

TIME FREQUENCY REPRESENTATION AND ITS ECONOMIC  
APPLICATIONS

A Master's Thesis

by

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APPLICATIONS

The Institute of Economics and Social Sciences  
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In Partial Fulfilment of the Requirements for the Degree of  
MASTER OF ARTS

in

THE DEPARTMENT OF  
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BİLKENT UNIVERSITY  
ANKARA

August 2009



I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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## ABSTRACT

### TIME FREQUENCY REPRESENTATION AND ITS ECONOMIC APPLICATIONS

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This thesis analyzes real oil price crises and US output gap by using time frequency representation. Firstly, time frequency representation is introduced by giving some basic definitions, formulations and illustrative examples. After that, frequency characteristics of demand-side driven and supply-side driven real oil price shocks are analyzed. Also, frequency characteristic of US output gap is analyzed by dividing the output gap series in three parts.

Keywords: Time Frequency Representation, Real Oil Price Crises, Output Gap

## ÖZET

### ZAMAN FREKANS GÖSTERİMİ VE EKONOMİK UYGULAMALARI

Akkoyun, Hüseyin Çağrı

Yüksek Lisans, İktisat Bölümü

Tez Yöneticisi: Yard. Doç. Dr. Ümit Özlale

Ağustos 2009

Bu çalışma, reel petrol fiyatı krizlerini ve ABD çıktı açığını, zaman frekans gösterimini kullanarak inceler. Önce, zaman frekans gösterimi birkaç temel tanım, formüller ve açıklayıcı örnekler vererek tanıtıldı. Bundan sonra, talep yönlü ve arz yönlü petrol fiyatı şoklarının frekans özellikleri incelendi. Ayrıca, ABD çıktı açığının frekans özellikleri, çıktı açığı serisi üç parçaya bölünerek incelendi.

Anahtar Kelimeler: Zaman Frekans Gösterimi, Reel petrol Fiyatı Krizleri, Çıktı Açığı

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# **CHAPTER 1**

## **INTRODUCTION**

In time series econometrics, a majority of the time series data is analyzed in time domain while we also observe studies that employ the frequency domain to understand the frequencies that govern a specific time series data.

In addition, to extract the data in a pre-specified frequency range, such as extracting the business cycle components of a time series, Fourier Transformation is employed to shift the data between time domain and frequency domain. Band-pass filtering serves that purpose.

On the other hand, a significant portion of the time series data exhibits non-stationarity, either in mean or in variance. That is mostly the case for the data with observations over a long period of time, which is likely to have structural breaks. Conditional volatility models are also good examples in this context, where we have time-varying variance.

However, when the data exhibits non-stationarity, Fourier transformation yields inaccurate results since it is formed of infinite

sinusoidal waves which have stationary characteristics. Thus, we cannot use band-pass filtering for a non-stationary data either in mean or in variance.

Based on the above discussion, this thesis describes and applies an alternative method, the time-frequency representation, which will not overcome the above-mentioned problems and offer a tractable way to analyze a time series data both in time domain and frequency domain simultaneously. Adopted from the field of signal extraction, time-frequency representation makes it possible to track the time-varying frequencies of a time series over time. By doing so, one can understand whether a specific point in time series is governed by low frequency or high frequency movements. Moreover, it is also possible to detect whether the dominating frequency of a time series change over time.

After providing detailed information about the time-frequency representation, two exercises follow. First, we concentrate on the real oil prices and apply TFR to understand whether the oil price episodes have different characteristics in terms of frequencies. We find that supply side drive shocks are dominated by low frequency cycles while the recent demand driven episode is governed by high-frequency cycles. Next we apply TFR to the NBER recession dates and find that expansions and recessions have different frequency dynamics.

As a result, this thesis introduces a tractable tool for non-stationary data to analyze the time domain and frequency domain characteristics in a simultaneous fashion.

## CHAPTER 2

### TIME FREQUENCY REPRESENTATION

#### 2.1. Introduction

Time frequency analysis was developed in the area of quantum mechanics by extending the Fourier transform. Before introducing the Fourier Transform, let us define two concepts.

**Time Representation:** Time representation of a signal is achieved by assigning a value to each time index of a signal. If the time index is discrete, then the signal can be called as time series data. Time representation is commonly used and easy to imagine.

**Frequency Representation:** Frequency representation is achieved by assigning a value to each frequency index of a signal where each frequency refers to an infinite sinusoidal wave in time domain and the value is directly proportional with the amplitude of the related sinusoidal wave. Frequency representation is generally used for special purposes such as band-pass filter and it is hard to imagine.

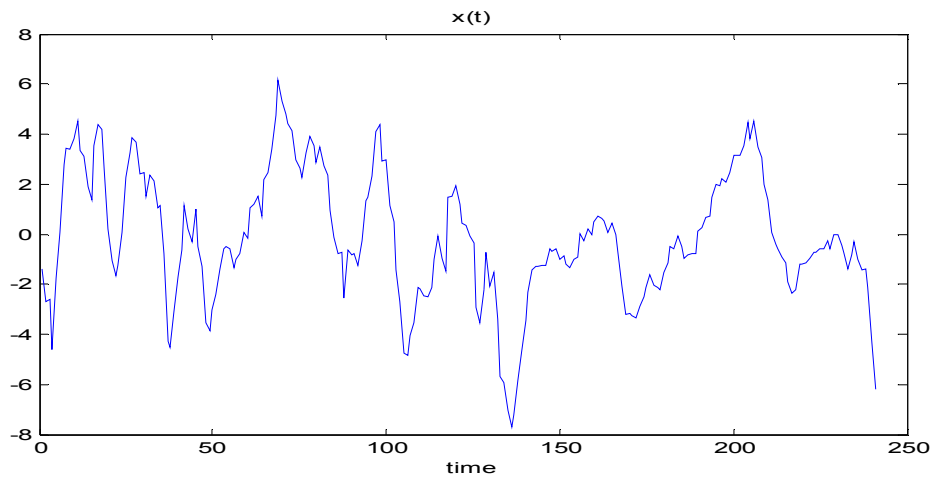
## 2.2. Fourier Transform

Frequency representation of a signal is achieved by the Fourier transform. The equation for Fourier transform:

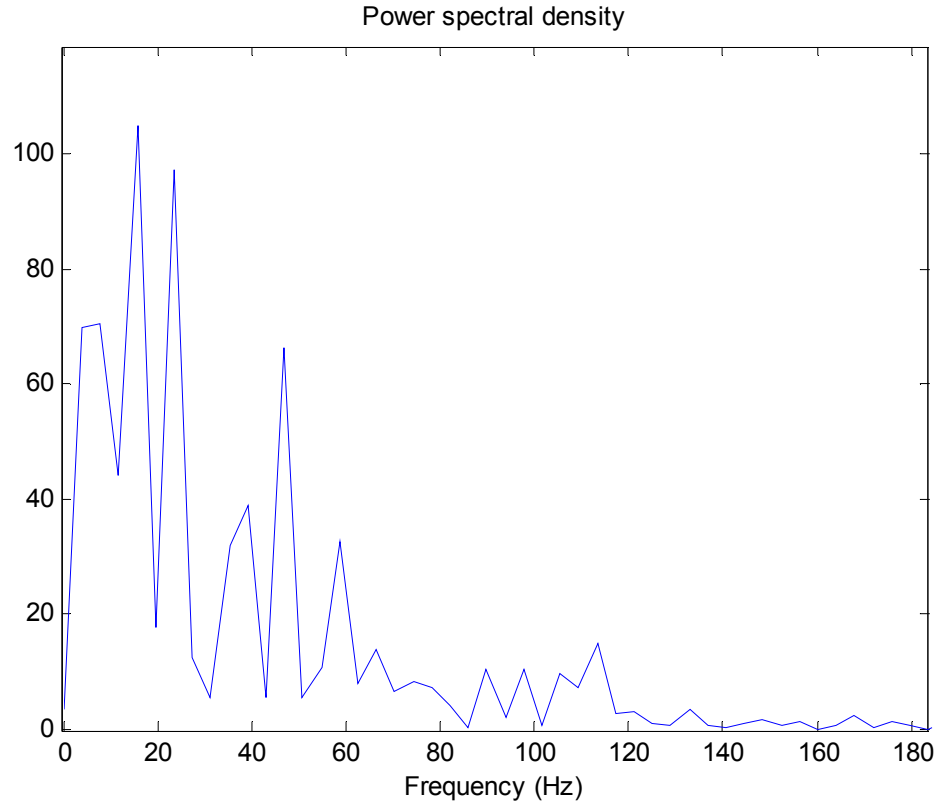
$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

where  $x(t)$  is the signal in time domain,  $X(f)$  is the frequency representation of the signal in frequency domain and the exponential part refers to sinusoidal waves of different frequencies, note that “ $e^{-j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$ ” where  $j = \sqrt{-1}$ .  $X(f)$  is a complex number with magnitude and phase components. The magnitude of  $X(f)$ ,  $|X(f)|$ , is commonly used in analysis since the phase component does not reflect a direct meaningful figure.

**Example 1:** let  $x(t)$  be a time series data with 241 observations. The graph of the signal in time domain is in Figure 2.1. , power spectrum,  $|X(f)|^2$ , is in Figure 2.2 and Figure 2.2. illustrates the fact that  $x(t)$  is generated mostly by the sinusoidal waves up to 120 Hz.



**Figure 2.1.**

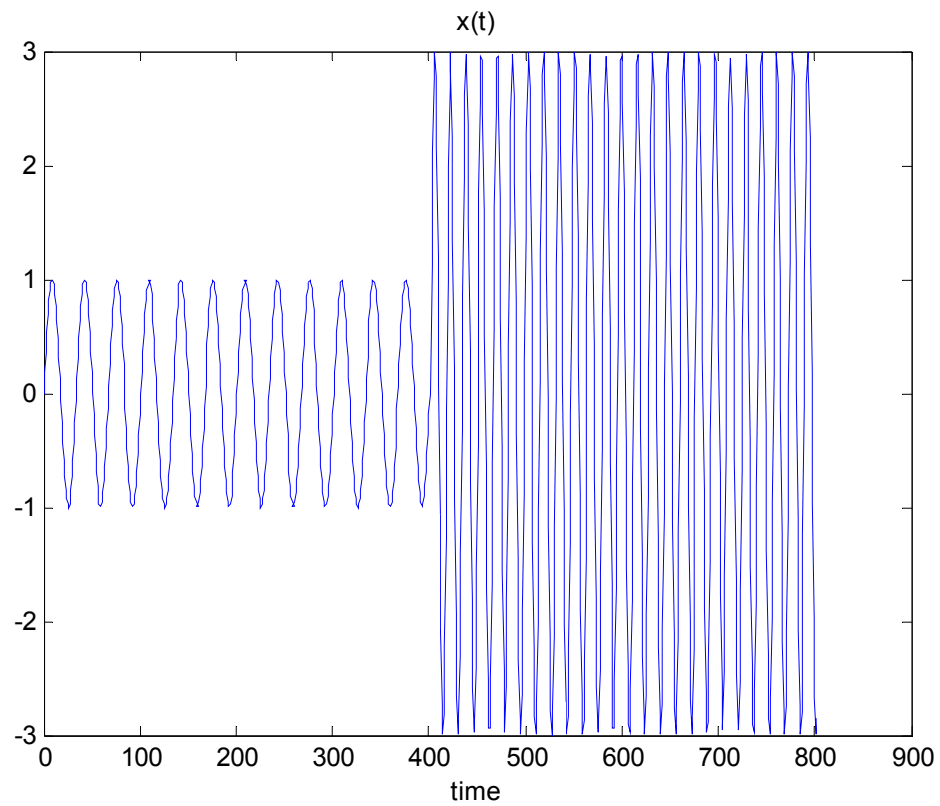


**Figure 2.2.**

Fourier transform is a tractable tool for filtering the desired frequencies of a signal. A signal in time domain can be transferred to frequency domain by using the Fourier transform. After the desired frequencies are filtered, remaining part can be transferred into time domain by using inverse Fourier transform algorithm. This operation is called as band-pass filtering. However, Fourier transform will not operate accurately when applied to nonstationary signals since the building blocks of the frequency representation,  $X(f)$ , are infinite sinusoidal waves which have stationary characteristics. An illustrative example is below.

