. Second, we obtained the open form of corresponding reward functions. For the Centralized System, we obtained first-order conditions for optimal order quantities and also showed that there are unique optimal order quantities under model assumptions by proving the concavity of the reward function.

In Section 3.2, we analyzed the Decentralized System. We, again, obtained the open form of the corresponding reward function. Then, we obtained the first order conditions which characterize the Nash Equilibrium by taking partial derivatives of the corresponding reward functions. We also showed that there is a unique Nash Equilibrium in the Decentralized System. Since we obtained the closed form solution and equilibrium, we conduct extensive numerical analysis to get insights by calculating a centralized solution and Nash Equilibrium in realistic parameter settings in Chapter 5.

Chapter 4

MATHEMATICAL MODEL WITH STOCHASTIC A RELIABILITY FACTOR

It is clear that when a relief agency sends its surplus inventory to other organizations after demand realizations, relief items do not reach the other organization intact or the arrival time of the aid is late in general. Therefore, the organization who faces stock-out after demand realization can not use all the items which come from the other organization. In Chapter 3, we defined ρ_{ij} as the portion of the transshipped items can be used by relief agency j, when agency i helps the casualties at location j. Note that, reliability factor in Chapter 3 was assumed as a deterministic variable between 0 and 1. On the other hand, estimating the reliability factor perfectly is almost impossible, because it depends on many random events. Therefore, we extend our mathematical model with a stochastic reliability factor to obtain a more realistic model. Then, we analyze our problem with stochastic reliability factor ρ_{ij} which has a probability distribution between 0 and 1. In Chapter 3, agencies do not have to optimize their transshipment amount because of the deterministic nature of the ρ_{ij}. However, they also need to optimize their T_{ij} to maximize their expected reward when the ρ_{ij} is a stochastic variable. Note that in Chapter 3, we defined six possible cases which agencies can face, and we wrote corresponding reward function for each case. When ρ_{ij} becomes a stochastic variable, we need to revise our reward function for agency i as follows for each case.

Case 1: $Q_i > X_i$ and $Q_j > X_j$

(Both stocking quantities are more than demand quantites)

$$\pi_i(Q_i, Q_j) = r_i X_i - c_i Q_i + s_i(Q_i - X_i)$$

In this case, there is no transshipment because both agencies face a leftover inventory.

Case 2: $Q_i > X_i$, $Q_j < X_j$, and $Q_i + Q_j < X_i + X_j$

(Stocking quantity for agency i is more than demand quantity in region i, stocking quantity for agency j is less than demand quantity in region j , and the total stocking quantity is less than total demand quantity)

$$\pi_i(Q_i, Q_j) = r_i x_i - c_i Q_i + (E[\rho_{ij}]r_i - \tau_{ij})(Q_i - X_i)$$

In this case, optimal transshipment amount for agency i is trivial. Since the leftover inventory of the agency i is smaller than the unmet demand at location j, agency i should send all of the leftover inventory.

Case 3: $Q_i > X_i$, $Q_j < X_j$, and $Q_i + Q_j > X_i + X_j$

(Stocking quantity for agency i is more than demand quantity in region i, stocking quantity for agency j is less than demand quantity in region j, and the total stocking quantity is more than total demand quantity)

$$\pi_i(Q_i, Q_j) = r_i x_i - c_i Q_i + r_{ij} E[min(x_j - Q_j, T^*_{ij}\rho_{ij})] + s_i(Q_i - X_i - T^*_{ij}) - \tau_{ij} T^*_{ij}$$

In this case, optimal transshipment amount is not trivial since the leftover inventory is bigger than the unmet demand and agency i may need to send more than the unmet demand to increase its expected reward. This case is analyzed in the next section to find optimal transshipment amount T_{ij}.

Case 4: $Q_i < X_i$, $Q_j > X_j$, and $Q_i + Q_j < X_i + X_j$

(Stocking quantity for agency i is less than demand quantity in region i, stocking quantity for agency j is more than demand quantity in region j, and the total stocking quantity is less than total demand quantity)

$$\pi_i(Q_i, Q_j) = r_i Q_i - c_i Q_i - p_i(X_i - Q_i - E[\rho_{ji}](Q_j - X_j))$$

In this case, the optimal transshipment amount for agency j can be easily found. Since the leftover inventory of the agency j is smaller than the unmet demand at location i, agency j should send all of the leftover inventory.

Case 5: $Q_i < X_i$, $Q_j > X_j$, and $Q_i + Q_j > X_i + X_j$

(Stocking quantity for agency i is less than demand quantity in region i, stocking quantity for agency j is more than demand quantity in region j, and the total stocking quantity is more than total demand quantity)

$$\pi_i(Q_i, Q_j) = r_iQ_i - c_iQ_i - p_iE[max(0, X_i - Q_i - T^*_{ji}\rho_{ij})$$

In this case, optimal transshipment amount is not trivial since the leftover inventory is bigger than the unmet demand and agency j may need to send more than the unmet demand to increase its expected reward. This case is analyzed in the next section to find optimal transshipment amount T_{ji}.

Case 6: $Q_i < X_i$ and $Q_j < X_j$

(Both stocking quantities are more than demand quantites)

$$\pi_i(Q_i, Q_j) = r_iQ_i - c_iQ_i - p_i(X_i - Q_i)$$

In this case, there is no transshipment because both agencies face an unmet demand.

Note that, the reward functions for Case 1,2,4 and 6 remained the same. For Cases 3 and 5, we need to find optimal T_{ij} and T_{ji} values to obtain open form of the reward functions. Also, note that optimal T_{ij} changes according to our system is centralized or decentralized. Therefore, we analyze the reward functions for both decentralized and centralized systems. Furthermore, in this chapter, agencies need to make a transshipment decision after the demand realizations. After the transshipment, the reliability factor is realized and agencies obtain their rewards according to the realized reliability factor. Therefore, our system is represented by a two-stage problem. In the first stage, the agencies need to make their stocking decisions. At the end of the first stage, demands are realized and the agencies decide their transshipment amounts after the demand realizations. At the end of the second stage, the reliability factor and the corresponding reward values are realized. The order of events illustrated in Figure 4.1.

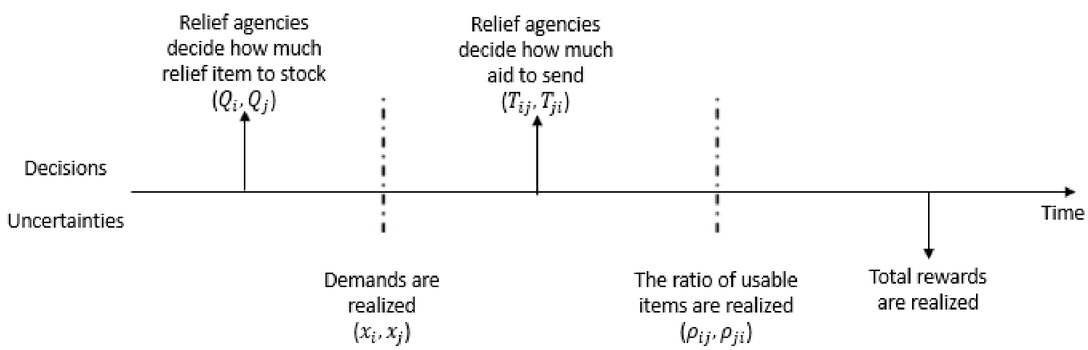


Figure 4.1: Order of events

4.1 The Centralized System

From the previous section, we know that the reward functions change in cases 3 and 5. Therefore, we need to find optimal transshipment amounts by constructing subproblem after demand realizations. As a subproblem, the central agency or the decision maker needs to determine optimal aid amount in cases 3 and 5. We can define the reward by sending aid from agency i to agency j after demand realizations as follows:

$$\pi_{ij} = E[(r_{ij} + p_j)min(T_{ij}\rho_{ij}, x_j - Q_j) - (\tau_{ij} + s_i)T_{ij}]$$

Open form of π_{ij} is as follows:

$$\pi_{ij} = (r_{ij}+p_j)\int_0^{\frac{x_j-Q_j}{T_{ij}}} T_{ij}xf_{\rho_{ij}}(x)dx+(r_{ij}+p_j)\int_{\frac{x_j-Q_j}{T_{ij}}}^{1}(x_j-Q_j)f_{\rho_{ij}}(x)dx-(\tau_{ij}+s_i)T_{ij}$$

where $f_{\rho_{ij}}(x)$ is the probability distribution function of the random variable ρ_{ij}.

By taking the partial derivatives of π_{ij},

$$\begin{aligned} \frac{\partial \pi_{ij}}{\partial T_{ij}} &= (r_{ij}+p_j)(\int_0^{\frac{x_j-Q_j}{T_{ij}}} xf_{\rho_{ij}}(x)dx + (-\frac{x_j-Q_j}{T_{ij}^2})T_{ij}(\frac{x_j-Q_j}{T_{ij}})f_{\rho_{ij}}(x) \\ & -(-\frac{x_j-Q_j}{T_{ij}^2})(x_j-Q_j)f_{\rho_{ij}}(x) - (\tau_{ij}+s_i) \\ &= (r_{ij}+p_j)\int_0^{\frac{x_j-Q_j}{T_{ij}}} xf_{\rho_{ij}}(x)dx - (\tau_{ij}+s_i) \end{aligned} \tag{4.1}$$

Then, the first order condition which characterizes optimal T_{ij}^* is as follows:

$$\int_0^{\frac{x_j-Q_j}{T_{ij}^*}} xf_{\rho_{ij}}(x)dx = \frac{\tau_{ij}+s_i}{r_{ij}+p_j} \tag{4.2}$$

Proposition 3 *There exists a unique optimal transhipment amount and first order optimality condition is sufficient.*

Proof.

To prove the uniqueness of the optimal solution , it is sufficient to show the concavity of π_{ij}. To show that, we obtain the second order partial derivative of π_{ij} by using equation 4.1.

$$\frac{\partial^2 \pi_{ij}}{\partial T_{ij}^2} = (r_{ij}+p_j)(\frac{x_j-Q_j}{T_{ij}}f_{\rho_{ij}}(\frac{x_j-Q_j}{T_{ij}}))\frac{Q_j-x_j}{T_{ij}^2} < 0$$

Note that $x_j > Q_j$, $T_{ij} > 0$, and $f_{\rho_{ij}}(\frac{x_j-Q_j}{T_{ij}}) > 0$. Therefore, the second derivative of the π_{ij} is always smaller than 0. Proof is done.

