Designing Time Contingent Shipping Fee Policies

by

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This is to certify that I have examined this copy of a master's thesis by

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and have found that it is complete and satisfactory in all respects,
and that any and all revisions required by the final
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To my parents

ABSTRACT

With the evolution of the internet in recent years, we have witnessed an increasing interest in online shopping. Along with this trend, a new concept arose: the online shipping fee charged by the internet seller replaces the fixed costs of the consumer in an offline setting such as time and travel costs. While marketing research has much insight about the ordering behavior in the offline setting, a great deal of uncertainty exists on the optimal ordering behavior of the consumer who is faced with this new barrier. In this paper, I model a rational online repeat buyer's ordering behavior being faced with time contingent shipping policies of the internet seller. Then, I demonstrate that by increasing the time threshold for slow shipping the seller may induce the shopper to increase their order size and to decrease their consumption rate. These implications of the model will give insight about designing time contingent shipping fee policies.

Keywords: The Internet Shopping, Online Shipping Policies, Shipping Fees

ÖZET

Son yıllarda internet'in yaygınlaşmasıyla, internet üzerinden online olarak yapılan alışverişin de ciddi olarak arttığını gözlemliyoruz. Bu yeni sistem, gene yeni bir kavram olan gönderi ücretini de beraberinde getirdi. İnternet satıcısı tarafından konulan sabit maliyet, bir anlamda eski sistemde müşterinin mağazaya giderken ulaşım ve zaman için sarf ettiği maliyetin yerini aldı. Pazarlama literatürüne baktığımızda, müşterinin mağazaya giderek yaptığı, geleneksel diye tabir edebileceğimiz alışveriş sistemi üzerine yapılmış pek çok çalışma mevcut olsa bile, elektronik alışveriş için oldukça az sayıda çalışmaya rastlıyoruz. Dolayısıyla, müşterinin siparişini nasıl verdiğiyle ilgili olarak ciddi bir belirsizlik söz konusudur. Bu makalede, bir internet satıcısının web sitesini düzenli olarak ziyaret eden ve sitenin zamana bağlı gönderi politikalarıyla karşılaşan bir kullanıcının sipariş davranışını nasıl gerçekleştirdiğini modelledik. Bulgularımıza bakacak olursak, internet satıcısı siparişin gönderi süresini uzattığında müşterisinin sipariş miktarını arttırabilmekte, bununla da kalmayıp müşterisinin tüketimini de azaltabilmektedir. Bulgularımız zamana bağlı gönderi politikaları oluşturan internet satıcıları için faydalı olacaktır.

Anahtar Kelimeler: Internetten Alışveriş, Gönderi Ücretleri

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1 Introduction

During the past decade, people have been using the Internet for online retailing more than ever. In 2006 alone, United States Department of Commerce reported that annual sales in Internet retailing exceeded $100 billion. This year the overall optimism of online shopping is supported by the sales number on "Cyber Monday", the ceremonial kick-off of the holiday online shopping season in the United States between Thanksgiving Day and Christmas. According to comScore data (2007) the revenue generated by online spending broke the threshold of $700 million in one day and 55% of office employees (68.5 million people) planned to shop online. The reason behind the magic of online shopping is it's quickness, easiness and the mere thought of the possibility of free shipping. On the other hand, the survey conducted by Ernst and Young (1999) demonstrates that 60% of online shoppers are dissatisfied by shipping fees and over 50% of consumers claims shipping fees as their main complaint about online shopping. As the surveys conclude, shipping fee policies are the most important of marketing decisions for online retailers; however the desing of these policies is a complex task. Many online retailers are trying to attract their consumers' attention with shipping related promotions. The retailers paid between $4 and $16 per order for the price of arousing a catching thought of discounted or free shipping (Barsh et al. 2000). The initial trend in shipping fee promotions aimed to encourage consumers to increase their order size to pass a monetary threshold for free shipping. Nowadays, marketers of many online retailing companies added time contingent shipping fee policies in their handbook.

The research in offline retailing demonstrates that the main factor affecting the ordering behavior of the shopper is the location of the store, since it determines the value of incurred fixed cost (Bell et al. (1998), Fotheringham (1988)). In an online setting, the key factor is the shipping policy of the internet seller. In most shopping sites, shoppers get the advantage of free shipping after satisfying some conditions such as ordering an amount that is higher than a monetary or waiting time threshold. According to Bell et al. (2006), this free shipping condition is the former one, whereas in this paper, it is the latter one.

This means the shopper should wait for a time period for the delivery of his order to take the advantage of free shipping. Otherwise, he pays a previously defined shipping fee to get the instant fast shipping option, or falls into stock shortage and confronts with the cost of it.

In this study, I construct an analytical model for an online repeat buyer of goods like non-perishable grocery and drug store items that require stockpiling. While we have much insight about the ordering behavior in an offline setting, the mechanism of online retailing is not clear. When we compare the online case with the offline one, we see that fixed costs of offline setting like travel and time do not exist in the online version; whereas, there is an additional cost charged for shipping. Our cost-minimizing shopper orders a non-durable good and puts the purchased quantity in his stock with positive cost of inventory. The time spent between consecutive orders determines whether he will choose slow shipping with or without shortages or expedited shipping. At each visit, he is faced with prices and shipping policies of the seller that he can not change, and decides how much to purchase along with which shipping policy. Our model determines the optimal order quantity of the shopper being faced with different values of the seller's time threshold. On the other hand, we attain an useful result supporting the fact that the seller can affect both the ordering behavior of the shopper and his optimal consumption level by simply changing the value of the time threshold. The remaining part of the paper is organized as follows: first I describe the three different modelling approaches used to analyze the ordering behavior of consumer. The initial model assumes that the order size of the consumer is deterministic and does not allow the consumer to remain in shortage. The second model integrates shortage cost and allows the consumer to remain in shortage for a period of time. Since the order size of the consumer can be affected from the promotions, campaigns and advertisements of the seller's site, the third model uses a stochastic component for the order size. The next section describes how the optimal consumption rate of the consumer changes under given parameters of the seller allowing the shopper's consumption rate endogenous. The final section concludes with a summary of managerial issues and suggestions for future research.