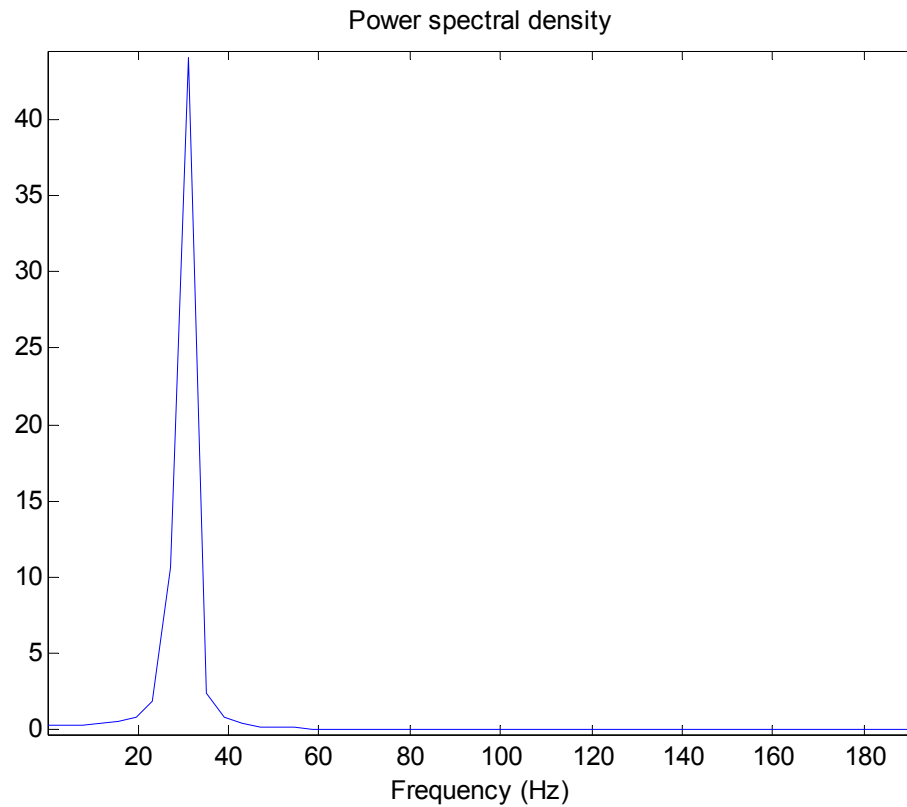
**Example 2:**  $x(t)$  is composed of two sinusoidal signals. The first 401 points of the  $x(t)$  is a sinusoidal wave with frequency of 30 Hz and amplitude of 1 and the last 401 points of the  $x(t)$  is a sinusoidal wave with frequency of 62

Hz and amplitude of 3. The graph of the signal in time domain is in Figure 2.3. and power spectrum of  $x(t)$  is in Figure 2.4.



**Figure 2.3.**

In this example, we expect to see two peaks around 30 Hz and 62 Hz. However, we only see a peak around 32 Hz. Even if we have seen two peaks around 30 Hz and 62 Hz, they will refer to infinite length sinusoidal waves and the sum of these waves will generate a signal which is different from the starting signal.



**Figure 2.4.**

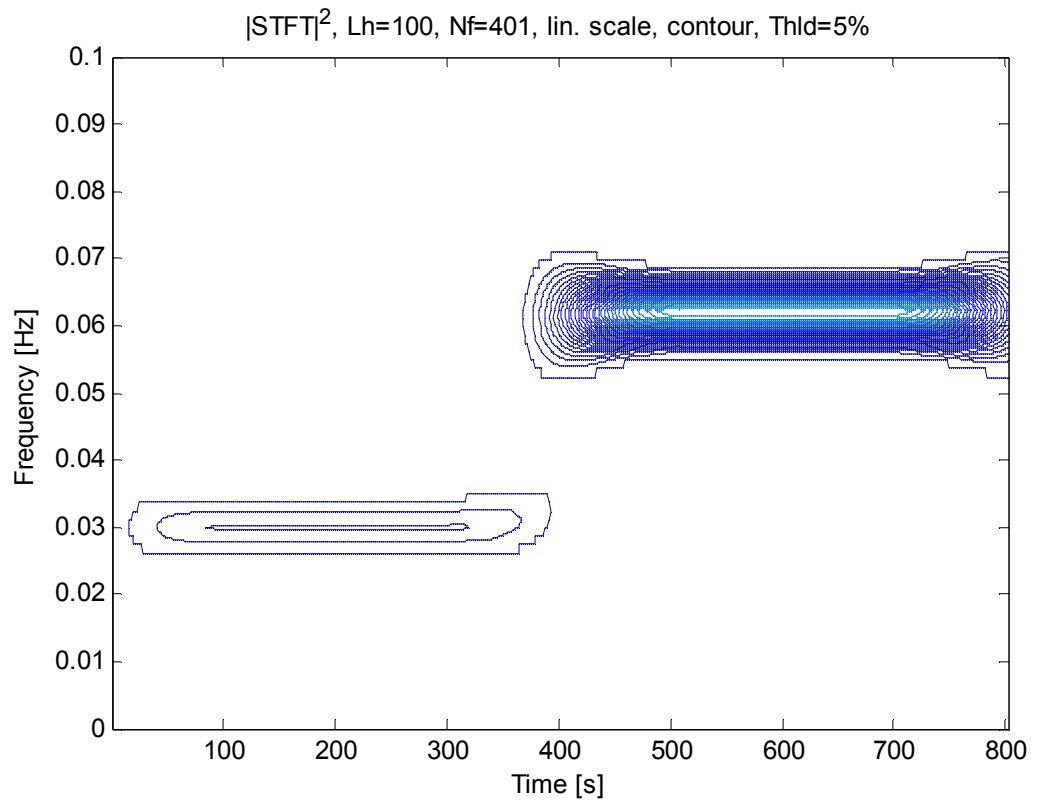
As a result, Fourier transform fails. However, time frequency representation (TFR) overcomes these failures. Before talking about TFR let us define “time frequency domain”.

**Time Frequency Domain:** Time frequency domain consists of three dimensions where the first dimension is time, the second dimension is frequency and the third dimension is the value as a function of energy, of a signal at corresponding time and frequency. For a constant time cross section at  $t^*$ , one can observe the effective frequencies at  $t^*$ . Also, for a constant frequency cross section at  $f^*$ , one can observe the time periods where  $f^*$  is effective.

### 2.3. Time Frequency Representation

TFR was developed by Eugene Wigner who won the Nobel Prize for physics in 1963 and Dennis Gabor who won the Nobel Prize for physics in 1971. It is mainly a mapping from a signal in time domain to time frequency domain. Different from Fourier transform, TFR includes time information for the frequencies. By the help of this property, nonstationary signals, which have time varying first and second moments, are best analyzed with TFR.

**Example 3:** Let  $x(t)$  be the same signal in example 2. TFR of  $x(t)$  is in Figure 2.5.

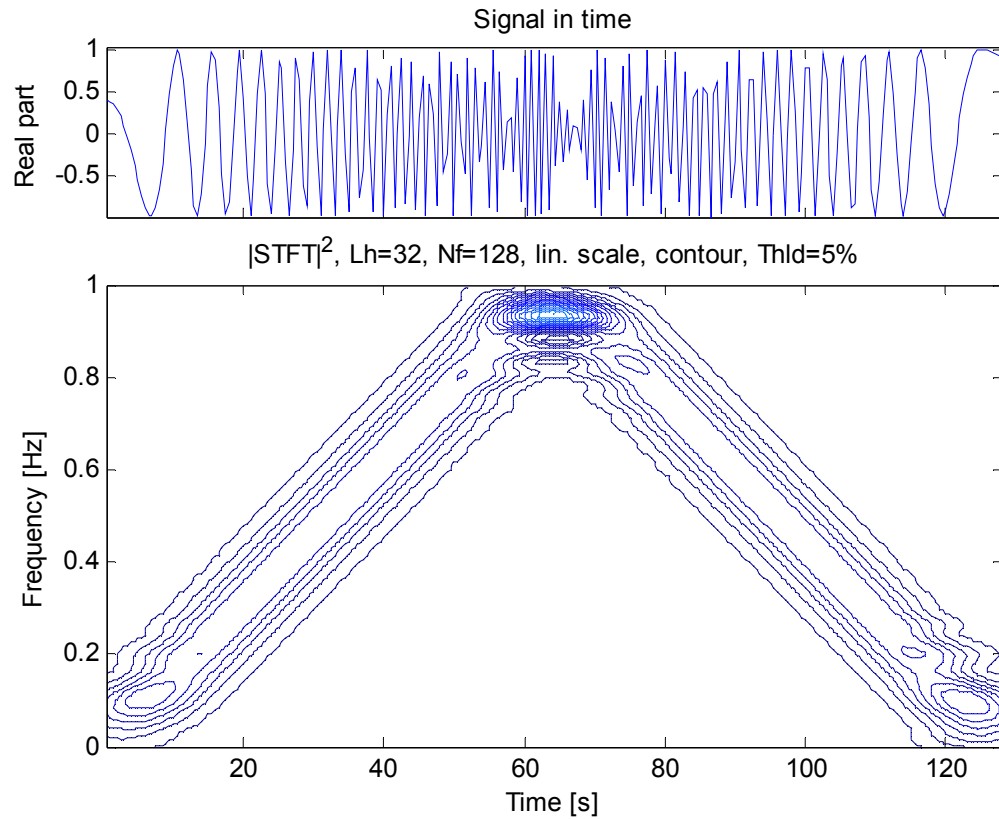


**Figure 2.5.**

In the above figure, there are two frequency bands. First one is around the normalized frequency of 0.030 and the second one is around the normalized frequency of 0.062, as expected. Also, the thresholds of the sinusoidal waves are very clear. The frequency band around 0.030 ends at 401<sup>th</sup> time point and other one starts. The value around the frequency band of 0.062 is greater than the frequency band of 0.030 which means that the variation of the sinusoidal wave with frequency of 62 Hz is higher than the variation of the sinusoidal wave with frequency of 30 Hz.

