DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

# MATHEMATICAL PROGRAMMING BASED HEURISTICS FOR PRODUCTION ROUTING PROBLEMS WITH VISIT SPACING POLICY AND TRANSSHIPMENT

by
Mustafa AVCI

June, 2019
İZMİR

# MATHEMATICAL PROGRAMMING BASED HEURISTICS FOR PRODUCTION ROUTING PROBLEMS WITH VISIT SPACING POLICY AND TRANSSHIPMENT

**A Thesis Submitted to the**
**Graduate School of Natural and Applied Sciences of Dokuz Eylül University**
**In Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Industrial Engineering, Industrial Engineering Program**

**by**
**Mustafa AVCI**

**June, 2019**
**İZMİR**

**Ph.D. THESIS EXAMINATION RESULT FORM**

We have read the thesis entitled **"MATHEMATICAL PROGRAMMING BASED HEURISTICS FOR PRODUCTION ROUTING PROBLEMS WITH VISIT SPACING POLICY AND TRANSSHIPMENT"** completed by **MUSTAFA AVCI** under supervision of **PROF. DR. ŞEYDA A. YILDIZ** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Doctor of Philosophy.

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## ACKNOWLEDGMENTS

First, I would like to thank my supervisor, Prof. Dr. Şeyda A. YILDIZ, for her support, guidance, and understanding. She not only served as my supervisor but also encouraged me throughout my graduate programs. She always gave me freedom in my research while helping me to overcome challenges and giving advices.

I would like to express my sincere thanks to the other members of my thesis committee: Prof. Dr. Efendi NASİBOĞLU and Assoc. Prof. Dr. A. Serdar TAŞAN for their constructive comments to improve the quality of this thesis.

Finally, I would like to express my special thanks to my family, Gonca, Emre, Muzaffer AVCI, Hülya, Kemal, Pınar YUNUSOĞLU, for their love, patience, encouragement, and endless support in my whole life.

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**MATHEMATICAL PROGRAMMING BASED HEURISTICS FOR PRODUCTION ROUTING PROBLEMS WITH VISIT SPACING POLICY AND TRANSSHIPMENT**

## ABSTRACT

The production routing problem (PRP) is an integrated operational planning problem that combines the two well-known optimization problems, vehicle routing problem (VRP) and lot-sizing problem (LSP). The purpose of this thesis is to integrate the visit spacing policy (VSP) and the concept of transshipments into the PRP. The reason behind considering the VSP in the context of the PRP is to ensure the service consistency. The VSP is especially important to provide higher quality service to the retailers. In this thesis, we extend the basic version of the PRP by taking into consideration the VSP. For its solution, an iterative matheuristic algorithm (MA) is proposed. The proposed MA is tested on a set of randomly generated problem instances with VSP as well as on standard PRP and the inventory routing problem (IRP) with the VSP benchmark instances. The results indicate the effectiveness of the algorithm. In order to achieve the second purpose of this thesis, we extend the classical version of the PRP by considering transshipments, either from supplier to retailers or between retailers, to further reduce the total cost. To solve the problem, we develop a mathematical programming heuristic. The algorithm is applied to two sets of randomly generated problem instances. The results show the effectiveness of the proposed approach. Moreover, an extensive computational study is performed to reveal the effect of the transshipments on the overall cost.

**Keywords:** Production routing problem, consistency, transshipment, visit spacing, matheuristics, heuristics

# ZİYARET ARALIKLANDIRMA POLİTİKALI VE AKTARMALI ÜRETİM ROTALAMA PROBLEMLERİ İÇİN MATEMATİKSEL PROGRAMLAMA TABANLI SEZGİSEL YÖNTEMLER

## ÖZ

Üretim rotalama problemi (ÜRP), araç rotalama ve parti büyüklüğü belirleme problemlerini birleştiren bir entegre operasyonel planlama problemidir. Bu tezin amacı ziyaret aralıklandırma politikası (ZAP) ve aktarma kavramının ÜRP kapsamında incelenmesidir. ZAP, perakendecilere daha kaliteli hizmet sağlayabilmek için özellikle önemlidir. Bu tez kapsamında, ÜRP'nin klasik versiyonu ZAP dikkate alınarak genişletilmiştir. Problemin çözümü için bir yinelemeli mat-sezgisel algoritma (MA) geliştirilmiştir. Önerilen MA rastgele türetilen problem örneklerinin yanı sıra, literatürde bulunan klasik ÜRP ve ZAP içeren envanter rotalama test problemlerine de uygulanmıştır. Elde edilen sonuçlar algoritmanın etkinliğini göstermektedir. Bu tezin ikinci amacı olan toplam sistem maliyetini düşürmek için, ÜRP'nin klasik versiyonu tedarikçiden perakendecilere veya perakendeciler arasındaki aktarmalar dikkate alınarak genişletilmiştir. İlgili problemi çözmek için, bir matematiksel programlama sezgiseli geliştirilmiştir. Önerilen algoritma, rastgele oluşturulmuş iki problem kümesine uygulanmıştır. Elde edilen sonuçlar önerilen çözüm yönteminin etkinliğini ortaya koymaktadır. Ayrıca, tıpkı ZAP'lı ÜRP için olduğu gibi, aktarmalı ÜRP için de ilgili kavramın maliyetler üzerindeki etkisini ortaya çıkarmak için geniş kapsamlı sayısal deneyler gerçekleştirilmiştir.

**Anahtar kelimeler:** Üretim rotalama problem, tutarlılık, aktarma, ziyaret aralıklandırma, mat-sezgiseller, sezgiseller

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# CHAPTER ONE
# INTRODUCTION

## 1.1 The Production Routing Problem and Its Importance

In a typical supply chain, the decision problems of production, inventory, distribution, and routing operations are generally optimized independently. For instance, the production lot-sizing operations are planned with the aim of minimizing the production and inventory holding costs at the production facility. Then, the distribution decisions are determined with respect to the lot-sizing decisions previously taken. Since a decision taken for a particular operation is used as an input for its successor operations, independent optimization of the problems does not give a satisfactory result for the entire system. Therefore, the integration of these decision problems has attracted research interest due to the benefits provided by the coordination of these operations. The advances in hardware technology and in optimization techniques have made possible to develop effective solution algorithms for these complicated problems. There exist many successful studies related to integrated inventory and distribution planning in the literature. For example, by implementing an integrated production and distribution planning system, the Kelogg company saves 4.5 million dollars in 1995 (Brown, Keegan, Vigus & Wood, 2001). In addition, it has been reported in Çetinkaya, Üster, Easwaran & Keskin (2009) that an integrated production, inventory, distribution, and routing system led to a 10% decrease in logistics costs in Frito-Lay. Other quantitative studies on the value of integrated decisions can be found in Chandra & Fisher (1994), Stålhane, Andersson, Christiansen & Fagerholt, K. (2014), Archetti & Speranza (2016), and Côté, Guastaroba & Speranza (2017).

The inventory routing problem (IRP) is one of the integrated problems which requires the combined optimization of the inventory, distribution, and routing problems to minimize the total cost. The production routing problem (PRP) is a generalized version of the IRP in which the production decisions are also taken into account. The practical applications of the PRP can be especially seen in vendor-managed inventory (VMI) systems. In a VMI system, the supplier manages inventory levels of the retailers, and makes replenishment decisions for each retailer. The ability to determine the time and size of all deliveries allows the supplier to combine the retailer orders efficiently. Moreover, the retailers benefit by being free from the inventory control process. The PRP integrates two difficult well-known problems, the lot-sizing problem (LSP) and the vehicle routing problem (VRP), which makes the problem very challenging (Adulyasak, Cordeau & Jans, 2015a).

## 1.2 Research Objectives

The purpose of this thesis is twofold. The first objective is to study an important consistency concept within the PRP to improve the service quality in VMI systems. The second one is to integrate the transshipment concept into the PRP to reduce the overall system cost.

Regarding the first research objective, although VMI policies tend to benefit both the vendor and the retailers, solving PRPs by aiming only cost minimization may not provide satisfactory results to both parties. To remain competitive, companies also need to take into consideration the service quality level, because high service quality establishes a bond between the retailer and the supplier. By this way, companies ensure the customer loyalty which leads to recurring revenues (Kovacs, Golden, Hartl & Parragh, 2014). In this respect, the consistency issues have been studied in the periodic vehicle routing problem (Kovacs et al., 2014) and in the IRP (Coelho, Cordeau & Laporte, 2012a; Coelho & Laporte, 2013) to reflect some service quality standards. These are mainly associated with the quantity and the frequency of the deliveries, fleet size, and vehicle load.

Among the consistency concepts, visit spacing policy (VSP) is especially important to provide higher quality service to the retailers. In the VSP implementation, visit times of the retailers are regulated by imposing a minimum and a maximum time interval between two consecutive visits to the same retailer. Thereby, possible inconveniences stemming from the visit time irregularities both to the retailer and to the supplier are prevented. This policy not only yields smoother operations by distributing the visits evenly over the planning horizon, but it also leads to the quantity consistency on deliveries. As indicated in Coelho et al. (2012a), the quantity of delivery is especially important for the customers. By imposing the minimum time constraint, frequent visits that lead to too small deliveries are prevented. On the other hand, the maximum time constraint prohibits long time intervals between consecutive visits, thereby preventing too large deliveries. The frequent deliveries take a lot of time and lead to inconveniences to the customers while too large deliveries may create congestion in the warehouse. A case study about the effect of the service consistency in a VMI system can be found in Day, Wright, Schoenherr, Venkataramanan & Gaudette (2009). The authors investigate the inventory replenishment problem of a company that provides carbon dioxide gas for carbonated soft drinks to restaurants, hospitals, chemical manufacturers, and various other businesses. Due to increased competition, the company wants to distinguish itself by a superior customer service. This is achieved by ensuring regularity on delivery times to the customers and by improving driver-customer familiarity. The VSP has been studied in the periodic VRP (Christofides & Beasley, 1984; Francis, Smilowitz & Tzur, 2008) and in the IRP (Coelho et al., 2012a) but it has not yet been considered in the PRP.

In this thesis, we address the PRP with the visit spacing policy (PRP-VSP) for the first time. Initially, two mixed integer programming (MIP) models are formulated for the PRP-VSP. Since the models can only solve small size problems, a matheuristic algorithm (MA) is developed for solving the PRP-VSP. The proposed MA is implemented in a multi-start scheme in which an MIP-based local search procedure is iteratively applied. The proposed algorithm is applied to a set of randomly generated PRP-VSP instances. The results indicate that the developed approach can produce high quality solutions to the PRP-VSP in reasonable computation times. Additionally, we perform a numerical investigation to observe the effect of the VSP on the overall system cost.

In order to achieve our second objective, we extend the standard PRP by considering transshipments. In this case, in addition to the regular deliveries, transshipments between retailers or between the supplier and a retailer can be made. These transshipments are performed by a subcontractor. Transshipments provide benefits both in deterministic and stochastic environments. In stochastic environments, transshipments between retailers belonging to the same chain make the unexpected demand variation problem easier to overcome (Axsäter, 1990; Dada, 1992; Lee, 1987; Nonås & Jörnsten, 2007). In deterministic environments, transshipments are made at predetermined moments in time to redistribute stock among the retailers to reduce the inventory holding costs. These type of transshipments are especially beneficial in the retail sector because the inventory holding costs are often dominant in that environment (Paterson, Kiesmüller, Teunter & Glazebrook, 2011). Moreover, deterministic subproblems can arise when solving a stochastic IRP/PRP in a rolling horizon framework in which demand forecasts for the next time periods are utilized as approximations of the unknown demand. A case study on an inventory-routing system used by a supermarket chain was presented in Mercer & Tao (1996). In the inventory routing system, deliveries are made from a factory to warehouses, and lateral transshipments can occur between warehouses. The results indicate that transshipments lower the number of sales lost. To the best of our knowledge, the concept of transshipments has been studied in the IRP (Coelho, Cordeau & Laporte, 2012b, 2014; Lefever, Aghezzaf, Hadj-Hamou & Penz, 2018), but it has not yet been considered in the PRP.

In this thesis, we introduce, and study, the PRP with transshipments (PRPT). Initially, an MIP model is formulated for the PRPT. Since the model can only solve small-size problems, a mathematical programming heuristic (MPH) is proposed to solve the problem. The performance of our MPH is compared with a general-purpose commercial solver. The results indicate that the developed approach can produce high-quality solutions to the PRPT in reasonable computation times. Moreover, we perform another numerical investigation to reveal the effect of the transshipments on the overall cost. The results show the benefits of allowing transshipments.

### 1.3 Outline of the Thesis

The rest of this thesis involves five chapters, and is organized as follows;

In Chapter 2, the classical PRP is explained in detail. Initially, formal problem description is given along with the MIP formulation of the problem. Next, the related literature of the PRP is presented. Finally, research gaps related to the PRP are stressed.

Chapter 3 introduces our study on the PRP-VSP with a single vehicle. Initially, the developed MIP formulations are presented. Next, the details of the proposed MA are given, and finally the computational results are presented.

In Chapter 4, we extend our work presented in Chapter 3 by considering multiple vehicles in the PRP-VSP. After giving the MIP formulations, the variants of MA proposed for the multi-vehicle version of the PRP-VSP are given. Next, the computational results obtained for the multi-vehicle problems are presented.

Chapter 5 presents our study on the PRPT. Initially, the MIP formulation of the PRPT is provided. After that, the details of the developed MPH and the computational results are successively given.

Finally, Chapter 6 summarizes the thesis and provides directions for future research.

# CHAPTER TWO
# THE PRODUCTION ROUTING PROBLEM

## 2.1 Introduction

In this chapter, the production routing problem (PRP) is explained and a review of the related literature is given. The chapter is organized as follows. In the next section, a formal problem description and a mixed integer programming (MIP) formulation are provided for the PRP. Next, we present a literature review for the problem. Finally, Section 2.5 summarizes the context of the chapter.

## 2.2 The Production Routing Problem

In this section, we initially give information about the integrated supply chain problems related with the PRP. Figure 2.1 presents the categories of integrated production-distribution problems with a discretized planning horizon. As seen from the figure, the lot-sizing problem (LSP) involves the decisions of production quantity, schedule, and inventory quantity. The integrated LSP with direct shipment (ILSPDR) (see Federgruen & Tzur, 1999; Jin & Muriel, 2009) takes into account the transportation of the products to the customers. The transportation costs of the products to the customers are assumed to be a fixed cost. The production, setup, inventory, and direct shipment costs are minimized over the planning horizon.

When the routing decisions are included and the production decisions are omitted, the ILSPDR transformed into the inventory routing problem (IRP) (see Andersson, Hoff, Christiansen, Hasle & Løkketangen, 2010; Coelho, Cordeau & Laporte, 2013). The classical IRP consists of a central distribution center, a production plant, and a set of retailers. Both the production plant and the retailers have their own warehouses. An inventory holding cost is incurred in every planning period for each unit of the product stored at the warehouses. In each period of the planning horizon, the demand for the product at each retailer must be satisfied. The production quantity which is ready at the beginning of each planning period is assumed to be known in advance. For the transportation of products, there exist a limited number of capacitated vehicles available at the beginning of each planning period. The aim of the IRP is to determine the quantities and schedules of the products to be delivered to the retailers as well as the set of vehicle routes that minimize the total cost which consists of the inventory holding costs and transportation costs.

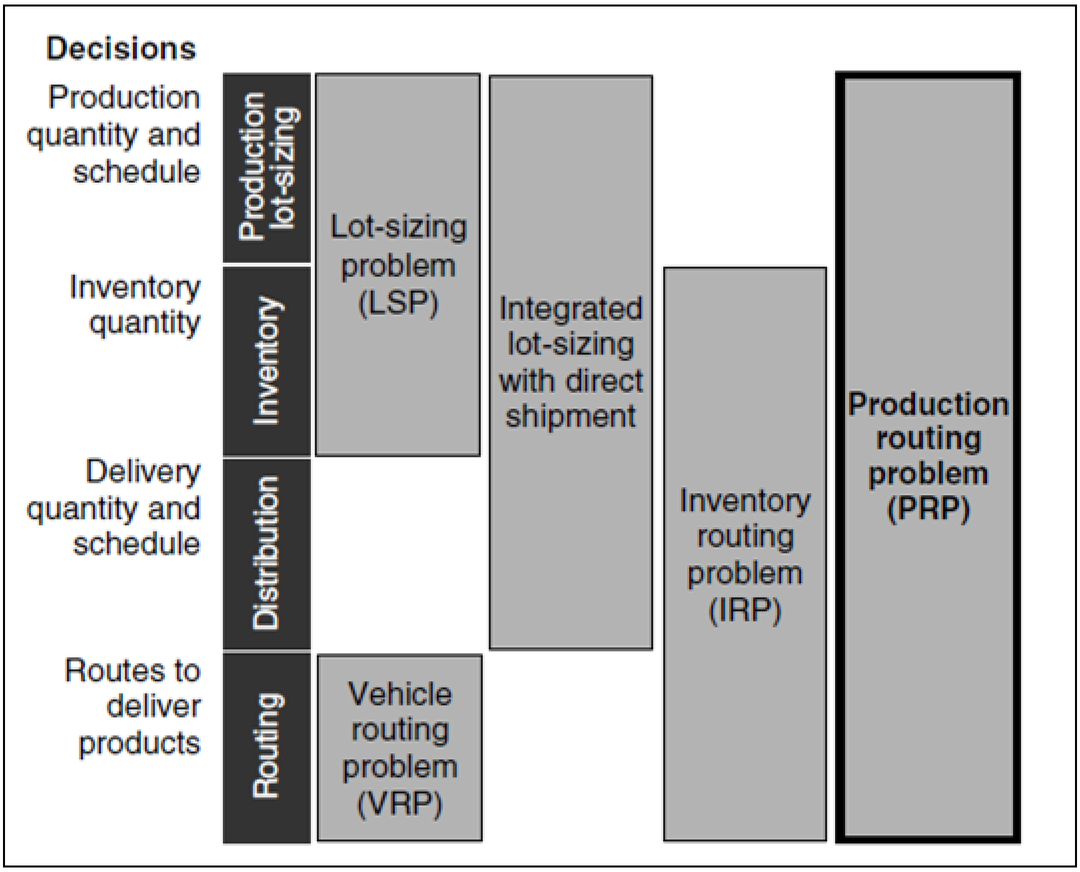


Figure 2.1 Supply chain planning problems (Adulyasak, Cordeau & Jans, 2014a)

The PRP is a generalized version of the IRP, where the production decisions are also taken into account. At the beginning of each planning period, the production plant must decide whether or not production is executed and determine the corresponding lot-size. If production is executed, a fixed setup cost and unit production costs are incurred. Moreover, the lot-size cannot exceed the production capacity. In the next section, we provide the problem description and a formulation of the standard PRP which is comprised of a network of a single product, a single production plant, and multiple retailers.

## 2.3 Mathematical Formulation for the PRP

The PRP is defined on a complete graph $G=(N,A),$ where $N=\{0,...,n\}$ is the node set and $A=\{(i,j):i,j\in N,i\neq j\}$ is the arc set. Node 0 represents the production plant, while the other nodes denote retailers. The plant manufactures a single product which is sold by the retailers along a planning horizon $T=\{1,...,H\}$. There is a set $K=\{1,...,m\}$ of homogeneous vehicles available for transportation at the beginning of each time period. The relevant notation for the model is given as follows:

<u>*Sets*</u>

$N^r$ : set of all retailers $\{1,...,n\}$

$N$ : set of all nodes $\{0,1,...,n\}$

$T$ : set of time periods $\{1,...,H\}$

$K$ : set of identical vehicles $\{1,...,m\}$

<u>*Parameters*</u>

$u$ : unit production cost

$f$ : production setup cost

$C$ : production capacity

$Q$ : capacity of a vehicle

$h_i$ : unit inventory holding cost at node $i$

$L_i$ : maximum inventory level at node $i$

$I_{i0}$ : initial inventory level at node $i$

$d_{it}$ : demand at retailer $i$ in period $t$

$c_{ij}$ : transportation cost from node $i$ to node $j$

$M_t$ : $\min\{\mathrm{C},\sum_{l=t}^{H}\sum_{i\in N^r} d_{il}\}$

$\bar{M}_{it}$ : $\min\{L_i,Q,\sum_{l=t}^{H} d_{il}\}$

*Decision Variables*

$p_t$ : production quantity in period $t$

$$y_t : \begin{cases} 1 & \text{if there is production at the plant in period } t \\ 0 & \text{otherwise} \end{cases}$$

$I_{it}$ : inventory level at node $i$ at the end of period $t$

$$x_{ijtk} : \begin{cases} 1 & \text{if vehicle } k \text{ travels directly from node } i \text{ to node } j \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$$

$q_{itk}$ : quantity delivered to retailer $i$ in period $t$ by vehicle $k$

$$z_{itk} : \begin{cases} 1 & \text{if retailer } i \text{ is visited in period } t \text{ by vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

$$z_{0tk} : \begin{cases} 1 & \text{if vehicle } k \text{ is dispatched in period } t \\ 0 & \text{otherwise} \end{cases}$$

The PRP can be formulated as follows:

$$Min\,Z = \sum_{t\in T}(u.p_t + f.y_t + \sum_{i\in N} h_i.I_{it} + \sum_{k\in K}\sum_{i\in N}\sum_{j\in N\setminus\{i\}} c_{ij}.x_{ijtk}) \tag{2.1}$$

subject to

$$I_{0t-1} + p_t = \sum_{i\in N^r}\sum_{k\in K} q_{itk} + I_{0t}, \forall t\in T \tag{2.2}$$

$$I_{it-1} + \sum_{k\in K} q_{itk} = d_{it} + I_{it}, \forall i\in N^r, \forall t\in T \tag{2.3}$$

$$p_t \le M_t.y_t, \forall t\in T \tag{2.4}$$

$$I_{0t} \le L_0, \forall t\in T \tag{2.5}$$

$$I_{it-1} + \sum_{k\in K} q_{itk} \le L_i, \forall i\in N^r, \forall t\in T \tag{2.6}$$

$$q_{itk} \le \bar{M}_{it}.z_{itk}, \forall i\in N^r, \forall t\in T, \forall k\in K \tag{2.7}$$

$$\sum_{k\in K} z_{itk} \le 1, \forall i\in N^r, \forall t\in T \tag{2.8}$$

$$\sum_{j\in N\setminus\{i\}} x_{jitk} = \sum_{j\in N\setminus\{i\}} x_{ijtk}, \forall k\in K, \forall i\in N, \forall t\in T \tag{2.9}$$

$$\sum_{j\in N\setminus\{i\}} x_{ijtk} = z_{itk}, \forall k\in K, \forall i\in N, \forall t\in T \tag{2.10}$$

$$\sum_{j\in N^r} q_{itk} \le Q_k.z_{0tk}, \forall k\in K, \forall t\in T \tag{2.11}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijtk} \leq \sum_{i \in S} z_{itk} - z_{etk}, \forall k \in K, \forall t \in T, \forall S \subseteq N^r : |S| \geq 2, \forall e \in S \quad (2.12)$$

$$p_t, I_{it}, q_{itk} \geq 0, \forall i \in N, \forall k \in K, \forall t \in T \quad (2.13)$$

$$x_{ijtk}, y_t, z_{itk} \in \{0,1\}, \forall i \in N, \forall j \in N, \forall k \in K, \forall t \in T \quad (2.14)$$

The objective function (2.1) minimizes the total production, setup, inventory, and routing costs. Constraint sets (2.2) – (2.6) represent the lot-sizing part of the problem. Constraint sets (2.2) and (2.3) are the inventory flow balance at the plant and retailers, respectively. Constraint set (2.4) comprises the setup forcing and production capacity constraints. The constraints force the setup variable to be one if production takes place in a given period and limit the production quantity to the minimum value between the production capacity and the total demand in the remaining periods. Constraint sets (2.5) and (2.6) limit the maximum inventory at the plant and retailers, respectively. Constraint set (2.7) allows a positive delivery quantity only if retailer $i$ is visited in period $t$. The delivered quantity $q_{itk}$ is limited by $\bar{M}_{it}$, which is the minimum of the retailer inventory capacity $L_i$, vehicle capacity $Q_k$, and total demand of retailer $i$ in periods from $t$ to $H$. Constraint set (2.8) imposes that each retailer can be visited by at most one vehicle. Constraint sets (2.9) and (2.10) are the vehicle flow conservation constraints. Constraint set (2.11) imposes the vehicle capacity on each vehicle. Constraint set (2.12) constitutes the subtour elimination constraints similar to those of the traveling salesman problem (TSP) and constraints (2.13) and (2.14) define the nature of the decision variables.

### 2.4 Literature Review

The PRP has not received much attention from the researchers due to its difficulty. However, in recent years, there has been a growing interest to the problem owing to the advantages of integrating the production and logistics decision problems. A comprehensive review of the papers related to the PRP can be found in Adulyasak et al. (2015a).

In Table 2.1, the solution algorithms developed for the PRP are given according to the classification made in Adulyasak et al. (2015a). The related works on the PRP can be categorized according to the problem characteristics and solution methods.

Regarding the production aspect, the number of plants and products and the production capacity of plants are considered as different problem characteristics of the PRP in the literature. As for the inventory aspect, there are two different inventory policies considered in the related papers. The first policy, called order-up-to level (OU), dictates that the amount of delivery to a visited retailer must increase its inventory level up to a predetermined stock level. In the second policy, called maximum level (ML), the delivery quantity to a retailer may be any positive value which does not make its inventory level bigger than the maximum stock level. Also, the structure of the vehicle fleet, the number of vehicles, and the vehicle capacities are the attributes that characterize the distribution characteristics in the related studies.

When the solution methods proposed for the PRP are analyzed, it is observed that many of them introduce heuristic solution approaches because of the complexity of the problem. The PRP was first introduced in Chandra (1993) and Chandra & Fisher (1994). In these papers, the PRP is decomposed into the capacitated lot-sizing and the distribution scheduling problem. The capacitated lot-sizing problem is solved by using the Silver-Meal algorithm (Silver & Meal, 1973) and the obtained results are used as inputs for the distribution scheduling problem which is solved by a simple heuristic algorithm. Another decomposition based heuristic solution approach was proposed by Lei, Liu, Ruszczynski & Park (2006). In this study, the authors considered the PRP with multiple production plants and heterogeneous fleet of vehicles. A two-phase heuristic solution approach was developed for the problem. In the first phase, a restricted mixed-integer linear model of the problem in which the routing constraints are excluded is solved. In the second phase, a routing heuristic is implemented to obtain the vehicle routes for each planning period.

Table 2.1 Classification of the papers on the PRP (Adapted from Adulyasak et al. (2015a))

| | Problem characteristics | | | | | | | | Solution method | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Production | | | Inventory | | Distribution | | | | |
| Author(s) | N.Plants | Product | Cap. | Policy | Cap. | Fleet | N.Vehs | Cap. | Type | Approach |
| Chandra (1993) | Single | Multiple | ✓ | ML | | Hom. | Unlimited | ✓ | H | Decomposition |
| Chandra & Fisher (1994) | Single | Multiple | ✓ | ML | | Hom. | Unlimited | ✓ | H | Decomposition |
| Fumero & Vercellis (1999) | Single | Multiple | ✓ | ML | | Hom. | Limited | ✓ | H | Lagrangian relaxation |
| Lei et al. (2006) | Multiple | Single | ✓ | ML | ✓ | Het. | Limited | ✓ | H | Decomposition |
| Boudia et al. (2007) | Single | Single | ✓ | ML | ✓ | Hom. | Limited | ✓ | H | GRASP |
| Boudia et al. (2008) | Single | Single | ✓ | ML | ✓ | Hom. | Limited | ✓ | H | Decomposition |
| Boudia & Prins (2009) | Single | Single | ✓ | ML | ✓ | Hom. | Limited | ✓ | H | Memetic |
| Bard & Nananukul (2009a) | Single | Single | ✓ | ML | ✓ | Hom. | Limited | ✓ | H | Tabu search |
| Solyalı & Süral (2009) | Single | Single | | OU | ✓ | Hom. | Limited | ✓ | H | Lagrangian relaxation |
| Bard & Nananukul (2009b, 2010) | Single | Single | ✓ | ML | ✓ | Hom. | Limited | ✓ | H | Branch-and-price |
| Ruokokoski et al. (2010) | Single | Single | | ML | | Hom. | Single | | E | Branch-and-cut |
| Armentano et al. (2011) | Single | Multiple | ✓ | ML | ✓ | Hom. | Limited | ✓ | H | Tabu search |
| Archetti et al. (2011) | Single | Single | | ML/OU | ✓ | Hom. | Single | ✓ | E/H | Branch-and-cut/MIP Heuristic |
| Adulyasak et al. (2014a) | Single | Single | ✓ | ML | ✓ | Hom. | Multiple | ✓ | H | ALNS |
| Adulyasak et al. (2014b) | Single | Single | ✓ | ML/OU | ✓ | Hom. | Multiple | ✓ | E/H | Branch-and-cut/ALNS |
| Absi et al. (2015) | Single | Single | ✓ | ML | ✓ | Hom. | Multiple | ✓ | H | Iterative MIP heuristic |
| Adulyasak et al. (2015b) | Single | Single | ✓ | ML | ✓ | Hom. | Multiple | ✓ | E | Benders-based branch-and-cut |
| Brahimi & Aouam (2016) | Single | Multiple | ✓ | ML | ✓ | Hom. | Single | ✓ | H | Relax and Fix |
| Kumar et al. (2015) | Single | Single | ✓ | ML | ✓ | Hom. | Limited | ✓ | H | PSO |
| Qiu et al. (2016) | Single | Single | ✓ | ML | ✓ | Hom. | Single | ✓ | H | Branch-and-price |
| Zhang et al. (2017) | Multiple | Multiple | ✓ | ML | ✓ | Hom. | Limited | ✓ | H | MIP-based heuristic |
| Solyalı & Süral (2017) | Single | Single | ✓ | ML | ✓ | Hom. | Multiple | ✓ | H | MIP-based heuristic |
| Agra et al. (2018) | Single | Single | ✓ | ML | ✓ | Hom. | Multiple | ✓ | H | SAA heuristic |
| Russell (2017) | Single | Single | ✓ | ML | ✓ | Hom. | Multiple | ✓ | H | MIP-based heuristic |
| Qiu, Ni et al. (2018) | Multiple | Single | ✓ | ML | ✓ | Het. | Multiple | ✓ | E | Branch-and-cut guided search |
| Qiu, Wang et al. (2018) | Single | Single | ✓ | ML | ✓ | Hom. | Multiple | ✓ | H | VNS |
| Miranda et al. (2018) | Single | Multiple | ✓ | ML | | Het. | Multiple | ✓ | H | Decomposition |
| Darvish et al. (2018) | Single | Single | | ML | ✓ | Hom. | Single | ✓ | E | Branch-and-cut |
| Li et al. (2019) | Single | Multiple | ✓ | ML | ✓ | Hom. | Multiple | ✓ | H | MIP-based heuristic |
| Chitsaz et al. (2019) | Single | Single | ✓ | ML | ✓ | Hom. | Multiple | ✓ | H | Decomposition Matheuristic |
| Neves-Moreira et al. (2019) | Single | Multiple | ✓ | ML | ✓ | Het. | Multiple | ✓ | H | Matheuristic |

*Notes*. Hom., homogeneous, Het., heterogeneous, E, exact, H, heuristic

Boudia, Louly & Prins (2008) developed two greedy heuristic solution approaches which are supplemented with local search procedures. In the first heuristic, called uncoupled approach, the production plan and the distribution plan are determined separately. In the second approach, which is a decomposition heuristic, two problems are solved simultaneously. The computational results indicate the superiority of the second algorithm in terms of the solution quality. Solyalı & Süral (2009) proposed a lagrangian relaxation based solution approach to solve the PRP with OU policy. Brahimi & Auoam (2016) studied the PRP with backordering which corresponds to a stockout situation where customer demand in a given period is satisfied from production/delivery of subsequent periods. A relax and fix heuristic combined with a local search process is suggested for the problem. Agra, Requejo & Rodrigues (2018) considered a PRP where the demand is uncertain and allowed to be backlogged. The authors suggested a sample average approximation (SAA) heuristic to solve the problem.

Metaheuristic solution approaches have been widely applied to the PRP. Boudia, Louly & Prins (2007) proposed a heuristic algorithm based on greedy randomized adaptive search procedure (GRASP) for the PRP. In the proposed approach, an initial solution is constructed by sequentially solving the production and delivery problems. After constructing the initial solution, 3-opt, inserting and swapping operations are applied in the local search phase. Additionally, a path relinking procedure is implemented to further improve the solution. Boudia & Prins (2009) developed a memetic algorithm for the PRP. In this study, the initial population is generated by using a heuristic procedure. In this procedure, the produced amount of goods at each period is initially set to the total demand. After that, the saving heuristic (Clarke & Wright, 1964) is applied to obtain the vehicle routes and the Wagner-Whitin algorithm (Wagner & Whitin, 1958) is implemented to calibrate the production plan. Then, a crossover mechanism is used to create new offsprings. After this process, the local search phase of Boudia et al. (2007) is adopted to improve the new chromosomes. Bard & Nananukul (2009a) developed a tabu search (TS) based heuristic solution approach in which a reactive tabu list is employed. In the proposed algorithm, an initial solution is created by solving the restricted PRP in which the routing constraints are dropped.

Then, the vehicle routes are constructed by using a TS based algorithm. A local search procedure consisting of swap and transfer moves is implemented to improve the obtained solutions. Another TS based algorithm was suggested by Armentano, Shiguemoto & Løkketangen (2011). In their study, Wagner-Whitin and the saving algorithms are sequentially used for the construction of the initial solution. After that, a local search procedure consisting of the transfer move is applied. Furthermore, a path relinking procedure is used for diversification. Adulyasak, Cordeau & Jans (2014a) developed an adaptive large neighborhood search (ALNS) (Ropke & Pisinger, 2006) algorithm for the PRP. In their algorithm, an enumeration scheme in which the production and distribution problems are solved is applied to obtain different initial solutions. After that, two types of operators, called selection and transformation, are implemented to the created solutions in sequence and the minimum cost flow problem is solved to determine the production, inventory and delivery decisions. A multi-objective version of the PRP with time windows constraints was studied by Kumar et al. (2016). In the study, not only the total operation cost but also the carbon emissions are taken into account in the objective function. A self-learning particle swarm optimization (PSO) algorithm is suggested for the solution of the problem. Qiu, Wang, Xu, Fang & Pardalos (2018) proposed a variable neighborhood search (VNS) algorithm to solve the PRP. In the solution method, the delivery and routing decisions are handled by local search methods based on skewed general VNS and guided variable neighborhood descent, respectively.

In addition to the metaheuristic algorithms, some heuristic methods combined with exact solution approaches are developed for the PRP. A lagrangian relaxation based approach for the PRP is proposed by Fumero & Vercellis (1999). The authors considered the PRP with multiple products, maximum level policy, and unit transportation costs. The PRP is decoupled into the LSP and the VRP subproblems and the vehicle capacity constraints and the plant inventory constraints are dualized. MIP based heuristic methods are presented by Archetti, Bertazzi, Paletta & Speranza (2011), Absi, Archetti, Dauzère-Pérès & Feillet (2015), Solyalı & Süral (2017), and Russell (2017) for the PRP. Archetti et al. (2011) proposed a heuristic algorithm to solve the decomposed lot-sizing and inventory routing subproblems. In their study, the lot-sizing problem is optimized by using MIP commercial solver and the inventory routing subproblem is solved by a remove-insert heuristic procedure. Absi et al. (2015) developed an iterative MIP heuristic for the PRP. In the developed heuristic, the production planning subproblem, where the transportation costs are assumed to be fixed costs, is initially solved by a commercial MIP solver. Then, the vehicle routes are determined by using the saving algorithm according to the production decisions. After that, the fixed transportation costs are updated according to the information received from routing subproblem and the production planning problem is solved again.

Solyalı & Süral (2017) developed a five-phase heuristic for the PRP. In the first phase of the heuristic, a traveling salesman problem (TSP) over all nodes is solved. The second phase is to solve a restricted PRP formulation in which the fixed sequence of nodes obtained in the first phase is imposed. In the third phase, the vehicle routes of the solution obtained from the second phase are updated. The fourth phase is to solve another restricted formulation of the PRP where the routes found in the third phase is dictated but insertion and removal of nodes from these routes are allowed. Finally, using the solution obtained in the fourth phase, the routes of vehicles are determined in the fifth phase by solving a TSP for each vehicle in each period. Russell (2017) proposed two mathematical programming based heuristics for the PRP. One of the heuristics employs set partitioning and other heuristic uses the concept of seed routes to determine an approximate solution.

Branch-and-price based heuristic algorithms were proposed in Bard & Nananukul (2009b) and Bard & Nananukul (2010) for the PRP. Qiu, Qiao & Pardalos (2016) presented a pollution PRP with carbon cap-and-trade, which incorporates carbon emissions into production, inventory, and routing decisions. A branch-and-price based heuristic was suggested to solve the problem. Another study that considers the carbon emissions in production and inventory routing problems were presented in Darvish, Archetti & Coelho (2018). In this study, the effect of operational decisions on costs and emissions are compared. Moreover, a branch-and-cut based exact algorithm is developed to solve the emission minimization problem. Zhang, Sundaramoorthy, Grossmann & Pinto (2017) introduced the multiscale PRP which involves multiple commodities and continuous production. In their study, the authors proposed a mixed integer formulation for the problem. Moreover, an iterative heuristic method in which an MIP model with a restricted set of candidate routes are solved at each iteration and the set of candidate routes are updated for the next iteration. Miranda, Cordeau, Ferreira, Jans & Morabito (2018) introduced a rich PRP motivated from a furniture manufacturer. The problem considers several important features, such as multiple products, sequence-dependent setup times, and heterogeneous fleet of vehicles. A decomposition heuristic was developed to solve the problem. Li, Chu, Chu & Zhu (2019) presented a multi-product PRP with outsourcing options. An MIP model and a three level heuristic were developed for the problem. Chitsaz, Cordeau & Jans (2019) introduced the assembly routing problem, which integrates the production planning with the inbound vehicle routing. A three-phase decomposition matheuristic was proposed to solve the problem. The algorithm is flexible and it was also applied to the PRP and the IRP. Neves-Moreira, Almada-Lobo, Cordeau, Guimarães & Jans (2019) addressed a multi-product PRP inspired by the supply chain of a meat producer. The introduced problem involves several real-life features such as product family setups, perishable products, and delivery time windows. A three-phase methodology to tackle the problem.

The first exact algorithm for the PRP was presented by Ruokokoski, Solyalı, Cordeau, Jans & Süral (2010). In their study, The PRP is considered with an uncapacitated plant and a single vehicle. The authors developed a branch-and-cut algorithm to solve the problem. Another branch-and-cut algorithm for the PRP was developed in Archetti et al. (2011). The PRP with multiple vehicles was first tackled by an exact algorithm in Adulyasak, Cordeau & Jans (2014b). The authors compared the performances of two different formulations. The first developed formulation involves vehicle indexes while the second one does not. The computational results indicated the superiority of the formulation with vehicle indexes. The same authors proposed a Benders decomposition based branch-and-cut algorithm for the PRP with demand uncertainty (Adulyasak, Cordeau & Jans, 2015b). Qiu, Ni et al. (2018) introduced the PRP with reverse logistics and remanufacturing. An MIP model with valid inequalities and a branch-and-cut guided search algorithm were proposed to solve the problem.

### 2.5 Chapter Summary

In this chapter, we explained the PRP in detail. Moreover, a literature review was presented for the problem. When the related literature is analyzed, it is seen that there is no study that considers the service consistency concepts in the PRP. Moreover, it is observed that the concept of transshipments has not yet been integrated within the PRP. Considering the unfulfilled research potential in this area, the purpose of this thesis is to study these two important concepts within the PRP.

# CHAPTER THREE
# SINGLE VEHICLE PRODUCTION ROUTING PROBLEM WITH VISIT SPACING POLICY

## 3.1 Introduction

This chapter presents our first study on the production routing problem with visit spacing policy (PRP-VSP). The chapter is organized as follows. In Section 3.2, the problem definition of the PRP-VSP and the developed MIP formulations are given. In Section 3.3, our matheuristic algorithm (MA) which is proposed for the PRP-VSP is explained in detail. The computational results are given in Section 3.4. Finally, the chapter is summarized in Section 3.5.

## 3.2 Problem Description and Formulations

The network of the PRP-VSP is defined on a graph $G = (N, A)$, where $N = \{0,\dots, n\}$ represents the set of nodes and $A = \{(i, j) : i, j \in N, i \neq j\}$ represents the set of arcs. Node 0 represents a production plant, while the other nodes denote retailers. The plant manufactures a single product which is sold by the retailers along a planning horizon $T = \{1,\dots, H\}$. Each retailer $i$ has a known demand $d_{it}$ for each time period $t$ and a storage area with a limited capacity $L_i$. An inventory holding cost $h_i$ is incurred at node $i$ for the time periods in which products are stored in the node. The production plant must decide in each period whether or not to make the product and determine the corresponding lot size. If production does take place, a fixed setup cost $f$ is resulted along with a unit production cost $u$ that is incurred for each produced unit. Additionally, the lot size cannot exceed the production capacity $C$. The distribution of the products to the retailers is made by a vehicle with a limited capacity $Q$. A transportation cost $c_{ij}$ is defined for each node pair $(i, j)$. The vehicle can perform at most one route in each time period, and the vehicle routes starts and ends at the production plant. Each retailer can be visited at most once in a route. Delivered quantities for each retailer are determined in accordance with the maximum level policy. At the beginning of the planning horizon, an initial inventory $I_{i0}$ is available at each node *i*. For each retailer *i*, the visiting sequence is arranged considering the minimum and maximum number of time periods, $min_i$ and $max_i$, allowed between the consecutive visits made to the retailer *i* one after the other. The aim of the considered PRP-VSP is to combine the decisions of production, inventory, and routing operations that minimize the total cost with respect to the stated constraints.

The first mathematical model presented here is based on the PRP formulation developed in Boudia et al. (2007) and the periodic VRP formulation introduced in Cordeau, Gendreau & Laporte (1997). In this formulation, we initially generated all possible visit combinations for all retailers and these visit combinations are used as an input in the model. Table 3.1 depicts the generation of visit combinations for a retailer over seven periods. In this example, the maximum and initial inventory levels of the retailer are respectively set to 40 and 36. Besides, the minimum and maximum time periods between the consecutive visits are determined as one and two, respectively. Considering the demand quantities given in the table, it can be inferred that the latest time period for the first visit to the retailer must be set to three. By utilizing these inputs, all possible visit combinations for the corresponding retailer are generated and represented in Table 3.1. In the table, each visit combination is sketched in a column consisting of binary values for each time period. Each binary value indicates either the retailer is visited at the time period or not. For example, if the first visit combination is selected for the retailer, it will be visited on periods 1, 3, 5, and 7, respectively. Moreover, in the last row of the table, the feasibility condition of each generated visit combination with respect to the specified maximum inventory level is depicted. The sequence of operations performed at any delivery location in each time period is the following: first the product is delivered, then the demand is satisfied, and the final inventory level is calculated. The inventory level of a retailer before the consumption must be less than or equal to the maximum level. Accordingly, in the sketched example, visit combinations 5, 8, and 10 are infeasible. In visit combinations 5 and 10 (resp. visit combination 8), the inventory level at the end of period 3 (resp. period 1) can be up to 22 (resp. 20). The fact that the total quantity of demands in periods 4 and 5 (resp. in periods 2 and 3) exceeds 22 (resp. 20) makes visit combinations 5 and 10 (resp. visit combination 8) infeasible. Hence, we disregard the infeasible visit combinations in our formulation.