■

Note that first order partial derivative of π_{ij} always decreasing. It is positive when $T_{ij} = x_j - Q_j$ and it is negative when T_{ij} approaches to the infinity which means that optimal T_{ij} is finite. By using this information, the general way for optimal T_{ij} as follows:

$$T^*_{ij} = \begin{cases} \int_0^{\frac{x_j - Q_j}{T^*_{ij}}} x f_{\rho_{ij}}(x)dx = \frac{\tau_{ij}+s_i}{r_{ij}+p_j} & \text{if } \int_0^{\frac{x_j - Q_j}{Q_i - x_i}} x f_{\rho_{ij}}(x)dx < \frac{\tau_{ij}+s_i}{r_{ij}+p_j} \\ Q_i - x_i & \text{otherwise} \end{cases} \tag{4.3}$$

After finding optimal T^*_{ij}, we can turn back the original problem and revise the open form of the reward function accordingly.

$$\begin{aligned}
\pi^c \quad = \quad & \int_0^{Q_i}\int_0^{Q_j}[r_i x_i - c_i Q_i + s_i(Q_i - x_i)] + [r_j x_j - c_j Q_j + s_j(Q_j - x_j)]f(x_i, x_j)dx_i dx_j \\
+ \quad & \int_0^{Q_i}\int_{Q_j+(Q_i-x_i)}^{\infty}[r_i x_i - c_i Q_i + (E[\rho_{ij}]r_i - \tau_{ij})(Q_i - x_i) \\
& +[r_j Q_j - c_j Q_j - p_j(x_j - Q_j - E[\rho_{ij}](Q_i - x_i))]f(x_i, x_j)dx_i dx_j \\
+ \quad & \int_0^{Q_i}\int_{Q_j)}^{Q_j+(Q_i-x_i)}[r_i x_i - c_i Q_i + r_{ij}E[min(x_j - Q_j, T^*_{ij}\rho_{ij})] + s_i(Q_i - x_i - T^*_{ij}) - \tau_{ij}T^*_{ij}] \\
& +[r_j Q_j - c_j Q_j - p_j E[max(0, x_j - Q_j - T^*_{ij}\rho_{ij})]f(x_i, x_j)dx_i dx_j \\
+ \quad & \int_{Q_i+(Q_j-x_j)}^{\infty}\int_0^{Q_j}[r_i Q_i - c_i Q_i - p_i(x_i - Q_i - E[\rho_{ji}](Q_j - x_j))]f \\
& +[r_j x_j - c_j Q_j + (E[\rho_{ji}]r_{ji} - \tau_{ji})(Q_j - x_j)]f(x_i, x_j)dx_i dx_j \\
+ \quad & \int_{Q_i}^{Q_i+(Q_j-x_j)}\int_0^{Q_j}[r_i Q_i - c_i Q_i - p_i E[max(0, x_i - Q_i - T^*_{ij}\rho_{ij})] \\
& +[r_j x_j - c_j Q_j + r_{ji}E[min(x_i - Q_i, T^*_{ji}\rho_{ji})] \\
& +s_j(Q_j - x_j - T^*_{ji}) - \tau_{ji}T^*_{ji}]f(x_i, x_j)dx_i dx_j \\
+ \quad & \int_{Q_i}^{\infty}\int_{Q_j}^{\infty}[r_i Q_i - c_i Q_i - p_i(x_i - Q_i)] + [r_j Q_j - c_j Q_j - p_j(x_j - Q_j)]f(x_i, x_j)dx_i dx_j
\end{aligned} \tag{4.4}$$

Also note that,

$$\begin{aligned}
E[min(x_j - Q_j, T^*_{ij}\rho_{ij})] &= \int_0^{\frac{x_j-Q_j}{T^*_{ij}}} T^*_{ij} x f_{\rho_{ij}}(x)dx + \int_{\frac{x_j-Q_j}{T^*_{ij}}}^1 (x_j - Q_j) f_{\rho_{ij}}(x)dx \\
&= T^*_{ij}(\frac{\tau_{ij}+s_i}{r_{ij}+p_j}) + (x_j - Q_j)(1 - F_{\rho_{ij}}(\frac{x_j-Q_j}{T^*_{ij}})) \quad (4.5)
\end{aligned}$$

$$E[max(0, x_j - Q_j - T^*_{ij}\rho_{ij})] = (x_j - Q_j)F_{\rho_{ij}}(\frac{x_j-Q_j}{T^*_{ij}}) - T^*_{ij}(\frac{\tau_{ij}+s_i}{r_{ij}+p_j}) \quad (4.6)$$

$$E[max(0, x_i - Q_i - T^*_{ji}\rho_{ji})] = (x_i - Q_i)F_{\rho_{ji}}(\frac{x_i-Q_i}{T^*_{ji}}) - T^*_{ji}(\frac{\tau_{ji}+s_j}{r_{ji}+p_i}) \quad (4.7)$$

$$E[min(x_i - Q_i, T^*_{ji}\rho_{ji})] = T^*_{ji}(\frac{\tau_{ji}+s_j}{r_{ji}+p_i}) + (x_i - Q_i)(1 - F_{\rho_{ji}}(\frac{x_i-Q_i}{T^*_{ji}})) \quad (4.8)$$

To obtain the first order conditions to find optimal quantities, we need to take the partial derivatives of Equation 4.4 with respect to Q_i. Note that

$$\int_0^{\frac{x_j-Q_j}{T^*_{ij}}} x f_{\rho_{ij}}(x)dx = \frac{\tau_{ij}+s_i}{r_{ij}+p_j}$$

By taking implicit differentation,

$$\begin{aligned}
- & (\frac{x_i-Q_i}{T^*_{ji}})f_{\rho_{ji}}(\frac{x_i-Q_i}{T^*_{ji}}) - (\frac{x_i-Q_i}{T^*_{ji}})(\frac{x_i-Q_i}{T^{*2}_{ji}})f_{\rho_{ji}}(\frac{x_i-Q_i}{T^*_{ji}})(\frac{\partial T^*_{ji}}{\partial Q_i}) = 0 \\
& \frac{\partial T^*_{ji}}{\partial Q_i} = -\frac{T^{*2}_{ji}}{x_i - Q_i} \quad (4.9)
\end{aligned}$$

Also,

$$\begin{aligned}
\frac{\partial((x_i - Q_i)F_{\rho_{ji}}(\frac{x_i-Q_i}{T^*_{ji}}))}{\partial Q_i} &= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i)\frac{\partial(\int_0^{\frac{x_i-Q_i}{T^*_{ji}}} f_{\rho_{ji}}(x)dx)}{\partial Q_i} \\
&= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i)f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}})\frac{\partial(\frac{x_i-Q_i}{T^*_{ji}})}{\partial Q_i} \\
&= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i)f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}})(\frac{-T^*_{ji} + (x_i - Q_i)\frac{T^{2*}_{ji}}{x_i-Q_i}}{T^{2*}_{ji}}) \\
&= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i)f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}})(1 - \frac{1}{T^*_{ji}}) \qquad (4.10)
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{\partial(E[max(0, x_i - Q_i - T^*_{ji}\rho_{ji})])}{\partial Q_i} &= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i)f_{\rho_{ij}}(\frac{x_i - Q_i}{T^*_{ji}})(1 - \frac{1}{T^*_{ji}}) \\
&+ \frac{T^{*2}_{ji}}{x_i - Q_i}(\frac{\tau_{ji} + s_j}{r_{ji} + p_i}) \qquad (4.11)
\end{aligned}$$

Since the above derivation depends on the Q_i, Q_i, x_i, and x_j, we can call the above derivatives as follows to simplify the notation:

$$\frac{\partial(E[max(0, x_i - Q_i - T^*_{ji}\rho_{ji})])}{\partial Q_i} = K^c_i(Q_i, Q_j, x_i, x_j) \qquad (4.12)$$

Note that,

$$E[min(x_i - Q_i, T^*_{ji}\rho_{ji})] = (x_i - Q_i) - E[max(0, x_i - Q_i - T^*_{ji}\rho_{ji})] \qquad (4.13)$$

Therefore,

$$\frac{\partial(E[min(x_i - Q_i, T^*_{ji}\rho_{ji})])}{\partial Q_i} = -(1 + K^c_i(Q_i, Q_j, x_i, x_j)) \qquad (4.14)$$

When we calculate the first order partial derivatives of the above equation, we obtain the expected value of a marginal unit of inventory at location i as

$$
\begin{aligned}
\frac{\partial \pi^c}{\partial Q_i} &= s_i Pr(x_i < Q_i, x_j < Q_j) \\
&+ \big(-\tau_{ij} + E[\rho_{ij}](r_{ij}+p_j)\big) Pr(x_i < Q_i, x_j > Q_j + Q_i - x_i) \\
&+ \big(s_i\big) Pr(x_i < Q_i, Q_j < x_j < Q_j + Q_i - x_i) \\
&+ (r_i + p_i) Pr(x_i > Q_i + Q_j - x_j, x_j < Q_j) \\
&+ (r_i) Pr(Q_i < x_i < Q_i + Q_j - x_j, x_j < Q_j) \\
&+ \int_{Q_i}^{Q_i+(Q_j-x_j)} \int_0^{Q_j} (-(p_i + r_{ji}) K_i^c(Q_i, Q_j, x_i, x_j) - r_{ji} + (\tau_{ji} - s_j)(\frac{T_{ji}^{*2}}{x_i - Q_i}) f(x_i, x_j) dx_i dx_j \\
&+ (r_i + p_i) Pr(x_i > Q_i, x_j > Q_j) - c_i
\end{aligned}
\tag{4.15}
$$

When we collect the terms, we obtain

$$
\begin{aligned}
\frac{\partial \pi^c}{\partial Q_i} &= (r_i + p_i)(1 - Pr(x_i < Q_i - Pr(Q_i < x_i < Q_i + Q_j - x_j)) \\
&+ (r_i) Pr(Q_i < x_i < Q_i + Q_j - x_j) \\
&+ (-\tau_{ij} + E[\rho_{ij}](r_{ij} + p_j)) Pr(Q_i + Q_j - x_j < x_i < Q_i) \\
&+ s_i(Pr(x_i < Q_i - Pr(Q_i + Q_j - x_j < x_i < Q_i)) - c_i \\
&+ \int_{Q_i}^{Q_i+(Q_j-x_j)} \int_0^{Q_j} (-(p_i + r_{ji}) K_i^c(Q_i, Q_j, x_i, x_j) - r_{ji} + (\tau_{ji} - s_j)(\frac{T_{ji}^{*2}}{x_i - Q_i}) f(x_i, x_j) dx_i dx_j
\end{aligned}
\tag{4.16}
$$

To simplfy the notation, for given inventory levels (Q_1, Q_2), we define events and associated probablities as shown in Table 4.1. Also let us define $\widehat{K_i^c}(Q_i, Q_j)$ to simply further as follows:

$$
\begin{aligned}
\widehat{K_i^c}(Q_i, Q_j) &= \int_{Q_i}^{Q_i+(Q_j-x_j)} \int_0^{Q_j} (-(p_i + r_{ji}) K_i(Q_i, Q_j, x_i, x_j) - r_{ji} \\
&+ (\tau_{ji} - s_j)(\frac{T_{ji}^{*2}}{x_i - Q_i}) f(x_i, x_j) dx_i dx_j
\end{aligned}
\tag{4.17}
$$

If we rearrange equation 4.16, we obtain first order conditions which characterize the optimal order quantity as follows;

Table 4.1: Cases and associated probability functions

Case	Description	Probability
E_i^1	$X_i < Q_i$	$A_i(Q_i)$
E_i^2	$Q_i + Q_j - X_j < X_i < Q_i$	$B_i(Q_i, Q_j)$
E_i^3	$Q_i < X_i < Q_i + Q_j - X_j$	$C_i(Q_i, Q_j)$

$$A_i(Q_i) - B_i(Q_i, Q_j)\left(\frac{-\tau_{ij} + E[\rho_{ij}](r_{ij} + p_j) - s_i}{r_i + p_i - s_i}\right) \tag{4.18}$$
$$+C_i(Q_i, Q_j)\left(\frac{p_i}{r_i + p_i - s_i}\right) + \frac{\widehat{K_i^c}(Q_i, Q_j)}{r_i + p_i - s_i} = \frac{r_i + p_i - c_i}{r_i + p_i - s_i}.$$

Since obtaining the open form of the hessian matrix of the π^c is not possible, we can not prove the uniqueness of the optimal quantities. We conducted extensive numerical analysis to understand the behavior of the optimal quantities with respect to stochastic behavior of the reliability factor in Chapter 5.

4.2 The Decentralized System

In the Decentralized model (DM) with stochastic reliability factor, we know that the reward functions change in cases 3 and 5 which defined at the beginning of Chapter 4. Therefore, we need to find optimal transshipment amounts by constructing sub-problem after demand realizations. As a subproblem, agency i and agency j need to determine optimal aid amount in cases 3 and 5. We can define the reward by sending aid from agency i to agency j after demand realizations as follows:

$$\pi_{ij} = E[r_{ij} min(T_{ij}\rho_{ij}, x_j - Q_j) - (\tau_{ij} + s_i)T_{ij}]$$

Open form of π_{ij} is as follows:

$$\pi_{ij} = r_{ij}\int_0^{\frac{x_j-Q_j}{T_{ij}}} T_{ij}xf_{\rho_{ij}}(x)dx + r_{ij}\int_{\frac{x_j-Q_j}{T_{ij}}}^1 (x_j-Q_j)f_{\rho_{ij}}(x)dx - (\tau_{ij}+s_i)T_{ij}$$

where $f_{\rho_{ij}}(x)$ is the distribution of ρ_{ij}.