**Example 4:** Let  $x(t)$  be a signal of 128 points. In first 64 points its frequency is increasing and in last 64 points its frequency is decreasing. The plot of the signal and TFR of  $x(t)$  are in Figure 2.6. Observe that in time frequency domain, frequency increases for the first 64 points and decreases for the last 64 points. This example shows that time frequency representation can capture even the very fast variations in a time series data.

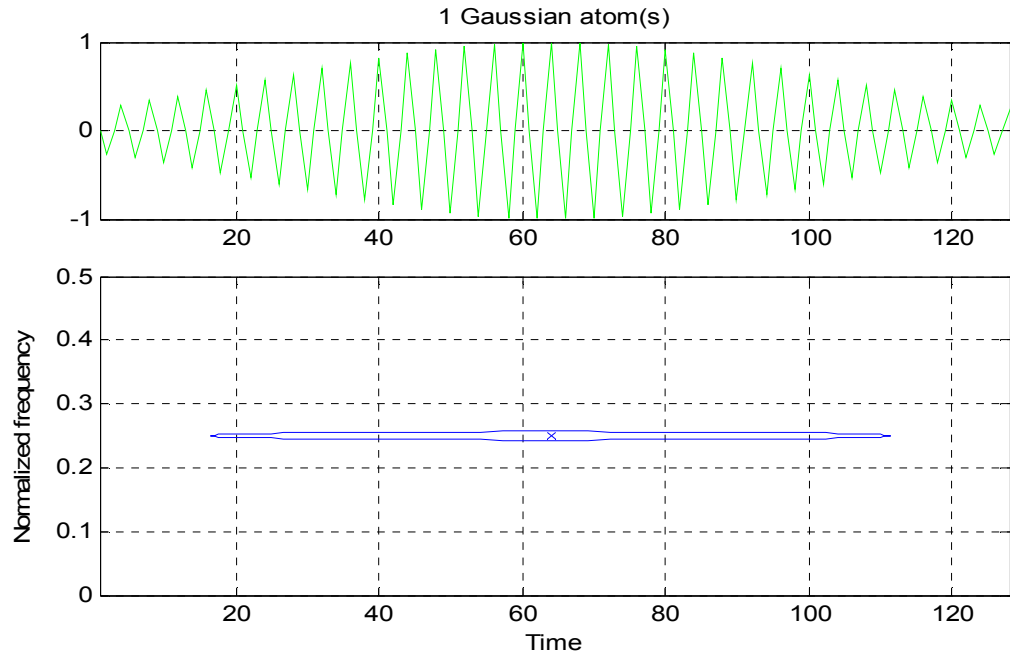
As it is mentioned in examples, TFR preserves the characteristics of time series data with time varying variance while Fourier transform fails in most cases since it can only observe infinite duration sinusoidal waves with different frequencies where the composition of these waves have stationary characteristics for different periods. However, the building blocks for time frequency transformations are not sinusoids but they are called atoms which can be seen in Figure 2.7. Thus, time frequency transformations can preserve nonstationary characteristics.



**Figure 2.6.**

Also, TFR shows the frequency evolution of a time series data that eases the observation of seasonal effects. Also, the corresponding value for specific time and frequency periods gives information about the energy of that region which is determined by these specific time and frequencies. This energy information is an increasing function of the variance of the signal. Note that, the integral of some TFRs over the entire time frequency plane gives the total energy of the signal.

Time Frequency Toolbox for MATLAB is a tool for time frequency analysis. It includes most of the time frequency transformations and a well formed tutorial with important definitions and clarifying examples. Now, some basic definitions will be introduced.



**Figure 2.7.**

**Heisenberg-Gabor Inequality:** In time domain, one has perfect resolution in time but no resolution in frequency. Similarly, in frequency domain, one has perfect resolution in frequency but no resolution in time. Heisenberg-Gabor Inequality:

$$\Delta t \cdot \Delta f \geq 1/4\pi$$

implies that one cannot have perfect resolution both in time and frequency simultaneously. If time resolution is increased i.e.  $\Delta t$  decreases then frequency resolution decreases i.e.  $\Delta f$  increases. Thus there are always some distortions or interference in time frequency representation.

**Analytical Signal:** Let  $x(t)$  be a signal and  $X(f)$  be the Fourier transform of  $x(t)$ . Define  $x_a(t)$  as the analytical signal associated with  $x(t)$  if:

$$X_a(f) = 0 \quad \text{for } f < 0$$

$$X_a(f) = X(0) \quad \text{for } f = 0$$

$$X_a(f) = 2X(f) \quad \text{for } f > 0$$

where  $X_a(f)$  is the Fourier transform of  $x_a(t)$ . The frequency representation of a real signal is symmetric around  $f=0$ . Analytical signal helps us to get rid of the unnecessary symmetric part and this decreases the interference.

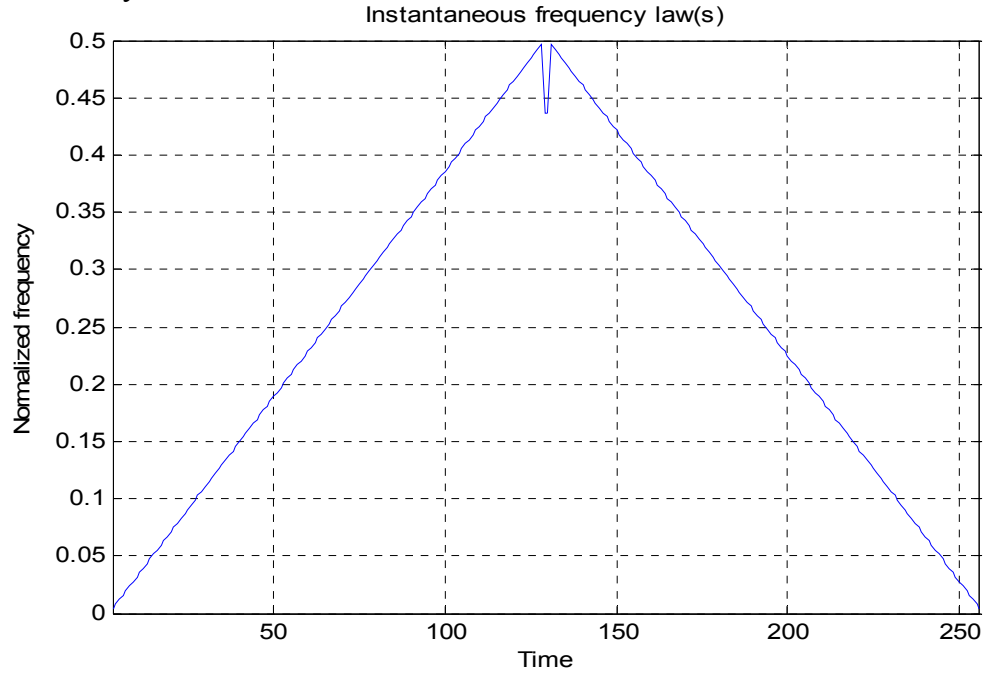
**Instantaneous Frequency:** The instantaneous frequency of  $x(t)$  estimates frequencies for  $x(t)$  as a function of time. There are different estimation methods for instantaneous frequency.

**Normalized Frequency:** Unit for frequency which is inversely proportional with the time period between two consecutive points.

**Example 5:** Let  $x(t)$  be the same signal in example 4. Instantaneous frequency plot of  $x(t)$  obtained by using Time Frequency Toolbox is in Figure 2.8. It is a better picture when compared with Figure 2.6. since it calculates an estimation for frequency at each instant in time.

**Group Delay:** Group delay is the dual of the instantaneous frequency. It estimates the average time behaviour as a function of frequency.

There are various types of time frequency transformations developed for different purposes and each transformation has its own properties. Some wide used transformations and their properties are given below. In all formulas,  $x(t)$  will be the signal and its time frequency representation will be denoted by  $F_x(t, f; \cdot)$ .



**Figure 2.8.**

## 2.4. Time Frequency Transformations

### 2.4.1. Short Time Fourier Transform (STFT)

The formula for the short time Fourier transform:

$$F_x(t, f; h) = \int_{-\infty}^{+\infty} x(u)h^*(u - t)e^{-j2\pi fu} du$$

where  $h(t)$  is a short time analysis window. STFT calculates the frequency transformation of the windowed signal for all time instants. If the length of

the window is increased i.e. time resolution is decreased, then frequency resolution increases. STFT is invertible and its MATLAB code is available.

The formula for the inverse short time Fourier transform:

$$x(t) = \frac{1}{E_h} \iint_{-\infty}^{+\infty} F_x(u, \xi; h) h(t-u) e^{j2\pi t \xi} du d\xi$$

where  $E_h = \int_{-\infty}^{+\infty} |h(t)|^2 dt$  is the energy of  $h(t)$ . This is a very important property. One can filter the noise and unwanted components in time frequency domain and transfer the remaining part from time frequency domain to time domain.

Also, STFT preserves frequency shifts and time shifts:

If  $y(t) = x(t) e^{j2\pi f_0 t}$  then  $F_y(t, f; h) = F_x(t, f - f_0; h)$  and,

If  $y(t) = x(t - t_0)$  then  $F_y(t, f; h) = F_x(t - t_0, f; h) e^{j2\pi t_0 f}$ .

#### 2.4.2. Wavelet Transform (WT)

The formula for the short time Fourier transform:

$$F_x(t, a; \Psi) = \int_{-\infty}^{+\infty} x(u) \Psi_{t,a}^*(u) du$$

where  $\Psi_{t,a}(u) = |a|^{-1/2} \Psi(\frac{u-t}{a})$ . The variable 'a' corresponds to a scale factor, in the sense that taking  $|a| > 1$  dilates the wavelet  $\Psi$  and taking  $|a| < 1$  compresses the wavelet  $\Psi$ . The basic difference between the WT and STFT is as follows: when the scale factor 'a' is changed, the duration and the bandwidth of the wavelet are both changed but window's shape remains the

same. In contrast to the STFT, which uses a single analysis window, the WT uses short windows at high frequencies and long windows at low frequencies. This partially overcomes the resolution limitation of the STFT.

The inverse transform is also available. The formula is as follows:

$$x(t) = \iint_{-\infty}^{+\infty} F_x(u, a; \Phi) \Psi_{u,a}(t) du \frac{da}{a^2}$$

where  $\Phi$  is the synthesis wavelet that satisfies  $\int_{-\infty}^{+\infty} \Psi(\xi) \Phi^*(\xi) \frac{d\xi}{|\xi|} = 1$ .