Table 3.1 An example for the visit combination generation

| | | Alternative Visit Combinations | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Period | Demand | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 20 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 2 | 13 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| 3 | 18 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 4 | 8 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 5 | 19 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 6 | 6 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 7 | 11 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| | Condition | F | F | F | F | I | F | F | I | F | I | F | F |

*Notes.* F, Feasible, I, Infeasible

The relevant notation for the model is summarized as follows:

*Sets*

$N^r$ : set of all retailers $\{1,\dots,n\}$

$N$ : set of all nodes $\{0,1,\dots,n\}$

$R$ : set of all visit combinations for all retailers

$R_i$ : set of visit combinations for retailer $i \in N^r$

$T$ : set of time periods $\{1,\dots,H\}$

*Parameters*

$u$ : unit production cost

$f$ : production setup cost

$C$ : production capacity

$Q$ : capacity of the vehicle

$h_i$ : unit inventory holding cost at node $i$

$L_i$ : maximum or target inventory level at node $i$

$min_i$ : minimum number of periods between the consecutive visits of retailer $i$

$max_i$ : maximum number of periods between the consecutive visits of retailer $i$

$fv_i$ : latest time period for the first visit to retailer $i$

$I_{i0}$ : initial inventory level at node $i$

$d_{it}$ : demand at retailer $i$ in period $t$

$c_{ij}$ : transportation cost from node $i$ to node $j$

$a_{rt} : \begin{cases} 1 & \text{if period } t \text{ belongs to visit combination } r \\ 0 & \text{otherwise} \end{cases}$

$M_t : \min\{C, \sum_{l=t}^{H} \sum_{i \in N^r} d_{il}\}$, maximum quantity that can be produced in period $t$

$TD_{it} : \min\{H, t + max_i\}$, the latest possible time period before the next visit to retailer $i$ after it is visited in period $t$

$DD_{it} = \sum_{j=t}^{TD_{it}} d_{ij}$, total demand in the periods between $t$ and $TD_{it}$

$\bar{M}_{it} : \min\{L_i, Q, DD_{it}\}$

*Decision Variables*

$p_t$ : production quantity in period $t$

$y_t : \begin{cases} 1 & \text{if there is production at the plant in period } t \\ 0 & \text{otherwise} \end{cases}$

$I_{it}$ : inventory level at node $i$ at the end of period $t$

$x_{ijt} : \begin{cases} 1 & \text{if the vehicle travels directly from node } i \text{ to node } j \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$

$q_{it}$ : quantity delivered to retailer $i$ in period $t$ by the vehicle

$z_{it} : \begin{cases} 1 & \text{if retailer } i \text{ is visited in period } t \text{ by the vehicle} \\ 0 & \text{otherwise} \end{cases}$

$z_{0t} : \begin{cases} 1 & \text{if the vehicle is dispatched in period } t \\ 0 & \text{otherwise} \end{cases}$

$s_{ir} : \begin{cases} 1 & \text{if visit combination } r \in R_i \text{ is assigned to retailer } i \\ 0 & \text{otherwise} \end{cases}$

The first PRP-VSP model (*Model 1*) can be formulated as follows:

$$(Model\,1)\; Min\, Z = \sum_{t \in T} (u.p_t + f.y_t + \sum_{i \in N} h_i.I_{it} + \sum_{i \in N} \sum_{j \in N \backslash \{i\}} c_{ij}.x_{ijt}) \qquad (3.1)$$

subject to

$$I_{0t-1} + p_t = \sum_{i \in N^r} q_{it} + I_{0t}, \forall t \in T \tag{3.2}$$

$$I_{it-1} + q_{it} = d_{it} + I_{it}, \forall i \in N^r, \forall t \in T \tag{3.3}$$

$$p_t \leq M_t . y_t, \forall t \in T \tag{3.4}$$

$$I_{0t} \leq L_0, \forall t \in T \tag{3.5}$$

$$I_{it-1} + q_{it} \leq L_i, \forall i \in N^r, \forall t \in T \tag{3.6}$$

$$q_{it} \leq \bar{M}_{it} . z_{it}, \forall i \in N^r, \forall t \in T \tag{3.7}$$

$$\sum_{j \in N \setminus \{i\}} x_{jit} = \sum_{j \in N \setminus \{i\}} x_{ijt}, \forall i \in N, \forall t \in T \tag{3.8}$$

$$\sum_{j \in N \setminus \{i\}} x_{ijt} = z_{it}, \forall i \in N, \forall t \in T \tag{3.9}$$

$$\sum_{i \in N^r} q_{it} \leq Q . z_{0t}, \forall t \in T \tag{3.10}$$

$$\sum_{r \in R_i} s_{ir} = 1, \forall i \in N^r \tag{3.11}$$

$$z_{it} - \sum_{r \in R_i} a_{rt} . s_{ir} = 0, \forall i \in N^r, \forall t \in T \tag{3.12}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijt} \leq \sum_{i \in S} z_{it} - z_{et}, \forall t \in T, \forall S \subseteq N^r : |S| \geq 2, \forall e \in S \tag{3.13}$$

$$p_t, I_{it}, q_{it} \geq 0, \forall i \in N, \forall t \in T \tag{3.14}$$

$$x_{ijt}, s_{ir}, y_t, z_{it} \in \{0,1\}, \forall i \in N, \forall j \in N, \forall r \in R, \forall t \in T \tag{3.15}$$

The objective function (3.1) minimizes the total production, setup, inventory, and routing costs. Constraint sets (3.2) - (3.6) represent the lot-sizing part of the problem. Constraint sets (3.2) and (3.3) are the inventory flow balance at the plant and retailers, respectively. Constraint set (3.4) comprises the setup forcing and production capacity constraints. The constraints force the setup variable to be one if production takes place in a given period and limit the production quantity to the minimum value between the production capacity and the total demand in the remaining periods. Constraint sets (3.5) and (3.6) limit the maximum inventory at the plant and retailers, respectively. Constraint set (3.7) allows a positive delivery quantity only if retailer *i* is visited in period *t*. The delivered quantity $q_{it}$ is limited by $\bar{M}_{it}$, which is the minimum of the retailer inventory capacity $L_i$, vehicle capacity $Q$ and $DD_{it}$. Here, $DD_{it}$ stands for the total demand in the periods between *t* and $TD_{it}$ which is the minimum of the number of time periods and the latest possible time for the next visit to retailer *i*. Constraint sets (3.8) and (3.9) are the vehicle flow conservation constraints. Constraint set (3.10) imposes the vehicle capacity. Constraint set (3.11) means that one feasible visit combination must be assigned to each retailer while constraint set (3.12) enforces that each retailer is visited only in the periods corresponding to the assigned combination. In constraint set (3.12), the decision variable $z_{it}$ equals to one if the selected visit combination $r$ for retailer $i$ ($s_{ir}$ = 1) covers time period $t$ ($a_{rt}$ = 1). Otherwise, it is set to zero. Constraint set (3.13) constitutes the subtour elimination constraints similar to those of the TSP and constraints (3.14) and (3.15) define the nature of the decision variables.

Instead of generating and using all possible visit combinations, the visit spacing constraints can be formulated in an alternative fashion which is utilized in Coelho et al. (2012a) and Hamzadayi, Topaloglu & Kose (2013). In order to make such a change in the formulation, constraint sets (3.16), (3.17), and (3.18) must substitute for constraint sets (3.11) and (3.12).

$$\sum_{m=0}^{min_i} z_{i,t+m} \leq 1, \forall i \in N^r, t = 1, ..., (H - min_i) \tag{3.16}$$

$$\sum_{m=0}^{max_i} z_{i,t+m} \geq 1, \forall i \in N^r, t = 1, ..., (H - max_i) \tag{3.17}$$

$$\sum_{m=0}^{fv_i - 1} z_{i,m+1} \geq 1, \forall i \in N^r \tag{3.18}$$

Constraint set (3.16) ensures that a retailer is not visited before the specified minimum number of periods pass since the last visit. Namely, in a set of times periods from $t$ to $t + min_i$, it is imposed that no more than one visit can be made to the retailer $i$. Constraint set (3.17) enforces that a retailer should be visited within the specified maximum number of periods since the last visit. In other words, in a set of time periods from $t$ to $t + max_i$, at least one visit must be made to the retailer $i$. Consequently, the visiting time windows for retailers are formed by Constraint sets (3.16) - (3.17). Finally, constraint set (3.18) ensures that the first visit to a retailer must be performed before the maximum number of periods elapses from the beginning of the planning horizon. With these new constraint sets, our second formulation (*Model 2*) consists of the equations (3.1) - (3.10), (3.13) - (3.15), and (3.16) - (3.18).

### 3.3 Matheuristic Algorithm (MA)

In this section, our proposed matheuristic algorithm (MA) for the PRP-VSP with single vehicle is introduced. Matheuristic algorithms can be generally defined as algorithms that consist of the integration of metaheuristic techniques with mathematical programming models (Fanjul-Peyro, Perea & Ruiz, 2017). In recent years, there has been a growing interest among researchers in developing matheuristic algorithms for combinatorial optimization problems owing to the advances in exact methods and technology. A recent survey on matheuristics for routing problems can also be found in Archetti & Speranza (2014). Some recent applications of matheuristic algorithms on routing problems can also be found in Bosco, Laganà, Musmanno & Vocaturo (2014), Archetti, C., Corberán, Á., Plana, I., Sanchis, J. M., & Speranza, M. G. (2015), Hemmati, Hvattum, Christiansen & Laporte (2016), Grangier, Gendreau, Lehuédé & Rousseau (2017), and Mancini (2016).

According to the classification made in Archetti & Speranza (2014), developed matheuristics can be divided into three classes: improvement heuristics, column generation based approaches, and decomposition approaches. In the improvement heuristics, mathematical programming models are utilized to improve a solution found by a different heuristic approach. In the column generation based approaches, the column generation phase is terminated prematurely, thus loosing the guarantee of optimality. Decomposition based approaches divide the problem into smaller subproblems and some or all of them are solved through mathematical programming. Our solution approach belongs to the class of decomposition based approaches. The developed MA decomposes the main problem into subproblems to reduce the complexity. In MA, while the distribution and routing subproblems are solved heuristically, the lot-sizing, inventory, and delivery quantity decisions are handled by utilizing a mathematical programming formulation.

From a general perspective, the proposed MA can be described as a multi-start heuristic search procedure based on an MIP-based iterative local search procedure. In the proposed algorithm, the construction of an initial solution consists of three stages.

In the first stage, a random visit combination $vc_i \in R_i$ is selected for each retailer *i*. By this way, the binary $s_{ir}$ and $z_{it}$ variables are fixed. After this stage, two problems which are the production-inventory (*PI*) and routing (*Route*) subproblems are sequentially solved to constitute the whole solution. The *PI* subproblem, which is solved through a standard mathematical programming solver, is used to determine the continuous $p_t$, $I_{it}$, and $q_{it}$ variables as well as the binary $y_t$ variables. The solution of *PI* subproblem is obtained by solving a restricted version of *Model 1*, where the routing costs $c_{ij}$ and variables $x_{ijt}$ are not included and the visit combination for each retailer has been already fixed. Finally, the remaining problem is the computation of routes for the vehicle for all time periods. In this stage, we employ an iterated local search (ILS) algorithm to construct the vehicle route for each time period. The utilized ILS is adapted from the second stage of the algorithm introduced in Avci & Topaloglu (2015) to solve the VRP with simultaneous pickup and delivery. Note that in initial solution construction, the *PI* subproblem may be infeasible with respect to the selected visit combinations since the demand of all retailers cannot be met due to capacity constraints. In the developed MA, the initial solution generation procedure is repeated until a feasible solution is obtained.

The local search procedure is developed based on the idea proposed in Archetti, Bertazzi, Hertz & Speranza (2012) and used in Coelho & Laporte (2013) and Solyalı & Süral (2017). The procedure consists of two stages. The first stage corresponds to solving another subproblem (*LS*) through a standard mathematical programming solver. The corresponding subproblem is formulated as an MIP where we impose the routes and visit combinations of the current solution but allow unemployed visit combinations to substitute for the current ones. For each unemployed visit combination *r* of retailer *i*, we define a parameter $\lambda_{ir}$ which stands for the approximate transportation cost of replacing the current visit combination, $vc_i$, of retailer *i* with visit combination *r*. Moreover, a tabu list which is used to store the visit combinations of the recently visited solutions is employed in the formulation to avoid cycling. After the restricted formulation is solved and new visit combinations are determined, the routing subproblem, *Route*, is solved to complete the new solution in the second stage of the procedure.

Employing the alternative visit combinations in the local search is especially beneficial to handle the visit spacing constraints. However, in instances with large visit windows and a long planning horizon, a large number of alternative visit combinations can exist for a retailer. Hence, considering all visit combinations in this procedure may increase the computational burden. In order to deal with this issue, we develop a working visit combination set (WVCS) generation mechanism. With this mechanism, for each retailer, the number of alternative visit combinations to be considered at each local search execution is limited to a specified number.

The main steps and the flowchart of MA are provided in Figures 3.1 and 3.2, respectively, where $X_{lb}$ and $X_{best}$ represent the best solution of the current multi-start iteration and the best solution obtained from all multi-start iterations, respectively. The input parameters *maxStart*, *maxIter*, and *maxSub* represent the total number of multi-start iterations, the maximum number of consecutive local search executions without improvement, and the maximum number of allowed visit combination substitutions for a single local search execution, respectively. At the beginning of each multi-start iteration, the WVCS generation mechanism, *WorkingSets*, is executed and WVCS for each retailer $i$, $WR_i$, is formed. In this mechanism, the parameters *Ms* and *Mr* are utilized to determine the number of visit combinations to be selected for each retailer.

After an initial solution, $X$, is constructed, the local search procedure is implemented iteratively. In Algorithm 1, f($X_{lb}$) and f($X$) represent the objective function values of $X_{lb}$ and $X$, respectively. At the end of each local search execution, if an improved solution is obtained, then the WVCSs are updated and the tabu list, *tabuList*, is initialized. Otherwise, the resulting solution is added to *tabuList* which is specifically employed to store the recently visited unimproved solutions. By utilizing *tabuList* in *LS* subproblem, it is ensured that the procedure does not revisit those unimproved solutions. Because the WVCSs are updated at the end of each successful local search execution, the necessity of storing the tabu solutions that are generated with the previous WVCSs no longer exists. Therefore, *tabuList* is initialized at the end of each successful local search implementation. Each multi-start iteration of MA is terminated when the value of $s$ becomes bigger than *maxIter*. Note that in the procedure, the *LS* subproblem may be infeasible with respect to the imposed *tabuList*. Whenever this case happens, the current multi-start iteration is stopped and the algorithm proceeds to the next multi-start iteration. In the following subsections, the *PI* and *LS* subproblem formulations and WVCS generation are described, respectively.

**Algorithm 1:** MA
**Input:** problem data, *maxStart*, *maxIter*, *maxSub*, *Ms*, *Mr*
**Output:** global best solution, $X_{best}$
$it \leftarrow 1$;
**while** $it \leq maxStart$
    determine working visit combination set $WR_i$ for each $i \in N^r$, *WorkingSets*(*Ms*, *Mr*);
    select a visit combination $vc_i \in WR_i$ randomly for each $i \in N^r$;
    solve the production-inventory subproblem, $PI(vc_i)$;
    solve the routing subproblem, $Route(vc_i)$, and complete the initial solution, $X$;
    calculate $\lambda_{ir}$ for each $i \in N^r$ and $r \in WR_i \setminus \{vc_i\}$;
    initialize *tabulist*; $s \leftarrow 0$; $X_{lb} \leftarrow X$;
    **if** $it$ is equal to 1
        $X_{best} \leftarrow X$;
    **end if**
    **while** $s \leq maxIter$
        solve $LS(X, tabuList, \lambda_{ir}, maxSub)$ subproblem and get $vc_i$ for each $i \in N^r$;
        solve the routing subproblem, $Route(vc_i)$, and complete the new solution, $X'$;
        $X \leftarrow X'$;
        calculate *new* $\lambda_{ir}$ values with respect to $X$;
        **if** $f(X) \leq f(X_{lb})$
            $X_{lb} \leftarrow X$; $s \leftarrow 0$;
            initialize *tabuList*;
            generate new working visit combination sets, *WorkingSets*(*Ms, Mr*);
        **else**
            $s \leftarrow s + 1$;
            add $X$ to *tabuList*;
        **end if**
    **end while**
    **if** $f(X_{lb}) < f(X_{best})$
        $X_{best} \leftarrow X_{lb}$;
    **end if**
    $it \leftarrow it + 1$;
**end while**

Figure 3.1 Main steps of MA

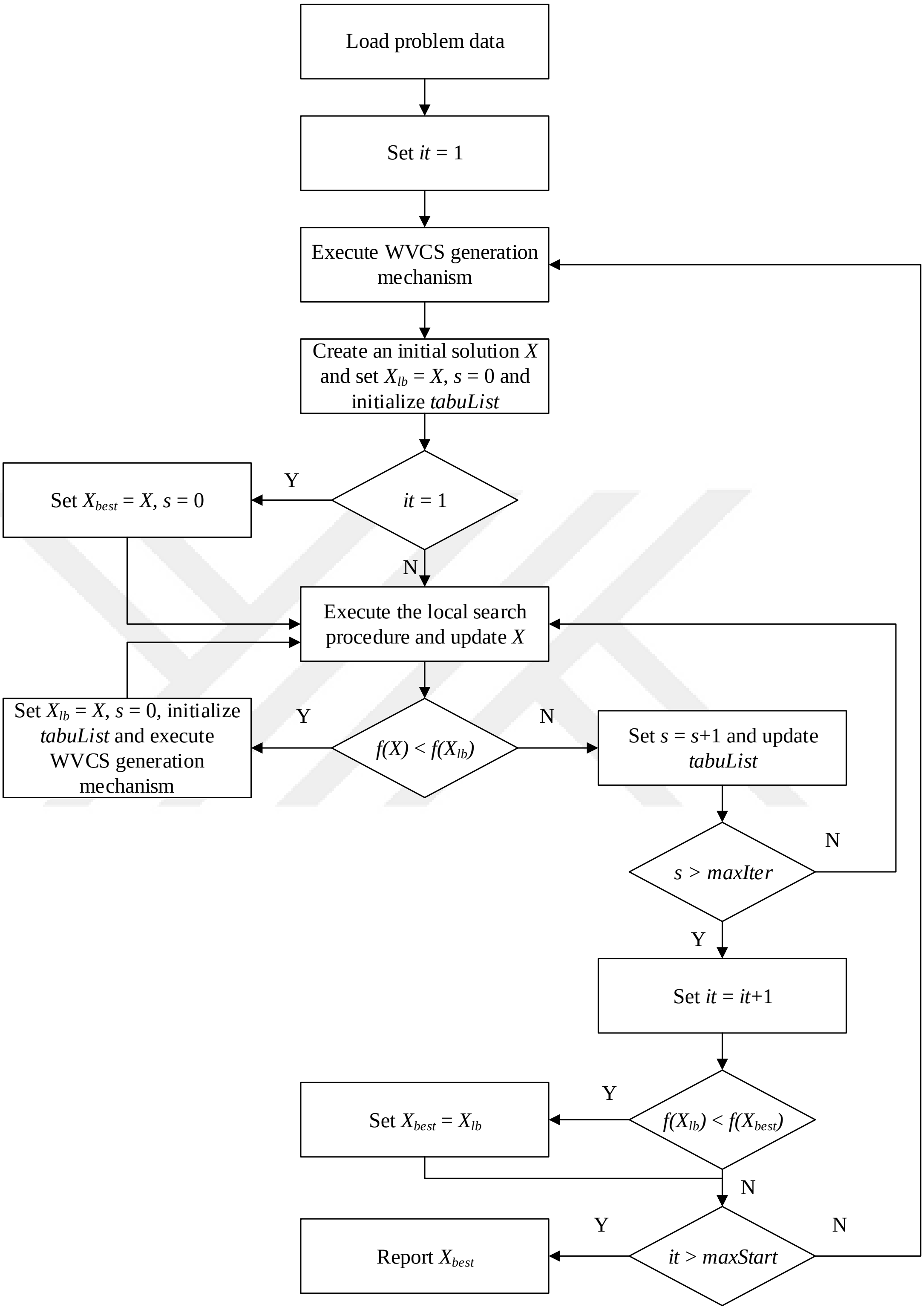


Figure 3.2 Flowchart of MA

### *3.3.1 A Brief Description of the Utilized Routing Algorithm*

In this section, we briefly describe the utilized ILS to solve the routing subproblems in our MA. The utilized ILS algorithm starts with a random initial solution. Then, the random initial solution is improved by a variable neighborhood descent (VND) (Mladenović & Hansen, 1997) based local search mechanism. If the corresponding subproblem is a TSP, only intra-route neighborhood structures are utilized in VND. If the routing subproblem requires more than one vehicle, both inter-route and intra-route neighborhood structures are implemented in the local search. At each iteration of ILS, a perturbation mechanism and the local search procedure are successively applied to obtain an improved solution. Whenever an improved solution is obtained, the current solution is updated. The perturbation mechanism has an adaptive nature and it provides calibrated perturbation moves according to the structure of the search space. The algorithm is finished when the number of successive iterations without an improvement reaches a specified number. In our executions, we terminate the algorithm after 40 failed consecutive iterations. Detailed information on the perturbation mechanism and the neighborhood structures can be found in Avci & Topaloglu (2015).

### *3.3.2 PI Subproblem Formulation*

In this subsection, we describe the mathematical model employed for the *PI* subproblem. In this phase, the decisions of when and how much to produce and how much to deliver are determined. To formulate the *PI,* we let $\beta_{it}$ be a parameter which is equal to one if retailer *i* is visited in period *t* according to its visit combination, $vc_i$, and zero otherwise. We also let $\beta_{0t}$ be a parameter being equal to one if the vehicle is dispatched in period *t* according to the selected visit combinations, and zero otherwise. The objective is to minimize production and inventory costs. The subproblem can be formulated as follows:

$$(PI) \quad Min\, Z = \sum_{t \in T} (u.p_t + f.y_t + \sum_{i \in N} h_i.I_{it}) \tag{3.19}$$

subject to (3.2) - (3.6), (3.14)

$$q_{it} \le \bar{M}_{it}.\beta_{it}, \forall i \in N^r, \forall t \in T \tag{3.20}$$

$$\sum_{i \in N^r} q_{it} \le Q.\beta_{0t}, \forall t \in T \tag{3.21}$$

$$y_t \in \{0,1\}, \forall t \in T \tag{3.22}$$

### *3.3.3 Approximate Visit Combination Substitution Costs ($\lambda_{ir}$) Calculations and LS Subproblem Formulation*

$\lambda_{ir}$ calculations are performed based on the possible insertion/removal costs of retailer *i* into/from the existing vehicle routes of *X*. For $i \in N^r$ and $r \in WR_i \setminus \{vc_i\}$, let $Ins_{ir}$ be the set of time periods belonging to *r*. For $i \in N^r$, let $Rem_i$ be the set of time periods belonging to the existing $vc_i$. For $i \in N^r$, and $t \in T$, we also let $ic_{it}$ (resp. $rc_{it}$) be the cost of inserting (resp. removing) retailer *i* into (resp. from) the route of the vehicle in period *t*. For each $i \in N^r$ and $r \in WR_i \setminus \{vc_i\}$, $\lambda_{ir}$ is calculated as follows:

$$\lambda_{ir} = \sum_{t \in Ins_{ir} \setminus \mathrm{Re}m_i} ic_{it} - \sum_{t \in \mathrm{Re}m_i \setminus \mathrm{Ins}_{ir}} rc_{it} \tag{3.23}$$

In Eq. (23), $ic_{it}$ is the cheapest insertion cost for inserting retailer *i* in the route of period *t* while $rc_{it}$ is associated with the cost decrease occurred when retailer *i* is removed from the route of period *t* and its predecessor is linked with its successor. In order to formulate *LS* subproblem, we define $\psi_{ir}$ as a binary variable being equal to one if visit combination $r \in WR_i \setminus \{vc_i\}$ is substituted for $vc_i$, and zero otherwise. Moreover, let $g_{it}$ be a parameter which equals to one if period *t* is covered by $vc_i$, and zero otherwise. Furthermore, we let $TL_i \subseteq WR_i \setminus \{vc_i\}$ be a set of visit combinations of retailer *i* that are employed in the solutions of *tabuList*. The MIP model used in this procedure is constructed as follows:

$$(LS) \quad Min\, Z = \sum_{t \in T} (u.p_t + f.y_t + \sum_{i \in N} h_i.I_{it}) + \sum_{i \in N^r} \sum_{r \in WR_i \setminus \{vc_i\}} \lambda_{ir}.\psi_{ir} \tag{3.24}$$

subject to (3.2) - (3.6), (3.10), (3.14)

$$q_{it} \le \bar{M}_{it}. \sum_{r \in WR_i \setminus \{vc_i\}} (a_{rt}.\psi_{ir}) + \bar{M}_{it}.g_{it}(1 - \sum_{r \in WR_i \setminus \{vc_i\}} \psi_{ir}), \forall i \in N^r, \forall t \in T \tag{3.25}$$

$$\sum_{r \in WR_i \setminus \{vc_i\}} \psi_{ir} \le 1, \forall i \in N^r \tag{3.26}$$

$$\sum_{\forall i \in N_c} \sum_{r \in WR_i \setminus \{vc_i\}} \psi_{ir} \geq 1 \tag{3.27}$$

$$\sum_{\forall i \in N_c} \sum_{r \in WR_i \setminus \{vc_i\}} \psi_{ir} \leq maxSub \tag{3.28}$$

$$\psi_{ir} = 0, \forall i \in N^r, \forall r \in TL_i \tag{3.29}$$

$$y_t, \psi_{ir} \in \{0,1\}, \forall t \in T, \forall i \in N^r, \forall r \in WR_i \setminus \{vc_i\} \tag{3.30}$$

The objective function (3.24) is the sum of fixed and variable production costs, inventory holding costs at plant and retailers, and the visit combination substitution costs. Constraint set (3.25) ensures that the delivered quantity to retailer *i* in period *t* can be positive if and only if that retailer is visited in period *t*. Constraint set (3.26) guarantees that at most one visit combination can be substituted for each retailer. Constraint (3.27) enforces that at least one visit combination substitution must be made while the maximum number of visit combination substitutions is limited in constraint (3.28). Constraint set (3.29) forbids the possible employment of tabu visit combinations. Finally, constraint set (3.30) defines nature of the decision variables.

### *3.3.4 WVCS Generation Mechanism (WorkingSets)*

The main steps of *WorkingSets* are given in Figure 3.3. *WorkingSets* starts with calculating a selectivity score, $SS_{ir}$, in a randomized way for each visit combination $r \in R_i$ of each retailer *i* having more alternative visit combinations than *Ms*. In Figure 3.3, $TCN_i$ is the total number of alternative visit combinations for retailer *i* and *Mr* is a number in the interval (0, 1]. The number of alternative visit combinations to be selected for each retailer *i* is the maximum of *Ms* and $SN_i$ where $SN_i$ is equal to $Mr \times TCN_i$. Throughout our preliminary experiments, it is observed that building the WVCSs predominantly from the visit combinations covering a small number of periods generally provides high-quality solutions for the PRP-VSP instances. Based on this observation, the $SS_{ir}$ calculation is designed to allow visit combinations covering fewer time periods to score higher. In order to ensure diversification in the $SS_{ir}$ calculation, the number of times the visit combination *r* has been added to $WR_i$, $fn_{ir}$, in the current multi-start execution is also taken into account. To calculate $SS_{ir}$, a random number $\rho_{ir}$ is generated in the interval [1, 1.5] for each $i \in N_c$ and $r \in R_i$ and $SS_{ir}$ is set to $(\rho_{ir} / fn_{ir}) / \sum_{t \in T} a_{rt}$ .

**Algorithm 2:** WorkingSets
**Input:** *Ms*, *Mr*
**Output:** WVCSs ($WR_i$ for each $i \in N^r$)
**for** $i \leftarrow 1 : n$
  $TCN_i \leftarrow$ the total number of visit combinations for retailer $i$;
  **if** $TCN_i > Ms$
    calculate the selectivity score for each alternative visit combination $r \in R_i$ of retailer $i$;
    $SN_i \leftarrow TCN_i \times Mr$
    **if** $Ms > SN_i$
      add the first *Ms* of the visit combinations with the highest selectivity scores to $WR_i$;
    **else**
      add the first $SN_i$ of the visit combinations with the highest selectivity scores to $WR_i$;
    **end if**
  **else**
    add all visit combinations to $WR_i$;
  **end if**
**end for**

Figure 3.3 Main steps of *WorkingSets*

After $SS_{ir}$ values are calculated, the visit combinations of each retailer are sorted in descending order of their scores and *Ms* or $SN_i$ of those at the top of the list are included to the WVCS of the corresponding retailer.

### 3.4 Computational Study

While the proposed MA was coded in MATLAB R2016b, the mathematical models were coded in IBM ILOG CPLEX 12.6. All experiments were executed on a 4.00 GHz intel core i5-760X computer with 32 GB RAM. Moreover, we utilized IBM ILOG CPLEX for MATLAB which is an extension to IBM ILOG CPLEX that allows the user to define and solve optimization problems within MATLAB to solve *LS* and *PI* formulations. The executions of the models and MA have been limited to 3600 CPU seconds.

Since there are, to our knowledge, no standard benchmarks for the PRP-VSP, we created randomly a set of single vehicle problem instances to test the performance of MA. These instances have also been utilized to observe the effect of the VSP on the overall system cost. Additionally, by applying small modifications, the proposed algorithm has been applied to the standard single vehicle PRP instances presented in Archetti et al. (2011). In the following subsection, details on these utilized problem sets are given.

### *3.4.1 Test Instances*

In this subsection, we initially give the details of our single vehicle PRP-VSP instance generation procedure. After that, general characteristics of the utilized standard PRP data set are explained.

#### *3.4.1.1 Single Vehicle PRP-VSP Instance Generation*

Our instance generation procedure is inspired from Archetti et al. (2011). The problems are divided into four classes according to their cost and capacity characteristics. The instances belonging to the first class are standard instances and the rest of the instances are generated by using these instances. The instances in the second class are comprised of low capacity values by halving the production, vehicle, and the plant inventory capacities of the first set instances. While the third class instances have higher production setup and unit production costs, the fourth class instances do not have any inventory holding costs at the retailers. For each problem class, there exist 56 instances containing 10 to 100 retailers and 10 to 20 time periods.

The problem instances are also classified according to the type of VSP applied for the retailers. The implemented VSPs are given in Table 3.2. In the table, each couple of numbers in parentheses indicates a visit time window. As can be seen from the table, when the VSP type number increases, larger visit time windows are employed in the problem instances. For example, for the problems with 10 periods and VSP type I, the minimum and maximum number of time periods between two consecutive visits are equal. For the instances with VSP type II, the visit time window of each retailer is taken as either (1, 2) or (2, 3).

Table 3.2 Visit windows applied for each class of instances

| | Planning Horizon | |
|---|---|---|
| VSP | 10 periods | 20 periods |
| VSP I | (1, 1), (2, 2) | (1, 1), (2, 2), (3, 3) |
| VSP II | (1, 2), (2, 3) | (1, 2), (2, 3), (3, 4) |
| VSP III | (1, 2), (2, 4) | (1, 2), (2, 4), (3, 5) |
| VSP IV | (1, 3), (2, 5) | (1, 3), (2, 5), (3, 7) |

The first class problem instances have been generated as follows: the coordinate values of the nodes are uniformly distributed within the interval [0, 100] while the demand quantities of retailers for all time periods are uniformly distributed within the interval [5, 25]. The transportation cost between two locations is five times the Euclidean distance between the locations. The inventory holding costs of the nodes are generated in the interval [1, 4]. Unit production cost is three times the inventory holding cost at the production plant, while the production setup cost 100 times the unit production cost. The inventory capacity at retailer $i$ is generated in the interval [$U_1$, $U_2$], where $U_1 = \max_{t=1..Tm-min_i}(\sum_{j=t}^{t+min_i} d_{ij})$ and $U_2 = \max_{t\in T}(\sum_{j=t}^{TD_{it}} d_{ij})$.

The production and vehicle capacities are set to the sum of five and four periods of retailer consumption on average, respectively. The inventory capacity at the plant is two times the production capacity. As already mentioned, the production, vehicle, and plant inventory capacities are reduced by half for the second class instances. For the third class instances, unit production cost is 10 times the inventory holding cost at the production plant. We divide all generated instances into two sets, S1 and S2. The first set (S1) is comprised of the instances with up to 20 retailers while the second one (S2) is comprised of the other instances. The first and the second sets consist of 96 (S1) and 128 (S2) instances, respectively.

*3.4.1.2 Standard PRP Instances*

In order to test the performance of MA on the basic PRP instances with a single vehicle, we utilize the data set, A1, which is introduced in Archetti et al. (2011). The data set consists of instances with 14 retailers and six time periods. The problems are characterized by constant demand, no production capacity, and no plant inventory capacity but with initial inventory at the plant and retailers. There are 96 instance types and five instances per type, for a total of 480 instances for the set. The problems are categorized into four classes according to their cost patterns. The problems in the first class (types 1-24) are standard instances while those in the second class (types 25-48) and the third class (types 49-72) are characterized with high production cost and large transportation cost, respectively. The problems in Class IV have no retailer inventory holding cost.

### *3.4.1 Computational Results on Single Vehicle PRP-VSP Instances*

In order to test the performance of MA with a heuristic algorithm on the generated PRP-VSP instances, we have adapted the iterative MIP heuristic (IM) of Absi et al. (2015) to the PRP-VSP. This version of the algorithm is marked as IM-VSP. Of the presented IM variants in Absi et al. (2015), we utilized IM-MS, which is implemented in a multi-start scheme with 20 restarts. The adapted version of IM contains the visit spacing constraints (3.16), (3.17) and (3.18) in the lot-sizing phase. Moreover, our implementation of IM differs from its original implementation in the routing phase, where we apply our routing procedure (*Route*) to generate the vehicle routes.

IM-VSP and MA are executed three times for each problem instance. After preliminary test results, the values of the parameters *maxStart*, *maxIter*, *Ms*, and *Mr* are set to 10, 10, 50, and 0.35 for MA, respectively. The value of *maxSub* is adjusted according to the number of retailers and is set to $\lceil n/2 \rceil$. Note that the *LS* formulation of MA and the lot-sizing phase of IM-VSP are solved with a time limit of $n$ seconds. As the models could not solve the instances in the second set, we made the performance comparison of MA, IM-VSP, and the models by using only the set S1.

The executions of the models and the heuristics have been limited to 3600 CPU seconds. In Appendix A, we also provide the data and optimal solution of a small size instance with 10 retailers and 10 periods.

The computational results obtained for the set S1 are summarized in Table 3.3. Moreover, detailed results of MA, IM-VSP and the models for the problem set are reported in Tables B1 through B4 in Appendix B. In Table 3.3, for each problem class, the average gap percentages of the algorithms with respect to the optimum results, when available, or with respect to the lower bounds produced by the models are presented along with the average computational times. '# best' shows the number of times a heuristic finds the best solution. From Table 3.3, it is seen that MA outperforms IM-VSP in terms of the average gap percentages with respect to the lower bounds for the first set. The average gap of MA and IM-VSP for all instances are found as 1.14 and 1.30, respectively. Moreover, MA and IM-VSP have reached the same solution in 64 instances. In the other 32 instances, MA outperforms IM-VSP in 21 out of 32 instances. These results indicate the superiority of MA over IM-VSP in the set S1. Furthermore, the overall average computational times are 878.2 and 115.7 seconds for IM-VSP and MA, respectively, which shows that MA is faster than IM-VSP.

Table 3.3 Summary of computational results for the set S1

| | IM-VSP | | MA | |
|---|---|---|---|---|
| Pr. Classes | Avg. Gap% | Avg. t. (s) | Avg. Gap% | Avg. t. (s) |
| Class I | 1.41 | 677.5 | **1.20** | 105.9 |
| Class II | 1.74 | 636.9 | **1.52** | 131.5 |
| Class III | 0.38 | 1203.0 | **0.31** | 112.0 |
| Class IV | 1.65 | 995.3 | **1.53** | 113.5 |
| Avg. | 1.30 | 878.2 | **1.14** | 115.7 |
| #Best | | 75 | | **85** |

When the detailed results for the set S1 are examined, it is observed that the models can produce optimal solutions for 48 out of 96 instances, and they fail to provide a feasible solution for the other instances within the specified computation time. For the 48 instances where the optimality is confirmed, MA and IM-VSP have reached the optimal solutions in 46 and 41 instances, respectively, and the average gaps of MA and IM-VSP with respect to the optimal results are 0.00% and 0.24%, respectively. It is also worth to remark that the computational times of MA are generally far less than those of the models. For the 48 instances with a known optimal solution, the smallest of the computational times spent by the models for finding the optimal solutions are averaged around 688.6 seconds, while the average computational time of MA for these instances is only 35.8 seconds. Furthermore, it is also noticeable that the gaps between the average and the best results of MA are averaged around 0.12%. This outcome indicates the robustness of the proposed approach. When the results of the models are compared, it is observed that there does not exist a significant difference between the performances of the models. *Model 1* and *Model 2* provide optimal solutions for 43 and 45 instances, respectively. Besides, the average computational times of the models are 2262.4 and 2255.9 seconds, respectively.

A summary of the computational results obtained for the set S2 is reported in Table 3.4. Detailed results are also given in Tables B5 and B6 in Appendix B. In Table 3.4, for each algorithm, the average gap percentages with respect to the best solutions and the average computational times are reported. It can be seen in the table that MA provided better results than IM-VSP in terms of the average gaps and the computational times for all classes. Moreover, as in the first set of instances, MA found more best solutions than IM-VSP. These results also confirm the superiority of MA over IM-VSP on the PRP-VSP instances.

Table 3.4 Summary of computational results for the set S2

| | IM-VSP | | MA | |
|---|---|---|---|---|
| Pr. Classes | Avg. Gap% | Avg. t. (s) | Avg. Gap% | Avg. t. (s) |
| Class I | 0.25 | 2651.7 | **0.11** | 1543.0 |
| Class II | 0.22 | 1946.9 | **0.12** | 1518.2 |
| Class III | 0.21 | 3002.2 | **0.03** | 1535.7 |
| Class IV | 0.56 | 2845.8 | **0.05** | 1431.2 |
| Avg. | 0.31 | 2611.6 | **0.08** | 1507.0 |
| #Best | | 58 | | **87** |

### *3.4.2 Analyzing the Effect of the VSP on the Overall Cost*

To observe the effect of the VSP on the PRP, we solve all problem instances in the sets S1 and S2 without imposing the visit spacing constraints. In order to solve the basic version of the PRP by MA, we initially created a new visit time window ($min_i$, $max_i$) for each retailer $i$ in each problem instance. The visit windows are formed as follows: for each retailer $i$, $min_i$ is set to zero, which makes it possible to visit the retailer on consecutive periods. The value of $max_i$ is set to the maximum number of time periods during which the retailer can meet the customer demands without any visit from the plant. After creating the new windows, we generated all the visit combinations for all the retailers. When $min_i$ and $max_i$ are respectively set to 0 and $H$, the number of alternative visit combinations grow at a rate proportional to $O(2^H)$. Specifically, allowing visits in consecutive time periods to a retailer gives rise to this exponential growth. Therefore, throughout the generation process, we observed that intractable number of visit combinations were created for the retailers in the instances with 20 periods, which makes these instances unsuitable for solving with MA. To deal with this issue, we have implemented IM-MS of Absi et al. (2015) to solve the instances with 20 periods. The implemented algorithm is the same as IM-VSP except the VSP constraints. In the instances with 20 periods, the cost comparison was conducted by using the results of IM-MS and IM-VSP for PRP and PRP-VSP, respectively. In 10-period instances, the results of MA for the PRP and PRP-VSP were utilized in cost comparison. The reason behind this application is to avoid errors arising from algorithm differences.

The average cost increase for each problem class and each VSP type is provided in Table 3.5. Moreover, detailed cost values for all instances are reported in Tables B7 through B14 in Appendix B. When the cost of imposing the VSP is analyzed, it is observed that the average increases on the total cost are higher in the first and the second problem classes compared to the third and fourth class instances. It is also noticeable that the biggest and the smallest average cost increases have occurred in the first and the fourth VSP type, respectively, for each problem class. Moreover, it is worth to remark that the average cost increase on the second class of instances is bigger than that on the first problem class of instances. This result indicates that as the capacities are tightened, bigger cost increases are observed for these instances.

The average cost values obtained by solving the instances with and without VSP are depicted in Table 3.6. When the results are analyzed, the changes on the inventory holding costs at the retailers (R. Inv.) are quite noticeable. Without the VSP policy, the average total inventory holding costs at the retailers constitute 8.07%, 7.72% and 4.80% of the average overall cost for the first, second, and third problem classes, respectively. It is observed that these values are increased to 19.93%, 19.57%, and 8.64% for the first, second, and third problem classes, respectively, when the VSP is applied. On the other hand, it is found that the VSP does not have any significant effect on the total production costs. The average cost increase is around 3.33% for the instances where there is no inventory holding cost at the retailers or the production costs are dominant. However, for the other problem instances, the average cost change increases to 8.67%. The results show that for the systems where the inventory holding costs are important, managers need to determine whether the VSP is bearable in its impact on the inventory holding costs before deciding to implement it. Moreover, since it does not lead to significant cost increases in the systems with low inventory holding costs, the managers of such systems can apply the VSP more easily.

Table 3.5 Average cost increase % for each class of instances between the standard PRP and PRP-VSP

| | VSP type | | | | |
|---|---|---|---|---|---|
| Pr. Classes | VSP I | VSP II | VSP III | VSP IV | Avg. |
| Class I | 9.62 | 8.32 | 8.50 | 6.08 | 8.13 |
| Class II | 11.90 | 8.65 | 9.14 | 7.19 | 9.22 |
| Class III | 3.65 | 2.74 | 2.96 | 2.06 | 2.85 |
| Class IV | 6.06 | 4.66 | 3.06 | 1.43 | 3.80 |
| Avg. | 7.81 | 6.09 | 5.91 | 4.19 | 6.00 |

### *3.4.3 Computational Results on Single Vehicle PRP Instances*

The performance of MA is compared with five heuristic algorithms which are the optimization based ALNS of Adulyasak et al. (2014a) (Op-ALNS), multi-start IM (IM-MS) of Absi et al. (2015), multi-phase heuristic (5P) of Solyalı & Süral (2017), VNS of Qiu, Wang et al. (2018), three level heuristic (TLH) of Li et al. (2019), and three-level decomposition matheuristic (CCJ-DH) of Chitsaz et al. (2019). Because it was mentioned in Solyalı & Süral (2017) that the heuristic proposed by Archetti et al. (2011) requires a correction in its implementation, the results of this heuristic have not been included into the comparison. The optimal solutions for the instances have been found using the branch-and-cut algorithm in Ruokokovski et al. (2010) (Solyalı & Süral, 2017).

Table 3.6 Average cost values obtained by solving the instances with and without the VSP

| | Prod. | | | P. Inv. | | | R. Inv. | | | Trans. | | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pr. Classes | PRP-VSP | PRP | Diff.% | PRP-VSP | PRP | Diff.% | PRP-VSP | PRP | Diff.% | PRP-VSP | PRP | Diff.% | PRP-VSP | PRP | Diff.% |
| Class I | 69672.6 | 70336.9 | -0.94 | 570.6 | 204.3 | 179.37 | 22177.1 | 8240.4 | 169.13 | 18843.9 | 23386.6 | -19.42 | 111264.3 | 102168.2 | 8.90 |
| Class II | 70460.1 | 70722.6 | -0.37 | 615.7 | 239.8 | 156.77 | 22045.9 | 7919.6 | 178.37 | 19503.3 | 23684.4 | -17.65 | 112625.0 | 102566.3 | 9.81 |
| Class III | 225492.0 | 226117.0 | -0.28 | 3120.8 | 2190.9 | 42.44 | 23379.4 | 12581.8 | 85.82 | 18486.6 | 21022.5 | -12.06 | 270478.8 | 261912.1 | 3.27 |
| Class IV | 68917.2 | 67835.1 | 1.60 | 683.6 | 483.8 | 41.30 | - | - | - | 17454.6 | 16224.7 | 7.58 | 87055.5 | 84543.6 | 2.97 |
| Avg. | 108635.5 | 108752.9 | 0.00 | 1247.7 | 779.7 | 104.97 | 22534.2 | 9580.6 | 144.44 | 18572.1 | 21079.6 | -10.39 | 145355.9 | 137797.6 | 6.24 |

*Notes.* Prod., total production cost, P.Inv., inventory holding cost at the plant, R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Diff.% = ((PRP-VSP - PRP) / PRP) x 100

It is important to note that in order to adhere to the PRP formulation given in Archetti et al. (2011), the maximum inventory level constraint set (3.6) are replaced with the constraint set (3.31) and $\bar{M}_{it}$ is the minimum of $\{L_i + d_{it}, Q, DD_{it}\}$ for these instances. By these changes, it is imposed that the level of inventory at retailer *i* is not greater than its maximum level after delivery and consumption. In constraint set (3.6), it was enforced that the inventory level at retailer *i* after delivery must be smaller than its maximum level.

$$I_{it} \leq L_i, \forall i \in N^r, \forall t \in T \tag{3.31}$$

The summary of the results are provided in Tables 3.7 and 3.8. Moreover, detailed results of MA are given in Table B15 in Appendix B. In the tables, In Table 3.7, '# optimal' shows the number of times a heuristic finds the optimal solution. As it is seen from Table 3.7, MA is the best performing algorithm in terms of the average gap percentages with respect to the optimal solutions for the data set. The average gap of MA for all instances is found as 0.02%. Moreover, the number of instances for which MA has reached the optimal solution is 398, which is the highest number among all compared heuristics. Furthermore, it can be inferred from Table 3.8 that the computational times of MA are reasonable for the standard PRP instances. Although not designed for the PRP, it becomes clear that the proposed MA also performs well on the standard PRP instances with a single vehicle.