By taking derivatives of π_{ij},

$$\frac{\partial \pi_{ij}}{\partial T_{ij}} = r_{ij}\int_0^{\frac{x_j-Q_j}{T_{ij}}} xf_{\rho_{ij}}(x)dx - (\tau_{ij}+s_i)$$

Then, the first order condition which characterizes the optimal T_{ij}^* is as follows:

$$\int_0^{\frac{x_j-Q_j}{T_{ij}^*}} xf_{\rho_{ij}}(x)dx = \frac{\tau_{ij}+s_i}{r_{ij}}$$

Proposition 4 *There exists a unique optimal transhipment amount and first order condition is sufficient for optimality.*

Proof.

To prove the uniqueness of the optimal solution, it is sufficient to show that the concavity of π_{ij}. To show that we can obtain the second derivative of the π_{ij} by taking the derivatives.

$$\frac{\partial^2 \pi_{ij}}{\partial T_{ij}^2} = r_{ij}(\frac{x_j-Q_j}{T_{ij}}f_{\rho_{ij}}(\frac{x_j-Q_j}{T_{ij}}))\frac{Q_j-x_j}{T_{ij}^2} < 0$$

Note that $x_j > Q_j$ and $T_{ij} > 0$. Therefore, the second derivative of the π_{ij} is always smaller than 0. Proof is done.

■

After finding optimal T_{ij}^*, we can turn back the original problem and revise the open form of the reward function accordingly.

$$
\begin{aligned}
\pi_i^d(Q_i,Q_j) &= \int_0^{Q_i}\int_0^{Q_j}[r_ix_i - c_iQ_i + s_i(Q_i - x_i)]f(x_i,x_j)dx_idx_j \\
&+ \int_0^{Q_i}\int_{Q_j+(Q_i-x_i)}^{\infty}[r_ix_i - c_iQ_i + (E[\rho_{ij}]r_i - \tau_{ij})(Q_i - x_i)]f(x_i,x_j)dx_idx_j \\
&+ \int_0^{Q_i}\int_{Q_j}^{Q_j+(Q_i-x_i)}[r_ix_i - c_iQ_i + r_{ij}E[min(x_j - Q_j, T_{ij}^*\rho_{ij})] \\
&+ s_i(Q_i - x_i - T_{ij}^*) - \tau_{ij}T_{ij}^*]f(x_i,x_j)dx_idx_j \\
&+ \int_{Q_i+(Q_j-x_j)}^{\infty}\int_0^{Q_j}[r_iQ_i - c_iQ_i - p_i(x_i - Q_i - E[\rho_{ji}](Q_j - x_j))]f(x_i,x_j)dx_idx_j \\
&+ \int_{Q_i}^{Q_i+(Q_j-x_j)}\int_0^{Q_j}[r_iQ_i - c_iQ_i - p_iE[max(0, x_i - Q_i - T_{ji}^*\rho_{ij})]f(x_i,x_j)dx_idx_j \\
&+ \int_{Q_i}^{\infty}\int_{Q_j}^{\infty}[r_iQ_i - c_iQ_i - p_i(x_i - Q_i)]f(x_i,x_j)dx_idx_j \qquad (4.19)
\end{aligned}
$$

Also note that,

$$
\begin{aligned}
E[min(x_j - Q_j, T_{ij}^*\rho_{ij})] &= \int_0^{\frac{x_j-Q_j}{T_{ij}^*}} T_{ij}^* x f_{\rho_{ij}}(x)dx + \int_{\frac{x_j-Q_j}{T_{ij}^*}}^1 (x_j - Q_j)f_{\rho_{ij}}(x)dx \\
&= T_{ij}^*(\frac{\tau_{ij}+s_i}{r_{ij}}) + (x_j - Q_j)(1 - F_{\rho_{ij}}(\frac{x_j - Q_j}{T_{ij}^*})) \qquad (4.20)
\end{aligned}
$$

Respectively,

$$
E[max(0, x_i - Q_i - T_{ji}^*)] = (x_i - Q_i)F_{\rho_{ji}}(\frac{x_i - Q_i}{T_{ji}^*}) - T_{ji}^*(\frac{\tau_{ji}+s_j}{r_{ji}}) \qquad (4.21)
$$

To obtain first order contition to find a Nash Equilibrium, we need to take the derivative of above function with respect to Q_i. Note that

$$
\int_0^{\frac{x_j-Q_j}{T_{ij}^*}} x f_{\rho_{ij}}(x)dx = \frac{\tau_{ij}+s_i}{r_{ij}}
$$

By taking implicit differentation,

$$
\begin{aligned}
- & (\frac{x_i - Q_i}{T^*_{ji}}) f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) - (\frac{x_i - Q_i}{T^*_{ji}})(\frac{x_i - Q_i}{T^{*2}_{ji}}) f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}})(\frac{\partial T^*_{ji}}{\partial Q_i}) = 0 \\
& \frac{\partial T^*_{ji}}{\partial Q_i} = -\frac{T^{*2}_{ji}}{x_i - Q_i} \qquad (4.22)
\end{aligned}
$$

Also,

$$
\begin{aligned}
\frac{\partial((x_i - Q_i)F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}))}{\partial Q_i} &= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i)\frac{\partial(\int_0^{\frac{x_i - Q_i}{T^*_{ji}}} f_{\rho_{ji}}(x)dx)}{\partial Q_i} \\
&= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i) f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}})\frac{\partial(\frac{x_i - Q_i}{T^*_{ji}})}{\partial Q_i} \\
&= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i) f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}})(\frac{-T^*_{ji} + (x_i - Q_i)\frac{T^{2*}_{ji}}{x_i - Q_i}}{T^{2*}_{ji}}) \\
&= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i) f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}})(1 - \frac{1}{T^*_{ji}}) \qquad (4.23)
\end{aligned}
$$

Then,

$$
\begin{aligned}
\frac{\partial(E[max(0, x_i - Q_i - T^*_{ji})])}{\partial Q_i} &= -F_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}}) + (x_i - Q_i) f_{\rho_{ji}}(\frac{x_i - Q_i}{T^*_{ji}})(1 - \frac{1}{T^*_{ji}}) \\
&+ \frac{T^{*2}_{ji}}{x_i - Q_i}(\frac{\tau_{ji} + s_j}{r_{ji}}) \qquad (4.24)
\end{aligned}
$$

Since the above derivation depends on the Q_i, Q_i, x_i , and x_j, we can call the above derivatives as follows to simplify the notation.

$$
\frac{\partial(E[max(0, x_i - Q_i - T^*_{ji})])}{\partial Q_i} = K^d_i(Q_i, Q_j, x_i, x_j) \qquad (4.25)
$$

Note that,

$$
E[min(x_i - Q_i, T^*_{ji}\rho_{ji})] = (x_i - Q_i) - E[max(0, x_i - Q_i - T^*_{ji}\rho_{ji})] \qquad (4.26)
$$

Therefore,

$$\frac{\partial(E[min(x_i - Q_i, T^*_{ji}\rho_{ji})])}{\partial Q_i} = -(1 + K^d_i(Q_i, Q_j, x_i, x_j)) \tag{4.27}$$

When we calculate the derivatives of the reward function, we obtain the expected value of a marginal unit of inventory at location i as

$$\begin{aligned}
\frac{\partial \pi^d_i}{\partial Q_i} &= s_i Pr(x_i < Q_i, x_j < Q_j) \\
&+ \big(-\tau_{ij} + E[\rho_{ij}]r_{ij}\big) Pr(x_i < Q_i, x_j > Q_j + Q_i - x_i) \\
&+ \big(s_i\big) Pr(x_i < Q_i, Q_j < x_j < Q_j + Q_i - x_i) \\
&+ (r_i + p_i) Pr(x_i > Q_i + Q_j - x_j, x_j < Q_j) \\
&+ (r_i) Pr(Q_i < x_i < Q_i + Q_j - x_j, x_j < Q_j) \\
&+ \int_{Q_i}^{Q_i+(Q_j-x_j)} \int_0^{Q_j} (-p_i K^d_i(Q_i, Q_j, x_i, x_j)) f(x_i, x_j) dx_i dx_j \\
&+ (r_i + p_i) Pr(x_i > Q_i, x_j > Q_j) - c_i
\end{aligned} \tag{4.28}$$

When we collect the terms, we obtain

$$\begin{aligned}
\frac{\partial \pi^d_i}{\partial Q_i} &= (r_i + p_i)(1 - Pr(x_i < Q_i) - Pr(Q_i < x_i < Q_i + Q_j - x_j)) \\
&+ (r_i) Pr(Q_i < x_i < Q_i + Q_j - x_j) \\
&+ (-\tau_{ij} + E[\rho_{ij}]r_{ij}) Pr(Q_i + Q_j - x_j < x_i < Q_i) \\
&+ s_i(Pr(x_i < Q_i - Pr(Q_i + Q_j - x_j < x_i < Q_i)) - c_i \\
&+ \int_{Q_i}^{Q_i+(Q_j-x_j)} \int_0^{Q_j} (-p_i K^d_i(Q_i, Q_j, x_i, x_j)) f(x_i, x_j) dx_i dx_j
\end{aligned} \tag{4.29}$$

Let us define $\widehat{K^d_i}(Q_i, Q_j)$ to simply the notation as follows:

$$\widehat{K^d_i}(Q_i, Q_j) = \int_{Q_i}^{Q_i+(Q_j-x_j)} \int_0^{Q_j} (-p_i K_i(Q_i, Q_j, x_i, x_j)) f(x_i, x_j) dx_i dx_j \tag{4.30}$$

If we rearrange 4.29, we obtain Nash Equilibrium which is given by (Q_1^d, Q_2^d) such that Equation 4.31 holds for *i,j=1,2*.

$$A_i(Q_i) - B_i(Q_i, Q_j)\left(\frac{E[\rho_{ij}]r_{ij} - s_i - \tau_{ij}}{r_i + p_i - s_i}\right)$$
$$+C_i(Q_i, Q_j)\left(\frac{p_i}{r_i + p_i - s_i}\right) + \frac{\widehat{K_i^d}(Q_i, Q_j)}{r_i + p_i - s_i} = \frac{r_i + p_i - c_i}{r_i + p_i - s_i}. \tag{4.31}$$

In this chapter, we revised our mathematical model by assuming the reliability factor as a stochastic parameter. After that, we revised the corresponding centralized and decentralized reward functions. Because of the stochastic nature of the reliability factor, agencies needed to optimize their relief aid amount after demand realizations in case 3 and 5. First, we find an analytical solution for optimal aid amount after demand realizations which depends on the stocking decisions and demand realizations. Second, we obtained revised reward functions which depend on the optimal aid amounts after demand realizations. Finally, we obtained the first order conditions which characterize the centralized solution and the Nash Equilibrium. However, because of the complexity of the revised mathematical model, we could not show the convexity of the centralized model or the uniqueness of the Nash Equilibrium in this chapter. On the other hand, we conducted numerical analysis to understand the characteristics of solutions for the centralized and decentralized models in Chapter 5 these analyses give valuable insights about the nature of the problem.

Chapter 5

NUMERICAL ANALYSIS

In this chapter, we conducted an extensive numerical analysis to get insights for the organizations which seek cooperation opportunity and to show the importance of the cooperation efforts before and after the disaster. Coskun et al. [13] showed the importance of sharing of the surplus inventory after demand realizations and concluded that both agencies can get more reward in this way. In this chapter, we analyzed the effect of the coordinated inventory decisions before disasters by comparing two cooperation scenarios. In many cases, we calculated the "price of anarchy" (PoA) which denotes the inefficiency of the system and is calculated as the ratio of the rewards between the coordinated system and the decentralized system. In Section 5.1, we analyzed the results of the mathematical model with deterministic reliability factor and in Section 5.2, we analyzed the results of the mathematical model with a stochastic reliability factor.

5.1 Results of the mathematical model with a deterministic reliability factor

In this section, we first assume that both agencies' parameters are identical and show that the effects of the variability of demands, the effect of the reliability factor, and the effect of the transshipment cost by changing the parameters in a symmetrical way (Subsection 5.1.1). On the other hand, clearly, there is no symmetry in real life and to get more meaningful results, we conduct an analysis with asymmetric parameters in Subsection 5.1.2.