The STFT and WT are linear transformations that decompose signal into its elementary components which are well localized in time and frequency. Now, some quadratic transformations that distribute energy of a signal on time frequency plane will be introduced.

### 2.4.3. Spectrogram

The formula for the spectrogram is as follows:

$$F_x(t, f; h) = \left| \int_{-\infty}^{+\infty} x(u) h^*(u - t) e^{-j2\pi f u} du \right|^2$$

It is the squared version of the STFT. Thus we obtain the energy of the windowed part of the signal localized in time frequency plane. Window  $h(t)$  is assumed to have unit energy and the integral of the spectrogram over the entire time frequency plane gives the energy of the signal:

$$E_x = \iint_{-\infty}^{+\infty} F_x(t, f) dt df$$

Similar to STFT, it preserves time and frequency shifts.

#### 2.4.4. Scalogram

A similar distribution to the spectrogram case can be defined in the wavelet case. The scalogram is the squared version of the WT and it preserves energy i.e. the integral of the scalogram over entire time frequency plane gives the energy of the signal.

#### 2.4.5. Wigner Ville Distribution (WVD)

The WVD is a quadratic distribution. The formula for the WVD is as follows:

$$F_x(t, f) = \int_{-\infty}^{+\infty} x\left(t + \frac{u}{2}\right) x^*\left(t - \frac{u}{2}\right) e^{-j2\pi f u} du$$

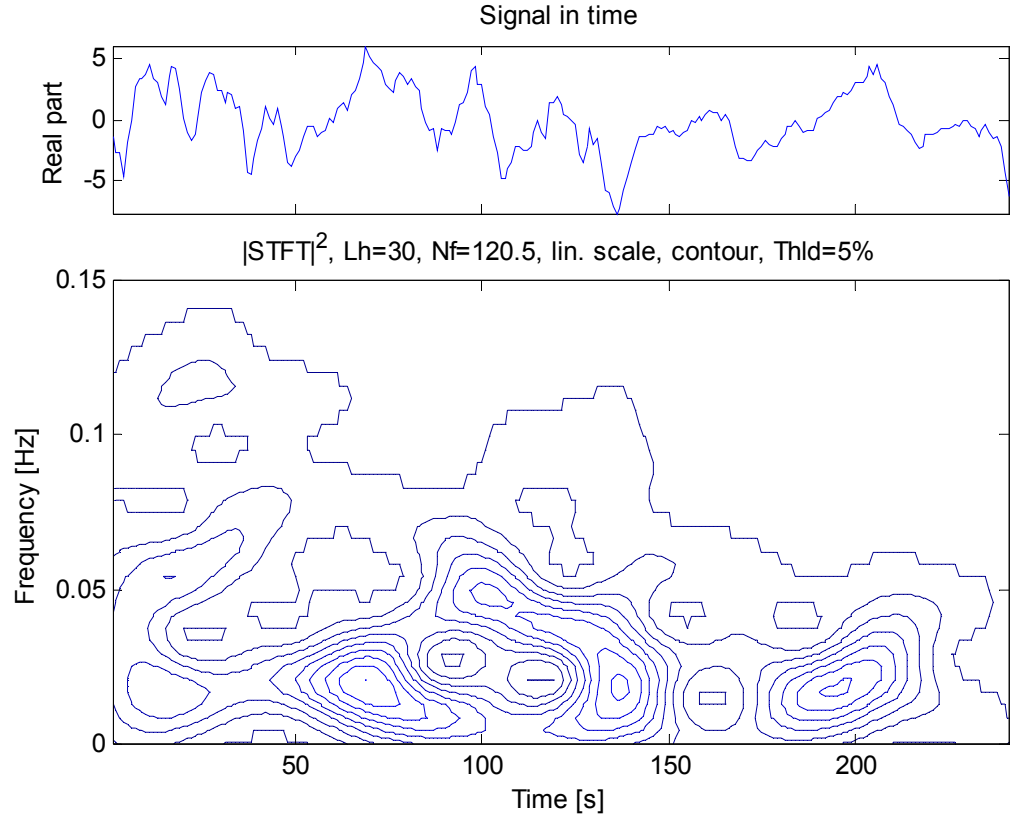
This distribution satisfies large number of desirable mathematical properties. The WVD is always real valued, it preserves time and frequency shifts. Also it preserves energy. Another important property is that the energy spectral density and the instantaneous power can be obtained as marginal distributions of the WVD:

$$\int_{-\infty}^{+\infty} F_x(t, f) dt = |X(f)|^2$$

$$\int_{-\infty}^{+\infty} F_x(t, f) df = |x(t)|^2$$

The instantaneous frequency and group delay of the signal  $x(t)$  can be recovered by using the first order moments of WVD in time and frequency domain respectively. Besides these properties, the WVD has interference terms and some methods, which are not discussed here, are developed to overcome these problems.

**Example 6:** The differences between the TFRs can easily be seen in below figures. STFT, scalogram and WVD of the same  $x(t)$  are given in Figure 2.9., Figure 2.10., Figure 2.11.



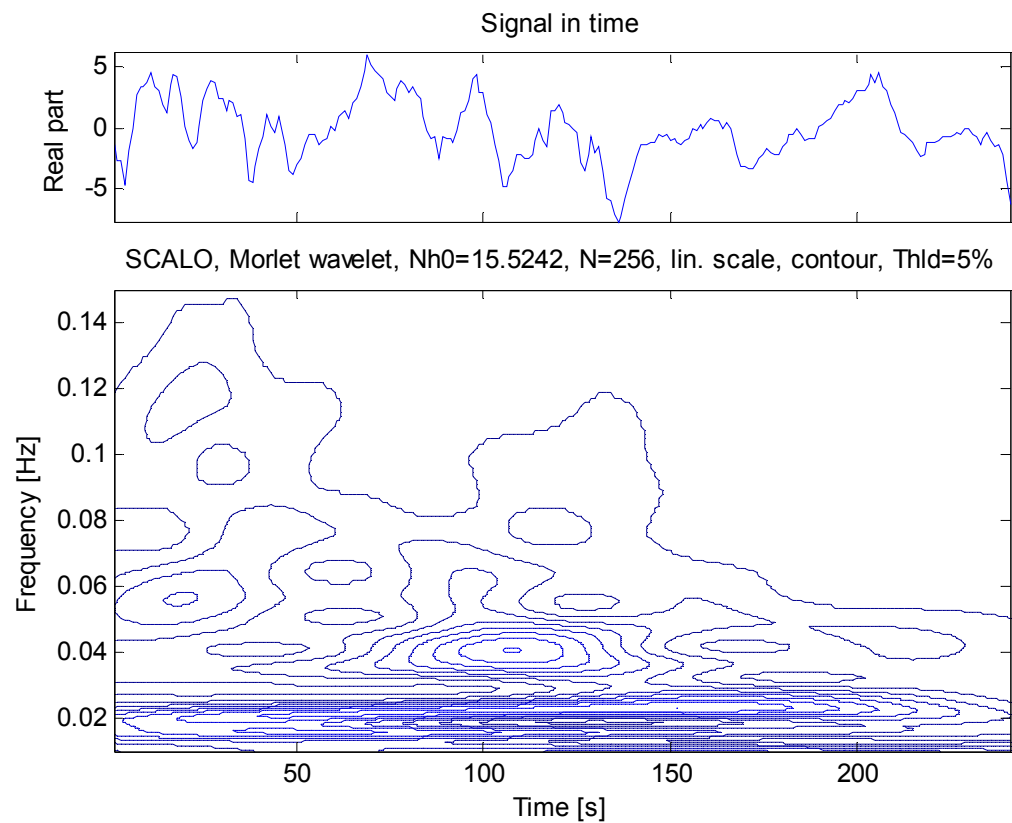
**Figure 2.9.**

Now, let interpret Figure 2.9. Until first 80 points, the variance of the series is mostly dominated by the low frequency component around 0.02, which corresponds to a cycle with period length of  $1/0.02 = 50$  points. Also, in the first 50 points, cycles up to normalized frequency of 0.15 are effective on the formation of the series i.e. the variance of the series in the first 50 points is explained by large number of cycles with various periods.

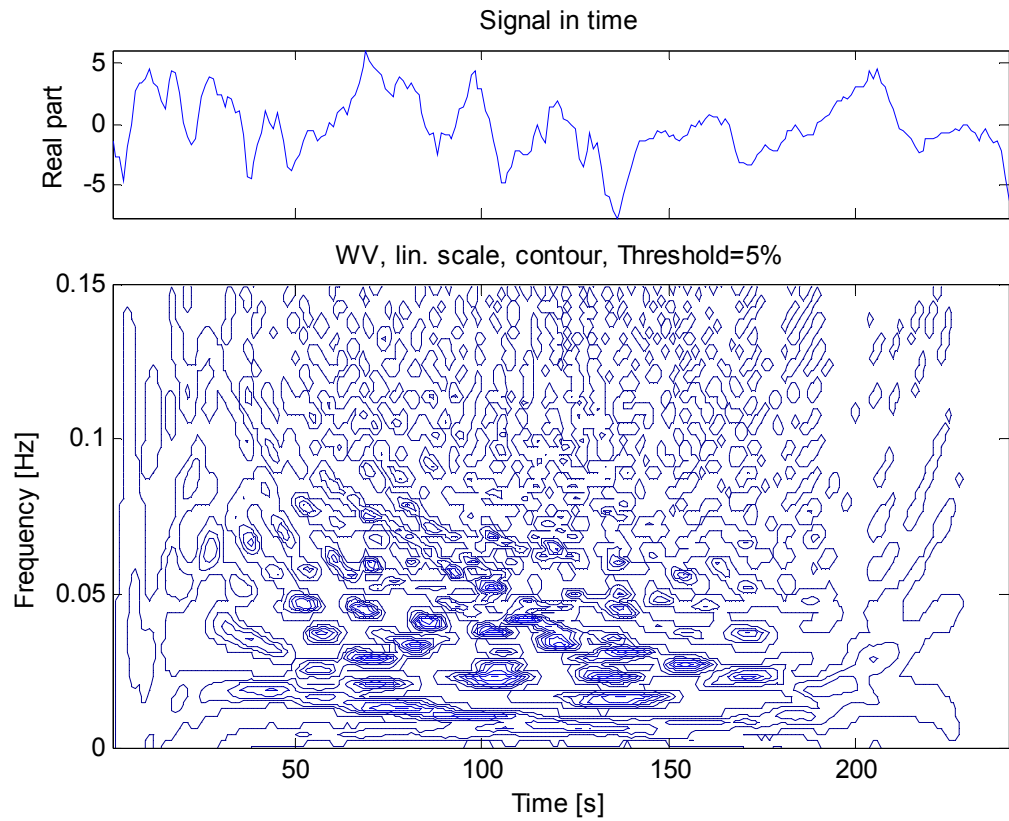
In the time period between 80 and 150, a new dominating frequency has occurred around the normalized frequency of 0.04 that corresponds to a

cycle with  $1/0.04 = 25$  points length in time. However, the cycle around the normalized frequency of 0.02 is more dominant with respect to the cycle around the normalized frequency of 0.04 since the amplitude is higher around the normalized frequency of 0.02.