2 Ordering Behavior

2.1 Deterministic Model Without Shortages

2.1.1 Model and Assumptions

I use the traditional economic order type (EOQ) model as in the paper of Blattberg et all. (1981) to analyze the ordering behavior of our rational cost minimizing shopper. In my model, the shopper faces with the parameters of the internet seller, which are the expedited shipping fee K, time threshold T and price of one unit of non-durable good p . The shopper's costs for the n^{th} order are transaction cost k that is the cost of time and effort of filling in forms and billing information for the order; expenditure cost pQ where Q is the quantity ordered; inventory holding cost $\frac{h}{2r}Q^2$ where h is inventory holding cost of one unit of commodity for one unit of time, Q/r is the time spent between consecutive orders when r is the consumption rate of the shopper, $\frac{Q}{2}$ is the average inventory level; and delivery cost D_n defined as

$$D_n = \begin{cases} 0 & \text{if } \frac{Q}{r} \geq T \\ K & \text{if } \frac{Q}{r} < T \text{ and the shopper chooses fast shipping} \end{cases} \tag{1}$$

So the total cost for the n^{th} visit will be

$$C_n = k + D_n + pQ + \frac{h}{2r}Q^2 \tag{2}$$

Starting with the assumption that, at $t = 0$, the shopper has the amount of Q in his inventory; the consumer who is faced with two different shipping options, expedited and slow shipping. In the case of choosing the slow option, the delivery time will be time threshold T, and in the case of the fast option the order is delivered instantly. If the shopper's consuming period for the n^{th} order is greater than the time threshold T, he can get the advantage of free shipping without any conditions. If it is less than time threshold, the consumer chooses fast shipping by paying shipping fee noted as K in the model. With the assumptions above, it is possible to formulate the cost of the consumer as follows

$$C_n = \begin{cases} k + pQ + \dfrac{h}{2r}Q^2 & \text{if } \dfrac{Q}{r} \geq T, \text{ or } Tr \leq Q \\ k + K + pQ + \dfrac{h}{2r}Q^2 & \text{if } \dfrac{Q}{r} < T, \text{ or } Tr > Q \end{cases} \tag{3}$$

Dividing the total cost to the time between consecutive orders, we can construct the expected long run average cost per unit time as

$$E_n(C_n) = (k + pQ + \frac{h}{2r}Q^2 + D)/(Q/r) \tag{4}$$

equivalently,

$$LR(Q) = \begin{cases} \dfrac{kr}{Q} + pr + \dfrac{hQ}{2} & \text{if } Tr \leq Q \quad \text{slow shipping} \\ \dfrac{(k+K)r}{Q} + pr + \dfrac{hQ}{2} & \text{if } Tr > Q \quad \text{fast shipping} \end{cases} \tag{5}$$

Solving first order conditions requires taking the derivatives of costs with respect to Q and setting equal to zero. The optimal order quantities (second order conditions are satisfied) for both slow shipping and fast shipping cases become

$$Q^*_{slow} = \sqrt{\frac{2kr}{h}} \text{ and } Q^*_{fast} = \sqrt{\frac{2(k+K)r}{h}} \tag{6}$$

Combining the equations above, we can define optimal long run costs in terms of order size as

$$LR^*_{slow} = hQ^*_{slow} + pr \text{ and } LR^*_{fast} = hQ^*_{fast} + pr \tag{7}$$

If $Q^*_{slow} \geq Tr$, then $Q^* = Q^*_{slow}$; because, when $Q \geq Tr$, the cost minimizing order size in slow shipping Q^*_{slow} will not result in shortage. Thus, shopper will always choose slow shipping shopping option, and Q^*_{slow} will minimize the long run average cost of slow shipping. When $Tr \geq Q^*_{slow}$, Q^*_{slow} will cause shortage and consumer will need to increase the order size. Hence, the shopper will start to order the value of Tr in this interval and get the advantage of free shipping. However, at some point because of higher inventory cost, the cost of ordering more will exceed the cost of the fast shipping option. Then our consumer starts to order with fast shipping and Q^*_{fast} becomes the optimal order quantity.

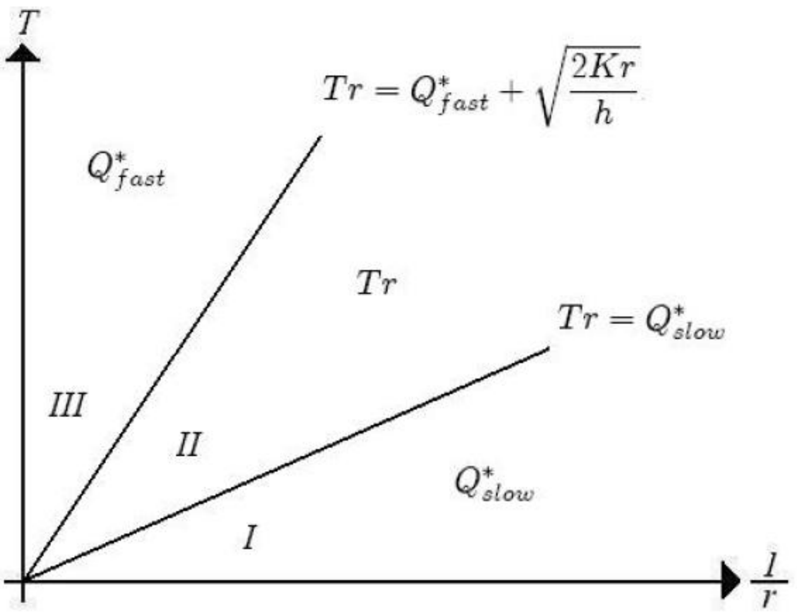

Figure 1 Optimal quantities of model without shortage for different combinations of T and 1/r.
(Check Appendix to see how boundaries are obtained.)