Table 5.1: Base model parameters with deterministic reliability factor and symmetric case

Parameter	Value
$\mu_1 = \mu_2$	272,953
$\sigma_1 = \sigma_2$	60,596
$r_1 = r_{12}$	4,000
$r_2 = r_{21}$	4,000
$c_1 = c_2$	1,000
$p_1 = p_2$	3,000
$\tau_{12} = \tau_{21}$	300
$\rho_{12} = \rho_{21}$	1

5.1.1 Numerical Analysis - Symmetric Case

In this subsection, we assume that the model parameters are identical and are presented in Table 5.1. Since we do not have any realistic parameter settings, we obtained these parameters from Coskun et al. [13]. The authors estimated the earthquake risk of Istanbul and depending on this risk, they also estimated the demand distribution for relief aid in Istanbul by analyzing the risk of every district of Istanbul. Note that throughout this subsection, we used these base parameter values. In this part, it is assumed that the distributions of the demand quantities are independent and truncated normal distribution with mean μ and variance σ^2.

An example to the best response curves

Figures 5.1 and 5.2 show the best response functions, the decentralized equilibrium, and centralized solutions, respectively. Since the problem is symmetric in this case, it is natural to have $Q_1^* = Q_2^*$ for both centralized and decentralized problems. In Figure 2, we observe that as the reliability factor decreases, the slope of the best response functions and the difference between solution values also decreases. It is clear that if $\rho = 0$, the centralized and decentralized solutions are identical because the organizations cannot help each other and they face their own newsvendor-variant problem.

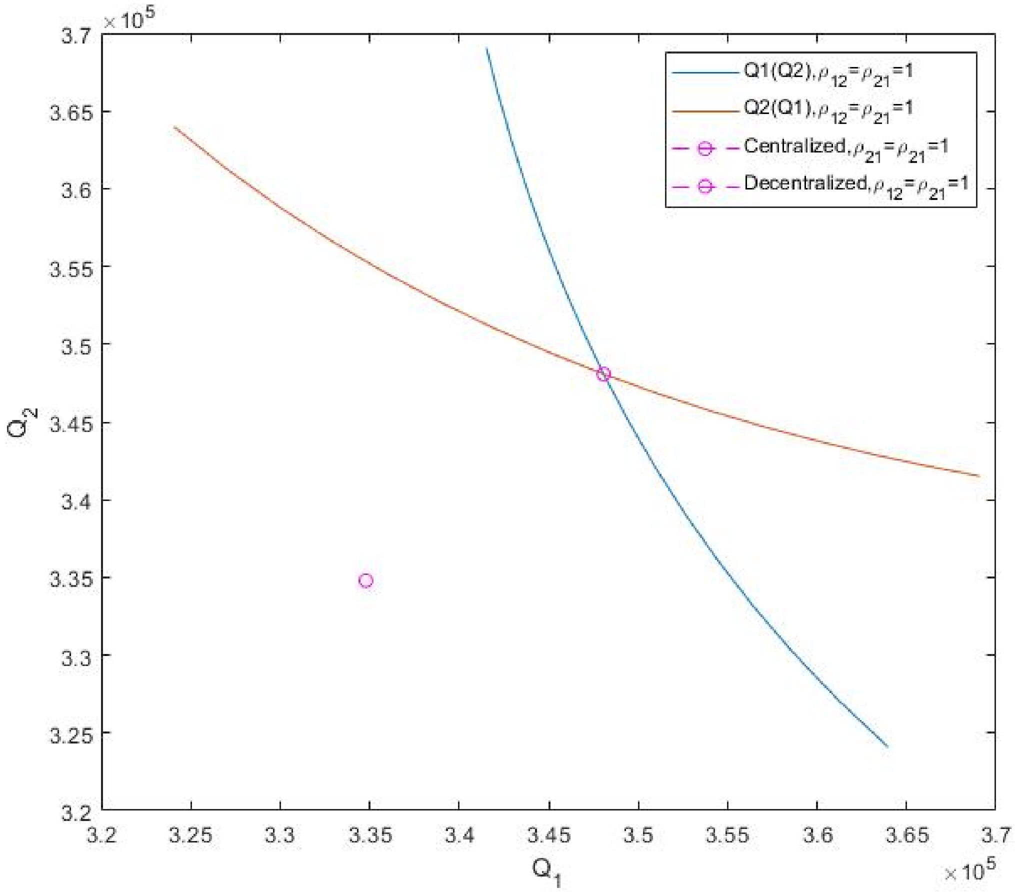


Figure 5.1: Best response curves and and the centralized solution

Effect of the variability of demands

As the variability of the demand distributions increases, both centralized and decentralized profits decrease as expected. Some countries face regular floods during rainy seasons and their demands for relief items has a small variance. On the other hand,

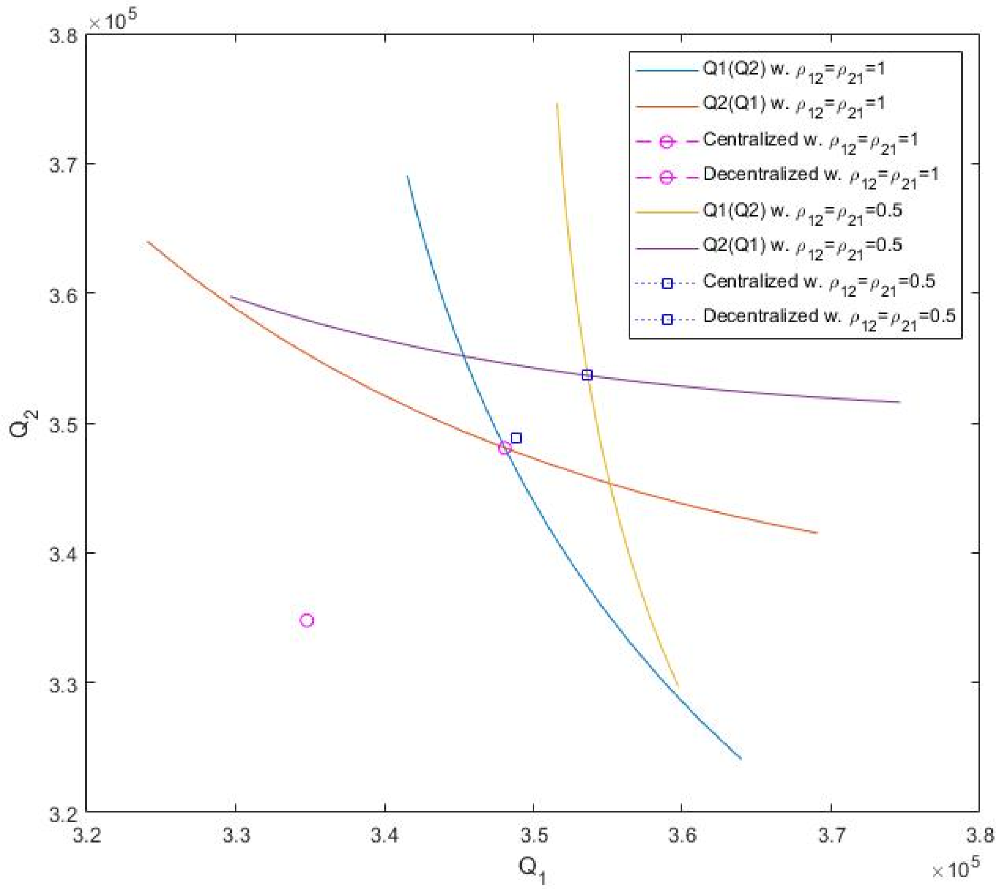
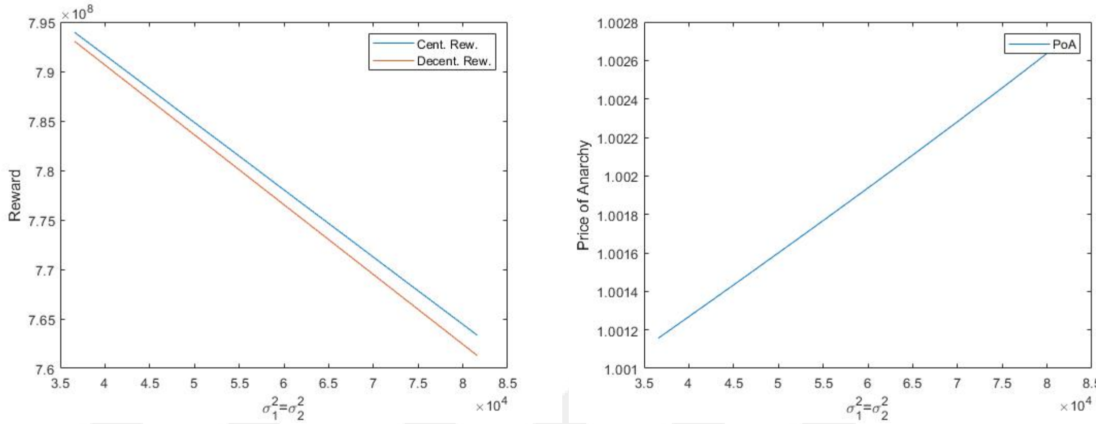


Figure 5.2: Best response curves and the centralized solution w. ρ_{ij}

some countries are prone to the risk of major disasters such as earthquakes with a low probability. Therefore, analyzing the effect of demand variability is of interest. In Figure 5.3, it can be seen that the importance of coordination increases even if total net rewards decrease as variability increases. It means that the coordinated inventory decisions become more viable as the variability of the demand distributions increases interestingly.



(a) Rewards vs. variance

(b) Price of anarchy vs. variance

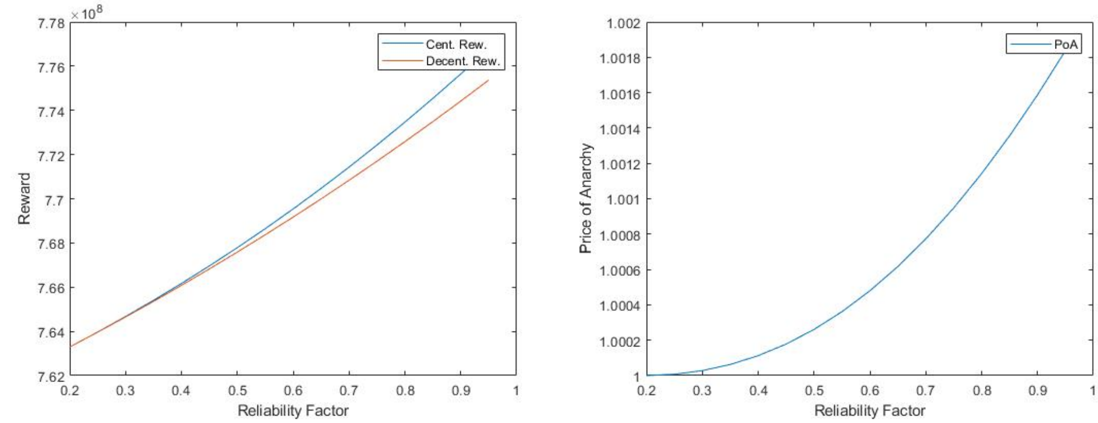
Figure 5.3: The effects of variance of demands

Effect of the reliability factor

Sometimes, relief items do not reach the other organization intact or the arrival times of the aids are late. Therefore, the organization who faces stock-out after demand realization can not use all the items which come from the other organization. In this paper, the ratio of total usable relief items to total relief items which are sent is defined as the reliability factor and the analysis of this factor can give useful insights to the humanitarian organizations. In Figure 5.4, the total net rewards for both centralized and decentralized cases increase as the reliability factor increases. This implies that organizations may want to invest to increase the reliability factor if the cost of investment is less than the reward gained by increasing it. Furthermore, the PoA increases with the reliability factor and it makes the coordination even more important for the agencies.