In the time period between 150 and 220, most of the variance is achieved by the cycle around the normalized frequency of 0.025, which corresponds to a period of 40 points length. The frequency of the dominant cycle is increased when compared with the first 80 points of the time series.



**Figure 2.10.**



**Figure 2.11.**

There many other TFRs which are specified for different purposes. Some of them are the Rihaczek distribution, the Margenau-Hill distribution, the Page distribution, the Choi-Williams distribution, the Born-Jordan distribution, Zhao-Atlas-Marks distribution, the Butterworth distribution.

## **CHAPTER 3**

### **ANALYSIS OF REAL OIL PRICE CRISES**

#### **3.1. Introduction**

This section applies the time-frequency representation techniques to real oil prices. In particular, using TFR techniques, we explain the changing volatility of real oil prices and investigate the differences between supply side driven oil price shocks of 1970's and the recent demand driven oil price shock. For this purpose, we check whether the dominating frequency of the real oil prices exhibit significant difference over time.

The upsurge and the following collapse of oil prices in the recent years intensified the discussion on oil price dynamics and their role on the world economy. Although there has been controversy about the role of oil prices on macroeconomic performance, a consensus has been reached on the changing dynamics of oil prices over time. Hamilton (2009) argues that changes in real oil prices have been difficult to predict, have a quite high standard deviation, and they are governed by different regimes at different points in time. He also reports that real oil prices seem to follow a random

walk without drift. Given the findings of Lee et al (1995) and Ferderer (1996) which find an important role of oil price volatility on macroeconomic performance, such an increase deserves special attention.<sup>1</sup>

About the changing dynamics of oil price dynamics, Hamilton (2009) compares previous supply side driven oil price shocks with the recent demand driven episode and finds differences on the causes of these shocks. On the other hand, Gately and Huntington (2002) mentions the role of global demand and argue that the cyclical behavior of oil prices may be heavily affected by demand dynamics, which also exhibit a volatile path. Thus, these studies suggest the existence of different volatility patterns, depending on the source of oil price shocks.

The above discussion implies that the frequency decomposition of oil prices could have significantly changed over time. While the variation in oil prices may have been mainly governed by its low frequency component in certain periods, high frequency components may have dominated certain subsamples. For example, the dominating frequency of oil price variation may be quite different between supply-side and demand-side driven oil price shocks. Similarly, the low frequency component of the oil prices may account for most of the variation in case of a shock to the trend while the high frequency movements can dominate the series during a temporarily high demand period.

Based on the above discussion, we employ time-frequency representation techniques to capture the changing frequency decomposition of oil prices through time. Such a methodology, for which the theoretical

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Also see Abosedra and Laopadis (1997) for a discussion of oil price volatility and its stochastic behavior.

foundations is given in the first chapter, makes it possible to detect whether and how the dominating cycle of a time series change over time. As it is mentioned before, adopted from the field of signal extraction, TFR methods have only been used in a limited number of studies, most probably due to their complexities.

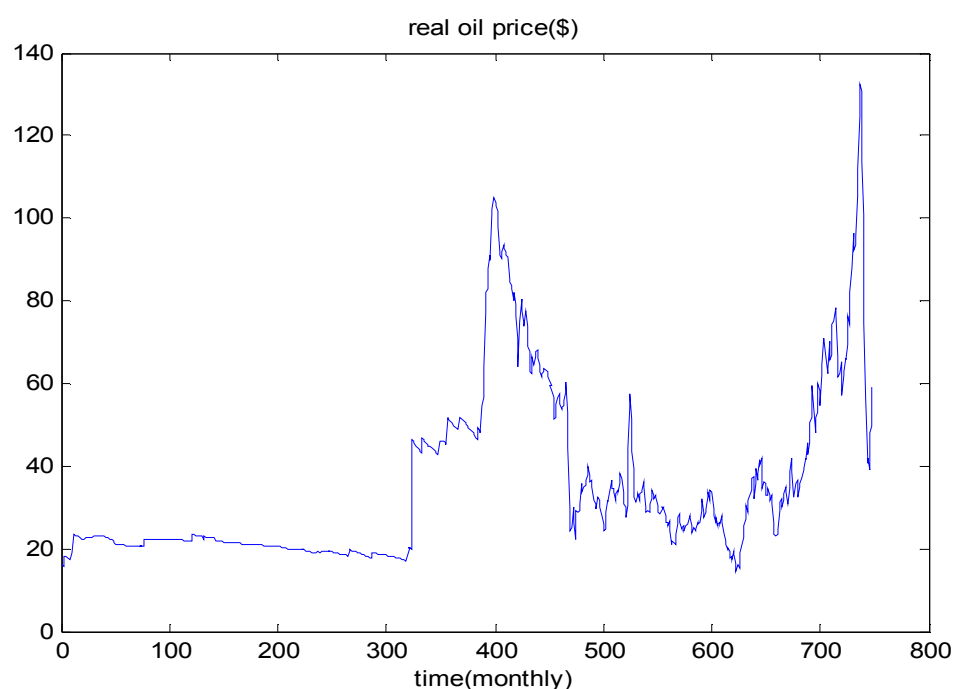
We perform the following exercise. Using monthly data from 1969:M1, we check whether the frequency decomposition of real oil prices has changed. In other words, we identify the periods with respect to the dominance of low frequency or high frequency components that govern the changes in the real oil prices. By doing so, we will be able to see whether the oil price episodes display significant differences in terms of volatility.

### **3.2. TFR Analysis**

As in Hamilton (2009), we derive real oil prices by calculating monthly average price (in dollars per barrel) of West Texas Intermediate for 1947:M1 through 2009:M5 divided by the ratio of the CPI for the previous month to the CPI in April 2009. The obtained series is shown in Figure 3.1.

Figure 3.1. clearly indicates that the series is non-stationary. The augmented Dickey-Fuller and KPSS tests confirm this finding. These results support the claim that real oil price follows a random walk with drift. Moreover, the variance does not stay constant over time.

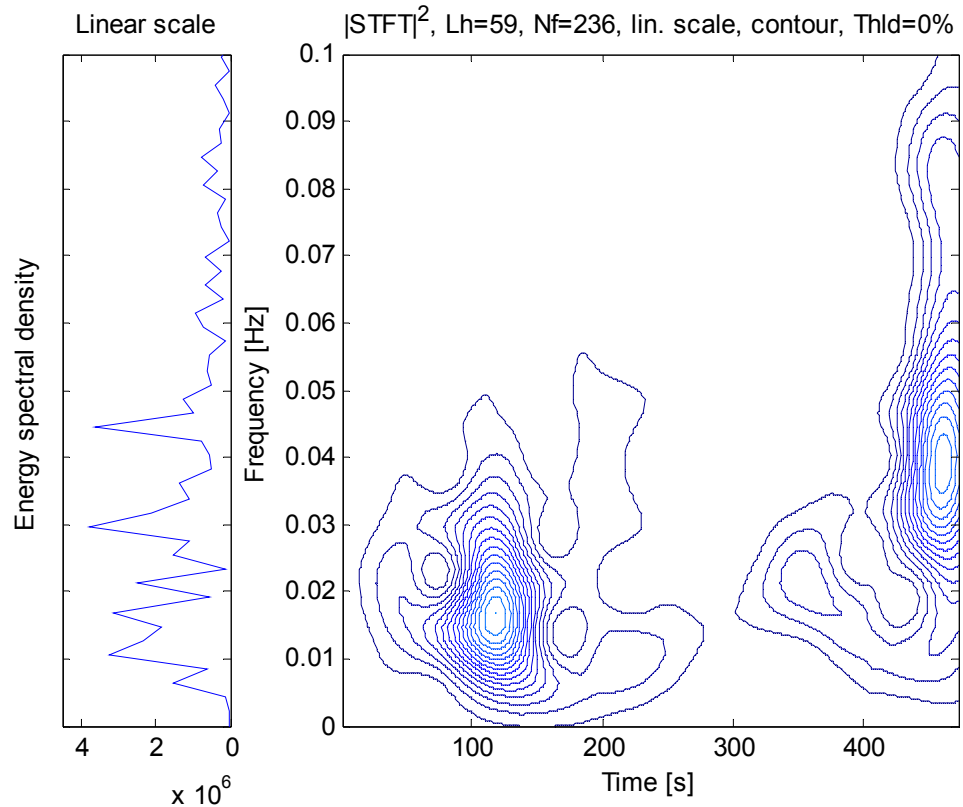
As it is mentioned in the first section, due to the non-stationary characteristic of the series, a Fourier transformation cannot be employed. Moreover, a frequency domain analysis would not provide information about the changing frequencies over time. To be more specific, even if we could transform the series from time to frequency domain, we would not be able to test whether there has been a change in the frequency decomposition over time. Thus, we would not be able to tell whether a fluctuation in a certain period of time is caused by high-frequency components or low-frequency components. Thus, we use TFR to analyze the series simultaneously in time domain and frequency domain. Also, it should be noted that the energy spectral density given at the left side of the Figure 3.2. does not give information about the timing of the dominant frequencies. Thus, it is impossible to determine the time varying dynamics of the series.



**Figure 3.1**

Before applying the Short-Time Fourier Transformation, which is our main TFR technique in this study, we have to filter out the series to remove the mean. It should be reminded that, the series should be stationary in mean to apply the TFR techniques. For this purpose, we use HP filtering for the series.

Figure 3.2. shows the time-frequency representation and energy spectral density of the series.