2.1.2 Implications of the Model Without Shortages

In the first region of Figure 1, time threshold T is so small compared to time to consume one unit of good. Therefore, the shopper can catch the threshold with holding a small amount of inventory. In this region, consumer always catches the threshold and choose slow shipping option, pay no shipping fee. The optimal order size will be Q^*_{slow} since this minimizes his average long run cost. In the second region, T starts to increase compared to $1/r$, Q^*_{slow} will not be enough anymore and consumer will order Tr to get the advantage of free shipping. Increasing order size causes inventory cost to increase and in the third region, the consumer will chose choose fast shipping option instead of increasing the order size.

The following figure summarizes optimal quantities for different combinations of T and $1/r$ that is the time to consume one unit of commodity.

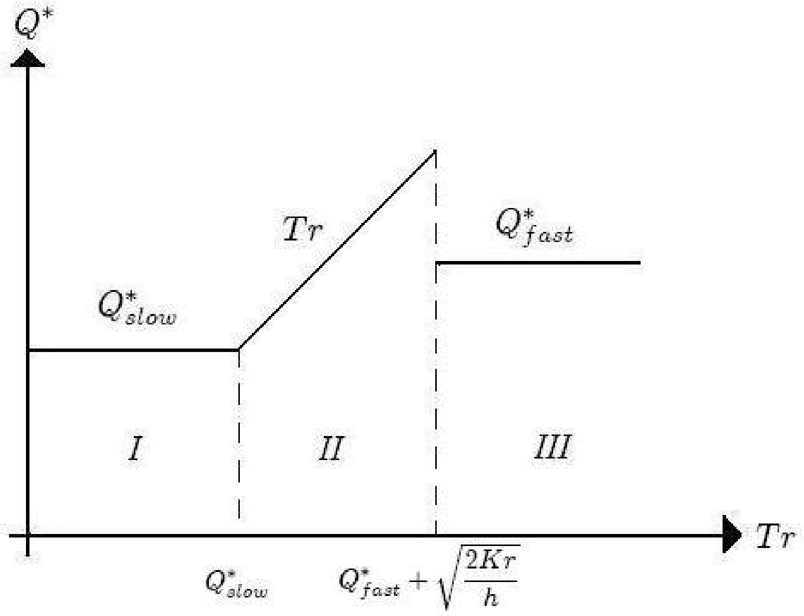

Figure 2 Optimal quantities of model without shortage at different Tr ranges for a given r.

As it seems from the graph, the maximum order size is attained in the right end of second region. The retailer will earn shipping fee in the third region but there will be a decrease in order size and increase in delivery costs. If we assume that the consumer will continue to order from the same retailer in next shopping and shipping fee is larger than the extra costs of expedited delivery, third region will be more profitable for the retailer since the consumption rate does not change. If shipping fee is smaller than the extra costs of expedited delivery, the retailer can force consumer to order an amount in second region. With the increasing probability of losing customer to another online retailing store, the order size will gain importance and the retailer will need to make consumer to order an amount close to the boundary in second region.

2.2 Deterministic Model With Shortages

2.2.1 Model and Assumptions

For the model allowing shortages, I continue to use traditional economic order type (EOQ) model. The aim of the consumer is again to minimize his average long run cost given the parameters of the seller, that are T, p and K. The shopper's costs for the n^{th} order are almost the same as the earlier setup except an additional cost of shortage S_n defined as

$$S_n = \begin{cases} 0 & \text{if } \frac{Q}{r} \geq T \\ (T - \frac{Q}{r})s & \text{if } \frac{Q}{r} < T \text{ and the shopper chooses slow shipping} \end{cases} \tag{8}$$

Different from the previous model, it is assumed that there is a quadratic cost structure that is very reasonable for the shortage S_n that is equal to $(T - \frac{Q}{r})^2 s$ where s is the cost of shortage per unit time and $(T - \frac{Q}{r})$ is the total time of shortage. Then, the cost of the n^{th} visit is

$$C_n = k + D_n + pQ + \frac{h}{2r}Q^2 + S_n \tag{9}$$

As in the benchmark model, we assume that, at $t = 0$, the shopper has the amount of Q in his inventory. His order size Q is deterministic; and two shipping policies he faces is as before. If the shopper's consuming period for the n^{th} order is greater than the time threshold T, he can get the advantage of free shipping without any conditions. If it is less than T, then shopper has two options, he can fall into shortages and face with the cost of shortage or choose instant fast shipping by paying K. The cost of the shopper for the n^{th} order is

$$C_n = \begin{cases} k + pQ + \frac{h}{2r}Q^2 & \text{if } \frac{Q}{r} \geq T \\ k + pQ + \frac{h}{2r}Q^2 + (T - \frac{Q}{r})^2 s & \text{if } \frac{Q}{r} < T \quad \text{and } K > (T - \frac{Q}{r})s \\ k + K + pQ + \frac{h}{2r}Q^2 & \text{if } \frac{Q}{r} < T \quad \text{and } K \leq (T - \frac{Q}{r})s \end{cases} \tag{10}$$

or equivalently,

$$C_n = \begin{cases} k + pQ + \frac{h}{2r}Q^2 & \text{if } Tr \leq Q \\ k + pQ + \frac{h}{2r}Q^2 + (T - \frac{Q}{r})^2 s & \text{if } Q < Tr < \frac{Kr}{s} + Q \\ k + K + pQ + \frac{h}{2r}Q^2 & \text{if } Tr \geq \frac{Kr}{s} + Q \end{cases} \tag{11}$$

Then, our expected cost per unit time becomes

$$E_n(C_n) = (k + S + pQ + \frac{h}{2r}Q^2 + D)/E(T_n) \tag{12}$$

where

$$E(T_n) = \begin{cases} Q/r & \text{slow shipping without shortages and expedited shipping} \\ T & \text{slow shipping with shortages} \end{cases} \tag{13}$$