Effect of the transshipment cost

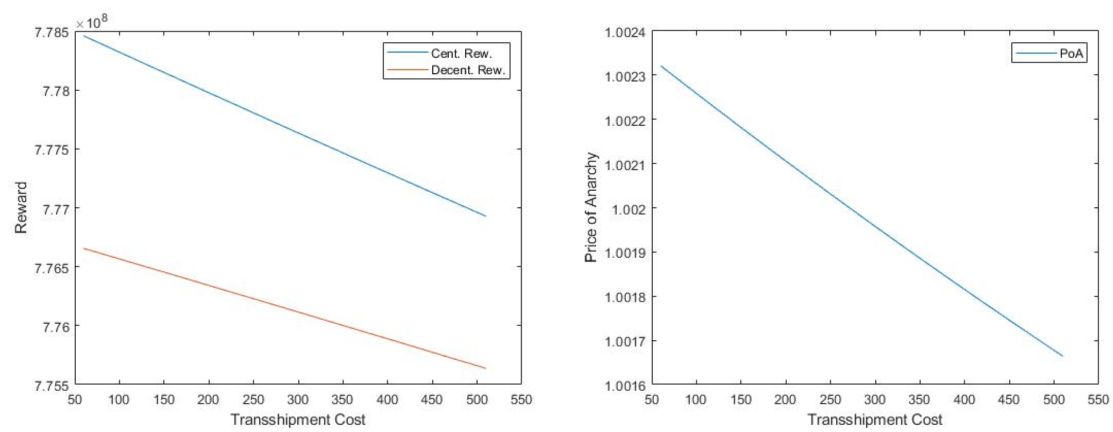
The transportation cost can be interpreted to be proportional to the distance between the warehouses of the two agencies. Figure 5.5 shows that as transportation cost



(a) Rewards vs. reliability factor (b) Price of anarchy vs. reliability factor

Figure 5.4: The effect of the reliability factor

increases, total profits and the importance of coordination decreases. It is clear that if the organizations' service regions are close to each other, the importance of the coordination should not be ignored.



(a) Rewards vs. transshipment cost (b) Price of anarchy vs. transshipment cost

Figure 5.5: The effect of the transshipment cost

Futhermore, the summary of the Subsection 5.1.1 can be seen in Table 5.2.

Table 5.2: Summary of Subsection 5.1.1

Parameter	$Q_1^c = Q_2^c$	$Q_1^d = Q_2^d$	$\pi^c(Q_1, Q_2)$	$\pi^d(Q_1, Q_2)$	PoA
$\sigma_1 = \sigma_2 \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\rho_{12} = \rho_{21} \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\tau_{12} = \tau_{21} \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$

Table 5.3: Base model parameters with deterministic reliability factor and asymmetric parameters

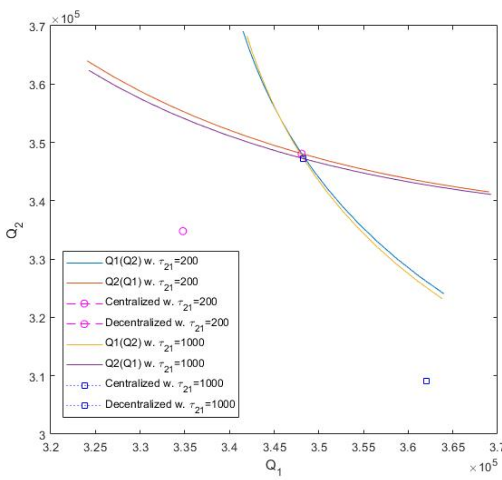
Parameter	Value
$\mu_1 = \mu_2$	272,953
$\sigma_1 = \sigma_2$	60,596
$r_1 = r_{12}$	4,000
$r_2 = r_{21}$	4,000
$c_1 = c_2$	1,000
$p_1 = p_2$	3,000
$\tau_{12} = \tau_{21}$	300
$\rho_{12} = \rho_{21}$	1

5.1.2 Numerical Analysis - Asymmetric Case

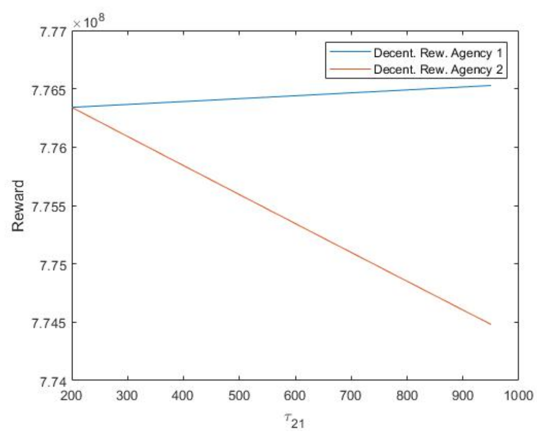
In this part, the asymmetric characteristics of the agencies are analyzed. In practice, the asymmetric case is most likely to occur and it is important to get some insights from asymmetric parameter settings. The initial model parameters are given Table 5.3. For every parameter analysis, we analyzed the effect of the asymmetric behavior of the examined parameter by changing it and the other parameters were the same with Table 5.3.

Effect of different transportation costs

In Figure 5.6, the effect of the transportation cost of agency 2 is analyzed. As the transportation cost of agency 2 increases, the decentralized reward of agency 2 decreases as expected but agency 1's reward increases interestingly. The following backward induction can be an interpretation of this analysis. As the transportation cost of agency 2 increases, agency 2 wants to keep less stocks because the gain to help agency 1 decreases. The stock-out risk of agency 2 increases as it keeps less stocks and agency 1 takes advantage of this stock-out risk and increases its expected profit by keeping more stocks. This intuitive explanation is also consistent with the left-hand-side of Figure 6.



(a) Best response curves and the centralized solution with $\tau_{12} = 200$

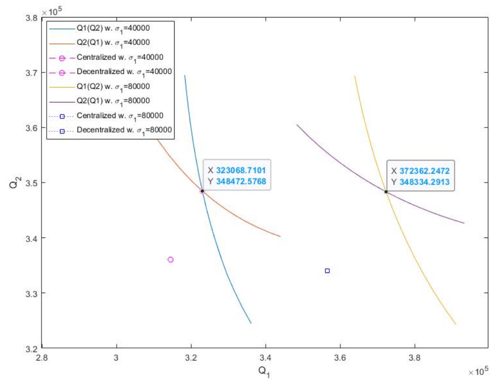


(b) Expected decentralized rewards with $\tau_{12} = 200$

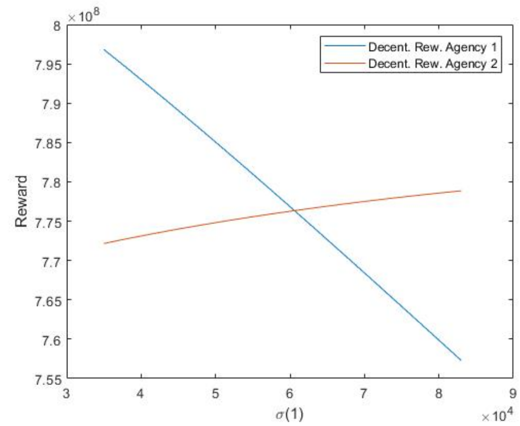
Figure 5.6: Effect of the transshipment cost

Effect of different variances

The effect of the variance is similar to varying the transportation cost. The similar intuitive explanation is also valid in this case and the change of the equilibrium points and the decentralized profits for each agency can be seen in Figure 5.7.



(a) Best response curves and the centralized solution with $\sigma_2 = 60,596$

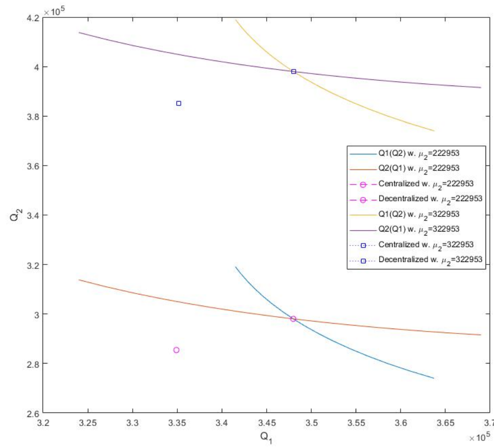


(b) Expected decentralized rewards with $\sigma_2 = 60,596$

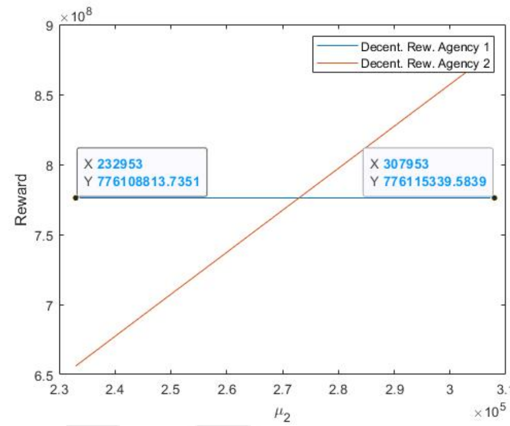
Figure 5.7: Effect of the variability

Effect of different means

Some countries try to decrease the impact of possible disasters by investing in their infrastructure. In such a case, the expected demand for relief item in that location may decrease significantly. Clearly, this situation affects the decision of the agency which tries to serve that location or country. Moreover, it is important to analyze what is the effect of this circumstance to the other agency as well. In Figure 5.8, the changes in the best response functions and equilibrium point can be observed. As the demand of agency 2 increases, the reward of agency 2 increases as expected. Surprisingly, the profit of agency 1 also increases. It is hard to estimate this change intuitively because both leftover inventory and stock-out risk change for agency 2. This is an interesting observation to note for which it is difficult to find an immediate interpretation.



(a) Best response curves and the centralized solution with $\mu_1 = 272,953$



(b) Expected decentralized rewards with $\mu_1 = 272,953$

Figure 5.8: Effect of the different means

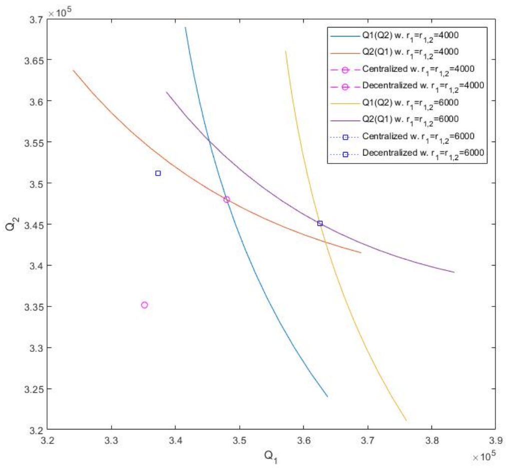
Effect of different rewards

Figure 5.9 shows the effect of reward parameter changes. As the reward of agency 1 increases, the expected reward of agency 1 increases and the expected profit of agency 2 decreases. Since agency 1 keeps more stocks, agency 2 earns less rewards from location 1 and therefore its expected reward decreases.

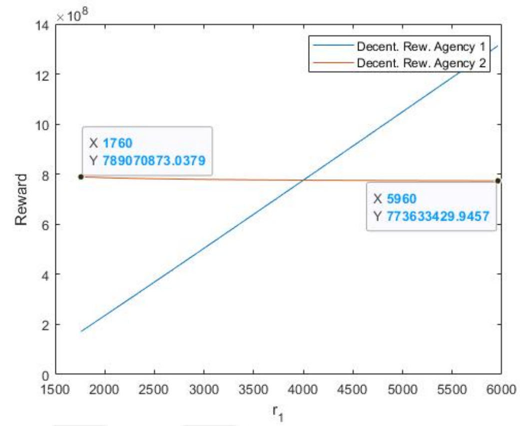
Futhermore, the summary of the Subsection 5.1.2 can be seen in Table 5.4.

Table 5.4: Summary of Subsection 5.1.2

Parameter	Q_1^c	Q_2^c	Q_1^d	Q_2^d	$\pi_1^d(Q_1, Q_2)$	$\pi_2^d(Q_1, Q_2)$
$\tau_{12} \leftrightarrow, \tau_{21} \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$
$\sigma_1 \uparrow, \sigma_2 \leftrightarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\mu_1 \leftrightarrow, \mu_2 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$r_1 = r_{12} \uparrow, r_2 = r_{21} \leftrightarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$



(a) Best response curves and the centralized solution with $r_2 = 4000$



(b) Expected decentralized rewards with $r_2 = 400$

Figure 5.9: Effect of different reward parameters

5.2 *Results of the mathematical model with a stochastic reliability factor*

In this section, we conduct numerical analysis on the case with a stochastic reliability factor. Since making the reliability factor stochastic does not change the expected reward functions significantly, we expect that the effect of the other parameters will not change in this model. On the other hand, it is important to understand how the solutions and the corresponding expected reward values change as the mean and the variance of the reliability factory changes. In Chapter 4, we defined the stochastic reliability factor parameter, ρ_{ij}, which has a probability density function that has a range between 0 and 1. In the numerical analysis, we use Beta distribution because of its flexibility as the distribution of the reliability factor. As in Section 5.1, we conducted several analyses with two cases having symmetric and asymmetric parameters.