**Figure 3.2.**

It can be seen that two major oil price shocks are nicely captured in Figure 3.1. First, the episode around 1980:M1 clearly shows that the volatility in this period is mainly governed by low frequency components. These components vary between the normalized frequency of 0.006 and 0.03, which

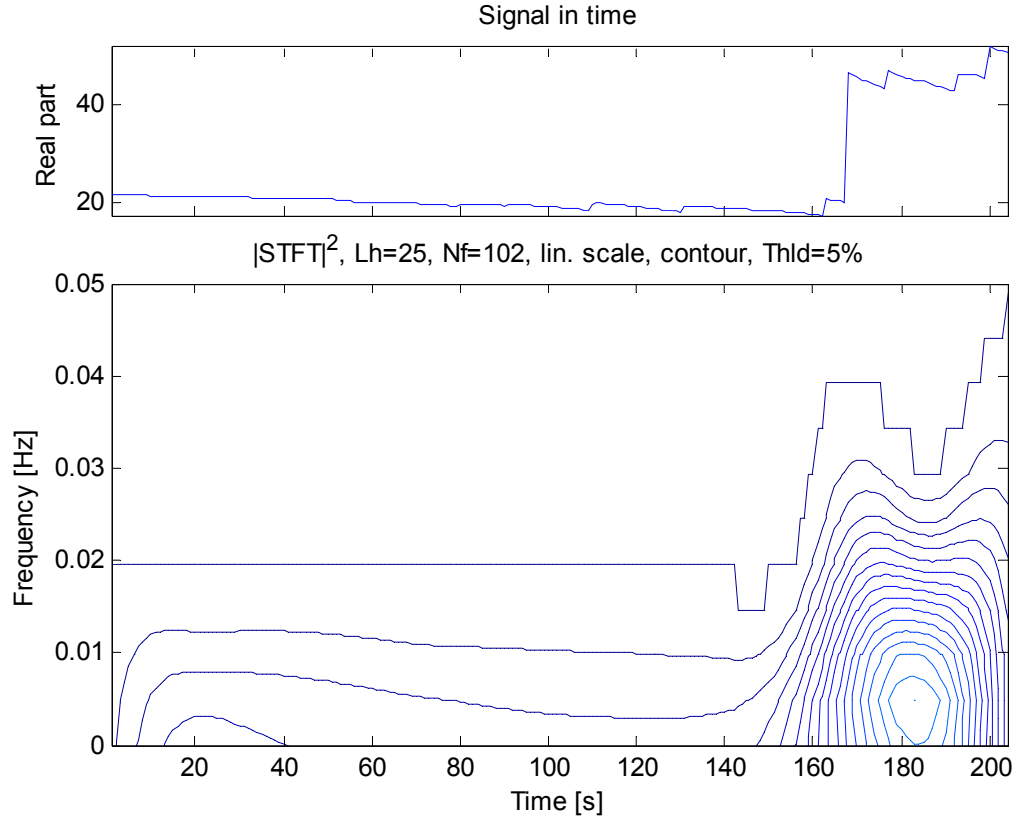
correspond the cycles with the period length between 13.9 years and 2.8 years respectively. And the most dominant component is around the normalized frequency of 0.017 corresponding a cycle with the period length of 4.9 years.

The second major oil price shock is observed at the end of the series, period between 2006:M10 and 2009:M5. The effective components vary between the normalized frequency of 0.02 and 0.09, which correspond the cycles with the period length between 4.2 years and 0.93 years, respectively. Clearly, high frequency components are effective in this period compared with the first major oil price shock. Also, the most dominant component is around the normalized frequency of 0.038 which corresponds to a cycle with the period length of 2.2 years.

Keeping in mind that the first major shock is supply-side driven while the second major shock is demand driven, the results above are promising. We find out that the low frequency components are effective for the 1979 episode. However, high frequency components are effective for the recent episode. These results show the significant difference of the source between the demand-side and supply-side driven oil price shocks.

Now, the series will be divided into its subparts to observe the frequency characteristics of the real oil price crises occurred in 1973 and 1990. The crisis in 1973 was caused by the embargo of the Arabian countries due to support of Israel by western countries and the crisis in 1990 was caused by the Gulf War. Thus both of the crises are supply-side driven shocks.

For the analysis of the crisis in 1973, the part between 1960:M1 and 1976:12 is taken into account. The plot of the series and STFT of the series is given in Figure 3.3.

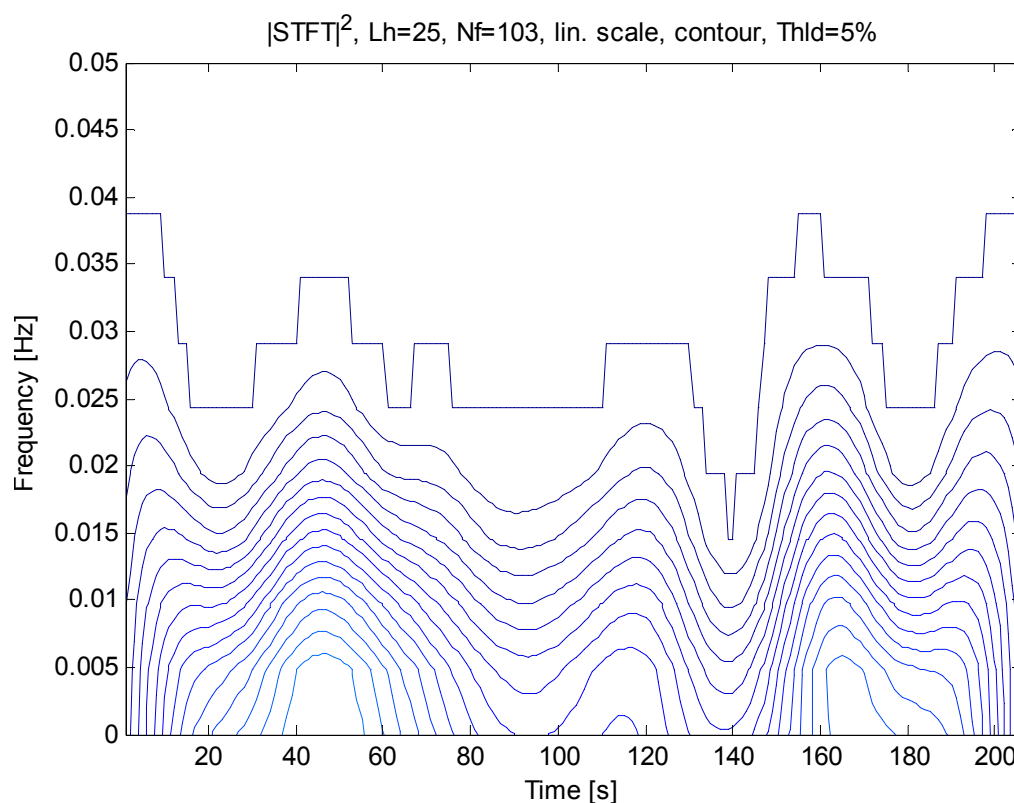


**Figure 3.3.**

In the period starting at 1973, effective frequencies are around the normalized frequency of 0.005 which corresponds to a cycle with 16.7 years period length and the upper bound for the effective frequencies is around the normalized frequency of 0.02 which correspond to a cycle with period length of 4.2 years. These results show that low frequencies are effective at 1973 real oil price crisis and following period.

The period between 1986:M12 and 2003:M11 is analyzed for the real oil price crisis in 1990. The TFR of the given period is in Figure 3.d. The 37<sup>th</sup> data point corresponds to 1990:M1 and the following 2 years period forms the

most of the variation. In this period low frequency components, around the normalized frequency of 0.01, are effective again as expected and also the most effective component is around the normalized frequency of 0.005 which corresponds to a cycle with 16.7 years period length.



**Figure 3.4.**

The analysis clearly shows that demand-side driven real oil price shocks are mostly formed by high frequency components where the supply-side driven real oil price shocks are dominated by low frequency components.

## **CHAPTER 4**

### **OUTPUT GAP ANALYSIS**

In this section output gap of the US is analyzed by using TFR to understand the time-frequency characteristics behind. Quarterly time series data between 1949(I) and 2009(I) are used. The data is divided into three subsamples to observe the frequency characteristics of short recession periods. Also frequency shifts are studied in expansion and recession periods.

#### **3.1. The Part between 1949-I and 1970-IV**

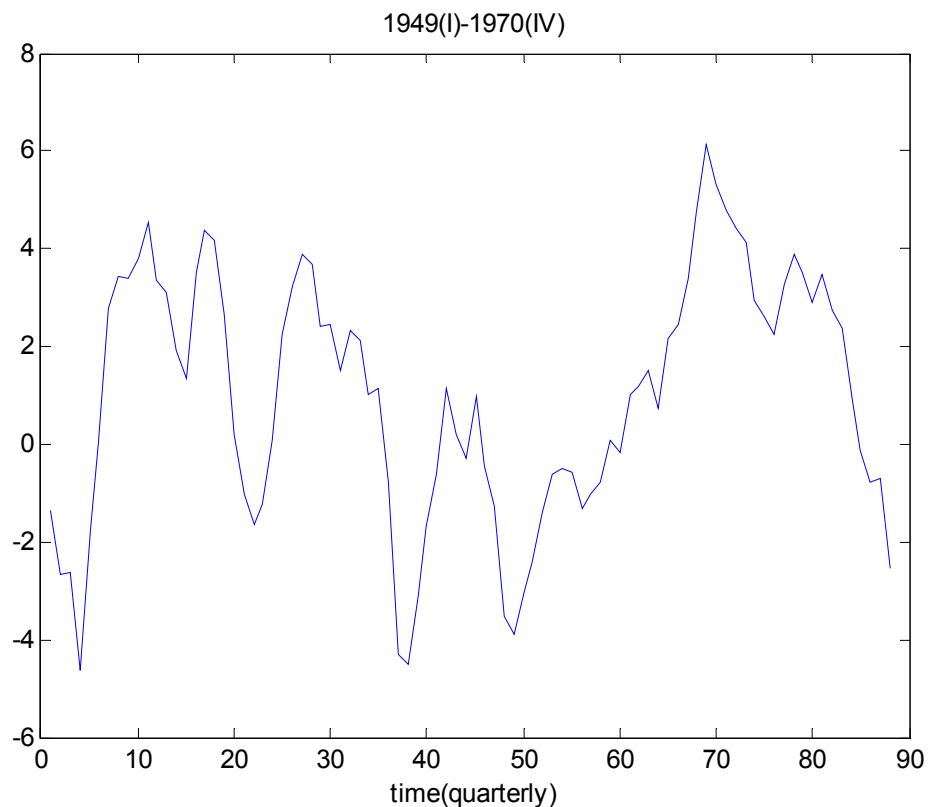
The plot of the output gap in the given period is given in Figure 4.1. and expansion and recession dates of NBER are listed below:

- 1949 (IV) - 1953 (II) , expansion with 45 months length
- 1953 (II) - 1954 (II) , recession with 10 months length
- 1954 (II) - 1957 (III) , expansion with 39 months length
- 1957(III) - 1958 (II) , recession with 8 months length