Table 3.7 Average Gap% with respect to the optimal solutions of A1

| Classes | Op-ALNS | IM-MS | 5P | VNS | TLH | CCJ-DH | MA |
|---|---|---|---|---|---|---|---|
| Class I | 1.70 | 0.09 | 0.03 | 0.25 | 0.15 | 0.47 | **0.01** |
| Class II | 0.36 | 0.01 | **0.00** | 0.04 | 0.03 | 0.08 | **0.00** |
| Class III | 8.43 | 0.57 | 0.18 | 1.56 | 1.52 | 2.20 | **0.04** |
| Class IV | 0.93 | 0.02 | 0.03 | 0.21 | 0.57 | 0.23 | **0.01** |
| Avg. | 2.86 | 0.17 | 0.06 | 0.51 | 0.57 | 075 | **0.02** |
| # Optimal | 2 | 307 | 290 | 113 | 168 | 45 | **398** |

Table 3.8 Average computational times for the set A1

| Classes | Op-ALNS | IM-MS | 5P | VNS | TLH | CCJ-DH | MA |
|---|---|---|---|---|---|---|---|
| Class I | 9.2 | 251.0 | 5.0 | 12.3 | 29.3 | 14.4 | 11.5 |
| Class II | 8.9 | 214.2 | 4.9 | 12.8 | 27.8 | 13.6 | 10.4 |
| Class III | 7.6 | 237.2 | 4.7 | 12.2 | 28.0 | 12.8 | 9.2 |
| Class IV | 8.7 | 216.9 | 5.2 | 12.6 | 25.1 | 14.9 | 9.4 |
| Avg. | 8.6 | 229.8 | 5.0 | 12.5 | 27.5 | 13.9 | 10.1 |

## 3.5 Chapter Summary

In this chapter, we introduce our study on the single vehicle PRP-VSP. Initially, two mathematical formulations have been developed for the PRP-VSP based on previous formulations. The models have been implemented on the generated test problems and it has been observed that it can generate feasible and optimal solutions to only small size instances. Therefore, in order to solve the PRP-VSP, a simple matheuristic algorithm (MA) is developed. The proposed MA is implemented in a multi-start scheme in which some subproblems are solved exactly using mathematical programming.

The performance of MA is tested on a set of randomly generated PRP-VSP instances. The results show that the developed MA can give high-quality solutions to the instances in reasonable computation times. Besides, MA is implemented to the standard PRP instances from the literature. The computational results indicate that the developed MA is effective in solving both the PRP. Additionally, to reveal the effect of the VSP on the overall cost, all problem instances are solved without imposing the visit spacing constraints. According to the results, it is observed that the VSP results in an increase on the overall cost of between 1.43% and 11.90%, on average. Furthermore, the results indicate that the VSP generally influences the inventory holding costs at the retailers significantly.

In the following chapter, we extend our study on the PRP-VSP by considering multiple vehicles.

# CHAPTER FOUR
# MULTI-VEHICLE PRODUCTION ROUTING PROBLEM WITH VISIT SPACING POLICY

## 4.1 Introduction

In this chapter, the production routing problem with visit spacing policy (PRP-VSP) is considered with multiple vehicles. In order to tackle the multi-vehicle problems, we develop two variants of our matheuristic algorithm (MA) presented in Chapter 3. The notation, models, and procedures of this chapter are strongly related to those presented in Chapter 3. For this reason, before proceeding to this chapter, we recommend readers read Chapter 3.

The performance of the developed MA variants are tested on a set of multi-vehicle PRP-VSP instances which are generated by utilizing the multi-vehicle PRP data set introduced in Boudia et al. (2007). In addition, the developed algorithms are applied to standard PRP instances introduced in Archetti et al. (2011) and the multi-vehicle inventory routing problem with visit spacing policy (IRP-VSP) instances solved in Coelho et al. (2012a).

The remainder of this chapter is organized as follows. In Section 4.2, the mathematical formulation of the multi-vehicle PRP-VSP is presented. In Section 4.3, the variants of MA are explained in detail. Section 4.4 presents the computational results obtained for the multi-vehicle problem instances. Finally, the context of the chapter is summarized in Section 4.5.

## 4.2 Mathematical Formulations for the Multi-Vehicle PRP-VSP

The PRP-VSP with multiple vehicles can be formulated by extending the mathematical models, *Model 1* and *Model 2*, presented in Chapter 3. Let $K = \{1, \ldots, m\}$ be the set of homogeneous vehicles. The multi-vehicle PRP-VSP formulations are written using the same variables $x$, $q$, $z$ with the same notation as in the models from the previous chapter, however, a vehicle index $k$ is added to these variables. The first multi-vehicle formulation that is the modified version of *Model 1* can be stated as follows:

$$Min\,Z = \sum_{t\in T}(u.p_t + f.y_t + \sum_{i\in N} h_i.I_{it} + \sum_{k\in K}\sum_{i\in N}\sum_{j\in N\setminus\{i\}} c_{ij}.x_{ijtk}) \tag{4.1}$$

subject to (3.4), (3.5), (3.11)

$$I_{0t-1} + p_t = \sum_{k\in K}\sum_{i\in N^r} q_{itk} + I_{0t}, \forall t \in T \tag{4.2}$$

$$I_{it-1} + \sum_{k\in K} q_{itk} = d_{it} + I_{it}, \forall i \in N^r, \forall t \in T \tag{4.3}$$

$$I_{it-1} + \sum_{k\in K} q_{itk} \le L_i, \forall i \in N^r, \forall t \in T \tag{4.4}$$

$$q_{itk} \le \bar{M}_{it}.z_{itk}, \forall i \in N^r, \forall t \in T, \forall k \in K \tag{4.5}$$

$$\sum_{j\in N\setminus\{i\}} x_{jitk} = \sum_{j\in N\setminus\{i\}} x_{ijtk}, \forall i \in N, \forall t \in T, \forall k \in K \tag{4.6}$$

$$\sum_{j\in N\setminus\{i\}} x_{ijtk} = z_{itk}, \forall i \in N, \forall t \in T, \forall k \in K \tag{4.7}$$

$$\sum_{i\in N^r} q_{itk} \le Q.z_{0tk}, \forall t \in T, \forall k \in K \tag{4.8}$$

$$\sum_{k\in K} z_{itk} - \sum_{r\in R_i} a_{rt}.s_{ir} = 0, \forall i \in N^r, \forall t \in T \tag{4.9}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijtk} \le \sum_{i\in S} z_{itk} - z_{etk}, \forall t \in T, \forall k \in K, \forall S \subseteq N^r : |S| \ge 2, \forall e \in S \tag{4.10}$$

$$\sum_{k\in K} z_{itk} \le 1, \forall i \in N^r, \forall t \in T \tag{4.11}$$

$$p_t, I_{it}, q_{itk} \ge 0, \forall i \in N, \forall t \in T, \forall k \in K \tag{4.12}$$

$$x_{ijtk}, s_{ir}, y_t, z_{itk} \in \{0,1\}, \forall i \in N, \forall j \in N, \forall r \in R, \forall t \in T, \forall k \in K \tag{4.13}$$

When $m$ = 1, constraint sets (4.2) – (4.10) are equivalent to (3.2) – (3.3), (3.6) – (3.10) and (3.12) – (3.13), respectively. Constraint set (4.11) imposes that each retailer can be visited by at most one vehicle in a period. Constraint sets (4.12) and (4.13) define the nature of the decision variables. The visit spacing constraints (3.16) – (3.18) of *Model 2* can be adapted to the multi-vehicle PRP-VSP as follows:

$$\sum_{k\in K}\sum_{l=0}^{min_i} z_{i,t+l,k} \le 1, \forall i \in N^r, t = 1,...,(H - min_i) \tag{4.14}$$

$$\sum_{k\in K}\sum_{l=0}^{max_i} z_{i,t+l,k} \ge 1, \forall i \in N^r, t = 1,...,(H - max_i) \tag{4.15}$$

$$\sum_{k \in K} \sum_{l=0}^{fv_i - 1} z_{i,l+1,k} \geq 1, \forall i \in N^r \tag{4.16}$$

With these new constraint sets, the multi-vehicle extension of *Model 2* consists of the equations (3.4), (3.5), (3.11), (4.1) - (4.8), (4.10) - (4.13), and (4.14) - (4.16).

## 4.3 Variants of MA for the Multi-Vehicle Cases

For the cases with more than one vehicle ($m > 1$), we develop two variants of MA. In this section, we present the modifications made in the steps of MA for the first and the second variant, respectively. Apart from these modifications, the algorithm is not changed.

### *4.3.1 Variant of MA with Aggregated Capacity Constraints for the Multi-Vehicle PRP-VSP*

The first variant of MA makes use of aggregated vehicle capacity constraints in its *PI* and *LS* subproblem formulations to tackle the multi-vehicle cases of the PRP-VSP. We call this variant MA-ACC. The aggregated vehicle capacity constraints are formulated in the *PI* and *LS* formulations of MA-ACC as follows:

$$\sum_{i \in N^r} q_{it} \leq Q.\beta_{0t}.m, \forall t \in T \tag{4.17}$$

With constraint set (4.17), the *PI* formulation of MA-ACC (*PI-ACC*) consists of the following equations: (3.2) – (3.6), (3.14), (3.19), (3.20), (3.22), and (4.17). The *LS* formulation of MA-ACC (*LS-ACC*) is comprised of the equations (3.2) – (3.6), (3.14), (3.24) – (3.30), and (4.17).

Constraint set (4.17) ensures that the total delivered quantity cannot exceed the available fleet capacity, however, it does not guarantee the feasibility of the solutions in terms of the number of vehicles utilized in a period. Because the split delivery is forbidden, it can be possible that all of the deliveries in a period can only be transported by using more than $m$ vehicles. In order to get rid of this issue, if $X_{lb}$ is infeasible at the end of a multi-start iteration, a repair procedure, *Repair*, is implemented to make $X_{lb}$ feasible. In this procedure, a modified version of *LS-ACC* formulation and the routing subproblems are successively solved to obtain a feasible solution. The modified *LS-ACC* formulation (*LSRepair*) does not involve the tabu solution constraints (3.29) and it contains a new aggregated vehicle capacity constraint set (4.18) instead of the set (4.17).

$$\sum_{i \in N^r} q_{it} \leq Q \,.\, \beta_{0t} .m.\theta_t \,, \forall t \in T \tag{4.18}$$

With these two modifications, *LSRepair* consists of the equations (3.2) – (3.6), (3.14), (3.24) – (3.28), (3.30), and (4.18). In (4.18), parameter $\theta_t$ is introduced to provide the feasibility of the routing subproblem by reducing the total delivered quantity in a period. The main steps of *Repair* are given in Figure 4.1. At the beginning of the procedure, $\theta_t$ is initialized to one for each $t \in T$. At each iteration of the procedure, *WorkingSets* procedure is executed and $\lambda_{ir}$ values are calculated, respectively. After that, $\theta_t$ values are updated according to the vehicle routes of $X_{lb}$. If the number of used vehicles in period $t$ exceeds $Km$, $\theta_t$ is decreased by 0.01. Otherwise, if the number of dispatched vehicles in period $t$ is less than $m$, $\theta_t$ is increased by 0.01, unless it is equal to one. This $\theta_t$ updating strategy is adopted from Absi et al. (2015). After $\theta_t$ values are updated, *LSRepair* and *Route* subproblems are solved sequentially and new $X_{lb}$ is formed. The procedure is repeated until a feasible $X_{lb}$ is found.

**Algorithm 3:** Repair
**Input:** problem data, *Ms*, *Mr*, $m$, $X_{lb}$
**Output:** $X_{lb}$
initialize $\theta_t$ for each $t \in T$;
**repeat**
  generate WVCSs, *WorkingSets*(*Ms*, *Mr*);
  calculate $\lambda_{ir}$ values with respect to $X_{lb}$;
  update $\theta_t$ values with respect to $X_{lb}$;
  solve *LSRepair*($X_{lb}$, $\lambda_{ir}$, $m$, $\theta_t$) and get $vc_i$ and $q_{it}$ for each $i \in N^r$ and $t \in T$;
  solve *Route*($vc_i$, $q_{it}$), and complete new $X_{lb}$;
**until** $X_{lb}$ becomes feasible

Figure 4.1 Main steps of *Repair*

### 4.3.2 The Second Variant of MA for the Multi-Vehicle PRP-VSP

In MA-ACC, the delivery quantity decisions are determined according to the aggregated vehicle capacity constraints. The fact that the *PI-ACC* and *LS-ACC* formulations do not involve decisions on the allocation of retailers to vehicles increases the efficiency of the algorithm significantly. However, in the problem instances with a tight vehicle capacity, it may decrease the accuracy of the visit combination substitution costs. In order to investigate this issue, we develop a second variant of MA, MA-SV, in which the *PI* and *LS* subproblems involve decisions on the clustering of retailers to vehicles.

In the *PI* subproblem formulation of MA-SV, the delivered quantity decision variable $q_{it}$ is replaced with the decision variable $q_{itk}$ that stands for the quantity delivered to retailer $i$ in period $t$ by vehicle $k$. Accordingly, the inventory balance constraint sets (3.2) and (3.3) and inventory capacity constraint set (3.6) are replaced with the constraint sets (4.2) – (4.4) from the multi-vehicle PRP-VSP formulation, respectively. Moreover, new delivered quantity constraint sets (4.19) and (4.20) are employed instead of the sets (3.20) and (3.21). Furthermore, another new constraint set (4.21) are utilized to forbid the split delivery. Consequently, the *PI* formulation of MA-SV, *PI-SV*, consists of the equations (3.19), (4.2) – (4.4), (4.12), and (4.19) – (4.22).

$$q_{itk} \leq \bar{M}_{it}.\beta_{it}.z_{itk}, \forall i \in N^r, \forall t \in T, \forall k \in K \tag{4.19}$$

$$\sum_{i \in N^r} q_{itk} \leq Q.\beta_{0t}.z_{0tk}, \forall t \in T, \forall k \in K \tag{4.20}$$

$$\sum_{k \in K} z_{itk} \leq \beta_{it}, \forall i \in N^r, \forall t \in T \tag{4.21}$$

$$y_t, z_{itk} \in \{0,1\}, \forall i \in N, \forall t \in T, \forall k \in K \tag{4.22}$$

In the *LS* subproblem of MA-SV, the insertion (resp. removal) cost $ic_{it}$ (resp. $rc_{it}$) is replaced with $ic_{itk}$ (resp. $rc_{itk}$) which is the cost of inserting (resp. removing) retailer $i$ into (resp. from) route $k$ in period $t$. The introduced parameters $ic_{itk}$ and $rc_{itk}$ are utilized in the objective function in place of the visit combination substitution costs $\lambda_{ir}$. Accordingly, the binary variable $\psi_{ir}$ is not employed in the new formulation. Instead, we make use of the $s_{ir}$ variable from *Model 1*. Additionally, we define $\gamma_{itk}$ (resp. $\delta_{itk}$) as a binary variable being equal to one if retailer $i$ is inserted (resp. removed) into (resp. from) route $k$ in period $t$, and zero otherwise. Moreover, parameter $g_{it}$ is also extended with a vehicle index $k$ ($g_{itk}$) and it equals to one if retailer $i$ is visited by vehicle $k$ in period $t$, and zero otherwise. As a result, the MIP model for the *LS* subproblem of MA-SV is formulated as follows:

$$(LS-SV)\,Min\,Z=\sum_{t\in T}(u.p_t+f.y_t+\sum_{i\in N}h_i.I_{it}) + \sum_{t\in T}(\sum_{i\in N^r}\sum_{k\in K}((1-g_{itk})ic_{itk}.\gamma_{itk}-g_{itk}.rc_{itk}.\delta_{itk})) \tag{4.23}$$

subject to (3.4) - (3.5), (4.2) – (4.4), (4.12)

$$q_{itk}\le \bar{M}_{it}(g_{itk}+(1-g_{itk})\gamma_{itk}-g_{itk}.\delta_{itk}),\forall i\in N^r,\forall t\in T,\forall k\in K \tag{4.24}$$

$$\sum_{i\in N^r}q_{itk}\le Q,\forall t\in T,\forall k\in K \tag{4.25}$$

$$\sum_{r\in WR_i\cup\{vc_i\}}s_{ir}=1,\forall i\in N^r \tag{4.26}$$

$$\sum_{k\in K}(g_{itk}+\gamma_{itk}-\delta_{itk})-\sum_{r\in WR_i\cup\{vc_i\}}a_{rt}.s_{ir}=0,\forall i\in N^r,\forall t\in T \tag{4.27}$$

$$s_{ir}=0,\forall i\in N^r,\forall r\in TL_i \tag{4.28}$$

$$\sum_{k\in K}(g_{itk}+(1-g_{itk})\gamma_{itk}-g_{itk}.\delta_{itk})\le 1,\forall i\in N^r,\forall t\in T \tag{4.29}$$

$$\sum_{i\in N^r}((1-g_{itk})\gamma_{itk}-g_{itk}.\delta_{itk})\le e,\forall t\in T,\forall k\in K \tag{4.30}$$

$$s_{ir},\gamma_{itk},\delta_{itk}\in\{0,1\},\forall i\in N^r,\forall t\in T,\forall k\in K,\forall r\in WR_i\cup\{vc_i\} \tag{4.31}$$

As in the *PI-SV* formulation, the $q$ variable has a vehicle index $k$. The objective function (4.23) minimizes the total cost. Constraint sets (4.24) is equivalent to the constraint set (3.25). Vehicle capacity constraints are dictated in constraint set (4.25). Constraint set (4.26) - (4.27) are the counterparts of the VSP constraint sets (3.11) and (4.9) from the multi-vehicle PRP-VSP formulation. Constraint set (4.28) is the tabu visit combination constraints and it is equivalent to the constraint set (3.29). Constraint set (4.29) prevents the split delivery. In the basic version of MA and MA-ACC, we limit the maximum number of visit combination substitutions to a specified number, *maxSub*. In MA-SV, we remove the parameter *maxSub* and introduce a new parameter $e$ in (4.30) for a similar purpose. Constraint set (4.30) enforces that the total number of insertions and removals in a vehicle route cannot be bigger than $e$. Finally, constraint set (4.31) is for binary restrictions.

In MA-SV, after the *PI-SV* and *LS-SV* formulations are solved, the vehicle routes are determined by applying a two-stage procedure. In the first stage, using the values of $q_{itk}$, a TSP is solved for each vehicle $k$ in each period $t$. Later, a VRP is solved for each period to obtain better vehicle routes.

## 4.4 Computational Study

For the multi-vehicle cases of PRP-VSP, we adapted the standard multi-vehicle PRP instances proposed in Boudia et al. (2007) to the PRP-VSP. Additionally, by applying small modifications, the proposed algorithms have been applied to the standard PRP instances presented in Archetti et al. (2011) as well as to the multi-vehicle (IRP-VSP) instances solved in Coelho et al. (2012a). The running platform and the software of the algorithms are the same with those presented in Chapter 3.

### *4.4.1 Test Instances*

#### *4.4.1.1 Multi-Vehicle PRP-VSP Instances*

The data set generated by Boudia et al. (2007) contains three problem sets, B1, B2, and B3, with respectively 50, 100, and 200 retailers and 20 periods. Each problem set involves 30 instances. The instances are also characterized with a time varying demand, limited production and storage capacities at the plant and a limited number of vehicles. In order to apply the VSP, we set the minimum and maximum number of periods between two consecutive visits to one and seven, respectively. However, the test instances cannot be made suitable for the VSP by only imposing the visit windows. For the data set, it is assumed that the initial inventory level of the production plant equals to the total retailer demand in the first period. Moreover, it is assumed that there is no production in period 1 and the demand in this period must be satisfied from the initial inventory at the plant. In this case, all solutions must include distribution plans for periods 1 and 2 each of which covers all retailers. To avoid the consecutive visits in periods 1 and 2, we set the initial inventory level of the production plant to zero and not assume that production is not be made in the first period for each instance. After these modifications, we call the modified data sets B1-VSP, B2-VSP, and B3-VSP, respectively.

#### *4.4.1.2 Standard PRP Instances*

The data set introduced in Archetti et al. (2011) involves problem instances with 14 (A1), 50 (A2), and 100 (A3) retailers and six time periods in three sets. The characteristics of the single-vehicle set A1 are described in Chapter 3. The instances in the sets A2 and A3 share the same problem characteristics, however, the vehicle fleet is unlimited in these sets.

#### *4.4.1.3 IRP with the VSP Instances*

There exist two data sets utilized to solve the IRP-VSP in Coelho et al. (2012a). The first data set (S-IRP-VSP) consists of 160 small single vehicle IRP instances originally proposed in Archetti, Bertazzi, Laporte & Speranza (2007). This data set contains 100 instances with five to 50 retailers and three periods and 60 instances with five to 30 retailers and six periods. The second set (L-IRP-VSP) involves 60 larger instances presented in Archetti et al. (2012), with from 50 to 200 retailers and six time periods. All instances are also classified according to whether their inventory holding costs are high or low. The instances of both sets were adapted to the multi-vehicle IRP in Coelho et al. (2012a) by dividing the original vehicle capacity by the number of vehicles considered. For the IRP-VSP, the authors have solved the instances with three vehicles by setting the minimum and maximum number of periods between two consecutive visits to one and two, respectively. After applying the VSP, we have observed that six small size instances with six time periods in the set S-IRP-VSP have become infeasible. Therefore, we have solved 154 instances in this data set.

### *4.4.2 Computational Results*

*4.4.2.1 Computational Results on Multi-Vehicle PRP-VSP Instances*

As we have done for the computational study on single vehicle instances, we have adapted a multi-vehicle variant of the iterative MIP heuristic (IM) of Absi et al. (2015), IM-VRP, to the multi-vehicle PRP-VSP to test the performances of the multi-vehicle variants of MA. We call this adapted version of the algorithm IM-MVVSP. As in IM-VSP, IM-MVVSP involves the VSP constraint sets (3.16) – (18) in the lot-sizing phase and our routing algorithm is utilized to construct the vehicle routes.

The value of the parameter *maxStart* is set to two for both variants of MA. Additionally, the parameter *e* of MA-SV is set to 10 after preliminary tests. Except from these changes, settings of the parameters of MA for the sets S1 and S2 kept unchanged for the multi-vehicle instance sets. Moreover, the executions of the heuristics have been limited to 7200 CPU seconds.

Computational results obtained for the sets B1-VSP, B2-VSP, and B3-VSP are summarized in Table 4.1. Detailed results on these data sets are reported in Tables C1 and C2 in Appendix C. In Table 4.1, the average gap percentages (Avg. Gap%) with respect to the best results are provided along with the average computation times (Avg. t. (s)) and the number of best solutions found (#Best) for each algorithm on each data set. The computational results indicate that MA-ACC is the best performing algorithm in terms of the average gap percentages with respect to the best solutions and the number of best solutions found for all data sets. The average gap of MA-ACC is found as 0.63%. IM-MVVSP is the second best performing algorithm for all data set with an average gap percentage of 0.99. The performance of MA-SV is significantly worse than other heuristics in terms of the average gap percentages and the average computational times. Besides, it is also noticeable that as the problem size increases, the effectiveness of MA-SV worsens. The reason for that outcome is that the time required to obtain a good solution by the *LS* formulation of MA-SV, *LS-SV*, greatly increases with the problem size. Multiple vehicle capacity constraints clearly complicate the solution of *LS-SV*.

Table 4.1 Summary of the computational results for the multi-vehicle PRP-VSP instances

| | IM-MVVSP | | | MA-ACC | | | MA-SV | | |
|---|---|---|---|---|---|---|---|---|---|
| *Set* | Avg. Gap% | Avg. t. (s) | #Best | Avg. Gap% | Avg. t. (s) | #Best | Avg. Gap% | Avg. t. (s) | #Best |
| B1-VSP | 0.95 | 1862.4 | 7 | **0.70** | 1148.6 | **18** | 2.69 | 4503.4 | 5 |
| B2-VSP | 0.69 | 5140.4 | 8 | **0.54** | 3828.9 | **21** | 4.85 | 7200.0 | 1 |
| B3-VSP | 1.32 | 7200.0 | 12 | **0.64** | 7200.0 | **18** | 5.42 | 7200.0 | 0 |
| All | 0.99 | 4734.3 | 27 | **0.63** | 4059.2 | **57** | 4.32 | 6301.1 | 6 |

#### *4.4.2.2 Computational Results on the Sets A2 and A3*

The fleet size is unlimited in the sets A2 and A3. As MA-ACC and MA-SV are designed to solve the PRP-VSP instances with a limited number of vehicles, we have not implemented these algorithms to these data sets. Instead, we have adapted the basic version of MA to the data sets by only removing the vehicle capacity constraint set (3.10) from the *LS* and *PI* formulations. The performance of MA is compared with five heuristic algorithms which are the optimization based ALNS of Adulyasak et al. (2014a) (Op-ALNS), multi-start IM variants (IM-MultiTSP and IM-VRP) of Absi et al. (2015), multi-phase heuristic (5P) of Solyalı & Süral (2017), VNS of Qiu, Wang et al. (2018), three level heuristic (TLH) of Li et al. (2019), and three-level decomposition matheuristic (CCJ-DH) of Chitsaz et al. (2019). The value of the parameter *maxStart* is set to five and two for the sets A2 and A3, respectively.

A summary of the results obtained for the sets A2 and A3 are reported in Table 4.2. The detailed results of MA on these data sets are provided in Tables C3 and C4, respectively, in Appendix C. In Table 4.2, '# best' shows the number of times a heuristic finds the best solution. As it is seen from Table 4.2, MA is the best performing algorithm in terms of the average gap percentages with respect to the best solutions for the set A2. The average gap of MA for all instances is found as 0.12%. Moreover, MA has obtained the best results for 136 instances, while improving 125 best known solutions. When the results obtained for A3 is analyzed, it is observed that MA has provided the best results for 124 instances, while improving 123 best known solutions. Although MA is the second best algorithm in terms of the average gap percentages with respect to the best solutions, the difference between the average gaps of 5P and MA is only 0.02%.

The average computational times obtained for A2 and A3 are provided in Table 4.3. A fair comparison of MA with the other methods in terms of computational time cannot be made because the algorithms were implemented on different platforms and software. However, our computational experiments indicate that the computational times of the proposed MA do not differ significantly from those of other compared methods. The average computational times are 82.2 and 188.8 seconds for A2 and A3, respectively. This outcome indicates that MA spends reasonable computation times to solve the PRP instances.

Table 4.2 Average Gap% with respect to the best solutions of A2 and A3

| | Classes | Op-ALNS | IM-VRP | IM-MultiTSP | 5P | VNS | TLH | CCJ-DH | MA |
|---|---|---|---|---|---|---|---|---|---|
| A2 | Class I | 1.23 | 0.27 | 1.19 | 0.15 | 0.15 | 0.27 | 0.19 | **0.07** |
| | Class II | 0.18 | 0.06 | 0.11 | **0.02** | 0.04 | 0.03 | 0.04 | **0.02** |
| | Class III | 3.88 | 1.34 | 3.08 | 0.44 | 0.39 | 1.16 | 0.68 | **0.30** |
| | Class IV | 0.26 | 0.17 | 0.53 | 0.13 | **0.05** | 0.11 | 0.07 | 0.08 |
| | Avg. | 1.39 | 0.46 | 1.23 | 0.19 | 0.16 | 0.39 | 0.24 | **0.12** |
| | # Best | 1 | 9 | 14 | 108 | 55 | 59 | 111 | **136** |
| A3 | Class I | 1.03 | 0.25 | 1.89 | 0.10 | 0.20 | 0.20 | 0.29 | **0.05** |
| | Class II | 0.14 | 0.03 | 0.12 | **0.02** | 0.06 | **0.02** | 0.03 | **0.02** |
| | Class III | 3.74 | 1.32 | 3.90 | **0.23** | 0.39 | 1.42 | 1.68 | 0.33 |
| | Class IV | 0.32 | 0.23 | 0.69 | 0.04 | 0.07 | **0.03** | 0.05 | 0.07 |
| | Avg. | 1.30 | 0.46 | 1.65 | **0.10** | 0.18 | 0.42 | 0.51 | 0.12 |
| | # Best | 0 | 28 | 8 | **142** | 29 | 86 | 65 | 124 |

Table 4.3 Average computational times for A2 and A3

| | | Op-ALNS | IM-VRP | IM-MultiTSP | 5P | VNS | TLH | CCJ-DH | MA |
|---|---|---|---|---|---|---|---|---|---|
| A2 | Class I | 50.2 | 25.6 | 338.5 | 16.4 | 25.2 | 43.1 | 328.8 | 89.2 |
| | Class II | 49.5 | 21.7 | 235.7 | 13.8 | 21.3 | 36.5 | 272.8 | 77.5 |
| | Class III | 42.7 | 22.6 | 317.9 | 15.8 | 24.5 | 39.3 | 252.6 | 78.0 |
| | Class IV | 44.1 | 27.7 | 375.8 | 25.4 | 28.4 | 42.1 | 271.5 | 84.1 |
| | Avg. | 46.6 | 24.4 | 317.0 | 17.9 | 24.9 | 40.3 | 281.4 | 82.2 |
| A3 | Class I | 228.5 | 85.5 | 514.2 | 323.5 | 84.6 | 168.6 | 1313.1 | 187.0 |
| | Class II | 217.6 | 76.1 | 497.4 | 50.6 | 73.4 | 131.9 | 1062.7 | 101.6 |
| | Class III | 197.8 | 75.1 | 509.3 | 349.6 | 72.8 | 144.5 | 1016.9 | 290.3 |
| | Class IV | 206.0 | 86.6 | 507.0 | 125.1 | 83.7 | 159.8 | 1056.6 | 176.2 |
| | Avg. | 212.5 | 80.8 | 507.0 | 212.2 | 78.6 | 151.2 | 1112.3 | 188.8 |

### *4.4.2.3 Computational Results on the IRP-VSP Instances*

Our algorithms can be easily modified to solve the IRP-VSP by fixing the production decisions. For $t \in T$, let $B_t$ be the production quantity made available in period $t$. First, the production setup variable $y_t$ and quantity variable $p_t$ are set to one and $B_t$, respectively, for each $t \in T$. Second, $\bar{M}_{it}$ is set to the minimum of $\{L_i, Q\}$ instead of $\{L_i, Q, DD_{it}\}$.

The aforementioned IRP with the VSP data sets, S-IRP-VSP and L-IRP-VSP, have been solved only in Coelho et al. (2012a), to the best of our knowledge. In this study, the authors have developed an ALNS algorithm to the multi-vehicle IRP with the consistency features. Hence, we compare the computational results of MA-ACC and MA-SV with the results of ALNS. The value of the parameter *maxStart* is set to five for both variants of MA.

The computational results on the set S-IRP-VSP are summarized in Table 4.4. Detailed computational results are provided in Table C5 through C8 in Appendix C. The first two columns in Table 4.4 give information about the characteristics of the set instances. The first column (‘Inv. C.’) indicates the type of the inventory holding costs of the instances while the second column (‘*H*’) shows the number of periods. For MA-ACC and MA-SV, we report the average gap percentages (Avg. Gap) with respect to the best results along with the average computation times (Avg. t. (s.)) obtained for each instance group. As in the other data sets, '# best' shows the number of times a heuristic finds the best solution. As the computational times of ALNS were not reported in Coelho et al. (2012a), we only give its average gaps. From the table, it is seen that MA-SV outperforms other algorithms in terms of the average gap percentages with respect to the best solutions. Moreover, MA-SV has managed to obtain the best solutions for 111 out of 154 instances, while improving 76 best known solutions.

Table 4.4 Summary of the computational results obtained for the set S-IRP-VSP

| | | ALNS | MA-ACC | | MA-SV | |
|---|---|---|---|---|---|---|
| Inv. C. | *H* | Avg. Gap | Avg. Gap | Avg. t. (s.) | Avg. Gap | Avg. t. (s.) |
| Low | 3 | 0.95 | 4.22 | 62.7 | **0.13** | 265.4 |
| High | 3 | 1.49 | 3.20 | 61.1 | **0.09** | 264.0 |
| Low | 6 | 0.75 | 3.74 | 92.4 | **0.31** | 395.9 |
| High | 6 | **0.22** | 2.74 | 90.4 | 0.44 | 454.8 |
| | Avg. | 0.96 | 3.55 | 76.7 | **0.20** | 345.0 |
| | #Best | 74 | 9 | | **111** | |

The performance of MA-ACC on the set S-IRP-VSP is far worse than MA-SV and ALNS, which is quite interesting. Specifically, as the problem size decreases, the gaps of the solutions of MA-ACC with respect to the best solutions become larger. Even on most of the small size instances with up to 10 retailers, MA-ACC has failed to reach the best known solutions. This issue is probably originated from tight vehicle capacity constraints on these instances. Because MA-ACC involves aggregated vehicle capacity constraints in its *LS-ACC* formulation, the delivered quantity values may become so large that retailer clusters expected to cause low transportation costs on the same route cannot be created due to low vehicle capacity. In order to investigate this issue, we perform a small numerical investigation on a subset of small size instances with low inventory holding costs under different vehicle capacity scenarios. In our study, we have resolved the selected group of instances by increasing the vehicle capacity $Q$ to $Q \times 2$, $Q \times 3$, and $Q \times 4$. The results are reported in Table 4.5. In the table, we also present the previously obtained results for the original instances (Base case scenario). When the average gaps between the results of MA-ACC and MA-SV are analyzed, it has been seen that as the vehicle capacity increases, the average gap decreases. While the average gap is 5.17% for the base case scenario, it decreases to 0.00% for Scenario 3. This result clearly shows the impact of the vehicle capacity constraints on the quality of the obtained solutions of MA-ACC.

Table 4.5 Results for different vehicle capacity scenarios of some instances with Inv. C.: Low and $H$ = 3 from the set S-IRP-VSP

| | | Scenarios | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Base case scenario | | | Scenario 1: $Q \rightarrow Q \times 2$ | | | Scenario 2: $Q \rightarrow Q \times 3$ | | | Scenario 3: $Q \rightarrow Q \times 4$ | | |
| inst. | n | MA-ACC | MA-SV | Gap% | MA-ACC | MA-SV | Gap% | MA-ACC | MA-SV | Gap% | MA-ACC | MA-SV | Gap% |
| 1 | 5 | 1540.57 | 1430.51 | 7.69 | 1395.68 | 1248.54 | 11.78 | 1247.68 | 1235.94 | 0.95 | 1234.68 | 1234.68 | 0.00 |
| 1 | 10 | 3100.87 | 2766.61 | 12.08 | 2038.82 | 2038.82 | 0.00 | 1683.82 | 1683.82 | 0.00 | 1649.82 | 1649.82 | 0.00 |
| 3 | 5 | 3497.96 | 3497.96 | 0.00 | 1884.02 | 1884.02 | 0.00 | 1652.02 | 1652.02 | 0.00 | 1568.02 | 1568.02 | 0.00 |
| 3 | 10 | 2666.69 | 2649.26 | 0.66 | 2096.10 | 2094.96 | 0.05 | 1955.10 | 1953.84 | 0.06 | 1871.10 | 1871.10 | 0.00 |
| 5 | 5 | 1677.21 | 1516.65 | 10.59 | 1220.21 | 1220.21 | 0.00 | 999.42 | 999.42 | 0.00 | 999.42 | 999.42 | 0.00 |
| 5 | 10 | 2648.07 | 2648.53 | -0.02 | 1904.96 | 1904.96 | 0.00 | 1837.96 | 1837.96 | 0.00 | 1683.96 | 1683.96 | 0.00 |
| | Avg. | | | 5.17 | | | 1.97 | | | 0.17 | | | 0.00 |

*Notes.* Gap% = ((MA-ACC – MA-SV) / MA-SV) × 100

A summary of the computational results obtained for the set L-IRP-VSP are presented in Table 4.6. Detailed results on this set can be found in Table C9 in Appendix C. The results on this data set show that MA-ACC and MA-SV perform far better than ALNS. Both variants of MA outperform ALNS on all of the instances. Unlike on the set S-IRP-VSP, MA-ACC is the best performing algorithm in terms of the number of best solutions found on this data set. The average gaps of MA-ACC and

MA_SV are very close to each other. However, the average computational times of MA-ACC are less than those of MA-SV, which indicates that MA-ACC is more efficient than MA-SV on the set L-IRP-VSP.

Table 4.6 Summary of the computational results obtained for the set L-IRP-VSP

| | ALNS | MA-ACC | | MA-SV | |
|---|---|---|---|---|---|
| Inv. C. | Avg. Gap | Avg. Gap | Avg. t. (s.) | Avg. Gap | Avg. t. (s.) |
| Low | 54.29 | **1.20** | 1554.5 | 1.34 | 11556.6 |
| High | 18.82 | 0.85 | 1634.0 | **0.63** | 11593.0 |
| Avg. | 36.55 | 1.03 | 1594.2 | **0.99** | 11574.8 |
| #Best | 0 | **36** | | 24 | |

### 4.5 Chapter Summary

In this chapter, we extend our previous study on the PRP-VSP by considering multiple vehicles. Two variants of MA have been proposed to tackle the multi-vehicle version of the problem. The performance of developed algorithms has been tested on randomly generated instances. The results indicate the effectiveness of the proposed approaches. Moreover, the performance of MA is tested on standard PRP and the IRP with the VSP instances from the literature. The computational results show that the developed algorithm generates competitive results with the most powerful algorithms developed so far, for the standard PRP and the IRP with the VSP. Especially, it has been observed that the developed approach has improved the best known solutions for several test instances.

The following chapter presents our work on the PRP with transshipments.

# CHAPTER FIVE
# THE PRODUCTION ROUTING PROBLEM WITH TRANSSHIPMENT

## 5.1 Introduction

In this chapter, we study the production routing problem with transshipment (PRPT). The concept of transshipments has been formally integrated within in the inventory routing problem (IRP) in Coelho et al. (2012b), Lefever et al. (2018), and Coelho et al. (2014). The IRP with transshipments was introduced in Coelho et al. (2012). In this paper, the authors presented a formulation and proposed an adaptive large neighborhood search (ALNS) for the problem. The developed ALNS changed the vehicle routes while the remaining problem of determining quantity decisions and transshipment moves is solved by using a network flow algorithm. The authors also performed an extensive numerical study to assess the effect of transshipments on the overall system cost. Lefever et al. (2018) proposed a branch-and-cut algorithm for the IRP with transshipments. In this study, new sets of valid inequalities for the problem are presented. Computational results indicate that the suggested valid inequalities greatly reduce the computational burden for solving the problem to optimality. The IRP with transshipments has been considered in dynamic and stochastic environments in Coelho et al. (2014). The authors suggested heuristic algorithms for different solution policies.

In this study, we address the PRP with the concept of transshipment for the first time. Initially, an MIP model is formulated for the PRPT. Although the formulated model performs well on small size instances, its performance tumbles in middle and large size instances. For this reason, we propose a mathematical programming heuristic (MPH) to solve the PRPT. In order to perform a computational study, some of the PRP benchmark instances introduced in Archetti et al. (2011) are adapted to the PRPT. In addition, a set of production routing problem with visit spacing policy (PRP-VSP) instances presented in Chapter 3 are adapted to the PRPT. The performance of our MPH is compared with a general-purpose commercial solver. The results indicate that the developed approach can produce high-quality solutions to the PRPT in reasonable computation times. In addition, an extensive study has been performed to reveal the effect of the transshipments on the overall cost. The results show the benefits of allowing transshipments.

The rest of this chapter is organized as follows. In Section 5.2, the formal problem description and the developed MIP model for the PRPT are given. In Section 5.3, we present our MPH algorithm. The computational results are given in Section 5.4. Finally, a summary of the chapter is given in Section 5.