5.2.1 Beta Distribution

The Beta distribution is one of the continuous probability distributions which is defined on the interval [0, 1] parametrized by two positive shape parameters, denoted by α and β. These two shape parameters control the shape of the beta distribution and the mean and the variance for beta distribution are known in terms of α and β (see Equation 5.1). Since we need to use the mean and the variance of the beta distribution in the numerical analysis, we obtained the α and β values in terms of mean and variance as follows.

First, let set μ and σ^2 for beta distribution,

$$\begin{aligned} \mu &= \frac{\alpha}{\alpha+\beta} \\ \sigma^2 &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \end{aligned} \tag{5.1}$$

After that, if we solve the Equation 5.1 for α and β, we obtain

$$\begin{aligned} \alpha &= (\frac{1-\mu}{\sigma^2} - \frac{1}{\mu})\mu^2 \\ \beta &= \alpha(\frac{1}{\mu} - 1). \end{aligned} \tag{5.2}$$

Throughout the numerical analysis, we calculate the shape parameters for a given mean and variance. Furthermore, we provide the Figure 5.10 to show the flexibility and suitability of the beta distribution as an example.

5.2.2 Numerical Analysis - Symmetric Case

In this subsection, we assumed that the model parameters are identical and are given Table 5.5 . We use the same parameter values except the stochastic reliability factor. We assume that the reliability factor has a Beta distribution with mean 0.5 and variance 1/12. Note that throughout this subsection, we use this base parameter values. In this part, it is, again, assumed that distributions of the demand quantities are independent and truncated normal distribution with mean μ and variance σ^2.

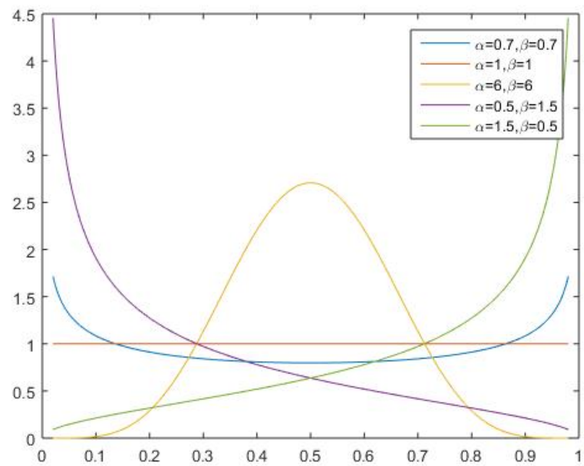


Figure 5.10: The probability density functions of the Beta distribution

Table 5.5: Base model parameters with stochastic reliability factor and symmetric parameters

Parameter	Value
$\mu_1 = \mu_2$	272,953
$\sigma_1 = \sigma_2$	60,596
$r_1 = r_{12}$	4,000
$r_2 = r_{21}$	4,000
$c_1 = c_2$	1,000
$p_1 = p_2$	3,000
$\tau_{12} = \tau_{21}$	300
$\mu_{\rho_{12}} = \mu_{\rho_{21}}$	0.5
$\sigma^2_{\rho_{12}} = \sigma^2_{\rho_{21}}$	1/12

An example to the best response curves

Figures 5.11 and 5.12 show the best response functions, the decentralized equilibrium, and centralized solutions, respectively. Since the problem is symmetric in this case, it is natural to have $Q_1^* = Q_2^*$ for both centralized and decentralized problems. In Figure 5.12, we observe that as the mean of the reliability factor decreases, the slope of the best response functions and the difference between solution values also decreases. This is because it is clear that if $\rho = 0$, the centralized and decentralized solutions are identical because the organizations cannot help each other and they face their own newsvendor-variant problem.

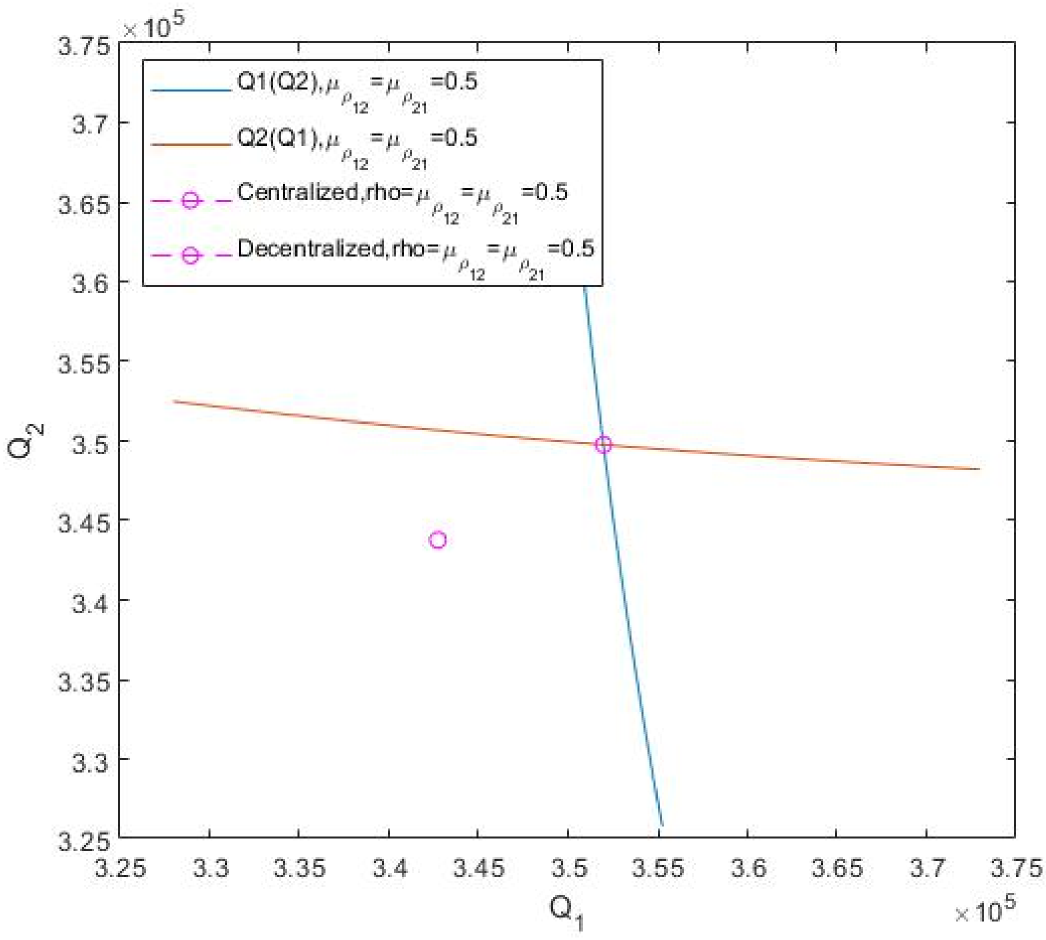


Figure 5.11: Best response curves and the centralized solution

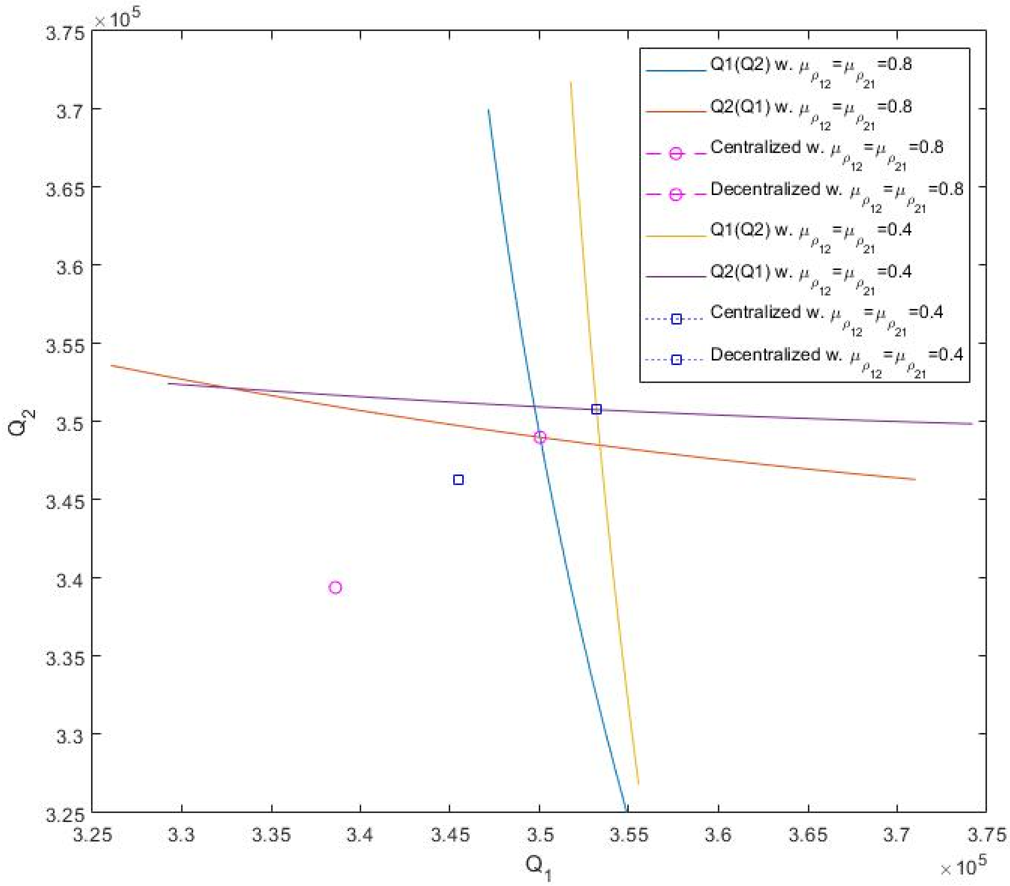
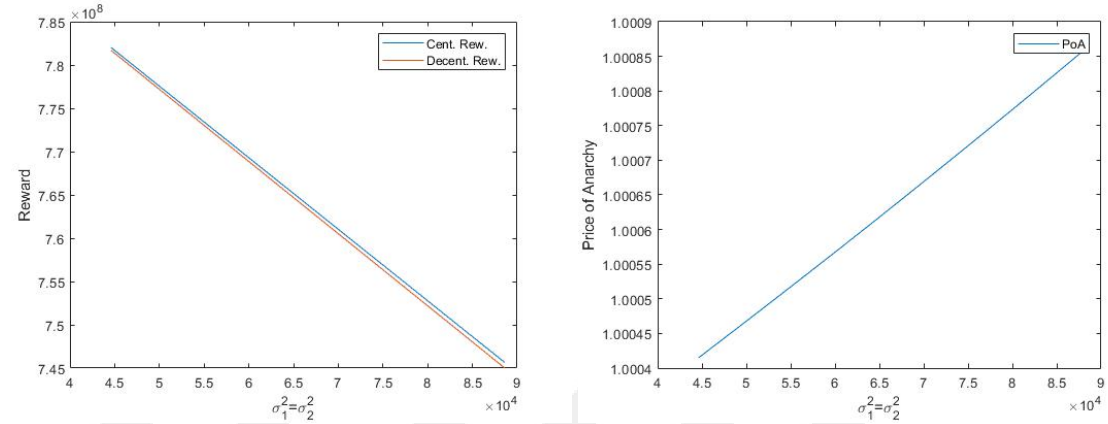


Figure 5.12: Best response curves and the centralized solution w. ρ_{ij}

Effect of the variability of demands

As mentioned at the beginning of Section 5.2, we expect to obtain the same results about the effect of the variability of demands. We conduct the same analysis with stochastic reliability factor and it can be seen that the importance of coordination increases even if total net rewards decrease as variability increases in Figure 5.13. On the other hand, the sensitivity of the price of anarchy with respect to the variability of demands decreases according to Figure 5.3, which is expected. Since the mean of the reliability factor is bigger in Figure 5.3, the opportunity coming from cooperation is bigger and correspondingly, the sensitivity effect is bigger.