After dividing expected cost to expected elapsed time, we get our average expected long run cost per unit time as

$$LR(Q) = \begin{cases} \frac{kr}{Q} + pr + \frac{hQ}{2} & \text{if } Tr \leq Q & \text{slow w/o shor.} \\ \frac{1}{T}(k + pQ + \frac{h}{2r}Q^2 + \frac{Q^2}{r^2}s) + Ts - 2\frac{Q}{r}s & \text{if } Q < Tr < \frac{Kr}{s} + Q & \text{slow w/ shor.} \\ \frac{(k+K)r}{Q} + pr + \frac{hQ}{2} & \text{if } Tr \geq \frac{Kr}{s} + Q & \text{fast} \end{cases} \tag{14}$$

After deriving first order conditions, we get the optimal order quantities and average long run costs per one unit of time (second order conditions are satisfied) for as

$$Q^*_{slow} = \sqrt{\frac{2kr}{h}},\ Q^*_{shortage} = \frac{2sTr - pr^2}{hr + 2s} \text{ and } Q^*_{fast} = \sqrt{\frac{2(k+K)r}{h}} \tag{15}$$

$$LR^*_{slow} = hQ^*_{slow} + pr \text{ and } LR^*_{fast} = hQ^*_{fast} + pr \tag{16}$$

After defining optimal long run costs for all three shipping cases, it is time to obtain global optimums that minimizes the cost of the shopper. If $Q^*_{slow} \geq Tr$, then $Q^* = Q^*_{slow}$; because, when $Q \geq Tr$, the shopper always chooses slow shopping option, and Q^*_{slow} minimizes the long run average cost of slow shipping. (Region I) When $Tr \geq Q^*_{slow}$ ordering Q^*_{slow} does not guarantee that he can get the advantage of free shipping. Hence, the shopper should order the value of Tr in this interval, and still get the advantage of free shipping. (Region II) When Tr continues to increase the shopper cannot increase his order size to catch the threshold, because of higher inventory cost. He faces with either the cost of shortage or the cost of fast delivery, for a while shortages is more profitable compared to paying fast shipping fee for the shopper and ordering $Q^*_{shortage}$ is the best option he can do. (Region III) When Tr becomes an unreachable amount, ordering with instant fast shipping is the best option for him, therefore he should order Q^*_{fast} as his optimal order quantity.(Region IV)

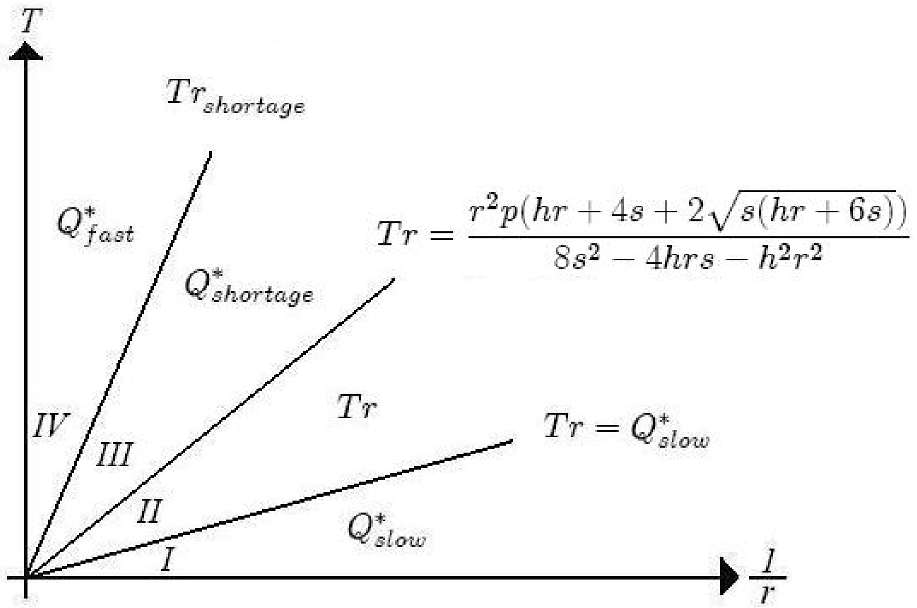

Figure 3 Optimal quantities of model with shortage for different combinations of T and 1/r. (Check Appendix to see how boundaries are obtained.)

The above figure summarizes optimal quantities for different combinations of T and

$1/r$ that is the time to consume one unit of commodity where

$$Tr_{shortage} = \frac{r\{pr^2h + 4prs + h^2rQ^*_{fast} + 2shQ^*_{fast} + \sqrt{A}\}}{2s(4s - hr)} \tag{17}$$

where A can be written as

$$A = p^2r^4h^2 + 6p^2r^3hs + 2pr^3h^3Q^*_{fast} + 12pr^2h^2sQ^*_{fast} + 24p^2r^2s^2 + 16prs^2hQ^*_{fast}$$
$$+h^4Q^{*2}_{fast}r^2 + 4h^3Q^{*2}_{fast}rs + 4h^2s^2Q^{*2}_{fast} + 4h^2r^2sk - 8hrs^2k - 32s^3k \tag{18}$$

2.2.2 Implications of Deterministic Model With Shortages

In the first region, time threshold T is very high compared to time to consume one unit of good. In other words, consumption rate r is so low; therefore, the shopper's inventory holding cost is low and his order size is small. In this region, the consumer can always choose slow shipping option and pay no shipping fee by choosing Q^*_{slow}. In the second region, the time threshold starts to increase and to get the advantage of free shipping and he orders Tr. When r increases, unit inventory cost (a function of $h/2$) decreases. In the third region, because of the inventory holding cost, consumer prefers shortage to get the advantage of free shipping by choosing $Q^*_{shortage}$ to minimize his cost. In the fourth region, falling into shortages becomes more costly than paying high shipping fee. As a result, consumer starts to choose fast shipping option and pays K. After discussing the results obtained from figure 3, we can construct the following figure that summarizes the global optimums for changing Tr values at a given consumption rate r.