### 5.2 Problem Description and Formulation

We consider a production-distribution system which is defined on a graph $G = (N, A)$, where $N = \{0,\ldots,n\}$ is the node set and $A = \{(i, j): i, j \in N, i \neq j\}$ is the arc set. We denote by 0 the production plant and by other nodes the retailers. A single product is manufactured in the production plant, and is sold by the retailers along a discretized planning horizon $T = \{1,\ldots,H\}$. The quantity demanded at retailer $i$, $i \in N \setminus \{0\}$, in time period $t$, $t \in T$, is indicated by $d_{it}$. At each location, the production plant or a retailer location, there exists a storage area with a limited capacity $L_i$. The inventory holding cost at node $i$, $i \in N$, is denoted by $h_i$. In each time period, a decision on whether to make the product is taken by the production plant. In case of the production, a fixed setup cost $f$ is resulted along with a unit production cost $u$ that is incurred for each produced unit. Moreover, the produced lot size is limited with a production capacity $C$. A single vehicle with a limited capacity $Q$ distributes the products to the retailers. The vehicle can perform at most one route in each time period, and the vehicle routes start and end at the production plant. Each retailer can be visited at most once in a route. A transportation cost $c_{ij}$ is associated with each arc $(i, j) \in A$. Delivered quantities for each retailer are determined according to the maximum level policy. An initial inventory $I_{i0}$ is defined for each node $i$, $i \in N$. A transshipment can originate from the depot or from any retailer on a special request basis. This can be done by subcontracting to a carrier who will pickup goods from any transshipment point. These outsourced deliveries are only made by direct shipping and the unit cost associated with transshipping products from $i$ to $j$ is $b_{ij}$. The aim of the considered problem is to combine the decisions of production, inventory, routing, and transshipment operations that minimize the total cost with respect to the stated constraints. In each time period in which both regular deliveries and transshipments are performed, the sequence of operations at the retailer locations is the following: first the regular deliveries are made by the vehicle, second transshipments are performed, then the demands of the retailers are satisfied. After that, the final inventory levels are calculated. To formulate the problem, the following notation is defined:

*Sets*

$N^r$ : set of all retailers $\{1,...,n\}$

$N$ : set of all nodes $\{0,1,...,n\}$

$T$ : set of time periods $\{1,...,H\}$

*Parameters*

$u$ : unit production cost

$f$ : production setup cost

$C$ : production capacity

$Q$ : capacity of the vehicle

$h_i$ : unit inventory holding cost at node $i$

$L_i$ : maximum or target inventory level at node $i$

$I_{i0}$ : initial inventory level at node $i$

$d_{it}$ : demand at retailer $i$ in period $t$

$c_{ij}$ : transportation cost from node $i$ to node $j$

$b_{ij}$ : unit transshipment cost of the product from node $i$ to node $j$

$M_t$ : $\min\{C, \sum_{l=t}^{H} \sum_{i \in N^r} d_{il}\}$, maximum quantity that can be produced in period $t$

$\bar{M}_{it}$ : $\min\{L_i, Q, \sum_{j=t}^{H} d_{ij}\}$, maximum quantity that can be delivered to node $i$ in period $t$

*Decision Variables*

$p_t$ : production quantity in period $t$

$y_t : \begin{cases} 1 & \text{if there is production at the plant in period } t \\ 0 & \text{otherwise} \end{cases}$

$I_{it}$ : inventory level at node $i$ at the end of period $t$

$x_{ijt} : \begin{cases} 1 & \text{if the vehicle travels directly from node } i \text{ to node } j \text{ in period } t \\ 0 & \text{otherwise} \end{cases}$

$q_{it}$ : quantity delivered to retailer $i$ in period $t$ by the vehicle

$z_{it} : \begin{cases} 1 & \text{if retailer } i \text{ is visited in period } t \text{ by the vehicle} \\ 0 & \text{otherwise} \end{cases}$

$z_{0t} : \begin{cases} 1 & \text{if the vehicle is dispatched in period } t \\ 0 & \text{otherwise} \end{cases}$

$w_{ijt}$ : amount of product transshipped from node $i, i \in N$, to node $j, j \in N^r$, in period $t$

$v_{it}$ : sum of the deliveries made by the vehicle in period $t$ after visiting retailer $i$

The PRPT can be formulated as follows:

$$(PRPT)\, Min\, Z = \sum_{t \in T} (u.p_t + f.y_t + \sum_{i \in N} h_i.I_{it} + \sum_{i \in N} \sum_{j \in N \setminus \{i\}} c_{ij}.x_{ijt} + \sum_{i \in N} \sum_{j \in N^r} b_{ij}.w_{ijt}) \quad (5.1)$$

subject to

$$I_{0t-1} + p_t = \sum_{i \in N^r} q_{it} + I_{0t} + \sum_{i \in N^r} w_{0it}, \forall t \in T \quad (5.2)$$

$$I_{it-1} + q_{it} + \sum_{j \in N / \{i\}} w_{jit} = d_{it} + I_{it} + \sum_{j \in N^r / \{i\}} w_{ijt}, \forall i \in N^r, \forall t \in T \quad (5.3)$$

$$p_t \le M_t.y_t, \forall t \in T \quad (5.4)$$

$$I_{0t} \le L_0, \forall t \in T \quad (5.5)$$

$$I_{it-1} + q_{it} + \sum_{j \in N \setminus \{i\}} w_{jit} \le L_i, \forall i \in N^r, \forall t \in T \quad (5.6)$$

$$q_{it} \le \bar{M}_{it}.\mathrm{z}_{\mathrm{it}}, \forall i \in N^r, \forall t \in T \quad (5.7)$$

$$\sum_{i \in N^r} q_{it} \le Q.z_{0t}, \forall t \in T \quad (5.8)$$

$$\sum_{j \in N \setminus \{i\}} x_{jit} = \sum_{j \in N \setminus \{i\}} x_{ijt}, \forall i \in N, \forall t \in T \quad (5.9)$$

$$\sum_{j \in N \setminus \{i\}} x_{ijt} = z_{it}, \forall i \in N^r, \forall t \in T \quad (5.10)$$

$$v_{it} - v_{jt} + Q.x_{ijt} \le Q - q_{jt}, \forall i \in N^r, \forall j \in N^r, \forall t \in T \quad (5.11)$$

$$q_{it} \le v_{it} \le Q, \forall i \in N^r, \forall t \in T \quad (5.12)$$

$$p_t, I_{it}, q_{it}, w_{ijt} \ge 0, \forall i \in N, \forall j \in N^r, \forall t \in T \quad (5.13)$$

$$x_{ijt}, y_t \in \{0,1\}, \forall i \in N, \forall j \in N, \forall t \in T \quad (5.14)$$

$$z_{it} \in \{0,1\}, \forall i \in N^r, \forall t \in T \quad (5.15)$$

The objective function (5.1) minimizes the total production, setup, inventory, routing, and transshipment costs. Constraint sets (5.2) and (5.3) are the inventory flow balance constraints at the plant and retailers, respectively. Constraint set (4) ensures the production setup variable to be one if product is made in a given period and limit the production quantity to the minimum value between the production capacity and the total demand in the remaining periods. Constraint sets (5.5) and (5.6) limit the maximum inventory at the plant and retailers, respectively. Constraint set (5.7) limits the delivered quantity $q_{it}$ to $\bar{M}_{it}$, which is the minimum of the retailer inventory capacity $L_i$, vehicle capacity $Q$, and the total demand of retailer $i$ in the remaining periods. Constraint set (5.8) dictates the vehicle capacity, and the vehicle flow conservation constraints are given in Constraint sets (5.9) and (5.10). The subtour elimination constraints are given in constraint sets (5.11) and (5.12). Finally, the constraint sets (5.13), (5.14), and (5.15) define the nature of the decision variables. The introduced PRPT is NP-hard since it contains a traveling salesman problem (TSP) that is known to be NP-hard (Boudia et al., 2007; Archetti et al., 2011; Adulyasak et al., 2014a). Due to the complexity of the problem, we develop a heuristic algorithm for the PRPT.

## 5.3 Solution Method

We propose a two-phase mathematical programming heuristic (MPH) to solve the PRPT. In the first phase, a feasible solution is initially constructed by sequentially solving two decomposed problems, the production-distribution and routing subproblems. After that, the obtained initial solution is updated by applying an MIP-based local search procedure iteratively. The local search procedure is based on the insertion and removal of the retailers from the vehicle routes. The feasible solution construction and local search procedures are implemented in a multi-start scheme, and each multi-start execution is repeated until a given number of local search iterations is reached. Throughout the first phase, the production setup and distribution decisions for all solutions, i.e., the production setup configurations and delivery schedules for the retailers, are recorded in schedule lists. These schedule lists are employed in the second phase.

The second phase of MPH is based on the a priori tour idea, which is introduced in Pınar & Süral (2006) and employed in Solyalı & Süral (2011) for the single vehicle IRP. The idea is to make the vehicle routing decisions easier by dictating a predetermined sequence of all retailers for all routes where the vehicle always follows the given sequence for the retailers to be visited and skips the non-visited ones. At the beginning of the second phase, the predetermined sequence of all retailers is obtained by solving a TSP over all nodes. Next, using the obtained sequence and information collected during the first phase, we formulate and solve a restricted version of the PRPT as an MIP. Specifically, utilizing the production and delivery schedule lists constructed during the first phase enables the algorithm to focus the search on the promising regions of the solution space. Moreover, we tighten the MIP formulations solved in the local search and in the second phase with some inequalities, which reduces the computational burden significantly. Main steps of our MPH is given in Figure 5.1. Next, we explain the components of our heuristic in detail.

### *5.3.1 First Phase*

As mentioned previously, the first phase of MPH consists of the multi-start executions of the initial solution construction and the local search procedures. We denote by *maxStart* and *maxIter* the number of multi-start executions and the number of local search iterations applied in a single multi-start execution, respectively. At the beginning of the first phase, an empty delivery schedule set, $K_i$, for each retailer $i \in N^r$, is created. In addition, an empty production setup configuration set, *Ps*, is created to store the production setup configurations. During the search process, whenever a new solution is obtained through either initial solution construction or local search, the delivery schedule of each retailer $i \in N^r$ (resp. the production setup configuration) in the new solution is added to $K_i$ (resp. *Ps*) if it does not exist in the current set.

**Algorithm 1:** MPH
**Input:** problem data, *maxStart*, *maxIter*, *pIter*
**Output:** best solution, $X_b$
// first phase of MPH //
create an empty set $K_i$ for each $i \in N^r$;
create an empty set *Ps*;
**for** $i \leftarrow 1$:*maxStart*
    construct an initial solution, $X$;
    update set *Ps* and set $K_i$ for each $i \in N^r$ by utilizing the solution $X$;
    **for** $j \leftarrow 1$:*maxIter*
        select a random retailer $\lambda$;
        calculate the insertion and removal costs $\Gamma_t$ and $\Delta_t$ for each period $t \in T$ for retailer $\lambda$;
        **if** $j$ is a multiple of *pIter*
            solve $LSM'(X, \lambda, \Gamma_t, \Delta_t)$ model and obtain a new solution $X'$;
            update set *Ps* and set $K_\lambda$ by using the solution $X'$;
        **else**
            solve $LSM(X, \lambda, \Gamma_t, \Delta_t)$ model and obtain a new solution $X'$;
            update set $K_\lambda$ by using the solution $X'$;
        **end if**
        $X \leftarrow X'$;
    **end for**
**end for**
// second phase of MPH //
solve a giant TSP over all nodes obtain the sets $\alpha_i$ and $\beta_i$ for each $i \in N$;
solve $SP(Ps, K_i, \alpha_i, \beta_i)$ model and obtain $X_b$;
update the vehicle routes of $X_b$ by solving a TSP for each period t $\in T$;

Figure 5.1 Main steps of MPH

#### *5.3.1.1 Initial Solution Construction*

Our initial solution construction procedure is developed based on the method introduced in Absi et al. (2015). In order to obtain a feasible solution, we initially solve a restricted version of the original PRPT, where the routing costs in the original model are replaced with fixed costs $SC_{it}$ representing the approximate cost of visiting retailer $i$ in period $t$. The value of $SC_{it}$ is calculated for each $i \in N^r$ and $t \in T$ as follows: a random number $\rho_{it}$ is generated from the interval [0.5, 1.5] and the visiting cost $SC_{it}$ is set to $\rho_{it} \times (c_{0i} + c_{i0})$. The resulting production-distribution subproblem (PDSP) is formulated as follows:

$$\text{(PDSP)} Min\, Z = \sum_{t \in T} (u.p_t + f.y_t + \sum_{i \in N} h_i.I_{it} + \sum_{i \in N} \sum_{j \in N^r} b_{ij}.w_{ijt} + \sum_{i \in N^r} SC_{it}.z_{it}) \quad (5.16)$$

subject to (5.2) – (5.8), (5.13) – (5.15)

This PDSP is solved to determine the continuous $p_t$, $I_{it}$, $q_{it}$, and $w_{ijt}$ variables as well as the binary $z_{it}$ and $y_t$ variables. Therefore, the remaining problem for each period is a standard TSP. In this stage, we employ the ILS algorithm utilized in previous chapters to construct the vehicle route for each time period.

#### *5.3.1.2 Local Search*

In the local search, we obtain a new solution by applying a neighbor solution generation procedure to the current solution at each iteration. The neighbor solution generation procedure is based on a solution improvement method proposed in Archetti et al. (2012). In this procedure, we solve an MIP for another restricted version of the PRPT in which we impose the production setup configuration and the vehicle routes obtained in the current solution and allow insertion and removal of only one randomly selected retailer to the vehicle routes. In addition, when the number of local search iterations is a multiple of a given parameter, *pIter*, the production setup configuration of the current solution is not dictated in the restricted formulation to produce a new production plan.

In order to formulate the restricted formulation solved in this phase, we initially select a random retailer, $\lambda$. After that, we define $s_t$ and $r_t$ as binary variables account for the insertion of retailer $\lambda$ into period $t$ and removal of retailer $\lambda$ from period $t$, respectively. The insertion and removal costs for each period $t \in T$ are represented as $\Gamma_t$ and $\Delta_t$, respectively. $\Gamma_t$ is the cheapest insertion cost for inserting retailer $\lambda$ in the route of period $t$ while $\Delta_t$ is associated with the cost decrease occurred when retailer $\lambda$ is removed from the route of period $t$ and its predecessor is linked with its successor. Based on the current solution, we let $a_{it}$ be a parameter being equal to one if retailer $i$ is visited in period $t$, and zero otherwise. We also let $\pi_t$ be a parameter which is equal to one if the production does take place in period $t$, and zero otherwise. The restricted model solved in a regular local search iteration can be formulated as follows:

$$Min\,Z = \sum_{t\in T}(u.p_t + f.y_t + \sum_{i\in N} h_i.I_{it}) + \sum_{t\in T}(\sum_{i\in N}\sum_{j\in N^r} b_{ij}.w_{ijt} + (1-a_{\lambda t}).\Gamma_t.s_t - a_{\lambda t}.\Delta_t.r_t) \quad (5.17)$$

subject to (5.2) – (5.6), (5.8), (5.13), (5.14)

$$y_t = \pi_t, \forall t \in T \quad (5.18)$$

$$q_{it} \le \bar{M}_{it}.a_{it}, \forall i \in N^r / \{\lambda\}, \forall t \in T \quad (5.19)$$

$$q_{\lambda t} \le \bar{M}_{\lambda t}.(a_{\lambda t} - a_{\lambda t}.r_t + (1 - a_{\lambda t}).s_t), \forall t \in T \quad (5.20)$$

$$\sum_{t\in T} r_t.a_{\lambda t} \ge 1 \quad (5.21)$$

$$r_t, s_t \in \{0,1\}, \forall t \in T \quad (5.22)$$

The objective function (5.17) is the sum of fixed and variable production costs, inventory holding costs at the plant and retailers, transshipment costs, and the insertion and removal costs of the randomly selected retailer, $\lambda$. Constraint set (5.18) imposes the current production setup configuration in the new solution. Constraint set (5.19) (resp. constraint set (5.20)) ensures that the quantity delivered to retailer $i$ (resp. $\lambda$) by the vehicle in period $t$ can be positive if and only if that retailer is visited in period $t$. Constraint (5.21) enforces that the retailer $\lambda$ must be removed from at least one vehicle route, which guarantees that the new solution to be obtained differs from the current one. Finally, constraint set (5.22) defines the nature of the decision variables.

We reduce the complexity of this model by implementing new sets of constraints. To this aim, we define $ST$ as a set of triplets $(i, j, t)$ where $i \in N \setminus \{\lambda\}$, $j \in N^r \setminus \{\lambda\}$, $t \in T$, and $w_{ijt}$ equals to zero in the current solution. Moreover, for each $i \in N^r \setminus \{\lambda\}$, let $TP_i$ be a set of time periods in which retailer $i$ is visited by the vehicle, i.e., $TP_i = \{t\text{: } t \in T,\ a_{it} = 1\}$. For each $i \in N^r \setminus \{\lambda\}$ and $t \in TP_i$, we define a parameter $\delta_{it}$ which is the latest time period before the next visit to retailer $i$, when there exists a time period $k$ such that $k > t$ and $a_{ik} = 1$. If $t$ is the latest time period in $TP_i$, $\delta_{it}$ is set to $H$. For each $i \in N^r \setminus \{\lambda\}$ and $t \in TP_i$, we also let $\gamma_{it}$ be a parameter which is the total demand of retailer $i$ in the periods from $t$ to $\delta_{it}$. In order to decrease the computational burden, the following constaint sets are added to model solved in the local search:

$$\sum_{(i,j,t)\in ST} w_{ijt} = 0 \tag{5.23}$$

$$I_{iH} = 0, \forall i \in N \tag{5.24}$$

$$I_{it-1} + q_{it} + \sum_{j\in N\setminus\{i\}} \sum_{k=t}^{\delta_{it}} w_{jik} - \sum_{j\in N\setminus\{i\}} \sum_{k=t}^{\delta_{it}} w_{ijk} \geq \gamma_{it}, \forall i \in N^r \setminus \{\lambda\}, \forall t \in TP_i \tag{5.25}$$

Constraint (5.23) eliminates all transshipment variables which do not related to retailer $\lambda$ and are equal zero in the current solution. Constraint set (5.24) sets the final inventory level of each node $i$, $i \in N$, to zero. As mentioned in Archetti et al. (2011), it is never convenient in a standard PRP to have a final inventory level greater than zero, both at the plant and the retailers. This is also valid for the PRPT. Finally, constraint set (5.25) imposes for each $i \in N^r \setminus \{\lambda\}$ and $t \in TP_i$ that the sum of the inventory level at retailer $i$ before time period $t$, quantity delivered to retailer $i$ by the vehicle in period $t$, and the total net quantity transshipped to retailer $i$ in periods from $t$ to $\delta_{it}$ must satisfy the total demand of retailer $i$ in periods from $t$ to $\delta_{it}$. With these new constraint sets, the restricted formulation (*LSM*) solved in the local search consists of the equations (5.2) – (5.6), (5.8), (5.13), (5.14), and (5.17) – (5.25). After *LSM* is solved, the new delivery schedule obtained for retailer $\lambda$ is added to its corresponding set $K_\lambda$ unless it has already been available in that set. The restricted formulation (*LSM´*) solved in every *pIter* iterations involves the same set of equations with those of *LSM* except for the constraint set (5.18). As mentioned earlier, the production setup configuration obtained by solving *LSM´* is added to the set *Ps* if it has not been created before.

### *5.3.2 Second Phase*

The second phase of MPH consists of three stages. In the first stage, a giant TSP over all nodes is solved to obtain the sequence of retailers to be visited in a period. Secondly, using the information coming from the first phase and the sequence of retailers, an MIP for a restricted version of the problem is formulated and solved. Finally, we solve a TSP for each period to improve the solution obtained by solving the MIP formulation.

At the beginning of the phase, after the giant TSP is solved, we form two sets, $\alpha_i$ and $\beta_i$, for each node $i \in N$ to save the sequence of retailers to be visited in a period. Here, $\alpha_i$ and $\beta_i$ are defined as sets of nodes which can be visited after and before node $i$ according to its order on the giant tour, respectively. Table 5.1 depicts the generation of the sets $\alpha_i$ and $\beta_i$ for a three-retailer instance.

Table 5.1 All $\alpha_i$ and $\beta_i$ sets for a 3-retailer instance on a given tour 0-1-2-3-0

| $i$ | $\alpha_i$ | $\beta_i$ |
|---|---|---|
| 0 | {1, 2, 3} | {1, 2, 3} |
| 1 | {0, 2, 3} | {0} |
| 2 | {0, 3} | {0, 1} |
| 3 | {0} | {0, 1, 2} |

In the MIP formulation, the production decisions in the PRPT are simplified by considering only the configurations stored in the set $Ps$. In the same manner, for each retailer $i \in N^r$, the delivery schedule of the retailer is selected among those in the set $K_i$, which reduces the complexity on the distribution part of the problem. Moreover, as mentioned earlier, imposing the a priori tour for the vehicles simplify the routing part of the PRPT. In order to formulate the restricted model, the set $TP_i$ and the parameters $\delta_{it}$, $\gamma_{it}$, and $\pi_t$ from the first phase are extended with a schedule index as follows:

$TP_{ik}$ : according to delivery schedule $k$, set of time periods in which retailer $i$ is visited by the vehicle, $i \in N^r$ and $k \in K_i$.

$\delta_{ikt}$ : according to delivery schedule $k$, the latest time period before the next visit to retailer $i$, $i \in N^r$, $k \in K_i$, and $t \in TP_{ik}$. If $t$ is the latest time period in $TP_{ik}$, $\delta_{ikt}$ equals to $H$.

$\gamma_{ikt}$ : the total demand of retailer $i$ in the periods from $t$ to $\delta_{ikt}$, $i \in N^r$, $k \in K_i$, and $t \in TP_{ik}$.

$$\pi_{rt} : \begin{cases} 1 & \text{if the product is made in period } t \text{ according to setup configuration } r, r \in Ps \\ 0 & \text{otherwise} \end{cases}$$

Additionally, for each $i \in N^r$, $k \in K_i$, and $t \in T$, let $g_{ikt}$ be one if time period $t$ is covered by delivery schedule $k$, and zero otherwise. Let $\phi_r$, $r \in Ps$, be one if setup configuration $r$ is selected, and zero otherwise, and let $\psi_{ik}$ be one if the delivery schedule $k \in K_i$ is selected for retailer $i \in N^r$. The restricted formulation solved in this phase can be formulated as follows:

$$Min\, Z = \sum_{t \in T} (u.p_t + f.y_t + \sum_{i \in N} h_i.I_{it} + \sum_{i \in N} \sum_{j \in \alpha_i} c_{ij}.x_{ijt} + \sum_{i \in N} \sum_{j \in N^r} b_{ij}.w_{ijt}) \qquad (5.26)$$

subject to (5.2) – (5.6), (5.8), (5.13), (5.24)

$$\sum_{r \in Ps} \phi_r = 1 \qquad (5.27)$$

$$y_t = \sum_{r \in Ps} \phi_r.\pi_{rt}, \forall t \in T \qquad (5.28)$$

$$\sum_{j \in \alpha_0} x_{0jt} = z_{0t}, \forall t \in T \qquad (5.29)$$

$$\sum_{j \in \beta_0} x_{j0t} = z_{0t}, \forall t \in T \qquad (5.30)$$

$$\sum_{j \in \alpha_i} x_{ijt} = \sum_{k \in K_i} g_{ikt}.\psi_{ik}, \forall i \in N^r, \forall t \in T \qquad (5.31)$$

$$\sum_{j \in \beta_i} x_{jit} = \sum_{k \in K_i} g_{ikt}.\psi_{ik}, \forall i \in N^r, \forall t \in T \qquad (5.32)$$

$$\sum_{k \in K_i} \psi_{ik} = 1, \forall i \in N^r \qquad (5.33)$$

$$q_{it} \le \bar{M}_{it}.\sum_{k \in K_i} g_{ikt}.\psi_{ik}, \forall i \in N^r, \forall t \in T \qquad (5.34)$$

$$I_{it-1} + \sum_{m=t}^{\delta_{ikt}} q_{it} + \sum_{j \in N \setminus \{i\}} \sum_{m=t}^{\delta_{ikt}} w_{jim} - \sum_{j \in N \setminus \{i\}} \sum_{m=t}^{\delta_{ikt}} w_{ijm} \ge \gamma_{ikt}.\psi_{ik}, \forall i \in N^r, \forall k \in K_i, \forall t \in TP_{ik} \qquad (5.35)$$

$$0 \le x_{ijt} \le 1, \forall i \in N, \forall j \in \alpha_i, \forall t \in T \qquad (5.36)$$

$$z_{0t}, y_t \in \{0,1\}, \forall t \in T \qquad (5.37)$$

$$\phi_r \in \{0,1\}, \forall r \in Ps \qquad (5.38)$$

$$\psi_{ik} \in \{0,1\}, \forall i \in N^r, \forall k \in K_i \qquad (5.39)$$

The objective function (5.26) is identical to (5.1) except for a minor change in the sum of the transportation costs. Constraint (5.27) guarantees the selection of one production setup configuration among the set *Ps*. Constraint set (5.28) imposes that the product is made only in the periods corresponding to the selected setup configuration. Constraint sets from (5.29) – (5.32) are the a priori tour constraints for the plant ((5.29) and (5.30)) and the retailers ((5.31) and (5.32)), respectively. These constraints enforce that if retailer $i \in N^r$ is visited in period $t \in T$, then it will be visited in the order imposed by the a priori tour. Note that the $z_{it}$ variables for the retailers in the original formulation are replaced with the $\psi_{ik}$ variables. Constraint sets (5.33) and

(5.34) are respectively the counterparts of the constraint sets (5.19) – (5.20) and (5.25) from the *LSM* formulation. Finally, constraint sets from (5.36) – (5.39) define the nature of the decision variables.

The presented restricted model is further tightened by employing the following inequalities:

$$x_{ijt} \leq z_{0t}, \forall i \in N^r, \forall j \in \alpha_i, \forall t \in T \tag{5.40}$$

The constraint set (5.40) is originally proposed in Archetti et al. (2007) for the IRP. It is also valid for the PRPT. In addition to (5.40), we eliminate the number of possible transshipments by utilizing the ideas proposed in Lefever et al. (2018). Let *SB* be a set of binaries ($i, j$) for all $i \in N^r$ and $j \in N^r$ such that $h_i + b_{0j} \leq h_0 + b_{ij}$. We also let *STR* as a set of triplets ($i$, $j$, $t$) for all $i \in N^r$, $j \in N^r$, and $t \in T$ such that $h_0 \leq h_i$, $b_{0j} \leq b_{ij}$, and $\sum_{s=1}^{t} d_{is} \geq I_{i0}$. The following inequalities are added to the formulation:

$$\sum_{(i,j)\in SB} \sum_{t\in T} w_{ijt} = 0 \tag{5.41}$$

$$\sum_{(i,j,t)\in STR} w_{ijt} = 0 \tag{5.42}$$

When $h_i + b_{0j} \leq h_0 + b_{ij}$, it is more or equally expansive to store a unit in the inventory of the plant and to transship a unit from retailer $i$ to retailer $j$ than to store a unit in the inventory of retailer $i$ and to transship a unit from the plant to $j$. Therefore, constraint (5.41) eliminates the transshipment variable $w_{ijt}$ for each $(i, j) \in SB$ and $t \in T$. Constraint (5.42) is based on the same idea, but it is only valid when the initial inventory is exhausted. Note that the ideas employed in constraints (5.41) and (5.42) are proposed for the IRP with unlimited capacity in the storage area of the plant. Although these constraints are not always valid in the cases with limited storage capacity in the plant, we add them to the formulation. Because these constraints significantly reduce the complexity on the transshipment decisions. Besides, it has been observed in preliminary experiments that the quality of the obtained solutions are not much affected by the utilization of these constraints.With these new constraints, the formulation solved in the second phase (*SP*) consists of the equations (5.2) – (5.6), (5.8), (5.13), (5.24), and (5.26) – (5.42). After *SP* is solved, the vehicle route is determined for each period by applying our ILS algorithm.

## 5.4 Computational Study

Our algorithm and the mathematical model were coded in C++ using Microsoft Visual Studio 2015. We use the callable library IBM ILOG CPLEX 12.8 with default settings to solve the formulations. All experiments were executed on a 4.00 GHz intel core i5-760X computer with 32 GB RAM.

### *5.4.1 Test Instances*

To evaluate the performance of our MPH, we have used the single-vehicle PRP instance set A1 presented in Archetti et al. (2011). Moreover, we have adapted a subset of generated PRP-VSP instances introduced in Chapter 3 to the PRPT.

As mentioned in Chapter 3, The PRP-VSP instances in the set S are categorized into four classes. In this study, we only adapt the 56 instances in the first class to the PRPT. For these instances, the visit windows are not applied for the retailers. The set of adapted test instances are denoted by S-PRPT. The set A1 comprises 96 instance types and five instances per type, for a total of 480 instances. In this study, we have utilized the first instance for each problem type resulting in 96 test problems for the PRPT. We call the set of utilized test instances as set A-PRPT.

All test instances have been adapted to the PRPT by employing a transshipment cost value $b_{ij}$ for each $i \in N$ and $j \in N^r \setminus \{i\}$. For all instances, each $b_{ij}$ value is set to $c_{ij} \times 0.1$. Table 2 gives an overview of the utilized benchmark instances.

### *5.4.2 Computational Results*

The proposed MPH is executed five times for each problem instance. After preliminary tests, the values of parameters *maxStart*, *maxIter*, and *pIter* are set to five,

$(n \times H) / 2$, and $\lceil n / 2 \rceil$, respectively. The executions of the model and MPH have been limited to 3600 CPU seconds.

#### *5.4.2.1 Computational Results on the Instance Set S-PRPT*

The computational results obtained for the set S have been reported in Tables 5.2 and 5.3. In the tables, the problem specifications including the number of retailers and the number of periods for each instances are given in the second and the third columns, respectively. In the fourth through the sixth columns, the results of CPLEX, i.e., the lower bound (L.B.), the upper bound (U.B.), and the computational time, are reported for each problem instance, respectively. For MPH, we report the best solution, average solution and the average computation time obtained from each instance.

When the results for the set S-PRPT is analyzed, it is seen that the model can provide optimal solutions for only five out of 56 instances. For each instance where the optimality is confirmed, MPH has also obtained the optimal result. When the average gap percentages with respect to the lower bounds are compared, it has been observed that MPH outperforms CPLEX. The average gaps of MPH and CPLEX for all instances are found as 5.99% and 80.03%, respectively. In addition, MPH and CPLEX have produced the same solution in 10 instances. In the other 46 instances, MPH outperforms CPLEX in 44 out of 46 instances. It is also worth to remark that the average computational times of MPH (1060 seconds) is far less than that of CPLEX (3287.8 seconds). These results indicate the superiority of MPH over CPLEX in the set S-PRPT.

#### *5.4.2.2 Computational Results on the Instance Set A-PRPT*

Summary of the computational results obtained for the set A-PRPT is given in Tables 5.4. Moreover, detailed results of MPH and CPLEX for the problem set are reported in Tables D1 and D2 in Appendix D. In Table 5.4, for each problem class, the average gaps with respect to the lower bounds and the average computational times of both methods have been presented. From the table, it has been observed that our MPH has provided better results than CPLEX in terms of the overall average gaps with respect to the lower bounds. The average gap percentage of MPH and CPLEX are 1.70 and 1.81, respectively. Besides, CPLEX has found the optimal solution on 41 out of 96 instances. For the instances where the optimality is confirmed, MPH has reached the optimal solution on 22 out of 41 instances. Although MPH cannot provide the optimal solution for 19 instances, the average gap of MA with respect to the optimal results is only 0.04%. Moreover, for the 41 instances with a known optimal solution, the computational times spent by CPLEX for finding the optimal solutions are averaged around 202.8 seconds, while the average computational time of MPH for these instances is only 9.4 seconds. Furthermore, the overall average computational time of MPH is far less than that of CPLEX as observed for the set S.

Table 5.2 Computational results obtained for the first and second class instances in the set S-PRPT

| | | | CPLEX | | | MPH | | |
|---|---|---|---|---|---|---|---|---|
| Ins. | *n* | *H* | L.B. | U.B. | t.(s) | Min. | Avg. | t.(s) |
| 1 | 10 | 10 | 24286.5 | 24519.0* | 3600 | 24519.0 | 24853.3 | 7 |
| 2 | 15 | 10 | 31129.6 | 32361.5* | 3600 | 32361.5 | 32614.0 | 10 |
| 3 | 20 | 10 | 40223.9 | 42385.5* | 3600 | 42417.0 | 42443.6 | 15 |
| 4 | 35 | 10 | 49000.6 | 52848.0* | 3600 | 51431.0 | 51728.2 | 48 |
| 5 | 50 | 10 | 82092.8 | 103811.0* | 3600 | 87825.5 | 88108.0 | 279 |
| 6 | 75 | 10 | 119400.0 | 234535.0* | 3600 | 124951.0 | 125291.4 | 632 |
| 7 | 100 | 10 | 111187.0 | 478875.0* | 3600 | 118852.0 | 119690.0 | 1050 |
| 8 | 10 | 20 | 44495.4 | 45487.0* | 3600 | 45426.5 | 45719.0 | 70 |
| 9 | 15 | 20 | 77643.3 | 79496.5* | 3600 | 79689.0 | 80003.7 | 131 |
| 10 | 20 | 20 | 79310.0 | 84721.5* | 3600 | 83272.5 | 83603.8 | 230 |
| 11 | 35 | 20 | 60440.0 | 99173.5* | 3600 | 70325.0 | 71973.9 | 1441 |
| 12 | 50 | 20 | 127424.0 | 220700.0* | 3600 | 141917.0 | 142491.9 | 3600 |
| 13 | 75 | 20 | 244735.0 | 948181.0* | 3600 | 267249.0 | 269955.8 | 3600 |
| 14 | 100 | 20 | 233109.0 | 757990.0* | 3600 | 252065.0 | 254948.8 | 3600 |
| 15 | 10 | 10 | 15002.5 | 15004.0 | 167 | 15004.0 | 15140.7 | 7 |
| 16 | 15 | 10 | 23891.2 | 24687.5* | 3600 | 24670.5 | 24784.6 | 10 |
| 17 | 20 | 10 | 46999.7 | 47689.0* | 3600 | 47569.0 | 47691.4 | 15 |
| 18 | 35 | 10 | 76693.9 | 78640.5* | 3600 | 78381.5 | 78869.9 | 41 |
| 19 | 50 | 10 | 62618.4 | 72960.5* | 3600 | 66185.5 | 66393.8 | 257 |
| 20 | 75 | 10 | 153601.0 | 166693.0* | 3600 | 159809.0 | 160178.4 | 628 |
| 21 | 100 | 10 | 113158.0 | 384549.0* | 3600 | 121633.0 | 122694.5 | 1120 |
| 22 | 10 | 20 | 35805.3 | 36366.5* | 3600 | 36188.5 | 36380.4 | 61 |
| 23 | 15 | 20 | 49955.8 | 53193.0* | 3600 | 52717.5 | 53040.1 | 123 |
| 24 | 20 | 20 | 74726.8 | 86446.0* | 3600 | 82738.5 | 83124.1 | 236 |
| 25 | 35 | 20 | 124717.0 | 146890.0* | 3600 | 132280.0 | 132793.2 | 1488 |
| 26 | 50 | 20 | 130155.0 | 307589.0* | 3600 | 145075.0 | 146141.0 | 3600 |
| 27 | 75 | 20 | 244005.0 | 687557.0* | 3600 | 264101.0 | 265482.9 | 3600 |
| 28 | 100 | 20 | 228274.0 | 821967.0* | 3600 | 247515.0 | 250217.2 | 3600 |

*Notes.* L.B., lower bound, U.B., upper bound,
* optimality is not confirmed after the specified CPU time

Table 5.3 Computational results obtained for the third and fourth class instances in the set S-PRPT

| | | | CPLEX | | | MPH | | |
|---|---|---|---|---|---|---|---|---|
| Ins. | *n* | *H* | L.B. | U.B. | t.(s) | Min. | Avg. | t.(s) |
| 29 | 10 | 10 | 18346.3 | 18348.0 | 124 | 18348.0 | 18512.8 | 7 |
| 30 | 15 | 10 | 38876.2 | 38880.0 | 45 | 38880.0 | 39132.7 | 10 |
| 31 | 20 | 10 | 20906.7 | 21479.0* | 3600 | 21665.5 | 21820.0 | 16 |
| 32 | 35 | 10 | 76825.7 | 83020.5* | 3600 | 79679.5 | 79945.4 | 44 |
| 33 | 50 | 10 | 59589.1 | 70873.5* | 3600 | 64469.0 | 64979.3 | 299 |
| 34 | 75 | 10 | 152586.0 | 203927.0* | 3600 | 159642.0 | 159919.4 | 686 |
| 35 | 100 | 10 | 112231.0 | 353528.0* | 3600 | 119633.0 | 120912.8 | 1200 |
| 36 | 10 | 20 | 46711.9 | 48043.0* | 3600 | 47771.5 | 48332.1 | 54 |
| 37 | 15 | 20 | 79483.7 | 81163.5* | 3600 | 81241.5 | 81438.3 | 114 |
| 38 | 20 | 20 | 75315.6 | 79917.5* | 3600 | 78863.0 | 79007.6 | 250 |
| 39 | 35 | 20 | 94953.1 | 113590.0* | 3600 | 105234.0 | 105693.4 | 1351 |
| 40 | 50 | 20 | 173224.0 | 390912.0* | 3600 | 189119.0 | 190811.7 | 3600 |
| 41 | 75 | 20 | 118074.0 | 453020.0* | 3600 | 134616.0 | 135730.2 | 3600 |
| 42 | 100 | 20 | 142683.0 | 933044.0* | 3600 | 163505.0 | 164654.9 | 3600 |
| 43 | 10 | 10 | 15130.0 | 15131.5 | 77 | 15131.5 | 15326.3 | 5 |
| 44 | 15 | 10 | 24483.1 | 24485.5 | 103 | 24485.5 | 24604.2 | 10 |
| 45 | 20 | 10 | 41224.0 | 41718.5* | 3600 | 41718.5 | 41957.9 | 11 |
| 46 | 35 | 10 | 60854.0 | 64983.0* | 3600 | 62772.5 | 63504.1 | 44 |
| 47 | 50 | 10 | 39778.4 | 59438.0* | 3600 | 44942.0 | 45236.2 | 279 |
| 48 | 75 | 10 | 87285.8 | 184274.0* | 3600 | 92674.5 | 93120.3 | 828 |
| 49 | 100 | 10 | 152790.0 | 430824.0* | 3600 | 161085.0 | 162929.4 | 1103 |
| 50 | 10 | 20 | 35985.8 | 36531.5* | 3600 | 36334.0 | 36741.9 | 62 |
| 51 | 15 | 20 | 51164.0 | 55131.0* | 3600 | 55031.0 | 55168.2 | 113 |
| 52 | 20 | 20 | 62072.8 | 64933.5* | 3600 | 64732.0 | 65225.0 | 246 |
| 53 | 35 | 20 | 93358.9 | 126626.0* | 3600 | 104643.0 | 105294.2 | 1564 |
| 54 | 50 | 20 | 127891.0 | 366364.0* | 3600 | 144126.0 | 145169.8 | 3600 |
| 55 | 75 | 20 | 115471.0 | 553781.0* | 3600 | 130227.0 | 130863.6 | 3600 |
| 56 | 100 | 20 | 405663.0 | 1290940.0* | 3600 | 435560.0 | 439988.8 | 3600 |

*Notes.* L.B., lower bound, U.B., upper bound,
* optimality is not confirmed after the specified CPU time

Table 5.4 Summary of the computational results obtained for the set A-PRPT

| | CPLEX | | MPH | |
|---|---|---|---|---|
| Classes | Avg. Gap(%) | Avg. Time (s) | Avg. Gap(%) | Avg. Time (s) |
| Class I | 0.47 | 1858 | 0.48 | 11 |
| Class II | 0.07 | 1714 | 0.07 | 10 |
| Class III | 6.20 | 2942 | 5.71 | 11 |
| Class IV | 0.52 | 2082 | 0.53 | 10 |
| Avg. | 1.81 | 2149 | 1.70 | 10 |

### *5.4.3 Analyzing the Effect of the Transshipments on the PRP*

In order to investigate the effect of allowing transshipments in the PRP, we solve all of the instances in the set A with different transshipment cost values. Additionally, the optimal results for the original PRP instances without transshipment have been reported in Solyalı & Süral (2017), which allows us to compare our obtained results with those obtained for the standard PRP.

The computational results obtained for this section are summarized in Table 5.5. Additionally, the corresponding detailed results are provided in Tables D3 through D6 in Appendix D. In Table 5.5, we report the overall average cost decreases for different transshipment cost scenarios for each problem class. It can be seen from the table that the average overall cost reduction percentage decreases from 2.35 to 0.29 when the transshipment cost $b_{ij}$ increases from $0.1 \times c_{ij}$ to $0.5 \times c_{ij}$. It has also been observed that the cost reduction significantly decreases when $b_{ij}$ increases from $0.1 \times c_{ij}$ to $0.15 \times c_{ij}$. Moreover, it can be seen that the cost reductions obtained for the instances in the Class I are significantly larger than those obtained for the instances in the other classes. Besides, the smallest average cost decrease occurs on the instances belonging to the Class IV. The results indicate the importance of the inventory holding cost values at the retailer locations for the benefits earned by transshipment. It is shown that for the systems where the inventory holding costs at the retailers are important, transshipments can lead to significant cost reductions.

Table 5.5 Average cost decrease (%) with respect to the PRP when $b_{ij} = \rho \times c_{ij}$

| Classes | $\rho$ = 0.10 | $\rho$ = 0.15 | $\rho$ = 0.20 | $\rho$ = 0.25 | $\rho$ = 0.30 | $\rho$ = 0.35 | $\rho$ = 0.40 | $\rho$ = 0.45 | $\rho$ = 0.50 | Avg. |
|---|---|---|---|---|---|---|---|---|---|---|
| Class I | 5.43 | 3.82 | 2.88 | 2.32 | 1.90 | 1.62 | 1.36 | 1.13 | 0.95 | 2.38 |
| Class II | 0.83 | 0.59 | 0.45 | 0.37 | 0.31 | 0.26 | 0.23 | 0.19 | 0.16 | 0.38 |
| Class III | 1.99 | 0.43 | 0.29 | 0.20 | 0.17 | 0.13 | 0.09 | 0.04 | 0.02 | 0.37 |
| Class IV | 1.13 | 0.56 | 0.34 | 0.22 | 0.14 | 0.09 | 0.05 | 0.03 | 0.02 | 0.29 |
| Avg. | 2.35 | 1.35 | 0.99 | 0.78 | 0.63 | 0.52 | 0.43 | 0.35 | 0.29 | 0.85 |

## 5.5 Chapter Summary

In this chapter, the production routing problem (PRP) has been considered with the transshipment options. A mathematical programming model has been developed for the PRP with transshipments (PRPT) based on the models proposed for the PRP. The model has been implemented to test instances and it has been observed that although it can provide high-quality solutions on small-size instances, its performance tumbles as the problem size increases. In order to obtain good solutions for large-size instances, we have developed a two-phase mathematical programming heuristic for the PRPT. Comparative tests on two sets of benchmark instances have shown that our algorithm can provide high-quality solutions to the PRPT in reasonable computing times. Additionally, we perform a numerical investigation to reveal the effect of the transshipments on the overall system cost. The results have indicated that the use of transshipments may lead to significant cost reductions.

# CHAPTER SIX
# CONCLUSION

## 6.1 Summary of the Thesis

In this thesis, we integrate the visit spacing policy (VSP) and the concept of transshipments into the production routing problem (PRP) for the first time. The PRP especially arises in vendor-managed inventory (VMI) systems. The VSP is an essential consistency concept in a VMI system to ensure service quality. On the other hand, the implementation of transshipments provides benefits in a VMI system by reducing the inventory holding costs.

Our study on the PRP with the VSP (PRP-VSP) has been presented in Chapters 3 and 4. In Chapter 3, the PRP-VSP is considered with a single vehicle. Two mathematical formulations are developed for the problem. The developed formulations differ in their way of modeling the VSP. In the first formulation, the VSP is modeled by employing binary decision variables for alternative visit combinations. In the second formulation, the VSP is modeled by implementing new constraint sets to ensure the specified minimum and maximum number of time periods between two consecutive visits for the retailers. The models have been implemented on the generated test problems and it has been observed that they cannot generate feasible solutions to middle and large size instances. Therefore, a matheuristic algorithm (MA) has been proposed to solve the problem. The proposed MA is implemented in a multi-start scheme in which some subproblems are solved exactly. In Chapter 4, our study given in Chapter 3 has been extended by considering multiple vehicles in the PRP-VSP. In order to solve the multi-vehicle PRP-VSP, two variants of MA have been developed. Extensive computational experiments have been performed for both versions of the PRP-VSP to assess the performance of the developed approaches as well as to analyze the effect of the VSP on the PRP.

The study of PRP with transshipment (PRPT) has been given in Chapter 5. Initially, we develop a mathematical formulation for the problem. The computational experiments show that the developed model cannot perform well on middle and large size PRPT instances. Hence, a two-phase mathematical programming heuristic (MPH) has been proposed for the problem. The algorithm has been applied to two sets of randomly generated problem instances. The results indicate the effectiveness of the proposed approach. Moreover, an extensive computational study has been performed to reveal the effect of the transshipments on the overall cost.

### 6.2 Contributions

This thesis makes several contributions to the relevant literature of the PRP. First of all, a consistency concept, the VSP, is investigated within the context of the PRP for the first time. Two different mathematical programming formulations are developed for the PRP-VSP. Besides, a powerful algorithm (MA) is developed to tackle the PRP-VSP. The developed MA can be applied not only to the PRP-VSP but also to the standard PRP and IRP with the VSP. The computational results indicate that the developed MA is effective in solving both the PRP and the IRP with the VSP. Specifically, it has been observed that MA improves several best known solutions of benchmark instances. Additionally, to reveal the effect of the VSP on the overall cost, all generated PRP-VSP instances are solved without imposing the visit spacing constraints. According to the results, it is observed that the VSP results in an increase on the overall cost of between 1.43% and 11.90%, on average. Furthermore, the results indicate that the VSP generally influences the inventory holding costs at the retailers significantly.

Another contribution of this thesis is to integrate the concept of transshipments into the PRP for the first time. A mathematical programming heuristic is proposed to solve the problem. Comparative tests on two sets of benchmark instances show that our algorithm can provide high-quality solutions to the PRPT in reasonable computing times. Additionally, the numerical investigation performed reveals the effect of the transshipments on the overall system cost. The obtained results show that the use of transshipments may lead to significant cost reductions.

### 6.3 Future Research Directions

There exist several future research directions originated from this thesis. Related to our study on the PRP-VSP, the proposed MA can be modified with a new working visit combination set procedure that does not need the generation of all visit combinations for the retailers. Such a procedure enables the algorithm to solve the PRP instances with 20 periods introduced in Boudia et al. (2007). Moreover, other consistency concepts may be handled in the PRP. Especially, the driver consistency can be incorporated into the PRP.