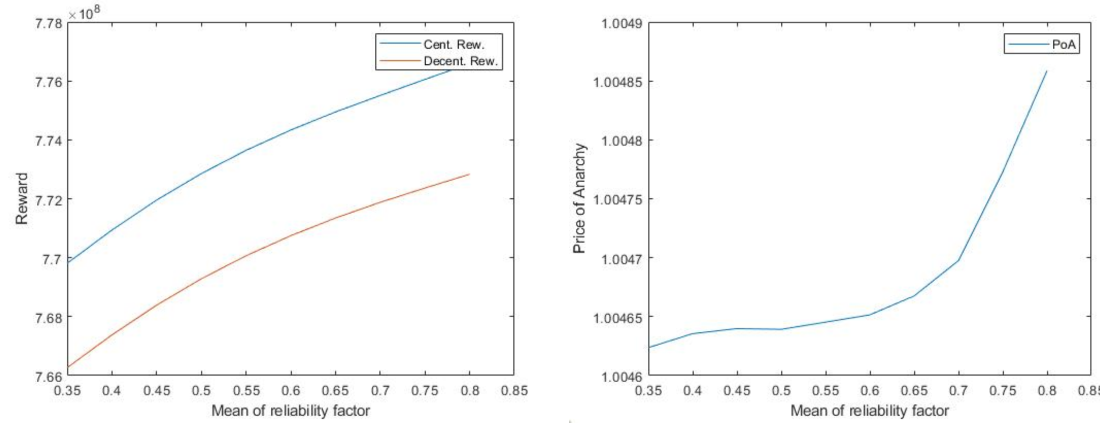


(a) Rewards vs. variance (b) Price of Anarchy vs. variance

Figure 5.13: The effects of variance of demands

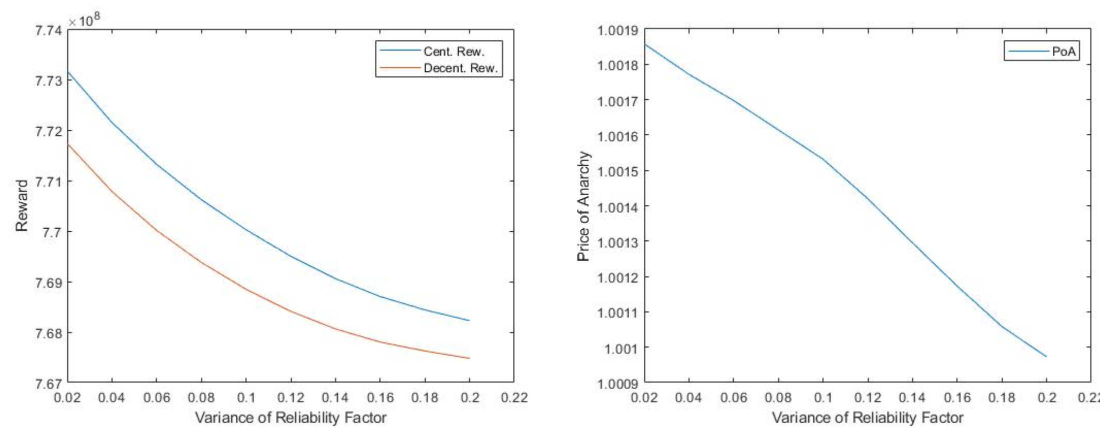
Effect of the reliability factor

In this subsection, we analyze both the effect of the mean of the reliability factor and the variance of the reliability factor. Note that the theoretical limit for the variance of the reliability factor is 0.25 from Popoviciu's inequality ([27]). As expected, the expected value of the centralized and decentralized rewards increases as the mean of the reliability factor increases (Figure 5.14). On the other hand, the expected value of the rewards decreases as the variance of the reliability factor increases (Figure 5.15). This is also an intuitive result because as the variance of the reliability factor increases, the reward which is coming from cooperation decreases. Furthermore, the changes in the price of anarchy with respect to the changes in the mean and the variance of the reliability factor can be seen in Figure 5.14 and 5.15. We may interpret this as follows. The relief agencies may want to invest to increase the expectation of the reliability factor or decrease the variance of reliability factor according to the corresponding rewards and costs of the probable policies.



(a) Rewards vs. reliability factor (b) Price of anarchy vs. reliability Factor

Figure 5.14: The effect of the mean of the reliability factor

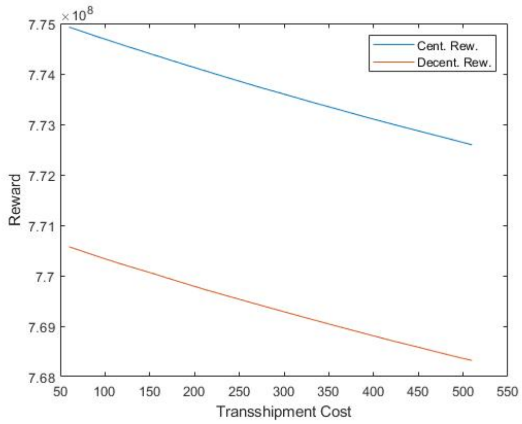


(a) Rewards vs. reliability Factor (b) Price of anarchy vs. reliability factor

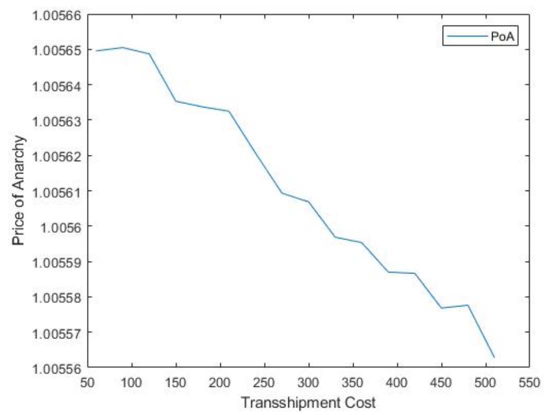
Figure 5.15: The effect of the variance of the reliability factor

Effect of the transshipment cost

The interpretation of the effect of the transshipment cost is almost the same with the deterministic version of the same problem as expected. It can be said that humanitarian organizations which are close to each other can get more rewards by coordinating their inventory decisions (Figure 5.16). Since the transshipment cost is relatively very low compared to the reward and penalty cost parameters in the humanitarian settings, its effect on the expected rewards and the price of anarchy is minimal. Therefore, it can be concluded that the organizations should try to cooperate even if they are located far away.



(a) Rewards vs. transshipment cost



(b) Price of anarchy vs. transshipment cost

Figure 5.16: Effect of the transshipment cost

Futhermore, the summary of the Subsection 5.2.2 can be seen in Table 5.6.

Table 5.6: Summary of Subsection 5.2.2

Parameter	$Q_1^c = Q_2^c$	$Q_1^d = Q_2^d$	$\pi^c(Q_1, Q_2)$	$\pi^d(Q_1, Q_2)$	PoA
$\sigma_1 = \sigma_2 \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\mu_{\rho_{12}} = \mu_{\rho_{21}} \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\sigma_{\rho_{12}} = \sigma_{\rho_{21}} \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\tau_{12} = \tau_{21} \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$

5.2.3 Numerical Analysis - Asymmetric Case

In this part, the asymmetric characteristics of the agencies are analyzed with the stochastic reliability factor. In practice, the asymmetric case is most likely to occur and it is important to get some insights from asymmetric parameter settings. The initial model parameters are presented in Table 5.5. For every parameter analysis, we analyzed the effect of the asymmetric behavior of the examined parameter by changing it and the other parameters were the same with Table 5.5 as we set their values in Subsection 5.1.2. Since the results of these analyses are the same with the case of the deterministic reliability factor of the same analysis, we provide the corresponding results in Appendix A. These figures show the effects of different transshipment cost, different mean and variance of demands, and different reward parameters and these effects are the same with Subsection 5.1.2.

5.3 Price of the uncertainty of the reliability factor

The difference between the models in Chapters 3 and 4 is the uncertainty of the reliability factor. In Chapter 3, we assume that the reliability factor is a deterministic variable. In Chapter 4, we extend the proposed model by relaxing this assumption so that the reliability factor is a stochastic variable. In real life, the relief agencies may ignore the uncertainty of the reliability factor and decide their optimum centralized quantities according to their assumption of the reliability factor. We analyze the effect of the assuming the reliability factor as a deterministic parameter while it is taken as stochastic in this section. In Figure 5.17, it can be seen that the agencies get approximately 1% less reward by assuming the reliability factor as a deterministic variable in the centralized model. Furthermore, as the variance of the reliability factor increases, the price of the uncertainty of the reliability factor also increases as expected. On the other hand, ignoring the uncertainty of the reliability factor does not cause a significant loss in the decentralized system. In the decentralized system, the agencies keep less stocks by assuming the reliability factor as deterministic and their stocking decisions close to the centralized system. As a result, they get almost the same reward in the decentralized system. This analysis shows the importance of the stochasticity of the reliability factor. Therefore, extending the mathematical model with a stochastic reliability factor is valuable and the agencies should consider this characteristics as well to maximize their expected reward value.

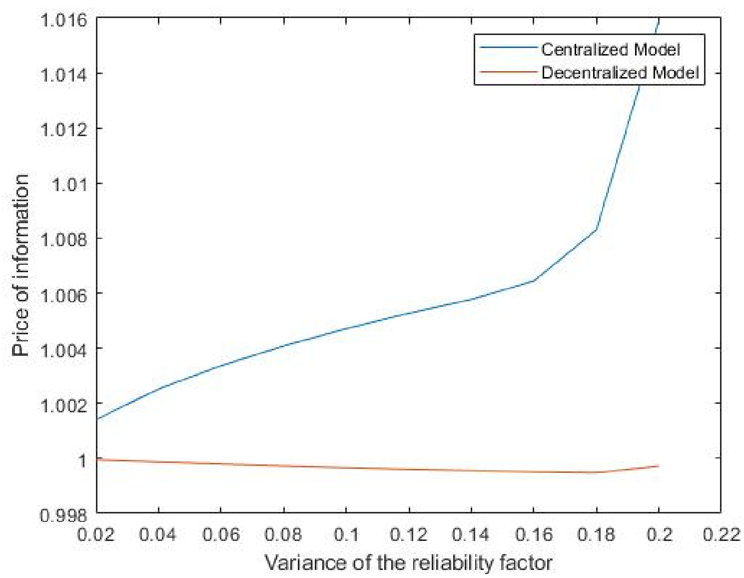


Figure 5.17: The effect of the variability of the reliability factor

5.4 Two realistic case scenarios for the Turkish Red Crescent

In addition to extensive numerical analysis, we analyze realistic case scenarios for two possible cooperation scenarios for Turkish Red Crescent. Possible cooperation partners for the Turkish Red Crescent are the Hellenic Red Cross in Greece and the Iranian Red Crescent in Iran. We choose these two organizations because they have significantly different demand characteristics. It is known that Greece has a low demand expectation with high demand variability because of the sudden-onset earthquake risk. On the other hand, Iran has a high demand expectation with low demand variability because of the predictable flood and storm seasons. In this section, we create two cooperation scenarios between Turkey-Iran and Turkey-Greece to

Table 5.7: Demand parameters for response agencies

Parameter	Turkey	Greece	Iran
μ	272,953	172,953	372,953
σ	60,596	90,596	30,596

Table 5.8: Results of the Turkey-Iran cooperation

Agency	Q_c^*	Q_d^*	π_c^*	π_d^*
Turkey	343,225	353,409	766,277,927	766,764,881
Iran	411,147	412,379	1,098,092,004	1,096,246,324

Table 5.9: Results of the Turkey-Greece cooperation

Agency	Q_c^*	Q_d^*	π_c^*	π_d^*
Turkey	345,056	351,907	772,084,589	770,805,155
Greece	278,590	288,149	444,787,796	444,008,708

analyze the effect of the coordinated inventory decisions. We use the same parameters with the ones in Section 5.2 to set the Turkish Red Crescent parameters. We use the same parameters for the Hellenic Red Cross and Iranian Red Crescent except demand distributions. In Table 5.7, we construct the demand parameters for all response agencies. Since we do not know the exact distributions for Iran and Greece, we choose the demand parameters according to Turkey's parameters and their demand characteristics.