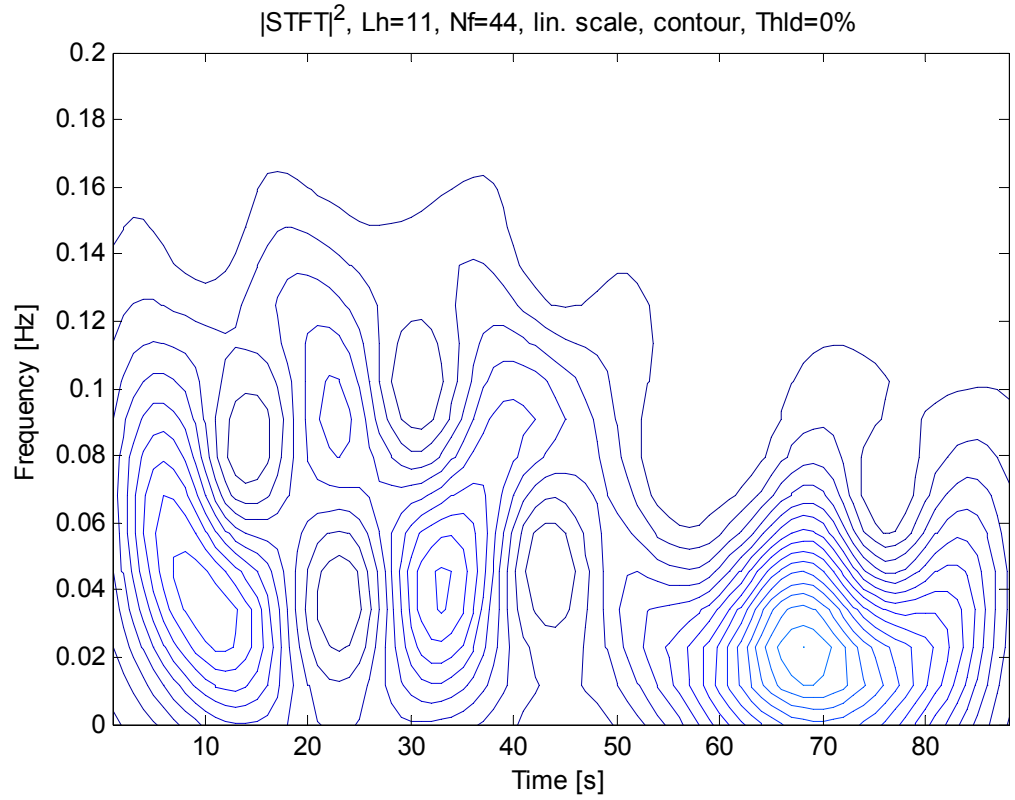
- 1958 (II) - 1960 (II) , expansion with 24 months length
- 1960 (II) - 1961 (I) , recession with 10 months length
- 1961 (I) - 1969 (IV) , expansion with 106 months length
- 1969 (IV) - 1970 (IV) , recession with 11 months length

where NBER defines a recession as “a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales”.

The STFT of the time series data is given in Figure 4.2.



**Figure 4.1.**



**Figure 4.2.**

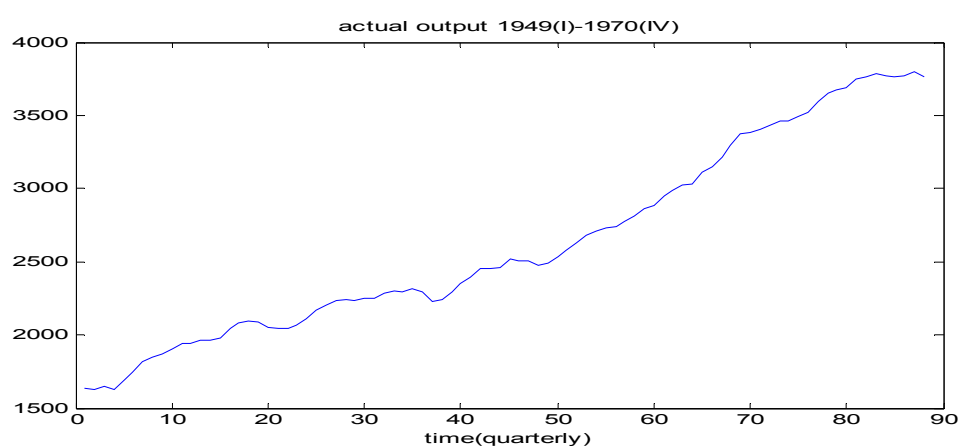
During the first expansion period between 1949(IV) and 1953(II), the normalized frequency decreased from 0.08 to 0.03 i.e. dominating cycle's period increased steadily while going at the end of the expansion period. During the following recession period between 1953(II) and 1954(II), two frequency components of normalized frequency 0.04 and 0.1 were effective i.e. variance of series in this period was mostly formed by a long term component with period length of 8.3 years and a short term component with period length of 3.3 years.

In the expansion period between 1954(II) and 1957(III), two frequency components of normalized frequency of 0.04 and 0.1 were effective again. However, the short term component was much less effective when compared with the recession period between 1953(II) and 1954(II) i.e.

long term component was much more effective. In the following recession period 1957(III) and 1958(II), the same frequency components as in the previous expansion period were effective.

In the expansion period between 1958(II) and 1960(II), dominating frequency increased from 0.04 from 0.09 gradually i.e. series became more volatile by time. In the following period of recession between 1960(II) and 1961(I), dominating frequency decreased around the normalized frequency of 0.02.

The last expansion period between 1961(I) and 1969(IV) formed the most of the variation of the whole time series data between 1949(I) and 1970(IV). The peak level of the variation is around the 69<sup>th</sup> data point which corresponds to 1966(I). Also, the dominating frequency was lower, around the normalized frequency of 0.02, when compared with the rest of the data. This points a significant dominating frequency shift. If the graph of the actual output in Figure 4.3. is investigated trend shift can easily be seen at the 69<sup>th</sup> data point and this illustrates the trend display characteristics of TFR.

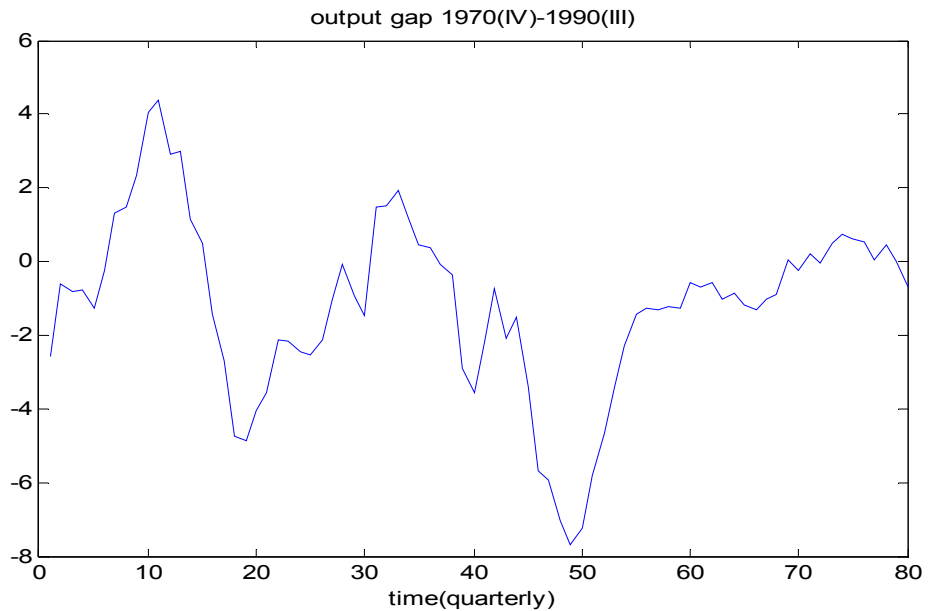


**Figure 4.3.**

#### 4.2. The part between 1970-IV and 1990-III

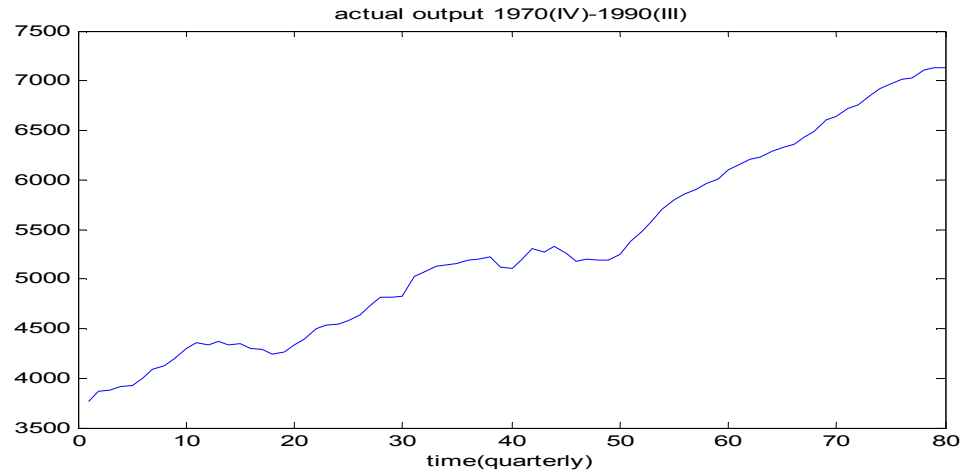
The plot of the output gap in the given period is given in Figure 4.4. and plot of the actual output is given in Figure 4.5. Expansion and recession dates of NBER 1970(III) and 1990(III) between are listed below:

- 1970 (IV) - 1973 (IV) , expansion with 36 months length
- 1973 (IV) - 1975 (I) , recession with 16 months length
- 1975 (I) - 1980 (I) , expansion with 58 months length
- 1980(I) - 1980 (III) , recession with 6 months length
- 1980 (III) - 1981 (III) , expansion with 12 months length
- 1981 (III) - 1982 (IV) , recession with 16 months length
- 1982 (IV) - 1990 (III) , expansion with 92 months length

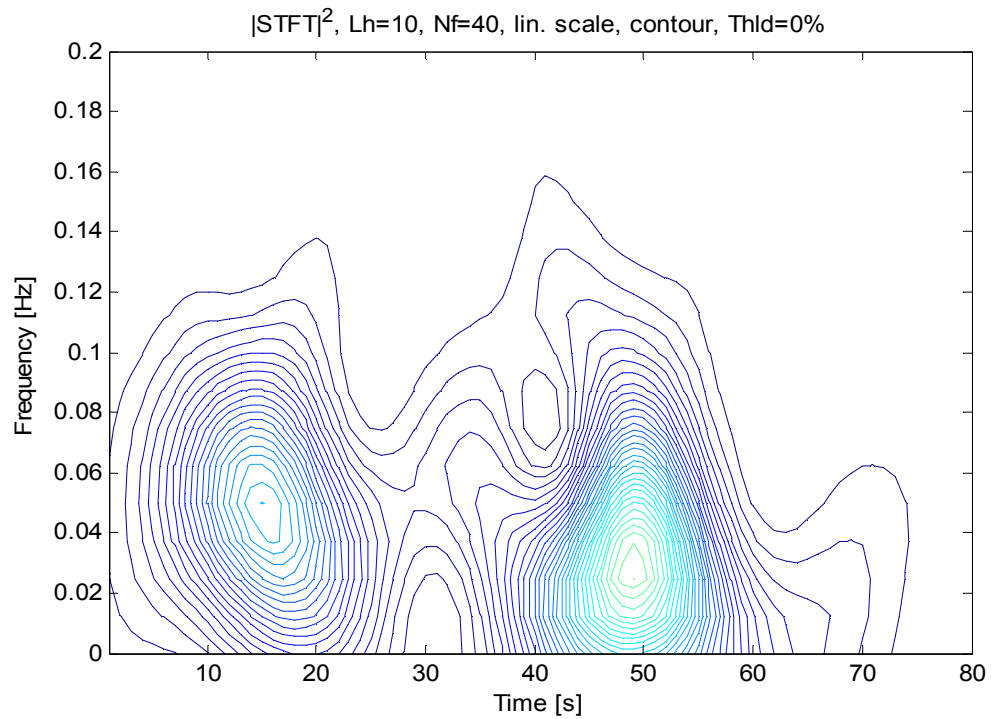


**Figure 4.4.**

Figure 4.4. shows that the most of variation of the series is mostly dominated by the first 56 points. Thus we expect to see that the most of the energy gather between 1<sup>st</sup> and 56<sup>th</sup> points in time frequency representation. The STFT of the output gap between 1970(IV) and 1990(III) is given in Figure 4.6.