In the further research of our work on the PRPT, it might be interesting to study the stochastic version of the problem. In stochastic environments, the use of transshipments can make the whole system more resistible to the unforeseeable demand variation problem. Moreover, the PRPT can be extended by considering multiple products and multiple vehicles.

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# APPENDICES

## APPENDIX A: An Illustrative Example for the PRP-VSP

In this section, a small PRP-VSP example consisting of 10 retailers and 10 periods is presented along with its optimal solution. We present the coordinate values, inventory parameters, visit windows, and demand values of each node in Table A1. The parameters *C*, *Q*, *f*, and *u* are 769, 615, 3000, and 30, respectively. Moreover, the transportation cost between two locations is five times the Euclidean distance between the locations (The Euclidean distances between the locations are rounded to the nearest integer).

The optimal solution of the problem, obtained by *Model 1*, is sketched in Table A2 and Figures A1, A2, and A3, respectively. The optimal value of the objective function is 29958. In Table A2, for each period, the production quantity, the inventory levels of the nodes, and the delivery quantities of the retailers are given. In the aforementioned figures, vehicle routes obtained for all periods are depicted.

Table A.1 Coordinates of the nodes, inventory parameters, visit windows, and demand values

| | Coordinates | | Inv. Parameters | | | Vis. Wind. | | $d_{it}$, $i \in N^r$, $t \in T$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *Nodes* | *Coor. x* | *Coor. y* | $L_i$ | $h_i$ | $I_{i0}$ | $min_i$ | $max_i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Plant | 83.58 | 96.55 | 1538 | 3 | 0 | - | - | - | - | - | - | - | - | - | - | - | - |
| Ret. 1 | 41.39 | 53.30 | 57 | 2 | 16 | 2 | 2 | 15 | 16 | 12 | 12 | 17 | 23 | 8 | 18 | 23 | 16 |
| Ret. 2 | 3.18 | 0.54 | 46 | 1 | 22 | 1 | 1 | 21 | 19 | 21 | 22 | 24 | 10 | 5 | 7 | 13 | 5 |
| Ret. 3 | 70.83 | 17.50 | 53 | 2 | 23 | 2 | 2 | 11 | 6 | 23 | 21 | 9 | 19 | 9 | 19 | 5 | 22 |
| Ret. 4 | 66.01 | 73.78 | 45 | 2 | 19 | 1 | 1 | 23 | 22 | 17 | 11 | 16 | 19 | 13 | 9 | 16 | 23 |
| Ret. 5 | 22.69 | 31.74 | 58 | 1 | 11 | 2 | 2 | 6 | 8 | 18 | 12 | 6 | 24 | 11 | 23 | 11 | 21 |
| Ret. 6 | 28.85 | 40.83 | 47 | 2 | 16 | 1 | 1 | 8 | 24 | 13 | 10 | 22 | 25 | 13 | 14 | 23 | 22 |
| Ret. 7 | 8.92 | 36.03 | 51 | 4 | 18 | 2 | 2 | 23 | 15 | 9 | 17 | 13 | 21 | 14 | 14 | 6 | 11 |
| Ret. 8 | 82.26 | 14.30 | 62 | 1 | 22 | 2 | 2 | 19 | 25 | 18 | 18 | 22 | 20 | 12 | 21 | 13 | 23 |
| Ret. 9 | 14.33 | 79.26 | 30 | 1 | 14 | 1 | 1 | 16 | 7 | 22 | 8 | 12 | 10 | 12 | 17 | 12 | 9 |
| Ret. 10 | 81.81 | 85.91 | 42 | 4 | 12 | 1 | 1 | 12 | 21 | 8 | 8 | 20 | 17 | 21 | 21 | 9 | 8 |

Table A.2 Optimal production and delivery quantities obtained for the problem

| | | $I_{it}$ ($q_{it}$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Periods | $p_t$ | Plant | Ret. 1 | Ret. 2 | Ret. 3 | Ret. 4 | Ret. 5 | Ret. 6 | Ret. 7 | Ret. 8 | Ret. 9 | Ret. 10 |
| 1 | 364 | 112 | 28(27) | 25(24) | 29(17) | 22(26) | 26(21) | 39(31) | 24(29) | 43(40) | 14(16) | 21(21) |
| 2 | - | 112 | 12 | 6 | 23 | 0 | 18 | 15 | 9 | 18 | 7 | 0 |
| 3 | - | 0 | 0 | 22(37) | 0 | 11(28) | 0 | 10(8) | 0 | 0 | 8(23) | 8(16) |
| 4 | 429 | 175 | 40(52) | 0 | 28(49) | 0 | 30(42) | 0 | 34(51) | 42(60) | 0 | 0 |
| 5 | - | 0 | 23 | 10(34) | 19 | 19(35) | 24 | 25(47) | 21 | 20 | 10(22) | 17(37) |
| 6 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 339 | 0 | 41(49) | 7(12) | 24(33) | 9(22) | 34(45) | 14(27) | 20(34) | 34(46) | 17(29) | 21(42) |
| 8 | - | 0 | 23 | 0 | 5 | 0 | 11 | 0 | 6 | 13 | 0 | 0 |
| 9 | 233 | 93 | 0 | 5(18) | 0 | 23(39) | 0 | 22(45) | 0 | 0 | 9(21) | 8(17) |
| 10 | - | 0 | 0(16) | 0 | 0(22) | 0 | 0(21) | 0 | 0(11) | 0(23) | 0 | 0 |

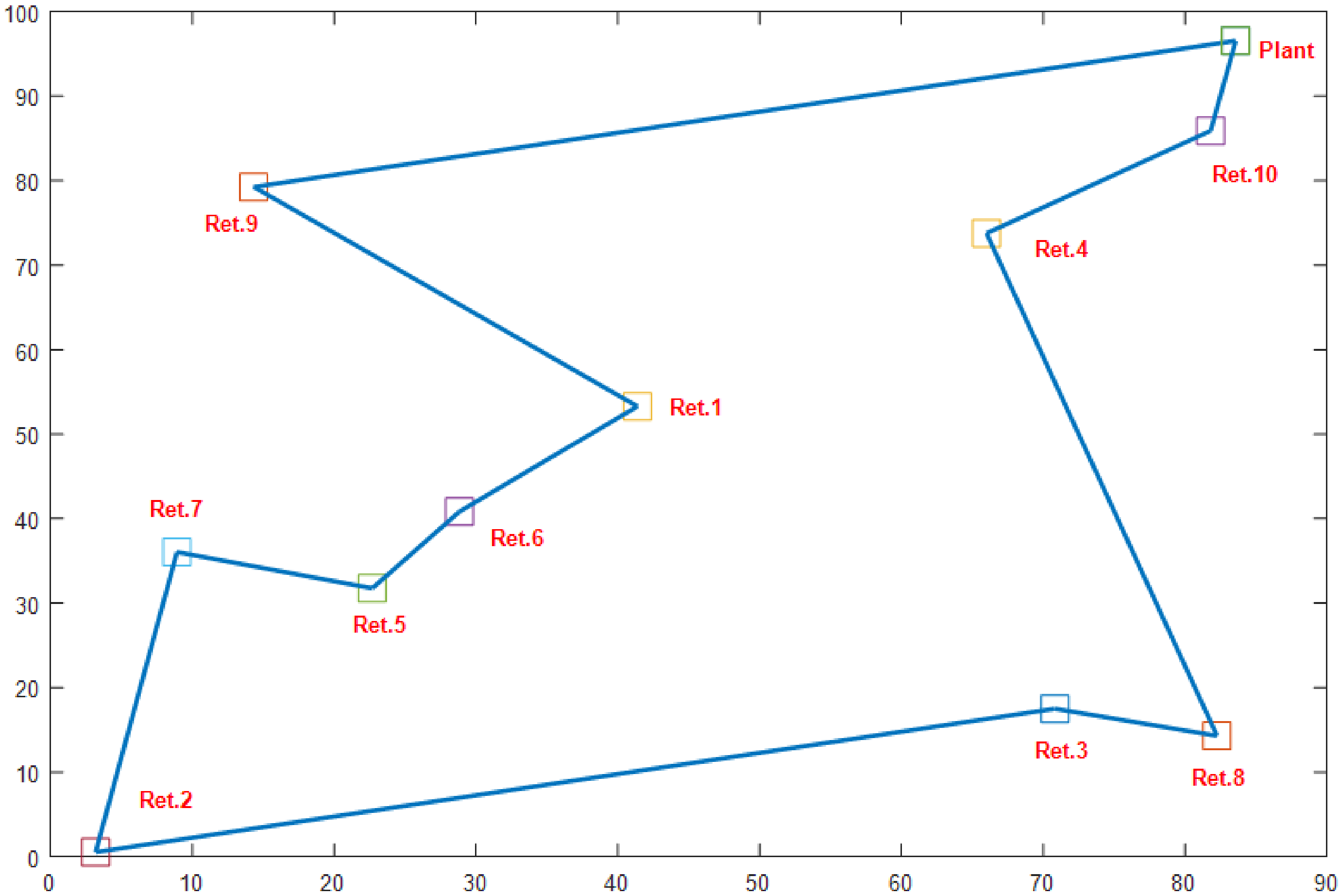

Figure A.1 Vehicle route traced by the vehicle in periods 1 and 8

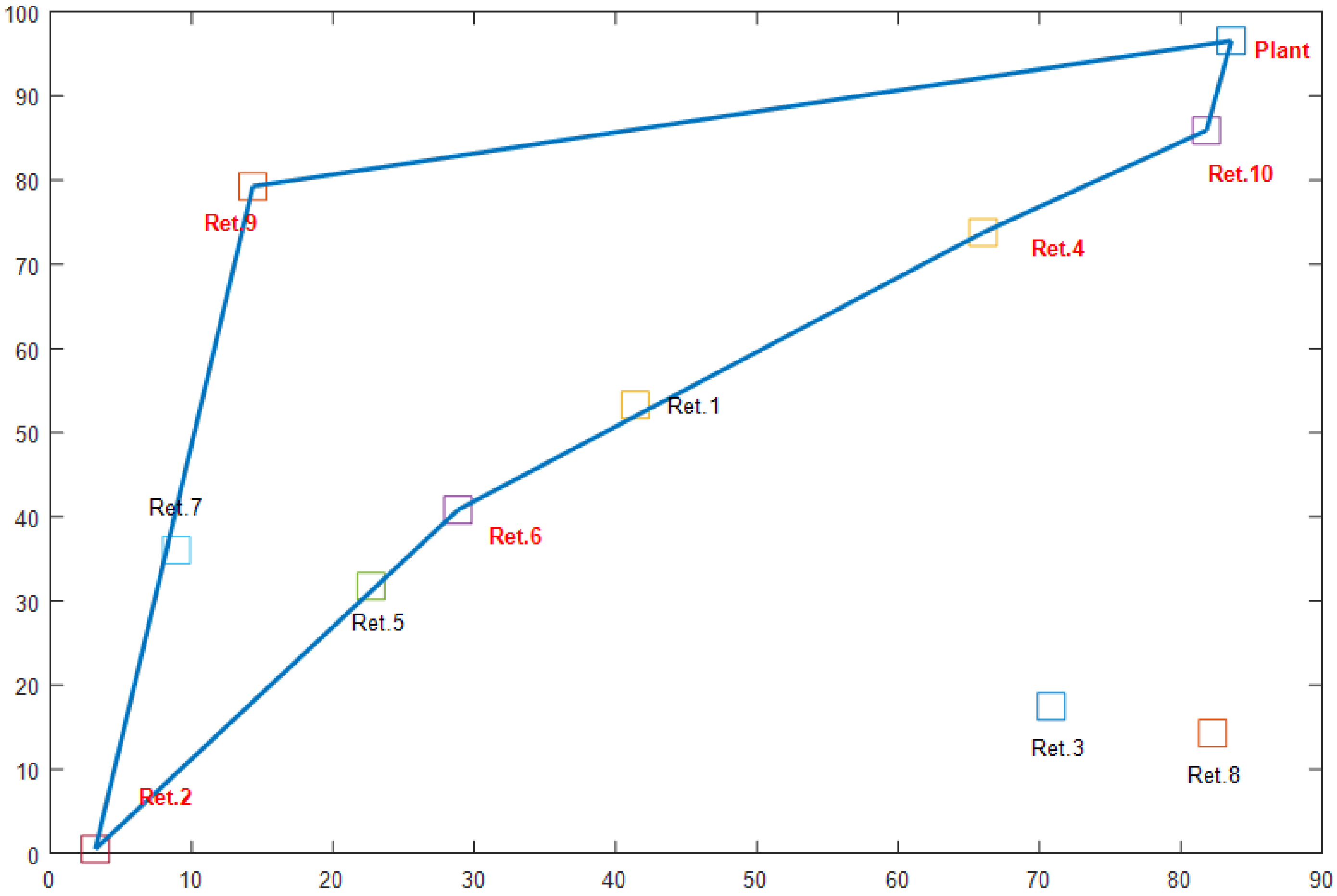


Figure A.2 Vehicle route traced by the vehicle in periods 4 and 10

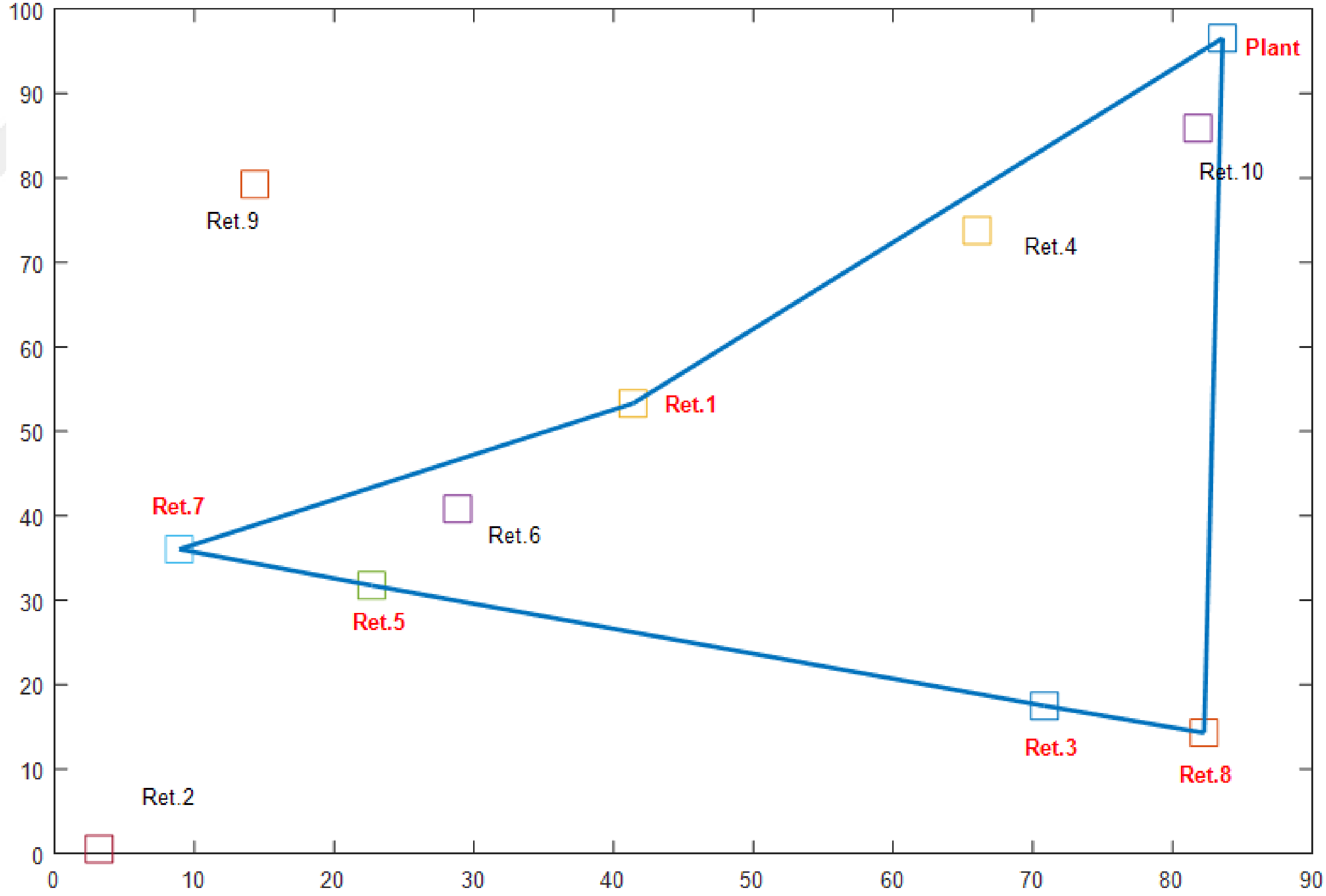


Figure A.3 Vehicle route traced by the vehicle in periods 3, 5, and 9

## APPENDIX B: Detailed Computational Results for Chapter 3

In this section, we provide the detailed results of the computational study presented in Chapter 3. In Tables B1 through B6, the computational results obtained for the sets S1 and S2 are reported, respectively. Each problem instance is labeled as 'X_Y_Z', where the numbers X and Y denote the problem class and VSP type of the instance, respectively. Here, the number Z corresponds to a specific retailer number-period number combination. The problem specifications including the number of retailers and the number of time periods for each problem instance are given in the second and the third columns, respectively. For MA and IM-VSP, we report the best solution, average solution and the average computation time obtained from each instance. Bold numbers indicate the algorithm has reached the best solution.

The detailed cost values obtained by solving the generated PRP-VSP instances with and without VSP are provided in Tables B7 through B14. The objective function values found by MA on the standard PRP instances are given in Table B15. In the table, bold numbers indicate that MA has reached the optimal solution.

Table B.1 Detailed computational results obtained for the set S1

| | | | Model1 | | Model2 | | | IM-VSP | | | | MA | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | Obj. | t(s) | Obj. | t(s) | L.B. | Best | Avg. | t(s) | Gap% | Best | Avg. | t(s) | Gap% |
| 1_1_1 | 10 | 10 | 29958 | 2582 | ** | 3600 | 29958 | **29958** | 30757 | 21 | 0.00 | **29958** | 29958 | 6 | 0.00 |
| 1_1_2 | 15 | 10 | 37837 | 33 | 37837 | 98 | 37837 | **37837** | 37837 | 44 | 0.00 | **37837** | 37837 | 6 | 0.00 |
| 1_1_3 | 20 | 10 | ** | 3600 | ** | 3600 | 46173 | **47423** | 47512 | 50 | 2.71 | **47423** | 47423 | 11 | 2.71 |
| 1_1_8 | 10 | 20 | 51740 | 24 | 51740 | 32 | 51740 | **51740** | 51740 | 47 | 0.00 | **51740** | 51740 | 5 | 0.00 |
| 1_1_9 | 15 | 20 | 93678 | 256 | 93678 | 748 | 93678 | **93678** | 93678 | 111 | 0.00 | **93678** | 93678 | 33 | 0.00 |
| 1_1_10 | 20 | 20 | 98224 | 47 | 98224 | 121 | 98224 | **98224** | 98224 | 133 | 0.00 | **98224** | 98224 | 45 | 0.00 |
| 1_2_1 | 10 | 10 | 17063 | 28 | 17063 | 23 | 17063 | 17258 | 17258 | 55 | 1.14 | **17063** | 17063 | 19 | 0.00 |
| 1_2_2 | 15 | 10 | ** | 3600 | ** | 3600 | 28266 | **28583** | 28676 | 63 | 1.12 | **28583** | 28583 | 21 | 1.12 |
| 1_2_3 | 20 | 10 | 52403 | 1846 | 52403 | 1422 | 52403 | **52403** | 52403 | 281 | 0.00 | **52403** | 52403 | 34 | 0.00 |
| 1_2_8 | 10 | 20 | 41872 | 2144 | 41872 | 1357 | 41872 | **41872** | 41872 | 522 | 0.00 | **41872** | 42017 | 85 | 0.00 |
| 1_2_9 | 15 | 20 | ** | 3600 | ** | 3600 | 61035 | **61335** | 61335 | 769 | 0.49 | **61335** | 61511 | 108 | 0.49 |
| 1_2_10 | 20 | 20 | ** | 3600 | ** | 3600 | 85698 | 92704 | 92704 | 2292 | 8.18 | **92501** | 92640 | 290 | 7.94 |
| 1_3_1 | 10 | 10 | 21002 | 15 | 21002 | 14 | 21002 | **21002** | 21002 | 34 | 0.00 | **21002** | 21002 | 11 | 0.00 |
| 1_3_2 | 15 | 10 | 43580 | 55 | 43580 | 29 | 43580 | **43580** | 43582 | 365 | 0.00 | **43580** | 43582 | 30 | 0.00 |
| 1_3_3 | 20 | 10 | ** | 3600 | ** | 3600 | 24594 | 24975 | 25076 | 135 | 1.55 | **24806** | 24862 | 59 | 0.86 |
| 1_3_8 | 10 | 20 | 52794 | 2213 | 52794 | 1852 | 52794 | **52794** | 52879 | 915 | 0.00 | **52794** | 53011 | 64 | 0.00 |
| 1_3_9 | 15 | 20 | ** | 3600 | ** | 3600 | 86276 | **88598** | 88598 | 2686 | 2.69 | **88598** | 88882 | 189 | 2.69 |
| 1_3_10 | 20 | 20 | ** | 3600 | ** | 3600 | 81983 | 86430 | 86446 | 3600 | 5.42 | **86145** | 86367 | 472 | 5.08 |
| 1_4_1 | 10 | 10 | 15647 | 8 | 15647 | 8 | 15647 | 16178 | 16178 | 40 | 3.39 | **15647** | 15657 | 13 | 0.00 |
| 1_4_2 | 15 | 10 | 28128 | 2388 | 28128 | 1806 | 28128 | 28307 | 28307 | 161 | 0.64 | **28128** | 28128 | 34 | 0.00 |
| 1_4_3 | 20 | 10 | ** | 3600 | ** | 3600 | 44887 | **45103** | 45103 | 515 | 0.48 | **45103** | 45103 | 50 | 0.48 |
| 1_4_8 | 10 | 20 | ** | 3600 | 38783 | 1614 | 38783 | **38783** | 38823 | 253 | 0.00 | **38783** | 39087 | 113 | 0.00 |
| 1_4_9 | 15 | 20 | ** | 3600 | ** | 3600 | 57201 | **58538** | 58906 | 968 | 2.34 | 58968 | 59304 | 317 | 3.09 |
| 1_4_10 | 20 | 20 | ** | 3600 | ** | 3600 | 68110 | **70630** | 70999 | 2201 | 3.70 | 71122 | 71325 | 526 | 4.42 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, L.B., lower bound, Gap% = ((Best Sol. - L.B.) / L.B.) × 100
** no feasible solution found

Table B.2 Detailed computational results obtained for the set S1 (cont.)

| | | | Model1 | | Model2 | | | IM-VSP | | | | MA | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | Obj. | t(s) | Obj. | t(s) | L.B. | Best | Avg. | t(s) | Gap% | Best | Avg. | t(s) | Gap% |
| 2_1_1 | 10 | 10 | 31157 | 3548 | 31157 | 3375 | 31157 | **31157** | 31157 | 34 | 0.00 | **31157** | 31157 | 10 | 0.00 |
| 2_1_2 | 15 | 10 | 39962 | 41 | 39962 | 61 | 39962 | **39962** | 40097 | 81 | 0.00 | **39962** | 39962 | 20 | 0.00 |
| 2_1_3 | 20 | 10 | ** | 3600 | ** | 3600 | 47075 | **47930** | 48225 | 87 | 1.82 | **47930** | 47930 | 29 | 1.82 |
| 2_1_8 | 10 | 20 | 57293 | 30 | 57293 | 29 | 57293 | **57293** | 57293 | 75 | 0.00 | **57293** | 57293 | 8 | 0.00 |
| 2_1_9 | 15 | 20 | 94770 | 109 | 94770 | 137 | 94770 | **94770** | 94770 | 160 | 0.00 | **94770** | 94770 | 33 | 0.00 |
| 2_1_10 | 20 | 20 | 101184 | 1711 | ** | 3600 | 101184 | **101184** | 101184 | 238 | 0.00 | **101184** | 101184 | 53 | 0.00 |
| 2_2_1 | 10 | 10 | 17173 | 46 | 17173 | 33 | 17173 | **17173** | 17242 | 81 | 0.00 | **17173** | 17207 | 20 | 0.00 |
| 2_2_2 | 15 | 10 | ** | 3600 | ** | 3600 | 28545 | **28954** | 29074 | 120 | 1.43 | **28954** | 28954 | 27 | 1.43 |
| 2_2_3 | 20 | 10 | ** | 3600 | ** | 3600 | 54084 | **54370** | 54781 | 607 | 0.53 | **54370** | 54405 | 47 | 0.53 |
| 2_2_8 | 10 | 20 | ** | 3600 | ** | 3600 | 43097 | **43431** | 43431 | 901 | 0.77 | 43456 | 43537 | 78 | 0.83 |
| 2_2_9 | 15 | 20 | ** | 3600 | ** | 3600 | 60601 | **61914** | 61914 | 895 | 2.17 | **61914** | 62046 | 124 | 2.17 |
| 2_2_10 | 20 | 20 | ** | 3600 | ** | 3600 | 87575 | 96138 | 96373 | 3600 | 9.78 | **96050** | 96159 | 409 | 9.68 |
| 2_3_1 | 10 | 10 | 22229 | 220 | 22229 | 104 | 22229 | **22229** | 22229 | 80 | 0.00 | **22229** | 22229 | 18 | 0.00 |
| 2_3_2 | 15 | 10 | 44428 | 514 | 44428 | 187 | 44428 | 44444 | 44444 | 256 | 0.04 | **44428** | 44428 | 32 | 0.00 |
| 2_3_3 | 20 | 10 | ** | 3600 | ** | 3600 | 24720 | 25100 | 25394 | 194 | 1.54 | **24859** | 24859 | 55 | 0.56 |
| 2_3_8 | 10 | 20 | ** | 3600 | ** | 3600 | 54631 | **55482** | 55705 | 779 | 1.56 | **55482** | 55593 | 81 | 1.56 |
| 2_3_9 | 15 | 20 | ** | 3600 | ** | 3600 | 88498 | **92070** | 92460 | 1364 | 4.04 | **92082** | 92264 | 224 | 4.05 |
| 2_3_10 | 20 | 20 | ** | 3600 | ** | 3600 | 83184 | 86843 | 86884 | 1360 | 4.40 | **86347** | 86758 | 505 | 3.80 |
| 2_4_1 | 10 | 10 | 16570 | 42 | 16570 | 39 | 16570 | 16829 | 17181 | 58 | 1.56 | **16570** | 16689 | 17 | 0.00 |
| 2_4_2 | 15 | 10 | ** | 3600 | ** | 3600 | 28637 | **28744** | 28905 | 299 | 0.37 | 28798 | 28820 | 35 | 0.56 |
| 2_4_3 | 20 | 10 | ** | 3600 | ** | 3600 | 45995 | **46467** | 46467 | 562 | 1.03 | **46467** | 46467 | 41 | 1.03 |
| 2_4_8 | 10 | 20 | ** | 3600 | ** | 3600 | 41507 | 42993 | 43055 | 741 | 3.58 | **42736** | 42958 | 228 | 2.96 |
| 2_4_9 | 15 | 20 | ** | 3600 | ** | 3600 | 59034 | 60064 | 60568 | 1359 | 1.74 | **59800** | 60098 | 327 | 1.30 |
| 2_4_10 | 20 | 20 | ** | 3600 | ** | 3600 | 68822 | 72550 | 72700 | 1355 | 5.42 | **71864** | 72190 | 736 | 4.42 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, L.B., lower bound, Gap% = ((Best Sol. - L.B.) / L.B.) × 100

** no feasible solution found

Table B.3 Detailed computational results obtained for the set S1 (cont.)

| | | | Model1 | | Model2 | | | IM-VSP | | | | MA | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | Obj. | t(s) | Obj. | t(s) | L.B. | Best | Avg. | t(s) | Gap% | Best | Avg. | t(s) | Gap% |
| 3_1_1 | 10 | 10 | ** | 3600 | ** | 3600 | 65321 | 66571 | 66571 | 52 | 1.91 | **65421** | 65421 | 7 | 0.15 |
| 3_1_2 | 15 | 10 | 87055 | 1275 | 87055 | 264 | 87055 | **87055** | 87055 | 110 | 0.00 | **87055** | 87055 | 11 | 0.00 |
| 3_1_3 | 20 | 10 | ** | 3600 | ** | 3600 | 111973 | **113288** | 113288 | 112 | 1.17 | **113288** | 113288 | 5 | 1.17 |
| 3_1_8 | 10 | 20 | 123005 | 91 | 123005 | 87 | 123005 | **123005** | 123005 | 104 | 0.00 | **123005** | 123005 | 5 | 0.00 |
| 3_1_9 | 15 | 20 | 224681 | 122 | 224681 | 126 | 224681 | **224681** | 224681 | 159 | 0.00 | **224681** | 224681 | 32 | 0.00 |
| 3_1_10 | 20 | 20 | 232071 | 155 | 232071 | 315 | 232071 | **232071** | 232071 | 292 | 0.00 | **232071** | 232071 | 52 | 0.00 |
| 3_2_1 | 10 | 10 | 29327 | 59 | 29327 | 29 | 29327 | 29383 | 29383 | 106 | 0.19 | **29327** | 29327 | 20 | 0.00 |
| 3_2_2 | 15 | 10 | ** | 3600 | ** | 3600 | 61042 | **61391** | 61391 | 103 | 0.57 | **61391** | 61391 | 19 | 0.57 |
| 3_2_3 | 20 | 10 | ** | 3600 | ** | 3600 | 134974 | **135037** | 135037 | 377 | 0.05 | **135037** | 135037 | 31 | 0.05 |
| 3_2_8 | 10 | 20 | 87596 | 654 | 87596 | 433 | 87596 | **87596** | 87596 | 980 | 0.00 | **87596** | 87596 | 49 | 0.00 |
| 3_2_9 | 15 | 20 | 129482 | 2495 | 129482 | 2421 | 129482 | **129482** | 129482 | 2010 | 0.00 | **129482** | 129618 | 139 | 0.00 |
| 3_2_10 | 20 | 20 | ** | 3600 | ** | 3600 | 218407 | **223366** | 223366 | 3600 | 2.27 | 223391 | 223406 | 265 | 2.28 |
| 3_3_1 | 10 | 10 | 44854 | 4 | 44854 | 5 | 44854 | **44854** | 44854 | 69 | 0.00 | **44854** | 44854 | 13 | 0.00 |
| 3_3_2 | 15 | 10 | 109844 | 42 | 109844 | 36 | 109844 | **109844** | 109844 | 422 | 0.00 | **109844** | 109844 | 26 | 0.00 |
| 3_3_3 | 20 | 10 | ** | 3600 | ** | 3600 | 46196 | **46306** | 46441 | 572 | 0.24 | 46371 | 46504 | 53 | 0.38 |
| 3_3_8 | 10 | 20 | 121191 | 2038 | 121191 | 1162 | 121191 | **121191** | 121337 | 2019 | 0.00 | **121191** | 121398 | 100 | 0.00 |
| 3_3_9 | 15 | 20 | ** | 3600 | ** | 3600 | 221971 | **222130** | 222130 | 3600 | 0.07 | **222130** | 222199 | 195 | 0.07 |
| 3_3_10 | 20 | 20 | ** | 3600 | ** | 3600 | 213149 | **214892** | 214996 | 3600 | 0.82 | 214914 | 214966 | 544 | 0.83 |
| 3_4_1 | 10 | 10 | 27176 | 8 | 27176 | 7 | 27176 | **27176** | 27527 | 74 | 0.00 | **27176** | 27176 | 15 | 0.00 |
| 3_4_2 | 15 | 10 | 59222 | 702 | 59222 | 434 | 59222 | **59222** | 59292 | 667 | 0.00 | **59222** | 59222 | 27 | 0.00 |
| 3_4_3 | 20 | 10 | ** | 3600 | ** | 3600 | 110106 | 110399 | 110399 | 2251 | 0.27 | **110374** | 110725 | 41 | 0.24 |
| 3_4_8 | 10 | 20 | ** | 3600 | 85804 | 663 | 85804 | **85804** | 85804 | 392 | 0.00 | 85808 | 85937 | 135 | 0.00 |
| 3_4_9 | 15 | 20 | ** | 3600 | ** | 3600 | 127785 | 128459 | 128854 | 3600 | 0.53 | **128387** | 128425 | 315 | 0.47 |
| 3_4_10 | 20 | 20 | ** | 3600 | ** | 3600 | 156122 | **157863** | 158031 | 3600 | 1.12 | 157881 | 158051 | 588 | 1.13 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, L.B., lower bound, Gap% = ((Best Sol. - L.B.) / L.B.) × 100

** no feasible solution found

Table B.4 Detailed computational results obtained for the set S1 (cont.)

| | | | Model1 | | Model2 | | | IM-VSP | | | | MA | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | Obj. | t(s) | Obj. | t(s) | L.B. | Best | Avg. | t(s) | Gap% | Best | Avg. | t(s) | Gap% |
| 4_1_1 | 10 | 10 | ** | 3600 | ** | 3600 | 27547 | **27627** | 28462 | 20 | 0.29 | **27627** | 27627 | 4 | 0.29 |
| 4_1_2 | 15 | 10 | 33477 | 33 | 33477 | 71 | 33477 | **33477** | 33477 | 40 | 0.00 | **33477** | 33477 | 6 | 0.00 |
| 4_1_3 | 20 | 10 | ** | 3600 | ** | 3600 | 40888 | **42159** | 42159 | 48 | 3.11 | **42159** | 42159 | 7 | 3.11 |
| 4_1_8 | 10 | 20 | 43957 | 50 | 43957 | 48 | 43957 | **43957** | 43957 | 44 | 0.00 | **43957** | 43957 | 4 | 0.00 |
| 4_1_9 | 15 | 20 | 78464 | 160 | 78464 | 334 | 78464 | **78464** | 78464 | 133 | 0.00 | **78464** | 78464 | 34 | 0.00 |
| 4_1_10 | 20 | 20 | 80461 | 74 | ** | 3600 | 80461 | **80461** | 80461 | 112 | 0.00 | **80461** | 80461 | 46 | 0.00 |
| 4_2_1 | 10 | 10 | 14264 | 175 | 14264 | 142 | 14264 | **14264** | 14264 | 44 | 0.00 | **14264** | 14264 | 22 | 0.00 |
| 4_2_2 | 15 | 10 | ** | 3600 | ** | 3600 | 23395 | **23777** | 23865 | 58 | 1.63 | **23777** | 23788 | 21 | 1.63 |
| 4_2_3 | 20 | 10 | ** | 3600 | ** | 3600 | 45588 | **45763** | 45763 | 829 | 0.38 | **45763** | 45763 | 31 | 0.38 |
| 4_2_8 | 10 | 20 | ** | 3600 | 31690 | 2872 | 31690 | **31690** | 31764 | 1202 | 0.00 | **31690** | 31702 | 74 | 0.00 |
| 4_2_9 | 15 | 20 | ** | 3600 | ** | 3600 | 45761 | **46838** | 46838 | 1218 | 2.35 | **46838** | 46838 | 123 | 2.35 |
| 4_2_10 | 20 | 20 | ** | 3600 | ** | 3600 | 67672 | **73634** | 73642 | 3600 | 8.81 | **73634** | 73888 | 374 | 8.81 |
| 4_3_1 | 10 | 10 | 18002 | 32 | 18002 | 21 | 18002 | **18002** | 18002 | 37 | 0.00 | **18002** | 18002 | 12 | 0.00 |
| 4_3_2 | 15 | 10 | 38670 | 108 | 38670 | 3329 | 38670 | **38670** | 38670 | 379 | 0.00 | **38670** | 38670 | 26 | 0.00 |
| 4_3_3 | 20 | 10 | ** | 3600 | ** | 3600 | 18099 | 18547 | 18554 | 362 | 2.48 | **18209** | 18276 | 41 | 0.61 |
| 4_3_8 | 10 | 20 | ** | 3600 | 40650 | 3428 | 40650 | **40650** | 40650 | 748 | 0.00 | **40650** | 40661 | 64 | 0.00 |
| 4_3_9 | 15 | 20 | ** | 3600 | ** | 3600 | 72134 | **73078** | 73078 | 3600 | 1.31 | **73078** | 73313 | 227 | 1.31 |
| 4_3_10 | 20 | 20 | ** | 3600 | ** | 3600 | 66071 | **71113** | 71258 | 3600 | 7.63 | 71121 | 71164 | 615 | 7.64 |
| 4_4_1 | 10 | 10 | 10395 | 10 | 10395 | 7 | 10395 | **10395** | 10519 | 26 | 0.00 | **10395** | 10395 | 11 | 0.00 |
| 4_4_2 | 15 | 10 | 21376 | 202 | 21376 | 146 | 21376 | **21376** | 21376 | 566 | 0.00 | **21376** | 21453 | 23 | 0.00 |
| 4_4_3 | 20 | 10 | ** | 3600 | ** | 3600 | 37003 | **37408** | 37408 | 930 | 1.09 | **37408** | 37408 | 38 | 1.09 |
| 4_4_8 | 10 | 20 | ** | 3600 | 30619 | 3501 | 30619 | 30842 | 30842 | 309 | 0.73 | **30621** | 30721 | 64 | 0.01 |
| 4_4_9 | 15 | 20 | ** | 3600 | ** | 3600 | 41464 | **41811** | 41964 | 2383 | 0.84 | **41811** | 42174 | 300 | 0.84 |
| 4_4_10 | 20 | 20 | ** | 3600 | ** | 3600 | 47733 | 52051 | 52305 | 3600 | 9.05 | **51891** | 52058 | 556 | 8.71 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, L.B., lower bound, Gap% = ((Best Sol. - L.B.) / L.B.) × 100
** no feasible solution found

Table B.5 Detailed computational results obtained for the set S2

| | | | IM-VSP | | | MA | | | | | | IM-VSP | | | MA | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | Best | Avg. | t(s) | Best | Avg. | t(s) | Ins. | N.Ret. | N.Per. | Best | Avg. | t(s) | Best | Avg. | t(s) |
| 1_1_4 | 35 | 10 | **54800** | 54807 | 166 | 54835 | 54845 | 19 | 2_1_4 | 35 | 10 | **55611** | 55616 | 270 | 55842 | 55847 | 72 |
| 1_1_5 | 50 | 10 | **93718** | 93723 | 236 | 93788 | 93810 | 15 | 2_1_5 | 50 | 10 | 96367 | 96367 | 410 | **96357** | 96422 | 122 |
| 1_1_6 | 75 | 10 | 136479 | 136496 | 400 | **134740** | 134785 | 183 | 2_1_6 | 75 | 10 | **136581** | 136640 | 693 | 136584 | 136675 | 250 |
| 1_1_7 | 100 | 10 | 132118 | 132171 | 710 | **132013** | 132097 | 442 | 2_1_7 | 100 | 10 | 132150 | 132192 | 958 | **132041** | 132098 | 424 |
| 1_1_11 | 35 | 20 | **78811** | 79467 | 209 | **78811** | 78816 | 89 | 2_1_11 | 35 | 20 | **82429** | 82550 | 359 | **82429** | 82567 | 115 |
| 1_1_12 | 50 | 20 | **153840** | 153857 | 413 | **153840** | 153876 | 161 | 2_1_12 | 50 | 20 | **156805** | 156805 | 610 | **156805** | 156882 | 216 |
| 1_1_13 | 75 | 20 | **290788** | 290846 | 822 | 290901 | 290969 | 309 | 2_1_13 | 75 | 20 | **297215** | 297355 | 1253 | 297437 | 297463 | 435 |
| 1_1_14 | 100 | 20 | **298352** | 298427 | 1181 | 298664 | 298769 | 686 | 2_1_14 | 100 | 20 | **297535** | 298144 | 1524 | 298115 | 298453 | 676 |
| 1_2_4 | 35 | 10 | **86386** | 86386 | 2434 | 86416 | 86421 | 122 | 2_2_4 | 35 | 10 | 87600 | 88025 | 3600 | **87565** | 87590 | 137 |
| 1_2_5 | 50 | 10 | 74036 | 74090 | 2049 | **72420** | 72477 | 277 | 2_2_5 | 50 | 10 | 73848 | 73983 | 1954 | **73755** | 73954 | 237 |
| 1_2_6 | 75 | 10 | 169975 | 170845 | 3600 | **169649** | 169708 | 634 | 2_2_6 | 75 | 10 | 172555 | 172618 | 3600 | **171576** | 171612 | 642 |
| 1_2_7 | 100 | 10 | 137355 | 137391 | 3600 | **136945** | 137098 | 733 | 2_2_7 | 100 | 10 | 137325 | 137385 | 2493 | **136919** | 137051 | 865 |
| 1_2_11 | 35 | 20 | **145677** | 146001 | 3600 | 145837 | 146199 | 1261 | 2_2_11 | 35 | 20 | **146121** | 146279 | 3600 | 147132 | 147362 | 1521 |
| 1_2_12 | 50 | 20 | **158694** | 160336 | 3600 | 160092 | 160361 | 3600 | 2_2_12 | 50 | 20 | 160575 | 161047 | 3600 | **158609** | 160339 | 3600 |
| 1_2_13 | 75 | 20 | 294111 | 294273 | 3600 | **293658** | 294016 | 3600 | 2_2_13 | 75 | 20 | **289369** | 290790 | 3600 | 293775 | 294664 | 3600 |
| 1_2_14 | 100 | 20 | **281815** | 282243 | 3600 | 282169 | 282448 | 3600 | 2_2_14 | 100 | 20 | **281577** | 281986 | 3600 | 282353 | 282662 | 3600 |
| 1_3_4 | 35 | 10 | **86350** | 86527 | 3600 | 86358 | 86362 | 154 | 2_3_4 | 35 | 10 | 86918 | 87402 | 3600 | **86905** | 86928 | 148 |
| 1_3_5 | 50 | 10 | 72001 | 72045 | 3600 | **71456** | 71519 | 445 | 2_3_5 | 50 | 10 | 72012 | 72093 | 3600 | **71307** | 71598 | 413 |
| 1_3_6 | 75 | 10 | 170706 | 170734 | 3600 | **169358** | 169431 | 2040 | 2_3_6 | 75 | 10 | 171802 | 171896 | 3600 | **169837** | 169877 | 1616 |
| 1_3_7 | 100 | 10 | 132802 | 132912 | 3600 | **132788** | 132838 | 844 | 2_3_7 | 100 | 10 | **132679** | 132744 | 3600 | 132854 | 132861 | 886 |
| 1_3_11 | 35 | 20 | **114476** | 114738 | 3600 | **114476** | 114709 | 1015 | 2_3_11 | 35 | 20 | **114794** | 115670 | 1359 | **114794** | 115023 | 1145 |
| 1_3_12 | 50 | 20 | **199982** | 200772 | 3600 | **199982** | 200113 | 3600 | 2_3_12 | 50 | 20 | **200535** | 203098 | 1366 | 200762 | 201766 | 3600 |
| 1_3_13 | 75 | 20 | **167072** | 167265 | 3600 | **167072** | 167218 | 3600 | 2_3_13 | 75 | 20 | 167206 | 167498 | 1384 | **166962** | 167036 | 3290 |
| 1_3_14 | 100 | 20 | 196123 | 196398 | 3600 | **195666** | 195944 | 3600 | 2_3_14 | 100 | 20 | 196535 | 196863 | 1387 | **195559** | 195875 | 3600 |
| 1_4_4 | 35 | 10 | **68598** | 68998 | 3600 | 68643 | 68701 | 303 | 2_4_4 | 35 | 10 | **69457** | 69822 | 1359 | 69602 | 69722 | 239 |
| 1_4_5 | 50 | 10 | 50054 | 50237 | 633 | **49595** | 49734 | 244 | 2_4_5 | 50 | 10 | 50135 | 50359 | 677 | **50048** | 50398 | 231 |
| 1_4_6 | 75 | 10 | 101228 | 102158 | 3600 | **101049** | 101697 | 1238 | 2_4_6 | 75 | 10 | 103020 | 103078 | 1371 | **102130** | 102186 | 964 |
| 1_4_7 | 100 | 10 | 173105 | 173554 | 3600 | **171579** | 172655 | 3600 | 2_4_7 | 100 | 10 | 173815 | 174524 | 1364 | **173462** | 174115 | 3085 |
| 1_4_11 | 35 | 20 | **119534** | 119881 | 3600 | **119534** | 119901 | 2161 | 2_4_11 | 35 | 20 | 120917 | 121517 | 1359 | **120585** | 120659 | 2052 |
| 1_4_12 | 50 | 20 | **154675** | 155216 | 3600 | 155394 | 155855 | 3600 | 2_4_12 | 50 | 20 | 154791 | 155939 | 1379 | **154518** | 155908 | 3600 |
| 1_4_13 | 75 | 20 | **154108** | 154295 | 3600 | 154445 | 154844 | 3600 | 2_4_13 | 75 | 20 | 154678 | 154916 | 1358 | **154206** | 154631 | 3600 |
| 1_4_14 | 100 | 20 | **464249** | 466957 | 3600 | 469822 | 471746 | 3600 | 2_4_14 | 100 | 20 | **471505** | 472263 | 1414 | 471996 | 472935 | 3600 |