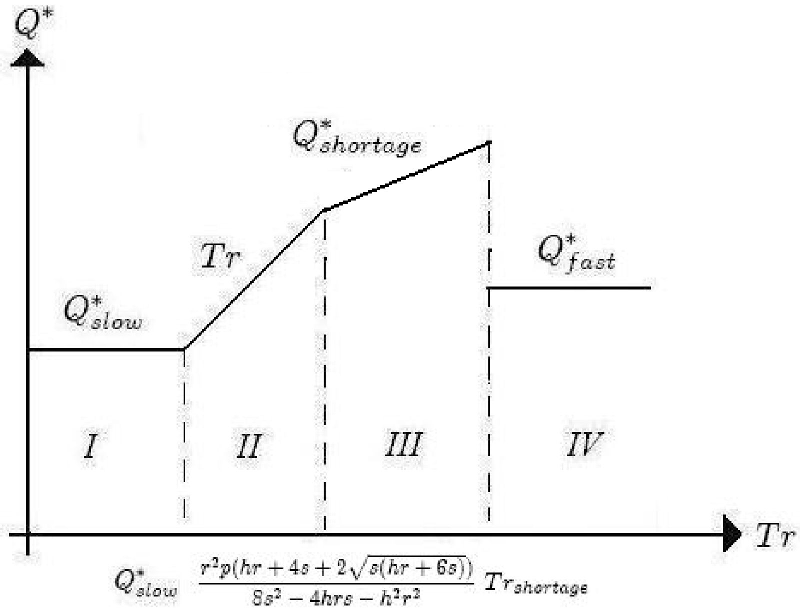

Figure 4 Optimal quantities of model with shortage at different Tr ranges for a given r.

As it is seen from the graph, in the third region order size is maximum and there is no extra delivery costs of expedited shipping for retailer. In the fourth region, the retailer starts to gain shipping fee. Again depending on the difference between shipping fee and extra delivery costs of expedited shipping, the retailer may prompt consumer in the left or right side of the boundary between third and fourth region. With the higher probability of repeat purchase, the retailer may not care about the order size and just look at the difference between shipping fee and extra delivery costs of expedited shipping.

2.3 Stochastic Quantity Model

2.3.1 Model and Assumptions

The steps that we will follow for this model is almost the same with the deterministic case without shortages; however, this time, our order size is stochastic, i.e, changing from order to order. The order size for the nth visit is $Q_n = Q + \epsilon_n$. This ϵ_n is our random error term with $U(-\beta, \beta)$. From the distribution of the error term $E(Q_n) = Q$ and by assumption $Var(Q_n) = \sigma^2$. When the shopper visits the seller's web site, he has a fixed quantity of order in his mind. The seller web site's campaigns like promotions, advertisements defines the final decision of his order size. With this set up our cost structure for the nth visit becomes

$$C_n = k + D_n + pQ_n + \frac{h}{2r}Q_n^2 \tag{19}$$

By using this cost structure we can find our cost for the nth visit as

$$C_n = \begin{cases} C_{n,slow} = k + pQ_n + \frac{h}{2r}Q_n^2 & Q_n/r \geq T \\ C_{n,fast} = k + K + pQ_n + \frac{h}{2r}Q_n^2 & Q_n/r < T \end{cases} \text{,i.e.,} \quad \begin{matrix} \varepsilon_n \geq Tr - Q \\ \varepsilon_n < Tr - Q \end{matrix} \tag{20}$$

By using this cost function we can obtain the optimal average long run costs as

$$LR(Q) = \frac{E(C_n)}{E(T_n)} = \frac{k + pQ + \frac{h}{2r}(Q^2+\sigma^2) + \psi(Tr - Q)K}{\frac{Q}{r}} \tag{21}$$

where ψ is the probability distribution of ε_n and $\varepsilon_n\tilde{}U(-\beta,\beta)$

$$LR(Q) = \begin{cases} L_{fast}(Q) = \{k + K + pQ + \frac{h}{2r}(Q^2+\sigma^2)\}/\frac{Q}{r} & \text{if } Q \leq Tr - \beta \\ L_{int}(Q) = \{k + pQ + \frac{h}{2r}(Q^2+\sigma^2) + K\frac{Tr - Q + \beta}{2\beta}\}/\frac{Q}{r} & \text{if } Q \in (Tr-\beta, Tr+\beta) \\ L_{slow}(Q) = \{k + pQ + \frac{h}{2r}(Q^2+\sigma^2)\}/\frac{Q}{r} & \text{if } Q \geq Tr + \beta \end{cases} \tag{22}$$

By using this long run cost function, we have the following optimal quantities for slow and fast shipping cases

$$Q^*_{slow} = \sqrt{\frac{2kr}{h} + \sigma^2} \text{ and } Q^*_{fast} = \sqrt{\frac{2(k+K)r}{h} + \sigma^2} \tag{23}$$

After some algebra it can be shown that optimal quantity for the intermediate branch is

$$Q^*_{int} = \sqrt{Q^{*2}_{slow} + \frac{Kr}{\beta h}(Tr + \beta)} = \sqrt{Q^{*2}_{fast} + \frac{Kr}{\beta h}(Tr - \beta)} \tag{24}$$

By using these optimal quantities we can redefine long run costs as

$$LR(Q) = \begin{cases} L_{fast}(Q) = \frac{h}{2}(\frac{Q^{*2}_{fast}}{Q} + Q) + pr & \text{if } Q \leq Tr - \beta \\ L_{int}(Q) = \frac{h}{2}(\frac{Q^{*2}_{int}}{Q} + Q) - \frac{Kr}{2\beta} + pr & \text{if } Q \in (Tr - \beta, Tr + \beta) \\ L_{slow}(Q) = \frac{h}{2}(\frac{Q^{*2}_{slow}}{Q} + Q) + pr & \text{if } Q \geq Tr + \beta \end{cases} \tag{25}$$

Then, optimal long run costs for slow, fast and intermediate branches as

$$LR(Q^*_{slow}) = hQ^*_{slow} + pr,\ LR(Q^*_{fast}) = hQ^*_{fast} + pr \text{ and } LR(Q^*_{int}) = hQ^*_{int} - \frac{Kr}{2\beta} + pr \tag{26}$$

After defining optimal long run costs for all three shipping cases, it is time to obtain global optimums that minimizes the cost of the shopper. If $Q^*_{slow} \geq Tr$, then $Q^* = Q^*_{slow}$; because, when $Q \geq Tr$, the shopper always chooses slow shopping option, and Q^*_{slow} minimizes the long run average cost of slow shipping. When $Tr \geq Q^*_{slow}$ ordering Q^*_{slow} does not guarantee that he can get the advantage of free shipping. Hence, the shopper should order the value of $Tr + \beta$ in this interval, and still get the advantage of free shipping. When Tr continues to increase the shopper cannot increase his order size to catch the threshold, because of higher inventory cost. In this third interval, he can choose both slow shopping and fast shopping from time to time. Thus, Q^*_{int} is the best option he can do. When Tr becomes an unreachable amount, ordering with instant fast shipping is the only option for him, therefore he should order Q^*_{fast} as his optimal order quantity.