**Figure 4.5.**



**Figure 4.6.**

Most of the energy is gathered in the first 56 points, as expected. For the first 26 points, the dominating frequencies fluctuate between a normalized frequency of 0.02 and 0.08, which corresponds to the cycles between 12.5 years and 3.12 years, respectively. The most effective cycle is around the normalized frequency of 0.05 i.e. the cycle is 5 years length. Time projection of the other high energy area is the time period around the 50<sup>th</sup> point. The effective frequency is roughly 0.03 that corresponds to 8.25 years in that period. The last 20 points do not have much effect on variation as can be seen in both Figure 4.4. and Figure 4.6.

In the expansion period between 1970(IV)-1973(IV) and the recession period between 1973(IV)-1975(I), the effective frequency components varied from normalized frequency of 0.02 to 0.08 with the most effective frequency component was at the normalized frequency of 0.05.

The fluctuation of the effective frequencies decreased and clustered around the normalized frequency of 0.05 in the expansion period between 1975(I) and 1980(I) i.e. number of effective cycles with different periods declined. And in the following recession period between 1980(I) and 1980(III), the dominating frequency decreased from 0.05 to 0.03. Thus recession period acted as a transition period between high frequency period and low frequency period.

The dominating frequency is around the normalized frequency of 0.03 in the expansion period between 1980(III)-1981(III) and the recession period

between 1981(III)-1982(IV). Although the period is small, the majority of the variance domination was realized here.

In the expansion period between 1982(IV) and 1990(III), the dominating frequency increased from normalized frequency of 0.03 to 0.08 i.e. the series became more volatile by time. Even the length of the sample was greater than the other expansion periods, the amount of the effect on variation was smaller.

#### **4.3. The Part between 1990-III and 2009-I**

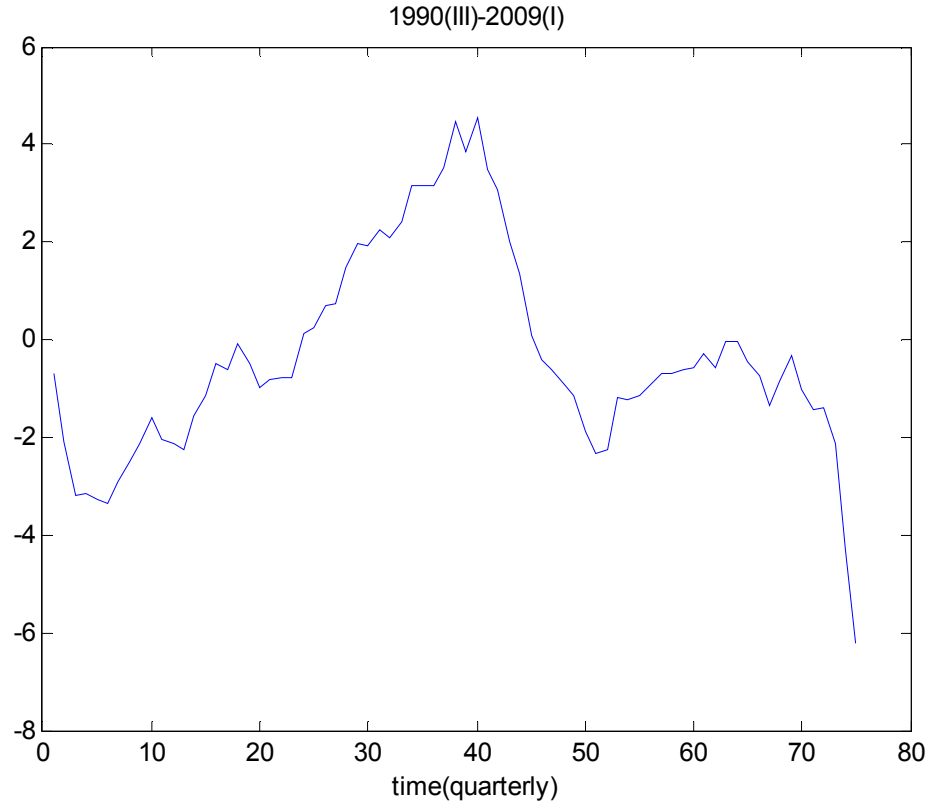
The plot of the output gap in the given period is given in Figure 4.7. and expansion and recession dates of NBER are listed below:

- 1990 (III) - 1991 (I) , recession with 8 months length
- 1991 (I) - 2001 (I) , expansion with 120 months length
- 2001(I) - 2001 (IV) , recession with 8 months length
- 2001 (IV) – 2007 (IV), expansion with 73 months length.

Note that at the end date of the last recession date has not been announced yet.

The STFT of the time series data for the given period is in Figure 4.8. and Figure 4.9. for better monitoring.

The normalized frequency decreased gradually from 0.08 to 0.02 in the recession period between 1990(III) and 1991(I) i.e. the period length of the effective cycle increased gradually from 3.12 years to 12.5 years.



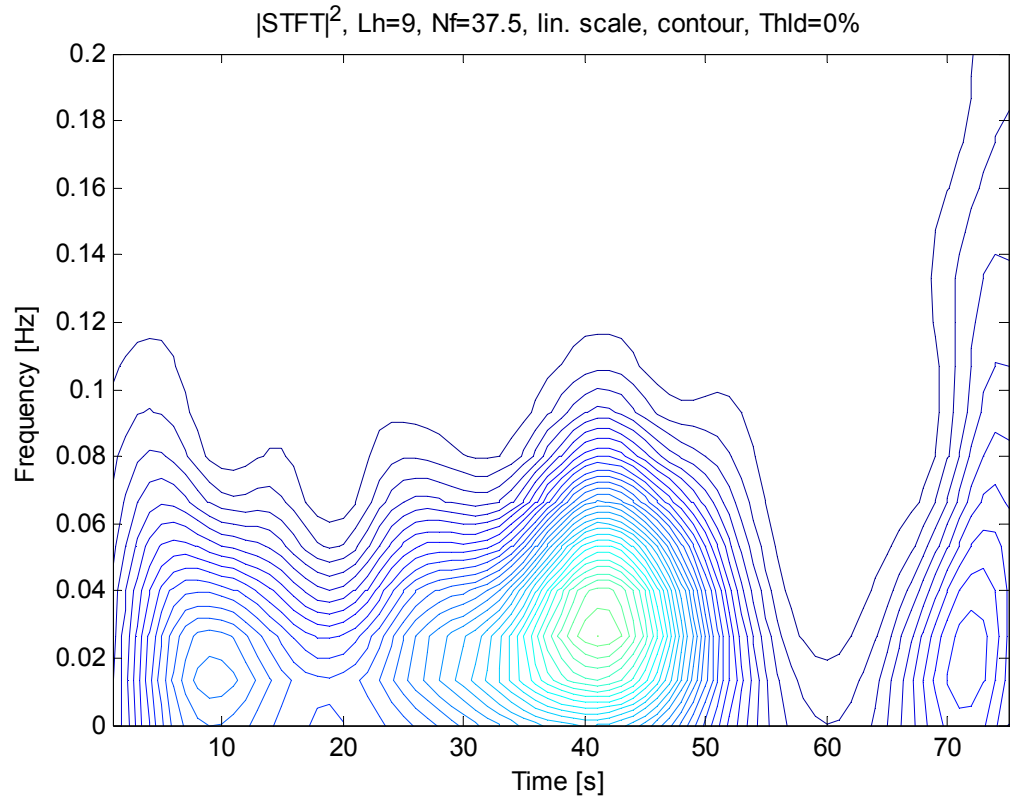
**Figure 4.7.**

In the expansion period between 1991(I) and 2001(I), the dominating frequency was around the 0.01 until the 32<sup>th</sup> data point corresponding 1998(III). After this point the effective frequency was around 0.025 until 2001(I).

In the recession period between 2001(1) and 2001(IV), the dominating frequency was around the normalized frequency of 0.025 due to the effect of the windowing of STFT and short length of the period.

Figure 4.8. and Figure 4.9. demonstrate that the expansion period between 2001(IV) and 2007(IV) had no effect on the variance of the series.

The recession period starting from 2007(IV) is composed of cycles up to normalized frequency of 0.14. If the whole analysis is considered, normalized frequency of 0.14 is very high i.e. series become much more volatile and short term components are very effective compared with the rest of the output gap.



**Figure 4.8.**

$|STFT|^2$ , Lh=9, NF=37.5, lin. scale, surf, Thld=0%

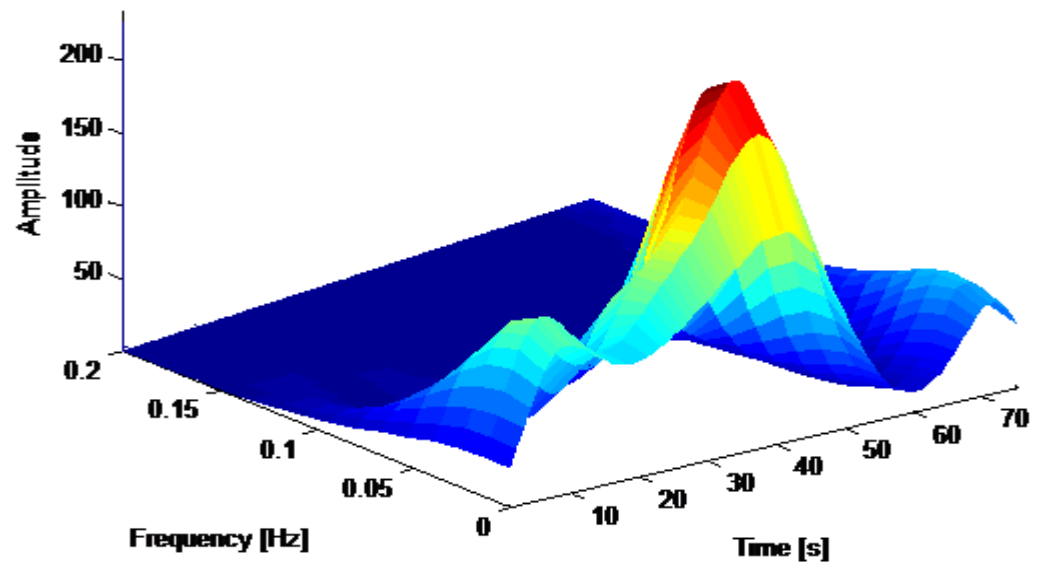


Figure 4.9.

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