Table B.6 Detailed computational results obtained for the set S2 (cont.)

| | | | IM-VSP | | | MA | | | | | | IM-VSP | | | MA | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | Best | Avg. | t(s) | Best | Avg. | t(s) | Ins. | N.Ret. | N.Per. | Best | Avg. | t(s) | Best | Avg. | t(s) |
| 3_1_4 | 35 | 10 | 130149 | 130149 | 367 | **129940** | 129960 | 28 | 4_1_4 | 35 | 10 | **46946** | 46948 | 142 | 46956 | 46981 | 16 |
| 3_1_5 | 50 | 10 | **242908** | 242913 | 481 | 242948 | 242973 | 24 | 4_1_5 | 50 | 10 | **81119** | 81122 | 220 | 81124 | 81176 | 19 |
| 3_1_6 | 75 | 10 | 357441 | 357458 | 737 | **354437** | 354475 | 62 | 4_1_6 | 75 | 10 | 116193 | 116213 | 328 | **115228** | 115416 | 152 |
| 3_1_7 | 100 | 10 | **324017** | 324050 | 1977 | 324127 | 324144 | 219 | 4_1_7 | 100 | 10 | 109095 | 109114 | 906 | **107228** | 108425 | 512 |
| 3_1_11 | 35 | 20 | **153285** | 153933 | 376 | **153285** | 153290 | 92 | 4_1_11 | 35 | 20 | **56084** | 56084 | 204 | **56084** | 56087 | 110 |
| 3_1_12 | 50 | 20 | **364900** | 364908 | 751 | **364900** | 365071 | 192 | 4_1_12 | 50 | 20 | **118884** | 118894 | 353 | **118884** | 118904 | 184 |
| 3_1_13 | 75 | 20 | **761422** | 761477 | 1381 | 761503 | 761561 | 332 | 4_1_13 | 75 | 20 | **236436** | 236464 | 715 | **236436** | 236474 | 383 |
| 3_1_14 | 100 | 20 | **699524** | 699602 | 3600 | 699656 | 699670 | 926 | 4_1_14 | 100 | 20 | 213648 | 213661 | 1797 | **213588** | 213693 | 872 |
| 3_2_4 | 35 | 10 | **225567** | 225567 | 3600 | **225567** | 225574 | 80 | 4_2_4 | 35 | 10 | **74139** | 74425 | 3600 | 74144 | 74148 | 139 |
| 3_2_5 | 50 | 10 | 171361 | 171418 | 3600 | **171310** | 171350 | 264 | 4_2_5 | 50 | 10 | 57582 | 57737 | 3600 | **57503** | 57576 | 188 |
| 3_2_6 | 75 | 10 | 468649 | 468706 | 3600 | **468617** | 468640 | 858 | 4_2_6 | 75 | 10 | 150441 | 150563 | 3600 | **147174** | 147543 | 584 |
| 3_2_7 | 100 | 10 | 338015 | 338044 | 3600 | **334344** | 334475 | 1389 | 4_2_7 | 100 | 10 | 105247 | 105378 | 3600 | **105003** | 105078 | 553 |
| 3_2_11 | 35 | 20 | **365833** | 365889 | 3600 | 365985 | 366011 | 1383 | 4_2_11 | 35 | 20 | 119067 | 119240 | 3600 | **117550** | 117699 | 1613 |
| 3_2_12 | 50 | 20 | 371770 | 372644 | 3600 | **371273** | 371497 | 3600 | 4_2_12 | 50 | 20 | 119770 | 120105 | 3600 | **119683** | 119849 | 3600 |
| 3_2_13 | 75 | 20 | 746763 | 747076 | 3600 | **746322** | 747205 | 3600 | 4_2_13 | 75 | 20 | 231985 | 232796 | 3600 | **228743** | 231060 | 3600 |
| 3_2_14 | 100 | 20 | **682056** | 682190 | 3600 | 682313 | 683386 | 3600 | 4_2_14 | 100 | 20 | 211749 | 212358 | 3600 | **210724** | 211514 | 3600 |
| 3_3_4 | 35 | 10 | **229282** | 229334 | 3600 | 229294 | 229298 | 92 | 4_3_4 | 35 | 10 | **75145** | 76129 | 3600 | 75150 | 75195 | 176 |
| 3_3_5 | 50 | 10 | 168126 | 168654 | 3600 | **167934** | 167986 | 600 | 4_3_5 | 50 | 10 | 56463 | 56775 | 3600 | **55948** | 56152 | 539 |
| 3_3_6 | 75 | 10 | 468622 | 468872 | 3600 | **468563** | 468578 | 1015 | 4_3_6 | 75 | 10 | 148141 | 148684 | 3600 | **147264** | 147346 | 650 |
| 3_3_7 | 100 | 10 | 327837 | 328420 | 3600 | **326901** | 326961 | 1419 | 4_3_7 | 100 | 10 | 104443 | 105011 | 3600 | **104428** | 104598 | 640 |
| 3_3_11 | 35 | 20 | **263771** | 264191 | 3600 | 263968 | 264085 | 1501 | 4_3_11 | 35 | 20 | 85443 | 85532 | 3600 | **85173** | 85386 | 1464 |
| 3_3_12 | 50 | 20 | 512516 | 512782 | 3600 | **511816** | 512191 | 3340 | 4_3_12 | 50 | 20 | 160527 | 161315 | 3600 | **158383** | 159625 | 3600 |
| 3_3_13 | 75 | 20 | 322659 | 323548 | 3600 | **322397** | 322949 | 3600 | 4_3_13 | 75 | 20 | 101507 | 101759 | 3600 | **99786** | 100326 | 3600 |
| 3_3_14 | 100 | 20 | **398317** | 399826 | 3600 | 400916 | 401905 | 3600 | 4_3_14 | 100 | 20 | 124416 | 124943 | 3600 | **123285** | 123846 | 3600 |
| 3_4_4 | 35 | 10 | 171826 | 171826 | 3600 | **171826** | 172027 | 174 | 4_4_4 | 35 | 10 | 55669 | 55669 | 3600 | **55669** | 55671 | 201 |
| 3_4_5 | 50 | 10 | 101180 | 101438 | 3600 | **100197** | 100455 | 399 | 4_4_5 | 50 | 10 | 35323 | 35393 | 3600 | **34491** | 35102 | 269 |
| 3_4_6 | 75 | 10 | 252831 | 253045 | 3600 | **250757** | 250855 | 1469 | 4_4_6 | 75 | 10 | **79863** | 80651 | 3600 | 80066 | 80197 | 701 |
| 3_4_7 | 100 | 10 | 462602 | 464100 | 3600 | **461756** | 461815 | 2480 | 4_4_7 | 100 | 10 | 142197 | 142479 | 3600 | **141707** | 141969 | 1584 |
| 3_4_11 | 35 | 20 | 268948 | 269480 | 3600 | **268702** | 269260 | 2003 | 4_4_11 | 35 | 20 | 86749 | 87096 | 3600 | **85920** | 86637 | 1849 |
| 3_4_12 | 50 | 20 | 357789 | 357990 | 3600 | **357549** | 357600 | 3600 | 4_4_12 | 50 | 20 | **108739** | 109279 | 3600 | 109270 | 109456 | 3600 |
| 3_4_13 | 75 | 20 | 308647 | 308878 | 3600 | **308066** | 308808 | 3600 | 4_4_13 | 75 | 20 | **92569** | 93028 | 3600 | 93041 | 93481 | 3600 |
| 3_4_14 | 100 | 20 | 1300192 | 1302841 | 3600 | **1282450** | 1284402 | 3600 | 4_4_14 | 100 | 20 | **382735** | 384409 | 3600 | 383988 | 384720 | 3600 |

Table B.7 Cost values obtaining by solving the PRP and the PRP-VSP for the set S1

| | | | Prod. | | P. Inv. | | R. Inv. | | Trans. | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | Gap% |
| 1_1_1 | 10 | 10 | 15885 | 15885 | 1476 | 771 | 2217 | 1932 | 10380 | 8575 | 29958 | 27163 | 10.29 |
| 1_1_2 | 15 | 10 | 21636 | 22536 | 1701 | 0 | 4045 | 2969 | 10455 | 8860 | 37837 | 34365 | 10.10 |
| 1_1_3 | 20 | 10 | 29673 | 28773 | 294 | 0 | 5201 | 3911 | 12255 | 10065 | 47423 | 42749 | 10.93 |
| 1_1_8 | 10 | 20 | 32103 | 31203 | 1716 | 381 | 7561 | 6707 | 10360 | 8810 | 51740 | 47101 | 9.85 |
| 1_1_9 | 15 | 20 | 61236 | 58836 | 1424 | 584 | 14463 | 9691 | 16555 | 13245 | 93678 | 82356 | 13.75 |
| 1_1_10 | 20 | 20 | 59895 | 58095 | 1593 | 402 | 17116 | 10269 | 19620 | 16675 | 98224 | 85441 | 14.96 |
| 1_2_1 | 10 | 10 | 5664 | 5364 | 39 | 211 | 2600 | 2600 | 8760 | 7795 | 17063 | 15970 | 6.84 |
| 1_2_2 | 15 | 10 | 14472 | 15072 | 920 | 114 | 4226 | 2718 | 8965 | 8935 | 28583 | 26839 | 6.50 |
| 1_2_3 | 20 | 10 | 35928 | 35928 | 1216 | 36 | 6234 | 5997 | 9025 | 7215 | 52403 | 49176 | 6.56 |
| 1_2_8 | 10 | 20 | 20574 | 21174 | 588 | 320 | 9670 | 5926 | 11040 | 10840 | 41872 | 38260 | 9.44 |
| 1_2_9 | 15 | 20 | 30774 | 31374 | 898 | 114 | 13423 | 6719 | 16240 | 16805 | 61335 | 55012 | 11.49 |
| 1_2_10 | 20 | 20 | 57510 | 57510 | 1461 | 447 | 17508 | 11774 | 16225 | 14840 | 92704 | 84571 | 9.62 |
| 1_3_1 | 10 | 10 | 10734 | 10734 | 520 | 504 | 2733 | 2096 | 7015 | 6215 | 21002 | 19549 | 7.43 |
| 1_3_2 | 15 | 10 | 29496 | 29496 | 1104 | 280 | 4405 | 3991 | 8575 | 7655 | 43580 | 41422 | 5.21 |
| 1_3_3 | 20 | 10 | 9528 | 9828 | 250 | 108 | 5698 | 3265 | 9330 | 10250 | 24806 | 23451 | 5.78 |
| 1_3_8 | 10 | 20 | 29313 | 31113 | 2064 | 843 | 11277 | 7176 | 10140 | 11700 | 52794 | 50832 | 3.86 |
| 1_3_9 | 15 | 20 | 57228 | 58428 | 3032 | 76 | 14973 | 12809 | 13365 | 11070 | 88598 | 82383 | 7.54 |
| 1_3_10 | 20 | 20 | 58365 | 58365 | 801 | 201 | 12769 | 8396 | 14495 | 14555 | 86430 | 81517 | 6.03 |
| 1_4_1 | 10 | 10 | 4941 | 4941 | 68 | 68 | 5238 | 5238 | 5400 | 5400 | 15647 | 15647 | 0.00 |
| 1_4_2 | 15 | 10 | 13326 | 14526 | 886 | 0 | 5911 | 3378 | 8005 | 8410 | 28128 | 26314 | 6.89 |
| 1_4_3 | 20 | 10 | 28998 | 28998 | 0 | 114 | 7330 | 4272 | 8775 | 10055 | 45103 | 43439 | 3.83 |
| 1_4_8 | 10 | 20 | 20340 | 20940 | 216 | 222 | 7717 | 6695 | 10510 | 10590 | 38783 | 38447 | 0.87 |
| 1_4_9 | 15 | 20 | 31176 | 30576 | 114 | 580 | 13983 | 11413 | 13265 | 13225 | 58538 | 55794 | 4.92 |
| 1_4_10 | 20 | 20 | 38886 | 38286 | 542 | 216 | 16462 | 12799 | 14740 | 15560 | 70630 | 66861 | 5.64 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, Prod., total production cost, P.Inv., inventory holding cost at the plant R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Gap% = ((PRP-VSP - PRP) / PRP) × 100

Table B.8 Cost values obtaining by solving the PRP and the PRP-VSP for the set S1 (cont.)

| | | | Prod. | | P. Inv. | | R. Inv. | | Trans. | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | Gap% |
| 2_1_1 | 10 | 10 | 16785 | 15885 | 1143 | 1053 | 2224 | 1891 | 11005 | 8820 | 31157 | 27649 | 12.69 |
| 2_1_2 | 15 | 10 | 22536 | 22536 | 2202 | 0 | 3959 | 2983 | 11265 | 8900 | 39962 | 34419 | 16.10 |
| 2_1_3 | 20 | 10 | 29673 | 28773 | 489 | 0 | 5573 | 4107 | 12195 | 10060 | 47930 | 42940 | 11.62 |
| 2_1_8 | 10 | 20 | 33903 | 33003 | 2973 | 807 | 7592 | 4869 | 12825 | 10455 | 57293 | 49134 | 16.61 |
| 2_1_9 | 15 | 20 | 61236 | 61236 | 1532 | 524 | 14742 | 6704 | 17260 | 15125 | 94770 | 83589 | 13.38 |
| 2_1_10 | 20 | 20 | 61695 | 58995 | 816 | 504 | 17783 | 9269 | 20890 | 17945 | 101184 | 86713 | 16.69 |
| 2_2_1 | 10 | 10 | 5664 | 5664 | 48 | 108 | 2701 | 2037 | 8760 | 9105 | 17173 | 16914 | 1.53 |
| 2_2_2 | 15 | 10 | 15072 | 15072 | 1000 | 390 | 4382 | 3174 | 8500 | 8510 | 28954 | 27146 | 6.66 |
| 2_2_3 | 20 | 10 | 37128 | 37128 | 1148 | 0 | 6729 | 3993 | 9365 | 8425 | 54370 | 49546 | 9.74 |
| 2_2_8 | 10 | 20 | 21774 | 21774 | 728 | 792 | 9174 | 5037 | 11755 | 11130 | 43431 | 38733 | 12.13 |
| 2_2_9 | 15 | 20 | 31374 | 31374 | 848 | 266 | 13257 | 7041 | 16435 | 17120 | 61914 | 55801 | 10.96 |
| 2_2_10 | 20 | 20 | 59310 | 60210 | 1521 | 141 | 17767 | 7944 | 17540 | 18150 | 96138 | 86445 | 11.21 |
| 2_3_1 | 10 | 10 | 10734 | 10734 | 914 | 882 | 3191 | 2023 | 7390 | 6530 | 22229 | 20169 | 10.21 |
| 2_3_2 | 15 | 10 | 30696 | 30696 | 960 | 20 | 4537 | 2718 | 8235 | 8695 | 44428 | 42129 | 5.46 |
| 2_3_3 | 20 | 10 | 9528 | 9528 | 274 | 343 | 5482 | 3659 | 9575 | 9500 | 24859 | 23030 | 7.94 |
| 2_3_8 | 10 | 20 | 33813 | 32913 | 438 | 798 | 10366 | 5537 | 10865 | 13375 | 55482 | 52623 | 5.43 |
| 2_3_9 | 15 | 20 | 63228 | 62028 | 824 | 776 | 13448 | 9776 | 14570 | 13515 | 92070 | 86095 | 6.94 |
| 2_3_10 | 20 | 20 | 58365 | 58365 | 1086 | 492 | 12677 | 7709 | 14715 | 15110 | 86843 | 81676 | 6.33 |
| 2_4_1 | 10 | 10 | 5541 | 5541 | 201 | 201 | 4188 | 4188 | 6640 | 6640 | 16570 | 16570 | 0.00 |
| 2_4_2 | 15 | 10 | 13926 | 14526 | 684 | 0 | 6118 | 3496 | 8070 | 8290 | 28798 | 26312 | 9.45 |
| 2_4_3 | 20 | 10 | 28998 | 28998 | 639 | 114 | 6685 | 5256 | 10145 | 9490 | 46467 | 43858 | 5.95 |
| 2_4_8 | 10 | 20 | 22740 | 22140 | 294 | 492 | 6009 | 5214 | 13950 | 14290 | 42993 | 42136 | 2.03 |
| 2_4_9 | 15 | 20 | 31776 | 31776 | 386 | 674 | 12922 | 7463 | 14980 | 16630 | 60064 | 56543 | 6.23 |
| 2_4_10 | 20 | 20 | 39486 | 40086 | 532 | 0 | 16752 | 11045 | 15780 | 16800 | 72550 | 67931 | 6.80 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, Prod., total production cost, P.Inv., inventory holding cost at the plant R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Gap% = ((PRP-VSP - PRP) / PRP) × 100

Table B.9 Cost values obtaining by solving the PRP and the PRP-VSP for the set S1 (cont.)

| | | | Prod. | | P. Inv. | | R. Inv. | | Trans. | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | Gap% |
| 3_1_1 | 10 | 10 | 49950 | 46950 | 2775 | 4590 | 2316 | 2566 | 10380 | 8735 | 65421 | 62841 | 4.11 |
| 3_1_2 | 15 | 10 | 69120 | 69120 | 3426 | 2457 | 4054 | 3881 | 10455 | 8690 | 87055 | 84148 | 3.45 |
| 3_1_3 | 20 | 10 | 89910 | 89910 | 6147 | 3573 | 5366 | 5807 | 11865 | 10820 | 113288 | 110110 | 2.89 |
| 3_1_8 | 10 | 20 | 98010 | 98010 | 6990 | 2937 | 7645 | 8507 | 10360 | 8930 | 123005 | 118384 | 3.90 |
| 3_1_9 | 15 | 20 | 184120 | 188120 | 8584 | 2344 | 15557 | 12737 | 16420 | 13555 | 224681 | 216756 | 3.66 |
| 3_1_10 | 20 | 20 | 187650 | 184650 | 6849 | 3873 | 18062 | 16068 | 19510 | 15450 | 232071 | 220041 | 5.47 |
| 3_2_1 | 10 | 10 | 16880 | 16880 | 1087 | 1017 | 2600 | 2255 | 8760 | 8730 | 29327 | 28882 | 1.54 |
| 3_2_2 | 15 | 10 | 46240 | 46240 | 2008 | 1932 | 4483 | 3914 | 8660 | 8185 | 61391 | 60271 | 1.86 |
| 3_2_3 | 20 | 10 | 115760 | 115760 | 3000 | 2144 | 7552 | 7439 | 8725 | 7790 | 135037 | 133133 | 1.43 |
| 3_2_8 | 10 | 20 | 64580 | 64580 | 2168 | 2422 | 9708 | 8171 | 11140 | 9795 | 87596 | 84968 | 3.09 |
| 3_2_9 | 15 | 20 | 94580 | 94580 | 4328 | 4552 | 14404 | 10646 | 16170 | 15075 | 129482 | 124853 | 3.71 |
| 3_2_10 | 20 | 20 | 185700 | 185700 | 3510 | 2367 | 18276 | 15745 | 15880 | 13660 | 223366 | 217472 | 2.71 |
| 3_3_1 | 10 | 10 | 33780 | 33780 | 1326 | 1176 | 2733 | 2586 | 7015 | 6180 | 44854 | 43722 | 2.59 |
| 3_3_2 | 15 | 10 | 94320 | 94320 | 2100 | 1064 | 4934 | 5025 | 8490 | 7835 | 109844 | 108244 | 1.48 |
| 3_3_3 | 20 | 10 | 29760 | 30760 | 1238 | 813 | 6533 | 4173 | 8840 | 9070 | 46371 | 44816 | 3.47 |
| 3_3_8 | 10 | 20 | 97710 | 97710 | 2064 | 2169 | 11277 | 10038 | 10140 | 10280 | 121191 | 120197 | 0.83 |
| 3_3_9 | 15 | 20 | 190760 | 194760 | 3032 | 76 | 14973 | 12809 | 13365 | 11070 | 222130 | 218715 | 1.56 |
| 3_3_10 | 20 | 20 | 179550 | 182550 | 5274 | 3519 | 15638 | 13093 | 14430 | 14140 | 214892 | 213302 | 0.75 |
| 3_4_1 | 10 | 10 | 16470 | 16470 | 68 | 68 | 5238 | 5238 | 5400 | 5400 | 27176 | 27176 | 0.00 |
| 3_4_2 | 15 | 10 | 44420 | 44420 | 886 | 2090 | 5911 | 3934 | 8005 | 8350 | 59222 | 58794 | 0.73 |
| 3_4_3 | 20 | 10 | 90660 | 90660 | 2604 | 2604 | 8220 | 8220 | 8890 | 8890 | 110374 | 110374 | 0.00 |
| 3_4_8 | 10 | 20 | 65800 | 65800 | 1034 | 1112 | 8215 | 8287 | 10755 | 10085 | 85804 | 85284 | 0.61 |
| 3_4_9 | 15 | 20 | 97920 | 97920 | 2670 | 3030 | 14869 | 12068 | 13000 | 12820 | 128459 | 125838 | 2.08 |
| 3_4_10 | 20 | 20 | 121620 | 123620 | 3592 | 1812 | 17381 | 14107 | 15270 | 15025 | 157863 | 154564 | 2.13 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, Prod., total production cost, P.Inv., inventory holding cost at the plant R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Gap% = ((PRP-VSP - PRP) / PRP) × 100

Table B.10 Cost values obtaining by solving the PRP and the PRP-VSP for the set S1 (cont.)

| | | | Prod. | | P. Inv. | | R. Inv. | | Trans. | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | Gap% |
| 4_1_1 | 10 | 10 | 15885 | 15885 | 1362 | 537 | - | - | 10380 | 8550 | 27627 | 24972 | 10.63 |
| 4_1_2 | 15 | 10 | 21636 | 20736 | 1386 | 1083 | - | - | 10455 | 9475 | 33477 | 31294 | 6.98 |
| 4_1_3 | 20 | 10 | 29673 | 28773 | 231 | 0 | - | - | 12255 | 9760 | 42159 | 38533 | 9.41 |
| 4_1_8 | 10 | 20 | 31203 | 30303 | 2394 | 366 | - | - | 10360 | 8760 | 43957 | 39429 | 11.48 |
| 4_1_9 | 15 | 20 | 60036 | 56436 | 2008 | 1208 | - | - | 16420 | 12460 | 78464 | 70104 | 11.93 |
| 4_1_10 | 20 | 20 | 59895 | 55395 | 1056 | 1341 | - | - | 19510 | 15820 | 80461 | 72556 | 10.90 |
| 4_2_1 | 10 | 10 | 5064 | 5364 | 530 | 68 | - | - | 8670 | 7760 | 14264 | 13192 | 8.13 |
| 4_2_2 | 15 | 10 | 14472 | 13872 | 760 | 472 | - | - | 8545 | 8435 | 23777 | 22779 | 4.38 |
| 4_2_3 | 20 | 10 | 35928 | 34728 | 1100 | 788 | - | - | 8735 | 8055 | 45763 | 43571 | 5.03 |
| 4_2_8 | 10 | 20 | 19974 | 19374 | 676 | 388 | - | - | 11040 | 9340 | 31690 | 29102 | 8.89 |
| 4_2_9 | 15 | 20 | 30174 | 28974 | 874 | 476 | - | - | 15790 | 14175 | 46838 | 43625 | 7.37 |
| 4_2_10 | 20 | 20 | 56610 | 55710 | 1179 | 1026 | - | - | 15845 | 13905 | 73634 | 70641 | 4.24 |
| 4_3_1 | 10 | 10 | 10134 | 10734 | 768 | 296 | - | - | 7100 | 5905 | 18002 | 16935 | 6.30 |
| 4_3_2 | 15 | 10 | 28296 | 28296 | 1884 | 1008 | - | - | 8490 | 7715 | 38670 | 37019 | 4.46 |
| 4_3_3 | 20 | 10 | 9528 | 8928 | 96 | 308 | - | - | 8585 | 8520 | 18209 | 17756 | 2.55 |
| 4_3_8 | 10 | 20 | 30213 | 29313 | 747 | 942 | - | - | 9690 | 9815 | 40650 | 40070 | 1.45 |
| 4_3_9 | 15 | 20 | 57228 | 57228 | 2540 | 1068 | - | - | 13310 | 11345 | 73078 | 69641 | 4.94 |
| 4_3_10 | 20 | 20 | 56565 | 55665 | 1293 | 693 | - | - | 13255 | 13420 | 71113 | 69778 | 1.91 |
| 4_4_1 | 10 | 10 | 4941 | 4941 | 54 | 54 | - | - | 5400 | 5400 | 10395 | 10395 | 0.00 |
| 4_4_2 | 15 | 10 | 13326 | 13326 | 460 | 342 | - | - | 7590 | 7640 | 21376 | 21308 | 0.32 |
| 4_4_3 | 20 | 10 | 28998 | 28098 | 0 | 219 | - | - | 8410 | 8835 | 37408 | 37152 | 0.69 |
| 4_4_8 | 10 | 20 | 19740 | 19740 | 272 | 186 | - | - | 10830 | 9900 | 30842 | 29826 | 3.41 |
| 4_4_9 | 15 | 20 | 29976 | 29376 | 310 | 566 | - | - | 11525 | 11665 | 41811 | 41607 | 0.49 |
| 4_4_10 | 20 | 20 | 37686 | 37686 | 580 | 240 | - | - | 13785 | 12730 | 52051 | 50656 | 2.75 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, Prod., total production cost, P.Inv., inventory holding cost at the plant R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Gap% = ((PRP-VSP - PRP) / PRP) × 100

Table B.11 Cost values obtaining by solving the PRP and the PRP-VSP for the set S2

| | | | Prod | | P. Inv | | R. Inv | | Trans | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | Gap% |
| 1_1_4 | 35 | 10 | 32676 | 32076 | 236 | 124 | 7788 | 5991 | 14135 | 12700 | 54835 | 50891 | 7.75 |
| 1_1_5 | 50 | 10 | 65106 | 66006 | 0 | 0 | 12527 | 6326 | 16155 | 17265 | 93788 | 89597 | 4.68 |
| 1_1_6 | 75 | 10 | 94698 | 96498 | 0 | 0 | 19482 | 7059 | 20560 | 25160 | 134740 | 128717 | 4.68 |
| 1_1_7 | 100 | 10 | 85248 | 84648 | 0 | 0 | 21545 | 7762 | 25220 | 29185 | 132013 | 121595 | 8.57 |
| 1_1_11 | 35 | 20 | 32739 | 32739 | 0 | 1234 | 22727 | 10252 | 23345 | 29645 | 78811 | 73870 | 6.69 |
| 1_1_12 | 50 | 20 | 91014 | 94014 | 0 | 0 | 34726 | 10823 | 28100 | 38150 | 153840 | 142987 | 7.59 |
| 1_1_13 | 75 | 20 | 201726 | 207126 | 0 | 0 | 54482 | 9985 | 34580 | 54285 | 290788 | 271396 | 7.15 |
| 1_1_14 | 100 | 20 | 177168 | 178368 | 814 | 0 | 76220 | 10342 | 44150 | 64645 | 298352 | 253355 | 17.76 |
| 1_2_4 | 35 | 10 | 59940 | 61140 | 1876 | 132 | 11580 | 6943 | 13020 | 12905 | 86416 | 81120 | 6.53 |
| 1_2_5 | 50 | 10 | 43602 | 43602 | 0 | 0 | 14078 | 8292 | 14740 | 15865 | 72420 | 67759 | 6.88 |
| 1_2_6 | 75 | 10 | 129036 | 131436 | 0 | 0 | 20733 | 7985 | 19880 | 25290 | 169649 | 164711 | 3.00 |
| 1_2_7 | 100 | 10 | 87300 | 87300 | 0 | 0 | 27340 | 5000 | 22305 | 32080 | 136945 | 124380 | 10.10 |
| 1_2_11 | 35 | 20 | 96192 | 96192 | 927 | 600 | 26628 | 13311 | 21930 | 25135 | 145677 | 135238 | 7.72 |
| 1_2_12 | 50 | 20 | 92442 | 94842 | 372 | 274 | 36745 | 12690 | 29135 | 38140 | 158694 | 145946 | 8.73 |
| 1_2_13 | 75 | 20 | 202122 | 203022 | 678 | 132 | 56201 | 13415 | 35110 | 51260 | 294111 | 267829 | 9.81 |
| 1_2_14 | 100 | 20 | 177354 | 177954 | 0 | 0 | 65406 | 11616 | 39055 | 59225 | 281815 | 248795 | 13.27 |
| 1_3_4 | 35 | 10 | 63492 | 62292 | 600 | 0 | 9476 | 6794 | 12790 | 11385 | 86358 | 80471 | 7.32 |
| 1_3_5 | 50 | 10 | 42798 | 42798 | 214 | 0 | 13534 | 7125 | 14910 | 16425 | 71456 | 66348 | 7.70 |
| 1_3_6 | 75 | 10 | 129504 | 131904 | 0 | 0 | 20159 | 7615 | 19695 | 24970 | 169358 | 164489 | 2.96 |
| 1_3_7 | 100 | 10 | 85806 | 85806 | 0 | 0 | 24197 | 6061 | 22785 | 30595 | 132788 | 122462 | 8.43 |
| 1_3_11 | 35 | 20 | 65604 | 65604 | 862 | 832 | 26480 | 12794 | 21530 | 26665 | 114476 | 105895 | 8.10 |
| 1_3_12 | 50 | 20 | 135675 | 140175 | 0 | 108 | 37172 | 10427 | 27135 | 39680 | 199982 | 190390 | 5.04 |
| 1_3_13 | 75 | 20 | 69807 | 70107 | 0 | 0 | 60560 | 11702 | 36705 | 52780 | 167072 | 134589 | 24.13 |
| 1_3_14 | 100 | 20 | 89100 | 89100 | 0 | 0 | 65818 | 12635 | 41205 | 62505 | 196123 | 164240 | 19.41 |
| 1_4_4 | 35 | 10 | 44757 | 45657 | 765 | 0 | 11826 | 6311 | 11295 | 12520 | 68643 | 64488 | 6.44 |
| 1_4_5 | 50 | 10 | 22035 | 22635 | 0 | 0 | 12615 | 6256 | 14945 | 18340 | 49595 | 47231 | 5.01 |
| 1_4_6 | 75 | 10 | 64638 | 65838 | 96 | 0 | 18915 | 6780 | 17400 | 23260 | 101049 | 95878 | 5.39 |
| 1_4_7 | 100 | 10 | 124893 | 126693 | 0 | 0 | 27016 | 8524 | 19670 | 29165 | 171579 | 164382 | 4.38 |
| 1_4_11 | 35 | 20 | 65856 | 68256 | 1230 | 1344 | 29738 | 8946 | 22710 | 31220 | 119534 | 109766 | 8.90 |
| 1_4_12 | 50 | 20 | 89874 | 92274 | 90 | 66 | 39321 | 13151 | 25390 | 37570 | 154675 | 143061 | 8.12 |
| 1_4_13 | 75 | 20 | 68214 | 68814 | 271 | 0 | 51688 | 11777 | 33935 | 50300 | 154108 | 130891 | 17.74 |
| 1_4_14 | 100 | 20 | 353568 | 355968 | 0 | 0 | 72436 | 24034 | 38245 | 53990 | 464249 | 433992 | 6.97 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, Prod., total production cost, P.Inv., inventory holding cost at the plant R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Gap% = ((PRP-VSP - PRP) / PRP) × 100

Table B.12 Cost values obtaining by solving the PRP and the PRP-VSP for the set S2 (cont.)

| | | | Prod | | P. Inv | | R. Inv | | Trans | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | Gap% |
| 2_1_4 | 35 | 10 | 32076 | 32076 | 1192 | 368 | 7784 | 6156 | 14790 | 13205 | 55842 | 51805 | 7.79 |
| 2_1_5 | 50 | 10 | 64206 | 66006 | 1932 | 0 | 12779 | 5832 | 17440 | 18025 | 96357 | 89863 | 7.23 |
| 2_1_6 | 75 | 10 | 95598 | 96498 | 651 | 0 | 19725 | 6887 | 20610 | 25425 | 136584 | 128810 | 6.04 |
| 2_1_7 | 100 | 10 | 85248 | 85248 | 0 | 0 | 21538 | 5989 | 25255 | 30460 | 132041 | 121697 | 8.50 |
| 2_1_11 | 35 | 20 | 33339 | 33939 | 657 | 538 | 22413 | 8304 | 26020 | 30705 | 82429 | 73486 | 12.17 |
| 2_1_12 | 50 | 20 | 92214 | 92814 | 1136 | 0 | 34195 | 13026 | 29260 | 36180 | 156805 | 142020 | 10.41 |
| 2_1_13 | 75 | 20 | 204426 | 207126 | 1572 | 0 | 53152 | 11389 | 38065 | 52035 | 297215 | 270550 | 9.86 |
| 2_1_14 | 100 | 20 | 177168 | 177768 | 470 | 0 | 76472 | 13795 | 43425 | 61685 | 297535 | 253248 | 17.49 |
| 2_2_4 | 35 | 10 | 63540 | 61140 | 280 | 460 | 10360 | 6674 | 13385 | 13445 | 87565 | 81719 | 7.15 |
| 2_2_5 | 50 | 10 | 43602 | 44202 | 438 | 58 | 14080 | 7032 | 15635 | 18140 | 73755 | 69432 | 6.23 |
| 2_2_6 | 75 | 10 | 130236 | 131436 | 0 | 0 | 21485 | 7778 | 19855 | 25170 | 171576 | 164384 | 4.38 |
| 2_2_7 | 100 | 10 | 87300 | 86700 | 0 | 0 | 27034 | 8052 | 22585 | 29370 | 136919 | 124122 | 10.31 |
| 2_2_11 | 35 | 20 | 97092 | 96192 | 456 | 240 | 25813 | 13492 | 22760 | 24605 | 146121 | 134529 | 8.62 |
| 2_2_12 | 50 | 20 | 94242 | 95442 | 456 | 88 | 36177 | 11713 | 29700 | 38490 | 160575 | 145733 | 10.18 |
| 2_2_13 | 75 | 20 | 199422 | 203022 | 0 | 0 | 55862 | 13231 | 34085 | 51120 | 289369 | 267373 | 8.23 |
| 2_2_14 | 100 | 20 | 177354 | 177354 | 0 | 0 | 65338 | 13995 | 38885 | 56220 | 281577 | 247569 | 13.74 |
| 2_3_4 | 35 | 10 | 63492 | 62292 | 680 | 132 | 10263 | 6857 | 12470 | 11805 | 86905 | 81086 | 7.18 |
| 2_3_5 | 50 | 10 | 42798 | 42198 | 132 | 0 | 13357 | 7933 | 15020 | 15530 | 71307 | 65661 | 8.60 |
| 2_3_6 | 75 | 10 | 129504 | 131904 | 0 | 0 | 21098 | 7922 | 19235 | 24685 | 169837 | 164511 | 3.24 |
| 2_3_7 | 100 | 10 | 85806 | 85806 | 0 | 0 | 24563 | 6258 | 22485 | 30565 | 132854 | 122629 | 8.34 |
| 2_3_11 | 35 | 20 | 66204 | 66204 | 298 | 388 | 26747 | 13758 | 21545 | 25680 | 114794 | 106030 | 8.27 |
| 2_3_12 | 50 | 20 | 133875 | 139275 | 765 | 0 | 40025 | 12513 | 25870 | 37525 | 200535 | 189313 | 5.93 |
| 2_3_13 | 75 | 20 | 69807 | 70107 | 0 | 0 | 60704 | 11809 | 36695 | 52665 | 167206 | 134581 | 24.24 |
| 2_3_14 | 100 | 20 | 88800 | 88800 | 206 | 205 | 66024 | 13694 | 41505 | 61345 | 196535 | 164044 | 19.81 |
| 2_4_4 | 35 | 10 | 46557 | 45657 | 243 | 246 | 10357 | 5845 | 12445 | 13855 | 69602 | 65603 | 6.10 |
| 2_4_5 | 50 | 10 | 21735 | 22635 | 245 | 161 | 13148 | 5672 | 14920 | 18645 | 50048 | 47113 | 6.23 |
| 2_4_6 | 75 | 10 | 65238 | 65838 | 188 | 184 | 18709 | 6872 | 17995 | 23120 | 102130 | 96014 | 6.37 |
| 2_4_7 | 100 | 10 | 125793 | 126693 | 0 | 0 | 26039 | 8467 | 21630 | 29300 | 173462 | 164460 | 5.47 |
| 2_4_11 | 35 | 20 | 67656 | 67056 | 152 | 982 | 29404 | 12940 | 23705 | 27770 | 120917 | 108748 | 11.19 |
| 2_4_12 | 50 | 20 | 89874 | 91674 | 410 | 0 | 38677 | 13741 | 25830 | 37190 | 154791 | 142605 | 8.55 |
| 2_4_13 | 75 | 20 | 68214 | 68814 | 238 | 0 | 51416 | 12331 | 34810 | 49710 | 154678 | 130855 | 18.21 |
| 2_4_14 | 100 | 20 | 358368 | 359568 | 0 | 0 | 71577 | 16407 | 41560 | 60045 | 471505 | 436020 | 8.14 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, Prod., total production cost, P.Inv., inventory holding cost at the plant R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Gap% = ((PRP-VSP - PRP) / PRP) × 100

Table B.13 Cost values obtaining by solving the PRP and the PRP-VSP for the set S2 (cont.)

| | | | Prod | | P. Inv | | R. Inv | | Trans | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | Gap% |
| 3_1_4 | 35 | 10 | 106920 | 106920 | 1112 | 0 | 7788 | 6158 | 14120 | 12805 | 129940 | 125883 | 3.22 |
| 3_1_5 | 50 | 10 | 211020 | 211020 | 3231 | 75 | 12572 | 9827 | 16125 | 14240 | 242948 | 235162 | 3.31 |
| 3_1_6 | 75 | 10 | 312660 | 309660 | 1785 | 222 | 19482 | 13897 | 20510 | 19305 | 354437 | 343084 | 3.31 |
| 3_1_7 | 100 | 10 | 276160 | 280160 | 1106 | 788 | 23281 | 9657 | 23580 | 28765 | 324127 | 319370 | 1.49 |
| 3_1_11 | 35 | 20 | 104130 | 105130 | 3083 | 2681 | 22727 | 11613 | 23345 | 28415 | 153285 | 147839 | 3.68 |
| 3_1_12 | 50 | 20 | 297380 | 297380 | 4618 | 1072 | 34682 | 18220 | 28220 | 31550 | 364900 | 348222 | 4.79 |
| 3_1_13 | 75 | 20 | 669420 | 666420 | 2652 | 6156 | 54620 | 18468 | 34730 | 50930 | 761422 | 741974 | 2.62 |
| 3_1_14 | 100 | 20 | 580560 | 576560 | 0 | 8198 | 78924 | 25158 | 40040 | 55170 | 699524 | 665086 | 5.18 |
| 3_2_4 | 35 | 10 | 195800 | 199800 | 3720 | 456 | 13142 | 9863 | 12905 | 12305 | 225567 | 222424 | 1.41 |
| 3_2_5 | 50 | 10 | 139340 | 141340 | 2490 | 386 | 14750 | 8670 | 14730 | 15785 | 171310 | 166181 | 3.09 |
| 3_2_6 | 75 | 10 | 422120 | 422120 | 4860 | 364 | 21837 | 14012 | 19800 | 20095 | 468617 | 456591 | 2.63 |
| 3_2_7 | 100 | 10 | 283000 | 287000 | 1636 | 744 | 28513 | 11142 | 21195 | 27675 | 334344 | 326561 | 2.38 |
| 3_2_11 | 35 | 20 | 305640 | 308640 | 8577 | 684 | 28456 | 22818 | 23160 | 21955 | 365833 | 354097 | 3.31 |
| 3_2_12 | 50 | 20 | 300140 | 302140 | 5152 | 3954 | 37273 | 17114 | 29205 | 35740 | 371770 | 358948 | 3.57 |
| 3_2_13 | 75 | 20 | 649740 | 649740 | 4563 | 2361 | 59390 | 26096 | 33070 | 38665 | 746763 | 716862 | 4.17 |
| 3_2_14 | 100 | 20 | 579180 | 577180 | 0 | 6412 | 66491 | 22421 | 36385 | 52950 | 682056 | 658963 | 3.50 |
| 3_3_4 | 35 | 10 | 203640 | 203640 | 2356 | 336 | 10928 | 10057 | 12370 | 11275 | 229294 | 225308 | 1.77 |
| 3_3_5 | 50 | 10 | 136660 | 138660 | 2472 | 480 | 14302 | 8356 | 14500 | 15295 | 167934 | 162791 | 3.16 |
| 3_3_6 | 75 | 10 | 423680 | 423680 | 3016 | 804 | 22472 | 17343 | 19395 | 20480 | 468563 | 462307 | 1.35 |
| 3_3_7 | 100 | 10 | 278020 | 282020 | 1202 | 1144 | 26524 | 10706 | 21155 | 28050 | 326901 | 321920 | 1.55 |
| 3_3_11 | 35 | 20 | 208680 | 210680 | 4788 | 2714 | 29278 | 19369 | 21025 | 23475 | 263771 | 256238 | 2.94 |
| 3_3_12 | 50 | 20 | 443250 | 440250 | 3114 | 1953 | 38917 | 23920 | 27235 | 28825 | 512516 | 494948 | 3.55 |
| 3_3_13 | 75 | 20 | 223690 | 224690 | 2297 | 5515 | 61382 | 15168 | 35290 | 50415 | 322659 | 295788 | 9.08 |
| 3_3_14 | 100 | 20 | 290000 | 295000 | 1447 | 1204 | 67800 | 14423 | 39070 | 60460 | 398317 | 371087 | 7.34 |
| 3_4_4 | 35 | 10 | 146190 | 149190 | 1305 | 240 | 13461 | 9010 | 10870 | 12315 | 171826 | 170755 | 0.63 |
| 3_4_5 | 50 | 10 | 71450 | 72450 | 898 | 824 | 13049 | 7341 | 14800 | 16965 | 100197 | 97580 | 2.68 |
| 3_4_6 | 75 | 10 | 211460 | 211460 | 2334 | 1782 | 19803 | 13218 | 17160 | 19170 | 250757 | 245630 | 2.09 |
| 3_4_7 | 100 | 10 | 407310 | 410310 | 2709 | 1752 | 31672 | 20650 | 20065 | 22290 | 461756 | 455002 | 1.48 |
| 3_4_11 | 35 | 20 | 211520 | 213520 | 3954 | 5678 | 32154 | 16159 | 21320 | 27210 | 268948 | 262567 | 2.43 |
| 3_4_12 | 50 | 20 | 283580 | 289580 | 7040 | 3906 | 43579 | 19494 | 23590 | 33570 | 357789 | 346550 | 3.24 |
| 3_4_13 | 75 | 20 | 219380 | 220380 | 2821 | 5239 | 52866 | 15806 | 33580 | 47890 | 308647 | 289315 | 6.68 |
| 3_4_14 | 100 | 20 | 1170560 | 1162560 | 7664 | 824 | 81188 | 41111 | 40780 | 44665 | 1300192 | 1249160 | 4.09 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, Prod., total production cost, P.Inv., inventory holding cost at the plant R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Gap% = ((PRP-VSP - PRP) / PRP) × 100