The following figure summarizes optimal quantities for different combinations of T and $1/r$ that is the time to consume one unit of commodity. In the appendix part, you can find how we obtain the boundaries separating these four regions.

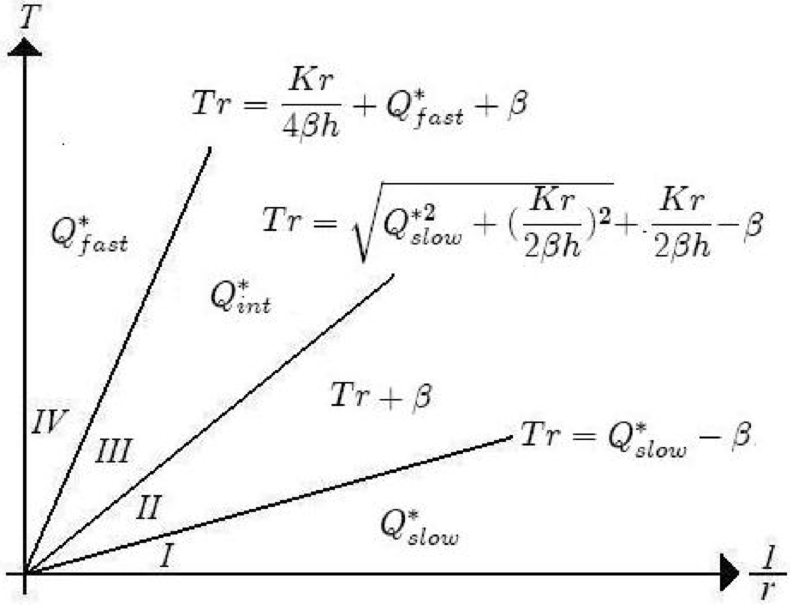

Figure 5 Optimal quantities of stochastic model for different combinations of T and 1/r. (Check Appendix to see how boundaries are obtained.)

2.3.2 Implications of the Stochastic Quantity Model

In the first region of Figure 5, the shopper can always catch the time threshold, since T is very small compared to time to consume one unit of commodity that is $1/r$. Therefore, the best strategy for the shopper is to order Q^*_{slow}. In the second region, the time threshold T starts to increase, thus to get the advantage of free shipping, he should increase his order size to $Tr + \beta$, otherwise he might go beyond the threshold. When r increases, his unit inventory cost, which is a function of $h/2r$, decreases. Thus, increasing order size is not create a trouble for the shopper in this region. In the third region, he can not catch T, because, inventory holding cost becomes a big deal for him. In this region, he starts to choose the expedited shipping according to his previous order size; i.e., sometimes either one of the shipping policies might be more appropriate. The best he can do is to choose Q^*_{int} to minimize his cost. In the fourth region, falling into shortages is more costly than paying high shipping fee. As a result of this, he always chooses fast shipping option and pays K. After discussing the results obtained from Figure 5, we can construct the following figure that summarizes the global optimums for changing Tr values at a given consumption rate r.

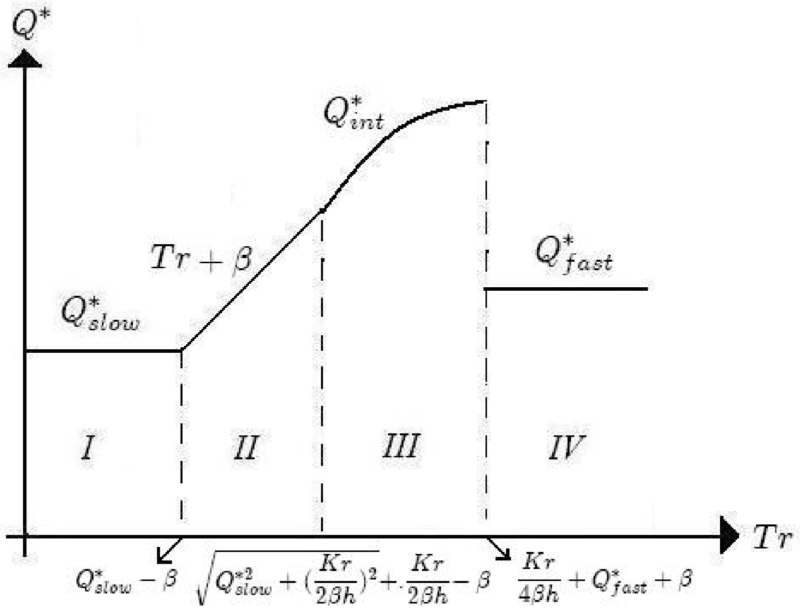

Figure 6 Optimal quantities of stochastic model at different Tr ranges for a given r.

3 Endogenous Consumption

Up to this point, I assume the shopper's consumption rate r is exogenous. Earlier empirical research by Bell et all. (2007) and Sun (2005) showed that it is not always the case, i.e., under given parameters of the seller, the shoppers consumption rate may be endogenous. To get the optimal consumption rate we concentrate on the second region of Figure 2, 4 and 6, where T is in moderate size.