When we analyze the results of both cooperation scenarios, it can be seen that Greece is the better option to cooperate in terms of stocking quantities and the expected reward value for the Turkish Red Crescent. Turkey needs to keep less stocks and gets more reward if it cooperates with Greece. Interestingly, Turkey gets more reward if it chooses decentralized cooperation scenarios with Iran even if it needs to keep more stocks. From these two analyses, it can be concluded that centralized cooperation with the Hellenic Red Cross is a better option for the Turkish Red Crescent.

5.5 Insights for the relief agencies

- The importance of coordination increases even if the total expected rewards decrease as variability increases. Therefore, if the predictability of the disaster decreases, the relief organizations would gain more by coordinating their inventory decisions.

- As the mean of the reliability factor increases, the expected rewards in both cooperation scenarios increases. This means that the agencies may want to invest to increase the reliability factor by considering the resulting opportunities.

- The importance of coordination and the expected rewards decrease as transportation costs increases as expected. However, the corresponding decrease in the expected rewards is very small because the importance of the transportation cost in humanitarian relief is very low compared to the penalty cost. Therefore, agencies should try to coordinate their inventory decisions even if they are located far away.

- If one of the agencies invests its infrastructure to decrease their demand variability, this increases the expected rewards of the organization who invested in its infrastructure as expected. Interestingly, this investment decrease the expected rewards of the other organization and the other organization should keep more stocks in this situation.

- Lastly, when we interpret the variability of the reliability factor, we can say that the effect of the variability of the reliability factor is the same with the effect of the variability of the demand. However, the magnitude of its effect is smaller than the variability of the demand.

Chapter 6

CONCLUSION

In this thesis, we consider humanitarian agencies which stock relief items to take proactive actions for unexpected disasters such as earthquakes or tsunamis. One of the main challenges of humanitarian operations is finding an appropriate level of inventory because if the organization chooses to stock a small amount of relief items, the risk of not being able to respond to the sudden-onset disasters will be high. On the other hand, keeping a large amount of stockpile clearly costs too much. Furthermore, there are many relief agencies which try to help people in case of a disaster and their lack of coordination before and after the disaster causes some implications, as well as inefficiency. In this study, we introduce two cooperation scenarios (centralized and decentralized) between relief agencies which aim to optimize their stocking decisions by means of a newsvendor-variant mathematical model.

In the first cooperation scenario, we assume that both of the organizations decide their inventory levels together and agree to share their left-over inventory to optimize their joint efforts. In the second scenario, we assume that they reach an agreement to share their residual inventory after demand realizations, but they decide on their own inventory level independently to optimize their own efforts. The effect of the inventory sharing after demand realizations in disaster management was studied by Coskun et al. [13] and they showed that inventory sharing can increase the efficiency of the system. In this study, we further show that the effect of the coordinated inventory decisions by comparing the two aforementioned cooperation scenarios. First, we develop a stylized newsvendor-based inventory model to reflect the real world by considering the reliability factor. We obtain the open form of the expected reward function. Furthermore, we obtain the optimal solution for the centralized model and the Nash

Equilibrium (NE) for the decentralized model analytically by assuming the reliability factor is deterministic. We also consider the same mathematical model with the stochastic reliability factor and obtain the first order conditions for optimal solutions. Because of the complexity of the model with stochastic reliability factor, we can not prove the uniqueness of the optimal solution and the NE.

We conduct extensive numerical analyses to show the effect of the parameters on the corresponding reward values and solutions, and to derive insights. For example, we calculate the effects of the variability of demands, the effect of the reliability factor, and the effect of the transshipment cost by changing these parameters symmetrically and asymmetrically. We show the importance of the coordinated inventory decisions by introducing the "price of anarchy" (PoA) concept, which denotes the inefficiency of the system and is calculated as the ratio of the rewards between the coordinated system and the decentralized system. Furthermore, we analyze the effect of the stochastic reliability factor by changing its variance and means. In each case, we obtain unique optimal quantities and the Nash equilibrium point which means that we can not find any contradiction for the uniqueness of the solutions. We identify the most promising cooperation scenarios and our numerical analyses provide many insights and guidelines to relief agencies which may ignore the importance of horizontal cooperation and doubt the resulting opportunities.

There are three possible extensions of our model. As a natural extension, we can further analyze the mathematical model with a stochastic reliability factor to show the uniqueness of the solutions. Secondly, when we consider more than two agencies in the proposed model, it is not clear that the stability of cooperation between all of the agencies exists. Even if the total expected rewards will be higher, some of the agencies may get smaller rewards in the cooperative solution [33]. In the theoretical point of view, the existence of the "core" can be shown by utilizing cooperative game theory. If there is a core in this mathematical model, it means that the cooperation between the relief agencies is always beneficial for each counterpart of this agreement. Finally, some relief agencies may want to keep some portion of their surplus inventory after demand realizations to be safe for another sudden-onset disaster and our study can also consider this feature of the model as an extension along with the case of more than two humanitarian organizations.

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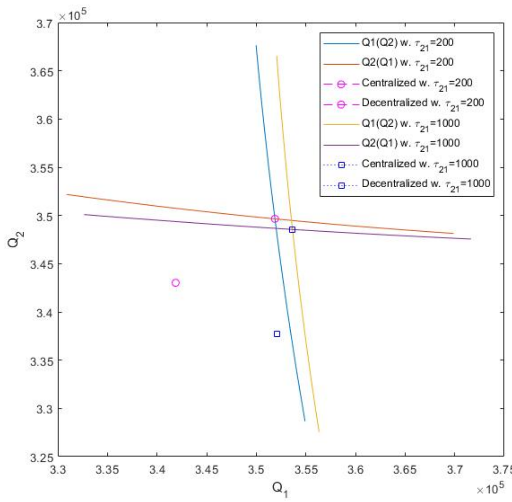
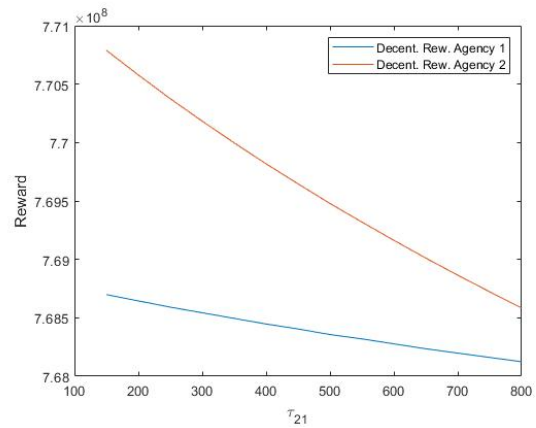
Appendix A

FIGURES IN SUBSECTION 5.2.3

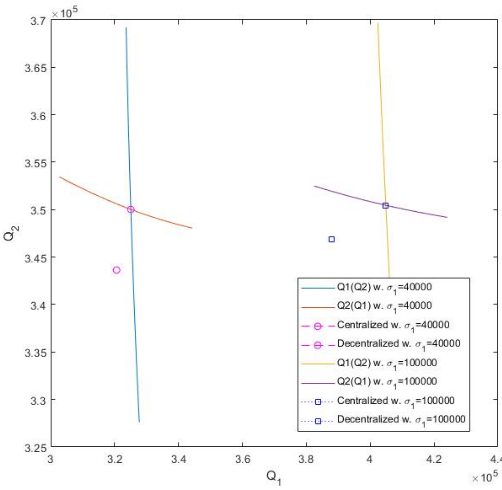


(a) Best response curves and the centralized solution with $\tau_{12} = 1000$

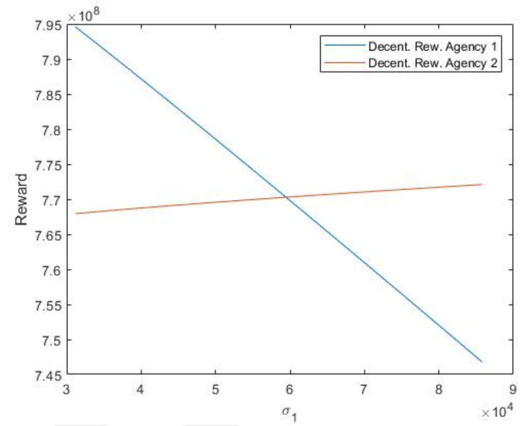


(b) Expected decentralized rewards with $\tau_{12} = 1000$

Figure A.1: Effect of the transshipment cost

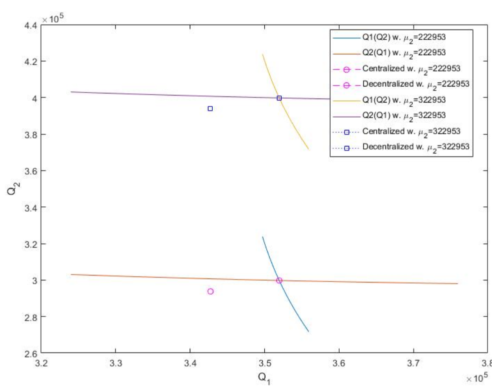


(a) Best response curves and the centralized solution with $\sigma_2 = 60,596$

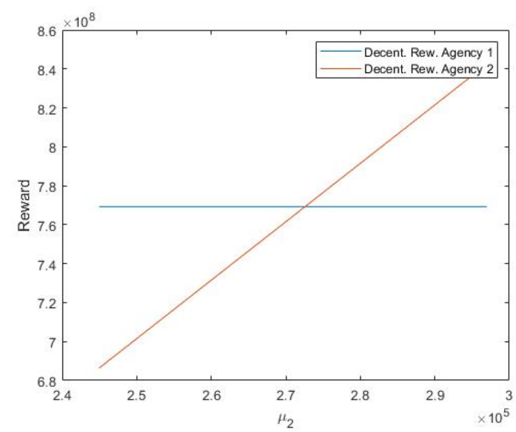


(b) Expected decentralized rewards with $\sigma_2 = 60,596$

Figure A.2: Effect of the variability

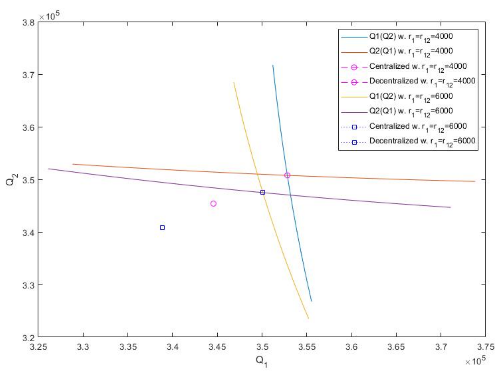


(a) Best response curves and the centralized solution with $\mu_1 = 272,593$

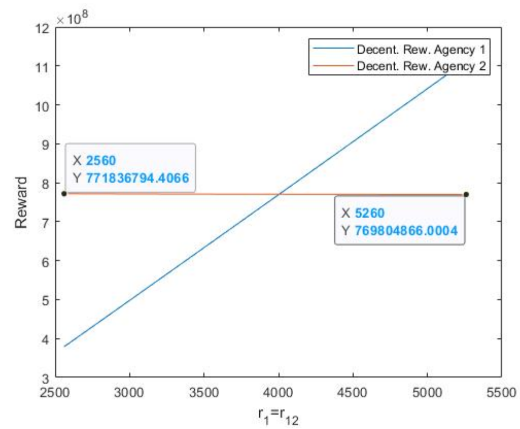


(b) Expected decentralized rewards with $\mu_1 = 272,593$

Figure A.3: Effect of the different means



(a) Best response curves and the centralized solution with $r_2 = 4000$



(b) Expected decentralized rewards with $r_2 = 4000$

Figure A.4: Effect of different reward parameters