Table B.14 Cost values obtaining by solving the PRP and the PRP-VSP for the set S2 (cont.)

| | | | Prod | | P. Inv | | R. Inv | | Trans | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | N.Ret. | N.Per. | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | PRP-VSP | PRP | Gap% |
| 4_1_4 | 35 | 10 | 32676 | 32076 | 170 | 58 | - | - | 14110 | 12390 | 46956 | 44524 | 5.46 |
| 4_1_5 | 50 | 10 | 64206 | 63306 | 828 | 0 | - | - | 16090 | 12885 | 81124 | 76191 | 6.47 |
| 4_1_6 | 75 | 10 | 94698 | 93798 | 0 | 48 | - | - | 20530 | 17945 | 115228 | 111791 | 3.07 |
| 4_1_7 | 100 | 10 | 83448 | 84648 | 0 | 0 | - | - | 23780 | 21880 | 107228 | 106528 | 0.66 |
| 4_1_11 | 35 | 20 | 32739 | 31539 | 0 | 783 | - | - | 23345 | 22170 | 56084 | 54492 | 2.92 |
| 4_1_12 | 50 | 20 | 91014 | 88614 | 0 | 856 | - | - | 27870 | 25960 | 118884 | 115430 | 2.99 |
| 4_1_13 | 75 | 20 | 201726 | 198126 | 0 | 1191 | - | - | 34710 | 32505 | 236436 | 231822 | 1.99 |
| 4_1_14 | 100 | 20 | 174168 | 174168 | 0 | 0 | - | - | 39480 | 39480 | 213648 | 213648 | 0.00 |
| 4_2_4 | 35 | 10 | 59940 | 59940 | 1324 | 204 | - | - | 12880 | 11425 | 74144 | 71569 | 3.60 |
| 4_2_5 | 50 | 10 | 43002 | 42402 | 296 | 12 | - | - | 14205 | 11920 | 57503 | 54334 | 5.83 |
| 4_2_6 | 75 | 10 | 127836 | 126636 | 228 | 548 | - | - | 19110 | 17680 | 147174 | 144864 | 1.59 |
| 4_2_7 | 100 | 10 | 85500 | 85500 | 108 | 112 | - | - | 19395 | 19155 | 105003 | 104767 | 0.23 |
| 4_2_11 | 35 | 20 | 95292 | 92592 | 1845 | 288 | - | - | 21930 | 18665 | 119067 | 111545 | 6.74 |
| 4_2_12 | 50 | 20 | 91842 | 90642 | 1118 | 306 | - | - | 26810 | 23830 | 119770 | 114778 | 4.35 |
| 4_2_13 | 75 | 20 | 199422 | 194022 | 513 | 486 | - | - | 32050 | 29790 | 231985 | 224298 | 3.43 |
| 4_2_14 | 100 | 20 | 176754 | 173754 | 0 | 784 | - | - | 34995 | 34085 | 211749 | 208623 | 1.50 |
| 4_3_4 | 35 | 10 | 62292 | 59892 | 808 | 1236 | - | - | 12050 | 11425 | 75150 | 72553 | 3.58 |
| 4_3_5 | 50 | 10 | 41598 | 41598 | 540 | 206 | - | - | 13810 | 12135 | 55948 | 53939 | 3.72 |
| 4_3_6 | 75 | 10 | 129504 | 127104 | 0 | 716 | - | - | 17760 | 17040 | 147264 | 144860 | 1.66 |
| 4_3_7 | 100 | 10 | 84606 | 83406 | 102 | 208 | - | - | 19720 | 19465 | 104428 | 103079 | 1.31 |
| 4_3_11 | 35 | 20 | 65004 | 63804 | 734 | 304 | - | - | 19705 | 17745 | 85443 | 81853 | 4.39 |
| 4_3_12 | 50 | 20 | 133875 | 131175 | 1152 | 915 | - | - | 25500 | 23845 | 160527 | 155935 | 2.94 |
| 4_3_13 | 75 | 20 | 68907 | 68907 | 485 | 119 | - | - | 32115 | 30320 | 101507 | 99346 | 2.18 |
| 4_3_14 | 100 | 20 | 88500 | 88200 | 196 | 379 | - | - | 35720 | 33985 | 124416 | 122564 | 1.51 |
| 4_4_4 | 35 | 10 | 43857 | 43857 | 1137 | 474 | - | - | 10675 | 10960 | 55669 | 55291 | 0.68 |
| 4_4_5 | 50 | 10 | 21435 | 21435 | 301 | 214 | - | - | 12755 | 12570 | 34491 | 34219 | 0.79 |
| 4_4_6 | 75 | 10 | 64038 | 63438 | 548 | 118 | - | - | 15480 | 14695 | 80066 | 78251 | 2.32 |
| 4_4_7 | 100 | 10 | 123993 | 123093 | 594 | 144 | - | - | 17120 | 16180 | 141707 | 139417 | 1.64 |
| 4_4_11 | 35 | 20 | 64656 | 64056 | 1118 | 528 | - | - | 20975 | 18810 | 86749 | 83394 | 4.02 |
| 4_4_12 | 50 | 20 | 87474 | 85674 | 660 | 954 | - | - | 20605 | 20245 | 108739 | 106873 | 1.75 |
| 4_4_13 | 75 | 20 | 67014 | 66114 | 125 | 442 | - | - | 25430 | 25815 | 92569 | 92371 | 0.21 |
| 4_4_14 | 100 | 20 | 351168 | 346368 | 792 | 784 | - | - | 30775 | 32200 | 382735 | 379352 | 0.89 |

*Notes.* N.Ret., retailer number, N.Per., number of periods, Prod., total production cost, P.Inv., inventory holding cost at the plant R.Inv., total inventory holding cost at the retailers, Trans., transportation cost, Gap% = ((PRP-VSP - PRP) / PRP) × 100

Table B.15 Objective function values found by MA for the set A1

| | Ins. | | | | | | Ins. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| N | 1 | 2 | 3 | 4 | 5 | N | 1 | 2 | 3 | 4 | 5 |
| 1 | **40390** | **35183** | **31100** | 37053 | **34814** | 49 | **66907** | **62818** | **52770** | **63968** | **61636** |
| 2 | **40600** | **35428** | **31296** | **37184** | **35159** | 50 | **69184** | **63767** | **53125** | **66660** | **64411** |
| 3 | **41278** | **35691** | 31447 | **37721** | 35853 | 51 | **73768** | **67283** | **55235** | 71501 | **73168** |
| 4 | **82787** | **70049** | **63090** | **75284** | **70799** | 52 | 108715 | **97610** | **83394** | **102006** | 97166 |
| 5 | **83212** | **70214** | **63118** | **75556** | **71156** | 53 | **110504** | **98575** | **84228** | 104203 | 99877 |
| 6 | **83954** | **70789** | 63396 | **76346** | 72237 | 54 | 115577 | **102278** | 86328 | **109312** | **108570** |
| 7 | **31889** | **27054** | **23605** | **28809** | **27444** | 55 | 57923 | **54615** | **43909** | **55531** | 53811 |
| 8 | **32162** | **27219** | **23633** | **29081** | **27801** | 56 | **59454** | **55580** | **44743** | 57728 | 56522 |
| 9 | **32904** | **27794** | 23918 | **29871** | **28878** | 57 | **64524** | **59012** | 46843 | 63056 | **65215** |
| 10 | **71289** | **59725** | **52806** | **64436** | **60580** | 58 | **97225** | **87355** | **74179** | **91371** | **87587** |
| 11 | 72110 | **60290** | **53113** | **65108** | **61301** | 59 | **99435** | **89045** | **74613** | **93740** | **89961** |
| 12 | **74674** | **61534** | **54266** | **66965** | **63943** | 60 | 106300 | **93702** | 77243 | **100627** | 100520 |
| 13 | **43735** | **38260** | **33879** | **40068** | **38112** | 61 | **95311** | **93545** | **75239** | **93609** | **90854** |
| 14 | 44642 | **38643** | 34157 | 40880 | **38810** | 62 | **99540** | **95210** | **76634** | **98326** | **96318** |
| 15 | **45510** | **39355** | **34486** | **41619** | **40387** | 63 | 108806 | **101114** | **80651** | **108028** | **113469** |
| 16 | **85781** | **73129** | **65387** | **78265** | **73722** | 64 | 137092 | **128233** | **105851** | **131624** | 126459 |
| 17 | **86269** | **73395** | **65467** | **78716** | **74352** | 65 | **140856** | **130028** | **107656** | 135869 | 131668 |
| 18 | **87516** | **74345** | **65926** | **80038** | 76362 | 66 | **150606** | **137216** | 111784 | **145967** | **148871** |
| 19 | **34731** | **30134** | **25902** | **31790** | **30367** | 67 | 86159 | **85238** | **66366** | 85235 | 82977 |
| 20 | **35219** | **30400** | **25982** | **32241** | **30997** | 68 | **89806** | **87033** | **68171** | **89172** | 88313 |
| 21 | **36466** | **31350** | **26441** | **33563** | **32958** | 69 | **99556** | **92447** | 72299 | **99492** | **105516** |
| 22 | **74460** | **62879** | **55550** | 67712 | **63860** | 70 | 125667 | **117978** | **96636** | **120989** | 116905 |
| 23 | **75137** | **63774** | **55682** | **68432** | **64497** | 71 | **129856** | **120650** | **98041** | **125392** | 121873 |
| 24 | **78236** | **65169** | 56825 | **70859** | **68023** | 72 | 141332 | **127137** | 102699 | **137282** | **140821** |
| 25 | **240619** | **201503** | **183110** | 218112 | 204860 | 73 | 26639 | **22692** | **20125** | **24406** | **22889** |
| 26 | 240856 | **201748** | **183306** | **218243** | **204989** | 74 | **26979** | **23158** | **20190** | **24787** | 23276 |
| 27 | **241078** | **202011** | 183457 | **218621** | 205683 | 75 | **28558** | **23862** | **20763** | **26043** | 25111 |
| 28 | **615739** | **513569** | **468450** | **557684** | **523679** | 76 | **65518** | **55110** | **49111** | **59636** | **55613** |
| 29 | 616016 | **513734** | **468478** | **557956** | **524036** | 77 | **66822** | **55802** | **49590** | **60477** | 56770 |
| 30 | **616754** | **514309** | **468746** | **558746** | **525113** | 78 | **70328** | **57582** | **51128** | 63102 | **60155** |
| 31 | **231689** | **193374** | **175615** | **209709** | **197274** | 79 | **29556** | **25767** | **22577** | **27423** | 26028 |
| 32 | 231966 | **193539** | **175643** | **209981** | **197631** | 80 | **30020** | **26341** | **22577** | **27994** | 26458 |
| 33 | **232704** | **194114** | **175911** | **210771** | **198708** | 81 | **32059** | **27494** | **23303** | **29815** | 29201 |
| 34 | **604089** | **503245** | **458166** | **546836** | **513460** | 82 | **68736** | **58415** | **51848** | **62904** | **58879** |
| 35 | 604910 | **503810** | **458473** | **547508** | **514181** | 83 | 70029 | **59383** | **52159** | **63937** | **59939** |
| 36 | **607474** | **505054** | 459636 | 549369 | **516823** | 84 | **73829** | **61214** | **53668** | **66899** | **64211** |
| 37 | 243964 | **204580** | **185889** | **221116** | **207942** | 85 | **226343** | **189012** | **172135** | **205306** | **192719** |
| 38 | **244442** | **204963** | 186167 | **221765** | **208640** | 86 | 226789 | **189478** | **172200** | **205687** | **193096** |
| 39 | 245439 | **205675** | **186496** | **222519** | **210217** | 87 | **228358** | **190182** | **172773** | **206943** | **194920** |
| 40 | 618605 | **516649** | **470747** | **560665** | **526602** | 88 | **598318** | **498630** | **454471** | **542036** | **508493** |
| 41 | **619069** | **516915** | **470827** | **561116** | **527232** | 89 | **599622** | **499322** | **454950** | **542877** | **509646** |
| 42 | **620316** | **517865** | **471286** | **562438** | 529209 | 90 | **603128** | **501102** | **456488** | **545463** | **513035** |
| 43 | 234555 | **196454** | **177912** | **212690** | **200197** | 91 | 229418 | **192087** | **174587** | **208323** | 195858 |
| 44 | **235019** | **196720** | **177992** | **213141** | **200827** | 92 | **229820** | **192661** | **174587** | **208894** | 196288 |
| 45 | 236273 | **197670** | **178451** | **214463** | 202804 | 93 | **231859** | **193814** | **175313** | **210715** | **198976** |
| 46 | **607260** | **506399** | **460910** | 549997 | **516740** | 94 | 601651 | **501935** | **457208** | **545304** | **511759** |
| 47 | **607937** | **507294** | **461042** | **550832** | **517377** | 95 | 602783 | **502903** | **457519** | **546337** | 512838 |
| 48 | **611036** | **508689** | **462166** | 553320 | 520919 | 96 | **606629** | **504734** | **459028** | **549299** | **517091** |

## APPENDIX C: Detailed Computational Results for Chapter 4

In this section, we provide the detailed results of the computational study presented in Chapter 4. Tables C1 and C2 present the detailed results obtained for the multi-vehicle PRP-VSP sets B1-VSP, B2-VSP, and B3-VSP. For each algorithm, the objective function values and the computational times are reported. The objective function values found by MA on the standard PRP instance sets A2 and A3 are given in Tables C3 and C4, respectively. Finally, the detailed computational results obtained for the IRP with the VSP data sets S-IRP-VSP and L-IRP-VSP are reported in Tables C5 through C9.

Table C.1 Detailed results obtained for the sets B1-VSP, B2-VSP, and B3-VSP

| | | | MA-ACC | | MA-SV | | IM-MVVSP | | | | | MA-ACC | | MA-SV | | IM-MVVSP | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| set | inst. | N.Ret. | Obj. | t(s) | Obj. | t(s) | Obj. | t(s) | set | inst. | N.Ret. | Obj. | t(s) | Obj. | t(s) | Obj. | t(s) |
| B1-VSP | 1 | 50 | 354493 | 935 | 361950 | 4302 | 362280 | 1930 | B2-VSP | 16 | 100 | 664715 | 3879 | 668094 | 7200 | 668227 | 4935 |
| B1-VSP | 2 | 50 | 353523 | 1357 | 356939 | 4770 | 354195 | 1896 | B2-VSP | 17 | 100 | 679435 | 3712 | 720564 | 7200 | 684583 | 4915 |
| B1-VSP | 3 | 50 | 351099 | 1356 | 365148 | 4592 | 344016 | 2023 | B2-VSP | 18 | 100 | 645739 | 3745 | 665051 | 7200 | 647995 | 5067 |
| B1-VSP | 4 | 50 | 361700 | 1013 | 354024 | 4320 | 355225 | 1862 | B2-VSP | 19 | 100 | 671184 | 3711 | 706980 | 7200 | 678858 | 5033 |
| B1-VSP | 5 | 50 | 355309 | 1307 | 373716 | 4479 | 360964 | 1640 | B2-VSP | 20 | 100 | 677584 | 4063 | 701509 | 7200 | 675376 | 5020 |
| B1-VSP | 6 | 50 | 361481 | 1270 | 364611 | 4350 | 365247 | 1652 | B2-VSP | 21 | 100 | 657896 | 4076 | 659655 | 7200 | 658230 | 5038 |
| B1-VSP | 7 | 50 | 369750 | 1194 | 392386 | 4421 | 367628 | 2017 | B2-VSP | 22 | 100 | 677219 | 3691 | 737405 | 7200 | 679375 | 5048 |
| B1-VSP | 8 | 50 | 357090 | 1333 | 359085 | 4358 | 356329 | 1749 | B2-VSP | 23 | 100 | 646200 | 3994 | 646279 | 7200 | 647253 | 4947 |
| B1-VSP | 9 | 50 | 370136 | 1329 | 373198 | 4402 | 373807 | 1658 | B2-VSP | 24 | 100 | 651576 | 3876 | 652665 | 7200 | 649877 | 5303 |
| B1-VSP | 10 | 50 | 353957 | 1348 | 348839 | 4753 | 354029 | 2023 | B2-VSP | 25 | 100 | 680640 | 4040 | 681431 | 7200 | 666042 | 5221 |
| B1-VSP | 11 | 50 | 349768 | 1166 | 354112 | 4344 | 351713 | 1668 | B2-VSP | 26 | 100 | 647901 | 3622 | 699394 | 7200 | 648478 | 5018 |
| B1-VSP | 12 | 50 | 363302 | 1226 | 363547 | 4424 | 353071 | 1986 | B2-VSP | 27 | 100 | 650904 | 3697 | 693159 | 7200 | 653398 | 5354 |
| B1-VSP | 13 | 50 | 356394 | 954 | 356191 | 4575 | 356347 | 1915 | B2-VSP | 28 | 100 | 649648 | 3674 | 669240 | 7200 | 644717 | 5302 |
| B1-VSP | 14 | 50 | 359136 | 861 | 372866 | 4486 | 364303 | 2046 | B2-VSP | 29 | 100 | 667171 | 3624 | 681291 | 7200 | 679407 | 5224 |
| B1-VSP | 15 | 50 | 370165 | 1066 | 358384 | 4734 | 360805 | 1740 | B2-VSP | 30 | 100 | 670053 | 3954 | 732921 | 7200 | 676252 | 5091 |
| B1-VSP | 16 | 50 | 342925 | 1214 | 368214 | 4670 | 349114 | 1994 | B3-VSP | 1 | 200 | 994708 | 7200 | 1047416 | 7200 | 1026366 | 7200 |
| B1-VSP | 17 | 50 | 373246 | 986 | 372415 | 4449 | 374462 | 1938 | B3-VSP | 2 | 200 | 1019617 | 7200 | 1066676 | 7200 | 1035239 | 7200 |
| B1-VSP | 18 | 50 | 363843 | 890 | 383437 | 4435 | 364467 | 1777 | B3-VSP | 3 | 200 | 990269 | 7200 | 1143386 | 7200 | 1040498 | 7200 |
| B1-VSP | 19 | 50 | 361859 | 1034 | 378271 | 4616 | 365491 | 1649 | B3-VSP | 4 | 200 | 1047368 | 7200 | 1051401 | 7200 | 984587 | 7200 |
| B1-VSP | 20 | 50 | 365073 | 1028 | 367011 | 4564 | 358196 | 1641 | B3-VSP | 5 | 200 | 973701 | 7200 | 1012574 | 7200 | 996817 | 7200 |
| B1-VSP | 21 | 50 | 358859 | 1276 | 365082 | 4457 | 360789 | 1952 | B3-VSP | 6 | 200 | 1024702 | 7200 | 1052496 | 7200 | 1018474 | 7200 |
| B1-VSP | 22 | 50 | 340218 | 1205 | 356867 | 4384 | 351356 | 1934 | B3-VSP | 7 | 200 | 1021448 | 7200 | 1026423 | 7200 | 1008916 | 7200 |
| B1-VSP | 23 | 50 | 367630 | 1354 | 374409 | 4662 | 367244 | 1962 | B3-VSP | 8 | 200 | 1003816 | 7200 | 1012699 | 7200 | 1004500 | 7200 |
| B1-VSP | 24 | 50 | 354893 | 1290 | 355904 | 4436 | 359166 | 1755 | B3-VSP | 9 | 200 | 982285 | 7200 | 983910 | 7200 | 967028 | 7200 |
| B1-VSP | 25 | 50 | 362603 | 1027 | 370325 | 4653 | 371432 | 1946 | B3-VSP | 10 | 200 | 1005594 | 7200 | 1051296 | 7200 | 1004703 | 7200 |
| B1-VSP | 26 | 50 | 368208 | 880 | 386872 | 4620 | 376290 | 1929 | B3-VSP | 11 | 200 | 1015393 | 7200 | 1076305 | 7200 | 999903 | 7200 |
| B1-VSP | 27 | 50 | 343183 | 1025 | 359527 | 4633 | 344539 | 1877 | B3-VSP | 12 | 200 | 1055493 | 7200 | 1086652 | 7200 | 1020726 | 7200 |
| B1-VSP | 28 | 50 | 368309 | 1194 | 367348 | 4389 | 347718 | 1986 | B3-VSP | 13 | 200 | 1019026 | 7200 | 1036274 | 7200 | 1028568 | 7200 |
| B1-VSP | 29 | 50 | 365646 | 968 | 371197 | 4365 | 379923 | 1732 | B3-VSP | 14 | 200 | 1005771 | 7200 | 1022423 | 7200 | 1005940 | 7200 |
| B1-VSP | 30 | 50 | 358096 | 1373 | 362848 | 4458 | 359726 | 1997 | B3-VSP | 15 | 200 | 1040542 | 7200 | 1141480 | 7200 | 1077524 | 7200 |

*Notes.* N.Ret., retailer number

Table C.2 Detailed results obtained for the sets B1-VSP, B2-VSP, and B3-VSP (cont.)

| | | | MA-ACC | | MA-SV | | IM-MVVSP | | | | | MA-ACC | | MA-SV | | IM-MVVSP | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| set | inst. | N.Ret. | Obj. | t(s) | Obj. | t(s) | Obj. | t(s) | set | inst. | N.Ret. | Obj. | t(s) | Obj. | t(s) | Obj. | t(s) |
| B2-VSP | 1 | 100 | 655166 | 3773 | 735474 | 7200 | 655337 | 5342 | B3-VSP | 16 | 200 | 997321 | 7200 | 1016606 | 7200 | 1003352 | 7200 |
| B2-VSP | 2 | 100 | 685367 | 4049 | 668348 | 7200 | 668162 | 5191 | B3-VSP | 17 | 200 | 1002969 | 7200 | 1012467 | 7200 | 1002050 | 7200 |
| B2-VSP | 3 | 100 | 648952 | 3974 | 645901 | 7200 | 644254 | 5303 | B3-VSP | 18 | 200 | 1008092 | 7200 | 1010016 | 7200 | 1004067 | 7200 |
| B2-VSP | 4 | 100 | 657327 | 3623 | 642497 | 7200 | 649827 | 5055 | B3-VSP | 19 | 200 | 1067445 | 7200 | 1227680 | 7200 | 1078796 | 7200 |
| B2-VSP | 5 | 100 | 654072 | 3958 | 682222 | 7200 | 668943 | 4939 | B3-VSP | 20 | 200 | 1017029 | 7200 | 1088448 | 7200 | 1026673 | 7200 |
| B2-VSP | 6 | 100 | 677680 | 3784 | 688882 | 7200 | 656773 | 5074 | B3-VSP | 21 | 200 | 1012489 | 7200 | 1019160 | 7200 | 1016421 | 7200 |
| B2-VSP | 7 | 100 | 649268 | 4055 | 718882 | 7200 | 672499 | 5208 | B3-VSP | 22 | 200 | 974060 | 7200 | 1010689 | 7200 | 1010003 | 7200 |
| B2-VSP | 8 | 100 | 646623 | 4080 | 705992 | 7200 | 669100 | 5262 | B3-VSP | 23 | 200 | 987213 | 7200 | 1026942 | 7200 | 988239 | 7200 |
| B2-VSP | 9 | 100 | 673768 | 3755 | 697933 | 7200 | 674087 | 5266 | B3-VSP | 24 | 200 | 1012213 | 7200 | 1042951 | 7200 | 1010279 | 7200 |
| B2-VSP | 10 | 100 | 660891 | 3878 | 721539 | 7200 | 662326 | 5264 | B3-VSP | 25 | 200 | 1001802 | 7200 | 1147629 | 7200 | 1090220 | 7200 |
| B2-VSP | 11 | 100 | 656159 | 3724 | 707751 | 7200 | 657128 | 4868 | B3-VSP | 26 | 200 | 993689 | 7200 | 1012973 | 7200 | 1008796 | 7200 |
| B2-VSP | 12 | 100 | 669323 | 3697 | 697820 | 7200 | 683933 | 5260 | B3-VSP | 27 | 200 | 1024673 | 7200 | 1030134 | 7200 | 995579 | 7200 |
| B2-VSP | 13 | 100 | 655331 | 3788 | 728714 | 7200 | 658880 | 5260 | B3-VSP | 28 | 200 | 1004801 | 7200 | 1013189 | 7200 | 997693 | 7200 |
| B2-VSP | 14 | 100 | 685968 | 3722 | 690854 | 7200 | 659576 | 5324 | B3-VSP | 29 | 200 | 976400 | 7200 | 1223207 | 7200 | 1009134 | 7200 |
| B2-VSP | 15 | 100 | 663137 | 3649 | 680738 | 7200 | 667374 | 5083 | B3-VSP | 30 | 200 | 983920 | 7200 | 1013465 | 7200 | 1007845 | 7200 |

*Notes.* N.Ret., retailer number

Table C.3 Objective function values found by MA for the set A2

| | Ins. | | | | | | Ins. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| N | 1 | 2 | 3 | 4 | 5 | N | 1 | 2 | 3 | 4 | 5 |
| 1 | 114352 | 110296 | 103840 | 117773 | 103054 | 49 | 148532 | 140375 | 133299 | 153956 | 137994 |
| 2 | 115281 | 110902 | 104497 | 119068 | 104026 | 50 | 153309 | 144266 | 137553 | 161385 | 146129 |
| 3 | 117199 | 112244 | 106429 | 121590 | 107116 | 51 | 164085 | 153580 | 147676 | 173619 | 161640 |
| 4 | 235088 | 228115 | 219884 | 251229 | 214357 | 52 | 263644 | 252306 | 246399 | 282172 | 246445 |
| 5 | 236003 | 229277 | 220865 | 252835 | 215767 | 53 | 268620 | 258067 | 251295 | 289753 | 253824 |
| 6 | 238277 | 231210 | 222741 | 255415 | 218956 | 54 | 280054 | 267574 | 260680 | 302738 | 269730 |
| 7 | 91285 | 87435 | 84003 | 96700 | 83632 | 55 | 119288 | 111406 | 110284 | 127394 | 115911 |
| 8 | 92239 | 88612 | 84987 | 98194 | 85080 | 56 | 125046 | 116726 | 115488 | 134398 | 123288 |
| 9 | 94673 | 90560 | 86940 | 100773 | 88255 | 57 | 135938 | 126828 | 124886 | 147930 | 138868 |
| 10 | 207774 | 202035 | 197104 | 225760 | 191832 | 58 | 236333 | 225916 | 223420 | 256762 | 223474 |
| 11 | 208784 | 203151 | 198085 | 227171 | 193322 | 59 | 241090 | 231319 | 227961 | 263394 | 231023 |
| 12 | 211045 | 205088 | 199983 | 229789 | 196523 | 60 | 252234 | 241475 | 237785 | 276719 | 247290 |
| 13 | 123830 | 119310 | 112067 | 128494 | 113336 | 61 | 182302 | 167858 | 167525 | 191967 | 178474 |
| 14 | 126073 | 120099 | 114073 | 131177 | 115791 | 62 | 196189 | 179437 | 175074 | 206312 | 193269 |
| 15 | 130127 | 123513 | 117575 | 136132 | 122366 | 63 | 216121 | 198596 | 195392 | 233071 | 225395 |
| 16 | 242190 | 234136 | 226522 | 258822 | 222330 | 64 | 298976 | 282412 | 279382 | 320655 | 286403 |
| 17 | 244128 | 236470 | 228483 | 261987 | 225264 | 65 | 309304 | 292934 | 288828 | 334872 | 301382 |
| 18 | 248729 | 240298 | 232245 | 267258 | 231625 | 66 | 332053 | 312671 | 308758 | 362042 | 333180 |
| 19 | 98439 | 93453 | 90593 | 104423 | 91549 | 67 | 152925 | 140649 | 141515 | 165722 | 156613 |
| 20 | 100357 | 95545 | 92486 | 107554 | 94611 | 68 | 164204 | 151667 | 151801 | 178641 | 169365 |
| 21 | 105239 | 99627 | 96406 | 112598 | 100932 | 69 | 187269 | 171578 | 171260 | 206822 | 203430 |
| 22 | 214970 | 208053 | 203693 | 233334 | 199822 | 70 | 271296 | 255467 | 256079 | 294821 | 261679 |
| 23 | 216935 | 210084 | 205711 | 236461 | 202825 | 71 | 282225 | 266062 | 265868 | 308368 | 278859 |
| 24 | 221408 | 214171 | 209543 | 241524 | 209185 | 72 | 303528 | 285691 | 285755 | 335901 | 309943 |
| 25 | 725222 | 710182 | 695271 | 796706 | 668406 | 73 | 76861 | 74585 | 74342 | 85038 | 72438 |
| 26 | 726356 | 710781 | 696163 | 797986 | 669794 | 74 | 77797 | 75717 | 75263 | 86298 | 74251 |
| 27 | 728136 | 712155 | 697849 | 800538 | 672740 | 75 | 80063 | 77606 | 77292 | 89234 | 77303 |
| 28 | 1862230 | 1825768 | 1795039 | 2060054 | 1720319 | 76 | 191966 | 188864 | 186329 | 214075 | 178975 |
| 29 | 1863243 | 1826965 | 1795998 | 2061203 | 1721831 | 77 | 192934 | 189997 | 187243 | 215444 | 180467 |
| 30 | 1865707 | 1829061 | 1797903 | 2064187 | 1724827 | 78 | 194811 | 191675 | 189097 | 218211 | 183343 |
| 31 | 700257 | 685376 | 672926 | 773643 | 647045 | 79 | 83769 | 80390 | 80800 | 92640 | 80578 |
| 32 | 701416 | 686320 | 673863 | 774876 | 648606 | 80 | 85986 | 82662 | 82689 | 95247 | 83769 |
| 33 | 703518 | 688629 | 675849 | 778181 | 652019 | 81 | 90393 | 86129 | 86704 | 101039 | 90178 |
| 34 | 1820615 | 1786791 | 1758762 | 2020094 | 1683552 | 82 | 199862 | 195018 | 193666 | 221562 | 187613 |
| 35 | 1821655 | 1787914 | 1759644 | 2021662 | 1685157 | 83 | 201763 | 197165 | 195599 | 224192 | 191074 |
| 36 | 1823593 | 1789649 | 1761618 | 2024026 | 1688069 | 84 | 205923 | 200980 | 199462 | 229959 | 196961 |
| 37 | 734549 | 719079 | 704110 | 806977 | 678721 | 85 | 679313 | 667023 | 658057 | 755820 | 630016 |
| 38 | 736891 | 720422 | 705740 | 809958 | 681221 | 86 | 680279 | 668097 | 659179 | 757228 | 631690 |
| 39 | 740956 | 723247 | 709481 | 814891 | 687343 | 87 | 682372 | 669835 | 661048 | 759827 | 634623 |
| 40 | 1868736 | 1831727 | 1801840 | 2067312 | 1728154 | 88 | 1798287 | 1767099 | 1742956 | 2001025 | 1665721 |
| 41 | 1871179 | 1833837 | 1803381 | 2070181 | 1731491 | 89 | 1799256 | 1768184 | 1743883 | 2002432 | 1667253 |
| 42 | 1875651 | 1838276 | 1807437 | 2076350 | 1737947 | 90 | 1801129 | 1769909 | 1745771 | 2005195 | 1670304 |
| 43 | 707089 | 691220 | 679647 | 781353 | 654849 | 91 | 686577 | 672892 | 664756 | 764323 | 637961 |
| 44 | 708793 | 693406 | 681583 | 784051 | 658534 | 92 | 688677 | 675339 | 666823 | 766399 | 641760 |
| 45 | 713864 | 697486 | 685072 | 790040 | 664696 | 93 | 692775 | 678846 | 670333 | 772603 | 647861 |
| 46 | 1828153 | 1793693 | 1765608 | 2028526 | 1691981 | 94 | 1806176 | 1774111 | 1750282 | 2009444 | 1674281 |
| 47 | 1830125 | 1795801 | 1767710 | 2030944 | 1695384 | 95 | 1807960 | 1776254 | 1752185 | 2012430 | 1677777 |
| 48 | 1834522 | 1799034 | 1771539 | 2036545 | 1701605 | 96 | 1812249 | 1779800 | 1756170 | 2017792 | 1683666 |

Table C.4 Objective function values found by MA for the set A3

| N | Ins. 1 | 2 | 3 | 4 | 5 | N | Ins. 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 200912 | 194850 | 197954 | 195800 | 205675 | 49 | 264260 | 252024 | 267296 | 249426 | 271254 |
| 2 | 203249 | 196838 | 200708 | 197404 | 208213 | 50 | 274540 | 263510 | 280946 | 259753 | 285075 |
| 3 | 207721 | 201169 | 206493 | 201502 | 213171 | 51 | 297811 | 285860 | 307518 | 279982 | 314278 |
| 4 | 429546 | 408066 | 420863 | 402445 | 436590 | 52 | 481360 | 457342 | 476681 | 445238 | 491164 |
| 5 | 431755 | 410203 | 423718 | 404058 | 439755 | 53 | 492797 | 467455 | 492141 | 455293 | 506306 |
| 6 | 436327 | 414444 | 429283 | 408233 | 444837 | 54 | 516710 | 489915 | 521182 | 477526 | 535109 |
| 7 | 167595 | 158784 | 163512 | 153725 | 170637 | 55 | 217439 | 204678 | 216849 | 196996 | 222865 |
| 8 | 169748 | 161055 | 166636 | 155713 | 173085 | 56 | 229071 | 216159 | 232027 | 207793 | 237450 |
| 9 | 174800 | 165316 | 172634 | 160190 | 178958 | 57 | 253362 | 239620 | 261112 | 227970 | 266383 |
| 10 | 386183 | 362350 | 376570 | 351253 | 391194 | 58 | 436222 | 408148 | 429898 | 393463 | 443837 |
| 11 | 388567 | 364574 | 379692 | 353333 | 394236 | 59 | 447826 | 419235 | 444471 | 404175 | 459055 |
| 12 | 393378 | 369036 | 385605 | 357807 | 399940 | 60 | 471918 | 441426 | 473923 | 426126 | 487420 |
| 13 | 218338 | 210802 | 216550 | 211424 | 224905 | 61 | 331030 | 316034 | 340483 | 305764 | 340015 |
| 14 | 222672 | 215473 | 221974 | 214827 | 229366 | 62 | 352770 | 339987 | 363260 | 324927 | 368112 |
| 15 | 232421 | 223778 | 233486 | 223001 | 239389 | 63 | 397858 | 380733 | 422832 | 366090 | 426562 |
| 16 | 444575 | 422820 | 436122 | 413134 | 451798 | 64 | 543769 | 515828 | 543438 | 498361 | 557039 |
| 17 | 449058 | 426735 | 441824 | 417164 | 457305 | 65 | 566991 | 536805 | 573730 | 518790 | 587681 |
| 18 | 458245 | 435079 | 453691 | 426101 | 468920 | 66 | 615157 | 581195 | 633458 | 563680 | 645613 |
| 19 | 179897 | 170264 | 177302 | 164295 | 182963 | 67 | 281225 | 267744 | 286673 | 250760 | 291838 |
| 20 | 184740 | 174391 | 182631 | 168408 | 189069 | 68 | 305030 | 294376 | 311850 | 270722 | 320575 |
| 21 | 194364 | 183309 | 194776 | 177232 | 200476 | 69 | 354639 | 330002 | 373192 | 316979 | 375791 |
| 22 | 398646 | 373756 | 389639 | 361882 | 404329 | 70 | 498201 | 465906 | 496581 | 446891 | 510798 |
| 23 | 403355 | 378235 | 395898 | 365992 | 410356 | 71 | 521402 | 487789 | 524672 | 470535 | 539699 |
| 24 | 412902 | 387231 | 407880 | 374886 | 421920 | 72 | 569968 | 533828 | 584765 | 510466 | 596715 |
| 25 | 1376008 | 1288609 | 1342969 | 1256037 | 1395577 | 73 | 143523 | 133786 | 141060 | 129120 | 145867 |
| 26 | 1378298 | 1290671 | 1345561 | 1257697 | 1397956 | 74 | 145959 | 135887 | 144045 | 131337 | 148853 |
| 27 | 1382940 | 1295163 | 1351520 | 1261556 | 1403036 | 75 | 150929 | 140287 | 149954 | 135596 | 154610 |
| 28 | 3557097 | 3318987 | 3466434 | 3221876 | 3601724 | 76 | 362346 | 337420 | 354322 | 326756 | 367260 |
| 29 | 3559217 | 3320663 | 3469609 | 3224285 | 3604659 | 77 | 364739 | 339622 | 357229 | 328901 | 370110 |
| 30 | 3563980 | 3325568 | 3475463 | 3228440 | 3610569 | 78 | 369658 | 344048 | 363247 | 333209 | 375970 |
| 31 | 1337668 | 1247321 | 1302045 | 1208652 | 1354300 | 79 | 155825 | 144977 | 154379 | 139762 | 158923 |
| 32 | 1340071 | 1249351 | 1305150 | 1210651 | 1357187 | 80 | 160549 | 149429 | 160253 | 143817 | 164862 |
| 33 | 1345159 | 1253937 | 1311099 | 1215047 | 1362723 | 81 | 170467 | 158459 | 171956 | 152476 | 176387 |
| 34 | 3485194 | 3243140 | 3392343 | 3141789 | 3526111 | 82 | 374448 | 348639 | 367354 | 337158 | 380368 |
| 35 | 3487385 | 3245291 | 3395022 | 3143827 | 3528861 | 83 | 379285 | 353057 | 373323 | 341263 | 385890 |
| 36 | 3492524 | 3249834 | 3401159 | 3148207 | 3534250 | 84 | 389108 | 362166 | 384991 | 350122 | 397811 |
| 37 | 1393113 | 1304588 | 1361054 | 1270421 | 1414017 | 85 | 1301115 | 1209319 | 1267580 | 1170799 | 1317334 |
| 38 | 1397183 | 1308340 | 1366931 | 1274859 | 1418689 | 86 | 1303428 | 1211407 | 1270499 | 1172828 | 1320013 |
| 39 | 1406686 | 1317410 | 1378199 | 1282699 | 1429620 | 87 | 1308302 | 1215890 | 1276253 | 1177025 | 1325804 |
| 40 | 3569288 | 3331037 | 3480270 | 3232937 | 3614672 | 88 | 3446576 | 3203434 | 3355964 | 3102149 | 3487852 |
| 41 | 3574197 | 3334785 | 3486108 | 3237304 | 3621197 | 89 | 3448879 | 3205437 | 3358558 | 3104177 | 3490737 |
| 42 | 3584379 | 3343947 | 3497973 | 3245532 | 3632829 | 90 | 3453951 | 3210128 | 3364797 | 3108604 | 3496064 |
| 43 | 1350434 | 1259199 | 1315454 | 1219626 | 1367609 | 91 | 1314127 | 1221469 | 1281904 | 1181664 | 1331502 |
| 44 | 1355312 | 1263821 | 1321837 | 1223783 | 1373371 | 92 | 1318961 | 1225756 | 1287380 | 1186084 | 1336934 |
| 45 | 1365224 | 1272980 | 1333597 | 1232342 | 1384941 | 93 | 1329141 | 1234677 | 1299023 | 1194833 | 1348922 |
| 46 | 3498661 | 3255264 | 3406913 | 3153461 | 3540907 | 94 | 3460422 | 3215876 | 3370637 | 3114226 | 3502873 |
| 47 | 3503262 | 3259985 | 3412606 | 3157591 | 3546508 | 95 | 3464908 | 3220466 | 3376105 | 3118187 | 3508548 |
| 48 | 3513243 | 3268626 | 3423995 | 3166113 | 3557667 | 96 | 3475055 | 3229320 | 3388066 | 3126900 | 3519634 |

Table C.5 Detailed Computational results for the set S-IRP-VSP

| | | | Low Inventory Holding Cost | | | | | High Inventory Holding Cost | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | MA-ACC | | MA-SV | | ALNS | MA-ACC | | MA-SV | | ALNS |
| inst. | N.Ret. | N.Per. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. |
| 1 | 5 | 3 | 1540.57 | 0.7 | 1430.51 | 3.1 | 1430.51 | 2410.89 | 0.7 | 2300.47 | 2.1 | 2300.47 |
| 2 | 5 | 3 | 1640.66 | 0.5 | 1582.69 | 1.4 | 1582.69 | 2398.72 | 0.4 | 2369.93 | 2.2 | 2369.93 |
| 3 | 5 | 3 | 3497.96 | 0.6 | 3497.96 | 2.9 | 3497.96 | 4704.32 | 0.7 | 4704.32 | 3.5 | 4704.32 |
| 4 | 5 | 3 | 2702.38 | 0.5 | 2553.83 | 2.7 | 2553.69 | 3259.82 | 0.5 | 3141.77 | 2.6 | 3140.09 |
| 5 | 5 | 3 | 1677.21 | 0.5 | 1516.65 | 1.6 | 1516.65 | 2829.36 | 0.5 | 2679.60 | 2.6 | 2679.60 |
| 1 | 10 | 3 | 3100.87 | 2.4 | 2766.61 | 10.3 | 2766.61 | 6055.53 | 2.2 | 5540.09 | 10.4 | 5540.09 |
| 2 | 10 | 3 | 3716.37 | 2.7 | 3692.16 | 9.4 | 3691.69 | 5960.79 | 2.2 | 5972.90 | 10.8 | 5960.79 |
| 3 | 10 | 3 | 2666.69 | 2.3 | 2649.26 | 10.9 | 2649.26 | 4919.04 | 1.7 | 4808.06 | 8.6 | 4808.06 |
| 4 | 10 | 3 | 3405.95 | 1.3 | 3405.95 | 6.0 | 3253.32 | 5586.58 | 1.3 | 5570.30 | 5.1 | 5417.62 |
| 5 | 10 | 3 | 2648.07 | 1.7 | 2648.53 | 6.3 | 2647.60 | 5423.29 | 1.7 | 5423.29 | 9.4 | 5423.29 |
| 1 | 15 | 3 | 2800.48 | 17.7 | 2800.48 | 77.1 | 2798.14 | 6263.58 | 24.2 | 6252.54 | 93.0 | 6252.54 |
| 2 | 15 | 3 | 3202.46 | 16.0 | 2757.82 | 90.7 | 2757.82 | 6193.13 | 17.7 | 6071.26 | 61.2 | 6071.26 |
| 3 | 15 | 3 | 3160.96 | 17.6 | 3072.84 | 76.1 | 3072.84 | 7007.25 | 15.4 | 6926.18 | 82.4 | 6926.18 |
| 4 | 15 | 3 | 3008.26 | 10.8 | 2900.32 | 48.7 | 2900.32 | 5879.33 | 15.6 | 5721.52 | 56.6 | 5721.52 |
| 5 | 15 | 3 | 3423.43 | 30.0 | 3405.31 | 133.7 | 3405.31 | 6182.91 | 20.8 | 6126.87 | 99.9 | 6126.87 |
| 1 | 20 | 3 | 3606.00 | 35.6 | 3648.36 | 170.0 | 3614.72 | 8168.65 | 55.2 | 8232.92 | 249.1 | 8166.94 |
| 2 | 20 | 3 | 3282.19 | 21.7 | 2908.48 | 94.8 | 2908.48 | 7657.05 | 43.7 | 7505.69 | 153.4 | 7499.50 |
| 3 | 20 | 3 | 3121.48 | 26.4 | 3064.78 | 146.8 | 3064.78 | 7967.14 | 28.3 | 7840.54 | 89.1 | 7840.54 |
| 4 | 20 | 3 | 4324.62 | 31.5 | 4323.32 | 114.5 | 4324.02 | 8256.71 | 24.7 | 8157.53 | 137.5 | 8171.53 |
| 5 | 20 | 3 | 4482.64 | 33.4 | 4303.24 | 147.7 | 4309.84 | 9520.80 | 33.2 | 9324.66 | 127.3 | 9324.66 |

*Notes.* N.Ret., retailer number, N.Per., number of periods

Table C.6 Detailed Computational results for the set S-IRP-VSP (Cont.)