3.1 Deterministic Model Without Shortages

Assume the shopper's gross value from consuming r units of product per unit time is given by the quadratic value function

$$V(r) = \frac{a^2 - (a - br)^2}{2b} = r(a - \frac{b}{2}r) \tag{27}$$

Hence, marginal benefit of increasing consumption by one unit of good per unit of time is $\frac{dV}{dr} = a - br$. Let us assume that total utility of consumption from consuming Q units of product at rate r is demonstrated by a separable utility function as below

$$U(Q, r) = V(r) - \bar{C}(Q, r) \tag{28}$$

To maximize this utility function, the shopper should minimize his expected average long run cost and obtain a value Q^*, then by using this optimal quantity, he should maximize his utility.

The optimal expected average long run cost in region II is

$$\bar{C}(r) = \frac{h}{2}Tr + pr + \frac{k}{T} \tag{29}$$

From this cost, marginal cost of increasing consumption is

$$d\bar{C} = (\frac{h}{2}T + p)dr \tag{30}$$

If we increase the consumption rate by one unit, average purchase cost increases by p units, and average inventory cost increases by $\frac{h}{2}T$ units. After obtaining the marginal cost, it is time to get the optimal consumption rate, that is

$$r^*(p, T) = \frac{1}{b}\left[a - p - \frac{h}{2}T\right] \tag{31}$$

From the equation above, it can easily be seen that the consumption rate of the consumer is decreasing with the time threshold for the slow shipping of the internet seller.

3.2 Deterministic Model With Shortages

For the second region of Figure 4, our analysis is the same as the former section except the value function assumption since, we would obtain the same result by quadratic value function. For the buyer, we again use a separable utility function that is $U(Q, r) = V(r) - \bar{C}(Q, r)$. However, this time, we assume to have a log value representation, thus our value function of consuming r units of the good is

$$V(r) = \ln(r) \tag{32}$$

For this value function marginal benefit of increasing consumption by one unit is $\frac{dV}{dr} = \frac{1}{r}$.

The optimal expected average long run cost in region II is

$$\bar{C}(r) = \frac{h}{2}Tr + pr + \frac{k}{T} \tag{29}$$

From this cost, marginal cost of increasing consumption is

$$d\bar{C} = (\frac{h}{2}T + p)dr \tag{30}$$

By using the separable utility assumption, it can be shown that

$$r^*(p, T) = (\frac{h}{2}T + p)^{-1} \tag{31}$$

Again, by increasing the time threshold T, the seller might force the shopper to decrease his consumption rate. This is intuitive, since to catch the new threshold, the shopper should decrease his consumption level. By doing that, he can continue to stay in the second region of Figure 4 where the size of the threshold is moderate, and does not have to pay the expedited shipping fee K.

3.3 Stochastic Quantity Model

For the second region of Figure 6, I again assume the shopper's gross value from consuming r units of product per unit time is given by the log-value function

$$V(r) = \ln(r) \tag{32}$$

For this value function marginal benefit of increasing consumption by one unit is $\frac{dV}{dr} = \frac{1}{r}$.

Again, by a separable utility function as below

$$U(Q, r) = V(r) - \bar{C}(Q, r) \tag{33}$$

To maximize this utility function, the shopper should minimize his expected average long run cost and obtain a value Q^*, then by using this optimal quantity, he should maximize his utility.

The optimal expected average long run cost in region II is

$$\bar{C}(r) = \frac{h}{2}(\frac{\frac{2kr}{h} + \sigma^2}{Tr + \beta} + Tr + \beta) + pr \tag{34}$$

From this cost, marginal cost of increasing consumption is

$$d\bar{C} = [\frac{h}{2}(\frac{\frac{2k}{h}}{Tr + \beta} - \frac{T(\frac{2kr}{h} + \sigma^2)}{(Tr + \beta)^2} + T) + p]dr \tag{35}$$

If we increase the consumption rate by one unit, average purchase cost increases by p units, and average inventory cost increases by $\frac{h}{2}T$ units. After obtaining the marginal cost, it is time to get the optimal consumption rate when $\beta \to 0$ (to obtain a closed form solution, I make this assumption), that is

$$r^*(p, T) = \frac{T + \sqrt{T^2 + T^2\sigma^2h^2 + 2T\sigma^2hp}}{T^2h + 2Tp} \tag{36}$$

From the equation above, it can easily be seen that the consumption rate of the consumer is decreasing with the time threshold for the slow shipping of the internet seller when $T > 1$.

4 Discussion

In this section, we discuss the results that we have found so far. Our first result was about the directly proportional relation between the time threshold for slow shipping and the order size of the buyer. To test this result, we can use and an OLS model that has a set of explanatory variables related to the basic characteristics of the internet buyer and a dummy variable for two different time thresholds; and a dependent variable that is the monetory value of the order size of the buyer. We expect the coefficient for the threshold dummy to be positive and significant.

Our second result was about the inversely proportional relation between the time threshold and the consumption rate of the consumer. We can test this by taking a time period, for instance six months, for different consumers of the same product while the threshold is constant and taking another period with the same group of buyers again for the same product, but with a different time threshold. We can construct an econometric model like the one above, but with the true value of the threshold as the threshold regressor and aggregate consumption for each period as the regressant. Our expectation from this model is to find the coefficient for the threshold variable to be negative and significant.

In addition to these analysis, we might work on two models for two goods such as hearth and vitamin pills that we have different consumption rates to study the effect of the time threshold on the monetary value of the consumption on these goods. The results of this study might be very interesting for the policy makers. The internet sellers can apply different time thresholds for different commodities to maximize their profits.

Finally, we can study two models again for two commodities that are perfect substitutes of each other, i.e., they have almost the same consumption rate. We can study the ordering behavior emperically for these goods under different shipping policies. The response of the customers for these goods under different time thresholds might be different. The internet company might again get the advantage of high revenues by appliying true shipment policies.