| | | | Low Inventory Holding Cost | | | | | High Inventory Holding Cost | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | MA-ACC | | MA-SV | | ALNS | MA-ACC | | MA-SV | | ALNS |
| inst. | N.Ret. | N.Per. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. |
| 1 | 25 | 3 | 3670.58 | 57.8 | 3507.06 | 318.7 | 3503.38 | 9388.52 | 79.3 | 8924.88 | 377.6 | 8893.82 |
| 2 | 25 | 3 | 3969.00 | 63.6 | 3966.44 | 326.1 | 3992.52 | 10008.62 | 46.4 | 9721.74 | 181.6 | 9747.74 |
| 3 | 25 | 3 | 4078.13 | 49.4 | 4068.66 | 268.1 | 4092.98 | 10424.03 | 56.6 | 10404.88 | 209.0 | 10427.80 |
| 4 | 25 | 3 | 3771.21 | 55.1 | 3679.93 | 277.4 | 3679.93 | 9232.10 | 69.8 | 9057.57 | 301.7 | 9071.01 |
| 5 | 25 | 3 | 4381.84 | 31.5 | 4120.26 | 118.5 | 4120.26 | 11839.96 | 39.3 | 11250.92 | 150.7 | 11253.90 |
| 1 | 30 | 3 | 4304.49 | 79.5 | 4251.64 | 274.2 | 4286.08 | 12947.12 | 85.5 | 12908.90 | 493.9 | 12919.60 |
| 2 | 30 | 3 | 4809.82 | 57.0 | 4155.38 | 291.4 | 4196.36 | 11835.50 | 73.1 | 11749.68 | 310.8 | 11783.80 |
| 3 | 30 | 3 | 4254.26 | 48.2 | 3820.66 | 165.5 | 3820.66 | 13175.76 | 39.7 | 12487.96 | 134.9 | 12555.40 |
| 4 | 30 | 3 | 4155.42 | 63.8 | 4066.60 | 275.1 | 4087.46 | 10451.86 | 71.9 | 10420.40 | 300.7 | 10447.40 |
| 5 | 30 | 3 | 3589.54 | 71.0 | 3588.64 | 352.9 | 3588.64 | 10454.13 | 61.6 | 10450.42 | 261.2 | 10450.40 |
| 1 | 35 | 3 | 4099.23 | 118.5 | 4084.52 | 421.5 | 4179.86 | 12435.35 | 153.5 | 12395.96 | 513.5 | 12492.60 |
| 2 | 35 | 3 | 4419.43 | 118.0 | 4269.06 | 643.5 | 4293.58 | 11577.45 | 95.2 | 11191.68 | 446.7 | 11195.80 |
| 3 | 35 | 3 | 4755.75 | 72.0 | 4727.58 | 221.9 | 4829.82 | 14759.72 | 69.2 | 14697.56 | 254.0 | 14783.10 |
| 4 | 35 | 3 | 4033.57 | 163.4 | 4012.26 | 543.5 | 4011.04 | 11125.04 | 103.9 | 11093.08 | 478.9 | 11114.60 |
| 5 | 35 | 3 | 4036.69 | 93.7 | 4035.58 | 278.1 | 4085.60 | 11582.46 | 102.9 | 11568.34 | 576.7 | 11633.90 |
| 1 | 40 | 3 | 5943.87 | 84.4 | 4532.88 | 402.0 | 4617.74 | 15480.58 | 70.5 | 14256.52 | 359.7 | 14378.40 |
| 2 | 40 | 3 | 4839.81 | 107.8 | 4687.81 | 620.5 | 4738.95 | 12320.87 | 121.9 | 12132.63 | 642.9 | 12213.10 |
| 3 | 40 | 3 | 4773.26 | 83.8 | 4310.24 | 249.2 | 4439.34 | 14071.12 | 88.1 | 14040.36 | 351.9 | 14153.40 |
| 4 | 40 | 3 | 3995.86 | 111.1 | 3994.38 | 598.6 | 4093.16 | 11905.67 | 91.9 | 11848.98 | 410.1 | 11996.80 |
| 5 | 40 | 3 | 4507.47 | 78.6 | 4423.36 | 255.6 | 4428.78 | 13916.56 | 106.4 | 13911.00 | 312.9 | 13933.80 |

*Notes.* N.Ret., retailer number, N.Per., number of periods

Table C.7 Detailed Computational results for the set S-IRP-VSP (Cont.)

| | | | Low Inventory Holding Cost | | | | | High Inventory Holding Cost | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | MA-ACC | | MA-SV | | ALNS | MA-ACC | | MA-SV | | ALNS |
| inst. | N.Ret. | N.Per. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. |
| 1 | 45 | 3 | 4567.55 | 117.6 | 4537.30 | 390.7 | 4592.30 | 15535.14 | 91.7 | 14770.98 | 491.4 | 14887.30 |
| 2 | 45 | 3 | 4899.51 | 84.2 | 4744.96 | 373.5 | 4961.36 | 14348.04 | 114.1 | 14270.08 | 319.9 | 14312.00 |
| 3 | 45 | 3 | 4272.32 | 133.8 | 3965.34 | 743.1 | 4356.06 | 15184.03 | 146.2 | 12088.54 | 446.0 | 15223.10 |
| 4 | 45 | 3 | 5586.15 | 79.1 | 4988.50 | 218.3 | 5108.60 | 14561.83 | 85.6 | 14549.88 | 472.3 | 14647.70 |
| 5 | 45 | 3 | 4143.28 | 144.3 | 4154.34 | 818.0 | 4190.13 | 14032.13 | 139.9 | 12563.59 | 667.0 | 14113.40 |
| 1 | 50 | 3 | 5265.32 | 101.2 | 5202.18 | 333.9 | 5304.94 | 16545.92 | 85.7 | 15734.80 | 428.4 | 15905.60 |
| 2 | 50 | 3 | 5745.90 | 118.3 | 5683.76 | 646.9 | 5772.86 | 16232.74 | 137.8 | 16201.86 | 459.0 | 16354.50 |
| 3 | 50 | 3 | 5269.60 | 128.7 | 5259.49 | 646.8 | 5430.33 | 16709.91 | 91.3 | 13082.63 | 316.7 | 16155.40 |
| 4 | 50 | 3 | 5307.67 | 141.2 | 5286.44 | 499.8 | 5380.90 | 17455.22 | 126.6 | 17419.46 | 409.2 | 17563.40 |
| 5 | 50 | 3 | 5089.13 | 223.4 | 5088.26 | 885.1 | 5135.26 | 16861.12 | 120.5 | 16683.88 | 432.3 | 16661.30 |
| 1 | 5 | 6 | 4953.67 | 1.8 | 4782.66 | 10.0 | 4782.66 | 7387.11 | 1.5 | 7384.86 | 8.6 | 7384.86 |
| 2 | 5 | 6 | 4444.36 | 1.3 | 4259.62 | 6.5 | 4259.62 | 7000.15 | 1.3 | 6583.67 | 6.6 | 6583.67 |
| 1 | 10 | 6 | 8122.73 | 3.6 | 7552.85 | 19.1 | 7552.85 | 12284.59 | 3.1 | 11868.63 | 12.8 | 11868.6 |
| 2 | 10 | 6 | 9385.96 | 3.8 | 9113.86 | 19.4 | 8813.06 | 12567.97 | 3.4 | 12356.48 | 17.2 | 12123.9 |
| 3 | 10 | 6 | 6532.59 | 2.8 | 6503.10 | 12.6 | 6443.6 | 11030.57 | 2.8 | 10324.39 | 12.7 | 10258.6 |
| 4 | 10 | 6 | 8354.71 | 4.6 | 8055.85 | 13.1 | 7997.35 | 12153.75 | 3.2 | 11754.52 | 15.9 | 11670.6 |
| 5 | 10 | 6 | 6461.45 | 3.4 | 6232.26 | 19.2 | 6202.56 | 11121.21 | 4.5 | 11149.65 | 24.9 | 11075.8 |
| 1 | 15 | 6 | 7123.69 | 51.5 | 7124.79 | 245.3 | 7123.69 | 14010.88 | 55.8 | 13772.73 | 311.1 | 13828.1 |
| 2 | 15 | 6 | 7722.32 | 44.7 | 7446.68 | 159.2 | 7464.56 | 14074.25 | 34.2 | 13883.16 | 161.1 | 13903.2 |
| 3 | 15 | 6 | 8624.62 | 32.5 | 8415.93 | 176.8 | 8468.16 | 16434.64 | 36.8 | 15870.31 | 167.3 | 15878.4 |
| 4 | 15 | 6 | 8267.77 | 57.3 | 7504.33 | 261.5 | 7525.67 | 13835.35 | 47.8 | 12670.97 | 267.1 | 12575.4 |
| 5 | 15 | 6 | 8577.65 | 27.8 | 8170.22 | 92.6 | 8026.59 | 13294.06 | 22.7 | 13194.89 | 144.9 | 13034.9 |

*Notes.* N.Ret., retailer number, N.Per., number of periods

Table C.8 Detailed Computational results for the set S-IRP-VSP (Cont.)

| | | | Low Inventory Holding Cost | | | | | High Inventory Holding Cost | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | MA-ACC | | MA-SV | | ALNS | MA-ACC | | MA-SV | | ALNS |
| inst. | N.Ret. | N.Per. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. |
| 1 | 20 | 6 | 9422.97 | 133.8 | 9109.52 | 613.4 | 9255.97 | 17720.76 | 107.3 | 17384.46 | 677.1 | 17216.9 |
| 2 | 20 | 6 | 7600.08 | 162.3 | 7324.34 | 603.8 | 7369.69 | 16154.67 | 102.8 | 15854.16 | 556.3 | 15838.5 |
| 3 | 20 | 6 | 9191.90 | 67.5 | 8823.94 | 325.8 | 8814.72 | 16750.15 | 106.1 | 16345.78 | 710.6 | 16227.5 |
| 4 | 20 | 6 | 10911.07 | 121.1 | 10457.63 | 341.9 | 10502.1 | 18058.60 | 109.6 | 17857.66 | 521.6 | 17802.5 |
| 5 | 20 | 6 | 11320.48 | 130.4 | 11209.75 | 638.1 | 11137.6 | 20290.28 | 99.6 | 20001.29 | 513.0 | 19703.4 |
| 1 | 25 | 6 | 8983.95 | 182.3 | 8913.98 | 878.7 | 8944.51 | 17516.07 | 185.8 | 17496.56 | 936.7 | 17289.4 |
| 2 | 25 | 6 | 10264.74 | 133.8 | 10061.95 | 518.4 | 10199.1 | 19987.82 | 102.9 | 19318.13 | 410.2 | 19477.7 |
| 3 | 25 | 6 | 11317.59 | 173.5 | 10771.42 | 785.3 | 11022.5 | 21794.74 | 227.0 | 21139.00 | 870.2 | 21338.8 |
| 4 | 25 | 6 | 9220.43 | 122.1 | 9240.01 | 456.1 | 9540.92 | 18402.16 | 132.8 | 18074.53 | 568.8 | 17956.2 |
| 5 | 25 | 6 | 11350.23 | 142.5 | 11071.57 | 516.5 | 11142.1 | 22748.05 | 144.7 | 22420.68 | 967.2 | 22541.9 |
| 1 | 30 | 6 | 11296.51 | 183.9 | 10875.24 | 666.7 | 11230.5 | 26068.96 | 214.8 | 25778.81 | 1027.2 | 25993.1 |
| 2 | 30 | 6 | 10782.80 | 209.7 | 9944.93 | 752.8 | 10177.5 | 22849.13 | 205.8 | 22333.58 | 827.0 | 22534.2 |
| 3 | 30 | 6 | 10079.48 | 171.6 | 9867.10 | 885.4 | 9958.46 | 24923.59 | 168.0 | 24942.18 | 699.2 | 25063.6 |
| 4 | 30 | 6 | 10158.03 | 151.1 | 9956.23 | 558.8 | 9944.73 | 20095.00 | 145.3 | 20033.95 | 590.2 | 19929.3 |
| 5 | 30 | 6 | 9679.76 | 175.4 | 9540.00 | 555.5 | 9714.93 | 21237.65 | 170.6 | 21040.68 | 1019.8 | 21211.9 |

*Notes.* N.Ret., retailer number, N.Per., number of periods

Table C.9 Detailed Computational results obtained for the set L-IRP-VSP

| | | Low Inventory Holding Cost | | | | | High Inventory Holding Cost | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | MA-ACC | | MA-SV | | ALNS | MA-ACC | | MA-SV | | ALNS |
| inst. | N.Ret. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. | Obj. Val. | Time (s) | Obj. Val. | Time (s) | Obj. Val. |
| 1 | 50 | 14773.27 | 292.8 | 13763.08 | 1940.5 | 15714.83 | 35108.25 | 318.7 | 34264.99 | 2074.8 | 41495.78 |
| 2 | 50 | 14951.61 | 276.1 | 14700.84 | 1809.0 | 15015.10 | 33892.64 | 373.6 | 34071.56 | 2846.5 | 41024.06 |
| 3 | 50 | 13600.69 | 333.0 | 13179.34 | 2907.3 | 14463.18 | 32352.40 | 360.6 | 33119.04 | 2974.4 | 39024.53 |
| 4 | 50 | 14098.89 | 262.3 | 14500.96 | 1781.0 | 16583.98 | 35611.58 | 379.7 | 35376.14 | 3024.1 | 36213.40 |
| 5 | 50 | 13607.33 | 388.2 | 13069.21 | 2357.1 | 15797.86 | 32997.64 | 292.9 | 32972.29 | 2202.4 | 33797.43 |
| 6 | 50 | 14189.62 | 406.8 | 14835.45 | 3424.5 | 16258.29 | 36534.40 | 415.0 | 36028.05 | 3527.2 | 37216.78 |
| 7 | 50 | 14843.95 | 299.8 | 14696.28 | 1959.7 | 16311.37 | 34723.77 | 332.7 | 33740.62 | 2737.9 | 41116.16 |
| 8 | 50 | 15483.32 | 340.7 | 15743.94 | 1980.9 | 17535.31 | 31249.97 | 277.7 | 31716.88 | 2096.5 | 35647.34 |
| 9 | 50 | 13572.11 | 321.1 | 13661.14 | 2150.0 | 15749.62 | 33805.12 | 351.6 | 34059.76 | 2474.5 | 36826.31 |
| 10 | 50 | 14310.04 | 350.5 | 13750.23 | 2204.2 | 15494.06 | 35257.34 | 389.9 | 35327.83 | 2431.1 | 41206.11 |
| 1 | 100 | 18566.69 | 974.9 | 18878.70 | 5880.0 | 29444.29 | 60285.31 | 964.5 | 60725.08 | 6780.4 | 61665.60 |
| 2 | 100 | 17369.90 | 863.2 | 17263.32 | 6194.1 | 31083.67 | 56311.97 | 873.4 | 55739.18 | 6605.9 | 58090.02 |
| 3 | 100 | 19074.07 | 850.7 | 19771.44 | 6696.2 | 27524.10 | 61896.87 | 886.8 | 62409.39 | 6969.3 | 73909.58 |
| 4 | 100 | 17152.17 | 841.7 | 17461.48 | 4921.6 | 32663.92 | 54549.97 | 907.0 | 54663.08 | 6970.3 | 60200.81 |
| 5 | 100 | 18261.35 | 883.8 | 18044.05 | 5935.8 | 34671.10 | 60459.28 | 1096.9 | 60500.64 | 6530.9 | 72183.32 |
| 6 | 100 | 19611.32 | 979.0 | 19407.66 | 7570.5 | 36721.44 | 59254.46 | 1071.7 | 59392.77 | 7885.0 | 72758.88 |
| 7 | 100 | 18633.03 | 773.4 | 17898.89 | 6370.0 | 30728.49 | 59038.27 | 825.0 | 58981.25 | 5761.0 | 61911.65 |
| 8 | 100 | 19228.11 | 920.2 | 19895.75 | 7994.4 | 29733.37 | 59248.27 | 848.7 | 59141.08 | 6727.9 | 70902.47 |
| 9 | 100 | 20137.87 | 740.0 | 20163.30 | 4981.9 | 32884.80 | 62888.11 | 831.3 | 63516.73 | 6117.4 | 71378.44 |
| 10 | 100 | 18417.34 | 1082.0 | 17693.36 | 8224.7 | 33995.74 | 59183.32 | 1099.1 | 59274.90 | 6667.0 | 70086.39 |
| 1 | 200 | 27500.99 | 3545.3 | 28513.80 | 25350.4 | 43560.41 | 103728.18 | 3692.9 | 101205.48 | 29304.0 | 126623.20 |
| 2 | 200 | 26021.28 | 3719.7 | 27905.77 | 23725.3 | 50848.68 | 101528.49 | 3612.1 | 102063.72 | 21747.9 | 135796.60 |
| 3 | 200 | 27723.62 | 3405.9 | 27752.52 | 23794.1 | 51612.19 | 98428.95 | 3783.0 | 99633.81 | 22381.1 | 134917.66 |
| 4 | 200 | 27312.18 | 3827.0 | 27449.49 | 26498.7 | 48402.07 | 99854.77 | 3604.6 | 103047.52 | 25527.6 | 129877.92 |
| 5 | 200 | 27068.19 | 3610.1 | 28244.32 | 30340.5 | 42484.71 | 98889.02 | 3488.2 | 100047.19 | 25743.0 | 129362.24 |
| 6 | 200 | 28354.84 | 2932.0 | 27992.83 | 25249.7 | 48852.28 | 112759.96 | 3506.6 | 99542.04 | 26395.8 | 129978.25 |
| 7 | 200 | 27582.53 | 3089.3 | 26997.29 | 26736.7 | 47496.06 | 90647.42 | 3564.7 | 89797.18 | 27692.8 | 123098.78 |
| 8 | 200 | 26549.21 | 3309.4 | 27074.67 | 28150.2 | 52301.34 | 93134.61 | 3605.8 | 94535.68 | 23351.9 | 123247.80 |
| 9 | 200 | 27464.77 | 3411.1 | 27986.57 | 28115.9 | 48182.84 | 95831.10 | 3546.6 | 97348.84 | 23886.9 | 117794.62 |
| 10 | 200 | 27379.62 | 3604.4 | 27380.60 | 21451.8 | 44648.12 | 100082.88 | 3718.8 | 101202.35 | 28353.5 | 116929.74 |

*Notes.* N.Ret., retailer number

## APPENDIX D: Detailed Computational Results for Chapter 5

In this section, we provide the detailed results of the computational study presented in Chapter 5. In Tables D1 and D2, the computational results obtained for the set A-PRPT are reported. In the tables, the problem specifications including the number of retailers and the number of time periods for each problem instance are given in the second and the third columns, respectively. For MPH, we report the best solution, average solution and the average computation time obtained from each instance. Detailed results of the computational study to reveal the effect of the transshipments on the overall system cost are provided in Tables D3 through D6.

Table D.1 Computational results obtained for the set A-PRPT

| | CPLEX | | | MPH | | |
|---|---|---|---|---|---|---|
| Ins. | L.B. | U.B. | t(s) | Min. | Avg. | t(s) |
| 1 | 36323.7 | 36327.3 | 57 | 36327.3 | 36344.4 | 10 |
| 2 | 36390.6 | 36508.8* | 3600 | 36508.8 | 36597.8 | 11 |
| 3 | 37583.1 | 37586.8 | 65 | 37586.8 | 37589.1 | 14 |
| 4 | 74927.1 | 74934.6 | 36 | 74934.6 | 75001.7 | 9 |
| 5 | 76002.6 | 76010.2 | 185 | 76029.7 | 76040.9 | 9 |
| 6 | 77328.7 | 77336.4 | 44 | 77336.4 | 77406.6 | 11 |
| 7 | 30152.4 | 30155.4 | 241 | 30178.4 | 30178.4 | 9 |
| 8 | 30620.1 | 30729.4* | 3600 | 30729.4 | 30729.4 | 11 |
| 9 | 31559.8 | 31939.4* | 3600 | 31943.8 | 32008.2 | 15 |
| 10 | 68616.4 | 68623.3 | 73 | 68623.3 | 68644.1 | 10 |
| 11 | 69467.1 | 69474 | 37 | 69474 | 69474.0 | 11 |
| 12 | 71257.9 | 71505.9* | 3600 | 71505.9 | 71553.7 | 15 |
| 13 | 41005.1 | 41009.2 | 164 | 41009.2 | 41102.4 | 10 |
| 14 | 41505.9 | 41742* | 3600 | 41742 | 41804.8 | 10 |
| 15 | 42117.9 | 42760.3* | 3600 | 42760.3 | 43010.7 | 12 |
| 16 | 79521.9 | 79529.8 | 71 | 79529.8 | 79629.6 | 10 |
| 17 | 81289.3 | 81742* | 3600 | 81728.9 | 81802.2 | 11 |
| 18 | 83431.3 | 84236.7* | 3600 | 84236.7 | 84283.4 | 13 |
| 19 | 33114.5 | 33117.8 | 179 | 33150.1 | 33150.1 | 9 |
| 20 | 33840.1 | 34106.9* | 3600 | 34118 | 34192.8 | 10 |
| 21 | 35082.4 | 36414.4* | 3600 | 36323.1 | 36327.5 | 14 |
| 22 | 71700.6 | 71707.8 | 237 | 71760.1 | 71760.1 | 10 |
| 23 | 73396.3 | 73527.7* | 3600 | 73612.3 | 73628.3 | 12 |
| 24 | 75829.3 | 76294* | 3600 | 76294 | 76337.9 | 15 |
| 25 | 236104 | 236127 | 125 | 236127 | 236146.0 | 7 |
| 26 | 236251 | 236309* | 3600 | 236309 | 236402.2 | 8 |
| 27 | 237363 | 237387 | 73 | 237387 | 237435.4 | 11 |
| 28 | 607674 | 607735 | 30 | 607735 | 607791.5 | 7 |
| 29 | 608749 | 608810 | 91 | 608810 | 608865.7 | 8 |
| 30 | 610075 | 610136 | 57 | 610193 | 610225.4 | 10 |
| 31 | 229932 | 229955 | 148 | 229978 | 229982.7 | 8 |
| 32 | 230465 | 230529* | 3600 | 230529 | 230539.9 | 9 |
| 33 | 231420 | 231739* | 3600 | 231762 | 231783.0 | 12 |
| 34 | 601363 | 601423 | 100 | 601423 | 601432.1 | 9 |
| 35 | 602214 | 602274 | 84 | 602274 | 602280.7 | 9 |
| 36 | 604130 | 604306* | 3600 | 604306 | 604340.3 | 12 |
| 37 | 240785 | 240809 | 115 | 240809 | 240929.1 | 9 |
| 38 | 241403 | 241542* | 3600 | 241542 | 241546.8 | 9 |
| 39 | 241923 | 242560* | 3600 | 242560 | 242782.6 | 10 |
| 40 | 612269 | 612330 | 88 | 612339 | 612449.8 | 9 |
| 41 | 614203 | 614529* | 3600 | 614542 | 614626.3 | 10 |
| 42 | 616327 | 617037* | 3600 | 617046 | 617140.4 | 12 |
| 43 | 232895 | 232918 | 194 | 232950 | 232950.0 | 9 |
| 44 | 233646 | 233907* | 3600 | 233907 | 233963.2 | 9 |
| 45 | 234962 | 236287* | 3600 | 236123 | 236141.7 | 12 |
| 46 | 604447 | 604508 | 273 | 604560 | 604571.1 | 9 |
| 47 | 606267 | 606328 | 166 | 606412 | 606414.0 | 10 |
| 48 | 608601 | 609094* | 3600 | 609094 | 609111.9 | 12 |

*Notes.* L.B., lower bound, U.B., upper bound,
* optimality is not confirmed after the specified CPU time

Table D.2 Computational results obtained for the set A-PRPT (cont.)

| | CPLEX | | | MPH | | |
|---|---|---|---|---|---|---|
| Ins. | L.B. | U.B. | t(s) | Min. | Avg. | t(s) |
| 49 | 64454.6 | 64461 | 397 | 64554 | 64718.6 | 8 |
| 50 | 65099.5 | 68058.3* | 3600 | 68103.3 | 68432.7 | 10 |
| 51 | 66717.3 | 73386* | 3600 | 73091.2 | 73389.8 | 12 |
| 52 | 103050 | 103060 | 420 | 103224 | 103250.0 | 9 |
| 53 | 105393 | 108073* | 3600 | 108118 | 108175.4 | 10 |
| 54 | 108548 | 114885* | 3600 | 114885 | 115203.9 | 13 |
| 55 | 55204.5 | 55210 | 595 | 55374 | 55474.9 | 9 |
| 56 | 54967 | 59667.5* | 3600 | 58587.5 | 58699.5 | 10 |
| 57 | 57326.8 | 64106* | 3600 | 64106 | 64558.4 | 14 |
| 58 | 93790.6 | 93800 | 358 | 93800 | 94070.8 | 9 |
| 59 | 95177.3 | 98414.5* | 3600 | 98414.5 | 98591.7 | 10 |
| 60 | 98952.8 | 106998* | 3600 | 105485 | 106034.0 | 13 |
| 61 | 90282.9 | 92124.1* | 3600 | 92378.1 | 92378.1 | 8 |
| 62 | 90906 | 99931.1* | 3600 | 99141.2 | 99404.3 | 10 |
| 63 | 94448.4 | 109301* | 3600 | 107381 | 108558.9 | 13 |
| 64 | 130710 | 130723 | 434 | 131048 | 131688.8 | 9 |
| 65 | 131810 | 140325* | 3600 | 139358 | 139629.7 | 11 |
| 66 | 137093 | 150614* | 3600 | 149186 | 150344.6 | 14 |
| 67 | 80476.2 | 82454.1* | 3600 | 82861.1 | 83106.8 | 8 |
| 68 | 80846.4 | 89276.6* | 3600 | 88308.2 | 88491.9 | 10 |
| 69 | 85113.6 | 99563.6* | 3600 | 98136.3 | 98823.0 | 14 |
| 70 | 120153 | 121149* | 3600 | 121551 | 121551.0 | 9 |
| 71 | 120801 | 129771* | 3600 | 128138 | 128796.0 | 10 |
| 72 | 126089 | 140878* | 3600 | 139813 | 140453.8 | 14 |
| 73 | 25889.1 | 25891.7 | 380 | 25903.7 | 25903.7 | 9 |
| 74 | 26460.5 | 26760.5* | 3600 | 26796.3 | 26825.9 | 11 |
| 75 | 27625.3 | 28079* | 3600 | 28061.1 | 28068.9 | 12 |
| 76 | 64302.3 | 64308.7 | 246 | 64308.7 | 64325.0 | 10 |
| 77 | 65080 | 65086.5 | 32 | 65086.5 | 65116.2 | 11 |
| 78 | 67146.2 | 67259.2* | 3600 | 67268 | 67272.2 | 13 |
| 79 | 28568.7 | 28571.5 | 204 | 28632.3 | 28632.3 | 9 |
| 80 | 29063.3 | 29822.6* | 3600 | 29822.6 | 29833.0 | 10 |
| 81 | 30523.9 | 31833.3* | 3600 | 31828 | 31882.5 | 13 |
| 82 | 67259.8 | 67266.5 | 650 | 67272.3 | 67277.9 | 9 |
| 83 | 68912.8 | 68919.7 | 409 | 68919.7 | 68919.7 | 11 |
| 84 | 71204.2 | 71973.4* | 3600 | 71973.4 | 71973.8 | 13 |
| 85 | 225669 | 225692 | 549 | 225704 | 225707.7 | 9 |
| 86 | 226304 | 226561* | 3600 | 226561 | 226618.0 | 10 |
| 87 | 227425 | 227866* | 3600 | 227861 | 227866.9 | 11 |
| 88 | 597049 | 597109 | 256 | 597109 | 597112.8 | 9 |
| 89 | 597827 | 597887 | 24 | 597887 | 597905.2 | 10 |
| 90 | 599970 | 600059* | 3600 | 600069 | 600080.3 | 11 |
| 91 | 228349 | 228372 | 178 | 228432 | 228432.0 | 9 |
| 92 | 228866 | 229739* | 3600 | 229623 | 229661.3 | 9 |
| 93 | 230299 | 231809* | 3600 | 231628 | 231656.8 | 11 |
| 94 | 599950 | 600067* | 3600 | 600072 | 600093.0 | 9 |
| 95 | 601660 | 601720 | 251 | 601720 | 601737.8 | 10 |
| 96 | 603991 | 604773* | 3600 | 604773 | 604792.3 | 11 |

*Notes.* L.B., lower bound, U.B., upper bound,
* optimality is not confirmed after the specified CPU time

Table D.3 Overall cost changes from ρ = 0.10 to ρ = 0.50

| | | PRPT: $b_{ij} = \rho \times c_{ij}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | PRP | $\rho$ = 0.10 | $\rho$ = 0.15 | $\rho$ = 0.20 | $\rho$ = 025 | $\rho$ = 0.30 | $\rho$ = 0.35 | $\rho$ = 0.40 | $\rho$ = 0.45 | $\rho$ = 0.50 |
| 1 | 40390 | 36327.3 | 37394.6 | 38322.6 | 38512.8 | 38681.3 | 38849.9 | 39018.4 | 39186.9 | 39355.5 |
| 2 | 40600 | 36508.8 | 37733.9 | 38550.6 | 38739.5 | 38902.3 | 39063.9 | 39225.4 | 39409.9 | 39548.5 |
| 3 | 41278 | 37586.8 | 38688.6 | 39086.6 | 39303.5 | 39496.6 | 39689.7 | 39870.4 | 40031.9 | 40193.5 |
| 4 | 82787 | 74934.6 | 76059.5 | 77069.6 | 78065 | 78884.3 | 79052.9 | 79214.4 | 79376 | 79537.5 |
| 5 | 83212 | 76029.7 | 77374 | 78571.2 | 79441.8 | 79610.3 | 79778.9 | 79947.4 | 80109 | 80270.5 |
| 6 | 83954 | 77336.4 | 79230.1 | 80735.6 | 81063.5 | 81296.3 | 81449.7 | 81640.4 | 81812.5 | 81984.5 |
| 7 | 31889 | 30178.4 | 30486.8 | 30697.2 | 30865.8 | 31034.3 | 31202.8 | 31364.4 | 31525.9 | 31687.5 |
| 8 | 32162 | 30729.4 | 31176.1 | 31411.6 | 31591.8 | 31760.3 | 31928.8 | 32107.4 | 32162 | 32162 |
| 9 | 32904 | 31943.8 | 32563.7 | 32848.2 | 32888.3 | 32914.2 | 32928 | 32911 | 32911 | 32928 |
| 10 | 71289 | 68623.3 | 69076.8 | 69287.2 | 69455.8 | 69624.3 | 69792.9 | 69954.4 | 70116 | 70277.5 |
| 11 | 72096 | 69474 | 70536.7 | 71037.4 | 71342.8 | 71545.3 | 71713.9 | 71882.4 | 72044 | 72096 |
| 12 | 74674 | 71505.9 | 72538.4 | 73475.2 | 74005.8 | 74531.9 | 74681 | 74695 | 74698 | 74698 |
| 13 | 43735 | 41009.2 | 41683.2 | 42021.6 | 42360 | 42688.2 | 43014.9 | 43340 | 43664.8 | 43735 |
| 14 | 44506 | 41742 | 42503.2 | 42841.6 | 43180 | 43508.2 | 43834.9 | 44161.6 | 44486.8 | 44654 |
| 15 | 45510 | 42760.3 | 43546.3 | 44317.4 | 45253 | 45405.5 | 45510 | 45539 | 45510 | 45539 |
| 16 | 85781 | 79529.8 | 81528.2 | 81861 | 82185.8 | 82510.5 | 82835.3 | 83160 | 83484.8 | 83809.5 |
| 17 | 86269 | 81728.9 | 82689.7 | 83060.6 | 83399 | 83725.5 | 84050.3 | 84375 | 84699.8 | 85024.5 |
| 18 | 87516 | 84236.7 | 85250.5 | 85840.4 | 86506.5 | 87299.2 | 87516 | 87516 | 87516 | 87516 |
| 19 | 34731 | 33150.1 | 33678.2 | 34011 | 34335.8 | 34731 | 34731 | 34731 | 34731 | 34731 |
| 20 | 35219 | 34118 | 34839.7 | 35227 | 35227 | 35227 | 35227 | 35227 | 35227 | 35227 |
| 21 | 36466 | 36323.1 | 36393.8 | 36453 | 36466 | 36466 | 36466 | 36466 | 36466 | 36466 |
| 22 | 74460 | 71760.1 | 72268.2 | 72601 | 72925.8 | 73250.5 | 73575.3 | 73900 | 74224.8 | 74460 |
| 23 | 75137 | 73612.3 | 74671.4 | 75117 | 75137 | 75137 | 75137 | 75137 | 75137 | 75137 |
| 24 | 78236 | 76294 | 77905 | 78210 | 78236 | 78236 | 78236 | 78236 | 78236 | 78236 |

Table D.4 Overall cost changes from ρ = 0.10 to ρ = 0.50 (cont.)

| | | PRPT: $b_{ij} = \rho \times c_{ij}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | PRP | $\rho$ = 0.10 | $\rho$ = 0.15 | $\rho$ = 0.20 | $\rho$ = 025 | $\rho$ = 0.30 | $\rho$ = 0.35 | $\rho$ = 0.40 | $\rho$ = 0.45 | $\rho$ = 0.50 |
| 25 | 240619 | 236127 | 237195 | 238123 | 238313 | 238481 | 238650 | 238818 | 238987 | 239156 |
| 26 | 240835 | 236309 | 237571 | 238389 | 238541 | 238737 | 238878 | 239035 | 239205 | 239384 |
| 27 | 241078 | 237387 | 238486 | 238887 | 239104 | 239297 | 239490 | 239670 | 239826 | 239983 |
| 28 | 615739 | 607735 | 608860 | 609870 | 610865 | 611684 | 611853 | 612014 | 612176 | 612338 |
| 29 | 616012 | 608810 | 610174 | 611444 | 612242 | 612410 | 612579 | 612747 | 612909 | 613071 |
| 30 | 616754 | 610193 | 612030 | 613536 | 613864 | 614057 | 614250 | 614440 | 614612 | 614785 |
| 31 | 231689 | 229978 | 230287 | 230497 | 230666 | 230834 | 231003 | 231164 | 231326 | 231488 |
| 32 | 231962 | 230529 | 230966 | 231211 | 231392 | 231560 | 231729 | 231897 | 231962 | 231962 |
| 33 | 232704 | 231762 | 232409 | 232648 | 232688 | 232714 | 232724 | 232711 | 232711 | 232711 |
| 34 | 604089 | 601423 | 601877 | 602087 | 602256 | 602424 | 602611 | 602754 | 602916 | 603078 |
| 35 | 604896 | 602274 | 603337 | 603837 | 604143 | 604345 | 604514 | 604682 | 604844 | 604896 |
| 36 | 607474 | 604306 | 605338 | 606275 | 606806 | 607411 | 607481 | 607495 | 607498 | 607498 |
| 37 | 243854 | 240809 | 241483 | 241822 | 242160 | 242488 | 242815 | 243140 | 243465 | 243790 |
| 38 | 244442 | 241542 | 242303 | 242642 | 242980 | 243308 | 243635 | 243962 | 244287 | 244442 |
| 39 | 245310 | 242560 | 243346 | 244218 | 244960 | 245318 | 245339 | 245310 | 245310 | 245310 |
| 40 | 618581 | 612339 | 614328 | 614661 | 614986 | 615311 | 615635 | 615960 | 616285 | 616610 |
| 41 | 619069 | 614542 | 615490 | 615861 | 616199 | 616526 | 616850 | 617175 | 617500 | 617825 |
| 42 | 620316 | 617046 | 618062 | 618640 | 619190 | 619988 | 620316 | 620316 | 620316 | 620316 |
| 43 | 234531 | 232950 | 233478 | 233811 | 234136 | 234461 | 234531 | 234531 | 234531 | 234531 |
| 44 | 235019 | 233907 | 234590 | 235019 | 235027 | 235027 | 235019 | 235027 | 235019 | 235027 |
| 45 | 236266 | 236123 | 236200 | 236266 | 236266 | 236266 | 236266 | 236266 | 236266 | 236266 |
| 46 | 607260 | 604560 | 605068 | 605401 | 605726 | 606051 | 606375 | 606700 | 607025 | 607260 |
| 47 | 607937 | 606412 | 607471 | 607917 | 607937 | 607937 | 607937 | 607937 | 607937 | 607937 |
| 48 | 611036 | 609094 | 610705 | 611010 | 611036 | 611037 | 611036 | 611036 | 611036 | 611036 |

Table D.5 Overall cost changes from ρ = 0.10 to ρ = 0.50 (cont.)

| | | PRPT: $b_{ij} = \rho \times c_{ij}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | PRP | $\rho$ = 0.10 | $\rho$ = 0.15 | $\rho$ = 0.20 | $\rho$ = 025 | $\rho$ = 0.30 | $\rho$ = 0.35 | $\rho$ = 0.40 | $\rho$ = 0.45 | $\rho$ = 0.50 |
| 49 | 66907 | 64554 | 66907 | 66907 | 66907 | 66907 | 66907 | 66907 | 66907 | 66907 |
| 50 | 69184 | 68103.3 | 69184 | 69185 | 69184 | 69185 | 69185 | 69185 | 69185 | 69185 |
| 51 | 73768 | 73091.2 | 73168.8 | 73246.4 | 73324 | 73401.6 | 73479.2 | 73556.8 | 73752.6 | 73712 |
| 52 | 108475 | 103224 | 105936 | 107553 | 108475 | 108475 | 108475 | 108475 | 108475 | 108475 |
| 53 | 110504 | 108118 | 110504 | 110504 | 110504 | 110504 | 110504 | 110504 | 110504 | 110504 |
| 54 | 115574 | 114885 | 114963 | 115040 | 115118 | 115196 | 115345 | 115434 | 115428 | 115506 |
| 55 | 57425 | 55374 | 57425 | 57425 | 57425 | 57425 | 57425 | 57425 | 57425 | 57425 |
| 56 | 59454 | 58587.5 | 59454 | 59454 | 59454 | 59454 | 59454 | 59454 | 59454 | 59454 |
| 57 | 64524 | 64106 | 63939.6 | 63990.4 | 64117.3 | 64206.1 | 64294.9 | 64383.8 | 64472.6 | 64456 |
| 58 | 97225 | 93800 | 96676.3 | 97225 | 97225 | 97225 | 97225 | 97225 | 97225 | 97225 |
| 59 | 99435 | 98414.5 | 99435 | 99435 | 99435 | 99435 | 99435 | 99435 | 99435 | 99435 |
| 60 | 106294 | 105485 | 105710 | 105760 | 105887 | 105916 | 105993 | 106071 | 106243 | 106226 |
| 61 | 95311 | 92378.1 | 95311 | 95311 | 95311 | 95311 | 95311 | 95311 | 95311 | 95311 |
| 62 | 99540 | 99141.2 | 99566 | 99540 | 99566 | 99540 | 99566 | 99566 | 99566 | 99566 |
| 63 | 108800 | 107381 | 107538 | 107696 | 107957 | 108137 | 108170 | 108327 | 108485 | 108643 |
| 64 | 136879 | 131048 | 136388 | 136879 | 136879 | 136879 | 136879 | 136879 | 136879 | 136879 |
| 65 | 140856 | 139358 | 140856 | 140856 | 140856 | 140856 | 140856 | 140856 | 140856 | 140856 |
| 66 | 150606 | 149186 | 149332 | 149490 | 149648 | 149806 | 150087 | 150267 | 150447 | 150437 |
| 67 | 85829 | 82861.1 | 85829 | 85829 | 85829 | 85829 | 85829 | 85829 | 85829 | 85829 |
| 68 | 89806 | 88308.2 | 89806 | 89806 | 89806 | 89806 | 89806 | 89806 | 89806 | 89806 |
| 69 | 99556 | 98136.3 | 98282.4 | 98496.6 | 98676.8 | 98755.8 | 99037 | 99217.2 | 99397.4 | 99562 |
| 70 | 125629 | 121551 | 125629 | 125629 | 125629 | 125629 | 125629 | 125629 | 125629 | 125629 |
| 71 | 129856 | 128138 | 129856 | 129856 | 129856 | 129856 | 129856 | 129856 | 129856 | 129856 |
| 72 | 141326 | 139813 | 140052 | 140267 | 140447 | 140627 | 140807 | 140841 | 141167 | 141332 |

Table D.6 Overall cost changes from ρ = 0.10 to ρ = 0.50 (cont.)

| | | PRPT: $b_{ij} = \rho \times c_{ij}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Ins. | PRP | $\rho$ = 0.10 | $\rho$ = 0.15 | $\rho$ = 0.20 | $\rho$ = 025 | $\rho$ = 0.30 | $\rho$ = 0.35 | $\rho$ = 0.40 | $\rho$ = 0.45 | $\rho$ = 0.50 |
| 73 | 26543 | 25903.7 | 26116.2 | 26236.2 | 26315.8 | 26395.3 | 26474.8 | 26543 | 26543 | 26543 |
| 74 | 26979 | 26796.3 | 26979 | 26979 | 26979 | 26979 | 26979 | 26979 | 26979 | 26979 |
| 75 | 28558 | 28061.1 | 28389 | 28483 | 28509.5 | 28519.4 | 28529.3 | 28534.4 | 28542.2 | 28550 |
| 76 | 65518 | 64308.7 | 64752.1 | 64893.2 | 64972.8 | 65052.3 | 65131.9 | 65211.4 | 65290.9 | 65370.5 |
| 77 | 66822 | 65086.5 | 66018 | 66651 | 66806.5 | 66834 | 66822 | 66822 | 66822 | 66822 |
| 78 | 70328 | 67268 | 68377.3 | 69065.8 | 69643 | 69883 | 70120 | 70265 | 70288.8 | 70305 |
| 79 | 29556 | 28632.3 | 29036.3 | 29278 | 29438.8 | 29556 | 29556 | 29556 | 29556 | 29556 |
| 80 | 30020 | 29822.6 | 30020 | 30020 | 30020 | 30020 | 30020 | 30020 | 30020 | 30020 |
| 81 | 32059 | 31828 | 31913 | 31946.8 | 31960 | 31984.3 | 32002.3 | 32007.4 | 32023.2 | 32039 |
| 82 | 68736 | 67272.3 | 67736.8 | 67948 | 68108.8 | 68269.5 | 68430.3 | 68591 | 68736 | 68736 |
| 83 | 69920 | 68919.7 | 69887 | 69920 | 69920 | 69920 | 69920 | 69920 | 69920 | 69920 |
| 84 | 73829 | 71973.4 | 73103 | 73559 | 73695 | 73741.7 | 73761.6 | 73777.4 | 73793.2 | 73809 |
| 85 | 226343 | 225704 | 225916 | 226036 | 226116 | 226195 | 226275 | 226343 | 226343 | 226343 |
| 86 | 226779 | 226561 | 226779 | 226779 | 226779 | 226779 | 226779 | 226779 | 226779 | 226779 |
| 87 | 228358 | 227861 | 228189 | 228283 | 228310 | 228319 | 228329 | 228334 | 228349 | 228350 |
| 88 | 598318 | 597109 | 597552 | 597693 | 597773 | 597852 | 597932 | 598011 | 598091 | 598171 |
| 89 | 599622 | 597887 | 598847 | 599451 | 599607 | 599634 | 599622 | 599622 | 599622 | 599622 |
| 90 | 603128 | 600069 | 601177 | 601866 | 602443 | 602683 | 602920 | 603068 | 603088 | 603105 |
| 91 | 229356 | 228432 | 228836 | 229078 | 229239 | 229356 | 229356 | 229356 | 229356 | 229356 |
| 92 | 229820 | 229623 | 229820 | 229820 | 229820 | 229820 | 229820 | 229820 | 229820 | 229820 |
| 93 | 231859 | 231628 | 231713 | 231744 | 231760 | 231776 | 231792 | 231819 | 231823 | 231851 |
| 94 | 601536 | 600072 | 600506 | 600748 | 600909 | 601070 | 601230 | 601391 | 601536 | 601536 |
| 95 | 602720 | 601720 | 602687 | 602720 | 602720 | 602720 | 602720 | 602720 | 602720 | 602720 |
| 96 | 606629 | 604773 | 605903 | 606359 | 606495 | 606542 | 606562 | 606577 | 606593 | 606622 |