5 Conclusion

In this paper, we modeled an internet buyer's shopping behavior who visits an internet seller's site repeatedly to buy a non-durable good. To find the optimal ordering behavior of the shopper, we applied traditional EOQ model. The shopper's aim is to minimize his average long run cost that has transaction, purchasing, shipping, inventory and shortage components under a given price and shipping policy of the seller. The shopper faces with two options to place his order, one is expedited shipping with a fixed shipping fee, and the other is slow and free shipping. The size of each order cycle in terms of time determines by which policy the shopper should order. The contributions to the marketing literature of this study can be summarized as

- By increasing the waiting time for slow shipping that is T the seller may induce the shopper to increase his order size;
- By increasing T, the seller may induce the shopper to decrease his/her consumption rate.

Further research may consider the revenue generated by increasing the order size of the consumer, the probability of repurchase, the shipping fee for expedited shipping and the extra costs of fast delivery for retailer to find the optimal level of T which maximizes the profit.

6 Appendix

6.1 Boundaries for Model Without Shortages

Boundary between Region I and II:

We know that $LR^*_{slow} = pr + hQ^*_{slow}$ and $LR_{slow}(Q) = pr + \frac{h}{2}(\frac{Q^{*2}_{slow}}{Q} + Q)$, hence, at the boundary, these two costs should be equal to each other, i.e., $hQ^*_{slow} = \frac{h}{2}(\frac{Q^{*2}_{slow}}{Tr} + Tr)$, or equivalently $Tr = Q^*_{slow}$.

Boundary between Region II and III:

We know that $LR_{slow}(Q) = pr + \frac{h}{2}(\frac{Q^{*2}_{slow}}{Q} + Q)$ and $LR^*_{fast} = pr + hQ^*_{fast}$, hence, at the boundary, these two costs should be equal to each other, i.e., $\frac{h}{2}(Tr) + \frac{kr}{(Tr)} = \sqrt{2(k+K)hr}$. Solving this equation yields $Tr = Q^*_{fast} + \sqrt{\frac{2Kr}{h}}$.

6.2 Boundaries For Model With Shortages

Boundary between Region I and II:

We know that $LR^*_{slow} = pr + hQ^*_{slow}$ and $LR_{slow}(Q) = pr + \frac{h}{2}(\frac{Q^{*2}_{slow}}{Q} + Q)$, hence, at the boundary, these two costs should be equal to each other, i.e., $hQ^*_{slow} = \frac{h}{2}(\frac{Q^{*2}_{slow}}{Tr} + Tr)$, or equivalently $Tr = Q^*_{slow}$.

Boundary between Region II and III:

We know that $LR_{slow}(Q) = pr + \frac{h}{2}(\frac{Q^{*2}_{slow}}{Tr} + Tr)$ and $LR^*_{shortage} = \frac{1}{T}(k + pQ^*_{shortage} + \frac{h}{2r}Q^{*2}_{shortage} + \frac{Q^{*2}_{shortage}}{r^2}s) + Ts - 2\frac{Q^*_{shortage}}{r}s$, hence, at the boundary, these two costs should be equal to each other. Solving this equation yields $Tr = \frac{r^2p(hr + 4s + 2\sqrt{s(hr+6s)})}{8s^2 - 4hrs - h^2r^2}$.

Boundary between Region III and IV:

We know that $LR^*_{shortage} = \frac{1}{T}(k + pQ^*_{shortage} + \frac{h}{2r}Q^{*2}_{shortage} + \frac{Q^{*2}_{shortage}}{r^2}s) + Ts - 2\frac{Q^*_{shortage}}{r}s$ and $LR^*_{fast} = pr + hQ^*_{fast}$, hence, at the boundary, these two costs should be equal to each other. Solving this equation yields the boundary Tr to be
$Tr = \frac{r\{pr^2h + 4prs + h^2rQ^*_{fast} + 2shQ^*_{fast} + \sqrt{A}\}}{2s(4s - hr)}$, where $A = p^2r^4h^2 + 6p^2r^3hs + 2pr^3h^3Q^*_{fast} + 12pr^2h^2sQ^*_{fast} + 24p^2r^2s^2 + 16prs^2hQ^*_{fast} + h^4Q^{*2}_{fast}r^2 + 4h^3Q^{*2}_{fast}rs + 4h^2s^2Q^{*2}_{fast} + 4h^2r^2sk - 8hrs^2k - 32s^3k$

6.3 Boundaries for the Stochastic Model

Boundary between Region I and II:

We know that $LR^*_{slow} = pr + hQ^*_{slow}$ and $LR_{slow}(Q) = pr + \frac{h}{2}(\frac{Q^{*2}_{slow}}{Q} + Q)$, hence, at the boundary, these two costs should be equal to each other, i.e., $hQ^*_{slow} = \frac{h}{2}(\frac{Q^{*2}_{slow}}{Tr+\beta} + Tr + \beta)$, or equivalently $Tr = Q^*_{slow} - \beta$.

Boundary between Region II and III:

We know that $LR_{slow}(Q) = pr + \frac{h}{2}(\frac{Q^{*2}_{slow}}{Q} + Q)$ and $LR^*_{int} = pr + hQ^*_{int} - \frac{Kr}{2\beta} + pr$, hence, at the boundary, these two costs should be equal to each other, i.e., $\frac{h}{2}(\frac{Q^{*2}_{slow}}{Tr+\beta} + Tr + \beta) = h\sqrt{Q^{*2}_{slow} + \frac{Kr}{\beta h}(Tr+\beta)} - \frac{Kr}{2\beta}$. Solving this equation yields $Tr = \sqrt{Q^{*2}_{slow} + (\frac{Kr}{2\beta h})^2} + .\frac{Kr}{2\beta h} - \beta$

Boundary between Region III and IV:

We know that $LR^*_{int} = hQ^*_{int} - \frac{Kr}{2\beta} + pr$ and $LR^*_{fast} = pr + hQ^*_{fast}$, hence, at the boundary, these two costs should be equal to each other. , i.e., $h\sqrt{Q^{*2}_{fast} + \frac{Kr}{\beta h}(Tr - \beta)} - \frac{Kr}{2\beta} = hQ^*_{fast}$. Solving this equation yields $Tr = \frac{Kr}{4\beta h} + Q^*_{fast} + \beta$.

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8 Vita

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